Master's thesis

NTNU
Norwegian University of Science and Technology
Faculty of Economics and Management
Dept. of Industrial Economics and Technology Management

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Michel Aleksander Evensen Norum

# An Optimization-Based Approach to the School Prioritization Problem in Trondheim Municipality

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# Preface

This thesis concludes our Master of Science in Industrial Economics and Technology Management at the Norwegian University of Science and Technology, Department of Industrial Economics and Technology Management. The work is based on the Specialization Project of the fall of 2020.

We wish to express our thanks to our supervisor, Professor Henrik Andersson, for valuable guidance, productive discussions and helpful feedback, throughout the project period. We also would like to thank Trondheim municipality, and especially Jan Albert Hårvik and Sveinung Øystein Eiksund for informative meetings and for providing first hand knowledge about the problem.

Trondheim, June, 2020
Sofie Rønvik Aslaksen, Michel Aleksander Evensen Norum

# Abstract

One of most important responsibilities the Norwegian municipalities is to provide its inhabitants with a well-organized school system. Rapidly increasing populations and continuous deterioration of school buildings make this a substantial challenge for several municipalities. The municipalities must therefore consider several school development projects to meet these challenges. This thesis aims to provide an unbiased decision tool for municipalities by studying an area in southern Trondheim.

Through an extensive literature study on school location problems, this thesis reveals that most facility location problems aim to minimize the travel costs and the opening or closing costs of a facility. Moreover, it is clear that integrating uncertainty and real-size data significantly increases the computational complexity of the problems. The literature study exposes three gaps: a precise mathematical description including all the aspects of Trondheim's problem, an objective function taking utilization and quality of the schools into account, and a solution method that can solve the problem within an acceptable time.

To close these gaps, this thesis proposes the School Prioritizing Problem with Alternatives (SPPA). This problem addresses an important question: Given a set of distinct possible projects what are the optimal projects to execute at what time? The SPPA aims to simultaneously minimize three terms: the length and hazard of the pupils' road to school, unwanted school capacity utilization, and inconveniences from poor school conditions that affect the educational environment. As the population development is uncertain, multiple future scenarios are taken into account. To manage the complexity of the model as the number of considered scenarios increase, a solution algorithm based on the branch and bound scheme is developed in two variations, the Execution Order Specific Branch and Bound (EOSBB) algorithm and Alternative Execution Order Specific Branch and Bound (AEOSBB) algorithm.

Preliminary tests on a deterministic variation of the SPPA find performance-enhancing extensions. The implementation of a maximum allowed school deterioration and distance to school improves the computational effort without causing significant alterations to the solutions. The performance of the solution algorithms is tested on real-size data provided by Trondheim municipality, in a 15-year planning horizon. The tests demonstrate that the AEOSBB algorithm successfully solves the SPPA to optimality for instances of up to 100 scenarios within an acceptable time. Consequently, the AEOSBB algorithm is the preferred solution method for the model.

The outputs from the SPPA demonstrate how the model can aid the municipalities as an unbiased tool in decision-making. The solution is an unambiguous school prioritization order, with coherent years of execution for each project. The solution also includes a completely new school district map.

This thesis successfully introduces a precise, mathematical model and demonstrates how the proposed solution method can be used to solve real-world problems for a municipality in Norway. As today's decision-making process is tedious and subjective, we believe that the contribution of this thesis can be of great value when planning a future school expansion strategy in the municipalities.

# Sammendrag

En av de viktigste oppgavene til norske kommuner er å tilby innbyggerne sine et godt organisert skolesystem. Raskt økende innbyggertall og kontinuerlig slitasje på skolebygninger gjør dette til en stor utfordring for flere kommuner. Kommunene må derfor vurdere flere skoleutviklingsprosjekter for å imøtekomme disse utfordringene. Denne masteroppgaven har som mål å tilby et objektivt beslutningsverktøy for kommuner ved å studere et område sør i Trondheim.

Gjennom en omfattende litteraturstudie avdekker denne masteroppgaven at de fleste anleggslokasjonsproblemer (Facility Location Problems) retter seg mot å minimere reisekostnader og kostnader ved å åpne eller stenge anlegg. Videre er det tydelig at det å integrere usikkerhet og realistisk data øker beregningskompleksisteten til problemene. Litteraturstudien avdekker tre mangler: en presis matematisk beskrivelse som inkluderer alle aspekter ved Trondheims problem, en objektivfunksjon som tar hensyn til utnyttelse og kvalitet ved skolene og en løsningsmetode som kan løse problemet innen akseptabel tid.

Med den hensikt å adressere disse manglene, foreslår denne masteroppgaven Skoleprioriteringsproblemet med alternativer (School Prioritizing Problem with Alternatives - SPPA). Dette problemet tar for seg et viktig spørsmål: Når det finnes et sett av distinkte mulige prosjekter, hva er de optimale prosjektene å gjennomføre, og til hvilken tid? SPPA minimerer tre uttrykk samtidig: lengden på, og mulige farer langs, elevenes skolevei, uønsket kapasitetsutnyttelse på skolene og ulempene fra dårlig tilstand på skolebyggene som påvirker læringsmiljøet. Siden befolkningsutviklingen er usikker må flere fremtidsscenarioer bli vurdert. For å håndtere kompleksiteten i modellen når antall scenarioer øker, er en løsningsalgoritme, basert på *branch and bound* metoden, utviklet i to variasjoner, den Utførelsesrekkefølgespesifikke branch and bound (EOSBB) algoritmen og den Alternative utførelsesrekkefølgespesifikke branch and bound (AEOSBB) algoritmen.

Innledende tester på en deterministisk variasjon av SPPA finner prestasjonsfremmende utvidelser. Implementeringen av en maksimal tillat forverring av skolene og lengde på elevenes skolevei forbedrer modellens kjøretid betraktelig, uten at det blir store endringer

i løsningene. Effektiviteten av løsningsalgoritmene er testet i et 15 års perspektiv på data i virkelig størrelse, gitt av Trondheim kommune. Tester demonstrerer at AEOSBB algoritmen lykkes med å løse SPPA til optimalitet for instanser med 100 scenarioer innen akseptabel tid. Derfor er AEOSBB algoritmen foretrukket løsningsmetode for modellen.

Resultatene fra SPPA demonstrerer hvordan modellen kan brukes som et objektivt beslutningsverktøy for å hjelpe kommuner. Løsningen er en utvetydig skoleprioriteringsrekkefølge med tilhørende år for ferdigstillelse av hvert prosjekt. I tillegg inkluderer løsningen et helt nytt kart over skoledistriktene.

Denne masteroppgaven lykkes med å introdusere en presis matematisk modell og demonstrerer hvordan den foreslåtte løsningsmetoden kan brukes for å løse problemer av virkelig størrelse i kommuner i Norge. Siden dagens beslutningsprosess er omfattende og subjektiv, tror vi at bidraget fra denne masteroppgaven kan være av stor verdi når fremtidens skolestrategi skal planlegges i kommunene.

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# Abbreviations

| | |
|---|---|
| **AEOSBB** | Alternative Execution Order Specific Branch and Bound |
| **B&B** | Branch and Bound |
| **CC** | Complete Calculation |
| **DM** | Deterministic Model |
| **EOSBB** | Execution Order Specific Branch and Bound |
| **EVPI** | Expected Value of Perfect Information |
| **FLP** | Facility Location Problem |
| **LP** | Linear Programming |
| **MIP** | Mixed Integer Problem |
| **MOP** | Multi-Objective Problem |
| **MSP** | Multi-Scenario Problem |
| **NAC** | Non-Anticipativity Constraint |
| **SO** | School-Order |
| **SOP** | Single-Objective Problem |
| **SPPA** | School Prioritization Problem with Alternatives |
| **SSP** | Single-Scenario Problem |
| **VSS** | Value of Stochastic Solution |

# 1 Introduction

Organized schooling in Norway dates back to the middle ages, but it was not until the end of the 18th century that education became a prioritized matter (Thune, T., 2019). This resulted in the People Act of 1889 which gave all Norwegian children the right to seven years of schooling. A major reorganization of the school system in the late 1990s incorporated the secondary schooling as a part of the Education Act. This act states that children of school age, hereafter denoted as pupils, have the right, and are obliged, to attend both primary and secondary school (Ministry of Education and Research, 1998). It is the municipalities that must provide the necessary capacity for all pupils in their area. This sets high demands for organized school structure in the municipalities. To obtain a robust school structure, it is critical that schools are of the right size and located in the right place, for a long-term perspective (Trondheim Kommune, 2019b). This implies that urgent action should be avoided to cope with minor changes in student numbers. Furthermore, the school facilities ought to be flexible and functional, to satisfy the content of new reforms and organizational changes. The school structure must also handle the uncertainty in population growth and the need for school capacity in the forthcoming years.

This thesis is motivated by the collaboration with Trondheim Municipality, hereafter denoted as Trondheim, and the challenges they face when determining the future school structure. In essence, Trondheim has a list of potential school projects that can be executed. For the existing schools, the list of projects includes renovation alternatives, expansion alternatives, and alternatives including both renovation and expansion. In addition, there exists a set of potential new schools that each can have several alternatives concerning size and location. Each of the possible alternatives is denoted as a project. The decision regarding which of the projects gets funding is political and is made in the Trondheim City Council, hereafter City Council, based on recommendations from the Chief Municipal Executive (CME). The CME creates a shortlist of school development projects and presents this to the City Council in a prioritized order (Trondheim Kommune, 2019b). Today, the CME has few objective tool to compare the need for, and potential gain of, the potential projects, resulting in that the political decisions are largely based on discretion. Trondheim is currently in a process where several new projects are considered. In this thesis data from an area in southern Trondheim is used.

In this thesis, the School Prioritization Problem with Alternatives (SPPA) is studied. The purpose is to use mathematical optimization to provide an unbiased decision tool to assist Trondheim in deciding which project to execute at what time. To achieve this, a multi-objective model is proposed. Traditionally, school location problems seek to minimize travel distance and facility costs, constrained by a maximum capacity and a demand. However, in a real-world setting, the travel distance is only a part of the entire perspective. Therefore, the model presented in this thesis focuses on three objectives: the length and hazards from pupils' road to school, unwanted school capacity utilization, and inconveniences from poor physical condition of school building and its impact on the education. These three factors constitute the objective function which is minimized in the model. The model then provides a preferred order of execution of the projects, in addition to allocate pupils to schools. To the best of our knowledge, addressing these three objectives simultaneously has never been done previously. Due to the complexity of the problem, it is computationally demanding to find solutions within acceptable computational time. Consequently, two versions of an Execution Order Specific Branch and Bound (EOSBB) algorithm is developed to find solutions to the real-world-sized problem. This is a unique approach and a new algorithm for solving the school prioritization problem.

The method and results presented in this thesis provide a considerable contribution to the available literature and research on school location problems. Furthermore, the results can grant decision-makers a better understanding of the outcomes given different decisions. The main contributions can be outlined as:

- A precise mathematical description of the SPPA that takes uncertainty in population growth and the need for school capacity into account. The model addresses three objectives simultaneously, which provides a real-world formulation of the problem.
- A new solution method is provided by developing the EOSBB algorithm. This allows the model to be applied with complex, real data in a manageable way.
- The practical usage of the model and solution method is demonstrated using a real case from southern Trondheim.

The ideas in this thesis are, to some extent, based on the foregoing Specialization Project conducted in the Fall of 2019 (Aslaksen and Norum, 2019). The conceptual perception of the problem is similar in both the Specialization Project and this thesis. The model presented in this thesis differs by including uncertainty in population growth. Moreover, a new perspective is added by the introduction of a solution method to the model. Also, the practical usage of the model and solution algorithm is demonstrated.

The thesis is arranged as follows. In Chapter 2, the main challenges faced by Trondheim, is introduced. Then, the topics concerning the SPPA are discussed in light of existing literature in Chapter 3. The thesis continues with a thorough description of the problem in Chapter 4. In Chapter 5, the mathematical elements, and structure of the SPPA are outlined. Next, the developed solution algorithm is presented in Chapter 6. Subsequently, the data instances are described in Chapter 7. Next, in Chapter 8, results from different tests of the suggested model are presented, and in Chapter 9, the practical application of the model is demonstrated. Finally, in Chapter 10 concluding remarks and future research opportunities are described.

# 2 Background

In Norway, the municipalities are the school owners of public elementary schools. According to the Education Act (Ministry of Education and Research, 1998), school owners are required to have a proper and sturdy system to ensure that schools are operated by existing laws and regulations. In this chapter, relevant background information for the thesis is presented. In this chapter Section 2.1, 2.3, and 2.4 are reproduced from (Aslaksen and Norum, 2019). First, a brief introduction to the history of school planning in Trondheim is given in Section 2.1. Second, the population development prognoses is presented in 2.2. Third, the main challenges in school planning today is considered in Section 2.3. Fourth, in Section 2.4 an overview of the process prioritizing and executing projects is provided. Lastly, in Section 2.5 the designated area is introduced.

## 2.1 A Brief History of Trondheim's School System

The oldest organized school in Trondheim still in operation is Ila school, where teaching started in 1770 (Trondheim Kommune, 2015). The number of children in the district increased swiftly, and in the 1830s the school was moved to a new site to provide the necessary capacity. Since then, the school system in Trondheim has been through numerous changes. In the period between 1970 and 1986, 19 new schools were established due to considerable growth in the number of pupils and changes in resident areas. Moreover, in the 1990s a new national curriculum was implemented, that included new requirements for school building design. This forced the municipality to execute several renovation projects to alter the schools to meet the standard of these requirements. Since 2000, over 40 major school projects have been executed in Trondheim. One of the main goals of these projects was to increase the capacity in specific areas in the municipality. Furthermore, it has been important to create a flexible school structure that can easily be adjusted to forthcoming changes.

## 2.2 Population Development Prognoses

The population growth in Trondheim is largely dependent on the construction of new homes and, thus, the population development prognoses are based on the estimated housing development (Trondheim Kommune, 2019a). These forecasts are based on a database containing all the potential residential building projects in Trondheim. The total number of potential residences is referred to as the building potential. The database is updated annually and provides information on how far each potential project is in the realization process. The currently known building potential is 45,000 units. A realization of all these units would result in a population increase of 85,000 people. The potential residences are divided into four categories: detached house, horizontally divided residences, vertically divided residences, and low-/mid-rise buildings, where each type respectively has an expected average number of pupils per housing unit.

Trondheim has arranged the building potential in three categories depending on the likelihood of realization (Trondheim Kommune, 2019b). The three categories have properties as follows:

**A** This category includes the projects that are given actual building permits. This involves projects that are ahead in the project process and the total building potential from these are 6,000 homes

**B** This category contains the building potential in projects where area and regulations plans are approved in the municipality. These projects are in an earlier stage than the projects in A, and hold a building potential at around 12,000 homes.

**C** This is the category with the highest building potential and comprises the projects that are only in the start phase of planning the projects, and where geographic areas are deposited for residential purpose. These are the projects that are in the earliest stage of the process. This is estimated to be 27,000 homes.

The population development in different parts of the municipality is thus dependent on the building potential of the area. Higher potential in an area increases the uncertainty in the future population. As category B and C contain a large number of potential projects, the population forecast is highly uncertain. This can result in a population development that is both lower or higher than projected.

## 2.3 Challenges in School Planning Processes

This section presents the current main challenges Trondheim faces in terms of school planning:

- Increased capacity demand
- Deterioration of existing schools
- Inconveniences on the road to school

The school capacity in Trondheim has been a challenge for many decades, and currently, 19 school districts do not have any spare capacity (Trondheim Kommune, 2019c). This means that several schools today have a capacity-utilization beyond the optimal amount. The red areas in Figure 2.1 illustrates these school districts. The realization of residential building projects and expected population growth in the municipality causes an urgent need for more capacity (Trondheim Kommune, 2015). At the same time, several school districts operate with capacity utilization below the optimal level. This indicates that the municipality struggles to evenly distribute the capacity utilization between the schools.

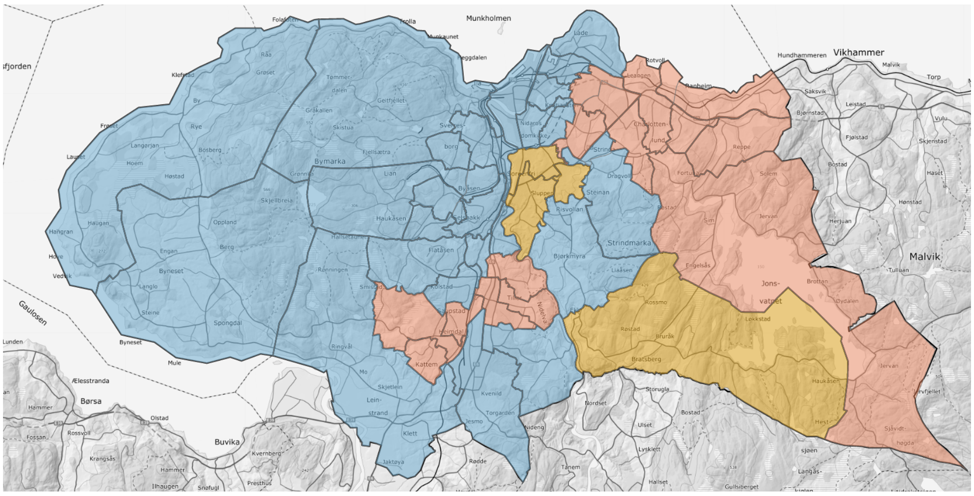



**Figure 2.1:** Capacity utilization of the school districts in Trondheim. Red areas highlights school districts where built capacity is reached, and the yellow areas are districts with limited capacity available.

Trondheim operates with two levels of school capacities: optimal and built. Trondheim defines the built capacity as the estimate of how many students a school facility can accommodate (Trondheim Kommune, 2017). However, due to fluctuations in the number of pupils, it is challenging to operate schools at the built capacity over time. Instead, Trondheim plans for an utilization at 90% of the built capacity. This is the level a school can operate without capacity problems over time and is denoted as the optimal capacity (Trondheim Kommune, 2017). Furthermore, Trondheim has established that schools in periods may have a higher number of pupils than the built capacity, to ensure that every pupil is allocated to a school. Consequently, to allow special considerations, this thesis introduces a third level, maximum capacity. This is the absolute maximum allowed utilization level. However, it is impossible to operate a school at maximum capacity while maintaining stable school districts.

It is determined by Trondheim that nearby schools must have available capacity for contractors to get a building permit in the area. This means that new housing units are not accepted and conducted before the school capacity problem is resolved. Consequently, the school capacity can both be a restricting factor for the city development and used as a tool for the municipality to control where and when new residential building projects should be executed. However, this is not strictly adhered to and exceptions have been made. This results in affecting the already pressured capacity at many of the schools in the municipality.

In addition to expanded capacity demand, several existing schools require maintenance to uphold the necessary standards. The Norwegian law states that the environment in schools should encourage health, well being, good social and environmental conditions and prevent sickness and injury (Helse- og omsorgsdepartementet, 2014). Currently, Trondheim is responsible for some schools with challenges concerning the physical condition (Trondheim Kommune, 2019b). In addition, some schools use temporary pavilions in poor shape, which further increases the need for rehabilitation actions. This again can affect the learning environment in these schools, and, therefore, it is important to upgrade schools that are in poor condition.

Another challenge for Trondheim is the pupils' road to school. It is desired that the pupils walk to their allocated school. Thus, by road to school, we consider the walking route from a pupil's home to his or her respective school. The Education Act (Ministry of Education and Research, 1998) states that every pupil has the right to attend the school they live geographically closest to or to the school they are allocated to through the school districts. However, the Act declares that dangerous roads to school and topography are relevant considerations as well when allocating pupils to a school (Utdanningsdirektoratet, 2014). For instance, a steep hill on the road to school or a forest can affect the layout of the school districts.

## 2.4 Project Prioritization and Execution

To create a school structure that takes on the challenges described above, the City Council in Trondheim decides on a prioritized execution order of the potential school projects

(Trondheim Kommune, 2019b). The ranking is based on three preferable assessment topics, in prioritized order:

1. Measures that solve significant challenges related to physical learning and/or work environment, either based on technical condition or sustained high student numbers.
2. Measures that increase capacity in areas where lack of school capacity limits the building of new houses.
3. Measures replacing pavilions that are in good condition.

In addition to the prioritization criteria presented above, Trondheim has some desired guidelines regarding school size. To achieve high utilization, each new school should have at least two parallels. This improves the sturdiness of the school districts as the schools are better equipped when facing fluctuation in the number of pupils.

When a project is selected the first step of the execution is to establish a total financial budget (Trondheim Kommune, 2015). Secondly, regulation permits are provided and contractors are selected. Approximately two years after the project is initialized, the actual construction of the project is started. The construction of a major project is estimated to last for two years. Thus, the budget of a school building project is often distributed over a four year period.

## 2.5 Area of Consideration

In this thesis, an area in southern Trondheim is considered. The area consists of the following current school districts:

| |
|---|
| Breidablikk |
| Flåtåsen |
| Huseby |
| Hårstad |
| Kattem |
| Okstad |
| Romolslia |
| Rosten |
| Sjetne |
| Stabbursmoen |
| Tonstad |
| Åsheim |

The designated area is representative of many of the same challenges as Trondheim. First and foremost this is an area with already limited capacity in some schools. Moreover, there is a high building potential in the area, where many of the projects are in category A. Thus, the population is expected to grow which further exposes the capacity challenges. In addition, several of the existing school buildings need renovation to meet the government's requirements. Figure 2.2 illustrates where the area is located in Trondheim.

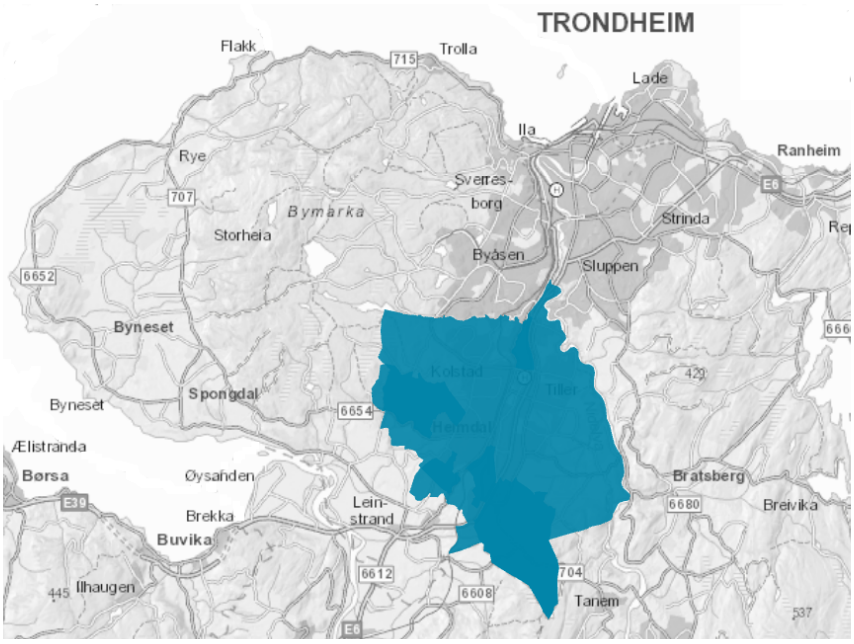



**Figure 2.2:** The highlighted area is the area in southern Trondheim that is considered in this thesis.

Several of the existing schools in the designated area are suitable for potential projects. Furthermore, there are two potential new schools, Lundåsen and Hallsteingård, with several alternatives for capacity. Table 2.1 shows the different types of projects that can be conducted at each school. The first column lists the schools and the second to the forth indicated what type of project that can be executed. The Alternative-column shows which school that have multiple alternatives considering a capacity project.

**Table 2.1:** The possible projects that can be executed at each school in the designated area.

| School | Capacity | Renovation | New School |
|---|---|---|---|
| Breidablikk | X | X | - |
| Flatåsen | - | X | - |
| Huseby | - | - | - |
| Hårstad | - | - | - |
| Kattem | X | X | - |
| Okstad | - | - | - |
| Romolslia | - | X | - |
| Rosten | X | - | - |
| Sjetne | X | - | - |
| Stabbursmoen | X | X | - |
| Tonstad | X | X | - |
| Åsheim | X | X | - |
| Hallsteingård | - | - | X |
| Lundåsen | - | - | X |

# 3 Literature Review

In this part, the literature related to the School Prioritization Problem with Alternatives (SPAA) is discussed. To the best of our knowledge, there is scant literature regarding school location with limited alternatives. Consequently, this literature review discusses different aspects and elements of the problem. The SPPA is a facility location problem (FLP) where the location sites can be chosen from a set of alternatives. Moreover, as the model consists of some conventional constraints, such as capacity and budget constraints, these are relevant to discuss as well. This chapter is recreated from Aslaksen and Norum (2019) with some alterations. To supplement, Section 3.1.6 and 3.2 are added.

First, FLP, with its most relevant extensions, are discussed in Section 3.1. Second, solution methods are presented in 3.2. Finally, in Section 3.3 the contribution from this thesis and how it differs from existing literature are presented.

## 3.1 School Location Problem and its Most Common Extensions

School location is a variation of facility location problem (FLP) where the site of schools is determined. The FLP has been widely discussed and there exists considerable amount of literature on the subject (Nickel and Gama (2015), Castillo-López and López-Ospina (2015), Daskin and Maas (2015)). A FLP can, in short, be described as a problem where the objective is to decide where, and possibly when, a facility should be located, to serve given customers. The model takes certain parameters into account, such as traveling cost, set up cost, and customer demand (Nickel and Gama, 2015). In the SPPA, schools are considered facilities, and the customers are pupils. Even so, various elements separate the SPPA from a FLP. For instance, in addition to the location of the facilities, there exist several alternatives concerning location site and facility size for each school. This results in many additional decisions to be made in every possible location, thereupon the complexity of the problem increases.

Another central problem in discrete location modeling is the $p$-median problem (Daskin and Maas, 2015). This problem aims to locate $p$ facilities which minimize the overall distance between the demand and its closest facility. An important property of the $p$-median problem, presented by Daskin and Maas (2015), is the decreasing cost of distance with each added facility. Moreover, the marginal improvement in distance cost is expected to decrease with additional facilities. Both of these properties exist to some degree in the SPPA. It is obvious that by increasing the number of schools from $x$ to $x + 1$ schools, the costs from traveling distance decrease or stay the same. However, as the SPPA seeks to optimize capacity utilization, additional facilities may degrade the capacity objective as excess capacity is unwanted. Furthermore, the utility, in travel distance, of adding one more school declines. For example, it is a larger improvement to increase from 0 to 1 school in an area, than from 6 to 7 schools. Concerning the SPPA, the budget constraint bounds the number of executed projects, thus it is a capacitated $p$-median problem.

Another relevant problem in facility location modeling is the cover location problem. A cover model is commonly used if the facilities provide a service that must lie within a maximum distance between the facility and customer, for instance, fire stations (García and Marín, 2015). If this requirement is fulfilled, the customer is considered *covered*. In cover problems, there are mainly two different forms: the model can either minimize the costs of covering all of the customers (set covering) or maximize the number of covered customers (maximal covering). One of the disadvantages of the set covering problem is that several customers (pupils) are covered by several facilities (schools). This creates higher facility costs due to extra facilities and may cause districting problems which are addressed in Section (3.1.5). As the SPPA requires that every pupil is allocated to a school, the characteristics of a set covering problem are recognizable. However, since the model contains a budget constraint, which restricts the amount of located schools, one can argue that the model is also related to the maximal cover problem.

### 3.1.1 Objective

In short, the objective function in FLP aims to minimize travel costs between facilities and demand nodes. Mattsson (1986) proposes a model where the aim is to reduce monetary transportation costs associated with assigning a child to a specific school, where the distance is multiplied with each allocated child. Schoepfle and Church (1991) has a similar approach where the travel distance for a child allocated to a school is measured as a cost and minimized. Hernández et al. (2012) present a facility location model intended for prison location, and as both prisons and schools are a public facility, the problems are comparable. In the model presented by Hernández et al. (2012), the travel distance is a term to minimize in the objective function, and a maximum transfer distance is included in the constraints.

Caro et al. (2004) and Correia and Melo (2017) both extend the initial perception of distance and travel cost. Correia and Melo (2017) propose an extensive model to minimize the cost of facilities that can be closed or expanded. The model adds a tardiness penalty cost to the travel cost, which applies in cases where the delivered goods are delayed in the demand period and the unit must be replaced later. Thus, the scope of the objective is broadened to capture several aspects of the practical problem. Differently, Caro et al. (2004) suggest a model that optimizes school districting. In the article, the distance parameter is extended with distance-equivalences from obstacles on the road to school. This is similar to elements from the SPPA where not only distance, but other important aspects on the road to school are considered.

