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Optimal Harvesting for Salmon Farmers During Harmful Algal Blooms

A Real Options Approach

Master's thesis in Industrial Economics and Technology Management
Supervisor: Maria Lavrutich
June 2020

Master's thesis

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Research Questions

We consider the optimal harvesting decision of a salmon farmer that faces the risk of harmful algal bloom and market uncertainty. The salmon farmer seeks to maximize the financial value of the fish farm by determining the optimal course of actions during the algal bloom, and the optimal time to harvest after the bloom. Specifically, we develop a framework to compare the options to perform an early harvest, or to wait in order to learn about the true algal risk. Later, we extend this framework by taking into account the option to move the salmon to an algal free location.

In this thesis we answer the questions of:

- What is the optimal harvesting strategy and value of managerial flexibility during a harmful algal bloom while receiving imperfect information about the true algal risk? How does the opportunity to move affect the course of actions?
- What should policy-makers do to facilitate optimal decision-making during harmful algal blooms?

To illustrate the results and investigate the robustness of our model, we present two case studies with realistic industry parameters from the Norwegian and Chilean salmon farming industries.

Preface

This thesis is conducted as part of achieving a Master of Science at the Norwegian University of Science and Technology (NTNU). The degree specialization is in Financial Engineering at the Department of Industrial Economics and Technology Management.

First and foremost, we would like to thank our supervisor, Associate Professor Maria Lavrutich, for all stimulating discussions, feedback, and guidance.

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Trondheim, June 11, 2020

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Abstract

Harmful algal blooms can cause fatal damage to farmed salmon. This forces salmon farmers who face the risk of algal blooms to make difficult decisions regarding what to do with their biomass. In 2019, an algal bloom in Norway caused great financial damage to both smaller and larger salmon farmers, which had dire effects on the local communities they support. This revealed the need for proper management tools to aid farmers facing algal risk. There exists some literature on optimal harvesting of salmon, but no models for helping salmon farmers make optimal decisions while facing the risk of losing their biomass. This master thesis introduces a novel decision-making evaluation method for flexibility in harvesting during harmful algal blooms. Here, we demonstrate that if salmon farmers can actively learn about the true risk of losing the biomass, the value of flexibility in the harvesting decision is significant. We use the Least Squares Monte Carlo approach together with two-factor price modeling, risk modeling, and learning through signals, in order to determine the optimal timing of early harvesting. Furthermore, we quantify the value of having flexibility in the timing of early harvest. In addition, we develop a framework for evaluating the possibility of moving the biomass and examine the effect this has on the harvesting decision. Through case studies in a Norwegian and Chilean setting, we ensure the robustness of our model. We find that the availability of reliable information heavily affects what decisions salmon farmers should make, with higher reliability yielding higher value for salmon farmers. When information is sufficiently reliable, it is worth taking the risk of losing the biomass in order to learn, and make a better-informed decision at a later stage. Therefore, it is of great importance that policy-makers, governments, and industry organizations facilitate communication between industry actors, the collection of reliable data, and implementation of mitigation strategies. This applies both during a harmful algal bloom and as preventative measures.

Sammendrag

Skadelige algeoppblomstringer kan føre til dødelighet hos oppdrettslaks. Dette tvinger lakseoppdrettere som står overfor risikoen for algeoppblomstring til å ta vanskelige beslutninger om hva de skal gjøre med biomassen. I 2019 forårsaket en algeoppblomstring i Norge store økonomiske tap for små og store lakseoppdrettere. De økonomiske tapene hadde alvorlige konsekvenser for de berørte lokalsamfunnene. Denne hendelsen avslørte behovet for et beslutningsverktøy som kan hjelpe lakseoppdrettere som står overfor algerisiko. Fra før finnes det en del litteratur om optimalt slaktetidspunkt for laks, men ingen modeller som hjelper lakseoppdrettere med å ta optimale beslutninger når de står overfor risiko for at biomassen går tapt. Denne masteroppgaven introduserer en ny evalueringsmetode for beslutningtaking med fleksibilitet i slaktetidspunkt under skadelige algeoppblomstringer. Vi demonstrerer at dersom lakseoppdrettere kan lære om den sanne risikoen for at biomassen går tapt, tilfører fleksibilitet i slaktetidspunktet betydelig verdi. Vi bruker metoden Least Squares Monte Carlo sammen med tofaktor prismodellering, risikomodellering og læring gjennom signaler for å bestemme det optimale tidspunktet for tidlig slakt. Videre kvantifiserer vi verdien av fleksibilitet for tidlig slakt. I tillegg utvikler vi et rammeverk for å evaluere muligheten for å flytte biomassen og undersøker hvilken effekt dette har på beslutningen om å slakte. Gjennom casestudier i en norsk og chilensk setting, sikrer vi modellens robusthet. Resultatene våre viser at beslutningene oppdrettere bør ta, påvirkes i stor grad av tilgjengeligheten av pålitelig informasjon. Videre viser resultatene at høyere pålitelighet øker verdien for lakseoppdretterne. Dersom informasjonen er tilstrekkelig pålitelig er det verdt å ta risikoen for å miste biomassen for å lære mer om den sanne risikoen og få et bedre beslutningsgrunnlag. Derfor er det viktig at politikere og bransjeorganisasjoner legger til rette for kommunikasjon mellom industriaktører, innsamling av pålitelig data og implementering av skadebegrensningstiltak. Dette gjelder både under en skadelig algeoppblomstring og som forebyggende tiltak.

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List of Abbreviations

DCF	Discounted cash flow
EH	Early Harvest
EH-model	Early Harvest model
EH-M-model	Early Harvest-Move model
FCR	Feed conversion ratio
FPI	Forward Price Index
GBM	Geometric Brownian motion
HAB	Harmful algal bloom
LSM	Least Squares Monte Carlo
MAB	Maximum allowed biomass
NPV	Net present value
PD	Pancreas disease
RO	Real options
SSB	Statistics Norway

Chapter 1

Introduction

In the spring of 2019, a harmful algal bloom (HAB) in Norway caused great damage to salmon farmers located in the counties Nordland and Troms. The HAB resulted in the loss of 14 500 tons of Atlantic salmon with economic consequences and ripple effects estimated between 2.3 to 2.8 billion NOK (Kontali, 2020). The risk of losing millions worth of revenues forced small and large salmon farmers to make swift decisions regarding how they should respond to the threat. At the same time, salmon farmers received information about the algal spread from research communities, as well as hearsay from nearby farms, which created an incentive to wait in order to learn about the risk and make a more informed decision (Directorate of Fisheries, 2020a; Karlsen et al., 2019).

During the HAB outbreak, there were mainly two actions that farmers took, namely to perform an early harvest or move the biomass. As an example, SalMar decided to harvest 1 000 tons of salmon weeks before the planned schedule[1]. Other large salmon companies with spatial diversification, such as Cermaq and Nordlaks, moved their fish away from the HAB to alternative locations in order to secure further salmon growth (Directorate of Fisheries, 2019b). The fundamental problem for farmers during HABs is to choose the right action at the right time. There exists a wide body of literature regarding optimal harvesting time of salmon, such as the work of Asche and Bjørndal (2011) and Ewald et al. (2017). However, none of these account for the risk of losing the biomass and the possibility to learn about the risk level. Thus, there is a need for models the salmon farmers can apply to find the optimal decisions in such a situation. This is the problem we address in this thesis.

Performing an early harvest entails losing the future growth of the biomass, and the possibility to harvest optimally at a later stage. Moving the biomass can be costly due to direct moving costs and indirect production costs as feed conversion ratios (FCRs) and salmon mortality increase, as a result of high fish density during wellboat transportation (Basrur et al., 2009; Calabrese et al., 2017). Moreover, the salmon farmers do not know the true risk that the algae will arrive at their farms. However, farmers receive imperfect signals from other farmers and research organizations about the true risk. Another challenge that

[1] https://e24.no/boers-og-finans/i/8mOkkG/salmar-slakter-tusen-tonn-laks-for-aa-sikre-seg-mot-alger

the farmers face is that the future salmon price is highly uncertain. Salmon spot prices tend to fluctuate in the short-run, but salmon farmers can observe the expected long-run price via forwards prices. Given all the uncertainties involved, the timing of the decision is crucial. Static valuation methods, such as the Net Present Value (NPV) approach, have a now-or-never nature and does not recognize the value of information or flexibility. In order to encapsulate the uncertainties and the value of waiting in order to learn, more advanced valuation tools are needed. We apply Real Options (RO) methodology to find the optimal timing and choice of strategy for salmon farmers facing HAB risk.

HABs are considered to be a global issue and is not an event exclusively experienced by Norwegian salmon farmers. Chile, the second largest salmon producer after Norway, has also experienced several dramatic HAB outbreaks. In 2016, a severe HAB oubreak in the southern parts of Chile killed 39 000 tons of Atlantic salmon and trout (Montes et al., 2018). More recently, in April 2020, Marine Farm in the Aysén region lost 43 tons due to a HAB[2]. In both Chile and Norway, the salmon farming industry supports many local communities along the coasts. Losses as a result of HABs can have large effects on these communities as the financial effects can cause bankruptcies and loss of livelihoods. The frequency and severity of HABs have increased dramatically on a global scale in recent decades, and this trend may continue due to climate change (Anderson, 2009; Sellner et al., 2003). There is also consensus among scientists that economic losses are increasing due to HABs (Anderson et al., 2012). Thus, better decision-making tools in the presence of HAB risk can contribute to securing the future of the local communities that are necessary for a sustainable salmon farming industry.

The aim of this master thesis is to identify optimal harvesting strategies for small and large salmon farmers when facing the risk of HAB arrival and stochastic prices. In order to do so, we develop three novel models using RO methodology. The first model, the General Single Rotation Model (GSR-model), finds the optimal time to harvest while facing uncertain prices without algal risk. Moreover, the GSR-model quantifies the value of flexibility in the operations of a salmon farmer, and is also used as input into the more advanced, subsequent models. The Early Harvest Model (EH-model) is the main focus of this thesis. The EH-model finds the optimal harvesting strategy and quantifies the value of harvesting flexibility during a time-limited HAB, while facing stochastic prices. The EH-model also accounts for the imperfect information farmers receive, which they use to learn about the true HAB arrival rate. The third and final model, the Early Harvest-Move Model (EH-M-model), extends the EH-model and allows companies with spatial diversification to jointly evaluate the decision between early harvesting and moving. We apply the EH-M-model to investigate if moving the biomass can bring additional value for farmers. We apply the models on two case studies, for Norway and Chile. This is of interest since the world's two largest producers of farmed salmon operate under different production conditions. We investigate these two cases in order to identify optimal strategies for salmon farmers from both parts of the world. All models use sophisticated price modeling, and are solved using state of the art simulation and regression methods.

Our thesis contributes to the literature in the following ways. We develop a novel optimal early harvesting model that incorporates both active and passive learning in an RO framework. Salmon farmers learn about the uncertain arrival rate of HAB through signals

[2] https://salmonbusiness.com/10000-harvest-size-salmon-die-from-red-tide/

from research organizations. Based on these signals, farmers can actively update their beliefs about the algal arrival rate in accordance with Bayes' rule. Additionally, farmers passively learn about the salmon price development by postponing their decision. To the best of our knowledge, we are the first to include HAB risk modeling in the context of optimal harvesting within the aquaculture literature. Our model could also be applied to other kinds of disease outbreaks that incorporates learning about the true rate of infection which is not known. We use realistic industry parameters and show that our model is robust to different geographical settings.

Furthermore, we provide novel insights for salmon farmers, policy-makers and industry organizations that can be summarized as follows. Firstly, we give salmon farmers a framework for making optimal harvesting decisions during HABs. Additionally, we offer recommendations for policy-makers on how they can facilitate optimal decision-making for salmon farmers during HABs.

Our results show that there is significant value in managerial flexibility in the harvesting decision both with and without HAB risk. The value of harvesting flexibility varies across the production cycle. In particular, the harvesting flexibility has little value in the early stages of the cycle, but increases in value for later stages. In other words, the current weight of the salmon largely affects the harvesting decision. As a consequence, if the HAB occurs early in the cycle when the biomass is low, salmon farmers should ignore the signals and perform no early harvest. However, if the HAB arrival intensity is sufficiently increased, we find that flexibility in the harvesting decision is valuable even for the early production cycle stages. Furthermore, we find that when the signals are sufficiently reliable, it is worth taking the risk of losing the biomass in order to learn more about the true risk.

The main insights of this thesis are that there is significant value in information when farmers have the possibility to actively learn about the true risk. The availability of reliable information heavily affects what decisions salmon farmers should make, with higher reliability yielding higher value for salmon farmers. Therefore, it is of great importance that policy-makers, governments, and industry organizations facilitate communication between industry actors, the collection of reliable data, and implementation of mitigation strategies, both during a HAB and as proactive measures.

The remainder of this thesis is organized as follows. Chapter 2 presents relevant aspects of the salmon farming industry in Norway and Chile with respect to our problem. Chapter 3 presents a review of the literature relevant to our research questions. The three models and the solution approaches are described in Chapter 4. In Chapter 5, we quantify parameters for a Norwegian and Chilean case study. Results and discussion of the case studies are presented in Chapter 6. Finally, Chapter 7 concludes the master thesis and provides suggestions for further work.

Chapter 2

Background

This chapter presents relevant aspects of the salmon farming industry with respect to the research questions. Section 2.1 gives an overview of the different phases of the production cycle and discusses relevant production costs in salmon farming. Section 2.2 presents background on HABs, including a review of recent events, the biological preconditions for HABs, and the actions available for salmon farmers during a HAB. Section 2.3 discusses salmon price characteristics and the salmon futures market.

2.1 Salmon Farming

The salmon farming production cycle can be broken down to the following phases:

i) **Egg and spawn production**. In hatcheries, eggs are fertilized and hatched.

ii) **Smolt production**. In a controlled freshwater environment on land, salmon are grown to a weight of around 100 to 150 grams. This usually takes between 10 to 16 months.

iii) **Sea phase**. The sea farming phase is where the salmon are transferred to sea water cages for further growth. This period lasts about 12 to 24 months and requires ideal water temperatures and sheltering from harsh weather. In this part of the production cycle, biological issues such as HABs become present to salmon farmers. Thus, it is the sea farming phase that is of interest in this thesis.

iv) **Harvesting and processing**. Towards the end of the production cycle, fish are transferred to a process plant for slaughtering and final processing. Transportation of salmon is usually conducted through the use of wellboats (Mowi, 2019).

The total average production cycle length is about three years. However, the production cycle in Chile is shorter compared to Norway. This is because the sea water temperatures are more ideal for salmon growth and the temperatures have fewer fluctuations. The average sea water temperature in Chile is about 12°C, while production regions in Norway average around 10°C (Mowi, 2019). The sea water temperature is considered to be a natural competitive advantage for Chilean salmon farmers as they operate with shorter cycles.

The sea farming phase is the most time consuming part of the production cycle. In this phase, the salmon is traditionally kept in open sea cages made of plastic, metal or rope nets. The cages are moored to the bottom of the ocean and kept afloat at the surface of the sea. This allows for free flow of water while keeping the salmon in place, as well as easy access for feeding and harvesting. At the same time, the sea pens must be sufficiently sheltered from harsh weather. The coastline and fjords of countries such as Chile and Norway, providing ideal production conditions, have consequently given them a competitive advantage within salmon farming.

As the salmon is exposed to the surrounding aquatic environment, the sea farming phase is the phase where major biological challenges arise. The issues of HABs, sea lice, and spread of diseases are some of the obstacles the aquaculture industry is seeking to overcome. The biological challenges have resulted in authorities imposing industry regulations and incentivizing technological innovation, seeking to achieve more sustainable production. Policy-makers in Norway have, e.g., set a maximum allowed biomass (MAB) of 780 tons per licence, except for the counties Finnmark and Troms where the MAB is 945 tons (Directorate of Fisheries, 2017). There is also a limit of 200 000 salmon per cage. In addition to regulations and biological issues, salmon farmers also face various production costs in the sea farming phase. The next section will look at the largest and most relevant production costs associated with this part of the production cycle.

Relevant Sea Phase Production Costs

The production costs for Norwegian salmon farmers have been steadily increasing since the early 2000s. The same can be said for salmon farmers located in Chile, but the production costs have been more variable. The average production costs for slaughtered and prepared salmon in 2018 were around 38 NOK per kilogram salmon produced in Norway[1]. When comparing against Chile for the same year, the average total production costs were 6.5% lower (Iversen et al., 2019). Aquaculture in Norway saw a significant increase in the costs from 2012 to 2016, mainly due to higher feed prices and issues related to lice and diseases.

Figure 2.1 below shows the production costs by cost category for 2018 in Norway.

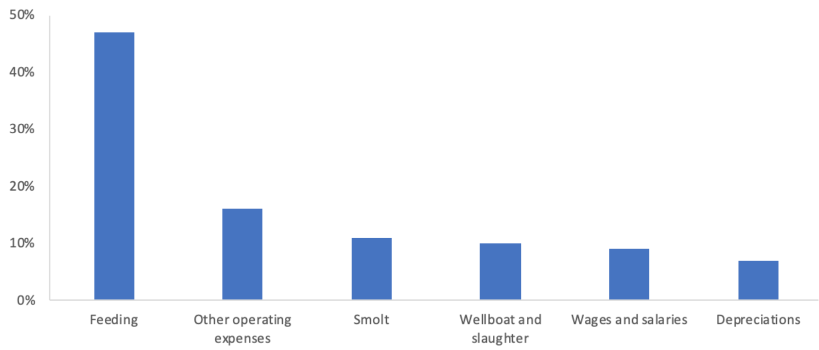


Figure 2.1: Illustration of the production costs, excluding financial expenses, for Norwegian salmon farmers during 2018. Other operating expenses includes costs related to services, maintenance, administration, etc (Iversen et al., 2019).

[1] https://www.barentswatch.no/havbruk/kostnader

It is evident from Fig. 2.1 that feeding costs alone account for around 50% of the total production costs and is thus the most important input factor in salmon aquaculture (Asche and Oglend, 2016). Feeding costs occur throughout the production cycle and is highly relevant for the sea farming phase. As harvesting the fish means that the farmer stops paying feeding costs, these costs have a great impact on the decision to harvest or to continue farming. As seen in Fig. 2.1, costs related to wellboat and slaughter make up about 10% of the production costs. Wellboat and slaughter costs are important when considering the possible actions during HAB outbreaks. This is because transportation of fish is necessary if the salmon farmer decides to harvest early or move the biomass to another production facility. We will not include costs regarding depreciation, wages and salaries, smolt, and other operating expenses. This is because these costs are not directly affecting the operation and the actions in the sea farming phase.

Feeding costs involve feeding the salmon to an optimal slaughter weight. The feeding costs can be decomposed into feed conversion ratio (FCR) and feed price. The FCR is a production indicator that indicates how effectively the fish is consuming the feed. The FCR is affected by relationships such as the biological feed utilization, feed waste, and lost biomass during production. These relationships are again affected by HABs, diseases, sea lice, sea water temperature, feed quality, feeding regime, etc.

Feeding costs and feed price movements in the salmon producing countries are historically quite similar. The similarities are due to the fact that the feed markets are international markets. Estimations conducted by Iversen et al. (2019) show that feed prices for salmon producing countries have steadily increased since the early 2000s. The average feed price per kilogram for Norwegian salmon farmers rose from 10.90 NOK/kg to 11.26 NOK/kg from 2017 to 2018 (Directorate of Fisheries, 2019a). As the feeding cost is the highest contributor to the total production costs and it occurs at a larger scale in the sea farming phase, we include this cost to account for the production costs in our models.

Wellboat and slaughter costs arise from transportation of salmon and harvesting at process plants. Transportation of salmon is usually conducted through the use of wellboats. Salmon companies often rent wellboats on long-term charter contracts, but the larger companies may have their own vessels. According to the Directorate of Fisheries (2019a), the average cost of wellboat and slaughter for Norwegian farmers was 3.79 NOK/kg in 2018. This cost level is considerably lower in Norway compared to other salmon producing nations. The reason for this is that Norway has invested into efficient and highly automated processing plants. Additionally, the Norwegian wellboats are of substantial size and the infrastructure is good. Therefore, the Norwegian aquaculture industry is able to reduce costs related to wellboat and harvest due to efficient logistics and automated processing plants (Iversen et al., 2019). The following section will look into how HABs develop, HAB outbreaks in recent times, and what actions salmon farmers can undertake to minimize economic losses due to HABs.

