Master's thesis

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Creating Robust and Cost-Efficient Line Plans for the Oslo Metro Using Combinatorial Benders' Decomposition

Master's thesis in Industrial Economics and Technology Management
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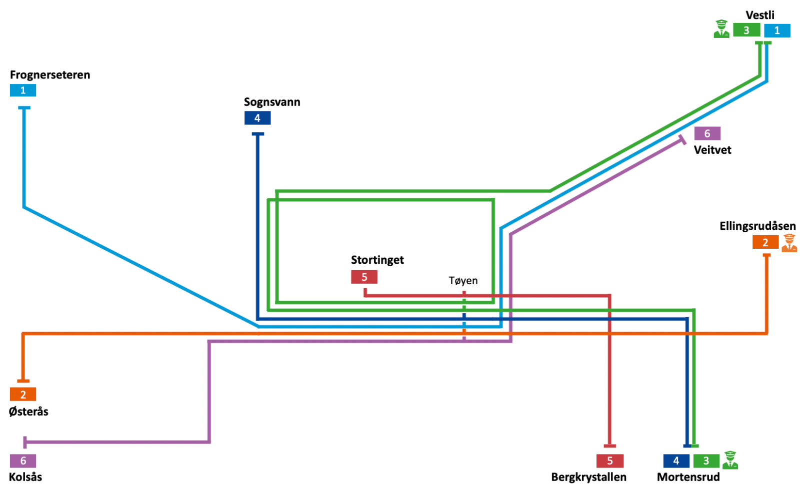

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Preface

This Master's Thesis concludes our Master of Science degree with a specialization in Managerial Economics and Operations Research at the Norwegian University of Science and Technology (NTNU). The research and the writing of the thesis are completed during the spring of 2020. The Master's Thesis (TIØ4905) is a continuation of the work by Harbo et al. (2019) in the specialization project in Managerial Economics and Operations Research. The thesis is carried out in collaboration with Sporveien AS.

We would like to thank our supervisor Associate Professor Magnus Stålhane (Department of Industrial Economics and Technology Management, NTNU) for contributing to the thesis with thorough feedback and helpful discussions. We are also grateful for the cooperation with the representative of Sporveien AS, Helge Holtebekk, who has provided us with valuable insights into the problem addressed in this thesis.

Trondheim, June 5, 2020

Summary

The Oslo Metro transported more than 119 million people in 2019, and the passenger demand is expected to increase in the years to come. The opportunities of reducing costs and the carbon footprint provide incentives to operate the Oslo Metro efficiently. A means to achieve operational efficiency is to minimize the costs related to the number of trains needed in operation. The current planning of the Oslo Metro is done manually based on experience and discretion using Excel. This makes it challenging to weigh all requirements and preferences concerning the structure and operations of the metro system, and might result in suboptimal operations.

This thesis is written in collaboration with Sporveien AS, which is the owner and administrator of the Oslo Metro. The common goal is to create a decision support tool to generate a line plan and a corresponding timetable that minimize costs. The problems of line planning and timetabling are mathematically formulated, and the resulting models are integrated and solved using Combinatorial Benders' Decomposition. Studying relevant literature, a similar problem with all the considered aspects has to the best of our knowledge not previously been investigated.

The input of the model is the structural and operational parameters that Sporveien AS currently operates with. Further, an analysis of historical data on travel times has been conducted, and the results are used as input to the model. The optimal solution of the model differs from the current line plan and timetable. It proves that significant cost reductions are enabled by utilizing the decision support tool created. The cost reduction is primarily due to a reduction of maximally three trains needed in operation. The results further indicate that the decision support tool generates solutions that balance the trade-off between cost-efficiency and robustness. An extended version of the model including soft constraints provides further analysis of this trade-off.

Future projects regarding changes to the operations and structure of the Oslo Metro are studied in this thesis. A planned operational change is the introduction of additional arrivals, which enhances the public transport service in the Oslo Metro. Furthermore, a planned structural change is the addition of a new stretch of railway tracks connecting Fornebu and the city center. The decision support tool created in this thesis proves to efficiently incorporate the addition of new infrastructure to the existing network of the Oslo Metro.

Sammendrag

I 2019 transporterte T-banen mer enn 119 millioner passasjerer i Oslo og Bærum kommune, og antall reisende er forventet å øke i årene som kommer. Mulighetene for kostnadsreduksjon og reduksjon i klimaavtrykk gjør det ønskelig å drifte T-banen mer effektivt. Driften av T-banen kan effektiviseres ved minimering av kostnader knyttet til antall tog. Dagens drift av T-banen utføres manuelt ved bruk av Excel, basert på erfaring og skjønn. Dette gjør det utfordrende å vekte alle krav og preferanser knyttet til strukturen og driften av T-banen, noe som kan medføre suboptimale løsninger.

Denne masteroppgaven er skrevet i samarbeid med Sporveien AS, som eier og administrerer T-banen. Formålet er å lage et beslutningsverktøy for å generere et linjekart og tilhørende tidstabell som minimerer kostnader. Linjeplanleggingsproblemet og tidstabellproblemet er matematisk formulert, og de resulterende modellene er integrert og løst ved bruk av Kombinatorisk Benders' Dekomponering. Etter omfattende litteratursøk konkluderer forfatterene av denne oppgaven med at problemstillingen som oppgaven tar for seg ikke er drøftet i litteraturen tidligere.

Modellen benytter de operasjonelle og strukturelle parameterene som Sporveien bruker i driften av T-banen i dag. I tillegg benyttes parametere som er estimert, basert på historisk data på kjøretidsavvik. Modellen genererer en løsning som er mer optimal enn T-banens nåværende linjekart og tidstabell. Det viser seg at signifikante kostnadsreduksjoner er mulig å oppnå ved bruk av beslutningverktøyet som er beskrevet i denne masteroppgaven. Denne reduksjonen skyldes primært en reduksjon på maksimalt tre tog relativt til dagens drift. Videre indikerer resultatene at verktøyet genererer løsninger som finner en god balanse i avveiningen mellom robusthet og kostnadseffektivitet. Ved å inkludere myke restriksjoner i modellen, oppnår man videre innsikt i avveiningen mellom robusthet og kostnadseffektivitet.

Problemet som studeres i masteroppgaven tar for seg planlagte prosjekter angående endringer i driften og strukturen av T-banen. En planlagt operasjonell endring vil resultere i ytterligere avganger, noe som forbedrer kollektivtilbudet i Oslo og Bærum. Videre er tilskuddet av en ny T-banestrekning, som knytter Fornebu til sentrum, en strukturell endring som Sporveien planlegger å gjennomføre. Resultater viser at beslutningsverktøyet presentert i denne masteroppgaven kan inkorpore ny infrastruktur til det eksisterende T-banenettet på en effektiv måte.

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Chapter 1

Introduction

The world's first metro was built in London in the 19th-century (Dugdale, 2019), and today cities around the world rely on the metro as the leading means of passenger transport. In Oslo, the metro was inaugurated in 1966. In 2019 it transported more than 119 million people (Sporveien AS, 2020c). With its five metro lines, the Oslo Metro makes up the backbone for passenger transport in the capital of Norway.

Trends indicate that the role of the Oslo Metro will be increasingly prominent in the years to come. This is partly due to the trend of population growth in Oslo. According to Statistics Norway (2020), the current population of close to 700 000 inhabitants is expected to reach 760 000 within 2030. Furthermore, there is an increasing focus on the environment and climate change. In Oslo, it has been developed a climate strategy to reduce the CO_2 emissions by 95% within 2030 (Oslo kommune Klimaetaten, 2018). A central part of the strategy consists of reducing the number of vehicles in the city center and shifting the passenger transport towards the public alternatives. These two trends have the potential to further boost the increasing demand for the efficient transportation offered by Oslo Metro, which is operated purely on electricity from renewable sources (Ruter, 2019).

This Master's Thesis is written in collaboration with Sporveien AS, which owns and administrates the Oslo Metro. Their mission is to provide a broad offer in public transportation at the lowest cost possible for the society and the environment. The costs related to operating the Oslo Metro amounted to 1.93 billion NOK in 2019 (Sporveien AS, 2020c). Due to the significant operational costs and the expected increase in demand, running the metro in an efficient manner is important to limit costs to a minimum. The investment costs of a train amount to 110 million NOK, and the costs of operating and maintaining the trains are substantial.

The purpose of this thesis is to study to which extent Sporveien AS can save costs by applying mathematical models and solution methods to its planning processes. To achieve this, the objective is to find the set of metro lines and the corresponding timetable that minimize costs.

These problems are referred to as the line planning problem and the train timetabling problem in the literature, respectively. The problems involve the decision of railway branch combinations into lines and the generation of a corresponding timetable. These decisions must comply with the structural and operational requirements of Sporveien AS. Today these problems are solved using trial-and-error in Excel, based on experience and discretion. Since this tool is unable to capture all requirements and evaluate all possible outcomes, it might result in suboptimal operations of the Oslo Metro.

Harbo et al. (2019) were the first to investigate this problem, and solved it with a similar version of the solution method applied in this thesis. The objective was the minimization of trains in operation. The problem studied in this thesis is an extension of the problem investigated by Harbo et al. (2019). New areas of conflict are added, and additional operational requirements such as triple-frequency areas are considered. This contributes to a model that accurately contemplates the operations of the Oslo Metro. The problem addressed in this thesis further includes the concept of step-back drivers, which is not addressed by Harbo et al. (2019). Therefore, the thesis aims to minimize costs related to the number of trains and step-back drivers applied. It further contributes to generating robust timetables, by the application of individually adapted buffer times to each train. An extended version of the model further enhances the robustness of the model through the application of soft constraints. Combinatorial Benders' Decomposition with optimality and feasibility cuts is the solution method used to solve the problem. This solution method integrates the problems of line planning and train timetabling. The results of the solution method prove that significant cost reductions can be made by utilizing the decision support tool developed in this thesis. The decision support tool further proves to generate more robust timetables relative to the timetable currently used by Sporveien. Through a case study of one of the future projects of Sporveien, the tool proves to efficiently incorporate the addition of new infrastructure to the existing metro network.

The remainder of this thesis is structured as follows. Chapter 2 provides the background for the Oslo Metro and describes concepts relevant for the problem studied. Chapter 3 provides a detailed description of the problem, while an overview of relevant literature is presented in Chapter 4. The problem described in Chapter 3 is decomposed and formulated in two mathematical models which are presented in Chapter 5. This chapter further presents an extended version of the model. The solution method is described in detail in Chapter 6. The parameters and data used in the model are provided and explained in Chapter 7. Chapter 8 contains a computational study of the solution method results. Chapter 9 states concluding remarks based on the computational study. Suggestions for future research and problem extensions are elaborated in Chapter 10.

Chapter 2

Background

This chapter introduces the company Sporveien AS and its subsidiary Sporveien T-banen AS which operates and maintains the metro system in Oslo, referred to as *Oslo Metro*. It further provides a description of Oslo Metro and its structure, and the different planning processes and how they are conducted today. This chapter is partially based on the work by Harbo et al. (2019).

In Section 2.1, Sporveien AS, its subsidiaries, and their cooperating company Ruter AS are presented. Section 2.2 describes Oslo Metro on a general level, while Section 2.3 explains in detail the structure of the metro system and how the process of line planning is currently conducted. In Section 2.4, the process of determining timetables in the Oslo Metro and its importance for efficient operations is elaborated. The concept of rolling stock scheduling and crew scheduling and how this is being performed in the Oslo Metro today is described in Section 2.5 and Section 2.6, respectively. Section 2.7 describes some of the future projects planned by Sporveien, concerning both operational and structural changes in the Oslo Metro.

2.1 Sporveien AS

Sporveien AS is the leading actor in public transport in Norway measured in the number of single journeys and operates mainly in Oslo (Sporveien AS, 2020a). Sporveien AS will from now on be referred to as *Sporveien*. Sporveien is owned solely by the municipality of Oslo and has four subsidiaries: Sporveien T-banen, Sporveien Trikken, Unibuss, and Sporveien Vognmateriell. Sporveien owns infrastructure related to the metro and tram system in Oslo, and develops and administrates it through the subsidiaries Sporveien T-banen and Sporveien Trikken, respectively. Sporveien's third subsidiary Unibuss delivers bus journeys by tenders. Sporveien Vognmateriell acquires and administrates the trains used by Sporveien. An organizational chart showing the structure of Sporveien and its relation to its cooperating company is given in Figure 2.1.

Ruter is the administrator of the public transport services in Oslo and Viken using the infrastructure owned by Sporveien. Sporveien receives most of its income from the contracts of sale contracted with Ruter. The municipality of Oslo owns 60% of Ruter and 40% is owned by the county of Viken (Sporveien AS, 2020d).

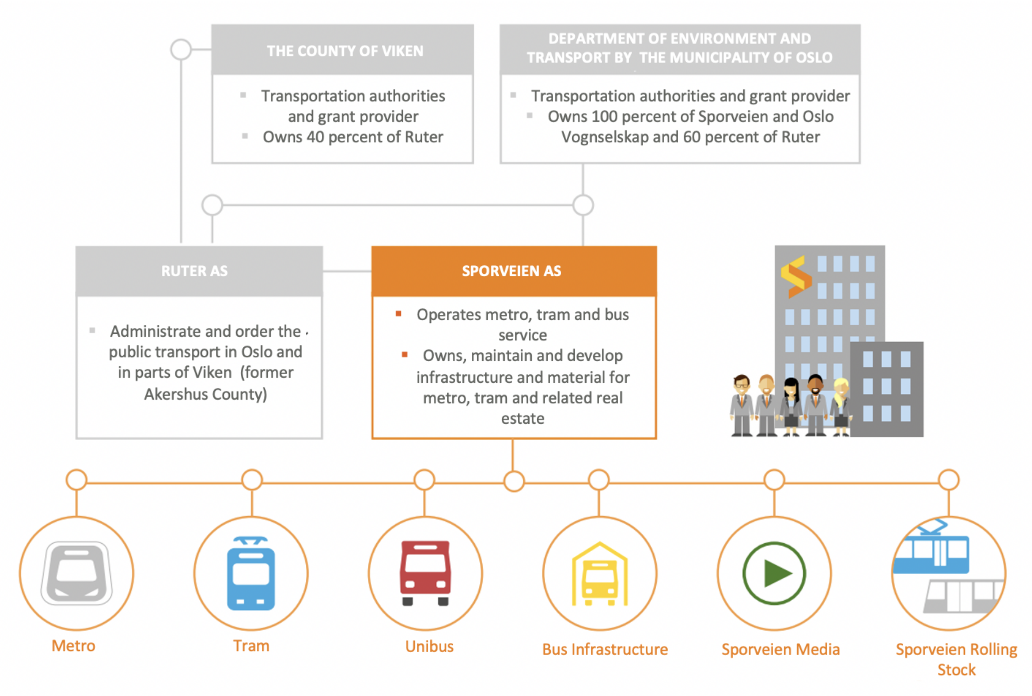

Figure 2.1: Organizational chart describing the structure of Sporveien and its relation to the cooperating company Ruter AS (Sporveien AS, 2020d).

According to Sporveien's yearly report based on operations in 2019, the company conducted 269 million single journeys through its subsidiaries and obtained a total income of over 4,78 billion NOK (Sporveien AS, 2020d). 65% of the public transportation in Oslo and Akershus is delivered by Sporveien, whereof 30% is by metro, 22% by bus, and 13% by tram.

2.2 Oslo Metro

Oslo Metro is owned and operated by Sporveien T-banen, and delivers all public transportation by metro in Oslo and Viken. The word "T-bane" is the Norwegian term for metro, where the T stands for "tunnel". It directly translates to "tunnel track", which it has been named although most of the tracks in the metro system are above ground level. In 2019, Oslo Metro transported over 119 million travellers and obtained an income of approximately 1,9 billion NOK (Sporveien AS, 2020d). This equals approximately 40% of Sporveien's total income the same year. The customer satisfaction rate in 2019 was 98%, which is reflected by the fact that Oslo Metro the previous year was ranked the third most efficient metro company among 36 metro systems in the world (Sporveien AS, 2020c; Asperud, C. Sporveien AS, 2018). This shows that Sporveien T-banen lives up to its social mission, which is "to provide a broad offer in public transportation at the lowest cost possible for the society and the environment" (Sporveien AS, 2020d).

2.3 The Infrastructure of Oslo Metro

Oslo Metro consists of 101 stations with associated tracks across Oslo and the neighboring county of Viken. Each station has a span of one to four platforms, which is where travellers can board and disembark the trains. There can only be one train present at a platform at a time. A map of the Oslo Metro can be viewed in Figure 2.2. The city center is the area in the map between the stations Majorstuen and Tøyen, and the tracks running between these stations are placed underground in a tunnel. The tracks connecting each terminal station to the city center are referred to as *branches*. As can be seen from Figure 2.2, Oslo Metro has eight branches, whereof four connect the city center and terminal stations in the eastern part of Oslo. The remaining four branches connect the city center with terminal stations in the western part of Oslo. The respective branches are referred to as *east branches* and *west branches*. The metro system of Western Oslo consists of the branches with terminal stations Sognsvann, Frognerseteren, Østerås, and Kolsås. The metro system of Eastern Oslo consists of the branches with terminal stations Vestli, Ellingsrudåsen, Mortensrud, and Bergkrystallen.

Line Planning

A *line* is formed by combining a west branch with an east branch. The process of determining the line structure is in the literature described as the process of *line planning*. In the Oslo Metro this process concerns the decision of which west and east branch that should be combined. This is further explained in Section 4.3. Today, the Oslo Metro consists of five lines. The five existing lines can be studied in Figure 2.2, in which the lines are distinguished by color and number.

In Oslo Metro, the process of line planning is a manual task performed in Excel by Sporveien, and it is based on a set of requirements and preferences. Sporveien aims to find a combination of branches that increases operational efficiency, satisfies the passenger demand, and reduces costs.

A consequence of all branches being connected in the city center is that all trains on every line traverse the city center through the same tunnel. The underground tunnel is as previously mentioned entered and exited at the stations Majorstuen and Tøyen, and contains two parallel tracks with trains running in opposite directions. All trains entering and exiting this tunnel therefore need to be carefully coordinated and scheduled to minimize the trains' waiting time and avoid *conflict*. Conflict is in this context defined as two trains simultaneously situated at the same location. Oslo Metro is today operated with a minimum

Figure 2.2: Map of the current line plan of Oslo Metro (Ruter, 2015).

headway time of 90 seconds to avoid conflict. Headway time is the distance in regards to time between two consecutive trains. The tunnel and its complications will be further addressed in Section 2.4 under the headline ”Conflicts”.

2.4 Timetabling in Oslo Metro

An important task of a metro operator is to generate a timetable for the trains in the metro system. This is in the literature referred to as the *train timetabling problem*. Timetable planning contributes to efficient operation of the metro system. The timetable provides information about arrival and departure times at all stations of each line. The process of timetable planning is also closely related to other planning processes, such as line planning, rolling stock scheduling, and crew scheduling. This is further elaborated in Section 4.2. The timetable process is in Oslo Metro performed using Excel through trial and error (H. Holtebekk, personal communication, 11/10/2019).

Sporveien operates with a cyclic timetable for the Oslo Metro. This is a timetable where all arrivals and departures are repeated at each station every cycle period. In a railway system, one cycle period could typically be one hour, while in a metro system it is normally shorter due to a higher departure frequency. A cycle period is referred to as a *ground period*. Concepts related to timetabling that are used in the remainder of this thesis are introduced and defined in the following paragraphs.

Frequencies

Frequency requirements are imposed on the branches. They determine how many trains depart from a station along a branch within the ground period. Oslo Metro operates with a ground period of 15 minutes, and each branch has a frequency requirement of either single or double frequencies. A *single frequency* represents one departure from a station along a branch within the ground period. A *double frequency* represents two departures from a station along a branch within the ground period. A double-frequency branch can either be served by two single-frequency lines or one double-frequency line. This relation will be further discussed in the problem description in Chapter 3. Today, the cooperating company Ruter determines the frequencies on each branch based on customer statistics. Sporveien uses these frequencies when designing the structure of the lines and the timetables (H. Holtebekk, personal communication, 11/10/2019).

Regulation Time

Regulation time is added to absorb possible delays that occur at an operational level. It hence contributes to making the timetable more *robust*. Robustness in timetables will be further explained in Section 4.4.4. Oslo Metro is today operated with a regulation time varying between approximately two and 12 minutes.

Dwell Time

Dwell time is defined as the time a train is stationary at a station while in operation. At all stations, dwell time includes the time needed for passengers to board and disembark the train. Oslo Metro operates with a minimum dwell time of 20 seconds at all stations for this purpose. At terminal stations, the dwell time also includes technical turnaround time, i.e. the time it takes to switch the driving direction of the train. The technical turnaround time is referred to as the *technical dwell time*. In addition, a break for the driver equal to five percent of the total driving time between two terminal stations in both directions is included. This break is taken at only one of the two terminal stations of a line. The regulation time is also a part of the dwell time at the terminal stations. The dwell time is hence significantly longer at terminal stations compared to the intermediate stations. At the terminal stations Sognsvann, Østerås, Bergkrystallen, Mortensrud, and Ellingsrudåsen specifically, the dwell time must be 13 minutes or less to avoid conflict.

Round-trip Time

The round-trip time of a line is the time it takes from a train starts its route at a certain point on the line until it returns at the same point in the same direction. The round-trip time of a line hence corresponds to the travel time and the dwell time in both directions on the line.

Precise, Punctual and Delayed Arrivals

Sporveien defines a train arriving within one minute after scheduled arrival time as precise. Arrivals later than one minute, but within three minutes of scheduled arrival time are defined as punctual. Arrivals later than three minutes after scheduled arrival time are defined as delayed.

Conflicts

Oslo Metro, which has only one track in each direction in the tunnel, is particularly exposed to the issue of conflicts. Hence, this tunnel creates the most significant bottleneck of the Oslo Metro. Conflicts must also be avoided at terminal stations where there can't be more than one train present at a time. Further, the trains at the branches with terminal stations at Østerås and Kolsås cross railway tracks at Smestad station. Conflict hence arises if there is not a sufficient distance between the trains running in opposite directions at Smestad.

2.5 Trains in Operation in Oslo Metro

In the Oslo Metro, the number of trains assigned to each line is dependent on the line's frequency and its round-trip time. This dependency can be illustrated by a line which has single-frequency and a round-trip time of two hours. The line hence has one arrival per ground period of 15 minutes, which equals four arrivals an hour. To maintain the required frequency, the line needs a total of eight trains serving it.

Sporveien currently operates with a total of 57 trains (H. Holtebekk, personal communication, 01/10/2019). The costs related to trains make up a substantial amount of the total costs of Sporveien. As mentioned in the introduction, the investment cost of a train is 110 million NOK. Further, the alignment costs per train amount to 110 million NOK, and are costs related to the technical train preparations. There are also operational costs and maintenance costs associated with each train in the rolling stock.

2.6 Crew Scheduling and Step-Back Drivers

Crew scheduling is a planning step that allocates the necessary crew to each train. Each train is serviced by one driver. Further, to reduce dwell time at the terminal stations and thereby the round-trip time, *step-back drivers* can be applied. A step-back driver is a driver that waits at the terminal station and is ready to operate arriving trains and release the arriving driver. This reduces the dwell time as the train continues operations without waiting for the driver to return from the break. Sporveien currently operates with step-back drivers on two lines. These lines are Line 4 with the break taken at Bergkrystallen, and on Line 5 at Sognsvann.

2.7 Future Projects

This section provides information about future projects planned by Sporveien. Section 2.7.1 addresses the addition of a ninth arrival through the tunnel each ground period. The addition of a new west branch to the metro system is described in Section 2.7.2, while Section 2.7.3 presents the incorporation of a new signalling system in the operations.

2.7.1 The Ninth Arrival

Sporveien has decided to introduce an additional arrival through the tunnel each ground period. This is done to increase the metro efficiency and meet the increasing public transport demand. The new arrival is planned to serve the stations between Veitvet and Stortinget. Veitvet is a station on the east branch with Vestli as its terminal station. This project is planned to be carried out during the time in which this thesis is written, i.e. spring 2020.

2.7.2 The Addition of the Fornebu Branch

The addition of a new west branch from the city center to Fornebu is planned to be completed within 2027. The branch is planned to be a double-frequency branch and to serve six new stations.

2.7.3 The Application of a New Signalling System

Sporveien plans to invest in a new signalling system with a technology called communication-based train control (CBTC) (Sporveien AS, 2020b). The system adjusts and optimizes the speed and headway time between the trains, which results in reduced travel time. It is estimated that the reduction is three percent on average on all lines (H. Holtebekk, personal communication, 15.05.2020).

Chapter 3

Problem Description

The problem studied in this thesis is the minimization of train and driver costs through optimization of the line plan in the Oslo Metro. It is an extension of the problem studied by Harbo et al. (2019). The optimal line plan is the combination of west and east branches that minimizes the total costs while producing a feasible timetable. The optimization of the line plan is therefore dependent on both the problems of line planning and train timetabling. These problems must be addressed while respecting the structural and operational restrictions elaborated below. As described in Section 2.7.1, a ninth arrival through the tunnel each ground period is implemented during spring 2020. As this change is conducted during the time this thesis is written, the project is included in the problem. The same applies to the application of the new signalling system which is described in Section 2.7.3.

The line planning concerns the decision of which west branch that should be combined with which east branch, based on their respective frequency requirements. Oslo Metro is today operated with a cyclic timetable. The cyclic timetable is constructed by repeating all scheduled arrivals in each ground period. To find a feasible timetable it is therefore sufficient to perform the train scheduling for one ground period.

Branches are defined as either single or double frequency branches, as described in Section 2.4. Sporveien has defined acceptable time intervals between the two arrivals on a double-frequency branch, which are referred to as *time interval distribution requirements*. The time interval has an upper and a lower bound. The sum of these bounds must equal the duration of the ground period to obtain an even distribution of arrivals. Further, at certain parts of the railway tracks, there are triple-frequency areas. At stations within these areas, there are three arrivals each ground period. These occur due to either three single-frequency branches sharing rails and platforms, or one single-frequency branch sharing rails with a double-frequency branch. There are two such areas in the metro system. The first area is situated between the stations Blindern and Ullevål stadion. This triple-frequency area occurs because three single-frequency branches share rails. The second area is between Majorstuen and Smestad, and the rails are shared by the single-frequency branch Kolsås and the double-frequency branch Østerås. Triple-frequency areas are also subject to time interval distribution requirements.

The combination of branches to form lines is dependent on whether the branches have single or double frequency requirements. A branch with double frequency can be combined in two ways. The first possibility is to combine it with one double-frequency branch, creating a line with double frequency. The other possibility is to combine it with two single-frequency branches, creating two lines with single frequency. In both cases, the time intervals between the trains serving the double-frequency branch must satisfy the time interval distribution requirements, as specified above. A branch with single frequency can only be combined with one other branch. Due to restrictions in platform length at the branch Frognerseteren, it is required that Frognerseteren must be combined with either Mortensrud, Bergkrystallen, Vestli or Veitvet.

To combine all branches into lines, the total frequency of the west branches must equal the total frequency of the east branches. The total frequency is the aggregated frequencies of the branches in a direction. If the east branches have a higher total frequency than the west branches, fewer arrivals are entering the tunnel from the west than from the east each ground period. The frequency mismatch is handled by having the excessive arrivals on the east branches turn at a station fitted for this purpose. One such station is Stortinget, which is located in the tunnel and could be used to turn for trains coming from an east branch heading west. Should the west branches have a higher total frequency than the east branches, the trains entering the tunnel from the west branches can turn at Helsfyr.

As previously mentioned, the tunnel is the main area of conflict in the Oslo Metro. The trains are therefore scheduled with a given headway time to avoid conflict. Another area of conflict is located at the station Smestad, at which the westbound trains on the branch Kolsås must cross the rails it shares with the branch Østerås. A sufficient time buffer is therefore needed between the westbound trains crossing the rails and the eastbound trains on the branch Østerås. The last addressed areas of conflict are at specific terminal stations, where two trains cannot be situated simultaneously. To avoid conflict, the trains must depart from the station in due time before the next arrival. Hence, there is a maximum limit on the dwell time at these terminal stations.

As explained in Section 2.5, the number of trains needed on a line is determined based on the round-trip time and the frequency of the line. Elements included in the round-trip time are dwell time and travel time between stations. The driver break included in the dwell time can only be taken at one of the two terminal stations of a line. Step-back drivers can be applied to the terminal stations to replace the driver break, and hence reduce the dwell time. The regulation times must also be chosen to account for robustness in the timetable.

In summary, the objective of this problem is to find the line structure and the corresponding timetable that minimize costs. The costs are related to the variable costs of trains and the variable costs associated with the use of step-back drivers. The principal decisions to be made concern the combination of west and east branches to form a line plan, and the scheduling of trains in the corresponding timetable. For each line, it must be decided which terminal station the drivers take their break. Further, the application of step-back drivers must be considered. Lastly, regulation time sufficient to make the timetable robust must be decided. The mentioned decisions are made to comply with the structural and operational requirements.

Chapter 4

Literature Review

This chapter presents previously published research and literature that are relevant for this thesis. This chapter is partially based on the work by (Harbo et al., 2019). The literature search strategy used to find relevant papers is explained in Section 4.1. In Section 4.2 the planning process in public rail transport is presented as a hierarchical structure of planning steps. Literature concerning the planning steps that are relevant for this thesis, which are line planning and train timetabling, are introduced in that respective order in Section 4.3 and Section 4.4. Section 4.5 presents literature addressing the problem of combining the line planning problem and train timetabling problem. Lastly, Section 4.6 highlights this thesis' contributions to the field of line planning and train timetabling.

4.1 Search Strategy

The literature review of this thesis contains the aspects of line planning and train timetabling. These problems are in the literature referred to as the line planning problem (LPP) and the train timetabling problem (TTP), respectively.

The literature search on TTP started on a more generic level and was then narrowed down to TTP with cyclic timetables, the Periodic Event Scheduling Problem (PESP), and robust optimization. In most metro systems considered in the literature, the trains on different lines do not share rails. This structure is more commonly found in literature concerning normal railway systems. Due to the structure of the Oslo Metro, the search for relevant literature was therefore focused on normal railway systems.

To the best of our knowledge, the relevant literature on the LPP in the field of metro optimization is scarce. We find that the focus is largely on the creation of a new railway structure, and on deciding the associated frequencies. Both the railway structure and the frequencies are predefined in the problem studied in this thesis. Therefore, due to the limited relevance, the scope of the search in the literature on the LPP is narrower than for the TTP. Search has also been conducted to find literature related to the combination of line planning and train timetabling, and robust timetables.

Table 4.1 shows the different terms that have dominated the search of relevant literature. The primary search engine is Google Scholar, which provides a broad specter of scholarly literature.

Table 4.1: Overview of search words used in Google Scholar in conjunction with TTP and LPP, as well as other search elements considered relevant.

TTP	**LPP**	**Other search elements**
Cyclic timetables	Line concept	PESP
Metro + cyclic timetables	Assignment problem	PESP + metro
Objectives	Objectives	Combined LPP and TTP
Train scheduling	Metro	Network design + train timetabling
Robust timetables		Soft constraints

4.2 The Planning Process in Public Rail Transport

The task of planning in the public rail transport is described as highly complex, with a broad set of elements which need to be managed simultaneously. Due to this, the planning process is divided into several steps which should be managed in a hierarchical order (Ghoseiri et al., 2004). Ghoseiri et al. (2004) presents the hierarchy of the planning process as given in Figure 4.1, and similar descriptions are discussed in Assad (1980) and Bussieck et al. (1997). The planning process as depicted here is applicable in all public transport but is in this chapter discussed in a railway context.

