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A stochastic dynamic programming approach

Master's thesis in Industrial Economics and Technology Management
Supervisor: Belsom, Einar
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Preface

This thesis is the final work of our Master of Science degree in Industrial Economics and Technology Management at the Norwegian University of Science and Technology (NTNU). The analyses in this thesis are calculated and plotted using the numerical software Matlab. We would like to express our sincere gratitude and appreciation to our supervisor Einar Belsom for his valuable guidance and support.

Abstract

We investigate problems in contracting between risk-neutral shareholders and a risk-averse CEO in a principal-agent framework. In particular, we examine how equity-based compensation, such as options and stocks, influence the risk-taking behavior of the CEO. Our approach relies on stochastic dynamic programming in discrete time intervals to model the preferred debt ratio (kappa) of the CEO and construct a three-dimensional kappa surface based on the constant relative risk aversion model (CRRA). We design both a single-period model and a more complex multi-period model spanning over multiple years with numerous option packages and vesting restrictions on stocks. Comparing the kappa surface of CEOs on different contracts, we find that the optimal debt ratio from the shareholders' perspective is substantially higher than what the CEO prefer. This is due to the fact that shareholders are more diversified, while CEOs are risk-averse and heavy time-discounters. We find that option-based compensation does not strictly lead to higher risk-taking behavior and thus only to some extent aligns the incentives between the CEO and shareholders. Moreover, we compare the certainty equivalent rate of return for the shareholders to the CEO's, and find that both the CEO and shareholders are better off having less weight in options and a higher weight in non-equity based compensation compared to what empirical studies suggest to be the standard for large American companies. Thus, our findings imply that there exist more efficient ways to structure compensation contracts in corporations.

Sammendrag

Vi undersøker problemer i avtaleforhold mellom risikonøytrale aksjonærer og en risikoavers administrerende direktør (adm. direktør) i et prinsipal-agent rammeverk. Vi undersøker hvordan aksjebasert avlønning, for eksempel gjennom opsjoner og aksjer, påvirker adm. direktørs risikovilje. Vår tilnærming til problemstillingen er å ta i bruk stokastisk dynamisk programmering i diskrete tidsintervaller for å modellere den foretrukne gjeldsgraden (kappa) til adm. direktør og konstruere en tredimensjonal kappaoverflate basert på en nyttefunksjon for en adm. direktør med konstant relativ risikoaversjon (CRRA). Vi utvikler både en enkeltperiodemodell og en mer kompleks multiperiodemodell som strekker seg over flere år med ulike opsjonsspakker og salgsrestriksjoner. Når vi sammenligner kappa-overflaten til adm. direktører med forskjellige kontrakter, ser vi at den optimale gjeldsgraden fra aksjonærenes perspektiv er vesentlig høyere enn hva adm. direktør foretrekker. Dette skyldes at aksjonærene er mer diversifiserte i forhold til adm. direktør som er risikoavers og har høy tidsdiskontering. Vi ser av funnene at opsjonsbasert avlønning ikke alltid fører til høyere risikotakende oppførsel hos adm. direktør, og bare til en viss grad samordner insentivene mellom adm. direktør og aksjonærer. Videre sammenligner vi sikkerhetsekvivalenter til aksjonærene og adm. direktør, og finner at både adm. direktør og aksjonærer er bedre stilt ved å ha mindre vekt i opsjonsavlønning og en høyere vekt i ikke-aksjebasert avlønning sammenliknet med hva empiriske studier antyder er standardkontrakten for store amerikanske selskaper. Dermed indikerer våre funn at det finnes bedre måter å strukturere lønnskontrakter på i selskaper.

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1. Introduction

"If Tesla & SpaceX go bankrupt, so will I. As it should be."
— Elon Musk (2019)

The wealth of Elon Musk has been estimated to a staggering $30 billion, but most of his wealth is tied to the success of the companies he manages through stock and option ownership (Kiersz and Borden, 2019). Elon Musk is one of many CEOs who is paid partly in stocks or options in the companies they lead. In this thesis, we intend to explore how such compensation schemes affect the risk-taking behavior of CEOs.

High CEO salaries have received a lot of public attention over the last decades and sparked intense debate. The CEO compensation structure is of great interest to the shareholders, with important implications for the functioning of financial markets. Owners wish to maximize return on their investment by designing compensation schemes that reward a CEO who behaves according to their interests, yet also want to avoid paying the CEO more than required. However, less attention has been attributed to the structure of CEO compensation than to the size of the CEO compensation, and Jensen and Murphy (1990) claim that it is not about how much to pay, but how. Even though the level of pay affects how skilled managers a firm can attract, it does not necessarily have anything to do with the CEO's incentives to run the firm in the shareholders' interests.

A shift began around 1990 in executive compensation moving toward more equity-based compensation to motivate the CEO to manage the firm more in line with shareholders' preferences (Kolb, 2012). Two common types of equity-based compensation packages are option-based compensation, commonly referred to as ESOs (employee stock options), and restricted stocks. They involve some attractive properties, but also have some shortcomings. While the compensation structures aim to provide incentives for the CEO to maximize shareholder value, they have also been demonstrated to provide the executive incentives to reduce firm volatility (Lambert et al., 1991; Carpenter, 2000). Moreover, equity-based compensation serves as a prevention against the CEO walking away from a destroyed firm with his or her wealth intact. As an illustration, the CEO of Lehman Brothers earned an estimated $22 million in 2007 right before the crash (Furhmann, 2019), but the bankruptcy cost him more than $710 million due to stock ownership (Kolb, 2012).

Empirical research shows that shareholders respond positively to incentive compensation plans by bidding up stock prices significantly when announcements of incentive plans reach the public (Brickley et al., 1985; Billett et al., 2010). However, it is generally recognized in the literature that no incentive schemes perfectly align the incentives between the principal and agent (Jensen et al., 2004; Kolb, 2012). Thus, the

CEO as an agent will to some extent behave in a way that contradicts the interests of the shareholders.

The separation of ownership and management in a firm can lead to conflict between the desires of the owners and the CEO they hire to manage the firm on their behalf. This conflict of interest is usually referred to in the literature as a principal-agent problem. The problem stems from the fact that the principal cannot observe and control the agent, only design an incentive scheme to induce the agent to act according to the principal's interests. Jensen and Meckling (1979, p. 5) define the agency theory and the resulting conflict of interest as follows:

> *We define an agency relationship as a contract under which one or more persons (the principal(s)) engage another person (the agent) to perform some services on their behalf which involves delegating some decision making authority to the agent. If both parties to the relationship are utility maximizers, there is good reason to believe that the agent will not always act in the best interests of the principal.*

Our focus is on the CEO as an agent. More generally, the problem of getting the agent to behave according to the principals' preferences exists in most corporations and organizations at every management level. Thus, the study of the conflict of interest between CEO and shareholders is of great interest and importance as the generalized problem can be applicable to other principal-agent problems as well.

From the perspective of the CEO, a larger share of equity-based compensation has a diminishing utility as it makes his or her wealth portfolio less diversified. The CEO can reduce the risk of his or her portfolio by dividing it between assets inside and outside the firm. To illustrate this, consider the case when Enron collapsed. Many employees had a large fraction of their salary in the form of Enron shares (Kolb, 2012). After the bankruptcy, the employees lost both their future income cash flow as well as their retirement savings. In other words, a CEO would prefer to not to have all compensation directly tied to the share price to prevent keeping all of the eggs in one basket.

One of the CEO's key decisions that affect expected value of a firm is related to capital-budgeting. We compare the desired debt ratio of the firm from the CEO's perspective to that of the shareholders'. The key research question is a matter of incentive compensation: how does compensation structure affect debt ratio preferences of the CEO with time-varying firm value, and what is the optimal incentive contract from a shareholder's perspective? We develop both a single-period model and a more complex multi-period model spanning over several years with multiple option packages and shares with vesting restrictions.

This paper makes several contributions to the literature. While the literature on executive compensation is extensive, few papers directly investigate the impact of the compensation structure on the risk-behavior of the CEO in terms of debt ratio preferences. When estimating the optimal capital structure, it is common in the literature on trade-off theory of capital structure to determine it at an initial date and let the level of debt remain unchanged subsequently; see for example Leland (1998). This is a simplification as firms have the flexibility to adjust the level of debt. In our model, we let the debt ratio vary each day to let the firm adjust the financial structure in response to stochastic changes in the economic environment or firm performance. In that manner, our approach to the agency problem differs from most of the literature on this topic as we will apply dynamic programming to investigate a dynamic environment.

There already exist several papers that investigate the optimal executive compensation when firm value follows a stochastic process (He, 2009; Edmans et al., 2012). In these papers, the CEO affects the stochastic process through an unobservable variable describing effort. However, in contrast to these models, all variables in our model are observable. Moreover, these papers do not consider option-based compensation in their model. We add option-based compensation to the wealth portfolio of the CEO as the options provide different incentives compared to stock-based compensation. To fully investigate the effect of option-based compensation on CEO behavior we must add a time dimension as behavior will not only depend on firm value, but also the expiration date of the options.

The conflict of interest concerning the risk-behavior in a principal-agent problem has been investigated in the hedge fund industry using dynamic programming (Hodder and Jackwerth, 2007; Asheim, 2014; Scheel et al., 2015). On the basis of the work of these authors regarding incentive contracts in the hedge fund industry, we adjust their dynamic programming approach to fit the corporate governance problem. We thus extend on current literature by developing a dynamic programming model to find the optimal debt ratio in corporations from a CEO point of view, given different incentive schemes. Principally, we should expect the preferred risk-behavior of a CEO in a firm to deviate from the behavior of a hedge fund manager as their incentives diverge due to different compensation structures. For instance, the option packages granted to the CEO are different from option-like high-water mark structures in the hedge fund industry, as option-based compensation in corporations is usually issued at-the-money. Moreover, due to vesting restrictions on stocks, the structure of the compensation payout differs for a CEO compared to a hedge fund manager. The non-equity based compensation also differs fundamentally as it is fixed over a longer period, while hedge fund managers receive a percentage of assets under management (AUM) which will vary from year to year.

The rest of the paper is structured as follows. In Chapter 2 we introduce the utility and wealth function of the CEO and provide the necessary background to get an understanding of CEO compensation. Next, in Chapter 3 we describe the single-period model that serves as a basis for our computations of the risk-behavior of the CEO in the subsequent chapter. Chapter 4 is devoted to presenting our results from the single-period model and investigating the conflict of interest between CEOs and shareholders in a firm. Next, we introduce the multi-period model in Chapter 5 where we consider a longer time horizon with a more complex option package structure and a vesting restriction on the stocks. In Chapter 6 we compute the optimal contract of these stakeholders by comparing differences in certainty equivalent returns and executing sensitivity analyses on risk aversion. Finally, Chapter 7 summarizes and concludes.

2. Utility, wealth and risk preferences

In our model, we assume that the CEO behaves in such a way as to maximize his or her personal utility. To investigate the debt ratio preferences of the CEO, we must identify the incentives of the CEO and to what extent they are linked to the shareholders' desire to increase firm value. Firstly, we introduce the utility function we use to describe the CEO's preferences in our model. The utility function depends on the wealth of the CEO, so in the subsequent section we construct the wealth function, and calibrate it based on common CEO compensation contracts. Finally, we reflect upon the wealth and risk preferences of the shareholders.

2.1 CRRA utility

Utility is a conceptual construct of pleasure or well-being broadly used in economics. Several models have been developed in the literature to describe utility mathematically. The Constant Relative Risk Aversion (CRRA) utility function is widely used in economic literature (Wakker, 2008). We use the CRRA utility function developed by Von Neumann and Morgenstern (1947) to model the CEO's risk preferences, representing constant relative risk aversion and expressed as

$$U(W) = \begin{cases} \dfrac{W^{1-\gamma} - 1}{1-\gamma} & \gamma \neq 1 \\ ln(W) & \gamma = 1 \end{cases}, \tag{2.1}$$

where

> W denotes the wealth of the CEO. The wealth function is described in detail later in Section 2.2.
>
> γ is the risk aversion coefficient of the CEO.

The risk aversion coefficient, γ, reflects the degree of preference for a certain outcome over an uncertain higher reward. As the degree of an individual's risk aversion increases, γ increases. $\gamma = 0$ corresponds to risk neutrality where the CEO is indifferent with regard to uncertainty as long as the expected value is the same. For instance, a risk neutral CEO will be indifferent between receiving \$5 with a probability of 100% or receiving \$10 with a probability of 50%. We set the risk-aversion coefficient $\gamma = 4$ in our model similarly to what previous studies in the hedge fund industry do (Hodder and Jackwerth, 2007; Scheel et al., 2015). The numerical value of our risk-aversion coefficient is justified in-depth in Appendix A.

There are several desirable properties related to the CRRA utility function. Firstly, the function is monotonic in wealth; more wealth always increase the utility of the CEO. Secondly, the function is consistent with diminishing marginal utility, meaning that utility increases more slowly when the wealth becomes larger for $\gamma > 0$. Furthermore, when following the CRRA utility function, risk-taking decisions are not dependent on the scale of the initial wealth, according to Arrow (1971) and Azar (2006).

One significant weakness of the CRRA utility function is that the risk aversion coefficient is held constant. Ideally, a utility function should reflect the risk aversion that truly characterize human behavior. According to the prospect theory developed by Tversky and Kahneman (1992) gains and losses are valued differently. This is not taken into account in the CRRA utility function as the risk aversion coefficient is held constant. The choice of model has been shown to yield different optimal compensation contracts in the literature (Dittmann and Maug, 2007; Dittmann et al., 2010). Thus, using the CRRA utility in our model has some shortcomings. However, using the CRRA utility function compared to prospect theory greatly simplifies computations due to the lack of path dependency. Prospect theory can be applied in our model by introducing a new state variable. Nevertheless, an extra dimension in the model's state space would affect the run-time greatly and make the model more complex, therefore we limit this thesis to CRRA utility and leave prospect theory out of our problem scope.

2.2 CEO wealth

We can divide CEO compensation into two distinct categories based on whether the compensation directly depends on share performance or not, namely equity-based compensation and non-equity based compensation. We hypothesize that these different components of wealth create different incentives that influence the risk-taking behavior of a CEO. In this paper, we focus on the most common types of executive compensation including stocks, options and non-equity based compensation.

Most attention in the public has been focused on the compensation packages in big corporations. Moreover, larger firms have the most data available. For these reasons, this paper concentrates on creating a model of executive compensation in such firms.

Equity-based compensation

A substantial fraction of the CEO compensation is directly linked to the share price in large firms (Russell and Williams, 2019). A common perception is that the larger fraction of the CEO wealth that depends on the firm value, the more aligned are the incentives with the shareholders'. From empirical analyses in the literature, it has been confirmed that CEOs do respond to risk-taking incentives in their compensation (Rajgopal and Shevlin, 2002; Coles et al., 2006). Jensen and Murphy (1990) investigated performance pay and incentives of CEOs empirically and found that CEO wealth on average changes $3.25 for every $1,000 change in shareholder wealth. Of this amount, they found that $2.65 stems from existing stocks and options in the CEO's wealth portfolio.

Option-based compensation, commonly referred to as executive stock options (ESOs), gives the CEO a right to buy stocks from the firm at a predetermined price (strike price) at a future date (expiration date). The structure of this type of compensation is illustrated in Figure 2.1a. Such contracts have many similarities to exchange-traded options. Kolb (2012) provides an extensive description of the features of so-called ESOs that deviate from exchange-traded options:

- **Issuance of new shares**: Options awarded to the CEO are usually issued as a warrant. In contrast to purchasing an option where an already existing share is the underlying asset, the company usually issues new shares when the CEO exercises the ESO.
- **Non-transferability**: The options are usually not transferable, meaning that the CEO cannot sell them before exercising.
- **Vesting requirement**: They generally have a vesting requirement specifying a period before the option can be exercised, not necessarily being the expiration date. Thus, the typical ESO is a Bermudan option.

The motivation to distribute option-based compensation to the CEO from a shareholders' perspective is related to the desire to increase the risk-taking behavior and effort that lead to greater firm value. The reason why the CEO would want to increase the risk is due to the fact that the option value increases when the volatility increases. Moreover, granting option-based compensation does not require any immediate cash outlay and is thus attractive from a liquidity perspective. For example in start-ups, option-based compensation is particularly attractive, since it partly substitutes cash salaries during times of cash starvation.

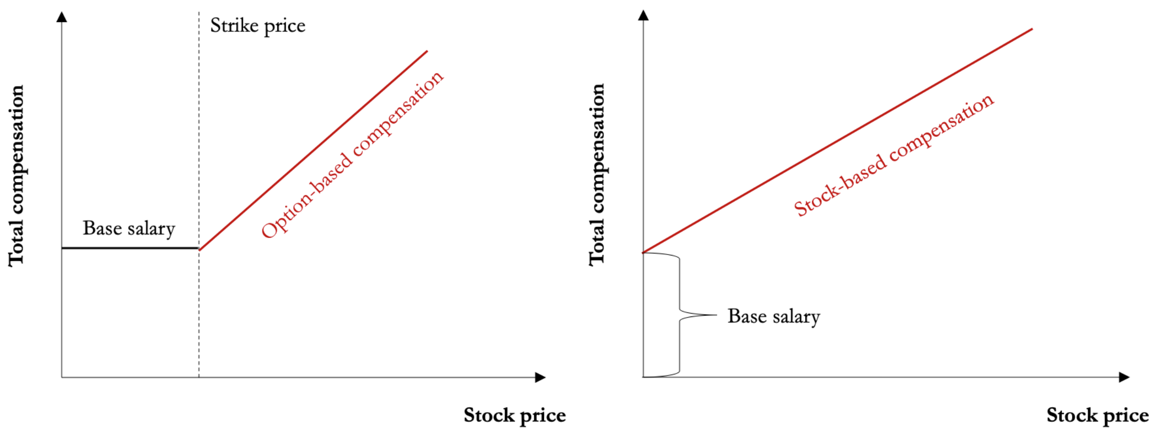
Option-based compensation schemes have been controversial since they are often accused of being a waste of shareholders' money, especially considering the dilution of shareholder value that comes from the issuance of new shares when an ESO is exercised. Moreover, critics of option-based compensation commonly claim that options have unlimited upside but no downside. As noted by Sappington (1991), when the agent (CEO) to some extent is insured against bad outcomes, he or she will make less effort to avoid bad outcomes. However, Hall (2000) and Carpenter and Yermack (2013) argue that this is incorrect, as options have a value equal to the option price, and when the stock price declines far below the strike price, the option becomes close to worthless. Based on this, we hypothesize that when the stock price falls significantly below the strike price of the option, the option does not influence the decision-making of the CEO. Similarly, when the stock price rises substantially above the strike price, we hypothesize that the option has similar impact on behavior as stocks.

Another problem is related to whether the CEO holds the share for a period after exercising the option or sells it right away. CEOs tend to sell the shares right after option exercise (Kolb, 2012) or receive a cash settlement instead of receiving the underlying asset upon exercise. Consequently, the incentives that the options provided disappear after exercise, and to restore this incentive, the firm must issue new options.

The most common compensation type in 2018 for the top 100 highest-paid executives in the US was stock grants, according to Russell and Williams (2019). With such a compensation type, the total compensation will vary linearly with the stock price as illustrated in Figure 2.1b. Thus, the CEO is rewarded for an increasing stock price, and similarly penalized for a declining stock price. It has been empirically found that CEOs' ownership in the firm constitutes a great part of their total personal wealth (Elsilä et al., 2013). Stock grants are usually restricted, meaning that the executive must fulfill a condition to receive the granted stocks, which is usually related to a commitment to stay in the firm for a particular amount of time. Such vesting restriction on stocks are typically spanning over three to four years (Kadan and Yang, 2005).