2.2 Harmful Algal Bloom

Algae are a diverse group of organisms that live in aquatic environments with the ability to perform photosynthesis (Vidyasagar, 2016). An algal bloom is a rapid increase in the population of algae in aquatic environments. Algae are important to the environment as they contribute with producing oxygen through photosynthesis. On the other hand, algae can be harmful to aquatic life. HABs can cause mortality, reduced welfare, and result in poor water quality for farmed salmon. It can inflict mechanical injury on the gills of the farmed salmon and cause suffocation through oxygen depletion (Anderson, 2009; Dale, 2020). For a HAB to take place, there must be enough nutrients and light for it to develop. HABs often occur in the spring, as nutrients build up over the winter and increased daylight provide ideal growth conditions. However, as algal blooms consume the nutrients it will naturally fade out after some time. This makes HABs inherently time-limited events. Sellner et al. (2003) argue that HABs mainly arise from anthropogenic loadings (human activities on nature) and natural processes (e.g., water circulation and upwelling). Anthropogenic loadings can lead to nutrient pollution where the seawater becomes overly enriched with nutrients and minerals. The frequency and severity of HABs have increased globally in recent times and the increase might continue due to climate change (Anderson, 2009; Sellner et al., 2003).

HABs have affected salmon farmers negatively, in terms of large losses of fish, in the biggest salmon producing countries for a long period of time. Blooms of *Alexandrium catanella* in 2002 and 2009 killed tons of fish in the southern production areas in Chile. Later, in 2016, high densities of *A. catenella* and *Pseudochattonella cf. verruculosa* caused deaths for 27 million salmon and trout which equaled a total biomass of 39 000 tons (Montes et al., 2018). Advection of greater nutrient-rich and saline seawater were the main reasons behind the HAB in 2016, which were made possible by changes in atmospheric and oceanographic conditions during the summer of the same year (León-Muñoz et al., 2018). More recently, in April 2020, a company based in the Aysén region communicated to the Chilean National Fisheries and Aquaculture Service that 43 tons were killed due to a HAB outbreak[2]. It is evident that HABs are reported frequently and vary in severity for salmon farmers in the aquaculture industry.

Similarily, salmon farmers along the Norwegian coastline have been struggling with various species of algae. *Gyrodinium aureolum* hit farmers located at Senja in 1982. Another species, *Chrysochromulina leadbeateri*, caused big economic losses of farmed and wild fish during the summer of 1991 in Lofoten and Vestfjorden. The same species had a smaller bloom in 1998 along the coast of Troms county, but had a bigger impact for salmon farmers when it came back in 2008 (Dale, 2020; Lorentzen and Pettersson, 2005). More recently, *Chrysochromulina leadbeateri* caused great damage to farmers located in the counties Nordland and Troms in the middle of May 2019. Production facilities that were hit especially hard were located in Astafjorden, Ofotfjorden, Vestfjorden, and Tysfjorden, and these farmers lost 14 500 tons of Atlantic salmon. The lost biomass represented approximately 2% of the biomass on a national level. Kontali has estimated that the economic consequences lie between 2.3 to 2.8 billion NOK, including lost profits, ripple effects, and lost taxes (Kontali, 2020). The economic losses during the 2019 HAB event were substantial for many salmon farmers and the corresponding local communities. This motivates the development of an economic decision tool for salmon farmers during HAB outbreaks to minimize economic losses.

In many cases, it is hard to foresee blooms, the reasons behind them, as well as their total duration. Even though there is always a possibility of a HAB developing at a facility

[2] `https://salmonbusiness.com/10000-harvest-size-salmon-die-from-red-tide/`

or reaching a facility, salmon farmers and authorities can look to various measures to minimize economic losses. Anderson (2009) mentions prevention, control, and mitigation as strategies stakeholders can undertake to deal with the threat of HABs. Prevention refers to the actions taken to limit the HABs from happening in the first place. The issue here is that we often lack knowledge about why HABs occur in certain areas, so this makes it harder to implement actions to regulate the outbreaks. Control strategies directly interrupt the bloom process by suppressing or completely destroying it. However, implementing control strategies is very costly and has a large negative impact on the marine environment. Finally, mitigation strategies involve dealing with an ongoing HAB and is about reducing the negative impacts. It is important for fish farmers to take immediate action if a HAB is reported nearby or if the fish behave abnormally. Therefore, we will focus on mitigation strategies for reducing the potential economic impacts due to HABs. In this thesis, we look at the following mitigation strategies:

i) **Early harvest**. This action serves as a tool for securing revenues from the biomass at the current spot price before potential salmon deaths. On the other hand, if the HAB does not reach the production facility and the salmon farmer undertakes an early harvest, the salmon farmer loses potential higher biomass. So there is a trade-off between securing biomass revenues through early harvesting and taking the risk to further grow the salmon biomass. SalMar was one of many salmon farming companies with production facilities in close proximity to the HAB outbreak in Nordland and Troms in 2019. SalMar decided to harvest 1 000 tons of salmon only a few weeks before the planned schedule[3]. The early harvest approach may not have been viable for SalMar if the salmon had recently entered the sea farming phase of the production cycle. Thus, the time since the start of the sea phase must also be taken into consideration.

ii) **Move biomass to an alternative location**. This can be an effective action for securing future biomass growth. However, transporting fish under higher densities gives rise to higher stress levels which can affect FCR and mortality rate negatively (Basrur et al., 2009; Calabrese et al., 2017). In addition, this action is only available for enterprises that operate multiple locations (i.e., companies with spatial diversification). Only four out of fourteen companies moved salmon to another location under the 2019 HAB outbreak in Norway. These were Cermaq, Nordlaks, Ellingsen Seafood, and Nordnorsk Stamfisk. A fifth company, Lerøy Aurora, considered to move their salmon, but upon receiving additional information about the HAB spread cancelled the move (Karlsen et al., 2019). Licences from the Directorate of Fisheries and the Norwegian Food Safety Authority are needed to perform a moving operation. The Directorate of Fisheries started to work on these licences swiftly after the HAB was detected which allowed several salmon companies to move salmon (Directorate of Fisheries, 2019b).

After the Norwegian Food Safety Authority and the Directorate of Fisheries were informed about the 2019 HAB outbreak, several organizations were brought in to assist with analyses and information to the salmon farmers. The research team consisted of SINTEF, Akvaplan-niva, and the Institute of Marine Research. In addition, the Norwegian Mete-

[3] `https://e24.no/boers-og-finans/i/8mOkkG/salmar-slakter-tusen-tonn-laks-for-aa-sikre-seg-mot-alger`

orological Institute provided forecasts on sea water streams which could bring the HAB to new locations (Directorate of Fisheries, 2019b). These organizations provided farmers with information regarding the current spread and density of the HAB. Additionally, farmers received forecasts on future spread. However, the information flow was not organized by a single organization. This meant that the salmon farmers received information from multiple sources at unknown intervals during the algal bloom. Furthermore, the degree of collaboration was high in the Norwegian aquaculture industry during the HAB outbreak, and salmon farmers shared resources and information with each other (Karlsen et al., 2019). The collaboration and involvement from research organizations gave salmon farmers the opportunity to make better-informed decisions.

2.3 Salmon Futures Market and Salmon Spot Price Index

In what follows, we briefly introduce the Fish Pool salmon futures market and the Fish Pool Index (FPI). We analyze the characteristics of the salmon price in order to highlight what important features are required to make our price modeling as realistic as possible.

Futures markets serve as a price risk management tool for market participants that are exposed to price risk from the underlying commodity or asset. Fish Pool ASA exchange market was established in Norway in 2005 and is the leading provider of futures contracts on farmed salmon. Fish Pool provides daily updates on futures prices for contracts with monthly maturities, with contract lengths up to five years. These contracts reflect the future price expectations of the registered trade members at Fish Pool for the coming months and years (Ankamah-Yeboah et al., 2017). In this thesis, we use weekly observations of forward prices from Fish Pool. The data retrieved from Fish Pool spans from week 14, 2013 to week 18, 2020 and we include contracts with one month and up to five years to maturity in our analysis.

There exist several salmon spot price indicators for Atlantic salmon in the market. Price indicators often used by analysts include the FPI, the Fish Pool European Buyers Index, the NASDAQ Salmon Index, the Kontali Farmers Index, and the export statistics from Statistics Norway (SSB) (Fish Pool ASA, 2020c; The Nasdaq Group, Inc., 2017). In this thesis, the FPI is the chosen index for the salmon spot price because it is used as a basis for financial settlement of all forward contracts at Fish Pool. The index is a synthetic market price, composed of both the NASDAQ Salmon Index and the Norwegian export statistics from SSB, weighted 95% and 5%, respectively. It is calculated using a weighted weekly average of sizes from 3-6 kg, head-on gutted salmon, following a fixed size distribution (Fish Pool ASA, 2020a). For our analysis, we study spot price history for the same period as for the forward contracts, spanning from week 14, 2013 to week 18, 2020.

The FPI spot and forward price history obtained from Fish Pool is shown in Fig. 2.2, where the blue line represents spot prices and the gray line 24-month forward prices.

Figure 2.2: Graph plotting the development of the Fish Pool Index spot price and 24 month forwards from 2013 week 14, to 2020 week 18 (Fish Pool ASA, 2020b; 2020c).

Figure 2.2 indicates a much higher volatility for the spot price than for the long-maturity forward. For example, during the HAB in Norway in 2019, prices dropped from 72.41 NOK to 42.50 NOK (i.e., a 41% reduction). However, the long-maturity forward indicates that the price fall was not expected to persist. Likewise, during the Covid-19 outbreak this year, we see once more that the spot price falls below the long-maturity forward. Comparing the volatilities of spot and 24-month forwards shows decreasing volatility as a function of maturity. This indicates mean-reversion in the salmon spot price process (Näsäkkälä and Fleten, 2005; Schwartz and Smith, 2000).

Mean reversion may occur as a result of delay in the production adjustments by salmon farmers to changing price levels. Intuitively, when the salmon spot price increases, existing salmon producers have incentives to increase their production and new entrants are attracted to the market, causing downward pressure on prices. Conversely, when the spot price decreases, there will be upward pressure on prices because some high-cost producers may be forced to exit and producers will temporarily lower their production. These adjustments are not instantaneous due to the biological characteristics of the production cycle. Thus, there will be periods with a temporary high or low spot price, which will revert towards a long-term equilibrium level.

In addition to mean-reverting traits, there appears to be uncertainty about the long-term equilibrium price to which the salmon spot price reverts to. Changes in the equilibrium price may come from fundamental changes in the market (e.g., new regulations or disruptive technologies). In order to capture both the effects of uncertainty about the long-term price and the mean reversion in prices, we adopt a two-factor model for the salmon price process.

Chapter 3

Literature Review

In this chapter, we look into the literature that is relevant for our problem of finding the optimal harvesting strategy during a harmful algal bloom while receiving imperfect information about the true algal risk. We begin by presenting literature in the context of optimal harvesting. Furthermore, we investigate literature on HABs and time-limited threats, in addition to signaling and learning. Next, we present valuation techniques for optimal harvesting decisions. We argue that real options (RO) methodology is most suitable to answer our research questions. Moreover, we study state-of-the-art solution procedures for real options analysis. Lastly, we examine state-of-the-art commodity price models.

There exists a wide body of literature in the context of optimal harvesting of salmon. Early work analyzes how different costs and growth curves affect the harvesting time, but does not include uncertainties in the model and salmon prices are assumed to be deterministic (Bjørndal, 1988). Further work extends this analysis. Arnason (1992) analyzes interdependence of optimal feeding schedule and harvesting time. Later, Forsberg (1999) develops a harvesting planning model that has the ability to take all production restrictions into consideration. The harvesting model in Forsberg (1999) is later used to find the value of price information, based on different price scenarios by Forsberg and Guttormsen (2006). Forsberg and Guttormsen (2006) extend former production planning models to also include forecasting of prices. Asche and Bjørndal (2011) add to the existing literature by providing systematic economic analyses based on more up-to-date Norwegian industry data. In a more recent study, Ewald et al. (2017) consider the optimal harvesting problem for both single and infinite production cycle rotations. They build on findings from Asche and Bjørndal (2011) and account for stochastic prices in a two-factor price model, using a large set of forwards contracts from Fish Pool exchange market to estimate prices. In line with Ewald et al. (2017), we adapt and apply a two-factor price model to our problem using latest price information. Moreover, we follow Asche and Bjørndal (2011) on their biomass growth assumptions, to be detailed in Chapter 4.

However, none of these studies have accounted for the risk of losing the biomass in the optimal harvesting models. Large efforts have been made in studying causes, detection, and economical impacts of HABs (see, e.g., Sellner et al. (2003), Lorentzen and Pettersson (2005), Hoagland and Scatasta (2006), Anderson et al. (2012), and Montes et al. (2018)).

However, to the best of our knowledge, HABs have not been studied with respect to finding optimal early harvesting strategies for salmon farmers. Pettersen et al. (2015) study the possible benefits of disease triggered early harvest due to pancreas disease (PD). They do not focus on the risk of PD arrival, but apply a partial budgeting approach to compare scenarios with and without PD outbreaks inside a sea pen. In their harvesting strategy, they assume that the salmon farmer adopts a diagnostic screening program to monitor the virus levels in the farm. This data is used to forecast a PD outbreak, which for certain thresholds trigger an early harvest to avoid disease losses. Different from the problem under consideration in Pettersen et al. (2015), salmon farmers facing HAB threat can not use a device to monitor algae levels inside the pen, because the salmon die shortly after the algal arrival.

As discussed in Section 2.2, HABs are examples of time-limited events. Such events have not been studied in optimal harvesting problems within the aquaculture literature, but appears, e.g., in the context of evacuation decisions in case of fire accidents in Reniers et al. (2007). Reniers et al. (2007) develop a decision aid model for the problem of whether or not, and when, to evacuate chemical installations threatened by fire. The probability that an initiating fire event escalates into a large-scale accident between the time of notification and the maximum duration of the threat is given by a Poisson arrival rate. Once the potential fire threat becomes zero, it will remain zero from then on, and the decision-maker will no longer consider evacuation of the facility. This is of similar characteristics in the case of HAB outbreaks: salmon farmers will no longer consider harvesting early once the HAB threat ceases. We adopt the approach of a time-limited event, described in Reniers et al. (2007), to correspond with our HAB problem in Chapter 4. Different from Reniers et al. (2007) is that during this time-limited threat, we also incorporate learning with respect to the perceived risk level.

Learning within the aquaculture literature tends to appear in the context of technology uncertainty and adaptation strategies. There are several papers that account for a passive learning-by-doing effect (see, e.g., Nilsen (2010) and Sandvold (2016)). Other papers, such as Hagspiel et al. (2018), incorporate passive learning in a wait-and-see manner. Within other areas of research, e.g., operational research, learning appears within the framework of information flow, where arriving signals are actively used to update ones beliefs about unknown parameters. Harrison and Sunar (2015) make use of a continuous-time Bayesian framework for updating a firm's beliefs of the unknown project value. Furthermore, the firm can choose between several costly learning modes. Each learning mode has a known cost and provides information of a known quality. Another example of Bayesian learning in an RO framework is Dalby et al. (2018), that study how investment behavior in renewable energy is affected by updating a subjective belief on the timing of a subsidy revision. In their setting, the two states of the world (good or bad) are not known to investors. These states indicate the duration of the subsidy scheme, and through Bayesian learning, the investor updates the belief about the transition rate between the subsidy regimes based on arrival of exogenous signals. Similar to Harrison and Sunar (2015), Dalby et al. (2018) assume that the signal arrival frequency regarding the state of the world is high enough to be modelled as a Brownian Motion (i.e., a continuous time process). Unlike Harrison and Sunar (2015) and Dalby et al. (2018), Thijssen et al. (2004) assume that signals arrive discretely through a Poisson birth process. Thijssen et al. (2004) consider a firm with the opportunity to invest in a project with uncertain profitability. Over time, the arrival of signals indicate the profitability of the project. The true state of the world is either good or bad, and this affects the profitability of the project. The firm uses the signals to update its belief that the state of the world is good in accordance with Bayes' rule. Ultimately, these signals affect the firm's valuation of the project and is used to form a decision rule. In our problem, signals do not arrive continuously, and farmers have free access to the different learning modes (e.g., phone-based information from nearby farmers, reports, and forecasts from research organizations). Moreover, the farmer can not influence the quality of the information, nor when the signals arrive. Therefore, we adopt the discrete-time signal process of Thijssen et al. (2004) to correspond to our HAB situation, where farmers receive imperfect information indicating the true HAB arrival rate. We are among the first to incorporate active learning in the context of risk modeling within the aquaculture literature. In what follows, we argue that RO analysis is the most suitable valuation method for our problem.

When decision-makers have to make a choice between several alternatives (e.g., harvesting or waiting), there exist many ways of conducting this choice. The method of discounted cash flows (DCF) is a well-known and widely used method. If the decision is whether to make an investment or not, DCF involves calculating the Net Present Value (NPV) of the expected future cash flows received by doing an investment or performing an action. If the choice is between doing an action or not, the DCF-method says that if the NPV is positive, the action should be done. If there exists multiple alternatives, the action with the highest resulting NPV should be chosen. The problem with this method is that it views investment opportunities and choices as now-or-never decisions and completely ignores the value of managerial flexibility. Thus, it will often lead to sub-optimal decisions (Mcdonald and Siegel, 1986). For problems such as the one we are studying, with managerial flexibility in the timing of the decision, a multitude of uncertainties, and the possibility to wait and learn, RO analysis is a better alternative (Dixit and Pindyck, 1994; Trigeorgis, 1996). A real option is the real-world counterpart of a financial option, and is the right, but not the obligation, to undertake an investment or decision. The two main solution approaches in RO analysis are contingent claims analysis and dynamic programming (Dixit and Pindyck, 1994). The two methods make different assumptions about financial markets and discount rates used to value future cash flows. Contingent claims analysis mainly derives its principles from financial theory. The valuation of an asset is performed by setting up a portfolio of existing traded assets with similar risk and return characteristics as the asset. Dynamic programming is a very general tool for dynamic optimization, and is especially useful when treating multiple sources of uncertainty (Dixit and Pindyck, 1994). The approach breaks up a whole series of decisions into a maximization problem with just two parts, known as the Bellman equation: the immediate decision and the continuation value. The continuation value is a function that incorporates the consequences of all subsequent decisions, given that they are optimal decisions. The approach originates from the work of Bellman (1956) and his principle of optimality.

One of the most applied methods for solving complex RO problems is a combination of simulation and dynamic programming techniques. An early example is that of Boyle et al. (1997), who show that simulation methods can be used to solve American-type option problems. Tsitsiklis and Van Roy (2001) propose a simulation and regression method, and provide proofs that such methods converge and are viable methods if the natural distribution of the underlying state process is simulated properly. A similar but more well-known method is the Least Squares Monte Carlo (LSM) approach developed by Longstaff and Schwartz (2001). LSM is based on least-squares regressions in which the explanatory variables are polynomials of the underlying variables. The essence of the LSM approach is that the regressions estimate the continuation value in the Bellman equation. We apply the LSM approach to solve our optimal harvesting problem. Analyses by Clément et al. (2002) proved the convergence of the LSM algorithm under general assumptions. Furthermore, Moreno and Navas (2003) found that it is robust to the choice of functions in the regression. Moreover, the LSM method has seen widespread use in the RO literature. A notable example is Cortazar et al. (2008), who extend the model of Brennan and Schwartz (1985) to include a realistic three-factor model for stochastic prices. Brennan and Schwartz (1985) use a finite-difference scheme to solve their model. Cortazar et al. (2008) find that the solution found by the LSM procedure converges to that found by the finite-difference method. Furthermore, they argue that LSM reduces the need for simplifying assumptions compared to other available methods, concluding that it is a better fit for real world problems. Another example of LSM within the RO literature is Gamba (2003), who extends the work of Longstaff and Schwartz (2001) by proposing a framework for evaluating several real options dependent on multiple state variables. Included in this framework is the evaluation of mutually exclusive options, which we apply for comparing the possibility to harvest and move during a HAB. Gamba (2003) provides numerical results to show the convergence of the algorithm, and applies it to real-life capital budgeting problems.