Ferland and Guenette (1990), on the other hand, disregard travel distance in the objective function. The article proposes a decision support system for school districting, but instead of minimizing distance, as Caro et al. (2004) suggest, Ferland and Guenette (1990) state that the children should be allocated to the nearest school. This applies if the given school has sufficient capacity, and where the closest students are allocated first. Delmelle et al. (2014) propose a capacitated median model that aims to minimize transportation cost where the costs are subject to a budget constraint. The model expresses the travel distance as an increasing, non-linear function if the distance is above a maximum allowed distance. This is an interesting approach as it allows the distance to exceed maximum, but the penalty increases strikingly if it does. The SPPA takes the travel cost into account as the model minimizes the non-monetary cost from the road to school. However, in order to ensure that every pupil is allocated to a school, there are no maximum distances in the SPPA.

Menezes and Pizzolato (2014) review the development of education systems in areas with extraordinary population growth rater. As with all of the articles presented in this thesis, the objective is to minimize travel cost, but it is formulated as a capacitated $p$-median and maximum cover model. The objective is to maximize the cover of as many users as possible within a determined distance and with $p$ located facilities. Caro et al. (2004) also present a maximum distance from pupils to schools and can be regarded as a cover problem in the same way as Menezes and Pizzolato (2014).

Some of the articles reviewed in this thesis formulate the model as a multi-objective optimization problem (MOP). López Jaimes et al. (2011) introduce MOP as a model containing an objective function where several objectives are to some degree in conflict with each other. In Castillo-López and López-Ospina (2015) this is prominent as the model seeks to minimize facility costs and travel costs for the students at the same time. These are clearly conflicting objectives as more facilities reduce travel expenditures, but increase facility costs. Similarly, Hernández et al. (2012) introduce two opposing objectives where the model aims to minimize the cost of expanding the facilities and simultaneously avoid overpopulation in the facilities. Also the SPPA can be considered as a MOP, as three different objectives are addressed at the same time. Minimizing the road to school cost and obtaining an optimal capacity utilization are especially conflicting objectives, as some pupils may be allocated to a school further away to share the capacity amongst the schools.

### 3.1.2 Multi-Period Model

The purpose of multi-period, or dynamic, models is to not only decide where, but also when, a facility should be sited to minimize facility and delivery cost from the facility to the customer in the planned time period (Nickel and Gama, 2015). One of the first articles on multi-period facility location is Wesolowsky (1973). The article proposes a general dynamic model allowing a location to change within the planned time horizon. The future change in cost and demand is forecasted and the optimal location is found accordingly. Nickel and Gama (2015) emphasize the importance of defining the planning horizon upfront as this is the time frame of the problem. In all of the articles with dynamic models referred to in this thesis, the time aspect is described as a discrete set, which is also the main focus in this thesis. As the SPPA considers facility location in multiple time periods and some input parameters can change over time, this model can be regarded as a dynamic facility location model.

Hernández et al. (2012) presents a multi-period location model where each time period has different scenarios due to uncertainty in future demand. This is separate from Delmelle et al. (2014) who propose a model without uncertainty and the time period has only one forecasted scenario. Mattsson (1986) on the other hand, uses the same model in three different time periods where the future parameters are estimated. This differs from the other two as the time is not a consideration in the actual model. Therefore, this model is only partly dynamic. Correia and Melo (2017) have a dissimilar approach where the time period is used the same way as Delmelle et al. (2014). However, the time periods are not uniform as the article defines two sets of time periods, one where decisions can be made and one where they can not. This exemplifies that the multiple period FLP can be expressed and used in different ways.

### 3.1.3 Capacity Constraint

All of the relevant articles present in this thesis are capacitated models with a maximum capacity constraint. As in the SPPA, the changes in demand are inMattsson (1986), Hernández et al. (2012), Delmelle et al. (2014), Menezes and Pizzolato (2014) and Castillo-López and López-Ospina (2015) caused by a rapidly growing areas that creates a need to expand the capacity. In most of the articles, capacity is restricted by a given maximum value (Caro et al. (2004), Hernández et al. (2012)). Castillo-López and López-Ospina (2015) on the other hand, propose a different model regarding capacity. This article addresses the need for school capacity in rural zones where the objective is to minimize travel costs. To assign all of the school children to a school, capacity is determined based on how many pupils are expected to be allocated to a school. Moreover, Delmelle et al. (2014) and Correia and Melo (2017) present a flexible capacity constraint where the maximum capacity can increase by renting extra units. This gives an additional cost for renting the units, but the capacity, on the other hand, is not strictly limited. Menezes and Pizzolato (2014) limit the capacity in the number of located facility, not in number of pupils at each school. This means that an expected capacity for each school is calculated and the model is restricted by the number of facilities.

### 3.1.4 Budget Constraint

Even though the basic parameters are not multi-periodic, constraints, such as the budget constraint, can create dynamic properties in the model. For example, if the budget constraint of installing new facilities exists per year, then locating facilities over time can be inevitable (Nickel and Gama, 2015). As mentioned in previous paragraphs, Delmelle et al. (2014) presents a model for location of school facilities with flexible capacity, but it is constrained by a budget constraint. This means that the costs of building new facilities or expanding capacity is restrained to the total available budget for the time horizon. Hernández et al. (2012) propose a different type of budget constraint. Instead of a separate constraint, the budget is expressed as the maximum number of built facilities which, again, is determined by the available budget for that period. In that way, the number of facilities becomes the budget constraint. The same approach is presented in Menezes and Pizzolato (2014) where $p$ facilities are considered the budget.

### 3.1.5 Districting Model

Kalcsics (2015) describes districting as the problem where small, geographic units are clustered together to a larger, contiguous, and balanced district. A district can, for example, be a zip code area or, in our case, a school district. A school district consists of several neighborhoods, which can be regarded as the small units being clustered together. Districting problems address the issue of how to best allocate the customer's demand to each located facility (Kalcsics, 2015). In the SPPA, this means allocating the pupils to schools so that the capacity utilization at the schools are balanced, and thus decide the school districts in a given time horizon.

One of the earliest articles on districting problems was Koenigsberg (1968). He proposes a general model where the objective is to create a school district with minimal travel distance. Later, Ferland and Guenette (1990), Schoepfle and Church (1991), and Caro et al. (2004) each present a districting model where the criteria they consider is to ensure continuity from elementary to secondary school. This means that the children from the same district should go to the same school as they advance to higher grades. Moreover, ethnic diversity and balance is another consideration in the districting problems discussed in Koenigsberg (1968), Schoepfle and Church (1991) and Caro et al. (2004). However, in all of the articles, minimum travel distance is the main objective and thus the most important consideration. Since ethnic diversity and compact districts can be conflicting objectives, this can be a difficult trade-off.

### 3.1.6 Uncertainty

The facility location problems with uncertainty are addressed in existing literature in various contexts (Correia and Saldanha-da-Gama, 2019). In FLP, uncertainty can for instance occur in demand, travel time, location of customers, or other model-specific parameters. The uncertain parameters are then represented as random variables. Each random variable has several possible outcomes and a set that represents all of these outcomes. A scenario is defined in King and Wallace (2012) as a possible future, with one outcome for each random variable. Snyder (2006) presents two main drawbacks by using scenarios. Firstly, it can be cumbersome to identify and determine the scenarios. Secondly, the number of scenarios must be limited for computational reasons, which can affect the range of future possible states. However, Snyder (2006) emphasizes that the scenario approach often results in more manageable models, and allows the parameters to be dependent over different time periods. This can be favorable especially since demand often is interdependent across time periods.

When modeling problems with uncertainty, Correia and Saldanha-da-Gama (2019) distinguish between three different frameworks: robust optimization, stochastic programming, and chance-constrained programming. Robust optimization was first introduced by Soyster (1973), and later extended in several studies such as Bertsimas and Sim (2004) and Ben-Tal et al. (2009). The idea behind robust optimization is to *immunize* the problem against uncertainty (Ben-Tal et al., 2009). This is obtained by *robust feasibility*, which means that the robust solution is feasible for every realization of the uncertain data. In the FLP it is appropriate to use robust optimization if the uncertainty is successfully captured by a given set of scenarios, and there are no accessible probabilistic information (Correia and Saldanha-da-Gama, 2019). If this is the case, Snyder and Daskin (2006) point out that there are two favored objectives: to minimize expected cost or to minimize regret. Regret in this context means the costs from worst-case scenarios. One of the main advantages of robust optimization is that it does not require a distribution function for the random parameters. However, as solutions found by robust optimization must be feasible in the worst-case scenarios, they can be very costly compared to other methods (King and Wallace, 2012).

The robust framework introduced by Soyster (1973) has been criticized for being too conservative as it is feasible for the most unlikely worst-case scenarios (Bertsimas and Sim, 2004). A less conservative approach is Stochastic programming. This framework is often applied if the uncertainty can be described by a given probability distribution (Correia and Saldanha-da-Gama, 2019). When this framework is used in a FLP, the problem is denoted as a stochastic facility location problem. This method requires that the distribution functions for the random variables are known or can be computed. By exploiting these probabilities, the model can find feasible solutions and provide valuable insight for decision-makers. A common way of expressing the stochastic programming problem is by a two-stage model. In the first stage, some decision is made given available information at that time. In the second stage, the problem is optimized with this decision when the outcome of the uncertainty is known (Snyder, 2006). This is demonstrated in Bieniek (2015), where a two-stage stochastic program with recourse is applied to locate facilities with a both discrete and continuous distribution of the demand. An advantage of stochastic programming that King and Wallace (2012) point out, is that the problem type does not change profoundly from the deterministic problem, which is an advantage when solving the model.

When it is allowed that one or several constraints may be exceeded in some scenarios, the chance constrained framework is a suitable approach (Correia and Saldanha-da-Gama, 2019). The idea is that the constraints must hold for a given amount of the scenarios, for instance as presented in Carbone (1974). In the article, $p$ facilities must be located concerning random, possibly correlated, demands. However, it is only required that the demand is met in a certain percentage of the outcomes. This means that the model attempts to ignore the worst extremes unless they are explicitly confronted (King and Wallace, 2012). The method is therefore appropriate for reliable problems, but can be difficult to solve as it often requires demanding computational effort. In addition, as in stochastic programming, a drawback in the chance constraint framework is the requirement that the probability distribution for the random variables is given or can be found.

In this thesis, the probability distributions for the random variables are estimated. This, in addition to the fact that the model considers different scenarios for future population growth, motivates a stochastic programming approach to solve the SPPA.

Several of the articles surveyed in this thesis consider uncertainty. Snyder and Daskin (2006) present a model for a general facility problem with uncertain demand and transportation cost. This is presented in a robust framework where a $p$-robust model is proposed that combines both minimizing expected cost and minimizing regrets. Laporte et al. (1994) also presents a model where facilities must be located to meet the uncertain demand for the future. Unlike Snyder and Daskin (2006), the problem is presented as a stochastic two-stage model where the first stage variables are binary and the second stage are continuous variables. Hernández et al. (2012) utilize the stochastic programming framework as well. In the article, uncertainty lies in the future demand for prison facilities. The uncertainty is modeled as a set of scenarios and a scenario tree scheme is used to illustrate possible states of future demand in prison capacity. Each scenario has a given probability that motivates a stochastic programming approach. Castillo-López and López-Ospina (2015) have a different approach, where a stochastic discrete choice model is introduced. The model proposes a choice probability that determines the probability that a student chooses to attend a given school.

## 3.2 Solution Methods

Like the FLP, the SPPA can be classified as a NP-hard problem. This means that finding the optimal solution often requires many, and sometimes highly ineffective, enumerations (Daskin and Maas (2015), Castillo-López and López-Ospina (2015)). As discussed in this chapter, the SPPA is composed of building blocks from a variety of optimization frameworks. This enables several possible solution methods for the problem. Some methods are briefly covered in this section.

### 3.2.1 Exact Solutions

Exact solution algorithm method means that, given necessary computational resources, the algorithm will provide an optimal solution. This can often be time-consuming and computationally demanding, and exact algorithms are therefore often applied if the model is tested on smaller instances or the computational time is not restricted. Menezes and

Pizzolato (2014) present exact solutions for two models for the FLP: the capacitated $p$-median model and the maximum covering location problem. By using AIMMS software and CPLEX solver, exact solutions are found and the characteristics of the models are compared. The school districting model, proposed in Caro et al. (2004), is solved with exact solution methods by using geographic information systems (GIS) tools. By using GIS, the input data can be prepared and interpreted in a way that reduces the complexity in the model. The exact solutions that are found can then provide valuable insight into the trade-offs involved in the problem.

The mathematical model presented in the specialization project Aslaksen and Norum (2019) was solved by an exact solution method. However, the computational effort was too strenuous, and an optimal solution could not be provided within an acceptable time. As the model in this thesis is extended, in addition to more complex data from real-world situations in Trondheim, the demand for computational resources increases. Therefore, the computational requirement for the problem exceeds the available resources, using an exact solution algorithm without alterations. Thus, a customized solution algorithm is developed to ease the computational effort.

### 3.2.2 Heuristic Approach

Due to the complexity in the FLP, heuristics are commonly used to find an acceptable solution (Daskin and Maas, 2015). One approach is the Tabu Search (TS) algorithm which is developed by Glover (1990). This is an iterative process that starts with an initial solution and neighbor operator, which defines the neighborhood that can be searched in each move. An example of such an operator is a flip neighborhood. This means that only one variable can change value in each search, thus the neighborhood becomes the solutions where this is prevailing. In a TS, either one or multiple operators can be used simultaneously. Further, the TS uses the best improvement search and moves to the best solution in the neighborhood. To avoid cycling, the search history is exploited. By creating a tabu list, the search is constrained and prevented from returning to the same solution. This approach is further presented in Castillo-López and López-Ospina (2015), which solves the nonlinear FLP by using a TS metaheuristic. In the article, the use of TS, in addition to a system of equations, evaluates each solution. This results in reduction strategies of the neighborhood size to the heuristic and enables larger problems. The TS is an example of an improvement heuristic, where the value of the heuristic depends on the presence of an initial solution. However, these solutions are often too cumbersome to compute, and the use of construction heuristics can be more appropriate.

A construction heuristic uses simple rules to develop solutions from scratch. Hernández et al. (2012) propose a multi-period stochastic model where complexity in the probabilistic scenarios, and the large-scale model incentives for a construction heuristic approach. To incorporate the uncertainty in the model, the solution method presented in Hernández et al. (2012) applies a scenario tree generation. The uncertainty is represented by the scenarios, and a branch-and-cluster coordination method is used to solve the problem. This is a method combined by a branch-and-fix coordination, proposed in Alonso-Ayuso et al. (2003), and a branch and bound (B&B) method. The solution method presented in Hernández et al. (2012) is a heuristic approach as the original problem is LP-relaxed to provide good, feasible solutions in affordable computation effort. Ferland and Guenette (1990) propose a more basic construction heuristic for the districting problem. In the article, the school districts are created by allocating a neighborhood to a zone where the closest neighborhood is allocated first. Subsequently, the zones are allocated in ascending order up until the outermost zone is allocated to a school. The mathematical model presented in this thesis has similarities with the problem described in Hernández et al. (2012). This further incentivizes the use of B&B approach as a preferred solution method.

### 3.2.3 Other Relevant Methods

Delmelle et al. (2014) explore the Pareto optimal solution space by solving the bi-criteria problem using the $\epsilon$-constraint and weighted sum method. The $\epsilon$-constraint method is a common method to solve MOPs as it is characterized by its simplicity and applicability (López Jaimes et al., 2011). In this approach, presented in López Jaimes et al. (2011), all but one objective are used as constraints bound by some allowable level $\epsilon$. This leaves one selected objective to be minimized and thus creates a single objective problem (SOP). In the weighted sum method, a weighting parameter is assigned to each objective and the model is then solved as a SOP. In Delmelle et al. (2014), this method provided new insight to the features in the model by exploring the relationship between the objectives.

The solution method used by Mattsson (1986) combines a tree search with a lower bound obtained by a Lagrangean relaxation. By solving the Lagrangean dual problem, a highest lower bound is found. The tree search is then terminated when the duality gap between the best found objective value and this bound is less than a certain value. This conveys that the difference between optimal and accepted solutions differ by, at most, this gap.

## 3.3 Our Contribution

As presented in this chapter, the FLP is a well-studied problem. The problem, in its simplest form, aims to minimize the travel and facility cost, when the facilities are required to meet customers' demands. However, this problem has been expanded in various ways, and several solution methods have been developed. The variations discussed in this study are summarized in Table 3.1. The decisions that have been made when creating the SPPA are based on what is found preferable considering the nature of the problem.

The SPPA provides a unique, new mathematical model where three objectives are addressed simultaneously. The model focuses on the conflicting objectives of both minimizing the costs from road to school and optimizing the capacity utilization, while also preventing costs from poor school conditions. With these considerations, the SPPA undertakes a complex representation of the real-world problem. In addition, instead of presenting the road to school cost entirely as a function of distance, the SPPA considers dangerous roads and change in topography as desired parameters to minimize as well. By implementing these considerations, the model can solve real-world school planning problems in a greater extent than what has been done before. Caro et al. (2004) propose a model with similarities in the travel cost, however, the SPPA focuses on the parameters more directly by implementing topography and dangerous roads as separate terms in the objective function.

As the future demand in the SPPA is unknown, the model incorporates uncertainty, which further effectuates the complexity of the model. As in Hernández et al. (2012), the demand is modeled as a set of scenarios where the aim is to minimize total cost. This provides a detailed and more realistic representation of the real-world problem, which again results in more reliable solutions.

Furthermore, the SPPA introduces a standout new way of considering the capacity constraint. Delmelle et al. (2014) and Correia and Melo (2017) propose a model with flexible capacity, where the capacity is expanded by renting extra units. In the SPPA, the capacity is presented as a non-monetary cost in the objection function. To guarantee that each pupil is allocated to a school, the built capacity can be exceeded. As this is an undesired scenario in a long-term perspective, the non-monetary cost increases distinctly if the utilized capacity exceeds the constructed maximum. Furthermore, the SPPA introduces an optimal level of capacity utilization, and deviation from this level in either direction is unwanted an thus costly. Therefore, the SPPA allows for fluctuations in the number of pupils and presents a more realistic capacity constraint.

The budget constraint is another area where the SPPA stands out from the articles surveyed in this thesis. Delmelle et al. (2014) propose a budget constraint that considers the entire time period. The SPPA on the other hand considers a budget for each year. This is due to the economic structure in Norwegian municipalities, where the budget for building new schools is distributed over the years of the planned building. Thus, the model does not incur costs for the whole project period in the first year. This is a new and precise way to consider the costs of a school project.

Finally, extended use of the branch and bound scheme is applied in the two variations of the Execution Order Specific Branch and Bound algorithm to solve the SPPA. This method has similarities with the branch-and-cluster approach presented in Hernández et al. (2012), for instance, scenario tree generation and fixing variables. The algorithms uses the structure of the branch and bound method in order to provide a sequence and time schedule for the execution of the projects, in addition to solving the school location problem. This is a unique way of solving a FLP that decreases the required computational effort.

Conclusively, this thesis presents a model that addresses three gaps in the existing literature: a precise, mathematical description that takes uncertainty into account, a model that undertakes three objectives simultaneously, and a new solution method provided by the algorithms. In addition, the SPPA considers multiple elements in minimizing the travel cost such as dangerous roads and topography. Moreover, a softened capacity constraint is introduced along with a budget constraint that is distributed over the planning horizon. With all of these elements connected in one model, decisions can be made, where several aspects can be weighted, and projects prioritized. Moreover, the algorithm provides a favorable scheme for solving the model. To our knowledge, this is the first time all of these considerations are joined in one model simultaneously.

**Table 3.1:** Comparison of the relevant articles and the SPPA

| Article | Application area | Facility location | MOP | Distriction problem | Multiple periods | Budget constraint | Uncertainty |
|---|---|---|---|---|---|---|---|
| Castillo-lopez | Schools | Yes | Yes | No | No | No | Yes |
| Caro | Schools | No | No | Yes | No | No | No |
| Hernandez | Prisons | Yes | No | No | Yes | Yes | Yes |
| Delmelle | Schools | Yes | Yes | No | Yes | Yes | No |
| Mattson | Schools | Yes | No | No | Partly | No | No |
| Koenigsberg | Schools | No | No | Yes | Yes | No | No |
| Correia | General | Yes | No | No | Yes | No | No |
| Ferland | Schools | No | No | Yes | No | No | No |
| Schoepfle | Schools | No | No | Yes | No | No | No |
| Wesolowsky | General | Yes | No | No | Yes | No | No |
| Snyder | General | Yes | No | No | Yes | No | Yes |
| SPPA | School | Yes | Yes | Yes | Yes | Yes | Yes |

# 4 School Prioritization Problem with Alternatives

The School Prioritization Problem with Alternatives (SPPA) is the problem of deciding what school projects to execute to best accommodate the school capacity in a community. Essentially, given a set of distinct possible projects, the SPPA is the problem of determining which projects to execute at what time to optimally meet uncertain future demand for school capacity. The goal of the SPPA is to provide an unbiased decision tool.

The SPPA is a multi-objective optimization problem, where a comprehensive assessment of all the objectives is of importance. In this model, the following elements are undesired and should be minimized:

- The cost from a long, hazardous and inconvenient road to school, from each pupil's home to their allocated school.
- The cost from non-optimal exploitation of school capacity.
- The cost from the inconvenience of pupils attending schools with low standards of the physical educational environment.

Each objective is weighted according to a desired importance-measure. In addition, each possible realization of the future population, denoted scenario, yields different values of the objectives. The objective value of each scenario is weighted with the coherent possibility of that scenario occurring. When the objectives above are optimized, a set of prioritized projects is returned.

The SPPA considers a designated region, divided into smaller zones. A zone is initially either inhabited with a distinct population or uninhabited. Each zone has a population development throughout the time horizon. Thus, each zone has a number of pupils that needs to be allocated to a school each year. The population development in each zone each year is uncertain and represented as random variables. The random variables represent scenarios that occur with a given probability. All pupils in a zone must attend a school every year, and it is undesired that pupils change their assigned school more than once during the planning period.

Initially, there exists a definite number of schools. Each initially existing school has three levels of capacity and a condition. The three capacity levels are: optimal, built, and maximum. Built capacity is the capacity the school is constructed to handle in case of an ideal utilization of its facilities. The optimal capacity is the capacity the schools can handle to ensure stability in the school districts in a real-world setting. Maximum capacity is the absolute maximum amount that can be allocated to the school. This level allows for some school to accept more pupils than the built level, to ensure that every pupil is allocated to a school.

Each school has an initial condition, related to the current physical state of the school building. During the planning horizon, the condition of the schools further deteriorates. A cost is related to the inconvenience of pupils attending a school in a poor condition as discussed in Chapter 2. The total cost of condition for a school is thus both dependent on the number of pupils at the school and the condition of the school. If a school is renovated the condition of the school resets to a good as new state.

There exists a list of potential projects that can be realized in the planning period. The SPPA distinguishes between schools that are suitable for possible projects and schools that are not. Schools that are not suitable for projects are schools with available capacity and satisfactory physical condition. For the schools that are suitable, each project is either a capacity expansion project, a renovation project, or a combination project. Each school may have several projects. There is also a set of potential new schools. Each of the new schools may have multiple alternatives regarding size and location. Each combination of location and size composes one project. The set of possible projects, including the opportunity of doing nothing, is denoted as the school's alternatives. Only one project for each school can be executed.

Each project has a given duration. As discussed in Chapter 2, the duration of a project includes the construction time, as well as the time it takes to get the necessary permits and find contractors. The monetary expenses of all the projects can be defined. The total expenditures for each project are distributed over the project's duration. The total expenditures for all projects under development are limited by a given budget for each year in the planning horizon.

The ideal way from a zone to a school is denoted as the road to school. The utility of the zone-to-school allocation is dependent on the distance between the school and the center of the zone. Furthermore, it is undesired if roads to school require the pupils to travel through difficult topography such as up or down a steep hill. Lastly, roads to school that require a crossing of dangerous sections, such as a heavily trafficked road or roads with high speed limits should be avoided. If a school is located in a zone, the pupils in that zone must be allocated to that school.

# 5 Mathematical Model

In this chapter, a mathematical model is proposed for the School Prioritization Problem with Alternatives (SPPA). The SPPA is a stochastic and scenario-based model, based on the mathematical formulation proposed in Aslaksen and Norum (2019). Section 5.1 introduces the notation used in the model. Section 5.2 presents the mathematical model with its objective and constraints. Lastly, Section 5.3 elaborates on some of the parameters of the model.

## 5.1 Notation

Sets:

| | |
|---|---|
| $\mathcal{S}$ | set of schools |
| $\mathcal{S}^E$ | set of existing schools without potential projects in the planning horizon (existing unchangeable schools) |
| $\mathcal{S}^C$ | set of existing schools with potential projects in the planning horizon (existing changeable schools) |
| $\mathcal{S}^P$ | set of potential new schools in the planning horizon |
| $\mathcal{A}_s$ | set of alternatives for school $s$ |
| $\mathcal{T}$ | set of time periods |
| $\mathcal{Z}$ | set of zones |
| $\mathcal{Z}_s$ | set of zones in which school $s$ is located |
| $\mathcal{E}$ | set of possible scenarios |

Indices:

| | |
|---|---|
| $s$ | school |
| $a$ | alternative |
| $i$ | zone |
| $i(s,a)$ | zone where school $s$ alternative $a$ exists |
| $t$ | time period |
| $e$ | scenario |

Parameters:

| | |
|---|---|
| $R_{sai}$ | total cost per pupil from undesired road to school $s$ alternative $a$ from zone $i$ |
| $E_{sat\tau}$ | the expense in time period $\tau$ of finishing school $s$ alternative $a$ in time period $t$ |
| $B_t$ | budget available for school projects in time period $t$ |
| $N_{ite}$ | number of pupils in zone $i$ in time period $t$ in scenario $e$ |
| $S^I_{sa}$ | 1 if school $s$ alternative $a$ initially exists |
| $C^{CON}_{sat}$ | condition cost of school $s$ alternative $a$ in time period $t$ |
| $\hat{Q}_{sa}$ | built capacity at school $s$, alternative $a$ |
| $Q^*_{sa}$ | optimal capacity at school $s$ alternative $a$ |
| $\bar{Q}_{sa}$ | maximum capacity at school $s$ alternative $a$ |
| $\hat{C}_{sa}$ | cost of having school $s$ alternative $a$ used at built capacity |
| $\bar{C}_{sa}$ | cost of having school $s$ alternative $a$ used at the maximum capacity |
| $p_e$ | the probability of scenario $e$ occurring |

Weighting parameters:

| | |
|---|---|
| $\alpha$ | weight of cost from road to school |
| $\beta$ | weight of utilization cost |
| $\gamma$ | weight of cost of condition of the school |

Variables:

| | |
|---|---|
| $x_{sate}$ | 1 if school $s$ alternative $a$ is finished in time period $t$ in scenario $e$ |
| $y_{sate}$ | 1 if school $s$ alternative $a$ exists in time period $t$ in scenario $e$ |
| $v_{ite}$ | 1 pupils in zone $i$ change their allocated school in time period $t$ in scenario $e$ |
| $w_{saite}$ | the amount of pupils in zone $i$ that is allocated to school $s$ alternative $a$ in time period $t$ in scenario $e$ |
| $q_{sate}$ | used capacity at school $s$ alternative $a$ at time period $t$ in scenario $e$ |
| $c(q_{sate})$ | cost from non-optimal capacity utilization of school $s$ alternative $a$ in time period $t$ in scenario $e$ |
| $z_{sate}$ | cost of inconvenience from poor school condition for school $s$ alternative $a$ in time period $t$ in scenario $e$ |

## 5.2 Complete Model

This section describes the objective function and the constraints in the SPPA. The model is presented in sections to ease the explanation. A compressed presentation of the model is given in Appendix A.

### Objective Function

$$\min \quad \sum_{e\in\mathcal{E}} p_e(\alpha \sum_{s\in\mathcal{S}}\sum_{a\in\mathcal{A}}\sum_{i\in\mathcal{Z}}\sum_{t\in\mathcal{T}} R_{sai}N_{ite}w_{saite} + \beta \sum_{s\in\mathcal{S}}\sum_{a\in\mathcal{A}}\sum_{t\in\mathcal{T}} c(q_{sate}) + \gamma \sum_{s\in\mathcal{S}^C}\sum_{a\in\mathcal{A}_s}\sum_{t\in\mathcal{T}} \hat{Q}_{sa}z_{sate}) \tag{5.1}$$

The objective function (5.1) specifies the intention of the model, i.e. to minimize the non-monetary cost from three terms given the possible realization of multiple scenarios. The first term consists of the costs from long, hazardous and inconvenient roads to school. The second term accounts for the utilization cost of the schools which are described as a function of the used capacity. The last term denotes the cost of inconvenient school conditions. Each term is weighted with its respective weighting parameter. The overall objective value of each scenario is given an importance-measure reflecting the probability of occurrence. $R_{sai}$ and $c(q_{sate})$ are elaborated on later in the chapter.

