



# Performing price scenario analysis and stress testing using quantile regression: A case study of the Californian electricity market



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## ABSTRACT

This paper uses quantile regression to demonstrate how electricity price distributions are linked to fundamental supply and demand variables. It investigates the California electricity market (zone SP15) for selected trading hours using data from January 8, 2013 to September 24, 2016. The approach quantifies a non-linear relationship between the fundamentals and electricity prices, just as predicted by the merit order curve. Natural gas, greenhouse gas allowance prices and load all have a positive effect on electricity prices, with the effect increasing with the quantiles. In contrast, solar production and wind production both have a negative effect on electricity prices. The effect of solar production increases with quantiles, whereas the effect of wind production decreases with quantiles. This paper also includes a stress testing case study in which a producer faces the risk of high solar and wind production, and investigates the effect on the lower tail of the price distribution. Overall, the results demonstrate how the proposed approach can be a helpful risk management tool for participants in the electricity market.

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## 1. Introduction

The recent transformation in renewable energy policies worldwide, and the steady expansion of renewable resources in electricity markets, is rapidly changing the dynamics of these markets. Market price uncertainty has increased the importance of methods to hedge risk, improved forecasting, and a better understanding of mechanisms driving electricity prices.

This paper quantifies the impacts of variable renewable electricity sources on electricity markets. We investigate the price dynamics at different trading hours at the SP15 zone in the California wholesale electricity market. Furthermore, we look at how key risk factors, such as load, generation capacity, solar and wind forecasts, gas prices, and greenhouse gas allowance prices influenced price formation in the period between January 8, 2013 and September 24, 2016. Electricity spot prices exhibit seasonality, mean-reversion, occasional price jumps and time-varying volatility. These features

are due to daily fuel-cost (oil, gas, coal, emissions) variations, hourly and seasonal weather conditions (wind, solar, precipitation) and load patterns, planned and forced outages, transmission constraints and other capacity restrictions. These characteristics of electricity prices are also due to the non-linear merit order curve that leads to a non-linear relationship between the price and its fundamental supply and demand variables. Until recently, this merit order curve in California has primarily consisted of natural gas and nuclear power plants.<sup>1</sup> Following the rapid increase in the share of renewable energy, the market price set by the merit order curve has been trending downwards. This is due to their low marginal costs, thereby resulting in renewable production being placed before traditional power plants in the merit order curve. The impacts of wind and solar production on prices are therefore important to analyze, in addition to fuel-based technologies. Extreme low prices can occur when there is a high production of solar and wind (that is difficult to store), together with a low load (for example mid-day, during winter/spring days with minimal

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<sup>1</sup> In addition, import and export also plays a large role in CAISO.

need for cooling or heating). Thus, in this paper we focus on the effects of renewables on prices, as well as the effects of classic fuel-based technologies.

Quantile regression models are well-suited for modeling electricity prices that depend non-linearly on supply and demand variables. In this paper, we estimate quantile regression models with the fundamentals for the different levels of electricity prices, price areas and trading periods. From these approaches, we find how the sensitivities to supply and demand variables vary over these dimensions. In addition to the analysis and interpretation of these sensitivities, the models we derive are useful in predicting price distributions, and hence Value at Risk (VaR) for given values of the independent variables.

The contribution of this paper is to show how such models can be applied in the California electricity market. This paper further shows how scenario analysis and stress testing can be performed using quantile regression. That is, it shows how a change in the value of one or more of the predictors influences the price distribution and thus the Value at Risk measures. This analysis is new in the context of the US power markets. Since each market has distinct characteristics through its input mix, transmission system, import/export restrictions, etc., it is important to build specific models for each market.

The rest of this paper is organized as follows: Chapter 2 presents a literature review on risk modeling of electricity prices. Chapter 3 reviews the California electricity market and the specific price areas we investigate. Chapter 4 describes the dataset and provides descriptive statistics for the variables applied in the analysis. Chapter 5 describes the methodology, while Chapter 6 reports the results. Chapter 7 shows how the estimated models can be used for scenario analyses, stress testing and risk management in general. Finally, in Chapter 8 we conclude and offer suggestions for further work.

## 2. Literature review

This work lies between the following strands of literature: (1) Value at Risk forecasting for energy commodities, and (2) An analysis of how supply and demand variables influence the electricity spot price formation.

Regarding Value at Risk forecasting for energy commodities, parametric approaches such as GARCH models with heavy tail error distributions, as well as long memory features, have been found to perform well in predicting upper and lower Values at Risk for a range of energy commodities. Giot and Laurent [1] argue for using GARCH or ARCH models with skewed student *t* error distributions. In their study, they use a 5-year hold out sample for a wide range of commodities (including energy) and test various risk models for 1 day value-at-risk forecasts. Aloui [2] finds that long memory models such as FIGARCH (with skewed *t* error distribution) provides the best value at risk prediction for main energy products. The results are conformed in Aloui and Mabrouk [3]. These authors also provides a detailed discussion on how this results can be applied in energy risk management and hedging. Most of the studies above concentrate on energy futures markets. Chilki et al. [4] also cover both spot prices of energy commodities and a broader set of seven linear and nonlinear GARCH-type. Again, their conclusion support the usage of long memory GARCH models with skewed *t* distributions.

VaR forecasts for very low and high quantiles can also be improved using Extreme Value Theory (EVT). Key references here are Byström [5] and Chan and Gray [6].<sup>2</sup> Byström [5] analyze hourly

spot prices on Nord Pool, and apply EVT to investigate the tails of the price change distribution. They find a good fit of the generalized Pareto distribution (GPD) along with an AR–GARCH filtered price change series, and accurate estimates as well as forecasts of extreme quantiles are produced. Chan and Gray [6] extend the analysis using an EGARCH-EVT specification. Compared to a number of other parametric models and simple historical simulation based approaches, the proposed EVT-based model performs well in forecasting out-of-sample VaR. As with Byström [5]; they suggest that GARCH EVT-based models are useful in forecasting VaR in electricity markets.

Non-parametric approaches such as historical simulation with mean and volatility filtering have been found to be successful in predicting price distributions (see Cabedo and Moyab [13]; Costello et al. [14]; and Gurrola-Perez and Murphy [15]. Cabedo and Moyab [13] analyze three VaR calculation methods: the historical simulation standard approach, the historical simulation with ARMA forecasts, and a variance–covariance method based on autoregressive conditional heteroskedasticity models forecasts. The results obtained indicate that the latter methodology provides the best VaR forecast, which fits the continuous oil price movements well and provides an efficient risk quantification. Costello et al. [14] find that a semi-parametric GARCH model generates VaR forecasts that are superior to the VaR forecasts from the ARMA with historical simulation and traditional GARCH models. Gurrola-Perez and Murphy [15] apply filtered historical simulation VaR models. This is an extension of historical simulation which provides the ability to incorporate information on recent market returns and thus produce risk estimates conditional on them. The authors finds that these estimates are superior to the unconditional ones produced by classical historical simulation. The paper explores the properties of various filtered historical simulation models and suggest to use them in producing VaR forecasts.

Lastly, there is also support for using semi-parametric approaches such as Quantile regression. VaR forecasting for commodities (including energy) using quantile regression and a comparison to other methods are found in Füss et al. [16]; Haugom et al. [17] and Steen et al. [18]. A general finding from these studies is that they perform very well relative to other models, and are easier to implement. Füss et al. [16] his paper examines the in- and out-of-sample performance of a wide range of value at risk models. The results suggest that the CAViaR models generally outperform the other models. A CAViaR model is an autoregressive model for the quantiles. The estimation is done through various quantile regression models with various specifications. Those models are able to incorporate the time-varying volatility adequately and are sensitive to changes in the return distribution over time as well. Haugom et al. [17] proposes a parsimonious quantile regression model for forecasting Value-at-Risk. The model uses various observable measures of volatility at different horizons as input into a quantile regression. They apply the model to a selection of financial assets, including oil. When subjected to formal coverage tests for out-of-sample VaR predictions, model performance is similar to more complicated models. The approach is however must easier to implement and understand. Steen et al. [18] implement this approach and a range of benchmark models for a wide range of commodity futures markets (including energy) and confirm the results.

