chapter title:
FROM RESEARCHER IN PURE MATHEMATICS TO PRIMARY SCHOOL MATHEMATICS TEACHER EDUCATOR

all authors’ names and affiliations:
Svein Arne Sikko and Yvonne Grimeland
Norwegian University of Science and Technology (NTNU, Norway)

corresponding author’s contact details (address and email):
Svein Arne Sikko
Department of Teacher Education
Faculty of Social and Educational Sciences
Norwegian University of Science and Technology (NTNU)
NO-7491 Trondheim
Norway

svein.a.sikko@ntnu.no

FROM RESEARCHER IN PURE MATHEMATICS TO PRIMARY SCHOOL MATHEMATICS TEACHER EDUCATOR

Svein Arne Sikko and Yvonne Grimeland

The Norwegian University of Science and Technology (NTNU, Norway)

In this chapter we investigate the transition from doing research in pure mathematics to becoming a mathematics teacher educator for prospective primary school teachers. The transition involves challenges at several levels, including both the teaching of students, and the nature and content of research in a different field. Central to this transition is the development from knowledge in the vanguard of pure mathematics to profound understanding of fundamental mathematics. What does a pure mathematician need to learn in order to become a mathematics teacher educator? How and in which contexts does the learning take place? Using a four-dimensional framework, we investigate questions about knowledge and learning, inquiry and reflection, insider and outsider, and individual and community.

Key words: mathematics teacher educator, boundary crossing, communities of practice, co-learning, Norwegian teacher education, self-study, inner research

INTRODUCTION

There is an emerging interest in learning more about who mathematics teacher educators (MTEs) are and how to become an MTE. This can, for instance, be evidenced by working sessions and discussion groups at recent conferences of the International Group for the Psychology of Mathematics Education (e.g., Beswick, Goos,

& Chapman, 2014). In their paper about challenges concerning being a mathematics teacher educator in China, Wu, Hwang and Cai (2017) also addressed this question, and pointed out that very little is known about the development of MTEs and what kind of challenges MTEs face in their work, but that it is important to investigate how MTEs develop into professionals. In the field of mathematics education research, student learning and understanding has been the focus of inquiry since the beginning, often building on constructivist or socio-cultural learning theories. There is an abundance of literature on what knowledge and learning means in mathematics and on how to work with students to help them build knowledge. Examples include, but are by no means limited to, Skemp's notions of relational and instrumental understanding (Skemp, 1976), Hiebert and colleagues' notions of conceptual and procedural knowledge (Hiebert, 1986), Freudenthal's theory of Realistic Mathematics Education (e.g., Freudenthal, 1991), and more recently also inquiry based learning (e.g., Artigue & Blomhøj, 2013).

Teacher learning, teacher professional development and what it means to be a mathematics teacher have been given growing attention in recent decades. Models describing mathematics teacher knowledge, such as Ball, Thames and Phelps' (2008) theory of mathematical knowledge for teaching or Rowland, Huckstep and Thwaites' (2005) notion of the knowledge quartet have received notable attention and generated considerable amounts of research. How teachers work to develop professionally has likewise received a great deal of attention, including research on lesson design studies (e.g., Gravemeijer, 2004; Simon, 1995) and lesson studies in different parts of the world (e.g,. Doig & Groves, 2011; Fernandez, 2005; Yang & Ricks, 2013).

However, much less is known about the learning and development of those who teach the teachers, the MTEs (Beswick & Chapman, 2013), and the theme is still underdeveloped.

Even (2008) found that “the education of mathematics teacher educators (of both prospective and practicing teachers) are rarely discussed in the scholarly literature” (p. 59). In addition, Even (2014) claimed that almost all published research in mathematics teacher education and professional development came from English speaking countries (p. 330). Thus, as Even (2014) put it, there is a need to better understand what educators working with teachers (also referred to as didacticians) need to learn, and when and how they should learn that (p. 332). As mathematics educators of prospective primary school teachers in a non-English speaking country, we will contribute to the field by addressing the needs raised by Even.

Goos (2014) additionally made it clear how little is known about ways in which MTEs are prepared for their role and how they learn and develop throughout their careers (p. 454). We address this gap by examining how a newly appointed MTE, the second author, made the transition into becoming an MTE, what she had to learn, and how.

In this chapter, we investigate the transition from being a pure mathematics researcher to becoming a primary school mathematics teacher educator (primary MTE). This transition involves moving into teaching both practising and prospective teachers, and it also involves moving into doing research in mathematics education. Murray and Male (2005) identified two key challenges when moving into teacher education, namely developing a pedagogy for teaching prospective teachers and becoming research active. While we investigate a transition from one field within higher education to another, the key areas where challenges are found are similar. This transition in both teaching and research is challenging on both a personal and an institutional level. Based on these observations, our research question is “What does a pure mathematician need to learn in order to become an MTE? How and in which contexts does s\he learn it?”