**Budget**

$$\text{s.t.} \sum_{s \in \mathcal{S}} \sum_{a \in \mathcal{A}} \sum_{t \in \mathcal{T}} E_{sat\tau} x_{sate} \leq B_\tau \qquad \tau \in \mathcal{T}, e \in \mathcal{E} \tag{5.2}$$

Constraint (5.2) stipulates that the expenses that occur in a time period cannot exceed the given budget for that same time period.

**Existing Unchangeable schools**

$$y_{sate} = S^I_{sa} \qquad s \in \mathcal{S}^E, a \in \mathcal{A}_s, t \in \mathcal{T}, e \in \mathcal{E} \tag{5.3}$$

Constraint (5.3) establishes that the existing unchangeable schools will remain in the initial state throughout the planning horizon.

**Existing Changeable Schools**

$$y_{sa1e} = S^I_{sa} \qquad s \in \mathcal{S}^C, a \in \mathcal{A}_s, e \in \mathcal{E} \tag{5.4}$$

$$y_{sa,t+1,e} \leq y_{sate} + x_{sa,t+1,e} \qquad s \in \mathcal{S}^C, a \in \mathcal{A}_s, t \in \mathcal{T} \setminus \{T\}, e \in \mathcal{E} \tag{5.5}$$

$$y_{sate} + \sum_{b \in \mathcal{A}_s | b \neq a} x_{sbte} \leq 1 \qquad s \in \mathcal{S}^C, a \in \mathcal{A}_s, t \in \mathcal{T}, e \in \mathcal{E} | S^I_{sa} = 1 \tag{5.6}$$

$$\sum_{a \in \mathcal{A}_s} y_{sate} = 1 \qquad s \in \mathcal{S}^C, t \in \mathcal{T}, e \in \mathcal{E} \tag{5.7}$$

$$\sum_{t \in \mathcal{T}} x_{sate} \leq 1 \qquad s \in \mathcal{S}^C, a \in \mathcal{A}_s, e \in \mathcal{E} \tag{5.8}$$

Constraint (5.4) ensures that the initial existing alternative for an existing changeable school is in fact the existing alternative in the first time period. According to constraint (5.5), if an alternative for an existing changeable school exists in a time period, it either existed in the previous time period or was completed in this period. Constraint (5.6) states that an alternative ceases to exist if another alternative for that school is completed. Constraint (5.7) ensures that a changeable existing school alternative exists in each time period. Finally, constraint (5.8) limits the number of upgrades and alternatives for a school to a maximum of one in the planning horizon.

**Potential New Schools**

$$y_{sa1e} = 0 \qquad s \in \mathcal{S}^P, a \in \mathcal{A}_s, e \in \mathcal{E} \tag{5.9}$$

$$y_{sa,t+1,e} \leq y_{sate} + x_{sa,t+1,e} \qquad s \in \mathcal{S}^P, a \in \mathcal{A}_s, t \in \mathcal{T}\backslash\{T\}, e \in \mathcal{E} \tag{5.10}$$

$$y_{sate} \leq y_{sa,t+1,e} \qquad s \in \mathcal{S}^P, a \in \mathcal{A}_s, t \in \mathcal{T}\backslash\{T\}, e \in \mathcal{E} \tag{5.11}$$

$$x_{sa,t+1,e} \leq y_{sa,t+1,e} \qquad s \in \mathcal{S}^P, a \in \mathcal{A}_s, t \in \mathcal{T}\backslash\{T\}, e \in \mathcal{E} \tag{5.12}$$

$$\sum_{a \in \mathcal{A}_s} y_{sate} \leq 1 \qquad s \in \mathcal{S}^P, t \in \mathcal{T}, e \in \mathcal{E} \tag{5.13}$$

Constraint (5.9) sets the initial state for each potential new school and thus makes sure that none exists in the first time period. Constraint (5.10) specifies that if an alternative for a potential new school exists, it either existed in the previous time period or was completed in this period. Constraint (5.11) ensures that if an alternative for a potential new school exists in a time period, then it must exist for the rest of the planning period. Constraint (5.12) guarantees that if a potential new school project is finished in the next time period, then the alternative also has to exist in the next period. Constraint (5.13) ensures that at most one alternative can be accepted during the time horizon for the potential new schools.

**Zone-to-School Allocation**

$$\sum_{s \in \mathcal{S}} \sum_{a \in \mathcal{A}} w_{saite} = 1 \qquad i \in \mathcal{Z}, t \in \mathcal{T}, e \in \mathcal{E} \tag{5.14}$$

$$w_{saite} \leq y_{sate} \qquad s \in \mathcal{S}, a \in \mathcal{A}_s, i \in \mathcal{Z}, t \in \mathcal{T}, e \in \mathcal{E} \tag{5.15}$$

$$\sum_{a \in \mathcal{A}_s} w_{sai,t+1,e} \leq \sum_{a \in \mathcal{A}_s} w_{saite} + v_{i,t+1,e} \qquad s \in \mathcal{S}, i \in \mathcal{Z}, t \in \mathcal{T}\backslash\{T\}, e \in \mathcal{E} \tag{5.16}$$

$$\sum_{t \in \mathcal{T}} v_{ite} \leq 1 \qquad i \in \mathcal{Z}, e \in \mathcal{E} \tag{5.17}$$

$$\sum_{i \in \mathcal{Z}} N_{ite} w_{saite} = q_{sate} \qquad s \in \mathcal{S}, a \in \mathcal{A}_s, t \in \mathcal{T}, e \in \mathcal{E} \tag{5.18}$$

$$w_{sa,i(s,a),te} = y_{sate} \qquad s \in \mathcal{S}, a \in \mathcal{A}_s, t \in \mathcal{T}, e \in \mathcal{E} \tag{5.19}$$

Equality constraint (5.14) ensures that pupils in each zone are allocated to exactly one school in each time period and every scenario. Constraint (5.15) states that pupils cannot be allocated to schools that does not exist. Constraint (5.16) stipulates that the same proportion of pupils in a zone must be allocated to the same school in the next time period unless they change schools. Constraint (5.17) enforces that a zone can redistribute the allocations of pupils only once in the planning horizon. Constraint (5.18) states that the used capacity of a school is equal to the number of pupils attending the school. Lastly, Constraint (5.19) ensures that zones that contain an existing school must be allocated to that school.

**School Quality**

$$z_{sate} = C^{CON}_{sat} y_{sate} \qquad s \in \mathcal{S}^C, a \in \mathcal{A}_s, t \in \mathcal{T} e \in \mathcal{E} \qquad (5.20)$$

Constraint (5.20) states the cost of condition of an alternative of a existing changeable school if the alternative exists in the time period.

**School Capacity**

As explained in Chapter 4, the capacity of schools needs to be flexible and thus expressed by soft constraints. The plot of the cost of the school capacity is illustrated in Figure 5.1, where there are three levels of capacity. At $Q^*_{sa}$, the capacity is at an optimal value and the corresponding cost is 0. The next level is $\hat{Q}_{sa}$ which indicates the built capacity. This implies a utility cost of $\hat{C}_{sa}$. At $\bar{Q}_{sa}$, the school cannot take another pupil and is considered completely full. As this is an undesired condition, this capacity comes with the highest cost, $\bar{C}_{sa}$. The cost of a completely empty school is equal to the cost of a school at maximum capacity, given that the school exists.

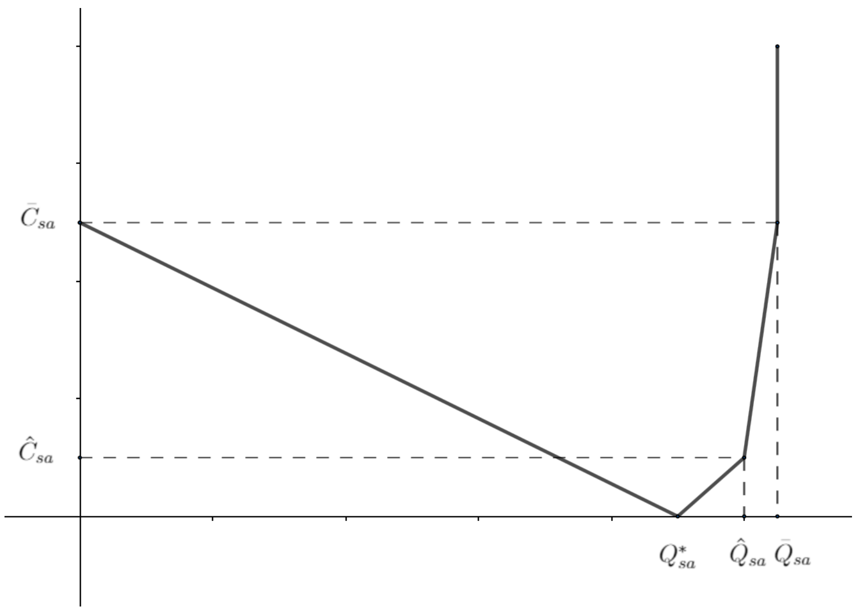



**Figure 5.1:** The plot of the cost of the school capacity.

Since the cost function is convex and piecewise linear, linear inequalities can be used to describe it. To simplify the inequalities, new parameters are introduced:

$$\theta_1 = \frac{\hat{C}_{sa}}{\hat{Q}_{sa} - Q^*_{sa}} \qquad \phi_1 = \frac{\hat{C}_{sa}}{\hat{Q}_{sa} - Q^*_{sa}} \cdot Q^*_{sa} \tag{5.21}$$

$$\theta_2 = \frac{\bar{C}_{sa} - \hat{C}_{sa}}{\bar{Q}_{sa} - \hat{Q}_{sa}} \qquad \phi_2 = \frac{\bar{C}_{sa} - \hat{C}_{sa}}{\bar{Q}_{sa} - \hat{Q}_{sa}} \cdot \hat{Q}_{sa} + \hat{C}_{sa} \tag{5.22}$$

$$\theta_3 = -\frac{\bar{C}_{sa}}{Q^*_{sa}} \qquad \phi_3 = \bar{C}_{sa} \tag{5.23}$$

$$\text{s.t.} \quad c_{sate} \geq \theta_1 q_{sate} - \phi_1 \qquad s \in \mathcal{S}, a \in \mathcal{A}_s, t \in \mathcal{T} \tag{5.24}$$

$$c_{sate} \geq \theta_2 q_{sate} - \phi_2 \qquad s \in \mathcal{S}, a \in \mathcal{A}_s, t \in \mathcal{T} \tag{5.25}$$

$$c_{sate} \geq \theta_3 q_{sate} + \phi_3 - \bar{C}_{sa}(1 - y_{sate}) \qquad s \in \mathcal{S}, a \in \mathcal{A}_s, t \in \mathcal{T} \tag{5.26}$$

$$q_{sate} \leq \bar{Q}_{sa} \qquad s \in \mathcal{S}, a \in \mathcal{A}_s, t \in \mathcal{T} \tag{5.27}$$

Maximum capacity is defined by constraint (5.24) and (5.25), where the cost of capacity must lie above these constraints. Constraint (5.26) prevents excess capacity if the school exists. Constraint (5.27) ensures that the capacity does not exceed the maximum capacity.

**Non-Anticipativity Constraint**

$$x_{sat} = x_{sate} \qquad s \in \mathcal{S}, a \in \mathcal{A}_s, t \in \mathcal{T}, e \in \mathcal{E} \tag{5.28}$$

The non-anticipativity constraint (5.28) assures that the model does not find solutions in time $t$ that are based on information that is not yet available.

**Non-Negativity/Binary Constraints**

$$x_{sate} = 0 \qquad s \in \mathcal{S}^E, a \in \mathcal{A}_s, t \in \mathcal{T}, e \in \mathcal{E} \tag{5.29}$$

$$x_{sate}, y_{sate} \in \{0,1\} \qquad s \in \mathcal{S}, a \in \mathcal{A}_s, t \in \mathcal{T}, e \in \mathcal{E} \tag{5.30}$$

$$v_{ite} \in \{0,1\} \qquad i \in \mathcal{Z}, t \in \mathcal{T}, e \in \mathcal{E} \tag{5.31}$$

$$w_{saite} \geq 0 \qquad s \in \mathcal{S}, a \in \mathcal{A}_s, i \in \mathcal{Z}, t \in \mathcal{T}, e \in \mathcal{E} \tag{5.32}$$

$$c_{sate}, q_{sate} \geq 0 \qquad s \in \mathcal{S}, a \in \mathcal{A}_s, t \in \mathcal{T}, e \in \mathcal{E} \tag{5.33}$$

$$z_{ste} \geq 0 \qquad s \in \mathcal{S}^C, t \in \mathcal{T}, e \in \mathcal{E} \tag{5.34}$$

Constraint (5.29) ensures that unchangeable schools will not be upgraded in any of the time periods in all scenarios. Non-negativity and binary conditions are determined by constraint (5.30)-(5.34).

## 5.3 Parameter Elaboration

To further include all of the aspects of the SPPA, the following elaborations of the parameters are proposed. These elaborations can be included in the model itself or be a part of a preliminary parameter development process.

**Road to School**

This section presents in detail how the value of the road to school parameter, $R_{sai}$, is calculated. $R_{sai}$ consists of input parameters that represent the distance from a zone center to a school, troublesome topography, and how dangerous the road to school is.

Parameters:

| | |
|---|---|
| $D_{sai}$ | distance between school $s$ alternative $a$ and zone $i$ |
| $A_{sai}$ | measurement of the topography between school $s$ alternative $a$ and zone $i$ |
| $F_{sai}$ | measure of how dangerous the road between school $s$ alternative $a$ and zone $i$ is |
| $P^D$ | penalty for distance |
| $P^A$ | penalty for topography |
| $P^F$ | penalty for dangerous roads |

Equality (5.35) determines the value of the road to school parameter as a result of distance, topography, and dangerous roads.

$$\mathrm{R}_{sai} = D_{sai} \cdot P^D + A_{sai} \cdot P^A + F_{sai} \cdot P^F \qquad s \in \mathcal{S}, a \in \mathcal{A}_s, i \in \mathcal{Z} \tag{5.35}$$

**Expenses of Projects**

The expenses of new schools or rehabilitation projects are distributed over the duration of the building process. The distribution is determined by a percentage for each project year.

Parameters:

| | |
|---|---|
| $E^{TOTAL}_{sat}$ | Total cost of completing school $s$ alternative $a$ in time period $t$ |
| $L_{t\tau}$ | the percentage of the total cost of a project finished in time period $t$ that is accounted for in time period $\tau$ |

The expenses from finishing a project in a time period are stated by (5.36) and are the product of the total cost of the project and the percentage distribution of the same period.

$$E_{sat\tau} = L_{t\tau} E^{TOTAL}_{sat} \qquad s \in \mathcal{S}, a \in \mathcal{A}_s, t \in \mathcal{T}, \tau \in \mathcal{T} \tag{5.36}$$

# 6 Solution Method

In this chapter, a new algorithm with two variations is outlined and proposed as a solution method for the School Prioritization Problem with Alternatives (SPPA). First, an overview of, and motivation for, the algorithm are presented in Section 6.1. Second, the notation and technicalities used in the algorithm are discussed in Section 6.2. Then, the different phases in the solution method are presented from Section 6.3 to 6.5. Lastly, the alternative formulation of the algorithm is presented in 6.6. The two complete variations of the algorithm are presented in Appendix B.

## 6.1 Execution Order Specific Branch and Bound

As discussed in Chapter 4, the uncertainty in the SPPA is modeled with a set of possible outcomes for population development. The complexity of this multi-scenario approach, in addition to the implementation of real-size data, motivates the need for a solution method that can reduce the computational effort of the model. As the number of considered scenarios grows, the need for an effective solution method emerges. Therefore, the Execution Order Specific Branch and Bound Algorithm is introduced. The idea of the algorithm is to introduce a process for fixating the execution order of the school projects, before solving the restricted multi-scenario SPPA with significantly reduced computational time. The algorithm is developed in two variations. In the standard Execution Order Specific Branch and Bound (EOSBB) algorithm, the set of projects at each school is aggregated. This means that the execution order is fixating school by school, not by individual projects. The Alternative Execution Order Specific Branch and Bound (AEOSBB) algorithm, presented in Section 6.6, fixates each of the individual projects instead.

The EOSBB algorithm is outlined in Figure 6.1. To solve the problem, the algorithm introduces a single-scenario phase, where the multi-scenario problem (MSP) is broken down into a set of single-scenario problems (SSP). The execution orders found in the single-scenario phase are then compared to each other in the multi-scenario phase. If a common execution order in all scenarios is found, the algorithm calculates the average objective value of the SSPs. Given that this average value is lower than the current best found objective value of previously solved MSPs, the MSP is solved with this common execution order as a restriction. If the objective value of the new MSP solution is lower than that of the previously best found solution, a new best solution is found.

If a common solution is not found, further branching on the execution order is necessary and new nodes are created. As long as there are unexamined nodes the search continues, and the procedure of finding single-scenario solutions is repeated with added restrictions to the allowed execution order. If there are no more unexplored nodes, the search is finished and the best found solution is returned as the optimal solution.

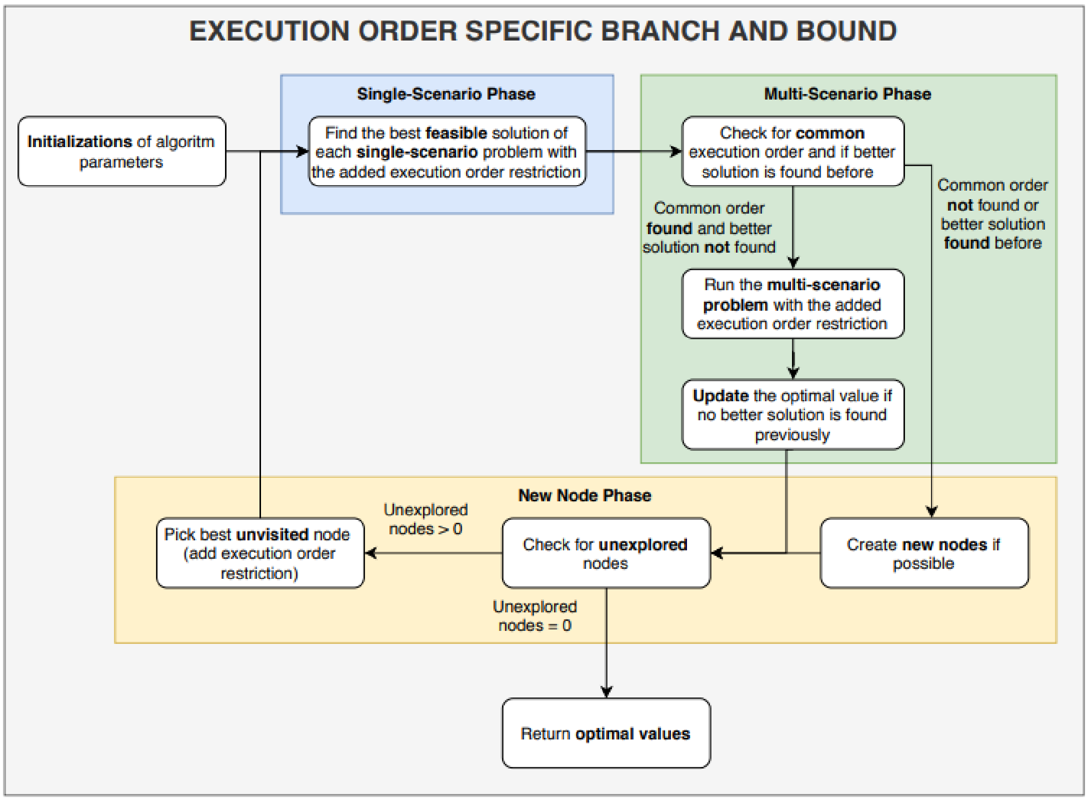



**Figure 6.1:** An outline of the EOSBB algorithm.

The extensive literature study in Chapter 3 presented various methods for solving the facility location problem. Particularly Hernández et al. (2012) successful use of a branch-and-cluster coordination algorithm scheme motivates the development of the Branch and Bound scheme of the EOSBB. Like Hernández et al. (2012), we propose a B&B algorithm where the original problem is relaxed. However, while Hernández et al. (2012) propose an LP-relaxation of the original problem, the EOSBB instead relaxes the original multi-scenario problem to a set of single-scenario problems. Then, when reapplying the restrictions of the execution order to the original problem, it still solves the problem to optimality. The EOSBB stands out as an algorithm not yet seen in existing research by providing a sequence and time schedule for execution of the projects.

## 6.2 Notation and Technicalities of the Solution Algorithm

To use the proposed solution algorithm outlined in Figure 6.1, new notation, and technicalities are introduced to the model. The added terms are introduced to enforce the execution order restriction, compare previously found solutions as well as store the necessary data.

### 6.2.1 The Execution Order Constraint

To have the SSPs and the MSPs comply with the current execution order requirements, new notation and a new constraint are introduced to the SPPA model. The restriction is enforced by introducing a new parameter, specifying the number of projects that need to be already finished before the projects of a given school can be completed.

$R_s$ The number of projects that need to be already finished before the projects of school $s$ can be completed.

Constraint (6.1) ensures that in order for a project at school $s$ to be completed in time-period $t$, the required number of completed projects prior to $t$ must be at least $R_s$.

$$\sum_{a \in \mathcal{A}_s} (1 + R_s) x_{sat} \leq 1 + \sum_{u \in \mathcal{S} | u \neq s} \sum_{a \in \mathcal{A}_u} \sum_{\tau=1}^{t-1} x_{ua\tau} \qquad s \in \mathcal{S}, t \in \mathcal{T} \qquad (6.1)$$

### 6.2.2 School-Order Matrix and Vector

The project completion variable, $x$, defined in Chapter 5, is the relevant variable when studying the order in which projects at schools are executed. To easily keep track of this order, a School-Order matrix (SO-matrix) is introduced. The school-order combination, ($s$,$o$), is 1 if any of the alternatives of school $s$ are finished as project number $o$, and 0 otherwise. If none of the projects at school $s$ are executed during the planning period, the school-order element at order $o = |\mathcal{S}| + 1$ is 1.

Table 6.1 displays an example of a SO-matrix for a made-up, five-school, situation. The table exhibits that the first school with a completed project is school 1. Further, we see that the second executed project is at school 3, and the third is at school 2. The bottom two cells in the rightmost column state that school 4 and 5 do not have any executed projects in the time horizon and thus, only three projects were executed in this example.

**Table 6.1:** School-Order matrix example

| School \Order | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 1 | 0 | 0 | 0 |
| 3 | 0 | 1 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 | 0 | 1 |
| 5 | 0 | 0 | 0 | 0 | 0 | 1 |

If more than one project is finished in a time period, the respective schools are considered to have the same order and the corresponding ($s$,$o$) elements are all 1. The number of simultaneously completed projects then decides the order of the next school with a completed project. For example, if two schools have projects completed simultaneously as order 1, the next school with a completed project have order 3. Thus, the output of running any given model is an integer SO-matrix. However, in the EOSBB it is necessary to compare the average values of all the single-scenario SO-matrices. The fractional average value then reveals how often a school-order combination occurs and is used as the basis for the branching, further discussed in Section 6.5.

Furthermore, a School-Order vector (SO-vector) is introduced. The SO-vector presents the information in the SO-matrix in a form that is easily comparable to the added number of already executed projects parameter, $R_s$. The vector is used to compare the active execution order restriction to previously solved SSPs. This is further elaborated on in Section 6.3. Each element in the vector represents the number of projects that are completed before projects at school $s$. For the schools without executed projects, the vector element is set equal to $|\mathcal{S}|$. Do note that for the SO-vector to be an accurate representation of the SO-matrix, the latter must have only integer values.

**Table 6.2:** School-Order vector example

| School | SO-vector value |
|---|---|
| 1 | 0 |
| 2 | 2 |
| 3 | 1 |
| 4 | 5 |
| 5 | 5 |

Table 6.2 shows the SO-vector of the example in Table 6.1. As a project at school 1 is completed first, no other projects have been completed and the value of the SO-vector

element is zero. Likewise, school 2 and 3 have the respective number of previously executed projects as their corresponding element in the vector. School 4 and 5 do not have any executed projects, and thus, the corresponding vector element is set to five.

### 6.2.3 The Branch and Bound Tree

The EOSBB algorithm branches on the execution order of the projects. To enable this, each node in the B&B tree stores an execution order restriction parameter, $R_s$. As the tree branches out, the restriction of the execution order tightens. This means that more and more schools are locked in to be conducted in a specific order. We use the term search depth to refer to the number of schools that are locked in. Each node store the average objective value of the SPPs of its parent node. This is used as a measure on the best possible objective value of the node and is thus used to determine what node to branch on. Furthermore, a node is either active or terminated. An unexamined node is active unless an MSP solution is found with an objective value lower than the stored average SSP value.

When initializing the EOSBB algorithm, the first node, node 1, is created without any restrictions regarding the execution order. This means that all elements in $R_s$ are equal to zero. When new nodes are created, a branching school and a branching order are required. The branching order is always set to one higher than the search depth of the parent node. For the initial node, the branching order is one, as a school needs to be locked to order one. The branching school is always one of the schools not yet locked to a specific order.

As the node branches, two new nodes, a downwards node, and a sideways node are created. In the downwards node, the branching school is locked to the branching order. Thus, one more school is locked to an order and the search depth increase. The sideways node restricts the same branching school from being completed at the branching order. No additional schools are locked in and the sideways node still has the same search depth as its parent node. Do note that locking in a school to an order in practice is achieved by restricting all other schools from being built at that order. That is, by increasing the $R_s$ element of all schools not previously locked to a school. The creation of new nodes is further discussed in Section 6.5.

In Table 6.3, a snapshot example of the information stored in the branch and bound tree nodes is presented.

**Table 6.3:** A snapshot example of the information stored in a node in the B&B tree.

| Identifier | Parent node | Avg. SSP obj. value (of parent node) | Search depth | Active / Terminated | Exec. ord. rest. 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | - | 0 | 0 | Terminated | 0 | 0 | 0 | 0 | 0 |
| 2 | 1 | 2.5 | 1 | Terminated | 1 | 1 | 0 | 1 | 1 |
| 3 | 1 | 2.5 | 0 | Terminated | 0 | 0 | 1 | 0 | 0 |
| 4 | 2 | 2.7 | 2 | Active | 2 | 1 | 0 | 2 | 2 |
| 5 | 2 | 2.7 | 1 | Active | 1 | 2 | 0 | 1 | 1 |
| 6 | 3 | 3.7 | 2 | Active | 1 | 0 | 1 | 1 | 1 |
| 7 | 3 | 3.7 | 1 | Active | 0 | 1 | 1 | 0 | 0 |

At the time of the snapshot, there exist 7 nodes. As nodes 1 to 3 already have been searched, they are terminated. The first still active node is node 4, which has node 2 as its parent node. We see from the information about node 4, that when the parent node was searched, the solving of the SSPs with the execution order restriction, yielded an average objective value of 2.7. Furthermore, it is easy to see the connection between the execution order of node 2 and node 4. In node 2, no other school than school 3 could complete the first project, and this also applies in node 4. However, node 4 is a further tightening as the execution order also restricts all other schools than school 2 from having the second completed project. Thus, as two schools are locked in, the search depth of node 4 is two.

### 6.2.4 The Set of Solutions to Single-Scenario Problems

It is desirable to reduce the overall number of required SSP runs in each node. The algorithm utilizes the fact that a cut in the solution space that does not remove the optimal solution still yields the same optimum. Therefore, only the SSP of the scenarios without previously found feasible solutions is solved. These are the scenarios with previous solutions that violate the current execution order restriction. Thus, it is necessary to store the execution order and objective value of each solved SSP. The execution order is stored as a SO-vector. The set of stored solutions is denoted $LIST$, and each scenario $e$ has such a set, $LIST_e$. If a solution in $LIST$ is feasible given the active execution order restriction, that solution is returned as the best solution for the SSP. Do note that when searching for feasible solutions for the SPPs of a node, only the nodes directly upwards in the tree can yield feasible solutions, and maximum one feasible solution can exist. By directly upwards in the tree, we mean all nodes that can be traced back parent by parent. Each set, $LIST_e$, is initialized as empty. The process of checking for feasible solutions is elaborated on in Section 6.3.

