Using Non-linear Integral Models in Automatic Control and Measurement Systems for Sensors' Input Signals' Recovery

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Abstract. The article is devoted to developing methods of dynamic correction of signals registered at the output of a non-linear measuring transducer for the purpose to recover its input signal. Dynamic correction leads to decreased inertia/time response of the measuring transducer and smoothes its non-linearity. Signal recovery problem is resolved based on the non-linear model of measuring transducer as a first kind, second degree polynomial Volterra integral equation. As the problem is ill-posed, the model regularization method is used. The article offers a differential regularization operator transforming the input polynomial integral equation into a polynomial integro-differential equation. For numerical realization of the given models' type an algorithm based on the difference and quadrature methods is offered. The key problem of limited modelling time when using the kernel, given by the tabular method, is resolved by applying a procedure of restarting of the calculation process with offset of the time interval the integral equation kernel defined on**.** To resolve the issue of significant short-period errors in calculations after the restart of the calculation process, we offer a method for obtaining solution from two parallel computing processes with tabular kernels offset in time at a half of the time interval they are defined on. A feature of integral models is their resistance to high-frequency interference, which are present in real engineering systems. The obtained results can be used in dynamic correction devices of measuring transducers of automatic control and measurement systems.

Keywords: dynamic correction of non-linear measuring transducer, polynomial integral models, tabular kernel, signal recovery.

1 Introduction

Intensive development of modern controlled engineering systems demands constant improvement of measuring devices as the basic components of such systems. Computerized measuring transducers play a special role in the development of controlled engineering systems. They are usually composed of a sensor, AD transducer and dynamic correction device [4, 8]. A dynamic correction device is designed based on mathematic models of the researched processes. To a great extent it depends on the difficulty of algorithmic signal processing relying upon these models. The difficulty also arises due to the fact that digital signals shall be processed in real time with limited hardware as such measuring systems are usually represented as mounted units [5].

The key issue arising in measurement process is recovering the input signals, which is classified as an inverse ill-posed problem [9, 10]. Models of different types can be used to describe the dynamic features of measuring transducers [8, 10]. To provide for the necessary level of adequacy of multiple practical problems, measuring transducer mathematic models shall account for different features, specifically, non-linearity of processes and space distribution of parameters. As such features are considered, models grow increasingly more complicated together with the dynamic correction device software. Using operator models in integral form with a range of positive features (multipurposeness, smoothing measuring data, efficient macromodels, etc. [2, 10]) appears promising. What concerns recovery, these models acquire the form of the first kind Volterra integral equation. Hence, the basis integral form of signal recovery model is

$$\int_0^t K(\xi,\zeta,t,s)\,x(\zeta,s)\,ds = y(\xi,t)\,, \tag{1}$$

where $K(\xi,\zeta,t,s)$ is the kernel of integral operator, $y(\xi,t)$ is the set input signal in point ξ, and $x(\zeta,t)$ is the sought input signal in point ζ, $t\in[0,T]$. Such models can be used only to describe linear processes; in its turn, to consider the non-linear features, the basic model may acquire a form of first kind polynomial Volterra integral equation [1, 6, 9]

$$\sum_{m=1}^{n}\underbrace{\int_0^t\ldots\int_0^t}_{m} K_m(\xi,\zeta,t,s_1,\ldots,s_m)\prod_{i=1}^{m}x(\zeta,s_i)\,ds_i = y(\xi,t)\,, \tag{2}$$

where $K_m(\xi,\zeta,t,s_1,\ldots,s_m)$ are multidimensional kernels, $x(\zeta,t)$, $y(\xi,t)$ are, correspondingly, the input and output signals applied at points ζ and ξ of the distributed object, n is a certain integer, and T is the transition process time.