We start by giving an overview of teacher education in Norway. This is necessary in order to understand how an increasing demand for MTEs has led to many research mathematicians moving into teacher education. We continue by reviewing relevant literature on the development of MTEs. The methodology used in this chapter is in the form of self-study and inner research, two terms that are next explained and justified. The transition from pure mathematician to primary MTE is then investigated as a boundary crossing using a four-dimensional framework.

TEACHER EDUCATION IN NORWAY

Ongoing reforms in teacher education in Norway have led to an increasing demand for mathematics teacher educators. The demand for pure mathematicians is, on the other hand, rather modest, resulting in people with a background in pure mathematics but none in teaching being drafted into mathematics teacher education posts. We will briefly outline the historical background to explain this situation.

Teacher education in Norway has traditionally been divided into two strands, with one strand catering for the education of primary school teachers and the other for secondary school teachers. Those who wanted to become teachers in primary school would attend teacher colleges, whereas prospective secondary school teachers would attend universities. Primary school teacher education focused on pedagogy and teaching methods. Unlike universities, teacher colleges did not focus on research. Staff at these colleges would typically be experienced teachers and not researchers, with people holding a PhD a rarity.

Throughout the last decades there has been an increasing focus, both in Norway and other countries, on developing teacher knowledge in mathematics. Partially this has been driven by international test scores in PISA and TIMSS that have been considered “disappointing”. Politicians and policy makers bluntly identified teachers as the “weak link” in the educational system, blaming the unsatisfactory results on teachers not having solid enough subject knowledge. This led to several reforms in teacher education.

In Norway, the primary school teacher education programme (grades 1-10) was increased from three to four years in 1992, including 15 mandatory credit points (ECTS) in mathematics. In 1998, this was increased to 30 ECTS. Since 2010 primary school teacher education has been divided into two strands, one for those wanting to become teachers for grades 1-7 and one for those wanting to become teachers for grades 5-10. For the 1-7 education, 30 ECTS of mathematics was kept as mandatory, while for the 5-10 education the mathematics requirement was increased to 60 ECTS. Finally, since 2017, teacher education, including preparation for primary school, has been offered through a five-year master’s programme. Parallel with the reforms in primary school teacher education, secondary school teacher education continued with the model where students first study content-based subjects, subsequently followed by a (now 1-year) course in pedagogy and subject didactics. In addition, a five-year programme has been introduced, where subject content and didactics are more integrated throughout.

Each reform described above has led to an increasing recruitment of MTEs. Concerning the calls for reform in mathematics education, Zaslavsky and Leikin (2004) commented that it seems that these calls are based on the assumption that there exists a supply of well-prepared MTEs who are ready to work with teachers in professional development programmes. The main source able to meet the demand for teacher educators would traditionally have been either experienced school teachers with a Master’s degree, or persons with a PhD in mathematics education. However, in the past, teacher education has not led to a Master’s degree. In addition, making the transition from being an experienced school teacher to become a university lecturer is financially not attractive in Norway. Furthermore, with PhD programmes in mathematics teacher education being relatively new in our country, the availability of well-prepared MTEs to handle the increasing number of pre-service teacher education students has become problematic. At the same time, the demand for research mathematicians has not increased to the same extent. There is limited availability of pure mathematics positions at universities, and the positions that are available are open to international competition, whereas in teacher education it is seen as advantageous to be Norwegian/Scandinavian. As a consequence, many people holding a PhD or Master’s degree in pure mathematics are now filling positions as MTEs in universities and colleges.

We next look at relevant literature on the transition into becoming an MTE which will help us to focus our study and identify gaps to be filled.

LITERATURE ON BECOMING A MATHEMATICS TEACHER EDUCATOR

The literature offers personal stories of people who make the transition from being a school teacher to becoming an MTE. Examples include the book edited by Russell and Korthagen (1995), and articles or chapters by Tzur (2001) and Krainer (2008). Dinkelman, Margolis and Sikkenga (2006) report on a study of how two classroom teachers made the transition to being teacher educators at the university. Likewise, the 28 teacher educators reported on by Murray and Male (2005) had a career background as teachers in primary or secondary school. There are also examples of mathematicians who have moved into teacher education, such as Hans Freudenthal, Alan Schoenfeld, John Mason, Lingyuan Gu (to name a very limited number of well-known mathematicians who have contributed significantly to mathematics education research); and autobiographical descriptions of such transitions given by Gill Hatch and Tim Rowland (Rowland & Hatch, 2006; Hatch & Rowland, 2006). So even if there are exceptions, most of the stories analysing the transition into mathematics teacher education describe people moving from being a school teacher to becoming a teacher educator. In fact, Dinkelman et al. (2006) claimed that “most practicing teacher educators were practicing teachers at some point” (p. 5). Less is thus known or written about those people who make the transition from being an active researcher in pure mathematics to becoming a teacher educator and mathematics education researcher.