## 6.3 Single-Scenario Phase

The input to the single-scenario phase is an active node with an execution order restriction parameter, $R_s$. For each scenario $e$, the algorithm initially checks if the current restriction is already fulfilled by a previous solution in $LIST_e$. This means it searches through all previously found solutions to find one with all elements in the SO-vector higher or equal to the corresponding element in the active $R_s$. If one or more of the elements is lower, the executed project at this school is completed before the required number of previously completed projects is obtained. Thus the current restriction is not fulfilled by that solution. If no previous solution fulfills the $R_s$, the SSP must be reoptimized for that scenario. When solved, the SO-vector is created and stored as a new element in $LIST_e$ with the corresponding objective value, $z_e$. However, if a feasible previous solution is found, the SSP is not reoptimized, and instead, the feasible solution is forwarded. The snapshot in Table 6.4 provides an example of the situation for a scenario in a five-school situation.

**Table 6.4:** A single-scenario phase example in a five-school situation and two previously found SSP solutions.

| | School | | | | | Comment |
|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | |
| $R_s$ | 0 | 2 | 1 | 2 | 2 | |
| $LIST$ 1 | 0 | 5 | 2 | 1 | 5 | Not feasible |
| $LIST$ 2 | 0 | 5 | 1 | 2 | 5 | Feasible |

In the example, the current execution order restriction ensures that a project at school 1 is executed first and a project at school 3 second. We see that this scenario's $LIST$ contains two previously found solutions. The first solution, $LIST$ 1, breaks the execution order restriction since school 4 is not allowed to have a project completed as the second project. However, $LIST$ 2 provides a feasible solution, as no schools have projects completed earlier than the restriction allows. Thus, the SSP does not need to be reoptimized. Instead, $LIST$ 2 is returned, as it is the optimal solution for this scenario.

At the end of the single-scenario phase, a set of active SSP solutions, one for each scenario, is forwarded to the multi-scenario phase. These solutions include the objective values and the SO-matrices, recreated from the stored vectors. If one or more of the single-scenario solutions do not yield a feasible solution given the current execution order restriction, the search from the node in the B&B tree is terminated in the multi-scenario phase. The single-scenario phase is outlined in Algorithm 1.

**Algorithm 1:** The single-scenario phase

**for** each scenario $e$ in $\mathcal{E}$ **do**<br>
    **if** $R_s$ is **not** already fulfilled by a solution in $LIST_e$ **then**<br>
        solve the SSP<br>
        generate the SO-vector<br>
        add the solution to $LIST_e$<br>
    **else**<br>
        return the feasible solution from $LIST_e$<br>
    **end-if**<br>
**end-do**

## 6.4 Multi-Scenario Phase

In the multi-scenario phase, the solutions found in the single-scenario phase are compared and evaluated. Initially, the average SO-matrix (SO-average), as well as the average objective value, $z^{avg}$, are computed from the set of active scenario solutions returned from the single-scenario phase. If one or more of the SSPs do not yield a feasible solution, the search from the current node is terminated and the multi-scenario phase is ended. The search from the node is also terminated if the algorithm has previously found a valid multi-scenario solution, $\bar{z}$, with a lower objective value than $z^{avg}$ as this branch can no longer provide a better MSP solution.

Subsequently, if the node is active, the SO-average is checked for integer values. If all elements are integers, the execution order in all of the SSP solutions is the same and a reasonable execution order restriction can be introduced to the MSP, which is then solved. However, all scenarios do not necessarily require the completion of the same number of projects. If the search depth of a node is equal to the number of executed projects of a scenario, further branching on the node does not lead to a common execution order. As the execution of more projects in that scenario would deteriorate the objective value, this is as deep as the search can go on the current branch. Thus, if the search depth of the node equals the number of executed projects for one or more SSP solutions, the MSP is solved with an execution order restriction equal to the execution order of the SSPs in question. If the found optimal objective value of the MSP, $z^{MSP}$, is lower than the previously found value, $\bar{z}$, the $z^{MSP}$ is the new best found value and the search from the node is terminated.

If a common order is not found and the search depth does not equal the minimum number of executed projects, the node is still active. The multi-scenario phase sends forward the status of the current node to the new-node phase, where active nodes are due for branching. The multi-scenario phase is outlined in Algorithm 2.

**Algorithm 2:** The multi-scenario phase

calculate the SO-average, and calculate $z^{avg}$
**if** one or more active single scenarios solutions not feasible **then**
    node terminated
**end-if**
**if** $z^{avg} \geq \bar{z}$ **then**
    node terminated
**end-if**
**if** the SO-average is integer **or**
search depth = minimum number of executed projects **then**
    run the MSP with added execution order restriction
    **if** $z^{MSP} \leq \bar{z}$ **then**
        $\bar{z} = z^{MSP}$
        node terminated
    **end-if**
**end-if**

## 6.5 New-Node Phase

In the new-node phase, there are two main processes: the creation of new nodes and the picking of a new node to examine. From the multi-scenario phase, the status of the current node is received. If the status of the node returned from the MSP is terminated, no new nodes are created. If the status is active, the node is branched on. The branching order is, as discussed in Section 6.2, related to the search depth of the current node. To find the best branching school, however, the SO-average is searched, looking for the school with the highest occurrence at the branching order. This is the school that in most of the SSP solutions was built at the branching order. The downwards and sideways nodes are then created.

An example of the node creation process is outlined in Table 6.5. The parent node displayed in Table 6.5b yields the SO-average presented in Table 6.5a. The parent node restricts all other schools than school 2 to have a project completed as the first, and all other schools than school 3 to have a project completed second. The search depth of the parent node is two and hence the branching order is three. Thus, the algorithm searches down the order-three column in the SO-average. The search shows that school 4 has the highest fractional value, highlighted in green. School 4 is therefore chosen as the branching school. Consequently, school 4 is locked to order three in the downward node, and school 1 and 5 are no longer allowed to have the third executed project. In the sideways node, however, school 4 is no longer allowed to have the third completed project. This means that only school 1 and 5 can have the third project. Each of the two nodes is then assigned the average SSP objective value, $z^{avg}$, of the parent node.

**Table 6.5:** Example of the SO-average and the creation of the downward and sideways nodes.

**(a)** The SO-average-matrix

| School \Order | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 0 | 0 | 0.2 | 0.8 |
| 2 | 1 | 0 | 0 | 0 |
| 3 | 0 | 1 | 0 | 0 |
| 4 | 0 | 0 | 0.6 | 0.4 |
| 5 | 0 | 0 | 0.2 | 0.8 |

**(b)** The execution order restriction of a parent node and the resulting nodes given the SO-average in (a)

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Parent node | 2 | 0 | 1 | 2 | 2 |
| Downward node | 3 | 0 | 1 | 2 | 3 |
| Sideway node | 2 | 0 | 1 | 3 | 2 |

After the creation of new nodes, the status of the parent node is changed to terminated. The tree is then searched to find the best yet active node with the lowest $z^{avg}$ value. If two nodes have the same value, the node with the deepest search depth is chosen.

The output of the new-node phase is a new node returned to the single-scenario phase. If no new nodes are found, the search is finished and the best found solution is returned as the optimal solution. If no solutions are found, the problem is infeasible. Algorithm 3 describes the steps of the processes.

**Algorithm 3:** The New-Node phase

```
1:  if current node is active then
2:        create two new nodes
3:        add the created nodes to the tree
    end-if
4:  if still active nodes then
5:        pick best node
6:        return to the single-scenario phase
    end-if
7:  return the optimal solution of the SPPA
```

## 6.6 Alternative Formulation of the Execution Order Specific Branch and Bound

The rationale behind introducing the EOSBB algorithm is to reduce the complexity of the MSP by transferring some of the computational challenge to the solving of simpler SSPs. To be of value the algorithm must reduce the computational time of the SPPA. This means that the time it takes to solve all the necessary SSPs, the restricted MSPs, and perform all the necessary tree mechanisms, must be shorter than the time it takes to solve the original MSP. The size of the B&B tree directly affects the number of SSPs that must be solved, and thus the size directly affects the performance of the algorithm.

In the algorithm presented in this chapter, the size of B&B tree is based on the number of existing schools, since only one execution order restriction exists per school. An alternative formulation of the algorithm is to branch on all the existing projects. As there potentially are several projects per school, this alternative formulation increases the number of execution order restrictions. Both the SSPs and MSPs still ensure the requirement that only one project can be executed per school. It is expected that the alternative formulation further increases the number of nodes of the B&B tree. However, it is also expected that a stricter formulation of the MSPs reduces the computational time. In this thesis, both formulations are tested to see whether a more substantial tree can further reduce the computational time of the SPPA.

This alternative formulation of the EOSBB is denoted as the Alternative Execution Order Specific Branch and Bound algorithm (AEOSBB). All mechanisms introduced and discussed in this chapter also apply and are used in the same way in the AEOSBB. However, some adjustments must be made. First, the SO-matrix and -vector now contains one element for each project instead of one element for each school. Second, a small alteration of the formulation of the execution order restriction parameter and constraint is necessary. The parameter is reformulated as follows:

$R_{sa}$ The number of projects that needs to be finished before school $s$ alternative $a$ can be completed.

Constraint (6.2) ensures that in order for school $s$ alternative $a$ to be completed in time-period $t$, the required number of projects completed prior to $t$ must be at least $R_s a$.

$$(1 + R_{sa})x_{sat} \leq 1 + \sum_{u \in \mathcal{S} | u \neq s} \sum_{b \in \mathcal{A}_u} \sum_{\tau=1}^{t-1} x_{ub\tau} \qquad s \in \mathcal{S}, a \in \mathcal{A}_s, t \in \mathcal{T} \tag{6.2}$$

# 7 Data Sets and Implementation

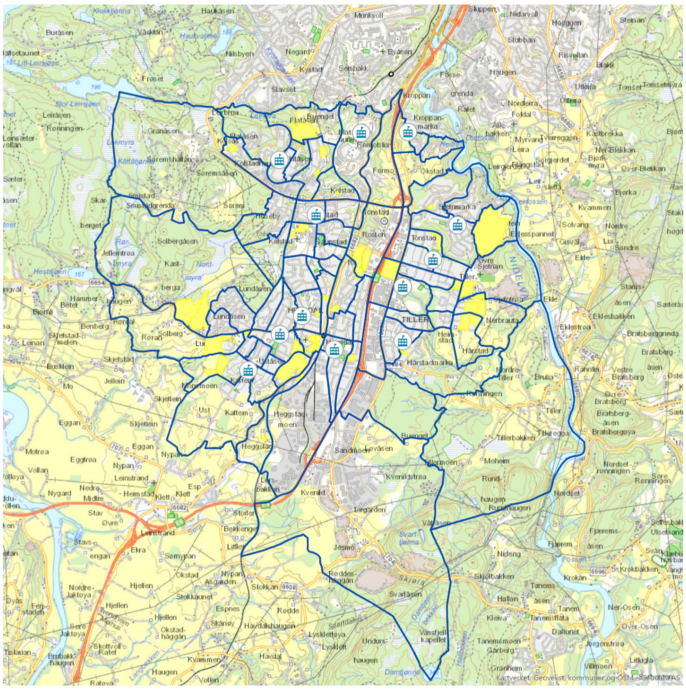
This chapter presents in detail the implementation of the data sets used in the computational study in Chapter 8. First, the designated area and its zones are discussed in Section 7.1. Second, the population development in the designated area is presented in Section 7.2, alongside the related uncertainty. Third, in Section 7.3 the distance from each zone to the schools is discussed.. Fourth, in Section 7.4 the existing schools and potential projects are outlined. Lastly, in Section 7.5 the weighting parameters are presented.

## 7.1 Area and Zones

For the case study, an area in southern Trondheim is considered. This area is defined by the current school districts and their respective population zones. The data is presented in ArcGis Pro, a geographic information system. ArcGis combines maps with spatial data and enables complex analysis of the information. The area consists of twelve current school districts, divided into 117 zones. In Figure 7.1a, the school districts, with the placement of their schools, are illustrated, and Figure 7.1b demonstrates the zones. In collaboration with Trondheim, this area is chosen as a representative selection of the municipality's current problems. The zones are determined and provided by Trondheim. These zones are made by clustering neighborhoods to simplify planning in the municipality. There are two types of zones. The first type is the zones that initially have a population, and where the number of residents stays the same throughout the planning period. These are referred to as unchangeable zones. The second type is the new zones without an initial population, and where residential building projects are planned and may be executed. Each new zone is related to exactly one residential building project. These zones are denoted as changeable zones. There are 74 unchangeable and 43 changeable zones.

Each unchangeable zone has two defined centers: a straightforward geographic center, and a center related to the location of residential homes in the zone. This center is the mean location of all the residential buildings in the zones. As the size of the zones is set sufficiently small, the use of centers is an equitable representation of the location of all

**(a)** Schools and school districts.



**(b)** Existing zones and changeable zones.

**Figure 7.1:** Illustration of school districts, zones, and existing schools in the area of consideration. The red lines indicate the school districts that are considered in this thesis. The blue symbols represent the location of existing schools. The blue lines illustrate the unchangeable zones that are considered in the study. The yellow areas are the location of the changeable zones.

the residents in that zone. Both centers are provided by Trondheim. The center for the location of homes is the most interesting, as this indicates where most people are located, and thus this center is used for the unchangeable zones.

As there are no current buildings in the changeable zones, the residential building center can not be found, and they are given a geographic mean instead. The center is found by using the ArcGis Pro tool *Mean Center*. These centers are the positions used for changeable zones in the model and referred to as the zone center for changeable zones.

## 7.2 Population

The population of a zone is defined as the number of pupils in the zone. The population development of a zone is dependent on whether the zone is changeable or unchangeable. In unchangeable zones, it is assumed that the number of newcomers and people moving out is equal, so the amount of pupils is constant throughout the planning horizon. The data for existing zones are provided by Trondheim and shows the number of pupils as of the 1st of January 2019. However, in zones with only one or two pupils, the amount is adjusted due to privacy protection regulations. Consequently, zones with one pupil are adjusted to zero, and zones with two pupils are set to three. Therefore, the population differs slightly from actual statistics.

Initially, the population of a changeable zone is zero. The population of the zone is then dependent on the realization of the residential building project. The location and potential amount of new residences are shown in Figure 7.2. As the Figure illustrates, the project size varies from approximately 30 to 900 residential buildings in the changeable zones.

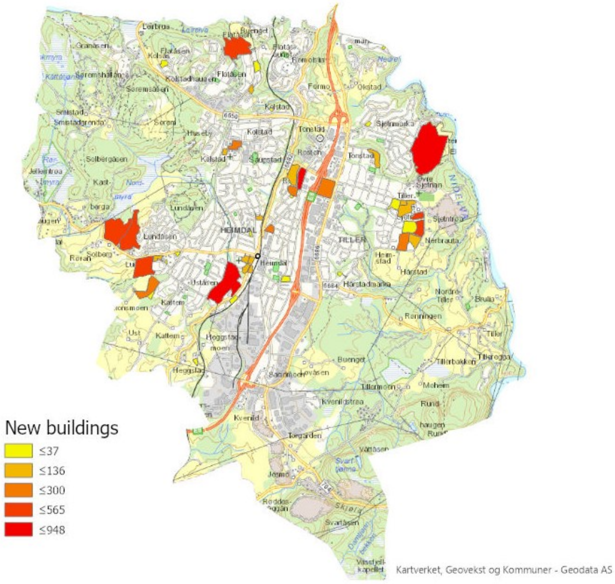



**Figure 7.2:** The location and potential amount of new residences in the changeable zones.

As presented in Chapter 2, the possible new residential buildings in Trondheim are categorized as detached houses, horizontally divided residences, vertically divided residences, and low-/mid-rise buildings. The amount of each building type is specified in all the possible residential building projects. Each of the different building types has an expected number of pupils. Table 7.1 presents the estimated number of pupils in one unit of each type of building.

**Table 7.1:** Average number of pupils in one unit, per type of residential building.

| Type of building | Number of pupils per unit | ID |
|---|---|---|
| Detached house | 0.27 | D |
| Horizontally divided | 0.13 | HD |
| Vertically divided | 0.33 | VD |
| Low-/Mid-rise building | 0.08 | L/M |

For each new residential building project, there is an anticipated year for when the first and the last residence is finished and move-in ready. Further, the number of finished buildings are assumed to be equally distributed in this time interval. An example of the relevant residential building project-specific data is shown in Table 7.2.

**Table 7.2:** Example of the relevant residential building projects specific data in a changeable zone. Building potential denotes the total number of residences that can be built in the changeable zone.

| Zone name | Building potential | Amount of building type | | | | First finished | Last finished |
|---|---|---|---|---|---|---|---|
| | | D | HD | VD | L/M | | |
| Hårstad Mindes veg | 180 | 0 | 0.2 | 0 | 0.8 | 2020 | 2023 |

Throughout the planning period, the population in the changeable zones is found by multiplying the number of finished residences with the respective number of pupils according to residence type. However, the residences that are utilizable during a year are not accounted for in the population register before the 1st of January the following year. Therefore, there is a one time period delay, from when a residence is finished to when its inhabitants are considered a part of the population. Table 7.3 shows an example of the population development in a changeable zone.

**Table 7.3:** Example of population development in the changeable zone Hårstad Mindes veg caused by the completion of building projects.

| Year | 2020 | 2021 | 2022 | 2023 | 2024 |
|---|---|---|---|---|---|
| Number of finished buildings | 45 | 90 | 135 | 180 | 180 |
| Population in pupils | 0 | 13 | 25 | 37 | 49 |

As discussed in Chapter 2, there are uncertainties associated with each of the residential building projects. First and foremost, it is uncertain whether the project will be executed or not. The probability that each project is executed is derived from the Regulatory Status Scheme administrated by Trondheim. This status scheme allocates the projects according to where they are located in the project process. For instance, projects with status 1 have only just been initialized, while a status 4 project has all necessary permissions to start the construction. Each of these statuses is assigned an associated probability of execution. A project's advancement in status is directly correlated to the progress of the planning process. This results in a higher probability of completion as the status increases. The Regulatory Status Scheme with coherent probabilities is presented in Table 7.4.

**Table 7.4:** The Regulatory Status Scheme for building projects in Trondheim.

| Status | Explaination | Probability |
|---|---|---|
| 1 | The project has not started planning work | 0.50 |
| 2 | Planning work is initiated | 0.60 |
| 3 | Complete plan proposal is ready to be presented for Trondheim | 0.70 |
| 4 | Trondheim has decided regulations | 0.85 |
| 5 | The building project is completed | 1.00 |

Second, it is uncertain whether the projects can begin at the scheduled time. Table 7.5 presents the estimated probabilities for a project being postponed by 0 to 5 years. For simplicity reasons, a delay only affects the start year, and the project duration remains unchanged. As the population growths in the changeable zones are correlated with the completion of the building projects, a delay in a project causes the same delay in population development.

**Table 7.5:** The probability of postponement of the projects in years.

| Delay in Years | Probability |
|---|---|
| 0 | 0.50 |
| 1 | 0.20 |
| 2 | 0.10 |
| 3 | 0.10 |
| 4 | 0.05 |
| 5 | 0.05 |

The same example presented in Table 7.3 is shown in Table 7.6, but with 1 year delay.

**Table 7.6:** Example of population development with 1 year delay in the new zone Hårstad Mindes veg.

| Year | 2020 | 2021 | 2022 | 2023 | 2024 | 2025 |
|---|---|---|---|---|---|---|
| Number of finished buildings | 0 | 45 | 90 | 135 | 180 | 180 |
| Population in pupils | 0 | 0 | 13 | 25 | 37 | 49 |

## 7.3 Distance

To calculate the accurate distance from each zone to each school, information from several data sources have been gathered and utilized. As presented in Chapter 2, it is desired that pupils walk to school. Thus, the shortest walking distance from the zone centers to the school is defined as the distance. A footpath network has been provided by Trondheim.

This enables walking distances to be calculated in ArcGis by using the built-in *Closest Faclity* tool. To avoid dangerous roads where footpaths are not available, ArcGis are restricted from allowing roads that are unsuited for pedestrians. The given footpath network is also expected to not include paths with height differences issues. Figure 7.3 illustrates the shortest walking distance from one of the zones to every school in the selected area. As shown in the figure, walkways are clearly taken into consideration. This is especially prominent when considering crossing dangerous roads, where the routes are taking underpasses into account.

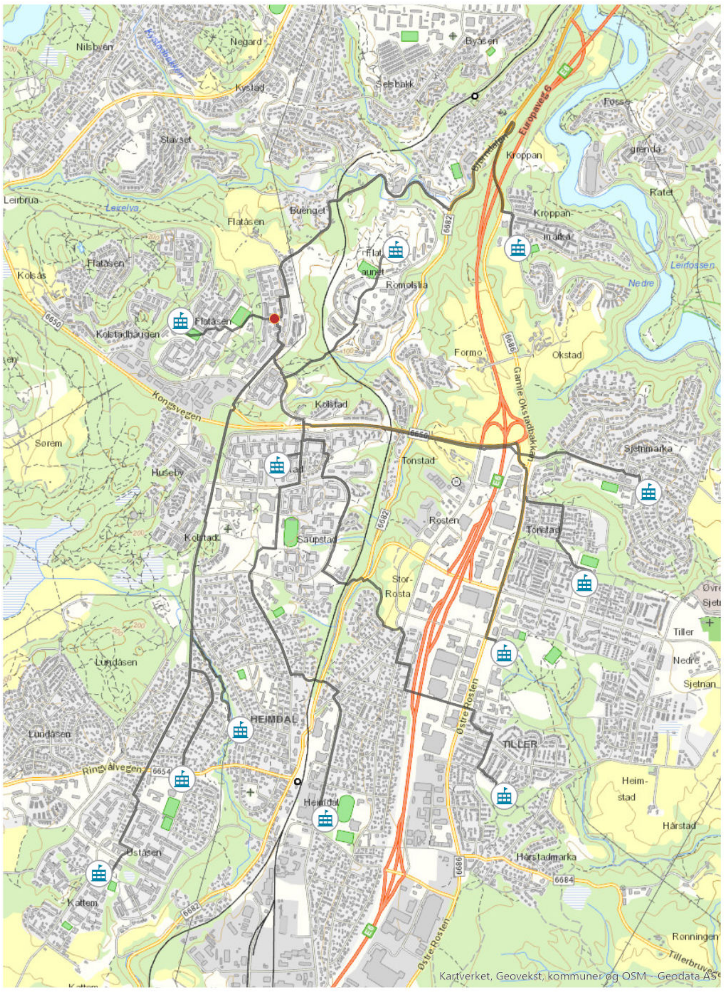



**Figure 7.3:** An example of walking distance from a zone to every school. The red dot illustrates the zone center and the black lines are the shortest possible walking routes to each distinct school.

## 7.4 Schools

In addition to the twelve already existing schools, the construction of two potential new schools are on the list of possible projects. All the schools have a set of common parameters. Each alternative for each school is assigned a value for built capacity. As discussed in Chapter 2 the optimal capacity level is given by Trondheim and set below the built capacity to handle fluctuations in the year to year amount of pupils. In addition, the maximum capacity value is established above the built capacity to allow for special considerations. The optimal and maximum capacity is dependent on the built capacity as shown in Table 7.7.

**Table 7.7:** Optimal and maximum capacity values as functions of built capacity.

| Capacity | Value |
|---|---|
| Optimal | $0.9 \cdot$ Built capacity |
| Maximum | $1.05 \cdot$ Built capacity |

Trondheim emphasizes that utilization above optimal capacity is undesired in a long-term perspective. Therefore a cost for capacity utilization deviating from the optimal level is set. Utilization above the built capacity level is even more undesirable and penalized accordingly. The parameters for cost at built capacity and cost at maximum capacity are presented in Table 7.8. The cost for under-utilization is set so that the cost of no students attending the school is equal to the penalty of capacity utilization at max capacity.

**Table 7.8:** Cost at maximum and built capacity.

| Parameter | Value |
|---|---|
| Cost at maximum capacity | 5 |
| Cost at built capacity | 1 |

#### Changeable Schools

Nine of the twelve already existing schools are changeable, meaning that there exist one or more potential projects for these schools. As discussed in Chapter 4, each project is either a renovation project, a capacity expansion project, or a combination of both. Table 7.9 shows each of the changeable schools with coherent projects.

**Table 7.9:** The number of each type of potential projects at each changeable school.

| School | Renovation projecs | Capacity projects | Combination projects | Sum |
|---|---|---|---|---|
| Breidablikk | 1 | 0 | 1 | 2 |
| Flatåsen | 1 | 0 | 0 | 1 |
| Kattem | 1 | 2 | 1 | 4 |
| Romolslia | 1 | 0 | 0 | 1 |
| Rosten | 0 | 1 | 0 | 1 |
| Sjetne | 0 | 3 | 0 | 3 |
| Stabbursmoen | 1 | 0 | 2 | 3 |
| Tonstad | 1 | 0 | 1 | 2 |
| Åsheim | 1 | 0 | 1 | 2 |
| Sum | 7 | 6 | 6 | 19 |

To calculate the cost of the different projects, a defined standard school is used as a reference. The standard school is provided by Trondheim and is a representation of the maximum possible size of a school in the municipality. The characteristics of the school are presented in Table 7.10.

**Table 7.10:** The characteristics of the standard school

| Built capacity [number of pupils] | Building costs [ mill NOK] | Built area [$m^2$] |
|---|---|---|
| 700 | 380 | 8,000 |

The cost for each capacity project is calculated by Equation 7.1. Here, *Added built capacity* refers to the resulting increase in the built capacity if the project is executed.

$$\text{Capacity project cost} = 380 \cdot \frac{\textit{Added built capacity}}{700} \tag{7.1}$$

The costs from renovation projects are further distinguished into two categories based on the nature of the renovation. For renovation projects that do not entail the replacement of pavilions, the renovation cost is estimated to be 60 % of the cost of building a new school of the same size. Thus, these projects are calculated by Equation 7.2, where the *Built capacity* is the built capacity at the school.

$$\text{Renovation project cost} = 0.6 \cdot 380 \cdot \frac{\textit{Built capacity}}{700} \tag{7.2}$$

If the projects require replacement of pavilions, this cost is calculated from the size of the replaced area, and Equation 7.3 is used. Here, *Square meters* is defined as the total square meters from the pavilions that need to be replaced.

$$\text{Pavillion replacement project cost} = 380 \cdot \frac{\textit{Square meters}}{8000} \tag{7.3}$$

The costs of the combination projects are found by summing the respective costs from the projects' parts.

All existing changeable schools have a condition state related to the physical condition of their buildings. This condition state is set to deteriorate throughout the time horizon until the school is upgraded by a renovation project. The expected condition and deterioration is provided by Trondheim and is outlined in a condition-matrix. If a renovation project is executed, the condition state is reset to zero and fixed for the rest of the planning period. Figure 7.4 presents the possible states.


**Figure 7.4:** The possible states for the changeable schools.

The condition cost of each school is given by the condition state so that the condition cost is equal to the square of the condition state. For instance, if a school is in state 3, the condition cost is set equal to 9. Figure 7.5 illustrates the condition cost as a function of the school's condition state.

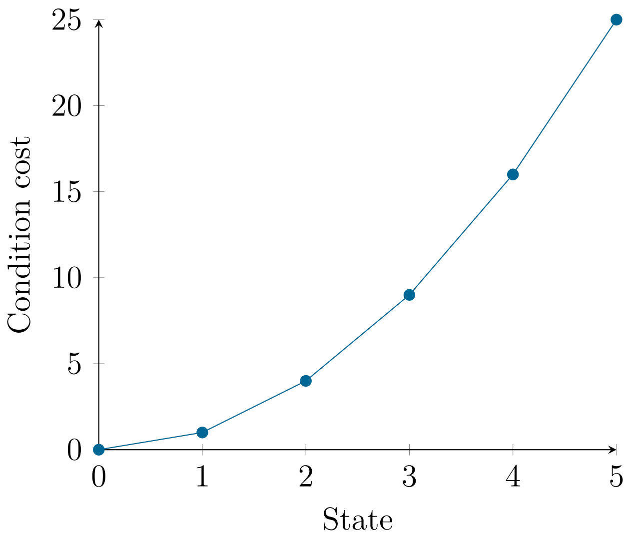



**Figure 7.5:** Condition cost as a function of the state the school is in.

### Unchangeable Schools

Three of the schools in the designated area are denoted as unchangeable schools. As discussed in Chapter 4, these are considered to be schools with available capacity and with satisfactory physical conditions. Therefore there do not exist any possible projects for these schools.

### Potential New Schools

The two potential new schools are each proposed with three capacity alternatives: small, medium, and large, to represent different possible realizations of the schools. The smallest size is set to represent the smallest new school size acceptable to Trondheim. As for the capacity projects, the building costs of the new schools are calculated by Equation 7.1. The built capacity and building cost are given for each project size in Table 7.11.

