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SHORT COMMUNICATION

Note on estimating bed shear stress caused by breaking random waves

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Abstract This note presents a method of how the bed shear stress caused by breaking random waves on slopes can be estimated. This is obtained by adopting the Sumer et al. (2013) bed shear stress formula due to spilling and plunging breaking waves on hydraulically smooth slopes combined with the Myrhaug and Fouques (2012) joint distribution of surf similarity parameter and wave height for individual random waves in deep water. The conditional mean value of the maxima of mean bed shear stress during wave runup given wave height in deep water is provided including an example for spilling and plunging breaking random waves corresponding to typical field conditions. Another example compares the present results with one case from Thornton and Guza (1983) estimating the wave energy dissipation caused by bed shear stress beneath breaking random waves.

The kinematics and dynamics of the fluid motion within the wave boundary layer near the seabed in shallow and intermediate water depth of wave-dominated areas are the dominant mechanism governing the flow and sediment transport. As the waves approach the surf and swash zones the flow is intensified by wave breaking leading to enhanced turbulence production. The bed shear stress represents an important flow component playing a crucial role affecting the sediment transport and morphology and therefore the stability of scour protections in coastal regions. Further details on the background and complexity together with a comprehensive literature review of breaking waves in coastal zones are provided in the recent textbook of Sumer and Fuhrman (2020).

The purpose of this article is to present a simple analytical method which can be used to give first estimates of the bed shear stress caused by breaking random waves

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on slopes. This is achieved by adopting the Sumer et al. (2013) bed shear stress formula for spilling and plunging breaking regular waves on hydraulically smooth slopes assuming that it is valid for individual breaking waves within a sea state of random waves. Then, the joint statistics of bed shear stress and wave height in deep water is derived by transformation of the Myrhaug and Fouques (2012) joint distribution of surf similarity parameter and wave height for individual random waves in deep water. Examples of calculating the conditional mean value of the maxima of mean bed shear stress during wave runup given the wave height in deep water corresponding to typical field conditions are provided.

The article is organized as follows. This introduction is followed by giving the theoretical background of the present approach. Then the statistical properties of the bed shear stress is derived by combining the Sumer et al. (2013) formula and the Myrhaug and Fouques (2012) joint probability density function (*pdf*) of surf similarity parameter and wave height. Two examples are provided by first demonstrating application of results in terms of the conditional mean value of the maxima of mean bed shear stress during wave runup given wave height in deep water for spilling and plunging breaking random waves corresponding to typical field conditions, and second by comparing the present model predictions with one case from Thornton and Guza (1983) estimating the wave energy dissipation due to bottom friction beneath breaking random waves. Finally, a summary and the main conclusions are provided.

The Sumer et al. (2013) empirical formula for the bed shear stress due to spilling and plunging breaking regular waves is

$$\frac{u_*}{\sqrt{gH_0}} = 0.085\,\xi_0^{\,0.6}, \quad 0.19 < \xi_0 < 1.42 \tag{1}$$

where the bed shear stress is given terms of the friction velocity $u_* = \sqrt{\max(\tau_{0\max})/\rho}$, i.e., associated with the maximum value of the mean bed shear stress due to the breaking wave (see their Eqs. (16), (17), (18) and Fig. 9). Here ρ is the fluid density, g is the acceleration due to gravity, H_0 is the deep water wave height, ξ_0 is the surf similarity parameter defined as $\xi_0 = m/\sqrt{s_0}$ where $m = \tan\beta$ is the slope with an angle β with the horizontal, $s_0 = H_0/((g/2\pi)T^2)$ is the wave steepness in deep water (i.e., using the linear dispersion relation), and T is the wave period. Eq. (1) is based on best fit to data obtained from the small scale laboratory experiments by Deigaard et al. (1991) and Sumer et al. (2013). The data represent flow conditions for regular breaking waves over a hydraulically smooth bed during the wave runup, and the types of breaking waves were classified based on those according to Galvin (1968):

$$\begin{array}{ll} \xi_0 < 0.5: & \text{spilling} \\ 0.5 < \xi_0 < 3.3: & \text{plunging} \\ \xi_0 > 3.3: & \text{surging} \end{array} \tag{2}$$

Further details on the background of these results are provided in Sumer et al. (2013) and Sumer and Fuhrman (2020). It should be noted that the surf similarity parameter was introduced originally by Iribarren and Nogales (1949) and later applied by Battjes (1974).

