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Himanshu Srivastav

Optimizing condition monitoring for dynamic health and risk management

Quantification of added value of condition information, optimization of inspection and monitoring strategies.

Doctoral thesis

NTNU
Norwegian University of Science and Technology
Thesis for the Degree of
Philosophiae Doctor
Faculty of Engineering
Department of Mechanical and
Industrial Engineering

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To my family and friends

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Preface

This thesis is submitted to the Norwegian University of Science and Technology (NTNU) for partial fulfillment of the requirements for the degree of Philosophiae Doctor.

This Ph.D. position was funded by a scholarship through the Centre for Innovation based Research (SFI) within subsea production and processing (SUBPRO). The research work was carried out at the Department of Mechanical and Industrial Engineering at NTNU, in Trondheim, Norway. Professor Anne Barros (current affiliation: Department of Industrial Engineering at CentraleSupélec, France) and Professor Mary Ann Lundteigen (current affiliation: Department of Engineering Cybernetics at NTNU) are the supervisor and co-supervisor, respectively. Both supervisors were faculty at the Department of Mechanical and Industrial Engineering at NTNU for the most duration of this Ph.D. and only for the last few months there was a change in affiliations. They ensured the smooth progress of the research submission of the thesis.

This work's target audience includes researchers and practitioners interested in the following areas: reliability engineering, maintenance engineering, automation engineering, and Oil and Gas industry.

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Abstract

This Ph.D. thesis explains the concepts of condition monitoring and associated challenges in maintenance modeling for subsea facilities. Thesis's main objective is to develop systematic frameworks to assess the performance of the subsea system considering its degradation phenomenon. The primary focus of this thesis is on modeling the degradation behavior of the subsea safety systems. We also extended the degradation modeling concepts to study the subsea production systems with components experiencing stochastic deterioration. We have addressed four research questions explicitly:

1. In the first research question, we developed a framework to assess the reliability of a safety instrumented system that is subjected to destructive periodic tests. We utilized a multi-phase Markov process (MMP) to model the degradation process of the SIS. The selection of a multi-phase Markov process is motivated by mainly two factors: (i) It allows us to have intermediate performance levels between perfectly working and uniquely failed levels. (ii)The impact of destructive testing is modeled by altering the transition rate of the degradation process. We developed a dynamic failure rate model that depends on the current degradation level and the number of tests experienced. We also performed the case study on Down-hole safety valves (DHSV) to determine the optimum number of periodic tests that maximize the average availability of DHSV in given mission time. A high frequency of tests will reduce the probability for DHSV to be in an undetected failed state and not to act on demand. On the other side, the cumulative stress experienced due to tests may degrade the performance to failure.

2. In the second research question, we extended degradation modeling techniques in the qualification of novel subsea technology. All-electric systems

are the novel subsea technology that is considered an upgrade of widely deployed electro-hydraulic systems. This novel technology promised more reliable equipment and a safer environment. The technology qualification requires the reliability assessment of new systems to provide sufficient evidence that the new technology is fit for the purpose without high risk. The current reliability assessment of such systems assumes perfect restoration during proof tests and no impact of degradation due to demands. Failure mode and effect analysis of all-electric actuation systems show that it is prone to degradation in performance due to power supply interruptions. These interruptions appear as random demands to the safety valves of the system. These valves may degrade its performance due to such demands. We utilized the MMP to model this situation. The impact of demand is modeled either by changing the initial condition of the MMP or by increasing the transition rates between two degraded states. We developed analytical formulae for realistic dynamic reliability assessment.

3. In the third research question, we studied the testing and maintenance strategies for a redundant SIS with imperfect detection of degraded state during proof tests. We studied the performance of redundant SIS under the combinations of staggered testing and simultaneous testing with preventive maintenance, corrective maintenance, and opportunistic maintenance. We developed analytical formulae for time-dependent unavailability and associated life cycle cost for finite mission time. This study's main purpose is to incorporate and balance system availability and life cycle costs. We performed a case study on the subsea high-integrity pressure protection system.

4. In the fourth research question, we extended the degradation modeling techniques in the domain of subsea production systems. Subsea production systems are operated very aggressively to extract hydrocarbon quickly and as much as possible. This causes premature wearing of the systems, which increases maintenance and repair costs. There exists a trade-off between high maintenance and repair costs versus high production profits. In this study, we addressed this trade-off by developing a method that integrates the deterministic control laws to the stochastically deteriorating components. We utilized non-homogeneous MMP on the component level to describe the system dynamics. We considered that transition rates of MMP are proportional to the operational loads. This assumption ensured that high operational loads would lead to faster deterioration of components. At the same time, we also assumed the productivity in each performance level of MMP is also propositional to the operational loads. This assumption will ensure that higher operational loads will lead to higher production. The resultant optim-

ization problem becomes a non-linear problem. We solved it numerically with the help of off-the-shelf non-linear problem solvers. The solver provides optimum values of operational load schedule, maintenance schedule, and maintenance efficiency, which maximizes the production. We performed a case study on the subsea compressor station to apply the developed method.

The frameworks developed in this thesis are meant to provide support to operators/engineers in informed decision-making. These decisions are based on the realistic performance assessment of the subsea system. It is assumed that the readers have familiarity with the general concepts in the domain of reliability theory.

Acknowledgement

गुरु गोविन्द दोऊ खड़े, काके लागू पाय।
बलिहारी गुरु आपने, गोविन्द दियो बताय।।

\- संत कबीर दास

Fifteenth century's famous Indian philosopher, 'Sant Kabir Das', coined the above verse to sing the glory of a Guru (guide or teacher) in life. In this verse he asks the question that, 'If both, Guru and God were to appear in front of you, whom should I worship first ?'. In the second line of the verse he answers, 'I should worship the Guru first, as I would never be able to recognize God without the guidance of the Guru'. I would like to acknowledge the unconditional love, care and support of my mother. She is my first Guru in life. The courage, to choose the right path and follow it, however hard the path may be, comes to me from my mother. Then, I would like to extend my deepest respect and gratitude to my father who cultivated my childhood with the virtues of hard work, patience and positive attitude. I would also like to thank my sister and her family for always being there for me. This journey to follow my dreams would not have been possible without the countless sacrifices made by my family.

In my professional life, first of all, I would express my sincere gratitude to my main supervisor: Prof. Anne Barros, for selecting me for this excellent research opportunity. I am grateful to her for creating an environment where I felt safe and free to express ideas. Her constant guidance and constructive feedback during

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discussions were phenomenal. Her energy levels are second to none. She has been an inspirational researcher. I have always looked up to her for qualities like integrity, humility, dedication. I feel fortunate to have her as my main supervisor. Not only did she transcend my journey as a researcher, but also influenced my way of life to become a stronger individual who now does not feel weak by accepting inherent vulnerability. I hope this association turns into a long-lasting collaboration.

I would like to thank my co-supervisor, Prof. Mary Ann Lundteigen. I was always impressed by her ability to stay organized, almost up to the order of zero entropy. Her ability to express an engineer's point of view for a particular research challenge is extraordinary. Her kind and able guidance during research discussions and feedback on the research papers augmented the research quality to the next level.

I would like to thank the SFI-SUBPRO for funding this Ph.D. project. I convey my gratitude to all members of SUBPRO center for their support and guidance. I would also like to thank all the administrative staff in the RAMS group at NTNU for making my life smooth during this long journey.

I would like to extend my humblest gratitude to all my fellow researchers at NTNU, who has left an impression in either my personal or professional life. I would like to convey my special thanks to Dr. Shareq Mohd Nazir. Shareq is my brother from another mother. Our friendship has seen ups and downs of life since the college days in India. The transformation in his personality from college days to now has been a remarkable journey and has always inspired me in life.

Last but not least, I would like to thank Prof. Millie Pant for being the source of inspiration, motivation since early college days in India. This journey would not have been possible without her constant support.

Thank you

Thesis Structure

This doctoral thesis is written in the format of a collection of articles, commonly known as compilation of thesis. There are two parts of this thesis: Part I (Main Report) and Part II (Articles).

Part I gives a brief introduction to the topics covered by the thesis, a presentation of research challenges, objectives, description of the research methods applied, the main results, and ideas for areas of further research. This part combines the main content of the publications found in Part II into a totality that serves to fulfill the objectives of the thesis. Additional details are found in the articles in Part II. The articles are stand-alone and can be read in any order.

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Publications

The list of research articles (that are included in the thesis) that have been submitted, published in international journals or in conference proceeding is mentioned the table 1.

Article ID	Page Num.	Title	Status
I	95	Optimization of periodic inspection time of SIS subject to a regular proof testing	Published
II	105	Modelling framework for performance analysis of SIS subject to degradation due to proof tests	Published
III	121	Introduction of degradation modeling in qualification of the novel subsea technology	Submitted
IV	139	Study of testing and maintenance strategies for redundant final elements in SIS with imperfect detection of degraded state	Published
V	155	Combined Maintenance Scheduling and Production Optimization	Published
VI	165	A Unified Approach for Simultaneous Optimization of Production and Maintenance Schedules	Revision submitted

Table 1: Overview of Articles included in the Ph.D. thesis

In the following, the details of the articles included in the thesis are presented together with Author's contribution.

Articles I:
Srivastav, Himanshu; de Azevedo Vale, Guilherme; Barros, Anne; Lundteigen, Mary Ann; Pedersen, Frank Børre; Hafver, Andreas; Oliveira, Luiz F(2018) Optimization of periodic inspection time of sis subject to a regular proof testing. Safety and Reliability – Safe Societies in a Changing WorldProceedings of ESREL 2018, June 17-21, 2018, Trondheim, Norway.

Contribution from the authors
Research problem were provided by DNV-GL as a case study to Second author for his master thesis. First (I) and Second author identified the state of art, and industrial practices on the topic. I and third author actively participated in framing the modelling assumptions. Based on that, I developed simulation based framework and presented associated numerical results. The results were then validated from fourth author and the industry partners.

Articles II:
Srivastav, Himanshu; Barros, Anne; Lundteigen, Mary Ann.(2019) Modelling framework for performance analysis of SIS subject to degradation due to proof tests. Reliability Engineering & System Safety. vol. 195 (106702)

Contribution from the authors
First(I) and Second author actively participated in conceptualizing the research idea. As a main author, I identified the state of art, and research gaps. Based on which, I framed modelling assumptions. Second and third author vetted the modelling assumptions. On approved modelling assumptions, I developed analytical framework and presented associated numerical results. I extended the developed framework on the case study of Down hole safety vavles (DHSV). The results of case study are vetted by second and third author.

Articles III:
Srivastav, Himanshu; Barros, Anne; Lundteigen, Mary Ann.(2020), Introduction of degradation modeling in qualification of the novel subsea technology, IEEE Transactions on Reliability

Contribution from the authors
First(I) and Second author actively participated in conceptualizing the research idea. As a main author, I identified the state of art, and research gaps. I also framed modelling assumptions. Second and third author vetted the modelling assumptions. On approved modelling assumptions, I developed analytical framework and presented associated numerical results. Second author proposed a suitable use-case. I generated numerical results for the use case. These results are vetted by second and third author.

Articles IV:
Zhang, Aibo; Srivastav, Himanshu; Barros, Anne; Liu,Yiliu(2020), Study of testing and maintenance strategies for redundant final elements in SIS with imperfect detection of degraded state, Reliability Engineering & System Safety

Contribution from the authors
First author and Second (I) author actively participated in conceptualizing the research idea. First author identified the state of art, and research gaps. I provided active support in formalize the research question mathematically, in framing modelling assumptions, in developing analytical expression and in discussing the numerical results. Results are vetted from remaining authors.