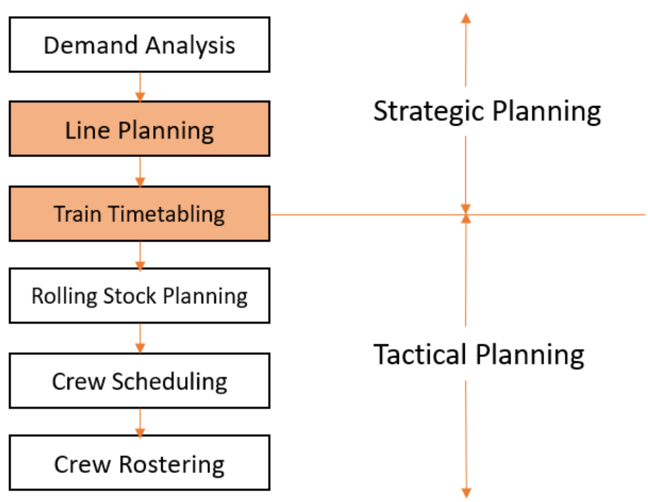

Figure 4.1: The hierarchical planning process in public rail transport.

The hierarchy consists of six sequential tasks (Ghoseiri et al., 2004). The tasks are further divided into two levels of planning, either *strategic* or *tactical*.

The first step in the hierarchical planning process is the task of demand analysis, where data on passenger demand is collected and interpreted (Ghoseiri et al., 2004). Based on the demand analysis, the composition of the lines and their departure frequencies are determined in the subsequent task of line planning. The third step is the task of train timetabling, also referred to as *train scheduling* in the literature. In this part of the planning process, the timetable with arrival and departure times for all stations of all lines is modeled. In the planning step of rolling stock planning, the trains are acquired and assigned to the lines. Personnel is assigned to each train to satisfy specific requirements of operations in the fifth step, crew scheduling. In the last planning step, crew rostering, rosters are constructed and assigned to each worker.

The six steps are strongly related, as the optimal solution found in one step will affect the optimal solution in the following steps by restricting the set of feasible decisions (Ghoseiri et al., 2004). It is therefore considered important that all calculations in the planning steps consider the objectives of the succeeding steps to find a globally optimal solution. The planning steps of relevance in this thesis are the colored steps in Figure 4.1, namely line planning and train timetabling. The literature further discussed in this chapter is therefore focused around these two planning steps, and thereby discusses planning on both a strategic and tactical level.

4.3 Line Planning Problem

The LPP is concerned with the determination of line structures and their respective routes and frequencies (Schöbel, 2011). Further, Kaspi (2013) refers to line planning as the task of deciding the routes, frequencies, and travel time on given railway infrastructure based on passenger demands. As mentioned in Section 4.1, we find that the literature on the LPP relevant to the problem in this thesis is scarce. The scope is therefore narrowed down to the different objectives used in the literature on the LPP. These are presented in the following section.

4.3.1 Objectives in the Line Planning Problem

In addition to finding a feasible line plan, the LPP has a twofold objective. On one hand, one wishes to reduce costs related to operations and maintenance. On the other hand, a high quality of service from the passengers' point of view is desired (Schöbel, 2011). Further, in the literature of line planning, models with different objectives are presented. Claessens et al. (1998) describe a model for cost-optimal railway line allocation, where the operational costs are subject to minimization. Other LPPs discussed in the literature aim at finding the best line plan from a passenger perspective. Bull et al. (2015) solve the LPP by minimizing the total trip time for passengers. The total trip time includes travel time as well as transfer time between lines. Furthermore, some models focus on both costs and passenger needs. Blanco et al. (2019) model the problem of line planning as a multi-objective problem. Both operational and maintenance costs as well as passenger-oriented costs such as quality of service are considered. The latter is measured in the cost of unmet demand.

4.4 Train Timetabling Problem

This section provides a thorough overview of the literature on TTP relevant to the problem studied in this thesis. Section 4.4.1 presents a more general overview of the TTP, while the more specific concepts of cyclic timetabling and PESP are assessed in Section 4.4.2. The application of cyclic TTP to metro systems in the literature is presented in Section 4.4.3. Robust train timetabling in the literature is presented in Section 4.4.4, while Section 4.4.5 discusses different objectives used in the solution of the problem.

4.4.1 A Historical View of the Train Timetabling Problem

As mentioned in Section 4.2, the TTP is concerned with constructing a timetable for a given set of lines to satisfy the requirements of departure frequency. The constructed timetables must comply with operational and infrastructural restrictions, such as capacity constraints (Ghoseiri et al., 2004). The rails are often shared by a large number of trains, making the synchronization of the trains a crucial task to optimize the operations of the railway network.

The problem of generating train timetables has for over a century been solved manually through trial and error (Ghoseiri et al., 2004). Mathematical programming methodology has been presented since then, but there are still actors in public rail transport who solve the problem manually. Amit and Goldfarb (1971) were the first to apply mathematical programming methodology to the TTP. Since then, the problem has been widely studied through the application of optimizations models with varying objectives. The variety of objectives will be further discussed in Section 4.4.5.

Szpigel (1973) was the first to model the TTP as a mixed-integer program to generate a timetable that minimizes total transit time. This solution approach applied the branch-and-bound solution framework as presented by Greenberg (1968) for a general job shop machine scheduling problem. Jovanović and Harker (1991) later presented a nonlinear integer programming model to minimize the deviation between planned and actual timetables. Brännlund et al. (1998) designed a Langrangian relaxation method to dualize track capacity constraints which they applied to an integer programming model.

Caprara et al. (2002) address the problem of timetabling, and their article is one of the most discussed on the subject of TTP in recent time, with a problem similar to the one of Brännlund et al. (1998). The TTP is solved using an Integer Linear Model (ILP) based on a graph-theoretic formulation, and a Lagrangian relaxation solution method is applied. The case of study is a single, one-track railway connecting two major stations, with several intermediate stations between them. The important lines between the two major stations are called ”corridors”, and consist of two independent one-way tracks carrying traffic in opposite directions. Because of high traffic density, the track resource is limited. The focus is on determining a feasible timetable for the trains in the corridor. As first presented by Jovanović and Harker (1991), each train in the network is assigned an *ideal timetable*, which is the most desirable timetable for each train without the track capacity constraints. The *actual timetable* is then generated by modifying the ideal timetable by applying the track capacity constraints. The objective is hence to maximize the profit by minimizing the deviation between ideal and actual timetable for all trains.

4.4.2 Cyclic Timetabling and Periodic Event Scheduling Problem

Dauscha et al. (1985) define cyclic scheduling as a finite set of tasks that has to be executed repeatedly, such that the time between two successive executions for the same task has a fixed period T between them. Further, a periodical or cyclic timetable is based on arrivals and departures along a line with fixed inter-arrival times within a given period. The period identically repeats itself, resulting in a whole cyclic timetable with fixed time intervals between arrivals and departures along a specific line (Bussieck et al., 1997). As seen from the two definitions, they both account for periodicity, with the first definition being more general than the latter.

When discussing cyclic timetabling, Serafini and Ukovich (1989) compare it to traditional non-cyclic scheduling problems. A major difference to the cyclic scheduling is the objective of the problems. Because of the lack of a fixed period, the traditional scheduling problem often aims to minimize the overall completion time of some activities. These activities are subject to requirements according to precedence. As the period in cyclic scheduling is fixed, it does not make sense to minimize the completion time. Cyclic scheduling also differs from traditional scheduling in computational complexity and its development. Most traditional scheduling problems can be solved in polynomial time, while cyclic problems are NP-complete, making the computation complex (Serafini and Ukovich, 1989). Further, cyclic timetabling has proven to work for large line networks (Kroon et al., 2009). Moreover, they are less complex as only one period needs to be scheduled (Peeters, 2003), easy to remember for customers (Barrena et al., 2014), and yield minimum waiting time (Larson and Odoni, 1981).

When modelling cyclic timetables, the PESP is often used. It was first introduced by Serafini and Ukovich (1989) and describes the problem of providing a timetable satisfying a set of periodical interval constraints (Lindner and Zimmermann, 2005). The objective of PESP is to determine at which time instant the periodic events, i.e. arrivals and departures, are to take place within a period. The basic version of PESP only deals with finding a feasible schedule and therefore does not account for an objective function. Lindner and Zimmermann (2005) argue why an objective function should be added to the original PESP problem.

Expansions to this model in TTPs are many, whereof some are introduced by Odijk (1996), Kroon et al. (1996), Nachtigall and Voget (1997) and Liebchen and Möhring (2007).

4.4.3 Cyclic Train Timetabling in Metro Systems

Cyclic timetabling is the most common form of timetabling in metro systems as they are easy for customers to memorize. Further, cyclic timetables are used in the metro system due to the high frequency of arrivals, which results in a narrow headway time (Halim et al., 2014). As the metro network usually consists of several lines (Halim et al., 2014), the coordination of them therefore quickly becomes complex (Kang et al., 2015).

The optimized Periodic Metro Scheduling (PMS) problem was introduced by Bampas et al. (2006). In this problem, exact algorithms are used to create a cyclic timetable which maximizes the minimum time between two successive trains. This is especially useful for systems where several trains use the same rail segment. Kang et al. (2015), Guo et al. (2017) and Li et al. (2018) optimize the train timetable of the metro network in order to secure synchronization between connected lines. This is done to align the transfer between the two lines. Kang et al. (2015) and Guo et al. (2017) present real-world case studies based on the Beijing Subway.

4.4.4 Robust Train Timetabling

A robust TTP builds a train schedule that aims to avoid delay propagation. This is done by accounting for possible operational delays in the planning phase (Cacchiani, 2012). This is most commonly performed through the insertion of buffer times in the schedule. Buffer times correspond to empty time slots, which are inserted into the timetable in the planning phase. Their purpose is to absorb delays occurring at an operational level. A timetable is considered robust if the trains can keep their scheduled time slots despite the occurrence of delays. Delays are by Kroon (2008) divided into primary and secondary delays. Primary delays occur due to external stochastic disturbances, while secondary delays occur due to knock-on effects of delays from one train to the next. Buffer times could absorb the primary delays and hence reduce the secondary delays.

The application of robustness to the TTP with a focus on finding robust yet efficient solutions has gained attention in the more recent years. The trade-off between a robust and cost-efficient timetable is frequently discussed in the literature. Carey (1994) states that a consequence of allocating too much buffer time to a process might lead to the realization of the process taking more time because more time is available. Cacchiani (2012) stresses the importance of deciding the length and timing of insertion of the buffer times which guarantees a good trade-off between efficiency and robustness.

Cacchiani (2012) aims at determining robust train timetables and considers six different approaches with a focus on their advantages and drawbacks. The approaches are applied to nominal timetables, i.e. timetables in which delays are not accounted for. This is done to further discuss the trade-off between the nominal objective, i.e. the efficient objective, and the robustness objective. The end goal is to find a train timetable that efficiently uses the railway infrastructure while guaranteeing solutions that remain feasible under small disturbances.

4.4.5 Objectives in the Train Timetabling Problem

A considerable part of the literature on the TTP focuses on maximizing the profit of the infrastructure manager, according to general charging rules. Brännlund et al. (1998) do this by maximizing the profit function for each train in service, which is then penalized by a per-minute cost for the unnecessary waiting of a train. Caprara et al. (2002), Caprara et al. (2006) and Cacchiani et al. (2016) represent a continuation of the models represented by Jovanović and Harker (1991) and Brännlund et al. (1998) by maximizing the profit for each train as if it were to operate according to its ideal timetable. They then penalize for any differences in departure times between the ideal and actual timetable. In all mentioned cases, the objective value is subject to the quality of the timetable. This is strongly related to the task of minimizing the potential delays of trains, where Kraay et al. (1991) and Higgins et al. (1996) represent models in crossing operational areas, minimizing both delays and cost.

A natural objective of TTPs is to minimize operational costs. Bussieck et al. (1997) and Lindner and Zimmermann (2005) focus on minimizing the operational cost of train timetables by using trains of different speed, size, and cost. Further, energy efficiency has proven an effective way of saving cost, resulting in this objective being widely used in the literature of more recent times. Li and Lo (2014) minimize the energy consumption, i.e. the operational cost, by jointly optimizing the timetable and the speed profile of the trains. Le et al. (2015) optimize the metro timetable to maximize the utilization of regenerative braking. Yin et al. (2017) on the other hand, present an approach to the TTP which minimizes the energy consumption and passenger waiting time.

Another objective used in the literature is the minimization of waiting time for passengers on the stations. Waiting times in railway systems often arise as transfer waiting time, i.e. the time a passenger needs to wait at a station when transferring from one line to another. This is addressed in the papers of Nachtigall and Voget (1997) and Barrena et al. (2014), in which the focus is to minimize waiting time by varying train speed and departure times. Tormos et al. (1996) and Zhou and Zhong (2007) present the objective of minimizing travel times for trains.

4.5 Integrating the Planning Steps of Line Planning and Train Timetabling

To the best of our knowledge, the relevant literature on this area is scarce. This is mainly due to how the problem of line planning is approached. Kaspi (2013) integrates the LPP and TTP and refers to line planning as the task of deciding the routes, frequencies, and travel time on a given railway infrastructure based on passenger demand. Schöbel (2017) solves the two problems iteratively and defines line planning as the problem of deciding which of the already existing lines to use, and their respective frequencies.

4.6 Our Contributions

This thesis studies the LPP and TTP of the real-world metro system Oslo Metro. The problem is initially concerned with line planning, and the TTP is solved to find a feasible timetable for the line combination resulting from the LPP. Hence, the problem concerns the optimization of both the LPP and TTP. The priority is to find efficient and convenient methods for the problem concerning Oslo Metro, specifically. However, we believe parts of the methods are sufficiently general to encompass problems with a similar railway structure. Our contributions to the field of LPP and TTP viewed as isolated problems are discussed in Section 4.6.1 and Section 4.6.2, respectively. Section 4.6.3 discusses the contributions to the integration of the LPP and the TTP.

4.6.1 Our Contributions to the Field of Line Planning

In the LPP studied in this thesis, the decision is concerned with the construction of the lines by combining predefined branches with prespecified frequency requirements. This results in a rather simple but unique problem to solve due to the already defined railway structure and branch frequencies. The method used to solve this problem by branch combination is, to the best of our knowledge, unique in the literature on the LPP. The method could hence inspire others who have a railway structure similar to the one of Oslo Metro, where lines can be rearranged to make operations more efficient. Further, the method can efficiently incorporate new railway branches, and find the optimal line plan based on both existing and planned railway infrastructure. The method is hence considered an efficient tool in the phase of planning the structure of a metro system.

As discussed in Section 4.3.1, the objective of LPPs studied in the literature typically concerns minimization of passenger waiting time (Bull et al., 2015), minimization of costs (Claessens et al., 1998), and minimization of passenger travel time as seen in Kaspi (2013); Schöbel (2017). The objective in the LPP studied in this thesis concerns minimization of the costs related to the number of trains and step-back drivers in operation. This includes investment costs, operational costs, driver costs, and maintenance costs. Objectives minimizing costs typically concerns operational costs and passenger inconvenience costs, like in Blanco et al. (2019), and not investment costs.

4.6.2 Our Contributions to the Field of Train Timetabling

The structure of the Oslo Metro makes the problem of finding a feasible timetable an intricate task. The cyclic TTP for the network of lines studied concerns the scheduling of trains where *all* lines share the same railway tracks and platforms along several stations. Further, the TTP studied in this thesis faces a high frequency of arrivals. This makes the scheduling of trains challenging, especially through the congested area. This thesis hence contributes to finding a timetable for such a structural and operational demanding network.

This is to the best of our knowledge unique in TTPs addressed in the literature concerning metro systems and regular railway systems. This might be because metro systems rarely have many lines sharing capacities, and regular railway systems typically have considerably lower frequencies than metro systems. The problem investigated in this thesis hence contains ”the worst of both worlds” in terms of complications in metro and railway systems. As opposed to Szpigel (1973), Jovanović and Harker (1991), Brännlund et al. (1998) and Caprara et al. (2002), who consider only a single line, our problem considers an entire railway network. Further, the mentioned articles do not consider frequency requirements, as the TTPs are not cyclic. Cacchiani et al. (2016) consider highly congested railway nodes, but do not use cyclic timetables nor considers frequencies. Halim et al. (2014) consider a cyclic train timetable in a metro network, and thereby line frequencies, but do not address problems where trains on different lines share railway capacities. Kang et al. (2015) address a cyclic TTP, but do not consider frequencies or the problem of trains on all lines sharing railway capacities.

Harbo et al. (2019) apply the same, constant regulation time to the branches to make the problem more robust. In this thesis, the application of robustness in a train timetable has been taken further. Each branch is given its own regulation time based on historical data on delays of scheduled arrivals. This makes the timetable more robust and ensures that the buffer times inserted are not higher than the individual need of each branch. This contributes to making the model robust and efficient. The inclusion of soft constraints in the model further contributes to finding a balance in the trade-off between a robust and cost-efficient timetable. A buffer time below the target value is penalized in terms of costs. A buffer time above the target value is rewarded with a cost reduction. The thesis hence contributes with a solution to finding timetables that are both robust and cost-efficient, with individually fitted buffer times per branch.

As discussed in Section 4.4.5, the typical objectives found in the literature investigating the TTP are maximization of profit (Caprara et al., 2002, 2006; Cacchiani et al., 2016), minimization of possible delays (Kraay et al., 1991; Higgins et al., 1996) and the minimization of operational costs (Bussieck et al., 1997; Lindner and Zimmermann, 2005). The objective of the TTP studied in this thesis concerns minimization of the variable costs related to trains. These costs include both investment costs and operational costs. This objective has not been found elsewhere in the literature related to this problem. The extended model further includes a penalty function, used to find timetables that are both robust and cost-efficient.

4.6.3 Our Contributions to Solving the Line Planning Problem in Combination with the Train Timetabling Problem

Another contribution of this thesis is a solution method for solving the LPP and TTP in combination. As mentioned in Section 4.2, the planning steps are highly correlated, making it easier to find a global optimum by solving the planning problems simultaneously. This solution method is called Combinatorial Benders' Decomposition, and it is further elaborated in Chapter 6. Harbo et al. (2019) apply feasibility cuts, while this thesis further includes optimality cuts to enhance the solution method.

To summarize, to the best of our knowledge the problem studied in this thesis has not before been investigated. This is mainly due to the uncommon structure of Oslo Metro, in which trains of all lines share the same rail capacities in addition to high departure frequencies.

Chapter 5

Mathematical Model

In this chapter, the mathematical formulations of the LPP and the TTP are presented. Section 5.1 presents the assumptions concerning both problems. Section 5.2 presents the model description and assumptions for the LPP specifically, followed by the mathematical formulation of the problem. The model description and assumptions for the TTP are presented in Section 5.3, followed by the mathematical formulation of the problem. An extension of the model is presented in Section 5.4.

The mathematical formulations of the LPP and TTP are extensions to the mathematical formulations as presented by Harbo et al. (2019). Several additional conflict areas and aspects are addressed to make the model more realistic in relation to the Oslo Metro. Examples are conflicts at the terminal stations and areas with triple-frequency demands. The project of the ninth arrival is also considered. Harbo et al. (2019) minimize the number of trains. In this thesis, the objective is to minimize variable costs related to the number of trains and step-back drivers. Further, the robustness of the model is considered by deciding individual buffer times for each branch. Finally, this thesis considers soft constraints with penalty and reward functions in the objective in an extended version of the model.

5.1 Common Assumptions for the LPP and TTP

Tøyen is chosen as a point of reference for the planning in both the LPP and the TTP. In the LPP, Tøyen corresponds to the starting point of all branches. A branch is defined based on the direction from which it is connected to Tøyen. Hence, a west branch is defined as the rails connecting Tøyen with a west terminal station. Similarly, an east branch connects Tøyen and an east terminal station. Tøyen is chosen as a reference point for the train scheduling in the TTP, as all trains traverse the station. Mind that any station in the tunnel east of Stortinget can be chosen, but as Tøyen is the first tunnel station it becomes a natural choice. Thus, if the timetable is feasible at Tøyen, it is feasible in all other stations in the network. The problem of finding a valid timetable is therefore addressed by determining arrival times for all trains of all lines at Tøyen station in both directions.

In both models, the frequency requirements are given for all branches. Further, the dwell time at all intermediate stations and the headway time are considered constant and equal for all arrivals. With given frequency requirements and dwell times, the travel time between Tøyen and a terminal station is assumed constant. The technical dwell times at terminal stations are also assumed constant for all arrivals on each branch.

5.2 Line Planning Problem

The model description and assumptions concerning the modelling of the LPP are presented in Section 5.2.1. In section 5.2.2, the mathematical formulation of the model is introduced.

5.2.1 Model Description and Assumptions

The combination of east and west branches is determined through a mathematical formulation similar to the formulation of the well known Assignment Problem. The binary decision variable x_{we} decides whether the west branch w is combined with the east branch e to form a line. F_g^I corresponds to the frequency requirement of the branch g. The maximum frequency of a line, F_{we}, is dependent on the frequencies of the branches that constitute it, which was explained in detail in Chapter 3. The cost parameter, C_{we}^{min}, is the minimum annual variable costs for the combination of branches w and e. The calculation of C_{we}^{min} for the different branch combinations is explained in Section 7.2.1. The modelling of specific branches and structures of the Oslo Metro is presented in the following sections.

Figure 5.1, shows how the branches are split into west and east branches dependent on the direction from which they are connected to Tøyen. The blue branches are defined as west branches, while the orange branches are defined as east branches.

Modelling of the Vestli Branches

As can be observed from Figure 5.1, there are two branches with Vestli as their terminal station. As described in Section 5.1, branches are defined as west and east branches based on the direction from which they are connected to Tøyen. Hence, one of the two branches is a west branch, while the other is an east branch. They are therefore modelled as the single-frequency branches Vestli West and Vestli East in the mathematical formulation. The two branches merge at the station Økern, where the trains serving them start to share rails. The stations between Økern and the terminal station Vestli therefore have double-frequency demands. This creates specific constraints in the mathematical formulation of the TTP due to the time interval distribution requirements. This is further described in Section 5.3.2.

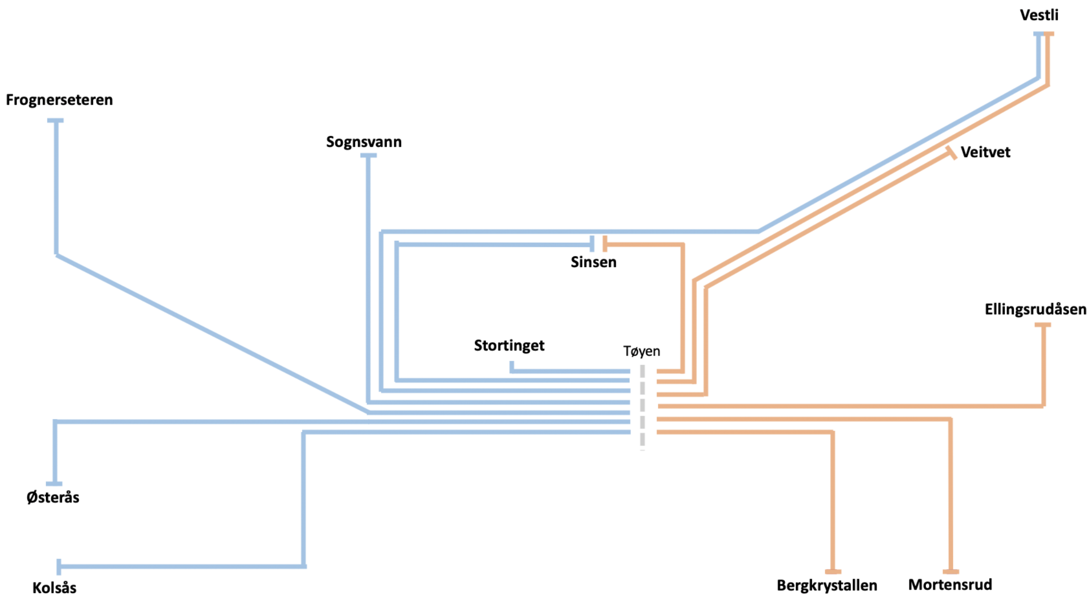

Figure 5.1: Illustration of the west and east branches of Oslo Metro.

Modelling of Ringen

Another curiosity of the structure of the Oslo Metro, is the circle line in the centre referred to as Ringen. From the map of the current operations of Oslo Metro, in Figure 2.2, one can see that the green line, Line 5, makes out Ringen as structured today. This is shown more specifically in Figure 5.2.

To model a similar structure, the line has been split into four branches. These are west branches Sognsvann and Ringen West, and the east branches Vestli East and Ringen East. Ringen East and Ringen West make out the full circle, both modelled to have Sinsen as their terminal station. This is shown in Figure 5.3. Be aware that this is an example of how Ringen would be split up in the current line system of Oslo Metro. Ringen East can be combined with any west branch, and Ringen West can be combined with any east branch in the system. Hence, they do not need to be combined with Sognsvann and Vestli East. The Ringen branches with terminal station Sinsen can be seen in Figure 5.1.

Ringen West and Ringen East are combined with another east and west branch respectively to form two new lines. These two constructed lines are then combined to form one single line which makes out the total line Ringen with a similar structure as the one of Line 5 in the current line plan.

Modelling of the Ninth Arrival

As stated in Chapter 2.7.1, Sporveien plans a ninth arrival through the tunnel each ground period. This arrival is planned to serve the stations between Veitvet and Stortinget. This

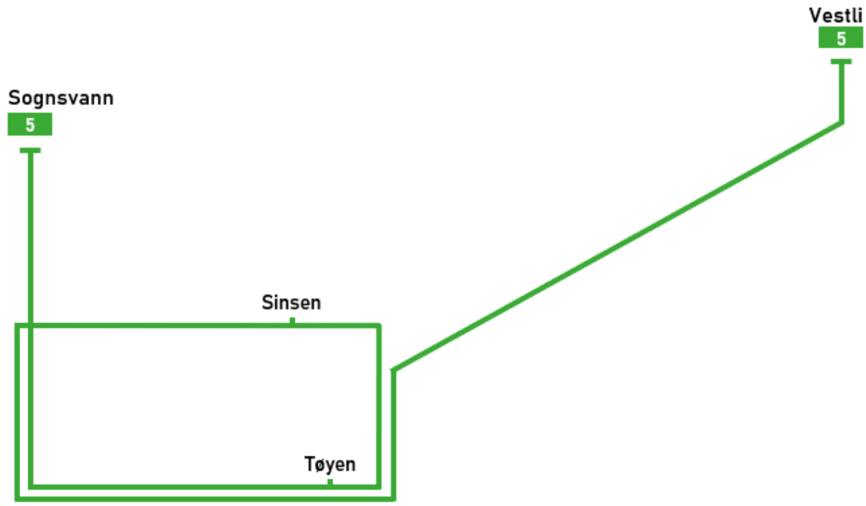

Figure 5.2: Map of Ringen as structured today.

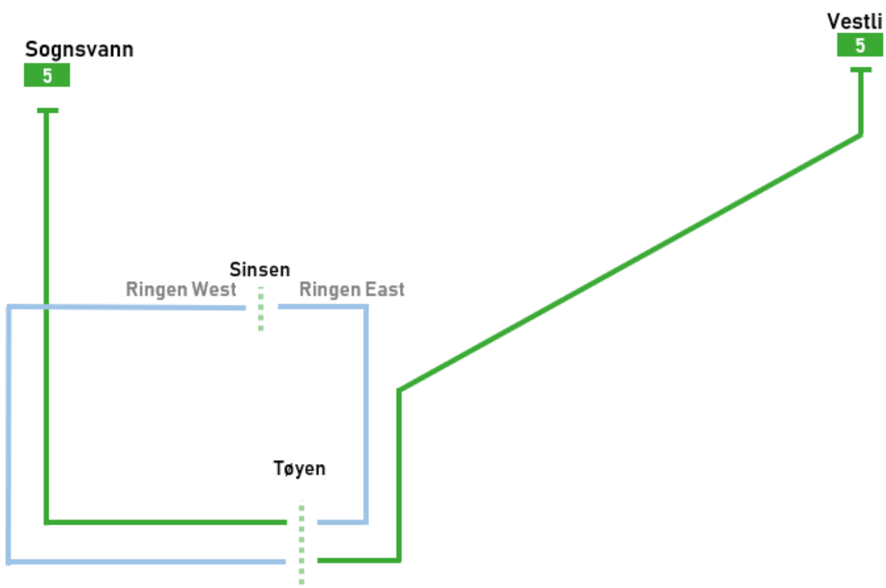

Figure 5.3: The line making up Ringen is split up in four branches for modelling purposes.

resolves to a single-frequency line with terminal stations at Stortinget and Veitvet in the line plan as planned by Sporveien. To model the additional arrival, the line is split into a west and an east branch. The new east branch has Veitvet as its terminal station, and the new west branch has Stortinget as its terminal station. The branches can be seen in Figure 5.1.

Introduction of the Stortinget Branch

In the current line plan of the Oslo Metro, the total frequency requirements of the east branches exceed the total frequency requirements of the west branches. With the inclusion of the Veitvet branch and the ninth arrival, two more arrivals are entering the tunnel at Tøyen from the east side each ground period. The frequency mismatch is solved by having these arrivals turn at Stortinget in the tunnel. A new west branch that connects Tøyen and the terminal station Stortinget is therefore defined. The branch is modelled as a double-frequency branch, to make the total frequency of the west side equal to the total frequency of the east side.

5.2.2 Mathematical Formulation of the Line Planning Problem

The mathematical formulation of the LPP is presented in this section, and it is a version of the well known Assignment Problem. Firstly, the sets, indices, parameters, and variables are introduced, followed by a description of the objective function and the constraints.

Definition of Sets, Indices, Parameters and Variables

Sets

G	Set of branches
G^W	Set of west branches, $G^W \subseteq G$
G^E	Set of east branches, $G^E \subseteq G$
G^{RW}	Set of west branches constituting the circle in Ringen, $G^{RW} \subseteq G^W$
G^{RE}	Set of east branches constituting the circle in Ringen, $G^{RE} \subseteq G^E$
G^{BMV}	Set of west branches which must be combined with one of the four branches Mortensrud, Bergkrystallen, Vestli East or Veitvet due to platform length, $G^{BMV} \subseteq G^W$
G^F	Set of east branches which can be combined with Frognerseteren, $G^F \subseteq G^E$

Indices

$g \in G$	Branch
$w \in G^W$	West branch
$e \in G^E$	East branch

Parameters

C_{we}^{min}	The minimum cost that can be achieved if branch w is combined with branch e
F_g^I	Required frequency imposed on branch g
F_{we}	Maximum frequency on the line composed of west branch w and east branch e

Decision variables

$$x_{we} \quad := \begin{cases} 1, \text{ if branch } w \text{ is combined with branch } e \\ 0, \text{ otherwise} \end{cases}$$

Objective Function

$$min\ z = \sum_{w \in W} \sum_{e \in E} C_{we}^{min} x_{we} \tag{5.1}$$

The objective function, as shown in constraint (5.1), aims to minimize the minimum costs that arise from connecting branch w with branch e. These minimum costs consist of the variable costs of the step-back drivers and trains necessary to satisfy the frequency requirements, F_{we}.