Sumer et al. (2013) stated that caution must be observed when Eq. (1) is used outside the indicated parameter range. This was followed by: “However, the range of the surf similarity parameter is sufficiently broad to cover, for the most part, most practical situations, namely spilling and plunging breaking conditions not only in the laboratory but also in the field.” Thus, motivated by this and taken as a first-order approximation, Eq. (1) is adopted and assumed to be valid for individual breaking random waves which in deep water has the wave height H_0 and wave period T. Then, for individual breaking random waves, Eq. (1) is rearranged to

$$u = \sqrt{h}\,\xi^d, \quad u_1 < u < u_2 \tag{3}$$

where

$$u = \frac{u_*}{c\,\xi_{rms}^d\sqrt{g H_{0rms}}}, \quad (c, d) = (0.085, 0.6) \tag{4}$$

and $h = H_0/H_{0rms}$, $\xi = \xi_0/\xi_{rms}$ are the normalized variables using the root-mean-square (*rms*) values H_{0rms} and ξ_{rms}, respectively. Thus, $u_1 = \sqrt{h}\,\xi_1^d$ with $\xi_1 = 0.19/\xi_{rms}$ and $u_2 = \sqrt{h}\,\xi_2^d$ with $\xi_2 = 1.42/\xi_{rms}$.

Here the joint *pdf* of u_* and H_0 is obtained from the joint *pdf* of ξ_0 and H_0 provided by Myrhaug and Fouques (2012) which was derived by transformation of a joint *pdf* of wave steepness s_0 and H_0 given by Myrhaug and Kjeldsen (1984). This empirically based joint *pdf* of s_0 and H_0 was a result of best fit to data from wave measurements with wave rider buoys made at three different deep water locations at sea on the Norwegian continental shelf. The Myrhaug and Fouques (2012) joint *pdf* of the normalized variables $\xi = \xi_0/\xi_{rms}$ and $h = H_0/H_{0rms}$ is

$$p(\xi, h) = p(\xi|h)p(h) \tag{5}$$

where the marginal *pdf* of h, $p(h)$, and the conditional *pdf* of ξ given h, $p(\xi|h)$, are given as a two-parameter Weibull *pdf* and a lognormal *pdf*, respectively, as

$$p(h) = \frac{2.39\,h^{1.39}}{1.05^{2.39}} \exp\left[-\left(\frac{h}{1.05}\right)^{2.39}\right]; \; h \ge 0 \tag{6}$$

$$p(\xi|h) = \frac{1}{\sqrt{2\pi}\sigma_\xi \xi} \exp\left[-\frac{(\ln\xi - \mu_\xi)^2}{2\sigma_\xi^2}\right] \tag{7}$$

The mean value μ_ξ and the variance σ_ξ^2 of $\ln\xi$ are given as, respectively,

$$\mu_\xi = \begin{cases} -0.048 + 0.5105h - 0.279h^2 & \text{for } h < 1.7 \\ -0.125\arctan[4(h-1.7)] + 0.0135 & \text{for } h > 1.7 \end{cases} \tag{8}$$

$$\sigma_\xi^2 = -0.0375\arctan[1.75(h - 1.20)] + 0.05625 \tag{9}$$

One should notice that here the function $\arctan\theta$ is defined for an angle θ in the range $-\pi/2 < \theta < \pi/2$. Furthermore, based on best fit to data the *rms* values H_{0rms} and ξ_{rms} are

$$H_{0rms} = 0.714\,H_s \tag{10}$$

$$\xi_{rms} = \frac{m}{\sqrt{0.7\,s_m}}, \quad s_m = \frac{H_s}{(g/2\pi)\,T_z^2} \tag{11}$$

where H_s is the significant wave height and s_m is a characteristic wave steepness for the sea state defined in terms of H_s and the mean zero-crossing wave period T_z. A plot of $p(\xi, h)$ is given in Fig. 6 in Myrhaug and Fouques (2012).

