Theoretical Physics Seminar in Trondheim
ISSN 0365-2459
No. 4, 1971

Covariant Quantum Electrodynamics in Terms of a Possible Ether Flow

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Abstract, updated 2021

This paper from 1971 has a special history. At that time the present author was working with the covariant quantization procedure for an electromagnetic field in a dielectric medium. In such a theory, the commutator for the potentials contains the medium's four-velocity V_μ. What appeared to be a natural idea, was to suggest that V_μ is not only a formal remedy (as in the Gupta-Bleuler theory), but that it reflects a concept of deeper physical significance, namely the four-velocity of the all-pervading ether. The idea was actually related to that put forward by Dirac in 1951-1953, although there in a somewhat different context. This proposal was written up in a paper published in the archival series "Theoretical Physics Seminar in Trondheim", No. 4, 1971. Submitting afterwards the paper to Il Nuovo Cimento, I received a report from a referee stating that all mention of an ether was pure nonsense, far outside of any reasonable physics. So I gave the idea up at that time. Some years have however elapsed since then, and as we know the ether concept is at present often used in theoretical physics, commonly called the Einstein-Dirac ether. I was probably far ahead of my time. I am posting an extract of this old paper on the arXiv now, since I was recently recommended to do so. A link to the original paper is given.

1. Introduction

In classical electrodynamics a free field may be described by the Lagrangian density

$$\mathcal{L} = -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} \quad , \tag{1}$$

where * $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. The variational equations obtained from (1) are

$$\Box A_\mu - \partial_\mu \partial^\nu A_\nu = 0 \quad , \tag{2}$$

which are equal to Maxwell's equations

$$\partial_\lambda F_{\mu\nu} + \partial_\mu F_{\nu\lambda} + \partial_\nu F_{\lambda\mu} = 0 \tag{3a}$$

$$\partial^\nu F_{\mu\nu} = 0 \tag{3b}$$

when these are solved in terms of the potentials.

In quantum electrodynamics one may start from the Fermi gauge Lagrangian density

$$\mathcal{L} = -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} - \tfrac{1}{2}(\partial^\mu A_\mu)^2 \quad . \tag{4}$$

Then the momenta canonically conjugate to A_μ become

$$\pi_\mu = \frac{\partial \mathcal{L}}{\partial \partial_0 A_\mu} = F_{\mu 0} - g_{\mu 0} \partial^\nu A_\nu \quad , \tag{5}$$

and the canonical quantization procedure can be carried out for all values of μ on the same footing. The variational equations obtained from (4) are

$$\Box A_\mu = 0 \quad , \tag{6}$$

so that the physical equations (2) will evidently be satisfied if the Lorentz condition

$$\partial^{\mu} A_{\mu} = 0 \tag{7}$$

can be imposed as an operator condition.

It is however well known that the Lorentz condition (7) runs into conflict with the canonical commutation rules. A discussion of the difficulties encountered is given by Källén in his Handbuch article[1]. Our first task in the following will be to trace out the d[illegible]r reasons for this problem, the gauge problem of electrodynam[illegible] One should not be content merely by considering the gauge problem as being due to an accidental conflict between the Lorentz condition and the canonical commutation rules. Rather, care should be exerted to trace out to what extent the gauge problem has its roots in Maxwell's equations themselves, which form the very core of electrodynamics. In a simple analysis in Sect. 2 it is shown that the general field-theoretical assumptions: (1) four-vector transformation properties for the potential A_{μ}, (2) invariance of the vacuum state, and (3) spectral condition, are <u>not</u> reconcilable with Maxwell's equations solved in the form (2). Therefore the specific canonical quantization method should not be kept responsible for the gauge problem; the problem is rather due to the fact that eqs. (2) do not fit into a general field theoretical scheme. Recent work by Strocchi[2], strictly within the framework of axiomatic field theory, confirms this statement.

After this analysis of the origin of the gauge problem our next task in the following will be to work out one specific quantization procedure which reconciles Maxwell's equations, solved in the form (2), with the general assumptions (1) - (3) above. In order to accomplish this we permit the existence of an extra parameter V_{μ} in the formalism, being of the nature of a four-velocity and thus satisfying the condition $V^2 \equiv V_\mu V^\mu = 1$. The formalism is constructed as compactly covariant as possible by expanding each Fourier component of the potential into a covariant k-dependent basis $e_\mu^{(\lambda)}(k)$. Thereby the components of the potential along the basis vectors turn out to be Lorentz invariant entities. We quantize only two of these components and retain the other two components as c-numbers. As the quantization is carried through in this way, the method bears some resemblance with the Coulomb gauge method. The difference is that we work in the Fermi gauge instead of in the Coulomb gauge and therefore have the freedom to perform restricted gauge transformations, $A_\mu \rightarrow A_\mu + \partial_\mu \chi$, where χ satisfies the relation $\Box \chi = 0$. Such a gauge transformation affects the physically unimportant c-number potential components and leaves the physically important components unaffected. The Hilbert (Fock) space gets a Lorentz invariant meaning, and there is no need of introducing an indefinite metric. The vacuum is defined in a gauge dependent way as the state in which both the number of physical photons and the c-number potential components are equal to zero. This corresponds in a most direct way to the definition of the vacuum in the Gupta-Bleuler theory as the state in which both the physical transverse photons and the unphysical longitudinal and scalar photons are absent. With this definition of the vacuum we obtain the same expression for the photon propagator as in the Coulomb gauge case.