Constraints

The constraints for the LPP can be divided into two categories. The first two sets of constraints, (5.2)-(5.3), concern the allocation of east branches to west branches, and vice versa, through frequency matching. The last two sets of constraints, (5.4)-(5.5), concern specific branch combination requirements. Binary constraints are given in constraints (5.6).

Allocation Constraints

$$\sum_{w \in G^W} F_{we} x_{we} = F_e^I \qquad e \in G^E \tag{5.2}$$

$$\sum_{e \in G^E} F_{we} x_{we} = F_w^I \qquad w \in G^W \tag{5.3}$$

Constraints (5.2) and (5.3) combine west branches with east branches to construct lines, dependent on the frequency requirement of the branches.

Specific Requirements Concerning Branch Combinations

$$\sum_{w \in G^{BMV}} \sum_{e \in G^F} x_{we} \geq 1 \tag{5.4}$$

Constraint (5.4) makes sure that the west branch Frognerseteren is combined with one of the east branches Mortensrud, Bergkrystallen, Vestli East or Veitvet.

$$x_{we} = 0 \qquad w \in G^{RW}, e \in G^{RE} \tag{5.5}$$

Constraints (5.5) make sure that the branches that make up the circle in Ringen are not combined as a line, as this would result in a loop without dwell time or driver break.

Binary Constraints

$$x_{we} \in \{0, 1\} \qquad w \in W, e \in E \tag{5.6}$$

5.3 Timetabling Problem

Section 5.3.1 provides the model description and assumptions concerning the modelling of the TTP. In Section 5.3.2 the mathematical formulation of the model is presented.

5.3.1 Model Description and Assumptions

The model of the TTP aims to generate an optimal timetable which minimizes the costs related to the number of trains and step-back drivers. As stated in Section 2.4, Oslo Metro operates with a cyclic timetable with ground period P^G. Therefore, it is adequate to schedule for the ground period when determining the arrivals at Tøyen. This means that if a train on a line is scheduled to arrive at Tøyen in the third minute within the ground period, this is repeated every ground period for the entire time of operation.

An important assumption when modelling the TTP for Oslo Metro, is that the travel times for all branches are constant. As a consequence, it is adequate to find the arrival times in both directions at Tøyen to obtain a feasible timetable. The arrival times at all other stations could then be found implicitly by simply adding or subtracting the time it takes for a train to travel from Tøyen to the respective stations.

Figure 5.4 illustrates the mathematical formulations used to model the timetable of a line. The sets, parameters and variables used in this illustration are explained in the following model description.

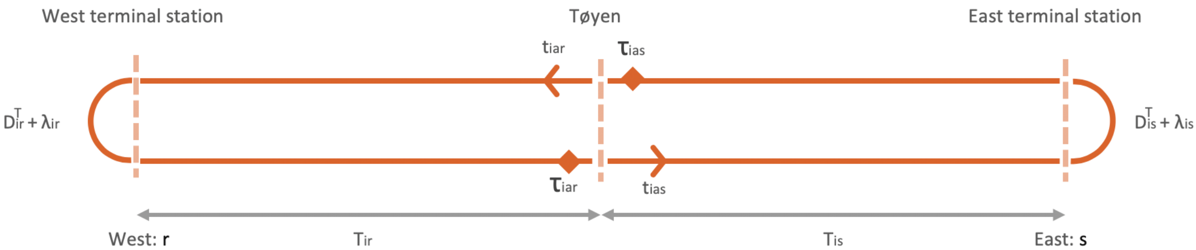

Figure 5.4: Illustration of the mathematical formulations used to model the timetable for a line.

The model outputs a timetable stating the arrival times at Tøyen for all arrivals a of all lines i in both directions r within a ground period P^G, which are represented by the variables t_{iar}. The variable t_{iar} is illustrated in Figure 5.4. As described in the problem description in Chapter 3, lines serving double-frequency branches or areas with triple-frequency requirements must satisfy the time interval distribution requirements.

The model defines a set R of the two directions, 1 and 2, which corresponds to $west$ and $east$ directions, respectively. The direction indexes r and s are used to describe the direction of the arrival at Tøyen, and the parameters for either a west or an east branch. For instance, the technical dwell time at a terminal station, D^T_{ir}, and the travel time, T_{ir}, are indexed with line number i and direction r, where the direction corresponds to the west or east branch of the specific line.

In order to connect an arrival at Tøyen in one direction, t_{iar}, with the arrival of the same train at Tøyen in the opposite direction, t_{ias}, the variable *time of return*, τ_{iar}, is used. The time of return defines the earliest possible time a train can be scheduled at Tøyen when returning in the opposite direction, t_{ias}, given t_{iar}. The time of return is given by the following equation in the mathematical formulation.

$$\tau_{iar} = t_{iar} + D^S + 2T_{ir} + (1 - \sigma_{ir})(2\Delta \sum_{s \in R} T_{is})\delta_{ir} + D^T_{ir} \qquad i \in I, a \in A_i, r \in R$$

The parameters and variables used in the equation are illustrated in Figure 5.4. The time of return includes the driver break, which equals the term $(2\Delta \sum_{s\in R} T_{is})$ from the equation above. The length of the driver break is equal to Δ percent of the time it takes to drive back and forth between two terminal stations. Further, the driver takes the driver break at one of the two terminal stations. Which terminal station of a line the break is being held is therefore included as a decision in the model, as it affects the time of return. The decision is made using the binary variable δ_{ir}, as seen from the equation above. Further, the binary variable σ_{ir} decides whether a step-back driver is applied to the terminal station at which the driver break is held. In the case of a step-back driver being applied, the dwell time is reduced by the duration of the driver break. Hence, the driver break is not included in the time of return. The regulation time, λ_{iar}, is described in the following section. Parameters which are not described in this section are defined in the parameter listing in Section 5.3.2.

Modelling a Robust Train Timetable for Oslo Metro

As described in Section 2.4, buffer times are added to timetables to absorb delays occurring at an operational level. The buffer times inserted in the timetable modelled in this thesis are the regulation times, λ_{iar}. They are decided for all arrivals a on all lines i in both directions r, except at the Ringen branches. As the Ringen branches do not have terminal stations, they are not assigned regulation times. To better understand how the regulation time is applied to the timetable, the two sets of constraints that involve this variable are provided below.

$$\tau_{iar} - z_{iar}P^G + \lambda_{iar} = t_{ias} \qquad g \in G \setminus \{G^{RW}, G^{RE}\}, i \in I_g, a \in A_i, r, s \in R | r \neq s$$

$$\lambda_{iar} \geq L_{ir} \qquad g \in G \setminus \{G^{RW}, G^{RE}\}, i \in I_g, a \in A_i, r \in R$$

The first set of constraints defines the time a train a can return to Tøyen, t_{ias}, in direction s on line i given the time of return variable, τ_{iar}. In addition to the time of return, τ_{iar}, the regulation time variable, λ_{iar}, is added. The regulation time, λ_{iar}, is the additional time spent dwelling at the terminal station, as shown in Figure 5.4. Its value is chosen to achieve the desired buffer time through the second set of constraints. The deviation parameter L_{ir} represents an estimated need of buffer of each branch and is a lower bound for the regulation time. The values of the deviation parameters, L_{ir} are discussed in Section 7.1. By deciding buffer times individually fitted to each branch, one can find a robust solution.

As mentioned, the regulation time is added to the terminal stations. Should a train be delayed upon arriving at the terminal station, the regulation time absorbs this delay, making it possible to leave the terminal station on scheduled departure time. Should the train be on time when arriving at the terminal station, the whole regulation time is spent dwelling as the train can not leave earlier than scheduled. The timetable is considered robust if the regulation times for all arrivals are sufficient to absorb the occurring delays.

Modelling of the Triple-Frequency Requirements

To distribute the three arrivals in a triple-frequency area evenly over the ground period, P^G, it is necessary to decide the order of the arrivals. The order of arrivals is decided at Tøyen, as the order remains the same on the triple-frequency areas due to the assumption of constant travel times. The order of the three arrivals constitutes a *pattern*. Assume Line i, Line j, and Line k to be the three single lines that share rails. Then the pattern $\{iar, jar, kar\}$ means that the arrival on Line i arrives Tøyen first, followed by the arrival on Line j and Line k in that respective order. The patterns are listed in the ordered set Q, which contains all P possible patterns. The ordered set Q is presented below.

$$Q = \{\{iar, jar, kar\}, \{iar, kar, jar\}, \{jar, iar, kar\},$$
$$\{jar, kar, iar\}, \{kar, iar, jar\}, \{kar, jar, iar\}\}$$

The specific pattern Q_p is further indexed using q', q'' and q''', where each index refers to an actual arrival a, along a line i in direction r. The pattern, Q_p can therefore be written as shown below.

$$Q_p = \{q', q'', q'''\}$$

The decision of which pattern of arrivals that should be chosen, is done trough the decision variable η_{pr}. This binary variable η_{pr} takes the value 1 if the specific pattern, Q_p, of arrivals is chosen for the direction r. Further, ζ_{yr} defines the time interval between the arrival y and the next arrival $y+1$, in direction r. The time intervals between the three arrivals must take values between $\underline{F^3}$ and $\overline{F^3}$ and sum up to the ground period, P^G, for an even distribution of arrivals.

Figure 5.5 shows the relation between the explained sets and set elements if the pattern {iar, jar, kar} is chosen for direction r.

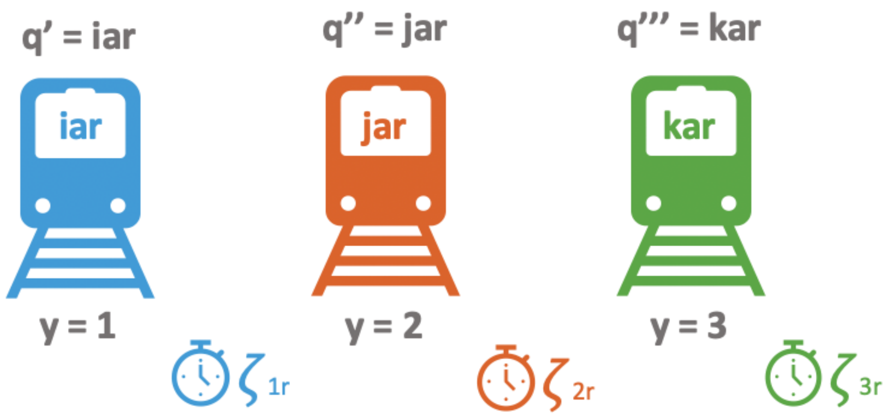

Figure 5.5: Illustration of the mathematical formulations used to model the triple-frequency requirements served by three single lines. The pattern $\{iar, jar, kar\}$ is represented, and the three arrivals have the time intervals ζ_{1r}, ζ_{2r} and ζ_{3r} between them.

A triple-frequency area can also be due to a double-frequency line and a single-frequency line sharing rails. This means that it is necessary to coordinate two arrivals along the same line with one arrival from a different line.

5.3.2 Mathematical Formulation of the Train Timetabling Problem

The mathematical formulation of the TTP is presented in this section. Firstly, the sets, indices, parameters, and variables are introduced. Secondly, the objective function is stated and described. Lastly, statements and descriptions of the constraints are provided. All time units are given in minutes.

Definition of Sets, Indices, Parameters and Variables

Sets

I	Set of lines
A_i	Set of arrivals within ground period P^G for line $i \in I$
R	Set of directions
R^W	Set of directions for west branches, $R^W \subseteq R$
R^E	Set of directions for east branches, $R^E \subseteq R$
G	Set of branches
G^1	Set of branches with double-frequency requirements served by one line, $G^1 \subseteq G$
G^2	Set of branches with double-frequency requirements served by two lines, $G^2 \subseteq G$
G^{WT}	Set of west branches with a maximum dwell time limit, $G^{WT} \subseteq G$
G^{ET}	Set of east branches with a maximum dwell time limit, $G^{ET} \subseteq G$
G^{XR}	Set of all branches except the ones that constitute Ringen, $G^{XR} \subseteq G$
G^{RW}	Set of west branches which constitute Ringen, $G^{RW} \subseteq G$
G^{RE}	Set of east branches which constitute Ringen, $G^{RE} \subseteq G$
G^{VW}	Set of west branch Vestli West, $G^{VW} \subseteq G$
G^{VE}	Set of east branch Vestli East, $G^{VE} \subseteq G$
G^K	Set of west branch Kolsås, $G^K \subseteq G$
G^O	Set of east branch Østerås, $G^O \subseteq G$
I_g	Set of lines that are composed of branch $g \in G$, $I_g \subseteq I$
Q	Set of possible patterns of arrivals at a triple-frequency area served by three single lines
P	Set of the number of possible patterns of arrivals at a triple-frequency area served by three single lines
Q_p	Set of the ordered arrivals at a triple-frequency area served by three single lines for pattern p
Y	Set of the number of ordered arrivals at a triple-frequency area served by three single lines
W	Set of possible patterns of arrivals at a triple-frequency area served by one double line and one single line

U	Set of the number of possible patterns of arrivals at a triple-frequency area served by one double line and one single line
W_u	Set of the ordered arrivals at a triple-frequency area served by one double line and one single line for pattern u
L	Set of the number of ordered arrivals at a triple-frequency area served by one double line and one single line

Indices

$i, j, k \in I$	Line
$a, b, c \in A_i$	Arrival
$g, h \in G$	Branch
$r, s \in R$	Direction
$p \in P$	Patterns for three single-frequency lines
$q', q'', q''' \in Q_p$	Arrival pattern for three single-frequency lines
$y \in Y$	Number of arrival pattern for three single-frequency lines
$u \in U$	Patterns for a double and single frequency line
$w', w'', w''' \in W_u$	Arrival pattern for a double and single frequency line
$l \in L$	Number of arrival pattern for a double and single frequency line

Parameters

T_{ir}	Travel time between Tøyen and the terminal station of line i in direction r. Includes dwell time at all intermediate stations
$T^{Smestad}$	Travel time between Tøyen and the station Smestad
D^S	Minimum dwell time at Tøyen
D^T_{ir}	Technical dwell time at terminal station of line i in direction r
$\overline{D}$	Maximum dwell time at specific terminal stations
Δ	Percentage of total driving time designated to driver break
H	Headway time
P^G	Ground period
L_{ir}	Deviation parameter on the branch in direction r on line i based on historical data from Oslo Metro

$\overline{F^2}$	Upper limit for the time interval distribution between two arrivals on a double-frequency branch
$\underline{F^2}$	Lower limit for the time interval distribution between two arrivals on a double-frequency branch
$\overline{F^3}$	Upper limit for the time interval distribution between three arrivals on parts of a branch
$\underline{F^3}$	Lower limit for the time interval distribution between three arrivals on parts of a branch
$B^{Smestad}$	Minimum time needed between the arrivals in opposite direction at Smestad to avoid conflict
C^{Train}	Variable costs of a train
C^{SB}	Variable costs of a step-back driver

Decision Variables

t_{iar}	Time of arrival at Tøyen for arrival a of line i, in direction r
t_r^{min}	Earliest time of arrival of a train at Tøyen, in direction r, within the ground period P^G
t_r^{max}	Latest time of arrival of a train at Tøyen, in direction r, within the ground period P^G
δ_{ir}	$:= \begin{cases} \text{1, if the break on line } i \text{ is taken at terminal station in direction } r \\ \text{0, otherwise} \end{cases}$
$\omega_{ia,jb,r}$	$:= \begin{cases} \text{1, if departure } a \text{ on line } i \text{ bounded for direction } r \text{ arrives earlier} \\ \text{than departure } b \text{ on line } j \text{ bounded for direction } r \\ \text{0, otherwise} \end{cases}$
η_{pr}	$:= \begin{cases} \text{1, if pattern } p \text{ of arrivals is chosen in direction } r \text{ on stations Blindern,} \\ \text{Forskningsparken and Ullevål Stadion} \\ \text{0, otherwise} \end{cases}$
μ_{ur}	$:= \begin{cases} \text{1, if pattern } u \text{ of arrivals is chosen in direction } r \text{ on stations Smestad and Borgen} \\ \text{0, otherwise} \end{cases}$
ζ_{kr}	Time interval k between arrivals in direction r on stations Blindern, Forskningsparken and Ullevål
ι_{lr}	Time interval l between arrivals in direction r on stations Smestad and Borgen
τ_{iar}	The earliest possible time of return at Tøyen of arrival a on line i in direction r, given t_{iar}

z_{iar} — Integer counting the number of ground periods P^G, that have passed between an arrival a of line i in direction r at Tøyen and the return of the same arrival at Tøyen in the opposite direction.

$$v_{iar,jbs} := \begin{cases} 1, \text{ if arrival } a \text{ of line } i \text{ in direction } r \text{ arrives earlier than arrival } b \\ \text{of line } j \text{ in direction } s \text{ at Vestli} \\ 0, \text{ otherwise} \end{cases}$$

n_r — Integer counting the number of ground periods needed to coordinate the two arrivals at Tøyen associated with Ringen in direction r

m_r — Integer counting the number of ground periods needed to coordinate the two arrivals at Vestli in direction r

o_{iar} — Integer counting the number of ground periods that have passed when arrival a on line i in direction r is situated at Smestad.

$$\sigma_{ir} := \begin{cases} 1, \text{ if step-back driver is used on line } i \text{ in direction } r \\ 0, \text{ otherwise} \end{cases}$$

λ_{iar} — The regulation time allocated to the terminal station of an arrival a along line i in direction r.

$$\phi_{ir} := \begin{cases} 1 \text{ , if the arrivals on the double line } i \text{ switch time slots in direction } r \\ 0, \text{ otherwise} \end{cases}$$

$$\rho_{ia} := \begin{cases} 1, \text{ if the arrival on Kolsås in direction west arrives Smestad before arrival } a \\ \text{on line } i \text{ on Østerås in direction east} \\ 0, \text{ otherwise} \end{cases}$$

Objective Function

$$min \sum_{i \in I} \sum_{a \in A_i} \sum_{r \in R} z_{iar} C^{Train} + \sum_{r \in R} n_r C^{Train} + \sum_{i \in I} \sum_{r \in R} \sigma_{ir} C^{SB} \tag{5.7}$$

The objective function in equation (5.7) minimizes the total cost associated with the number of trains and step-back drivers necessary to satisfy the requirements of the timetable. The integer z_{iar} counts the number of ground periods between the arrivals of the same train at Tøyen in opposite directions. Therefore it corresponds to the number of trains needed to maintain the frequency of one arrival per ground period P^G. z_{iar} hence corresponds to the round-trip time of arrival a on line i in direction r divided by the frequency of line i, rounded up to the closest integer. The variables z_{iar} take positive values for all lines i, on all arrivals a in both directions r, except for the lines and directions corresponding to the Ringen branches. These specific variables are set to zero. The number of trains needed on the Ringen branches is calculated separately by summation of n_r because these specific branches are modelled differently from the rest. By adding z_{iar} and n_r, the total number of trains needed to maintain the frequencies on all the lines in the metro network is obtained. The variable costs of a train, C^{Train}, is multiplied by z_{iar} and n_r to find the total variable costs of all trains needed in operation. The last term of the function represents the total variable costs of step-back drivers, based on the number of step-back drivers used in the operations and the associated cost per driver C^{SB}.

Constraints

Headway Time Constraints

$$t_{iar} \leq P^G \qquad i \in I, a \in A_i, r \in R \tag{5.8}$$

Constraints (5.8) guarantee that the time of arrival of a train at Tøyen is within the ground period P^G.

$$t_{iar} \geq t_r^{min} \qquad i \in I, a \in A_i, r \in R \tag{5.9}$$

$$t_{iar} \leq t_r^{max} \qquad i \in I, a \in A_i, r \in R \tag{5.10}$$

$$t_r^{max} + H - P^G \leq t_r^{min} \qquad r \in R \tag{5.11}$$

Constraints (5.9) - (5.11) ensure that the first and last train arriving Tøyen in the ground period P^G have sufficient time between them, i.e. the specified headway time H. This is due to the periodicity of the problem, where the ground period repeats itself throughout the timetable.

$$t_{iar} + H \leq t_{jbr} + (1 - \omega_{ia,jb,r})(P^G + H) \qquad i, j \in I, a \in A_i, b \in A_j, r \in R |\ i \neq j \tag{5.12}$$

$$t_{jbr} + H \leq t_{iar} + \omega_{ia,jb,r}(P^G + H) \qquad i, j \in I, a \in A_i, b \in A_j, r \in R |\ i \neq j \tag{5.13}$$

Constraints (5.12) and (5.13) secure that when a train along a line i arrives at Tøyen before a train from another line j, then the headway time between the trains is respected.

$$t_{iar} + H \leq t_{ibr} + (1 - \omega_{ia,ib,r})(P^G + H) \qquad i \in I, a, b \in A_i, r \in R |\ a \neq b \tag{5.14}$$

$$t_{ibr} + H \leq t_{iar} + \omega_{ia,ib,r}(P^G + H) \qquad i \in I, a, b \in A_i, r \in R |\ a \neq b \tag{5.15}$$

To ensure that also two arrivals a and b serving the same line i will have at least the headway time between them, constraints (5.14) and (5.15) are used.

Frequency Constraints On Double-Frequency Branches

$$t_{ibr} - t_{iar} \geq \underline{F^2} - (1 - \omega_{ia,ib,r})(P^G + \underline{F^2}) \qquad g \in G^1, i \in I_g, a, b \in A_i, r \in R|\ a \neq b \tag{5.16}$$

$$t_{ibr} - t_{iar} \leq \overline{F^2} + (1 - \omega_{ia,ib,r})(P^G + \overline{F^2}) \qquad g \in G^1, i \in I_g, a, b \in A_i, r \in R|\ a \neq b \tag{5.17}$$

$$t_{iar} - t_{ibr} \geq \underline{F^2} - \omega_{ia,ib,r}(P^G + \underline{F^2}) \qquad g \in G^1, i \in I_g, a, b \in A_i, r \in R|\ a \neq b \tag{5.18}$$

$$t_{iar} - t_{ibr} \leq \overline{F^2} + \omega_{ia,ib,r}(P^G + \overline{F^2}) \qquad g \in G^1, i \in I_g, a, b \in A_i, r \in R|\ a \neq b \tag{5.19}$$

Constraints (5.16)-(5.19) ensure that the two arrivals in a double-frequency line arrive at Tøyen with sufficient time interval between them, i.e. a minimum of $\underline{F^2}$ minutes and a maximum of $\overline{F^2}$ minutes. The constraints also take into consideration whether arrival a of line i arrives earlier than arrival b of the same line i, and vice versa. This is done through the decision variable $\omega_{ia,ib,r}$.

$$t_{jbr} - t_{iar} \geq \underline{F^2} - (1 - \omega_{ia,jb,r})(P^G + \underline{F^2}) \qquad g \in G^2, i, j \in I_g, a \in A_i, b \in A_j, r \in R|\ i \neq j \tag{5.20}$$

$$t_{jbr} - t_{iar} \leq \overline{F^2} - (1 - \omega_{ia,jb,r})(P^G + \overline{F^2}) \qquad g \in G^2, i, j \in I_g, a \in A_i, b \in A_j, r \in R|\ i \neq j \tag{5.21}$$

$$t_{iar} - t_{jbr} \geq \underline{F^2} - \omega_{ia,jb,r}(P^G + \underline{F^2}) \qquad g \in G^2, i, j \in I_g, a \in A_i, b \in A_j, r \in R|\ i \neq j \tag{5.22}$$

$$t_{iar} - t_{jbr} \leq \overline{F^2} - \omega_{ia,jb,r}(P^G + \overline{F^2}) \qquad g \in G^2, i, j \in I_g, a \in A_i, b \in A_j, r \in R|\ i \neq j \tag{5.23}$$

Constraints (5.20)- (5.23) make sure that the branches with double-frequency requirements which are served by two single-frequency lines, maintain the required time interval distribution of between $\underline{F^2}$ and $\overline{F^2}$ minutes. Also, the arrival order of line i and line j is taken into account through the decision variable $\omega_{ia,jb,r}$. The factors $(P^G + \underline{F^2})$ and $(P^G + \overline{F^2})$ in constraints (5.16)-(5.23) are big-M notations, ensuring that a constraint never becomes binding if the event it represents does not occur.

Frequency Constraints for Triple-Frequency Requirement Areas

Three Single-Frequency Branches Sharing Railway Tracks

$$t_{q'} + \zeta_{1r} \leq t_{q''} + P^G(1 - \eta_{pr}) \qquad p \in P, q' \in Q_p(1), q'' \in Q_p(2), r \in R \tag{5.24}$$

$$t_{q'} + \zeta_{1r} \geq t_{q''} - P^G(1 - \eta_{pr}) \qquad p \in P, q' \in Q_p(1), q'' \in Q_p(2), r \in R \tag{5.25}$$

$$t_{q''} + \zeta_{2r} \leq t_{q'''} + P^G(1 - \eta_{pr}) \qquad p \in P, q'' \in Q_p(2), q''' \in Q_p(3), r \in R \tag{5.26}$$

$$t_{q''} + \zeta_{2r} \geq t_{q'''} - P^G(1 - \eta_{pr}) \qquad p \in P, q'' \in Q_p(2), q''' \in Q_p(3), r \in R \tag{5.27}$$

$$t_{q'''} + \zeta_{3r} - P^G \leq t_q + P^G(1 - \eta_{pr}) \qquad p \in P, q \in Q_p(1), q''' \in Q_p(3), r \in R \tag{5.28}$$

$$t_{q'''} + \zeta_{3r} - P^G \geq t_q - P^G(1 - \eta_{pr}) \qquad p \in P, q \in Q_p(1), q''' \in Q_p(3), r \in R \tag{5.29}$$

$$\underline{F^3} \leq \zeta_{yr} \leq \overline{F^3} \qquad y \in Y, r \in R \tag{5.30}$$

Constraints (5.24) - (5.30) ensure that the time interval requirements between consecutive trains at triple-frequency areas served by three single lines are maintained. The requirements are time intervals between $\underline{F^3}$ and $\overline{F^3}$ minutes between consecutive arrivals.

$$\sum_{y \in Y} \zeta_{yr} = P^G \qquad r \in R \tag{5.31}$$

$$\sum_{p \in P} \eta_{pr} = 1 \qquad r \in R \tag{5.32}$$

Constraints (5.31) ensure that the time intervals between the arrivals are distributed on the ground period, P^G. Constraints (5.32) make sure that only one pattern of arrivals is chosen in each direction.

$$t_{w'} + \iota_{1r} \leq t_{w''} + P^G(1 - \mu_{ur}) \qquad u \in U, w' \in W_u(1), w'' \in W_u(2), r \in R \quad (5.33)$$

$$t_{w'} + \iota_{1r} \geq t_{w''} - P^G(1 - \mu_{ur}) \qquad u \in U, w' \in W_u(1), w'' \in W_u(2), r \in R \quad (5.34)$$

$$t_{w''} + \iota_{2r} \leq t_{w'''} + P^G(1 - \mu_{ur}) \qquad u \in U, w'' \in W_u(2), w''' \in W_u(3), r \in R \quad (5.35)$$

$$t_{w''} + \iota_{2r} \geq t_{w'''} - P^G(1 - \mu_{ur}) \qquad u \in U, w'' \in W_u(2), w''' \in W_u(3), r \in R \quad (5.36)$$

$$t_{w'''} + \iota_{3r} - P^G \leq t_{w'} + P^G(1 - \mu_{ur}) \qquad u \in U, w' \in W_u(1), w''' \in W_u(3), r \in R \quad (5.37)$$

$$t_{w'''} + \iota_{3r} - P^G \geq t_{w'} - P^G(1 - \mu_{ur}) \qquad u \in U, w' \in W_u(1), w''' \in W_u(3), r \in R \quad (5.38)$$

$$\underline{F^3} \leq \iota_{lr} \leq \overline{F^3} \qquad l \in L, r \in R \quad (5.39)$$

Constraints (5.33) - (5.39) ensure that the time interval requirements between consecutive trains at triple-frequency areas served by one double-frequency line and one single-frequency line are maintained. The requirements are time intervals between $\underline{F^3}$ and $\overline{F^3}$ minutes between consecutive arrivals.

$$\sum_{l \in L} \iota_{lr} = P^G \qquad r \in R \quad (5.40)$$

$$\sum_{u \in U} \mu_{ur} = 1 \qquad r \in R \quad (5.41)$$

Constraints (5.40) ensure that the time intervals between the arrivals will be distributed on the ground period, P^G. Constraints (5.41) make sure that only one pattern of arrivals is chosen in each direction.

Time of Return Constraints

$$\tau_{iar} = t_{iar} + D^S + 2T_{ir} + (1 - \sigma_{ir})(2\Delta \sum_{s \in R} T_{is})\delta_{ir} + D^T_{ir} \qquad g \in G^{XR} \backslash \{G^1\}, i \in I_g, a \in A_i, r \in R \tag{5.42}$$

$$\tau_{iar} = t_{iar} + D^S + 2T_{ir} + (1 - \sigma_{ir})(2\Delta \sum_{s \in R} T_{is})\delta_{ir} + D^T_{ir} \qquad g \in G^{RW}, i \in I_g, a \in A_i, r \in R^E \tag{5.43}$$

$$\tau_{iar} = t_{iar} + D^S + 2T_{ir} + (1 - \sigma_{ir})(2\Delta \sum_{s \in R} T_{is})\delta_{ir} + D^T_{ir} \qquad g \in G^{RE}, i \in I_g, a \in A_i, r \in R^W \tag{5.44}$$

$$\sum_{r \in R} \delta_{ir} = 1 \qquad i \in I \tag{5.45}$$

$$\delta_{ir} - \sigma_{ir} \leq 0 \qquad i \in I, r \in R \tag{5.46}$$

Constraints (5.42) define the time of return at Tøyen τ_{iar} for all branches g except for the branches in Ringen. The Ringen branches are excluded because the arrivals on the branches are coordinated in separate constraints. These are given and explained further in constraints (5.57)-(5.58). To ensure that the time of return is defined for the branches which are combined with the Ringen branches, constraints (5.43)-(5.44) are imposed. Constraints (5.45) ensure that the driver break is only taken at one of the two terminal stations of the line. Constraints (5.46) ensure that step-back drivers are used only at terminal stations where the driver break is taken.

