Master's thesis

NTNU
Norwegian University of Science and Technology
Faculty of Economics and Management
Department of Economics

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Coskewness as a Priced Risk Factor in the Norwegian Stock Market

Master's thesis in Financial Economics
Supervisor: Snorre Lindset
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Preface

This Master's thesis has been conducted at the Norwegian University of Science and Technology and concludes our Master of Science in Financial Economics. We would like to thank Snorre Lindset for contributing to our thesis and helping define the research question. Furthermore we would like to thank our families for support during this process. This thesis was written in cooperation by Johannes Løset and Jens Erlend Ølberg.

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Abstract

Our goal with this thesis was to determine if, and to what extent coskewness was a priced risk factor in the Norwegian stock market from 1999-2019, and to see if it could be a useful addition to the Capital Asset Pricing Model. To implement coskewness as a priced risk factor we introduced investors with cubic utility functions. After introducing cubic utility functions we maximised expected wealth of the investor and showed that coskewness could be a priced risk factor.

We cleaned our data, and stocks with fewer than 12 months of data were removed along with non-stocks such as Exchange Traded Funds and Exchange Traded Notes. Stocks defined as "new" and "old" were also removed.

Portfolios were constructed based on decile rankings of beta and coskewness of stocks and then used as a basis for Fama-Macbeth two-stage regressions. The two-stage regressions first estimates beta and coskewness, and in the second stage estimates their respective risk premiums. We also tested adding Fama-French factors and calculating their risk premiums.

Our results indicate that adding coskewness and Fama-French factors to the Capital Asset Pricing Model increases the explanation power, $\overline{adj.R^2}$, from 0.19 up to 0.40. However, we could not achieve the same results as Kraus and Litzenberger (1976) with regards to coskewness. Our estimated risk premiums are not statistically significant with absolute t-values being under 1.2 for all regressions. Our estimations for the risk premium for coskewness yield lower t-values when including Fama-French factors.

We conclude that coskewness does not adequately improve the model, and

that the Capital Asset Pricing Model is not an adequate model for asset pricing in the Norwegian stock market.

Sammendrag

Målet med denne masteroppgaven var å undersøke hvorvidt koskjevhet (coskewness) har vært en priset risikofaktor i det norske aksjemarkedet i perioden 1999-2019, og om det kunne være en nyttig utvidelse til kapitalverdimodellen (CAPM). I tillegg har vi sett på den kubiske nyttefunksjonen for å se på investorers nytte ved å implementere koskjevhet i modellene våre. Deretter maksimerte vi forventet formue og viste at koskjevhet kan brukes som en priset risikofaktor som kan forklare porteføljeavkastning.

Etter å ha gått gjennom datasettet, fjernet vi aksjer som har vært notert i færre enn 12 måneder. I tillegg fjernet vi verdipapirer som ikke kan betraktes som selskapsaksjer, som for eksepel ETF'er og ETN'er. Aksjer som var registrert som "nye" eller "gamle" ble også fjernet fra datasettet.

Porteføljene ble generert basert på en desilfordeling av beta og koskjevhet av aksjene. Deretter ble porteføljene brukt for å kjøre Fama-Macbeth's to-stegsregresjon. To-stegsregresjonen estimerte i første omgang beta og koskjevhet, og deretter, andre omgang, estimerte den faktorenes risikopremie. I tillegg testet vi Fama-French faktorer og kalkulerte tilhørende risikopremier.

Resultatene våre indikerer at det å legge til koskjevhet og Fama-French faktorer til kapitalverdimodellen øker forklaringsverdiene, $\overline{adj.R^2}$, til modellene med en forklaringskraft fra 0.19 til 0.40. Likevel evnet vi ikke å samsvare våre resultater med hva Kraus and Litzenberger (1976) fant i sine analyser når det kommer til koskjevhet. Med t-verdier under 1.2 for alle våre modeller, er ikke de estimerte risikopremiene statistisk signifikante. Dessuten får vi lavere t-verdier i risikopremien for koskjevhet når vi inkluderer Fama-French faktorer.

Vi konkluderer med at koskjevhet ikke er tilstrekkelig til å forbedre modellen vår. I tillegg viser vi at kapitalverdimodellen ikke er tilstrekkelig for aksjeprising i det norske aksjemarkedet.

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1 Introduction

Asset returns can be explained using numerous models, the most known model being the Capital Asset Pricing Model (CAPM), which attempts to explain the returns on assets through systematic risk. There have been several attempts to expand the CAPM. Notable examples are Fama and French who attempted to expand CAPM by including extra factor betas, and Kraus and Litzenberger who argued that factor betas work as a proxy for another type of risk. We aimed to find if, and to what extent, Kraus and Litzenberger's risk factor is a beneficial addition to CAPM in the Norwegian stock market using data from TITLON (Financial data for Norwegian academic institutions).

Kraus and Litzenberger's risk factor is usually referred to as coskewness. The intuitive explanation of skewness is that it helps describe the asymmetry of the distribution of returns. Coskewness is to skewness what covariance is to variance. Just like covariance shows the contribution of an asset's variance to another asset's variance, coskewness shows the contribution of one asset's skewness to a well diversified portfolio's.

The structure of this thesis is hierarchical. Chapter two covers both previous literature and the most central theories, which forms the theoretical foundation for this study. The methodological basis is presented in the third chapter. Chapter four describes our data processing, choice of proxy for risk-free rate, and descriptive statistics of individual stocks. In chapter five, the empirical results of the study are presented and analysed. Our conclusion is presented in chapter six.

2 Theoretical Framework

Our goal was to examine if and to what extent coskewness helps explain cross-sectional variations of monthly asset returns in the Norwegian stock market. Well established asset pricing models in financial theory like Markowitz (1952) and Black and Scholes (1973) assume that the asset returns are normally distributed and thus distributed symmetrically around their expected returns.

The original idea for pricing an asset as a combination of its empirical risk and its risk-free return comes from Markovitz's paper "Portfolio Selections" from 1952, a model that has since become a staple within asset pricing theory. Due to the simplicity, effectiveness, and ease of use, CAPM has become a staple introduction to asset pricing in finance. Further research of mean-variance performance composites such as CAPM have later been criticised by Ang and Chua (1979), stating that "(...) the performance measures may be inadequate because of the failure to consider higher moments of the distributions of investment returns.". A paper published by Kraus and Litzenberger (1976) took return asymmetry into consideration by incorporating the effects of coskewness in asset pricing. Kraus and Litzenberger expanded CAPM by allowing investor's utility functions to not only depend on expected wealth and variance, but also skewness. Kraus and Litzenberger's seminal paper(1976) gave a theoretical basis for including n order statistical moments in asset pricing and an empirical basis for 3rd order statistical moments.

In this section we aim to give an understanding of how we price coskewness theoretically, and how we apply this to real-life data.

2.1 Volatility

Volatility is often referred to as risk in the financial world, and as Bhowmik (2013) put it, "the higher the volatility, the riskier the security is". Volatility is often high in times of turmoil e.g. financial crisis and wars. We encounter volatile markets when there is uncertainty about future outcomes which can create instabilities in the capital market.