Articles V:
Verheyleweghen, Adriaen; Srivastav, Himanshu; Barros, Anne; Jäschke, Johannes, (2019), Combined Maintenance Scheduling and Production Optimization. Proceedings of the 29th European Safety and Reliability Conference(ESREL), 22 – 26 September 2019 Hannover, Germany

Contribution from the authors
First author and Second (I) author actively participated first in establishing the research collaboration the research idea, then conceptualizing the research idea. First author identified the state of art, and research gaps. I provided support to include degradation modelling in the existing framework. Then, I extended support in verification of modelling assumption, in improving framework, in discussing the numerical results, and in cross verification of numerical results.

Articles VI:
Verheyleweghen, Adriaen; Srivastav, Himanshu; Barros, Anne; Jäschke, Johannes.(2020) A Unified Approach for Simultaneous Optimization of Production and Maintenance Schedules, IEEE Transactions on Reliability

Contribution from the authors
First author and Second (I) author actively participated conceptualizing the research idea. First author identified the state of art, and research gaps. I provided support to framing modelling assumptions. Then, I extended support in discussing the numerical results, and in cross verification of numerical results.

There are several other research opportunities, I received during the tenure of this Ph.D.. In some of them, the contribution was significant still, they are not included in the thesis. In the following, the details of such articles alongwith the reasons for excluding them in the thesis are mentioned.

Articles VIII:
Welte, Thomas Michael; Vatn, Jørn; Sanz-Bobi, Miguel A.; Srivastav, Himanshu.(2019)Lifetime and maintenance modelling utilizing monitoring data Monitor X report L8. 2019.ISBN 978-82-436-1066-8 Publikasjon (Energi Norge) (447-2019)

Contribution from the authors
This is basically the technical report on a project namely ‘ Monitor X’ from SINTEF and several other industry partners. In this report, I contributed to proposed a twin geometric Browning motions(GBM) to model degradation of rotating equipment such as bearings. Then, I estimated the parameters of twin GBM from the degradation data available and estimated Remaining Useful Life of bearings.

Reasons for exclusion

Although, it was very relevant contribution with reference to the degradation modelling but the studies performed in this report didn't pertain to the domain of Oil & Gas sector. Hence, it is not included in this thesis.

Articles VII:
Islam, Abu Md Ariful; Srivastav, Himanshu; Vatn, Jørn; Barros, Anne; Lundteigen, Mary Ann.(2019) Time-dependent Unavailability Assessment of Final Element of Safety Instrumented Systems- an Application of Multiphase Markov Process. Proceedings of the 29th European Safety and Reliability Conference(ESREL) 22 – 26 September 2019 Hannover, Germany

Contribution from the authors
In this article, first author conceptualizes the research problem and arranged the literature review and research gap. I provided modelling support both for analytical and simulations. I also contributed in discussing results and associated optimization problem. Remaining authors vetted the overall article quality.

Reasons for exclusion
Although, the paper is quite relevant with respect to subject of this thesis. Since, I didn't participate in conceptualization of the research idea, I chose not to include the paper as a part of thesis.

Contents

List of Tables

List of Figures

List of Abbreviations

ABAO	As Bad As Old
AGAN	As Good As New
ASV	Autonomous Surface Vehicle
AUV	Autonomous Underwater Vehicles
BRU21	Better Resource Utilization in 21st Century
CBM	Condition-based Maintenance
CM	Condition Monitoring
CPM	Condition Performance Monitoring
CPM	Condition and Performance Monitoring
FPSO	Floating Production Storage and Offloading Oils Platform
IMR	Inspection, Maintenance and Repair
MCS	Monte Carlo Simulations
PDF	Probability Density Function
PHM	Prognostics and Health Management
RAMS	Reliability, Availability, Maintainability, and Safety
RBI	Risk Based Inspection
RBM	Risk Based Maintenance

RCM	Risk Centered Maintenance
ROTs	Remote-Operated Tools
ROVs	Remotely Operated Vehicles
RUL	Remaining Useful Life
SCM	Subsea Control Module
SIS	Safety-Instrumented-Systems
SUBPRO	Subsea Production and Processing
USV	Unmanned Surface Vehicles/Vessels

Part I

Main Report

Chapter I.1

Introduction

This chapter provides the background for this Ph.D. thesis in order to build a context. First, we provide a brief introduction to the research center that hosted this research project in section I.1.1. Then, section I.1.2 presents the global picture of the research project. This section also briefly references the previously executed research project at the research center, whose extension resulted in this research project. Finally, we describe the project focus in section I.1.3.

I.1.1 Project Background: the SFI SUBPRO

Subsea production and processing technology is a key enabler for the exploitation of Norwegian and international oil and gas resources. Norwegian oil companies and foreign oil companies with a basis in Norway, with the strong support of Norwegian-based suppliers and manufacturing companies, have been at the forefront of developing subsea fields.

Subsea production technologies have developed significantly in the past three decades. Current subsea practices involve the well installations from the topside to the seabed. The focus of research today is to target underwater resources that are farther from land, deeper in the water, and in a more demanding environment, such as the Arctic areas or the Gulf of Mexico. The industry needs to propose new subsea solutions to enhance the recovery factor of existing fields on the Norwegian Continental Shelf, reduce the cost of subsea installations and interventions and make subsea operations more cost-efficient without compromising the safety of stakeholders. Norwegian oil companies, together with Norwegian-based suppliers and manufacturers in the field of subsea, are taking efforts for new and innovative solutions to develop efficient new technology. SUBPRO (Subsea Production and Processing) and BRU21(Better Resource Utilization in the 21st century) are examples of such initiatives.

SUBPRO is a center for research based-innovation (SFI). It is established in August 2015 with a planned eight years duration. This center's objective is to accelerate the level of innovation within the subsea oil and gas industry by a collaboration of academia and industry. It is funded by the Research Council of Norway and 7 industrial partners (namely: **AkerBP**, **AkerSolutions**, **DNV.GL**, **equinor**, **Lundin Norway**, **Neptune Energy**, **KONSBERG**).

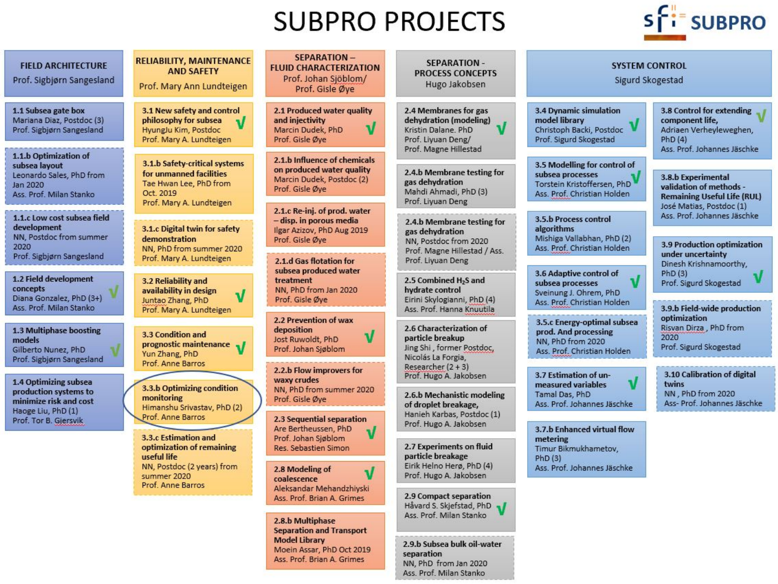


Figure I.1.1: Research Area wise projects at SUBPRO [111]

Research at SUBPRO is categorized under research areas of Field Architecture, Reliability, Availability, Maintenance, and Safety (RAMS), Separation-Fluid characterization, Separation-Process concepts, System Control. The research projects executed/ongoing are shown by Figure I.1.1. This Ph.D. project is serial numbered as 3.3b in the list and is identified with the circle around in the shown projects. This Ph.D. project is a natural extension of an already executed research project at serial number 3.3 'Condition and Prognostic based management'. The main objective [128] of project 3.3 was to develop frameworks (algorithmic) to optimize the maintenance, inspections, and re-configurations of subsea systems. In project 3.3, the research scholar developed CBM (Condition-based Maintenance) models for a single-unit for subsea systems (mainly stochastic process based). It uses a top-down approach to develop CBM models for multi-unit subsea systems. This research was mainly dedicated to the phase of condition-based decision making (as shown as Step 3 of Figure I.1.2).

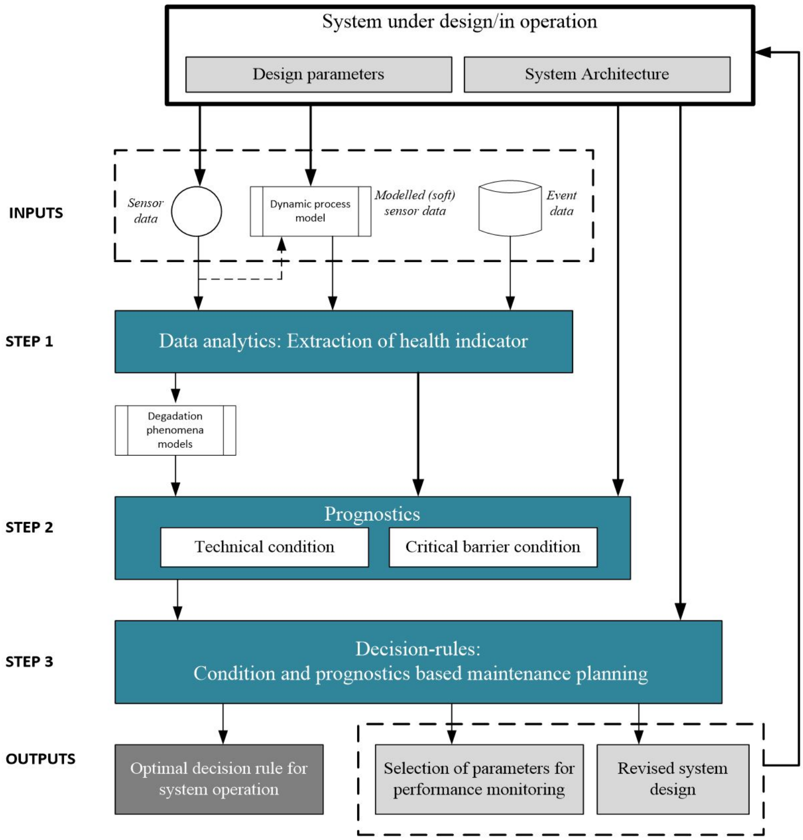


Figure I.1.2: Flowchart for Condition based Decision Making

Subsea systems need to be of very high reliability as the subsea climates are much harsher [73]. During the designing phase manufacturers need to consider that interventions like inspection maintenance are very expensive at deeper waters.

I.1.2 Global picture

Condition-based decision making (CBM) implementation relies on several interacting steps including, data collection, data processing, prognostics, and decision-making optimization.Before giving a global picture by explaining how these different steps interact and associated main challenges, we present relevant definitions.

I.1.2.1 Definitions

European standards (NS-EN 13306 [81]) for maintenance and related terminology define Condition-based maintenance (CBM) as'*Preventive maintenance which includes the assessment of physical conditions, analysis, and the possible ensuring maintenance actions*'. In the standards, Condition monitoring (CM) is defined as '*Activity either manually or automatically, intended to measure at predetermined intervals the characteristics and parameters of the actual physical state of an item*'.