Constraints Connecting the Same Arrival in Both Directions at Tøyen

$$\tau_{iar} - z_{iar}P^G + \lambda_{iar} = t_{ias} \qquad g \in G^{XR} \setminus \{G^1\}, i \in I_g, a \in A_i, r, s \in R | r \neq s \tag{5.47}$$

$$\tau_{iar} - z_{iar}P^G + \lambda_{iar} = t_{ias} \qquad g \in G^{RW}, i \in I_g, a \in A_i, r \in R^E, s \in R | r \neq s \tag{5.48}$$

$$\tau_{iar} - z_{iar}P^G + \lambda_{iar} = t_{ias} \qquad g \in G^{RE}, i \in I_g, a \in A_i, r \in R^W, s \in R | r \neq s \tag{5.49}$$

Constraints (5.47) make sure that the arrival of a train a of line i in direction r cannot arrive earlier at Tøyen than the time of return τ_{iar}. The integer variable z_{iar} is used to place the arrival time t_{ias} within the ground period P^G, i.e. it functions as a modulo operator for the cyclic planning process. The subsets Ringen branches are excluded in the subset G^{XR}, as the arrivals in Ringen, as mentioned in the constraints above, must be coordinated separately. This is further explained and performed in constraints (5.57)-(5.58). The subset G^1 is excluded, as it is accounted for in the following constraints concerning double-frequency lines. Constraints (5.48)-(5.49) are imposed to make sure that the arrivals in direction r and direction s are coordinated at Tøyen for branches combined with the Ringen branches. λ_{iar} is the regulation time, and corresponds to the time difference between the earliest time an arrival can return to Tøyen, τ_{iar}, and the actual return time, t_{ias}. Thus, the regulation time is the buffer added to the dwell time at the terminal station to make the timetable feasible and robust.

$$\tau_{iar}-z_{iar}P^G+\lambda_{iar} \leq t_{ibs}+P^G(1-\phi_{ir}) \qquad g \in G^1, i \in I_g, a, b \in A_i, r, s \in R | a \neq b, r \neq s \tag{5.50}$$

$$\tau_{iar}-z_{iar}P^G+\lambda_{iar} \geq t_{ibs}-P^G(1-\phi_{ir}) \qquad g \in G^1, i \in I_g, a, b \in A_i, r, s \in R | a \neq b, r \neq s \tag{5.51}$$

$$\tau_{iar}-z_{iar}P^G+\lambda_{iar} \leq t_{ibs}+P^G\phi_{ir} \qquad g \in G^1, i \in I_g, a, b \in A_i, r, s \in R | a \neq b, r \neq s \tag{5.52}$$

$$\tau_{iar}-z_{iar}P^G+\lambda_{iar} \geq t_{ibs}-P^G\phi_{ir} \qquad g \in G^1, i \in I_g, a, b \in A_i, r, s \in R | a \neq b, r \neq s \tag{5.53}$$

The constraints (5.50) - (5.53) allow arrivals on double lines to change time slots. That is, the arrival arriving first at Tøyen in one direction may arrive secondly at Tøyen in the opposite direction. The binary variable ϕ_{ir} indicates whether there is a time slot change on line i in direction r, or not. Constraints (5.50) and (5.51) are active when there is a change in time slot. Constraints (5.52) and (5.53) are active when there is no time slot change. These latter constraints correspond to constraints (5.47).

Constraints Concerning Regulation Time

$$\lambda_{iar} \geq L_{ir} \qquad g \in G^{XR}, i \in I_g, a \in A_i, r \in R \tag{5.54}$$

$$\lambda_{iar} \geq L_{ir} \qquad g \in G^{RW}, i \in I_g, a \in A_i, r \in R^E \tag{5.55}$$

$$\lambda_{iar} \geq L_{ir} \qquad g \in G^{RE}, i \in I_g, a \in A_i, r \in R^W \tag{5.56}$$

Constraints (5.54) make sure that the regulation time λ_{iar} imposed to withstand delays is larger than the delay parameter L_{iar} on the relevant branch. The buffer time is given per arrival traversing the branch in direction r on line i. This allows for the model to adapt the regulation time according to the need for a buffer for each arrival on each branch. The Ringen branches are excluded in these constraints. This is because the regulation time is not defined for these branches, as explained in Section 5.3.1. To ensure that the constraints are valid for the branches combined with the Ringen branches, constraints (5.55)-(5.56) are imposed.

Constraints concerning Ringen

$$\begin{gathered} t_{iar} + D^S + T_{ir} + T_{js} - n_r P^G = t_{jbr} \\ g \in G^{RW}, h \in G^{RE}, i \in I_g, j \in I_h, a \in A_i, b \in A_j, r \in R^W, s \in R^E \end{gathered} \tag{5.57}$$

$$\begin{gathered} t_{jbs} + D^S + T_{js} + T_{ir} - n_s P^G = t_{ias} \\ g \in G^{RW}, h \in G^{RE}, i \in I_g, j \in I_h, a \in A_i, b \in A_j, r \in R^W, s \in R^E \end{gathered} \tag{5.58}$$

$$\delta_{ir} = 0 \qquad g \in G^{RW}, i \in I_g, r \in R^W \tag{5.59}$$

$$\delta_{ir} = 0 \qquad g \in G^{RE}, i \in I_g, r \in R^E \tag{5.60}$$

Constraints (5.57) and (5.58) require that the train leaving Tøyen along Ringen West, must be back at Tøyen after the exact time it takes to traverse the circle. This must be done as the real life Ringen is just one line, and not two lines as constructed in the LPP. A train arriving Sinsen serving Ringen West, should not turn at Sinsen but simply continue its journey, now as a train serving Ringen East. This also applies to the opposite directions, i.e. a train going from Ringen East to Ringen West. Because of this, it is necessary to forbid the model to decide that a driver should take her or his driver break at Sinsen, as this is just a modelled terminal station. This is avoided through constraints (5.59) and (5.60).

Constraints Concerning the Vestli Branches

$$0 \leq t_{iar} + D^S + T_{ir} - m_r P^G \leq P^G \qquad g \in G^{VW}, i \in I_g, a \in A_i, r \in R^W \tag{5.61}$$

$$0 \leq t_{iar} + D^S + T_{ir} - m_r P^G \leq P^G \qquad g \in G^{VE}, i \in I_g, a \in A_i, r \in R^E \tag{5.62}$$

$$(t_{jbs} + T_{js} - m_s P^G) - (t_{iar} + T_{ir} - m_r P^G) \geq \underline{F^2} - (1 - v_{iar,jbs})(P^G + \underline{F^2})$$

$$g \in G^{VW}, h \in G^{VE}, i \in I_g, j \in I_h, a \in A_i, b \in A_j, r \in R^W, s \in R^E \tag{5.63}$$

$$(t_{iar} + T_{ir} - m_r P^G) - (t_{jbs} + T_{js} - m_s P^G) \geq \underline{F^2} - v_{iar,jbs}(P^G + \underline{F^2})$$

$$g \in G^{VW}, h \in G^{VE}, i \in I_g, j \in I_h, a \in A_i, b \in A_j, r \in R^W, s \in R^E \tag{5.64}$$

$$(t_{jbs} + T_{js} - m_s P^G) - (t_{iar} + T_{ir} - m_r P^G) \leq \overline{F^2} + (1 - v_{iar,jbs})(P^G + \overline{F^2})$$

$$g \in G^{VW}, h \in G^{VE}, i \in I_g, j \in I_h, a \in A_i, b \in A_j, r \in R^W, s \in R^E \tag{5.65}$$

$$(t_{iar} + T_{ir} - m_r P^G) - (t_{jbs} + T_{js} - m_s P^G) \leq \overline{F^2} + v_{iar,jbs}(P^G + \overline{F^2})$$

$$g \in G^{VW}, h \in G^{VE}, i \in I_g, j \in I_h, a \in A_i, b \in A_j, r \in R^W, s \in R^E \tag{5.66}$$

Constraints (5.63)- (5.66) ensure that the trains serving the lines bound for Vestli arrives at the terminal station with an time interval distribution between $\underline{F^2}$ and $\overline{F^2}$ minutes. This provides sufficient frequency of arrivals on the Vestli branch. Constraints (5.61) and (5.62) keep the ground period adjustment variable m_r from taking on too large values, to assure a time slot within the ground period.

Constraints Concerning the Area of Conflict at Smestad Station

$$0 \leq t_{iar} + D^T + T^{Smestad} - o_{iar} P^G \leq P^G \qquad g \in G^K, i \in I_g, a \in A_i, r \in R^W \tag{5.67}$$

$$0 \leq t_{iar} - T^{Smestad} + o_{iar} P^G \leq P^G \qquad g \in G^O, i \in I_g, a \in A_i, r \in R^E \tag{5.68}$$

$$(t_{iar} + D^T + T^{Smestad} - o_{iar} P^G) - (t_{jbs} - T^{Smestad} + o_{jbs} P^G) \geq B^{Smestad} + \rho_{ib} P^G$$

$$g \in G^K, h \in G^O, i \in I_g, j \in I_h, a \in A_i, b \in A_j, r \in R^W, s \in R^E \tag{5.69}$$

$$(t_{jbs} - T^{Smestad} + o_{jbs}P^G) - (t_{iar} + D^T + T^{Smestad} - o_{iar}P^G) \geq B^{Smestad} + (1 - \rho_{ib})P^G$$

$$g \in G^K, h \in G^O, i \in I_g, j \in I_h, a \in A_i, b \in A_j, r \in R^W, s \in R^E \qquad (5.70)$$

Constraints (5.67)-(5.70) concern the conflict at Smestad station, where the westbound arrivals along branch Kolsås must cross the railway tracks which they share with the arrivals on branch Østerås. Constraints (5.67) and (5.68) ensure that the time of arrivals at Smestad are kept within the ground period. Conflict at Smestad station is avoided through constraints (5.69) and (5.70), which assure a minimum time buffer $B^{Smestad}$ between the arrivals in opposite directions.

Constraints Concerning Conflicts At the Terminal Stations

$$D_{ir}^T + (1 - \sigma_{ir})2\Delta \sum_{s \in R} T_{is}\delta_{ir} + \lambda_{iar} \leq \overline{D} \qquad g \in G^{WT}, i \in I_g, a \in A_i, r \in R^W \quad (5.71)$$

$$D_{ir}^T + (1 - \sigma_{ir})2\Delta \sum_{s \in R} T_{is}\delta_{ir} + \lambda_{iar} \leq \overline{D} \qquad g \in G^{ET}, i \in I_g, a \in A_i, r \in R^E \quad (5.72)$$

Constraints (5.71)-(5.71) ensure that the time spent dwelling at terminal stations on specific west and east branches does not exceed a maximum threshold $\overline{D}$. The time spent at a terminal station consists of the technical dwell time, D_{ir}^T, the potential driver break, and the regulation time λ_{iar}.

Binary and Non-negativity Constraints

$$\delta_{ir} \in \{0, 1\} \qquad i \in I, r \in R \qquad (5.73)$$

$$\omega_{ia,jb,r} \in \{0, 1\} \qquad i, j \in I, a \in A_i, b \in A_j, r \in R \qquad (5.74)$$

$$\upsilon_{iar,jbs} \in \{0, 1\} \qquad i, j \in I, a \in A_i, b \in A_j, r, s \in R \qquad (5.75)$$

$$\eta_{pr} \in \{0, 1\} \qquad p \in P, r \in R \qquad (5.76)$$

$$\mu_{sr} \in \{0, 1\} \qquad s \in S, r \in R \qquad (5.77)$$

$$\sigma_{ir} \in \{0,1\} \qquad i \in I, r \in R \tag{5.78}$$

$$\phi_{ir} \in \{0,1\} \qquad g \in G^1, i \in I_g, r \in R \tag{5.79}$$

$$\rho_{ia} \in \{0,1\} \qquad g \in G^K \cap G^0, i \in I_g, a \in A_i \tag{5.80}$$

$$t_{iar} \geq 0 \qquad i \in I, a \in A_i, r \in R \tag{5.81}$$

$$t_r^{min} \geq 0 \qquad r \in R \tag{5.82}$$

$$t_r^{max} \geq 0 \qquad r \in R \tag{5.83}$$

$$\zeta_{kr} \geq 0 \qquad k \in K, r \in R \tag{5.84}$$

$$\iota_{lr} \geq 0 \qquad l \in L, r \in R \tag{5.85}$$

$$\lambda_{iar} \geq 0 \qquad g \in G \setminus \{G^{RW}, G^{RE}\} i \in I_g, a \in A_i, r \in R \tag{5.86}$$

$$\tau_{iar} \geq 0 \qquad i \in I, a \in A_i, r \in R \tag{5.87}$$

$$z_{iar} \in \mathbb{Z}^+ \qquad i \in I, a \in A_i, r \in R \tag{5.88}$$

$$m_r \in \mathbb{Z}^+ \qquad r \in R \tag{5.89}$$

$$n_r \in \mathbb{Z}^+ \qquad r \in R \tag{5.90}$$

$$o_{iar} \in \mathbb{Z}^+ \qquad g \in G^K \cap G^0, i \in I_g, a \in A_i \tag{5.91}$$

5.4 Extended Mathematical Model

In this section, extensions of the mathematical model given in the previous Section 5.3.2 are presented.

As explained in Section 5.3.2, constraints (5.54) ensure that the regulation time imposed to withstand delays is larger than the deviation parameter on the relevant branch. In the extended model, these constraints are soft. Soft constraints allow a deviation from the preferred conditions, and the deviation is penalized in the objective function (Lundgren, 2010). This means that the constraints (5.54) may be violated but at a given penalty. Another extension to the model is that the amount of regulation time exceeding the deviation parameter for a given line is rewarded in the objective function.

In Section 5.4.1, the penalty function and the reward in the extended model are described. The mathematical formulation of the extensions is presented in Section 5.4.2.

5.4.1 Penalty Function and Reward in the Extended Model

Piecewise Linear Penalty Function

The extended model is implemented with a piecewise linear penalty function, as defined in Lundgren (2010). The piecewise penalty function is illustrated in Figure 5.6. The graph is discontinuous in the breakpoints e. A bound Π_e, a weight ω_e, and a penalty parameter P_e are associated with each breakpoint. The weight ω_e corresponds to the coefficient of the penalty parameter P_e in the penalty function. As can be observed in Figure 5.6, the penalty function is linear in each interval, and the slope of the curve differs for each interval. Thus the violation v is penalized linearly according to which interval it belongs to.

Equation 5.92 defines the penalty function. It sums over the set of breakpoints.

$$f(v) = \sum_{e \in E} P_e w_e \tag{5.92}$$

Hence, the penalty function is a weighted sum of the penalty parameter of the breakpoints.

Reward of Robustness

The positive difference, $(\lambda_{iar}$ - $L_{ir})$, between the regulation time and the deviation parameter is rewarded in the objective function. Since the problem is a minimization problem, the reward is subtracted from the objective function. If this difference is negative, it is not rewarded.

An upper limit to the regulation variable, $\overline{\lambda}$, is introduced in the extended model to prevent the model from becoming unbounded. Since the difference $(\lambda_{iar}$ - $L_{ir})$ is subtracted from the objective function, this term must be prevented from taking infinitely large values.

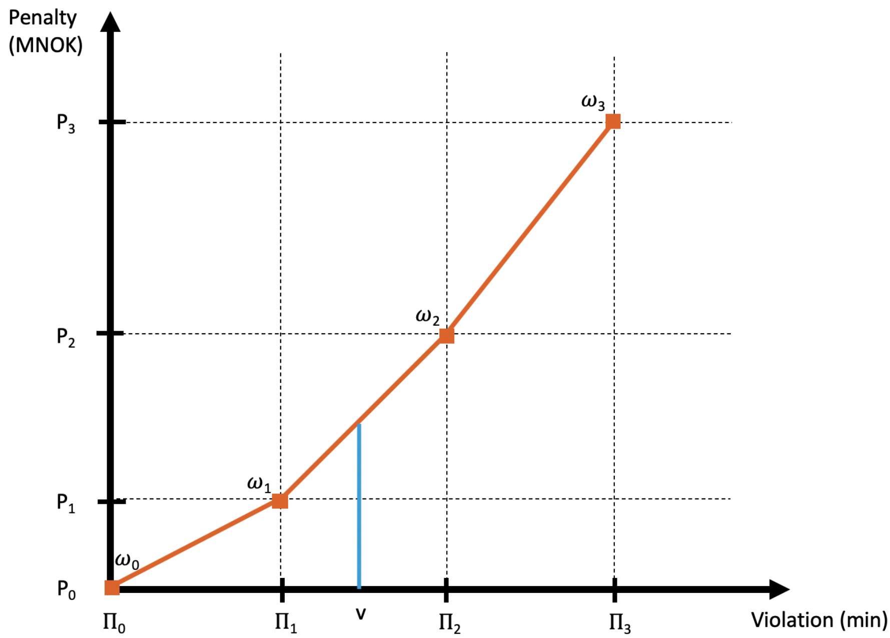

Figure 5.6: The piecewise linearity of the penalty function. In this example, ω_1 and ω_2 equals 0.5 for the violation variable v. The other weights equal 0.

5.4.2 Mathematical Formulation of the Extended Model

The extended model is based on the mathematical model described in Section 5.3.2. In addition, it contains the sets, parameters, variables, constraints, and objective function terms defined in this section.

As explained in Section 5.3.2, the branches of Ringen have no terminal stations. Therefore the constraints of the extended model which concern regulation time, are not defined for the lines including the Ringen branches.

Sets

E	Set of break points
J	Set of penalty intervals
J_e	Set of intervals having break point e as either lower or upper bound

Indices

$e \in E$	Breakpoint
$j \in J$	Penalty interval

Parameters

Π_e	Bound of breakpoint e
P_e	Penalty value associated to breakpoint e
S	Reward per unit α_{iar}
$\overline{\lambda}$	Upper limit for the regulation time
$\underline{L}$	Value of the lowest deviation parameter

Variables

d_{iar}	Violation of constraint (5.54) for arrival a on line i in direction r
w_{eiar}	Weight of break point e for deviation d_{iar} for arrival a on line i in direction r

$$\kappa_{jiar} \quad := \begin{cases} 1 \text{ , if the deviation } d_{iar} \text{ is within interval } j \text{ for arrival } a \text{ on line } i \text{ in direction } r \\ 0\text{, otherwise} \end{cases}$$

$$\beta_{iar} \quad := \begin{cases} 1\text{, if regulation time } \lambda_{iar} \text{ is larger than deviation } L_{ir} \text{ for arrival } a \text{ on line } i \text{ in direction r} \\ 0\text{, otherwise} \end{cases}$$

$$\alpha_{iar} \quad := \begin{cases} (\lambda_{iar} \text{ - } L_{ir})\text{, if violation } \lambda_{iar} \text{ is larger than deviation } L_{ir} \text{ for arrival } a \text{ on line } i \text{ in direction } r \\ 0\text{, otherwise} \end{cases}$$

Objective function

$$\sum_{i \in I} \sum_{a \in A_i} \sum_{r \in R} (f(d_{iar}) - S\alpha_{iar}) \tag{5.93}$$

The penalty function $f(d_{iar})$, depending on the violation parameter d_{iar}, is added to the objective function. For each unit of positive difference, α_{iar}, the reward parameter S is subtracted from the objective function.

Constraints

$$\lambda_{iar} + d_{iar} \geq L_{ir} \qquad g \in G^{XR}, i \in I_g, a \in A_i, r \in R \tag{5.94}$$

The constraints (5.54) are softened by introducing the violation variable d_{iar} as done in equation (5.94). By assigning d_{iar} a positive value, the regulation time λ_{iar} may take lower values than the deviation parameter L_{ir}.

The constraints (5.95) - (5.98) are associated to the penalizing of the violation variable according to the penalty function described in Section (5.4.1).

$$\sum_{e \in E} \Pi_e w_{eiar} - d_{iar} = 0 \quad g \in G^{XR}, i \in I_g, a \in A_i, r \in R \tag{5.95}$$

$$\sum_{e \in E} w_{eiar} = 1 \quad g \in G^{XR}, i \in I, a \in A_i, r \in R \tag{5.96}$$

Constraints (5.95) ensure that the weight variables w_{eiar} take on values according to the violation variable d_{iar}. The weight variables w_{eiar} are continuous and the constraints (5.96) ensure that they sum up to one.

$$w_{eiar} \leq \sum_{j \in J_e} \kappa_{jiar} \quad e \in E, g \in G^{XR}, i \in I_g, a \in A_i, r \in R \tag{5.97}$$

Constraints (5.97) ensure that the binary variables κ_{jiar}, indicating which interval d_{iar} is within, take values according to the weights w_{eiar}.

$$\sum_{e \in E} \kappa_{jiar} = 1 \quad g \in G^{XR}, i \in I_g, a \in A_i, r \in R \tag{5.98}$$

Constraints (5.98) ensure that the violation variable, d_{iar}, is within exactly one interval.

The constraints (5.99) - (5.104) concern the reward of the difference α_{iar}. The big-M value ($\overline{\lambda}$ - $\underline{L}$) is the largest value the difference (λ_{iar} - L_{ir}) can take.

$$\lambda_{iar} \leq \overline{\lambda} \quad g \in G^{XR}, i \in I_g, a \in A_i, r \in R \tag{5.99}$$

Constraints (5.99) ensure that the regulation time variable is lower than the upper limit for regulation time.

$$\lambda_{iar} - L_{ir}\beta_{iar} \geq 0 \quad g \in G^{XR}, i \in I_g, a \in A_i, r \in R \tag{5.100}$$

$$\lambda_{iar} - L_{ir} \leq (\overline{\lambda} - \underline{L})\beta_{iar} \quad g \in G^{XR}, i \in I_g, a \in A_i, r \in R \tag{5.101}$$

Constraints (5.100) and (5.101) make β_{iar} take value 1 if λ_{iar} is larger than L_{ir}, and 0 otherwise.

$$\alpha_{iar} \leq \lambda_{iar} - L_{ir} + (\overline{\lambda} - \underline{L})(1 - \beta_{iar}) \quad g \in G^{XR}, i \in I_g, a \in A_i, r \in R \tag{5.102}$$

$$\alpha_{iar} - (\overline{\lambda} - \underline{L})\beta_{iar} \leq 0 \quad g \in G^{XR}, i \in I_g, a \in A_i, r \in R \tag{5.103}$$

$$\lambda_{iar} - L_{ir} - \alpha_{iar} + (\overline{\lambda} - \underline{L})\beta_{iar} \leq (\overline{\lambda} - \underline{L}) \qquad g \in G^{XR}, i \in I_g, a \in A_i, r \in R \quad (5.104)$$

Constraints (5.102) - (5.104) ensure that α_{iar} takes value (λ_{iar} - L_{ir}) if λ_{iar} is larger than L_{ir}, and 0 otherwise.

The value of the parameters of the extended model are presented in Section 7.2.2.

Binary and Non-negativity Constraints

$$\kappa_{jiar} \in \{0,1\} \qquad j \in J, i \in I, ainA_i, r \in R \quad (5.105)$$

$$\beta_{iar} \in \{0,1\} \qquad i \in I, ainA_i, r \in R \quad (5.106)$$

$$\alpha_{iar} \geq 0 \qquad i \in I, a \in A_i, r \in R \quad (5.107)$$

$$d_{iar} \geq 0 \qquad i \in I, a \in A_i, r \in R \quad (5.108)$$

$$w_{eiar} \geq 0 \qquad e \in E, i \in I, a \in A_i, r \in R \quad (5.109)$$

Chapter 6

Applying Combinatorial Benders' Decomposition as a Solution Method

As explained in Section 4.2, the solution of a planning step in the planning process shown in Figure 4.1 affects the optimal solution in the successive planning step. The problem investigated in this thesis concerns two of these steps. Solving these two problems separately can lead to sub-optimal solutions. This is because the set of lines that minimizes the number of trains in the LPP does not guarantee the same objective value when given as input to the TTP. In the solution method in this thesis, the LPP is integrated with the TTP to enhance the overall planning process. This solution method is a version of Combinatorial Benders' Decomposition.

The most important features of Benders' Decomposition are presented from a theoretical point of view in Section 6.1. In Section 6.2, the application of Combinatorial Benders' Decomposition as a solution method for the problem of Oslo Metro is elaborated. This approach is similar to the approach described by Harbo et al. (2019), but with certain modifications.

6.1 Benders' Decomposition

Before elaborating on how Benders' Decomposition is used in the solution of this problem, the theory of the method is introduced in Section 6.1.1. Section 6.1.2 provides a more detailed explanation of the Combinatorial Benders' Decomposition, as this is the version of Benders' Decomposition used as the solution method to solve the problem investigated in this thesis.

6.1.1 Benders' Decomposition

Benders' Decomposition, introduced by Benders (1962), decomposes a problem called the *full problem* into a *Master Problem* (MP) and a *Subproblem* (SP). The variables of the full problem are divided into two subsets. The MP solves the problem over the first subset of variables. The SP treats this subset as fixed according to the solution from the MP, and solves the problem over the second subset. Hence, Benders' Decomposition models the full problem as a nested optimization problem.

If the solution from the MP results in an infeasible SP, the SP generates a *feasibility cut*. A feasibility cut corresponds to a constraint forbidding the given solution. The cut is added to the MP, which is resolved. In case the SP returns an objective value which differs from that of the MP, an *optimality cut* is generated. Optimality cuts affect the objective value associated with solutions previously found by the MP. In effect, for the solutions returned for the second time by the MP, the objective value is based on the optimality conditions of the SP (Taşkın, 2011).

Hence, the MP and the SP are resolved alternately, exchanging information. The information from the MP concerns the solution, and the information from the SP is related to optimality and feasibility conditions.

6.1.2 Combinatorial Benders' Decomposition

Combinatorial optimization is used to find the optimal solution within a finite set of possible solutions. The Assignment Problem is an example of a problem that can be solved using combinatorial optimization techniques.

Codato and Fischetti (2006) suggests Combinatorial Benders' Cuts for Mixed Integer Linear Programming (MILP) problems. In this case, a basic 0-1 Integer Linear Programming (ILP) problem is on the form

$$min\{c^T x : Fx \leq g, x \in \{0,1\}^n\} \tag{6.1}$$

This simple structure is disturbed when introducing additional continuous variables y. These variables do not occur in the objective function, but are included in the restrictions such that

$$a_i^T y \geq b_i, i \in I \tag{6.2}$$

if and only if

$$x_{ij} = 1, i, j \in I \tag{6.3}$$

Also, the y-variables have a set of constraints on the form

$$Dy \geq e \tag{6.4}$$

Since the constraints (6.4) only include y, and not x, it is possible to split the variables such that the MP is solved in x-space and the SP is solved in y-space. The idea is that the SP returns constraints that work on the x-variables only:

$$\sum_{i \in C} x_{j(i)} \leq |C| - 1, \quad C \subseteq I \tag{6.5}$$

In constraints (6.5), C is the *minimal infeasible subsystem*. The constraints are referred to as the Benders Feasibility Cut (BFC). The Benders' Decomposition algorithm will produce a sequence of BFCs that ensure that the MP does not return a solution that has previously resulted in an infeasible SP.

Furthermore, for each feasible solution found by the SP, a Benders Optimality Cut (BOC) is generated. This cut corresponds to a constraint added to the MP. The BOCs are in the form:

$$c^T x \geq q, x \in \{0, 1\}^n \tag{6.6}$$

where q is the objective value of the SP when solved with x as input. If the MP returns a given solution more than once, it is forced to take the objective value found by the SP for that specific solution input.

6.2 Combinatorial Benders' Decomposition in the Planning Process of Oslo Metro

In Section 6.2.1, the motivation for using the solution method of Combinatorial Benders' Decomposition in the case of Oslo Metro is elaborated. In Section 6.2.2, the application of the method is described.

6.2.1 The Motivation for the Application of Combinatorial Benders' Decomposition in the Case of Oslo Metro

Firstly, the structure of the problem of this thesis is suitable for the application of Combinatorial Benders' Decomposition. The variables of the LPP are binary, and for each pair of branches they indicate whether that pair is combined to form a line or not. The TTP on the other hand, has integer, binary and continuous variables. These variables do not appear in the objective function. Furthermore, the constraints involving the variables of the TTP are only included in case line i is in fact in the set of lines suggested. Hence, the structure of the problem in this thesis is similar to the problem described in Section 6.1.2. Therefore the Combinatorial Benders' Decomposition is a suitable solution method for our problem. The LPP and TTP are used as MP and SP, respectively.

Secondly, the objective value of the LPP is an optimistic bound for the full problem, and the objective of the TTP gives a pessimistic bound. Sections 5.2.2 and 5.3.2 describe how the LPP and the TTP minimize the variable costs of trains and step-back drivers.

The TTP has additional operational constraints compared to the LPP. Hence, the LPP is a relaxation of the full problem. Furthermore, the cost parameters are chosen such that the costs computed in the LPP are equal to or lower than the costs of the TTP. This is further elaborated in Section 7.2.1. Therefore, the LPP provides a dual bound for the problem, and the SP provides a primal bound. Hence, for any given line plan, the objective value of the LPP is lower than or equal to the objective in the TTP.

Consequently, the structure of the problem and the optimality of the LPP and TTP make the Combinatorial Benders' Decomposition an effective solution method for the problem studied in this thesis.

6.2.2 The Application of Combinatorial Benders' Decomposition in the Oslo Metro

Algorithm 1 Combinatorial Benders' Decomposition.

1: initialize $continue$:= **true**
2: initialize counter $k := 0$
3: **while** $continue$:= **true** **do**
4:    increment counter $k := k + 1$
5:    solve MP
6:    solve SP based on MP output
7:    **if** SP infeasible **then**
8:       add BFC to MP
9:    **end if**
10:    **if** SP feasible **then**
11:       **if** objective values of MP and SP are different **then**
12:          add BOC to MP
13:       **else**
14:          $continue$:= **false**
15:       **end if**
16:    **end if**
17: **end while**
18: **return** MP and SP solutions

The pseudocode for the solution method is presented in Algorithm 1. For each iteration k, the MP generates a set of constructed lines, as seen in line five. This solution is used as input when solving the SP in line six. If the SP is infeasible, a BFC is generated and added to the MP, as seen in line eight. The BFC corresponds to a constraint forbidding the current set of lines, as explained in Section 6.1.2. In case the SP finds a solution with a different objective value than the one in the MP, a BOC is generated and added to the MP. This is shown in line 12. This cut forces the objective function of the MP to take the value of the objective found in the SP, when the SP was last solved with the same input. The iterative solving of the models terminates when the objective value in the MP and the SP are equal, as seen in line 14. In line 18, the optimal solutions of the MP and the SP are returned after termination.

The computation of an irreducibly inconsistent subsystem (IIS) has the potential of tightening the relaxation of the MP, and thus reduce the run time (Codato and Fischetti, 2006). However, preliminary testing showed that the computations of the IIS were more computationally demanding than the solving of the model without the inclusion of it. Therefore, the use of IIS is not included in this thesis.

Chapter 7

Model Input Data

This chapter presents and explains the parameters used in the mathematical formulations of the model. In Section 7.1, the historical data on travel time deviations are presented. The section further explains how the data is processed to estimate the deviation parameters. An overview of the given parameters from Sporveien and the estimated parameters is given in Section 7.2.

7.1 Data on Travel Times

This section presents the historical data on travel times, and how it is processed to estimate the minimum needed regulation time on each branch in the Oslo Metro. Section 7.1.1 discusses the motivation for the use of historical data to estimate individual buffer times for all branches. Section 7.1.2 explains the processing of the data received by Sporveien. In Section 7.1.3 the historical data on deviations in travel times in the Oslo metro is analysed using two different approaches. This is done to find suiting parameters concerning deviations. The estimation of parameters on travel times of the branch Veitvet is discussed and calculated in Section 7.1.4. Implications of disregarding correlations in the data used to estimate the deviation parameters are discussed in Section 7.1.5.