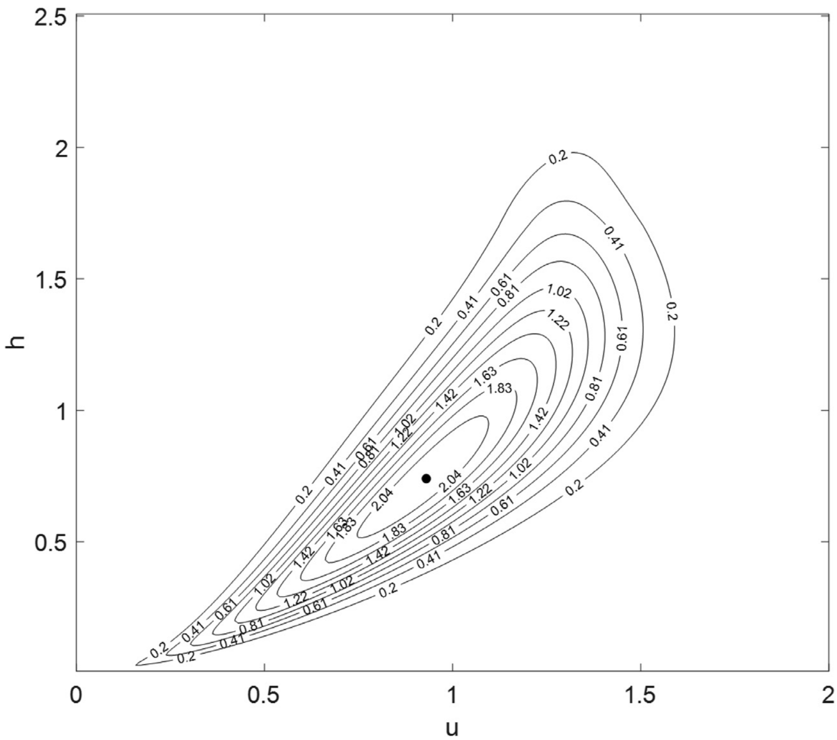

Figure 1 Isocontours of $p(u, h)$ with the peak value $p_{max} = 2.24$ located at $u = 0.93$ and $h = 0.74$.

The joint *pdf* u and h is obtained from the joint *pdf* of ξ and h by a change of variables from ξ, h to u, h yielding

$$p(u, h) = p(u|h)\, p(h) \tag{12}$$

where $p(h)$ is given in Eq. (6). This change of variable from ξ to u only affects $p(\xi|h)$ since $\xi = u^{\frac{1}{d}} h^{-\frac{1}{2d}}$, and by using the Jacobian $|d\xi/du| = u^{\frac{1}{d}-1} h^{-\frac{1}{2d}}/d$, this gives the following conditional lognormal *pdf* of u given h

$$p(u|h) = \frac{1}{\sqrt{2\pi}\sigma_u u} \exp\left[-\frac{(\ln u - \mu_u)^2}{2\sigma_u^2}\right] \tag{13}$$

Here μ_u and σ_u^2 are the mean value and the variance, respectively, of $\ln u$, given by

$$\mu_u = \frac{1}{2}\ln h + d\mu_\xi \tag{14}$$

$$\sigma_u^2 = d^2\, \sigma_\xi^2 \tag{15}$$

where μ_ξ and σ_ξ^2 are given in Eqs. (8) and (9), respectively.

Figure 1 shows the isocontours of $p(u, h)$ with the peak value $p_{max} = 2.24$ located at $u = 0.93$ and $h = 0.74$. Overall, it appears that u increases as h increases up to about $h = 1.5$ above which the *pdf* is nearly symmetric with respect to u of about 1.25.