In their paper on the significance of mathematical knowledge in teaching prospective elementary mathematics teachers, Zazkis and Zazkis (2011) wrote that the mathematical knowledge of MTEs is often taken for granted (p. 249). However, what actually constitutes relevant mathematical knowledge is often not made explicit. Zazkis and Zazkis addressed this gap by illustrating cases in which mathematical knowledge is beneficial. In their interviews with five mathematics teachers (all deemed to have solid mathematical background, i.e., with Master’s degrees in mathematics or a bachelor’s degree supplemented with graduate-level courses), it is apparent how the teachers’ mathematics background gave them self-confidence to work with mathematical problems and problem solving with their prospective teacher students. An important aspect was that these teachers saw their mathematical background as supporting their efforts to help students acquire a view of mathematics as an interconnected web of knowledge and not a more or less random set of formulas and procedures (p. 260). This account parallels the story of Gill Hatch (Hatch & Rowland, 2006), who found that her strong grasp of the mathematics meant that she could suggest more ways of approaching different mathematical themes, even without herself having tried them out in the classroom.

Artigue (1998) saw mathematics education research (mathematics didactics) as a field within applied mathematics, and stressed the importance of tight bonds between mathematics and didactics. She emphasized that didacticians should have a strong mathematical background, but also pointed out that didacticians coming from pure mathematics "have to try to preserve their present place within the world of mathematics production and mathematics education" (p. 483).

Our ongoing study aims to gain further insight into the particular communities of mathematicians and mathematics teacher educators on the one hand, and the boundary relations and boundary crossings between these two communities of practice on the other hand. Where and how is the boundary located, when does one cross the border from being a mathematician to becoming an MTE, and can you be both? Is who you are dependent on your research, where or what you publish?

Having conducted a thorough review of the literature on boundary crossing and boundary objects, Akkerman and Bakker (2011), concluded that the claims on boundary and learning found in the literature are of a general nature and that hardly any explication on how or what kind of learning takes place can be found (p. 133). The central questions to their research were 1) what is the nature of the boundaries between domains, and 2) what dialogical learning mechanisms take place at boundaries. They identified four potential learning mechanisms that can take place at boundaries: identification (coming to know what the diverse practices of different communities are about in relation to each other), coordination (creating coordinated and routinized exchanges between practices), reflection (expanding your perspective on the practices of your own and others' communities), and transformation (collaboration and co-development of new practices) (p. 150).

Goos and Bennison (2018) explored the potential for learning at the boundaries between communities of disciplinary mathematicians and MTEs in pre-service teacher education. They point to workload formulas, financial models and cultural differences between the disciplines as hindrances to broader collaboration. Of these, the cultural differences may be seen as the most difficult to overcome since they "are grounded in epistemological differences between the disciplines" (p. 272). Goos and Bennison (2018) also found that the physical separation of discipline and education academics (in different buildings) caused a striking hindrance to more interdisciplinary collaboration (p. 266).

As we have seen, several authors have discussed problems and challenges concerning mathematics teacher education and the development of MTEs. Within the literature, we have found that our own paths from being active *research mathematicians* to becoming active *research MTEs* is rarely analysed. It is our intention that this chapter can contribute to the development of knowledge in this field. To investigate this transition we apply the method of self-study and inner research.

METHODOLOGY: INNER RESEARCH AND SELF-STUDY

Krainer (2008) outlined four possible options for research on mathematics teacher educators: 1) self-reflection by MTEs on their own learning; 2) a survey of MTEs conducted by a team of researchers; 3) an MTE writes about other MTEs' development; and 4) a commission or organisation collects data on MTEs on a mandate from a government or university authority. In this chapter, we report on work that is a combination of options 1) and 3).

Quoting Feyerabend's (1991, p.141) thesis that all you can do if you want to be truthful is to tell a story, Mason (1994) contrasted mathematics with mathematics education. Whereas in mathematics, knowledge is built by adding new theorems to old, education is a journey of self-discovery where each new traveller has to re-experience, re-learn, re-express and re-integrate what previous generations have learned. Mason claimed that what researchers find out most about is themselves. By interrogating our own experiences, and addressing the questions on how to support teacher education students and teachers in developing their knowledge of mathematics and teaching, we report on transformations in ourselves. This "inner research" is about developing new types of "sensitivity", to the mathematical ideas, to the pedagogical and didactical possibilities and to the students we are working with. The transformation arising from moving into a new field means noticing different things, since as members of a research community, we notice what we are attuned to notice (Mason, 1998, p. 368). The mathematician notices mathematical structures and concepts, whereas the mathematics educator may notice the struggle to come to terms with the concepts. The use of "the particular" in the form of ourselves, thus resonates with Mason's notion of research from the inside, and may still contribute to knowledge in the field at large.

As Mason (1998) wrote, research in education is different from research in other fields in that it is about being sensitive to others and transformation of other people than oneself. Therefore the "only certain place to stand is in the most unlikely place: ourselves" (p. 360). Our approach is thus to use ourselves, mainly the second author, as examples to shed light on the processes of transition from being a mathematician to becoming an MTE. With Mason's words in mind, we believe our approach may contribute to extending knowledge and raising new questions regarding the development of mathematics teacher education and MTEs. At the same time, we acknowledge Mason's thesis that "in mathematics education everything remains problematic" (p. 358).