From the definitions above CBM is interpreted as a subcase of preventive maintenance where the decision criteria to perform maintenance actions are mainly the condition or health of the system. A CM system needs to be in place to perform CBM. A CM system collects all information required to build a health indicator of a system/process. This information can either be gathered through sensors available with the equipment/process or through manual inspection. Another related term is Condition and Performance Monitoring (CPM) system. The CPM system takes a holistic approach and focuses on the global picture. It takes inputs from the system's health, equipment's health, system's performance, and environmental parameters. Then, it utilizes this information to find the overall performance and technical health of the monitored system [104, 41]. Based on the available global picture of health and performance, a CPM system proposes maintenance and interventions. Experts then discuss and analyze the output from the CPM system and make decisions in [106].

I.1.2.2 Implementation of CBM

In summary, the successful implementation of a CPM system requires an effective CBM strategy which requires an efficient CM system in place. Figure I.1.2 explains various steps required for condition-based decision making for a particular system/process under consideration. Step 1 is to build a health indicator from the available information. This step requires inputs from sensors (a current measurement of some physical quantities), design parameters, and event data. Then, available information is fed to a dynamic process model, which then reflects the current health of the system. Step 2 is to build criteria for prognostic from the available resources and information. This requires the understanding of degradation phenomena of health indicators, technical conditions, and critical barrier conditions of the equipment. Step 3 is to define decision rules to plan condition-based and prognostic based maintenance activities. In the operational phase, these flowcharts help to develop the optimal decision rules. In the re-design phase, this may suggest the design changes if the expected performance is not up to the expected level and which parameters are essential for performance monitoring. In the flowchart the CM overlaps between steps 1 and 2, whereas the CBM is limited to step 3.

I.1.2.3 Main challenges

There are several challenges in implementing CM systems/models, specifically in the domain of Oil & Gas industry. The main ones are listed below.

Challenges in the operational phase
Currently, there is a lack of knowledge and methods to support the efficient use of already available data, including sensor readings and data provided by state prediction models, in subsea systems. Data and models are often generated at the unit or component level, for separate phenomena and different purposes. The connection of the data information to the system level is insufficient to fully take advantage of available data and models at different levels. The main challenges in the operational phase are (i) What are the most efficient ways to build models that utilize existing data collected from subsea equipment for predictive decisions? (ii) What kind of models can be developed based on data made available from industry partners to optimize decisions? (iii) How can data not intentionally collected for condition monitoring (such as operating data) be utilized in such models?

Challenges the design phase
In this phase, the main challenge is the selection of sensors and associated technology. In the case of expensive sensors or expensive installation/operation associated with the sensors, the main problem is to optimize the placement, the number of redundancies, choice of technology. Whereas, in the case of cheap sensors and sensor installations, the main challenges are the integration of sensors, choice of communications networks, and possible data analytic performances. The consequent decisions should be optimized according to the highest return of investment for prognostics and predictive decisions.

I.1.3 Project focus

Condition monitoring techniques for subsea systems has gained popularity in recent years. The main reason is the development of sensor technology. This development enabled a large amount of information available about the health of the system. Hernæs et al. [41] discussed the role of condition monitoring and maintenance to increase the design lifetime of the subsea system from 25 years to 50 years. Friedemann et al. [34] discussed the applicability of condition-monitoring technologies to subsea infrastructure in the oil production industry. The authors presented relevant case studies from the domain of energy and rail transport industry to utilize the experience from other industries on this topic. In Oil & Gas industry, production monitoring has always been of prime importance. It has become more effective due to the development of diagnostic techniques.

This improvement is due to better feedback about the condition monitoring and availability of relevant operational data. Nowadays, operators not only want to keep an eye on production levels of the fields but also have high interest in minimizing repair and inspection to cut down the high costs associated with it. Soosaipillai et al. [106] discussed that maintenance could be significantly reduced by investigating and monitoring the equipment's condition. Serene et al. [101] advocated that condition monitoring based diagnosis increases the availability of subsea systems. Further development in sensor technology and better and cheaper ways to communicate condition monitor data from remote locations has improved the diagnostic and prognostic actions performed on the equipment [130]. Vaidya [116] addressed that the modeling framework required to take condition-based design should be able to handle degradation modeling, uncertainty in the data from the sensors and should be able to incorporate expert opinion.

The research in this Ph.D. is dedicated to the first layer of Figure I.1.2, which is responsible for CM. This project aims to provide methods and models to optimize the CM of subsea equipment. In the operational phase, it encompasses the optimization of inspection periods for subsea equipment and the optimization of replacement strategies. In the design phase, the focus is to provide methods to quantify the effect of degradation with respect to a risk level and to demonstrate the added value of CM for novel subsea technologies. In this regard, a case study on all-electric actuation system is presented. Equipment/systems from subsea safety-control systems and production systems are central in this study.

The remainder of Part I of this thesis is organized as follows: chapter I.2 and chapter I.3 provide a relevant research background from industrial and academic points of view, respectively. These chapters build a preface for a better understanding of research questions. Chapter I.4 formally defines the research question, objectives, and delimitation of this thesis. Chapter I.5 discusses the scientific approach and methods utilized to perform the research carried out in this thesis. Chapter I.6 summarizes the contributions made to each research question. Finally, chapter I.7 concludes the Ph.D. thesis and presents final considerations.

Chapter I.2

Industrial Background

This Ph.D. project has its focus on applying CM techniques in the domain of Oil & Gas industry, as discussed in section I.1.3. This topic has gained much popularity in the subsea industry recently. In this chapter, we present an overview of the relevant background from an industrial perspective. We first discuss the industry's motivation for having CM systems in section I.2.1; we present the current status of the CM in the subsea industry in section I.2.2. CM systems help plant operators to make informed decisions. In a subsea environment, inspection, maintenance, and repair are typical decisions an operator can make. We discuss the current industrial practices of such activities and corresponding challenges in section I.2.3. In section I.2.4, we explain the standard procedure to implement the CM system. Finally, we conclude this chapter in section I.2.5.

I.2.1 Recent motivation for CM in subsea industry

Smedstad et al. [105] discuss that traditionally subsea fields are set up in a manner in which equipment (to produce hydrocarbon or inject water) are designed to be "over-engineered" with the given historical understanding of operational loads and failure modes of equipment. It is actually believed in the community that the equipment are over-engineered, and there is no real need for condition monitoring except for operational control purposes. This conventional thinking has been however challenged mainly because of the following factors listed below.

- *Greater Focus on Equipment Integrity*
  TechnipFMC[105] explained that some failures in offshore projects (example batch failure of subsea control module (SCM)) questioned the conventional thinking of equipment as 'over-engineered'. Root cause analysis of these

failures showed that they were non-structural failures in which equipment stopped functionality. Further, it is found out that these failures were electrical penetrator failures, subsea actuator failures. It is also established that there were enough early signs about the loss of equipment performance if the right information was made available for insight. Subsequently, rather than deploying highly engineered equipment, the industry started pushing for a better understanding of the degradation of subsea equipment. This knowledge was then coupled with the information available from field instrumentation to develop health indicators reflecting on the equipment's critical degradation. Another example is that post Macondo spill: rather than adopting for increasing the wall thickness, heavier equipment, and higher safety factors, operators chose to deploy better condition monitoring systems to monitor critical systems, establishing rigorous control to prevent unintended operation.

- *Cost Saving*
  Oil prices crashed to its half during 2014, which forced the industry to re-evaluate the pricing of equipment and services. Subsequently, the industry focused on optimizing field layouts. Conventionally, oil fields had layouts with the perspectives of geology, geophysics, drilling. They were not optimized as a whole. As a result of global optimization, subsea production manifolds become drastically smaller and lighter, and in-turn, cost-effective. The focus was then on the leaning of infrastructure but without compromising on integrity monitoring. Thus, instruments of condition monitoring were still in place. Indeed, the instrumentation part of condition monitoring has not changed a lot from before, but the new techniques (such as acoustic monitoring) are utilized to ensure the level of integrity monitoring.

- *Improved Regulatory Vigor*
  After the Macondo accident, the industry started becoming more concerned about the potential risks and costs of environmental accidents. These concerns resulted in the fact that today all new subsea fields are equipped with a 'data-collector' for bookkeeping of essential parameters for performing CM. Before this accident, very few operators (such as Statoil(Equinor now), Shellfrom Ormen Lange, Total and Gaz de France) installed condition performance monitoring (CPM) systems. There is also a strong push from the relevant authorities like American Petroleum Institute to formalize requirements for the equipment supplier including (i) a better understanding of failure or wear out (degradation) (ii) a strategy to place instrumentation to monitor the use conditions (iii) Tracking of accumulated fatigue for critical equipment (iv) Advance notification about the remaining useful life of the equipment.

- *Harsh environment and maintenance planning*
  Subsea systems are exposed to severe environments and harsh conditions, unlike systems placed onshore or on a platform. These challenges make them more prone to deterioration and in-turns towards internal degradation [76]. Complex and challenging subsea environment makes maintenance activities much more difficult compared to onshore equipment. Remote Operated Vehicle (ROV), specialized personnel, subsea tools, service vessels are generally required to perform subsea maintenance. The repair time is generally quite lower than the planning and arranging of subsea maintenance. For example, replacement of the subsea pump module typically takes a day, but planning the maintenance takes around a month of [33].

  In the event of a critical failure (which causes a sudden shutdown), the system can be repaired/maintained after the preparations of such activities are already in place. Then the operators need to observe whether the weather is favorable for maintenance activity. Until the maintenance is done, the system has zero productivity, which implies enormous economic losses and some reputation loss for the oil and gas production industry. To avoid such a situation, it becomes important for the operators of the subsea field:

  - To have indicators that monitor/observe deterioration of critical component of the subsea field
  - To arrange scheduling of maintenance in a manner to achieve maximum availability.

I.2.2 Status of CM in subsea industry

Condition monitoring (CM) techniques begin to be introduced to address such issues in the subsea field. In CM, various sensors are installed at key positions in the system to measure some physical quantities. In principle, the information received from sensors can be translated as an indicator of the health/condition of the system. Then, the remaining useful life (RUL) is predicted based on the knowledge gained about these indicators, failure propagation from past experiences. However, Oil & Gas being a relatively new industry to CM techniques has several challenges ahead to implement CM efficiently. Liu [62] mentioned some of them to be:

- A lack of specialized CM technique (both hardware and software) for subsea applications

- Requirements of specialized sensors due to the complex and harsh environment subsea

- Lack of recorded data about the condition of subsea equipment since available subsea data is mostly the flow process data
- The remote location of subsea platforms makes it challenging to communicate with equipment. Calibrations, up-gradation, and maintenance of subsea equipment becomes enormously expensive

In the existing literature, two main topics related to the implementations of CM in the subsea industry are arising: (i) What can be done in the framework of Prognostics and Health Management (PHM) (ii) What should be achieved for subsea control systems. These two main topics are covered hereafter.

PHM based Condition monitoring Friedemann et al. [34] explained that the challenges faced by subsea industry are similar to the ones of the more conventional industries like Energy and Railways. The authors list the challenges faced by all these industries as follows:

- The frequent occurrence of unplanned outage increases the cost and reduces revenues.
- Variable operating condition, operational modes, aging of infrastructure/equipment lead to new failure modes which makes it difficult to understand the failure mechanism and in-turn making it challenging to predict the useful lifetime.