7.1.1 Motivation for the Use of Historical Data

As explained in Section 5.3.1 the deviation parameter sets a lower limit on the regulation time of each branch. Well-defined deviation parameters can thus be used to allocate sufficient regulation times at each branch, to make the system more robust with respect to delays. To achieve deviation parameters as close to actual operations as possible, real historical data is considered and processed. Sporveien has comprehensive operational data concerning each train when in operation. Therefore, it is possible to find the deviations from the planned arrival of each train in the system. This can also be called travel time deviations and is referred to as *deviations*. This deviation data can be used to estimate the deviation parameters of each branch, which results in regulation times that are individually fitted to all branches. They are therefore more likely to fit their individual need of buffer to withstand delays, which enhance the robustness of the timetables. Further, deviation parameters based on real operational data can contribute to more realistic and useful results of the mathematical model.

In the estimation of the deviation parameters, correlations in the data on travel time deviations are disregarded. The data is hence assumed independent, but it is acknowledged that the travel times in the actual operations of Oslo Metro are likely to be correlated.

A robust timetable is not necessarily the timetable with the highest total regulation time. The regulation time must be seen in relation to the need for buffer at the specific branches. Regulation times that are not individually fitted can contribute to unnecessary high regulation times at some branches, and considerably lower regulation times at others. This could lead to an unstable timetable, as some branches are significantly more prone to delays than others. This emphasizes the importance of well-chosen values of the deviation parameters.

7.1.2 Processing of Data on Travel Times Obtained from Sporveien

Sporveien has supplied data for almost every single departure from a terminal station and the intermediate station Tøyen during 2019. This information is formatted in Excel and results in a file of about 750 MB in size. The file contains almost 1 million rows of information, where each row corresponds to information about the arrival or departure of a train at either a terminal station or at Tøyen. Each column represents different aspects and characteristics concerning a departure. As a first step of the data processing, data which is considered irrelevant is removed. Further, because the raw data is given on the lines in the current system, it is necessary to sort the data on branches. As a consequence, the processed data contains information about the deviation of an arrival at Tøyen, assuming that it started on time from the terminal station of the branch. This results in information concerning the departure of every train along all branches at Tøyen, and whether it is on time, early or delayed. This is the source data on which the analysis in Section 7.1.3 is based on. Table 7.1 shows the contents of a row in Excel concerning a train along the Frognerseteren branch.

Table 7.1: Example of an Excel row with data concerning Frognerseteren.

First station	**Direction**	**Start time**	**Current station**	**Time of departure**	**Time of planned departure**	**Deviation (sec)**
FRS	East	05.01.2019 16:37:00	TØY	05.01.2019 17:17:34	05.01.2019 17:17:00	34

An important part of the processing of data concerns the removal of outliers. If a train is forced to stop due to an unforeseen event, this will have a large effect on the deviation at Tøyen. Analysis of smaller subsets of the data is especially affected by such events. The outliers are removed by choosing a threshold and remove all deviations that exceed this value.

7.1.3 Travel Time Deviations on Branches in Oslo Metro

Two approaches are used when investigating the deviations at Tøyen of each branch in the metro system. The first approach is to find the average deviation of the departures at Tøyen serving a specific branch. This is done by splitting each operational day into half-hour intervals, and finding the average deviation of every departure at Tøyen along a branch within this interval. This makes it possible to look at the effect of rush hours and normal passenger demand hours on the deviations. Figures 7.1 and 7.2 shows the average deviation throughout the day, with data from every departure in 2019, of the branches Frognerseteren and Ellingsrudåsen. These branches are chosen as they represent a west branch and an east branch, with single and double frequency, respectively. Mind that a positive deviation indicates a delay, e.g. a deviation of positive 40 seconds corresponds to a 40-second delay. A negative deviation value means that a train arrived early at Tøyen.

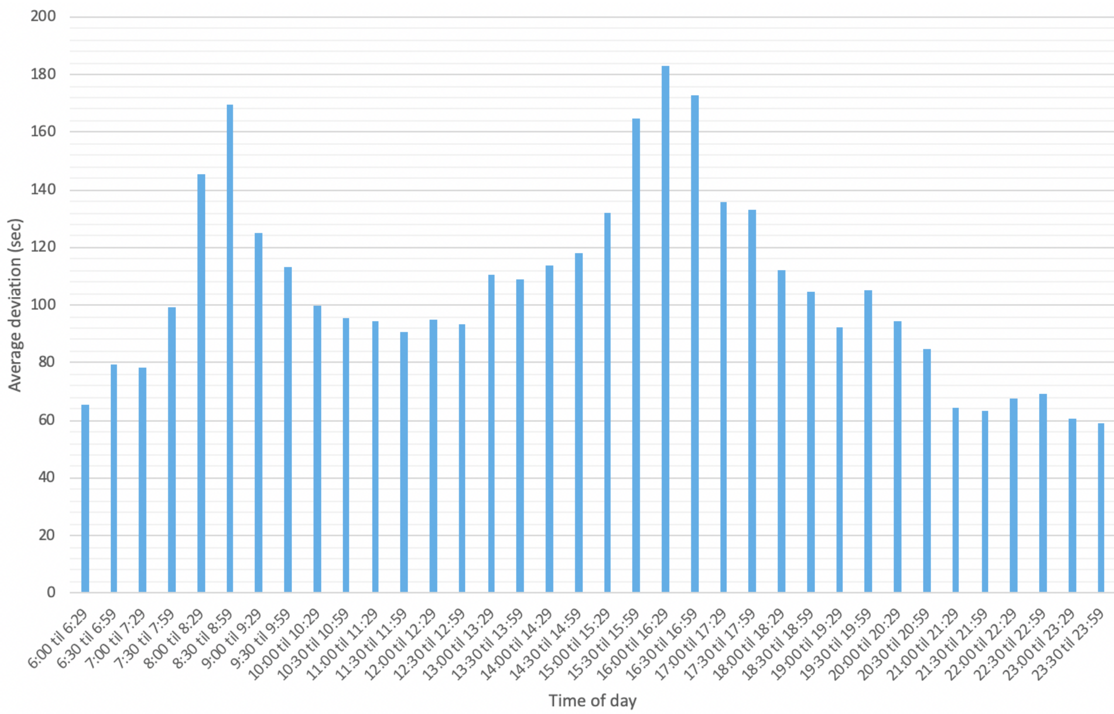

Figure 7.1: Average delay on the Frognerseteren branch throughout the day based on data from 2019. The x-axis represent the time of day, while the y-axis shows the average deviation of all departures at Tøyen along a specific branch at the given time interval.

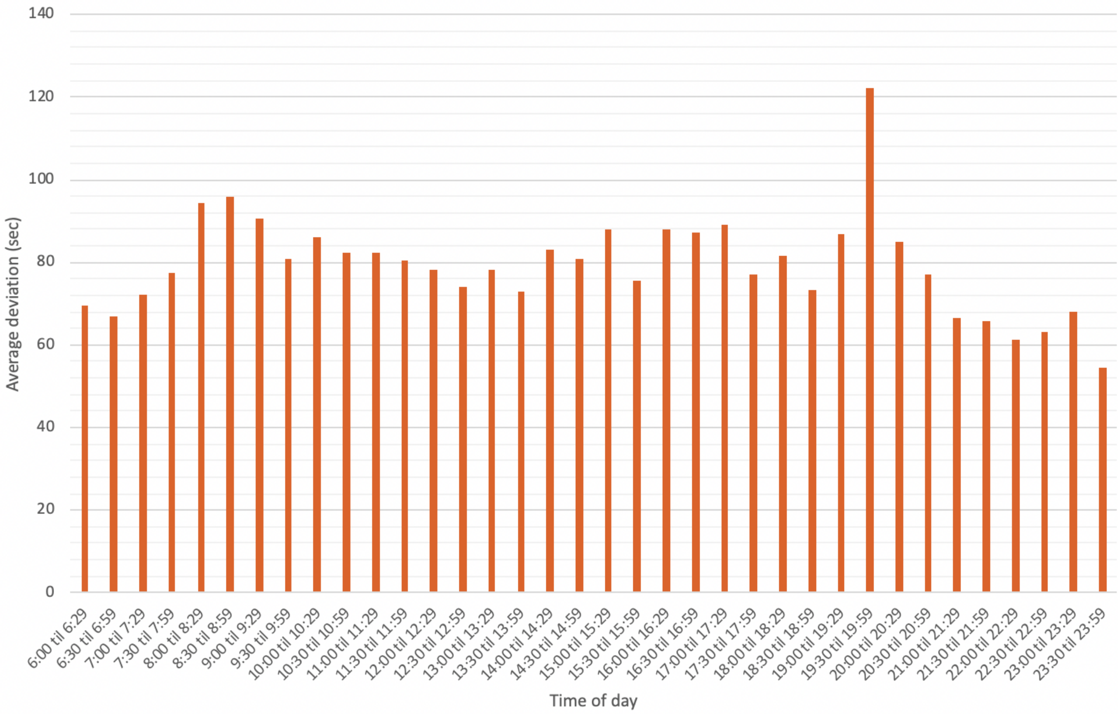

Figure 7.2: Average delay on the Ellingsrudåsen branch throughout the day based on data from 2019. The x-axis represent the time of day, while the y-axis shows the average deviation of all departures at Tøyen along a specific branch at the given time interval.

As can be observed from Figures 7.1 and 7.2, the travel times are sensitive to daily variations, like rush hours. The average delay from 2019 in the two time intervals from 08.00 to 09.00 and 16.00 to 17.00 is considerably higher than for the rest of the day.

A further motivation of the data analysis is to explore the sensitivity of travel times to yearly variations, like different seasons. Therefore, the arrival data is split into the month in which it appears, to investigate the deviations in separate months. From this analysis, the month of March stands out as the most delayed month for a majority of the branches. This can be due to external factors like unpredictable weather combined with generally high passenger demand. Figures 7.3 and 7.4 show the average deviations in the month of March of both branches Frognerseteren and Ellingsrudåsen.

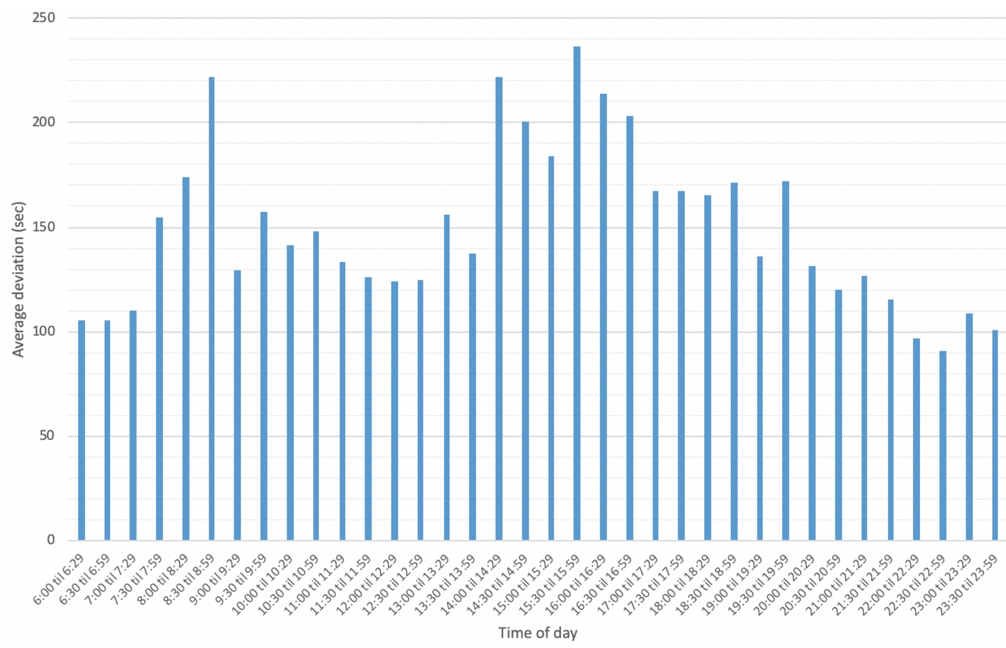

Figure 7.3: Average delay on the Frognerseteren branch in March 2019.

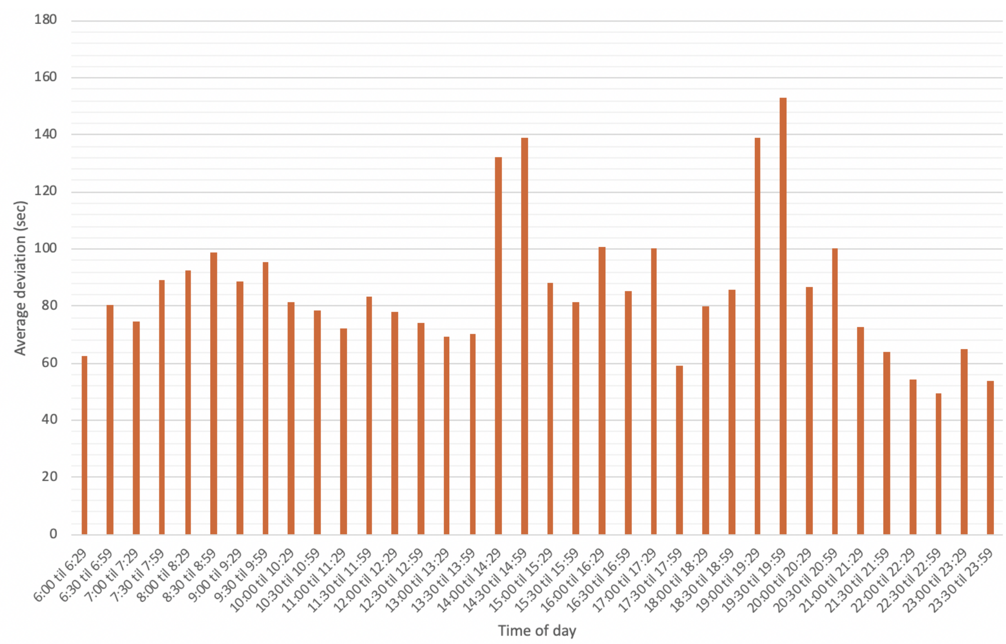

Figure 7.4: Average delay on the Ellingsrudåsen branch in March 2019.

It is from this analysis the deviation parameter, L_{ir}, is found. To construct a robust model, buffers that handle the delay in the most hectic parts of the day is necessary. Therefore, the average delay for the rush hour from 16.00 to 16.30 in March is chosen as the deviation parameter. As seen from Figures 7.3 and 7.4, this results in average delays, and thereby deviation parameters, of about 3.5 minutes and 1.5 minutes on the branches Frognerseteren and Ellingsrudåsen, respectively. As the average delays in March are higher than the average delays in the other months, this contributes to a timetable that is constructed to handle the passenger demand throughout the year. This is also the reason why the deviation data from March is chosen as a data basis for further analysis concerning deviations and robustness in Chapter 8. The numerical values of the estimated deviation parameters are provided in Table 7.3 in Section 7.2.1.

The corresponding standard deviations of the monthly average delay of March, are displayed in Figures 7.5 and 7.6. It can be observed that the standard deviations of both monthly measurements are relatively high, and in some cases even higher than the corresponding average. This shows that the data is subject to fluctuation. The fluctuations can be caused by effects in the normal operations like high passenger demand, by unforeseen events such as errors that cause trains to stop and by external factors like weather.

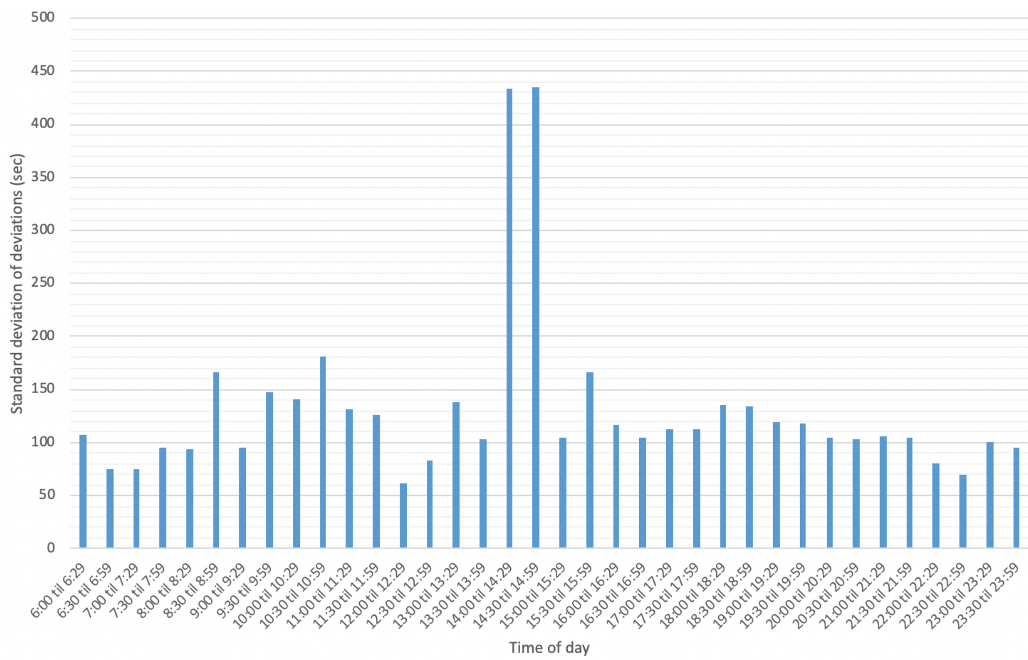

Figure 7.5: Standard deviation of the deviation measurements of the Frognerseteren branch in March 2019.

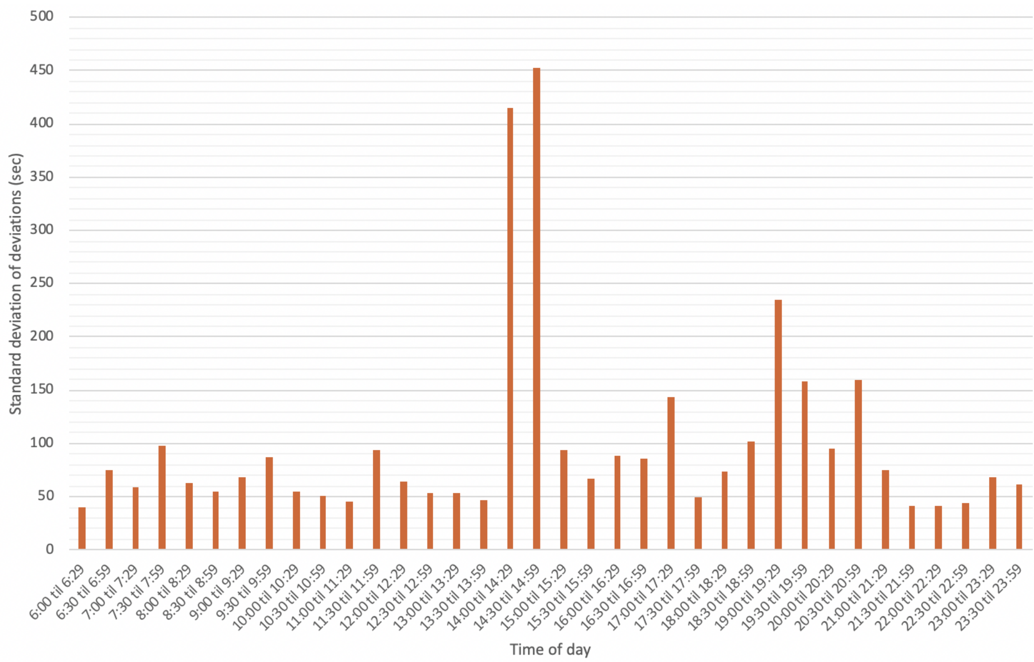

Figure 7.6: Standard deviation of the deviation measurements of the Ellingsrudåsen branch in March 2019.

The second approach to data investigation is to find the distribution of deviations of each branch in the system. This means how often a deviation occurs on a specific branch throughout the year. This is used to substantiate the choice of penalty time intervals in the extended model, which is further discussed in Section 7.2.2. Figure 7.7 and 7.8 show the distribution of deviations of each arrival at Tøyen serving the west branch Frognerseteren and the east branch Ellingsrudåsen.

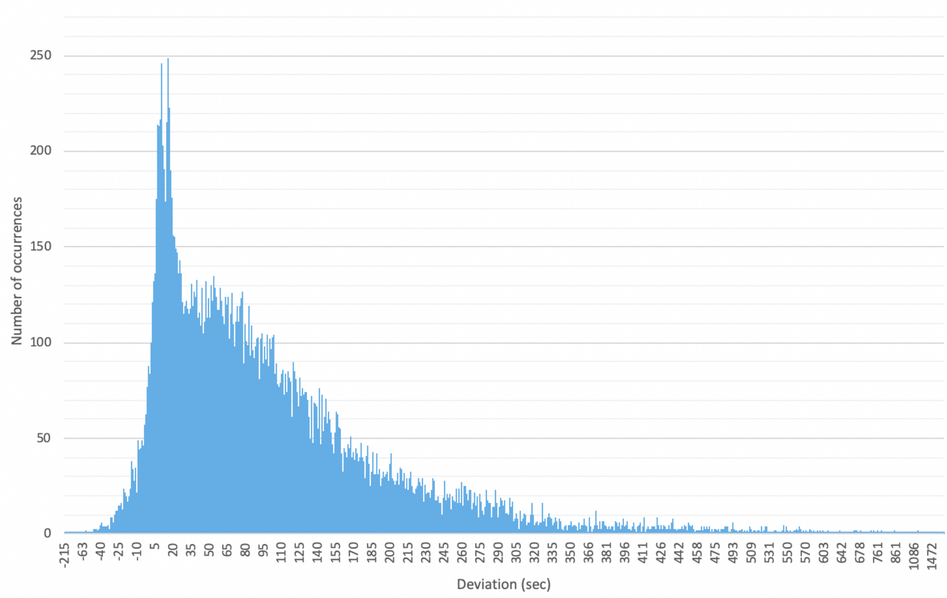

Figure 7.7: Distribution of deviation occurrences on the Frognerseteren branch in 2019. The x-axis shows the different deviations values, while the y-axis shows how many times they have occurred.

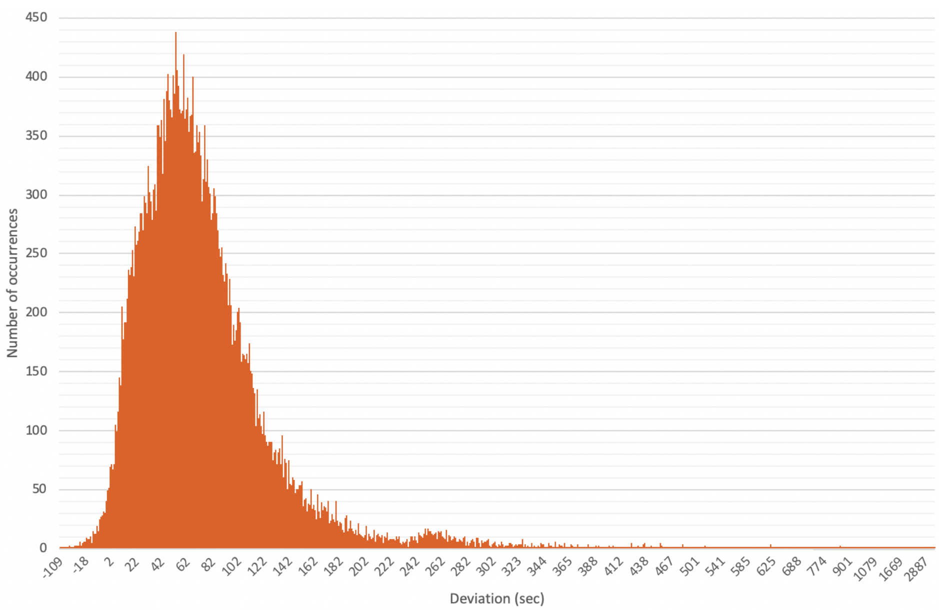

Figure 7.8: Distribution of deviation occurrences on the Ellingsrudåsen branch in 2019. The x-axis shows the different deviations values, while the y-axis shows how many times they have occurred.

As seen from Figure 7.7, the most common deviation of the departures at Tøyen from Frognerseteren is a delay of 16 seconds. The most common deviation of the departures at Tøyen from Ellingsrudåsen, as seen from Figure 7.8, is a delay of 53 seconds. Both cases are within the range of a punctual departure, as described in Section 2.4. As can be observed, the distribution of deviations on the Ellingsrudåsen branch has considerably more data points than the distribution of the deviations on the Frognerseteren branch. This is because the Ellingsrudåsen branch is a double-frequency branch, with two departures every ground period. The Frognerseteren branch has one departure every ground period. It can also be seen that the Ellingsrudåsen branch has higher deviations than the Frognerseteren branch. One of the reasons might be that the Ellingsrudåsen branch shares rails with another branch for a longer stretch than the Frognerseteren branch. As soon as a train along the Frognerseteren branch leaves the tunnel at Majorstuen, it does not share rails with any other branches. Two or more branches sharing rails can lead to knock-on delays, as explained in Section 4.4.4. This knock-on effect might also occur at the Ellingsrudåsen branch due to double-frequency departures, which might explain some of the increased deviations on this branch.

Figures 7.9 and 7.10 show the distribution of delays in March. This is used as a data basis in the sensitivity analysis concerning robustness in Section 8.2.1. As seen from the figures, some delays occur more frequently. For the Frognerseteren branch, these delays range from roughly 33 to 186 seconds. For the Ellingsrudåsen branch, they range from about 11 seconds to 102 seconds. The use of these findings will be further discussed in Section 8.2.1.

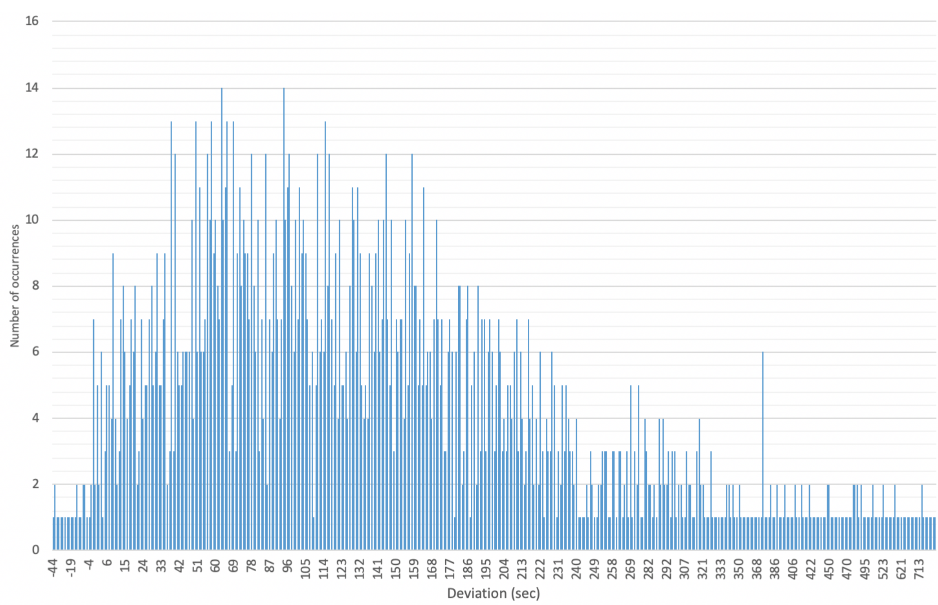

Figure 7.9: Distribution of deviation occurrences on the Frognerseteren branch in March 2019.

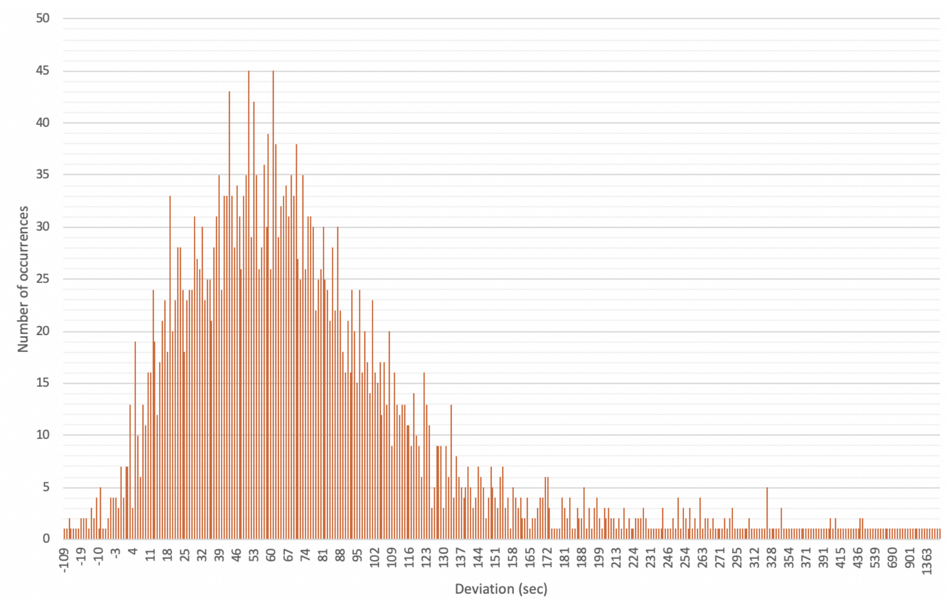

Figure 7.10: Distribution of deviation occurrences on the Ellingsrudåsen branch in March 2019.

7.1.4 Estimation of the Deviation Parameter for the Veitvet Branch

Due to the project of the ninth arrival, a new branch between Tøyen and Veitvet is introduced in the problem investigated in this thesis. As this branch is not operative yet, there is no historical data concerning the deviations on the branch. Therefore, the deviation parameter of this branch must be calculated. Veitvet is originally an intermediate station on the Vestli East branch. Therefore, the deviation parameter of Veitvet can be calculated from the deviations on this branch.

According to Sporveien, most of the deviations occur at the intermediate stations, while the actual travel time between them is less subject to fluctuations (H. Holtebekk, Personal communication, 11/10/2019). Therefore, a reasonable method of estimating the deviation of the Veitvet branch is to use the ratio between the number of stations along the Veitvet branch and the total number of stations along the Vestli East. This is shown in equation 7.1.

$$\text{Deviation parameter} = \frac{\text{Stations in new branch}}{\text{Total number of stations}} * \text{Total deviation} \tag{7.1}$$

Table 7.2 shows the deviation parameters and their components.

Table 7.2: The estimation of deviation parameters along new branches in the system.

New branch	Intermediate stations	Total number of stations	Station ratio	Total deviation (min)	Estimated deviation parameter (min)
Veitvet	12	23	0.52	1.41	0.73

7.1.5 Implications of Correlations in the Data on Travel Time Deviations

Correlations in the data on travel time deviations are disregarded in the estimation of the deviation parameters, as stated in Section 7.1.1. Although disregarded, it is acknowledged that correlations between the deviations are likely to exist due to knock-on delays on the branches which share rails. An implication of disregarding the correlations, is that it is not possible to conclude that the estimated deviation parameters represent the correct need for buffer at each branch. The deviations parameters could hence be regarded as approximations of the buffer times needed on the branches. One can still conclude that the use of buffer times reduces the occurrence of delays despite the correlations in the data. Further, the use of historical data to find individually fitted buffers is considered to be more accurate than the approach of Harbo et al. (2019), who use one global value for all branches.

7.2 Parameters of the Model

In this section, the numerical values of the parameters defined in Chapter 5 are presented. This includes the parameters of the base case and the extended model. The numerical values of the parameters used in the case studies presented in Section 8.3 are also provided. In section 7.2.1 the base case is defined and the associated parameters are presented. In Section 7.2.2 the parameters which are specific for the extended model are provided and explained. Lastly, in Section 7.2.3 the parameters of the case study are presented.