We are thus situating ourselves within the paradigm of self-study, which has a strong history in teacher education research. Borko, Liston and Whitcomb (2007) identified self-study as one of the sub-genres of practitioner research, and one of four genres of empirical research in teacher education: “Practitioner research examines practice from the inside” (p. 5). LaBoskey (2004) identified five characteristics of self-study: 1) it is self-initiated and focused, 2) it is improvement-aimed, 3) it is interactive, 4) it uses multiple, mainly qualitative, methods, and 5) it defines validity as a process based on trustworthiness. We next describe how we fit ourselves within these five characteristics, and by so doing we make our methodology transparent.

First, our research is self-initiated and focused. Both authors have PhD degrees in pure mathematics, more precisely in the subfield of abstract algebra called representation theory of Artin algebras. Both have made the transition into the field of primary mathematics teacher education. The first author made this transition two decades ago, the second author much more recently. In this paper, we focus on the second author’s transition. The second author studied mathematics and informatics over a period of 10 years at the Norwegian University of Science and Technology (NTNU), one of the largest and most research active universities in Norway, gaining a Master’s degree in mathematics leading to a PhD in September 2014. Immediately after graduating, she was offered a full-time position as primary MTE at a local teacher college where she stayed for 1.5 years before returning to NTNU, this time to the department of teacher education as a primary MTE. During her Master’s and PhD studies, she worked to a limited extent as a teaching assistant in undergraduate mathematics courses. This work involved tutoring groups of students during exercise sessions and assessing student assignments. During her PhD studies she also undertook some substitute lecturing in pure mathematics courses. However, she had no experience of teaching at primary school level.

Regarding improvement, LaBoskey (2004) describes self-study methodology as “designed to understand and improve our professional practice settings” (p. 845). By engaging in reflective inquiry into our own experiences we aim to improve our own practices and contribute to the learning of novice MTEs.

The third characteristic of self-study is its interactive nature. Interaction for us takes multiple forms. First, the two authors have collaborated directly for the writing of this chapter, of which more is detailed below. A second aspect of interaction is the discussions the authors have had with other colleagues in our institution. Thirdly, we experience interaction with our own students, both directly in the classroom with all students, in meetings with selected students taking part in reference groups discussing the teaching and learning of the courses, and also through anonymous student course evaluations in the form of questionnaires. In addition, the interaction with texts in various forms, such as educational research literature on mathematics, has been an important part of our work.

LaBoskey (2004) pointed out that self-study methodology uses multiple, mainly qualitative methods. Such methods were evident in our use of narrative inquiry, taking the story of the second author’s journey into teacher education as the starting point. Through dialogue and conversation between the two of us, we identified steps in the transition from pure mathematician to MTE that one or both of us found particularly prominent. During these meetings, notes were made and our experiences compared and contrasted to what we could find in the literature, also leading to literature searches. To ensure that the stories emerging from the dialogues would be more reliable, we consulted “artefacts” from our past that could substantiate our data. These included course plans and lecture notes, including PowerPoint presentations, from courses one or both of us had been teaching; meeting notes from faculty meetings and seminars; notes from literature study sessions for new faculty (also called "reading groups"), including mentors' lecture notes/PowerPoints, handwritten notes made in margins of the articles/chapters provided as readings, and evaluation reports from the reading group; reflective notes and reports from school visits following up prospective teachers, and reports from school mentors. From biweekly meetings throughout one semester (autumn 2017) a narrative emerged that subsequently was made into an organised text (the first version of this chapter).

Finally, in self-study methodology validity is defined as a process based on trustworthiness. Hamilton, Smith and Worthington (2008) claim that "triangulation of data establishes trustworthiness" (p. 21). Yin (2018) proposed that at least four types of triangulation are possible. While we did not make use of *methods* triangulation, our multiple data sources, as outlined above, constitute a form of *data* triangulation. In addition, two investigators looking at the same phenomenon, in our case the transition from pure mathematics to teacher education, constitutes a form of *investigator* triangulation. An important part of our inquiring dialogues was looking at our experiences through different theoretical lenses. These included the (expanded) teacher educators' triad (Leikin, Zazkis, & Meller, 2017), cultural historical activity theory (e.g., Yamagata-Lynch, 2010), and Valsiner's zone theory (e.g., Goos, 2014). Having different theoretical perspectives on the same data set constitutes *theory* triangulation. Trying to view our data through different theoretical lenses all within a sociocultural frame helped us zoom in on what kind of analytical frame that was best for analysing and presenting our data.

In the end we made the decision to analyse the data using a four-dimensional framework proposed by Jaworski (2003). Within each of the four dimensions Jaworski suggested questions that might be addressed. During our discussions we kept coming back to these questions as they provided a kind of guide through our travels along each of the dimensions. In the next section we go into detail about this and present our analysis.