**Table 7.11:** Built capacity in number of pupils and building cost in millions, of the three possible sizes of potential schools.

| Alternative | Built capacity | Building cost |
|---|---|---|
| Small | 350 | 190 |
| Medium | 450 | 244 |
| Large | 550 | 299 |

### Cost Distribution

The cost of executing the discussed projects is distributed over the project duration, as discussed in Chapter 4. The cost distribution is given in Table 7.12 and is estimated from real values provided by Trondheim, as presented in Chapter 2.

**Table 7.12:** Cost distribution for the projects.

| Years to finish | Cost distribution |
|---|---|
| 0 | 0.04 |
| 1 | 0.16 |
| 2 | 0.65 |
| 3 | 0.14 |

### Budget Parameter

The budget available for executing the school projects in the considered area is estimated from the yearly budget in Trondheim. It is budgeted yearly with approximately 380 million for school maintenance and development projects. Of this, 90% is designated for significant projects such as the ones discussed in this thesis. As the designated area includes roughly 1/4 of the schools and the population, the area-specific budget is estimated to 80 million NOK each year.

Strictly following the yearly budget disable the execution of many projects as their costs exceed the yearly budget. This leads to an unrealistically strict budget constraint. In practice, the expenditures in some years can exceed the yearly budget if they in other years are below it. Thus, the budget constraint, (5.2), must be reformulated to allow for accumulation over time periods. Also, a new parameter is introduced:

$\mathcal{Y}$ The number of years the budget is accumulated over.

Then, constraint (7.4) ensures that the expenses that occur in a series of consecutive time periods cannot exceed the accumulated budget for that same time period.

$$\sum_{\tau=\lambda}^{\lambda+\mathcal{Y}-1} \sum_{s\in\mathcal{S}} \sum_{a\in\mathcal{A}} \sum_{t\in\mathcal{T}} E_{sat\tau} x_{sate} \leq \sum_{\tau=\lambda}^{\lambda+\mathcal{Y}-1} B_{\lambda} \qquad \lambda \in \mathcal{T}, e \in \mathcal{E} | \lambda \leq (|\mathcal{T}| - \mathcal{Y} + 1) \tag{7.4}$$

Furthermore, the accumulation of projects is not meant to enable many small projects to be executed simultaneously. Constraint 7.5 ensures that at most one project can be executed in each time period.

$$\sum_{s\in\mathcal{S}} \sum_{a\in\mathcal{A}_s} x_{sate} \leq 1 \qquad t \in \mathcal{T}, e \in \mathcal{E} \tag{7.5}$$

In order to allow for the execution of each project, $\mathcal{Y}$ is set to 4.

## 7.5 Weight Parameters

As discussed in Chapter 4, the weight parameters reflect the desired importance-measure of the objectives. For simplicity, the weight parameters are set so that they are equal to one when summed. The weight parameters are presented in Table 7.13.

**Table 7.13:** The value of the weight parameters.

| Weight | Value |
|---|---|
| $\alpha$ | 0.80 |
| $\beta$ | 0.05 |
| $\gamma$ | 0.15 |

Trondheim emphasizes the importance of providing pupils with a short and safe road to school. As this is also a requirement by law, this is the most important objective and thus, the road to school objective weight, $\alpha$, is given the highest value. Since measures that solve significant challenges related to the physical learning environment of the schools is one of Trondheim's top priorities, the condition cost weight parameter, $\gamma$, is given the second-highest value. Lastly, the capacity utilization objective weight, $\beta$, is weighted to ensure satisfactory school capacity utilization.

# 8 Computational Study

In this chapter, the performance of the model suggested in Chapter 5 is tested with data instances as described in Chapter 7. The goal of the computational study is to test and improve the computational performance of the School Prioritization Problem with Alternatives (SPPA) by implementing various measures and the Execution Order Specific Branch and Bound (EOSBB) algorithms presented in 6. A complete list of hardware and software that are used to perform the tests in this computational study are described in Table 8.1. The test instances are run and solved in Mosel Xpress.

**Table 8.1:** List over used hardware and software in the computational study

| | |
|---|---|
| Hardware | Lenovo M5 |
| Processor | 2 x Intel E5-2670v3 |
| Memory | 64 Gb RAM |
| Operating System | CentOS 7.8.2003 |
| Fico Optimization Suite | v8.8 |
| Xpress Mosel Version | 64-bit v5.0.3 |

First, Section 8.1 presents the testing of a deterministic version of the SPPA, and is based on Aslaksen and Norum (2019). Second, in Section 8.2, the solution methods are tested and analyzed. Third, the stability of the model is tested in Section 8.3. Fourth, the value of information is discussed in Section 8.4. Lastly, an analysis of alterations in the non-anticipativity constraint is provided in Section 8.5.

## 8.1 Deterministic Model Testing

In this section, a deterministic model variation of the SPPA, denoted DM, is studied. The DM is a representation of the SPPA in its simplest form, as a single-scenario problem. In this context, deterministic means that the random population variables are replaced by their expected value, based on the probabilities in Chapter 7. The goal of the DM testing is to improve the computational performance of the SPPA, without cutting optimal, reallife feasible, solutions. This is achieved by implementing measures and restrictions specific for the real case study of Trondheim. As discussed in Chapter 3, the stochastic problem and the deterministic problem are not profoundly different, and implementations on the DM can thus easily be implemented on the stochastic SPPA. Throughout the DM testing, the maximal computational time is set to one hour.

In Section 8.1.1 we establish a base case version to the DM. Then, Section 8.1.2 presents a relaxation of the school change variable. Further, a maximum allowed school deterioration restriction is implemented in Section 8.1.3, and a set of zone-to-school allocation cuts are examined in Section 8.1.4. The section concludes with Section 8.1.5, which presents the final deterministic model.

### 8.1.1 The Base Case

As presented in Chapter 5, the objective function is divided into three objective terms: inconvenience from road to school, inefficient capacity utilization, and poor condition of the physical buildings. To compare the cost of the three objectives, we first need to establish a non-dominant objective function. In a non-dominant objective function, the objective values should be approximately equal if the weighting parameters in the objective function, (5.1), are assigned the same value. As the values of the objectives are unbalanced, each objective is assigned a divisor that, if applied, gives them equal significance. When the divisors as well as the objective weights discussed in Chapter 7 are applied, the problem is denoted the base case.

The set of balancing divisors is found by introducing a single-objective problem (SOP) for each objective. The objective values of the SOPs are then set as that respective objective's divisor. Consequently, by implementing the divisors, the best possible solution for each objective is 1. All later objective values are presented as a comparison to the divisor values found in the SOPs. The divisors are presented in Table 8.2. In the tables in this chapter, the road to school objective is abbreviated to RtS, the capacity utilization to Cap, and the school condition to Cond. The process of deciding the set of divisors is further presented in Appendix C.

**Table 8.2:** The objectives with their respective divisors

| Objective | Divisor |
|---|---|
| RtS | 48,160,000 |
| Cap | 2.73 |
| Cond | 208,577 |

The divisors yield information about the cost of each objective. However, these terms are represented as non-monetary costs and provide somewhat limited information about the practical implication of the solution. Therefore, we introduce a second set of measures to increase the practical understanding. These measures are average zone-to-school distance, average capacity utilization, and average school building condition.

To minimize the objective function, the average road to school and school condition ought to be as low as possible. The capacity term, however, is penalized if the utilization is either above or below the optimal capacity. This signifies that the performance of a solution can be illustrated by how low the average road to school and condition state is, and how close the average capacity utilization is to 90%. The values of the solutions of each SOP is given in Table 8.3.

**Table 8.3:** The results of the SOP tests, where the green areas represents the best possible average from each objective.

| | Avg. RtS [m] | Avg. Cap [%] | Avg. Cond |
|---|---|---|---|
| RtS | 678.6 | 86.3 | 2.00 |
| Cap | 2954.5 | 89.7 | 1.65 |
| Cond | 3239.4 | 99.1 | 1.29 |

The highlighted cells represent the best possible average road to school (Avg. RtS) and average school condition (Avg. Con), as well as the best found average capacity utilization (Avg. Cap). Note that if the three objectives are completely uncorrelated, the diagonal values are the optimal solutions for the multi-objective SPPA.

To find the base case solution, the DM is solved with the discussed divisors and with objective weights, as discussed in Chapter 7. Table 8.4 displays the solution values found when the base case is solved.

**Table 8.4:** Solution values of the multi-objective base case

| Integer Solution | Best Bound | Gap [%] | RtS | | Cap | | Cond | |
|---|---|---|---|---|---|---|---|---|
| | | | Obj. | Avg. [m] | Obj. | Avg. [%] | Obj. | Avg. |
| 1.4563 | 1.2286 | 15.6 | 1.19 | 807 | 4.01 | 89.7 | 2.00 | 1.51 |

From the table, we see that the three objectives are correlated, as the objective values are all worse than the values of the SOP solutions. For example, the road to school objective is 1.19 times higher than the divisor value. It is interesting to point out that whilst the capacity utilization objective has a value of 4.01, which indicates a worse capacity utilization, the average capacity utilization remains at the same level of 89.7%. This can be explained by investigating the utilization of each school individually. For instance, if a school is using 85% capacity in half of the time periods, and 95% in the other half, the average utilization will be 90%. However, the objective value is significantly worse than if the utilization was 90% in all of the periods. This illustrates that the average utilization can tell a different story than the objective value. Thus, it is necessary to study both to get a complete understanding of the solution.

Furthermore, Table 8.4 shows a gap between the best found integer solution and the best bound. This means that the DM cannot find the optimal solution within the maximum allowed computation time. As the DM is already a simple single-scenario version of the SPPA, the multi-scenario version is anticipated to struggle even more to find the optimal solution. It is therefore necessary to test measures that can reduce the computational time of solving the problem without deteriorating the solution.

### 8.1.2 Relaxation of the School Change Variable

When running the deterministic model, the maximum one change constraints, (5.16) and (5.17), stick out as heavily demanding constraints. The combination of considerable row generation and the binary variable $v$, increases the computational complexity considerably. In an attempt to improve the run time, we introduce two alternatives for relaxed formulations of the school change variable, $v$. The rationale for the relaxations is the significant road to school weighting parameter which causes deviations from the shortest path to be deeply unfavorable. Therefore, it is expected that the number of school changes remains limited even with the more relaxed formulations of the school change variable.

The first relaxation suggestion is to introduce a small adjustment factor, allowing minor zone-to-school changes without activating the maximum one change constraint. This means replacing constraint (5.16) with constraint (8.1). Constraint (8.1) allows a small amount of the pupils in a zone to change schools whilst still ensuring maximum one major school change throughout the time horizon. We denote this variation as the minor change model.

$$\sum_{a \in \mathcal{A}_s} w_{sai,t+1,e} \leq \sum_{a \in \mathcal{A}_s} w_{saite} + 0.1 + v_{i,t+1,e} \qquad s \in \mathcal{S}, i \in \mathcal{Z}_t^C, t \in \mathcal{T} \setminus \{T\}, e \in \mathcal{E} \quad (8.1)$$

The second suggestion is to introduce a relaxed $v$ model, where the binary requirement on the $v$ variable is relaxed. This relaxation means no longer restricting the number of changes, but instead the amount of change. This allows smaller changes between the schools whilst still restricting the overall amount of changes allowed during the planning period. To obtain this, the variable $v$ is redefined as below. The relaxed $v$ model is also implemented with constraint (8.1).

$v_{ite}$ how much of zone $i$ that changes its allocated school in time period $t$ in scenario $e$

To assess the performance of the two relaxations, multiple performance criteria are studied. It is obvious that a positive effect on run time is required and that this effect must be weighed against the deviation of the objective value. However, the performance of the relaxed models is also affected by the variation in multiple measures regarding zone to school allocation. Significant increases in the number of school changes or the number of existing zone-to-school combinations are undesirable and must be weighed against the impact on computational time. Similarly, a considerable reduction in the number of entire zones allocated to the same school during the entire planning period, zone-to-one-school zones, is undesirable. Table 8.5 shows the objective values and measures for the base case and the suggested relaxation models.

**Table 8.5:** The results when the deterministic base case model is optimized with relaxations of the school change variable.

| | Integer solution | Best bound | Time [s] | RtS | | Cap | | Cond | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | Obj. | Avg.[m] | Obj. | Avg.[%] | Obj. | Avg. |
| Base case | 1.4563 | 1.2286 | Max | 1.19 | 807 | 4.01 | 89.7 | 2.00 | 1.51 |
| Minor change | 1.4563 | 1.2286 | Max | 1.19 | 807 | 4.01 | 89.7 | 2.00 | 1.51 |
| Relaxed $v$ | 1.3208 | 1.3208 | 1948 | 1.09 | 742 | 2.96 | 89.7 | 1.96 | 1.45 |

We see that the minor change relaxation has no detectable effect on the performance. However, the relaxation of the variable $v$ drastically reduces the run time. Not surprisingly, the solution found with the relaxed $v$ model gives a better solution for all the objectives than the base case. Even so, we see that the objective value of the relaxed $v$ model solution is within the gap of the base case model solution. This indicates that the relaxation still provides a solution that is somewhat similar to the solution of the base case problem.

To verify the value of the solution of the relaxed $v$ model, the found set of executed projects is re-implemented in the base case. The result of this re-implementation on the objective value is present in Table 8.6 and compared to the solution of the base case.

**Table 8.6:** The solution of the base case with fixed project execution from the relaxed $v$ testing.

| | Integer Solution | Best Bound | Time [s] | Rts | Cap | Con |
|---|---|---|---|---|---|---|
| Base case | 1.4563 | 1.2286 | Max | 1.19 | 4.01 | 2.00 |
| Fixed executed projects | 1.4191 | 1.3382 | Max | 1.17 | 3.73 | 1.96 |

From the results, it is clear that the re-implementation of the set of executed projects yields a similar solution to the solution of the base case itself. In fact, the fixed executed projects problem is able to find a better integer solution within the time limit. Furthermore, even though the best bound of the base case is lower, we know from the result of the relaxed $v$, that the solution of the base case must be equal to or higher than 1.3208 as that is the solution of the relaxed problem. Thus the re-implementation of the fixed executed projects confirms the validity of applying the relaxation of $v$.

However, before concluding on the implementation of the relaxation, it is interesting to study the effect on the zone-to-school variable, $w$. Firstly, the amount of zones that is allocated to the same school throughout the planning period, zone-to-one-school, gives a measure of how stable the zone-to-school allocation is. Further, the number of non-zero $w$ elements can tell us about the number of divided zones, where pupils in the zone attend different schools. A high number of divided zones is also a measure of instability, as these zones are the most likely to change allocation from one time period to the next. Table 8.7 provides some key figures for the relaxed $v$ model compared to the base case.

**Table 8.7:** The effect from relaxing $v$ on the zone-to-school allocation. The three rightward columns present how many zones that are divided and allocated to different schools in the base case and with the relaxed $v$.

| | Zones-to-one-school [%] | Number of zones [in %] where the amount of pupils attending the same school is: | | |
|---|---|---|---|---|
| | | 100% | >50% | <25% |
| Base case | 49.1 | 63.4 | 79.0 | 13.1 |
| Relaxed $v$ | 38.2 | 55.4 | 74.5 | 14.2 |

From the table, we see that the relaxation of the school change variable $v$ reduces the number of zones-to-one-school allocations. However, we see that in both approaches, the majority of the pupils are allocated to the same school in most of the zones. This indicates that the relaxation of $v$ only causes minor modifications to the number of non-zero $w$ variables. An implementation of a strict maximum road to school distance constraint can further offset this increase. This is discussed in Section 8.1.4. Consequently, the positive effects of the drastic reduction in computational time surmount the negative affects on $w$, and the relaxation of $v$ is implemented.

### 8.1.3 Maximum Allowed School Deterioration

To further reduce the solution space of the model and hence improve the computational time, a restriction to the maximum school deterioration is introduced. This is achieved by forcing schools to be renovated within five years of reaching condition stage 5. The rationale behind this extension is that the schools in stage 5 are in an unacceptable physical condition and measures must be taken within five years to improve the condition. This is pursuant to the law of a good physical learning environment, presented in Chapter 2 as well as Trondheim's prioritization of school renovations projects.

The maximum allowed deterioration is added as a new constraint, where state 5 is the highest maximum allowed state. In practice, this is obtained by restricting the coherent cost of condition as presented in Constraint (8.2). Moreover, the condition-matrix discussed in Chapter 7 is altered such that a school state changes to artificial state 6 after five years in state 5.

$$z_{sate} \leq 25 \qquad\qquad s \in \mathcal{S}, a \in \mathcal{A}_s, t \in \mathcal{T}, e \in \mathcal{E} \tag{8.2}$$

An outline of the results from tests with the added constraint is presented in Table 8.8.

**Table 8.8:** A comparison of tests with and without a maximum allowed state of school condition.

| | Objective | Time [s] | RtS | Cap | Cond |
|---|---|---|---|---|---|
| Relaxed $v$ | 1.3208 | 1948 | 1.09 | 2.96 | 1.96 |
| Restricted $z$ | 1.3208 | 674 | 1.09 | 2.96 | 1.96 |

From the table, we observe that the model with the implemented new school deterioration constraint provides equal results to those without the cut, but with a 65% decrease in computation time. This indicates that the only effect from the restriction is a feasible cut in the solution space. Thus, the restricted value of $z$ can beneficially be implemented into the model, without deteriorating the quality of the solutions.

### 8.1.4 Road To School Cuts

The zone-to-school variable, $w$, is by far the variable with most elements. The complexity and size of the model are significantly influenced by the 4,329 zone-to-school combinations that exist in each time period and each scenario. This far exceeds the number of realistic combinations in the real-world situation.

As discussed in Chapter 2, Norwegian law declares strict regulations considering the distance between a zone and its allocated school. Thus, to prevent the model from spending unnecessary time on obviously unrealistic zone-to-school combinations, three types of maximum distance cuts are proposed. The three types are:

1. **Equal cuts**: All zones are restricted to the same maximum distance (i.e. 3500 meters).
2. **Shortest distance based cuts**: Zones are restricted based on their distance to the closest initially existing school.
   - **Extra meters**: An additional length is allowed (i.e. shortest distance + 500 meters).
   - **Multiplier**: An additional percentage is allowed (i.e. shortest distance $\cdot$ 1.5).
3. **Closest schools cuts**: Zones are restricted to go to one of its nearest schools.
   - **Two closest**: The zone must be allocated to its nearest or second-to-nearest initially existing school.
   - **Three closest**: The zone must be allocated to one of its three nearest initially existing schools.

The simplest type of cuts are the equal cuts, where the same maximum distance is allowed for each zone. These are easy to implement and cut the worst zone-to-school combinations, with the longest distances. To somewhat adjust for and exploit the location of the zone, the shortest distance based cuts are implemented. These give strict cuts for the urban zones that have several schools close by. At the same time, the cut allows for some flexibility for the rural zones located further away from its nearest schools. The extra meter cut ensures this by allowing a certain extra distance, but this distance is independent of the urbanity of the zones. In comparison, the extra distance allowed in the multiplier cut is determined by the distance of the closest school. This allows for individual zone differences.

Furthermore, the closest school cut is completely customized for each zone, as the maximum distance is solely based on that zone's distances to the schools. The shortest distance here means the shortest distance to an initially existing school. Likewise, the second closest and third closest schools are given by the initially existing schools. This means that if a new school is built closer than the second-to-nearest or third nearest school respectively, zones can potentially be allocated to one additional school. Table 8.9 displays an example of the value of the maximum allowed distance for the different cuts.

**Table 8.9:** An example of distance allowed in a specific urban zone and a rural zone, as a function of the closest school. The values are given in meters.

| | Closest | Equal | Shortest distance based | | Closest school | |
|---|---|---|---|---|---|---|
| | | | Extra meters (500m) | Multiplier (1.5) | Two closest | Three closest |
| Urban zone | 267 | 3500 | 767 | 401 | 801 | 907 |
| Rural zone | 2635 | 3500 | 3135 | 3953 | 3571 | 3804 |

All three types of cuts are tested and the complete calculations can be found in Appendix C. The multiplier cut, with an additional allowed distance of 50%, as well as the two closest and three closest cuts, provided the most promising results considering the computational time and objective values. The effect of these implementations is studied further. Table 8.10 outlines the objective values and measures of the model when implementing the cuts.

**Table 8.10:** Results from implementing a restriction of maximum zone-to-school distance.

| | Obj. value | Time [s] | RtS | | Cap | | Cond | |
|---|---|---|---|---|---|---|---|---|
| | | | Obj. | Avg. [m] | Obj. | Avg. [%] | Obj. | Avg. |
| Restricted $z$ | 1.3208 | 674 | 1.09 | 741 | 2.97 | 89.7 | 1.96 | 1.45 |
| Multiplier (1.5) | 2.7591 | 40 | 1.06 | 718 | 31.8 | 89.6 | 2.11 | 1.50 |
| Two closest | 1.8391 | 51 | 1.09 | 737 | 13.1 | 89.3 | 2.06 | 1.51 |
| Three closest | 1.3572 | 135 | 1.12 | 757 | 3.33 | 89.7 | 1.96 | 1.45 |

The table shows that implementing either of the three proposed cuts significantly improves the computational time, especially the multiplier and the two closest cuts. However, both of the corresponding solutions to these cuts yield significantly deteriorated objective values. Furthermore, we see that the increased objective value can be traced to the capacity utilization objective. Before deciding on whether to include either of the three proposed cuts, a closer study of the capacity utilization is needed. In Table 8.11, a selection of the studied schools are presented with coherent capacity utilization.

In the multiplier test, we see an unwanted result where the capacity is unevenly distributed amongst the schools. This is especially prominent at Okstad skole where the capacity utilization is at 61% and at Sjetne skole where it is 96% utilization. Moreover, the

**Table 8.11:** The capacity utilization of a selection of schools. All the values are given in %.

| School | Restricted $z$ | Multiplier (1.5) | Two Closest | Three Closest |
|---|---|---|---|---|
| Huseby skole | 89 | 95 | 89 | 88 |
| Hårstad skole | 90 | 94 | 91 | 90 |
| Okstad skole | 90 | 61 | 90 | 90 |
| Sjetne skole | 91 | 96 | 94 | 91 |

implementation of the two and three closest school cuts results in some deviations from the desired 90% utilization, but to a lesser degree.

The introduction of road to school cuts are expected to affect the allocation of zones to school, and thus the number of zones-to-one-school and divided zones. Once again, it is relevant to study the zone-to-school variable $w$. Table 8.12 displays the impact on the variable from the three cuts.

**Table 8.12:** The effect from implementing maximum distance cuts on the zone-to-school allocation.

| | Zones-to-one-school [%] | Number of zones [in %] where the amount of pupils attending the same school is: | | |
|---|---|---|---|---|
| | | 100% | >50% | <25% |
| Restricted $z$ | 38.2 | 55.2 | 74.5 | 14.4 |
| Multiplier (1.5) | 47.6 | 57.6 | 74.9 | 13.1 |
| Two closest | 38.2 | 64.4 | 81.9 | 10.6 |
| Three closest | 27.4 | 57.7 | 76.5 | 12.3 |

It is clear that the implementation of the multiplier cut increases the number of zones-to-one-school zones. However, we see that the number of zones in which all the pupils attend the same school only increases slightly. This indicates that the multiplier cut does not seem to considerably decrease the number of divided zones. The two closest cut does not increase the number of unchanged zones, but instead, we see a significant positive effect in the number of undivided zones. Lastly, the three closest cut does neither improve the amount of zones-to-one-school cuts or the number of undivided zones substantially.

The improvement in computational time, and the reduction in the number of divided zones, incentivize the implementation of the two closest schools cut. The multiplier cut leads to an undesirably large increase in the objective value. The objective value increase of the second closest cut can be viewed as only a mathematical deterioration, as the increase can be justified by practical considerations and obligations stated in Norwegian Law. Even though the negative effect on the objective value is considerably smaller in the three closest school cut compared to the two other, the comparatively high computational time, and the lack of improvement in the zone-to-school variable, reduces the value of the three closest cut.

### 8.1.5 Final Deterministic Model

The final deterministic model denotes the deterministic model with applied extensions and relaxations as discussed in this section. The forthcoming studies are based on the stochastic equivalent of the final deterministic model. A comparison of the final deterministic model and the base case is presented in Table 8.13.

**Table 8.13:** An overview of the building blocks that the base case and final model consists of.

| | Base Case | Final Model |
|---|---|---|
| (5.1) | X | X |
| (5.3) - (5.34) | X | X |
| (7.4) - (7.5) | X | X |
| Divisors | X | X |
| Relaxation of $v$ | - | X |
| Maximum $z$ constraint | - | X |
| Two closest schools cut | - | X |

## 8.2 Comparison of the Solution Methods

In this section, the computational performance of the proposed solution methods to the SPPA are tested and compared. The EOSBB and AEOSBB algorithms are compared with the Complete Computation method (CC). The CC method solves the problem by a direct implementation of the model as presented in Chapter 5, with the extensions presented in Section 8.1.

As discussed in Chapter 6, the two proposed algorithms are exact solution methods. Therefore, all three methods yield the same objective values when tested on the same data set. The results show that the EOSBB and AEOSBB algorithms provide the same solutions as the CC method in instances that the CC solves to proven optimality.

To evaluate the performance of the three solution methods, they are each tested on several scenario instances with varying sizes. The aim is to obtain insight into each method's scaleability, i.e. the method's ability to solve larger data instances. Figure 8.1 shows the computational time of different scenarios sizes and is given as the average value of five instances of each size. In this test, the maximum computation time is set to 25,000 seconds.

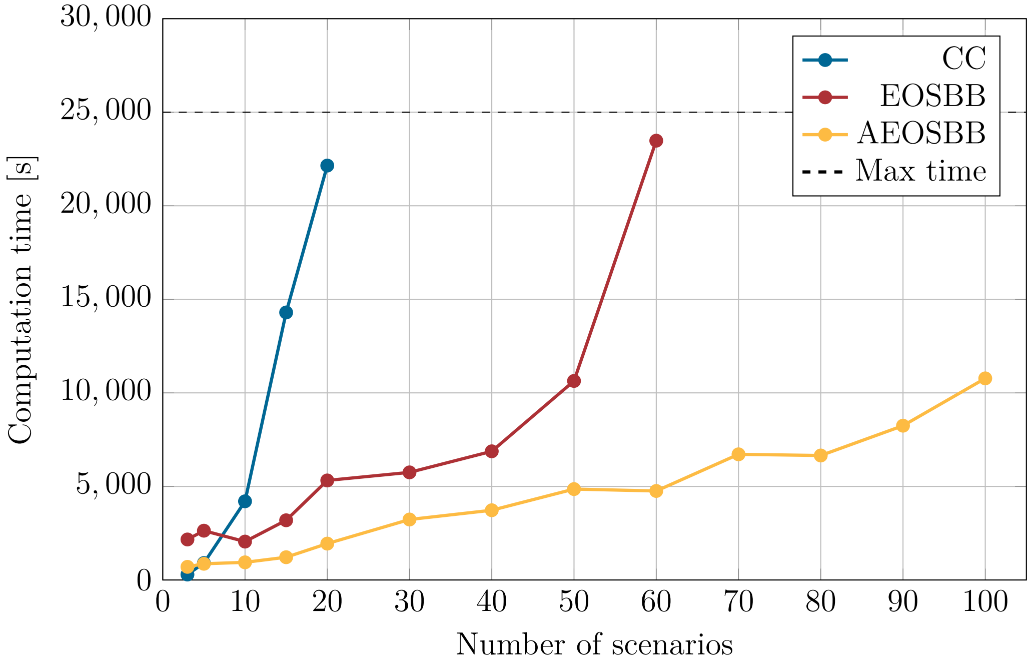



**Figure 8.1:** The average computational time is plotted against the number of scenarios for the CC, EOSBB and AEOSBB.

From the figure, it is clear that when the number of scenarios in the instances increases, the computational performance of the CC method significantly deteriorates. When the method is tested on instances of more than 20 scenarios, the computational time exceeds the given time limit.

The plot clearly illustrates that both of the suggested algorithms handle the increase in the number of scenarios decisively better than the CC method. Thus, both solution algorithms could beneficially be implemented as they have significantly higher scalability. Furthermore, we see that the AEOSBB algorithm manages to solve large instances decisively faster than the EOSBB. For instance, the AEOSBB algorithm solves the fifty scenarios instances on average in half the time of the EOSBB, and the computational time of the EOSBB continues to rapidly increase after this. Consequently, we can conclude that the reduction of the MSP solution space, as discussed in Chapter 6, is decisive for the computational performance.

As the AEOSBB algorithm provides optimal solutions in the shortest time, this is the preferred algorithm for solving the SPPA on real-size data. Thus, the solution algorithm is implemented in further studies.

## 8.3 Sample Stability Testing

In this section, the stability of the model and scenario generation process is tested. The concept of testing stability relates to determining that when we test our optimization model, the results are not just dependent on the scenario generation procedure, but on the properties of the model itself (King and Wallace, 2012). This section aims to verify the utilized scenario generation process of the model and thereafter decide on an appropriate scenario size of the instances used in later studies of the model. All theory presented in this section is from King and Wallace (2012).