To measure the volatility of an asset, we often calculate its standard deviation or variance based on historical changes in asset prices. Volatility, σ, can be described with the following equation

$$\sigma = \sqrt{\frac{1}{(N-1)} \sum_{t=1}^{N} (r_t - \overline{r})^2}, \tag{1}$$

where

r_t is the return at time t,

$\overline{r}$ is the mean return in the period, and

N is the number of observations.

From equation (1), we can see that the more an asset's pricing deviates from its mean, the higher the volatility.

Another measure of volatility often used in finance is the beta coefficient. The beta coefficient is a measure of an asset's change in return compared to the market's change in return, and will be discussed further in section 2.3.1.

2.2 Skewness and coskewness

Traditional financial theory often assumes that asset returns are normally distributed when estimating risk, and when using standard deviation as a proxy for asset risk. Kraus and Litzenberger relaxed the assumption of normally distributed returns by including skewness in the utility function of investors.

Skewness is a measure used to quantify the asymmetry of a distribution of an asset's return, which is beneficial as few return distributions are normally distributed (Harvey & Siddique, 1999). Coskewness is a measure of how the inclusion of an asset to a well-diversified portfolio will affect the total skewness of the portfolio.

Estimated skewness is defined mathematically as

$$\hat{Skew} = \frac{\frac{1}{T}\sum_{t=1}^{T}(r_t - \overline{r})^3}{\hat{\sigma}^3}, \tag{2}$$

where

r_t is the return in period t,

$\overline{r}$ denotes the average return, and

$\hat{\sigma}^3$ denotes the estimated standard deviation of the portfolio cubed.

When the skewness of a portfolio has a value of zero, the distribution is symmetrical and the returns are expected to be normally distributed. For positively skewed distributions the probability of observing extreme positive returns is higher and the probability of observing extreme negative returns is lower, while the opposite holds true for negatively skewed distributions. To calculate the skewness of a portfolio see appendix A.

Boyer, Mitton, and Vorkink analysed investors preferences for skewness and found that "(...) skewness-preferring investors may be willing to accept a stock with higher idiosyncratic volatility and lower expected returns in return for a chance at an extreme winner". These types of investors were described as "lotto"-investors, since they are willing to pay a premium to be able to catch some extreme positive returns. More traditional investors will pick stocks that maximise the Sharpe-ratio [1] of their portfolio. Figure 1 illustrates the returns of right and left skewed assets in comparison to normally distributed returns.

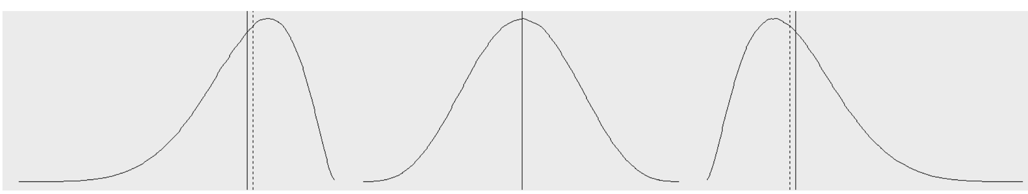

Figure 1: Three distributions: left/negative skewed, normal and right/positive skewed. The dotted lines indicate the distributions median, while the solid line indicates their mean.

Coskewness on the other hand, is a measure of how three random variables move together. In our analysis, coskewness refers to how one asset's return moves in relation to the market's squared return. Intuitively, coskewness shows how adding one asset to the market portfolio will change the skewness of the market portfolio.

[1] The Sharpe-ratio measures how a portfolio performs in respect to its risk. It is expressed as: $Sharperatio = \frac{r_p - r_f}{\sigma_p}$, where $r_p - r_f$ is the portfolios excess return over the risk free rate and σ_p is a measurement of the portfolios risk

Coskewness of stock i, γ_i, is defined as

$$\gamma_i = \frac{E[(r_i - E(r_i))(r_m - E(r_m))^2]}{E[(r_m - E(r_m))^3]}. \tag{3}$$

2.2.1 Investors preference

To establish an exact preference of risky portfolios using the first three moments of portfolio return, we require that investors have a cubic utility function (Kraus & Litzenberger, 1976). A cubic utility function by itself is not sufficient to create a three-moment valuation model, and we have therefore imposed additional restrictions on the utility function.

From Pratt (1964) we have that the necessary traits for an investors utility function are

(a) positive marginal utility of wealth,

(b) decreasing marginal utility, and

(c) non-increasing absolute risk aversion.

Absolute risk aversion (ARA) is defined as

$$ARA(W) = -\frac{U''(W)}{U'(W)},$$

where

$U(W)$ denotes utility as a function of wealth,

$U'(W)$ denotes the first derivative of the utility function with respect to wealth, and

$U''(W)$ is the second derivative with respect to wealth.

Therefore

$$\frac{\partial ARA(W)}{\partial W} = \frac{-U^{'}(W)U^{'''}(W) + U^{''}(W)^2}{U^{'}(W)^2} \leq 0, \tag{4}$$

where

$\frac{\partial ARA(W)}{\partial W}$ denotes the partial derivative of ARA with respect to

wealth, and

$U'''(W)$ is the third derivative of the utility function with respect

to wealth.

We know $U'(W)^2$ is positive, and since we demand positive marginal utility of wealth ($U'(W) > 0$) we then have that marginal utility must always be decreasing, which is expressed as

$$\frac{\partial^2 U(W)}{\partial W^2} \leq 0. \tag{5}$$

Using (5) we can rewrite (4) as

$$U^{'''}(W) \geq \frac{U^{''}(W)^2}{U^{'}(W)} > 0.$$

Since we know that $U'(W)$ is positive and we assume that $U^{''}(W)$ is a real number, it follows that skewness preference must be acknowledged when composing an optimal portfolio for investors with a cubic utility function.

Some of the functions that can be used and display the wanted attributes are, logarithmic, power, and negative exponential functions(Kraus & Litzenberger, 1976).

We expand the utility function as a Taylor series and then the investors expected utility at the end of the period, $E(U(\tilde{W}))$, can be written as

$$E(U(\tilde{W})) = U(\overline{W}) + \frac{[U''(\overline{W})]}{2!}\sigma_W^2 + \frac{[U'''(\overline{W})]}{3!}m_W^3 + R_n,$$

where

$\overline{W} = E(\tilde{W})$,

$\sigma_W^2 = E[(\tilde{W} - \overline{W})^2]$,

$m_W^3 = E[(\tilde{W} - \overline{W})^3]$, and

R_n is an error term representing the sum of the higher order moments.

From previous research by Kraus and Litzenberger (1976) we optimise the investors utility function given a constraint such that the Lagrangian, L, is

$$L = \Phi(\overline{W}, \sigma_W, m_W) - \lambda(\sum_{i=1}^{N} q_i + q_F - W_0),$$

where

q_i is the amount invested into security i,

q_f is the amount invested in the risk free asset, and

ϕ is denoted as the investors utility function dependent on expected wealth, standard deviation of wealth, and skewness of wealth.