Many publications [2, 10] dwell upon solving inverse problems based on the (1), with regularization methods being the most efficient. Researches [1, 9] are devoted to developing methods for solving the equations (2), which usually based upon direct or iterative methods. A range of factors complicate the use of the current methods to develop mathematical means and software for dynamic correction devices of measuring transducers, particularly, accumulation of calculations during numerical implementation of multidimensional integral models and ill-posedness of the problem, which does not allow to sufficiently recover the signals in the case of input data noise interference.

Thus, the task of developing methods for solving polynomial integral equations, which will serve as a foundation for software of computerized dynamic correction devices for measuring transducers, becomes topical.

2 Methods for solving polynomial integral equations

Let us consider a partial case (2) as a model of non-linear sensor – first kind, second degree polynomial Volterra integral equation

$$\int_0^t K_1(\xi,\zeta,t,s)x(\zeta,s)ds + \int_0^t\int_0^t K_2(\xi,\zeta,t,s_1,s_2)x(\zeta,s_1)x(\zeta,s_2)ds_1 ds_2 = y(\xi,t), \tag{3}$$

where $y(\xi,t)$ is the value obtained from the sensor, $x(\zeta,t)$ is the measuring value sought, $K_1(\xi,\zeta,t,s)$ is the first order kernel, and $K_2(\xi,\zeta,t,s_1,s_2)$ is the second order kernel. Integral equation kernels (3) depending on the used method may have different representation: analytical representation when using analytical methods for transformation of other models' types; tabular representation when using the identification methods [3, 7]. In our case, kernel $K_1(\xi,\zeta,t,s)$ is presented as a table (vector) and kernel $K_2(\xi,\zeta,t,s_1,s_2)$ is presented as a matrix.

Let us consider a method for solving the (3) based on model regularization.

2.1 Model regularization

As the signal recovery problem is ill-posed, and the use of known methods and algorithms does not provide a solution of necessary accuracy in case of interference of input data in limited time, it is proposed to create regularization algorithms using the differential regularization operator

$$Dx(\zeta,t) = \rho_2\alpha^2 \frac{d^2x(\zeta,t)}{dt^2} + \rho_1\alpha \frac{dx(\zeta,t)}{dt} + \rho_0\alpha x(\zeta,t), \tag{4}$$

where α is regularization parameter and ρ_i are regularization coefficients defining the operator order; they can equal 0 or 1.

The model experiments' method revealed that the first order differential regularization operator yields the best results. Thus, solving first kind Volterra integral equation is reduced to solving a polynomial integro-differential equation:

$$\alpha \frac{dx(\zeta,t)}{dt} + \int_0^t K_1(\xi,\zeta,t,s)x(\zeta,s)\,ds + \\ + \int_0^t \int_0^t K_2(\xi,\zeta,t,s_1,s_2)x(\zeta,s_1)x(\zeta,s_2)\,ds_1 ds_2 = y(\xi,t), \tag{5}$$

where α is a regularization parameter proposed to be found based on the model experiments' method [10].

2.2 Algorithms for solving polynomial integral equations

It is proposed to solve the given task by replacing the integrals with quadrature sums, which provides several advantages, in particular simple implementation and high stability of computational algorithms due to regularization features of sampling interval selection [10].

By introducing fixed-rate sampling ($i = \overline{1..n}$, $h = t_i - t_{i-1}$), applying the method of trapezoids and first order difference formula to the (5) and grouping the sought $x(\zeta,t_i)$ by degrees, we obtain the following equation:

$$\begin{aligned}
&\frac{1}{4}h^2 K_2(\xi,\zeta,t_i,t_{i,},t_i)\,x(\zeta,t_i)\,x(\zeta,t_i) + \\
&+\left(\frac{1}{4}h^2\left(K_2(\xi,\zeta,t_i,t_i,t_0) + K_2(\xi,\zeta,t_i,t_0,t_i)\right)x(\zeta,t_0) + \right.\\
&+\frac{1}{2}h^2\sum_{j=1}^{i-1}\left(K_2(\xi,\zeta,t_i,t_{i,},t_j) + K_2(\xi,\zeta,t_i,t_{j,},t_i)\right)x(\zeta,t_j) + \\
&\left.+\frac{1}{2}hK_1(\xi,\zeta,t_i,t_i) + \frac{\alpha}{h}\right)x(\zeta,t_i) + \sum_{j=1}^{i-1} hK_1(\xi,\zeta,t_i,t_j)\,x(\zeta,t_j) + \\
&+\frac{1}{2}hK_1(\xi,\zeta,t_i,t_0)\,x(\zeta,t_0) + \frac{1}{4}h^2K_2(\xi,\zeta,t_i,t_0,t_0)\,x(\zeta,t_0)\,x(\zeta,t_0) + \\
&+\frac{1}{2}h^2\sum_{j=1}^{i-1}\left(K_2(\xi,\zeta,t_i,t_{0,},t_j) + K_2(\xi,\zeta,t_i,t_{j,},t_0)\right)x(\zeta,t_0)\,x(\zeta,t_j) + \\
&+h^2\sum_{j=1}^{i-1}\sum_{g=1}^{i-1}K_2(\xi,\zeta,t_i,t_{j,},t_g)\,x(\zeta,t_j)\,x(\zeta,t_g) - \frac{\alpha}{h}x(\zeta,t_{i-1}) - y(\xi,t_i) = 0.
\end{aligned} \tag{6}$$

Let us introduce the following notation:

$$A_i = \frac{1}{4}h^2 K_2(\xi,\zeta,t_i,t_{i,},t_i)\,, \tag{7}$$

$$B_i = \frac{1}{4}h^2\left(K_2\left(\xi,\zeta,t_i,t_i,t_0\right)+K_2\left(\xi,\zeta,t_i,t_0,t_i\right)\right)x\left(\zeta,t_0\right)+$$
$$+\frac{1}{2}h^2\sum_{j=1}^{i-1}\left(K_2\left(\xi,\zeta,t_i,t_{i,},t_j\right)+K_2\left(\xi,\zeta,t_i,t_{j,},t_i\right)\right)x\left(\zeta,t_j\right)+\frac{1}{2}hK_1\left(\xi,\zeta,t_i,t_i\right)+\frac{\alpha}{h}, \tag{8}$$

$$C_i = \sum_{j=1}^{i-1} hK_1\left(\xi,\zeta,t_i,t_j\right)x\left(\zeta,t_j\right)+$$
$$+\frac{1}{2}hK_1\left(\xi,\zeta,t_i,t_0\right)x\left(\zeta,t_0\right)+\frac{1}{4}h^2K_2\left(\xi,\zeta,t_i,t_0,t_0\right)x\left(\zeta,t_0\right)x\left(\zeta,t_0\right)+$$
$$+\frac{1}{2}h^2\sum_{j=1}^{i-1}\left(K_2\left(\xi,\zeta,t_i,t_{0,},t_j\right)+K_2\left(\xi,\zeta,t_i,t_{j,},t_0\right)\right)x\left(\zeta,t_0\right)x\left(\zeta,t_j\right)+ \tag{9}$$
$$+h^2\sum_{j=1}^{i-1}\sum_{g=1}^{i-1}K_2\left(\xi,\zeta,t_i,t_{j,},t_g\right)x\left(\zeta,t_j\right)x\left(\zeta,t_g\right)-\frac{\alpha}{h}x\left(\zeta,t_i\right)-y(\xi,t_i).$$

Hence, (6) considering the notation of (7), (8), (9) has the following form

$$A_i x\left(\zeta,t_i\right)^2 + B_i x\left(\zeta,t_i\right)+C_i = 0\,. \tag{10}$$

Quadratic equations of the system (10) are solved in sequence, and for selection of one of the routes, the following algorithms is used: if $\left|x\left(\zeta,t_{i-1}\right)-x\left(\zeta,t_{i_1}\right)\right| \geq \left|x\left(\zeta,t_{i-1}\right)-x\left(\zeta,t_{i_2}\right)\right|$, then $x_i = x_{i_2}$, else $x_i = x_{i_1}$.