7.2.1 Base Case Parameters

The base case is defined as the set of parameters used when solving the LPP and the TTP for the case of the Oslo Metro. These parameters consist of parameters given by Sporveien and estimated values as explained in Section 7.1.4.

Structural and Operational Parameters

The estimated values of the deviation parameters are presented for all branches in Table 7.3. Ringen West and Ringen East are not included because these branches have no regulation time. This is explained in Section 5.3.1.

Table 7.3: Estimated values for the deviation parameters for all branches, based on average delay.

Branch $g \in G$	**Deviation parameter** L **(min)**
Sognsvann	3.71
Frognerseteren	3.57
Østerås	4.23
Kolsås	3.63
Vestli West	5.23
Stortinget	1.33
Ellingrudåsen	1.68
Mortensrud	1.57
Bergkrystallen	2.90
Vestli East	1.41
Veitvet	0.73

Values provided by Sporveien are presented in Tables 7.4 and 7.5 for west and east branches, respectively. The frequency, travel time and technical dwell time are provided for each branch.

Table 7.4: Numerical values of operational parameters concerning west branches. T and D^T are rounded to two decimals.

West Branch $w \in W$	**Frequency** F^I	**Travel time** T **(min)**	**Technical dwell time** D^T **(min)**
Ringen Vest	Single	20.37	0.00
Sognsvann	Single	21.34	2.33
Frognerseteren	Single	38.80	4.75
Østerås	Double	24.25	2.33
Kolsås	Single	33.95	2.33
Vestli Vest	Single	39.77	6.58
Stortinget	Double	3.88	1.50

Table 7.5: Numerical values of operational parameters concerning east branches. T and D^T are rounded to two decimals.

East Branch $e \in E$	**Frequency** F^I	**Travel time** T **(min)**	**Technical dwell time** D^T **(min)**
Ringen Øst	Single	5.82	0.00
Ellingrudåsen	Double	18.43	2.33
Mortensrud	Double	21.34	2.33
Bergkrystallen	Double	16.49	2.33
Vestli Øst	Single	22.31	6.58
Veitvet	Single	11.64	6.58

Other parameters related to the operations provided by Sporveien are listed in Table 7.6.

Table 7.6: Numerical values of operational parameters which are not branch specific.

Operational parameter	**Numerical value**
Dwell time D^S (min)	0.33
Maximum dwell time at terminal stations $\overline{D}$	13.0
Driver break share Δ	0.05
Headway time H (min)	1.50
Groundperiod P^G (min)	15.0
Travel time between Tøyen and the station Smestad $T^{Smestad}$	13.9
Minimum time between opposite running arrivals at Smestad $B^{Smestad}$	2.00
Minimum time at terminal stations B^{Term}	2.00
Double-frequency requirements $[\underline{F^2}, \overline{F^2}]$	[6, 9]
Triple-frequency requirements $[\underline{F^3}, \overline{F^3}]$	[3, 10]

Cost Parameters

The objective of Harbo et al. (2019) is to minimize the number of trains. In this thesis, the number of trains and the use of step-back drivers are the subjects of minimization. To compare the number of trains and the use of step-back drivers, costs are used as a common measure. The costs are modelled as annual variable costs with a time horizon of 30 years, as this is the expected lifetime of a train. In the computation of the annual variable costs, only the most significant components are considered. These include investments costs, alignment costs, preparation costs and driver costs. Investment costs are considered as variable costs, as they change in proportion to the number of trains used in the operations. Consequently, the costs computed in this thesis are an approximation of the annual variable costs of a train and of a step-back driver. The main purpose of the costs is to compare the solutions of the model, and not to reflect the actual variable costs of Sporveien. The cost parameters are found in Tables 7.7 and 7.8.

Table 7.7: The numerical values of the parameters used to compute annual variable costs of a train.

Parameters composing the variable costs of a train	**Numerical value**
Investment cost per train C^I (MNOK)	110.0
Alignment cost per train C^A (MNOK)	110.0
Daily preparation cost per train C^P (MNOK)	0.005
Hourly costs associated to a driver (MNOK) C^D	0.001
Expected lifetime of a train V (years)	30
Days of operation of the Oslo Metro per year N^O	365
Driver hours per year per train N^H	3000

Table 7.8: The numerical values of the parameters used to compute annual variable costs of a step-back driver.

Parameters composing the variable costs of step-back driver	**Numerical value**
Hours per year per step-back driver N^{SBH}	3000
Hourly costs associated with a step-back driver C^{SBH} (MNOK)	0.001

The annual variable cost of a train, C^{Train}, is computed according to equation (7.2).

$$C^{Train} = (C^I + C^A)/V + C^P N^O + C^D N^H \tag{7.2}$$

As the variable costs of a train are modeled as annual, the costs considered as nonrecurring are evenly distributed on the expected lifetime of a train V. This concerns the investment costs, C^I, and alignment costs, C^A.

The annual variable cost of a step-back driver, C^{SB}, is computed according to equation (7.3).

$$C^{SB} = N^{SBH} C^{SBH} \tag{7.3}$$

Based on equations (7.3) and (7.2) the values of the variable costs are computed and presented in Table 7.9

Table 7.9: The annual variable costs of a train and step-back driver. The values are given in MNOK, rounded to two decimals.

Annual variable cost of a train	**Annual variable cost of a step-back driver**
12.16	3.00

Computation of Costs Specific to the LPP

As explained in Section 6.2.1, the MP must be optimistic for the Combinatorial Benders' Decomposition to be a suitable solution method. Consequently, the optimal cost in the LPP must be equal to or lower than the optimal cost in the TTP. The variable C_{we}^{min} is therefore introduced in the LPP.

The minimum costs that can be achieved if branch w is combined with branch e, is denoted C_{we}^{min}. For each possible branch combination, C_{we}^{min}, takes the minimum value of two possible values. The first value is computed with the driver break included in the round-trip time. The second value excludes the driver break from the round-trip time, and a step-back driver is included. In the latter case, the step-back driver enables the exclusion of the driver break in the round-trip time, as explained in Section 2.6. This might lead to a reduction of one train. The equation used to compute C_{we}^{min} is shown in equation (7.4).

$$C_{we}^{min} = min\{T_{we}^{IN} C^{Train}, \ \ T_{we}^{EX} C^{Train} + C^{SB}\} \tag{7.4}$$

T_{we}^{IN} is the number of trains needed if west branch w is combined with east branch e when driver break is included in the round-trip time. T_{we}^{EX} is the number of trains needed if west branch w is combined with east branch e, when driver break is excluded in the round-trip time. This exclusion inflicts the cost of a step-back driver, C^{SB}. T_{we}^{EX} and T_{we}^{IN} are computed using equation (7.5) and equation (7.6), respectively.

$$T_{we}^{EX} = \left\lceil \frac{\Gamma^{EX}}{\frac{P^G}{F_{we}}} \right\rceil \tag{7.5}$$

$$T_{we}^{IN} = \left\lceil \frac{\Gamma^{IN}}{\frac{P^G}{F_{we}}} \right\rceil \tag{7.6}$$

The driver break is excluded from the round-trip time, Γ^{EX}, while it is included in the round-trip time Γ^{IN}. The equations used to compute Γ^{EX} and Γ^{IN} are equation (7.7) and equation (7.8), respectively.

$$\Gamma^{EX} = \sum_{r \in R} (2T_{ir} + D_{ir}^T) + 2D^S \tag{7.7}$$

$$\Gamma^{IN} = \sum_{r \in R} (2T_{ir} + D_{ir}^T) + 2D^S + 2\Delta \sum_{r \in R} T_{ir} \tag{7.8}$$

The frequency F_{we} is the maximum frequency of the line formed by west branch w and east branch e. This parameter is obtained using equation (7.9).

$$F_{we} = min\{F_w^I, F_e^I\} \qquad w \in G^W, e \in G^E \tag{7.9}$$

Deviation Parameters used in the Regulation Time Sensitivity Analysis

In Section 8.2.1 the sensitivity of the optimal solution to changes in the regulation time is investigated. This is done using the epsilon constraint method. As explained by Jaimes (2011), the epsilon constraint method is one of the most common methods used to solve multiobjective problems. Jaimes (2011) further explains that in the respective method, one of the objectives is minimized, while the other is used as constraints bound by different levels, i, of epsilon, ϵ_i. In the analysis presented in Section 8.2.1, the different values of ϵ_i corresponds to varying values of the deviation parameter, L_{ir}. As thoroughly explained in Section 8.2.1, the values of the deviation parameter are chosen corresponding to how much delay it should cover. The values of the deviation parameters and their corresponding percentage coverage used in the analysis are shown in Table A.1 in Appendix A.

7.2.2 Parameters of the Extended Model

The extended model is presented in Section 5.4. This section presents the numerical values of the parameters of the extended model and elaborates on the motivation for the chosen values. In the extended model, a penalty term and a reward term are added to the objective function. These costs are included to model the trade-off between robustness and optimality. However, Sporveien has no equivalent cost items in their operations. Therefore, the objective value of the extended model does not reflect the actual variable costs of Sporveien.

The Value of the Penalty Bounds

As explained in Section 5.4, the extended model includes a piecewise linear penalty function. This means that the penalty is linear within intervals. The bounds for the intervals correspond to Π_e for each extremity e.

Sporveien defines an arrival as precise, punctual or delayed, as explained in Section 2.4. The penalty intervals in the piecewise linear penalty function are defined according to this categorization. Interval 1 is associated with precise arrivals. Interval 1 has lower penalty bound Π_0, and upper penalty bound Π_1, and contains all cases of the arrival of a train within one minute after scheduled arrival time. Interval 2 contains the punctual arrivals, and this interval is lower bounded by Π_1 and upper bounded by Π_2. Interval 2 contains the cases of arrivals which arrive between one and three minutes after scheduled arrival time. Arrivals within interval 3 are defined as delayed, and the lower and upper bounds are Π_2 and Π_3 respectively. This interval contains the arrivals arriving between three and six minutes after scheduled arrival. The intervals with corresponding bounds and categories are summarized in Table 7.10.

Table 7.10: The penalty intervals, their upper and lower bounds, and the categorization of trains arriving within the intervals.

Interval	Lower bound (min)	Upper bound (min)	Category
1	$\Pi_0 = 0$	$\Pi_1 = 1$	Precise
2	$\Pi_1 = 1$	$\Pi_2 = 3$	Punctual
3	$\Pi_2 = 3$	$\Pi_3 = 6$	Delayed

The Value of the Penalty Parameters

In the piecewise penalty function as seen in Figure 5.6, the penalty per unit violation may differ depending on the value of the violation. Furthermore, a small violation of constraints (5.54) has minor effects on the operations, and is desirable if it reduces the number of trains needed in the metro system. Large violations, however, are less desirable as they might cause a considerable reduction in the robustness of the arrivals, and hence the timetable. Therefore it is desirable that the penalty per unit violation increases with the value of the violation variable.

Figure 7.7 in Section 7.1 is modified and provided in Figure 7.11, and shows the deviation value and their number of occurrences for all arrivals on the Frognerseteren branch in 2019.

Further, the penalty intervals 1, 2 and 3 are indicated in green, red and yellow, respectively. It can be observed that interval 1 contains the deviations that occurred most frequently. The data analysis shows that this is the case for most of the branches. As Sporveien prefers the deviations in interval 1 over deviations in interval 2 and 3, it is reasonable to penalize deviations in the two latter intervals harder than the deviations in interval 1.

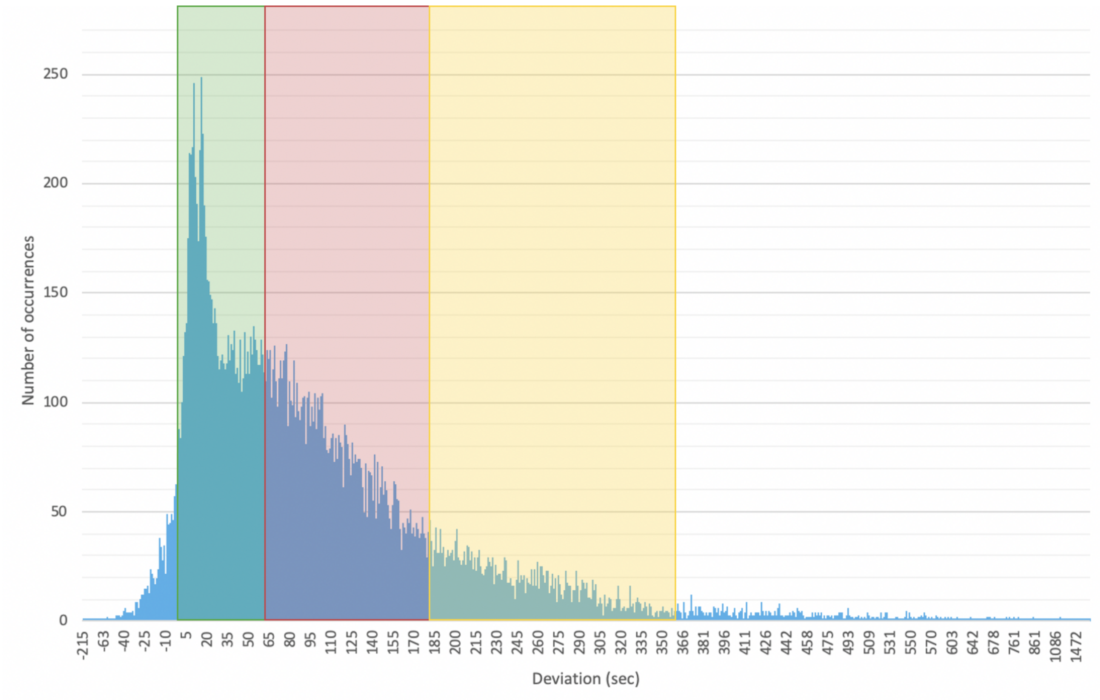

Figure 7.11: A modified version of Figure 7.7, which shows the distribution of the deviation values on the Frognerseteren branch in 2019. The penalty intervals 1, 2 and 3 are indicated in green, red and yellow, respectively.

Based on these observations as well as empirical results from solving the model with different parameter values, the parameters of the extended model are decided. The numerical values of the penalties, P_e, and corresponding bounds, Π_e, are seen in Table 7.11.

Table 7.11: The extremities in the piecewise linear penalty function, with corresponding bounds and penalties.

Extremity e	**Bound** Π_e	**Penalty** P_e
0	0	0
1	1	C^{Train} - 6
2	3	5(C^{Train} - 6)
3	6	10(C^{Train} - 6)

For interval 1, the penalty is between the penalty for extremity 0, P_0, and the penalty for extremity 1, P_1, depending on where in the interval the violation variable is situated. For arrivals with a deviation value in interval 2, the penalty is between P_1 and P_2. For arrivals with a deviation within interval 3, the penalty is between P_2 and P_3. Compared to P_1, the penalties P_2 and P_3 are five and ten times larger, respectively. This means that the penalty per unit violation is larger for interval 2 and 3, than for interval 1. Penalizing deviations according to the intervals shown in Table 7.11 results in a larger number of occurrences of violation variables in interval 1, and fewer occurrences of violation variables in interval 2 and 3. This is according to the preferences of Sporveien.

The sensitivity of the model solution to changes in the penalty parameters is analysed in Section 8.4.2.

The Value of the Reward Parameter

The regulation time must be seen in context with the need for buffer at a specific branch. Branches with a large deviation parameter need larger regulation times to obtain the same level of robustness as branches with smaller deviation parameters. Therefore, only the part of the regulation time exceeding the deviation parameter for a given branch is rewarded.

The reward parameter S corresponds to the reward per unit of difference between the regulation time and the deviation parameter, $(\lambda_{iar}$ - $L_{ir})$. The purpose of the reward is to have the model select the most robust optimal solution, in case several solutions have identical objective values. By assigning a value to S, which is considerably smaller than the other terms in the objective function, the reward functions as a differentiator between solutions that are equally optimal and unevenly robust. Therefore the value of the reward, S, is set to 0.001 MNOK.

The Value of the Upper Bound for Regulation Time

As explained in Section 5.4.2, the regulation time is assigned an upper bound, $\overline{\lambda}$, to prevent the model from becoming unbounded. This value is set to 15 minutes, which corresponds to the length of the ground period P^G.

7.2.3 Parameters Related to the Fornebu Branch

This section provides parameters related to the Fornebu branch. The parameters concern frequency, travel time, technical dwell time, and deviation parameter. These are provided by Sporveien, except the deviation parameter. Due to the lack of historical data on travel time deviations on the Fornebu branch, the deviation parameter is chosen based on the parameter value on branches with similar structure and operations. The Sognsvann branch is chosen for this purpose as it has a similar travel time and number of stations. The numerical values for the mentioned parameters are provided in Table 7.12. The computational study of this additional branch to the metro system is conducted in Section 8.3.2.

Table 7.12: The numerical values of the parameters related to the Fornebu branch.

Frequency	**Travel time (min)**	**Technical dwell time (min)**	**Deviation parameter (min)**
Double	21	2.33	3.71

Chapter 8

Computational Study

In this chapter, a computational study of the results obtained from solving the problem using Combinatorial Benders' Decomposition is presented. The parameters used in the computational study and their numerical values are presented in Section 7.2.

In Section 8.1 the results from the base case are presented. Sensitivity analyses on operational parameters in the base case are conducted and discussed in Section 8.2. Section 8.3 presents results from two case studies. The first case study concerns the solving of the TTP using the current line plan of Oslo Metro. The second case study concerns the inclusion of the Fornebu branch in the line plan. In Section 8.4, the results from solving the extended model are presented and discussed. This section further analyses how changes in the penalty value affect the optimal solution. The cost and utilization performance of the model based on the results are discussed in Section 8.5.

Our model is written in Python 3.7 and implemented using Spyder. The code is run using NTNU's online cluster Solstorm. The node type used in the cluster is Lenovo M5, with processor 2 x Intel E5-2670v3, 2.3 GHz, and a 64 Gb memory.

8.1 Base Case Results

This section presents and discusses the results obtained from the base case. The parameters used are presented in Section 7.2.1. The results are compared to the current line plan and operation, including the ninth arrival. This is further explained in Section 2.7 and in Section 7.2.1. Sporveien plans to operate with a total of either 60 or 61 trains with the ninth arrival included. This is 3-4 more than in the current operation with eight arrivals at Tøyen each ground period. As mentioned in Section 2.6, Sporveien currently operates with two step-back drivers. These are used at Bergkrystallen on Line 4 and Sognsvann on Line 5.

Table 8.1 presents the key numbers from the base case in terms of iterations and feasible subproblems. A subproblem corresponds to the TTP in the solution method. The base case returns 114 feasible solutions, i.e. 114 line plan solutions with corresponding feasible timetables.

Table 8.1: Key numbers from the running of the model.

Iterations	Feasible sub-problems	Infeasible sub-problems	Run time (sec)
517	114	403	857.85

The key results in terms of costs, trains and step-back drivers are given in Section 8.1.1. Section 8.1.2 presents and discusses the optimal line plan and the operational results. Finally, the resulting individual regulation times per arrival are presented in Section 8.1.3, and the robustness of the timetable is discussed.

8.1.1 Key Results in Numbers

Table 8.2 presents the key numerical results in terms of annual variable costs, and the number of trains and step-back drivers found in the optimal solution. The annual costs of trains and step-back drivers are given in the table. They add up to the total annual variable costs of 714.18 MNOK.

Table 8.2: Overall key results of the base case.

Number of trains	Annual cost of trains (MNOK)	Number of step-back drivers	Annual cost of step-back drivers (MNOK)	Annual variable costs (MNOK)
58	705.18	3	9	714.18

As seen from Table 8.2, the total number of trains found in the optimal solution is 58. The new line plan hence equals a reduction of 2-3 trains compared to the operational plans of Sporveien when the ninth arrival is included. The optimal solution of the base case further includes the application of three step-back drivers, which is one more than the two step-back drivers Sporveien currently uses.

8.1.2 The Optimal Line Plan and Operational Results

The resulting optimal line plan of the base case is presented in Table 8.3, and is illustrated in Figure 8.1. When comparing this solution to the current line plan of Oslo Metro in Figure 2.2, one finds that only Line 2 is composed of the same branches in both line plans. Figure 8.1 further illustrates which lines that use a step-back driver, and at which terminal station the step-back driver replaces the driver break.

It is observed that in the optimal line plan depicted in Figure 8.1, Ringen, the west branch Vestli West and the east branch Mortensrud are now combined into one single line, Line 3. This line has two arrivals at Tøyen during the ground period in both directions. The arrivals at Tøyen bound for either Vestli West or Mortensrud, are referred to as ”3 Vestli”

Table 8.3: The optimal branch combinations of the base case, with corresponding line numbers.

Line	West branch	East branch
1	Frognerseteren	Vestli East
2	Østerås	Ellingsrudåsen
3	Ringen West	Mortensrud
3	Vestli West	Ringen East
4	Sognsvann	Mortensrud
5	Stortinget	Bergkrystallen
6	Kolsås	Veitvet

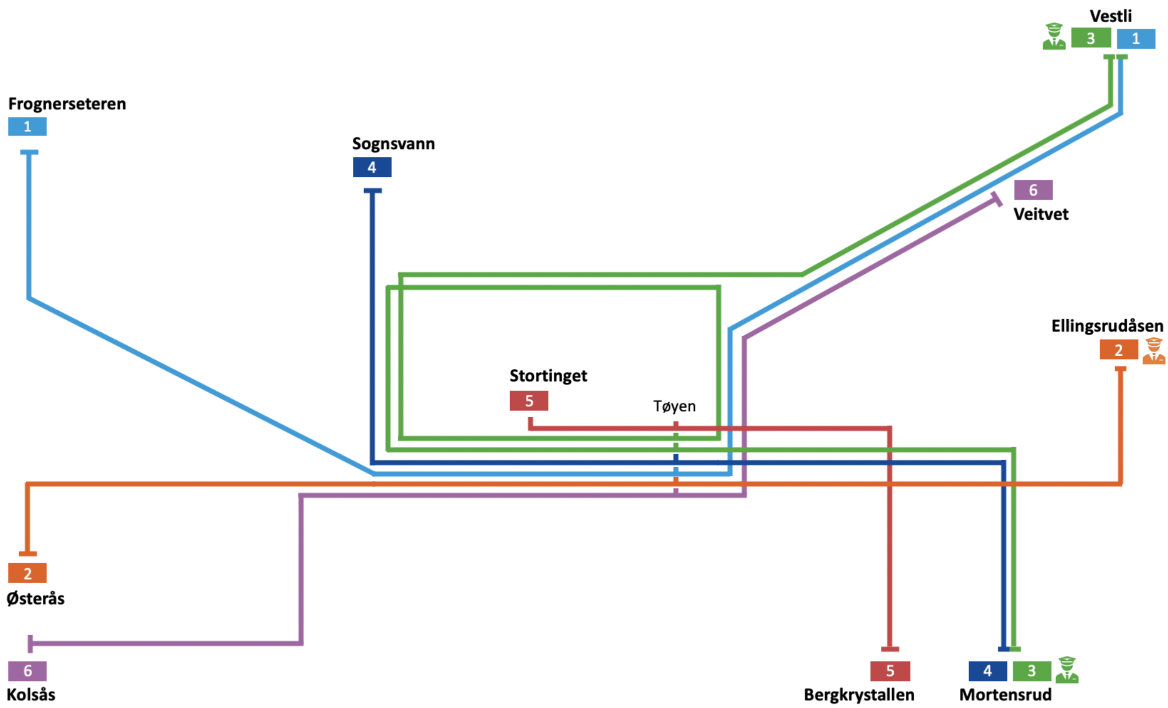

Figure 8.1: The optimal line plan of the base case.

and "3 Mortensrud", respectively. The arrivals at Tøyen entering Ringen are referred to as "3 Ringen" in both directions. These arrivals are displayed in Figure 8.2, which depicts the described structure of Line 3.

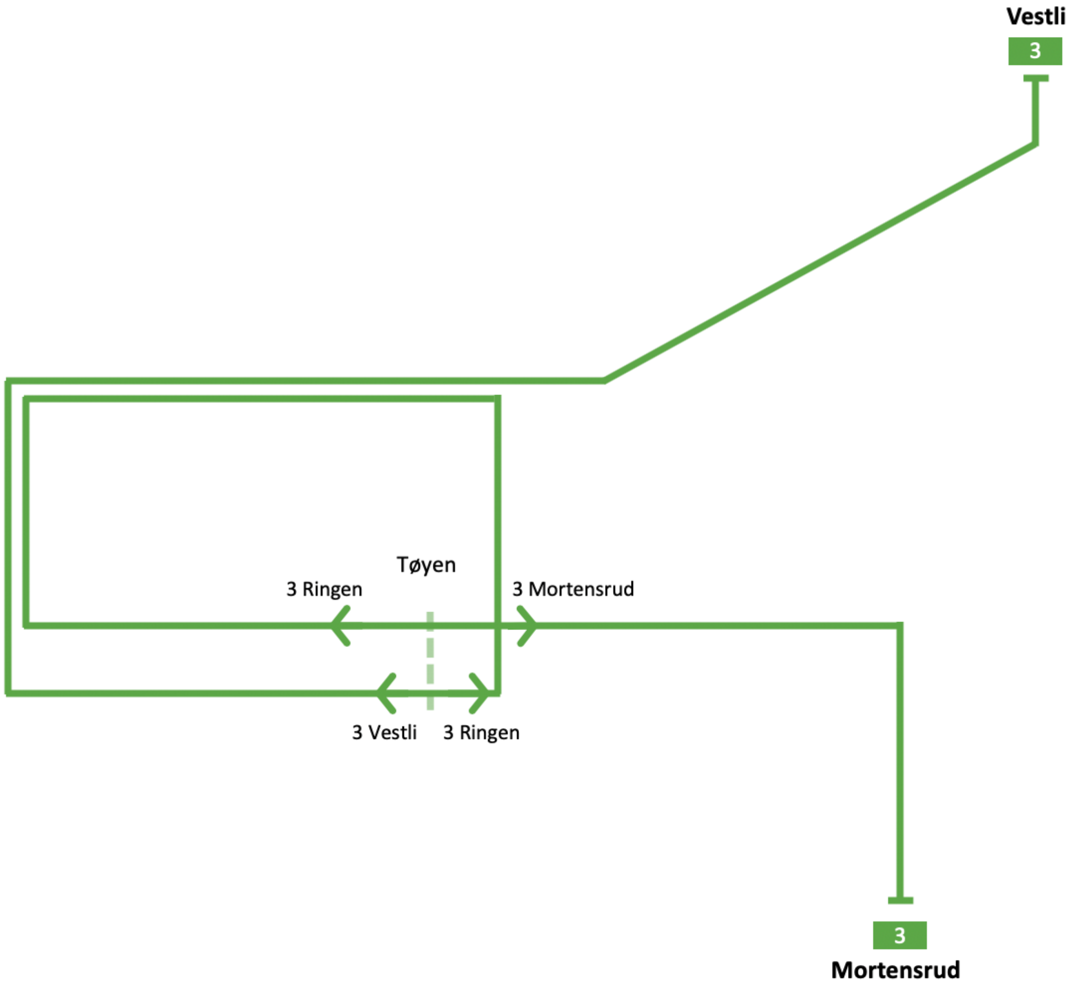

Figure 8.2: The arrivals at Tøyen of the line serving Ringen.

Table 8.4 shows how the 58 trains are distributed on the six lines in the optimal line plan. The table further presents the frequency of the lines, the use of step-back drivers, and the time saved by using them. It can be observed that the number of trains varies between five and 13 for each line. As previously discussed, the total number of trains needed per line is dependent on the frequency of the line and the round-trip time. The round-trip time of a line is dependent on the travel times of the branches that constitute it and the length of the dwell time. As mentioned in Chapter 3, the length of the driver break equals a certain percentage of the time it takes to travel back and forth between the two terminal stations on a line. The value is five percent, as given in Table 7.6. Using step-back drivers to shorten the dwell time of a line, and hence the round-trip time, could contribute to reduce the number of trains needed on a line. Table 8.4 provides the value of the dwell time reduction due to the application of a step-back driver. These values hence equal the extra time the trains should have spent dwelling while the driver had his/her break.

The use of step-back drivers and their location is one aspect of how the total number of trains can be reduced relative to the current operations. Another important aspect is the reduction of the travel times by three percent on all branches, due to the new signalling system Sporveien plans to apply. This is explained in Section 2.7. The new signalling system reduces the travel time between 7.2 seconds and 1.23 minutes on the different branches. This reduces the round-trip time, which in turn could reduce the number of trains needed to meet the frequency demands on each line.

Table 8.4: Results of the base case for each line in the new line plan.

Line	**Frequency**	**Use of a step-back driver indicated by X**	**Time saved by applying a step-back driver (min)**	**Number of trains**
1	Single	-	-	10
2	Double	X	4.27	13
3	Single	X	4.17	5
3	Single	X	4.56	8
4	Single	-	-	7
5	Double	-	-	7
6	Single	-	-	8

The optimal timetable for westbound and eastbound arrivals at Tøyen, corresponding to the optimal line plan, is presented in Appendix B, in Table B.1 and B.2, respectively.

8.1.3 The Regulation Times and the Overall Robustness of the Timetable

The values of the regulation time for each arrival in the optimal solution of the base case are presented in Table 8.5. As described in Section 5.3.1, the regulation times are imposed on the timetable to absorb occurring delays. The deviation parameters for all branches are also given in the table. These represent the minimum regulation time given for each branch, which is explained in Section 7.1.3. The robustness of a branch is hence strengthened if its regulation time exceeds the value of its deviation parameter.

From Table 8.5 one can observe that the majority of the arrivals take on regulation time values that exceed the deviation parameter. These excesses vary between 31.2 seconds at Kolsås on Line 6, and 10.06 minutes at Veitvet on the same line. The optimal solution is hence more robust than the minimum requirements, which contributes to enhancing the timetable's ability to absorb delays. The total regulation time of the timetable amounts to 68.68 minutes.

Table 8.5 further provides the regulation time of each branch in the timetable currently used by Sporveien. Although the timetable generated in the TTP and the timetable currently used in the Oslo Metro correspond to different line plans, the regulation time of the arrivals on the branches are comparable. The regulation times of the second arrival on the Stortinget branch and the arrival on Veitvet are not included in the comparison. This is because there does not exist any data on the regulation time on the future project of the ninth arrival. In the data from Sporveien, the regulation times with value zero represent the branches at which the step-back drivers are applied in the current system. The sum of the comparable regulation times is 56.57 minutes in the TTP and 54.18 minutes in the current timetable. Further, the timetable constructed in this thesis has a total of seven arrivals with regulation times exceeding the regulation times of the arrivals on the corresponding branches in the current timetable. These excesses are between 33 seconds and 5.30 minutes. For the remaining seven arrivals, the regulations times of Sporveien exceed the regulations time of the TTP. These excesses range between 25.8 seconds and 9.59 minutes.

Based on the comparison of regulation times, it can be concluded that the robustness of the timetable generated in the base case is approximately the same as for the timetable currently used by Sporveien. Thus, the results from the base case give a reduction in the number of trains needed in the operation and indicate a timetable equally robust to the timetable currently used. The base case hence results in a solution that is both cost-efficient and robust.