We then maximise expected utility of the investor's utility function by Lagrange optimising and end up with the following general solution

$$\overline{R}_i - R_F = -\frac{\phi_{\sigma_W}}{\phi_{\overline{W}}}\beta_{iP}\sigma_P - \frac{\phi_{m_W}}{\phi_{\overline{W}}}\gamma_{iP}m_P,$$

where

β_{iP} is the beta between stock i and porfolio P, defined as $\beta_{iP} = \frac{E[(R_i - \overline{R}_i)(R_P - \overline{R}_P]}{\sigma_P^2}$,

$-\frac{\phi_{\sigma_W}}{\phi_{\overline{W}}}$ is the investor's marginal rate of substitution between expected wealth and standard deviation, and

-$\frac{\phi_{m_W}}{\phi_{\overline{W}}}$ is the investor's marginal rate of substitution between expected wealth and skewness.

The marginal rates of substitution can normally be seen as shadow prices in the context of Lagrange optimisation as they cannot be observed.

According to Kraus and Litzenberger (1976) "(...) investors are found to have an aversion to variance and a preference to positive skewness", meaning that investors would prefer holding assets with a few large gains and frequent small losses over an asset with a few large losses and frequent small gains.

2.3 Factor models

2.3.1 The Capital Asset Pricing Model

CAPM is a model initially developed by Treynor (1961), Sharpe (1964), Lintner (1965), and Mossin (1966) that estimates the expected rate of return of an asset, $E(r_i)$, in excess to the risk free rate, r_f, based on asset sensitivity in relation to market fluctuations, β, times the expected return on the market portfolio, $E(r_M)$, in excess of the risk free rate. The CAPM is a single factor model and can be expressed as

$$E(r_i) - r_f = \beta_i[E(r_M) - r_f],$$

where the beta coefficient, β_i, is the covariance between an asset's return and the market return divided by the variance of the market return,

$$\beta_i = \frac{Cov(r_i, r_M)}{\sigma_M^2}.$$

2.3.2 The Three Factor Model

Fama and French (1993) developed an extension of CAPM by including two additional factors to help explain asset return.

These two factors are Small-Minus-Big (SMB) and High-Minus-Low (HML). SMB represents the excess returns of small market capitalisation over companies with high market capitalisation, and HML represents the returns of companies with high book-to-price ratio over growing companies, also known as companies with low book-to-price ratios.

2.3.3 Three Moment Pricing Model

In the assumption that expected returns cannot be explained by the market beta alone, Kraus and Litzenberger (1976) extended CAPM by incorporating the effect of coskewness as an explanatory variable. Kraus and Litzenberger's paper shows that systematic skewness could be relevant to market valuation.

3 Methodology

In this section, we present how we identified and quantified the risk premiums associated with coskewness. Our approach was based on previous papers regarding the subject, where the research of Kraus and Litzenberger (1976), Harvey and Siddique (1999), and Moreno and Rodríguez (2009) were the most influential.

3.1 Benchmark portfolio

The market portfolio consists of the aggregated holdings of all investors in the stock market. The market portfolio can serve as a benchmark against other portfolios or stocks, to analyse their performance. Since we were investigating the behaviour of coskewness in Norwegian stocks, it was natural to use a Norwegian reference index to describe the market portfolio. A commonly used market index used in Norway is the OSEBX (Oslo Børs Benchmark Index), and will serve as our reference to the market portfolio. The OSEBX was introduced in 1996 and includes some of the most traded stocks registered on the Oslo Stock Exchange. The OSEBX is revised semiannually and is adjusted for dividends.

3.2 Calculating the factors

3.2.1 Rate of return

Our data consists of daily registered stock data, such as opening price, closing price, and high and low prices. In addition, the data set includes logarithmic daily returns. The logarithmic returns are based on adjusted price, which reflects the stock's closing price accounting for corporate actions such as dividends, stock splits, and stock dilution. Kraus and Litzenberger (1976)'s paper includes calculations based on simple stock returns. Since log returns aggregate better over time and simple returns aggregate best across assets, we used both methods depending on which calculations were performed.

When calculating the returns of an asset over periods like a month or a decade we used the logarithmic returns, denoted r_t, and sum these returns for the period to end up with the periodic logarithmic return. Log returns can be expressed as

$$r_t = log(P_t/P_{t-1}) = log(P_t) - log(P_{t-1}), \tag{6}$$

where

P_t is the assets price at time t, and

P_{t-1} is the price in the previous period.

The logarithmic return, from period 1 to T, is then

$$R_{log} = \sum_{t=1}^{T} r_t. \tag{7}$$

By summarising the daily returns of a stock for a specific month, we calculated the monthly return.

On the other hand, when creating portfolios consisting of multiple stocks, we converted the logarithmic returns into simple returns, denoted R, using the expression

$$R = \exp(r) - 1.$$

We then calculated the arithmetic mean of the simple returns of the assets included in the given portfolio for each period t. These portfolios are known as "equally weighted portfolios".

3.2.2 Constructing the coskewness component

Over the years, several ways to estimate coskewness have been introduced. The first method is described in equation 3.

The second method, created by Harvey and Siddique (1999), defines the estimated coskewness, $\hat{\gamma}_i$, as

$$\hat{\gamma}_i = \frac{E[\epsilon_{i,t+1}\epsilon^2_{M,t+1}]}{\sqrt{E[\epsilon^2_{i,t+1}]}E[\epsilon^2_{M,t+1}]}, \tag{8}$$

where

$\epsilon_{i,t+1}$ is the residual for asset i in period $t+1$ after running a regression of excess return of asset i on the contemporaneous market excess return, and

$\epsilon^2_{M,t+1}$ is the squared residual for the market in period $t+1$ after running a regression of excess market return over its mean.

The third method for estimating coskewness, pioneered by Moreno and Rodríguez

(2009), is to run a regression using a quadratic model, where $\hat{\gamma}_i$ is the coskewness estimate for asset i,

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_i[r_{M,t} - r_{f,t}] + \gamma_i[r_{M,t} - \overline{r}_M]^2 + \epsilon_{it}. \tag{9}$$

where

α_i is

After evaluating all of the above methods for estimating coskewness in our data, we concluded that the method from equation (9) was the most practical, widely used, and the least computationally expensive one.

3.3 Forming portfolios

We formed portfolios to better illustrate the behaviour of assets that contain different degrees of coskewness. Kraus and Litzenberger (1976) form portfolios using the decile rankings of stock's beta and coskewness. Harvey and Siddique (1999) rank stocks by their coskewness and form three value-weighted portfolios, where one contains the assets with the 30 percent most negative coskewness, another contains the middle 40 percent, and the final consists of the top 30 percent. We form our portfolios based on Kraus and Litzenberger's approach.

By sorting stocks into equally weighted portfolios based on their beta and gamma, we formed ten portfolios based on each factor, for a total of 20 portfolios. All stocks were divided into deciles, 10 deciles were formed based on beta, and 10 deciles were formed based on gamma. The stocks with the 10% highest beta were placed in portfolio 1, 10-20% into portfolio 2, etc.