Most of the studies discussed above involve energy commodities such as oil and gas futures, but very few look at electricity spot prices. Hence, there is a need for more research in this direction. Nowotarski and Weron [19] point out that the majority of the studies on electricity spot price forecasting are point forecasting, and that very few involve distributional forecasts. Our paper will therefore contribute in this context, investigating distributional

<sup>2</sup> Additional references using extreme value error distributions are [7–12].

forecasts of electricity spot prices.

Turning to the literature investigating the formation of spot electricity prices using fundamental supply and demand variables, the aim of the papers mentioned below is to capture features such as mean reversion, time varying volatility, seasonality, spikes and the non-linear relationships to fundamentals. Variables used in these models include market concentration, demand forecasts, the forecasting of reserve margins and prices of fuels. As for the UK day-ahead electricity market, approaches such as state space, regime switching, logistic smooth transition models and quantile regression models appear in the literature (see Refs. [20–23]. For the Nordic region, Huisman et al. [24,25] investigated how fundamentals influence the electricity price using various non-linear models. Similar studies for the German market exist that use state space models, logistic regression and quantile regression (see Ref. [23,26,27]. The first insight from these studies is that the complex non-linear relationship between fundamentals and the price of electricity needs to be taken into account when choosing an appropriate econometric model. In addition, including fundamentals will generally increase the performance of the models, both in and out of the sample.

For the California market, there are several papers that provide key insights regarding market structure and the deregulation process over the last decades. Examples include Borenstein and Bushnell [28]; Borenstein et al. [29] and Borenstein et al. [30]. Over the past few years, a surge in subsidized renewable generation, combined with low natural gas prices, has driven wholesale prices steadily lower. According to the papers mentioned above, the role of variable renewable generation at both wholesale and distributed level is likely to continue to dominate the economics and policy of the industry in the future. This will be a great challenge for the producers of electricity, and creates a need for quantifying the price effects of renewables. There are a few papers studying the relationship between fundamentals and their implications for the price dynamics of the California electricity market. Woo et al. [31] investigate the effect on prices from a nuclear plant shutdown. They perform a regression analysis of hourly real-time price data from the California Independent System Operator (CAISO) from April 2010–December 2012. Their analysis indicates that the 2013 shutdown of the state's San Onofre plant raised the CAISO real-time hourly market prices by \$6/MWh to \$9/MWh, and that the price increases could have been offset by a combination of demand reduction, thereby increasing both solar generation wind generation. Woo et al. [32] studied the effect of virtual bidding in the CAISO market. They found that virtual bidding has reduced the volatility of the state's day-ahead hourly forward premiums, and that rising wind generation has altered the level and volatility of the premium. These findings suggest virtual bidding has improved market-price convergence in California's day-ahead and real-time markets. Woo et al. [33] investigate the merit order effects of renewable energy in the CAISO's day-ahead and real-time markets, as well as the relationship between the prices. Using a sample of approximately 21,000 hourly observations of CAISO market prices and their fundamental drivers from 2012 to 2015, they document statistically significant estimates of the so-called merit order effect, i.e., the price reduction caused by low marginal cost renewables, in the day-ahead and real-time markets. Woo et al. [34] also quantified the effect of California's CO<sub>2</sub> cap-and-trade program on the wholesale electricity prices of four interconnected market hubs in the Western US. A recent paper from Wiser [35] provides a comprehensive analysis on the effects of growth in variable types of renewable energy with respect to bulk power system assets, pricing and costs for different regions in the US. The paper discusses how power system planners, operators, regulators and policymakers will continue to be challenged to develop methods to smoothly and

cost-effectively manage the reliable integration of these new and growing sources of electricity supply.

Most of the studies mentioned above use linear models, while the approach in this paper using quantile regression will better capture the non-linearities between the electricity prices and independent variables. This paper also extends the use of independent variables to explain price formation.

### 3. The CAISO (California Independent System Operator) electricity market

#### 3.1. Market structure

The CAISO market operates within the Western Electricity Coordinating Council (WECC) that encompasses the western regions of Canada and the United States, as well as parts of northern Mexico. CAISO is the only restructured market in this region; the remainder of electricity trading in the WECC takes place through bilateral contracts. The CAISO market consists of both a day-ahead market (DAM) and a real-time market with locational marginal prices (LMPs) calculated for each system node based on a detailed model of the physical power system network. The development of the CAISO system is also impacted by California energy and environmental policy initiatives driving changes in the electric grid. These include goals of a 60% renewable electricity generation by 2030, greenhouse gas emission reduction to 1990 levels by 2020, and policies to increase distributed generation (for more information, see Refs. [36,37].<sup>2</sup>

The CAISO system consists of three main regions: NP15, SP15 and ZP26 - each region having one or more load-serving entities. In this paper, we focus on the SP15 price zone. Fig. 1 shows the geographical locations of the zones and their major load-serving entities.

#### 3.2. Generation mix

The electricity generation mix in CAISO has changed substantially over the past few years, primarily due to the ambitious renewable energy goals set by the California Energy Commission. Still, natural gas is the main source of electricity generation, supplemented by nuclear and a large share of imports (about 30%). In-state coal and oil contribute less than 1%. The proportion of renewable energy generation has increased rapidly since 2010, with hydro, solar, and wind being the main sources of renewables, see Fig. 2 below. For more information, see California Energy Commission [38] and CAISO [37].

#### 3.3. Supply curve

Figs. 3 and 4 show CAISO supply curves for the years 2013 and 2016, respectively (covering the start and end years of our dataset). The demand for electricity is rather inelastic for normal price ranges in the short run. The supply function reflects the merit order of short-run marginal costs, which increase steeply as the plants move from base load to peak load. This non-linear relationship indicates the necessity to use a non-linear model to capture the relationship between fundamental variables and the price of electricity. The figures illustrate how increased renewables from 2013 to 2016 have shifted the supply curve further "to the right." This has also led to lower average prices, which we show in the next section. The marginal cost of electricity when demand is moderate to high is usually set by the price of natural gas. Natural gas prices therefore have a significant impact on price when there is high demand.

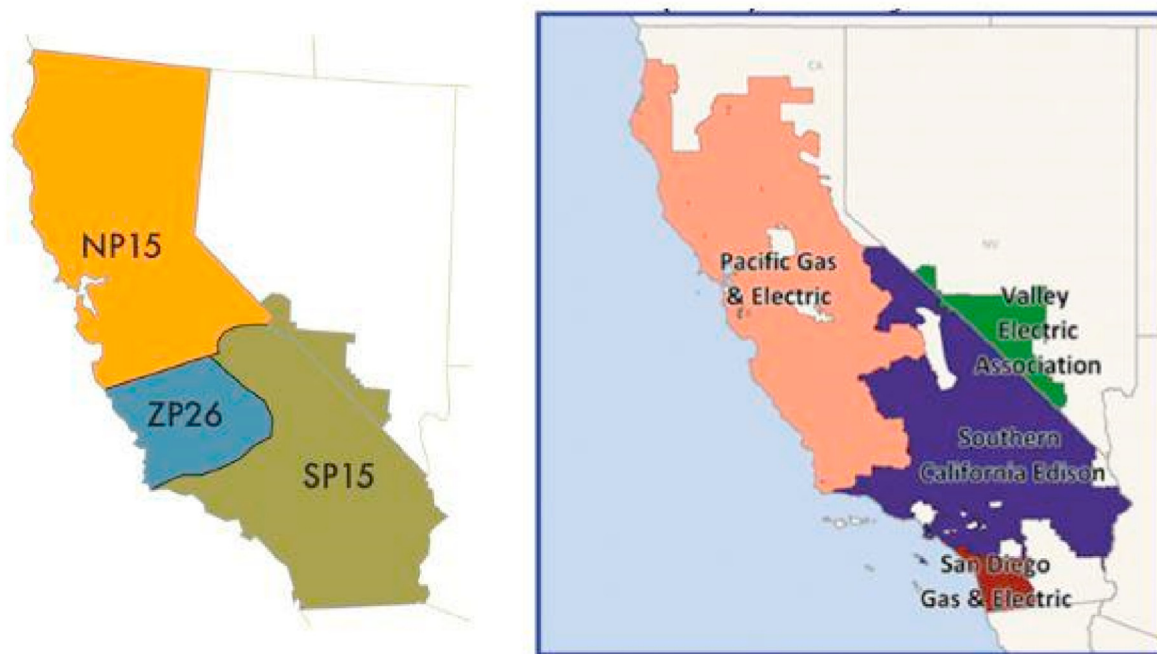


Fig. 1. CAISO price zones and local distributional companies (sources: CAISO [37] and FERC [36]).