In the aquaculture literature, Ewald et al. (2017) apply the LSM method for their optimal harvesting model. As previously mentioned, we add several components to our model compared to Ewald et al. (2017). The decisions for salmon farmers during HABs are affected by multiple sources of uncertainty, and it is of importance that all the dimensions of the problem are taken into account. The LSM framework allows us to make our models as realistic as possible. We apply the extensions proposed by Gamba (2003) in order to capture the joint effects of having the option to early harvest and the option to move.

In what follows, we review existing literature on the modeling of salmon prices. In early literature, commodity price processes were assumed to follow a stochastic process described by a geometric Brownian motion (GBM) (see, e.g., Brennan and Schwartz (1985) and Paddock et al. (1988)). In a price process that follows a GBM, there is a constant growth rate and the variance in future prices is increasing in proportion to time. Later, others argued the use of mean-reverting price models are more appropriate for commodities, because such prices might fluctuate randomly up and down in the short run, but ought to be drawn back towards some ”normal” price level in the long run. Such a ”normal” level could, e.g., be the long run marginal cost of production of the respective commodity (see, e.g., Laughton and Jacoby (1993, 1995), Dixit and Pindyck (1994), Cortazar and Schwartz (1994), and Smith and McCardle (1998)).

Section 2.3 describes characteristics of the salmon prices. To capture both the effect of mean reversion and uncertainty in the equilibrium price, we adopt a two-factor price model in line with Schwartz and Smith (2000). The work of Schwartz and Smith (2000) builds upon the former article of Schwartz (1997). In their model, Schwartz and Smith (2000) let the crude oil equilibrium price evolve according to a Brownian motion with drift and the short-term deviations are assumed to follow a mean-reverting process. To prevent negative prices, log transformations are made. Moreover, the state variables in the two-factor model are not directly observable and must be estimated using oil spot prices and/or oil futures contracts. Standard Kalman filtering techniques are commonly applied to estimate these state variables.

Within the aquaculture literature, Ewald et al. (2017) make use of futures from Fish Pool to estimate parameters in their adopted two-factor model which is strongly linked to Schwartz (1997). They use the two-factor price model to study the optimal harvesting problem and compute arbitrage free prices for lease and ownership of fish farms. Furthermore, they investigate the importance of a salmon futures market, such as Fish Pool, for price risk management. Similar to Ewald et al. (2017), we make use of salmon price information from Fish Pool, but from a more recent time period and for a wider range of contract maturities. Ewald et al. (2017) find that presence of seasonality in salmon futures only marginally affects the parameter estimates, and hence do not include a seasonality function. Schwartz and Smith (2000) do not incorporate seasonality and the findings in Ewald et al. (2017) justify our decision to use a non-seasonal price process as well. Different from Ewald et al. (2017) is that our two-factor model is based on the Schwartz and Smith (2000) model which does not explicitly consider stochastic convenience yields. Furthermore, we extend the optimal harvesting problem in Ewald et al. (2017), by including presence of harmful algal risk and learning via signals.

To summarize, we contribute to the existing body of literature on optimal harvesting by developing novel models that incorporates both active learning related to a time-limited threat, and passive learning about uncertain prices.

Chapter 4

The Models

In this chapter, we develop three realistic real options models for the problems of (i) finding the optimal time to harvest in a basic, single production cycle while facing stochastic prices, (ii) finding the optimal time to harvest while also facing uncertain HAB arrival risk, and (iii) finding the optimal course of actions when allowing for both early harvesting and moving the biomass.

Section 4.1 presents the General Single Rotation Model (GSR-model). In this model, the optimal harvesting time of salmon and the value of having the option to harvest optimally is found. We take into account production costs, harvesting costs, biomass growth, and uncertain salmon prices. For the price modeling, we employ a sophisticated two-factor price model based on observations of spot and forward prices. In order to find the optimal time of harvesting, we use a least squares Monte Carlo (LSM) approach.

Section 4.2 presents the solution approach for the GSR-model in detail. The LSM approach is a state-of-the-art solution technique for RO models. Results from the GSR-model are both used as input into our more advanced models and as a benchmark for results.

In Section 4.3, we extend the problem under consideration in the GSR-model by introducing time-limited HAB-risk. Salmon farmers now face an uncertain risk of a HAB arriving at their farm. During the algal outbreak, salmon farmers receive a flow of information about the algal spread coming from several sources, as discussed in Section 2.2. Based on these signals, farmers can actively update their beliefs of the algal arrival rate in accordance with Bayes' rule. In addition to actively learning from the arriving signals, farmers passively learn about the salmon price development by postponing their decision. These two elements combined let salmon farmers undertake a better-informed harvesting decision. However, the benefits of waiting for more information about the algal arrival rate and possibly a higher salmon price, must be weighted against the risk of losing the current biomass altogether due to algae arriving. Section 4.4 presents the method to quantify the value of harvesting flexibility under the risk of HAB arrival. This model is called the Early Harvest Model (EH-model) and is the primary focus in this thesis.

In Section 4.5, we extend the main model to also include the option to move the fish. We do so in order to emphasize that, as an alternative to early harvesting, some farmers may have the opportunity to move their fish to another location free of algal risk. In other words, salmon farmers now hold two mutually exclusive options, i.e., the option to harvest early and the option to move. To take into account the interaction between the two options, we present the Early Harvest-Move Model (EH-M-model). The solution procedure is built upon the framework of Gamba (2003) for mutually exclusive options, and completes this chapter.

4.1 General Single Rotation Model

We consider a salmon farmer who seeks to maximize the value of his farm's salmon biomass during a single production cycle. At each point in time, the salmon farmer must decide whether to harvest the fish now or to grow it further. By harvesting the fish, the farmer pays a one-time harvesting cost and receives the revenue from the harvested biomass. The fish farmer will make a profit of $B(t)\Big(S_t - C_H\Big)$ at the time of harvest, where $B(t)$ denotes total salmon biomass at time t, S_t is the salmon price at time t, and C_H represents the fixed harvesting cost per kilogram fish.

The total biomass $B(t)$ is the product between the number of fish in the pen, denoted $R(t)$, and the average individual weight of the fish, given by a weight curve $W(t)$. We denote the number of fish at time $t = 0$ by R_0, and assume that $W(t)$ follows a deterministic process described by a von Bertalanffy's growth function,

$$W(t) = w_\infty \left(a - be^{-c\left(\frac{t+t_{\text{sea}}}{365}\right)}\right)^3,$$

where w_∞ is the asymptotic average weight of an individual fish, a, b, and c are constants, and t_{sea} is the time since the fish was introduced to the sea pen. The von Bertalanffy's growth function is commonly applied to model fish growth, see for instance Asche and Bjørndal (2011) and Ewald et al. (2017).

Since the salmon is not reproducing in pens, it is common to introduce a fixed mortality rate, M, to model a decreasing number of fish over time. Following Asche and Bjørndal (2011), we find the number of fish in the pen at time t by solving

$$R(t) = R_0 e^{-Mt}.$$

Hence, we can estimate the total biomass $B(t)$ at time t by solving

$$B(t) = R(t)W(t) = R_0 e^{-Mt}\left(w_\infty \left(a - be^{-c\left(\frac{t+t_{\text{sea}}}{365}\right)}\right)^3\right). \tag{4.1}$$

Alternatively, the salmon farmer can postpone harvesting and continue growing the fish and potentially receive a higher salmon price in the future. The harvesting profit of $B(t)\Big(S_t - C_H\Big)$ is compared against the option to harvest at a later point in time, while paying the production costs $C_p(t)$ in the meantime. As discussed in Section 2.1, feeding cost is the main cost driver for salmon farmers during the sea phase. Therefore, we assume that the variable production costs consists of feeding costs only.

The total feed quantity required at time t is the amount of feed needed per fish multiplied with the amount of fish. To find this quantity, we multiply the FCR, f_r, and the weight growth of the fish, $W'(t)$, together with the amount of fish, $R(t)$, i.e., $f_r W'(t)R(t)$. Then for a given feed price per kilogram, C_f, the production costs at time t is

$$C_p(t) = f_r W'(t)R(t)C_f.$$

The optimal harvesting time, τ, of the GSR-model is thus the solution of the following optimal stopping problem,

$$\sup_{\tau} \mathbb{E}\left[B(\tau)\Big(S_\tau - C_H\Big)e^{-r\tau} - \int_0^\tau C_p(t)e^{-rt}dt\right]. \tag{4.2}$$

Equation (4.2) consists of two main parts. The first part consists of the cash flow received from selling the biomass at the optimal time, less the cost of harvesting, discounted to time zero. The second part is the discounted production costs paid from time zero to the optimal harvesting time, τ. In Eq. (4.2), S_t denotes the salmon spot price at time t. Our analysis of the salmon price characteristics in Section 2.3 motivate the use of a two-factor model for simulations of salmon spot prices. Therefore, we follow the seminal work of Schwartz and Smith (2000) on commodity price modeling.

In line with Schwartz and Smith (2000), we decompose the logarithm of the salmon spot price into the sum of two stochastic factors, i.e.,

$$\ln(S_t) = \chi_t + \xi_t, \tag{4.3}$$

where χ_t represents short-term deviations in salmon prices and ξ_t the equilibrium price level at time t.

Changes in the short-term deviations, χ_t, represent temporary changes in salmon prices and are assumed to revert to zero following an Ornstein-Uhlenbeck process,

$$d\chi_t = -\kappa\chi_t dt + \sigma_\chi dz_{\chi,t}. \tag{4.4}$$

Changes in the equilibrium price, ξ_t, represent fundamental changes that are expected to persist and are assumed to follow an arithmetic Brownian motion process,

$$d\xi_t = \mu_\xi dt + \sigma_\xi dz_{\xi,t}. \tag{4.5}$$

The Brownian motion increments of $dz_{\chi,t}$ and $dz_{\xi,t}$ are correlated with $\rho_{\chi\xi}dt = dz_{\chi,t}dz_{\xi,t}$. Parameter κ is a mean-reversion coefficient describing the rate at which short-term deviations are expected to dissipate, σ_χ represents the short-term volatility, μ_ξ the equilibrium drift rate and σ_ξ the equilibrium volatility.

Given χ_0 and ξ_0, and following from Eqs. (4.3)-(4.5), the logarithm of future spot prices are normally distributed, with expected value being

$$\mathbb{E}[\ln(S_t)] = e^{-\kappa t}\chi_0 + \xi_0 + \mu_\xi t,$$

and variance

$$\mathrm{Var}[\ln(S_t)] = (1 - e^{-2\kappa t})\frac{\sigma_\chi^2}{2\kappa} + \sigma_\xi^2 t + 2(1 - e^{-\kappa t})\frac{\rho_{\chi\xi}\sigma_\chi\sigma_\xi}{\kappa}.$$

The salmon price S_t will then be log-normally distributed and its expected price can be calculated solving

$$\mathbb{E}[S_t] = \exp(\mathbb{E}[\ln(S_t)] + \frac{1}{2}\mathrm{Var}[\ln(S_t)]).$$

In the two-factor price model, the short-term deviations, χ_t, and the equilibrium price level, ξ_t, are unobservable. Kalman filtering is a recursive procedure that allows us to compute estimates for the short-term deviations and for the equilibrium price based on observations of spot and forward prices. These estimates give us more sophisticated salmon price simulations. Details behind this procedure can be found in Appendix A. Our resulting state variables and model parameter estimates are presented in Section 5.1. For a careful description of the two-factor modeling approach and corresponding proofs we refer to Schwartz and Smith (2000), as these derivations and properties are not the main focus of this thesis.

Completing this section, we note that the closed-form solutions for the optimal stopping problem does not exist. Therefore, we solve Eq. (4.2) numerically by applying the LSM approach described in Section 4.2 below.

4.2 Solution Approach for the General Single Rotation Model

In this section, we outline our solution method for the GSR-model. We employ the LSM approach described by Longstaff and Schwartz (2001) to find the optimal time of harvesting. The LSM approach was originally created for approximating the value of American call options by simulation, and has been shown to be applicable in solving complex multidimensional real option problems (Cortazar et al., 2008; Gamba, 2003).

As stated in Eq. (4.2), the optimal stopping problem the farmer faces in the GSR problem is when to stop paying the production costs and harvest the fish, in order to get the cash flow from selling it:

$$\sup_{\tau} \mathbb{E}\left[B(\tau)\Big(S_\tau - C_H\Big)e^{-r\tau} - \int_0^\tau C_p(t)e^{-rt}dt\right].$$

In order to solve the problem using the LSM-approach, we need to recast the optimal stopping problem as an American option with a finite time-horizon. In theory, the GSR problem can be seen as an infinite time-horizon problem, but in reality the length of the rotation is rarely longer than two years, see Section 2.1. Therefore, we define a maximum length of the sea phase, T^{sp}, which is the latest time a farmer can harvest the salmon in the single rotation problem.

We start by splitting the problem into sub-problems which the farmer evaluates at K discrete points in time from the start of the option, $t = 0$, to the end of the option, $t = T^{sp}$: $0, t_1, t_2, ..., t_{K-1}, t_K = T^{sp}$. The short time elapsed between each time point is denoted $\Delta t = \frac{T^{sp}}{K}$, see Fig. 4.1.

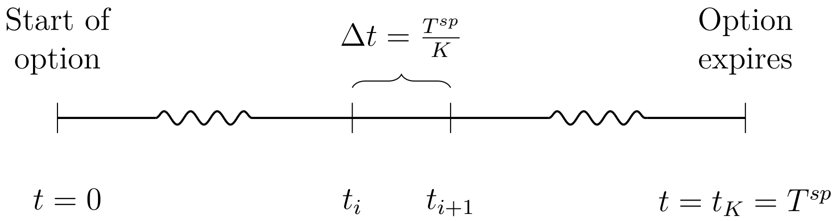


Figure 4.1: Illustration of time discretization in the GSR-model.

At every time point t_i, we simulate N realizations of the salmon spot price process. We simulate the price with Δt time between each time point. From Eq. (4.3) we find that the natural logarithm of the spot price at time $t + \Delta t$ is given by,

$$\ln(S_{t+\Delta t}) = \chi_{t+\Delta t} + \xi_{t+\Delta t}. \tag{4.6}$$

In order to simulate the two-factor price process needed in our option valuations, we follow the discretizations proposed by Davis (2012). The exact discretization of the short-term deviations in Eq. (4.4) is,

$$\chi_{t+\Delta t} = \chi_t e^{-\kappa \Delta t} + \sigma_\chi \sqrt{\frac{1 - e^{-2\kappa\Delta t}}{2\kappa}} \omega_t \tag{4.7}$$

where ω_t is an identically distributed random draw from a normal distribution with mean $\rho_{\chi\xi}\epsilon_t$, and variance $1 - \rho_{\chi\xi}^2$, i.e.,

$$\omega_t \sim N(\rho_{\chi\xi}\epsilon_t, 1 - \rho_{\chi\xi}^2).$$

Furthermore, the discretization of the equilibrium price in Eq. (4.5) is given by,

$$\xi_{t+\Delta t} = \xi_t + \mu_\xi \Delta t + \sigma_\xi \sqrt{\Delta t} \epsilon_t, \tag{4.8}$$

where $\epsilon_t \sim N(0, 1)$ is an independent and identically distributed random draw from the standard normal distribution.

The salmon price in Eq. (4.6) is then simulated using the discretizations in Eqs. (4.7) and (4.8), together with the estimated model parameters and state variables from the Kalman filter procedure.

The state vector $X_{t_i}(\omega)$ describes the simulated state of the price process at time t_i for realization ω. X_{t_i} contains the values of the stock price S_{t_i}, the equilibrium price ξ_{t_i} and the short-term deviation χ_{t_i}. Thus, we assume that the farmer can observe the equilibrium price, which according to Schwartz and Smith (2000) is unobservable. However, a farmer can easily estimate the value of the equilibrium price at any time, either by using the Kalman Filtering approach or with a spreadsheet of forward curves as Schwartz and Smith propose in their paper.

The value of harvesting, denoted Π, is a function of time and the state vector,

$$\Pi(t_i, X_{t_i}) = B(t_i)\Big(S_{t_i} - C_H\Big),$$

with biomass $B(t_i)$ as in Eq. (4.1) and t_{sea}, R_0 and M given as inputs to the model.

The value of the option to harvest optimally in the future, evaluated at time t_i with state vector X_{t_i}, is denoted $F_{GSR}(t_i, X_{t_i})$. The value of continuing, $\Phi(t_i, X_{t_i})$, is then the discounted expected value of the option at the next time step, less the cost of producing in the time period between the two time steps, i.e.,

$$\Phi(t_i, X_{t_i}) = \mathbb{E}_{t_i}[e^{-r\Delta t} F_{GSR}(t_{i+1}, X_{t_{i+1}})] - C_p(t_i) \times \Delta t.$$

At every time step t_i, the farmer chooses the alternative that maximizes the value of his farm: continuing or harvesting. The value of the option, at any time, is thus described by the following Bellman equation,

$$\begin{aligned} F_{GSR}(t_i, X_{t_i}) &= \max\Big\{\Pi(t_i, X_{t_i}),\ \Phi(t_i, X_{t_i})\Big\} \\ &= \max\Big\{B(t_i)\Big(S_{t_i} - C_H\Big),\ \mathbb{E}_{t_i}[e^{-r\Delta t} F_{GSR}(t_{i+1}, X_{t_{i+1}})] - C_p(t_i) \times \Delta t\Big\}. \end{aligned} \tag{4.9}$$

The next step is to determine the correct continuation value. The LSM procedure uses least squares regression to approximate the expected continuation value at each time step. We start at the expiration of the option, at $t = t_K$, where there is no possibility to continue, i.e., $\Phi(t_K, X_{t_K}) = 0$. Then, for every simulated realization ω, the value of the option at expiration is known,

$$F_{GSR}(t_K, X_{t_K}(\omega)) = \max\Big\{B(t_K)\Big(S_{t_K}(\omega) - C_H\Big), 0\Big\} = B(t_K)\Big(S_{t_K}(\omega) - C_H\Big).$$

Initially, we let the optimal stopping time τ for every realization ω be $\tau(\omega) = t_K$. Next, we work backwards from t_{K-1} to t_0. At every time point t_i, the expected continuation value is estimated following this procedure:

- First, we find the realized discounted cash flow for each realization ω. This is the cash flow received by harvesting at time $\tau(\omega)$, less the production costs incurred between time t_{i+1} and $\tau(\omega)$, discounted back to t_i.
- We define a basis function on the state variables in X_{t_i}, denoted $F(X_{t_i})$. We choose a set of simple powers and combinations of the state variables, as this type of basis functions gives accurate results (Longstaff and Schwartz, 2001). Therefore, we let $F(X_t) = \beta_0 + \beta_1 S_t + \beta_2 S_t^2 + \beta_3 S_t \xi_t + \beta_4 S^2 \xi_t^2$.

- Then, we fit a least squares regression with the realized cash flows of each realization as the dependent variable onto this basis function, $F(X_t)$. In order for the regression to be a reasonable estimation of the continuation function, the amount of realizations must be sufficiently high. The regression will converge in mean square and in probability to the true value of $F(X_t)$ as the number of simulations N goes to infinity. For computational efficiency, we only use the realizations which are in-the-money. The fitted basis function, $\hat{F}(X_{t_i})$, is then the approximation of the expected continuation value $\mathbb{E}_{t_i}[e^{-r\Delta t}F_{GSR}(t_{i+1}, X_{t_{i+1}})]$.
- For every realization ω, we estimate the continuation value at time t_i, including the cost of producing, and if the value of harvesting is greater than the value of continuing we update the optimal stopping time of the realization:

$$\tau(\omega) = t_i \quad \text{if} \quad \Pi\Big(t_i, X_{t_i}(\omega)\Big) \leq \hat{F}\Big(X_{t_i}(\omega)\Big) - C_p(t_i)\Delta t.$$

  If this is not the case, $\tau(\omega)$ is not updated.