The perceived main advantage of stock-based compensation is that it gives the CEO incentives to increase the long-term value of the firm by inducing a feeling of commitment towards the firm as equity holders. Furthermore, granting stocks to the CEO as part of the compensation gives him or her something to lose, since a decreasing stock price has a negative impact on the wealth of the CEO.

A natural question arises related to whether stock-based compensation or option-based compensation is the most effective from a shareholder's perspective. Hall (2000) argue that options have a larger downside risk compared to stocks in terms of return. Using the CRRA utility function, Dittmann and Maug (2007) found that the CEO should receive no options and more stocks in the compensation package. Jenter (2002) concludes similarly that stock grants are a more effective compensation form. However, in subsequent studies, when taking into account loss aversion, Dittmann et al. (2010) found that the more averse the agent is to losses, the more options are used in the optimal contract. Their analysis implies that options have a smaller downside risk compared to stocks when you take into account loss aversion.



(a) Option-based compensation. (b) Stock-based compensation.

Figure 2.1: Salary structure of equity-based compensation

An objection against equity-based compensation is that such incentives are not filtered for external factors, potentially rewarding or penalizing the CEO for random macroeconomic events. Moreover, incentive compensation may influence the truthfulness of the way CEOs report the results of their firms to the public and thus might lead to corporate dishonesty (Bebchuk and Fried, 2003).

Non-equity based compensation

Non-equity based compensation of the CEO is not directly linked to the share price of the firm, and mainly consists of base salary and bonus. The base salary of the CEO is usually stated as a fixed annual amount, and is usually the most visible portion of total compensation. Bonuses are the largest fraction of the non-equity based compensation (Russell and Williams, 2019). Bonuses can be fixed amounts, but are typically linked to performance measures such as share price or other KPIs. A valid question is therefore why we classify bonuses as non-equity based compensation. An annual bonus is typically based on overcoming a hurdle defined in accounting terms, for example growth in sales of a certain percentage, which is likely to be correlated to the share price. If the correlation is high, it will have an option-like payoff structure and the value of the bonus will depend on the probability that the performance target will be met. However, this is more indirectly linked to the share price compared to equity-based compensations such as stocks and options. Thus, we consider bonus to be fixed in our model and independent of share price.

There are other ways to compensate executives, for example pension plans, retirement accounts, departure payments and post retirement benefits. The exact magnitude of such compensations can be difficult to measure, and therefore we do not consider such compensations in our analysis for the sake of simplicity in model calibration.

Wealth held outside the firm

In addition to having wealth tied to the firm, the CEO will also hold wealth outside the firm. Elsilä et al. (2013) investigated empirically the total personal wealth of CEOs of listed Swedish firms, and found that personal wealth that is not related to the firm constitutes approximately 26 % of total personal wealth. The magnitude of the fraction of wealth held outside the firm should in principle affect how the CEO manages the firm since a larger fraction of wealth will be independent of firm performance. We will discuss the implications of a CEO that has a large amount of wealth outside the firm compared to the compensation further in Section 6.4.

Other literature regarding the relation between risk-taking and the agent's personal wealth, like Hodder and Jackwerth (2007), assume that the agent has no further personal wealth other than compensation from the firm. However, they argue that this assumption can readily be relaxed, but do not reflect any further on the impact of neglecting personal wealth on their results. In our model, we do not take into account wealth outside the firm, as we focus on the effect of stocks and options, but relaxing this assumption is a potentially interesting extension of our work in future studies. Also, as previously mentioned, the risk preferences when using CRRA utility is not dependent on the scale of the initial wealth.

CEO wealth function

We assume that the maturity date of the options is equal to the end of the period in our model. The total wealth of the CEO at the end of the period can then be expressed as follows:

$$W = a(1-b)S_0 + bS_T + n(1-b)max(0, S_T - K) \quad (2.2)$$

where

a denotes the total pay that occurs independent of the stock price, including base salary and bonus as a fraction of firm value at $t = 0$

b denotes the CEO's fraction of share ownership

S_0 is the stock price at the beginning of period 1

S_T is the stock price at maturity $t = T$

n is the amount of options as a fraction of total shares issued

K is the option strike price, which is set equal to S_0

Based on observed compensation contracts in the literature, we set the stock ownership $b = 1.6\%$, option ownership $n = 0.42\%$ and fraction of non-equity based compensation $a = 0.03\%$ of total firm value. The values are based on empirical analyses in executive compensation in the literature (Balsam, 2002; Jensen and Murphy, 1990). The numerical values of the parameters in our wealth function are further justified in Appendix A.

The behavior of the CEO in our model is affected by the compensation structure in terms of the weight of non-equity based compensation relative to equity-based compensation. We limit the equity-based compensation to a mix of options and stocks. The options are modeled as European due to their simple structure. In our model we also assume that ESOs are usually issued at-the-money (Kolb, 2012). When the stock price increases, equity-based compensation will constitute a larger portion of the wealth portfolio as the value of stocks and options will increase.

We multiply the compensation based on S_0 by a factor $(1-b)$, since the CEO owns a fraction of the firm equal to bS_0. We therefore subtract the amount equal to abS_0 in the first part of the function, otherwise the wealth function would have compensated the CEO with money he already owns. Not adjusting for this would effectively be to move money from the CEO's right side pocket to the CEO's left side pocket. The same logic applies to the wealth function's option term.

We model the compensation as being paid outside of the firm in order to avoid introducing path dependency in our model. This means that the magnitude of the compensation does not affect the firm value. If the compensation was to be subtracted from the company, the value of the company would not only depend on the current state, but also everything that has happened previously. In reality, a higher compensation to the CEO results in less value to the shareholders. For further analyses, the cost of the compensation for shareholders can easily be calculated and subtracted from their wealth. Introduction of a new state variable, allowing compensation to be subtracted directly from the firm, would also have solved the path dependency problem. However, it would increase the run-time of our model. With regard to the purpose of our model we believe that holding compensation payment outside the firm is a better solution. In addition, the value of the compensation is minor compared to the firm value, and does not have a significant impact.

2.3 Shareholders' wealth and risk preferences

The wealth of the shareholder is strictly increasing with the market value of the firm. Wealth can also be distributed to shareholders by paying cash dividends. During a dividend payout, the value of the outstanding shares drops by the amount of the dividend payout. This is likely to affect the incentives of the CEO who has a large fraction of option-based compensation. Studies have shown that stock options in executive compensation provide incentives to reduce dividends when they are not dividend protected (Lambert et al., 1989). However, most ESOs today are dividend protected and therefore we only consider such options (Zimmermann, 2016).

Hodder and Jackwerth (2007) and Scheel et al. (2015) assume that the principals have the same degree of risk aversion as the agent. In contrast to these papers, we assume that the shareholders are risk-neutral, since they are able to diversify their risk by investing in alternative investments. On the other hand, CEOs have a large proportion of their wealth tied to the firm and cannot diversify and will therefore tend toward risk-aversion. Therefore, we consider this to be a more realistic assumption.

The relation between firm value and the choice of debt ratio has been devoted significant attention in the literature. Modigliani and Miller (1958) pioneered on this topic by proposing that the firm value is independent of capital structure in the absence of bankruptcy costs and tax subsidies on interest payments in perfect capital markets with no agency costs. This implies that shareholders would be indifferent to the firm's debt ratio if we do not take into account these costs and benefits related to debt in perfect capital markets.

However, in the real world the markets are inefficient with agency costs, bankruptcy costs and tax subsidies, and thus the Modigliani-Miller assumptions do not hold. The theory on corporate capital structure has advanced significantly in the aftermath of Modigliani and Miller (1958). Jensen and Meckling (1979) remarked that this theory is insufficient in several ways. Among other things, it implies that no debt should be issued when there are no tax subsidies if bankruptcy costs are positive. This contradicts the fact that debt was commonly used prior to the introduction of tax subsidies. In our model, we take into account the benefits of tax subsidies related to interest payments. Consequently, issuing new debt will have two outcomes: it will increase the tax savings as long as the firm survives, but it will also reduce the probability of the firm's survival due to increased interest costs. Distress costs are taken into account through higher interest costs when debt ratio increases. However, we acknowledge that increasing interest costs may be an imperfect proxy for distress costs.

3. Model setup and solution methodology

If a part of the CEO's compensation is equity-based, it follows that a CEO's optimal risk-taking must depend on the development of the firm's share price. With that in mind, we present a dynamic model for investigating a CEO's optimal risk-taking. The optimal risk-taking is modeled to change with the development of the share price over a time period. Our model allows us to change the component weights of the compensation, so the CEO's risk-taking can be analyzed in light of different incentive contracts. Debt ratio is used as the metric for risk-taking. We consider this a problem of optimal control of a stochastic process, and use discrete time dynamic programming theory based on Markowitz (1959). In this chapter we present the setup for our single-period model, and justify the model assumptions. An overview of the model assumptions and parameters presented in this chapter is included in Appendix A.

3.1 The stochastic process for enterprise value

We decompose the enterprise value, EV_t, at each time state, t, into the market capitalization and the net interest bearing debt,

$$EV_t = S_t + D_t \tag{3.1}$$

where

S_t denotes the market capitalization

D_t is the net interest bearing debt

We consider financing an enterprise with debt as a strategy to increase the risk-taking of the firm. Seen from an enterprise point of view, debt financing increases risk since the debt issuers can petition for bankruptcy if the company fails to meet the debt covenants or fails to pay interest. An enterprise with debt is more vulnerable to bankruptcy and costs related to a liquidity squeeze. The higher the debt ratio, the higher interest rates claimed by the creditors. A financing strategy where the assets are financed partly by debt is a riskier financing strategy compared to an all-equity financed firm. This is why we consider debt ratio as an appropriate metric for an enterprise's risk-taking.

We consider the CEO's dynamic risk strategy as solely determined by the balance of equity and debt, and the enterprise value to be dependent on their movements only. The debt ratio, our metric for risk taking, is denoted kappa and is defined as

$$\kappa = \frac{D_t}{EV_t} \tag{3.2}$$

The return on market capitalization and net interest bearing debt follow two different processes. Debt contributes to the enterprise value with negative returns, better known as interest. The cost of interest is lowered when corporate tax is taken into account. The tax shield lowers the interest rate with a factor of (1 - τ), where τ is the corporate tax rate. Furthermore, when the debt ratio of a company increases, the interest rate on debt increases due to an increased bankruptcy risk. When adding distress costs to interest, we implicitly take bankruptcy costs into account. We define a distress multiplier, $M_{distress}$, based on empirically observed relations between debt ratio and interest rates on debt. This yields a function for $M_{distress}$ that increases exponentially. The multiplier is only defined for kappa values between 0 and 1. The computation and data for this function can be found in Appendix A.

$$M_{distress} = \begin{cases} 1 & \text{if } \kappa \in [0, 0.2] \\ 0.002781e^{12.05\kappa} + 0.969 & \text{if } \kappa \in (0.2, 1) \end{cases} \tag{3.3}$$

Accordingly, the continuously compounded yearly effective interest rate will be

$$r_e = r_i(1 - \tau)M_{distress} \tag{3.4}$$

where

r_i denotes the interest rate of AAA bonds

τ denotes the corporate tax rate

We acknowledge that the interest rate model imposes unrealistically high costs for high kappa values, but in our base case the model never suggests kappa values that high, even when we do not take the distress multiplier into account.

The future value of the market capitalization is considered a stochastic variable. The process of the market capitalization is the discrete time equivalent to the continuous stochastic process of a geometric Brownian motion (GBM) as shown in 3.5.

$$dS_t = (\mu_{r_{EV}} + (\mu_{r_{EV}} - r_e) \cdot \frac{\kappa}{1-\kappa})S_t dt + \sqrt{\sigma^2_{r_{EV}}(1 + \frac{\kappa^2}{(1-\kappa)^2})}\sqrt{dt}S_t dZ \tag{3.5}$$

where

$\mu_{r_{EV}}$ denotes the expected return on the enterprise value

$\sigma_{r_{EV}}$ denotes the standard deviation of the enterprise value

r_e denotes the effective interest rate

Z is a standard normal distributed stochastic variable

The derivation of Equation 3.5 is found in Appendix D. Since S_t follows a GBM, the value of S_t is log-normally distributed, and the log-returns of S_t, $\Delta ln(S_t)$, is normally distributed with mean and variance as follows for each discrete time step of length Δt:

$$\begin{aligned} \mu_{\Delta ln(S_t)} &= (\mu_{r_{EV}} + (\mu_{r_{EV}} - r_e)\frac{\kappa}{1-\kappa}) - \frac{1}{2}\sigma^2_{r_{EV}}(1 + \frac{\kappa^2}{(1-\kappa)^2})\Delta t \\ \sigma^2_{\Delta ln(S_t)} &= \sigma^2_{r_{EV}}(1 + \frac{\kappa^2}{(1-\kappa)^2})\Delta t \end{aligned} \tag{3.6}$$

The equations for the expected return and variance of the log-returns of S_t are important when creating the probability lookup matrix, which we will return to in Section 3.2. However, the assumption of normally distributed log-returns has some well known shortcomings in financial markets theory, as share prices tend to have a skewed distribution with fat tails.

Asset volatility, $\sigma_{r_{EV}}$, is assumed to be constant in our model. This is a simplification as it has been empirically shown that the asset volatility varies with leverage (Choi and Richardson, 2016). Asset volatility should increase when debt ratio increases, but modeling the asset volatility is left out of our problem scope. We have performed a sensitivity analysis on the asset volatility which is included in Appendix B.

We allow continuous re-balancing of debt in each time step. Effectively, this equals a CEO who has a mandate to alter the debt ratio strategy once each trading day, and sticks to this debt ratio choice until next trading day. If the market capitalization changes dramatically intraday, the enterprise can buy back shares or pay off loans to uphold the debt ratio. The alternative, which is to re-balance at discrete time states only, would make the net interest bearing debt constant, and the debt ratio would consequently change as the market capitalization changes. However, not having the debt ratio constant between the time states would result in a process for the market capitalization that does not follow a geometric Brownian motion. This causes numerical problems in our model, as this for example enables an enterprise to go bankrupt between two time states. For the market capitalization to follow the process of a GBM, there must be a constant debt ratio between the time states, which implies continuous re-balancing. Considering the fact that firms have access to overdraft facilities, and the possibility to buy and sell its own shares every trading day, we consider our continuous re-balancing assumption as plausible.

A key assumption in our model is that the stock price evolves independently of CEO performance. In reality, one should to some extent assume that CEO performance affects the stock price. There are models where the development of the stock price is affected by the effort level of the CEO. For example, Dittmann and Maug (2007) let the firm value depend on the level of effort of the CEO when they compute the optimal contract. However, a weakness of such a model is that the level of effort of the CEO is unobservable and difficult to quantify.

3.2 Model setup

In short, our model constructs a matrix of expected utilities given optimal behavior, by recursively searching for the kappa value that gives the highest expected utility. The expected utility is calculated based on the possible utilities in future scenarios and the probability of each scenario happening. The highest expected utility and corresponding debt ratio of this optimal choice is found in each time state. This pair of expected utility and debt ratio is stored in the matrix and used to calculate the next optimal utility and kappa choice moving one time step backwards. This searching algorithm finds the optimal debt ratio from a CEO's perspective for any reasonable market capitalization development, at all discrete time states in the period.

Market capitalization returns and kappa values

We design a discrete matrix of all possible developments of market capitalization, with continuously compounded returns. In our model, the continuously compounded equity return can vary from $(-500C)$ to $(500C)$, where C is the constant spacing, equal to $C = \Delta ln(S_i) = 0.005$. Accordingly, the market capitalization is allowed to take on values within the range $S_0 \cdot e^{\pm 500C}$. From this point forward, the market capitalization is denoted $S_i = S_0 \cdot e^{iC}$, where the variable i ranges from -500 to 500. The possible values of the market capitalization is the same for all time states, t, being interpreted as trading days in one year. In order to have 252 time steps, we need 253 time states, ranging from time state 0 to time state 252. The matrix of the change from the initial market capitalization therefore has a dimension of [1001, 253] and the 253 columns are identical.

$$\text{Matrix for change in market capitalization} = \begin{matrix} \\ i=500 \\ \vdots \\ i=-500 \end{matrix} \begin{matrix} t=0 & \cdots & t=252 \\ \left(\begin{matrix} 500C \\ \vdots \\ -500C \end{matrix}\right. & \begin{matrix} \cdots \\ \ddots \\ \cdots \end{matrix} & \left.\begin{matrix} 500C \\ \vdots \\ -500C \end{matrix}\right) \end{matrix}$$

Secondly, we design a discrete debt ratio vector with 2000 values for kappa ranging from 0 to 0.9995, having a spacing of 0.0005 between the kappa values and a dimension of [2000, 1].

$$\vec{\kappa} = \begin{pmatrix} 0 \\ 0.0005 \\ \vdots \\ 0.9990 \\ 0.9995 \end{pmatrix}$$

The debt ratio can only converge towards 1, but not reach 1, since our model does not allow the CEO to choose a capital structure with no market capitalization.

Lookup-matrix for jump probabilities of the market capitalization

In order to calculate expected utilities, we need the probabilities of moving from one market capitalization value at time state, t, to all possible market capitalization values that can be reached from the node in the next time state $t + 1$. We have chosen

100 grid steps up and 100 grid steps down in the next time state. The up and down steps are denoted j. This number of possible reachable nodes should not be lower in order to capture a wide enough part of the probability distribution. We have seen that 100 up and down steps are sufficient to capture the whole probability distribution for kappa values below 0.9. For higher kappa values, the model loses information from both tails of the distribution.

A discrete jump probability matrix will do the expected utility calculations in a run time efficient manner. Furthermore, if the jump probability matrix is sufficiently granular, the numerical inaccuracies will not be of any notable significance. In our model, the jump probability matrix gives a good approximation of the actual probability distribution for kappa values up to 0.9. For higher kappa values, the probability matrix is unable to capture the whole probability distribution. This makes the expected utilities for these kappa values incorrect, since the probability distribution would be skewed. However, our model does not find optimal kappa values above 0.9, and this weakness is thus considered a negligible problem.

As seen previously in Equation 3.6, $\Delta ln(S_t)$ is normally distributed and the expected value and volatility of S_t is solely dependent on kappa. The discrete probability of each level of log-return on the market capitalization given all 2000 kappa values, is found by the formula for discrete point probability of a normally distributed variable, divided by a normalization constant to make the sum of probabilities equal to one for the possible steps of each kappa value.

$$p_{j,\kappa} = \frac{\frac{1}{\sigma_S\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{jC-\mu_S}{\sigma_S})^2}}{\sum_{m=-100}^{100} \frac{1}{\sigma_S\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{mC-\mu_S}{\sigma_S})^2}} \tag{3.7}$$

where

$p_{j,\kappa}$ is the point probability of moving j nodes for each debt ratio κ

m denotes the number of up and down steps

3.3 Finding an optimal debt ratio grid by dynamic programming

To find the optimal debt ratios, we first calculate the terminal utilities at the terminal time state, $t = 252$, using the values from the matrix of change in market capitalization as input to our wealth function (Equation 2.2) and then calculating the corresponding utility in each node using the utility function (Equation 2.1), resulting in 1001 known utility values at the terminal nodes. The utility for the nodes at the last time period is defined as $U(W(S_{i,t=252}))$.