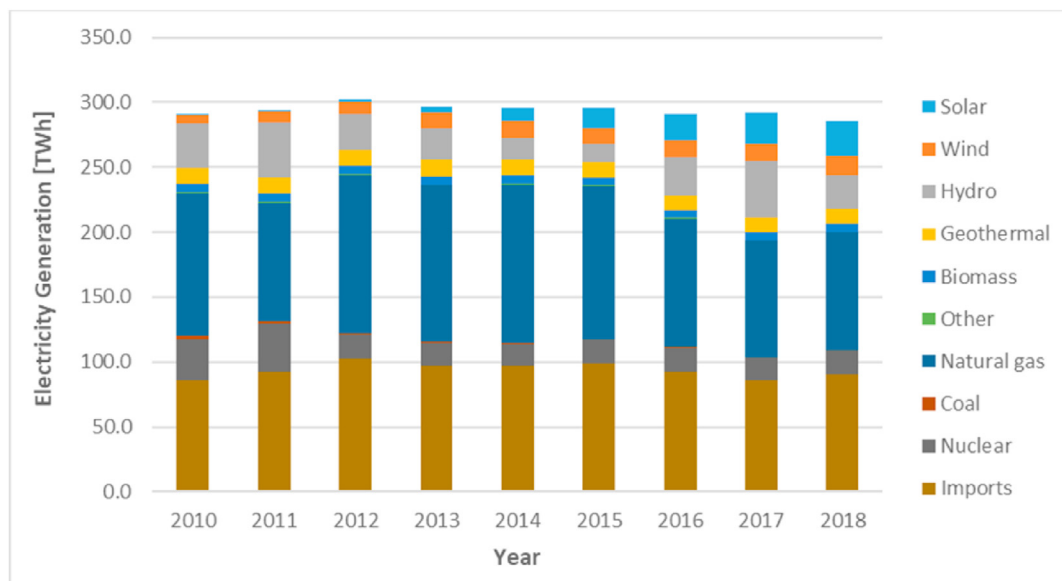


Fig. 2. Electricity generation mix in California, 2010–2018. (Source: California Energy Commission, Energy Almanac).

#### 4. Data and descriptive statistics

This section analyzes the characteristics of different variables and trading periods in the CAISO zone SP15, with all fundamental variables considered summarized in Table 1. The analysis is of the hourly DAM price over the timespan from January 8, 2013 to September 24, 2016. All data are from LCG Consulting [39] and CAISO’s OASIS database [40]. The timespan considered is appropriate due to the increase in installed solar capacity over this time period (0.92% in 2012 increasing to 7.67% in 2015 as reported by the California Energy Commission [38]). Furthermore, data prior to 2013 was not available to download for all of the variables considered.

Table 2 shows a summary of descriptive statistics for the DAM price for selected periods in SP15. Hour 4 represents an off-peak night hour, while hour 9 allows us to investigate a trading period where there is substantial ramping, as well as an increase in renewable energy production. Hour 12 contains both the maximum renewable energy production and high load. Hours 17 and 20 present steep ramping needs, as well as the occurrence of peak load. Fig. 5 displays the data graphically. From the figure, the known features of electricity prices are evident: spikes, seasonality and time-varying volatility. Late afternoon prices seem to be the most erratic. The highest average prices are in hour 20, whereas the lowest is in hour 4. The maximum price is at hour 17 when there is a



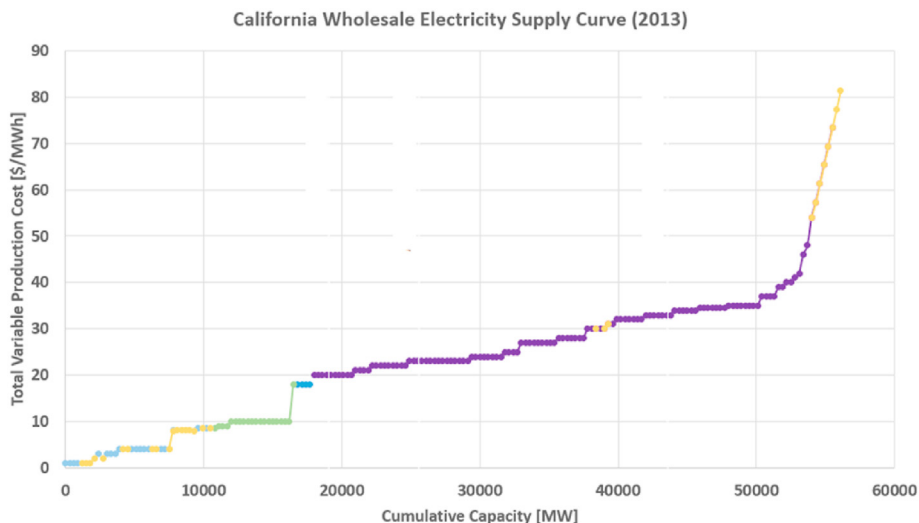


Fig. 3. Illustration of California wholesale electricity supply curve 2013 (Source: CA energy Almanac).

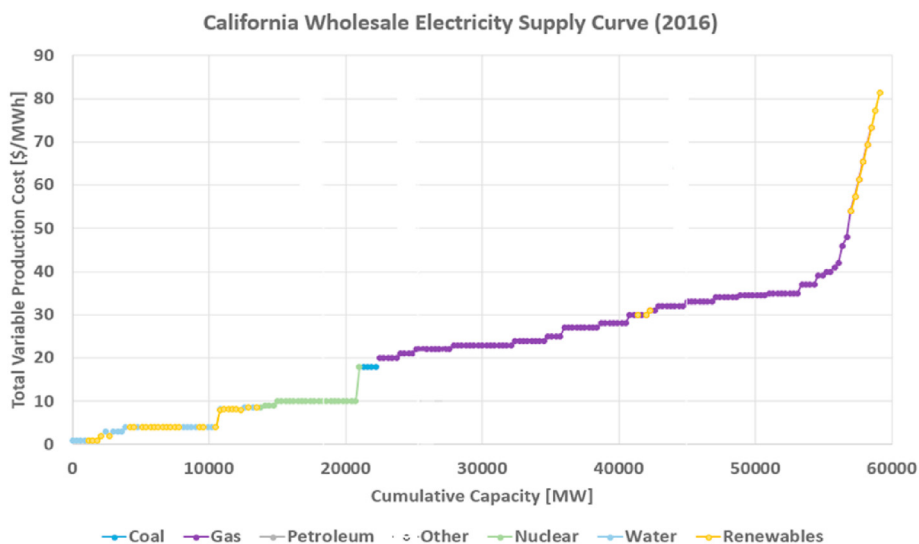


Fig. 4. Illustration of California wholesale electricity supply curve 2016 (Source: CA energy Almanac).

high demand, with the minimum at hour 12 when there could be a high production of wind and solar power. For hour 12, there are also occurrences of negative prices. The skewness and kurtosis of the price distributions vary over hours, but fat tails appear in all series. Price distributions are skewed to the right. We have also performed a Jarque-Bera test (not shown in the table), thus rejecting a normality assumption for all series.

Tables 3–7 show a summary of descriptive statistics for the fundamentals (independent variables) for hours 4, 9, 12, 17 and 20 of the SP15 price. Note that the gas and greenhouse gas (GHG) allowance price remains the same for all hours (these are daily prices), while wind, solar and load forecasts are specific for each hour. The gas price is positively correlated (as expected) with the DAM prices in the range between 0.55 and 0.68. The effect of the GHG allowance price is relatively small, and even negative for some hours. Regarding wind production, the average production is highest in hour 20 and lowest in hour 12. As expected, there is a negative relationship between wind production and prices in the range between  $-0.14$  and  $-0.33$ . The relationship seems to weaken monotonically over the day. On average, solar production is highest

midday and insignificant for hour 4 and hour 20. As expected, the relationship is negative for the hours with production. On average, load is highest for hour 17 and has (as expected) a positive correlation with prices. In general, the correlation increases over the course of the day.

Further analysis shows that the correlation of the independent variables also varies with the level of electricity price (not shown in the tables). This is expected, given the non-linear supply function. In order to capture these non-linear relationships, we therefore estimate quantile regression models for each of the selected hours in our analysis.