Finally, when this procedure has been performed for every time step, we have an optimal exercise time for every realization. The value of the option at time t_0 is the average of the discounted realized cash flows received, less the production costs paid, in every realization when following the optimal strategy found by the algorithm.

4.3 Introducing Harmful Algal Bloom Risk and Imperfect Signals

In what follows, we extend the problem under consideration in the GSR-model. First, we incorporate the risk of a HAB arriving as a time-limited event with an uncertain arrival rate. Then, we include the flow of information available to salmon farmers about the algal spread. Based on imperfect signals, farmers can form beliefs about the true algal arrival rate.

As discussed in Section 2.2, HABs have disastrous effects. At the same time, the duration of a HAB is limited due to its biological characteristics. To account for these two elements, we model the risk of HABs as time-limited events with an uncertain arrival rate at any given time. This approach is novel in the context of risk modeling in the aquaculture literature.

At time $t = 0$, a HAB is reported by a nearby farm. We let the time period from $t = 0$ to $t = T$ represent the maximum duration of the HAB. In the case of the HAB arriving at our farmer's location, we denote the arrival time t^{HAB}.

The presence of algal bloom at the farm is modeled as a binary variable, Γ_t. We let $\Gamma_t = 0$ if there is no algal bloom at the farm at time t, and $\Gamma_t = 1$ at the time of the HAB arrival and for all subsequent times, $t \geq t^{HAB}$:

$$\Gamma_t = \begin{cases} 0 & \text{before a HAB arrival,} \\ 1 & \text{during and after the HAB arrival,} \end{cases}$$

$$\text{and } \Gamma_0 = 0.$$

As described in Section 2.2, the effect of the HAB on the harvesting profit is that, if $\Gamma_t = 1$, the salmon dies and cannot be sold, giving zero revenues.

In other words, the value of the fish stock immediately goes to zero if the HAB arrives at the farm. The farmer now needs to take this risk into account when choosing the optimal harvesting time. If the HAB risk is endured, i.e., $\Gamma_T = 0$ indicating the HAB did not arrive, the farmer may continue to grow the fish and harvest at the optimal weight and price. In such a case, the optimal harvesting time is given by the GSR-model in Section 4.1.

As previously mentioned, the true risk of receiving a HAB is difficult to predict. However, during the HAB threat from $t = 0$ to $t = T$, there is a steady flow of information arriving to the salmon farmer about the algal spread. We assume signals are coming from word of mouth from neighboring farms and from research organizations. Based on these signals farmers can form beliefs about the true algal arrival rate.

To account for this flow of information we adapt a signal process similar to that of Thijssen et al. (2004). We introduce two states describing the risk of getting the HAB, the high risk state H, and the low risk state L. The world can only be one of these states, i.e., it is either high or low during the whole HAB. The states have corresponding Poisson arrival rates which are known to the farmer, λ_H and λ_L. The risk of HAB arrival in H is higher than in L, i.e., $\lambda_H > \lambda_L \geq 0$. The farmer does not know which risk state the farm is in when the HAB is first reported at $t = 0$. However, the salmon farmer has a prior belief that the true state is H: $P(H) = p_0$.

The signals are arriving at irregular intervals which indicate the true state of the risk. The signals are either good, denoted l and signalling that the true state is L, or bad, denoted h, signalling that the true state is H. We denote the cumulative sums of good and bad signals that have arrived up until time t as l_t and h_t, respectively. Moreover, the signals are known to be imperfect, and the farmer considers the probability of a signal being correct to be $P_{cs} > 0.5$, see Table 4.1.

Table 4.1: Probability of a signal indicating high or low HAB risk, given the true state of the world.

Risk/signal	h	l
H	P_{cs}	$1 - P_{cs}$
L	$1 - P_{cs}$	P_{cs}

The signals arrive according to a Poisson birth process with intensity $\mu > 0$. This is a realistic representation of how signals arrive, since research reports and news arrive at unknown intervals. Whenever a signal arrives, the farmer updates his belief of what the true state is via Bayesian updating. We introduce k_t as the amount of bad signals in excess of good signals that has arrived from time 0 to time t, i.e., $k_t = h_t - l_t$. By following Bayes' rule, the belief that the world is in the high risk state H, can be formulated as a function of k_t:

$$p(k_t) = \frac{P_{cs}^{k_t}}{P_{cs}^{k_t} + \frac{1-p_0}{p_0}(1 - P_{cs})^{k_t}}. \tag{4.10}$$

We refer to Appendix B for derivation of Eq. (4.10).

At time $t = T$, the HAB risk becomes zero and the signals stop arriving. In this case, the optimal harvesting problem is reduced to the problem under consideration in the GSR-model in Section 4.1. For $t < T$, i.e., during the algal bloom, salmon farmers receive a flow of information about the algal spread. Based on these signals, farmers are actively updating their beliefs of the algal arrival rate in accordance with Bayes' rule. In addition to actively learning from signals arriving, farmers are passively learning about the salmon price. Thus, farmers have an incentive to wait for more information in order to make a better-informed decision. However, the benefits of waiting for more information about the algal arrival rate and possibly a higher price, must be weighted against the risk of losing the current biomass altogether due to algae arriving. This is a realistic representation of the dilemma faced by salmon farmers during a real HAB. Hence, the farmer may decide to harvest the fish early in order to secure his profits, given the HAB has not yet arrived. In what follows, we will present a modeling approach that takes into consideration the HAB arrival risk and the information flow available to salmon farmers.

4.4 The Early Harvest Model

This section presents the method to quantify the value of flexibility of harvesting while facing the risk of HAB arrival. As a harvests during HABs often are referred to as early harvests, we name the model the Early Harvest model (EH-model). The solution technique for the EH-model is similar to the GSR-model, but with appropriate changes coming from the introduction of algal risk and its related signals.

With the model extensions described in Section 4.3, the optimal harvesting problem is now quite different than in the GSR case. While facing algal risk, the farmer must decide whether (and when) to secure biomass revenues by harvesting early, or risk losing the value altogether while waiting for more signals. Hence, the GSR-model must be extended to account for the HAB. We let the biomass weight function follow Eq. (4.1) as in the GSR-model, with t_{sea}, R_0 and M given as inputs to the model. The production cost, harvesting cost and harvesting profit are also the same, but with the addition that they are reduced to zero should the HAB arrive. The optimal stopping problem the farmer faces can now be formulated as

$$\sup_{\tau} \mathbb{E}\left[\Big(B(\tau)\Big(S_\tau - C_H\Big)e^{-r\tau} - \int_0^\tau C_p(t)e^{-rt}dt\Big) \times \Big(1 - \Gamma_\tau\Big)\right], \tag{4.11}$$

where τ is the optimal harvesting time. This optimal stopping problem is similar to Eq. (4.2), with the cash flows received and production costs paid at the time of optimal harvest, discounted from the optimal harvest time τ to time zero. The production costs paid until the optimal time of harvest are also discounted to time zero. Additionally, the last parenthesis ensures that if the HAB has arrived, the value is zero.

If the farmer endures the HAB-event without the HAB arriving at his location, i.e., $\Gamma_\tau = 0$ and $\tau \geq T$, he may continue to grow the fish and harvest at the optimal weight and price. In such a case, Eq. (4.11) is reduced to Eq. (4.2).

4.4.1 Solution Approach for the Early Harvest Model

Similar to the procedure in Section 4.2, we split the problem into K sub-problems, but now from the first report of the algal bloom at $t = 0$ to the maximum duration of the algal bloom $t = T$. See Fig. 4.2 below.

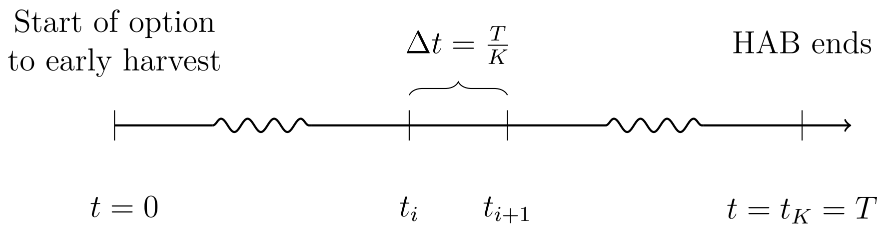


Figure 4.2: Illustration of time discretization in the EH-model. If the HAB is endured, the farmer may continue to grow the fish and harvest at the optimal weight and price.

At every time point t_i, we simulate N realizations of the two-factor price process. Furthermore, given the prior belief of being in state H, p_0, we simulate $N \times p_0$ realizations of the algal process and signals, assuming the true state being H. In the same way, we simulate another $N \times (1 - p_0)$ realizations, but now assuming the true state being L. Thus at every time step, we will have N realizations of the price, algal and signal processes.

In order to include HAB risk and signals in our EH-model, we introduce $\widetilde{X}_{t_i}(\omega)$ as the state vector of realization ω at time t_i. We let $\widetilde{X}_{t_i}(\omega)$ contain the simulated values of the price process, S_{t_i}, χ_{t_i} and ξ_{t_i}, together with the binary variable for HAB presence, Γ_{t_i}, and the current state of signals, k_{t_i}. We note that $X_{t_i}(\omega)$ from Section 4.2 is a subset of $\widetilde{X}_{t_i}(\omega)$ introduced here.

In order to find the optimal strategy for the EH-model, we start by evaluating the option at the very end of the HAB. At t_K, the option to harvest early is reduced to the GSR problem. Thus, we need to solve the GSR-model for all realizations where the algae did not arrive, i.e., where $\Gamma_{t_K}(\omega) = 0$. Denoting the resulting values $F_{GSR}\Big(t_K, X_{t_k}(\omega)\Big)$, we solve for the initial state being $X_{t_K}(\omega) = \Big\{S_{t_K}(\omega), \chi_{t_K}(\omega), \xi_{t_K}(\omega)\Big\}$ and for initial values $t_{sea}^{GSR} = t_{sea} + T$ and $R_0^{GSR} = R(T)$.

Having calculated $F_{GSR}\Big(t_K, X_{t_k}(\omega)\Big)$, we can now derive the option value at the time of expiry:

$$F\Big(t_K, X_{t_K}(\omega)\Big) = \max\Big\{0, F_{GSR}\Big(t_K, X_{t_k}(\omega)\Big)\Big(1 - \Gamma_{t_K}(\omega)\Big)\Big\}.$$

Here, $F\Big(t_K, X_{t_K}(\omega)\Big)$ represents the value of having the option to harvest early at the time of expiry in realization ω, in the EH-model.

Next, the optimal stopping time is initialized to $\tau(\omega) = t_K$. Then, we start working our way backwards using a similar LSM procedure as in Section 4.2. At every time t_i, the value of harvesting is zero if the HAB has arrived. Otherwise, it is

$$\Pi_{EH}\Big(t_i, \widetilde{X}_{t_i}(\omega)\Big) = B(t_i)\Big(S_{t_i} - C_H\Big) \times \Big(1 - \Gamma_{t_i}\Big).$$

Furthermore, the value of harvesting is compared to the value of continuing. The LSM procedure approximates the expected continuation value using a least squares regression. This is done similarly as described in Section 4.2, except for the addition of the signal state k_t to the basis function. In other words, the expected continuation value is now also dependent on the information given by the signals. For the regression, we define the new basis function below,

$$F(X) = \beta_0 + \beta_1 S_t + \beta_2 S_t^2 + \beta_3 \xi_t + \beta_4 S_t \xi_t + \beta_5 \xi_t^2 + \beta_6 k_t + \beta_7 k_t^2 + \beta_8 k_t S_t + \beta_9 k_t S_t \xi_t.$$

The value of continuing, $\Phi\Big(t_i, \widetilde{X}_{t_i}(\omega)\Big)$, is the discounted expected value of the option at the next time step, less the cost of producing in the period between the two time steps:

$$\Phi\Big(t_i, \widetilde{X}_{t_i}(\omega)\Big) = \Big[\mathbb{E}_{t_i}\Big[e^{-r\Delta t} F\Big(t_{i+1}, \widetilde{X}_{t_{i+1}}(\omega)\Big)\Big] - C_p(t_i) \times \Delta t\Big] \times \Big(1 - \Gamma_{t_i}\Big).$$

We decide on the optimal strategy by comparing the value of harvesting and the estimated continuation value. The farmer always chooses the alternative with the highest value,

$$F\Big(t_i, \widetilde{X}_{t_i}(\omega)\Big) = \max\Big\{\Pi\Big(t_i, \widetilde{X}_{t_i}(\omega)\Big), \Phi\Big(t_i, \widetilde{X}_{t_i}(\omega)\Big)\Big\},$$

and the optimal stopping time τ for every realization ω is updated if it was optimal to exercise the option to harvest early.

Finally, when the LSM procedure has been performed for every time step, we have an optimal exercise time for every realization. The value of the option at time t_0 is again the average of the discounted realized cash flows received, less the production costs paid, in every realization when following the optimal strategy found by the algorithm.

4.5 Extension to the Early Harvest Model

In this section, we extend the EH-model above by including the option to move the fish. We do so in order to emphasize that in some cases, farmers may have the opportunity to move their fish to another location without HAB threat, as an alternative to early harvesting. We denote the extended model the EH-M-model. The option to move is typically restricted to only the larger farmers with several farming locations. For these farmers, both options need to be taken into consideration in an optimal harvesting strategy. First, we describe the characteristics of the option to move and formulate its optimal stopping problem. Next, we present the solution approach for the extended model. Solving for the option to move follows the exact same approach as for the option to harvest early, but for a different payoff function. To take into account the interaction between the two options, we follow the procedure outlined in the Gamba (2003) framework of mutually exclusive options.

As an alternative to early harvesting, some farmers will have the opportunity to move their fish to another location without algal risk. This allows the farmer to harvest at the optimal time at the new location, similar to the approach in the GSR-model. However, as discussed in Section 2.2, the move is stressful for the salmon resulting in a higher mortality rate and a higher FCR. The mortality rate at the new location is denoted M_M, and the FCR is denoted f_{rM}. Furthermore, the move comes at a direct cost of renting a wellboat for fish transportation. We denote the total moving costs per kilogram fish by C_M.

As a result, the option value of moving is the value of harvesting optimally at the new location (without HAB risk, but with the higher mortality and FCR), less the moving cost. This option value is compared to the value of waiting and receiving more information about the algal risk. The optimal stopping problem the farmer now faces can be formulated as

$$\begin{aligned}\sup_{\tau_1} \mathbb{E}\Bigg[\Bigg(\bigg(\sup_{\tau_2} \mathbb{E}\Big[B(\tau_2)\Big(S_{\tau_2} - C_H\Big)e^{-r(\tau_2-\tau_1)} - \int_{\tau_1}^{\tau_2} C_p(t)e^{-r(t-\tau_1)}dt\Big] - C_M B(\tau_1)\bigg)e^{-r\tau_1} \\ - \int_0^{\tau_1} C_p(t)e^{-rt}dt\Bigg) \times \Big(1 - \Gamma_{\tau_1}\Big)\Bigg], \quad \tau_1 < \tau_2,\end{aligned} \tag{4.12}$$

where τ_1 is the optimal moving time and τ_2 is the optimal harvesting time. Finding τ_2 is similar to solving the optimal stopping problem in Eq. (4.2), starting from τ_1. The innermost supremum of Eq. (4.12) represents the value of producing at the new location and harvesting optimally at time τ_2, discounted back to the time of moving, τ_1. Next, the cost of moving the biomass at time τ_1 is subtracted, before discounting the derived value back from the time of moving. Then, production costs from time zero to τ_1 are subtracted. Lastly, we take into account, Γ_{τ_1}, indicating whether the algal bloom reached the farm before moving.

4.5.1 Solution Approach for the Early Harvest-Move Model

Farmers with both the option to harvest early and to move the fish, are holding two mutually exclusive options. Either one of them will be exercised or none will. Effectively, the farmer can choose between the optimal stopping problem in Eq. (4.11) and the one in Eq. (4.12) at every time step. In order to study this interaction, we adapt the framework of Gamba (2003) for evaluating mutually exclusive options. We denote the value of the option to undertake either of these actions as G. The problem now includes not only whether to continue or stop, but also what stopping mechanism to choose. To accommodate this, we extend the optimal stopping time τ to also include the optimal choice at the optimal time, either EH for early harvesting or M for moving. The new τ is denoted τ_G.

As before, we find the value of early harvesting by solving

$$\Pi_{EH}\Big(t_i, \widetilde{X}_{t_i}(\omega)\Big) = B(t_i)\Big(S_{t_i} - C_H\Big) \times \Big(1 - \Gamma_{t_i}\Big).$$

The value of moving is the value of the option to harvest optimally at the new location, less moving costs:

$$\Pi_M\Big(t_i, \widetilde{X}_{t_i}(\omega)\Big) = \Big[F^M_{GSR}\Big(t_i, X_{t_i}(\omega)\Big) - C_M B(t_i)\Big] \times \Big(1 - \Gamma_{t_i}\Big).$$

Here, we have defined F^M_{GSR} to be the value we get by solving the ordinary F_{GSR} of Eq. (4.9), but with the new mortality rate M_M and the new feed conversion ratio f_{rM} that arise from moving the fish, as described above. Moreover, we calculate $F^M_{GSR}\Big(t_i, X_{t_i}(\omega)\Big)$ with the initial state being $X_{t_i}(\omega) = \{S_{t_i}(\omega), \chi_{t_i}(\omega), \xi_{t_i}(\omega)\} \subseteq \widetilde{X}_{t_i}(\omega)$, and initial values $t^{GSR}_{sea} = t_{sea} + t_i$ and $R^{GSR}_0 = R(t_i)$.

Furthermore, the value of continuing is the discounted expected value of the option at the next time step, less the cost of producing for another time step:

$$\Phi(t_i, \widetilde{X}_{t_i}) = \Big[\mathbb{E}_{t_i}[e^{-r\Delta t} G(t_{i+1}, \widetilde{X}_{t_{i+1}})] - C_p(t_i) \times \Delta t\Big] \times \Big(1 - \Gamma_{t_i}\Big).$$

The expected continuation value of the option is estimated by least squares regression as previously described. The optimal choice in each realization ω is given by the following Bellman Equation,

$$G\Big(t_i, \widetilde{X}_{t_i}(\omega)\Big) = \max\Big\{\Pi_{EH}\Big(t_i, \widetilde{X}_{t_i}(\omega)\Big), \Pi_M\Big(t_i, \widetilde{X}_{t_i}(\omega)\Big), \Phi\Big(t_i, \widetilde{X}_{t_i}(\omega)\Big)\Big\}.$$

The optimal stopping time is then updated according to this rule,

$$\tau_G(\omega) = \begin{cases} \text{Unchanged} & \text{if } \; G\Big(t_i, \widetilde{X}_{t_i}(\omega)\Big) = \Phi\Big(t_i, \widetilde{X}_{t_i}(\omega)\Big), \\ (t_i, EH) & \text{if } \; G\Big(t_i, \widetilde{X}_{t_i}(\omega)\Big) = \Pi_{EH}\Big(t_i, \widetilde{X}_{t_i}(\omega)\Big), \\ (t_i, M) & \text{if } \; G\Big(t_i, \widetilde{X}_{t_i}(\omega)\Big) = \Pi_M\Big(t_i, \widetilde{X}_{t_i}(\omega)\Big). \end{cases}$$

Finally, when this procedure has been performed for every time step, we have an optimal action and exercise time for every realization. The value of the option at time t_0 is the average of the discounted realized cash flows received, less the production costs paid, in every realization when following the optimal strategy found by the algorithm.