Having worked out the quantization procedure we shall then, as the third part of our work, discuss the significance of the parameter V_μ. One might simply consider V_μ as a mathematical quantity by means of which one can make the theory more appealing from a physical point of view, as the Maxwell equations become valid when solved in terms of the potentials, and as the formal covariance is maintained together with a positive definite metric in the

Hilbert space. It is tempting, however, to go a step further and inquire whether V_μ may have a much deeper physical significance. In Sect. 5 we venture to discuss the possibility that V_μ is the four-velocity of the all-pervading ether. With this we do not mean to go back to the picture of the ether that one had in the 19th century; the ether idea must be introduced in accordance with the principle of relativity and also in accordance with quantum mechanics. As we shall see, it is just by picturing the ether as a quantal object that we are able to introduce it without thereby coming into conflict with the principle of relativity according to which all inertial systems are equivalent. In this context we make use of ideas that have been put forward earlier by Dirac(3), although he introduced them in a different connection. It is thus found possible to imagine an ether without thereby coming into conflict with the fundamental principles of relativity and quantum mechanics. However, the identification of the four-velocity of the ether with our parameter V_μ cannot be anything else than a pure conjecture at present. Further, we are not aware of any possibility to make a direct experimental test of whether there is an ether. The theory is nonlocal, but the effect of this nonlocality disappears in all expressions that can be compared with experiment. Yet it lies very close at hand to assume that the ether, if it exists, is intimately connected with the observable properties of the vacuum, as for instance the vacuum fluctuations. The vacuum properties make it natural to imagine that the vacuum is composed of some kind of matter. In view of these features we have therefore found it desirable to put forward the ether hypothesis, as an idea that seems to be worth some attention.

Finally we give some references to earlier works in this field. As far as the discussions in Sections 2 and 5 are concerned, the connections with the earlier treatments by Strocchi and Dirac, respectively, have already been pointed out. The technique with the covariant expansion of the potentials shown in Sect.4 has been made use of before, in an analysis of phenomenological electrodynamics(4). Otherwise, we find that the recent work by Schmutzer(5) is the one whose spirit comes closest to our own. Some resemblance is also found with the quantization method proposed by Mathews(6), since he quantizes only two Lorentz invariant polarization components of the potential. The difference from our kind of approach consists essentially in the fact that he does not make use of the four-vector V_μ, and therefore is met with some ambiguities in the distinction between the various polarization components. We shall briefly return to this point in Sect.4. Other covariant treatments have been given by Evans and Fulton(7) and by Moses(8), the latter using a group-theoretical language.

2. On the Origin of the Gauge Problem

The straightforward explanation for the difficulties encountered in the quantization of the electromagnetic field is that one carries through the canonical quantization procedure for all the polarization components of π_μ and A_μ, without regard to the Lorentz condition, and therefore accidentally happens to run into conflict with the latter when it is introduced in the theory later on. However, the gauge problem has very deep roots. We may first note that the conventional canonical procedure is based upon one particular covariant gauge, namely, the Fermi gauge as corresponding to the Lagrangian density (4). It is just in the Fermi gauge that the gauge condition, which is being introduced in the theory in order to attain correspondence with the Maxwell field, takes on the simple form (7). In view of this, it lies at hand first to inquire whether there may exist some other covariant gauge, corresponding to different variational equations and a different gauge condition, which is able to yield results more easily reconcilable with the Maxwell theory. However, there is no sign which indicates that any simplification can be obtained in this way. Specifically, B. Lautrup(9) has examined the formulation of quantum electrodynamics in a whole class of covariant gauges corresponding to the following Lagrangian density:

$$\mathcal{L} = -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} - \Lambda \partial^{\mu} A_{\mu} + \tfrac{a}{2} \Lambda^2 \quad . \tag{8}$$

Here $\Lambda = \Lambda(x)$ is a Lagrange multiplier field which is introduced to take care of the gauge condition, and a is the gauge parameter. The Fermi gauge corresponds to the case $a = 1$. The variational equations for A_μ obtained from the expression (8) are

$$\Box A_\mu - \partial_\mu \partial^\nu A_\nu = -\partial_\mu \Lambda \quad , \tag{9}$$

and the gauge condition takes on the general form

$$\partial^\mu A_\mu = a\Lambda \quad . \tag{10}$$

Lautrup has shown that although a consistent formulation of quantum electrodynamics can be given in any of the covariant gauges considered here, the Fermi gauge is by far the simplest choice. For instance, the field energy is not diagonalizable in any other gauge than the Fermi gauge. Thus we must conclude that the core of the gauge problem is not connected with the choice of the specific Fermi gauge in the conventional formulation of quantum electrodynamics.

It is now natural to raise the following question: Instead of proceeding in the somewhat indirect canonical way where the field equations (2) are divided into two separate sets of equations, respectively the variational equations and the gauge condition, is it possible to make a more direct analysis of how eqs.(2) fit into a general field-theoretical framework? Obviously, a solution of eqs.(2) need not satisfy eqs.(6) and (7) separately; the requirement that a physical solution shall satisfy eqs.(6) and (7) may prove to be unnecessarily strong.

It turns out that the above question has an affirmative answer. Specifically, one can show that eqs.(2) run into conflict with any conventional quantum field theory which is constructed on the basis of the following general postulates:

(1) Existence of a unitary representation of the Poincaré group, $\{a,\Lambda\} \to U(a,\Lambda)$ such that

$$U(a,\Lambda)A_\mu(x)U^{-1}(a,\Lambda) = \Lambda_\mu^{-1}{}_\nu A^\nu(\Lambda x+a) \quad . \tag{11}$$

(2) Existence and invariance of the vacuum state:

$$U(a,\Lambda)|0\rangle = |0\rangle \quad .$$

The vacuum state is assumed to possess zero energy and momentum.

(3) Spectral condition, i.e. positivity of the energy of every physical state in every inertial frame.