### 5. Method (quantile regression)

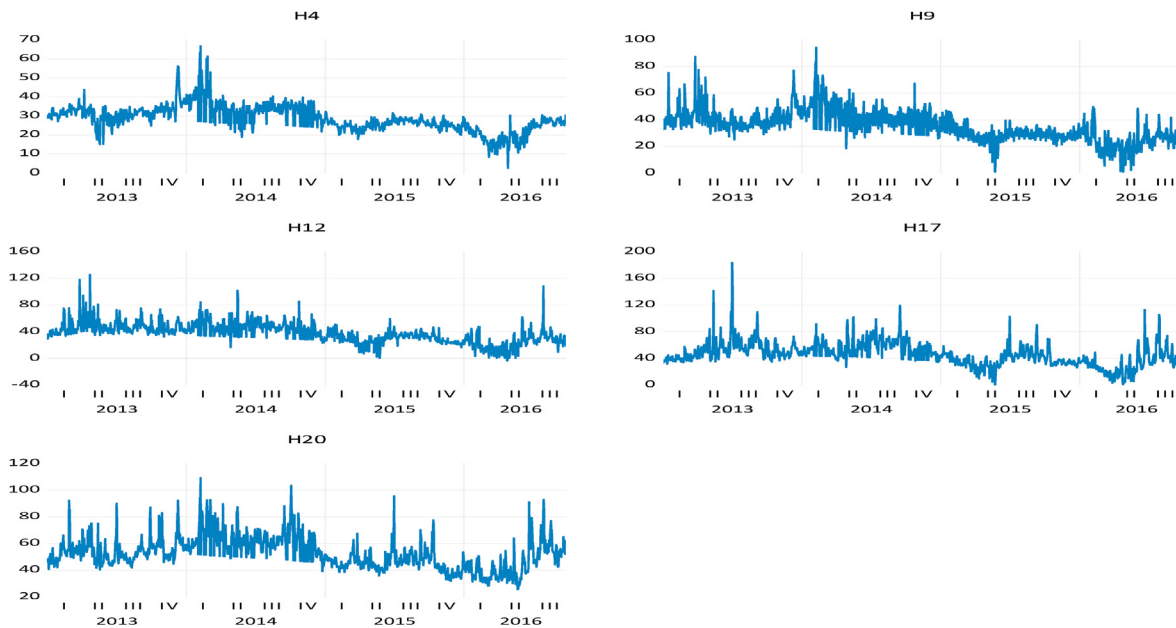
In order to attempt to capture the non-linear relationship between fundamentals and the electricity price, we apply a quantile regression model (see Ref. [41,42]; and [43] for more details). In ordinary regression, one finds the conditional mean of the electricity price given the set of explanatory variables by minimizing the squared residuals. In quantile regression, one finds a set of

**Table 1**  
Description of explanatory variables used in model. More details are in LCG consulting [39] and OASIS [40].

Variable	Granularity	Hourly	Daily	Unit	Source
DAM Price SP15	X			USD/MWh	LCG Consulting
Gas Price			X	USD/mmBtu	OASIS
GHG Allowance Price			X	USD/Mt CO2e	OASIS
Wind Forecast	X			MW	OASIS
Solar Forecast	X			MW	OASIS
Load Forecast	X			MW	OASIS

**Table 2**  
Descriptive statistics of the DAM price SP15 (\$/MWh) for hours 4, 9, 12, 17 and 20. Data for the period from January 8, 2013 to September 24, 2016 All data is retrieved from LCG consulting [39] and OASIS [40].

	Hour 4 (\$MWh)	Hour 9 (\$MWh)	Hour 12 (\$MWh)	Hour 17 (\$MWh)	Hour 20 (\$MWh)
Mean	28.61	35.78	37.69	45.90	52.58
Standard deviation	6.90	11.42	14.81	17.61	12.07
Excess Kurtosis	2.69	1.38	2.18	5.26	0.99
Skewness	0.56	0.48	0.46	1.06	0.73
Maximum	66.67	94.30	125.74	183.73	109.23
Minimum	2.39	0.58	-3.90	0.11	25.41



**Fig. 5.** DAM Price SP15 (\$/MWh) for hours 4, 9, 12, 17 and 20. Data for the period from January 8, 2013 to September 24, 2016 All data is retrieved from LCG Consulting [39] and OASIS [40].

**Table 3**  
Descriptive statistics for the independent variables for hour 4 in the price area SP15. Correlation with the hour 4 price is also included. Data for the period from January 8, 2013 to September 24, 2016 All data is retrieved from LCG consulting [39] and OASIS [40].

H4	Gas Price (USD/mmBtu)	GHG Allowance Price (USD/Mt CO2e)	Wind Forecast (MW)	Solar Forecast (MW)	Load Forecast: (MW)
Mean	4.27	12.80	897	0.02	11363
Standard deviation	1.01	0.86	615	0.07	990
Excess Kurtosis	3.44	0.93	-0.73	40.86	-0.05
Skewness	0.90	1.16	0.46	5.40	0.81
Maximum	12.44	16.45	2634	0.57	15008
Minimum	2.26	11.60	28.00	0.00	9428
correlation with H4	0.65	-0.15	-0.33	0.19	0.08

quantile lines where the weighted sum of absolute residuals is minimized according to the specific quantiles  $q$ . For example, in a 20% quantile line, the absolute residuals above this line are weighted 80%  $(1-q)$ , whereas the absolute residuals below this line

are weighted 20%  $(q)$ . In this way, all observations can be used to calculate the conditional distribution given the set of independent variables.

Using this approach, we can also investigate the non-linear

**Table 4**

Descriptive statistics for the independent variables for hour 9 in the price area SP15. Correlation with the hour 9 price is also included. Data for the period from January 8, 2013 to September 24, 2016 All data is retrieved from LCG consulting [39] and OASIS [40].

H9	Gas Price (USD/mmBtu)	GHG Allowance Price (USD/Mt CO2e)	Wind Forecast (MW)	Solar Forecast (MW)	Load Forecast (MW)
Mean	4.27	12.80	586	1558	13960
Standard deviation	1.01	0.86	523	1026	1640
Excess Kurtosis	3.44	0.93	1.00	-1.04	-0.09
Skewness	0.90	1.16	1.26	0.34	0.05
Maximum	12.44	16.45	2510	3747	19108
Minimum	2.26	11.60	24.34	62.93	10031
Correlation with H9	0.57	-0.05	-0.21	-0.64	0.27

**Table 5**

Descriptive statistics for the independent variables for hour 12 in the price area SP15. Correlation with the hour 12 price is also included. Data for the period from January 8, 2013 to September 24, 2016 All data is retrieved from LCG consulting [39] and OASIS [40].

H12	Gas Price (USD/mmBtu)	GHG Allowance Price (USD/Mt CO2e)	Wind Forecast (MW)	Solar Forecast (MW)	Load Forecast (MW)
Mean	4.27	12.80	535	2528	15348
Standard deviation	1.01	0.86	514	1446	2497
Excess Kurtosis	3.44	0.93	2.00	-1.15	0.18
Skewness	0.90	1.16	1.56	0.04	0.71
Maximum	12.44	16.45	2782	5383	23942
Minimum	2.26	11.60	21.94	122	10404
Correlation with H12	0.56	0.03	-0.25	-0.58	0.40

**Table 6**

Descriptive statistics for the independent variables for hour 17 in the price area SP15. Correlation with the hour 17 price is also included. Data for the period from January 8, 2013 to September 24, 2016 All data is retrieved from LCG consulting [39] and OASIS [40].

H17	Gas Price (USD/mmBtu)	GHG Allowance Price (USD/Mt CO2e)	Wind Forecast (MW)	Solar Forecast (MW)	Load Forecast (MW)
Mean	4.27	12.80	904	1472	16680
Standard deviation	1.01	0.86	658	1321	3649
Excess Kurtosis	3.44	0.93	-0.40	-1.04	0.00
Skewness	0.90	1.16	0.63	0.54	0.90
Maximum	12.44	16.45	3027	4304	28717
Minimum	2.26	11.60	20.36	2.00	11399
Correlation with H17	0.55	-0.07	-0.19	-0.21	0.61

**Table 7**

Descriptive statistics for the independent variables for hour 20 in the price area SP15. Correlation with the hour 20 price is also included. Data for the period from January 8, 2013 to September 24, 2016 All data is retrieved from LCG consulting [39] and OASIS [40].