The Norwegian stock market is characterised by four large capitalisation companies. As of ultimo 2019, Equinor, DNB, Telenor, and Mowi made up 54.82% of the total market capitalisation of stocks listed on the Oslo Stock Exchange. If investors wanted to invest in a value-weighted portfolio consisting of the stocks in the Oslo Stock Exchange, over half of the returns would be generated from the aforementioned stocks and investors could expect a rather stable return with lower volatility. Conversely, by investing in an equally weighted portfolio, they could capture both very large and very small returns that are more often generated by smaller companies. We argue that creating equally weighted portfolios more clearly illustrates the effects of implementing coskewness to the models. Due to the returns of the Norwegian stock exchange being dominated by a few large companies, using equally-weighted portfolios would allow smaller companies to have a larger impact on our results. To avoid under diversification of our portfolios, we attempted to keep at least 8-10 stocks in each portfolio at any time to properly diversify (Odegaard, 2017) thereby reducing the idiosyncratic risk of each portfolio down to its minimum.

3.4 Fama-Macbeth ordinary least squares

In this section, we will demonstrate how we estimated beta and coskewness, by following the two-step cross-sectional regression approach developed by Fama and MacBeth (1973).

The first step is to regress excess portfolio returns on the risk factors by running a time-series regression to estimate the portfolio's beta for the specific risk factors

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_{i1} f_{1t} + ... + \beta_{iK} f_{iK} + \epsilon_{it}, \tag{10}$$

for $t = 1, ...T,,$

$fori = 1, ..., N,,$

where

α_i equals the model intercept,

$r_{i,t} - r_{f,t}$ equals the excess asset return less the risk free rate in

period t,

f_{iK} equals the estimated factor K for portfolio i,

ϵ_{it} is the disturbance of random errors,

N is the number of assets, and

T is the number of time series observations in months.

Equation (10) served as a tool to compute the necessary coefficients needed to estimate the corresponding risk premiums. We regressed each portfolio's return on the risk factors to calculate each factor's beta, or the relationship between an asset's return and its corresponding factor. The estimated coefficient, β_{iK}, is a representation of how the mean change in factor K affects the predicted return. By extending the model, we have created a quadratic model that exhibits the nonlinear relationship with relation to market return

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_i[r_{M,t} - r_{F,t}] + s_i SMB_t + h_i HML_t \\ + \gamma_i[r_M - \overline{r}_{M,t}]^2 + \epsilon_{it}, \tag{11}$$

for $t = 1, ..., T$,

where

$r_{i,t} - r_{f,t}$ denotes the excess return of asset i on the risk free

rate in period t,

SMB_t denotes the Fama-French factor for size effect in period t,

HML_t denotes the Fama-French factor for value effect in period t,

$r_{M,t} - r_{F,t}$ is the excess return of the market on the risk free rate in

period t, and

β_i, s_i, h_i and γ_i are the estimated factor exposures to the risk

factors from the regression.

Model (3.4) is the extended Fama-French 3-factor model, which includes the market return at period t less the mean market return squared.

In the second step we regressed the returns of the constructed portfolios against the factor loadings on the estimated risk exposures, β_{i1}, that we estimated from the first regression for each period t. By doing so, we were able to determine the risk premiums for the corresponding factors in the model,

$$r_i - r_f = \lambda_0 + \lambda_\beta \hat{\beta}_i + \lambda_{SMB} \hat{s}_i + \lambda_{HML} \hat{h}_i + \lambda_\gamma \hat{\gamma}_i + \epsilon_i, \tag{12}$$

for $i = 1, ..., N$,

where

$\lambda_\beta, \lambda_{SMB}, \lambda_{HML}$, and λ_γ are the estimated risk premiums of the associated factors.

The risk premiums were estimated for each period t, and then appended to a matrix. We proceeded by averaging the risk premiums over the time period, thereby receiving the mean risk premium associated with the factors.

3.5 Robust regression

Green and Martin (2017) demonstrates that "(...) a very small fraction of outliers, in the returns and/or the factors, often distorts least squares cross-sectional regression estimates sufficiently enough to result in misleading conclusions as to whether a risk factor is priced." Therefore, to compensate for potentially extreme outliers or other influential observations, we ran robust regressions, and compared to regular OLS regressions.

By reducing the number of extreme observations, we could ensure that outliers did not affect the result as much as they would have when running an ordinary regression. To compensate for outliers, we ran a robust regression estimator called the *regression MM-estimate* derived by (Yohai, 1987) and implemented in R as the "lmrob"-function created by (Maechler et al., 2021).

Another option was to manually remove stocks that had generated extreme returns, but this provided an easy and less labour-intensive way of doing so.

We chose this regression because it claims to be efficient if our data contains outliers both in $y(returns)$ and the factors. The regression should also be more efficient for small samples compared to other robust estimators, which we believe is appropriate in our analysis, where we end up with relatively small samples when running our regressions, because of the natural limitations of the Norwegian stock market and the fact that our sample consist of just 20 unique portfolios.

4 Data

In this section, we describe how the data was accessed and what was done to fit the data to our requirements. This section also addresses limitations with the data and how these limitations were dealt with.

4.1 Fetching of data

Our data was sourced from TITLON, and consists of detailed daily observations from every company listed on the Oslo Stock Exchange in the time period from the start of 1980 to the end of 2019. The data used includes daily stock returns for all stocks, including de-listed ones, daily OSEBX returns, daily risk-free rates, SMB estimates, and HML estimates.

4.2 Simple returns vs log returns

Although the original data set was given in log returns, it became easier to calculate the return of portfolios using simple returns. Arithmetic average is easier to calculate using simple returns than logarithmic when averaging returns of a portfolio while the portfolio is continuously re-balanced every time any stock either exits or joins the stock exchange.

We calculated the original beta and gamma coefficients using log returns to save computing time, as it yields almost identical results as using simple returns. All stock returns were converted from daily logarithmic to monthly simple returns. The stocks were then sorted into portfolios using the procedure outlined in section 3. The mean return for each period t for all stocks in a portfolio was then calculated and the process repeated for all portfolios at all periods.

We used the portfolio returns to calculate beta and gamma for each portfolio by regressing the excess market return and the market return less the average market return squared upon each individual portfolio's return. The second stage regression was then used to estimate the factor premiums, by regressing portfolio returns on the estimated betas and gammas in each period and taking the mean of the estimated premiums. The format of the regression can be found in equation (9)

4.3 Data sorting

Starting with over two million observations, we began digging into the data to find weaknesses or missing values that had to be corrected for. We discovered that some companies had seemingly random trading days of stock data missing. We were unable to find out why this data was missing, but it can be inferred that there had been trading stops during these days, or that the data was mistakenly not gathered by TITLON. We assumed that the missing values did not affect our estimations in any substantial way due to the efficient market hypothesis which states that all stocks should trade at their fair market price and if there was a trading stop it would be compensated for by the next trading opportunity. Assuming the omitted data was not caused by trading stops, but human error, we assumed omissions were randomly distributed.