INVESTIGATION OF MTE LEARNING WITHIN A FOUR-DIMENSIONAL FRAMEWORK

The analytical framework we use to investigate the process of becoming a MTE is influenced by Wagner's (1997) discussion of cooperation between researchers and practising teachers. Wagner put forward co-learning agreements as one such form of cooperation where the roles of the participants are more ambiguous than in more traditional forms of cooperation. University researchers are outside and practising teachers are inside the school, but at the same time researchers are working inside and practitioners outside the university. While both the researchers and the practitioners are engaged in action and reflection and might learn something about the world of the other, it is equally important that "each may learn something more about his or her own world and its connections to institutions and schooling" (p. 16).

The transition from pure mathematics to mathematics teacher education involves several communities, researchers and practitioners of different kinds. In each of the communities people interact with both people inside the same community and people outside. A mathematician is part of the community of mathematicians, but also interacts with different types of students as part of his or her teaching, faculty from other departments as part of cooperation or administrative work, etc. These interactions may be seen in terms of being an insider in some situations and an outsider in other, and such interactions also involve learning from the different perspectives of those involved. Wagner's (1997) notions of co-learning and the insider-outsider perspective are therefore potentially useful in analysing the communities and their interactions. Jaworski (2003) extended Wagner's co-learning concept to include what she referred to as “insider researchers”: practitioners who also engage in research into teaching, and hence develop their own teaching. Jaworski stated that this situation often will mean teacher-researchers, but she pointed out that it can also include educator-researchers exploring processes and practices in teacher education. The latter is the case in this chapter where we investigate the transition from being a mathematics researcher to a primary MTE.

Jaworski (2003) proposed a four-dimensional framework (p. 263) that can be applied to research on development of mathematics teaching from insider or outsider perspectives. Each of the four dimensions consists of a reflexively-related pair: Knowledge and learning; Inquiry and reflection; Insider and outsider; and Individual and community. Jaworski emphasised that the elements of the framework are deeply related and interlinked. Our own journeys from research mathematician to MTE involve development of not only mathematics teaching, but also understanding of what mathematics teaching is. At different locations along the journey, we have experienced different insider and outsider roles. Our own knowledge has developed along with what it means to learn and what to learn. What a mathematician and an MTE inquire into and reflect upon differs. We thus find Jaworski’s four-dimensional framework useful in analysing the transition from being a mathematics researcher to a primary MTE. This can be seen as a way of doing inner research in the sense of Mason (1998) on the transitional phase.

We overlay our analysis using Jaworski’s (2003) framework with consideration of learning mechanisms at the boundary between mathematics and mathematics education. The first learning mechanism identified by Akkerman and Bakker (2011) was identification, which concerns learning how the diverse practices of each community or domain relate to one another, “defining one practice in light of another, delineating how it differs from the other practice" (p. 142). To realize and explicate the differences between the two practices entails learning something new about both practices, and can lead to reflection through perspective making and perspective taking as another of the four learning mechanisms involved in boundary crossing. In our analysis along each of Jaworski's four dimensions we therefore start by clarifying the practices of the two communities between which we have moved.

Knowledge and learning

For a researcher in pure mathematics, including PhD students, the focus is to develop new knowledge in mathematics itself, concentrating on solving open problems in a (usually) small subdomain of mathematics, unintelligible to those outside the particular subdomain. The second author's PhD work, for example, concerned classification problems for special biserial and gentle algebras (Grimeland, 2014).

In the community of MTEs the knowledge in focus is how individuals, and in particular pupils, learn and gain understanding of mathematics. Pure mathematics lessons concern conveying the mathematical content itself, and only to a lesser extent how this knowledge can be applied. By contrast, the mathematical concepts discussed in a session with prospective primary school mathematics teachers, are concepts in, or directly related to, the primary school curriculum. The focus of the teaching and learning is to help prospective teachers develop their mathematical knowledge for teaching (Ball et al., 2008). This includes helping them to develop a "profound understanding" (Ma, 2010) of the mathematical concepts, and also to develop knowledge of how children can work with and understand the concepts. The set of rational numbers is, for instance, a standard example used to shed light on certain algebraic structures such as fields of fractions, and an example that both authors have studied in depth as mathematics students. However, this type of treatment does not give any immediate insight into how pupils can build a meaningful understanding of what a fraction is, what it can mean, how it can be used in different situations, and how operations on fractions can come to make sense for the pupils: What are useful interpretations, and appropriate representations, for thinking about division or addition of fractions?