Chapter 5

Model Parametrization

In this chapter, we quantify and motivate the input values for the GSR- and HAB-models. We present two case studies, for Norway and Chile. This is of interest since the world's two largest producers of farmed salmon operate under different production conditions. We investigate these two cases in order to identify optimal strategies for salmon farmers from both parts of the world. Parameter estimations are based on relevant aquaculture studies from both regions, in addition to input from Chilean industry experts from our field trip to Puerto Montt[1]. The industry experts we have spoken to in Chile have several years of experience, some of whom with experience from both the Norwegian and Chilean salmon industry. Therefore, we consider our sources of information to be reliable. However, we keep in mind that this information is not necessarily representative for all salmon farmers, but it still provides reasonable assumptions. It should be noted that our assumptions for the Chilean case study are mostly based on insights from these industry experts, whereas the assumptions for the Norwegian case study are based on public available data and industry reports. Price process parameters are derived from historical spot and forward prices from Fish Pool. Thus, overall, we believe our quantifications reflect realistic levels for the two industries.

First, we discuss the input values for the GSR-model. These are related to discount rate, production costs, harvesting costs, mortality, weight development, and price estimation. Next, we discuss the variables that appear in the HAB-models. These are related to the signal and algal processes and to the consequences of moving the fish. Table 5.1 below summarizes the relevant input variables.

[1]The field trip to Puerto Montt was made possible by sufficient funding and we express our gratitude to Tekna and Legat til Henrik Homans minne for giving us this opportunity.

Parameter	Symbol	Norwegian case study	Chilean case study
GSR-model			
Discount rate	r	6%	8%
Feed conversion ratio	f_r	1.3	1.2
Price per kilogram feed	C_F	12 NOK/kg	12 NOK/kg
Harvesting cost per kilogram fish	C_H	3.9 NOK/kg	4.5 NOK/kg
Mortality rate	M	15%	13%
HAB-models			
HAB arrival intensity in state i	λ_i	0.07	0.07
Arrival rate of signals	μ	2	1.5
Probability of signal being correct	P_{cs}	0.75	0.60
Moving cost per kilogram fish	C_M	3.0 NOK/kg	2.8 NOK/kg
Feed conversion ratio after moving	f_{rM}	1.50	1.40
Mortality rate after moving	M_M	20%	18%

Table 5.1: Summary of relevant Norwegian and Chilean input variables for the GSR- and HAB-models.

5.1 General Single Rotation Model Parameters

Discount Rate (r)

The discount rate is set to 6% for the Norwegian case study and 8% for the Chilean. The discount rate displays the risk embedded in a project where an increase reflects a project with higher risk. The project we are considering in this thesis is the normal operation of growing salmon. The choice of discount rate is based on the assumption that the risk inherent in normal operations in aquaculture is relatively low. We also consider the applied discount rate by key players in the industry. For example, SalMar operates with a discount rate before tax of 8.3% (i.e., 6.5% after tax for a corporate tax rate of 22%) for all of their cash flow generating units and projects in the year of 2019 (SalMar ASA, 2019). Cermaq uses the WACC approach for determining discount rates. Their assumptions for the discount rate before tax for 2018 were 4.8% and 8.1% for Norway and Chile, respectively (Cermaq, 2018). Morten Nærland, the CFO of Cermaq Chile, confirmed this relationship during our discussions at their offices in Puerto Montt: Cermaq operates with a higher discount rate in Chile than in Norway.

Feed Conversion Ratio (f_r)

We use a FCR of 1.3 for the Norwegian case study and 1.2 for the Chilean case study. The numbers are based on the average FCR reported by the Directorate of Fisheries and Nofima during the last ten production years in Norway. The average FCR in Norway was 1.26 in 2018 according to the Norwegian Directorate of Fisheries, whereas Nofima reported a FCR of 1.29 (Directorate of Fisheries, 2019a; Iversen et al., 2019). Chile has historically had a higher and more variable FCR compared to Norway. However, the Chilean FCR has moved below Norwegian salmon farmers for the production year of 2018 (Iversen et al., 2019). Thus, we have chosen a lower parameter value for the Chilean case study.

Price per Kilogram Feed $\left(C_F\right)$

The feed price is set to 12 NOK/kg for both of our case studies. The choice is determined based on the current feed price level and the increasing price trend over time. Elevated competition between feed suppliers, MOWI's introduction of own feed factories, and focus on sustainable feed are factors explaining the continued growth in feed prices. The average feed price per kilogram for Norwegian salmon farmers rose from 10.90 NOK/kg to 11.26 NOK/kg from 2017 to 2018 (Directorate of Fisheries, 2019a). Estimations conducted by Iversen et al. (2019) show that prices for salmon producing countries have steadily increased since the early 2000s. In 2003, the prices ranged from 5 to 7.5 NOK/kg. For 2018, this range was between 11.5 and 13 NOK/kg. Feed price in Chile were lower for a longer period of time, but the price levels are now approaching the ones in Norway. Discussions with Pablo Ibarra, the General Manager of ATC Patagonia Research Center, support the fact that Chilean salmon farmers have given nutrition projects more attention in the last couple of years which can partly explain similar price levels.

Harvesting Cost per Kilogram Fish $\left(C_H\right)$

Harvesting cost is estimated to be 3.9 NOK/kg for Norway and 4.5 NOK/kg for Chile. These estimates are based on available cost studies undertaken by the Directorate of Fisheries (2019a). According to the Norwegian Directorate of Fisheries, the average cost of slaughter and transportation for Norwegian farmers was 3.79 NOK/kg in 2018. From 2011 to 2018, the average costs of slaughter and wellboats have made up about 10.44% of the total production costs for Norwegian salmon farmers[2] (Directorate of Fisheries, 2019a). This is further supported by economic analyses conducted by Kontali Analyse. They estimate that total production costs have seen a yearly average increase of 4.89% from 2011 to 2018, measured in 2018 real values (Kontali, 2020). This leads to the following estimation:

$$\begin{aligned} C_H^{\text{Norway}} &= \text{Total production costs 2020} \times \text{Avg. \% cost of slaughter and wellboat} \\ &= 37.27 \text{ NOK/kg} \times 10.44\% \approx 3.9 \text{ NOK/kg.} \end{aligned}$$

Furthermore, we assume 15% higher harvesting costs in Chile because of lower degree of automation at processing plants and lower standard on infrastructure. This yields an estimated cost of 4.5 NOK/kg for Chile. These assumptions are in line with discussions with industry experts in Puerto Montt.

Mortality Rate $\left(M\right)$

For the Norwegian case study a mortality rate of 15% is used, and for the Chilean case study we assume a slightly lower rate of 13%. The mortality rate for Norwegian salmon farmers is determined by looking at fish mortality and losses in production. The statistics on wastage in production is gathered by the Directorate of Fisheries and is open to the

[2]See Fig. 2.1 in Section 2.1 for total production cost percentages per cost category.

public. From 2005 to 2019, the yearly loss percentages have varied between 15% to 20% of the total number of fish (BarentsWatch, 2020; Directorate of Fisheries, 2020b). According to Iversen et al. (2019), wastage in production per generation in Chile has varied more than for any other salmon producing country. However, in 2016 and 2017, the mortality rate in Chile was lower than in Norway. Because of this, we assume a slightly lower mortality rate for the Chilean case study.

Salmon Weight Function $\big(W(t)\big)$

We assume that the salmon weight function, $W(t)$, follows the deterministic process described in Section 4.1. The von Bertalanffy's function is commonly applied in the literature, and for the Norwegian case study we follow Ewald et al. (2017) who find a, b, and c to be 1.113, 1.097, and 1.430 respectively. We set the asymptotic average weight, w_∞, to be 5.5 kg. With these values, the salmon will reach a weight of approximately six kilogram after two years in the sea, which is a realistic representation of the growth (The Norwegian Seafood Federation, 2020). This yields the following Norwegian weight function,

$$W_t^{\text{Norway}} = w_\infty\left(a - be^{-c\left(\frac{t+t_{\text{sea}}}{365}\right)}\right)^3 = 5.5\left(1.113 - 1.097e^{-1.430\left(\frac{t+t_{\text{sea}}}{365}\right)}\right)^3,$$

where t_{sea} is the time since the fish was introduced to the sea pen.

As discussed in Section 2.1, the salmon grows slightly faster in Chile due to higher water temperatures. In order to model a steeper weight curve, we adjust weight function constant b to $b = 1.000$ and get the following Chilean weight function,

$$W_t^{\text{Chile}} = 5.5\left(1.113 - e^{-1.430\left(\frac{t+t_{\text{sea}}}{365}\right)}\right)^3.$$

Time Since Start of Sea Phase $\big(t_{sea}\big)$

As described in Section 2.2, HABs can develop at any time during the sea phase. In order to study how the optimal strategy is affected by the fish weight at the start of an algal outbreak, we introduce the auxiliary parameter t_{sea}, the time since the start of the sea phase. This lets us estimate the current biomass at the start of a HAB. We let t_{sea} be either 200, 400 or 600 days, representing the early, middle and late stage of the sea phase, respectively.

Number of Fish Recruits $\big(R_0\big)$

We assume that the farmer wants to grow the biomass to the limit of what is allowed. In Norway, the maximum allowed biomass (MAB) is 780 tonnes per licence. As such, we choose the number of recruits so that the total biomass reaches the MAB at the end of the sea phase (assumed to be 24 months). Given our weight function, the MAB is reached at the end of the sea phase by introducing 165 000 salmon recruits at the start of the sea phase. Table 5.2 below shows the number of fish in the pen for different values of t_{sea}, including corresponding individual fish weight and total biomass. These estimates will be used in both case studies to investigate optimal policy and other relationships.

		Norwegian case study		Chilean case study	
t_{sea}	R_0	**Weight**	**Total biomass**	**Weight**	**Total biomass**
200 days	151 981	1.27 kg	193 372 kg	1.55 kg	236 215 kg
400 days	139 989	3.81 kg	533 707 kg	4.07 kg	569 474 kg
600 days	128 943	5.65 kg	728 205 kg	5.80 kg	747 506 kg

Table 5.2: Number of fish in the pen, R_0, for different values of t_{sea}, including corresponding estimates for individual fish weight $W(0)$ and total biomass $B(0)$. Estimates are presented for both case studies.

Maximum Length of Sea Phase $\left(T^{sp}\right)$

We set the maximum length of a single rotation sea phase to be two years, or 730 days, in both case studies. This is mainly to account for the fact that no farmers operate on a single rotation basis. Keeping a single generation of fish for longer than two years is not viable economically, as harvesting and putting out a new generation is better when considering more than a single rotation case. This means that with $t_{sea} = 200, 400$ and 600, the expiration of the option to harvest optimally are $T^{sp} = 530, 330$ and 130, respectively.

Parameter Estimation for the Two-Factor Salmon Spot Price Model

In the estimation of the two-factor model, we make use of historical salmon spot prices and forward contracts to estimate the model's unknown parameters (κ, σ_χ, μ_ξ, σ_ξ, $\rho_{\chi\xi}$, $\chi_{t=0}$ and $\xi_{t=0}$). Our data consists of weekly observations of spot prices and forwards from Fish Pool, spanning week 14, 2013 to week 18, 2020. The synthetic FPI spot prices are updated weekly. For the forwards, we let the closing price of the last trading day of a week represent the week's closing price. We include forwards with maturities ranging from one month up to a year, and with 18 months, two, three, four and five years to maturity. This gives a total of 368 weekly price observations of spot prices and 17 different forward contracts as input for our parameter and state variables estimation.

We identified the set of parameters that maximizes the log-likelihood function in Appendix A by rerunning the Kalman filter for different initial parameter values. Table 5.3 presents the maximum-likelihood parameter estimates.

Table 5.3: Maximum-Likelihood Parameter Estimates for Two-Factor Price Model.

Parameter	Description	Estimate	Std. dev.
κ	Short-term mean-reversion rate	2.7	0.18
σ_χ	Short-term volatility	0.49	0.10
μ_ξ	Equilibrium drift rate	0.07	0
σ_ξ	Equilibrium volatility	0.07	0.04
$\rho_{\chi\xi}$	Correlation in short-term and equilibrium process increments	-0.37	0.06
$\chi_{t=0}$	Log short-term deviation at time $t = 0$	-0.26 NOK/kg	-
$\xi_{t=0}$	Log equilibrium price level at time $t = 0$	4.18 NOK/kg	-

The parameter estimates in Table 5.3 serve as input into our price simulations, described in Section 4.2. For simplicity, we assume the price dynamics are the same for both Norway and Chile. Hence, we use the same price simulations in both case studies. At the start time of our option valuation, i.e. $t = 0$, the short-term (log) price deviation $\chi_{t=0}$ is estimated being -0.63, whereas the equilibrium (log) price level $\xi_{t=0}$ is estimated being 4.18. In other words, the two-factor model estimates an equilibrium price of 65.51 NOK/kg, with an estimated spot price of 34.84 NOK/kg for week 18, 2020. However, the observed spot price was in fact 50.53 NOK/kg (Fish Pool ASA, 2020c). In order to start our price simulations from a realistic starting point, we adjust the short-term deviation accordingly, giving instead $\chi_{t=0} = -0.26$.

The market is said to be in contango when spot prices lie below expected forward prices. In light of HAB events, a salmon price correction is typical in the short-run, but for the long-run the prices are usually expected to go up[3]. Thus, we find it reasonable to employ these parameter estimations for the price simulations in the option valuation.

5.2 Harmful Algal Bloom Models Parameters

HAB Arrival Intensity in State i (λ_i)

In the low risk state, we set the algal risk to be zero during a HAB: $\lambda_L = 0$. In the high risk state, we set it to $\lambda_H = 0.07$. This makes the expected arrival time $\frac{1}{0.07} = 14.3$ days, and the probability of arrival in one day is $1 - e^{-0.07} = 6.7\%$. The probability of getting an arrival of HAB within the maximum length of 21 days is $1 - e^{-0.07 \times 21} = 77\%$. The arrival rates of HABs in both states are assumed to be equal in both case studies. Professor Godoy at St. Sebastian University mentioned during our meeting that small HAB outbreaks are detected in Chile every year, and that they are unable to predict when or where it happens. Due to the large uncertainty in this parameter we examine the effects it has on decision making during HABs in Chapter 6.

Arrival Rate of Signals (μ)

The arrival intensity of signals is set to 2 for the Norwegian case study and 1.5 for the Chilean. This means that Norwegians expect to receive 2 signals per day, and Chileans expect 1.5 signals. See Table 5.4 for the probability of receiving a given amount of signals in one day.

Table 5.4: Probabilities of receiving different amount of signals in one day for the two case studies.

Case	μ	0 signals	1 signal	2 signals	3 signals	4 or more signals
Norway	2.0	13.5%	27.1%	27.1%	18.0%	14.3%
Chile	1.5	23.2%	33.5%	25.1%	12.6%	5.6%

From a meeting with Trine Dale, a HAB expert at the Norwegian Institute for Water

[3] https://e24.no/boers-og-finans/i/9v8Jld/lakseprisfall-etter-algeslakt-fortsetter-det-er-kaos

Research, we learned that farmers received daily signals from various research organizations and the Directorate of Fisheries (2020a) during the 2019 HAB event. These signals included results from water samples and algal spread simulations. Additionally, most farmers also performed their own sampling and communicated with neighboring farmers about their view of the situation (Karlsen et al., 2019). For the Chilean case study, however, we assume a slightly lower arrival rate of signals. During our field trip to Puerto Montt several industry experts claimed it was less collaboration among Chilean salmon farmers, and between governmental organizations and farmers, compared to Norway.

Probability of Correct Signals $\left(P_{cs}\right)$

The probability of a signal being correct is set to 75% for Norway and 60% for Chile. Trine Dale emphasized that the results from the water samples and simulations are subject to relatively high uncertainty. It was made clear to the farmers that they should not interpret the signals as the the truth, but rather as a guideline. Furthermore, Nofima finds in its report on the 2019 algal bloom that water samples were conducted and treated differently (Karlsen et al., 2019). This may also adversely affect the tests' validity. For the Chilean case study, we once again assume that the collaboration among key industry actors is less, compared to Norway. Thus, the probability of correct signals in Chile is assumed to be lower. The information received and the accuracy of the information is of course of great importance for a farmer wanting to choose the optimal action. Therefore, we will examine the effects of signal reliability and arrival rate in Chapter 6.

Moving Cost per Kilogram Fish $\left(C_M\right)$

For the Norwegian case study, we assume total moving costs to be 3 NOK/kg. This estimation is based on assumptions for costs related to the use of wellboats. For the Chilean case study, we use a slightly lower moving cost of 2.8 NOK/kg due to cheaper labor costs. We assume that moving the biomass to an alternative location is conducted through the use of wellboats. Liu et al. (2016) estimate in their study on economic performance, that harvesting costs (including the use of wellboat) make up about 12% of the total production costs for traditional open net pen salmon farming. This is similar to our estimates above. Furthermore, they estimate wellboat costs alone to be about 4% of the total production costs. There is, however, little data available on costs related to urgent moving of fish during a HAB. We assume that the cost of an urgent move during a HAB is twice the cost of a planned move operation. As a result, we get the following estimations by using our estimate for the total production costs for the year of 2020:

$$\text{Estimated moving cost in a planned operation} = 37.27 \text{ NOK/kg} \times 4\% = 1.49 \text{ NOK/kg.}$$

$$\text{Unplanned moving cost during algal outbreaks} = 1.49 \text{ NOK/kg} \times 2 \approx 3.0 \text{ NOK/kg.}$$

Feed Conversion Ratio After Moving $\left(f_{rM}\right)$

The FCR is assumed to increase approximately by 15% after moving the biomass to another facility. Therefore, we set the value of FCR after moving, f_{rM}, to 1.5 and 1.4 for Norway and Chile, respectively. The rationale behind this is that higher fish densities during transportation, loading, and unloading is a source of increased stress levels which can cause higher FCRs. This is supported by Basrur et al. (2009), who conducted a growth study for two groups of Atlantic salmon where one was subject to a weekly crowding stressor and the other was under controlled conditions. Higher stress and a 38% increase in FCR was observed for the group with higher density. Similarly, Calabrese et al. (2017) found that increasing stocking density for post-smolt had a negative effect on FCR.

Mortality Rate After Moving $\left(M_M\right)$

The mortality rate after moving is set to 20% and 18% for Norway and Chile, respectively. We assume that the mortality rate after transportation is increased by 5 percentage points, due to potential disease spread as a result of higher densities and stress levels during the move. This adverse effect of moving is supported by Calabrese et al. (2017), who find that stocking densities above $100\,\mathrm{kg/m^3}$ give notable negative effects on external welfare and fin damage[4].

Maximum Duration of Algal Bloom $\left(T\right)$

We set the maximum duration of the algal bloom to 21 days in our case studies. This is the same duration as the 2019 Norwegian algal bloom (Karlsen et al., 2019).

[4]The maximum density in sea pens in Norway is $25\,\mathrm{kg/m^3}$. We assume a density of $150\,\mathrm{kg/m^3}$ during wellboat transportation (Calabrese et al., 2017).

Chapter 6

Results and Discussion

In this chapter, we present and discuss results from the Norwegian and Chilean case studies of the GSR-, EH- and EH-M-models. We use the parameter values specified in Table 5.1 as base case. In addition, we investigate the effects of changing selected parameters. The objective of this chapter is to identify optimal choices for salmon farmers in Norway and Chile under the various models. Optimal harvesting strategies are investigated for three different stages of the sea phase. In addition, we quantify the added value of flexibility under the GSR-, EH-, and EH-M-models and discuss measures that policy-makers may use to facilitate better decision making in presence of HAB risk.

First, we present results from the GSR-model. We present the optimal harvesting time and investigate if the option to harvest optimally uncovers excess value compared to planned harvest at the end of the production cycle. Additionally, we study exercise boundaries for the case studies.

Second, we study results from the EH-model. Unlike in the GSR-model, where the decision concerns when to harvest optimally during the whole cycle, the EH-model rather focuses on when and if to harvest early during a time-limited HAB. We research the optimal harvesting strategy and exercise boundaries for the days during the HAB outbreak. The results of the EH-model are especially important to small-scale farmers that do not have the resources to move salmon to another location. We examine what effects selected parameters have on both the optimal harvesting strategy and the value of flexibility. Implications for policy-makers in the aquaculture industry are discussed for facilitating an early harvest strategy during HAB outbreaks and for improving signal frequency and quality.