On the other hand, we observed some companies with several data points on the same day. We found that this was due to being registered in several different sectors which resulted in duplicate data points. We removed the duplicates using a simple R script to make sure there was only one observation per day per stock. We also discovered securities that were not traditional stocks, such as Exchange Traded Funds (ETF) and Exchange Traded Notes (ETN). The ETF's and ETN's were excluded from our data as they were not traditional stocks and did not fit into the scope of our thesis. We observed that some of the companies listed had changed their name and their ticker symbol during their time on the Norwegian stock exchange. We found that sorting by the ISIN-number (International Securities Identification Number) was the best way to sort data by a specific stock. During our analysis we found stocks that included "new shares" in their name that possessed extreme movements in their returns. What type of shares these "new shares" were remains unclear, but due to their extreme fluctuations and relatively short listing period, we chose to exclude these as well.

Since the Norwegian Overnight Weighted Average-rate (NOWA-rate) was first registered in 1983, we had to exclude all data prior to 1983 to make our estimations consistent. In addition, data from the market portfolio, specifically the OSEBX-index, was missing on several dates. The OSEBX-index also included some extreme daily returns at some points. Based on these shortcomings, we did not continue to use all the data we first gathered but rather downloaded the market portfolio data separately. This data set was found in a different section in TITLON's data base and merged with the rest of the data based on timestamps.

4.4 Risk-free rate

The risk-free rate is the return an investor earns by placing their money in securities that are believed to be completely free of risk. Our data set includes the NOWA and Norges Bank's 3 month treasury bill rate. The NOWA-rate is the interest rate on unsecured overnight loans between banks that are active in the Norwegian overnight market. The treasury bills are government securities that are given as monthly averages. Evidently, the 3 month treasury rate is constant in the whole period. We decided to continue by using the NOWA-rate as a proxy for the risk-free rate to achieve a more fluid and correct estimate. Since the returns are given in daily observations, we found it reasonable to use the NOWA-rate as our risk-free rate, as it is easy to apply when calculating other required factors. Due to being registered on a daily basis, it was difficult to see how the NOWA-rate compared to the rate of a government bond, which is a more traditional reference to the risk-free rate. To conclude that the NOWA-rate was indeed a fitting reference to the risk-free rate, we calculated the NOWA-rate on a yearly basis and compared it to the registered 12-month annual average treasury bill rate. Comparing the two we got the results shown in table 1.

Year	NOWA-rate	12-month annual average treasury bill	Difference
2019	1,16	1,18	-0,02
2018	0,58	0,72	-0,14
2017	0,49	0,42	0,07
2016	0,56	0,50	0,06
2015	1,04	0,73	0,31
2014	1,48	1,29	0,19
2013	1,50	1,52	-0,02
2012	1,55	1,53	0,02

Table 1: A comparison of NOWA-rate and annual 12-month treasury rate.

Observing table 1, we can see that the difference between the NOWA-rate and the treasury rate is rather small. Based on the small difference, we decided to continue using the NOWA-rate as a reference to the risk-free rate. The daily market excess return in log, r_{RP}, can then be expressed as

$$r_{RP} = r_{OSEBX} - r_{NOWA},$$

where

r_{OSEBX} is the daily return of the OSEBX_index, and

r_{NOWA} is the daily NOWA-rate.

4.5 Appending factor values to the data

The data set from TITLON contained the previous month's beta values for each stock, but this data was also found to be missing observations and to be insufficient for our needs. We assume that the missing values was due to the lack of OSEBX-data in the original data set, and since these values are essential in our thesis, we had to recalculate the beta values to fit the data. We used the remaining beta values from the old data set as comparison to the ones estimated, and found them to be near identical, which strengthened our belief that our estimations were correct.

To complete our calculations and be able to begin our analysis and to be able to run regressions so that we could fetch the coskewness factor, we had to implement a column that calculated the squared monthly market returns less the mean of the market return, by regressing the excess stock return on this factor, we obtained an estimate for coskewness, as mentioned in the methodology section. To convert daily returns into monthly returns, we added a column with the year and month based on the "date"-column already in the data set. We sorted the data by month and summarised the logarithmic returns in each month, then added these to a new data frame only containing monthly observations.

4.6 Descriptive statistics

After cleaning the data and converting it into monthly observations, we calculated the monthly simple returns from log returns to be able to compute the monthly portfolio returns appropriately. The summary statistics for the stock returns, market returns, and the risk-free rate are presented in table 2 given as percent. On average, stocks have achieved a rate of return of 1.043 % per month in the period. The market portfolio achieved somewhat lower returns with 0.845 % a month. However, looking at the standard deviation of the two, we see that stock returns have been, unsurprisingly, more volatile. Calculating the Sharpe-ratio, which is a measure of excess return per unit of volatility, we estimated a Sharpe Ratio of 2.74% monthly for the stock returns, and 14% monthly for the market portfolio. Assuming that the Sharpe Ratio was the only information of interest, we would expect that a rational investor would prefer investing in the Market Portfolio. The reason for the mean of the stock returns being higher is that the asset returns become equal weighted and the smaller companies with large returns therefore contribute a larger amount.

The data consists of 251 months, with a total of 516 different stocks, making up a total of 56,444 observations.

Table 2: Descriptive statistics of monthly returns of the cleaned data given in percent in the period from January 1999 to December 2019.

Statistic	Mean(%)	St. Dev.	Min(%)	Median(%)	Max(%)
Stock Returns	1.043	0.3803	-96.000	0.000	5780
Market Return	0.8451	0.0581	-35.915	1.276	24.7656
Risk-free Rate	0.297	0.218	0.002	0.215	0.757

5 Results

5.1 Expected Returns of test portfolios

In this section, we look at the relationship between the returns of our portfolios and their beta, β, and if there is a systematic relationship between realised returns and their coskewness, γ. The portfolio characteristics are presented in tables 3 and 4, where we present the summary statistics of each of the portfolios.

In table 3, we showed the portfolios constructed based on each portfolio's movement of returns in relation to the market portfolio, called the beta-portfolios. The portfolios were created by their decile ranking on the basis of beta, as described in section 3. Similar to previous findings by Kraus and Litzenberger (1976), we observed a clear relationship between the mean return and mean beta, where portfolios that on average had achieved the lowest returns, were the ones with the smallest betas.

Interestingly, beta portfolio 1 exhibited a negative beta of -0.242. This indicated that beta portfolio 1 moved in the opposite direction of the market. When the market experienced negative returns, beta portfolio 1 would generate positive returns, and vice versa. By analysing the composition of beta portfolio 1, we found that the negative mean was affected by a few stocks which were seemingly listed on the exchange for a relatively short span in relation to the whole period, affecting our calculations.

While the portfolio with the lowest beta had an average return of 1.28% per month, the portfolio with the highest beta achieved an average return of

3.74%. There appeared to be a pattern where the average return increased in line with the betas, and we formally tested this by performing a paired t-test to compare the means of beta portfolio 1 and 10.

$$t - statistic = \frac{m}{s/\sqrt{n}} \tag{13}$$

where

m is the differences between the estimated means,

n is the sample size, and

s is the estimated standard deviation of the differences.

By calculating the t-statistic from equation (13), we ended up with a t-statistic of -1.024, and thus can not reject the null hypothesis that the means of beta portfolio 1 and 10 are significantly different.