From these postulates one can derive the following general expression for the two-point function:

$$\langle 0|A_\mu(x)A_\nu(y)|0\rangle = -\frac{1}{(2\pi)^3}\int dp\ \theta(p)(\rho_1(p^2)g_{\mu\nu} + \rho_2(p^2)p_\mu p_\nu)e^{-ip\cdot(x-y)} \quad , \tag{12}$$

where ρ_1, ρ_2 are spectral functions. Use of eqs.(2) leads to the relation $\rho_1 = 0$. Thus

$$\langle 0|A_\mu(x)A_\nu(y)|0\rangle = \frac{1}{(2\pi)^3}\partial_\mu\partial_\nu\int dp\ \theta(p)\rho_2(p^2)e^{-ip\cdot(x-y)} \quad , \tag{13}$$

and this leads to[9]

$$\langle 0|F_{\mu\nu}(x)F_{\rho\sigma}(y)|0\rangle = 0 \quad . \tag{14}$$

This result is expected not to hold for a real physical theory. Eq.(14) would for instance imply the unacceptable consequence that the field operators were commutative even for light-like separations, $(x-y)^2 = 0$. Thus we must conclude that the gauge problem is primarily not a consequence of the fact that the Maxwell equations, solved in the form (2), conventionally are replaced by variational equations and a gauge condition. The problem arises already on the level of eqs.(2) themselves, as these equations get into conflict with the above field-theoretical postulates. Note that in order to obtain the result (14) it is not necessary to assume that eqs.(2) hold on operator form. It is sufficient that they hold when applied to the vacuum state:

$$(\Box A_\mu - \partial_\mu \partial^\nu A_\nu)|0\rangle = 0 \quad . \tag{15}$$

The above results are confirmed by the results recently obtained by Strocchi[2] in the framework of axiomatic field theory. He assumed the field operators to be operator-valued distributions (not necessarily tempered distributions) in the Hilbert space. The field-theoretical postulates were chosen somewhat different from the postulates (1)-(3) above. Instead of using the spectral condition (our condition (3)), he assumed in ref.2a that the two-point function for the fields, $W_{\mu\nu\rho\sigma}(x-y) = \langle 0|F_{\mu\nu}(x)F_{\rho\sigma}(y)|0\rangle$, can be regarded as the boundary value of an analytic function $W_{\mu\nu\rho\sigma}(z)$, analytic in the forward tube. In ref.2b he gave up the postulate that A_μ be a four-vector under the Lorentz group and replaced it by the postulate of weak local commutativity for the potential components. In each case he was able to derive the result (14), showing the conflict between eqs.(2) and the set of postulates chosen.

In view of these features we are therefore on safe ground when asserting that eqs.(2) cannot be fitted in with general postulates about relativistic field theory.

3. Quantization: Preliminary Remarks

From the above discussion it is clear that any realistic method chosen in the quantization of the electromagnetic field will have to possess one of the following two properties: either, that

(α) Maxwell's equations, solved in terms of the potentials, are abandoned,

or, that

(β) some change is made in the conventional set of postulates about relativistic quantum field theory.

(The additional possibility that exists to construct a theory which possesses both properties (α) and (β) is not an actual possibility in an attempt to bridge electrodynamics and quantum field theory.) The well known Gupta-Bleuler method[10] belongs to the type (α). In this case eqs.(6) are required to hold as operator equations while the Lorentz condition (7) is replaced by the nonlocal condition

$$\partial^{\mu} A_{\mu}^{(+)} |\psi\rangle = 0 \quad , \tag{16}$$

where $\partial^{\mu} A_{\mu}^{(+)}$ is the positive frequency part of the operator $\partial^{\mu} A_{\mu}$. As eqs.(2) must be expected to hold when the mean values of the potentials are taken, it follows that all states in the Hilbert space cannot be physically realizable as this would lead us back to eqs.(2) on operator form. Accordingly, the Hilbert space becomes equipped with an indefinite metric and contains state vectors also with negative norm. The advantages of the Gupta-Bleuler method are that the postulates (1)-(3) in the preceding Section can be maintained, and further that the potentials are commutative for space-like separations. We may also refer to the result of axiomatic field theory that A_μ can be defined as a weakly local and Lorentz covariant operator-valued distribution only in a Hilbert space with an indefinite metric[11].

However, although the Gupta-Bleuler method gives a consistent description of quantum electrodynamics, it is clear that the indefinite metric and the corresponding unphysical (longitudinal and scalar) photons are features of the theory that are not quite satisfactory. We shall not study the various implications of this method here (see ref. 2b), but turn our attention to methods of the type (β) above. In this case some change has to be made in the basic assumptions about quantum field theory, and one usually chooses to give up the requirement that A_μ be a four-vector under the Lorentz group. The best known example of this type is the transverse Coulomb gauge method[12], giving the following transformation equation for the potential under the homogeneous Lorentz group:

$$U(0,\Lambda)A_\mu(x)U^{-1}(0,\Lambda) = \Lambda^{-1}_{\mu\nu}A^\nu(\Lambda x) + \partial_\mu\Phi(x,\Lambda) \quad . \tag{17}$$

Here $\Phi(x,\Lambda)$ is an operator gauge function by means of which the gauge condition $\nabla\cdot\underline{A} = 0$ can be made covariant. The advantages of the transverse Coulomb gauge are that only physical photons appear in the formalism, that eqs. (2) are kept valid, and that the metric of the Hilbert space is positive definite. Another method of the type (β), although less known, is to make use of the Valatin gauge[13] and work with <u>three</u> kinds of photons.

The method considered in the next Section also belongs to the type (β), in so far as the field equations (2) are required to hold, and as we change the basic postulates of quantum field theory in the sense that we permit the existence of an extra four-vector V_μ in the formalism. Thereby the above discussed conflict between eqs.(2) and the postulates (1)-(3) given in Sect.2 will no longer be present. We work in the Fermi gauge. The discussion about the physical significance of V_μ shall be postponed until Sect.5; for the present, we look upon V_μ merely as a classical four-velocity.

In our opinion the theory constructed in this way has some definite advantages from a physical point of view. For the sake of convenience we summarize the following properties of the theory:

(i) The potential A_μ transforms like a four-vector under Lorentz transformations. $A' = \Lambda A$.

(ii) Equations (6) and (7), and accordingly also eqs.(2), are satisfied as operator equations.

(iii) Each Fourier component of the potential is decomposed into a covariant basis $e_\mu^{(\lambda)}(k)$ so that the potential components along the basis vectors become Lorentz invariants. Only two of these potential components are subject to quantization. The quantized components are unaffected by restricted gauge transformations.

(iv) The potential is nonlocal, a property which is, however, of no physical significance.