The terminal nodes are used to calculate the rest of the matrix. For each node we consider a multinomial forward move. A multinomial forward move means that each node has more than two possible outcomes. In our model we have chosen 201 possible outcomes, meaning 100 grid steps up and 100 grid step down in the next time state, $t + 1$, as previously mentioned. The transition probabilities from the node to the

201 possible subsequent grid points, was calculated in the jump probability matrix. The expected utility at each node is then calculated by summing the product of the probability of reaching each of the next j up and down nodes and the expected value of the utility, see Equation 3.8. We calculate the utilities at earlier time states as an expectation of utilities found at the following time state, $t + 1$, to calculate the remaining utility grid.

$$E(U_{i,t,k}) = \begin{cases} \sum_{j=-100}^{100} p_{j,\kappa} U_{i+j,t=252} & \text{if } t = 252 - 1 \\ \sum_{j=-100}^{100} p_{j,\kappa} E(U_{i+j,t+1}) & \text{if } t < 252 - 1 \end{cases} \tag{3.8}$$

We loop through all the kappa values to calculate the expected utilities, $E(U_{t,i,k})$, with t and i constant, and find the kappa value that yields the highest expected utility, as shown in equation 3.9.

$$E(U_{i,t}) = \max_{\kappa \in (0,0.9995)} E(U_{i,t,\kappa}) \tag{3.9}$$

When the optimal $E(U_{i,t})$ is found for a $S_{i,t}$ node, the algorithm proceeds to the next $S_{i+1,t}$ node, and repeats the procedure from Equation 3.8 and 3.9. When all kappa values for a node in a given time state and market capitalization value is looped through, the algorithm moves backwards to the preceding time state, and repeats the process to calculate the expected utilities and optimal kappas here.

Figure 3.1 illustrates how each utility in the up and down nodes j and -j, is multiplied with its probability, $p_{j,\kappa}$, represented by the arrows, to find the expected utility. When the expected utility for one kappa value is found, it finds the expected utility for the next kappa value until iterated through all kappa values. Then the searching algorithm proceeds to the next node, S_i, in the same time state, and finds the expected utility for all kappa values again. When iterated through all nodes, S_i, in one time state, the searching algorithm moves one time step backwards, and starts at the top node, just below the upper boundary, and iterates through the same procedure.

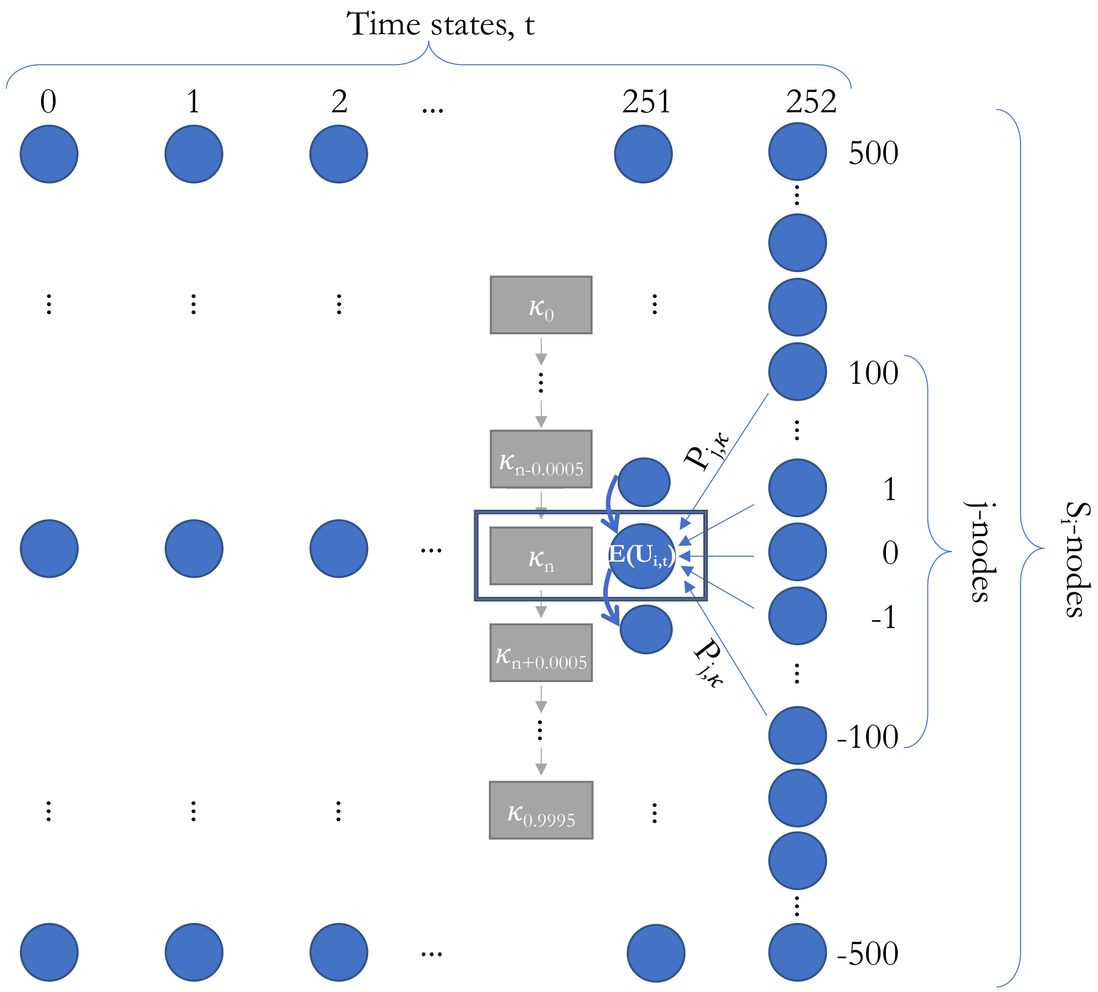


Figure 3.1: Finding the optimal κ for all utility values in each time state. The decision for the first time step is made at time state 0, and the decision for the final time step is made at time state 251.

The optimal debt ratio matrix is calculated simultaneously as the utility matrix by assigning the optimal kappa, giving the optimal utility in each node, to the corresponding node in the optimal debt ratio matrix. Our final result therefore illustrates the relationship between debt ratio, the stock price and the remaining time of the period.

3.4 Boundaries

Since we calculate utilities and debt ratios based on nodes above and below the current node, we need to set upper and lower boundaries in our model to limit the computations.

At the lower boundary, we assume that the CEO leaves the firm. We have observed that this is a common assumption at the lower boundary in other agency models, see for example He (2009). Taking into account quits and firings complicates risk-behavior as the CEO might leave if the firm value is too low; firings may for example provide an additional source of incentives. The assumption of CEO termination at a lower boundary is in line with prior studies documenting a negative relationship between firm performance and the probability of CEO tenure (Weisbach, 1988; Warner et al., 1988; Goyal and Park, 2002). However, Warner et al. (1988) show that this relationship only applies when performance is remarkably bad. This implies that we should set the lower boundary sufficiently low to make the model more realistic. We set the lower boundary to be 0.3 of the initial stock price.

We do not take into account the chance of departure before the firm value hits the liquidation boundary, as the CEO only quits at the lower boundary. This is indeed a simplification as the CEO as an individual has control over when he or she leaves the firm. Kolb (2012) set a constant chance of departure to 7 percent each year when valuing ESOs. This means that the CEO might quit to pursue other career opportunities with a probability of 7 percent. Taking into account the chance of departure in option valuation reduces the option value as the option will be worthless if the CEO quits before the expiration date. Considering the possibility that the CEO at any point might decide to leave is an interesting future model extension.

Since our focus is on large corporations, the chance of bankruptcy is substantially lower, justifying not considering bankruptcy scenarios in our model. Moreover, the likelihood of replacing the CEO should be higher in large corporations, since the equity share of the CEO is lower and thus also his or her influence regarding decisions of termination. When a CEO has a larger fraction of stock ownership in the firm, it is more difficult to remove him or her (Weisbach, 1988).

In our model, we allow the lower boundary to be crossed before the CEO has to exit. Upon hitting the termination boundary, the wealth received by the CEO would be equal to the current value of the stocks and the fixed compensation for the time period up until the exit. The current value of the options would be close to zero if the current stock price is low enough to trigger an exit in our model, and are not included.

The wealth function in Equation 3.10 is used to calculate the corresponding utility in node i at time t below the lower boundary which is subsequently used to calculate the expected utility in the preceding time states.

$$W_{i,t} = a(1-b)S_0\frac{t}{252} + bS_{i,t} \tag{3.10}$$

Most stocks that are awarded to the CEO have vesting restrictions, meaning that the CEO must work in the firm for a certain period to keep the stocks. When the CEO leaves the firm, we assume that the stocks are converted to cash for the remaining period. In our model the CEO cannot buy back the stocks, and the cash cannot be put to alternative investments. The implicit loss of not having any yield on the cash is also a part of the penalty. This is a simplification as CEOs that leave the firm get to keep their stocks for a period after resignation if the vesting requirement is fulfilled. Nevertheless, one should expect costs related to the termination (Spear and Wang, 2005). In our model, we do not exogenously impose costs associated with the event of termination. However, converting the stock holdings to cash at termination serves as a termination cost in our model that punishes the CEO at termination. This would impose an extra cost for the CEO upon termination. This is due to the fact that if the CEO would stay in the job, the stocks would have a positive expected return in the future. Thus, by selling the stocks at termination the CEO loses the expected returns on the stocks from the date of termination until the end of the period. This is not optimal, but imposes a cost of termination to the CEO.

A higher upper boundary will give more accurate results, but is set to limit the number of calculations. We choose an upper boundary that indicates a market capitalization of 7.4 times the initial market capitalization. This is well above the starting node, and is highly unlikely to be reached within one time period if starting in the middle node with market capitalization value of 1. For all nodes above the upper boundary, the wealth is set equal to the value of the stocks and options and the fixed compensation for the entire period. This gives a slightly lower wealth than the actually expected wealth in these nodes, but as long as the boundary is high enough, this should not affect the solution for the relevant parts of the grid.

The wealth function in Equation 3.11 is used to calculate the corresponding utility in node i above the upper boundary.

$$W_{i,t} = a(1-b)S_0 + bS_{i,t} + n(1-b)max(S_{i,t} - K, 0) \tag{3.11}$$

4. Single-period analysis

In this chapter, we present the results of the single-period model. We consider a time span of one year with one option package only. The aim of the single-period analysis is to investigate the conflict of interest between the shareholders and the CEO in regards to debt ratio preferences. First, we find the optimal debt ratio from the shareholders' perspective. Second, we construct a base case scenario of the risk-profile of the CEO based on observed contracts in the literature. Third, we go in-depth on the effect of equity-based compensation by altering the mix of stocks and options in the wealth portfolio of the CEO.

During our analyses, we will classify the different regions in the CEO's risk-taking profile, also called the kappa surface, inspired by the terminology introduced by Hodder and Jackwerth (2007). Although we investigate another principal-agent problem, our problem has many similarities to the problem in the hedge fund industry and thus some of the terminology is applicable to our analyses as well. The kappa surface will be divided into the following regions with distinct characteristics:

- **Option ridge**: This corresponds to the area where the stock price is close to the option strike price.
- **Valley of prudence**: This is the region close to the liquidation boundary where the CEO gets terminated.
- **Risk plateaus**: This region is characterized by constant risk preferences. In Hodder and Jackwerth (2007) flat regions are referred to as "Merton flats" as they investigate the hedge fund industry. We introduce a new terminology since we are concerned with debt ratios in corporations, as "Merton flats" is a reference to a an analysis by Merton (1969) concerning the hedge fund industry.

We present the preferred debt ratio of the CEO in a three-dimensional space. The x-axis and y-axis represent the market capitalization and time. The z-axis is the optimal debt ratio for the corresponding market capitalization (market cap) and time. As a result, we construct a kappa surface.

4.1 Optimal debt ratio of shareholders

In this section, we calculate the debt ratio that maximizes the expected return on equity. From a well-diversified and risk neutral shareholder's point of view, this should be equal to the optimal debt ratio of the firm. Thus, we find the preferred debt ratio of the firm from the shareholder's perspective.

In order to find the preferred debt ratio of shareholders, we simply calculate the expected return on equity with the model previously introduced, for each debt ratio. We use the expected return on equity from Equation 4.1 which is derived in Appendix D and the same values for all the parameters as previously introduced in Chapter 3.

$$\mu_{r_S} = \mu_{r_{EV}} + (\mu_{r_{EV}} - r_e)\frac{\kappa}{1-\kappa} \tag{4.1}$$

The optimal debt ratio of the shareholders should be constant and independent of the current share price, as the future possible outcomes are independent of past history when following a geometric Brownian motion.

Maximizing expected return on equity, we find that from a shareholder's perspective, a debt ratio of approximately 0.43 is optimal. Empirical studies on the average market debt ratio in US corporations indicate an interval of 0.22-0.38 for the observed debt ratio (Welch, 2004; Choi and Richardson, 2016; Ferris et al., 2018). Thus, our estimate is slightly above the observed market debt ratio. One possible explanation is that the interest tax shield from debt is not as valuable to some firms (Brigham and Ehrhardt, 2013). For instance, larger corporations tend to have lower taxes by for instance shifting profits to low-tax jurisdictions, which would reduce the tax shield benefit. Consequently, they get a smaller benefit than we model in our analysis. Moreover, we would not expect all shareholders to strive towards achieving the optimal debt ratio, as there may be influential shareholders that have other motives other than maximizing the expected return on equity. For example, some shareholders might have some degree of risk aversion if they are not sufficiently diversified. Also, as we observe later, the CEO tend to prefer a lower debt ratio than the shareholders and might therefore contribute to lowering the observed debt ratios.

4.2 Base case of CEO risk preferences

In this section, we investigate the optimal kappa surface of the CEO based on the numerical values of the compensation package presented in Section 2.2 and the model presented in Chapter 3. Figure 4.1 shows the output. From Figure 4.1 it is evident that in contrast to shareholder's constant risk preferences, the risk preferences of the CEO depends on the stock price and remaining time to option expiration.

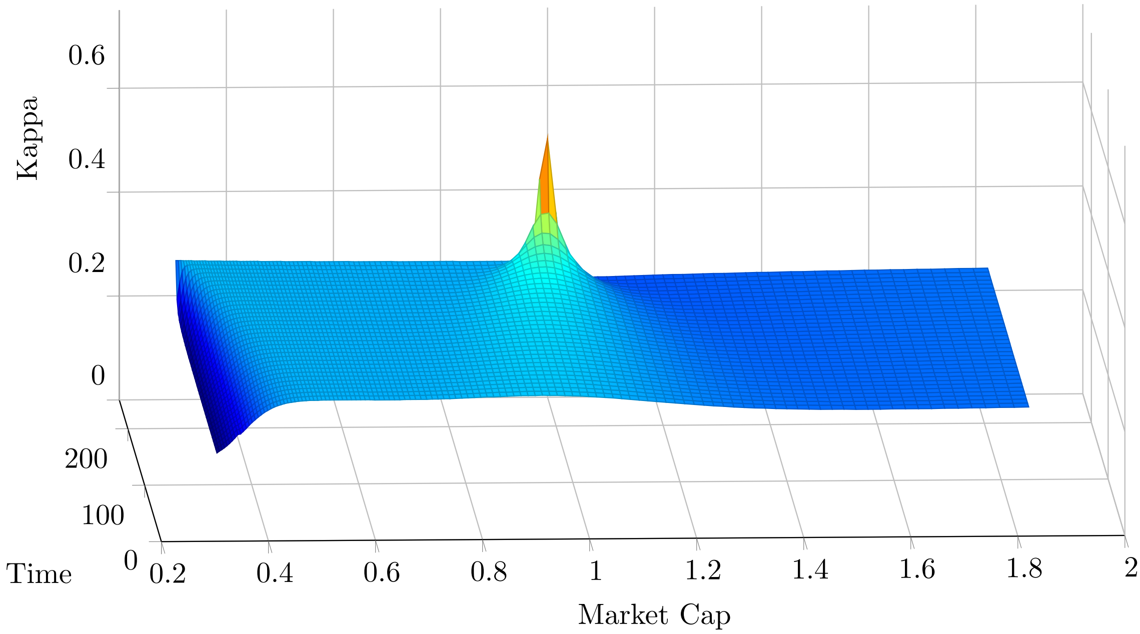


Figure 4.1: Kappa surface of our base case contract with non-equity based compensation, stocks and options. Non-equity based compensation a = 0.03%. Stock ownership b = 1.6%. Option ownership n = 0.42%. Option strike price K = 1

Comparing the optimal debt ratio of the shareholders from Section 4.1 to the results in Figure 4.1, we observe that most of the optimal kappa surface of our model is significantly lower than what the shareholders would prefer. This could lead to sub-optimal debt ratios for the shareholders in many scenarios. The lower debt ratio observed in Figure 4.1 would be optimal for a risk averse individual, such as the CEO modeled here. This is due to the assumption that the CEO has most of his or her wealth invested in the firm. As a consequence of this, the CEO would, in contrast to other shareholders, not have the wealth invested in a diversified portfolio, but would be highly exposed to the risks of a single firm, which is also the current source of income. Therefore, his or her wealth would have a much higher volatility and a risk averse actor would then prefer to have a lower debt ratio in order to decrease the risk of the portfolio.

As previously discussed, the shareholders on the other hand can diversify their portfolio, lowering their total risk. Thus, they can tolerate higher firm-specific risk if this leads to a higher expected return, since the increased risk is to a large extent offset by their other positions. Consequently, the shareholders would prefer a higher risk level, compared to a risk averse CEO behaving as modeled in Figure 4.1.

In the following, we will investigate the characteristics of the different regions of the kappa surface in depth.

Option ridge

When the stock price is below and close to the strike price and the time approaches the exercise date, the risk-preferences change drastically. The CEO will increase the risk in a ”last minute bet” in order to get the option in-the-money. In this event, kappa hits its maximum. Thus, the last minute bets observed in the hedge fund problem introduced by Hodder and Jackwerth (2007), also occur for our principal-agent problem between shareholder and CEO. However, gearing up firms is a much slower process than that of a hedge fund. There exists a lag between when a CEO wants to change debt ratio, and when the CEO is actually able to access new capital. However, we only look at what debt ratio the CEO would prefer if he or she could choose freely. We observe that the option ridge is the only part of the surface where the preferred debt ratio of the CEO reaches the same level as the shareholders prefer or even surpass it. Thus, it seems like options are a necessity in order to align the incentives of the CEO and the shareholders as the preferred debt ratio of the CEO would be significantly lower if stock ownership and fixed compensation were the only sources of wealth. However, we can also observe that when the option is slightly in-the-money, the preferred debt ratio is lower than in the case without options as the CEO seeks to lock-in the earnings, thus options are not necessarily increasing the risk willingness of the CEO for all underlying share prices. The stock price is likely to be in the area just above the strike price, slightly in-the-money, and a too low debt ratio here could be just as non-optimal for the stockholders as a too high risk level.

Valley of prudence
At the lower boundary, no more compensation is awarded to the CEO and the stocks are converted to cash at termination, as described in Section 3.4. Consequently, kappa declines rapidly right above the lower boundary as the CEO wants to prevent reaching the lower boundary, due to costs related to leaving the firm, by reducing the firm risk. This implies that if the CEO is afraid of being terminated due to a poorly performing share price, the risk-taking behavior will be further misaligned with the shareholders' preferences. Thus, the risk of getting fired can create undesired outcomes to the shareholders, and the job security should probably depend on alternative metrics.