Each scenario in this thesis is generated based on discreatization of random variables. The stability tests verify that the optimization model is not subject to systematic errors from poor discretization. There are two main types of stability tests: in-sample and out-of-sample. In-sample stability is a measure of the internal consistency of the model. That is, a test of the scenario generation process itself. On the other hand, out-of-sample tests are extended by testing the quality of the model as well. In stochastic programming, many different solutions can provide the same objective value. Thus, stability is measured by similarities in the objective values, not in the solutions.

### 8.3.1 In-sample Stability

The scenario generation process produces different scenario instances based on the same data. When testing in-sample stability, the objective values of the optimal solutions of different scenario instances are compared. If these objective values are approximately the same for all instances, we say that we have in-sample stability. In short, if we have perfect in-sample stability, the objective value is independent of the used instance.

To find the required scenario-size that provides in-sample stability for the SPPA, we solve the model for five different instances of each of the scenario-sizes. The average, minimum, and maximum found objective value of each size is presented in Figure 8.2. The figure clearly shows that an increase in scenario-size reduces the variation in the objective value of the instances. The improvement in stability is prominent, especially up to 20 scenarios.

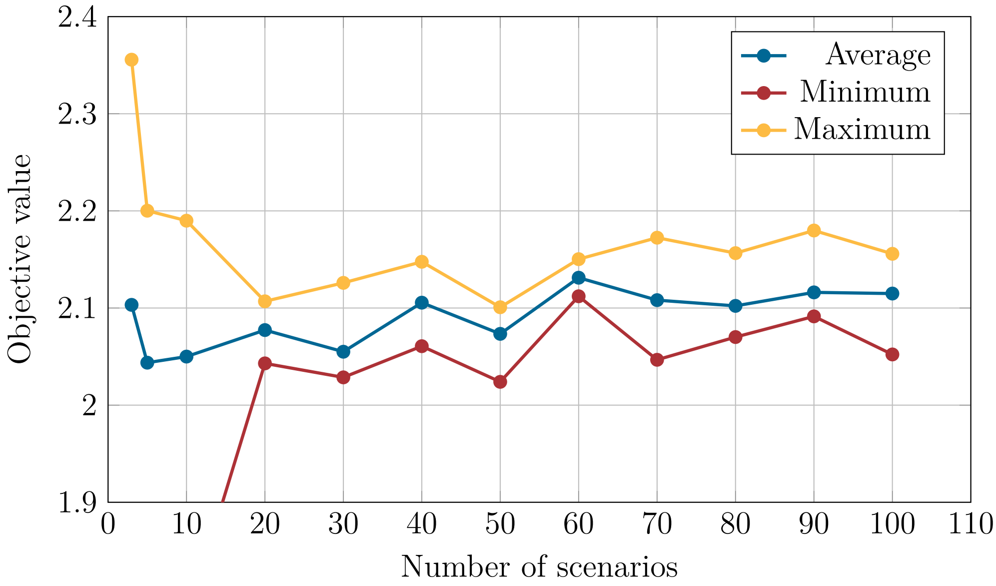



**Figure 8.2:** In-sample stability tests of the SPPA

The standard deviation, $\sigma^2$, and coefficient of variance, CV, from the tests are presented in Appendix D. The CV values determine the relative deviation in the tests' results and can thus be an indicator of in-sample stability. As we can see from the results, the CV value is generally low, especially when the scenario-size exceeds 20 as the, CV value stabilizes at below 2%. To further decide on a reasonable scenario-size, a comparison of the values for the 5, 20, and 50 scenario instances are presented in Figure 8.3.

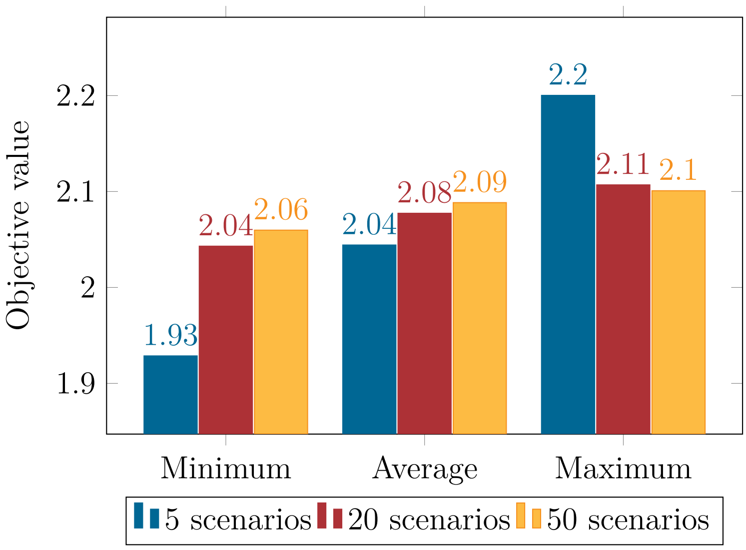



**Figure 8.3:** The minimum, average and maximum objective value from five runs of for the 5, 20, and 50 scenario instances.

From the figure, we see a significant improvement from the 5 scenario instances to the 20 scenarios instances. However, the improvement from 20 to 50 is marginal. For computation performance reasons, it is desirable to keep the number of scenarios as low as possible. Thus 20 is a reasonable number of scenarios as it provides sufficient in-sample stability.

### 8.3.2 Out-of-sample Stability

In out-of-sample stability tests, we want to test if the solutions of the instances yield the approximately same true objective value. By the true objective value, we mean the objective value of the true problem, in which all possible scenarios are taken into account. Out-of-sample stability testing reveals if there is false stability in the scenario generation procedure, as it takes the worst possible outcomes into account.

In practice, it is impossible to solve the true problem. Instead, the out-of-sample tests are performed on an objective function with 1,000 scenarios, and with fixed first stage variables, to simulate the corresponding true objective value. In the SPPA, the execution of school projects, variable $x$, is considered the first stage variable. The first stage variable of the simulated true objective function is fixed as the first stage solution of each of the scenario instances. The standard deviation of the objective values of each scenario size then provides a measure of the out-of-sample stability.

The out-of-sample stability is tested for 5 scenario instances of sizes ranging from 3 to 100. The decisions of which and when projects are executed in every instance, the $x$-variable, are fixed as the first stage variable in the simulated true objective function. The standard deviation of the true objective values for each instance size is presented in Figure 8.4.

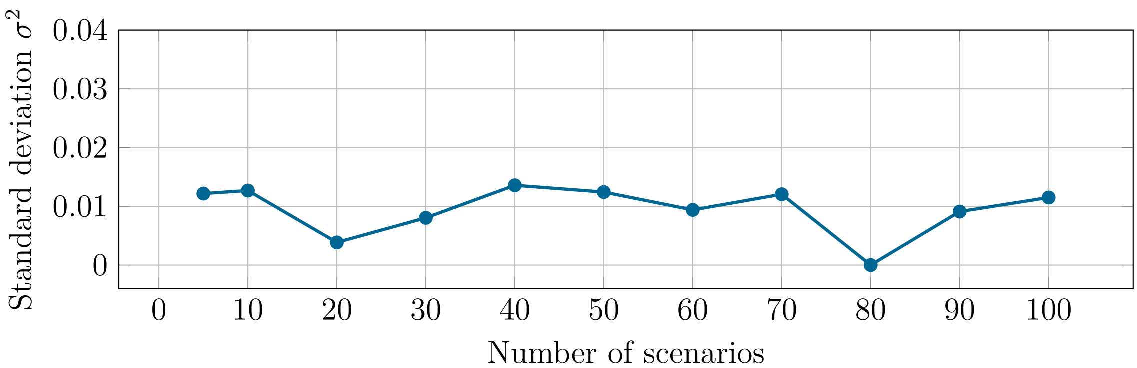



**Figure 8.4:** The standard deviation $\sigma^2$ from the simulated true objective function solved with corresponding first stage variables.

The true objective value of the three scenario instances did not yield feasible solutions to the true problem. However, the figure shows that even from the five scenario-size, the standard deviation of the true objective value is low. Increasing the scenario-size shows little effect on the standard deviation value. This implies that we have out-of-sample stability even with a relatively small number of scenarios.

### 8.3.3 Conclusion From Stability Testing

The results from in-sample and out-of-sample stability demonstrate stability in the model and the scenario generation process. However, the lack of variations in the true objective values also indicates that there is a somewhat restricted solution space. There seems to be a limited set of first stage variable solutions that yields feasible solutions to all the scenarios. While the tests on out-of-sample stability show stable results for both low and high numbers of scenarios, the in-sample tests imply stability as the number of scenarios escalates past 20. Consequently, we suggest that 20 scenarios are sufficient to obtain stability.

## 8.4 The Value of Information

In this section, the value of information about the uncertain future is studied. First, the value of considering uncertainty is evaluated. Second, the value of having perfect information about the future is assessed. The theory used in this section is rendered from Birge and Louveaux (2011).

A fundamental difference between a stochastic and a deterministic model is the consideration of uncertainty. This allows the stochastic model to take in more accurate information about the future. A measure that quantifies this value is the Value of the Stochastic Solution (VSS). The VSS is found by accessing the solutions found by the DM, in a stochastic setting. VSS for a minimization problem is given by Equation 8.3.

$$VSS = EEV - SP \tag{8.3}$$

Here, EEV denotes the expected result of using the expected value of the random variables. The idea is to find the expected objective value when the first stage variables are fixed to the $x$ variables of the deterministic problem solution. Furthermore, the SP is the objective value found by the stochastic model. A high VSS value indicates that the solution of the stochastic model is more suited to handle the possible future realizations. As the DM consistently avoids the edges of the distribution, it does not take extremes into account. The VSS is thus expected to be positive as the value of EEV is dependent on individual scenarios.

When solving the SPPA with the first stage solutions of the final model, the model is incapable of finding feasible solutions. In theory, this means that the VSS is $\infty$, as the stochastic model actually provides feasible solutions. However, in practical usage, infeasible solutions in which pupils are not allocated to a school are never allowed and unlimited resources would be used to avoid this. Consequently, the deterministic approach in a real-world setting is feasible, but often very expensive and inefficient. It is therefore cumbersome to find an actual value of the VSS. Nevertheless, the tests indicate that the stochastic solution is valuable as it addresses the worst-case scenarios and avoids comprehensive measures if the extremes occur.

In addition to the VSS, we want to find the Expected Value of Perfect Information (EVPI). This measurement quantifies the willingness to pay for obtaining information about future states. The EVPI is found by Equation (8.4).

$$EVPI = SP - WS \tag{8.4}$$

Here, the WS is the solution of the wait-and-see approach. In this approach, the optimal solution of each scenario realization is found and the average objective value is calculated. Five instances of the SPPA are tested, and the EVPI, as well as the SP and WS values, of each of the five test instances, are found. The results are presented in Table 8.14.

| Instance | SP | WS | EVPI |
|---|---|---|---|
| 1 | 2.078 | 1.905 | 0.174 |
| 2 | 2.107 | 1.951 | 0.156 |
| 3 | 2.105 | 1.968 | 0.138 |
| 4 | 2.060 | 1.886 | 0.174 |
| 5 | 2.104 | 1.935 | 0.169 |
| Average | 2.091 | 1.929 | 0.162 |

**Table 8.14:** The results from EVPI tests with coherent results from stochastic model tests and the wait-and-see approach.

As the table shows, the EVPI is generally low in all the test instances. The average objective value with access to perfect information only improves by 7.75% compared with the stochastic solution. In practice, this means that Trondheim should avoid costly measures to obtain accurate information on future population development.

## 8.5 Non-Anticipativity Constraint Alterations

The non-anticipativity constraint (NAC), (5.28), ensures that all projects must be executed in the same order and at the same time in all the scenarios. In practice, this is the equivalent of Trondheim deciding today on the projects to execute for the entire 15-year planning period. This rejects the possibility of altering the plan to the realization of the population development. By relaxing the NAC, we allow for alterations dependent on the future population. In this section we study the implementation of the following NAC relaxations:

- **Same Order:** The projects are executed in the same order, but not at the same time in all scenarios. For each individual scenario, it is not required that all projects in the order must be completed. Furthermore, no additional projects can be executed.

- **First Four:** The first four projects are executed in the same order and at the same time in all scenarios. The execution of all later projects is dependent on each scenario.

In the Same Order alteration, individual adjustments are allowed, while the long-term perspective of the planning is maintained. For Trondheim, this means the possibility to decide on a common building order, that is somewhat adjustable to the actual population realization. By this, we allow for the execution of more projects in scenarios with high overall population growth and the opposite in the scenarios with low growth. Consequently, we hope to alter the model to provide a better capacity-to-population fit in the later time periods.

The First Four relaxation, is designed to illustrate a realistic decision process for Trondheim. Here, Trondheim makes a decision only for the first few years, and later decisions are made as the actual population development is known. In this relaxation, only the first four projects are locked, which allows Trondheim to make scenario-specific decisions in the later years.

The implementation of the alternative NAC formulations is expected to yield lower objective values as well as significantly influence what school projects are executed. In practice, solving the Same Order alteration, with the proposed solution algorithm, eliminates the need for solving the MSP. Instead, single scenario problems must be solved until the search depth is equal to the maximum number of executed projects. That is, the number of executed projects in the scenario in which most projects are executed. Thus, the Same Order alteration is expected to somewhat reduce the computational time of the SPPA. The First Four alterations on the other hand are implemented by running the MSP each time four schools have been locked in to an order. Thus, the execution order specific branch and bound tree will be significantly reduced. However, the MSP is significantly less restricted, and thus the run time of the First Four is expected to increase as restricting the MSP has been proved key to reduce computational time. Table 8.15 displays the effect on the objective value of the two proposed NAC relaxations, in addition to the average number of executed projects. The original formulation is also presented to compare the three NAC variations.

**Table 8.15:** Objective value and the average number of executed projects for each instance of twenty scenarios.

| Instance | Original | | Same Order | | First Four | |
|---|---|---|---|---|---|---|
| | Obj. Value | # Projects | Obj. Value | # Projects | Obj. Value | # Projects |
| 1 | 2.0430 | 9 | 2.0008 | 7.6 | 1.9525 | 8.3 |
| 2 | 2.1069 | 9 | 2.0773 | 7.8 | 2.0437 | 8.5 |
| 3 | 2.1053 | 9 | 2.0752 | 8.15 | 2.0375 | 8.4 |
| 4 | 2.0599 | 9 | 2.0236 | 7.9 | 1.9834 | 8.25 |
| 5 | 2.0713 | 9 | 2.0366 | 8.1 | 1.9886 | 8.3 |
| Average | 2.0773 | 9 | 2.0427 | 7.91 | 2.0011 | 8.35 |

The fact that both relaxations improve the objective value of all the instances indicates that Trondheim can profit from adapting their decisions to the scenarios. Furthermore, we can see from the table that the First Four relaxation yields the best objective values. This shows that by freeing the common building order, more beneficial decisions can be made.

Moreover, it is interesting to study the correlation, or lack thereof, between the objective value and the average number of executed projects. Initially, from Table 8.15, there does not seem to be any clear correlation between them. The Same Order relaxation has a lower number of executed projects than the two other variations, but not the best objective value. Likewise, the original formulation yielded the highest number of executed projects, but also the worst objective value. It is therefore interesting to take a deeper look into the stature of the executed projects. As the capacity utilization objective penalizes unwanted capacity utilization in both directions, it is especially important to study the executed projects with regards to increased capacity. Table 8.16 shows the average increased capacity in the area throughout the time horizon.

We see from the table that the First Four alteration on average results in a smaller capacity increase than both the Same Order relaxation and the original formulation. This indicates that adding the extra flexibility of the First Four alteration allows for a better fit between the realized actual demand and supply of school capacity. Thus, there seems to be a correlation between the total increased capacity from the executed projects and the objective value, not the number of executed projects in itself.

Furthermore, we study the two alterations' effect on each of the three objectives, to find

**Table 8.16:** The average increased capacity in the area when considering the original formulation, Same Order and First Four as NAC.

| Instance | Increased built capacity | | |
|---|---|---|---|
| | Orginal | Same Order | First Four |
| 1 | 1000 | 884 | 863 |
| 2 | 1000 | 920 | 912 |
| 3 | 1000 | 943 | 923 |
| 4 | 1000 | 925 | 900 |
| 5 | 1000 | 888 | 883 |
| Average | 1000 | 912 | 896 |

where the positive effect can be traced. In Table 8.17 the objective value is presented for each of the three objectives.

**Table 8.17:** The three objective terms in each of the NAC tests.

| Objective | Original | Same Order | First Four |
|---|---|---|---|
| RtS | 1.08 | 1.08 | 1.08 |
| Cap | 17.83 | 17.12 | 16.32 |
| Cond | 2.06 | 2.14 | 2.07 |

We see clearly that the improvement of the two alterations can be traced to a reduction in the capacity objective. This emphasizes the value of flexibility, where the available capacity can be altered to the realized scenario. Furthermore, the road to school objective remains unchanged for all the instances and scenarios. This indicates that the alterations do not noteworthy change the zone-to-school allocation. However, as the NAC is relaxed the condition objective slightly deteriorates.

At first glance, it is not obvious why the Same Order alteration has this negative effect on the condition objective. However, a deeper look into the execution orders in question shows that the last project in the order of all the instances is a significant renovation project. Moreover, to fulfill the order restriction, a significant capacity expansion project must be conducted before the renovation project. This leads to significant over-capacity and therefore the renovation project is not favorable to execute. As a result, it is never conducted exactly eight projects in the Same Order alteration, even though both seven and nine projects are conducted numerous times.

It is important to point out that the first four projects are conducted at the same time in both of the two NAC variants, and in the original formulation. Furthermore, it is important to note that this order is different from the order of the deterministic solution. This indicates that the original model is useful for Trondheim even though they do not make all the decisions for the 15-year planning period today. It is therefore worth studying the alterations affect on the computational time of the model, presented in Table 8.18.

**Table 8.18:** The computational time of each of the NAC tests.

| | Original | Same Order | First Four |
|---|---|---|---|
| Avg. Time [s] | 1,944 | 1,903 | 12,435 |

We see that the original formulation and the Same Order average computational time are approximately equal. This shows that the solving of the additional SSPs takes approximately that same time as the solving of MSPs of the original formulation. The First Four objective value, however, is significantly deteriorating the computational time. This means that the additional time it takes to solve the less restricted MSP is far greater than the time saved on reduced tree generation.

As both of the alterations yields the execution of the same first four projects as original formulation, the First Four does not add sufficient additional value to weigh up for the extra computational time. The original formulation and the Same Order variations are both providing useful insight for Trondheim, but as the original formulation gives a common result for all the scenarios it will be used in the studies in Chapter 9. Do note, that the lack of variation in the first four executed projects can indicate that the solution of the SPPA is guided by the set of defined projects used as input data. It is possible that these first four projects are forced to be executed to provide feasible solutions.

# 9 Practical Usage of the SPPA

The overall idea of this thesis is to create a model and solution method that can aid municipalities like Trondheim in school development planning. Therefore, this chapter provides an analysis of the model's practical impact on Trondheim. As previously mentioned in Chapter 7, the data used as input to the model is from an area in southern Trondheim. In this chapter, we thoroughly study the impact of the solutions found in Chapter 8, in this area. The results presented, represent the optimal solution found by solving the SPPA to optimality with one 20-scenario instance. To add value to the results, it is meaningful to involve the decision-makers in the evaluation of the results. Therefore, the results' value and functionality are discussed in meetings with Trondheim.

First, Section 9.1 addresses the executed projects. Then, Section 9.2 presents a sensitivity analysis on the budget parameter. Next, Section 9.3 discusses the location of a new school and Section 9.4 address the divided zones. Lastly, Section 9.5 presents the new school districts.

## 9.1 Executed Projects

As presented in Chapter 2, one of the main challenges for Trondheim is to decide on what projects to execute and when to execute them. Therefore, one of the key findings from the SPPA is the set of completed projects with the corresponding year of completion. An overview of these projects is provided in Figure 9.1.

The figure presents the changeable and possible new schools along the planning horizon. Out of the two new schools, only Lundåsen is built. The construction of this new school is the first executed project and is completed in year 4. The situation at Lundåsen is studied further in Section 9.3. The first initially existing school to be upgraded is Rosten, with a capacity expansion completed in time period 5. Also Sjetne and Kattem are each subject to a capacity project during the planning period. Stabbursmoen, Åsheim, and Romolslia are all renovated while Tonstad and Breidablikk are expanded as well as renovated.

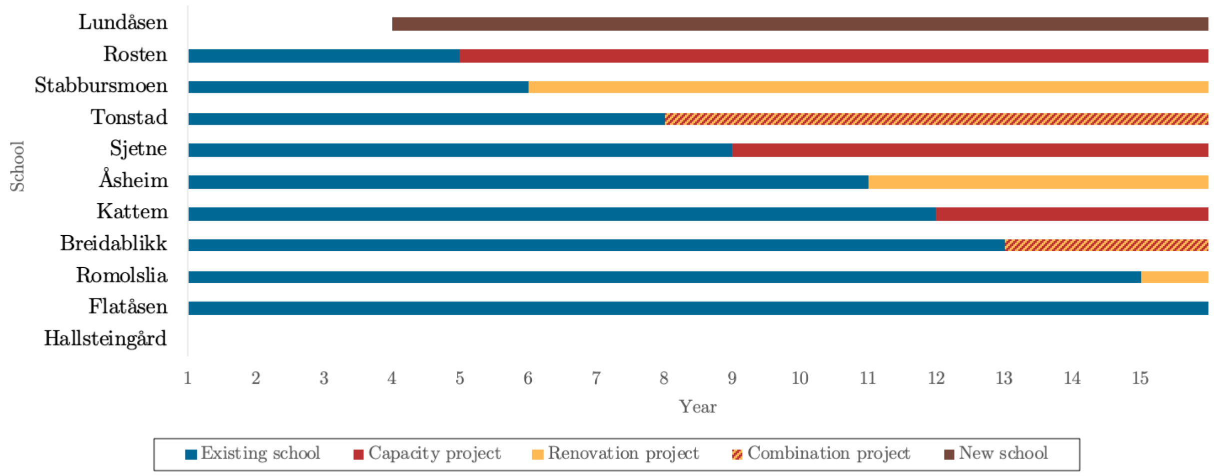



**Figure 9.1:** Timeline of when the changeable and possible new schools exists and when projects are executed. A change in color represents a completion of a project.

It is worth noticing that all but one school with a potential upgrade undergoes a change throughout the time horizon. This underlines the need for investing in schools in this area of Trondheim. Furthermore, we see that there is an even distribution of the project type in the completed projects. Thus, there are no obvious trends in the types of conducted projects.

For some of the schools, there exist several projects with different capacity expansion sizes. It is therefore interesting to examine the characteristics of the prioritized projects of these schools to see if there any clear trends of preferred size. Table 9.1 presents the selected project for the schools with several possible size alternatives.

**Table 9.1:** The chosen initialized project at schools with several possible size alternatives.

| School | Type of project | Selected alt. |
|---|---|---|
| Lundåsen | New school | Small |
| Sjetne | Capacity project | Small |
| Kattem | Capacity project | Small |

As the table shows, there is a clear tendency of choosing the smallest capacity alternative. Thus, with model parameters as described in this study, the results indicate that Trondheim should execute the projects with the least capacity expansion. However, it is also possible that the economy of scale principle is underestimated in the data instance, causing smaller projects to be more favorable.

## 9.2 Sensitivity Analysis of the Budget Parameter

This section studies how changes in the budget parameter affect the solution of the SPPA. As the budget in municipalities can vary each year, it is interesting to study how an increase or decrease in the budget affects the project prioritization and objective terms. The aim is to enlighten Trondheim on the causality between the value of the parameter and the results.

We introduce a budget weight as a factor describing the deviation from the expected yearly budget presented in Chapter 7. The minimum budget weight is the lowest value that still provides feasible solutions for all instances. This is found, by preliminary tests, to be 0.875. The value is then incrementally increased until the budget is doubled. Finally, the weight is set to 100 with the aim to simulate a no-budget situation. In each iteration, the objective value and executed projects are registered. Five instances are tested for each incremental budget weight.

### 9.2.1 Impact on Objective Values

It is interesting to observe how alterations in the budget affect the overall value of the found solution. The average objective values of each budget weight are presented in Appendix (E). In Figure 9.2, the average objective function is plotted as a function of the budget weight values.

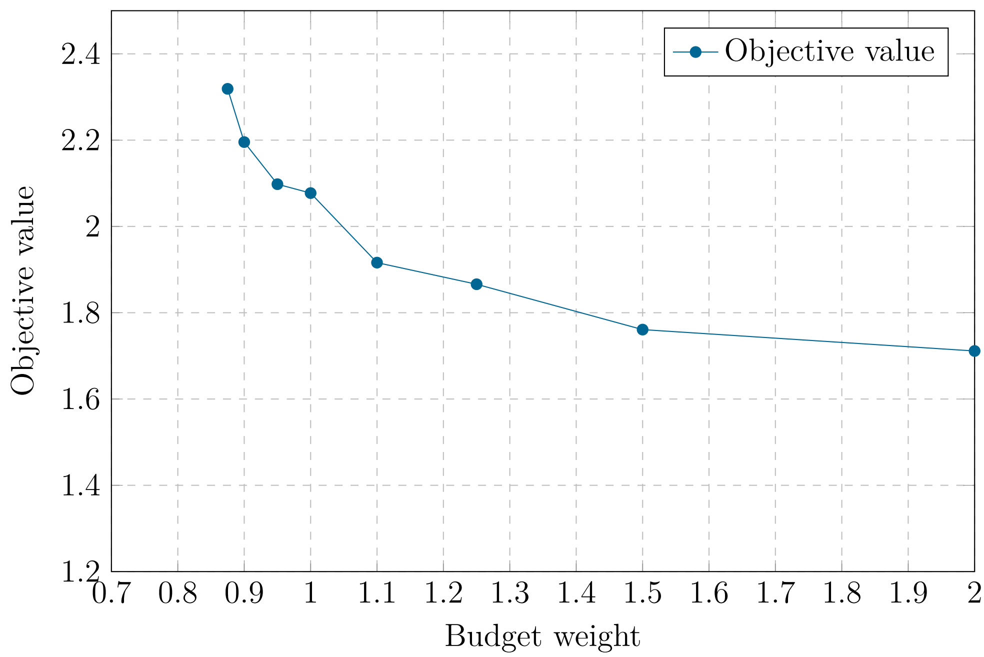



**Figure 9.2:** The objective value dependent on different budget weights

As we can see from the figure, even small reductions in the budget substantially deteriorate the objective value. Likewise, we see that the objective value improves considerably with the first incremental increases in the original budget. Budget increases up to 1.5 have clear positive effects but after this point the development curbs and flattens out. From the appendix, we see that even with the simulated, non-existing budget, the objective value is the same as when the budget is doubled. This shows that further increases in the budget have a marginal effect on the objective value as the budget grows.

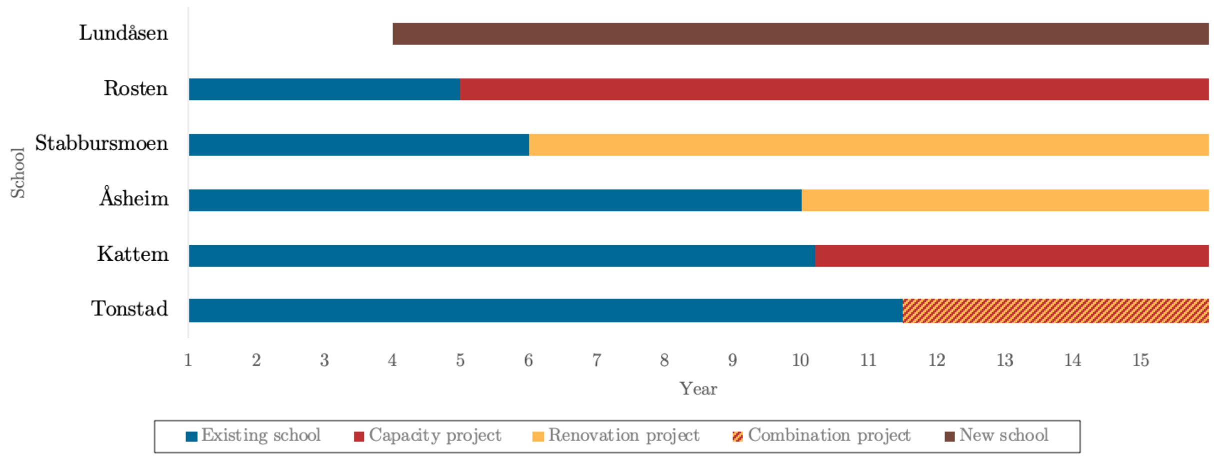
To understand the practical consequence of the budget alterations, a study of the three objective terms is conducted. From the test results in Appendix (E), we notice that as the budget grows, the improvements in the objective value seem to be linked to more optimal capacity utilization. At the same time, the condition objective also improves with the increasing budget. When the budget is doubled, the cost of capacity is reduced by 25% and the reduction in the condition cost is almost 50%. A further discussion of the budget's impact on the executed projects is provided in Section 9.2.2. The road to school objective, however, is only negligibly affected. This indicates that the zone-to-school allocation provided by today's budget is similar to what the allocation would be like with higher budgets.