Figure 2 depicts the conditional *pdf* of u given h for $h = 0.5$, 1.0, 1.4 and 2.1, i.e. corresponding to wave heights equal to $H_{0rms}/2$, H_{0rms}, H_s and $1.5H_s$, respectively (since $H_0 = hH_{0rms}$ and by using Eq. (10)). From Figure 2 it is observed that the peak values are shifted to higher values of u with varying peak values as h increases. The *pdfs* for $h = 1.4$ and 2.1 are nearly symmetric with respect to u. The features observed here are consistent with those observed in Figure 1.

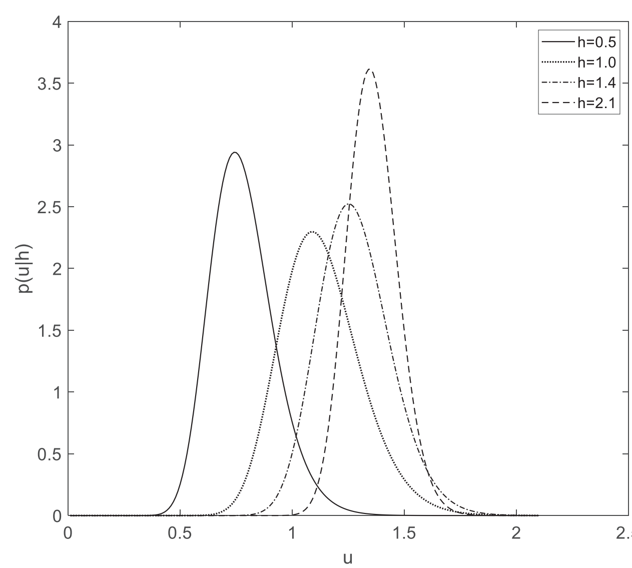

Figure 2 $p(u|h)$ versus u for h= 0.5, 1.0, 1.4 and 2.1.

According to Eq. (3), u is defined within a finite interval, and thus the conditional *pdf* of u given h follows the truncated lognormal *pdf*:

$$p_t(u|h) = \frac{1}{N_1} p(u|h), \quad u_1 < u < u_2 \tag{16}$$

$$N_1 = \Phi\left[\frac{\ln u_2 - \mu_u}{\sigma_u}\right] - \Phi\left[\frac{\ln u_1 - \mu_u}{\sigma_u}\right] \tag{17}$$

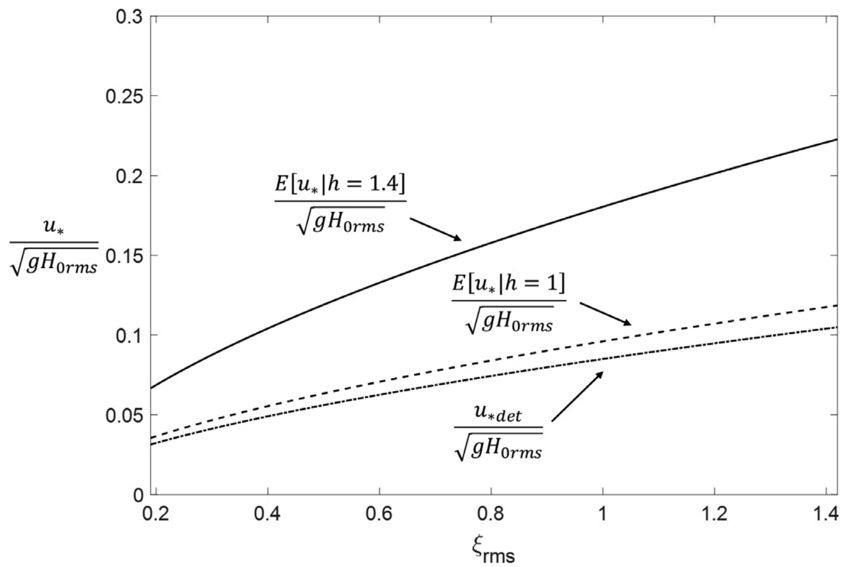

Figure 3 $u_{*\,\det}/\sqrt{gH_{0rms}}$, $E[u_*|h=1]/\sqrt{gH_{0rms}}$ and $E[u_*|h=1.4]/\sqrt{gH_{0rms}}$ versus $\xi_{rms}$from the lower to the upper lines, respectively.

where Φ is the standard Gaussian cumulative distribution function (*cdf*) given by

$$\Phi(\gamma) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\gamma} e^{-t^2/2}\, dt \quad (18)$$

It should be noted that the results in Figures 1 and 2 are modified for the truncated *pdf* in Eqs. (16) and (17) since the results depend on the interval limits u_1 and u_2. That is, the results will vary depending on the beach slope and the wave conditions as demonstrated in the subsequent example. However, overall Figures 1 and 2 exhibit the main features of the joint *pdf* of u and h.