The authors proposed a framework to utilize the information received through condition monitoring (on unit level or subsystem level) to enable optimal decision-making for the entire system, especially in the domain of operation and maintenance. The framework is based on the techniques of Prognostics and Health Management(PHM) models. PHM models generally consist of two layers i.e., health assessment ('P' layer) and health management ('HM' layer). The health assessment layer focuses on estimating the RUL on unit-level (subsystem, component), whereas HM takes these RULs as input to optimize the overall operational yield, efficiencies, maintenance, revenues. However, it is important to understand that both layers are not independent; information received from the 'P' layer affects the strategies proposed by the 'HM' layer and vice-versa.

It is proposed to decompose the entire subsea system into the downhole subsystem, the subsea subsystem, the surface subsystem, and the field subsystem (as shown in Figure I.2.1) to use PHM techniques. Then, better data collection at downhole, erosion detection at subsea, better debottlenecking at the surface, and computing better-optimized recovery strategies at field level may lead to enhanced overall recovery and revenues.

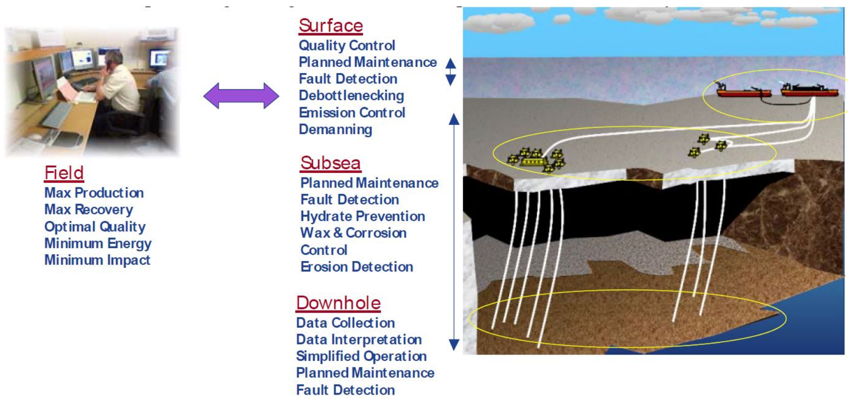


Figure I.2.1: Decomposition of subsea production field for PHM [34]

CM for subsea control system Neri et al. [80] proposed condition monitoring based subsea architecture for the subsea control system to increase recovery from the reservoir. The authors presented a study of process solutions developed by 'Aker Kværner'. These solutions are based on the models involving four consecutive steps: surveillance, analysis, optimization, and advanced control, and remote operations. The surveillance step is dedicated to instrumentation to get real-time data from subsea. The analysis step is responsible for gathering information from the measurements received from the surveillance step. The optimization and advanced control step ensure the implementation of the subsequent optimal operational actions. The final step delivers a complete solution, including strategies and operational actions based on all three steps.

FigureI.2.2 depicts the implementation of these four steps. Typically condition monitoring falls into first and second in this approach.

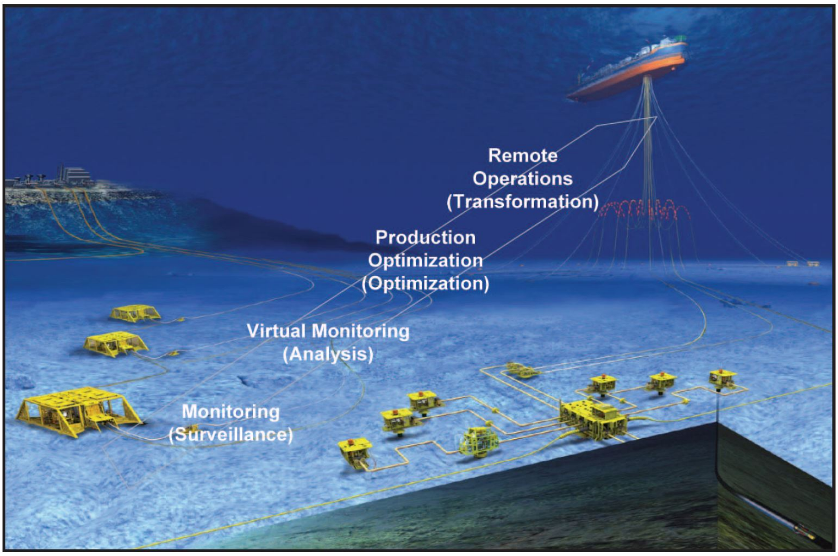


Figure I.2.2: Consecutive steps proposed by 'Aker Kværner' [80]

I.2.3 Status of Subsea Inspection, Maintenance and Repair (IMR)

This section aims to give an overview of how inspections, maintenance tasks, and repairs (IMR) are performed in the subsea industry, either there is CM available or not. It is demonstrated that the subsea area specificity makes IMR challenging, and CM is seen to overcome such challenges. In conclusion, current challenges and the future for subsea IMR are highlighted.

I.2.3.1 Practical framework for IMR

Let us start with some practical constraints about the subsea environment. A typical classification of subsea water depths is Shallow water (up to 500m), Deep water (between 500m to 1500m), and Ultra deep water (higher than 1500m). Subsea projects in Western Africa, Gulf of Mexico, Brazil, North Sea, and Norwegian Sea are typically at water depths more than 2000m [89]. Most subsea equipment are placed on the seabed, which are out of reach human divers. Thus, for such complex inspection, maintenance, and repair (IMR) operations, there are requirements of specialized vessels and remotely operated vehicles (ROVs) [77]. Apart from this, such activities require additional planning resources due to the severe weather conditions subsea. Operators also need to consider that shutting down plants for unplanned IMR operations will lead to production loss, resulting in massive revenue loss. IMR operations become very important and expensive in a subsea environment considering all these factors [99]. Even for smaller interventions, there arises the need for ROVs[73]. The cost related to ROVs usage is increasing and is predicted to be more expensive[72]. We give hereafter more details about practices and practical constraints for inspections on one side and maintenance/repair on the other side.

Subsea Inspection

It is a general practice in the subsea industry to perform visual inspections. It detects visible defects and failures, sometimes verify the physical state of the equipment [9]. Subsea inspections can also detect drastic environmental changes. ROVs inspection detects external abnormalities, but they can detect sound waves (which may be due to vibration of rotating equipment like pumps). Autonomous underwater vehicles (AUVs) are used to perform pipeline inspection [128].

Subsea Maintenance and Repair

The tools, specialized vessels, and vehicles are required to perform the subsea intervention. These are generally not owned by operators of the field. These services are often rented from other contractors. Hence, these activities need to be planned in advance. It is a general practice to place contracts with service companies during the design phase of the field. Decisions related maintenance strategies are also decided in the design phase[77]. The in-situ repair can be made by ROVs and Remote-Operated Tools (ROTs)without retrieving the equipment to the surface, while modular replacement will have to be supported by guided tools, wirelines and ROVs. Comprehensive recording and testing need to be completed before restarting the subsea systems [128].

There are three stages of the maintenance program for a subsea field. Each stage requires different types of condition monitoring data.

Stage 1 **Process monitoring and corrective maintenance**

Soosaipillai et al. [106] explained that in the beginning of the life cycle of an oil field, the focus of monitoring activity is to monitor wells and the reservoir's performance. 'Knowing how a well produces and how the reservoir performs is more important than knowing the integrity of a downhole pressure transmitter or a choke'.

In the stage parameters like flow rate, pressure, temperature, etc are monitored. If the observed data shows a significant deviation from the base trend, it is decided that there are some abnormalities in the production well. Then, the failed module would be pulled out from the sea and replaced. This is an example of reactive maintenance as it is performed on the failure of components [104].

These repairs are extremely expensive as they are not well planned, and there is enormous production loss cost associated with it. In such types of repair, the repair personnel may spend significant time on fault finding and the root cause of failure. This also increases the operational costs.

Stage 2 **Calendar and age based maintenance**

It is a general practice in the Oil & Gas industry to have periodic inspection and testing as the components degrade over time [7], [35]. DNV-GL developed an Integrity Management Process with requirements of both long term and short term planning for inspections and planning execution of inspection, monitoring, and testing activities[35]. This practice generally followed for components of subsea safety systems and subsea control systems.

A study from [79] showed that it might not be the most efficient way to plan IMR activities. The study concluded that 63% of maintenance work resulted in no value-added work, and only 4% are for failed components. One possible reason is that different components experience degradation differently according to their material property, operational loads, and environmental conditions to which they are exposed. On the contrary, it is reasonable to have a plan for maintenance as unforeseen failures of the subsea components may cause downtime of several months resulting in colossal production loss [33]. For example, a replacement of a subsea pump requires 48 hours, given the tools and trained personnel are available. Planning of such maintenance may take up to 30 days.

Stage 3 **Condition Based Maintenance**

The limitations of the maintenance strategies mentioned above have pushed industries to find ways to keep track of the condition of the systems [41]. Such information can be used efficiently utilized to make a maintenance-related decision, such as when to retrieve the module. Developments of sensor technology has resulted in better monitoring of the subsea condition data particularly for choke, pipelines, wells, etc. Subsea processing systems (i.e., compressors, separators, and boosters) are areas of high interest for which condition monitoring techniques are deployed [41]. Current practice is to perform diagnostic at the component level from the available condition data. Companies are now looking for ways to combine this information to provide a holistic point of view on the health of the system [128]. This approach will enable the combined maintenance of several components in a single maintenance slot [26], which will reduce maintenance costs drastically.

A CPM (Condition and Performance Monitoring) system, as discussed briefly in Section I.1.1, is meant to provide holistic information about the system's health for which the CPM system is deployed. Then, this information is used to propose maintenance and interventions. However, it is then up to operators and experts to analyze the decision suggested by the CPM system [106].

I.2.3.2 Current challenges of subsea IMR

Beyond the applied literature and the practical constraints described beforehand, some challenges are often highlighted in the academia regarding IMR implementations. There are mainly about the lack of data in the operational phase and the modularization/new technology in the design phase.

Lack of data

Aspen [9] discussed some of the current challenges faced by the Oil & Gas industry concerning IMR. The author mentioned that one of the industry's biggest challenges these days is the availability of sufficient reliable operational data. Mostly available data is a testing data of service companies before the deployment. Catastrophic and/or unforeseen failures occur without warning. Subsea data on such failures is non-monitorable. CM is generally deployed to monitor the progressive, gradual degradation, and is unable to monitor unforeseen failures [32].Eriksson et al. [33] advocated the prognostics modeling improvement to fully utilize the existing subsea data to perform just-in-time maintenance for the subsea system.

Modularization and new technology

The modularization of subsea equipment/systems consists of optimizing how different items of a system are put together to make a system as compact as possible and as easy to maintain as possible. The main idea is that the less reliable item should be the easiest to retrieve. Lima et al. [61] discussed the optimization problem associated with the concept of modularization as huge modules reduce the reliability of module, whereas tiny modules increase the complexity (due to more number of connectors process, power, and control components). Soosaipillai et al. [106] discussed that the added value of new technology in subsea monitoring needs to be calculated as the implementation of such new technologies will require intervention and loss of operation time.

I.2.3.3 Future challenges for subsea IMR

Regarding the future, there is an essential focus in applied literature on the use of autonomous vehicles for IMR. National Research Council of America published studies that advocated autonomous vehicles in the fields related to sea as they can achieve lower project costs [14]. There has been significant development in the subsea industry to deploy technologies to achieve lower project costs.