Table 3: A summary of excess monthly returns of portfolios created sorted by their calculated β-coefficient over the sample period. Each portfolio consists of up 61 stocks.

beta-portfolio	Mean Return(%)	St. Dev.	Min(%)	Median(%)	Max(%)	beta (β)	Coskew (γ)
1 *low*	1.283	0.120	-23.539	0.130	148.676	-0.242	-3.831
2	0.586	0.029	-11.513	0.633	11.510	0.145	-0.994
3	0.958	0.039	-15.311	1.007	17.561	0.336	-1.065
4	0.683	0.054	-20.722	0.772	18.080	0.532	-1.280
5	0.472	0.057	-21.242	0.575	25.166	0.686	-0.562
6	0.690	0.066	-26.393	1.033	20.508	0.822	-0.464
7	0.579	0.070	-25.330	0.824	22.389	0.985	0.007
8	0.648	0.083	-28.744	0.995	26.747	1.198	0.671
9	0.420	0.104	-29.436	0.117	47.557	1.424	1.420
10 *high*	3.741	0.356	-35.821	-0.109	391.769	2.100	2.406

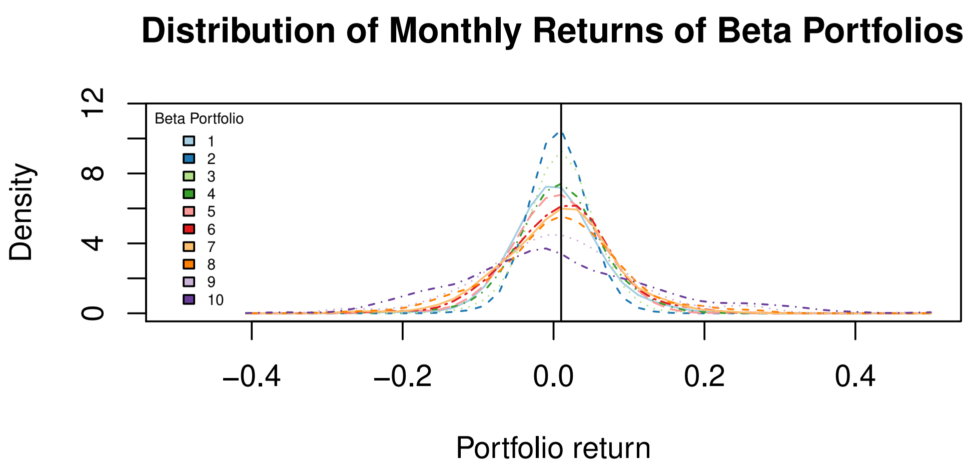

Figure 2: Portfolio returns are given on the x-axis, density is given on the y-axis. The vertical line represent the mean return across all portfolios

To give an illustration of the skewness of the portfolios we plotted the returns of the beta and gamma portfolios. Table 4 presents the relationship of returns in portfolios based on coskewness-deciles. The portfolio containing the lowest average coskewness, provided astonishing returns of 10.05% on average per month. The portfolio with the highest coskewness provided an average monthly return of 1.05%. The very high average monthly returns in gamma portfolio 1 have been influenced by a few strong outliers, affecting the portfolio performance as a whole. Even though there seem to be a huge difference in the mean returns in gamma portfolio 1 and 10, we can see that by running a paired t-test(*t-stat: 1.140*), their means are not significantly different. In comparison to table 3, we can see that the volatility of the portfolios in table 4 appeared to be more evenly spread. The beta-portfolios appeared to consist of riskier stocks the higher the beta, while the coskewness portfolios had higher amounts of risky stocks in both low and high coskewness portfolios. Relatively high volatility may not be surprising, as the portfolios with the mean coskewness further away from zero should include more extreme returns.

Comparing our results to those of (Kraus & Litzenberger, 1976), where the mean return grew as coskewness increased, our results indicated otherwise. We observed that as coskewness increased there was a corresponding decrease in mean return.

Table 4: A summary of excess monthly returns of portfolios created sorted by their calculated γ-coefficient over the sample period. Each portfolio consists of 122-123 stocks.

gamma-portfolio	Mean Return(%)	St. Dev.	Min(%)	Median(%)	Max(%)	beta (β)	Coskew (γ)
1 *low*	10.052	1.231	-49.611	-0.337	1902.316	0.028	-10.475
2	0.285	0.086	-42.195	0.483	33.510	0.277	-4.721
3	0.278	0.071	-37.302	0.652	19.364	0.670	-2.853
4	0.712	0.052	-23.392	1.067	14.140	0.527	-1.701
5	0.665	0.048	-21.346	0.756	17.461	0.546	-0.807
6	0.831	0.053	-19.156	1.064	19.334	0.704	-0.294
7	1.161	0.101	-23.409	0.799	121.714	1.008	0.628
8	0.767	0.089	-20.134	-0.136	54.822	1.115	1.690
9	-0.062	0.105	-23.938	-1.578	51.790	1.390	3.924
10 *high*	1.048	0.172	-52.922	-1.050	121.832	1.548	5.678

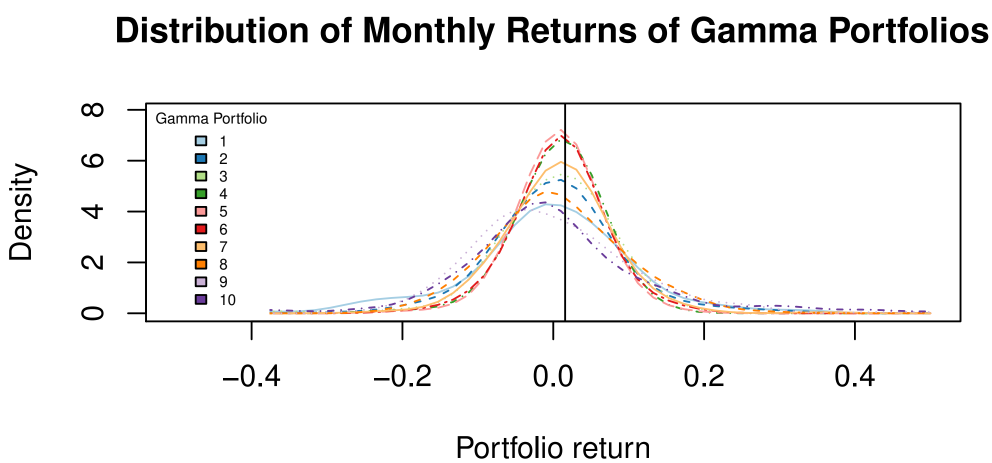

Figure 3: Portfolio returns are given on the x-axis, density is given on the y-axis.The vertical line represent the mean return across all portfolios

5.1.1 The Factors

Table 5 shows the descriptive statistics of the explanatory variables. We can see that the monthly average excess return since January 1999 has been about 0.48% a month. The SMB-factor tells us that small companies have outperformed large companies with an average of 0.46% per month. From the HML-factor, we can see that on average, value stocks have outperformed growth stocks by 0.24% per month.

Table 6 presents the correlation matrix for the variables used in the first stage regression illustrating their relative independence of each other.