2.3 Solving polynomial integral equations with tabular-type kernels in an infinite period

Model (5) is distinguished by integral kernels that set tabular (vector and matrix), which imposes time limits for the model. This issue is resolved by restart of the computational process with offset of the time interval the integral model's (5) kernels defined on. Kernel displacement operation is mathematically ill-posed as the restarted computational process does not account for the initial conditions while calculating the first discretion value of solution on the offset time interval. In this connection, at the beginning of the restart, there are significant tolerance in solution calculation. However, due to regularization feature the tolerance is rapidly reduced to the level of systematic error. To obtain a solution for infinite time interval with the acceptable tolerance not exceeding systematic error, it is proposed to use an additional computational process for the equation (5). The peculiarity of the additional computational process is time-offset of the kernels, that set tabular, by half time interval at which they are identified. Thus, one can obtain solutions with the acceptable accuracy on the intervals with significant tolerances at the restart of the main calculation process. The resulting solution is a combination of solutions obtained from two calculation processes in two threads; it also accounts for the fragments of the obtained results with stable convergence of the solution. Fig. 1 provides for the diagrams of the obtained solutions for the type (5) equations with restarts of the time-offset computational processes. Fig. 2, shows a solution diagram obtained from combining the solutions fragments of different threads.

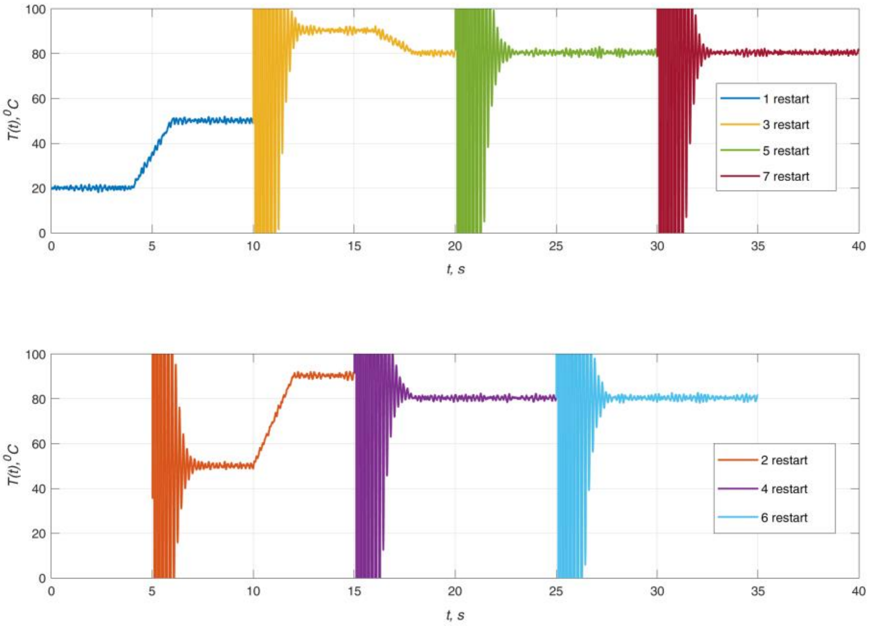

Fig. 1. Diagrams of the solutions of type (5) equations with restarts of calculation processes.