Table 8.5: Regulation time for each branch in the optimal solution and for the current operations of Oslo Metro.

Branch	**Line**	**Arrival**	**Regulation time TTP (min)**	**Deviation parameter (min)**	**Regulation time Sporveien (min)**
Frognerseteren	1	1	4.69	3.57	2.40
Vestli East	1	1	4.99	1.41	5.42
Østerås	2	1	4.23	4.23	5.67
Østerås	2	2	5.30	4.23	3.67
Ellingsrudåsen	2	1	1.68	1.68	2.27
Ellingsrudåsen	2	2	2.43	1.68	4.27
Mortensrud	3	1	2.71	1.57	5.67
Vestli West	3	1	7.46	5.23	2.42
Sognsvann	4	1	5.30	3.71	0.00
Mortensrud	4	1	4.75	1.57	2.67
Stortinget	5	1	2.41	1.33	12.00
Stortinget	5	2	1.33	1.33	-
Bergkrystallen	5	1	3.30	2.90	2.75
Bergkrystallen	5	2	3.42	2.90	0.00
Kolsås	6	1	3.90	3.63	4.97
Veitvet	6	1	10.79	0.73	-

8.2 Assessment of Sensitivity to Changes in Base Case Parameters

In this section the model's sensitivity to changes in the operational requirements is assessed. This is done by analysing the effect of changes in the base case parameters on the objective value. In Section 8.2.1 the robustness related to regulation time is discussed. In Section 8.2.2 the consequences of changes in the time interval distribution between arrivals along branches and areas with double and triple frequency requirements are investigated.

8.2.1 Model Sensitivity to Regulation Time

A delay is defined as the event of a train arriving at a station behind the scheduled arrival time. In this thesis, robustness is defined as the ability to withstand delays. In this section, the effect of changes in the deviation parameter on the robustness of the model, and the relation between them, is discussed. As explained in Section 7.1.5, the deviation data is prone to some uncertainty, as it is collected from the current line system. This must be taken into account when reading the results of this analysis.

As explained in Section 5.3.1, the deviation parameter representing the delay sets a lower limit on the regulation time on each branch, which aggregated represent the total buffer of the model. With an increasing deviation parameter, the buffer of the model also increases. Thus, the model becomes less sensitive to delays, and the robustness of the model is strengthened. However, adding regulation time to the model also increases the round-trip time, which may cause a raise in the number of trains or the use of step-back drivers, as discussed in Section 2.5 and 2.6. This will, in turn, increase the total costs and represents a negative effect on the optimal objective value of the model.

It is desired to minimize the total variable costs associated with trains and step-back drivers in operation. In addition, a robust solution is desirable. Therefore, an analysis of the relation between the deviation parameter, thereby the regulation time, and the objective value is carried out. This is done by changing the deviation parameter per branch. The result is the minimum cost for selected values of the deviation parameter. Each time the deviation parameter is changed, the problem is resolved and a new solution is generated. As discussed in Section 7.1.2, the deviation parameter is chosen from the month with the highest average delays, in the busiest part of the day. This corresponds to the average delay in the month of March in the time interval 16.00 to 16.30. This period forms the data basis of the analysis. The deviation parameter is changed according to how many of the departures at Tøyen from each branch it should account for. This means that with a buffer time steered by this deviation parameter, the model should be able to cover a given percentage of the delayed departures at Tøyen. In practice, this means that all arrivals should be able to arrive at Tøyen at the scheduled time for the given percentage of the time. In Table A.1, the deviation parameter covering different percentage shares of the delayed departures is shown.

To generate the results as shown in Figure 8.3, the epsilon constraint method is used. The method is explained in Section 7.2.1. In this analysis, the minimization of costs can be considered as one objective, while the maximization of regulation time can be considered as the other. The different values of the epsilon, ϵ_i, correspond to the different values of the average deviation, L_{ir}. The results of the epsilon constraint method are found in Table C.1. The findings are shown in Figure 8.3. The different colours covering one or several points represent how many trains are used in that specific solution. Further, the solutions outlined in grey make up parts of the Pareto optimal front, assuming increments of five percentage points. This means that the solutions cover five percent more of the delays for each increment. As only a selection of percentage values are considered, only parts of the Pareto optimal front are generated. The symbols next to the data points included in the Pareto front show the number of step-back drivers used in that specific solution.

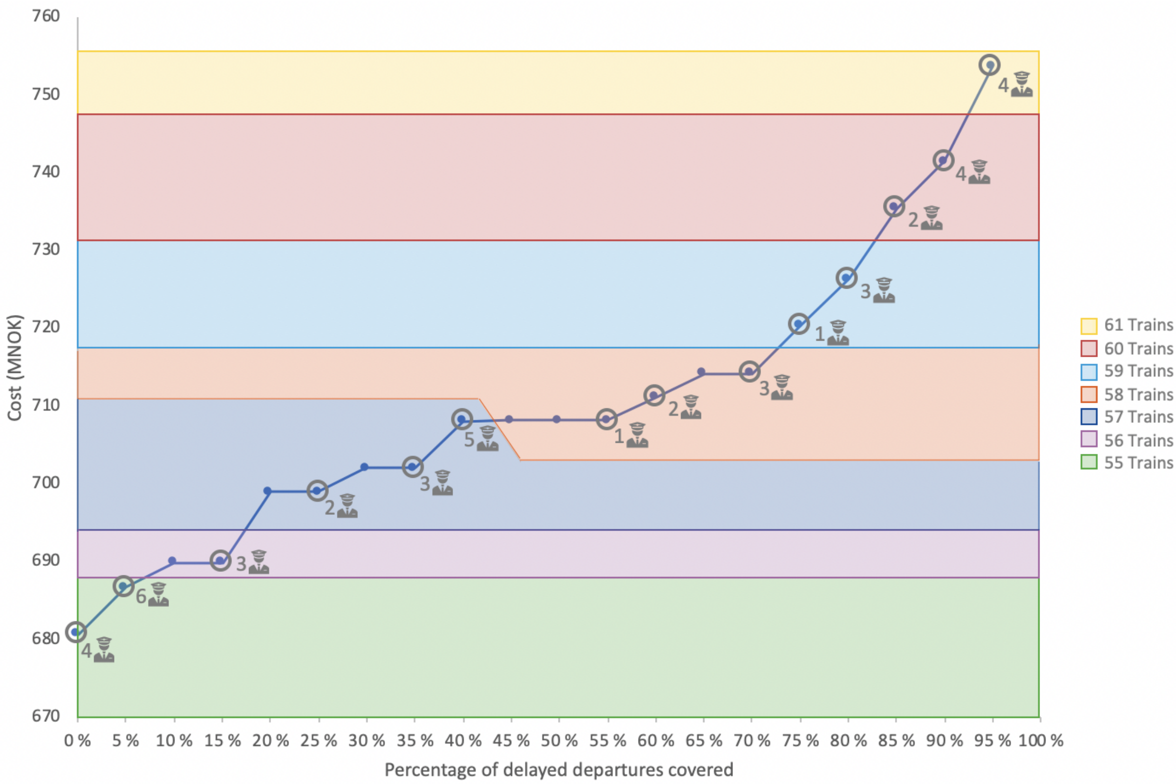

Figure 8.3: The sensitivity of the objective value to changes in the deviation parameter. The x-axis shows the percentage of delayed departures which are covered by the deviation parameter. The y-axis shows the optimal objective value from solving the model with the given deviation parameter.

For a given objective value, the solution on the Pareto front is the most robust one. Figure 8.3 shows the close relation between the deviation parameter and objective value. A solution rarely becomes more robust without increasing the cost. This shows that robustness comes with a cost, which is why the majority of the data points of the chosen increments are on the Pareto optimal front. This is especially the case in the low percentages, from 0% to about 20%, and in the high percentages, from 70% and upwards. In these parts of the curve, there is a rapid increase in cost as the solution becomes more robust. Increasing the coverage from 20% to 70%, i.e. an increase of 50 percent points, induce an increase of costs of only 12 MNOK. When increasing the coverage from 70% to 95%, an increase of only 25 percent points, the cost increases with as much as 39 MNOK. This means that increasing the coverage, and thereby the robustness of the model, from 20% to 70% will induce a relatively low cost. This further indicates that operating with a coverage close to 70% might be desirable, as this results in relatively high coverage with low induced costs.

The shape of the curve might have its explanation in the data basis. Even though the percentage shares increase with fixed increments along the x-axis, the deviation parameter does not. An increase in coverage of five percent might induce different increases in the deviation parameter of each branch, e.g. one minute at one branch and 40 seconds at another. Further, the distribution of delays, as illustrated in Figures 7.9 and 7.10, shows that delays within certain intervals happen more frequently than others. This means that when moving a percentage increment in this interval of the distribution, the deviation parameter is not considerably affected. This is because there are occurrences of delays of about equal size, meaning that a coverage of five percent more of these points do not lead to a significant increase in the deviation parameter. Hence, the costs do not increase significantly. This explains the flattening of the curve between 20% and 70% coverage. As the coverage goes beyond 70%, the differences in the data points become larger, as the delays here are not as frequently occurring. This leads to large changes in the deviation parameters with one increase in percentage increments.

The logic behind the flattening of the curve is further confirmed by the base case deviation parameter, using the average delay of the month of March between 16.00 and 16.30. For all branches, this parameter corresponds to a deviation parameter that would cover between 50% to 80% of the delayed departures. As the average delay is located in this percentage interval, so does the most frequent deviations. This is further reflected in the objective value of the base case of 714.18 MNOK, which is the same as the objective value of the solutions with 65% and 70% coverage. This means that there are two equally optimal solutions, where one guarantees coverage of 70% of the delayed departures while the other is adjusted to the average delay at each branch. The distribution of the deviation parameter, based on the average delay of each branch, over the five percent increments is shown in Figure C.1 in Appendix C. This shows that most of the deviation parameters used in the base case would cover between 70% to 75% of the delayed departures.

To investigate the robustness due to the chosen deviation parameters, they are compared to the deviation data from the whole year of 2019. This is done by examining how high percentage of the total number of departures in 2019 each percent limit in this analysis would cover. As March is considered as a hectic month, the deviation parameter covering a percentage in this month should be able to cover a higher percentage of the departures throughout the year. The branches Frognerseteren and Ellingsrudåsen are chosen for comparison, as in Section 7.1.3. The results are shown in Table 8.6.

Table 8.6: The coverage of delayed departures of the Frognerseteren and Ellingsrudåsen branch based on the given percent limits.

Percentage coverage in March	**Coverage in 2019 Frognerseteren**	**Coverage in 2019 Ellingsrudåsen**
100	98.78	99.17
95	98.31	98.54
90	96.95	96.03
85	95.89	94.18
80	93.75	92.70
75	92.75	89.41
70	91.85	82.90
65	90.14	77.20
60	88.43	71.26
55	87.42	65.28
50	87.01	61.14
45	85.11	56.66
40	83.68	50.85
35	78.85	44.73
30	70.83	36.24
25	66.48	30.09
20	64.72	22.61
15	58.30	18.58
10	54.46	10.92
5	41.66	5.70

From Table 8.6 it can be observed that a 70% coverage of the delayed departures in March between 16.00 to 16.30 would cover 91.85% of all departures of Frognersetern throughout 2019. It would cover 82.90% of all departures of Ellingsrudåsen in the same year. This indicates that choosing deviation parameters from March contributes to buffers in the model that would handle high percentages of the delayed departures throughout the year. This enhances the robustness of the timetable.

Another interesting observation from the analysis, as seen in Figure 8.3, is the use of step-back drivers. The number of step-back drivers continually increases for the solutions that result in the same number of trains as the coverage increases. For example, for the solutions that result in 57 trains, the number of step-back drivers increases from two at a coverage of 20% to five at a coverage of 40%. This is because when the coverage increases, so does the round-trip time. When the round-trip increases, more lines are forced to use step-back drivers to obtain the scheduled arrival times. As step-back drivers inflict a cost on Sporveien, the total costs increase as well. When the coverage is further increased, the round-trip time increases to a point where it is cheaper to invest in a new train, or because it is the only alternative to avoid infeasibility.

To investigate the robustness due to the chosen deviation parameters in this analysis, they are compared to the deviation data from the whole year of 2019. This is done by examining how high percentage of the total number of departures in 2019, each percent limit in this analysis would cover. As March is considered as a hectic month, the deviation parameter covering a percentage in this month, should be able to cover a higher percentage of the departures throughout the year. The results are shown in Table 8.6.

In summary, this analysis shows that small adjustments in regulation time affect the objective value of the problem. However, both the objective value and the robustness of the model should be considered when choosing the regulation time. Further, the analysis suggests that a regulation time that is close to 70% coverage might be desirable, as this results in relatively high robustness with low induced costs compared to lower coverage rates.

8.2.2 Changes in Time Interval Distribution Requirements

This section presents a sensitivity analysis of changes in time interval distribution requirements. This is done both on branches with double-frequency requirements, and on the parts of the railway network with triple-frequency requirements.

Double-Frequency Branches

In the optimal line plan of the base case, four branches with double-frequency requirements are served by double-frequency lines. These lines are Line 2 and Line 5, with two arrivals within the ground period. This leaves only one double-frequency branch served by two single-frequency lines. This branch is Mortensrud, which is served by Line 3 and Line 4. Table 8.7 lists the time interval between the arrivals on the double-frequency branches. The time intervals on the double-frequency lines are given for both directions on the line. The two arrivals serving the two single-frequency lines on Mortensrud are only coordinated in the direction of Mortensrud branch, i.e. in east direction at Tøyen.

Table 8.7: Time intervals between arrivals at Tøyen serving branches with double-frequency requirements. The time intervals are given in minutes.

Lines serving double-frequency branches	Time interval in direction west	Time interval in direction east
Line 2	[6.59, 8.41]	[7.34, 7.66]
Line 5	[6.90, 8.10]	[7.02, 7.98]
Line 3 and Line 4	-	[6.48, 8.52]

As discussed in Chapter 3, Sporveien has defined an allowable time interval between arrivals on branches with double frequency served by either one or two lines. This interval is defined as [6, 9] minutes, meaning that two arrivals from these specific lines must have a time difference of minimum 6 minutes and maximum 9 minutes, distributed on the ground period. However, it would be desirable to have time intervals between arrivals as close to 7.5 minutes as possible, i.e. [7.5, 7.5], as this would ensure an even distribution of arrivals.

From Table 8.7 it is observed that most lines are closer to a [7, 8] distribution than a [7.5,7.5] distribution. A test is therefore conducted to explore the possibilities of restricting the allowable time intervals from a distribution of [6, 9] to the even distribution of [7.5, 7.5]. This is done by varying the time intervals with increments of 0.5 minutes, while simultaneously recording the resulting optimal objective value. The results are shown in Table 8.8. For the optimal solution to each time interval, the table presents the relative change in time windows, the number of trains, the number of step-back drivers, and the annual variable costs. The time interval used in the base case, [6, 9] is the point of reference for the relative change in time windows.

Table 8.8: Sensitivity analysis of changes to the time interval between arrivals at Tøyen for lines serving double-frequency branches.

Time interval (min)	**Relative change (min)**	**Trains**	**Step-back drivers**	**Annual variable costs (MNOK)**
[7.5, 7.5]	[1.5, -1.5]	-	-	Infeasible
[7, 8]	[1.0, -1.0]	59	3	726.34
[6.5, 8.5]	[0.5, -0.5]	58	3	714.18
[6, 9]	[0.0, 0.0]	58	3	714.18
[5.5, 9.5]	[-0-5, 0.5]	58	3	714.18
[5, 10]	[-1.0, 1.0]	58	3	714.18

It is observed from Table 8.8 that when varying the lower and upper limit of the time interval, the problem becomes infeasible somewhere between the interval [7.5, 7.5] and [7, 8]. The time interval [6.5, 8.5] results in the same amount of costs, trains, and step-back drivers as the base case, although the line plan is different. This shows that there exist several optimal line plans for the problem. Further testing shows that the most even distribution of time intervals that provides a feasible timetable is [7.34, 7.66]. Hence, relative to the time interval [6, 9], an increase in the lower limit and a decrease in the upper limit exceeding 1 minute and 20 seconds result in an infeasible solution. This time interval is close to the time interval [7.5, 7.5] preferred by Sporveien, but it comes with a cost. This time interval results in a total cost of 747.66 MNOK, having 61 trains in operation and two step-back drivers. This is a cost increase of 4.7% relative to the base case using time interval [6, 9], and is a consequence of narrowing down the solution space by restricting the limits on the time interval.

Sporveien also allows a distribution of arrivals of [5, 10] minutes if considered necessary. To investigate how a relaxation of the time interval from [6, 9] to [5, 10] affects the optimal solution, the model is solved with time intervals [5.5, 9] and [5, 10]. These results are also provided in Table 8.8. Both time intervals [5.5, 9.5] and [5, 10] result in the same solution as in the optimal one of the base case. The only differences are that the timetables are slightly changed. Hence, the relaxations do not lead to a reduction in objective value compared to the base case. Further testing shows that no reduction in optimal objective value is obtained through further relaxation of the time interval distribution.

Triple-Frequency Areas

There are two stretches in the railway network with three arrivals per ground period. These are coordinated with a triple-frequency time interval distribution requirement. As described in Section 3, the stations served by three arrivals in the ground period are between the stations Majorstuen and Smestad, and the stations between Majorstuen and Ullevål Stadion on the west part of Ringen. The first mentioned area is served by the double-frequency branch Østerås, and the single-frequency branch Kolsås. The second area is served by the branches Ringen West, Sognsvann and Vestli West, all of them single-frequency branches.

To provide good service for the passengers, Sporveien aims to have the time intervals between the three arrivals as even as possible. It is hence preferable to have either intervals of [4,5,6] or [5,5,5] minutes, i.e. a distribution of maximum [4, 6] minutes on each interval. However, this is rarely achievable in practice, and the challenge of achieving an even distribution was also encountered in the modelling of the base case. To be able to achieve feasible time tables, a distribution of [3 ,9] minutes per time interval was chosen for the base case. Examples of possible time intervals for the three arrivals are hence [3,6,6] or [5,7,3].

In the optimal solution of the base case, the double-frequency line Line 2 serves Østerås, while the single-frequency line Line 6 serves Kolsås. The first triple-frequency area is hence served by these two lines. Further, the branches Sognsvann, Ringen West and Vestli West are served by Line 3 and Line 4. Line 3, which is Ringen, has two arrivals per ground period in this area. These two arrivals are 3 Ringen and 3 Vestli. These lines hence serve the second triple-frequency area. The resulting time intervals between the three arrivals serving the two different triple-frequency areas are provided in Table 8.9.

Table 8.9: Time intervals between arrivals at Tøyen serving areas with triple-frequency requirements. The time intervals are given in minutes.

Lines serving triple-frequency areas	**Time intervals in direction west**	**Time intervals in direction east**
Line 2 and Line 6	[3.00, 3.59, 8.41]	[3.37, 3.97, 7.66]
Line 3 and Line 4	[3.00, 3.48, 8.52]	[3.00, 3.48, 8.52]

From studying the results in Table 8.9, one can see that the time intervals are not the preferred distribution of [4,5,6] or [5,5,5]. It is therefore desirable to investigate the sensitivity of changes to the time interval distribution requirements. In this analysis, the distribution for the time intervals used in the base case, i.e., [3, 9], is both restricted and relaxed by one unit in the execution of the base case. The restriction is done to see whether it is possible to get closer to the desired time intervals and at what cost. The relaxation is performed to see how large costs can be saved by making the time intervals less strict. For the optimal solution to each time interval, Table 8.10 presents the relative change in time windows, the number of trains and step-back drivers, and the annual variable costs. A selection of time intervals between [4, 6] and [1, 13] is chosen to obtain a wide-ranging analysis. Not all possible intervals are investigated, as for many time intervals the solution space is unchanged. For instance, the time interval [4, 8] is not used, as it does not result in a larger solution space than the time interval [4, 7].

Table 8.10: Sensitivity analysis of changes in the time interval between arrivals at Tøyen serving areas with triple-frequency requirements.

Time interval (min)	**Relative change (min)**	**Trains**	**Step-back drivers**	**Annual variable costs (MNOK)**
[4, 6]	[1.0, -3.0]	-	-	Infeasible
[4, 7]	[1.0, -2.0]	-	-	Infeasible
[3, 7]	[0.0, -2.0]	-	-	Infeasible
[3, 8]	[0.0, -1.0]	59	1	720.34
[3, 8.5]	[0.0, -0.5]	58	3	714.18
[3, 9]	[0.0, 0.0]	58	3	714.18
[2, 8]	[-1.0, -1.0]	58	3	714.18
[2, 9]	[-1.0, 0.0]	57	5	708.02
[2, 10]	[-1.0, 1.0]	57	5	708.02
[1, 10]	[-2.0, 1.0]	57	4	705.02
[1, 13]	[-2.0, 4.0]	57	4	705.02

From Table 8.10 one can observe that the model cannot find a feasible timetable somewhere between the time intervals [3, 8] and [3, 7]. It is therefore not possible to achieve the desired time interval [4, 6]. The time interval [3, 8], having the upper limit decreased by one minute relative to the base case, results in a total of 59 trains, three step-back drivers, and a cost increase of 0.86 %. The time intervals [3, 8.5] and [2, 8] result in the same solution in regards to costs, trains, and step-back drivers as the base case solution having time interval [3, 9]. However, the line plans generated are different. This shows that small changes to the time interval can result in several optimal solutions.

When relaxing the time interval distribution requirements further than [2, 8], one can save between 0.86-1.28% in costs. Nevertheless, this results in an uneven distribution of arrivals, which provides poorer passenger service. Therefore it must be considered whether the cost reduction is worth a lower passenger satisfaction, which could also have negative economical consequences.

8.3 Case Studies

This section presents the computational results of the case studies. Section 8.3.1 discusses the results obtained when solving the TTP for the existing line plan of Oslo Metro. In Section 8.3.2, the results obtained when including the west branch Fornebu are presented and discussed. Both computational studies are performed based on the base case.

8.3.1 Solving the TTP Using the Current Line Plan of Oslo Metro

To compare our model with the current methods used by Sporveien in the timetabling process, the current line plan of Oslo Metro is inserted into the TTP. This is done by fixing the line combination of the LPP to the current line plan, and solve the TTP with the base case parameters. The current line plan is shown in Figure 2.1. In addition, the ninth arrival is included in the study, as a line between Veitvet and Stortinget. The key results from the solving of the TTP with the current line plan are provided in Table 8.11.

Table 8.11: Key results from the solving of the TTP using the current line plan as input.

Number of trains	**Number of step-back drivers**	**Annual variable costs (MNOK)**
59	2	723.34

As seen in Table 8.11, the case study results in a total of 59 trains and two step-back drivers, with a total cost of 723.34 MNOK. As Sporveien plans to operate with a total of 60-61 trains based on the same line plan, the TTP model returns 1-2 fewer trains needed in operation. The number of step-back drivers remains the same, but they are located at different terminal stations. In the current operations, Sporveien uses step-back drivers on Line 4 at Bergkrystallen, and on Line 5 at Sognsvann. In the optimal solution of the TTP, step-back drivers are applied to Line 2 at Ellingsrudåsen, and on Line 4 at Vestli West.

The results indicate that there exists a more optimal timetable than the one Sporveien operates with today. The utilization of the solution method presented in this thesis hence proves to be more efficient relative to the current manual planning. Furthermore, a contributing factor for the reduction is that the model applies individual fitted buffer times for each arrival based on historical data. This could reduce the use of unnecessary buffer times. It must be taken into account that the reduction in the number of trains might be influenced by the inclusion of the new signalling system in the TTP. This reduces the travel times by three percent relative to the travel times in the current operations.

To further investigate the robustness of the TTP, the regulation times in the TTP solution and in the operations of Oslo Metro today are compared. The ninth arrival is excluded in the comparison. Table 8.12 shows the regulation time for each arrival on each branch in the two solutions. The regulation times of the arrivals at Bergkrystallen and Sognsvann are zero in the current timetable. The reason for this is explained in Section 8.1.3.

Table 8.12: Comparison of the regulation times for each arrival in the timetable constructed by the TTP model and by Sporveien.

Branch	Line	Arrival	Regulation time TTP (min)	Regulation time Sporveien (min)
Frognerseteren	1	1	3.61	2.40
Bergkrystallen	1	1	7.54	2.75
Østerås	2	1	5.62	5.67
Østerås	2	2	4.66	3.67
Ellingsrudåsen	2	1	1.68	2.27
Ellingsrudåsen	2	2	1.68	4.27
Kolsås	3	1	3.73	4.97
Mortensrud	3	1	9.84	5.67
Stortinget	4	1	13.71	12.00
Mortensrud	4	1	3.84	2.67
Vestli West	5	1	5.37	2.42
Bergkrystallen	5	1	7.54	0.00
Sognsvann	6	1	5.72	0.00
Vestli East	6	1	2.38	5.42

The summation of all the regulation times in the two timetables results in a total of 76.92 minutes in the timetable of the TTP and a total of 54.18 minutes in the timetable used by Sporveien. The timetable constructed in the TTP hence has a total buffer which is 22.74 minutes larger than in the timetable currently used. A total of nine arrivals in the TTP have a regulation time exceeding the regulation time of the corresponding arrivals in the timetable of Sporveien. These excesses are between 59.4 seconds and 7.54 minutes. The difference in regulation time for the five remaining arrivals which have smaller regulation times in the TTP than in the timetable of Sporveien ranges between three seconds and 3.04 minutes. Therefore, the timetable constructed in this case study can be considered more robust than the timetable Sporveien operates with today. This is relative to both the total regulation time and the regulation time on each branch.

8.3.2 The Inclusion of the Fornebu Branch in the Oslo Metro

As mentioned in Section 2.7.2, a new branch is planned in the metro system as an extension of the service provided by Sporveien. This branch is defined as a west branch that connects the terminal station Fornebu to Tøyen. The new branch is a double-frequency branch. This means that each of the two arrivals that turn at Stortinget in the base case is now connected to a west branch. In other words, the Fornebu branch replaces the double-frequency Stortinget branch. As the travel time at the Fornebu branch is longer than the travel time of the Stortinget branch, more trains are needed in the system to meet the frequency requirements. This case study is an example of how the optimization model, by the inclusion of a branch and its associated parameters, can make structural and operational changes in the Oslo Metro more efficient. The additional parameters for the Fornebu branch concerning frequency, travel time, technical dwell time and the deviation parameter are given in Section 7.2.3.

Results of the Fornebu Case Study

The key numbers from the results of the Fornebu case study are given in Table 8.13. The resulting line plan is presented in Table 8.14, and is illustrated in Figure 8.4. Further results concerning the operation of the lines are provided in Table 8.15.

Table 8.13: Key results from the optimal solution of the Fornebu branch case study.

Number of trains	Number of step-back drivers	Annual variable costs (MNOK)
62	6	771.82

Table 8.14: The optimal line plan with the inclusion of the Fornebu branch.

Line	West branch	East branch
1	Frognerseteren	Mortensrud
2	Fornebu	Ellingsrudåsen
3	Ringen West	Mortensrud
3	Vestli West	Ringen East
4	Østerås	Bergkrystallen
5	Kolsås	Vestli East
6	Sognsvann	Veitvet

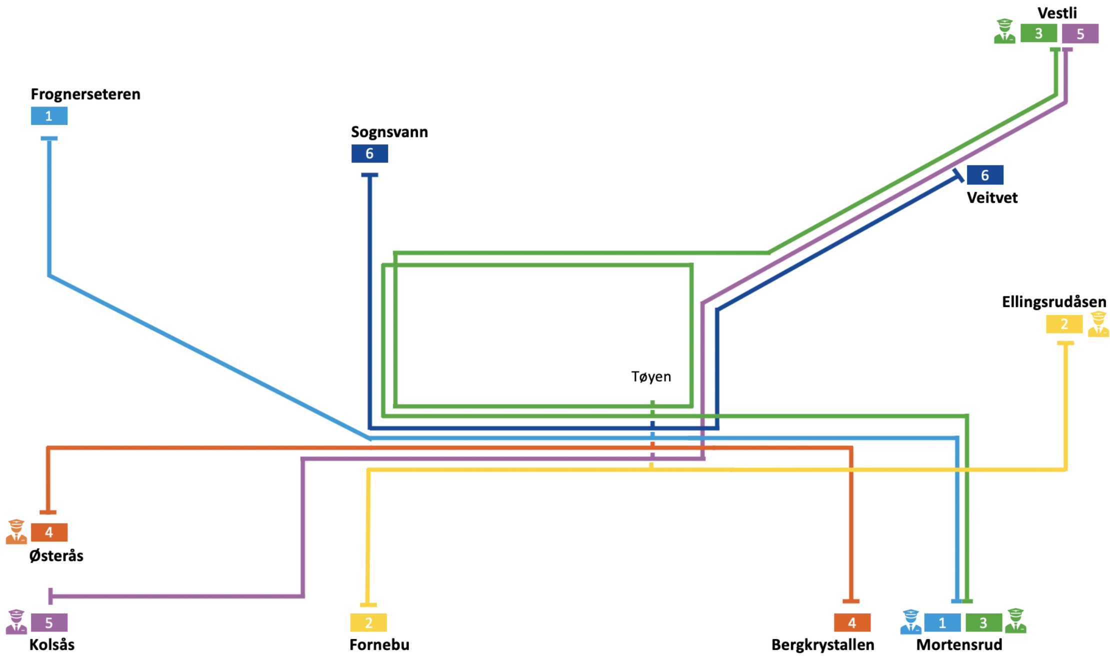

Figure 8.4: The optimal line plan in the case study of Fornebu.

Table 8.15: Results of the Fornebu case study for each line in the new line plan.

Line	Frequency	Use of step-back driver indicated by X	Direction of driver break	Number of trains
1	Single	X	West	9
2	Double	X	West	12
3	Single	X	East	5
3	Single	X	West	8
4	Double	X	East	13
5	Single	X	West	9
6	Single	-	East	6

As seen in Table 8.13, the inclusion of the Fornebu branch in the metro system results in a total of 62 trains. This means that the addition of the Fornebu branch gives four more trains relative to the base case. The resulting line plan is provided in Table 8.14. Compared to the line plan solution of the base case, only the line Ringen has the same branch combinations, with Vestli West and Mortensrud as its terminal stations. The distribution of the 62 trains on the six lines can be read from Table 8.15. Further, the table provides the frequencies of the lines, the use of step-back drivers, and the direction of the driver breaks. The corresponding optimal timetables for westbound and eastbound arrivals, and the resulting regulation times per arrival can be read from Tables D.1-D.3 in Appendix D.

Comparing the Results From the Case Study with the Plans of Sporveien Concerning the Addition of the Fornebu Branch

Sporveien plans to operate with a total of 64 trains when the Fornebu branch is included in the metro system. This case study hence returns two fewer trains in operation. It is not known how many step-back drivers Sporveien plans to use, as Sporveien has not yet started the operational planning phase for the Fornebu project. The step-back drivers in this case study are applied to all lines except at Line 6 in the optimal solution, as seen in Table 8.15. The line plan with the number of trains per line and their respective frequencies as planned by Sporveien are given in Table 8.16. When comparing the two line plans, the Fornebu case study results in new branch combinations for all lines.

Table 8.16: Line plan with the number of trains per line and frequency as planned by Sporveien for the inclusion of the Fornebu branch.