Likewise, reductions in the budget highly affect the capacity utilization and the average condition of the school. The cost from capacity utilization increases by 23% when the original budget is reduced to the minimum and the condition cost is only affected by 8%. Hence, altering the budget seems to affect the capacity utilization the most.

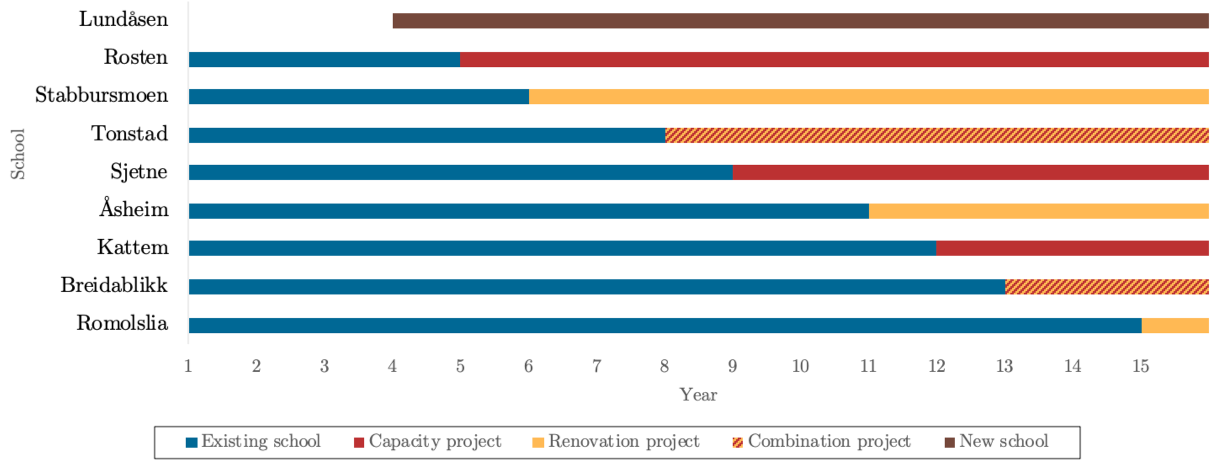
### 9.2.2 Impact on Project Prioritization

To elaborate on the effect of relaxing and restraining the budget parameter, we study how the model prioritizes projects differently when the budget changes. By studying the restrained budget solution, we get an understanding of which projects that must be executed to handle the future population. Likewise, the relaxed budget solution provides insight into the optimal set of projects when the budget is less of a factor. To obtain this, the two extreme situations with budget weights of 0.875 and 2 are considered and compared to the solution of the original budget.

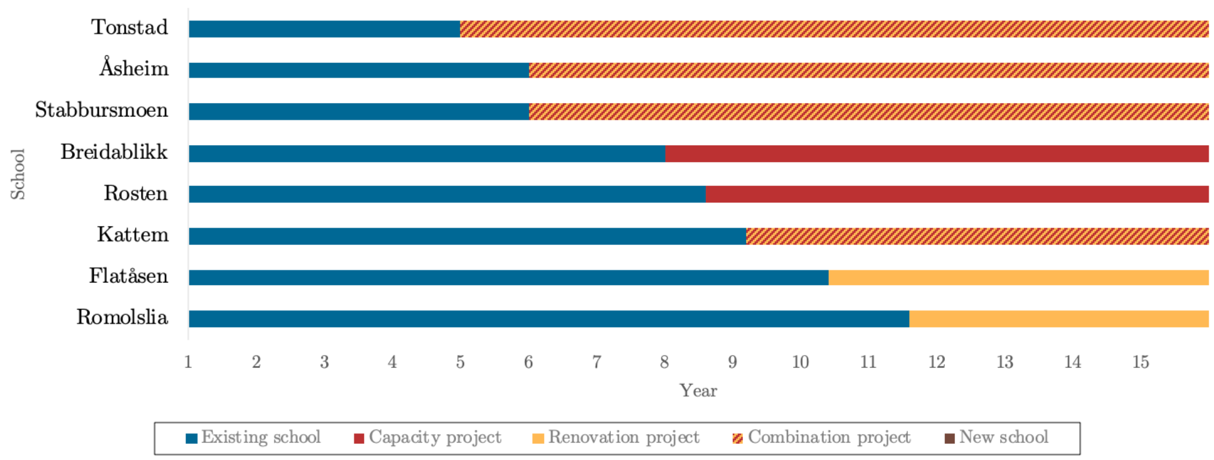
To understand the prioritization mechanism of the SPPA, we study the set of projects that are executed in multiple instances. It is expected that the most important projects are executed in all instances, as these projects handle the realization of most scenarios. Therefore, the projects that are executed in all of the studied instances are the ones considered in the comparison of the budget parameter. The timeline of the average year of completion for these projects is presented in Figure 9.3.




**(a)** Executed projects with 0.875× budget.




**(b)** Executed projects with 1× budget.




**(c)** Executed projects with 2× budget.

**Figure 9.3:** Illustration of executed projects with different budget parameters over the planning horizon. The year of completion is the average year over all the tested scenarios. Do note that only the changeable and new schools with completed projects in all the instances are presented.

When the budget weight is only 0.875, the utmost important projects are prioritized and executed. By comparing Figure 9.3a with the original solution presented in Figure 9.3b, we see that the same three projects are completed first. This underlines the urgency and importance of these three projects. One interesting change is the alternative that is prioritized at Kattem. As presented in Table 9.1, the smallest alternative is preferred with the original budget. However, with the reduced budget, the medium capacity alternative is chosen, hence, doubling the capacity expansion at Kattem. This is caused by the restricted budget limiting the number of projects that it is possible to execute. As a consequence, the capacity expansion project at Breidablikk is no longer executed, causing an increased capacity demand at Kattem. Thus, the model generally prioritizes fewer, but bigger capacity projects over many small ones, when the budget is restricted.

Figure 9.3c illustrates the executed projects with the doubled original budget. This figure presents several changes from the original budget. The first major difference is the absence of Lundåsen. As Lundåsen is executed in all instances with the smaller budget, it is surprising that the school is only built in 3 out of 5 scenarios with the enlarged budget. Instead of the new school, Åsheim and Stabbursmoen are prioritized with both capacity and renovation projects. This capacity expansion probably enables these two schools to handle the increasing demand in the area that Lundåsen would accommodate if built. This indicates that the construction of Lundåsen may not be optimal with an infinite budget, but is the preferred solution when the monetary resources are restricted.

Another clear observation is that the amount of executed combination projects is significantly higher with the doubled budget. This is natural as an increase in renovation solely improves the objective function. However, as excess capacity is undesired, the increased budget does not exaggerate the number of capacity expansion projects.

The sensitivity analysis that is conducted in this section can prove to be more useful today than first anticipated, due to the COVID-19 virus. Trondheim communicates that the extraordinary situation may affect the available budget for school projects and thus reduce the room for decisions. Moreover, as the future budgets in the municipality are uncertain, it is beneficial to be aware of the practical consequences of both an increase and a decrease in the budget. Therefore, this analysis can be of guidance in the forthcoming years.

## 9.3 A Study of Lundåsen

As previously stated, of the two possible new schools presented in Chapter 7, only Lundåsen school is built. To understand the factors contributing to this decision we study the area in close proximity to the school site.

Today, the pupils in this area are attending either Åsheim or Kattem. From the capacity map, presented in Chapter 2, we see that both of these schools have pressured capacity. For this area, Stabbursmoen is the third closest school. However, this school also experiences capacity issues, and thus, the situation in the area is already strained.

To make matters worse, three major residential building projects, denoted A, B, and C, are planned or initiated, in the area. From year 1 to year 5, 565 new residences are planned to be completed in project A. From the type of housing that is built, based on the calculations presented in Chapter 7, we estimate that 84 pupils are moving into these new homes. Project B is already ongoing, and by the planned completion in year 3, the project is estimated to increase the number of pupils in the area by 48. Moreover, in year 3 to year 8, the third building project, project C, at Lundåsen is finished. The building potential at this site is 450 and the amount of new pupils is 95.

Chapter 7, presents how each project is given a status that indicates the probability of completion. From that scheme, project A and B are given status 4 which indicates an 85% chance of completion. Project C is earlier in the regulation process and is given status 2, and thus a 60% chance of completion. Hence, the residential building projects are constructed in the majority of the scenarios.

When this population development occurs, the capacity demand of the already pressured Åsheim, Kattem, and Stabbursmoen school increases. In addition, the location of the residential building projects relocates the overall center of gravity of the population away from the already existing schools. This results in a less favorable road to school for the pupils.

The capacity issues can be solved either by increasing the capacity of one or more of the three already existing schools or by building the new school at Lundåsen. However, Lundåsen provides a better fit for the new center of the population, and the building of the school can significantly shorten the roads to school in the area. Consequently, a new school at Lundåsen is considered the best solution to resolve the issues in the area. Furthermore, in almost every scenario, Lundåsen is the first project to be executed. This indicates that this new school is one of the most important, and urgent, projects in southern Trondheim. Figure 9.4 shows the school district of Lundåsen in year 15 given that it is completed.

Furthermore, it is interesting to study if the construction of Lundåsen is solely based on the new population caused by the completion of projects A, B, and C, or if it is built regardless. Therefore, we test the SPPA on instances where the residential building

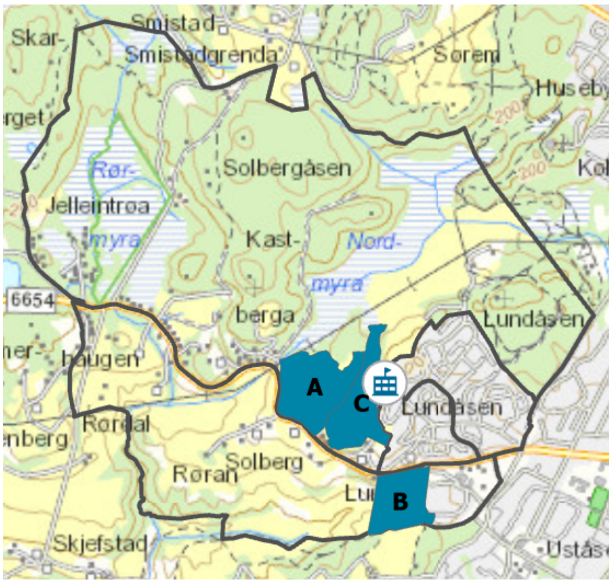



**Figure 9.4:** The school district for the new Lundåsen school in the final year of the planning horizon. The blue areas in the figure indicate the changeable zones, where the new homes have been built, and the labels denote each building project. Do note that some of the zones in Lundåsen school district are shared with other schools, as addressed in Section 9.5

projects are not executed. Not surprisingly, the results show that the new school is not built in these cases. With only the current population, the pupils are allocated to Stabbursmoen, Kattem, and Årstad, which are then able to accommodate them. We can therefore conclude that it is the residential building projects in the area that are the root cause of the construction.

Consultations with Trondheim indicate that the construction of Lundåsen is an option excessively discussed in the municipality. Several previously conducted analysis initiated by Trondheim have found it more economically favorable to expand Åsheim, Stabbursmoen or Kattem, instead of building a new school. Trondheim expresses that this fact makes the results from the model even more interesting, as the SPPA may evaluate more options and aspects than their previous studies. Further, Trondheim emphasizes some important elements that are not considered in the model as it is described in this thesis. For instance, since a new school means increased available school capacity, more entrepreneurs can get their building projects approved by the municipality. As the limited school capacity can be used as an instrument for controlling the city development, this issue needs to be investigated further. An important aspect to consider is how the construction of a new school leads to increased demand of infrastructure. It is undesirable to develop areas of the city that increase the need for transportation by car, and therefore it is important to organize suitable transportation options. These factors are some of the important elements that need to be further discussed to make beneficial alterations to the results.

## 9.4 Divided Zones

As previously described in Chapter 4, pupils in a zone are allowed to attend different schools. Thus, the model returns a set of zones in which the pupils are attending different schools, denoted divided zones. This allows us to identify zones that beneficially could be separated or arranged differently. The rationale behind this is that many zones are organized based on historical school districts and outdated factors, not necessarily on the situation today. The value of identifying these zones is especially prominent in zones where two schools are located at approximately the same distance from the zone center, and where dangerous roads do not affect the road to school. Figure 9.5 illustrates a zone that is divided in all tested scenarios.

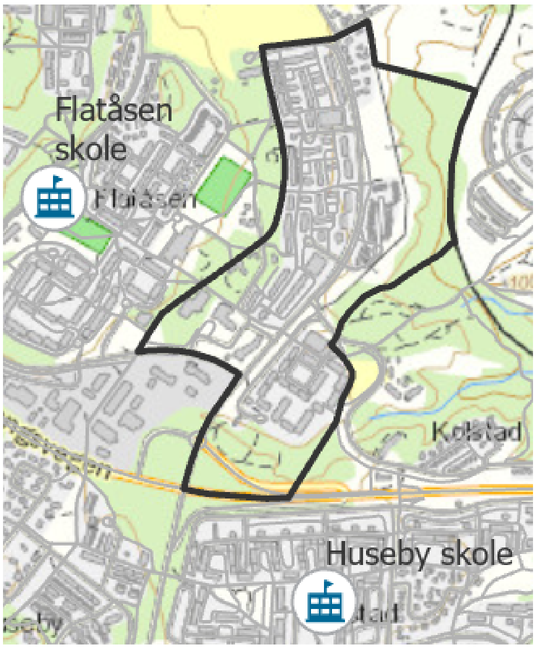



**Figure 9.5:** A zone with two nearby schools in about the same distance from the zone center. The model splits the zone and about half of the pupils are assigned to each school.

On average, 55% of the pupils in the zone are allocated to Flatåsen school and 45% are allocated to Huseby. With this allocation, the average capacity utilization at Flatåsen is 89% and Huseby is 91.6% which is close to optimal utilization. This indicates that this zone can beneficially be organized in a different way than it is today.

Another advantage of dividing the zones is that it allows for a more efficient sharing of the inconveniences. This means that in scenarios where one of the schools is concerned with high capacity demand, the nearby school can share some of the capacity and improve the utilization at both schools. Trondheim points out that this is a favorable result as the school districts can share the capacity across the current school districts to a greater extent than what is done today. By reorganizing the zones, the overall capacity utilization can be improved significantly. However, the higher the number of divided zones, the more postprocessing work is required to fully utilize and understand the results of the SPPA.

## 9.5 School Districts

The school districts found by the SPPA are not bounded by today's school districts. Therefore, the school districts proposed in the optimal solution involve some changes we want to study in detail. However, as the new districts are based on the existing zones, some similarities are to be expected.

### 9.5.1 New School Districts in the First Year

It is interesting to study how the model designs the optimal school districts in Trondheim today. This provides insight into what prioritization mechanisms are prevailing in the SPPA. Further understanding is obtained by comparing the new districts to the actual school districts in the area.

Figure 9.6 presents the optimal school districts given today's population, i.e. the optimal solution found by a one time period variation of the SPPA. In the figure, zones with the same color represent a school district where all the zones attend the school that lies within that colored area. The two-colored zones have pupils attending two different schools. Do note that as no projects are executed, the condition objective does not affect the found solution.

From the map, it is clear that zones, in general, are allocated to their nearest school. Around the majority of the schools, there are distinct clusters of zones. Despite this, if we only consider the distance, some zone-to-school allocations can seem fallacious. For instance, Okstad school lies seemingly close to residences in Romolslia school district. However, along the edges where the two districts meet, the E6 highway is located, and there are little crossing options for pedestrians. Thus, the school districts are divided as illustrated. This demonstrates that dangerous roads are considered when allocating the pupils. The same principle is also noticeable in the nearly separated zone that is allocated to Sjetne. Although this zone and Sjetne school also are separated by E6, there exist suitable underpasses that can be used, and the dangerous road is not crossed on the way to school.

By comparing the new map to the current school district map presented in Chapter 2, it is clear that the school districts are similar in several ways. As the school districts today are largely based on the same criteria that are effectuated in the SPPA, this result enhances the quality of the model. This indicates that the model provides applicable and valuable solutions. Moreover, we see that there are some differences, found mainly along the edges of the districts. These zones are the most troublesome to allocate, as several zone-to-school assignments can be equally good. Consequently, it is logical that it is in these zones that the differences occur and where we find the value of the SPPA as a school districting tool.

When taking a closer look into the numbers behind the new districts, we get an understanding of how the SPPA solution affects the capacity utilization in the area. The least beneficial capacity utilization in the first year is Romolslia with 72% utilization and Huseby with 77%. This is despite the fact that the new school districts for the two schools are expanded compared to the current school district. This shows that the SPPA is trying to distribute the excess capacity to share the inconveniences. This indicates that the districts found by the SPPA are somewhat prepared for population increases in most areas of southern Trondheim.

### 9.5.2 School Districts in the Final Year

To fully understand the impact of the executed projects, we want to study the found optimal school districts at the end of the planning horizon. The goal is to obtain a long-term understanding of the school districts in Trondheim, and how the realization of population development and the execution of projects form the future of Trondheim.

The complete map of the new school districts for Trondheim in year 15 is presented in Figure 9.7. The map illustrates the optimal situation and considers the zone-to-school allocations that occur in the majority of the scenarios of one 20 scenario instance.

As in the map for the first year, the new school districts in the final year clearly reflect the considerations regarding road to school in the objective function presented in Chapter 5. It is clear that most zones are allocated to what seems to be a suitable school concerning both road to school distance and hazards.

We observe that the school districts in the map have several similarities to the school districts in Trondheim today. This indicates that even with a population development, the distribution of inhabitants is somewhat the same. However, it is clear that the area has developed. The most prominent change is the new district assigned to Lundåsen school. This new district accommodates several of the zones previously allocated to Åsheim. As Åsheim is one of the schools with critical capacity today, a reduction in its school district is reasonable. Thus, the construction of the new school results in near-optimal capacity

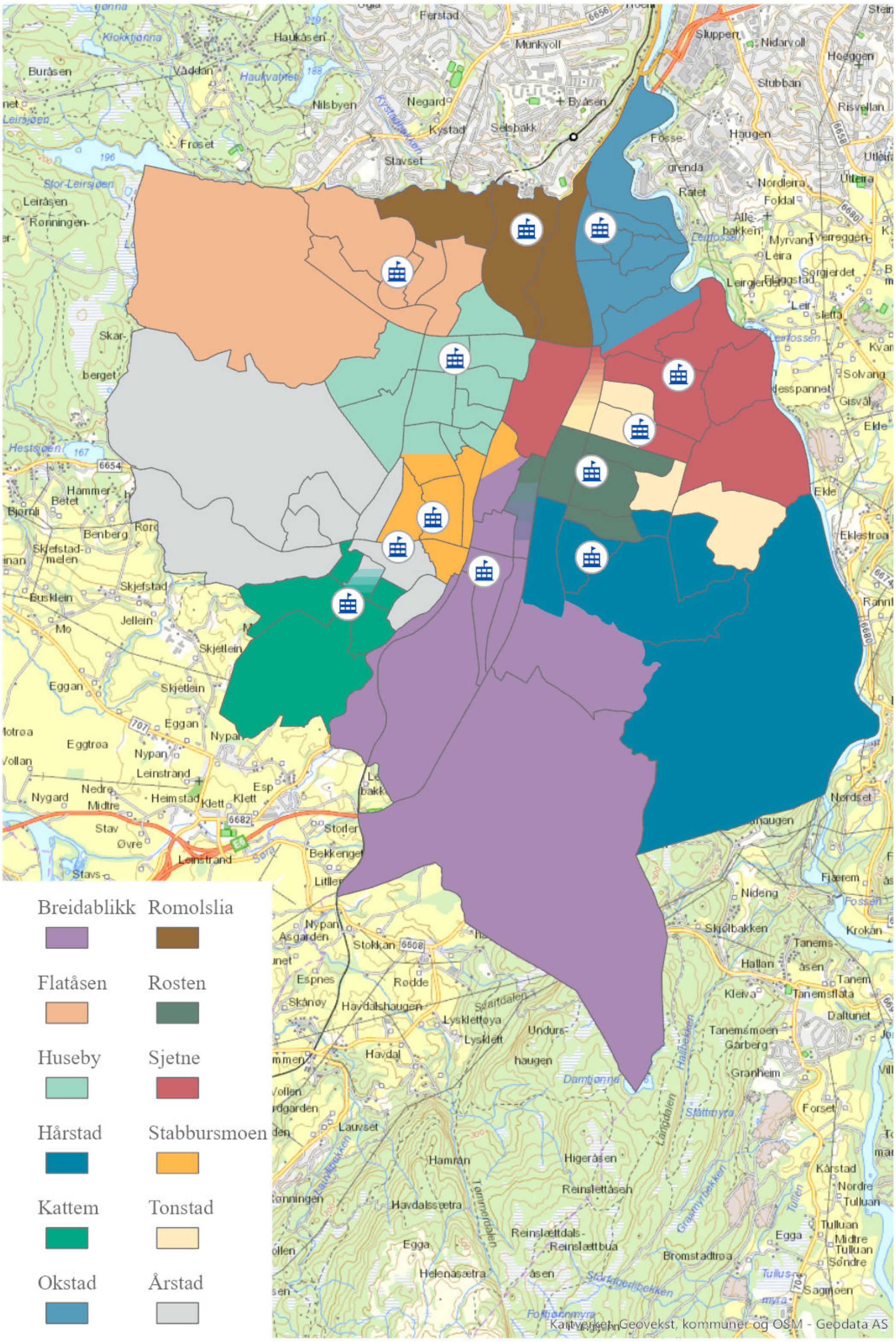



**Figure 9.6:** School districts for Trondheim in year 1.

utilization at both Åsheim and Lundåsen.

Another change is the expanded size of the school district assigned to Breidablikk. Today, Breidablikk is one of the school districts with under-utilization, and the expansion of the school district results in a new utilization of approximately 90% in all scenarios. This demonstrates that the new districts are based on the requirement for optimal utilization. Even though the district seems disproportionately big, the population of the zones in the area is low. This is the reason why the construction of a new school in the area is not an alternative even though it could reduce the road to school of the pupils living in this part of southern Trondheim.

As presented in the previous section, Romolslia and Huseby have spare capacity in the first year. However, in the final year, the average capacity at the two schools is 90% and 91.6% respectively. This illustrates that the model's ability to evenly distribute the capacity utilization already in earlier years provides relatively stable school districts as the population increases. Thus, by expanding the districts initially, the zones do not have to change schools later, and the capacity utilization is close to optimal at both schools.

Feedback from Trondheim implies that they find the outline of optimal school districts in year 15 especially interesting, as this enables long-term strategies for the districts. The organizing of school districts is not only a cumbersome process for the municipality, but also an important factor for the residents of Trondheim. As the school districts in Trondheim today are, at any given time, based on the current population, the districting process does not consider the long-term perspective. Therefore, these results can be used as suggestions for possible future districts and be useful for several stakeholders.

Lastly, it is important to emphasize that this is a decision tool for Trondheim and not an unambiguous solution. For instance, as the map shows, the model finds it best to allocate a small zone to Stabbursmoen, even though the surrounding zones are allocated to Breidablikk. This is explained by the fact that the zones are allocated based on the center of its population. However, this is not necessarily the best solution when considering social factors, which are not accounted for in the model. Consequently, the model can easily be used as a foundation when planning new districts, but further discussions are essential to obtain the best practical solution for Trondheim and its residents.

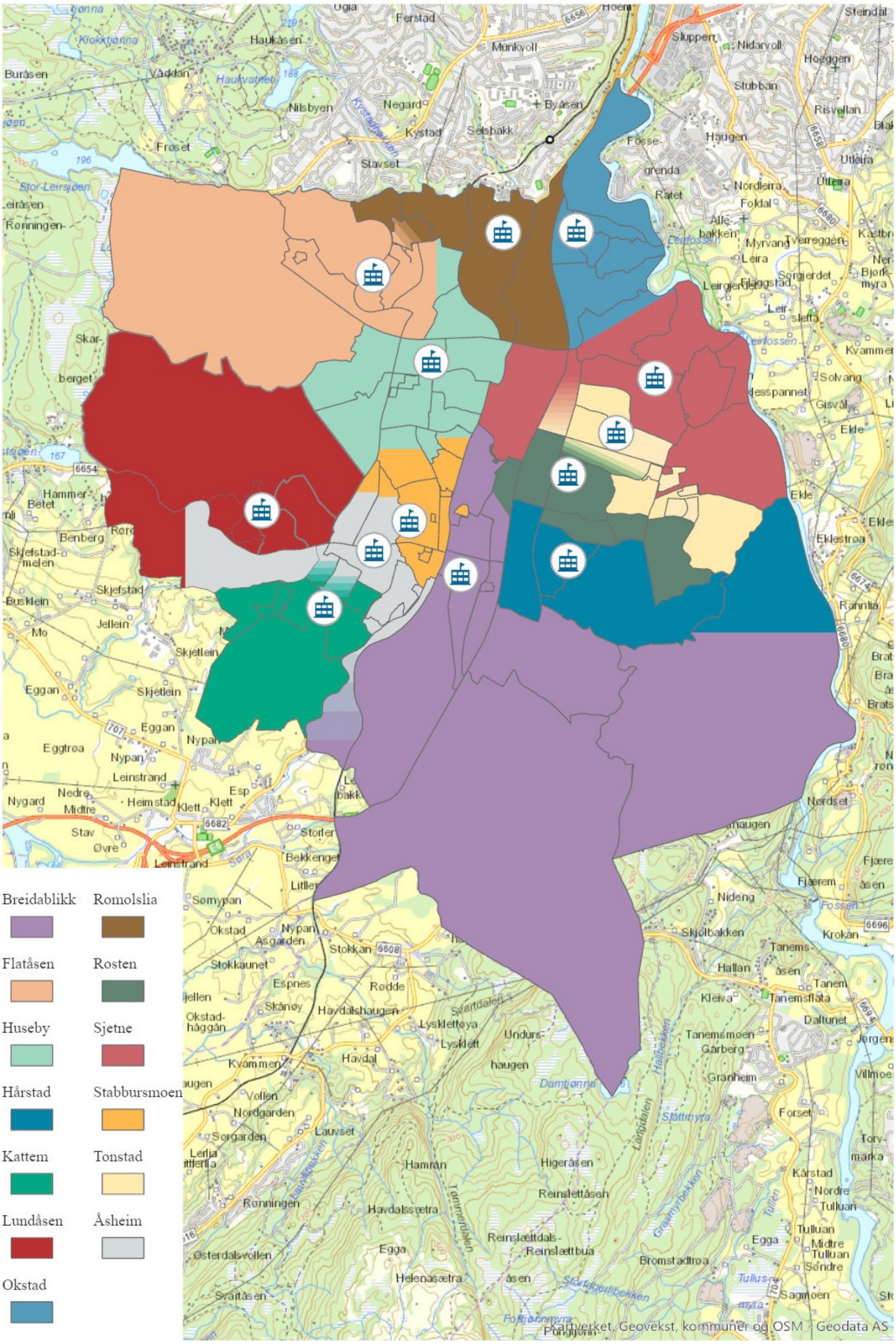



**Figure 9.7:** School districts for Trondheim in year 15.

# 10 Concluding Remarks and Future Research

In this chapter, the significant findings of this thesis is presented in Section 10.1 and future research opportunities are addressed in Section 10.2.

## 10.1 Concluding Remarks

This thesis presents a mathematical formulation of the School Prioritization Problem with Alternatives (SPPA). Furthermore, two variations of the Execution Order Specific Branch and Bound algorithm, EOSBB and AEOSBB, are proposed as solution methods to the problem.

The SPPA is the problem of deciding on what school projects to execute to best accommodate the school capacity in a community. This location problem addresses an important question: Given a set of distinct possible projects what are the optimal projects to execute at what time? The problem arises from the uncertainty in future capacity demand, wearing of existing school buildings, and the current lack of an objective decision tool. Therefore, there is an economic and social value from the introduced mathematical model and solution method that can aid municipalities, like Trondheim, in deciding school strategies for the future.