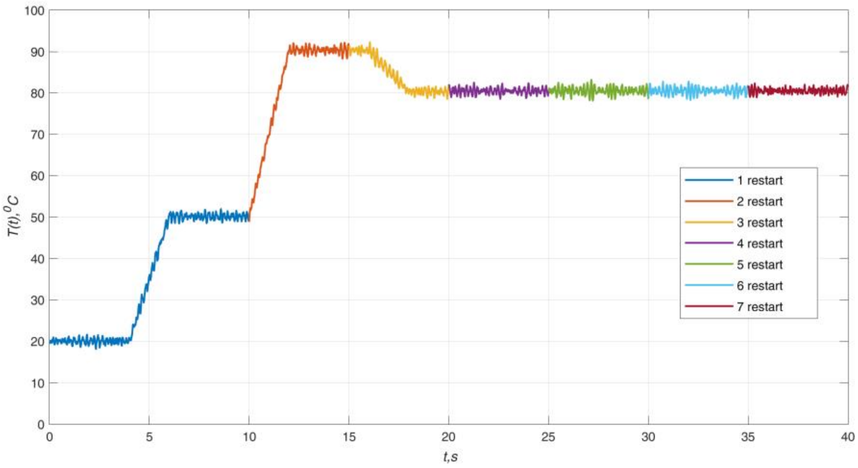

Fig. 2. Combined solutions' fragments.

3 Results and practical application of the offered methods

The research results were used in solving the problem of the temperature modes' operative control of computer network trunking and switching equipment chips. The experiment revealed that the "chip–thermosensor" measurement system is inertial and non-linear. Having applied the identification method [7], we obtained a model represented as a partial sum of the Volterra integro-power series:

$$R(t) = k_0 + \int_0^t K_1(s)T(t-s)ds + \int_0^t \int_0^t K_2(s_1, s_2)T(t-s_1)T(t-s_2)ds_1 ds_2 \,, \qquad (11)$$

where $R(t)$ is thermistor resistance value, $T(t)$ is chip temperature, k_0 is dimensionless factor, $K_1(s)$ is first order kernel given in tabular form (vector), $K_2(s_1, s_2)$ is second order kernel given in tabular form (matrix). To recover the temperature offset signal inside the chip based on model (11), we conducted its regularization by introducing a differential regularization operator. It resulted in the model:

$$\alpha \frac{dT}{dt} + \int_0^t K_1(s)T(t-s)ds + \int_0^t \int_0^t K_2(s_1, s_2)T(t-s_1)T(t-s_2)ds_1 ds_2 = R(t) - k_0 \,, \qquad (12)$$

where α is the regularization parameter.

Model (12) is the basis of the software component of the computer subsystem for controlling the chips' temperature modes, which allows to minimize the delay of thermal sensor reaction to chip temperature change. Temperature value is defined based on the thermal sensor resistance digital data sent by the computer network to the computerized dynamic correction and control system. Fig. 3 provides for diagrams of: a) the registered signal used as a basis for recovery of the signal of temperature change inside the chip; b) the result of signal recovery (recovered and actual).

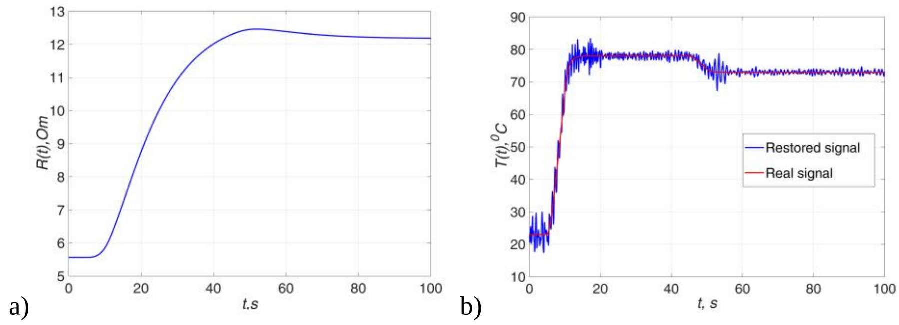

Fig. 3. Diagrams of the signals: a) registered at the thermal sensor output; b) recovered by the dynamic correction system

4 Conclusions

Thus, the developed method for solving first kind, second degree polynomial integral equations based on differential regularization operator and mechanism of restarting the computational process with offsetting the time interval the equation kernels are defined on, made it possible to solve the problem of recovery of the input signal of a non-linear measuring transducer in real time at high-frequency noise interference of input signal. A perspective direction for further research is to determine the optimal time interval the integral model kernels and regularization parameters are defined on, which provides the maximum rate of computational process convergence at restart.

5 References

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