Here the results will be exemplified by considering the conditional expected value of u given h as calculated from the truncated *pdf* in Eqs. (16) and (17) as (Bury, 1975)

$$E[u|h] = \int_{u_1}^{u_2} u\, p_t(u|h)\, du = \frac{N_2}{N_1} \exp\left(\mu_u + \frac{1}{2}\sigma_u^2\right) \quad (19)$$

$$N_2 = \Phi\left[\frac{\ln u_2 - (\mu_u + \sigma_u^2)}{\sigma_u}\right] - \Phi\left[\frac{\ln u_1 - (\mu_u + \sigma_u^2)}{\sigma_u}\right] \quad (20)$$

An alternative to the present stochastic method is to apply Eq. (1) for breaking random waves by using it for an equivalent sinusoidal wave, i.e. by replacing H_0 with H_{0rms} and T with T_z. By referring to this as the deterministic method, the result is

$$u_{*\,\det} = 0.085\, \xi_{rms}^{0.6} \sqrt{gH_{0rms}} \quad (21)$$

Figure 3 shows $u_{*\,\det}/\sqrt{gH_{0rms}}$ versus ξ_{rms} in the range $0.19 < \xi_{rms} < 1.42$ according to Eq. (21). The other two lines depict $E[u_*|h=1]/\sqrt{gH_{0rms}} = 1.1303 u_{*\,\det}$ and $E[u_*|h=1.4]/\sqrt{gH_{0rms}} = 2.1228 u_{*\,\det}$ versus ξ_{rms}, which are based on that $E[u|h=1] = 1.1303$ and $E[u|h=1.4] = 2.1228$, respectively, using the non-truncated *pdf*, i.e. Eqs. (19) (with $N_2/N_1 = 1$), (8), (9), (14) and (15). Thus, Figure 3 shows the relative differences between u_* using the deterministic method and u_* given the wave heights corresponding to H_{0rms} and H_s, respectively. As referred to regarding the results in Figures 1 and 2, the results in Figure 3 will also be modified using the truncated *pdf* in Eqs. (19) and (20) depending on the beach slope and the wave conditions, i.e. also affecting the range of validity.

One should notice that the bed shear stress is not a function of the local water depth. However, as demonstrated in the example by comparing the present results with one case from Thornton and Guza (1983) (hereafter referred to as TG83), the wave energy dissipation due to bottom friction beneath breaking random waves given in Eq. (36) contains the local water depth h_d. Moreover, another factor not accounted for is the bed roughness. The results are valid for hydraulically smooth flow for which the roughness Reynolds number $u_* k_s/\nu < 5$, $k_s = 2.5 d_{50}$ is Nikuradse's bed roughness, d_{50} is the median grain size diameter, and ν is the kinematic viscosity of the fluid (Soulsby, 1997). This aspect is addressed further in the subsequent example.

The present method should be validated by comparing predictions with data from measurements. Data from bed shear stress measurements for spilling and plunging breaking random waves on slopes associated with well-defined random wave conditions in deep water are required for making a proper validation of the method. To the authors' knowledge such data are not available in the open literature. Thus, first an example is included to demonstrate the application of estimating the bed shear stress caused by breaking random waves on slopes over hydraulically smooth beds, and second by comparing the present model predictions with one case from TG83 estimating the wave energy dissipation due to bottom friction beneath breaking random waves.