Finally, the results of the extended EH-M-model are discussed. The extension includes the possibility for salmon farmers to move their biomass to an alternative location for further growth before harvesting optimally without HAB threat. Spatial diversification is needed to carry out this strategy, so the model is mostly relevant to larger enterprises or those with other available locations. We present the optimal choices during HAB outbreaks and quantify the added value of flexibility. We also investigate a special case for increased moving cost to see how the optimal choice between the two respective mitigation strategies changes and how it affects the value of the options.

6.1 General Single Rotation Model Results

Table 6.1 presents the GSR-model's option values based on 200 000 price simulations and with a weekly harvesting decision ($\Delta t = 7$). Results are presented for different stages of the production cycle, listed under t_{sea}, representing early, middle, and late stages of the sea phase. We do this for both the Norwegian and the Chilean parameter sets. Planned harvest shows the harvesting value when it must be performed on the last day of the production cycle. The last two columns quantify the additional value of managerial flexibility in the harvesting timing, in comparison to planned harvest.

Table 6.1: GSR-model results for the Norwegian and Chilean case studies. There is a decreasing, positive value of managerial flexibility during the entire sea phase.

t_{sea}	Case	Planned Harvest	GSR-Model	Value of Flexibility	Percentage increase
200 days	Norway	38.70 MNOK	41.27 MNOK	2.57 MNOK	6.64 %
400 days	Norway	42.96 MNOK	44.73 MNOK	1.77 MNOK	4.18 %
600 days	Norway	42.38 MNOK	42.65 MNOK	0.26 MNOK	0.61 %
200 days	Chile	40.25 MNOK	43.15 MNOK	2.90 MNOK	7.20 %
400 days	Chile	43.97 MNOK	45.92 MNOK	1.95 MNOK	4.44 %
600 days	Chile	42.81 MNOK	43.07 MNOK	0.26 MNOK	0.61 %

It is evident from Table 6.1 that there is significant value in having the flexibility to optimally time the harvesting decision during the production cycle. This holds for all three stages of the sea phase in both of the case studies. We see that the value of flexibility is decreasing towards the end of the production cycle. This is reasonable as there is less time to take advantage of the flexibility. As the price will have less time to evolve from its initial level, it is less likely to reach the high levels required for harvesting to be optimal before the expiration, thus reducing the value of flexibility.

Figure 6.1 below shows the distribution of optimal harvesting time for the Norwegian and Chilean case studies. We see that the majority of the harvests still occur towards the end of the production cycle. We observe that salmon farmers in Chile generally will use the option to harvest earlier than planned more often than Norwegian farmers. This explains the slightly higher values of flexibility for the Chilean case study observed in Table 6.1.

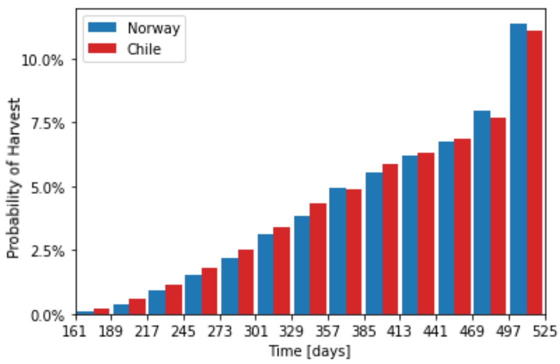


Figure 6.1: Probability of optimal harvest time in the GSR-model for Norwegian and Chilean case studies with $t_{sea} = 200$. Important to be noted is that the very last bin is left out. On the day of expiry, the probability is 45% for Norway and 43.5% for Chile. This last bin is thus several times larger than the others and is left out for visualization purposes.

The Norwegian and Chilean exercise boundaries are represented in Fig. 6.2 by blue and red curves, respectively. The boundaries separate the two regions where one should harvest (on and above the lines) and where the farmers should continue farming (below the lines). Differences between these boundaries come from the fact that the salmon grows faster earlier in the sea phase in Chile compared to Norway, which makes the value of harvesting higher. At the same time, the value of continuing growing the biomass will be lower in the later stages.

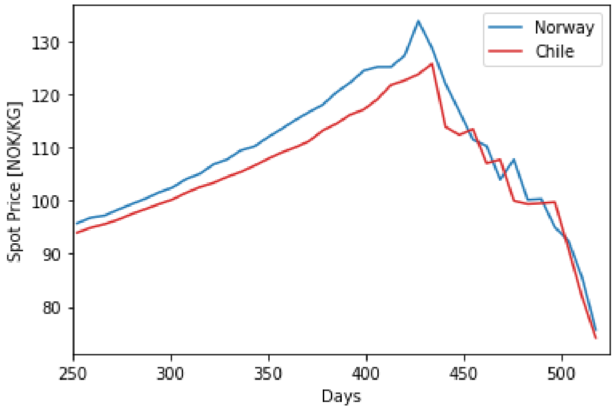


Figure 6.2: Exercise boundaries over time in the GSR-model for Norwegian and Chilean cases with $t_{sea} = 200$ and with fixed equilibrium price at the starting level (65.36 NOK/kg). The lines separate where the farmer should continue farming (below) and where the farmer should harvest (on and above). For visualization purposes the plot does not include the day of expiry, as harvesting will be exercised for any spot price.

The peaks of the lines coincide with peaks of the biomass growth. Shortly thereafter, the exercise boundaries fall quickly, and on the day of expiry (day 530) the salmon is harvested for any spot price. This explains why the majority of harvesting occur towards the end of the production cycle. This result indicates that under normal production conditions without algal risk, harvesting is mostly optimal very late in the cycle, except if the price is very high and expected to drop.

6.2 Early Harvest Model Results

Results from the EH-model are presented in Table 6.2[1]. Immediate Harvest represents the value when the salmon farmer decides to harvest immediately after the HAB occurs (i.e., on the first day of the HAB outbreak). No Early Harvest represents the value if the salmon farmer has no possibility to harvest during the HAB. Alternatively, this can be interpreted as the value if the farmer ignores the signals and accepts the risk of losing the biomass to the HAB. The next column, EH-model, is the value of harvesting optimally during and after the HAB. Finally, the Value of Early Harvest is the excess value of having the option to harvest during a HAB compared to not having any managerial flexibility during the HAB, and is calculated by subtracting the value of No Early Harvest from

[1] The results are based on 15 000 simulations of price, signals and HAB. Furthermore, the results are obtained with $\Delta t = 1$, meaning that the farmer makes a daily decision during the HAB. The compound option values are found by 10 000 simulations each, and $\Delta t = 14$.

the value of the EH-model. We use the term option value and value of early harvest interchangeably throughout this chapter.

Table 6.2: EH-model results showing the values of harvesting immediately, doing no early harvests during the HAB, and following the strategy found by the EH-model. The value of flexibility is the added value by following the EH-model compared to No Early Harvest.

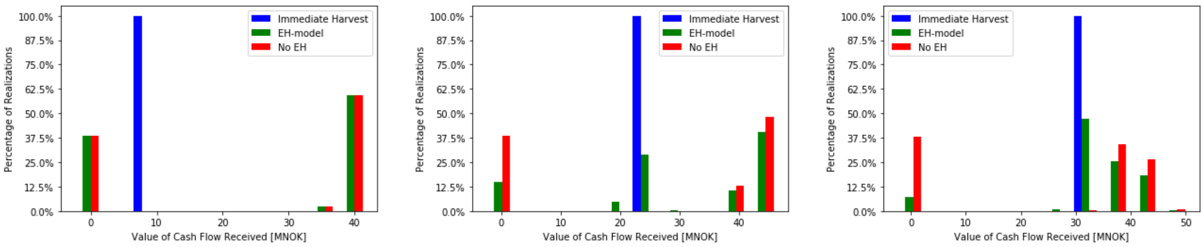
t_{sea}	Case	Immediate Harvest	No Early Harvest	EH-model	Value of Early Harvest
200 days	Norway	8.91 MNOK	24.99 MNOK	24.99 MNOK	0 MNOK
400 days	Norway	24.74 MNOK	27.33 MNOK	31.24 MNOK	3.91 MNOK
600 days	Norway	33.82 MNOK	25.90 MNOK	35.02 MNOK	9.12 MNOK
200 days	Chile	10.84 MNOK	26.23 MNOK	26.23 MNOK	0 MNOK
400 days	Chile	26.14 MNOK	28.15 MNOK	29.47 MNOK	1.32 MNOK
600 days	Chile	34.31 MNOK	26.22 MNOK	34.31 MNOK	8.09 MNOK

It is evident from Table 6.2 that the EH-model gives positive values of early harvest for $t_{sea} = 400$ and $t_{sea} = 600$ for both case studies. This tells us that there can be added value of performing early harvest in these stages of the sea phase. However, the value of early harvest is zero for $t_{sea} = 200$. This counter-intuitive result can be explained as follows. Early on in the sea phase, the biomass has the biggest potential for further growth. Furthermore, recall that in the High risk state the HAB arrival rate is $\lambda_H = 0.07$, which means that there is a 23% chance of surviving the HAB in the High state[2]. So, even if the farmer is perfectly certain that the true risk state is High at the beginning of the HAB, there is 23% chance of getting the value of harvesting optimally at a later stage, which amounts to a higher expected value than the value of immediate harvest[3]. This explains why one early on in the sea phase would be marginally better off by not early harvesting the salmon. Later, we will see that if we increase the probability of HABs arriving in the High state, i.e., increasing λ_H, early harvest does have value in the early sea phase as well.

Figures 6.3 and 6.4 below visualize the distribution of cash flows received from Immediate Harvest, No Early Harvest and under the EH-model for the Norwegian and Chilean case studies from Table 6.2. The blue, red and green bars represent the values of cash flows received from immediate harvesting, from ignoring the HAB risk and from following the EH-model, respectively. Note that on the y-axis, we plot the percentage of realizations that according to our simulations end up with that cash flow by following the given strategy. This percentage is equivalent to the probability of ending up in a given bin at the outset of the HAB-duration, and we use the terms probability and percentage of realizations interchangeably throughout this chapter.

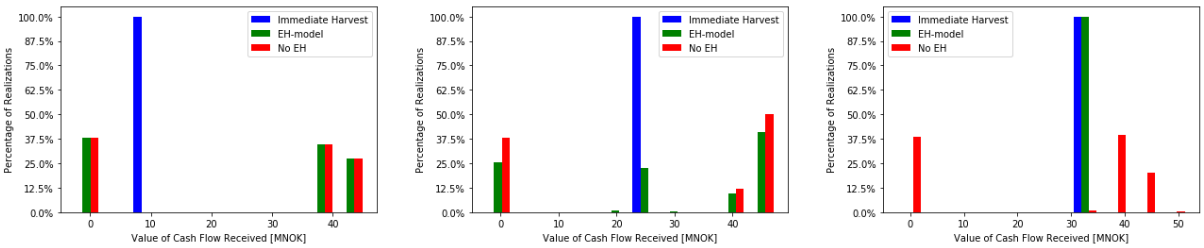
[2] $P(\text{No HAB arrival in 21 days}|\text{High state}) = e^{-0.07\times 21} = 0.23$.

[3] 0.23×42.27 MNOK $= 9.5$ MNOK > 8.91 MNOK.



(a) $t_{sea} = 200$ **(b)** $t_{sea} = 400$ **(c)** $t_{sea} = 600$

Figure 6.3: Histogram showing the distribution of Cash Flows Received with different strategies for the Norwegian case study.



(a) $t_{sea} = 200$ **(b)** $t_{sea} = 400$ **(c)** $t_{sea} = 600$

Figure 6.4: Histogram showing the distribution of Cash Flows Received with different strategies for the Chilean case study.

For $t_{sea} = 200$ in Figs. 6.3a (Norway) and 6.4a (Chile), we observe that the optimal choice found in the EH-model coincides with No Early Harvest. Consequently, we observe that the value of early harvest is 0 MNOK for $t_{sea} = 200$ in Table 6.2. For $t_{sea} = 400$ and 600, the probability of losing the biomass to the HAB when following the EH-model is reduced compared to doing No Early Harvest. The probability of getting the highest payoffs is also slightly reduced. However, the value of the cash flow received as a result of early harvesting outweighs the slight reduction in probability of getting the highest cash flows. Thus, for the middle and late stages of the production cycle, following the EH-model yields higher values than by ignoring the HAB risk. In Fig. 6.4c for Chile, we observe that the EH-model suggests harvesting immediately at the report of a HAB. This occurs as a result of lower signal arrival rate and less reliable signals, meaning that it is more valuable to harvest early in the late stage of the cycle, than to risk losing the biomass to HAB while waiting for more information.

Table 6.3 below shows the the probabilities of different outcomes under the EH-model. The table shows the probabilities of losing the biomass to HAB, performing an early harvest, and enduring the HAB (i.e., survive the HAB event and optimally harvest after the HAB period)[4]. These are prior probabilities at the time of the first report of HAB, i.e., at time t_0 in the EH-Model with no signals received.

[4]HABs are expected to arrive in 38.5% of the realizations in both case studies ($P(H) \times P(HAB|H) = 0.5 \times (1 - e^{-0.07\times 21}) = 38.5\%$).

Table 6.3: Probabilities of losing biomass to HAB, performing an early harvest, and enduring the HAB for the two case studies at different stages of the sea phase.

t_{sea}	Case	Lose to HAB	Early Harvest	Endure
200 days	Norway	38.5%	0%	61.5%
400 days	Norway	15.4%	34.0%	50.6%
600 days	Norway	7.9%	48.0%	44.1%
200 days	Chile	38.5%	0%	61.5%
400 days	Chile	24.5%	25.5%	50.0%
600 days	Chile	0%	100%	0%

Table 6.3 shows that by following the EH-Model, decision-makers can drastically reduce the probability of losing biomass to HABs, and that there is added value for risk management actions. For $t_{sea} = 200$, the results are equal for the two cases. For $t_{sea} = 400$, the probability of losing biomass to HABs for the Norwegian case study is lower than for the Chilean case study. The Norwegian salmon farmers also conduct early harvesting more often during the HAB outbreak for this sea phase stage. Given the baseline parameter sets for the two countries, Norwegian salmon farmers receive information more often and the accuracy of the signals is higher because of better collaboration in the aquaculture industry. Therefore, they are able to make better-informed decisions compared to Chilean farmers. It is evident from Table 6.3 that there is 0% chance of the losing biomass to HAB for the Chilean case study for $t_{sea} = 600$, as the optimal choice is to harvest immediately. This result is again related to the availability of reliable information where the Chilean salmon farmers are worse off. As they can expect to learn less from the signals and receive fewer signals, the value of waiting is lower than the value of harvesting immediately. Therefore, we conclude that the degree of information is an important factor when determining whether to wait or harvesting early.

Figure 6.5 indicates the distribution of optimal early harvests on a given day during the HAB duration of three weeks for both case studies when $t_{sea} = 400$.

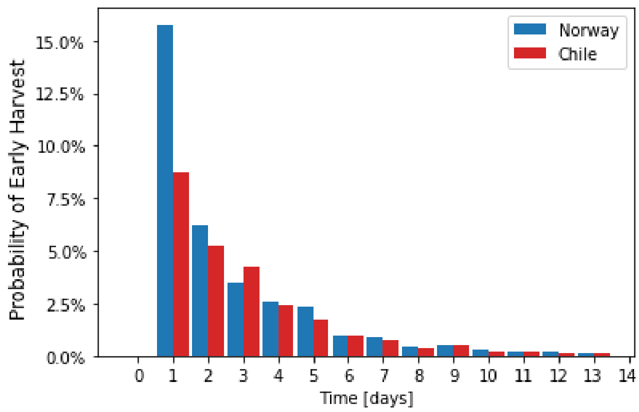


Figure 6.5: Probability of early harvests on a given day with $t_{sea} = 400$ for the Norwegian and Chilean case studies.

We see that Norwegian salmon farmers are more likely to perform an early harvest the first days after HAB detection compared to Chilean salmon farmers. As previously discussed, salmon farmers in Norway receive more signals which is a better foundation for decision making. Therefore, they have smaller probability of losing biomass to HAB and the option value is higher in this particular sea phase stage (see Table 6.3 and Table 6.2, respectively). Figure 6.5 shows that there is a decreasing probability of performing an early harvest as the number of days increases. Thus, our model suggests that most of the decisions for early harvesting should be conducted in the beginning of the HAB duration. This emphasizes that correct information from research organizations and collaboration within the industry play an important part for salmon farmers when deciding on the optimal early harvesting strategy.

Figure 6.6 illustrates exercise boundaries for the middle and late stage of the sea phase for the Norwegian case study. The exercise boundaries are plotted for a range of spot prices, equilibrium prices, and the signal state, k, which is the number of bad signals in excess of good received by the farmer at any given time. We drop the time subscript for ease of notation.

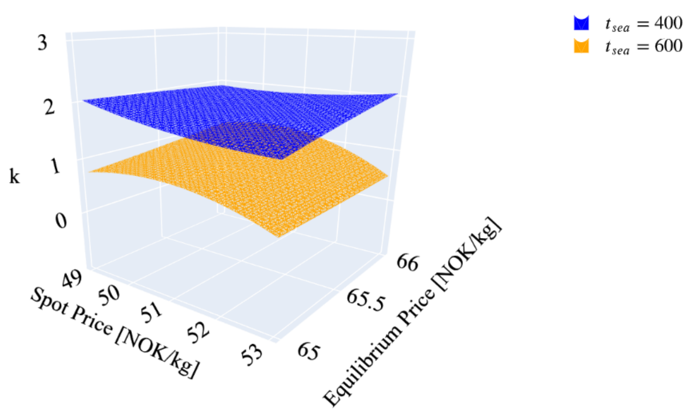
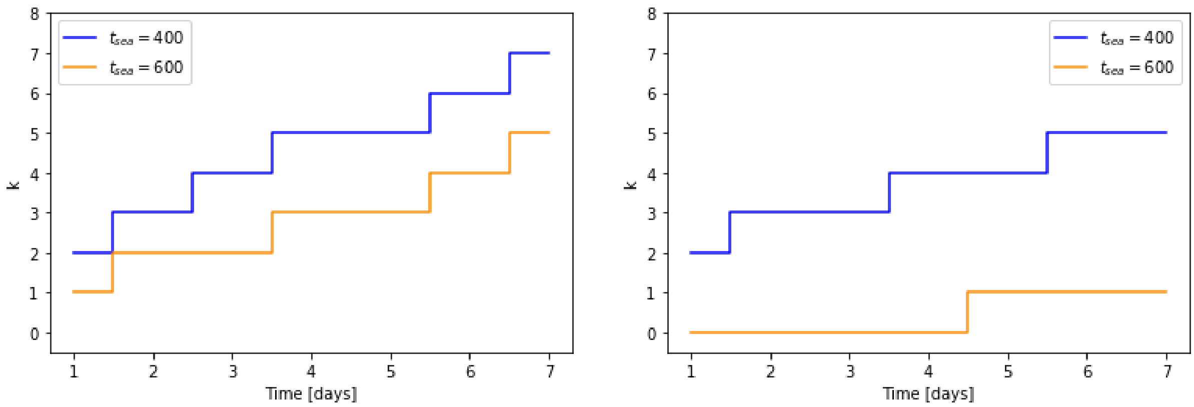


Figure 6.6: Separating planes for the first day of HAB for the Norwegian case study. The planes separate where the farmer should wait (below) and perform an early harvest (on and above). The planes are plotted in the range between the 10th and 90th percentiles for spot and equilibrium prices. Note that $t_{sea} = 200$ is not plotted, as the optimal strategy is to never perform an early harvest for any level of k.

Our model indicates that earlier in the sea phase, salmon farmers need to be more certain that they are in the High risk state when determining whether to harvest early or delaying the decision compared to later in the sea phase. This is reasonable as the salmon farmer has more to gain by growing the biomass further early in the sea phase.

In Figs. 6.7a (Norway) and 6.7b (Chile) we study how the exercise boundary evolves over time while keeping spot and equilibrium prices fixed. We fix the spot price to the last observed spot price from Fish Pool and the equilibrium price to the estimated equilibrium level from the two-factor price model.