Thus, in making the transition from pure mathematics researcher to primary MTE, both authors found it challenging to understand how to handle this changed lesson content. In her first years as a primary MTE it gradually became clear to the second author that there were more issues than she had expected that needed attending to by a primary MTE. To support this realisation, two boundary practices were particularly helpful: reading mathematics education literature and discussing mathematics education with colleagues. An example concerns communication patterns in mathematics classrooms and how to lead productive mathematical discussions. Coming from research mathematics, the mathematical content did not pose any challenges, but how to work with prospective teachers on orchestrating productive mathematical discussions in the classroom is something for which she - as a mathematician - was not prepared. Since, to begin with, she lacked a background in the mathematics education literature, it was not clear to her either how this could be done in a classroom with pupils or how to work with prospective teachers on developing insight into this pedagogical strategy. As a result of lacking familiarity with the literature her teacher education sessions were not founded on research in the mathematics education field, but rather were informed by trial and error. This is one example of how the second author gradually became aware that, for the students to become mathematics teachers, it is not sufficient to focus on the mathematics content and try gaining deep understanding of the mathematics itself. She thus needed to develop her own awareness of the diversity of mathematical knowledge for teaching and sensitivity to both student teachers and pupils. This process included re-experiencing and re-learning the need for the diverse aspects of knowledge inherent in teaching.

Inquiry and reflection

The focus of inquiry for the research mathematician is to try to describe concepts, connections and relationships that are not already known. Reflection is on the mathematical content, and how different concepts are related. For the primary MTE the focus of inquiry centres on how the learning of mathematics takes place and how it can be facilitated. Reflection is on which actions can be taken in a mathematics classroom and how these actions support pupils' learning of mathematics. Furthermore, a primary MTE also needs to attend to how prospective teachers learn what they need to learn in order to become teachers. Therefore, a primary MTE also inquires into the nature of mathematics teacher knowledge, and how it can be developed.

Through the discussions between the two authors about what it means to be a mathematician and what it means to be an MTE, the second author recognised that her areas of inquiry and reflection have changed, in the sense of having expanded. The reflections about mathematical concepts continue to be a part of the everyday activity of an MTE, but the reflections are now to a large extent centred around fundamental mathematical concepts that are related to the primary school curriculum. In this sense, the MTE's understanding of fundamental concepts grows, but in the process an enhanced understanding is developed of more abstract concepts related to the particular fundamental concepts. An example is the concept of division, where neither of the authors was aware of the distinction between partitive and quotitive models of division prior to moving into teacher education. For the mathematician, this distinction is not important; the question would rather be whether we are working within a division ring or not. For the teacher, on the other hand, concepts like division ring are not interesting; the question is rather how to be able to help pupils extend their understanding of division from division of integers to division involving fractions, and which representations and models that are helpful in this extension. Reflection on different models of division may include thinking about which models are appropriate in which situations and within which number sets, and thereby gaining deeper insight into the number systems themselves. Reflection may also lead us to think more closely on the connection between division and multiplication, the concept of inverse, both in the sense of inverse operation and inverse element, and thereby to develop other insights into group-, ring-, field-, and function-theory. This is helpful even if group theory is not the explicit topic, as the MTE Rachel mentions in the study conducted by Zazkis and Zazkis (2011, p. 257). These renewed insights into the practices of the two communities result from the learning mechanism of identification (Akkerman & Bakker, 2011, p. 142), and as such constitute renewed sense making within the separate communities rather than an overcoming of discontinuities between them (p. 143).

Inquiry is the norm in mathematics, even if this process has not always been the norm in school mathematics. It also has to be confessed that the inquiry of a mathematician rarely extends to inquiring into the teaching of mathematics. Having a background as a pure mathematician does not automatically provide an advantage going into mathematics teacher education. On the other hand, for an MTE the capacity and disposition towards inquiry is fundamental. As an MTE you need to inquire into your own practice. This includes inquiring into the choice of models and representations, trying out new approaches, not being “locked” into one particular way of doing things but instead continuing to reflect upon your own practice. It may also include letting students inquire “freely”, not being afraid they might get lost. As mathematicians we know that "getting lost" is also a sometimes necessary part of the learning process.

The background of a pure mathematician is not a disadvantage when stimulating prospective teachers to have an inquiring mind, hopefully resulting in school mathematics becoming more inquiry based. A person with a PhD in mathematics has both a deep and broad knowledge of mathematics, and also an understanding of the structure and coherence of mathematics. This understanding includes concepts or procedures that prospective teachers may not see as problematic or challenging because of their so far superficial knowledge. The knowledge a mathematician has may assist "students in acquiring a view of mathematics as an interconnected web of knowledge rather than a collection of unrelated facts and procedures" (Zazkis & Zazkis, 2011, p. 260)

In addition, the importance of knowing that mathematics is a living and developing field should not be underestimated. Pupils and students are often, surprisingly, unaware that mathematics is not a subject fully developed in ancient Greece, maybe due to the kinds of mathematics they have met in school and how they have worked with that mathematics. A researcher in pure mathematics, on the other hand, has firsthand experience in extending the field of knowledge within the subject, and knows that the field is "continuously" developing. This realisation gives an insight into the kinds of questions that can be raised and explored by students. It also includes knowledge about conjectures that are still open in the field of mathematics itself, including those that can be formulated at a level intelligible to prospective teachers and their future pupils, such as Goldbach's conjecture or the twin prime conjecture.