H20	Gas Price (USD/mmBtu)	GHG Allowance Price (USD/Mt CO2e)	Wind Forecast (MW)	Solar Forecast (MW)	Load Forecast (MW)
Mean	4.27	12.80	1053	51.45	16823
Standard deviation	1.01	0.86	649.95	92.86	2454
Excess Kurtosis	3.44	0.93	-0.78	2.77	0.82
Skewness	0.90	1.16	0.22	1.95	1.09
Maximum	12.44	16.45	2752	402.2	25976
Minimum	2.26	11.60	25.19	0.00	12598
Correlation with H20	0.68	-0.25	-0.14	-0.01	0.38

effect of the different independent variables. This is crucial, as the supply curve implies a non-linear relationship between the electricity price and its fundamentals. The sensitivity of natural gas prices is expected to be higher with higher levels of electricity prices, while the same will be expected for emission prices and load/demand. On the other hand, we expect a negative relationship between prices and renewables, with its highest effect when prices are low. We also control for remaining price dynamics not captured by the fundamentals, such as lagged prices and the volatility of prices.

The following quantiles are modeled in this paper: 1%, 5%, 10%, 20%, 30%, 40%, 50%, 60%, 70%, 80%, 90%, 95% and 99%. The linear quantile regression model is given by:

$$\ln(P_{h,t}^q) = \alpha_h^q + \beta_{1,h}^q \ln P_{h,t-1} + \beta_{2,h}^q \ln P_{h,t-7} + \beta_{3,h}^q \ln VOL_{h,t} + \beta_4^q \ln GAS_t + \beta_5^q \ln GHG_t + \beta_{6,h}^q \ln LOAD_{h,t} + \beta_{7,h}^q \ln SOLAR_{h,t} + \beta_{8,h}^q \ln WIND_{h,t} + \varepsilon_{h,t}^q \tag{1}$$

Here,  $q$  represents the quantile,  $t$  is time, and  $h$  is the hour under investigation (i.e. hours 4, 9, 12, 17, and 20).  $P_{t-1}$  and  $P_{t-7}$  are the lagged prices at lag 1 and 7, respectively. Prices are in \$/MWh.  $VOL_t$  refers to the exponentially weighted volatility using information up to  $t$ , and is the variation in relative price changes as a percent. Other features of seasonality and volatility dynamics are captured by the other independent variables.  $GAS_t$  is the natural gas price at  $t$  in

\$/MMBtu.  $GHG_t$  is the greenhouse gas emission price at  $t$  in \$/ton  $CO_2$ .  $LOAD_t$  is the load forecast for  $t+1$  available at  $t$  in MWh.  $SOLAR_t$  is the solar production forecast for  $t+1$  available at  $t$  in MWh.  $WIND_t$  is the wind production forecast for  $t+1$  available at  $t$  in MWh.  $\varepsilon_t$  is the error term.  $GAS$  and  $GHG$  are not specific for any hour, while the rest of the variables are specific for hour  $h$ . All variables are transformed by the natural logarithm. The reasons for applying this transformation are: 1) smoothing out spikes in the data, and 2) being able to interpret all betas as elasticities.

Model (1) allows us to estimate the price for a given quantile and hour with a specific set of elasticities to the independent variables. For example, for hour 4 and quantile 1% we expect weaker sensitivity to gas (as demands/prices tend to be lower at night and gas is only in use when there is high demand). If we look at hour 20 and quantile 99% ( $h = 20$  and  $q = 0.99$ ), we might have stronger sensitivity to gas (as demand/prices tend to be higher in the afternoon). Similar justifications can be made for other choices of  $h$  and  $q$ .

The estimation of (1) minimizes the weighted sum of absolute error deviations where the weights are  $q$  or  $1-q$ . We used EViews software for the estimation using the qreg procedure. We obtain the standard errors of the parameters using the Huber Sandwich Method. This method is robust when residuals are not independently and identically distributed, see Koenker [43] for more details. The Pseudo R-square (see Refs. [42]) is used to measure the goodness-of-fit of the models for each period and quantile.

## 6. Results and discussion

After running the model (1) for the price area SP15 for hours 4, 9, 12, 17 and 20, the results are as follows. Table 8 shows the coefficients for the selected trading periods and hours. Pseudo R2 is in the range 0.59–0.73 for the different models.<sup>3</sup> Moreover, most of the variables are significant at 5% or lower over the hours and quantiles (a significance below 5% is indicated with bold letters).

In the results for the log-log quantile regression model in Table 8, we can interpret all coefficients as elasticities. E.g. Hour 17, quantile 99% in Table 8 gives a coefficient of 0.904 for  $\ln Load$ . That means if load goes up 1% for Hour 17, the 99% quantile of electricity prices will go up 0.904%. Another example is Hour 4, 1% quantile and  $\ln Wind$ . Here the coefficient is  $-0.072$ , meaning that if wind production goes up 1% for Hour 4, the 1% quantile of electricity prices goes down by 0.072%. From Table 8 we see that the most significant variable overall is.

We have included a serial-correlation of prices as controls, since not all of the price persistence features can be captured by the independent variables. In general, there is a consistent positive serial-correlation in prices for both lag 1 and lag 7 ( $\ln P_{h,t-1}$  respectively  $\ln P_{h,t-7}$ ). The effects do not vary much over hours and quantiles. A positive serial-correlation is an indication of price adaption (high prices one day followed by high prices the next day), which can be due to the adaptive behavior of the market participants (see Bunn et al. [22], for a discussion of this). Another control variable used is the volatility of the prices (measured by exponential weighted moving average volatility with a smoothing parameter of 0.97). In many commodity markets, we have the so-called “inverse-leverage” effect. That is, volatility goes more up when markets go up than down. Here, we find no clear pattern in the effect of volatility ( $\ln Vol_{h,t}$ ), and for many of the hours (e.g. hour 4) it is not

significant. This could be because the model captures much of the effect of volatility through other independent variables.

As expected, the natural gas prices ( $\ln Gas_t$ ) overall have a positive and non-linear effect on the electricity price. In general, the effect is higher with higher quantiles, as expected from the supply curve. However, some results are not so clear. There are negative (unexpected) coefficients for low quantiles at hour 4, 9, 12, and 17. It might be that at low quantiles, gas power is not used, resulting in “strange” elasticities. Keep also in mind that all elasticities are indirect effects, controlling for the effect of all other variables.

Regarding GHG allowance prices, there is a mixture of negative and positive effects which is unexpected and hard to interpret. For hour 4 the impact of GHG allowance prices are in general not significant.

A higher load ( $\ln Load_{h,t}$ ) is an indication of a higher demand; hence, the observed positive effect is expected. For most hours the effect is also higher with higher quantiles. For hours 4 and 9, the opposite occurs and is hard to explain. There are also negative signs for some hours and quantiles.

Increased solar production should lead to lower prices, which is also generally the case. The effect is higher with higher quantiles. There are however also some positive values which are perplexing. For hours 4 and 20, solar production has (as expected) a very low (close to 0) effect.

Increased wind production should also lead to lower prices, which is generally the case in our results. There is however no clear pattern to indicate that the effect increases over the quantiles.

It is important to note that sensitivities vary according to the quantile (level of electricity price), as well as hours. Some fundamental factors are significant for low quantiles and certain hours (e.g. wind power at hour 12), whereas others are significant for high quantiles and certain hours (e.g. natural gas at hour 9). Some fundamentals have an increasing effect with quantiles (e.g. natural gas), while others have a decreasing effect (e.g. wind power production). These non-linear influences are built into the quantile regression models. In such a way, we can predict the whole price distributions given the set of fundamentals for each specific hour.<sup>4</sup> The next chapter describes applications of the model within the area of risk management.

<sup>4</sup> We have also performed a bucket regression. The detailed results of this can be found by contacting the authors. Bucket regression are performed in many ways to capture the non-linearities between the dependent variable and the independent variables. One application is to split the data into bins for the dependent variable and run separable regressions on the independent variable using the conditional data. For example, one might select the data according to the highest 1% of the data for the dependent variables (the 99% quantile) and run a regression on the corresponding set of values for the independent variables in the dataset. Similarly, one can select data between the 99% and 95% quantile for the dependent variable and run a regression the independent variables accordingly. And so on, for all the selected quantiles/buckets. What is the difference using quantile regression instead of splitting the data in quantiles and calculating multiple linear regressions? In the bucket regression, a subsample of data is used and parameters are found minimizing the sum of squared residuals for the specific regression line. In quantile regression, all data is used and parameters are found minimizing the weighted sum of absolute residuals for the specific regression line. The weights are the values according to the specific quantile. For example in the 99% quantile line, the absolute value of the residuals above this line is weighted 99%, absolute value of residuals below 1%. Again, all the data is used, not just the 1% of the highest values according to the dependent variable. Hence, Quantile regression provides more robust estimation technique in the presence of outliers. Bucket regression will (please contact authors for more details) provide insignificant parameter values in the tails as few observations is used. In quantile regression however, many parameter values are significant for the tail regressions. As the aim of our paper is to estimate pricing models for very low and high electricity prices, we therefore choose quantile regression as the proposed method.