(a) Norwegian Case Study

(b) Chilean Case Study

Figure 6.7: Exercise boundaries for the first week of the HAB for Norwegian and Chilean case studies. The boundary is plotted with respect to k (bad signals in excess of good signals received), with the spot price (50.40 NOK/kg) and equilibrium price (65.36 NOK/kg) fixed to the initial values. The farmer should perform an early harvest if observed k at a given day is on and above the line. If observed k is below, the farmer should continue and wait for more signals. Note that $t_{sea} = 200$ is not plotted, as the optimal strategy is to never perform an early harvest for any level of k.

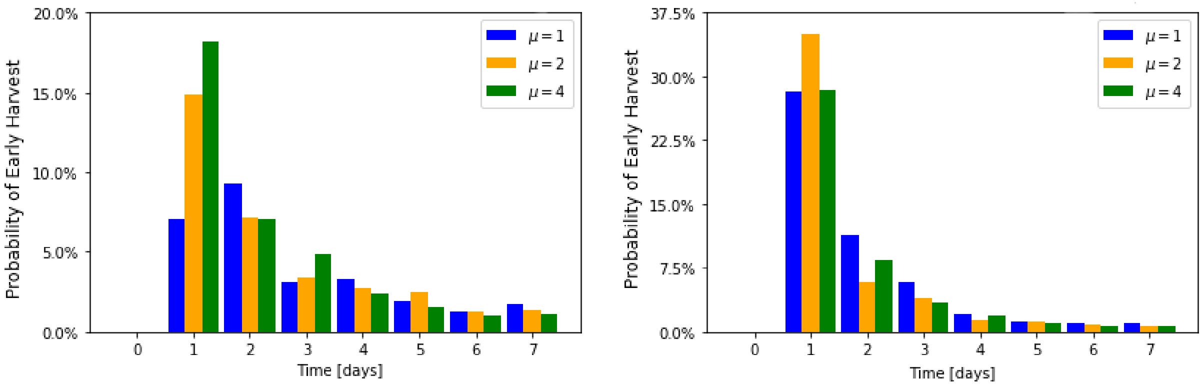
Now, we more clearly observe the optimal strategies for Norwegian and Chilean salmon farmers during the first week of a HAB outbreak, for different stages of the sea phase. For example, in Norway one should perform an early harvest on day four in the middle stage of the production cycle ($t_{sea} = 400$) if k is five or above (given the same spot and equilibrium prices). Whereas for the same situation in Chile, one would perform an early harvest if k is four or above. Note that the late stage ($t_{sea} = 600$) exercise boundary for Chile is at $k = 0$ for the first four days. This indicates that it is optimal to harvest immediately in this case, as the initial value of k is zero (no signals has arrived), which coincides with the results presented in Tables 6.2 and 6.3. This illustrates how the EH-model may be used to produce optimal strategies for different scenarios. In what follows, we will investigate how the signal arrival rate, the signal reliability, and the HAB arrival intensity affect the EH-model results.

Examining the Effect of Signals and HAB-risk

Here, we examine the effects of changing selected parameters related to HABs. First, we focus on the arrival rate of signals and reliability of signals. These parameters are highly interesting as industry organizations and policy-makers can directly influence the flow and certainty of information. An increase in information and collaboration will allow salmon farmers to make better decisions. Therefore, we discuss some implications for policy-makers and the aquaculture industry as a whole. In addition, we look at the HAB arrival intensity as this is a highly uncertain parameter. We use a range of values for these parameters to examine the effects on optimal strategy, cash flows received, and the value of flexibility. For tractability, the effects of changing selected parameter values are only examined for the Norwegian case study, as similar conclusions follow for the Chilean case study.

Changing the Arrival Rate of Signals

First, we study the effects of doubling and halving the parameter value for the arrival rate of signals in the Norwegian case study. Figure 6.8 presents the distribution of early harvests per day during the first week of the HAB outbreak. Salmon farmers do not receive any signals before day one.

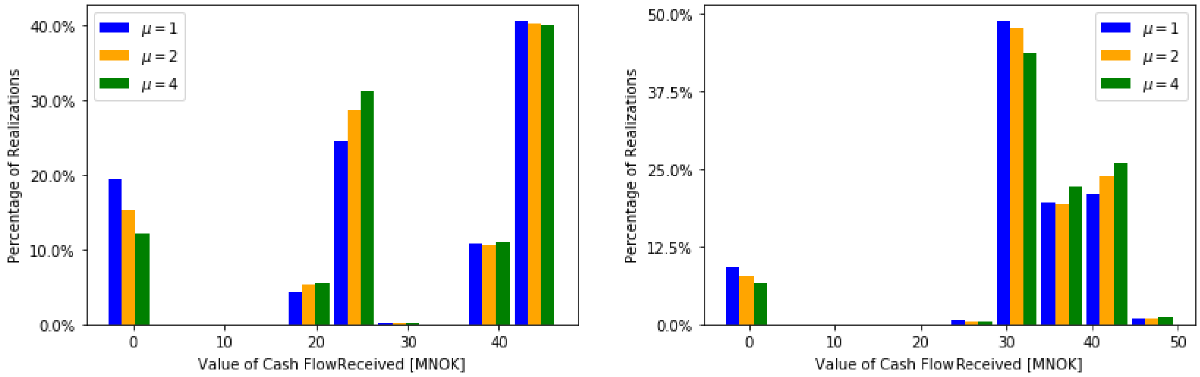


(a) $t_{sea} = 400$ **(b)** $t_{sea} = 600$

Figure 6.8: Probability of early harvests on a given day with different arrival rate of signals μ. The scale of the y-axis is different for visualization purposes. $t_{sea} = 200$ is not included as no early harvests should be performed for any μ.

By observing Fig. 6.8, it is clear that the majority of early harvests should happen at day one for the middle and late sea phase stages. The probability of early harvest being optimal decreases the following days of the HAB outbreak. For the middle sea phase stage, it is notable that an increasing arrival rate of signals leads to a higher probability of optimal early harvest during the first days of the HAB period. This does not hold when $t_{sea} = 600$. According to our model, the salmon farmers do not rely so much on the arrival rate of signals for the later stages in the sea phase where the early harvest choice is conducted with higher probability. From this we conclude that it is vital for salmon farmers to receive signals about the HAB risk as early as possible. Policy-makers should enhance collaboration between salmon farmers themselves and companies that posses knowledge about detecting HABs. Additionally, reporting HAB events should be centralized by, for example, the Directorate of Fisheries. The Directorate of Fisheries can then notify the surrounding production facilities efficiently. National or regional monitoring programs should also be encouraged to improve HAB detection.

Figure 6.9 below shows the probability of receiving a cash flow for the Norwegian salmon farmer in the different sea phase stages for a varying amount of signals.



(a) $t_{sea} = 400$ **(b)** $t_{sea} = 600$

Figure 6.9: Distribution of cash flows received in the EH-model with different arrival rate of signals μ. The scale of the y-axis is different for visualization purposes. $t_{sea} = 200$ is not included as the distribution for any μ is equivalent to Fig. 6.3a.

The scenarios that result in around 0 NOK for the salmon farmer are the cases where a HAB kills the salmon in the sea pen[5]. Figure 6.9 shows that the probability of losing biomass decreases for an increase in μ. Thus, salmon farmers are better off economically when receiving more signals about the true state of the HAB risk. For $t_{sea} = 600$, the cash flows received are generally higher than for the middle sea phase stage as the total biomass of the generations are higher. The option values are also higher for $t_{sea} = 600$. This is visible in Table 6.4. The table shows that the value of early harvest increases for higher arrival rates of signals, except for in the early sea phase stage, in which it is optimal to perform no early harvest for any signal arrival rate.

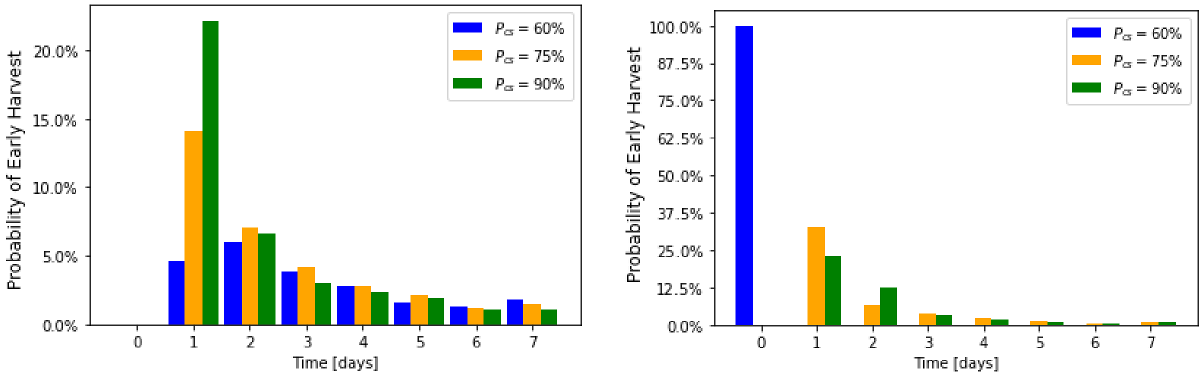
Table 6.4: Value of early harvest in the EH-model with different arrival rate of signals μ for the Norwegian case study.

t_{sea}	$\mu = 1$	$\mu = 2$	$\mu = 4$
200 days	0 MNOK	0 MNOK	0 MNOK
400 days	3.02 MNOK	3.91 MNOK	4.60 MNOK
600 days	8.09 MNOK	9.12 MNOK	9.74 MNOK

Changing the Reliability of Signals

This section studies how a change in the signal reliability affects the optimal strategy and added value of early harvest. We investigate the relationships for $P_{cs} = 60\%$, $P_{cs} = 75\%$, and $P_{cs} = 90\%$ under the different sea phase stages. Figure 6.10 shows the number of harvesting decisions during the first week of the HAB duration.

[5]These cash flows are actually slightly negative, as the farmer pays production costs until the HAB arrives.



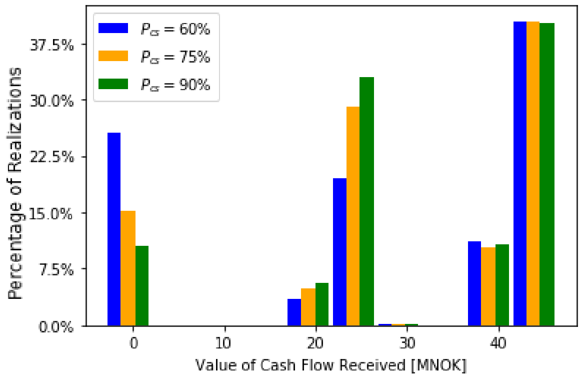
(a) $t_{sea} = 400$ **(b)** $t_{sea} = 600$

Figure 6.10: Probability of early harvest on a given day with different reliability of signals P_{cs}. The scale of the y-axis is different for visualization purposes. $t_{sea} = 200$ is not included as no early harvests should be performed for any P_{cs}.

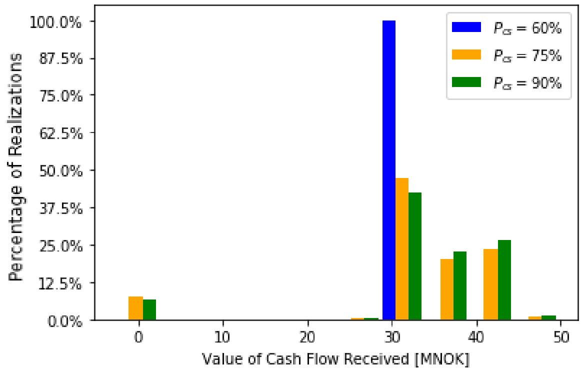
It is evident from Fig. 6.10 that the reliability of signals have a great impact on the early harvesting decision. The harvesting decisions vary considerably, especially when $P_{cs} = 60\%$. The probability of early harvesting is low for this signal reliability when $t_{sea} = 400$, but this is not the case when $t_{sea} = 600$. We see that the farmer should early harvest on day zero for this particular signal quality and sea phase stage. This means that it is not worth it for the salmon farmer to wait for any signals when the reliability of signals is this low for the late sea phase stage. This is the same result as we saw for the Chilean case study in Table 6.3.

When the reliability of signals is set to $P_{cs} = 75\%$ or $P_{cs} = 90\%$, the trend of early harvesting decisions become clearer. If early harvest is optimal, the salmon farmer will most probably early harvest in the first days of the HAB outbreak. The probability of making a harvesting decision decrease as the days go by. With a higher P_{cs} the farmer can make a better-informed decision earlier in the HAB event. If the HAB event happens during the middle stage of the sea phase, a higher reliability means making more early harvests in the first days. In the late stage of the sea-phase, this is reversed. Intuitively, this is because the relationship between the cost of making a mistaken early harvest, relative to the benefit of doing a correct one, shifts from favoring restrictive early harvesting in the middle stages of the sea phase to excessive early harvesting in the late stages of the sea phase.

Figure 6.11 below shows the salmon farmers' probability of receiving cash flows for different P_{cs}.



(a) $t_{sea} = 400$



(b) $t_{sea} = 600$

Figure 6.11: Distribution of cash flows received in the EH-model with different reliability of signals P_{cs}. The scale of the y-axis is different for visualization purposes. $t_{sea} = 200$ is not included as the distribution for any P_{cs} is equivalent to Fig.6.3a.

The effect of changing P_{cs} is similar to that observed when changing the arrival rate of signals. In particular, the probability of losing biomass to HAB decreases as signal reliability increases. Furthermore, the value of early harvest is lower when the reliability of signals is low. This is depicted in Table 6.5 where the value of early harvest increases for improved signal reliability. Again, in the early sea phase stage, it is optimal to never perform early harvest for any signal reliability.

Table 6.5: Value of early harvest in the EH-model with different reliability of signals for the Norwegian case study.

t_{sea}	$P_{cs} = 0.60$	$P_{cs} = 0.75$	$P_{cs} = 0.90$
200 days	0 MNOK	0 MNOK	0 MNOK
400 days	1.44 MNOK	3.91 MNOK	4.88 MNOK
600 days	7.12 MNOK	9.12 MNOK	10.02 MNOK

Interestingly, our model shows that when the signals are sufficiently reliable, it is worth taking the risk of losing the biomass in order to learn more about the true risk. The conclusion can also be drawn by looking at Fig. 6.11b. Since it is optimal to harvest at day zero when $P_{cs} = 60\%$, the cash flows received equal around 30 MNOK. For $P_{cs} = 75\%$ and $P_{cs} = 90\%$, the salmon farmer is able to achieve higher cash flows by learning about the true state of the HAB risk, but in order to do so risks losing the biomass and receiving no cash flow at all.

Numerous possibilities exist to improve the reliability of signals for the salmon farmers. One solution can be to improve the knowledge about HAB testing for the salmon companies themselves. Alternatively, the regional capacity can be improved to boost the testing ability. If the salmon farmers are able to conduct their own testing and analyses, they will benefit from faster and more efficient results. We know from the 2019 incident in Norway that the presence of deadly algae were noticed mostly by the salmon farmers themselves by observing changes in behavior and increased mortality rates (Karlsen et al., 2019). Implementing this measure may therefore be effective and the salmon farmers will not depend on other organizations to learn about the HAB risk.

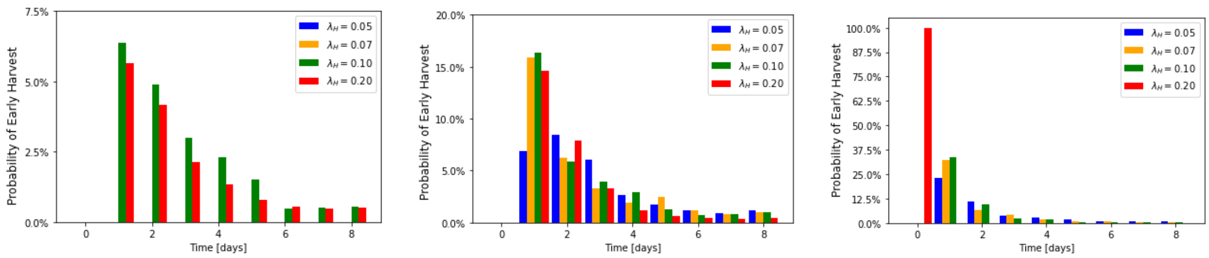
Changing the HAB Arrival Intensity

HAB arrival intensity is an uncertain parameter which depends on several factors in the natural environment as discussed in Section 2.2. Therefore, the choice of parameter value is highly uncertain. This motivates the choice of performing a sensitivity analysis for this particular parameter. We investigate how a range of different λ_H affect our research questions. See Table 6.6 for the chosen parameter values, expected time of arrival, and probability of HAB arrival in the High state.

Table 6.6: Different arrival rates with corresponding expectations regarding arrival time and probabilities of getting the HAB if the true risk state is High.

λ_H	Expected time of arrival	$P(HAB\|H)$
0.05	20.0 days	65%
0.07	14.3 days	77%
0.10	10.0 days	88%
0.20	5.0 days	98%

Figure 6.12 depicts the distribution of early harvests in the first week of the HAB outbreak for different HAB intensities.

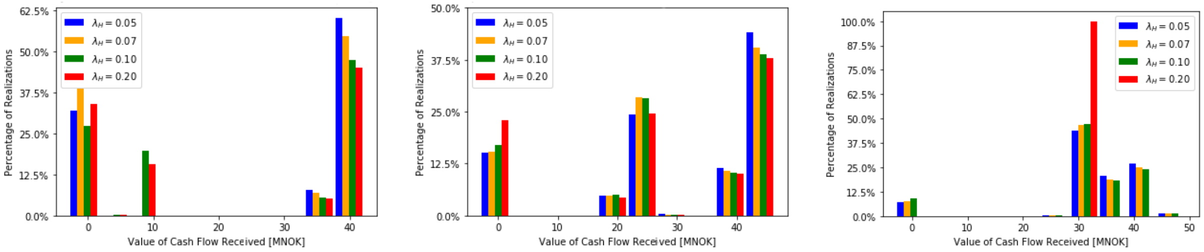


(a) $t_{sea} = 200$ **(b)** $t_{sea} = 400$ **(c)** $t_{sea} = 600$

Figure 6.12: Probability of early harvest on a given day for different arrival rates of HAB in the High risk state, λ_H. The scale of the y-axis is different for visualization purposes.

From Fig. 6.12 we see that the probability of early harvests decrease over the HAB duration. This result is true for all of the chosen HAB arrival rates. Interestingly, we observe that for the two highest values of λ_H, it can be viable to perform early harvests in the early sea phase. We also see that with higher λ_H, the probability of early harvest being optimal increases. An interesting result is obtained in Fig. 6.12c for $\lambda_H = 0.20$ where it is optimal to early harvest at day zero. This means salmon farmers should always conduct early harvests late in the production cycle if the HAB arrival intensity is very high.

The corresponding probability distribution of cash flows received by the salmon farmer is displayed in Fig. 6.13.



(a) $t_{sea} = 200$ **(b)** $t_{sea} = 400$ **(c)** $t_{sea} = 600$

Figure 6.13: Distribution of cash flows received in the EH-model with different arrival rates of HAB in the High risk state, λ_H. The scale of the y-axis is different for visualization purposes.

Figure 6.13 shows that the probability of losing biomass increases for higher λ_H, with the exception of $t_{sea} = 600$ as it is optimal to early harvest immediately. The value of early harvest in the different sea phase stages under various λ_H are illustrated in Table 6.7.

Table 6.7: Value of early harvest in the EH-model with different HAB arrival intensity for the Norwegian case study.

t_{sea}	$\lambda_H = 0.05$	$\lambda_H = 0.07$	$\lambda_H = 0.10$	$\lambda_H = 0.20$
200 days	0 MNOK	0 MNOK	0.38 MNOK	1.25 MNOK
400 days	2.18 MNOK	3.91 MNOK	5.57 MNOK	6.05 MNOK
600 days	6.86 MNOK	9.12 MNOK	10.53 MNOK	11.30 MNOK

We see from Table 6.7 that the option value is substantially higher for increasing HAB arrival intensity. We also see that the value of early harvest becomes positive for the highest arrival intensities for $t_{sea} = 200$. As the optimal actions and values of performing early harvests are highly dependent on the actual risk of getting the HAB, it is essential for policy-makers and aquaculture organizations to contribute with accurate and frequent signals about HAB risk.