MTEs in university are expected to do research as part of their profession. A particular difficulty experienced by the authors is the distance between the questions asked in mathematics and those asked in mathematics education. Questions in mathematics concern how to extend or develop a particular concept, how to build on already proven theorems in order to push the frontiers of research further. In mathematics education, questions may be about how to explain or help students and pupils explore a particular concept, and how to help them build on their prior knowledge. And mathematics education is not an axiomatic-deductive discipline – unlike in mathematics there are no theorems! However, in mathematics education research, questions similar to those in mathematics research also arise, for example, how to extend or develop concepts, and thereby push the frontiers of research. However, the nature of the research is different, leaning towards methods in the social sciences or the humanities. To be able to take part in research in mathematics education, it is important to know what the relevant literature is, and where to find it. The research fields of mathematics and mathematics education seem to be separate at the level of independent searchable databases giving access to the up-to-date research literature, making the transition between disciplines less smooth. Thus the second author struggled to find resources similar to MatSciNet (American Mathematical Society, 2018), or arXiv.org (Cornell University Library, n.d.) in mathematics education, resources that ideally could function as boundary objects connecting the two fields.

Insider and outsider

In Norway, the community of research mathematicians and the community of primary MTEs are typically separated. Traditionally, education of primary school teachers was undertaken in "teacher colleges" separate from the universities. Even today, when primary school teacher education is also undertaken in universities, particularly after the recent university reforms in Norway, the preparation of teachers takes place in teacher education departments separate from the pure mathematics departments. This separation has been highlighted in the study described by Goos and Bennison

(2018). Traditionally, then, an insider in the mathematics research domain remains an outsider to the teacher education world, and vice versa.

Research in mathematics researcher itself is either carried out alone (more rarely so today) or in collaboration with other research mathematicians (more common today). In this sense, doing mathematics research is an insider activity within the community of research mathematicians, being carried out outside of teacher education and not related to teaching. The purpose of this research is to expand mathematical knowledge within a well-defined area of pure mathematics. However, teaching mathematics at university often includes teaching mathematics courses for engineering students or others who need mathematics as a tool in their future studies or job. In this situation, a mathematician can be regarded as an outsider to the engineering programme while being an insider in the mathematics community. The mathematics being taught in such lessons is rarely related directly to the mathematician's research, and the mathematician does usually not conduct research on this teaching.

By contrast, in the Norwegian context, MTEs collaborate with practising teachers in several contexts. This collaboration can arise as part of prospective teachers' professional placements in schools, where groups of typically three or four students spend two or three weeks in the classroom of a practising teacher, supervised on a daily basis by this teacher but also by a university-based MTE. This experience creates opportunities for the learning mechanism of coordination (Akkerman & Bakker, 2011), whereby there is a communicative connection between the university and the school. Meetings at the boundary between school and university address one of the difficulties identified by Wu et al. (2017), concerning inconsistencies between the university teacher education courses and the prospective teachers' experiences during school practice (p. 1381).

Professional development for in-service teachers has been given high priority during the last decade in Norway, implying that MTEs and teachers meet at courses at the university and/or in schools. In addition, schools may host research and development projects supervised by university faculty. In these settings, the MTE has a dual role as an insider and outsider. The role of an MTE, both as an educator and as a researcher, is thus distinguished from the role of a mathematics researcher, which is more well-defined as pure researcher.

The MTE conducting research in schools and in cooperation with school teachers can make use of experiences and data from this research in his/her own teaching at the university. The purpose of the MTE's research in mathematics education is to expand knowledge about the teaching of mathematics and the professional development of mathematics teachers, and push the frontiers of mathematics education. An MTE working closely with school teachers does not only conduct research into other people's practice but also creates a community of practice of which s/he her-/himself is a part. This collaboration and co-development of new practices exemplifies the learning mechanism at the boundary that Akkerman and Bakker (2011) refer to as transformation.

Making the transition from mathematician to MTE, one is at first an outsider in the MTE community, as experienced by the second author. Primary school prospective teachers are not necessarily interested in or intrigued by mathematics *per se* (which is the strength of the mathematician), but are often more concerned with learning methods that may work in school, with which the mathematician is not so familiar. Such an experience constitutes a confrontation that is the start of a transformation process (Akkerman & Bakker, 2011, p. 146). Familiarizing herself with mathematics education research was a particularly important contribution to the transition from outsider to insider in this respect for the second author. In that way, she experienced a change, from the initial sense of being an outsider giving lessons to prospective teachers, and not having enough to offer, to being an insider, who actually has knowledge from which prospective teachers can benefit. Gaining knowledge about the literature on mathematics education research led the second author to reflect more on her own teaching, changing her teaching in ways that are informed by and more aligned with literature.