Line	West branch	East branch	Frequency	Number of trains
1	Frognerseteren	Veitvet	Single	8
2	Østerås	Ellingsrudåsen	Double	14
3	Fornebu	Mortensrud	Double	14
4	Sognsvann/Ringen West	Ringen East/Bergkrystallen	Single	10
5	Kolsås	Bergkrystallen	Single	8
6	Vestli West	Vestli East	Single	10

8.4 Extended Model

The extended model is defined in Section 5.4. In Section 8.4.1 the computational results of the model are presented. In Section 8.4.2, a sensitivity analysis of the penalty parameters is performed.

8.4.1 Results from the Extended Model

In this section, the results from the model extended with soft constraints are presented. The input parameters are the same as in the base case, in addition to the specific parameters of the extensions. As described in Section 5.4, the soft constraints permit the model to violate the constraints (5.54). Constraints (5.54) ensure that the regulation time must exceed the deviation parameter. The soft constraints are stated in constraints (5.94). The parameters which are specific for the model extensions are explained in Section 7.2.2.

Table 8.17 shows the optimal line plan of the extended model. It can be observed that the lines 1, 3 and 6 are composed of different branch combinations than in the optimal solution of the base case. The other lines are identical. The table also shows an overview of the use of step-back drivers on each line. Compared to the base case solution, two additional step-back drivers are included. These are used on Line 1 and Line 6.

Table 8.17: The optimal branch combination of the extended model, with corresponding line numbers and the use of step-back drivers.

Line	West branch	East branch	Use of step-back driver indicated by X
1	Frognerseteren	Mortensrud	X
2	Østerås	Ellingsrudåsen	X
3	Vestli West/Ringen West	Ringen East/Mortensrud	X
4	Stortinget	Bergkrystallen	-
5	Kolsås	Vestli East	X
6	Sognsvann	Veitvet	X

Table 8.18 presents the key results of the optimal solution of the extended model. Compared to the base case solution, the number of trains is reduced by one, whereas the number of step-back drivers is increased by two. The total violation aggregated over all lines is 0.90 minutes i.e. 54 seconds. As the value of the violation variable is positive, the constraints (5.54) from the base model are violated. Hence, the optimal solution of the extended model is infeasible with respect to the constraints of the base case. Further investigation shows that the violation occurs in its entirety on the second arrival on Line 5, westbound from Vestli to Kolsås. This means that the violation variable of all other arrivals is zero. A positive deviation variable of 0.90 minutes means that the regulation time is 0.90 minutes lower than the deviation parameter. The regulation time for this arrival is 0.43 minutes, while its deviation parameter is 1.33 minutes. Thus, the ability of the arrival to absorb eventual delays is lower than for the other arrivals. This specific arrival is hence more prone to delays. The total regulation time in the optimal solution of the extended model is 65.33 minutes. This is 3.35 minutes less than the total regulation time in the optimal base case solution.

Table 8.18: Key figures for the optimal solution of the extended model.

Trains	Step-back drivers	Total violation (min)	Total regulation time (min)
57	5	0.90	65.33

Table 8.19 presents the decomposition of costs in the optimal objective of the extended model. In the extended model, the penalty and the reward are included in the objective function. In the table, the reward is given with a negative sign as it is subtracted from the costs. This is explained in Section 5.4.1. The total costs amount to a total of 713.54 MNOK, which is 0.64 MNOK lower than the optimal objective value of the base case. As mentioned in Section 7.2.2, the objective value of the extended model does not reflect the actual variable costs of Sporveien. The value is used to compare solutions found by the model.

Table 8.19: Decomposition of costs for the optimal solution of the extended model, in MNOK rounded to two decimals.

Cost of trains	Cost of step-back drivers	Penalty	Reward	Annual variable costs
693.02	15.00	5.54	-0.02	713.54

Table 8.20 shows the key numbers from the solving of the extended model. The number of feasible solutions is 195, which is 81 more than in the base case. This is a result of the relaxation of the base case induced by the soft constraints in the extended model.

Table 8.20: Overall key results of the solving of the model.

Iterations	Feasible solutions	Infeasible solutions
517	195	322

For more results concerning the optimal solution of the extended model, see Table E.2. The Tables E.3 and E.4 present the timetable for the westbound and eastbound arrivals, respectively.

In summary, the extended model results in a significant increase in the number of feasible solutions. The optimal solution found in the extended model is infeasible with respect to the base case. It results in one less train, and two more step-back drivers, saving costs of 0.64 MNOK per year in the defined time horizon of 30 years. In the optimal solution, a positive deviation occurs once, and it is less than one minute.

8.4.2 Sensitivity of the Optimal Solution to Changes in the Penalty Values

In this section, the sensitivity of the optimal solution to changes in the penalty value is analysed. The results from solving the extended model with three different penalty values are studied. The focus of the sensitivity analysis is the variation in objective value and level of robustness. Hence, only selected values concerning costs and regulation time are presented for each solution.

The values of the penalties are presented in Table 8.21. The categories are defined as low, medium, and high. Figure 8.5 shows how the penalty values are distributed on the interval [0, C^{Train}]. The cost, C^{Train}, represents the annual variable costs related to a train. The medium category corresponds to solving the extended model with the parameters presented in Section 7.2.2.

Table 8.21: The categories of the penalties and their respective values.

Penalty category	Penalty value
Low penalty	C^{Train} - 12
Medium penalty	C^{Train} - 6
High penalty	C^{Train} - 1

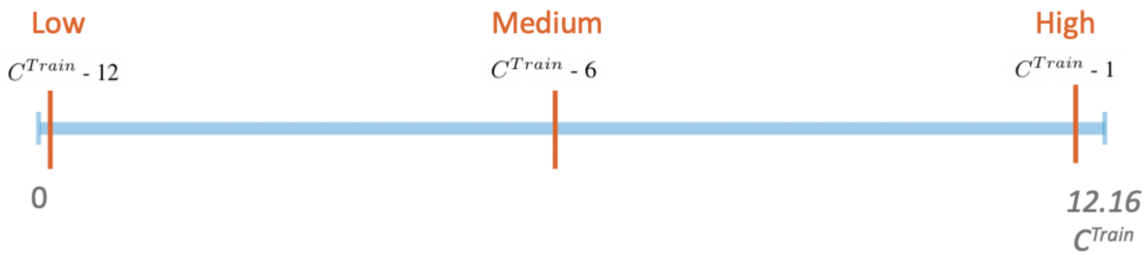

Figure 8.5: The penalty values distributed on the interval [0 , C^{Train}]. C^{Train} equals 12.16 MNOK.

The solving of the model with the low, medium and high penalty values are referred to as the *low case*, *medium case* and *high case*, respectively. Table 8.22 shows the number of iterations, and the number of feasible and infeasible solutions for the low, medium, and high case.

Table 8.22: Overall key results of the low, medium and high case.

Penalty category	Iterations	Feasible solutions	Infeasible solutions
Low	204	95	109
Medium	517	195	322
High	517	195	322

It can be observed from Table 8.22 that the low case has considerably fewer iterations than the medium and high case. This is because the Combinatorial Benders' Decomposition stops iterating if the subproblem finds a better solution than the master problem, as explained in Section 6.2.2. In the low case, this occurs in iteration 204. The medium and the high case are solved after 517 iterations.

Table 8.23 shows that the chosen penalty value affects the optimal solution of the model. The number of trains and step-back drivers differ in each of the cases. The total excess represents the summation of the excess that occurs when the regulation time exceeds the deviation parameter at a branch. The total excess in the metro system for each penalty value is also shown in Table 8.23.

Table 8.23: Key figures for the optimal solution of the extended model with low, medium and high penalty.

Penalty category	**Trains**	**Step-back drivers**	**Total violation (min)**	**Total excess** $(\lambda - L)$ **(min)**
Low	55	4	18.68	31.43
Medium	57	5	0.90	24.54
High	58	3	0.00	27.00

Choosing the high penalty value results in no positive violations. Thus the solution is the same as the base case solution. The low case, on the other hand, has significantly lower objective value and larger violation values. In the low case, there are two fewer trains and one less step-back driver than in the medium case. It can further be observed that both the total violation and the total excess are larger in the low case, than in the medium and high case. This indicates that certain branches have considerably larger regulation time, and other branches have a regulation time that is considerably lower than the deviation parameter. Consequently, some branches are more prone to delays than others. Further, a delay on one branch might induce a delay on other branches in the system due to knock-on effects. Knock-on effects are described in Section 4.4.4. These factors contribute to an unstable metro system with uneven distribution of buffer times. As a comparison, all the branches have larger regulation time than the deviation parameter in the optimal solution of the high case. Hence this line plan is less sensitive to knock-on effects. Table 8.24 presents the values of the components in the objective function for each of the cases.

Table 8.24: Decomposition of costs in the optimal solution of the extended model, in MNOK, rounded to two decimals.

Penalty category	**Cost of trains**	**Cost of step-back drivers**	**Total penalty**	**Total reward**	**Annual variable costs**
Low	668.71	12.00	4.35	-0.05	685.05
Medium	693.02	15.00	5.54	-0.02	713.54
High	705.18	9.00	0	-0.03	714.16

The optimal solution of the low case has an objective value of 685.05 MNOK. This value is considerably lower than the objectives in the medium and high cases, which are 713.54 MNOK and 714.16 MNOK, respectively. This difference is largely due to the difference in the number of trains.

Despite that the total violation is considerably larger for the low case than the medium case, as shown in Table 8.23, it can be observed that the total penalty in the low case is lower than the total penalty in the medium case. This is because the penalty value p_e is considerably lower in the low case than the medium case. The reward makes up a small part of the objective value for each of the solutions. This is due to the reward parameter, which is assigned a low value. As explained in Section 7.2.2, the purpose of the reward is to choose the most robust solution in case several solutions have identical objectives.

In summary, the extended model is sensitive to variations in the penalty value. Assigning the penalty a value close to the variable costs of a train results in a conservative solution. Assigning the penalty a low value, on the other hand, results in a reduction of two trains compared to the medium case. The timetable is in this case more prone to delays. The preferred line plan depends on how Sporveien evaluates the trade-off between robustness and cost-efficiency.

8.5 Cost and Utilization Performance

The optimal solution of the base case, as presented in Section 8.1, gives a new line plan with 58 trains and three step-back drivers. It reduces the number of trains in operation by 2-3 trains compared to what Sporveien plans to operate with after the inclusion of the ninth arrival. The number of step-back drivers is increased by one, as Sporveien plans to operate with only two step-back drivers. Applying an extra step-back driver induce a cost of about 3 MNOK a year. Saving 2-3 trains, on the other hand, would lead to significant cost savings surpassing the expenses of an extra step-back driver. In this thesis, costs related to trains concern investment cost, alignment cost, and operational costs. The investment cost of a train amounts to 110 MNOK. Further, Sporveien spends 110 MNOK per train in alignment cost. These two cost components constitute the most significant costs associated with trains in the Oslo Metro. Other operational costs, like maintenance and driver costs, must also be taken into account. Therefore, Sporveien could save millions by reducing their rolling stock for the Oslo Metro.

The mission of Sporveien, as mentioned in Section 2.2, indicates that the issue of balancing cost-efficiency and quality of service is central for the company. This corresponds to the trade-off between the minimization of costs and the robustness of the model as discussed in this thesis. A robust model results in a more reliable timetable. This means that the risk of delays is reduced, which can be considered as a model with a high quality of service. On the other hand, large regulation times could result in cost inefficiency. In this thesis, the minimum regulation time needed for each arrival is found based on historical data on travel time deviations. The regulation times on all individual branches are therefore more likely to fit their individual need of buffer to withstand delays. The optimal timetable in the solution of the TTP using the current line plan proved to be more robust than the timetable currently used by Sporveien. The generated timetable further resulted in a reduction in trains needed in operation. The timetable generated is therefore considered more robust and cost-efficient than the timetable used in the current operations of Oslo Metro.

The decision concerning the trade-off between robustness and the objective value also depends on the cost of delays compared to the cost of trains. The cost of delays depends on several factors, such as customer and employee satisfaction and reputation. Therefore these factors have the potential of affecting future profit. The monetary value is however difficult to estimate. This makes it challenging to determine the value of the trade-off. For instance, as discussed in Section 8.2.1, fixing the regulation time to zero results in a total of 55 trains. This is a solution that reduces total costs by approximately 35 MNOK compared to the results of the base case. However, this solution has a low level of robustness and thereby a low quality of service. Thus it might not be a preferred solution for Sporveien.

This is further illustrated in the sensitivity analysis conducted in Section 8.4.2 concerning the penalty value in the extended model version. In the high penalty case, the model chooses not to violate the minimum buffer required, as the cost of violating this constraint is considered too high.

Implementing a new branch to the metro system demands large structural and operational changes. Decisions regarding branch combinations and a corresponding timetable are difficult and time-consuming when performed by trial-and-error in Excel. The case study in Section 8.3.2 shows how efficiently the optimization model finds a new line plan and corresponding timetable with the inclusion of the branch. Taking use of optimization tools in the planning phase could hence save both time and costs. This further shows that the model handles fundamental changes in the structure of the Oslo Metro. This indicate that the model can be useful in the planning of structural changes in future projects concerning the Oslo Metro.

Chapter 9

Concluding Remarks

In this thesis, the problems of line planning and train timetabling are investigated. These problems are integrated and solved using Combinatorial Benders' Decomposition. A mathematical model that aims to find an optimal line plan for the Oslo Metro, with a corresponding optimal timetable, is proposed. What makes this problem unique, is the rare structure of the Oslo Metro in which all lines share the same rails through the city center. This makes the relevant literature to this problem scarce. To the best of our knowledge, the problem of integrating the LPP and the TTP with all the aspects included in this thesis has never before been studied.

The thesis is written in collaboration with Sporveien. Sporveien's mission is to provide reliable and accessible public transportation, to the lowest cost possible both for society and the environment. Therefore, they wish to investigate whether a rearrangement of the existing railway branches can result in more efficient operations, and reduce costs. The problem involves the considerations of Oslo Metro in regards to structure and operations and is based on operational data. Sporveien has supplied all data and information used in the formulation of the model.

Harbo et al. (2019) were the first to investigate this problem. Their objective is to minimize the number of trains in operation. As an extension of their problem, the objective of this thesis concerns the minimization of costs related to the number of trains and step-back drivers. Train-related costs involve investment costs, alignment costs, maintenance costs, and driver costs. This provides a more nuanced picture of the operations and enables the inclusion of step-back drivers to the problem. The problem investigated in this thesis extends the problem studied by Harbo et al. (2019) through the consideration of new areas of conflict and operational requirements. This contributes to a model which to a larger extent captures the actual operations in the Oslo Metro. Extensions concern the consideration of triple-frequency areas and the avoidance of conflict at Smestad station, and specific terminal stations. Further extensions are the inclusion of step-back drivers and allowing arrivals on double-frequency lines to change time slots.

A further extension to the work by Harbo et al. (2019) is the use of historical data on travel time deviations to estimate the minimum need of a buffer on all branches. The regulation times are hence individually fitted to each arrival in the model solution. This results in a model that values both robustness and cost-efficiency, contributing to a balance in the trade-off between the two. The trade-off between robustness and cost-efficiency is further investigated by the addition of soft constraints in the extended model.

The research done in the computational study proves that the solution method presented in this thesis enables considerable cost savings. This is primarily enabled through a reduction in the total number of trains needed in operation. The optimal solution of the base case results in a reduction of 2-3 trains compared to the current operations. Moreover, this solution is equally robust compared to the timetable of Sporveien, in terms of total buffer and the buffer on each branch. This indicates that the model finds a better trade-off between robustness and cost-efficiency. Furthermore, a case study is conducted in which the TTP is solved with the current line plan as input. This results in 1-2 fewer trains than the number of trains Sporveien currently operates with, and the corresponding timetable is significantly more robust. This emphasises that the model finds robust solutions that are also cost-efficient. The extended model enables further investigations of the trade-off between robustness and cost-efficiency. By including soft constraints, the model rewards the excess of buffer time and penalizes the absence of it. The results of the extended model show that there is significant potential for cost reduction if robustness is neglected. If customer service is highly valued, more conservative solutions are preferred. Excluding buffers from the timetable would hence not be a preferable alternative for Sporveien.

In the case study concerning the Fornebu branch, the model proves to be an efficient planning tool for incorporating new branches to the existing metro system. This means that the model could be a helpful planning tool for railway systems with a similar structure. This accounts for both existing systems and the incorporation of new infrastructure.

We conclude that the solution method integrating the planning steps of line planning and timetabling could contribute as a helpful decision-making tool for Oslo Metro and similar railway networks. The application of mathematical models and solution methods to the integrated planning process increases insights and helps to weigh all requirements and wishes regarding the structure and operations of the metro system. The model could also make the application of structural and operational changes to the Oslo Metro easier, being a generalized framework for the planning process of LPP and TTP. The planning tool developed in this thesis hence contributes to an efficient and robust operation of the metro system. As Oslo Metro is the leading actor within the public transport of Oslo, the planning tool would enhance the overall offer of public transport.

Chapter 10

Future Research

The metro network in Oslo is a complex structure, with numerous aspects that could be taken into consideration. Although the problem discussed in this report takes many of these considerations into account, there are still aspects that could be implemented. A possible extension to the problem is to include all intermediate stations in the timetabling problem. This could contribute to more accurate modelling of the operations. An example is the allocation of buffer times. In the problem addressed in this thesis, the buffer times on all branches are inserted at the terminal stations. The inclusion of the intermediate stations to the timetabling problem enables the possibility of allocating the buffers to more specific locations prone to delays. This contributes to the generation of robust and cost-efficient timetables.

The uncertainty in the travel times could be further considered through the use of stochastic models or simulations. Scenario generation could be used to evaluate the uncertainty in travel times, and hence contribute to finding more correct buffer times. Simulations can be used to evaluate the correct insertion of the buffer times. Another interesting extension to the problem is the inclusion of Machine Learning to reduce the probability of delays occurring, and mitigate their impact on the operations.

The problem studied in this thesis concerns the inclusion of soft constraints to enhance the robustness of the arrivals. Further preference constraints can be modeled as soft in a future study. Preference constraints are in this thesis considered as constraints based on preferences from Sporveien concerning the operations of Oslo Metro. It is desirable that these constraints are complied with, but not necessarily at any cost. The constraints concerning frequency requirements are examples of preference constraints that could be modeled as soft constraints in a future study.

A further extension of the problem is the integration of the planning step of demand analysis, as described in Section 4.2. This is the first step in the hierarchical planning process and concerns the interpretation of data on passenger demand. Demand analysis enables the model to take the decision of branch frequencies into account. As stated in Section 4.2, solving planning steps simultaneously can contribute to finding better overall solutions. The demand analysis could further be used to find trends in travel routes for metro passengers. This information could be applied to the LPP to find optimal branch combinations based on the passengers' travel habits. This could enhance the service level of both the line plan and the timetable.

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Appendix

Appendix A

Deviation Parameters used in the Regulation Time Sensitivity Analysis

Table A.1: Deviation parameter, L_{ir}, for the percentage shares of delayed departures at Tøyen.

Percentage	RINGW	SOG	FRS	ØSÅ	KOL	VESW	STT	RINGE	ELÅ	MOR	BKR	VES	VEI
0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0.82	0.98	1.17	0.70	0.77	0	0	0.22	0.07	0.52	0.05	0.03
10	0.11	1.10	1.42	1.33	1.70	1.50	0.17	0	0.35	0.23	0.65	0.28	0.15
15	0.37	1.30	1.57	1.95	1.80	2.22	0.22	0	0.52	0.28	0.80	0.47	0.24
20	0.49	1.62	1.67	2.43	1.97	2.68	0.60	0	0.60	0.45	0.93	0.53	0.28
25	0.66	1.77	1.92	2.87	2.17	2.83	0.67	0	0.73	0.52	1.07	0.63	0.33
30	0.76	1.88	2.13	3.25	2.22	3.20	0.82	0	0.83	0.58	1.32	0.78	0.41
35	0.83	2.05	2.63	3.55	2.43	3.28	0.93	0	0.97	0.68	1.43	0.88	0.46
40	0.89	2.42	3.08	3.63	2.67	3.55	1.12	0	1.07	0.78	1.55	1.00	0.52
45	1.00	2.50	3.25	3.82	2.78	4.05	1.15	0.02	1.17	0.90	1.72	1.07	0.56
50	1.23	2.58	3.48	4.05	3.18	4.53	1.25	0.02	1.25	1.02	1.80	1.12	0.58
55	1.45	2.95	3.53	4.13	3.50	4.75	1.35	0.04	1.33	1.12	2.03	1.17	0.61
60	1.68	3.00	3.68	4.32	3.68	5.00	1.55	0.05	1.47	1.18	2.12	1.23	0.64
65	1.89	3.25	3.95	4.47	3.90	5.12	1.60	0.07	1.63	1.27	2.23	1.35	0.70
70	2.19	3.50	4.30	4.70	4.03	5.15	1.70	0.11	1.83	1.40	2.45	1.37	0.71
75	2.48	4.78	4.47	5.17	4.13	5.22	1.75	0.15	2.22	1.80	3.03	1.82	0.95
80	2.92	6.25	4.70	5.62	4.47	5.48	1.80	0.15	2.55	2.00	3.88	2.07	1.08
85	4.01	7.52	5.43	6.43	4.85	5.95	2.17	0.16	2.80	2.30	5.37	2.20	1.15
90	4.10	7.90	6.20	6.92	5.92	6.38	2.55	0.18	3.32	3.20	6.37	2.58	1.35
95	5.05	9.13	7.50	8.55	8.82	6.78	2.70	0.22	5.23	5.08	7.05	2.92	1.52
100	7.53	10.98	8.20	12.52	13.80	7.48	4.88	0.70	7.05	15.98	15.87	8.98	4.69

Appendix B

Timetable Corresponding to the Optimal Line Plan of the Base Case

Table B.1: Arrivals at Tøyen in westbound direction, corresponding to the optimal line plan.

Line	Arrival	Time at Tøyen
1 Frognerseteren	1	13.07
2 Østerås	1	0.00
2 Østerås	2	6.59
3 Ringen	1	11.57
3 Vestli	1	8.09
4 Sognsvann	1	5.09
5 Stortinget	1	1.50
5 Stortinget	2	9.60
6 Kolsås	1	3.00

Table B.2: Arrivals at Tøyen in eastbound direction, corresponding to the optimal line plan.

Line	**Arrival**	**Time at Tøyen**
1 Vestli	1	1.55
2 Ellingsrudåsen	1	10.39
2 Ellingsrudåsen	2	3.05
3 Mortensrud	1	8.52
3 Ringen	1	12.00
4 Mortensrud	1	15.00
5 Bergkrystallen	1	5.52
5 Bergkrystallen	2	13.50
6 Veitvet	1	7.02

Appendix C

Model Sensitivity to Changes in the Deviation Parameter

Table C.1: Model sensitivity to changes in regulation time.

Percentage	Total costs (MNOK)	Number of trains	Number of step-back drivers
0	680.71	55	4
5	686.87	56	2
10	689.87	56	3
15	689.87	56	3
20	699.03	57	2
25	702.03	57	3
30	702.03	57	3
35	702.03	57	3
40	708.03	57	5
45	708.18	58	1
50	708.18	58	1
55	708.18	58	1
60	711.18	58	2
65	714.18	58	3
70	714.18	58	3
75	720.34	59	1
80	726.34	59	3
85	735.50	60	2
90	741.50	60	4
95	753.66	61	4
100	Infeasible	Infeasible	Infeasible

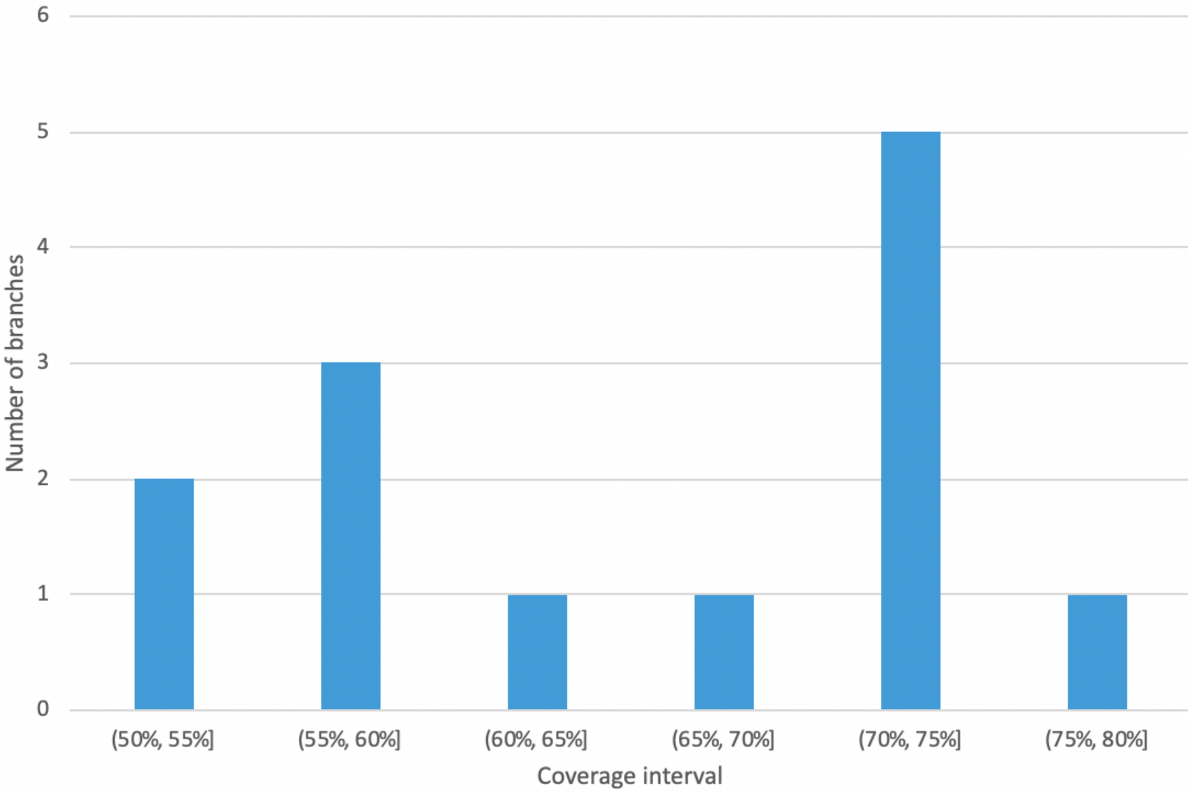

Figure C.1: The distribution of the deviation parameter, which is based on average delay, over the coverage percentages. The figure shows the number of branches with average delay within the given percentage intervals.

Appendix D

Results of the Fornebu Case Study

Table D.1: Arrivals at Tøyen in westbound direction, corresponding to the optimal line plan in the Fornebu case study.

Line	Arrival	Time at Tøyen (min)
1 Frognerseteren	1	13.50
2 Fornebu	1	9.70
2 Fornebu	2	1.50
3 Ringen	1	6.48
3 Vestli	1	3.00
4 Østerås	1	12.00
4 Østerås	2	4.50
5 Kolsås	1	7.98
6 Sognsvann	1	0.00

Table D.2: Arrivals at Tøyen in eastbound direction, corresponding to the optimal line plan in the Fornebu case study.

Line	Arrival	Time at Tøyen (min)
1 Mortensrud	1	10.50
2 Ellingsrudåsen	1	4.50
2 Ellingsrudåsen	2	13.50
3 Mortensrud	1	2.52
3 Ringen	1	6.00
4 Bergkrystallen	1	7.50
4 Bergkrystallen	2	15.00
5 Vestli	1	12.00
6 Veitvet	1	9.00

The buffer time, i.e. the regulation time, on all branches for all arrivals are presented in Table D.3. The delay parameters used for each branch are provided for comparison.

Table D.3: Regulation time for all arrivals on all lines, in both directions in the optimal solution of the Fornebu case study.

Branch	Line	Arrival	Regulation time (min)	Deviation parameter (min)
Frognerseteren	1	1	4.32	3.57
Mortensrud	1	1	2.66	1.57
Fornebu	2	1	5.40	3.71
Fornebu	2	2	4.60	3.71
Ellingsrudåsen	2	1	2.48	1.68
Ellingsrudåsen	2	2	1.68	1.68
Mortensrud	3	1	3.62	1.57
Vestli West	3	1	6.55	5.23
Østerås	4	1	4.34	4.23
Østerås	4	2	4.34	4.23
Bergkrystallen	4	1	6.36	2.90
Bergkrystallen	4	2	6.36	2.90
Kolsås	5	1	8.46	3.63
Vestli East	5	1	4.45	1.41
Sognsvann	6	1	8.66	3.71
Veitvet	6	1	2.51	0.73

Appendix E

Results of the Extended Model

Table E.1: Regulation time and violation for all arrivals on all lines, in both directions in the optimal solution of the extended model.

Branch	**Line**	**Arrival**	**Regulation time (min)**	**Deviation parameter (min)**	**Reward (MNOK)**	**Violation (min)**	**Penalty (MNOK)**
Frognerseteren	1	1	5.12	3.57	0.002	0	0
Mortensrud	1	1	1.86	1.57	0.000	0	0
Østerås	2	1	4.23	4.23	0.000	0	0
Østerås	2	2	5.14	4.23	0.001	0	0
Ellingsrudåsen	2	1	2.59	1.68	0.001	0	0
Ellingsrudåsen	2	2	1.68	1.68	0.000	0	0
Vestli West	3	1	6.55	5.23	0.001	0	0
Mortensrud	3	1	3.62	1.57	0.002	0	0
Stortinget	4	1	2.41	1.33	0.001	0.000	0
Stortinget	4	2	0.43	1.33	0.00	0.90	5.54
Bergkrystallen	4	1	3.30	2.90	0.000	0	0
Bergkrystallen	4	2	4.32	2.90	0.001	0	0
Kolsås	5	1	8.46	3.63	0.005	0	0
Vestli East	5	1	4.45	1.41	0.003	0	0
Sognsvann	6	1	8.66	3.71	0.005	0	0
Veitvet	6	1	2.51	0.73	0.002	0	0

Table E.2: Frequency, use of step-back drivers and number of trains in the optimal solution of the extended model.

Line	Frequency	Use of step-back driver indicated by X	Direction of driver break	Number of trains
1	Single	X	West	9
2	Double	X	East	13
3	Single	X	West	8
3	Single	X	East	5
4	Double	-	East	7
5	Single	X	East	9
6	Single	-	East	6

Table E.3: Arrivals at Tøyen in westbound direction, corresponding to the optimal line plan in the extended model.

Line	Arrival	Time at Tøyen
1 Frognerseteren	1	12.70
2 Østerås	1	4.61
2 Østerås	2	11.20
3 Vestli West	1	3.00
3 Ringen West	1	6.48
4 Stortinget	1	1.50
4 Stortinget	2	9.48
5 Kolsås	1	7.98
6 Sognsvann	1	0.00

Table E.4: Arrivals at Tøyen in eastbound direction, corresponding to the optimal line plan in the extended model.

Line	Arrival	Time at Tøyen
1 Mortensrud	1	10.5
2 Ellingsrudåsen	1	7.50
2 Ellingsrudåsen	2	15.00
3 Ringen East	1	9.00
3 Mortensrud	1	2.52
4 Bergkrystallen	1	13.50
4 Bergkrystallen	2	4.50
5 Vestli East	1	12.00
6 Veitvet	1	9.00

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