Risk plateaus
Risk plateaus are observed both deep in-the-money and deep out-of-the-money. The risk plateau below the strike price stems from the fact that the firm value is far enough from the liquidation boundary such that the possibility of getting terminated has no impact on the behavior of the CEO, and far enough from the strike price as to motivate the CEO to increase risk, to tilt the stock price above the strike price. For a stock price above the strike price, we observe a ramp up to a new risk plateau deep in-the-money. When the stock price is sufficiently deep in-the-money, the option only plays a minimal role in the CEO's decision-making, as owning a deep in-the-money option is almost similar to owning a share.

The most important takeaways from the base case scenario are as follows. Firstly, the risk preferences of the CEO clearly deviate from the optimal debt ratio of the shareholders. Secondly, option-based compensation contributes to increasing the risk-taking behavior of the CEO when the stock price is close to the strike price. This indicates that option-based compensation only has the desired impact on the risk-taking behavior when the share price is close to the strike price. This finding is in line with our hypothesis that option-based compensation fails to motivate the CEO when the stock price is far-off from the strike price. Third, if the CEO is fired when the stock price performs poorly, the risk-taking behavior drops drastically at this boundary. Our results indicate that if the CEO is aware of potential sanctions related to a poorly performing stock price, this can reduce the risk-taking behavior drastically when the share price is sufficiently below the strike price, described as a valley of prudence. As a consequence, linking the job security to the share price can be value-destructive for the shareholders.

4.3 The effect of equity-based compensation

So far, we have investigated the risk-taking behavior of the CEO based on a typical compensation contract as a base case scenario. An important factor in motivating CEOs to adopt the right level of risk from the shareholders' perspective is the choice of compensation structure. We will investigate how the mix of options, stocks and non-equity based compensation affects the risk-taking behavior of the CEO by considering alternative compensation contracts. We ask the following question: how does varying the proportion of options in the compensation package affect the decision-making of the CEO?

Higher portion of option-based compensation

First, we study the effect of a larger weight of options in the compensation package. Increasing the portion of options in the compensation package increases the exposure of the wealth portfolio of the CEO to changes in the stock price of the firm when the stock price is close to the strike price of the options. Figure 4.2 shows the kappa surface of a CEO with a high portion of option-based compensation.

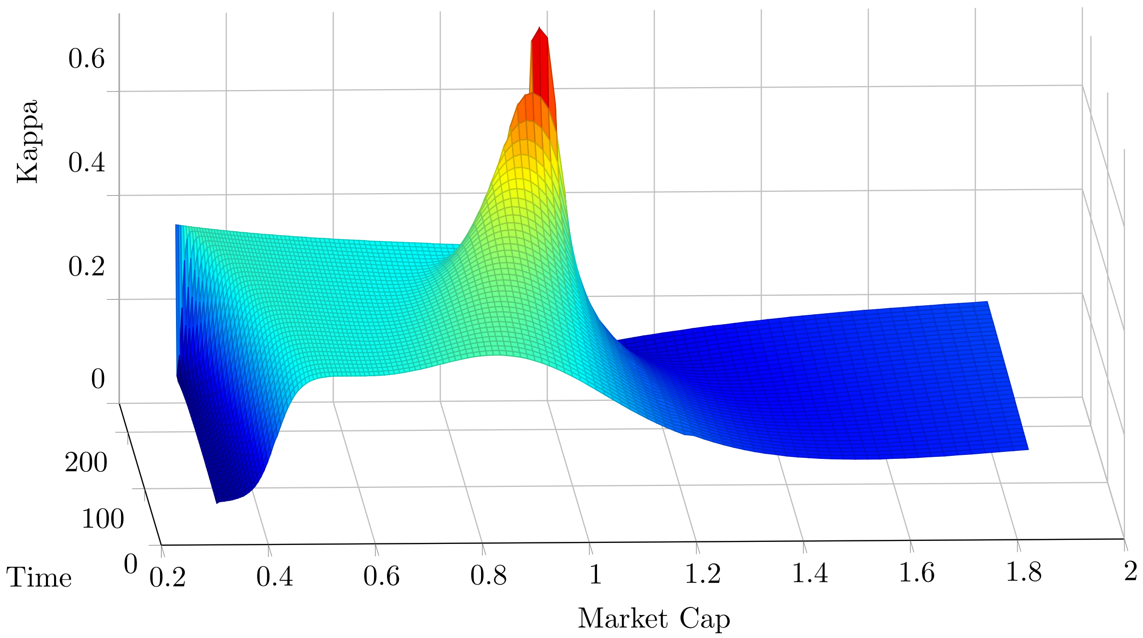


Figure 4.2: Kappa surface of a CEO with a higher proportion of options. Non-equity based compensation a = 0.03%. Stock ownership b = 0.10%. Option ownership n = 0.42%. Option strike price K = 1

Based on Figure 4.2, we see the following characteristics of a compensation structure consisting of mostly options and non-equity based compensation.

Option ridge
As we increase the weight of options in the wealth portfolio, the preferred debt ratio will increase drastically at the option ridge. We also observe that the option ridge becomes broader compared to the base case scenario, meaning that the options influence the risk-taking behavior of the CEO for a larger share price interval. Moreover,

when slightly in-the-money, the debt ratio would decrease significantly more drastically compared to the base case. Thus, we see that an increasing amount of options in a single-period leads to a less even kappa surface, and larger misalignments with the preferred kappa surface of the stockholders, in both directions. If the stock price is slightly in-the-money, the CEO's incentives will change as we get the "lock in" effect. To ensure that the CEO receives the option payoff, the CEO will likely want to manage the firm in a conservative manner. This is due to the fact that risky projects may lead to failure, leading to a drop in the stock price and consequently taking the CEO's options out-of-the-money.

Valley of prudence
At the lower boundary, the preferred debt ratio decreases drastically. The motivation to augment risk is especially significant when the CEO's stock option is deeply out-of-the money because the likelihood of being worthless at expiration is high unless something about the firm changes drastically.

Risk plateaus
With a higher mix of options, the ramp-up to the risk plateau deep in-the-money has a steeper increase, because the CEO has more to lose if the options are out-of-the money and thus the lock-in effect is stronger. Valle and Pavlik (2009) find similarly that managers with stock-options in-the-money emphasize safer corporate-investment decisions, which is consistent with an inclination to protect the value of the ESOs that is already impounded.

To summarize, incorporating more options in the compensation package of the CEO greatly amplifies the option ridge resulting in higher risk preferences that will exceed the optimal debt ratio from a shareholder's perspective. On the other hand, the CEO will behave even more carefully when the options are "locked in" as he or she will have more to lose. Moreover, the region where the ESOs have a large impact on the risk-taking behavior of the CEO increases compared to the base case.

No option-based compensation

Next, we set the option weight to be zero, meaning that the CEO only gets compensated with non-equity based compensation and stocks. Figure 4.3 shows the kappa surface for a CEO who has such a compensation structure. At the option ridge the kappa surface decreases compared to the base case, while in the area where the options were previously slightly in-the-money, the surface increases. Wright et al. (2007) find similarly that a small fraction of stock-based compensation in the CEO's portfolio increases the risk-taking behavior of the CEO, but for large holdings of stocks there is a tendency for the CEO to constrain or reduce the risk of the firm. We see that without any option based compensation, the kappa surface of the CEO would always be significantly lower than the optimal level for the stockholders, thus options seems necessary to induce the amount of risk-taking desired by the stockholders.

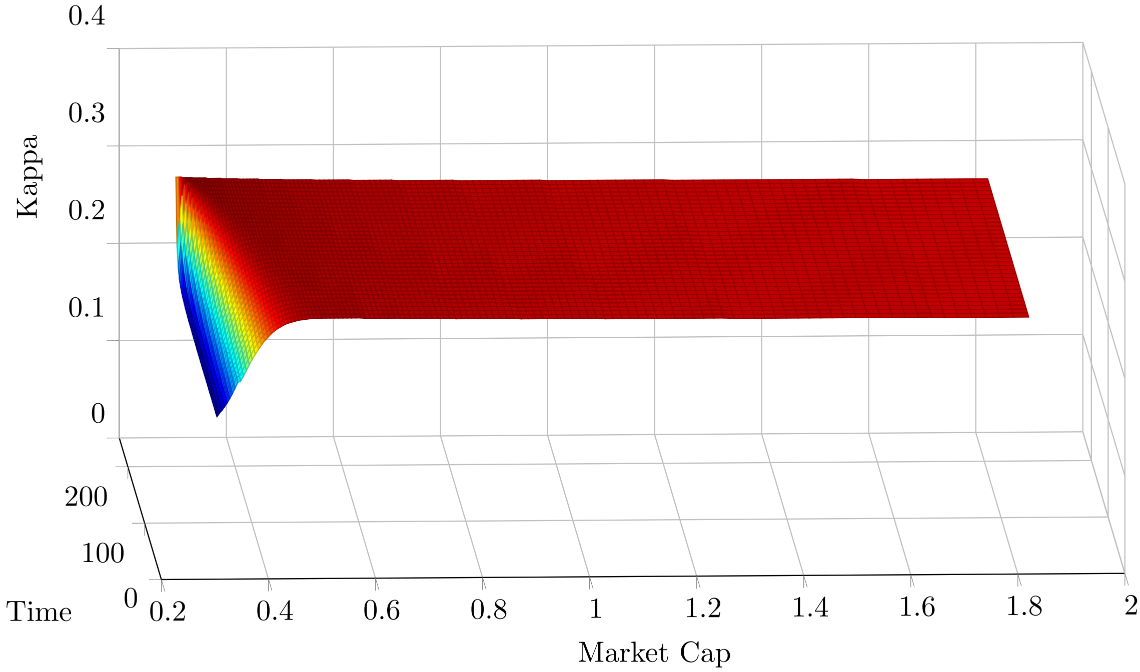


Figure 4.3: Kappa surface of a CEO with a wealth portfolio with no options. Non-equity based compensation a = 0.03%. Stock ownership b = 1.6%. Option ownership n = 0%. Option strike price K = 1

Option ridge
The most significant change is that the option ridge disappears in the absence of options. Thus, the last minute bets are not observed in such compensation packages. Since the option ridge disappears in the absence of options, the optimal debt ratio of the CEO will not reach the preferred level of the shareholders at any point.

Valley of prudence
As the stock price decreases from the original value, the value of the stocks in the wealth portfolio declines. Consequently, the non-equity based compensation constitutes a higher proportion of the wealth portfolio. Therefore, the wealth portfolio of the CEO is more diversified and less volatile as a higher proportion of the wealth is tied to the less risky non-equity based compensation. This leads to the CEO preferring a slightly riskier debt ratio at first as the stock price decreases. However, when the lower boundary gets closer, the significant costs of having to leave will be more important and the kappa surface declines quickly close to the lower boundary.

Risk plateaus
Most of the kappa surface is dominated by a risk plateau in the case of no options. As the stock price increases significantly above the base stock price of 1, the kappa surface is similar to the surface in a wealth portfolio only consisting of stocks, and at a constant level. In the scenario with only stocks, the CEO would prefer a risk level of approximately 0.3 at the risk plateau. This is exactly the level of the risk plateau in Figure 4.3. This makes sense, as the value of the non-equity based compensation diminishes compared to the value of the stocks as the stock price increases. Consequently, the CEO will behave as he or she only owns stocks.

As a concluding remark, we have demonstrated that in the absence of options, the risk-taking behavior of the CEO will depend less on the stock price and time. This implies that the risk preferences will be more stable, but substantially lower than the optimal debt ratio from the shareholder's perspective.

5. Multi-period analysis with multiple option packages

In the model considered so far, we have focused on a single option as part of the wealth portfolio of the CEO. Since firms typically issue option grants each year, a CEO's wealth portfolio is more complex in the real world (Kolb, 2012). Moreover, various option grants can be issued at various exercise prices and have different exercise dates. Thus, the CEO could hold a portfolio of options with different characteristics, implying various incentives for risk-taking behavior. Also, shares owned by the CEO might have a vesting period of multiple years, prohibiting the CEO from selling the stocks after only one year as in the single-period model. Therefore, we extend our model into a multi-period setting with multiple strike prices for the option-based compensation and longer vesting periods for the shares.

5.1 Model extension from single-period to multi-period

In the multi-period model, the wealth function at the terminal step of a period is augmented by adding a value of continuation. The value of continuation is the certainty equivalent of the expected wealth from starting at the same stock price in the same time, 0, in the next period $p + 1$. Consider a scenario with P periods, where the first period is period 0, the next period is period 1 and the last period is period $P - 1$. Since our optimization algorithm calculates utilities starting at the terminal time state in the last period, the last period $P - 1$ is modeled exactly like the single-period case.

Each period starts at time state 0, and this time state for a period p is the same date as time state 252 in the previous period $p - 1$. An example to illustrate this would be that a CEO receives the remuneration at the last day of the year, and then makes a kappa choice before the first day of the next year. At time state 252, the CEO makes the kappa choice for the time state 0 in the next period. This is illustrated by Figure 5.1.

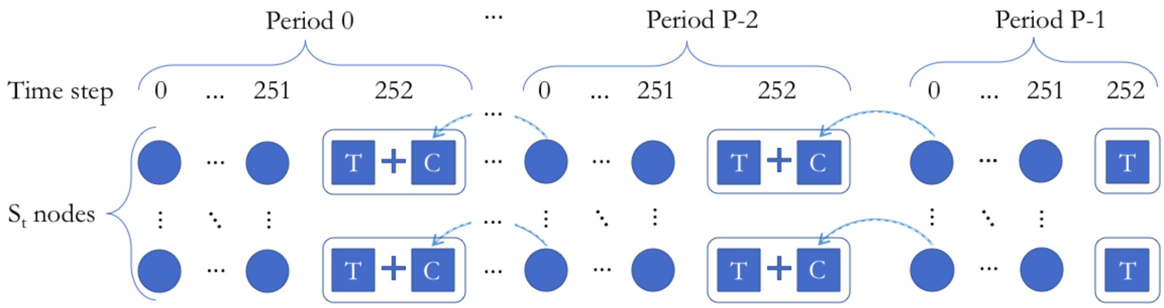


Figure 5.1: A certainty equivalent of the continuation wealth is added to the terminal wealth in each period. The blue circles indicate the expected utilities for each time state before the terminal time state for all possible market capitalization values. The squares, where T is terminal wealth and C is continuation wealth, illustrate the parts that comprise the wealth from which the utility is calculated in the terminal time state of the periods.

We calculate the continuation wealth by using the inverse of the utility function. We find the expected utility for the first time state in the next period, $E(U_{i,t=0,p+1})$, from the matrix of expected utilities $E(U_{i,t,p+1})$. Since the utility at the terminal time state, for the periods $p = 0$ to $p = P-2$, is dependent on the compensation the CEO expects to get the next year, the need for an appropriate time discount rate arises. Our model discounts the continuation wealth with the temporal discount rate $R = 30\%$, based on Pepper (2019) who argues that executives discount time heavily. The certainty equivalent of the continuation wealth is therefore:

$$W^{CEq}_{i,t=0,P-n-1} = \frac{(E(U_{i,t=0,P-n})(1-\gamma)+1)^{\frac{1}{1-\gamma}}}{1+R} \tag{5.1}$$

The certainty equivalent found in Equation 5.1 is simply the present value of the inverse of the expected utility from Equation 2.1. Since utilities cannot be added because of the concavity of the utility function, the next step is to add the the certainty equivalent of the continuation wealth, $W^{CEq}_{i,t=0,P-n-1}$, to the terminal wealth, $W_{i,t=252,P-n-1}$. The sum of wealth in the terminal time state is converted back to utility:

$$U_{i,t=252,P-n-1} = \frac{(W_{i,t=252,P-n-1} + W^{CEq}_{i,t=0,P-n-1})^{1-\gamma} - 1}{1-\gamma} \tag{5.2}$$

where

P is the number of periods

n is an integer $\in [1, P-1]$

$W_{i,t=252,P-n-1}$ is the wealth at the terminal time state in period $P-n-1$

$W^{CEq}_{i,t=0,P-n-1}$ is the continuation wealth

5.2 Multiple options with different expiration dates and strike prices

In this section, we introduce multiple options with different expiration dates and strike prices to our multi-period model. Let o denote the different option packages, with the total number of packages being O, then the expected utility for each option package o given the debt ratio κ, time state t and stock price node S_i will be

$$E(U_{i,t,o,\kappa}) = \sum_{j=-100}^{100} p_{j,\kappa} E(U_{i+j,t+1,o,\kappa}) \tag{5.3}$$

where

$p_{j,\kappa}$ is from the probability matrix described in Section 3.2

$E(U_{i,t,o,\kappa})$ is the expected utility dependent on option package o and kappa κ

As before, the expected utility is converted to the certainty equivalent wealth, CEq. Taking the multi-option case into account, the certainty equivalent of each option package will be

$$W^{CEq}_{i,t,o,\kappa} = (E(U_{i,t,o,\kappa}) \cdot (1-\gamma) + 1)^{\frac{1}{1-\gamma}} \tag{5.4}$$

The certainty equivalents are then summed to find the total expected utility for all kappa values, now with multiple options packages taken into account.

$$E(U_{i,t,\kappa}) = \frac{(\sum_{o=1}^{O} W^{CEq}_{i,t,o,\kappa})^{(1-\gamma)} - 1}{1-\gamma} \tag{5.5}$$

Similarly to the single period model, we find the kappa value giving the highest expected utility.

The option's strike price and time to expiration are input parameters to the model. We set the different strike prices and time to expiration for the options beforehand. This effectively means that no more options are issued endogenously while running the model, and that the options were issued in previous years only. In other words, we have created a scenario of what might have happened before the time window we analyze starts, by picking reasonable strike prices exogenously before running the model.

If we consider a four period perspective, our remuneration contract would be equal to the case where one option was issued every year before the time count starts at the beginning of period 0. At the beginning of year -4 an option expiring at the end of year 0 is issued, in year -3 an option expiring at the end of year 1 is issued and so on, until the option issued at the beginning of year 0 expires in year 3, being the last period. This is illustrated in Figure 5.2.

The wealth in the multi-period model is decomposed into wealth packages. There exist as many wealth packages as there exist different options. A wealth package consists of one of the options, T_O, in addition to a part of the non-equity based compensation, T_F, and the stock compensation, T_S. The non-equity based compensation component in the package is the total non-equity based compensation within the year, divided by the number of option packages in the model. Non-equity based remuneration is distributed equally throughout each option package. This is done to avoid the numerical problem of wealth being zero for an option package at expiration, see Figure 5.2. The stock compensation component is subject to vesting restrictions. The vesting contract we have chosen in our base case is one where the CEO only receives ownership of the stocks in the last period. In short, the option is exercised upon expiration if it is in-the-money, the non-equity based compensation is paid out at the end of every year for all wealth packages, and lastly the stocks are paid out at the end of the last period.