Note in particular that (i) and (ii) now become compatible properties. The reason for this is evidently the presence of the four-velocity V_μ in the theory. Specifically, the most general expression for the vacuum expectation value of the product of two potential components will now, apart from the terms shown on the right hand side of eq.(12), also contain terms of the form $p_\mu V_\nu$,

$p_\nu V_\mu$, and $V_\mu V_\nu$. Therefore we will no longer find the strong result (14). Finally, the quantization method chosen implies that the metric of the Hilbert space is positive definite, and no unphysical particles appear in the theory.

4. Quantization

We shall now quantize the free electromagnetic field, by imposing the fundamental commutation rules on the Fourier components of the potential. Our first task is then to construct a Fourier expansion which exhibits the Lorentz covariance in a compact way. To this end we find it convenient to make use of the same method as we have utilized before in connection with phenomenological electrodynamic theory of the radiation field within a material medium(14,4), viz., to select a certain inertial frame $\overset{o}{K}$ in which the field quantities are required to satisfy boundary conditions at the walls of a large box with volume $\overset{o}{\mathcal{V}}$. The frame $\overset{o}{K}$ is a completely arbitrary frame; this is an essential point of departure from the earlier phenomenological treatments wherein $\overset{o}{K}$ was naturally taken to be the rest frame of the medium. In another inertial frame K , with respect to which $\overset{o}{K}$ moves with the velocity $\underline{v}$, the periodicity conditions at the walls are in general lost. Instead, the Fourier component of the potential which corresponds to the wave vector $\underline{k}$ becomes periodic at the walls of a fictitious "box" with the volume

$$\mathcal{V}_k = \overset{o}{\mathcal{V}}[\gamma(1+\underline{v}\cdot\overset{o}{\underline{k}}/|\overset{o}{\underline{k}}|)]^{-1} , \tag{18}$$

where $\overset{o}{\underline{k}}$ is the wave vector in $\overset{o}{K}$ and $\gamma = (1-v^2)^{-\frac{1}{2}}$. If we introduce the Fourier expansion

$$A_\mu(x) = \sum_{\underline{k}} (2k_0 \overset{}{\mathcal{V}}_k)^{-\frac{1}{2}} (\exp[-ik\cdot x] a_\mu(k) + \exp[ik\cdot x] a_\mu^\dagger(k)) \qquad (19)$$

where $k_0 = |\underline{k}|$ it is clear that, because of the four-vector property of A_μ and the Lorentz invariance of the expression $2k_0 \mathcal{V}_k = 2\overset{o}{k}_0 \overset{o}{\mathcal{V}}$, the quantity $a_\mu(k)$ transforms like a four-vector under Lorentz transformations. Actually, we shall omit the normalization factor in (19) and work with the simple expansion

$$A_\mu(x) = \sum_{\underline{k}} (\exp[-ik\cdot x] a_\mu(k) + \exp[ik\cdot x] a_\mu^\dagger(k)) \quad . \qquad (20)$$

In connection with this expansion it must be borne in mind that the one-photon volume is now equal to $1/(2k_0)$ in any inertial frame. Further, in order to transform a sum over discrete values of $\underline{k}$ into an integral over $\underline{k}$ (as usual in the final step of actual calculations) we have to make use of the substitution

$$\sum_{\underline{k}} \longrightarrow (2\pi)^{-3} \int d\underline{k}/(2k_0) \quad .$$

Each Fourier component in (20) is now decomposed into a k-dependent basis:

$$a_\mu(k) = \sum_{\lambda=0}^{3} e_\mu^{(\lambda)}(k) a^{(\lambda)}(k) \quad , \qquad (21)$$

and we exploit the presence of the distinguished frame $\overset{o}{K}$ to introduce a set of covariant vectors $e_\mu^{(\lambda)}$. Namely, let $e_\mu^{(2)}$ be a real four-vector which satisfies the relations

$$V\cdot e^{(2)} = k\cdot e^{(2)} = 0 \qquad (22a)$$

$$e^{(2)}\cdot e^{(2)} = -1 \quad , \qquad (22b)$$

where V_μ is the four-velocity of $\overset{o}{K}$ with respect to K. Further, define the (pseudo) four-vector

$$e_\mu^{(3)} = -\frac{\varepsilon_{\mu\nu\rho\sigma} k^\nu V^\rho (e^{(2)})^\sigma}{k\cdot V} \tag{23}$$

where $\varepsilon_{0123} = 1$, satisfying

$$V\cdot e^{(3)} = k\cdot e^{(3)} = 0 \tag{24a}$$

$$e^{(3)}\cdot e^{(3)} = -1 \quad , \qquad e^{(2)}\cdot e^{(3)} = 0 \ . \tag{24b}$$

It should be noted that

$$\varepsilon_{\mu\nu\rho\sigma}(e^{(2)})^\rho (e^{(3)})^\sigma = -\frac{k_\mu V_\nu - V_\mu k_\nu}{k\cdot V} \quad ,$$

which in the frame $\overset{o}{K}$ implies

$$\underline{e}^{(2)} \times \underline{e}^{(3)} = \overset{o}{\underline{k}}/|\overset{o}{\underline{k}}| \ . \tag{25}$$

In accordance with this relation we shall see that $e_\mu^{(2)}$, $e_\mu^{(3)}$ can be interpreted physically as the covariant polarization vectors for the photon field. Now we shall introduce two four-vectors more which, however, cannot be interpreted as polarization vectors. Let us define

$$e_\mu^{(0)} = V_\mu \quad , \qquad e^{(0)}\cdot e^{(0)} = 1 \tag{26}$$

$$e_\mu^{(1)} = \frac{k_\mu - V_\mu k\cdot V}{k\cdot V} \quad , \qquad e^{(1)}\cdot e^{(1)} = -1 \ . \tag{27}$$

It can readily be verified that these vectors satisfy the following relations $(k^2=0)$:

$$g^{\mu\nu} e_\mu^{(\lambda)} e_\nu^{(\lambda')} = g^{\lambda\lambda'}$$

$$\sum_{\lambda,\lambda'=0}^{3} g_{\lambda\lambda'} e_\mu^{(\lambda)} e_\nu^{(\lambda')} = g_{\mu\nu} \ . \tag{28}$$

Moreover, it can be verified that the vectors $e_\mu^{(\lambda)}$ are all mutually orthogonal. Since one of them is time-like and the other three space-like, it follows[15] that they are linearly independent. It should be borne in mind that in eqs.(23) and (27) we have assumed that $k^2 = 0$.