First, results are exemplified for $h = 1$, i.e. a deep water wave height corresponding to H_{0rms}. Thus, substitution of this in Eqs. (8), (9), (14) and (15) yields

$$\mu_u = 0.1101 \quad (22)$$

$$\sigma_u^2 = 0.0248 \quad (23)$$

The given flow conditions are:

- $H_s = 2\,\text{m},\ T_z = 6.5\,\text{s}.$

This gives $H_{0rms} = 1.4\,\text{m}$ from Eq. (10) and $s_m = 0.03,\ \xi_{rms} = 6.9\,m$ from Eq. (11).

- Beach slopes $m = 0.05,\ 0.10.$

For these slopes, $\xi_{rms} = 0.345$ and 0.69, respectively. Thus, by taking Eq. (2) to be valid for random waves replacing ξ_0 with ξ_{rms}, these two values of ξ_{rms} correspond to spilling and plunging breakers, respectively.

For spilling breakers ($m = 0.05$) this gives:

$$u_1 = (0.19/0.345)^{0.6} = 0.6991 \tag{24}$$

$$u_2 = (1.42/0.345)^{0.6} = 2.3371 \tag{25}$$

which substituted in Eqs. (17), (19) and (20) yields

$$N_1 = 0.9985,\ N_2 = 0.9991 \tag{26}$$

$$E[u|h = 1] = \frac{0.9991}{0.9985} \cdot 1.1303 = 1.1310 \tag{27}$$

For plunging breakers ($m = 0.10$) this gives:

$$u_1 = (0.19/0.69)^{0.6} = 0.4613 \tag{28}$$

$$u_2 = (1.42/0.69)^{0.6} = 1.5419 \tag{29}$$

which substituted in Eqs. (17), (19) and (20) yields

$$N_1 = 0.9798,\ N_2 = 0.9710 \tag{30}$$

$$E[u|h = 1] = \frac{0.9710}{0.9798} \cdot 1.1303 = 1.1201 \tag{31}$$

It should be noted that for the non-truncated *pdf* the result is $E[u|h = 1] = 1.1303$, i.e. the effect of truncation is insignificant in this example.

Finally, based on the truncated *pdf*, the conditional expected value of the friction velocity for spilling breakers using Eqs. (4) and (27) becomes

$$E[u_*|H_0 = H_{0rms}] = 1.1310 \cdot 0.085 \cdot 0.345^{0.6}\sqrt{9.81 \cdot 1.4} = 0.188\,\text{m/s} \tag{32}$$

Similarly, the conditional expected value of the friction velocity for plunging breakers becomes

$$E[u_*|H_0 = H_{0rms}] = 1.1201 \cdot 0.085 \cdot 0.69^{0.6}\sqrt{9.81 \cdot 1.4} = 0.282\,\text{m/s} \tag{33}$$

Furthermore, by using Eq. (21) it follows that the stochastic to deterministic ratio, $E[u_*|H_0 = H_{0rms}]/u_{*\text{det}}$, becomes 1.1310 and 1.1201 for spilling and plunging breakers according to Eqs. (32) and (33), respectively, i.e. ratios of about 1.1 in this example.

As referred to the present method is valid for hydraulically smooth flow, i.e., for $u_* d_{50}/\nu < 2$, and consequently for $d_{50} < 2\nu/u_*$. Thus, by taking $\nu = 1.36 \cdot 10^{-6}\ \text{m}^2/\text{s}$, it follows by substituting for u_* from Eqs. (32) and (33) that the results are valid for $d_{50} < 15$ mm and $d_{50} < 10$ mm for spilling and plunging breakers, respectively. These upper values of grain sizes correspond to gravel representing pebble (Soulsby, 1997).

Second, the wave energy dissipation due to bottom friction beneath breaking random waves is estimated and compared with one case from TG83. TG83 calculated the average frictional energy dissipation by first time-averaging and then averaging over all Rayleigh-distributed wave heights yielding (see their Eq. (40))

$$E_{DTG} = \rho c_f \frac{1}{16\sqrt{\pi}} (0.42\sqrt{gh_d})^3 \tag{34}$$

where c_f is the bed friction coefficient, and h_d is the water depth. This result is based on assuming shallow water waves and that $H_{rms} = 0.42 h_d$.