Anderson et al. [6] advocated that significant cost reduction can be achieved in Oil & Gas operations by the use of surface systems such as unmanned surface vehicles(USVs) and autonomous surface vehicles(ASVs) . The authors presented case studies to show that cost reduction is achieved without any loss in data quality. The case studies are from offshore projects in Europe, the Gulf of Mexico, the Mediterranean sea, and Alaska. It is also proposed that these vehicles are best suited for Exploration and production (E& P), inspection, maintenance, and repair (IMR), and survey operations. USVs and ASVs are boats that operate on the surface of the water without a crew [126].

Vincent et al. [119] discussed that Autonomous Underwater Vehicles(AUVs) showed promise to challenge the traditional methods to perform IMR activities. It is proposed that novel autonomous vehicle platform namely uROV (untethered ROV) may be efficient on these three fronts when compared to traditional methods: (i) Cost reduction (ii) integration with latest sensor technology with the platform (iii) digital enablement through better data process automation and visualization.

I.2.4 Standardization

The literature review in this chapter is very applied and taken from the private domain without explicit reference to standards. However, CM procedure is standardized. We present here an overview of this standard with a discussion about the terms taken from academia. This gives more clear steps to justify the degradation modeling proposed in this Ph.D. project.

ISO 17359:2011 (revised in 2018)[48] provides guidelines for the general procedures to be considered when setting up a condition monitoring program for machines and includes references to associated standards required in this process. The detailed procedure to implement CM is shown in Figure I.2.3. Procedure discuss in several steps in detail. These steps are answers of following questions

- Is it beneficial to implement the CM system? (Cost-benefit analysis)
- For which system the CM needs to be implemented? (Identification of system)
- How to assess the reliability of the selected system? (Reliability and criticality audit)
- What type of maintenance strategies are selected? (Selection of maintenance strategy)

- How to monitor the health of the system? (Monitoring technology, Data acquisition system and analysis)
- Determine the condition-based maintenance action? (Prognosis or diagnosis)
- Is CM implementation effective? (Review)

It is also important to note that several feedback loops ensure the improvement in CM's implementation on several levels. Two important terms diagnosis and prognosis are used in the flowcharts, which need more clarifications.

Diagnosis Diagnosis aims to determine the presence, position, and severity of the defect or damage in the machine. It includes several steps, including pre-processing of health data, feature extraction, fault detection, fault isolation, and fault classification. ISO 13379-1:2012 (ISO (a))[47] proposes data-driven approaches and knowledge-based approaches for diagnosis. The basic idea of both is to build a base model that describes the normal conditions of the machine based on data or based on knowledge, and then look at the difference between the real measured value and the estimated value from the base model. If the difference lies in the unacceptable range, it may reveal something wrong with the machine.

Data-driven approaches, as the name suggests, are regression models from available condition data and event data. These models may not necessarily represent any physical phenomena of the machine. Knowledge-based approaches are mostly formulated from the understanding of the physics of failure of the machine. It may require inputs from operators and the experts of fields.

In this thesis, diagnosis is not in the scope, so we end the discussion on diagnosis here.

Prognosis Prognosis can be defined as the forecast of the remaining useful life (RUL), future condition, or probability of reliable operation of equipment based on the acquired condition data. RUL estimation is critically important since it impacts the planning of maintenance activities, spare parts provision, operational performance, and profitability [103]. The main difference between prognosis and diagnosis is that diagnosis answers the questions like whether the unit is faulty or not, location of the fault, the severity of faults. In contrast, prognosis tries to answer questions like given the current condition and based on the understanding of future usage of the unit, when is next failure likely to happen.[12, 62].

Lee et al. [60] made an attempt to provide a clear explanation between two terms as following '*Diagnostics is conducted to investigate or analyze the cause or nature of a condition, situation, or problem, whereas prognostics is concerned* *with calculating or predicting the future as a result of rational study and analysis of available pertinent data. In terms of the relationship between prognostics and diagnostics, diagnostics is the process of detecting and identifying a failure mode within a system or sub-system; while prognostics is the process of generating a rational estimation of the remaining useful life and/or remaining performance life until complete failure occurs. Prognostics, in its simplest form, is to monitor and detect the initial indications of degradation in a component, and be able to consistently make accurate predictions*'

Typically, the Oil & Gas industry has huge production loss costs in case of shutdown or catastrophic consequences due to critical failures. In such a situation, the prognosis becomes very useful and vital as it can predict the expected time to failure with a certain confidence, or survival probability until next maintenance. Heng et al. [40] showed that accurate prognosis significantly reduced expensive downtime and maintenance labor costs, helped in efficient spares inventory management, reduced hazardous conditions.

There is a similar word prognostics often used in the same context. Prognostics is a word newly coined by the scientific community to address the combination of diagnosis and prognosis [10]. ISO 13381-1[45] defines 'Prognostics is the estimation of the life before failure and the estimation of the risk of existence or the risk of the future apparition of one or several failure modes'. Prognostics is performed by integrating the sensor data and prediction models to assess the online degradation of products [112]. Poor operational subsea data, inability to monitor condition of subsea equipment the real-time degradation are main bottlenecks that applying prognostics techniques.

I.2.5 Conclusion

In this chapter, we presented the industrial background of the thesis. In this chapter, we stipulated practical factors, that channelised Oil & Gas industry's growth towards the implementation of CM. We also presented some examples of CM system from the Oil & Gas domain. We presented current industrial practices and challenges related to inspection, maintenance and repair activities. Finally, we discussed the standardized procedure to implement CM systems.

With this industrial background, we are now ready to move to chapter I.3 in which we will present the academic background of the key aspects of thesis such as degradation modeling and maintenance modeling.

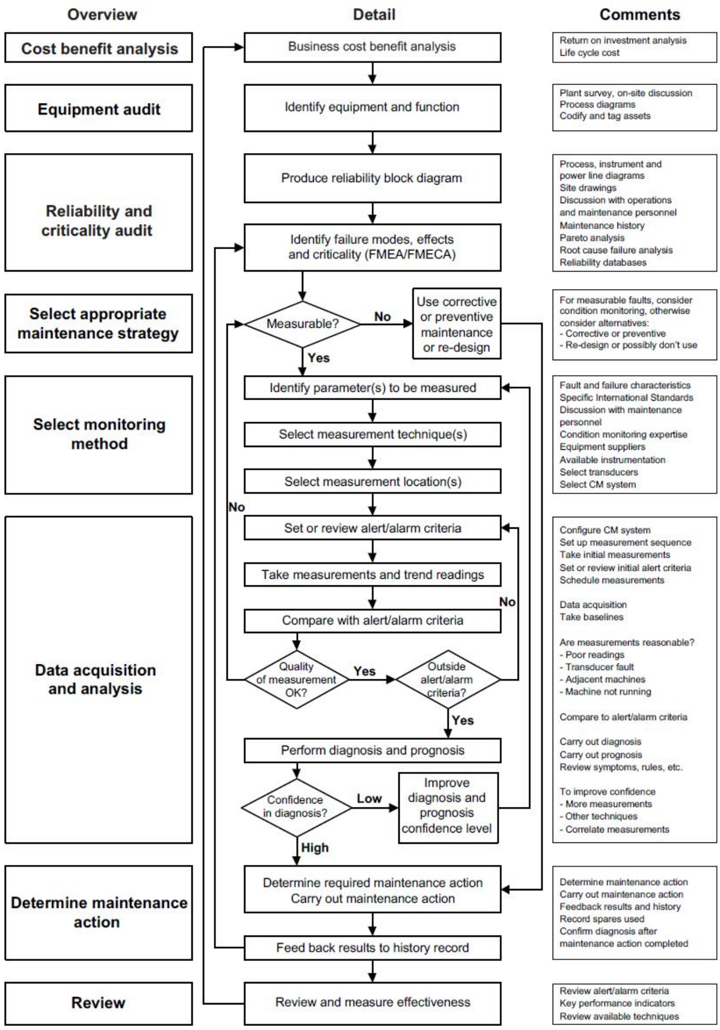


Figure I.2.3: The flowchart for implementing a condition monitoring [48]

Chapter I.3

Academic Background

This chapter covers the academic background necessary for the thesis. It is assumed that the reader understands the concepts of reliability theory.

I.3.1 Introduction

As discussed, in Section I.2.4, prognostics is an essential step in implementing the CM based system or procedure. According to ISO 13372 [46], prognostics is the 'analysis of the symptoms of faults to predict future condition and residual life within design parameters'. For scientists from the aviation industry, 'Prognostics is the ability to predict the future condition of a machine based on the current diagnostic state of the machinery and its available operating and failure history data' [18]. Another book from the aviation field [51] defines prognostics as 'predicting the time at which a component will no longer perform its intended function'.

The above prognostics definitions rely on methods to model the state of a component/system (says its degradation) and predict how much time is left until the component/system will reach a failed state. It is generally assumed while deploying prognostics methods that some information or data in terms of i) past/future operating condition of the system, ii) past/current system degradation levels are available.

The time left until failure is called remaining useful time (RUL). As the name suggests, it is a measure of useful life left with the component. The term *useful* is quite ambiguous and needs a definition on case to case basis. For example, for a subsea safety system, *useful* may be related to the availability of the safety system, whereas, for a subsea production system, it relates to the profitability of operations. Another vital aspect after having an estimation of the RUL (with some confidence) is to make a decision based on this estimate. In a subsea environment, operators make typical decisions such as scheduling the maintenance or selecting maintenance strategy, and its effectiveness.

The remaining of this chapter is organized as follows: section I.3.2 discuss in brief about the existing techniques to model degradation, section I.3.3 presents essential concepts on maintenance, and section I.3.4 concludes this chapter by correlating these sections with respect to research presented in this thesis.

I.3.2 Degradation modeling

I.3.2.1 Definition

In an engineering environment, degradation is the phenomena of irreversible accumulation of damages over the lifetime, which eventually leads to failure[15]. It may be attributed to reasons like corrosion, fatigue, wear, and tear, etc. Rui et al. [96] proposed an analogy between *occurrence of degradation in a system* with *the entropy of a system* based on the second law of thermodynamics. The second law of thermodynamics states, '*the entropy of an isolated system (disorder) increases with time*'. This analogy claims the inevitability of occurrence of degradation phenomena for the real systems.

Degradation is a physical phenomenon that can be detected with physical symptoms. Wear of tires, loss of strength of bridge beam, the outdoor withering of the coating system, the increase of vibration amplitude of bearings are some of the symptoms of several degradation phenomena [122, 42, 125, 97]. In order to model the degradation states and make predictions, the detection of symptoms is not enough. It has to be complemented by estimation or quantification of the degradation states. In some cases, degradation can be quantified directly by measuring a physical quantity such as the thickness of tire in case of tire wear. In many other cases, it may not be possible directly to measure the physical deterioration but only the *symptoms* or to say it, in other words, the *effects* of the degradation [76]. In such cases, based on the current understanding of developments of these effects of degradation and available history of information on these effects, some meaningful relationship is established to quantify the degradation. Such quantification often depends on expert opinions and judgements; hence it is often partly subjective in nature.