The SPPA is formulated as a multi-objective problem. The objective function aims to simultaneously minimize the overall cost from three terms: undesired roads to school, under- or over-utilized capacity, and inconvenience of poor school condition. These objectives are exposed to uncertainty in future demand and multiple scenarios are considered. The model differs from existing literature and closes three gaps in the literature: a precise mathematical description including all the aspects of Trondheim's problem, an objective function taking utilization and quality of the schools into account, and a solution method that can solve the problem within an acceptable time.

As the complexity of the problem increases with the number of considered scenarios, the computational effort of the multi-scenario problem proves demanding. The introduction of the solution algorithms addresses this problem by presolving each scenario separately as single-scenario problems, to then reduce the number of required calculations in the multi-scenario problem by limiting the execution order. As a consequence, the SPPA, with real-size data, is solvable within an acceptable time.

The SPPA has been tested on a part of southern Trondheim consisting of 74 existing zones and 43 changeable zones in 15 time periods. There are 12 initially existing schools as well as 2 possible new schools. A list of all possible projects in the given time horizon is developed and geographic information systems (GIS) are used to find optimal routes from a zone to each school. Furthermore, several scenarios of the population of the changeable zones are created. The SPPA is tested on instances of scenarios ranging from 3 to 100.

Preliminary, a deterministic variation of the SPPA is tested. The relaxation of some of the binary variables proves to have minor impact on the solution, but causes major improvements in computational time. In addition, the implementation of a maximum allowed deterioration and cuts in the allowed distance to school reduce the computational effort further. This results in a set of performance enchanting extensions which are included in the final model.

To test the performance of the solution algorithms, the effectiveness is compared with the performance of a Complete Computation (CC) approach. The results show that during a set time, both the EOSBB and the AEOSBB algorithms can solve significantly larger instances of the SPPA than the CC. Moreover, as the number of scenarios exceeds 20, the CC is not capable of solving the problem to optimality within 25,000 seconds. However, the EOSBB algorithm also struggles when the number of scenarios increases, and thus, the AEOSBB algorithm proves to be the preferred solution method. The tests strongly substantiate the outstanding performance of the new solution method and indicate that the method is suitable for larger data instances.

The SPPA successfully provides a detailed overview of which projects to conduct at what time. In all scenarios, the new Lundåsen school is the first executed project, which indicates that this is a project Trondheim beneficially can investigate further and consider executing. A completely new school district map for the final year is presented. This provides an outline of the optimal future school districts of the designated area. Moreover, the sensitivity analysis of the budget parameter shows that small adjustments on the available budget have a great impact on the overall quality of the schools in the designated area. Therefore, it is clear that the model easily can be used as a practical tool in decision-making.

However, it is important to emphasize that subjective considerations must be conducted alongside the objective solutions presented in the model. The found results can beneficially be used as a foundation for school development. However, as all decisions regarding schools have substantial repercussions, social factors such as neighborhoods, previous school districts, and affiliation to areas, are necessary to contemplate. Furthermore, the presented results are somewhat optimistic as some assumptions and simplifications are made. Lastly, the SPPA has been tested on limited data instances. This means that further studies need to be executed to evaluate if the model and solution method can be generalized.

In conclusion, this thesis has demonstrated that the SPPA successfully can be used as an unbiased decision tool for aiding Trondheim in school planning and project prioritization. Also, a solution method is developed that significantly improves the performance of the model. We believe that with the unique formulation of the SPPA and the increased computational performance provided by the AEOSBB algorithm, this thesis is a considerable contribution to the field of school location planning.

## 10.2 Elements for Future Research

In this section, some elements for further research and model improvements are presented. In Section 10.2.1, further research topics are discussed. Section 10.2.2 addresses possible extensions to the problem. Lastly, Section 10.2.3 presents other relevant solution methods that can be used when solving the SPPA.

### 10.2.1 Further Research Topics

An interesting new research opportunity is to study how a municipality can affect the location of new buildings. As the building projects are dependent on available school capacity, an increase of capacity in one part of the municipality can determine where new zones are located. This allows the municipality to use school planning to actively direct urban development in an area. This is a new approach to the school location problem and can be worth studying further.

The model and solution method presented in this thesis is based on problems faced by Trondheim. A possible future research topic is therefore to verify if the SPPA can be generalized to other municipalities. The model can easily be adapted to other Norwegian municipalities that face the same challenges as Trondheim. However, in municipalities with, for instance, decreasing population growth, the value of the SPPA is not proven. Furthermore, as the situations in some municipalities might require larger data instances, this could further increase computational demand and the performance of the SPPA. In addition, it can be interesting to examine if the model and solution method can be adjusted to be applicable to a general facility location problem.

### 10.2.2 Possible Extensions of the Problem

There are some elements regarding school planning, not included in this thesis, that may be interesting to implement in future studies. For instance, the temporary allocation of pupils when their allocated school is renovated can be a next topic to investigate. A possible way to implement this is to install pavilions at the existing school sites. This can be implemented in the model as either a non-monetary cost for the inconvenience of utilizing pavilions or a monetary leasing cost, as presented in Delmelle et al. (2014).

Another interesting expansion of the model is to include the location of secondary schools. In Norway, there is a strong interrelationship between the primary and secondary schools that pupils are allocated to. In addition, some of the potential future projects involve schools that have both primary and secondary schools at the same site. Thus, this is an extension that covers an additional part of the real-world school prioritization problem.

In this thesis, some simplifications have been made. For instance, the population is clustered to zones, and the zone centers are used when calculating the walking distance to a school. This means that some pupils, especially those located in the zone edges, have a longer road to school distance than what is considered in this thesis. Consequently, a more detailed distance measure can be obtained by a finer partitioning of the area. However, the right balance must be maintained between the positive effect of more detailed zones and the additional effort of acquiring more precise data.

### 10.2.3 Improvement of the Solution Algorithm

This thesis presents the EOSBB and AEOSBB algorithms as possible solution methods to the SPPA. However, as the size of the data instances increase, so does the computational complexity. Consequently, it is natural to assume that the solution method must be evolved as well. Heuristic methods are extensively used in facility location problems, as presented in Chapter 3. For instance, different types of genetic algorithms can be developed and may further improve the computational effort. Thus, a heuristic can be developed to simplify the computation when the size of the problem increases.

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# Appendices

# A Compressed Model

Sets:

| | |
|---|---|
| $\mathcal{S}$ | set of schools |
| $\mathcal{S}^E$ | set of existing schools without potential projects in the planning horizon (existing unchangeable schools) |
| $\mathcal{S}^C$ | set of existing schools with potential projects in the planning horizon (existing changeable schools) |
| $\mathcal{S}^P$ | set of potential new schools in the planning horizon |
| $\mathcal{A}_s$ | set of alternatives for school $s$ |
| $\mathcal{T}$ | set of time periods |
| $\mathcal{Z}$ | set of zones |
| $\mathcal{Z}_s$ | set of zones in which school $s$ is located |
| $\mathcal{E}$ | set of possible scenarios |

Indices:

| | |
|---|---|
| $s$ | school |
| $a$ | alternative |
| $i$ | zone |
| $i(s,a)$ | zone where school $s$ alternative $a$ exists |
| $t$ | time period |
| $e$ | scenario |

Parameters:

| | |
|---|---|
| $R_{sai}$ | total cost per pupil from undesired road to school $s$ alternative $a$ from zone $i$ |
| $E_{sat\tau}$ | the expense in time period $\tau$ of finishing school $s$ alternative $a$, in time period $t$ |
| $B_t$ | budget available for school projects in time period $t$ |
| $N_{ite}$ | number of pupils in zone $i$ in time period $t$ in scenario $e$ |
| $S^I_{sa}$ | 1 if school $s$ alternative $a$ initially exists |
| $C^{CON}_{sat}$ | condition cost of school $s$ alternative $a$ in time period $t$ |
| $\hat{Q}_{sa}$ | built capacity at school $s$ alternative $a$ |
| $Q^*_{sa}$ | optimal capacity at school $s$ alternative $a$ |
| $\bar{Q}_{sa}$ | maximum capacity at school $s$ alternative $a$ |
| $\hat{C}_{sa}$ | cost of having school $s$ alternative $a$ used at built capacity |
| $\bar{C}_{sa}$ | cost of having school $s$ alternative $a$ used at the maximum capacity |
| $p_e$ | the probability of scenario $e$ occurring |
| $D_{sai}$ | distance between school $s$ alternative $a$ and zone $i$ |
| $A_{sai}$ | measurement of the topography between school $s$ alternative $a$ and zone $i$ |
| $F_{sai}$ | measure of how dangerous the road between school $s$ alternative $a$ and zone $i$ is |
| $P^D$ | penalty for distance |
| $P^A$ | penalty for topography |
| $P^F$ | penalty for dangerous roads |
| $E^{TOTAL}_{sat}$ | Total cost of completing school $s$ alternative $a$ in time period $t$ |
| $L_{t\tau}$ | the percentage of the total cost of a project finished in time period $t$ that is accounted for in time period $\tau$ |

Weighting parameters:

| | |
|---|---|
| $\alpha$ | weight of cost from road to school |
| $\beta$ | weight of utilization cost |
| $\gamma$ | weight of cost of condition of the school |

Variables:

| | |
|---|---|
| $x_{sate}$ | 1 if school $s$ alternative $a$ is finished in time period $t$ in scenario $e$ |
| $y_{sate}$ | 1 if school $s$ alternative $a$ exists in time period $t$ in scenario $e$ |
| $v_{ite}$ | 1 if pupils in zone $i$ change their allocated school in time period $t$ in scenario $e$ |
| $w_{saite}$ | the amount of pupils in zone $i$ that is allocated to school $s$ alternative $a$ in time period $t$ in scenario $e$ |
| $q_{sate}$ | used capacity at school $s$ alternative $a$ at time period $t$ in scenario $e$ |
| $c(q_{sate})$ | cost from non-optimal capacity utilization of school $s$ alternative $a$ in time period $t$ in scenario $e$ |
| $z_{sate}$ | cost of inconvenience from poor school condition for school $s$ alternative $a$ in time period $t$ in scenario $e$ |

**Objective Function**

$$\min \quad \sum_{e\in\mathcal{E}} p_e(\alpha \sum_{s\in\mathcal{S}}\sum_{a\in\mathcal{A}}\sum_{i\in\mathcal{Z}}\sum_{t\in\mathcal{T}} R_{sai}N_{ite}w_{saite} + \beta \sum_{s\in\mathcal{S}}\sum_{a\in\mathcal{A}}\sum_{t\in\mathcal{T}} c(q_{sate}) + \gamma \sum_{s\in\mathcal{S}^C}\sum_{a\in\mathcal{A}_s}\sum_{t\in\mathcal{T}} \hat{Q}_{sa}z_{sate})$$

**Constraints**

$$\text{s.t.} \quad \sum_{s\in\mathcal{S}}\sum_{a\in\mathcal{A}}\sum_{t\in\tau} E_{sat\tau}x_{sate} \le B_\tau \qquad \tau\in\mathcal{T}, e\in\mathcal{E}$$

$$y_{sate} = S^I_{sa} \qquad s\in\mathcal{S}^E, a\in\mathcal{A}_s, t\in\mathcal{T}, e\in\mathcal{E}$$

$$y_{sa1e} = S^I_{sa} \qquad s\in\mathcal{S}^C, a\in\mathcal{A}_s, e\in\mathcal{E}$$

$$y_{sa,t+1,e} \le y_{sate} + x_{sa,t+1,e} \qquad s\in\mathcal{S}^C, a\in\mathcal{A}_s, t\in\mathcal{T}\setminus\{T\}, e\in\mathcal{E}$$

$$y_{sate} + \sum_{b\in\mathcal{A}_s|b\neq a} x_{sbte} \le 1 \qquad s\in\mathcal{S}^C, a\in\mathcal{A}_s, t\in\mathcal{T}, e\in\mathcal{E}|S^I_{sa}=1$$

$$\sum_{a\in\mathcal{A}_s} y_{sate} = 1 \qquad s\in\mathcal{S}^C, t\in\mathcal{T}, e\in\mathcal{E}$$

$$\sum_{t\in\mathcal{T}} x_{sate} \leq 1 \qquad s \in \mathcal{S}^C, a \in \mathcal{A}_s, e \in \mathcal{E}$$

$$y_{sa1e} = 0 \qquad s \in \mathcal{S}^P, a \in \mathcal{A}_s, e \in \mathcal{E}$$

$$y_{sa,t+1,e} \leq y_{sate} + x_{sa,t+1,e} \qquad s \in \mathcal{S}^P, a \in \mathcal{A}_s, t \in \mathcal{T}\backslash\{T\}, e \in \mathcal{E}$$

$$y_{sate} \leq y_{sa,t+1,e} \qquad s \in \mathcal{S}^P, a \in \mathcal{A}_s, t \in \mathcal{T}\backslash\{T\}, e \in \mathcal{E}$$

$$x_{sa,t+1,e} \leq y_{sa,t+1,e} \qquad s \in \mathcal{S}^P, a \in \mathcal{A}_s, t \in \mathcal{T}\backslash\{T\}, e \in \mathcal{E}$$

$$\sum_{a\in\mathcal{A}_s} y_{sate} \leq 1 \qquad s \in \mathcal{S}^P, t \in \mathcal{T}, e \in \mathcal{E}$$

$$\sum_{s\in\mathcal{S}} \sum_{a\in\mathcal{A}} w_{saite} = 1 \qquad i \in \mathcal{Z}, t \in \mathcal{T}, e \in \mathcal{E}$$

$$w_{saite} \leq y_{sate} \qquad s \in \mathcal{S}, a \in \mathcal{A}_s, i \in \mathcal{Z}, t \in \mathcal{T}, e \in \mathcal{E}$$

$$\sum_{a\in\mathcal{A}_s} w_{sai,t+1,e} \leq \sum_{a\in\mathcal{A}_s} w_{saite} + v_{i,t+1,e} \qquad s \in \mathcal{S}, i \in \mathcal{Z}, t \in \mathcal{T}\backslash\{T\}, e \in \mathcal{E}$$

$$\sum_{t\in\mathcal{T}} v_{ite} \leq 1 \qquad i \in \mathcal{Z}, e \in \mathcal{E}$$

$$\sum_{i\in\mathcal{Z}} N_{ite} w_{saite} = q_{sate} \qquad s \in \mathcal{S}, a \in \mathcal{A}_s, t \in \mathcal{T}, e \in \mathcal{E}$$

$$w_{sa,i(s,a),te} = y_{sate} \qquad s \in \mathcal{S}, a \in \mathcal{A}_s, t \in \mathcal{T}, e \in \mathcal{E}$$

$$z_{sate} = C^{CON}_{sat} y_{sate} \qquad s \in \mathcal{S}^C, a \in \mathcal{A}_s, t \in \mathcal{T} e \in \mathcal{E}$$

$$c_{sate} \geq \theta_1 q_{sate} - \phi_1 \qquad s \in \mathcal{S}, a \in \mathcal{A}_s, t \in \mathcal{T}$$

$$c_{sate} \geq \theta_2 q_{sate} - \phi_2 \qquad s \in \mathcal{S}, a \in \mathcal{A}_s, t \in \mathcal{T}$$

$$c_{sate} \geq \theta_3 q_{sate} + \phi_3 - \bar{C}_{sa}(1 - y_{sate}) \qquad s \in \mathcal{S}, a \in \mathcal{A}_s, t \in \mathcal{T}$$

$$q_{sate} \leq \bar{Q}_{sa} \qquad s \in \mathcal{S}, a \in \mathcal{A}_s, t \in \mathcal{T}$$

$$x_{sat} = x_{sate} \qquad s \in \mathcal{S}, a \in \mathcal{A}_s, t \in \mathcal{T}, e \in \mathcal{E}$$

$$x_{sate} = 0 \qquad s \in \mathcal{S}^E, a \in \mathcal{A}_s, t \in \mathcal{T}, e \in \mathcal{E}$$

$$x_{sate}, y_{sate} \in \{0, 1\} \qquad s \in \mathcal{S}, a \in \mathcal{A}_s, t \in \mathcal{T}, e \in \mathcal{E}$$

$$v_{ite} \in \{0, 1\} \qquad i \in \mathcal{Z}, t \in \mathcal{T}, e \in \mathcal{E}$$

$$w_{saite} \geq 0 \qquad s \in \mathcal{S}, a \in \mathcal{A}_s, i \in \mathcal{Z}, t \in \mathcal{T}, e \in \mathcal{E}$$

$$c_{sate}, q_{sate} \geq 0 \qquad s \in \mathcal{S}, a \in \mathcal{A}_s, t \in \mathcal{T}, e \in \mathcal{E}$$

$$z_{ste} \geq 0 \qquad s \in \mathcal{S}^C, t \in \mathcal{T}, e \in \mathcal{E}$$

**Parameter Equalities**

$$R_{sai} = D_{sai} \cdot P^D + A_{sai} \cdot P^A + F_{sai} \cdot P^F \qquad s \in \mathcal{S}, a \in \mathcal{A}_s, i \in \mathcal{Z}$$

$$E_{sat\tau} = L_{t\tau} E_{sat}^{TOTAL} \qquad s \in \mathcal{S}, a \in \mathcal{A}_s, t \in \mathcal{T}, \tau \in \mathcal{T}$$

$$\theta_1 = \frac{\hat{C}_{sa}}{\hat{Q}_{sa} - Q^*_{sa}} \qquad \phi_1 = \frac{\hat{C}_{sa}}{\hat{Q}_{sa} - Q^*_{sa}} \cdot Q^*_{sa}$$

$$\theta_2 = \frac{\bar{C}_{sa} - \hat{C}_{sa}}{\bar{Q}_{sa} - \hat{Q}_{sa}} \qquad \phi_2 = \frac{\bar{C}_{sa} - \hat{C}_{sa}}{\bar{Q}_{sa} - \hat{Q}_{sa}} \cdot \hat{Q}_{sa} + \hat{C}_{sa}$$

$$\theta_3 = -\frac{\bar{C}_{sa}}{Q^*_{sa}} \qquad \phi_3 = \bar{C}_{sa}$$

# B Building Order Algorithms

Algorithm 4 displays an outline of the entire EOSBB and Algorithm 5, the AEOSBB solution methods for solving the SPPA. The algorithms solve the SPPA to optimality.

**Algorithm 4:** Execution Order Specific Branch and Bound (EOSBB)

0: Initialize:
  $\bar{z} = \infty$
  $R_s = 0$ for all $s \in \mathcal{S}$
  $LIST_e$ as an empty set for each scenario $e$

**Single-scenario phase**

1: **for** each scenario $e$ in $\mathcal{E}$ **do**
2:   **if** $R_s$ is **not** already fulfilled by a solution in $LIST_e$ **then**
3:     solve the SSP
4:     generate the SO-vector
5:     add the solution to $LIST_e$
  **else**
6:     return the feasible solution from $LIST_e$
  **end-if**
**end-do**

**Multi-scenario phase**

7: calculate the SO-average, and calculate $z^{avg}$
8: **if** one or more active single scenarios solutions not feasible **then**
9:   node terminated
**end-if**
10: **if** $z^{avg} \geq \bar{z}$ **then**
11:   node terminated
**end-if**
12: **if** the SO-average is integer **or**
search depth = minimum number of executed projects **then**
13:   run the MSP with added execution order restriction
14:   **if** $z^{MSP} \leq \bar{z}$ **then**
15:     $\bar{z} = z^{MSP}$
16:     node terminated
  **end-if**
**end-if**

**New-node phase**

17: **if** current node is active **then**
18:   create two new nodes
19:   add the created nodes to the tree
**end-if**
20: **if** still active nodes **then**
21:   pick best node
22:   return to the single-scenario phase
**end-if**
23: return the optimal solution of the SPPA

**Algorithm 5:** Alternative Execution Order Specific Branch and Bound (AEOSBB)

0: Initialize:
  $\bar{z} = \infty$
  $R_{sa} = 0$ for all $s \in \mathcal{S}$
  $LIST_e$ as an empty set for each scenario $e$

**Single-scenario phase**

1: **for** each scenario $e$ in $\mathcal{E}$ **do**
2:   **if** $R_{sa}$ is **not** already fulfilled by a solution in $LIST_e$ **then**
3:     solve the SSP
4:     generate the SO-vector
5:     add the solution to $LIST_e$
  **else**
6:     return the feasible solution from $LIST_e$
  **end-if**
 **end-do**

**Multi-scenario phase**

7: calculate the SO-average, and calculate $z^{avg}$
8: **if** one or more active single scenarios solutions not feasible **then**
9:   node terminated
 **end-if**
10: **if** $z^{avg} \geq \bar{z}$ **then**
11:   node terminated
 **end-if**
12: **if** the SO-average is integer **or**
 search depth = minimum number of executed projects **then**
13:   run the MSP with added execution order restriction
14:   **if** $z^{MSP} \leq \bar{z}$ **then**
15:     $\bar{z} = z^{MSP}$
16:     node terminated
  **end-if**
 **end-if**

**New-node phase**

17: **if** current node is active **then**
18:   create two new nodes
19:   add the created nodes to the tree
 **end-if**
20: **if** still active nodes **then**
21:   pick best node
22:   return to the single-scenario phase
 **end-if**
23: return the optimal solution of the SPPA

# C Deterministic Model Testing

## C.1 Determining the Divisors

Each single scenario problem is solved with a maximum run time of 3600, and yield the following results.

**Table C.1:** The objective value, best bound and run time from each SOP.

| Objective | Integer solution | Best bound | Run time |
|---|---|---|---|
| RtS | 48,160,000 | N/A | 90 |
| Cap | 10.24 | 2.73 | MAX |
| Con | 208,577 | N/A | 1974 |

Both the road to school and school condition SOP is solved to optimality. The divisor for these two objectives is thus set equal to the found objective value.

The capacity utilization SOP, however, had a significant gap between the best bound and the found integer solution. The difference of moving forward with the best bound versus the integer solution value is equal to valuing capacity utilization approximately four times higher. In order to reduce the possibility of moving forward with the wrong divisor, a second set of test are conducted on the capacity utilization SOP in an attempt to reduced the gap. Several added restrictions, cutting the obviously worst solutions, are attempted. The following table presents the solution of the cut that yielded the smallest gap.

**Table C.2:** The solution with the lowest gap for the addition capacity utilization SOPs.

| Objective | Integer solution | Best bound | Run time |
|---|---|---|---|
| Original Cap | 10.24 | 2.73 | MAX |
| Cut Cap | 5.68 | 2.73 | 1974 |

The best bound remained unchanged, and was chosen as the divisor for the capacity utilization objective. However the value is uncertain, and the best bound is picked as to not undervalue the capacity utilization objective.

## C.2 Road To School Cuts

Results from various tests with the road to school cuts discussed in Chapter 8.

### C.2.1 Equality Cuts

**Table C.3:** Results from implementing equal cuts to the zone-to-school distance.

| Cut distance | Obj. value | Time [s] | RtS | | Cap | | Cond | |
|---|---|---|---|---|---|---|---|---|
| | | | Obj. | Avg. [m] | Obj. | Avg. [%] | Obj. | Avg. |
| 2750 | 1.3217 | 415 | 1.10 | 743 | 2.97 | 89.7 | 1.96 | 1.45 |
| 3000 | 1.3214 | 395 | 1.10 | 742 | 2.97 | 89.7 | 1.96 | 1.45 |
| 3250 | 1.3210 | 665 | 1.10 | 742 | 2.97 | 89.7 | 1.96 | 1.45 |
| 3500 | 1.3208 | 515 | 1.10 | 742 | 2.97 | 89.7 | 1.96 | 1.45 |
| 3750 | 1.3208 | 809 | 1.10 | 742 | 2.97 | 89.7 | 1.96 | 1.45 |
| 4000 | 1.3208 | 674 | 1.10 | 742 | 2.97 | 89.7 | 1.96 | 1.45 |

### C.2.2 Shortest Distance Based Cuts

#### Extra Meters

**Table C.4:** Results from implementing shortest distance cuts, with allowed extra meters, to the zone-to-school distance.

| Extra meters | Obj. value | Time [s] | RtS | | Cap | | Cond | |
|---|---|---|---|---|---|---|---|---|
| | | | Obj. | Avg. [m] | Obj. | Avg. [%] | Obj. | Avg. |
| 450 | 2.94 | 36 | 1.05 | 711 | 35.6 | 89.7 | 2.12 | 1.50 |
| 500 | 2.88 | 37 | 1.05 | 712 | 34.3 | 89.7 | 2.12 | 1.50 |
| 550 | 2.83 | 55 | 1.05 | 712 | 33.4 | 89.7 | 2.12 | 1.50 |
| 600 | 2.76 | 58 | 1.06 | 717 | 31.8 | 89.7 | 2.12 | 1.50 |
| 700 | 2.44 | 73 | 1.09 | 735 | 24.6 | 88.4 | 2.26 | 1.65 |
| 800 | 2.33 | 120 | 1.09 | 740 | 22.7 | 88.4 | 2.15 | 1.55 |
| 900 | 1.42 | 183 | 1.10 | 744 | 4.66 | 89.9 | 2.01 | 1.50 |
| 1000 | 1.39 | 236 | 1.10 | 745 | 4.12 | 89.9 | 2.01 | 1.50 |

**Multiplier**

**Table C.5:** Results from implementing shortest distance cuts, with allowed extra meters by a multiplier, to the zone-to-school distance.

| Multiplier | Obj. value | Time [s] | RtS | | Cap | | Cond | |
|---|---|---|---|---|---|---|---|---|
| | | | Obj. | Avg. [m] | Obj. | Avg. [%] | Obj. | Avg. |
| 1.35 | 3.01 | 47 | 1.05 | 706 | 37.1 | 90.1 | 2.15 | 1.53 |
| 1.40 | 3.00 | 53 | 1.05 | 712 | 37.0 | 90.1 | 2.05 | 1.50 |
| 1.45 | 2.96 | 39 | 1.05 | 712 | 35.9 | 89.7 | 2.12 | 1.50 |
| 1.50 | 2.76 | 40 | 1.06 | 718 | 31.8 | 89.7 | 2.12 | 1.50 |
| 1.60 | 2.72 | 59 | 1.05 | 711 | 30.9 | 89.2 | 2.26 | 1.62 |
| 1.70 | 2.33 | 65 | 1.09 | 736 | 22.3 | 89.4 | 2.28 | 1.70 |
| 1.80 | 2.22 | 54 | 1.09 | 739 | 20.0 | 89.4 | 2.28 | 1.70 |
| 1.90 | 2.01 | 65 | 1.13 | 761 | 15.8 | 89.5 | 2.15 | 1.59 |

### C.2.3 Closest Schools Cuts

**Table C.6:** Results from implementing closest schools cuts.

| | Obj. Value | Time [s] | RtS | | Cap | | Cond | |
|---|---|---|---|---|---|---|---|---|
| | | | Obj. | Avg. [m] | Obj. | Avg. [%] | Obj. | Avg. |
| Two closest | 1.8391 | 51 | 1.09 | 737 | 13.1 | 89.3 | 2.06 | 1.51 |
| Three closest | 1.3572 | 135 | 1.12 | 757 | 3.33 | 89.7 | 1.96 | 1.45 |

# D In-sample Stability

**Table D.1:** Standard deviation $\sigma^2$ and coefficient of variance from in-sample stability tests.

| Number of Scenarios | $\sigma^2$ | CV |
|---|---|---|
| 3 | 0.1448 | 6.85% |
| 5 | 0.0733 | 3.59% |
| 10 | 0.1640 | 8.00% |
| 20 | 0.0282 | 1.36% |
| 30 | 0.0408 | 1.99% |
| 40 | 0.0346 | 1.64% |
| 50 | 0.0171 | 0.82% |
| 60 | 0.0156 | 0.73% |
| 70 | 0.0495 | 2.35% |
| 80 | 0.0323 | 1.54% |
| 90 | 0.0385 | 1.73% |
| 100 | 0.0417 | 1.97% |
| Average | 0.0738 | 3.55% |

# E Sensitivity Analysis

**Table E.1:** The results from sensitivity analysis of the budget constraint. The budget is relaxed from its minimum value and to a value that is considered as infinity. The results are plotted and further explained in Chapter 9.

| Budget weight | Objective value | RtS | Cap | Con |
|---|---|---|---|---|
| 0.875 | 2.319 | 1.094 | 22.004 | 2.232 |
| 0.90 | 2.195 | 1.103 | 19.153 | 2.316 |
| 0.95 | 2.098 | 1.084 | 17.939 | 2.168 |
| 1.00 | 2.077 | 1.084 | 17.826 | 2.064 |
| 1.10 | 1.916 | 1.091 | 14.838 | 1.955 |
| 1.25 | 1.866 | 1.097 | 14.495 | 1.705 |
| 1.50 | 1.765 | 1.082 | 13.269 | 1.518 |
| 2.00 | 1.711 | 1.100 | 13.295 | 1.061 |
| 100 | 1.711 | 1.100 | 13.295 | 1.061 |

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