The advantage of introducing the vectors $e_\mu^{(\lambda)}$ is evidently, in accordance with eq.(21), that the Fourier components $a^{(\lambda)}$ become <u>invariants</u> under restricted homogeneous Lorentz transformations, which are the only transformations of interest here. Hence, in the classical case the relation

$$a^{(\lambda)}(\Lambda k) = a^{(\lambda)}(k) \tag{29}$$

holds for $\lambda = 0,1,2,3$. Moreover, in the frame $\overset{o}{K}$ the three-dimensional vectors $\underline{e}^{(\lambda)}$, $\lambda = 1,2,3$, form an orthonormal set. Note, however, that the $\underline{e}^{(\lambda)}$ are in general not orthogonal in other inertial frames.

Now let us turn our attention to the quantization. Instead of postulating the usual canonical commutation rules for all directions λ,λ' we shall rather quantize only those two components which in $\overset{o}{K}$ are transverse to $\overset{o}{\underline{k}}$, i.e.

$$\begin{aligned} [a^{(\lambda)}(k), a^{(\lambda')\dagger}(k')] &= \delta_{\underline{k}\underline{k}'}\,\delta_{\lambda\lambda'} \quad , \quad \lambda = 2,3 \\ [a^{(\lambda)}(k), a^{(\lambda')\dagger}(k')] &= 0 \quad\quad\quad , \quad \lambda = 0,1 . \end{aligned} \tag{30}$$

The other commutators are put equal to zero. Since $a^{(0)}, a^{(1)}$ commute with all field quantities they shall simply be regarded as c-numbers. It should be noted that because of the covariance of the vectors $e_\mu^{(\lambda)}$ the components $a^{(0)}$, $a^{(1)}$ become Lorentz invariants, i.e. they transform according to eq.(29) and never get mixed up with the operator components $a^{(2)}$, $a^{(3)}$. The relation between $a^{(0)}$ and $a^{(1)}$ follows from the requirement that the Lorentz condition (7) shall hold:

$$a^{(0)} = a^{(1)} \quad . \tag{31}$$

Further, as we have assumed that $k_0 = |\underline{k}|$ in the Fourier expansions it is evident that eqs.(6), and therefore also the field equations (2), are valid.

Now the c-number components $a^{(0)}$, $a^{(1)}$ are unphysical quantities whose magnitude can be changed by a restricted gauge transformation $A_\mu \rightarrow A_\mu + \partial_\mu \chi$, where χ satisfies $\Box\chi = 0$. For in the Fourier space this transformation can be written as

$$a^{(\lambda)}(k) \rightarrow a^{(\lambda)}(k) - ik^{(\lambda)}\chi(k) \quad ,$$

where $k_\mu = \Sigma\, e_\mu^{(\lambda)} k^{(\lambda)}$. As the invariant components $k^{(2)}$, $k^{(3)}$ vanish in $\overset{o}{K}$ and hence also in any other inertial frame, it is clear that the gauge transformation changes the components $a^{(0)}$, $a^{(1)}$ and leaves the transverse components unchanged. And as a restricted gauge transformation leaves eqs.(6) and (7) invariant, it follows that the magnitude of the components $a^{(0)}$, $a^{(1)}$ is of no physical importance. In fact, a certain value of these components corresponds to a certain mixture of longitudinal and scalar photons in the Gupta-Bleuler theory. We have obviously the possibility to put $a^{(0)} = a^{(1)} = 0$, in which case we would obtain a covariant picture of the transverse Coulomb gauge in $\overset{o}{K}$. We shall not, however, restrict the gauge in this way.

As regards the physical components $a^{(2)}$, $a^{(3)}$ we first note that their Lorentz invariance is expressed conveniently as

$$a^{(\lambda)}(\Lambda k) = U(0,\Lambda)a^{(\lambda)}(k)U^{-1}(0,\Lambda) \quad , \quad \lambda = 2,3 \; , \tag{32}$$

so that the Lorentz invariance of the commutation rules can be expressed in a very compact manner as

$$[a^{(\lambda)}(\Lambda k), a^{(\lambda')^\dagger}(\Lambda k')] = [a^{(\lambda)}(k), a^{(\lambda')^\dagger}(k')] \quad . \tag{33}$$

Next, let us give the expression for the four-momentum operator of the field:

$$P_\mu = \sum_{\underline{k}} k_\mu \sum_{\lambda=2}^{3} a^{(\lambda)\dagger}(k) a^{(\lambda)}(k) \quad , \tag{34}$$

where the zero point contribution has been omitted. It is clear that the present formalism displays the Lorentz covariance of the theory in a very compact way. The most general state vector of the operator (34) can be written as

$$\prod_{\underline{k}} \prod_{\lambda=2}^{3} |N^{(\lambda)}(k)\rangle = \prod_{\underline{k}} \prod_{\lambda=2}^{3} \frac{[a^{(\lambda)\dagger}(k)]^{N^{(\lambda)}(k)}}{[N^{(\lambda)}(k)!]^{\frac{1}{2}}} |0\rangle \quad , \tag{35}$$

where $|N^{(\lambda)}(k)\rangle$ is the eigenstate for the number $N^{(\lambda)}(k)$ of photons with momentum $\underline{k}$, energy $k_0 = |\underline{k}|$, and transverse polarization λ :

$$a^{(\lambda)\dagger}(k) a^{(\lambda)}(k) |N^{(\lambda)}(k)\rangle = N^{(\lambda)}(k) |N^{(\lambda)}(k)\rangle \quad , \tag{36}$$

and the vacuum state $|0\rangle$ is the state of no physical photons. As we work only with the physical photons there is no need of introducing an indefinite metric in the Hilbert space. Further, each occupation number becomes an invariant under Lorentz transformations:

$$N^{(\lambda)}(\Lambda k) = N^{(\lambda)}(k) \tag{37}$$

(note that the value of λ is unchanged). Thus the Lorentz invariance holds not only for the operator component $a^{(\lambda)}$, but also for the eigenvalue of the physical bilinear combination $a^{(\lambda)\dagger} a^{(\lambda)}$. Note that this invariance property for each value of λ is intimately connected with the use of covariant polarization vectors $e_\mu^{(\lambda)}$.