The present method is used to estimate the wave energy dissipation by first considering that for regular waves as $E_D = \frac{1}{T}\int_0^T \tau(t)u(t)\,dt$ where $\tau(t) = \rho {u_*}^2 u(t)|u(t)|$ is the time-varying bed shear stress, t is the time, $u(t) = U_0 \sin\omega t$ is the horizontal regular wave-induced velocity with the amplitude U_0, and $\omega = 2\pi/T$ is the angular wave frequency. Substitution of this gives $E_D = (4/3\pi)\rho {u_*}^2 {U_0}^3$. Further, in shallow water $U_0 = (H/2)\sqrt{g/h_d}$, giving

$$E_D = \rho {u_*}^2 \frac{1}{6\pi} (H\sqrt{g/h_d})^3 \tag{35}$$

Then, application of this for breaking random waves in shallow water with $H = H_{rms} = 0.42 h_d$, the deep water wave height as $H_0 = H_{0rms}$, and taking u_* as $E[u_*|h = 1]$, yields

$$E_D = \rho \frac{1}{6\pi} (0.42\sqrt{gh_d})^3 u_{*\text{det}}^2 (E[u|h = 1])^2 \tag{36}$$

The present result is compared with that by TG83 by taking the ratio of Eq. (36) (using Eq. (21)) and Eq. (34) giving

$$\frac{E_D}{E_{DTG}} = \frac{g}{c_f} \frac{8}{3\sqrt{\pi}} 0.085^2 \xi_{rms}^{1.2} H_{0rms} (E[u|h = 1])^2 \tag{37}$$

Now the wave conditions representative for Torrey Beach, California during November 1978 used by TG83 is adopted: $c_f = 0.01$, beach slope $\beta = 0.02$, $H_{0rms} = 0.5$ m, spectral peak period $T_p = 14$s. However, the present method uses T_z, and by assuming a Pierson-Moskowitz wave amplitude spectrum (Tucker and Pitt, 2001) $T_z = 0.7T_p = 9.8$s, which gives $\xi_{rms} = 0.35$ from Eqs. (10) and (11), i.e., spilling breakers. By taking $E[u|h = 1] = 1.13$ from the previous example, substitution in Eq. (37) gives the ratio $E_D/E_{DTG} = 1.9$. TG83 estimated the frictional dissipation to be less than 3% of the dissipation due to breaking for depths larger than 0.2 m within the surf zone, which was similar to results for laboratory beaches. Thus, the present method estimates the frictional dissipation to be less than 5% to 6% of the dissipation due to breaking.

A summary and the main conclusions of this work are as follows:

A simple analytical method for estimating the bed shear stress caused by breaking random waves on slopes using deep water wave statistics is presented. The results are achieved by adopting the Sumer et al. (2013) bed shear stress formula for spilling and plunging breaking regular waves on hydraulically smooth slopes during wave runup assuming it to be valid for breaking individual waves within a sea state of random waves. The statistical properties of the bed shear stress and the wave height in deep water is then derived by transformation of the Myrhaug and Fouques (2012) joint distribution of surf similarity parameter and wave height for individual random waves in deep water. Results are given in terms of the conditional mean value of the maxima of mean bed shear stress during wave runup given the wave height in deep water.

Example of results are provided for spilling and plunging breaking random waves corresponding to typical field conditions. For this particular example the present method yields bed shear stress values which are about ten percent larger than those obtained by using the Sumer et al. (2013) formula for breaking regular waves replacing the wave height and surf parameter with their corresponding *rms* values.

Another example compares the present model predictions with one case from Thornton and Guza (1983) by estimating the wave energy dissipation due to bottom friction beneath breaking random waves representing field conditions. The present method estimates the frictional dissipation to be a factor 1.9 larger than that given by Thornton and Guza.

As stated by Sumer et al. (2013) caution must be observed when using the present formula outside its range of validity, which also is the case for the presented method. However, comparison with data are required in order to validate the method. Meanwhile this approach should enhance the possibilities of assessing further the bed shear stress for spilling and plunging breaking random waves on slopes in laboratory and field conditions.

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