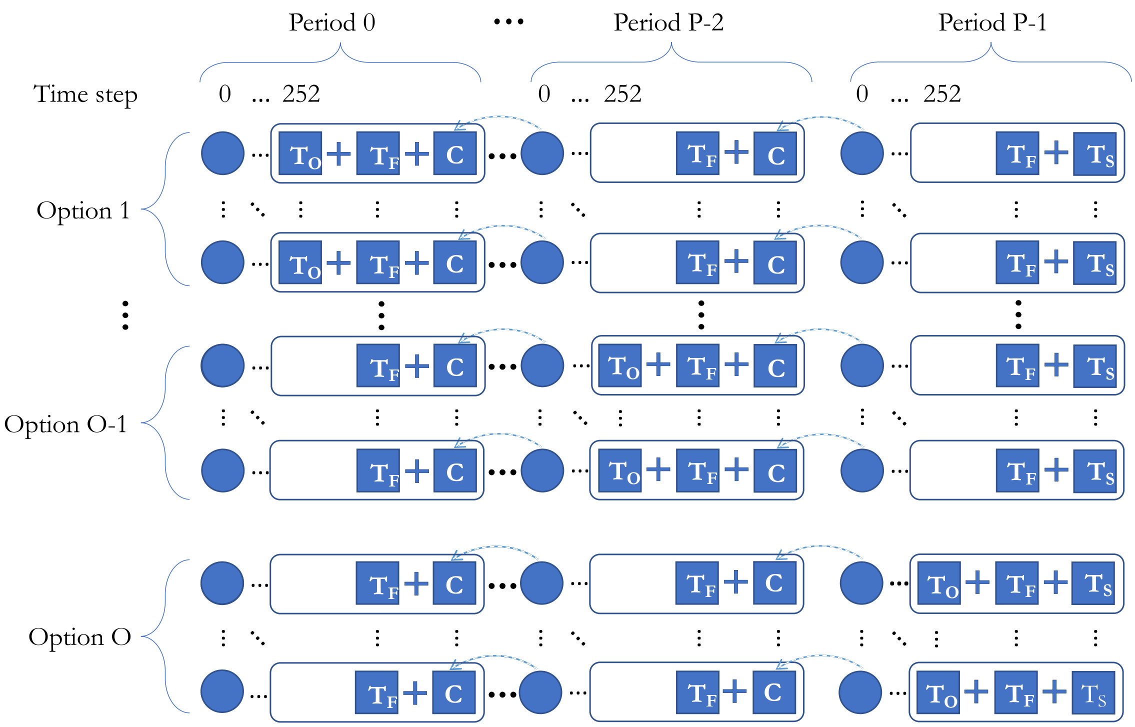


Figure 5.2: Utility matrix with multiple options. An example with $O = P$ options, where one option expires each year from period 0 to period $P-1$. The larger dots represent a leap in numerous periods, while the smaller dots represent leaps in time states or S_i nodes. Wealth packages comprise T_O, T_F and T_S and the added certainty equivalent of continuation, C. As previously, circles illustrate utilities and the box around the squares indicates the utility based on the wealth components in $t = 252$

5.3 Results from the multi-period model

In this section, we present the results from three different multi-period scenarios, and examine how the multi-period perspective affects the preferences in the first period.

We compare the results from the first period of multi-period cases to the single-period case. First, we present the two period case and a four period case to illustrate the main differences between single and multi-period. We also examine the case where no options expire in the first period, only including options expiring in the following periods.

We present graphical results for three scenarios; a two periods and two options scenario illustrated by Figure 5.3, a four periods and four options scenario illustrated by both Figure 5.4 and 5.5, and lastly a scenario where there are four periods and three options, shown in Figure 5.6, where none of the options expire in the first period. All figures depict the first period of each scenario.

Multi-period results with two periods and options

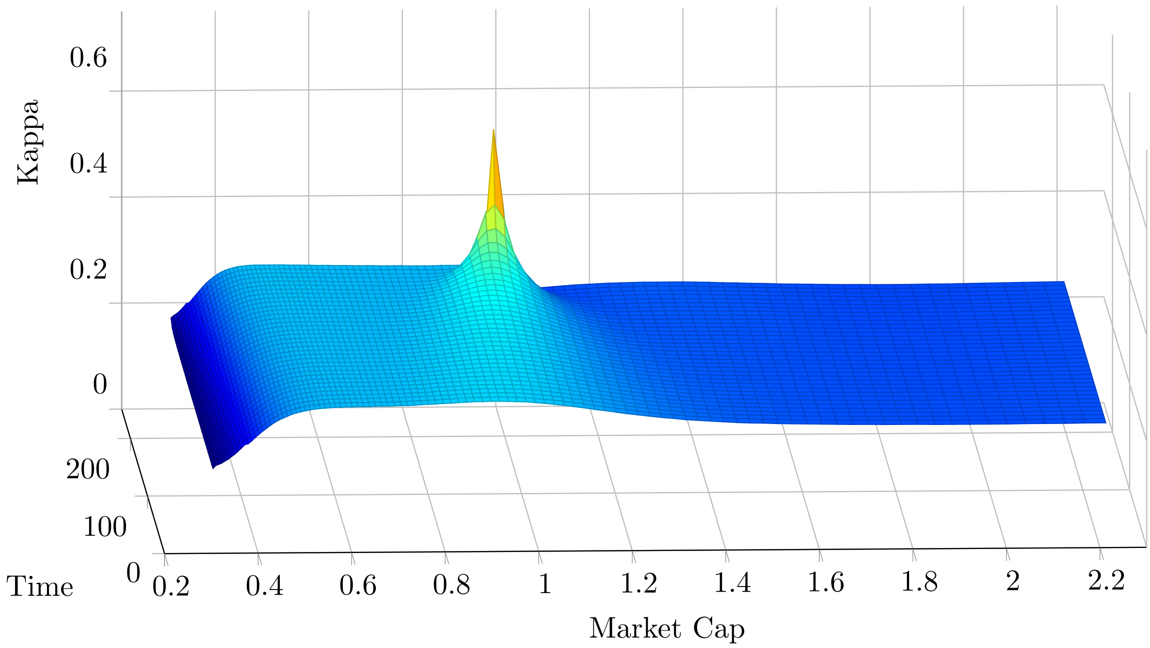


Figure 5.3: The first period of two periods with two options. Option strikes = [1, 1.3]. Options on 0.42% of the company's market cap. The remaining parameters are equal to the base case.

Higher peaks and lower valleys
As illustrated in Figure 5.1, the stocks are only converted to wealth in the last period, and do not contribute directly to the wealth function for any of the other periods. This causes the options to have a larger weight, and affects the risk-taking even more in a multi-period case than in a single-period case. The result is a more extreme option ridge peak, known as a last minute bet, in the first period. Consequently, we also see a more extreme lock-in effect, as the risk-taking after the option ridge lowers more rapidly than in the single-period case.

Also, as more periods are added to the model, the value of the stocks would be gradually lower compared to the value of all the option packages and the non-equity based compensation from all periods. This also causes the more extreme preferred debt ratios due to the option packages being a more prevalent part of the CEO's wealth, as the expected value of all the option packages increases with increasing number of option packages granted, while the CEO still owns the same amount of stocks in the company.

A faster increase towards risk plateaus
Since the risk-taking falls dramatically after the option ridge in the multi-period case, we accordingly observe a faster growth towards the risk plateaus, compared to the single-period case. Starting at a lower kappa value at the lock-in effect area, the market capitalization will need to increase more in the multi-period case than in the single-period case for the kappa value to reach the same risk plateau.

A less steep valley of prudence
For the multi-period case, we observe that the region close to the liquidation boundary descend more gradually than in the single-period case. The longer time perspective in the multi-period case causes the CEO to lower the kappa value at higher market capitalization values compared to the single-period. This is as expected, since the cost for the CEO to miss out on the potential option payments and other payments in the following periods is higher in the multi-period model. Thus, as the possibility of being terminated is higher when the market capitalization decreases, the CEO wants to take lower risk to increase the chances of retaining the job.

Effect of the discount rate
Since we have assumed the stocks to be available for the CEO only in the last period, the value of the stocks in period 1 will be very low compared to the value of options and non-equity based compensation due to discounting of the future value of the stocks over many years. Thus, the choices in the first period become more extreme in the multi-period model than in the single-period model. The model is checked for discount rate sensitivity, and the results remain almost the same for a discount rate of 15% as for 30%.

Effect of options that expire in the future
In the first period, we observe very little impact from the options that terminate in the following periods. A reason for this is that the volatility of the market cap is significant, especially several years in the future if the model's volatility of enterprise value is set to 16%. Even in the one period scenario, at the beginning of the period we observe only small signs of an option ridge effect around the strike prices of the option expiring at the end of the period. As a result of this, the uncertainty about

whether the option expiring in future periods are going to be in-the-money is so high that the desired kappa choice would depend more on the generally desired risk level, and not the possibility of exercising an in-the-money option in the future. In addition, the value of future option payments are also heavily discounted due to a CEO's myopia. These observations also remain if we lower the discount factor from 30% to 15%, which implies that multiple options in the future do not make the CEO alter the kappa choices in the first period significantly.

From two periods and options to four periods and options

When we augment the multi-period case from two periods with two options to four periods with four options, we observe that the effects described above appear stronger the more periods the model takes into account. This is illustrated by Figure 5.4 and Figure 5.5, where the last figure only depicts a broader view, showing kappa values for even higher market caps compared to the first figure. The top of the option ridge becomes higher, and the lock-in effect becomes more extreme, taking even lower kappa values. When looking at the first of four periods, as Figure 5.4 illustrates, the shares are discounted heavily and are consequently worth much less compared to the option payment. Therefore we see more extreme effects when augmenting with more periods, because the value of the stocks becomes smaller when the vesting contract pays out stocks only at the last period. Compared to the single-period case, the kappa values rise more drastically after the lock-in-effect in the multi-period case, since the lock-in-effect is more extreme, which is illustrated in Figure 5.5 below.

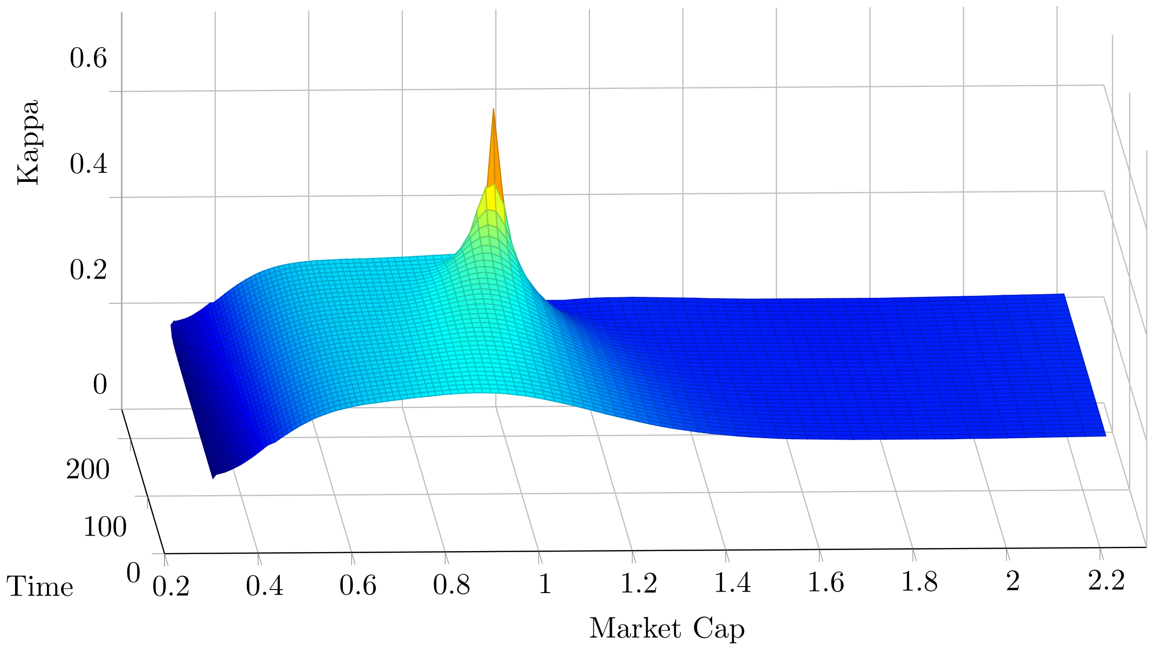


Figure 5.4: Four periods, four options. Option strikes = [1, 1.1, 1.2, 1.3] Options on 0.42% of the company's market cap. The remaining parameters are equal to the base case.

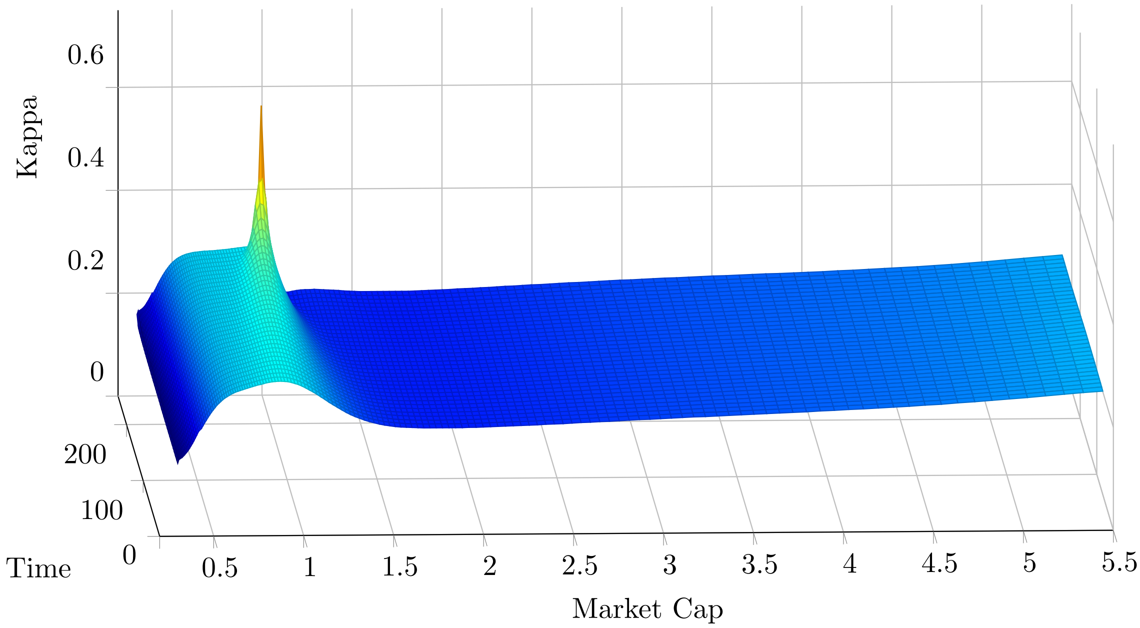


Figure 5.5: Four periods, four options. Option strikes = [1, 1.1, 1.2, 1.3]. Options on 0.42% of the company's market cap. The remaining parameters are equal to the base case. A wider framing, showing market caps up till 5.5.

Adjusting the vesting contracts

We also developed a model with a different structure of the vesting period of the stocks, intending to balance the weight of options and stocks more evenly for the multi-period case than previously described. If we assume that the stocks owned by the CEO have vesting periods that follow the same structure as the options, the stocks are granted gradually throughout all periods. For example, stocks can be granted every year, with a vesting period equal to the time to expiration for the option issued the same year. When the vesting period is over and the option expires, the CEO will receive the value of both the relevant stocks and options in that year's compensation package. In the scenario with four periods, 25% of the stocks would be freed at the end of each year. Such a compensation contract reduces the heavy discounting of the stocks we have observed when the stocks are collected in the final period, several years in the future. However, this does not have much impact on the results as most of the stocks are still paid out years later.

Option package with a one year cliff

After assuming in Section 5.2 that options in a four period scenario where issued in year -3, -2, -1 and 0, we look at the scenario where all options expire in the future, and none of them in the current period. This would be equal to the scenario with 4 periods, but only with options granted in year -2, -1 and 0. Figure 5.6 illustrates that in such a scenario, the kappa surface is the most even we have seen for a CEO in this study, because of the absence of options that expire in the current period.

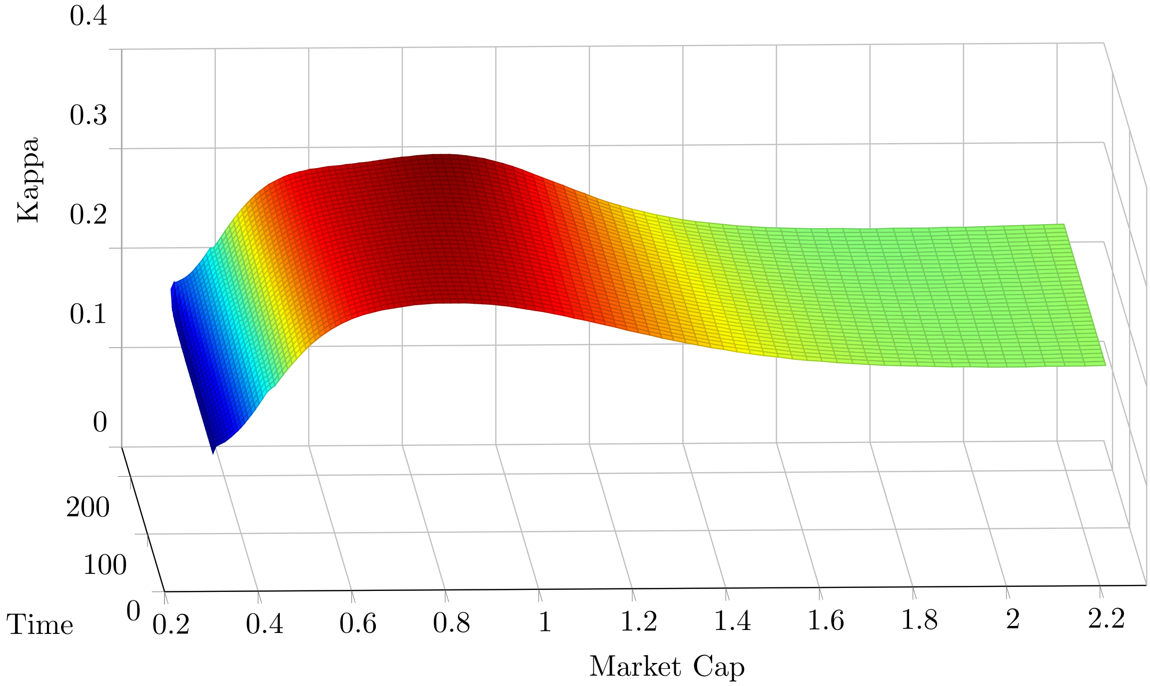


Figure 5.6: Four periods, three options. Option strikes = [1, 1.1, 1.2]: Each option package consists of options on 0.42% of the company's stock. No options expire in the first period. The remaining parameters are equal to the base case.

If there are no options that expire in the first examined period, but there are several options that expire in the future periods, the option ridge is much less extreme compared to the other multi period cases, which is shown in Figure 5.6. Compared to the single-period case, we observe lower overall kappa values, as the stocks are still worth less. This is because we assume a base case vesting contract where the stocks are available for the CEO only in the last period.

5.4 Model discussion

An important feature in our model is temporal discounting. According to standard economic theory, executives are assumed to be rational utility maximizers. A rational utility maximizer would discount future compensation at discount rates that are consistent with the rate on equally risky future cash flow. In contrast, according to temporal discounting theory, humans are irrational and discount time at much higher rates than predicted. There is empirical evidence that supports this view, indicating a median of 33% among executives (Pepper and Gore, 2014). A short-sighted view on compensation leads to a gap between the economic value of the compensation and the perceived value of the compensation by the CEO, and this value gap represents value destruction from a shareholder's perspective. We have investigated how the results change if we set the temporal discount rate lower to reflect a rational CEO, and we find that the kappa surface qualitatively does not change remarkably. The certainty equivalent returns for the CEO and shareholders, which we will present later in Chapter 6, decrease slightly with higher temporal discount rate, but not notably. The results from the sensitivity analysis are included in Appendix B.

We have set the temporal discount rate to be constant, which is a simplification according to behavioral economics theory. Ainslie (1991) postulates that discount functions are approximately hyperbolic, characterized by a high discount rate over short horizons and lower discount rates for longer horizons. However, if we incorporated a hyperbolic discount rate, uncertainty about determining the parameter that governs the degree of discounting arises. Moreover, since our model is not highly sensitive to the choice of temporal discount rate, we let the rate be fixed.