Let us now find the covariant commutator for the potentials. In this context it is convenient to introduce the projection operator

$$\tau_{\mu\nu} = g_{\mu\nu} + \frac{\partial_\mu \partial_\nu}{(V\cdot\partial)^2} - \frac{V_\mu \partial_\nu + V_\nu \partial_\mu}{V\cdot\partial} \quad , \tag{38}$$

which picks out the quantized transverse part $\mathcal{A}_\mu$ of the potential:

$$\mathcal{A}_\mu = \tau_{\mu\nu} A^\nu \quad , \quad \mathcal{A}_\mu = \tau_{\mu\nu} \mathcal{A}^\nu \quad .$$

The operator $\tau_{\mu\nu}$ is assumed to act on a Fourier expansion for which $k_0 = |\underline{k}|$. As only the quantized part contributes to the commutator we readily find

$$[A_\mu(x), A_\nu(y)] = [\mathcal{A}_\mu(x), \mathcal{A}_\nu(y)] = -i\tau_{\mu\nu} D(x-y) \quad , \tag{39}$$

where $D(x-y)$ is the singular function corresponding to mass zero particles(1). As the c-numbers $a^{(0)}$, $a^{(1)}$ have no influence upon this result it is clear that we obtain the same result as in the covariant Coulomb gauge case, i.e. when $a^{(0)} = a^{(1)} = 0$. In particular, in the frame $\overset{0}{K}$ we find for space-like separations the same nonvanishing result for the spatial components as in the transverse Coulomb gauge case:

$$[A_i(x), A_j(y)] = \frac{i(x_0 - y_0)}{4\pi} \left[\frac{\delta_{ij}}{(\underline{x}-\underline{y})^3} - \frac{3(x_i - y_i)(x_j - y_j)}{(\underline{x}-\underline{y})^5} \right] \quad . \tag{40}$$

However, in any inertial frame the two last terms in the expression (38) have no net effect upon the covariant commutator for the <u>field</u> strengths, and we are left with the usual local expression

$$[F_{\mu\nu}(x), F_{\rho\sigma}(y)] = i(g_{\mu\rho}\partial_\nu\partial_\sigma - g_{\nu\rho}\partial_\mu\partial_\sigma + g_{\nu\sigma}\partial_\mu\partial_\rho - g_{\mu\sigma}\partial_\nu\partial_\rho) D(x-y) \quad . \tag{41}$$

We close this section by deriving a covariant expression for the photon propagator

$$D_{\mu\nu}(x-y) = \tfrac{1}{2}\langle 0| \{ A_\mu(x), A_\nu(y) \} |0\rangle + \tfrac{1}{2}\varepsilon(x-y)\langle 0| [A_\mu(x), A_\nu(y)] |0\rangle \quad . \tag{42}$$

Here the anticommutator term requires special attention. In connection with eq.(35) we defined the vacuum state as the state where no physical photons are present. We shall now extend the definition of the vacuum state in the sense that we require the c-numbers $a^{(0)}$, $a^{(1)}$ to be <u>zero</u> for a vacuum. This is a nontrivial gauge-dependent requirement which excludes the possibility to apply restricted gauge transformations to the vacuum. In fact, this just corresponds to the requirement one imposes in the Gupta-Bleuler theory that the vacuum state shall contain no longitudinal or scalar photons.

For the anticommutator term we now find

$$\langle 0 | \{ A_\mu(x), A_\nu(y) \} | 0 \rangle = -\tau_{\mu\nu} D^{(1)}(x-y) \quad , \tag{43}$$

where $D^{(1)}(x-y)$ is the usual anticommutator function.[1] Thus

$$D_{\mu\nu}(x-y) = -\tfrac{1}{2}\tau_{\mu\nu} D^{(1)}(x-y) - \tfrac{i}{2}\varepsilon(x-y)\tau_{\mu\nu} D(x-y) \quad .$$

Here we want to commute $\varepsilon(x-y)$ with $\tau_{\mu\nu}$. This is not possible when $\tau_{\mu\nu}$ is given by the expression (38). However, we shall see that these quantities commute if $\tau_{\mu\nu}$ is expressed in a form which does not presuppose that it is acting on a Fourier expansion for which $k_0 = |\underline{k}|$:

$$\tau_{\mu\nu} = g_{\mu\nu} - V_\mu V_\nu + \frac{(\partial_\mu - V_\mu V\cdot\partial)(\partial_\nu - V_\nu V\cdot\partial)}{(V\cdot\partial)^2 - \Box} \quad . \tag{44}$$

This expression reduces to the expression (38) when the operator $\Box$ is replaced by zero. (Equation (44) corresponds to the new expressions one obtains for $e_\mu^{(3)}$ and $e_\mu^{(1)}$ by replacing the denominator $k\cdot V$ in eqs.(23) and (27) by $[(k\cdot V)^2 - k^2]^{\frac{1}{2}}$.) Now we see that $\varepsilon(x-y)$ commutes with $\tau_{\mu\nu}$ in the frame $\overset{0}{K}$, and as the former quantity is a scalar and the latter quantity a tensor it follows that they commute in any inertial frame. Thus

$$D_{\mu\nu}(x-y) = -\tfrac{1}{2}\,\tau_{\mu\nu}\left(D^{(1)}(x-y) + i\varepsilon(x-y)D(x-y)\right) \quad ,$$

which gives the propagator in the Fourier space as

$$D_{\mu\nu}(k) = \frac{-i}{k^2+i\varepsilon}\left[g_{\mu\nu} - V_\mu V_\nu + \frac{(k_\mu - V_\mu V\cdot k)(k_\nu - V_\nu V\cdot k)}{(V\cdot k)^2 - k^2}\right] \quad . \qquad (45)$$

This is the same expression as the one obtained in a covariant formulation of the transverse Coulomb gauge in $\overset{0}{K}$ ($a^{(0)} = a^{(1)} = 0$).(16) The result is what we should expect, in view of our definition of the vacuum. The c-numbers $a^{(0)}$, $a^{(1)}$ can in no way affect the propagator.