<sup>3</sup> For hours with negative values of the dependent variables, a constant is added to the variable value. This ensures a positive value of the variable, such that  $\ln(\text{variable})$  is possible to perform. When making predictions, we revert the process.



Table 8

Quantile regression results for selected trading periods in price area SP15. The following models have been applied for hour 4, 9, 12, 17 and 20 for quantiles ranging from 1% to 99% OLS estimates are also included. Bold numbers indicate significance at 5% or lower. Pseudo R<sup>2</sup> is in the range of 0.59–0.73 for the different models.  $\ln(P_{h,t}^q) = \alpha_h^q + \beta_{1,h}^q \ln P_{h,t-1} + \beta_{2,h}^q \ln P_{h,t-7} + \beta_{3,h}^q \ln VOL_{h,t} + \beta_{4,h}^q \ln GAS_t + \beta_{5,h}^q \ln GHG_t + \beta_{6,h}^q \ln LOAD_{h,t} + \beta_{7,h}^q \ln SOLAR_{h,t} + \beta_{8,h}^q \ln WIND_{h,t} + \varepsilon_{h,t}^q$ .

Hour	Quantile	0.01	0.05	0.1	0.2	0.3	0.4	0.5	OLS	0.6	0.7	0.8	0.9	0.95	0.99
4	$\alpha_h^q$	-7.663	-6.355	-3.566	-2.097	-1.465	-0.7	-0.569	-4.052	-0.221	0.208	0.686	1.121	1.628	1.164
	$\ln P_{h,t-1}$	<b>0.628</b>	<b>0.566</b>	<b>0.612</b>	<b>0.678</b>	<b>0.671</b>	<b>0.639</b>	<b>0.63</b>	<b>0.246</b>	<b>0.594</b>	<b>0.545</b>	<b>0.462</b>	<b>0.426</b>	<b>0.388</b>	<b>0.338</b>
	$\ln P_{h,t-7}$	<b>0.364</b>	<b>0.247</b>	<b>0.249</b>	<b>0.165</b>	<b>0.178</b>	<b>0.195</b>	<b>0.191</b>	<b>0.394</b>	<b>0.191</b>	<b>0.188</b>	0.16	0.131	0.108	0.099
	$\ln VOL_{h,t}$	-0.037	-0.026	-0.024	-0.003	0.004	0.014	0.024	0.000	0.028	0.028	0.033	0.032	0.029	0.044
	$\ln GAS_t$	<b>-0.297</b>	<b>0.004</b>	<b>0.012</b>	<b>0.067</b>	<b>0.081</b>	0.085	<b>0.125</b>	<b>0.336</b>	<b>0.162</b>	<b>0.218</b>	<b>0.346</b>	<b>0.461</b>	<b>0.555</b>	<b>0.685</b>
	$\ln GHG_t$	0.028	0.109	-0.098	-0.093	-0.015	-0.062	0.039	<b>0.560</b>	0.04	0.038	0.117	0.192	0.35	0.726
	$\ln LOAD_{h,t}$	<b>0.885</b>	<b>0.744</b>	<b>0.475</b>	<b>0.32</b>	<b>0.226</b>	0.164	0.122	<b>-0.037</b>	0.097	<b>0.065</b>	<b>0.01</b>	<b>-0.043</b>	<b>-0.132</b>	<b>-0.167</b>
	$\ln SOLAR_{h,t}$	<b>0.000</b>	<b>-0.001</b>	<b>-0.002</b>	<b>-0.001</b>	0.000	0.000	0.001	<b>-0.025</b>	0.001	<b>0.001</b>	<b>0.001</b>	<b>0</b>	<b>0</b>	<b>0.008</b>
	$\ln WIND_{h,t}$	<b>-0.072</b>	<b>-0.059</b>	<b>-0.045</b>	<b>-0.041</b>	<b>-0.034</b>	<b>-0.03</b>	<b>-0.026</b>	<b>-0.008</b>	<b>-0.027</b>	<b>-0.031</b>	<b>-0.031</b>	<b>-0.031</b>	<b>-0.032</b>	<b>-0.021</b>
9	Quantile	0.01	0.05	0.1	0.2	0.3	0.4	0.5	OLS	0.6	0.7	0.8	0.9	0.95	0.99
	$\alpha_h^q$	<b>-27.488</b>	<b>-10.971</b>	<b>-5.242</b>	<b>-2.847</b>	<b>-1.74</b>	<b>-0.889</b>	<b>-0.212</b>	<b>-3.448</b>	<b>0.458</b>	<b>0.812</b>	<b>1.110</b>	<b>1.823</b>	<b>1.824</b>	<b>1.289</b>
	$\ln P_{h,t-1}$	<b>0.699</b>	<b>0.848</b>	<b>0.58</b>	<b>0.512</b>	<b>0.389</b>	<b>0.323</b>	<b>0.261</b>	<b>0.240</b>	<b>0.234</b>	<b>0.194</b>	<b>0.165</b>	<b>0.132</b>	<b>0.084</b>	<b>0.025</b>
	$\ln P_{h,t-7}$	<b>1.073</b>	<b>0.5</b>	<b>0.336</b>	<b>0.345</b>	<b>0.365</b>	<b>0.367</b>	<b>0.33</b>	<b>0.385</b>	<b>0.296</b>	<b>0.236</b>	<b>0.145</b>	<b>0.091</b>	<b>0.076</b>	<b>0.069</b>
	$\ln VOL_{h,t}$	<b>-0.112</b>	<b>-0.082</b>	<b>-0.02</b>	<b>-0.005</b>	0.003	0.012	0.019	-0.003	0.021	0.028	<b>0.036</b>	<b>0.032</b>	<b>0.023</b>	<b>-0.011</b>
	$\ln GAS_t$	<b>-0.583</b>	<b>-0.446</b>	<b>-0.034</b>	<b>0.036</b>	<b>0.086</b>	<b>0.129</b>	<b>0.186</b>	<b>0.337</b>	<b>0.246</b>	<b>0.373</b>	<b>0.53</b>	<b>0.593</b>	<b>0.666</b>	<b>0.89</b>
	$\ln GHG_t$	<b>1.433</b>	<b>0.299</b>	<b>-0.082</b>	<b>-0.242</b>	<b>-0.315</b>	<b>-0.315</b>	<b>-0.397</b>	<b>0.596</b>	<b>-0.425</b>	<b>-0.368</b>	<b>-0.359</b>	<b>-0.341</b>	<b>-0.308</b>	<b>-0.132</b>
	$\ln LOAD_{h,t}$	<b>2.186</b>	<b>0.971</b>	<b>0.639</b>	<b>0.451</b>	<b>0.403</b>	<b>0.344</b>	<b>0.344</b>	<b>-0.061</b>	<b>0.307</b>	<b>0.281</b>	<b>0.265</b>	<b>0.255</b>	<b>0.276</b>	<b>0.282</b>
	$\ln SOLAR_{h,t}$	<b>0.103</b>	<b>0.05</b>	<b>-0.022</b>	<b>-0.032</b>	<b>-0.051</b>	<b>-0.062</b>	<b>-0.085</b>	<b>-0.027</b>	<b>-0.099</b>	<b>-0.105</b>	<b>-0.12</b>	<b>-0.151</b>	<b>-0.171</b>	<b>-0.181</b>
$\ln WIND_{h,t}$	<b>-0.011</b>	<b>-0.043</b>	<b>-0.037</b>	<b>-0.03</b>	<b>-0.029</b>	<b>-0.031</b>	<b>-0.031</b>	<b>-0.313</b>	<b>-0.03</b>	<b>-0.029</b>	<b>-0.027</b>	<b>-0.027</b>	<b>-0.026</b>	<b>-0.019</b>	
12	Quantile	0.01	0.05	0.1	0.2	0.3	0.4	0.5	OLS	0.6	0.7	0.8	0.9	0.95	0.99
	$\alpha_h^q$	<b>-7.241</b>	<b>-7.039</b>	<b>-4.258</b>	<b>-2.414</b>	<b>-1.436</b>	<b>-0.848</b>	<b>-0.701</b>	<b>-3.258</b>	<b>-1.033</b>	<b>-1.179</b>	<b>-1.205</b>	<b>-1.009</b>	<b>-1.207</b>	<b>-1.761</b>
	$\ln P_{h,t-1}$	<b>2.073</b>	<b>0.861</b>	<b>0.867</b>	<b>0.63</b>	<b>0.533</b>	<b>0.474</b>	<b>0.391</b>	<b>0.322</b>	<b>0.28</b>	<b>0.183</b>	<b>0.146</b>	<b>0.088</b>	<b>0.031</b>	<b>0.034</b>