I.3.2.2 Classification

Degradation models can be classified according to the nature of the state space. Barros [12] presents a representation of degradation models, as depicted in Figure

I.3.1. In this figure, three classes are proposed depending on the degradation space is (i) Binary (ii) Multi-state (iii) Continuous. Binary degradation space is heavily utilized in Reliability, Availability, Maintainability, and Safety (RAMS) community for lifetime analysis at the component level. These analyses considered that a component is either in a working state or in a failed state. Then based on the available understanding of the system, the time to failure is modeled using lifetime distribution with a constant and/or non-constant failure rate. The multi-state degradation space approach is generally used for system-level analysis; degradation levels are then representing the combinations of failed components. The underlying idea is that the systems are degraded when some components in a redundant structure fail, but it is still able to perform its function. The degraded states can correspond to working states with degraded performance. Markov processes are often adopted to model such cases. Continuous degradation space approach is generally utilized when the understanding of physical phenomena is well known and established as an observable continuous process. In such cases, the degradation behavior may be represented by physical laws or mathematical laws. In an ideal situation, if it is possible first to establish the continuous degradation space and then to continuously monitor it, all failures can be avoided.

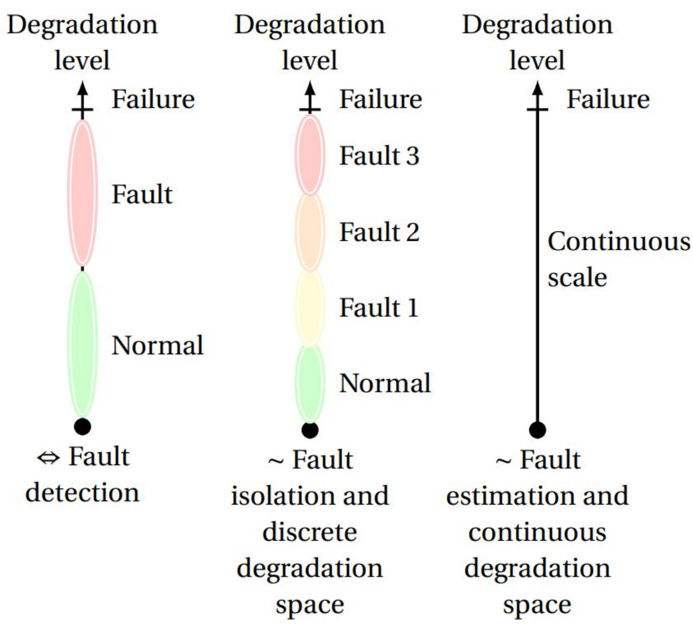


Figure I.3.1: Perspective on degradation modeling [12]

Another way to classify degradation models is presented in Zio and Compare [131], where three classes are proposed (i) Physical model (ii) Stochastic Models (iii) Experience-Based models. *Physical Models* establish the degradation behavior based on empirical and semi-empirical laws. An example is the Paris-Erdogan model of crack propagation in mechanical components. These models require in-depth knowledge about the failure mechanism. *Stochastic Models* inherently assume that degradation is a time-dependent random process that follows specific statistical properties. These models rely on the stochastic process to model degradation phenomena. *Experience-Based models* use a fuzzy logic theoretical framework to model degradation behavior based on the experts' guidance. These models are quite useful when there is no degradation data available.

Several other classifications presented in the literature are made according to the prognostics approaches, with some variations. Heng et al. [40] classify prognostics approaches into *data-driven* and *physics-based*. Vachtsevanos [115] added one more class *probability-based* to this classification in the book on Intelligent fault diagnosis and prognosis for engineering systems. In this approach, historical failure data and operational data are used to postulate probability density function (PDF). According to the authors,the *physics-based* approach is part of a more comprehensive *model-based* approach. Johnson et al. [51] presented five different approaches, which are extensions of the three aforementioned classes i.e. *experienced-based*, *evolutionary*, and *physics-based*.

There are several other ways to address the classification of the degradation models. Gorjian et al. [36] presented a thorough review of the classification of the degradation models used in reliability analysis. Rui et al. [96] presented a very recent literature review, where a general classification with two approaches for degradation modeling is used (i) Model-driven (based on the cognitive experience of mankind) (ii) Data-driven (using data learning techniques without forming a hypothesis).

Conclusion Some of the classifications are overlapping and, in some cases, contradicting other ways of classifications. However, the core of almost all classifications is based on two primary factors: (i) the understanding of failure mechanism (in the core sense of understanding of physical laws defining the degradation process) and (ii) the availability of failure data, field data, condition data.

I.3.2.3 Multi-state approaches

Multi-state approaches are widely used in practice when no continuously measured health indicator on the system degradation is directly available. For example, inspections are performed visually, and then a subjective decision is used to describe the condition of the component/system. The system degradation is then classified into few degraded states according to expert judgement, or according to the synthesis of some qualitative and quantitative measures. In short, multi-state modeling approaches give freedom to have a smooth (relatively easy) transition from qualitative to quantitative methods. Given the industrial background of subsea systems and the feedback provided by SUBPRO partners, it has been decided to use this modeling framework for this Ph.D. project. We provide here a short review of multi-state approaches in different application areas.

Welte et al. [124] used multi-state degradation models for hydro-power components. The objective of the study was to optimize maintenance and renewal. There were four states considered in the increasing order of degradation. These were (1) No indication of degradation, (2) Some indication of degradation, (3) Serious degradation, and (4) Critical. The same classification is considered for condition monitoring of wind turbines by the Norwegian Electricity Industry Association (EBL) to model deterioration of components [30]. The same number of states with the classification of (initial, minor deterioration, major deterioration, and Fail) are used in Risk-based asset management of power systems[31], [5] , [4].

Kallen and Van Noortwijk [52, 53] utilized a multi-state degradation model to build maintenance models for bridges. The condition of bridges was defined by seven states. Van Winden and Dekker [118] used six states (excellent, good, reasonable, moderate, bad, and very bad) condition model to rationalize maintenance costs for building.

In the field of water utilities, for example, [50] proposed a model using a Hidden Markov Method (HMM) to represent the degradation of Rapid Gravity Filters (RGF). The system condition is presented in five states: Excellent, Good, Acceptable, Poor, and Awful. Since the information about the condition will not be precise, they specified a belief distribution and algorithm.

In the oil and gas sector, Lundtofte and Solibakke [70] presented a case-study performed Reliability, Availability, and Maintainability (RAM) study on a Floating Production Storage and Offloading oils platform (FPSO). In this study classification of states was based on the production capacity of the FPSO. Three states were used in the decreasing order of capacity 100%, 50%, 0%. Liu and Lv [63] used a multi-state model to model the degradation of twin-screw in a booster system. The authors classified four states with no wear in the screw, slight wear, medium wear, serious wear. Vinod et al. [120] proposed a comprehensive framework to evaluate the reliability of piping under erosion-corrosion for risk-informed inspections. A system with four states is proposed to classify the degradation. The used states were defined on the measurement of corrosion depth in the piping.

I.3.3 Maintenance

In this section, the standard concepts related to maintenance are discussed. It covers relevant definitions of maintenance, objectives of maintenance, maintenance strategy and effectiveness, and evolution of maintenance over time. In the later part of this section, techniques on maintenance modeling and optimization are also covered. It is essential to mention that the main focus of the thesis is on degradation modeling. Since maintenance is associated decision variable with degradation modeling, maintenance is also a key feature in all the papers presented in this research. In this section, the main objective is to provide the relevant background about the topic of maintenance for a better understanding of research papers.

I.3.3.1 Definition

There are several definitions available for the term '*Maintenance*'.

1. European standards for Maintenance related terminology (NS-EN 13306) [81] defines maintenance as 'Combination of all technical, administrative and managerial actions during the life cycle of an item intended to retain it in, or restore it to, a state in which it can perform the required function'

2. International Electro-technical Commission (IEC 60050-190) [20] provides following definition of the term '*Maintenance*': 'Combination of all technical and management actions intended to retain an item in, or restore it to, a state in which it can perform as required'

3. Moubray [78] defines the term *Maintenance* as collection of activities which ensure that physical assets continue to do what their users want them to do.

4. Høyland and Rausand [43] provide a definition of *Maintenance* from management perspective. According to which it is reverse engineering activities, where the decision process is dependent on the technical and mechanical education of the maintenance staff and their hands-on expertise.

Maintenance encompasses activities that ensure EUC performance as required. For the contextual understanding of the maintenance, the terms such as '*perform as required*' and '*perform the required function*' should be well defined.

Generally, the inspection activity has the same objective as the maintenance activity to reduce the risk. So, the general maintenance encompasses maintenance activities and inspection activities.

I.3.3.2 Objective of Maintenance

The maintenance activities are performed to achieve certain objectives. NS-EN 13306 [81] defines following objectives for any maintenance activities:

- Ensure the availability of the item to function as required, at optimum cost
- Consider the safety, the persons, the environment and any other mandatory requirements associated with the item
- Consider any impact on the environment
- Uphold the durability of the item and/or the quality of the product or service provided considering cost

Inspections consist of activities that are performed to provide information about the status of the component under study [24]. In general, it is assumed that if inspections are perfect, they reveal the true degraded state of the component. Inspections share the same objectives as maintenance. In this thesis, inspection activities are considered as an integral part of maintenance. So, when the term maintenance is referred, it means inspection and maintenance.

I.3.3.3 Maintenance Strategy and effectiveness

An operator answers two main questions while deciding about performing maintenance: (i) When to perform maintenance? (ii) What to do? The maintenance strategy answers the question *when*. The answer to the question *what* is given by maintenance effectiveness. In this section, we discuss these questions.

Maintenance Strategy

The term *Maintenance strategy* is defined as a management method used to achieve maintenance objectives [81]. There are several types of maintenance strategies. Figure I.3.2 shows a classification of maintenance based on the standard designations. In corrective maintenance, an operator takes a maintenance action after the fault/failure is detected. Typically, an operator can choose to repair or replace the faulty/failed component or can switch operation to standby unit. Corrective maintenance is also called breakdown maintenance or run-to-failure maintenance. Aspen [9] summed up the corrective maintenance process in the following five steps: (i) Identification of the failure, (ii) Localization of the failure, trace it to a specific equipment/part in the system, (iii) Diagnosis of failed components, (iv) Replace or repair failed component, (v) ensuring the system is back in an operating state by testing.

Preventive maintenance (PM) is used when operator based on certain criteria (age, condition or clock) choose to perform the maintenance activities before failure. The main idea of preventive maintenance is to prevent future failures and/or reduce the probability of a future failure. If a failure occurs prior to the PM, the system is restored to a functioning state, and the PM strategy is continued as if there had been no failure [43]. In clock-based PM, pre-defined calendar dates are the criteria for the preforming the maintenance actions. In age-based PM, operator perform maintenance as soon as the machinery completes certain operational age. In this maintenance strategy, it is ensured that a new replaced component gets repaired only after it has fully utilized its operational life. It makes this strategy more efficient in terms of components, but less effective in terms of planning as the maintenance plan get shifted after a failure [43]. In Condition-based maintenance (CBM), the physical condition of machine forms the basis for maintenance action. In this strategy, condition data are utilized to predict the failure time and schedule the maintenance accordingly [100]. Condition monitoring of components or systems are required to perform meaningful prediction.