Moving from being an outsider to an insider in research in mathematics education is a necessary goal for a mathematician who is becoming a MTE in a university, and naturally takes some time, not least since the relevant research methods in pure mathematics and mathematics education are so different. Both authors have experienced this as a gradual and difficult path that requires severe effort and does not happen overnight.

Individual and community

The mathematics researcher is part of the community of mathematicians, sharing insights with other mathematicians. Moving into mathematics education involves a change in the community to which one (most strongly) belongs. Even if one tries to adhere to Artigue's statement (1998) that didacticians from a mathematical background should preserve their place within mathematics, doing research in either field is too complex to do "part-time", and therefore most of us have to make a choice of doing either one or the other. The second author experienced clear expectations from the community of mathematicians in the mathematics department that her interest should remain in doing research in pure mathematics, even after she had made the transition into the teacher education department. This expectation clashed with the expectations of the community of teacher educators who expect faculty holding a PhD, no matter in which field, to do research in education. The second author found it impossible to continue doing pure mathematics as her research activity at the expense of building knowledge as a primary MTE. The conflict between expectations on the two sides of the boundary echoes the findings of Dinkelman et al. (2006) that the two novice teacher educators in their study retained elements of their classroom teacher identities while struggling to construct their identity as teacher educators.

To overcome the challenges involved in becoming a primary MTE and mathematics education researcher, the second author found two "phenomena" (boundary objects) particularly helpful upon joining the department of education at NTNU. Attending an organised "reading group" on topics of mathematics education research, and research methods in the field, made a big contribution to her understanding of the nature of research in mathematics education and about relevant questions in mathematics education research. The reading group was organised to help newcomers in the mathematics section in the department of teacher education to gain insight into mathematics education research. The group was led by "more knowledgeable others" in the form of more experienced colleagues, including the first author. These "mentors" thus acted as brokers (Goos & Bennison, 2018, p. 260) facilitating the boundary crossing, and the readings in the form of journal papers and book chapters played a role as boundary objects. In this way, the second author became a participant in a community in which she was able to build a basis of knowledge that would have taken much longer to develop in a less organised setting, as experienced by the first author. Both authors found the reading group an opportunity to discuss research literature at the appropriate level in a community open to questions of any kind, providing learning for both the newcomers and the mentors.

The other helpful community "offering" was an archive, available to the mathematics section faculty, containing previously developed lesson plans. These plans had been developed by colleagues in the mathematics section of the teacher education department, to be used in a first-year mathematics education course for prospective primary school prospective teachers. Based on the lesson plans in the archive, colleagues would work collaboratively on redesigning course plans. Working in this manner gave the second author the opportunity to focus on developing knowledge about pupils' understanding of the particular topic of each lesson, and which activities are relevant and possible to use with prospective teachers, using the lesson plans as a boundary object So there was a dual type of learning, on the one hand learning about how pupils learn mathematics, and on the other hand learning about how to work with prospective teachers.

CONCLUSION

What does a pure mathematician need to learn in order to become a mathematics teacher educator? How and in which contexts does she learn it? As Murray and Male (2005) found, there are two paths of learning that need to be built. The first path concerns teaching, which involves teaching of prospective teachers, but it may also involve teaching of practising (in-service) teachers at further education courses (which has been given significant and continuing priority by the government in Norway the last decade). The second challenge is to do mathematics education research, which by its nature is very different from research in pure mathematics.

How do you learn to be a mathematics teacher educator? There are few systematic programmes aimed at educating the educators. This is not unique to mathematics teacher education or even to teacher education in general. In fact, in any university-level discipline you are traditionally left to yourself to figure out how to do your teaching and your research. Traditionally, therefore, there are few boundary objects and brokers to help with the transition. Building learning communities that are open and inviting to newcomers, making co-learning partnerships, and working together on lesson planning and research, makes it easier to understand the meaning of knowledge and learning in teacher education. A central theme in mathematics teacher education, at all levels, is that of developing a profound understanding of the mathematics being taught. The MTE needs this understanding him/herself, and also needs to help prospective teachers develop it. Likewise, both the MTE and her students need to develop knowledge of the diverse aspects inherent in being a mathematics teacher.

Collaborating with colleagues was important for the second author to develop understanding of what was going on in teacher education. The reading group community was important in building knowledge of what research in mathematics education may involve. The archive of lesson plans and the collaboration with colleagues on teaching contributed in essential ways to knowledge about what teaching and learning constitute in teacher education.

Joining projects and learning communities, with partner schools and fellow researchers, is a way to understand the shift in what it is relevant to inquire into and what research in mathematics education contains. A systematic approach, like a “reading group” as described here, is one way to start addressing this. An awareness of what constitutes research in mathematics teacher education is one of the traits of the transition. Our reflections on this case point to ways of understanding and facilitating the transition.

ACKNOWLEDGEMENT

The authors wish to thank Professor Tim Rowland for helpful comments and suggestions during the preparation of the manuscript. We also thank our editor Merrilyn Goos and the two anonymous reviewers for their comments and suggestion on the first draft of this chapter.

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