Table 5: The excess return given in percent of the respective monthly returns less the monthly risk free rate.

Statistic	Mean(%)	St. Dev.	Min(%)	Median(%)	Max(%)
r_M - r_F	0.475	0.059	-29.703	1.296	14.517
SMB	0.460	0.043	-16.259	0.603	15.549
HML	0.248	0.069	-29.287	-0.340	20.816
$(r_M - \overline{r}_M)^2$	0.348	0.903	0.001	0.103	9.119

Table 6: Correlation matrix of the explanatory variables for the first-stage regressions.

	$r_M - r_F$	SMB	HML	$(r_M - \overline{r}_M)^2$
$r_M - r_F$	1	-0.072	0.105	-0.451
SMB	-0.072	1	-.062	0.008
HML	0.105	-0.062	1	-0.057
$(r_M - \overline{r}_M)^2$	-0.451	0.008	-0.057	1

5.2 Fama-Macbeth Analysis

In the first subsection, we showed the results of using the Fama-Macbeth procedure on individual stocks. In the following subsection we describe the results of the test for the portfolios we created.

5.2.1 Fama-Macbeth regression on individual stocks

In this section, we describe the results achieved by testing different pricing models directly on all stocks instead of creating portfolios, demonstrating the need for said portfolios. The models we tested were the traditional CAPM-model, the Fama-French 3-factor model (FF3), and the extended FF3 model which includes coskewness-risk. The results are presented in table 7 and demonstrated the inadequacy of running any variation of CAPM on individual stocks.

Table 7: Cross-sectional regressions using the Fama-Macbeth procedure on individual stocks. The columns report the intercepts and the estimated risk premium given in percent and the t-statistic given in parentheses for each factor.

Model	λ_0	λ_β	λ_{SMB}	λ_{HML}	λ_γ	$\overline{Adj.R^2}$
CAPM	-0.318	1.419				0.031
	(-0.369)	(1.229)				
FF3	-0.493	1.580	-0.005	1.811		0.067
	(-0.587)	(1.315)	(-0.043)	(1.232)		
CAPM + coskew	-0.998	2.083			-0.366	0.045
	(-0.821)	(1.342)			(-1.124)	
FF3 + coskew	-0.940	2.108	0.195	1.231	-0.323	0.081
	(-0.798)	(1.323)	(0.152)	(0.979)	(-1.268)	

5.2.2 Fama-Macbeth regression on portfolios

Table 8 presents the results by running the Fama-Macbeth regression on our portfolios on different models. A factor is priced if there is a linear relationship between a factor and the portfolio returns. We expected the risk premiums for beta to be consistently positive, and negative for γ. For the risk premium estimations involving gamma we saw a negative coefficient in the CAPM + coskewness model, while in the Fama French 3 factor + coskewness model we see a positive gamma. Due to the inconsistency for gamma, we believe there is uncertainty in the gamma estimate. The risk premium for beta is in positive for all regressions, which is preferable. Using a one-tailed t-test, we determine whether the risk premium estimates for beta are greater than zero and if the risk premium estimates for gamma is less than zero. The t-statistics are given in table 9. From the table we can see that no estimate is significant enough to verify our expectations that beta is positive and gamma is negative. None of the portfolios constructed seemed to have any significant predictive power, meaning the factors were not associated with the changes in expected portfolio returns.

From table 8 we see that the CAPM model had an $\overline{adj.R^2}$ of about 0.194, compared to the original CAPM results of 0.85-0.90 (Sharpe, 1964). This was quite a step down and that we can not draw the same conclusion based on our data. The results do indicate that the explanation power of the models increases when adding factors, but not sufficiently so. However, the results have led us to believe that FF3 are indeed important pricing factors in the Norwegian stock market, since by adding these factors we see a generally see a greater explanatory power(R^2).

Table 8: Cross-sectional regressions using the Fama-Macbeth procedure. The columns report the intercepts and the estimated risk premium given in percent and the t-statistic for each factor in parentheses.

Model	λ_0	λ_β	λ_{SMB}	λ_{HML}	λ_γ	$\overline{adj.R^2}$
CAPM	0.622	0.779				0.194
	(1.290)	(0.919)				
FF3	-0.376	0.282	-2.55	5.44		0.342
	(-0.432)	(0.300)	(-1.034)	(1.103)		
CAPM + coskew	-1.750	3.170			-0.785	0.292
	(-0.903)	(1.250)			(-1.220)	
FF3 + coskew	-0.244	0.0816	-2.66	5.81	0.0471	0.395
	(-0.298)	(0.068)	(-1.0142)	(1.156)	(0.271)	

Table 9: The corresponding one-tailed t-statistics of beta being greater than zero and gamma being less than zero.

Model	beta>0	gamma<0
CAPM	-0.720	
FF3	0.461	
CAPM + coskew	1.248	-1.220
FF3 + coskew	0.0679	0.271

Our results did not seem to directly correspond with Kraus and Litzenberger (1976), although we did share their end result in that we believe CAPM alone is not an adequate model to explain stock returns.

We also tested the same models using robust regressions to control for unforeseen heteroscadicity and outliers, however it must be noted that coskewness aims to price these outliers in some way as outliers have a direct effect on skewness and coskewness alike and a robust regression could, in theory, detract from this pricing, by removing extremely negative or positive values thereby changing to the coskewness estimate.

Nevertheless, robust regressions provided an interesting aside as we saw the pricing errors (alphas) become consistent with statistically significant t-values ranging from 3.1 all the way to 4, as seen in the appendix B. We also found that there is a small, but consistent increase in the explanation power of the models presented. Unfortunately, we also saw that the expected signs of the coefficients changed and that our expectations did not hold true to our estimations. Our expectations based on the theory section implied that a risk-averse investor would prefer negative coskewness and that we would therefore find a negative risk-premium associated with this factor.

5.3 Robustness

In this section we will provide our results from running a robust regression on the first stage of the Fama-Macbeth regression.

Deciding to include additional factors to a model could erroneously be rejected due to outliers negatively affecting the new model. Even though the data may inhibit errors, it could just as well be the cause of rare events, such as turmoil in markets, acquisition of a company or if the company is filing for bankruptcy protection. If we are able to manage these outliers, we expect that the estimates will be more efficient in "normal" times.

We believe that extreme values should be observed more frequently in our sample consisting of equally-weighted portfolios than if we created value-weighted portfolios. Smaller companies, which fluctuate more, have a greater influence of the overall portfolio performance in equally-weighted portfolios than in value-weighted portfolios. As we can see from table 2, the highest observed monthly return of 5700% and the lowest of -96%, are indeed extreme observations. Even though the stocks that have generated the most extreme returns are placed in portfolios, the magnitude of these observations will affect the portfolio performance significantly. Extreme outliers in returns can also be observed in the portfolio returns from table 3 and 4, where the most extreme returns can be observed in the lowest and highest ranking of both beta and gamma portfolios.