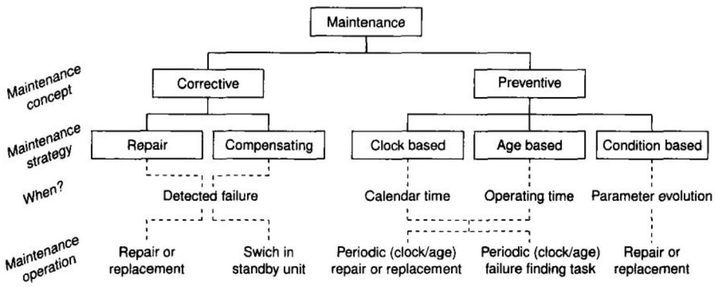


Figure I.3.2: Classification of maintenance [43]

Maintenance Effectiveness

The maintenance effectiveness defines the level up to which the system is restored after the maintenance. There exists a classification of the maintenance activities based on maintenance effectiveness. Pham and Wang [92] presented the following classification:

- **Perfect maintenance**: the system or component is either replaced with new component with original performance level or maintained to the limit where it's performance can be considered As Good As New (AGAN).
- **Minimal maintenance**: the system or component is restored to the state which it had just before the failure. It is also known as As Bad As Old(ABAO) maintenance.

- **Imperfect maintenance**: the system or component is restored between the AGAN and ABAO maintenance.

- **Worse maintenance**: the system or component's performance degrades after maintenance. Fox example: After the maintenance the failure rate increases compared to failure rate before the maintenance.

- **Worst maintenance**: the system or component un-deliberately fail due to the maintenance action.

I.3.3.4 Evolution of Maintenance

Moubray [78], in the book *Reliability-centered maintenance*, mentioned that since 1930's evolution of maintenance could be categorized through three generations. Dunn [27] added the fourth generation in the existing classification. These generations are the chronological representation of growing expectations from maintenance. Figure I.3.3 shows the evolution of expectation over the years. The growing expectations from maintenance led to subsequent conceptual development in the field of maintenance management. Arunraj and Maiti [8] discussed maintenance concepts developed in each generation. These are presented in Figure I.3.4.

The *first-generation* pertains to the time up to World War II, as industrialization has just begun during that time, the importance of maintenance is not well known. The concept of systematic maintenance was not prevalent, as equipment were uncomplicated and easy to repair. At this time, the maintenance philosophy was that *if it ain't broke, don't fix it*.

The *second-generation* pertains to the duration from the 1950s to the 1980s. Due to world war II, the demands of all types of goods increased dramatically, which led to the mechanization of industries. Increasing dependence on machines caused a significant reduction in the usage of manpower. The methods to reduce the downtime of machines are sought for higher production. This led to further developments of the technology and preventive maintenance philosophies [78].

The *third-generation* is considered from the mid-seventies to the early 21st century. In this period, industrialization picked up an accelerated growth. Industries moved towards just-in-time systems, which brought a greater emphasis on system availability and reliability. Simultaneously, the better QHSE standards, the higher demands on efficiency and profitability, the keen global competition, the increasing level of automation, etc. accelerated the development of the third-generation of maintenance management [67]. In this generation, philosophies related to predictive maintenance developed in the great detail.

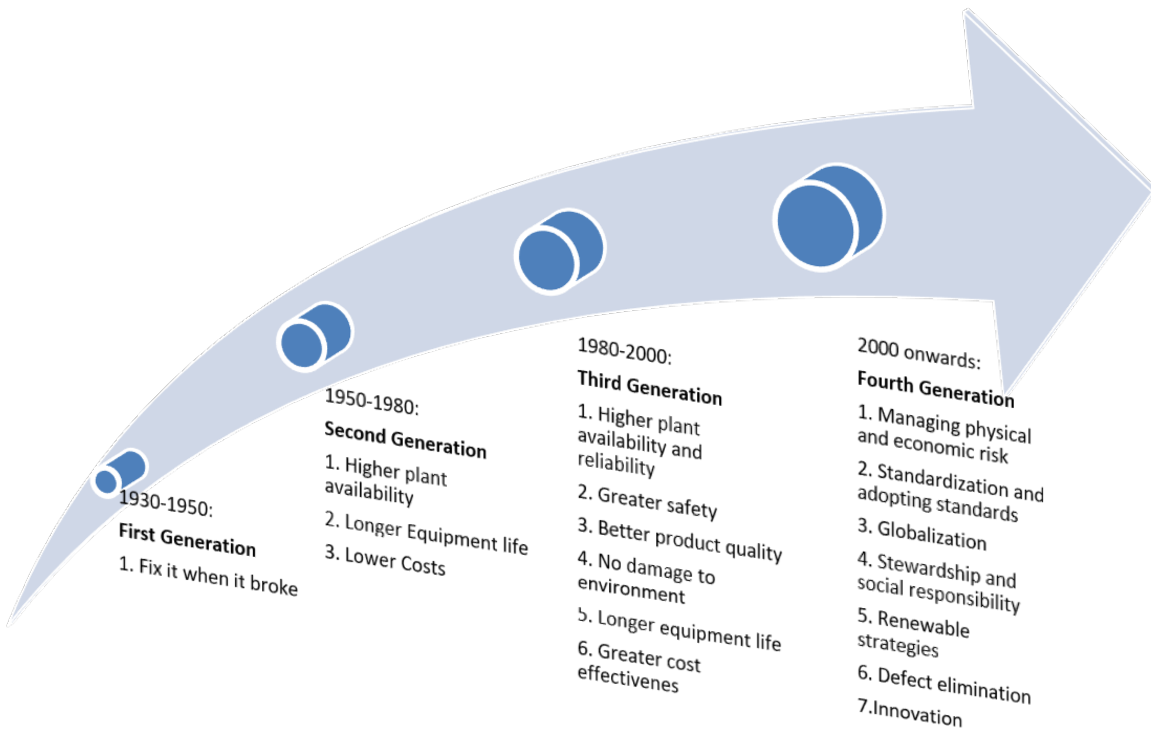


Figure I.3.3: Increasing expectations from maintenance [78, 27]

The *fourth-generation* pertains to the time since the early 21^{st} century. In this generation, the main focus shifted towards integrity management by combining maintenance and safety. There is more awareness related to risks of consequences of failures of equipment, system. It is expected that this generation will provide solutions for better management of physical and economic risks, early detection of fault to avoid catastrophic events, standardization. In this generation, concepts like risk-based inspection (RBI), risk-based maintenance (RBM), reliability centered maintenance (RCM), and condition-based monitoring gained much popularity.

I.3.3.5 Maintenance modeling and optimization

The most significant challenges in the domain of maintenance are selecting a maintenance modeling technique and addressing the inherent optimization problem. It is an extensively researched topic across the scientific fields because of its utility [3, 2, 102]. The main objective of these studies is to optimize maintenance strategies and effectiveness. There are several techniques available to implement maintenance modeling.

Maintenance modeling

A researcher may have to choose the maintenance model, which is continuous or discrete wrt maintenance action, dynamic or static wrt state-space, deterministic or probabilistic in terms of failure rate, constrained or unconstrained, and single-objective optimization or multi-objective optimization [93]. Cui [22] discussed dominating factors, such as maintenance strategy, system structure, dependencies among components, optimization criteria, etc., that need to be considered while selecting the maintenance models.

Barros [12] categorizes maintenance models approaches into scenario-based and state-transition approaches. In a scenario-based approach, maintenance models are developed by considering all possible sequences of events that may occur in the mission time. The major challenge with this approach is to list all possible sequences of events. In the state-transition approach, all the relevant states of the system, which affect the assessment wrt maintenance, are described. Then, a suitable sojourn time distribution is used to express the transition among the states. Markov process based maintenance models is the example of such an approach. The major challenge with this approach is that as the system's complexity increases, the relevant numbers of possible combinations of states may be difficult to compute.

Maintenance Optimization

Welte et al. [124] define the objective of maintenance optimization models as *find the maintenance and renewal strategy where the total costs of repair, inspections,*

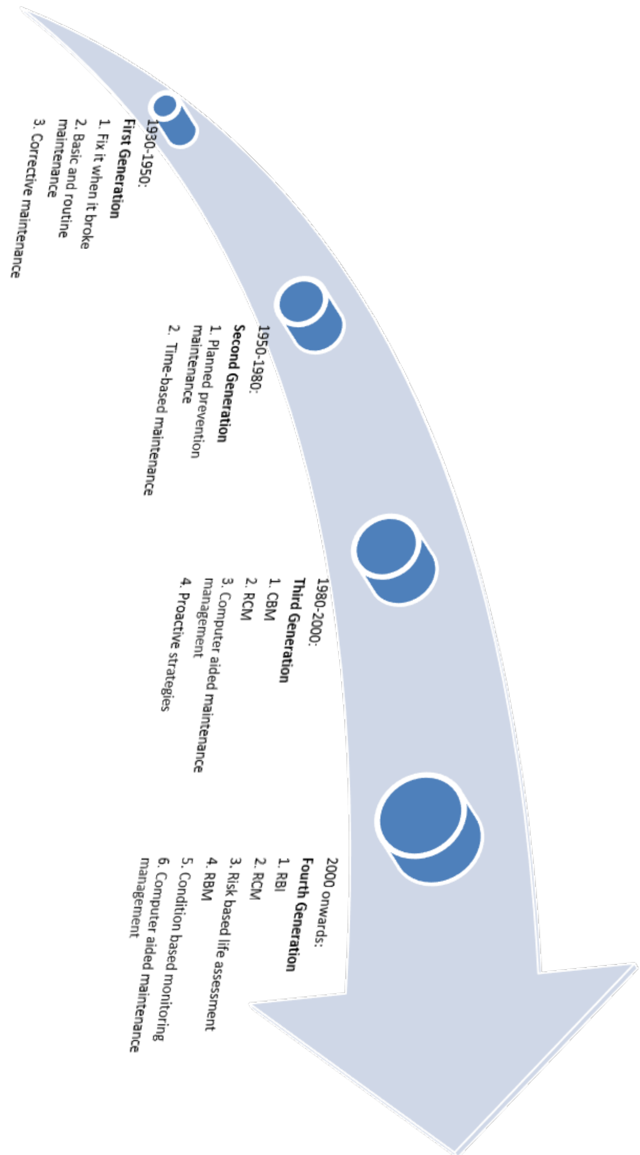


Figure I.3.4: Developments of concepts (generation-wise) [8] *production losses, and other consequences are minimal.*

Dekker [23] (1996) presented a review of the maintenance optimization models and considered the following challenges as key in further developing these models.

- Unavailability of useful data and associated analysis
- Need for the development of generic modeling such that standard models can be used
- Most concepts allow various interpretations; there is a need for proper formulation of the problem

Since then, the field has developed a lot. Van Horenbeek et al. [117] presented a framework (as shown in Figure I.3.5) based on the thorough literature review. It is detailed and almost covers all aspects of maintenance modeling optimization. In this framework authors considered several criteria such different types of maintenance policies (such as: CBM, time/use based maintenance, predictive maintenance etc.), maintenance concepts (such as life cycle costing, total productive maintenance, reliability centered maintenance etc.), maintenance actions (such as corrective replacement or maintenance, preventive replacement or maintenance etc.), data source (failure data, operational data, cost data, expert knowledge, etc.), system information (such as dependence, complete, technical system etc.), modeling techniques (such as continuous or discrete, deterministic or probabilistic, multi objective or single objective, etc.), maintenance effectiveness (perfect, imperfect, minimal, worst etc.), system configurations (such as single-unit, multi-unit, k out of N etc.), maintenance optimization criteria (such as maintenance cost, quality, reliability, availability etc.), and optimization algorithms(such as analytical, numerical, dynamic programming, evolutionary algorithm,simulation etc.).