Our model only consists of options already issued, as introduced earlier. When all strike prices are already set and known at the beginning of the investigated time interval, optimal behavior can easily be recursively calculated, as the decisions in the terminal nodes are not dependent on events happening between the beginning and the end of the time interval. A possible expansion of the model could be to introduce dynamic future option grants. This would be interesting, as it would show the behavior of CEOs who know they will receive new option grants in the following years. However, this introduces path dependency, because if options were to be granted at a later date, after the beginning of the first period in the model, the strike price would be stochastically determined and dependent on the stock price at the time of the option grant. Dynamic option grants could probably be included in the model, but would drastically increase the complexity and be very computationally expensive to handle in the dynamic programming context.

Another possible extension of the multi-period model could also be to issue new at-the-money options, with a time to expiration of one year at the beginning of each year. This would also introduce some path dependency to the model, but it is solvable in the same way as Hodder and Jackwerth (2007). However, the results from such an option scheme are not significantly different from our one period model, as the new options are granted at-the-money. When the CEO receives new options in the future, the performance today would not really change the value of these options as the CEO starts anew with a clean slate each year. Thus, the CEO would act almost as if only the current period is relevant, and very similarly as in the single-period model.

6. Optimal contract analysis

In Chapter 4 and Chapter 5 we have presented the preferred risk-taking behavior of the CEO and shareholders, and qualitatively discussed misalignments between these stakeholders by comparing the kappa surface of the CEO to the optimal debt ratio from a shareholder's perspective. We found no remuneration which aligned the risk preferences of the CEO and the shareholders, given our model assumptions. We extend on this by quantifying the misalignment of satisfaction to measure the conflict of interest. It is interesting to investigate if there exists a compensation contract that is better for both the CEO and the shareholders, compared to our base case compensation contract. A Pareto optimal contract is found if both the CEO and the shareholders certainty equivalent rate of return cannot be increased without either party being worse off. We search for such a contract by altering the weights of the compensation components and finding the CEO's and shareholders' certainty equivalent rates of return. These certainty equivalent returns are greatly influenced by our risk aversion assumptions for the CEO and shareholders, and therefore we also perform a sensitivity analysis on the risk aversion of the CEO.

6.1 Defining certainty equivalent rates of return

In order to calculate the effect of different option packages on the wealth of the shareholders and CEOs, we develop a metric that illustrates this in monetary terms, that is the certainty equivalent rate of return. The certainty equivalent return of shareholders is the expected value of their stock holdings at the end of the period less their initial value as a rate of return on their initial investment. Similarly, for the CEO the certainty equivalent return is the certainty equivalent of their final wealth less their initial ownership value as a rate of return on their initial ownership value. Note that the shareholders' expected wealth and the shareholders' certainty equivalent wealth are equivalent since we have assumed that shareholders are risk neutral. In addition, the definition of the certainty equivalent return for the single-period case and for the multi-period case must be such that the metrices are comparable. Therefore, we will later distinguish on how the certainty equivalent return is calculated for the single- and multi-period model respectively.

The certainty equivalent returns for the CEO and shareholders are separately calculated by running recursively through the grid, calculating the expected utilities and expected wealth. This is done using the CEO's preferred kappa-surface, and corresponding probability distributions for different scenarios of CEO compensation. The expected utility of the terminal wealth of the CEO is calculated using the same equations as before (Equation 2.1 and Equation 2.2) and the lookup-matrix for jump probability introduced in Section 3.2. Meanwhile, the expected terminal wealth of the shareholders is the terminal value of all stocks minus the compensation paid to the CEO and the CEO's stock holdings. The calculation of their expected wealth based on the kappa-surface is then straightforward.

Single-period calculations
Based on the CEO's and shareholders' expected utility and wealth, the certainty equivalent returns are calculated as shown in Equation 6.1 and Equation 6.2, dividing by the corresponding initial wealth.

$$\Pi_{CEO} = \frac{(E(U_{CEO}) \cdot (1-\gamma) + 1)^{\frac{1}{1-\gamma}} - bS_0}{bS_0} \tag{6.1}$$

$$\Pi_{SH} = \frac{E(S_T - W_{CEO,T}) - (1-b)S_0}{(1-b)S_0} \tag{6.2}$$

Multi-period calculations
When calculating the certainty equivalent return of the CEO and shareholders for the multi-period scenarios, it is important to choose a metric directly comparable to the metric used in the single-period case. To achieve this, we look at the expected returns obtained during the first period in the multi-period scenarios, using the kappa-surface as calculated in a multi-period scenario. The certainty equivalent return of the CEO is defined similarly as for a single-period. The wealth at the end of the first period is calculated as the value of the stocks, in addition to the value of the non-equity based compensation for the first period and the value of the options that expire at the end of the first period. The initial wealth is the stock holdings at the beginning of the first period, as previously. The expected value of options that expire in later periods or non-equity based compensation to be received later is not included, as this would hinder comparisons between scenarios with different numbers of periods. The wealth at the end of the first period for the shareholders is similarly to the single-period case defined as the value of all shares minus the CEO's part of the shares and compensation.

The value for S_0 is intuitively set to 1 for the single-period case. However, deciding what S_0 should be for the multi-period case is not that simple. We therefore consider both S_0=1 and S_0=the last strike price, where the latter is the stock price at which an option was issued just before the beginning of the period, which would be equal to issuing the option at-the-money. We also chose S_0=1 to easily compare the single-period case to the multi-period cases, but one should bear in mind that this S_0 choice makes the result dependent on the strike price scenario we have chosen.

6.2 Certainty equivalent returns for all scenarios

As in Chapter 5, when we compare scenarios with different number of periods, we examine only the first period. The results for the certainty equivalent returns for different scenarios are presented in Table 6.1.

Scenarios	Π_{CEO}	Π_{SH}
Single-period, base case	**4.84%**	**7.59%**
Single-period, κ optimal for shareholders	3.42%	8.52%
Single-period, no options in the CEO's compensation	3.58%	7.62%
2 periods, 2 options, strike prices: [1, 1.1], S_0= 1	4.83%	7.61%
3 periods, 3 options, strike prices: [1, 1.1, 1.2], S_0= 1	4.83%	7.63%
4 periods, 4 options, increasing strike prices: [1, 1.1, 1.2, 1.3], S_0= 1	4.82%	7.63%
4 periods, 4 options, constant strike prices: [1, 1, 1, 1], S_0= 1	4.82%	7.58%
4 periods, 4 options, decreasing strike prices: [1, 0.9, 0.8, 0.7], S_0= 1	4.83%	7.45%
4 periods, 4 options, increasing strike prices: [1, 1.1, 1.2, 1.3], $\frac{1}{4}$ stocks freed each year, $S_0 = 1$	4.83%	7.63%
2 periods, 2 options, strike prices: [1, 1.1], S_0= 1.1	5.80%	7.49%
3 periods, 3 options, strike prices: [1, 1.1, 1.2], S_0= 1.2	7.00%	7.34%
4 periods, 4 options, increasing strike prices: [1, 1.1, 1.2, 1.3] , S_0= 1.3	8.30%	7.17%

Table 6.1: Certainty equivalent returns for CEO and shareholders for different scenarios.

First, we compare the certainty equivalent returns from the base case single-period, where the CEO follows his or her optimal decisions, with the certainty equivalent returns as if the CEO chose the shareholders' optimal kappa values. As expected, Π_{CEO} is highest for the base case and significantly lower, -29.3% lower, for the shareholders' optimal scenario. Furthermore, Π_{SH} reaches its optimal value with respect to all other considered scenarios when the optimal kappa choice of 0.43 is upheld for the whole period. This is not surprising, as all other considered scenarios were run with the CEO's optimal kappa choices. When the options are excluded, Π_{CEO} also lowers dramatically, with -26%, and the shareholders' return, Π_{SH}, is slightly higher. The size of the CEO's stock holdings and compensation is small compared to the shareholders' ownership share, therefore we will see less dramatic changes in Π_{SH}, compared to Π_{CEO}, for all scenarios.

By comparing the results from the single-period and multi-period cases with $S_0 = 1$ and constantly increasing option strikes we observe, as before, that the multi-periodic effects are small. The preferred risk level in the first period of the options expiring in future periods only changes slightly. The inclusion of future periods results in slightly lower values of Π_{CEO}. At the same time, we observe that including a longer time horizon for the CEO makes him or her act more in line with the preferences of the shareholders, with Π_{SH} increasing when adding more time periods.

The four period scenario presented in Section 5.3 where the stocks have vesting periods following the same structure as the options, meaning that a fourth of the stocks can be sold each year, are also included in the table. In this scenario we observe that the results are very similar as when all the stocks have to be held over all four years. The CEO is only slightly better off, while the shareholders are mostly indifferent.

The high return for the CEO, Π_{CEO}, when $S_0 > 1$ stems from the CEO's options being much deeper in-the-money at the starting point, therefore they should not be compared to the results when $S_0 = 1$. We present the certainty equivalent return for both $S_0 = 1$ and $S_0 > 1$ because it is realistic that S_0 starts at the strike price of the last granted option. However, choosing different S_0 makes the comparison between the certainty equivalent returns difficult.

6.3 Altering the weight of contract parameters

In this section, we change the mix of non-equity based compensation and option based compensation in the wealth portfolio of the CEO to investigate how it impacts the return on the CEO's and shareholders' certainty equivalent wealth. We do this to examine whether there are more efficient ways to structure the compensation, or if the current base case compensation contract is close to Pareto efficiency.

Table 6.2 shows a certainty equivalent return matrix for the CEO and shareholders in the single-period case. The rows and columns correspond respectively to varying non-equity based compensation from 0% to 0.12% of total firm value and option-based compensation from 0% to 0.70%. The base case scenario from the single-period analysis is highlighted in grey (0.03%, 0.42%). We observe that for the scenarios where non-equity based compensation is increased and the weight of options is decreased, both the shareholders and the CEO are better off. It is important to remember that the CEO also owns 1.6% of the stocks in the company for all the calculated results in Table 6.2. Even though the option-based compensation is 0%, the CEO is still exposed to changes in the stock price.

Non-equity based weight		Options weight 0.00%	0.14%	0.28%	0.42%	0.56%	0.70%
0.00 %	Π_{CEO}	1.62%	2.05%	2.46%	2.86%	3.25%	3.62%
	Π_{SH}	7.64%	7.62%	7.61%	7.60%	7.58%	7.57%
0.01 %	Π_{CEO}	2.27%	2.71%	3.12%	3.52%	3.91%	4.28%
	Π_{SH}	7.63%	7.62%	7.61%	7.59%	7.58%	7.56%
0.02 %	Π_{CEO}	2.93%	3.36%	3.78%	4.18%	4.57%	4.95%
	Π_{SH}	7.62%	7.62%	7.60%	7.59%	7.58%	7.56%
0.03 %	Π_{CEO}	3.58%	4.01%	4.43%	4.84%	5.23%	5.61%
	Π_{SH}	7.62%	7.61%	7.60%	7.59%	7.57%	7.55%
0.04 %	Π_{CEO}	4.23%	4.66%	5.09%	5.50%	5.89%	6.27%
	Π_{SH}	7.62%	7.61%	7.60%	7.58%	7.57%	7.55%
0.05 %	Π_{CEO}	4.88%	5.32%	5.74%	6.15%	6.55%	6.93%
	Π_{SH}	7.62%	7.60%	7.59%	7.58%	7.56%	7.55%
0.06 %	Π_{CEO}	5.52%	5.97%	6.39%	6.81%	7.21%	7.59%
	Π_{SH}	7.61%	7.60%	7.59%	7.58%	7.56%	7.54%
0.07 %	Π_{CEO}	6.17%	6.62%	7.05%	7.46%	7.86%	8.25%
	Π_{SH}	7.61%	7.60%	7.59%	7.57%	7.56%	7.54%
0.08 %	Π_{CEO}	6.82%	7.27%	7.70%	8.12%	8.52%	8.91%
	Π_{SH}	7.61%	7.60%	7.59%	7.58%	7.56%	7.54%
0.09 %	Π_{CEO}	7.47%	7.92%	8.35%	8.77%	9.18%	9.57%
	Π_{SH}	7.60%	7.59%	7.58%	7.57%	7.55%	7.54%
0.10 %	Π_{CEO}	8.12%	8.57%	9.00%	9.43%	9.83%	10.23%
	Π_{SH}	7.59%	7.58%	7.57%	7.56%	7.55%	7.53%
0.11 %	Π_{CEO}	8.76%	9.22%	9.66%	10.08%	10.49%	10.89%
	Π_{SH}	7.59%	7.58%	7.57%	7.56%	7.54%	7.52%
0.12 %	Π_{CEO}	9.41%	9.87%	10.31%	10.73%	11.15%	11.55%
	Π_{SH}	7.59%	7.58%	7.57%	7.55%	7.54%	7.52%

Table 6.2: Change in certainty equivalent returns for CEO and shareholders as non-equity based compensation weight and option compensation weight are altered. The grey cell is the base case. Blue cells are the compositions where the CEO is better off. Green cells are the compositions where the shareholders are better off. Turquoise cells are compositions where both the CEO and the shareholders are better off. The non-equity based weight is expressed in terms of percentage of initial firm value S_0, as is the option compensation.

Based on Table 6.2, we first consider the sensitivity to non-equity based compensation by examining the certainty equivalent returns. We observe that the certainty equivalent return for the CEO is strictly increasing, while the shareholders' certainty equivalent return is strictly decreasing. This is intuitive as non-equity based compensation will linearly add to the CEO's wealth in the same way as this compensation subtracts linearly from the shareholders' return.

When varying the weight of options in the CEO's wealth portfolio, we also observe that the certainty equivalent returns are strictly increasing and strictly decreasing for the CEO and shareholders respectively. This is the same tendency as with altering non-equity based compensation. However, in contrast to the non-equity based compensation, the increase of the CEO's certainty equivalent return flattens out for higher weights of options.

The blue cells in Table 6.2 indicate the compositions of compensation where the CEO is better off, while the shareholders are not worse off. The green cells indicate the compositions of compensation where the shareholders are better off, while the CEO is not worse off. The grey cell is the base case. All cells between these cells, marked in turquoise, indicate a composition of options and non-equity based compensation where both the CEO and the shareholders are better off than in our base case compensation structure. This means that the optimal compensation mix that satisfies both shareholders and CEO lies in this area, and the current base case contract is not Pareto efficient, as there are many contracts that could make both the CEO and the shareholders better off.

The goal of the shareholders is to find how the compensation package of the CEO can be structured as to increase their expected return. If there is a well-functioning market for CEOs, they will likely demand a certain amount of compensation to accept the job, thus the value of the compensation should not be lower than in the base case. The optimal structure for the shareholders must therefore be such that they maximize their return, while the CEO is not worse off than in the base case. These solutions must lie on the boundary of green cells in Table 6.2. Looking at these cells, it is evident that the optimal structure for the shareholders would be a structure with 0% options weight and 0.05% non-equity based weight. The CEO would be more satisfied with a less risky compensation structure, with more of the safer non-equity based compensation and less of the risky options, while also acting more in line with the interests of the shareholders.

As a final note, we have earlier seen that a higher proportion of options lead to a more extreme kappa surface, with higher highs and lower lows. In addition to this, the penalty of the expected return on market capitalization by having a too high debt ratio is higher than the penalty of having a too low kappa. This means that in the choice between being a bit more or a bit less risky compared to the optimal kappa, a shareholder would rather choose the less risky strategy. Thus, a high proportion of options might be sub-optimal for the shareholder, even if it raises the risk willingness of the CEO in some instances, since it can increase the risk willingness too much.

6.4 Risk aversion sensitivity

In the absence of any incentives to increase risk, risk aversion will make the CEO reluctant to take on more risk even though it can increase shareholder value. From the firm's perspective, that is a profoundly undesirable outcome. So far, we have held the risk aversion coefficient, γ, fixed. However, as pointed out in the previous section, it is of great interest to perform a sensitivity analysis on the risk aversion coefficient as the degree of risk-aversion is individual-specific and will likely vary over time. More specifically, we investigate how the degree of risk aversion affects the certainty equivalent return of the CEO and the expected return for the shareholder. We base this sensitivity analysis on the single-period model, with compensation structure as in our base case.

Figure 6.1 and Figure 6.2 show respectively how the degree of CEO risk aversion impacts his or her certainty equivalent return and the shareholders' expected return.

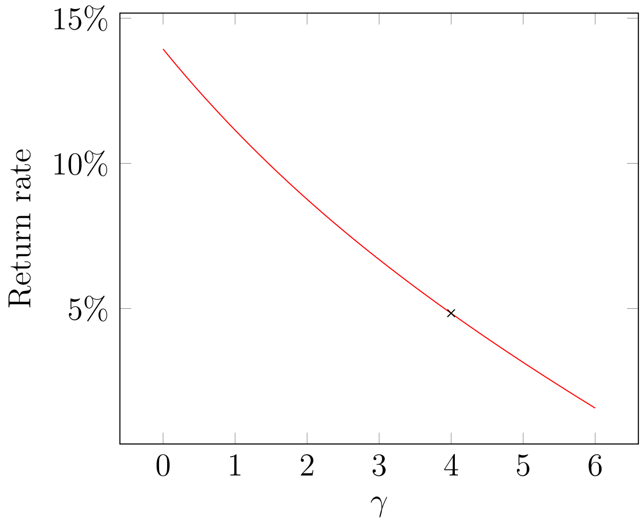


Figure 6.1: CEO's certainty equivalent return as a function of the risk-aversion coefficient. The cross marks the base case scenario at $\gamma = 4$.

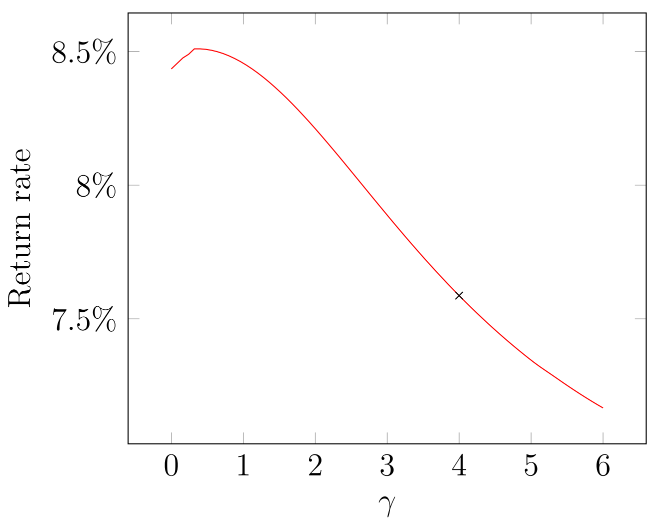


Figure 6.2: Shareholders' expected return as a function of the risk-aversion coefficient. The cross marks the base case scenario at $\gamma = 4$.