6.3 Early Harvest-Move Model Results

As previously discussed, moving the salmon will lead to an increased mortality rate and feed conversion ratio. Consequently, more salmon will die and production costs increase after moving. This obviously leads to a decrease in the value of harvesting optimally at a later stage, if the farmer has moved the biomass compared to endured the HAB at the existing location. Still, the possibility to move the salmon is found to be an attractive option with the introduction of HAB risk. The reason for this is the risk of HABs killing the fish and thus making the biomass worthless. Table 6.8 below shows the EH-M-model results based on 10 000 simulations of price, HAB, and signal processes, with another 7 500 compound GSR-model simulations at every time step.

Table 6.8: EH-M-model results presented for different stages of the production cycle, represented by different values for t_{sea}, for both the Norwegian and Chilean case studies. Immediate Harvest and Immediate Move shows the value of harvesting or moving immediately at the report of a HAB outbreak, No Move or EH shows the value when not allowing for flexibility to move or early harvest. EH-M-model shows the value obtainable when following the EH-M-model's strategy, and Value of Move or EH denotes the added value from the flexibility to move or early harvest.

t_{sea}	Case	Immediate Harvest	Immediate Move	No Move or EH	EH-M-model	Value of Move or EH
200 days	Norway	8.91 MNOK	35.61 MNOK	24.99 MNOK	35.61 MNOK	10.62 MNOK
400 days	Norway	24.74 MNOK	40.59 MNOK	27.33 MNOK	40.59 MNOK	13.26 MNOK
600 days	Norway	33.82 MNOK	39.55 MNOK	25.90 MNOK	39.55 MNOK	13.65 MNOK
200 days	Chile	10.84 MNOK	36.82 MNOK	26.16 MNOK	36.82 MNOK	10.66 MNOK
400 days	Chile	26.14 MNOK	41.66 MNOK	28.15 MNOK	41.66 MNOK	13.51 MNOK
600 days	Chile	24.41 MNOK	41.01 MNOK	26.22 MNOK	41.01 MNOK	14.79 MNOK

For all stages of the sea phase, the EH-M-model finds it optimal to move immediately at the report of a HAB outbreak, hence the equal values in EH-M-model and Immediate Move columns. This means that it is not worth risking losing the biomass while waiting for signals if the farmer has the option to move. As seen in Table 6.8, the derived value of flexibility to move or early harvest is relatively large. This implies that it is highly beneficial for salmon farmers with alternative farming sites available to move salmon. Furthermore, we find these values to be very similar for Norwegian and Chilean salmon farmers.

These findings imply that salmon farmers should seek to enable spatial diversification if possible. Typically, only the larger salmon farmers have multiple farming sites in different regions and hence the possibility to move their fish during HABs. For policy-makers, this has two implications. First, processing of applications for moving the fish during a HAB needs to be prioritized to allow for swift moving. Second, decision-makers should look into the possibility to establish a number of reserve fish farming sites in order to support small salmon farmers in the most endangered HAB areas. This is of importance to secure workplaces for smaller salmon farming companies and local communities.

The EH-M-model allows for joint consideration of early harvesting and moving, which is especially relevant for larger farmers. Compared to Table 6.2 showing EH-model results, we find that the option to move gives much higher option values, thus being the more attractive option, see Table 6.8. As a consequence of a relatively large moving profitability for our baseline case, we find that the option to move is dominating the option to early harvest and to wait in the EH-M-model. Therefore, it is of interest to investigate special cases in which an immediate move is not optimal. First, we look into how an increased cost of moving affects the relationship between moving and harvesting. Figure 6.14 shows the distribution of optimal moves and early harvests in the EH-M-model with varying cost of moving with the parameters from the Norwegian case study, and with $t_{sea} = 600$. We choose to examine the latest stage of the sea phase because it is at this time that the value of early harvest is closest to the value of moving (see Table 6.8).

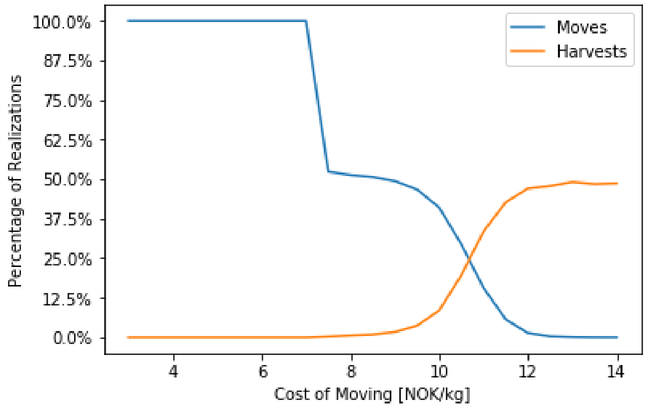


Figure 6.14: Plot showing the distribution of early harvests and moves performed during the HAB under the EH-M-model with different costs of moving. This is from the Norwegian case study with $t_{sea} = 600$.

It is evident from Fig. 6.14 that the option to move dominates the option to early harvest for move costs much higher than what we have estimated. It is first when the move cost is more than twice the original level, at 7 NOK/kg, that moving should not be done immediately. This means that the conclusion to move immediately holds for moving costs up to 7 NOK/kg. This implies that salmon farmers would be willing to pay a lot more than they do today for a moving operation, even when they have the option to early harvest.

We observe from Fig. 6.14 that the amount of moves is equal to early harvests performed for a break-even level of move cost, $C_M = 10.68$. Intuitively, the values of the option to move and the option to early harvest evaluated individually should be similar at this move cost. We examine the effect of evaluating them jointly in Table 6.9, which shows the values obtained by evaluating the options to move or early harvest individually, and the value found by the EH-M-model.

Table 6.9: Values found by evaluating the option to Move individually, the option to Early Harvest individually (i.e., the EH-model), and the EH-M-model which evaluates the option to Move or Early Harvest jointly. Obtained values are for the Norwegian case study with cost of moving increased to $C_M = 10.68$.

Option to Move	Option to Early Harvest	EH-M-Model
35.02 MNOK	35.02 MNOK	35.20 MNOK

As expected, the values of the individual options are roughly equal. However, evaluating the options to move or early harvest as mutually exclusive uncovers 200 000 NOK in excess value compared to when evaluating the options individually in this spesific case. The excess value comes from being able to choose the optimal choice between moving and harvesting.

Figure 6.15 plots the values for different move costs close to the break-even level.

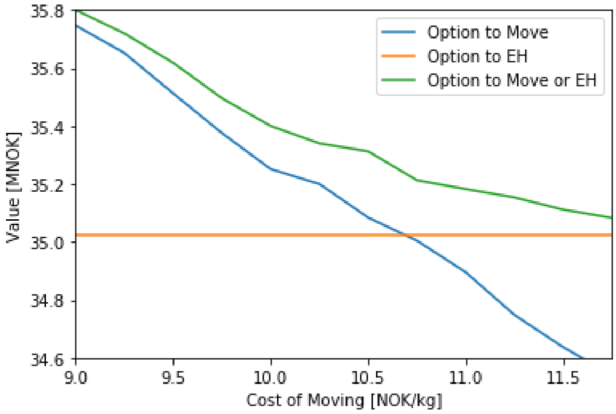


Figure 6.15: Plot showing the value of the option to move and early harvest evaluated individually, and as mutually exclusive options found by the EH and EH-M models with different moving costs. The roughness in the option to Move and the Option to Move or EH stems from randomness in the Monte Carlo simulations. The option to early harvest is not affected by the cost of moving. This is from the Norwegian case study with $t_{sea} = 600$.

Figure 6.15 illustrates that when one of the mutually exclusive options is worth a lot more than the other, the value of evaluating them jointly converges to the option with the highest value. However, there is excess value in evaluating them together. This applies especially when the values of the individual options are close to each other. This implies that farmers that have both options should evaluate them jointly in order to maximize the value of their biomass. Otherwise, it can lead to sub-optimal choices. This applies especially in cases where the values of harvesting and moving are relatively similar, such as very late in the sea phase and with high costs of moving.

We conclude this chapter by remarking that HABs are highly unpredictable and nearly impossible to influence by a policy-maker, see Section 2.2. Therefore, we recommend that policy-makers facilitate implementation of possible mitigation strategies available to salmon farmers when facing HABs. This facilitation includes permissions to move salmon biomass to alternative locations or use available processing plants for early harvesting. Salmon farmers themselves should discuss the terms for allocation of resources during HAB outbreaks, such as the use of wellboats. Communication will be more efficient if these terms are clarified before a potential HAB outbreak. Lastly, human monitoring and control can be increased at the salmon farms to notice changes in fish behavior and increased mortality rates, or with underwater camera technology.

With respect to our results, we understand that there are several uncertain factors interacting at once. These are related to the price, algal, and signal processes. To reduce these underlying uncertainties, we do a reasonable amount of simulations of the respective processes. We observe slightly different option values for each model run, coming from the nature of randomness of the Monte Carlo simulations. However, our results are very similar for each model run, ensuring that our conclusions are robust.

Lastly, our results hold when the market is in contango, i.e., when the spot price lies below expected future salmon prices. We have not looked into when the market is in backwardation and spot prices are expected to drop in the future. However, we find that contango is a reasonable market assumption with respect to HAB events. Thus, we believe our conclusions are valid with respect to our research questions.

Chapter 7

Conclusion

This thesis identifies optimal harvesting strategies for small and large salmon farmers when facing the risk of HAB arrival and stochastic prices. More specifically, we study optimal harvesting decisions and quantify the value of managerial flexibility when allowing for early harvesting, while receiving imperfect information about the true algal arrival rate. In an extension to this framework, we introduce the opportunity to move the fish and investigate how this affects the optimal course of actions. We present two case studies, for Norway and Chile, and discuss their results for different stages of the salmon farming sea phase. Realistic model parameters have been identified from interviews with Chilean industry experts and by studying relevant aquaculture studies on both industries. Our results are found to be robust to both geographical settings. Based on the results, we offer recommendations for policy-makers on how they can facilitate optimal decision-making for salmon farmers during HABs. In what follows, we will present the main findings of this thesis, our contributions to the existing body of literature, and suggestions for further research.

First, in the GSR-model, we find optimal harvesting time while facing stochastic prices. Our model uncovers excess value when accounting for managerial flexibility in the harvesting decision. The value of flexibility is decreasing towards the end of the production cycle. Conversely, we find that the probability of harvesting increases as a function of time. We find that with harvesting flexibility, the probabilities of using this flexibility and of waiting until the last possible harvest day are approximately equal. For salmon farmers this implies that under no algal risk, harvesting is most likely optimal at the end of the production cycle. However, fluctuating prices may lead to optimal harvesting before this, thus having the flexibility to change the time of harvesting is valuable.

Second, the EH-model allows for early harvesting when facing risk of HAB arrival and stochastic prices. We find that by following the EH-model, the probability of losing salmon to the HAB is strongly reduced. However, the results show that the value of harvesting flexibility varies across the production cycle. In particular, the harvesting flexibility has little value in the early stages of the sea phase, but increases in value for later stages. In other words, the current sea phase stage largely affects the harvesting decision. As a consequence, if the HAB occurs early in the sea phase stage when the biomass is low, salmon farmers should ignore the signals and perform no early harvest. However, if the

HAB arrival intensity is sufficiently increased, we find that flexibility in the harvesting decision is valuable even for the early sea phase stages.

Furthermore, we find that when the signals are sufficiently reliable, it is worth taking the risk of losing the biomass in order to learn more about the true risk. Conversely, if the signals are unreliable, salmon farmers late in the sea phase should not take this risk. Instead, they should harvest immediately on the report of a nearby HAB event. We conclude that increased HAB signal reliability leads to better-informed decisions and thus larger option values. Policy-makers may contribute to improved signal reliability by boosting regional testing capabilities and by improving HAB testing knowledge among salmon farming companies themselves.

We find that the value of flexibility in the harvesting decision increases, as the signal arrival rate increases. Thus, we conclude that more signals enables salmon farmers to make optimal decisions. For this reason, policy-makers should enhance collaboration between salmon farmers and organizations that posses knowledge about detecting HABs. Moreover, if the HAB reporting responsibility is centralized to, for example, the Directorate of Fisheries, all industry stakeholders may be informed efficiently at the same time.

Third, in the EH-M-model, our results suggest that salmon farmers having the opportunity to move the fish out of HAB risk areas, should do so immediately. Should current costs of moving significantly increase in the future, both the options of early harvesting and moving needs to be considered in order to avoid sub-optimal decisions. For policy-makers, the results imply two things. First, processing of applications for moving the fish during a HAB event needs to be prioritized to allow for swift moving of fish. Second, decision-makers should look into the possibility to establish a number of reserve fish farming areas in order to support small salmon farmers in the most endangered HAB areas.

Our work offers several promising directions for further research. Salmon farming companies organize their value chains differently, giving various production and moving costs across the industry. This can easily be accounted for by transforming our production and moving cost parameters into cost functions with relevant cost components as input. Developing more sophisticated cost functions would allow salmon farming companies to estimate even more accurate results, accommodating their specific cost settings.

Moreover, we do not include biomass insurance as a scalable risk management tool. Biomass insurance lets salmon farmers hedge against some of the potential losses and would be an interesting extension to our model. It would also be of interest to extend the GSR-model to account for multiple rotations. In the EH-model, one could allow for the possibility to start a new rotation after a given time in order to make the model even more realistic.

Lastly, we assume the price dynamics are the same for both Norway and Chile, even though they serve different markets and are exposed to different currencies. Further work should put additional efforts into obtaining historical Chilean price data and investigate whether the price processes differ geographically. Moreover, we do not account for seasonality in salmon spot prices. Extensions to the price model could make the two-factor price process even more sophisticated by incorporating seasonality.

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Appendix A

Kalman Filter Procedure

In our two-factor price model, the short-term deviations, χ_t, and the equilibrium price level, ξ_t, are unobservable and must be estimated from spot and forwards prices from Fish Pool. The following transition equation describes the evolution of state variables,

$$\mathbf{x}_t = \mathbf{c} + \mathbf{G}\mathbf{x}_{t-1} + \boldsymbol{\omega}_t, \qquad t = 1, ..., n, \tag{A.1}$$

where n is the number of sets of price observations and

$$\mathbf{x}_t = \begin{bmatrix} \chi_t \\ \xi_t \end{bmatrix} \qquad \text{is a } 2 \times 1 \text{ vector of the state variables,}$$

$$\mathbf{c} = \begin{bmatrix} 0 \\ \mu_\xi \Delta t \end{bmatrix} \qquad \text{is a } 2 \times 1 \text{ vector,}$$

$$\mathbf{G} = \begin{bmatrix} e^{-\kappa \Delta t} & 0 \\ 0 & 1 \end{bmatrix} \qquad \text{is a } 2 \times 2 \text{ matrix,}$$

and $\boldsymbol{\omega}_t$ is a 2×1 vector of disturbance terms with zero expected value and covariance matrix as following,

$$\mathbf{W} = \mathrm{Cov}[(\chi_{\Delta t}, \xi_{\Delta t})] = \begin{bmatrix} (1 - e^{-2\kappa t})\frac{\sigma_\chi^2}{2\kappa} & (1 - e^{-\kappa t})\frac{\rho_{\chi\xi}\sigma_\chi\sigma_\xi}{\kappa} \\ (1 - e^{-\kappa t})\frac{\rho_{\chi\xi}\sigma_\chi\sigma_\xi}{\kappa} & \sigma_\xi^2 t \end{bmatrix}.$$

The time difference between each price observation is Δt. Furthermore, the following measurement equation describes the relationship between the price observations and the state variables, χ_t and ξ_t, i.e.,

$$\mathbf{y}_t = \mathbf{d}_t + \mathbf{F}\mathbf{x}_t + \mathbf{v}_t, \qquad t = 1, ..., n, \tag{A.2}$$

where

$$\boldsymbol{y}_t = \begin{bmatrix} \ln(F_{T_1}) \\ \vdots \\ \ln(F_{T_N}) \end{bmatrix},$$

is an $N \times 1$ vector of log forwards observations with time maturities T_j, $j = 1, ..., N$, where N is the number of forward contracts,

$$\boldsymbol{d}_t = \begin{bmatrix} A(T_1) \\ \vdots \\ A(T_N) \end{bmatrix}, \qquad \text{is an } N \times 1 \text{ vector in which}$$

$$A(T) = \mu_\xi T + (1 - e^{-2\kappa T})\frac{\sigma_\chi^2}{4\kappa} + (1 - e^{-\kappa T})\frac{\rho_{\chi\xi}\sigma_\chi\sigma_\xi}{\kappa} + \frac{\sigma_\xi^2}{2}T,$$

$$\boldsymbol{F} = \begin{bmatrix} e^{-\kappa T_1} & 1 \\ \vdots & \vdots \\ e^{-\kappa T_N} & 1 \end{bmatrix}, \qquad \text{is an } N \times 2 \text{ matrix,}$$

and $\boldsymbol{v}_t$ is an $N \times 1$ vector of disturbance terms with zero expected value and a diagonal covariance matrix with diagonal elements $(s_1^2, ..., s_N^2)$.

In the Kalman filter procedure, the short-term deviations and the equilibrium price level must be jointly estimated with the other model parameters, i.e., κ, σ_χ, μ_ξ, σ_ξ and $\rho_{\chi\xi}$, together with standard deviations for these estimates. Schwartz and Smith (2000) propose finding these model parameters using maximum likelihood estimation, but do not explicitly state any likelihood function. However, by using transition Eq. (A.1), measurement Eq. (A.2) and following the procedure in Chapter 3.4 of Harvey (1989), we derive the following likelihood function,

$$\log L = -\frac{Nn}{2}\log 2\pi - \frac{1}{2}\sum_{t=1}^{n}\log|\mathbf{F}_t| - \frac{1}{2}\sum_{t=1}^{n}\mathbf{v}_t'\mathbf{F}_t^{-1}\mathbf{v}_t. \tag{A.3}$$

We want to find the set of parameters (κ, σ_χ, μ_ξ, σ_ξ and $\rho_{\chi\xi}$, together with standard deviations for these estimates) that maximizes this likelihood function. To ensure a global maximum is reached, the optimization procedure must be repeated for a variety of initial parameter values. Our resulting parameter estimates and state variables are presented in Section 5.1. For an in-depth explanation of the Kalman filtering procedure and corresponding proofs, we refer to Harvey (1989) and West and Harrison (1996).

Appendix B

Derivation of High-Risk State Belief Function

The number of bad signals received (indicating true risk is High, H) is denoted h, and the number of good signals received (indicating true risk is Low, L) is denoted l. The farmer has prior belief that there is p_0 probability of being in the High state. Furthermore, he believes that the probability of a given signal being correct is P_{cs}. We denote the belief that the farmer is in the High state as a function of the number of bad and good signals received $P(h, b)$. By following Bayes rule, we get that the belief is given by:

$$P(h,l) = \frac{P(h,l \mid H) \times P(H)}{P(h,l \mid H) \times P(H) + P(h,l \mid L) \times P(L)} =$$

$$\frac{P_{cs}^h \times (1-P_{cs})^l \times p_0}{P_{cs}^h (1-P_{cs})^l p_0 + (1-P_{cs})^h P_{cs}^l (1-p_0)},$$

by multiplying numerator and denominator by $\frac{P_{cs}^{-l}}{p_0(1-P_{cs})^l}$, we get

$$\frac{P_{cs}^{h-l}}{P_{cs}^{h-l} + \frac{1-p_0}{p_0}(1-P_{cs})^{h-l},}$$

, we then introduce $k = h - l$, the number of bad signals (signaling High risk) in excess of good signals (signaling low risk) and get the belief function

$$p(k) = \frac{P_{cs}^k}{P_{cs}^k + \frac{1-p_0}{p_0}(1-P_{cs})^k}.$$

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