Terms proportional to k_μ or k_ν in the propagator give no contribution to the physical amplitudes, although it should in this context be pointed out that Bialynicki-Birula(17) recently has pointed out that the earlier conventional arguments given to derive this fact are incomplete. Next, the term proportional to $V_\mu V_\nu$ in the propagator is in general cancelled by a Coulomb interaction term(16), and we are thus left with the same effective propagator expression as in the Gupta-Bleuler theory.

It should be borne in mind that it is first after the choice of a distinguished frame $\overset{0}{K}$ that we have the extra parameter at our disposal which is necessary in order to be able to define the basis vectors $e_\mu^{(\lambda)}$ and hence the operator components $a^{(\lambda)}$ in an unambiguous way. This does not imply a conflict with the principle of relativity, however, because $\overset{0}{K}$ is a completely arbitrary inertial frame. All inertial systems can still be regarded as being completely equivalent. Now it is possible to construct a covariant formalism essentially of the type considered above, but without introducing the four-velocity of the frame $\overset{0}{K}$ in the formalism. This is Mathews' method of approach.(6) As in such a case one has no extra parameter at one's disposal in the construction of the vectors $e_\mu^{(\lambda)}$, one has to treat a whole class of permissible expressions for the $e_\mu^{(\lambda)}$ on the same footing, and will thereby be confronted with a certain kind of ambiguity in the commutation rules for the $a^{(\lambda)}$.

5. Discussion. The Ether Concept

So far the quantity V_μ has merely played the role as a mathematical parameter in the formalism, giving the four-velocity of a different, although arbitrary, inertial frame $\overset{o}{K}$ with respect to the frame K in which the fields are expressed. We have already noted that the use of V_μ implies certain advantages from a physical point of view, in particular that the validity of eqs.(2) is maintained along with the four-vector property of A_μ and the positive definite metric in the Hilbert space. [It is reasonable to assume that the Maxwell equations form the very core of electrodynamics, in view of the gauge invariance of any expression that is directly comparable with experiment. Further, the introduction of the four-potential A_μ seems to be an almost unavoidable step in the quantization of the electromagnetic field; in the general interacting case we are not even able to write down a local expression for the interaction Lagrangian in terms of the fields alone. Therefore the solutions (2) of Maxwell's equations in the free field case may be expected to conform closely to the physical reality.] Now we might be content by looking upon V_μ simply as a formal remedy by means of which the formalism can be given a physically more attractive appearance than what is the case in the Gupta-Bleuler theory. However, it would be of great fundamental interest here to go a step further and inquire if not V_μ has a much deeper physical significance. It might be that the necessity of using V_μ in order to obtain the above physically satisfactory features of the theory actually reflects the fact that V_μ is the four-velocity of a physical object. In the following we shall venture to discuss the radical possibility that V_μ is the four-velocity of the all-pervading ether.

Now the conception of an ether is not unique, so that we must specify what we mean when speaking about the ether. Let us first focus our attention on the ether in the traditional sense of the word, viz. the absolute ether appearing in Lorentz' theory of electrons.[18] According to Lorentz there is a certain system of inertia, the absolute system, wherein the ether is at rest. The ether is assumed not to be dragged along by moving bodies. Let the absolute system be denoted by $\overset{o}{K}$ and let a particular point P have the coordinates $\overset{o}{t}, \overset{o}{\underline{x}}$ in this system; then the coordinates of P in another inertial system K with respect to which $\overset{o}{K}$ moves with the uniform velocity $\underline{v}$ are assumed to be given by the Galilean transformation

$$t = \overset{o}{t} \quad , \qquad \underline{x} = \overset{o}{\underline{x}} + \underline{v}\overset{o}{t} \quad . \tag{46}$$

It is just the assumption of the Galilean transformation equations (46) which makes the Lorentz theory amenable to experimental tests. We know that in this area there has been performed numerous experiments, which by now have reached a very high degree of accuracy, and which all speak in disfavour of the Lorentz ether hypothesis. Let us give some examples. Concerning the classical Michelson-Morley experiments[19] the accurate experiment performed by Joos in 1930[19d] provided an upper limit for the ether drift of 1.5 km/s. Later on the ether-drift experiments have been repeated with the use of modern experimental technique involving, for instance, the use of lasers, and one has been able to push the upper limit considerably further down.[20] Highest sensitivity seems to have been obtained with the use of the Mössbauer effect[21]; Isaak[21c] reports that an experiment of this type carried out at Birmingham showed that the ether velocity in the laboratory did not exceed 5 cm/s. In view of these facts we therefore have to reject the ether hypothesis in its particular Lorentz form.