	$\ln P_{h,t-7}$	<b>1.029</b>	<b>0.715</b>	<b>0.371</b>	<b>0.336</b>	<b>0.276</b>	<b>0.203</b>	<b>0.167</b>	<b>0.280</b>	<b>0.123</b>	<b>0.056</b>	<b>0.044</b>	<b>0.032</b>	<b>0.031</b>	<b>0.025</b>
	$\ln VOL_{h,t}$	<b>-0.285</b>	<b>-0.092</b>	<b>-0.045</b>	<b>-0.008</b>	0.000	0.007	0.012	0.003	0.02	0.018	<b>0.022</b>	<b>0.017</b>	<b>0.014</b>	<b>0.01</b>
	$\ln GAS_t$	<b>-1.951</b>	<b>-0.408</b>	<b>-0.208</b>	<b>-0.01</b>	<b>0.08</b>	<b>0.186</b>	<b>0.317</b>	<b>0.410</b>	<b>0.497</b>	<b>0.686</b>	<b>0.741</b>	<b>0.825</b>	<b>0.862</b>	<b>0.939</b>
	$\ln GHG_t$	<b>-0.218</b>	<b>0.749</b>	<b>0.227</b>	<b>-0.103</b>	<b>-0.235</b>	<b>-0.283</b>	<b>-0.246</b>	<b>0.603</b>	<b>-0.145</b>	<b>-0.052</b>	0.013	0.133	0.164	0.325
	$\ln LOAD_{h,t}$	<b>-0.249</b>	<b>0.3</b>	<b>0.321</b>	<b>0.333</b>	<b>0.341</b>	<b>0.347</b>	<b>0.359</b>	<b>-0.098</b>	<b>0.418</b>	<b>0.462</b>	<b>0.465</b>	<b>0.451</b>	<b>0.523</b>	<b>0.537</b>
	$\ln SOLAR_{h,t}$	<b>0.317</b>	<b>0.12</b>	<b>0.037</b>	<b>-0.018</b>	<b>-0.054</b>	<b>-0.081</b>	<b>-0.096</b>	<b>-0.031</b>	<b>-0.116</b>	<b>-0.138</b>	<b>-0.145</b>	<b>-0.174</b>	<b>-0.227</b>	<b>-0.222</b>
$\ln WIND_{h,t}$	<b>-0.06</b>	<b>-0.108</b>	<b>-0.086</b>	<b>0.056</b>	<b>-0.048</b>	<b>0.039</b>	<b>-0.033</b>	<b>-0.317</b>	<b>-0.03</b>	<b>-0.031</b>	<b>-0.027</b>	<b>0.022</b>	<b>-0.015</b>	<b>-0.023</b>	
17	Quantile	0.01	0.05	0.1	0.2	0.3	0.4	0.5	OLS	0.6	0.7	0.8	0.9	0.95	0.99
	$\alpha_h^q$	<b>-5.525</b>	<b>-3.761</b>	<b>-3.069</b>	<b>-2.178</b>	<b>-1.96</b>	<b>-1.791</b>	<b>-2.024</b>	<b>-5.869</b>	<b>-2.375</b>	<b>-2.765</b>	<b>-3.373</b>	<b>-3.972</b>	<b>-4.702</b>	<b>-7.442</b>
	$\ln P_{h,t-1}$	<b>0.565</b>	<b>0.508</b>	<b>0.55</b>	<b>0.585</b>	<b>0.57</b>	<b>0.55</b>	<b>0.524</b>	<b>0.443</b>	<b>0.49</b>	<b>0.423</b>	<b>0.375</b>	<b>0.336</b>	<b>0.346</b>	<b>0.313</b>
	$\ln P_{h,t-7}$	<b>0.255</b>	<b>0.235</b>	<b>0.161</b>	<b>0.137</b>	<b>0.112</b>	<b>0.093</b>	<b>0.072</b>	<b>0.080</b>	<b>0.075</b>	<b>0.064</b>	0.027	0.027	0.027	-0.094
	$\ln VOL_{h,t}$	<b>-0.087</b>	<b>-0.051</b>	<b>-0.029</b>	<b>-0.02</b>	<b>-0.01</b>	<b>-0.003</b>	0.004	-0.004	0.008	<b>0.016</b>	<b>0.02</b>	<b>0.027</b>	<b>0.035</b>	<b>0.013</b>
	$\ln GAS_t$	<b>-0.129</b>	<b>0.023</b>	<b>0.088</b>	<b>0.098</b>	<b>0.166</b>	<b>0.208</b>	<b>0.277</b>	<b>0.420</b>	<b>0.316</b>	<b>0.405</b>	<b>0.479</b>	<b>0.532</b>	<b>0.555</b>	<b>0.713</b>
	$\ln GHG_t$	<b>-0.092</b>	<b>0.081</b>	<b>0.164</b>	<b>0.04</b>	<b>0.03</b>	<b>-0.005</b>	<b>0.053</b>	<b>0.754</b>	<b>0.086</b>	<b>0.178</b>	<b>0.316</b>	<b>0.416</b>	<b>0.489</b>	<b>0.631</b>
	$\ln LOAD_{h,t}$	<b>0.711</b>	<b>0.501</b>	<b>0.413</b>	<b>0.343</b>	<b>0.332</b>	<b>0.337</b>	<b>0.36</b>	<b>-0.021</b>	<b>0.398</b>	<b>0.442</b>	<b>0.499</b>	<b>0.551</b>	<b>0.611</b>	<b>0.904</b>
	$\ln SOLAR_{h,t}$	<b>-0.04</b>	<b>-0.029</b>	<b>-0.025</b>	<b>-0.021</b>	<b>-0.019</b>	<b>-0.02</b>	<b>-0.019</b>	<b>-0.044</b>	<b>-0.02</b>	<b>-0.022</b>	<b>-0.027</b>	<b>-0.031</b>	<b>-0.029</b>	<b>-0.043</b>
$\ln WIND_{h,t}$	-0.055	-0.053	-0.045	-0.033	-0.03	<b>-0.029</b>	<b>0.03</b>	<b>0.107</b>	<b>-0.028</b>	<b>-0.032</b>	<b>-0.029</b>	<b>0.027</b>	<b>-0.036</b>	<b>-0.015</b>	
20	Quantile	0.01	0.05	0.1	0.2	0.3	0.4	0.5	OLS	0.6	0.7	0.8	0.9	0.95	0.99
	$\alpha_h^q$	<b>-3.646</b>	<b>-2.173</b>	<b>-1.305</b>	<b>0.07</b>	<b>-0.089</b>	<b>0.068</b>	<b>-0.089</b>	<b>-1.238</b>	<b>-0.116</b>	<b>-0.496</b>	<b>-0.369</b>	<b>-0.534</b>	<b>-1.096</b>	<b>-3.149</b>
	$\ln P_{h,t-1}$	<b>0.357</b>	<b>0.397</b>	<b>0.432</b>	<b>0.573</b>	<b>0.574</b>	<b>0.615</b>	<b>0.623</b>	<b>0.559</b>	<b>0.635</b>	<b>0.639</b>	<b>0.616</b>	<b>0.596</b>	<b>0.604</b>	<b>0.578</b>
	$\ln P_{h,t-7}$	<b>0.105</b>	<b>0.137</b>	<b>0.14</b>	<b>0.099</b>	<b>0.128</b>	<b>0.106</b>	<b>0.104</b>	<b>0.077</b>	<b>0.117</b>	<b>0.151</b>	0.137	0.134	0.131	-0.044
	$\ln VOL_{h,t}$	-0.033	-0.022	-0.019	-0.006	-0.001	0.007	0.01	0.008	0.012	0.021	<b>0.036</b>	<b>0.054</b>	<b>0.064</b>	<b>0.066</b>
	$\ln GAS_t$	<b>0.122</b>	<b>0.148</b>	<b>0.184</b>	<b>0.182</b>	<b>0.177</b>	<b>0.164</b>	<b>0.168</b>	<b>0.173</b>	<b>0.161</b>	<b>0.158</b>	<b>0.215</b>	<b>0.263</b>	<b>0.246</b>	<b>0.454</b>
	$\ln GHG_t$	<b>0.593</b>	<b>0.109</b>	<b>-0.055</b>	<b>-0.195</b>	<b>-0.128</b>	<b>-0.14</b>	<b>-0.087</b>	<b>0.307</b>	<b>-0.095</b>	<b>0.007</b>	<b>0.039</b>	<b>0.064</b>	<b>0.118</b>	<b>0.558</b>
	$\ln LOAD_{h,t}$	0.403	<b>0.365</b>	<b>0.306</b>	<b>0.162</b>	<b>0.151</b>	<b>0.136</b>	<b>0.138</b>	<b>-0.004</b>	<b>0.135</b>	<b>0.135</b>	<b>0.127</b>	0.152	0.214	0.38
	$\ln SOLAR_{h,t}$	<b>0.002</b>	<b>0.002</b>	<b>0.001</b>	<b>0.001</b>	<b>0.001</b>	<b>0.001</b>	<b>0.000</b>	<b>-0.015</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.001</b>	<b>0.000</b>	<b>0.002</b>
$\ln WIND_{h,t}$	<b>-0.02</b>	<b>-0.026</b>	-0.032	<b>-0.026</b>	<b>-0.024</b>	<b>-0.024</b>	<b>-0.023</b>	<b>-0.153</b>	<b>-0.022</b>	<b>-0.019</b>	<b>-0.02</b>	<b>-0.024</b>	<b>-0.037</b>	<b>-0.042</b>	