In the figures, we observe that when the CEO's risk aversion declines from our base case, $\gamma = 4$, to approximately $\gamma = 0.35$, the expected return for all stakeholders increase. A value of $\gamma = 0$ means that the CEO makes optimal choices, solely based on his or her expected return, as he or she is risk-neutral. Thus, the shareholders can also expect a higher return with a less risk-averse CEO. Nevertheless, it is only the certainty equivalent return of the CEO that is monotonically increasing with decreasing values of gamma. As the risk aversion approaches $\gamma = 0$ the expected return for shareholders in Figure 6.2 starts to drop. We hypothesize that this happens because as the CEO becomes more or less risk-neutral, he makes rational choices as a shareholder which benefits all of the other shareholders as well, but when γ approaches zero the marginal effect as a shareholder of lower risk-aversion declines. Even though the CEO will make rational choices as a shareholder, he will still have other interests compared to the shareholders due to the option component in his remuneration. When he makes rational decisions on his behalf, maximizing the value of his options, he will make more extreme choices and benefit at the other shareholders' expense. Thus, his kappa surface will not be equal to the shareholders' flat kappa surface even though he is completely risk-neutral due to his option ownership. When $\gamma = 0$ and the CEO is risk-neutral, the optimal compensation from the shareholders' perspective would be for the CEO to only own shares in the company. In this case, the CEO would act to maximize the expected return, and have exactly similar incentives as the shareholders, because the CEO is just a shareholder as well.

It is also interesting to note that the maximum value of the expected return for shareholders is 8.51% when γ is approximately 0.35. This is just slightly lower than what the expected return would be if the CEO behaved exactly as the shareholders would want, which was found to be 8.52%. Thus, when the CEO is close to being risk-neutral he or she will to a large extent manage the firm in line with the interests of the shareholders.

Due to the CEO's significant risk-aversion, more options in the wealth portfolio leads to higher volatility of the personal wealth and unwanted uncertainty, but no options results in too low kappa choices compared to the shareholders' optimal risk level. The initial differences in risk aversion, assuming CEOs and shareholders to have a γ value of 4 and 0 respectively, make a CEO compensation contract that perfectly aligns risk preferences impossible. This problem stems, among other factors, from our initial assumption that the CEO does not have wealth outside the firm. If the CEO had more of his or her wealth outside the firm, the volatility of the total wealth would be smaller and the CEO would be more willing to take risk in the firm in question. The implication of our results indicate that the CEO should be more diversified, with more wealth outside the firm to act more in line with the shareholders' preferences.

To summarize our results, we find that both the CEO and shareholders will be better off with the CEO having lower risk-aversion. Based on this, the firm should take into account the CEO's risk aversion when determining the composition of the compensation package. The question is whether it is possible to reduce the risk-aversion of the CEO. It has been argued in the literature that it is possible to change the risk-aversion of an individual (Lovallo et al., 2020). The CEO could possibly be less risk-averse if he or she is more diversified. This implies that the shareholders might be better off by having a CEO that invests in securities and assets outside the firm to have a more diversified wealth portfolio. As a result, he would behave more as a well-diversified investor which would align the incentives to a greater extent. Alternatively, one could design compensation contracts that are less risky for the CEO by linking the payoff to the relative stock price compared to the competitors. In that way, they would be exposed to less risk, but still be exposed to the share price. For example, the Norwegian energy company Equinor links the bonus to the relative total shareholder return based on the return compared to peers (Equinor, 2019).

7. Conclusions

We analyze an environment in which the principal (shareholder) controls the agent (CEO) through the construction of a compensation package consisting of both equity-based compensation and non-equity based compensation to alter his or her risk-taking behavior. In line with previous literature related to executive compensation (Kolb, 2012; Jensen and Murphy, 1990; Kløve and Valholm, 2011), our results support the existence of misalignment of incentives in regards to risk preferences between CEO and shareholders. Our contribution lies in a new approach involving dynamic programming in a discrete time framework. This approach enables us to consider this principal-agent problem from a new perspective by constructing a three-dimensional kappa surface to observe how risk preferences change with the evolution of time and firm value.

The preferred risk level of the CEO is found to be typically lower than the optimal debt ratio from the shareholder's perspective, except for the time right before option expiration if the share price is close to the strike price. This is due to the fact that CEOs are more risk-averse than shareholders, because their wealth is not well-diversified. If the option-based compensation has a large weight in the wealth portfolio of the CEO, this can lead to excessive risk-taking that greatly exceeds the preferred risk level of the shareholders. At the same time, when the share price is slightly in-the-money, a lock-in effect can occur, leading to a substantial reduction in risk-taking behavior from the CEO. We find that by taking into account vesting restrictions on stocks in a multi-period model, the kappa surface is amplified making the last minutes bets even riskier and the lock-in effect greater.

In addition, we also find that when the stock price is deeply out-of-the money, the option-based compensation loses much of its power to influence the decision-making behavior of the CEO, which is in line with what we hypothesized. Our results can be used to further investigate how compensation should be re-balanced to address problems that are manifested in recent crises, such as stock price crashes, and implies that the practice of repricing stock options to deal with "underwater" stock options has some desirable effects on the risk-taking behavior of the CEO. However, it should be noted that we have not investigated the effect of repricing stock options, but that would be an interesting future model extension.

Finding the right mix of option-based compensation can in some cases motivate the CEO to choose higher debt ratios that are more aligned with the desires of the shareholders. On the other hand, for some share prices near the option's strike price it can also lead to excessive or insufficient risk-taking compared to what the shareholders desire. Thus, we cannot conclude that options align the incentives of CEOs and shareholders. Even with a Pareto efficient combination of the remuneration components, the preferred risk level of the firm for CEOs and shareholders will deviate greatly, as long as the CEO and the shareholders have widely different preferences towards risk.

We extend on the kappa surface analyses by quantifying the satisfaction misalignment in monetary terms with the introduction of the certainty equivalent metric. As presented in Chapter 6, we find that a kappa value too high compared to the shareholders' optimal kappa value results in a greater penalty in expected return than a too low kappa value deviating by the same amount. Even so, we find that the certainty equivalent rate of return of both the CEO and shareholders will be higher with less weight in option-based compensation and more weight in non-equity based compensation. This stems from the fact that there is a value gap between the cost of options from a shareholders' perspective and how the CEO values the options due to the CEO's risk aversion, as well as the fact that the options causes the CEO to make decisions that is beneficiary for him- or herself, but sub-optimal for the shareholders. Nevertheless, the key takeaway from this is that the base case compensation structure is not Pareto efficient. Our results imply that there are compensation structures where both the CEO and the shareholders would be better off.

The common assumption that a CEO's risk aversion can be offset by compensation that is more valuable for riskier corporate finance strategies, hereby options, is not completely true. Option-based compensation only functions as intended for a limited stock price interval. We suggest that a better way to align CEO and shareholder incentives is by looking towards the risk aversion. When CEOs are not well-diversified, which seems to be the standard case, a satisfying alignment of risk preferences can only be achieved by addressing the misconception around equity-based compensation and managerial risk aversion. In our model the CEO should ideally be as diversified as the shareholders, and own securities in other firms and industries. The rest should be a non-equity based remuneration that is high enough to compete with the industry standard. We have not considered wealth outside the firm in our analysis, but a future model extension could examine how diversified the CEO should be to have more aligned interests with the shareholders. From another point of view, the risk aversion of the CEO could for example be reduced by designing compensation contracts with alternative metrics where the payoff is dependent on the share price relative to competitors.

It is important that shareholders have an understanding of how the construction of executive compensation packages affects the way the CEO manages the firm and herein lies our main contribution. However, it should be noted that there are three important perspectives related to motivation and executive compensation that our model is unable to capture. Firstly, we have not considered intangible wealth in terms of for example managerial career concerns and personal reputation as this is difficult to quantify. Moreover, the motivation of the CEO is assumed to be driven by external rewards related to wealth. In the literature, motivation that arises from such rewards is classified as extrinsic motivation. Intrinsic motivation, which refers to doing something because it is inherently interesting, is usually regarded as more effective (Ryan and Deci, 2000). Since external rewards impose costs to the shareholder, both the shareholder and CEO would benefit from maximizing intrinsic motivation. Thus, our analyses do not provide the whole picture in regards to how to effectively motivate the CEO. Secondly, when analysing the effects of ESOs, our focus has been on how they impact the risk-taking behavior of the CEO. We have not reflected upon the less desirable consequences of ESOs such as the impact on earnings manipulation and other types of destructive behavior within the firm. It has been addressed in the literature that ESOs have the power to induce motivation to commit earnings fraud and misreporting (Burns and Kedia, 2006; Efendi et al., 2007). Thirdly, we have not taken into account that increased effort from the CEO can positively affect the firm value as the development of firm value is modeled as a stochastic process with no link to CEO effort.

In regards to future research, we propose some interesting potential model extensions that we did not incorporate due to complications related to path dependency. First, the options in our multi-period model are predetermined and set manually. Incorporating dynamic future option grants would be interesting as the CEO would be aware of receiving new options in the future which could impact his or her incentives. Next, we have assumed that the CEO has constant risk aversion. It would be interesting to investigate how taking into account prospect theory with loss aversion would influence our results. Finally, adding transaction and re-balancing costs when changing the capital structure would make our model more realistic and would therefore serve as an interesting future model extension.

References

Ainslie, G. (1991). Derivation of "rational" economic behavior from hyperbolic discount curves. *The American Economic Review*, 81(2):334–340.

Arrow, K. J. (1971). The theory of risk aversion. *Essays in the theory of risk-bearing*, pages 90–120.

Asheim, F. A. (2014). Hedge fund manager-investor conflicts of interest: A numerical analysis with loss-aversion. Master's thesis, Institutt for industriell økonomi og teknologiledelse.

Azar, S. A. (2006). Measuring relative risk aversion. *Applied Financial Economics Letters*, 2(5):341–345.

Balsam, S. (2002). *An introduction to executive compensation.* Academic Press.

Beaton, N. J. (2010). *Valuing early stage and venture-backed companies*, volume 503. John Wiley & Sons.

Bebchuk, L. A. and Fried, J. M. (2003). Executive compensation as an agency problem. *Journal of economic perspectives*, 17(3):71–92.

Billett, M. T., Mauer, D. C., and Zhang, Y. (2010). Stockholder and bondholder wealth effects of ceo incentive grants. *Financial Management*, 39(2):463–487.

Brickley, J. A., Bhagat, S., and Lease, R. C. (1985). The impact of long-range managerial compensation plans on shareholder wealth. *Journal of accounting and economics*, 7(1-3):115–129.

Brigham, E. F. and Ehrhardt, M. C. (2013). *Financial management: Theory & practice.* Cengage Learning.

Burns, N. and Kedia, S. (2006). The impact of performance-based compensation on misreporting. *Journal of financial economics*, 79(1):35–67.

Carpenter, J. and Yermack, D. (2013). *Executive compensation and shareholder value: theory and evidence*, volume 4. Springer Science & Business Media.

Carpenter, J. N. (2000). Does option compensation increase managerial risk appetite? *The journal of finance*, 55(5):2311–2331.

Chetty, R. (2006). A new method of estimating risk aversion. *American Economic Review*, 96(5):1821–1834.

Choi, J. and Richardson, M. (2016). The volatility of a firm's assets and the leverage effect. *Journal of Financial Economics*, 121(2):254–277.

Coles, J. L., Daniel, N. D., and Naveen, L. (2006). Managerial incentives and risk-taking. *Journal of financial Economics*, 79(2):431–468.

Damodaran, A. (2016). Bond ratings, cost of debt and debt ratios. `http://people.stern.nyu.edu/adamodar/podcasts/cfspr16/Session18.pdf`. Accessed: 2020-04-14.

Dittmann, I. and Maug, E. (2007). Lower salaries and no options? on the optimal structure of executive pay. *The Journal of Finance*, 62(1):303–343.

Dittmann, I., Maug, E., and Spalt, O. (2010). Sticks or carrots? optimal ceo compensation when managers are loss averse. *The Journal of Finance*, 65(6):2015–2050.

Edmans, A., Gabaix, X., Sadzik, T., and Sannikov, Y. (2012). Dynamic ceo compensation. *The Journal of Finance*, 67(5):1603–1647.

Efendi, J., Srivastava, A., and Swanson, E. P. (2007). Why do corporate managers misstate financial statements? the role of option compensation and other factors. *Journal of financial economics*, 85(3):667–708.

Elsilä, A., Kallunki, J.-P., Nilsson, H., and Sahlström, P. (2013). Ceo personal wealth, equity incentives and firm performance. *Corporate Governance: An International Review*, 21(1):26–41.

Equinor (2019). Annual report 2019. Accessed: 2020-05-19.

Ferris, S. P., Hanousek, J., Shamshur, A., and Tresl, J. (2018). Asymmetries in the firm's use of debt to changing market values. *Journal of Corporate Finance*, 48:542–555.

Furhmann, R. (2019). 5 ceos with the biggest payouts during the global financial crisis bailouts. `https://www.investopedia.com/investing/5-ceos-biggest-payouts-during-global-financial-crisis-bailouts/`. Accessed: 2020-03-11.

Goyal, V. K. and Park, C. W. (2002). Board leadership structure and ceo turnover. *Journal of Corporate finance*, 8(1):49–66.

Hall, B. J. (2000). What you need to know about stock options. `https://hbr.org/2000/03/what-you-need-to-know-about-stock-options?fbclid=IwAR2L5RGzbp02rZyTVbOa_wyZEnmMh6dMzUhsP6-FulYXn3OFzBqwnwfD62Y`. Accessed: 2020-03-24.

He, Z. (2009). Optimal executive compensation when firm size follows geometric brownian motion. *The Review of Financial Studies*, 22(2):859–892.

Hodder, J. E. and Jackwerth, J. C. (2007). Incentive contracts and hedge fund management. *Journal of Financial and Quantitative Analysis*, 42(4):811–826.

Jensen, M. C. and Meckling, W. H. (1979). Theory of the firm: Managerial behavior, agency costs, and ownership structure. In *Economics social institutions*, pages 163–231. Springer.

Jensen, M. C. and Murphy, K. J. (1990). Performance pay and top-management incentives. *Journal of political economy*, 98(2):225–264.

Jensen, M. C., Murphy, K. J., and Wruck, E. G. (2004). Remuneration: Where we've been, how we got to here, what are the problems, and how to fix them.

Jenter, D. (2002). Executive compensation, incentives and risk. MIT Sloan Working Paper.

Kadan, O. and Yang, J. (2005). Executive stock options and earnings management-a theoretical and empirical analysis. In *AFA 2006 Boston Meetings Paper*.

Kiersz, A. and Borden, T. (2019). Elon musk fact-checked wikipedia and said he'd go bankrupt if his companies failed. he could be right — and it shows how complicated ceo compensation has become. `https://www.businessinsider.com/elon-musk-tesla-compensation-explanation-2019-6?r=US&IR=T`. Accessed: 2020-02-12.

Kløve, B. and Valholm, S. S. (2011). Optimal executive incentives in a principal agent framework: The effects of risk aversion modelling choices. Master's thesis, Institutt for industriell økonomi og teknologiledelse.

Kolb, R. W. (2012). *Too much is not enough: Incentives in executive compensation*. Oxford University Press.

Lambert, R. A., Lanen, W. N., and Larcker, D. F. (1989). Executive stock option plans and corporate dividend policy. *Journal of financial and quantitative analysis*, 24(4):409–425.

Lambert, R. A., Larcker, D. F., and Verrecchia, R. E. (1991). Portfolio considerations in valuing executive compensation. *Journal of Accounting Research*, 29(1):129–149.

Leland, H. E. (1998). Agency costs, risk management, and capital structure. *The Journal of Finance*, 53(4):1213–1243.

Lovallo, D., Koller, T., Uhlaner, R., and Kahneman, D. (2020). Your company is too risk-averse. `https://hbr.org/2020/03/your-company-is-too-risk-averse`. Accessed: 2020-05-14.

Markowitz, H. M. (1959). *Portfolio Selection: Efficient Diversification of Investments*. Yale University Press.

Merton, R. C. (1969). Lifetime portfolio selection under uncertainty: The continuous-time case. *The review of Economics and Statistics*, pages 247–257.

Merton, R. C. (1974). On the pricing of corporate debt: The risk structure of interest rates. *The Journal of finance*, 29(2):449–470.

Miller, M. H. and Modigliani, F. (1961). Dividend policy, growth, and the valuation of shares. *the Journal of Business*, 34(4):411–433.

Modigliani, F. and Miller, M. H. (1958). The cost of capital, corporation finance and the theory of investment. *The American economic review*, 48(3):261–297.

Musk, E. (2019). "if tesla & spacex go bankrupt, so will i. as it should be.". `https://twitter.com/elonmusk/status/1208831048026509312`. Tweeted 2019-12-22.

Nikolova, S. M. (2003). The informational content and accuracy of implied asset volatility as a measure of total firm risk. *University of Florida*, pages 33–42.

Pepper, A. (2019). *Agency Theory and Executive Pay*. Springer.

Pepper, A. and Gore, J. (2014). The economic psychology of incentives: An international study of top managers. *Journal of World Business*, 49(3):350–361.

Rajgopal, S. and Shevlin, T. (2002). Empirical evidence on the relation between stock option compensation and risk taking. *Journal of Accounting and Economics*, 33(2):145–171.

Russell, K. and Williams, J. (2019). The highest-paid c.e.o.s of 2018: A year so lucrative, we had to redraw our chart. `https://www.nytimes.com/interactive/2019/business/highest-paid-ceos-2018.html`. Accessed: 2020-02-14.

Ryan, R. M. and Deci, E. L. (2000). Intrinsic and extrinsic motivations: Classic definitions and new directions. *Contemporary educational psychology*, 25(1):54–67.

Sappington, D. E. (1991). Incentives in principal-agent relationships. *Journal of economic Perspectives*, 5(2):45–66.

Schaefer, S. M. and Strebulaev, I. A. (2008). Structural models of credit risk are useful: Evidence from hedge ratios on corporate bonds. *Journal of Financial Economics*, 90(1):1–19.

Scheel, C. F., Herud, T. A., and Rime, E. F. (2015). Managerial risk profile in hedge funds with multiple high-water marks-numerical modelling and fund structure analysis. Master's thesis, NTNU.

Spear, S. E. and Wang, C. (2005). When to fire a ceo: Optimal termination in dynamic contracts. *Journal of Economic Theory*, 120(2):239–256.

Tversky, A. and Kahneman, D. (1992). Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and uncertainty*, 5(4):297–323.

Valle, M. and Pavlik, R. M. (2009). Predicting the investment decisions of managers under the influence of stock option incentives. *Journal of Management Research*, 9(3):133–141.

Von Neumann, J. and Morgenstern, O. (1947). Theory of games and economic behavior, 2nd rev.

Wakker, P. P. (2008). Explaining the characteristics of the power (crra) utility family. *Health economics*, 17(12):1329–1344.

Warner, J. B., Watts, R. L., and Wruck, K. H. (1988). Stock prices and top management changes. *Journal of financial Economics*, 20:461–492.

Weisbach, M. S. (1988). Outside directors and ceo turnover. *Journal of financial Economics*, 20:431–460.

Welch, I. (2004). Capital structure and stock returns. *Journal of political economy*, 112(1):106–131.

Wright, P., Kroll, M., Davis, P., and Jackson, W. T. (2007). The influences of the chief executive officer's stock and option ownership on firm risk taking: an examination of resource allocation choices. *Academy of Strategic Management Journal*, 6.

Yahoo (2020). Interest rates indicators. `https://ycharts.com/indicators/categories/interest_rates`. Accessed: 2020-04-14.

Zimmermann, P. (2016). The fallacy of fully dividend-protected stock options and convertible bonds. *The Journal of Derivatives*, 23(3):61–72.

Appendices

A. Model assumptions and parameters

In this section, we summarize the underlying assumptions in our model. We also present the numerical values of the model parameters and provide a brief justification for the parameters.