However, it should be made clear that the above result does not compel us completely to avoid the ether idea per se. It is possible to imagine an ether which behaves in accordance with relativity. By replacing eqs.(46) by the Lorentz transformation equations we will no longer have the above mentioned conflict with the experiments. In classical terms one might then imagine the ether as some sort of classical fluid, distributed all over the space and defining one distinguished frame $\overset{o}{K}$ as the frame in which the ether would be at rest. (From a cosmological point of view it would for instance be a possibility to associate the frame $\overset{o}{K}$ with the frame in which the 3^{o} blackbody radiation in the universe appears to be isotropic.[22]) However, we do not find such a picture of the ether to be very appealing, for the reason that it would destroy the principle of isotropy of space in a vacuum. In a complete vacuum such an ether would in an arbitrary inertial frame define one preferred spatial direction, viz. the direction of the three-dimensional vector $\underline{V}$. The principle of isotropy of space seems to be a rather fundamental principle in physics — cf., for instance, the close connection between the isotropy property and the conservation law for angular momentum which is displayed by Noether's theorem — and it should not readily be given up. But now there is indeed one way in which the idea of an ether can be made compatible with the principle of isotropy of space, namely, to picture the ether as a quantal object. The necessity of introducing the ether hypothesis in accordance with quantum mechanics has been pointed out before, by Dirac.[3] According to this quantal picture the velocity of the ether at a particular point in space-time is no longer a well-defined quantity, but is assumed to be distributed over various possible values according to a probability law given by the square of the modulus of some wave function. We may assume that the four components V_μ, now taken to be operators, are commuting objects. Because of the relation $V^2 = 1$ only three of the components V_μ can be specified independently; we let these components be the spatial members $V_i = (V_1, V_2, V_3)$ while the fourth member then is determined as $V_0 = [1+\underline{V}^2]^{\frac{1}{2}}$. We can set up an orthogonal representation in which the basic vectors are simultaneous eigenvectors of the three components V_i, and we may assume that for each space-time point $\underline{x}, t$ the probability of finding the ether within the volume element $d\underline{V}$ in the velocity space is proportional to $d\underline{V}/V_0$. This probability is isotropic in the velocity space. Further, it is a Lorentz invariant, so that the isotropy of the probability distribution for the ether velocity applies to <u>every</u> inertial frame. In this description all inertial frames become completely equivalent, in accordance with the principle of relativity. It should only be borne in mind that from the standpoint of an arbitrary frame K the pertinent frame $\overset{o}{K}$ is no well-defined frame; $\overset{o}{K}$ may be in <u>any</u> translational motion with respect to K. This quantal uncertainty of the frame $\overset{o}{K}$ apparently fits well in with the arbitrariness of $\overset{o}{K}$ in the formalism in the previous Section.

We also meet the problem of how to describe the interaction between the operator fields A_μ and V_μ. We shall here deliberately choose the simplest solution and assume that they are commuting fields.

Now having seen that there is a possibility in principle to have an ether, behaving in accordance with relativity and quantum mechanics, we have to examine the question whether it is possible to make an experimental test of its existence. We are not aware of any possibility to make a direct measurement of V_μ. Further, the velocity-dependent terms in the photon propagator (45) have been shown to be of no experimental significance. But now there are certain effects in quantum electrodynamics which apparently can be connected with the idea of a fluctuating ether, viz. the observable effects of the vacuum. According to the usual formalism the electromagnetic quantities fluctuate vigorously even in a complete vacuum and give rise to an infinite zero point energy. Typical observed quantal effects, such as the attractive force between two conducting parallel plates (the Casimir effect[23]), can be calculated by starting from the expression for the zero point energy. It is not inconceivable that the existence of fluctuating electromagnetic quantities in a vacuum is closely connected with the existence of a fluctuating ether, although the laws of interaction of the electromagnetic field with the ether are not known. As the zero point energy emerges as a simple consequence of the commutation rules for the potentials, it can be expected that the ether must have some deep influence upon these commutation rules.

Notice that the above analysis concerns inertial systems only. An ether hypothesis of the kind considered here cannot say anything about the particular role played by the inertial systems in comparison with more general coordinate systems. The hypothesis therefore does not give any explanation of the fact that for Newton was the main reason for introducing the notion of an absolute space, viz. that only the inertial systems show the property that the equations of motion for a mechanical system depend exclusively on the physical state of the mechanical system itself. (It is clear that once the notion of an absolute space has been introduced, it is very natural to imagine an ether always being at rest in this space.) Nor are we in a position to say anything about the role of the ether in the theory of gravitation, as the gravitational forces, according to the equivalence principle, are put on an equal footing with the fictitious forces appearing in accelerated coordinate systems.

Rounding off these speculations about the ether we have to remark that our intention has not been to insist upon the actual presence of an ether. The evidence is at present too weak to enable us to draw any decisive conclusion. Even though one might be inclined to accept the presence of an ether, in view of the compatibility of the ether hypothesis with relativity and quantum mechanics, one should bear in mind that the identification of the ether velocity with our parameter V_μ cannot be anything else than a pure conjecture. Nevertheless we have found it worthwhile to give the above discussion, as a presentation of a radical possibility that seems to be worth some attention. It ought to be mentioned that Dirac introduced his ether hypothesis in an entirely different connection, viz. as a possible consequence of his new theory of electrodynamics according to which the square of the four-potential is required to be a universal constant.(24) However, the essentials of the analysis remain the same. Dirac has also discussed the problem of normalizing the ether wave function.

Let us now leave the ether hypothesis and as a final remark in connection with the use of the four-velocity V_μ say a few words about the nonlocality, which is a characteristic property of the formalism in the previous Sections. In electrodynamics this nonlocality will never lead to difficulties, as the physically important field strengths are local, cf. eq.(41). (Even in the typical Aharonov-Bohm effect(25), in which the potentials appear to play an important role, the quantity of experimental interest can be expressed as the integral $\oint \underline{A} \cdot d\underline{l}$ around the magnetic equipment which produces the potential. This integral can evidently be transformed into a surface integral involving the magnetic field $\underline{B}$.) However, one might wonder whether the nonlocality is of importance in other and more general field theories, which do not show the gauge invariance property. We know that the main results from field theory rest heavily upon the postulate of locality. However, we shall be content only by mentioning the problem here, without pursuing the subject further.

*

I wish to thank Professor H. Wergeland and Dr. K. Mork for discussions and reading of the manuscript.

Footnotes and References

* We employ the coordinates $x_\mu = (x_0, x_1, x_2, x_3) = (t, \underline{x})$, the diagonal components of the metric $g_{\mu\nu} = (1,-1,-1,-1)$, and choose units such that $\hbar$ and c become equal to unity.

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