7. Risk management case study: application of the model

It is of interest to discuss how the proposed model applies to scenario analysis and stress testing. As a starting point, consider a producer of electricity concerned with low prices. The hour of interest is hour 17. The risk the producer is facing is a scenario with a lower demand than expected together with a higher production of wind and solar than expected. As a base scenario, we can use the last values of the data points in our analysis.

On September 24, 2016 we had the following observations of the dependent variables in levels and log levels for hour 17 (Table 9):

A risk manager might ask, “What is the 5% Value at Risk given this base scenario for our independent variables?” In other words, what is the value such that 95% of the time, price will be higher than this value (in 5% of the occasions the price will be lower)?

From the basis scenario above, we can calculate directly the 5% VaR using the parameters for quantile 5% and hour 17, and using the quantile regression model with the parameters given in Table 8:

$$\ln(P_{h=17,t}^{5\%}) = - 3.761 + 0.508 \ln P_{12,t-1} + 0.235 \ln P_{12,t-7} - 0.051 \ln VOL_{12,t} + 0.023 \ln GAS_{h,t} + 0.081 \ln GHG_t + 0.501 \ln LOAD_{12,t} - 0.029 \ln SOLAR_{12,t} - 0.053 \ln WIND_{12,t}$$

$$\ln(P_{h=17,t}^{5\%}) = - 3.761 + 0.508*3.636 + 0.235*3.558 - 0.051*(- 1.542) + 0.023*(1.840) + 0.081*(2.562) + 0.501*(9.812) - 0.029*(8.310) - 0.053*(5.270) = 3.608$$

Taking  $e^{3.608}$ , we get a price of 36.89 \$/MWh. This means that it is 95% likely that prices will be 36.89 \$/MWh or higher. Prices will be lower than 36.89 \$/MWh 5% of the time.

The risk manager might also be concerned about possible (“stressed”) scenarios for hour 17 prices. One such situation could

**Table 9**  
Observations of dependent variables for September 24th, 2016 at hour 17.

Dependent variable	Level	ln
$P_{h=17,t-1}$	37.93	3.64
$P_{h=17,t-7}$	35.09	3.56
$VO_{h=17,t-7}$	0.21	-1.54
$Gas_t$	6.30	1.84
$GHG_t$	12.96	2.56
$Load_{h=17,t-7}$	18254	9.81
$Solar_{h=17,t-7}$	4065	8.31
$Wind_{h=17,t-7}$	194	5.27

**Table 10**  
Range of solar and wind production values in MWh for stress testing.

Solar	Wind
10	10
100	100
500	250
1000	500
1500	1000
2000	1250
2500	1500
3000	1750
3500	2000
4000	2500
4500	3000

be a scenario of a high production of solar and wind. What will be the 5% Value at Risk under such a scenario (keeping everything else equal)?

The min/max observed range of solar and wind in our dataset for hour 17 are 2.00–4304 MWh and 20.36 to 3026MWh, respectively. (Theoretically, these bounds could be even wider.) The model can be recalculated using inputs for wind and solar following a range of values for solar and wind, keeping all the rest of the other variables at the values for the base scenario above.

We first need to select a range of solar and wind production. As an example, we can consider the values in Table 10, ranging from the minimum to the maximum in our dataset:

When solar and wind are both at 10 MWh, we get:

$$\ln(P_{h=17,t}^{5\%}) = -3.761 + 0.508*3.636 + 0.235*3.558 - 0.051*(-1.542) + 0.023*(1.840) + 0.081*(2.562) + 0.501*(9.812) - 0.029*(\ln(10)) - 0.053*(\ln(10)) = 3.939$$

Taking  $e^{3.939}$ , we get the 5% VaR at 51.39 \$/MWh.

Continuing in this way, one obtains the following results in Table 11:

The higher production of solar and wind (assuming load and other variables remaining the same) will decrease the 5% Value at Risk substantially from 51.39 \$/MWh to 31.82 \$/MWh.

**Table 11**  
The 5%VaR for hour 17 at increasing levels of solar and wind production.

Solar	Wind	5% VaR H17
10	10	51.39
100	100	42.55
500	250	38.68
1000	500	36.55
1500	1000	34.82
2000	1250	34.12
2500	1500	33.58
3000	1750	33.13
3500	2000	32.75
4000	2500	32.24
4500	3000	31.82

In addition, one could look at scenarios in which both load, and perhaps gas and GHG prices, also changed individually or simultaneously. Ultimately, quantile regression allows us to investigate how fundamental supply and demand variables influence the whole price distribution and to evaluate risk measures such as Value at Risk.

## 8. Conclusions

This paper explored the dependence of electricity spot price distributions on fundamentals in the California electricity market using a quantile regression model. Natural gas, GHG allowance prices and load all have a positive effect on electricity prices, which increases with quantiles. Solar production and wind production have a negative effect on electricity prices. The effect of solar production increases with quantiles, while the effect of wind production decreases with quantiles. In our model, we quantify these effects over different hours and quantiles.

This paper also demonstrated how to use this framework for scenario analysis and stress testing in risk management. Results of the case study show the impact of increasing wind and solar on the 5% VaR. Performing such scenarios and quantifying the effect on risk are crucial for effective risk management.

Further research could extend the analysis to include more variables/proxies for supply and demand. For example, it would be of interest to investigate how reserve capacity and import/export conditions influence prices. This will be an important factor - in particular as CAISO increasingly relies on import and export from neighboring states for flexible capacity to help integrate California's increasing renewable generation. It would also be interesting to extend the analysis to other price areas, such as NP15, as other areas have a different input mix and thus different sensitivities to fundamentals. Lastly, investigating price formation and risk in CAISO's real-time market and energy imbalance market using the approach in this paper would also be of interest.

## Credit author statement

Sjur Westgaard (writing all parts, reviewing and editing, quality control of data and regression results, corresponding author). Stein-Erik Fleten (reviewing). Ahlmahz Negash (reviewing, input on market description). Audun Botterud (reviewing, input on market description). Katinka Bogaard (Initial data analysis and reports, research student work). Trude Haugsvaer Verling (Initial data analysis and reports, research student work).

## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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## Appendix A. Supplementary data

Supplementary data to this article can be found online at <https://doi.org/10.1016/j.energy.2020.118796>.

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