A.1 Model assumptions

The key model assumptions are as follows:

- Market capitalization is a stochastic variable with log-normal returns.
- The market capitalization evolves independent of CEO performance.
- Volatility of the return on enterprise value is constant.
- The CEO is fired or quits at the lower boundary only.
- No farewell package is included for the CEO at termination.
- Re-balancing and transaction costs are ignored.
- Dividends are neglected as we assume that the options are dividend-protected.
- The funding of the CEO compensation package is external, meaning that the compensation does not affect total firm value.
- The risk-aversion of the CEO is constant.
- The CEO must convert the stocks into cash when leaving the firm.

A.2 Model parameters

The values of the model parameters are summarized in Table A.1.

Core model		
Time to maturity (years)	T	1
Time step	Δt	1/252
Grid step for the return on market capitalization	C	0.0050
Possible debt ratios (kappa)	κ	(0:0.0005:0.9995)
Initial stock value	S_0	1.0
Grid size	G	1001
Lower boundary (fraction of firm value)	L	0.30
Upper boundary (fraction of firm value)	U	7.4
Multi-period model		
Multi-period discount rate (%)	R	30
Vesting period on stocks (years)	V	4.0
Asset return and volatility		
Enterprise value, annual expected return (%)	$\mu_{r_{EV}}$	6.0
Enterprise value, annual standard deviation (%)	$\sigma_{r_{EV}}$	16
Tax and interest rate		
Tax shield rate (%)	τ	21
Interest rate of AAA bonds (%)	r_i	2.6
CRRA utility model		
Risk aversion coefficient	γ	4.0
Wealth function		
Stock ownership fraction (%)	b	1.6
Option ownership (%)	n	0.42
Option strike price	K	1.0
Non-equity based compensation (%)	a	0.03

Table A.1: Numerical values of model parameters

Core model parameters

The time step in our model is set to a daily frequency such that $\Delta t = 1/252$. In the single-period model, we consider a time span of one year $T = 1.0$. We construct a grid with size 1001, corresponding to 500 up and down nodes with constant spacing $C = 0.0050$.

The lower boundary in our model is set to 30% of initial stock value. The reason why we do not consider lower values in our model is of practical concerns related to the reduction in computational speed with lower boundaries. Nevertheless, the choice of lower boundary only impacts the risk-taking behavior near the boundary. The lower we set the boundary, the higher is the preferred risk-taking of the CEO right before reaching the valley of prudence.

The upper boundary is set to an arbitrary sufficiently high value to ensure accuracy. In our model, the upper boundary is set to 7.4 of initial stock value.

Multi-period parameters

In our multi-period analysis, we must take into account temporal discounting, which is a phenomenon in behavioral economics describing the rate an individual devalue delayed rewards. According to Pepper (2019), executives are very high temporal discounters and typically devalue incentives at a rate of 30 percent. Thus, we set the temporal discount rate $R = 30\%$.

In our four period model, the stocks have a vesting restriction equal to four years, $V = 4$. The numerical parameter is set to four years since stocks usually have a vesting period of three to four years (Kadan and Yang, 2005). This means that the CEO must stay in the company for fours year before he or she has full ownership of the shares. If the CEO is terminated before the vesting period ends, he or she loses the shares. This is a simplification as vesting restrictions on stocks are usually scheduled to be payed out annually. For a vesting period of four years, the CEO would typically get full ownership of 25% of the restricted stocks each year and thus have full ownership over 100% of the shares at the end of year 4.

Asset return and volatility parameters

The return on a firm's assets, the enterprise value return, can be represented as a weighted average of the return of its underlying financial claims assuming the theorem of Miller and Modigliani (1961). Using US data on firms with an asset value of at least \$100 million, Choi and Richardson (2016) calculated the average annual asset return to be in the region 5.3-6.2%. Thus, we set the continuously compounded return to be $\mu_{r_{EV}} = 6.0\%$ in our model.

The asset volatility is generally unobservable due to the lack of comprehensive data on debt. The equity volatility can be observed and could thus be calculated directly. However, as noted by Beaton (2010), the equity volatility would be inappropriate as an input in our model since the firm's leverage affects the equity volatility. Moreover, Choi and Richardson (2016) report that equity volatility is significantly more persistent and asymmetric compared to asset volatility. With a mathematical relationship between asset volatility and equity volatility, this relationship could be used to calculate the implied asset volatility. Merton (1974) showed that there exists such a relation given by the equation

$$\sigma_{r_{EV}} = \frac{1}{N(d_1)} \times \frac{S}{EV} \times \sigma_{r_S} \tag{A.1}$$

where

$\sigma_{r_{EV}}$ is the asset volatility

$N(d_1)$ is a familiar term from the Black Scholes equation describing the cumulative normal density evaluated at d_1[1]

S is the market capitalization value

EV is the enterprise value

σ_{r_S} is the volatility of the market capitalization

Asset volatility has been investigated empirically using this relation and found to be in the range 16 to 35 % for large US firms (Choi and Richardson, 2016; Nikolova, 2003; Schaefer and Strebulaev, 2008). We use an annual volatility $\sigma_{r_{EV}} = 16$ % in our model.

Tax and interest rate parameters

The two factors that are relevant to consider when computing the optimal debt ratio are bankruptcy costs and the tax benefits of debt. Higher interest rates when debt increases is a good proxy for bankruptcy costs, and we thus construct an exponential function to let the interest rate depend on kappa.

The tax shield is set to the corporate tax rate in the US, currently at a rate of 21 percent as of June 2020.

Credit ratings affect the cost of borrowing and hence interest rates. If the debt ratio is higher, the corresponding interest rate will likely also be higher since the risk increases. To take these distress costs into account, we let the interest rate depend exponentially on the debt ratio for debt ratios greater than 0.2. For debt ratios below 0.2, the interest rate is fixed at the rate of AAA rated corporate bonds. To approximate the exponential function, we use data on the long-term US Corporate Effective yield accessed from Yahoo (2020) on bonds with different credit ratings, and corresponding debt ratio provided by Damodaran (2016). The numerical values are presented in Table A.2. The long-term average of AAA rated corporate bonds $r_i = 2.6$, and the multipliers in Table A.2 are calculated by dividing each interest rate with r_i.

[1]According to the Black Scholes formula, assuming no dividends, $d_1 = \frac{ln(S_0/K)+t(r+\sigma^2/2)}{\sigma\sqrt{t}}$, where S_0 is the underlying share price, K is the strike price, σ is the volatility, r is the continuously compounded risk-free interest rate and t is the time until expiration.

Credit rating	Debt ratio	Interest rate	Multiplier
AAA	0	2.6%	1
AAA	0.2	2.6%	1
B-	0.5	5.4%	2.1
CCC	0.6	12.3%	4.8

Table A.2: Numerical values to approximate the exponential function of the distress multiplier. The interest rate is the continuously compounded yearly rate.

Figure A.1 shows how the interest rates, before adjusting for tax shield benefits, depend on kappa. We acknowledge that the interest rate model imposes unrealistic high costs at high kappa values. We only consider kappa values less than 0.7. Since our solution area is within kappas up to 0.7, the high values for the distress multiplier above 0.7 are never considered by the model and thus the distress multiplier for kappa values above 0.7 does not affect our solution.

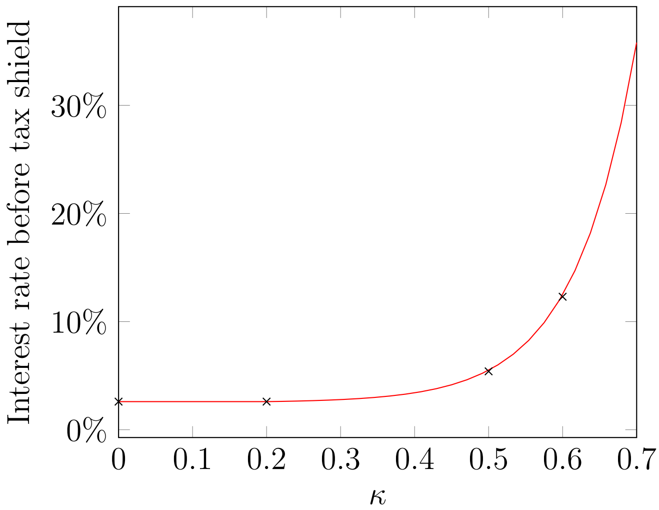


Figure A.1: The effect of debt ratio on interest rate

CRRA utility model parameters

The value of the risk-aversion coefficient is disputed in the literature due to limitations in estimating it empirically (Chetty, 2006). The risk-aversion coefficient has been set to $\gamma = 4$ when modeling the utility of a hedge fund manager in previous literature (Hodder and Jackwerth, 2007; Scheel et al., 2015). Even though we expect the risk aversion of a CEO to deviate somewhat from the risk aversion of a hedge fund manager, this value for γ is commonly used in the literature. Azar (2006) estimated the risk aversion coefficient for a general individual and found it to be between 4.2 and 5.4, which is not far off from the γ value we choose.

Wealth function parameters

As a base case, we start our analysis by constructing a wealth function based on empirical analyses in the literature related to common equity ownership structures and compensation. Balsam (2002) investigated CEO equity ownership as a percentage of shares outstanding in the US and found that for the largest corporations, average stock ownership was 1.6 percent and average option ownership was 0.42 percent.

When it comes to the non-equity based component of the compensation, Jensen and Murphy (1990) found that bonus and salary constitute approximately 0.03 percent of total firm value, so the non-equity based compensation in our wealth function will be fixed at 0.030 percent of initial firm value and remain constant throughout the period. We thus set the parameters in the wealth function to these values, before tweaking with the parameters to investigate the shift in risk-behavior of the CEO.

The strike price of the ESOs issued to the CEO is initially set at-the-money, as ESOs are usually issued at-the-money (Kolb, 2012). Thus, the strike price is equal to the initial firm value.

B. Sensitivity and robustness tests

In this section, we include sensitivity and robustness tests on the input parameters asset volatility and the temporal discount rate. We use the multi-period model for both analyses with the base case contract presented in Chapter 2 as illustrated in Figure 5.4 with four periods and four options.

Asset volatility, $\sigma_{r_{EV}}$

The asset volatility varies across firms, industries and countries, and it is therefore of great interest to check how sensitive our model is to this input parameter. In Table B.1, we show how the certainty equivalent return varies with different asset volatilities based on the multi-period model. The upper range of asset volatilities that we examine is set based on Choi and Richardson (2016) who estimate asset volatilities using a comprehensive data set on US firms.

$\sigma_{r_{EV}}$	10%	16%	22%	28%	34%
Π_{CEO}	9.00%	4.82%	-0.21%	-6.14%	-12.80%
Π_{SH}	8.29%	7.63%	7.03%	6.69%	6.52%

Table B.1: Model sensitivity to the asset volatility parameter. The grey column highlights the asset volatility used in the multi-period model. The compensation contract has the following parameters: non-equity based compensation $a = 0.03\%$, stock ownership $b = 1.6\%$, option ownership $n = 0.42\%$ and option strike prices $K = [1, 1.1, 1.2, 1.3]$.

We observe that as the asset volatility increases, the certainty equivalent return of the CEO decreases quite fast and turns negative. The certainty equivalent return turns negative as the uncertainty becomes very large because the certain wealth the CEO would be willing to accept now, instead of receiving the uncertain wealth in the future, becomes smaller than the value of his stocks as the uncertainty increases. The expected return for the shareholders also decreases slightly when the volatility increases, as the CEO makes less desirable choices from the shareholders' perspective.

It should be noted that this thesis examines risk preferences in terms of debt ratio, but our results are also applicable for investigating risk preferences in terms of asset volatility. In cases where a change in the CEO's incentives result in a lower preferred debt ratio, we also expect the CEO to prefer a lower asset volatility.

Temporal discount rate, R

As the temporal discount rate is individual-specific, we investigate how the results vary with the choice of the temporal discount rate. Table B.2 shows how the certainty equivalent return of the CEO and shareholders change with different temporal discount rates in the multi-period model. The range we consider in this analysis is based on Pepper and Gore (2014), who measure time preferences of executives from all around the world, finding results ranging from approximately 15% to 45% temporal discounting.

R	15 %	30%	45%
Π_{CEO}	4.83%	4.82%	4.80%
Π_{SH}	7.64%	7.63%	7.62%

Table B.2: Model sensitivity to the temporal discount rate parameter. The grey column highlights the temporal discount rate used in the multi-period model. The compensation contract has the following parameters: non-equity based compensation $a = 0.03\%$, stock ownership $b = 1.6\%$, option ownership $n = 0.42\%$ and option strike prices $K = [1, 1.1, 1.2, 1.3]$.

Based on the results from Table B.2 we can conclude that our model is not very sensitive to the choice of the temporal discount rate.

C. Analysis of optimal total return

In addition to the calculations of the certainty equivalent returns of the CEO and the shareholders in Section 6.3, we have also examined how the desired risk-taking of the CEO, with different compositions of the compensation affects the total return of all stakeholders of the firm. With total return, we mean the expected increase in value of the firm before any of the return is divided among the stakeholders. In a way, this shows the socioeconomic benefit of the firm and does not consider how the benefit is divided among the stakeholders.

The definition of total return will simply be as follows:

$$\Pi_{TR} = \frac{E(S_T) - S_0}{S_0} \tag{C.1}$$

Where S_T represents the market cap before any compensation is paid to the CEO, and S_0 represents the initial market cap.

The results of this analysis are shown in Table C.1. The rows show the non-equity based compensation altering from 0 to 0.13%. The columns indicates options altering from 0 to 0.84%. The grey cell indicates today's base case. The other cells are colored similarly to the corresponding cells in Table 6.2.

		Options weight						
		0.00 %	0.14 %	0.28 %	0.42 %	0.56 %	0.70 %	0.84 %
Non-equity based	0.01 %	7.641%	7.646%	7.650%	7.653%	7.653%	7.653%	7.651%
	0.03 %	7.654%	7.659%	7.663%	7.665%	7.666%	7.666%	7.664%
	0.05 %	7.666%	7.671%	7.675%	7.678%	7.679%	7.679%	7.677%
	0.07 %	7.677%	7.683%	7.688%	7.691%	7.692%	7.692%	7.690%
	0.09 %	7.688%	7.695%	7.700%	7.703%	7.704%	7.704%	7.703%
	0.11 %	7.699%	7.707%	7.712%	7.715%	7.717%	7.716%	7.715%
	0.13 %	7.710%	7.718%	7.724%	7.727%	7.729%	7.728%	7.727%

Table C.1: Total return rates, Π_{TR}. Representing the expected increase in value of the firm before any of the return is divided among the stakeholders. The cells are colored similarly to the corresponding cells in Table 6.2.

In Table C.1 the total return rates, Π_{TR}, increase monotonically as the non-equity based compensation increases. This is in line with previous results, as the CEO is more willing to take risk when the better decisions from a risk-neutral economic view. With regards to the option weight, Π_{TR} increases until an option weight of 0.56%, but decreases from 0.70% and onward. This could indicate that approximately 0.56%

is an optimum, incentivizing the CEO to make decisions closer to the optimum on average as the options raise the kappa surface in some instances, when the share price is close to the strike price of the option. Also, we observe that the contracts that are more optimized for both the CEO and the shareholders are not the most socioeconomically optimal.

Figure C.1 shows how the total return varies as the risk aversion of the CEO, γ, is changed. This is for the single-period model with the base case compensation structure, as in Section 6.4. The return rate here shows the same characteristics as in Figure 6.2

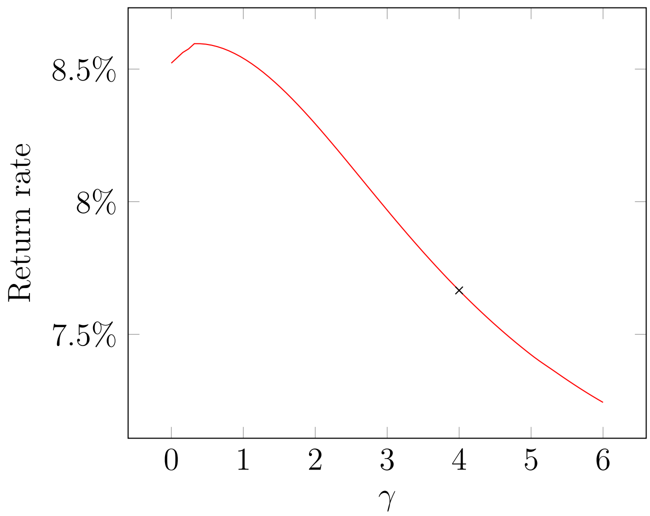


Figure C.1: Total return

D. Derivation of the process of the market capitalization

In this section we derive the process of the market capitalization, which we referred to as Equation 3.5. First, we define the return on the market capitalization in terms of the return on enterprise value, effective interest rate, size of debt and size of market cap. The return on the market capitalization is:

$$r_S = r_{EV} + (r_{EV} - r_e)\frac{D_t}{S_t} \tag{D.1}$$

where

r_{EV} denotes the return on enterprise value

r_e denotes the effective interest rate defined in Equation 3.3

D_t denotes the size of debt

S_t denotes the size of market capitalization

The expected value of the return on the market capitalization is thus:

$$E[r_S] = \mu_{r_S} = \mu_{r_{EV}} + (\mu_{r_{EV}} - r_e)\frac{D_t}{S_t} \tag{D.2}$$

where

μ_{r_S} denotes the yearly expected return on the market cap

$\mu_{r_{EV}}$ denotes the yearly expected return on the enterprise value

The variation of the return on market capitalization is accordingly:

$$Var[r_S] = \sigma^2_{r_S} = \sigma^2_{r_{EV}}(1 + (\frac{D_t}{S_t})^2) \tag{D.3}$$

where

σ_{r_S} denotes the yearly standard deviation on the market capitalization

σr_{EV} denotes the yearly standard deviation on the enterprise value

We want to express $\frac{D}{S}$ by κ:

$$\frac{D}{S} = \frac{D}{EV - D} = \frac{D/EV}{EV/EV - D/EV} = \frac{\kappa}{1 - \kappa} \tag{D.4}$$

We insert $\frac{D}{S} = \frac{\kappa}{1-\kappa}$ in the expressions for μ_{r_S} and σ_{r_S}.

$$\mu_{r_S} = \mu_{r_{EV}} + (\mu_{r_{EV}} - r_e)\frac{\kappa}{1-\kappa} \tag{D.5}$$

$$\sigma^2_{r_S} = \sigma^2_{r_{EV}}(1 + (\frac{\kappa}{1-\kappa})^2) \tag{D.6}$$

When we have found expressions for μ_{r_S} and σ_{r_S}, we insert them in the geometric Brownian motion equation.

$$dS_t = \mu_{r_S} S_t dt + \sigma_{r_S} S_t dW \tag{D.7}$$

$$dS_t = (\mu_{r_{EV}} + (\mu_{r_{EV}} - r_e) \cdot \frac{\kappa}{1-\kappa})S_t dt + \sqrt{\sigma^2_{r_{EV}}(1 + \frac{\kappa^2}{(1-\kappa)^2})} S_t dW_t \tag{D.8}$$

where

dt is the time differential

dW_t is the Wiener process

The Wiener process, dW_t, is defined to follow the normal distribution with mean 0 and standard deviation $\sqrt{dt}$, and therefore we can rewrite $dW_t = \sqrt{dt} dZ_t$ where $dZ_t \sim \mathcal{N}(0,1)$. This yields

$$dS_t = (\mu_{r_{EV}} + (\mu_{r_{EV}} - r_e) \cdot \frac{\kappa}{1-\kappa})S_t dt + \sqrt{\sigma^2_{r_{EV}}(1 + \frac{\kappa^2}{(1-\kappa)^2})} \sqrt{dt} S_t dZ_t \tag{D.9}$$

where

dZ_t is a normal distributed variable, $dZ_t \sim \mathcal{N}(0,1)$

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