In table 10, we have provided summary statistics of the beta and gamma estimates by running a least squared regression and the robust MM-regression. In the beta column, we can see that the minimum and maximum values are slightly reduced, while their means do not differ by a lot. By running a paired t-test, we do not reject the null hypothesis and conclude that the means are not statistically different (*t-stat: 0.288)*. Looking at the gamma column, however, we can see a bigger difference in estimates. Both the outliers and the mean from the robust regression are smaller compared to those from the least squared regression, indicating that outliers may have had an impact on the gamma estimations. Running a paired t-test on the two means, we do not reject the null-hypothesis, and conclude that the means are not statistically different(*t-stat: -0.689*).

Table 10: Summary statistics of Robust slopes on beta and gamma.

	beta		gamma	
	LS	Robust	LS	Robust
Minimum	-0.241	-0.035	-10.478	-4.573
Median	0.695	0.688	-0.513	-0.534
Mean	0.790	0.776	-0.631	-0.403
Maximum	2.100	1.790	5.678	3.675

Comparing our regression results from the least squares regression and the MM-regression, we can see from table 12 a clear difference in the coefficients, indicating that our data has been heavily influenced by outliers. The factors' t-statistics from the robust regression are still low in general just as in the least squared regression. None of the estimates are statistically significant leading us to not reject the null hypothesis that their mean is different from zero. The intercept now appears to be statistically different from zero, strengthening our conclusion that our factors' may not be well suited to explain the portfolio returns in this analysis. From the robust regression we can see that the estimate of gamma's risk premium is positive, which is contrary to our expectations. Testing whether gamma is negative (*t-stat: 0.115*), we can not reject the null-hypothesis. By looking at the difference in average adjusted R-squared, we can see that the variance in returns explained by the robust model is almost identical to the least squared regression.

Table 11: Comparison between estimated risk premiums using robust regressions for the 1st stage and and using OLS for both stages, where factor premiums and intercepts are given in percent, with the corresponding t-stat given in parenthesis below.

Method	λ_0	λ_β	λ_{SMB}	λ_{HML}	λ_γ	$\overline{adj.R^2}$
LS	-0.244	0.082	-2.657	5.810	0.047	0.395
	(-0.300)	(0.068)	(-1.014)	(1.156)	(0.271)	
Robust	0.753	0.238	1.093	2.651	0.011	0.400
	(3.340)	(0.335)	(0.777)	(1.1725)	(0.116)	

6 Conclusion

In this thesis, we have investigated whether coskewness is an adequate factor in explaining stock returns in the Norwegian stock market. By running a robust regression, we could clearly see that our model was affected by outliers in the first stage. These outliers were presented in table 11 showing a statistically significant pricing error with robust regressions, with near identical explanation power. Depending on the model used, we were able to capture from 19 to 40 percent of the variance in expected returns of our portfolios.

With t-values of the estimated risk premiums using regular OLS hovering around an absolute value of 1, we could not reject the null hypothesis for any of the coefficients. For robust regressions in both stages, we did however estimate statistically significant alphas with t-values going as high as 4.055 in the case of the CAPM + coskewness model, giving us proof that our model was insufficient in explaining portfolio rate. The existence of statistically significant alphas indicates the existence of a consistent pricing error in the model. The risk premiums estimated for the FF3 + coskewness model were frequently 1 to 2 orders of magnitude lower than other estimated premiums and consistently demonstrated exceptionally small t-values, which led us to the conclusion that coskewness was not a priced risk factor in the Norwegian market.

Given the low explanation power and the fact that none of our regressions gave us any coefficients with significant t-values, we conclude our thesis by stating that CAPM is clearly not sufficient in explaining stock returns in the Norwegian market, and that adding coskewness and/or FF3 factors to the model is not enough to make it statistically significant. We did, however, see a benefit from adding the additional factors, and believe further research is warranted into multifactor Fama and French models.

A Skewness of a portfolio

$$Skew(X) = \mu_p = E(\frac{X-\mu}{\sigma}) = \frac{\mu^3}{\sigma^3} = E[\frac{(x-\mu)^3}{(E[(x-\mu)]^2)^{\frac{3}{2}}}]$$

Assuming μ and σ are finite then;

$$\mu_3 = \frac{E(X^3) - 3\mu * E(X^2) + 3\mu^2 * E(X) - \mu^3}{\sigma^3}$$

$$\mu_3 = \frac{E(X^3) - 3\mu * \sigma^2 - \mu^3}{\sigma^3}$$

Given that :

$$m_1 = W^T * \mu = \sum_{i=1}^{N} W_i \mu_i = \mu_p$$

$$m_2 = \sigma_p^2 + m_1^2$$

$$m_2 = W^T \Sigma W + \mu_p^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \sigma_{ij} + \mu_p^2$$

$$m_3 = \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{k=1}^{N} w_i w_j w_k S_{ijk}$$

Where W is a vector representing the weight in each security, + μ is a vector representing the expected return on each security. Σ represents the covariance matrix for all securities, and finally S_{ijk} is the co-skewness between securities i,j,and k.

Where co-skewness is defined as:

$$S_{ijk} = \sum_{t=1}^{T} \frac{(r_{ti} - \overline{r}_i)(r_{tj} - \overline{r}_j)(r_{tk} - \overline{r}_k)}{T}$$

It then follows that:

$$Skew_p = \frac{m_3 - 3(m_2 m_1) + 2m_1}{\sigma_p^3}$$

Which expands in to:

$$\begin{aligned} Skew_p = \frac{1}{(\sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij})^{\frac{3}{2}}} [\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N} w_i w_j w_k S_{ijk} \\ -3(\sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij} + (\sum_{i=1}^{N} w_i \mu_i)^2)(\sum_{i=1}^{N} w_i \mu_i) \\ +2(\sum_{i=1}^{N} w_i \mu_i)^3] \end{aligned}$$

B Robust regression tables

Table 12: Cross-sectional robust regressions where factor premia and intercepts are given in percent, with the corresponding two-tailed t-statistic given in parenthesis below.

Methodology	λ_0	λ_β	λ_{SMB}	λ_{HML}	λ_γ	$\overline{adj.R^2}$
CAPM	0.679	-0.329				0.265
	(3.13)	(-0.704)				
FF3	0.771	0.350	1.293	2.880		0.331
	(3.319)	(0.681)	(1.050)	(1.144)		
CAPM + coskew	0.794	-0.479			0.068	0.310
	(4.055)	(-1.016)			(0.889)	
FF3 + coskew	0.753	0.238	1.093	2.651	0.011	0.400
	(3.340)	(0.335)	(0.777)	(1.1725)	(0.116)	

Table 13: Risk premiums for associated factors using robust regressions during the first stage, and ordinary least squares for second stage.

Methodology	λ_0	λ_β	λ_{SMB}	λ_{HML}	λ_γ	$\overline{adj.R^2}$
CAPM	1.051	0.307				0.195
	(1.937)	(0.460)				
FF3	2.070	0.837	4.097	-15.902		0.279
	(1.949)	(0.994)	(1.449)	(-1.406)		
CAPM + coskew	-1.732	3.462			-0.828	0.276
	(-0.837)	(1.181)			(-1.149)	
FF3 + coskew	0.242	7.136	12.786	-26.332	-1.076	0.386
	(0.384)	(1.231)	(1.286)	(-1.34)	(-1.18)	

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