In this thesis, we focused on some maintenance optimization problems. These are related to both single-unit and multi-unit. The components are stochastically deteriorating, and the deterioration is dependent on the various external factors such imperfect maintenance, usage and operation load. In optimization problem for safety instrumented systems the optimization criteria like maximizing the availability and reducing life cycle costing are considered. For subsea production system, we considered expected net production profit as the optimization criteria. These are covered in detail in the upcoming chapters.

I.3.4 Conclusion

In this thesis, the focus is to develop frameworks that integrate the interaction between maintenance and degradation processes. Degradation is a natural phe-

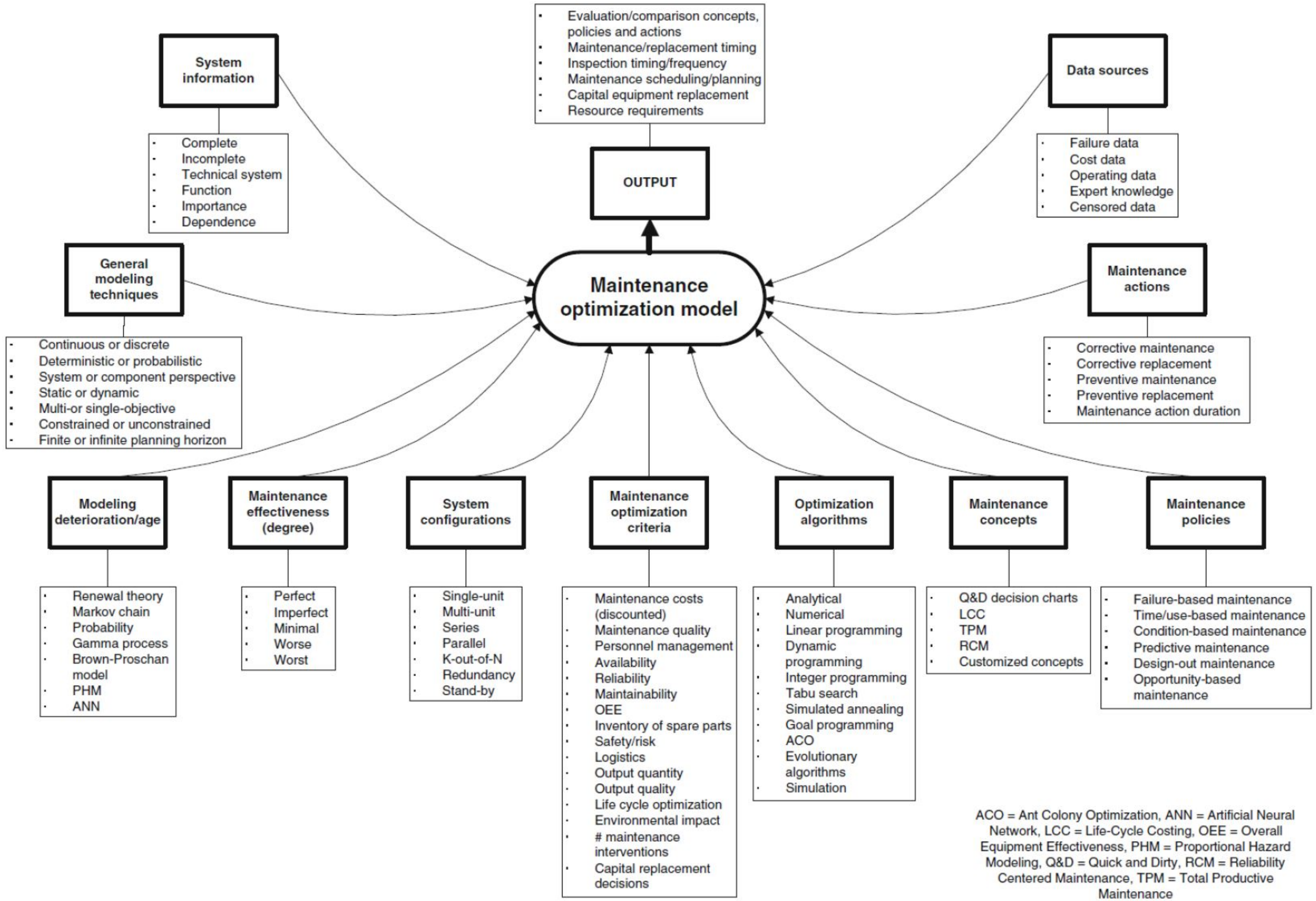


Figure I.3.5: Maintenance optimization classification framework [117] nomenon with a subsea system due to various factors, as discussed in section I.3.2 where maintenance is a decision variable that may (most likely) and may not interfere with the internal degradation. Most of the time, the standards of the domain consider the underlying assumption that whenever the maintenance takes place, it sets the system to the state of AGAN. This assumption is convenient to keep the calculations simple; however, it may not always be economically beneficial to perform such maintenance strategies in practice. In such a condition, it is vital to develop an understanding of the effects of various maintenance strategies on the internal degradation process.

Another critical assumption followed in the standards is that by default, almost all the components are considered to have a binary state. These states are uniquely working state and uniquely failed states. Then, on a case-to-case basis, a suitable lifetime model is used for useful prediction and further analysis. This assumption also helps for easy calculation of the relevant reliability measures. However, to develop a better understanding of component failures, the possibilities of states with degraded performance need to be considered. As discussed in section I.3.2, the research work carried out in this thesis is dedicated to multi-state degradation models. However, it is not possible in the application area of the thesis (subsea systems) to perfectly reveal the actual state of the component at the time of inspection. The framework built is based on this assumption that actual degradation remains hidden during the inspections. Inspection only reveals whether a component is failed or working.

With these understanding, we move forward to chapter I.4, in which specific research challenges are discussed in detail.

Chapter I.4

Research Questions, Objectives and Delimitation

In this chapter, we will discuss the specific research questions addressed in this Ph.D. thesis. These specific research questions stem from industrial experts' discussions, interdisciplinary discussions on familiar topics, and scientific discussions with the supervisors. Each question is based on a specific literature review from both industrial and academic points of view. The detailed literature review and relevant technical information are discussed in the attached papers specific to each research question. Here, we present a brief context about the research questions, descriptions of the research questions, and the research papers linked to these questions. There is a total of four research questions addressed in this thesis. Three are based on degradation phenomena experienced by a subsea safety system. The remaining research question is based on the degradation phenomena experienced by the subsea production system.

I.4.1 Research Question 1 - Degradation due to destructive periodic tests

I.4.1.1 Context

This research question originated as a result of discord between the offshore industry and regularity authority on the topic of the frequency of testing for some component of the blowout preventer (BOP). BOP is a safety barrier that ensures the safety of the platform against blowouts. It consists of a large valve used to seal, control, and monitor oil and gas wells [25]. Pressure testing is a way to ensure that BOP performs its safety function in case of demand situations. Precisely, in the development of well control rules [16], the offshore industry has an opinion that pressure testing of BOP's components should be performed after every 21 days as recommended in the standard American Petroleum Institute (API) 53 [110]. To which, the US Bureau of Safety and Environmental Enforcement(BSEE) asserted that,'*BSEE is not aware, however, of any new data that justifies increasing the BOP pressure testing interval for all BOPs from* 14 *days to* 21 *days*'.

The Offshore industry's argument is based on the understanding that the pressure testing of components of BOP may induce stress in them as discussed in the report from BSEE [16], which over time may accumulate and may induce failure due to the stress generated. However, the BSEE argument is based on the subsea domain's general practice, that frequent testing will ensure the lower probability of failure (PFD) as testing reveal information about the condition of the components. PFD on demand in the relevant measure to assess the reliability of BOP.

It is pertinent to mention that reducing the frequency, as suggested by the offshore industry, will have significant financial implications. For example: If the operational cycle for BOP is considered as 5 years, then, if the changes proposed in testing frequency by the offshore industry is accepted, it will reduce their operational cost from USD 400 million (considering the testing frequency of every second week) to USD 150 million (considering the testing frequency of every third week) [16].

This case got the attention of the Research and Development group of DNV-GL Rio. They collaborated with the Group Technology and Research group of DNV-GL Oslo. The team of Dr. Luiz Fernando Oliveira from DNV-GL Rio, Dr. Frank Børre Pedersen and Dr. Andreas Hafver from DNV-GL Oslo extended the collaboration, under the framework of SUBRPO, to include NTNU in this research challenge. The support sought through academia is limited to the degradation modeling of subsea safety systems. It was also required to analyze the inherent optimization problem between information gained by frequent testing versus the harmful impact of frequent testing. However, the financial aspects are not covered in the analysis.

I.4.1.2 Brief description of scientific problem

Safety-Instrumented System (SIS) is required to ensure the safety of equipment under consideration (EUC). SIS performs the associated safety functions as and when there is a demand situation. SISs are classified into three modes of operations based on the average frequency of the demand situation as per the standard of the Oil & Gas industry [44]. These are (i) low-demand mode of operation (demand frequency is less than 1 per year), (ii) high-demand mode of operation (demand frequency is greater than 1 per year), (iii) continuous mode of operation (demand frequency is greater than 1 per year, and safety function also operates as a continuous control function). Most of the SIS in the Oil & Gas industry are pertaining to low-demand mode of operation. In the low-demand mode of operation, the final elements (consists of mechanical components) of the SIS remain idle most of the time and are activated only in demand situations. SIS is periodically tested (namely proof tests) to confirm that they can act on demand. Their performance is then quantified by their mean downtime per unit of time between two proof tests (commonly named average probability of failure on demand, PFD_{avg}).

Performance analysis of SIS is a widely discussed topic in the research community due to its practical criticality [17, 28, 29, 37, 39, 59, 69, 95]. Most of the existing literature assumes 'non-destructive testing'. An important challenge raised by the industry is that the proof tests are stressful for the mechanical components of SIS and can degrade its condition [87]. There is some literature on this topic in the Nuclear industry, which is contemporary to the Oil & Gas industry in terms of safety requirements [75, 74].

The available literature in both the nuclear and the Oil & Gas sector considers binary state (unique 'working' and unique 'failed' state) on component level. Then, based on the case in hand, it improves the lifetime model associated. Discussions with industry experts such as Dr. Luiz Fernando Oliveira and Dr. Frank Børre Pedersen suggested that it is reasonable to assume more than two performance levels for such components. For example, partial proof tests of DHSV sometimes detect leakage, which is within the acceptable limits. This can be classified as a functioning state with degraded performance.

The existing methods are insufficient to assess the performance of such SIS. Hence, there is a need for further research on this topic. We formulate the following research questions in this regard:

1. Build other forms of failure rates, which can be time-dependent but also condition dependent.

2. Implement condition-based maintenance policies and optimize inspection strategies.

3. Define other kinds of functional decomposition (e.g., degraded mode) and associated performance measures.

Related Publications

This question is discussed in detail in the following research articles:

- Srivastav, Himanshu; de Azevedo Vale, Guilherme; Barros, Anne; Lundteigen, Mary Ann; Pedersen, Frank Børre; Hafver, Andreas; Oliveira, Luiz F(2018) Optimization of periodic inspection time of sis subject to a regular proof testing. Safety and Reliability – Safe Societies in a Changing WorldProceedings of ESREL 2018, June 17-21, 2018, Trondheim, Norway. *-Article I at page Number* 95.

- Srivastav, Himanshu; Barros, Anne; Lundteigen, Mary Ann.(2019) Modelling framework for performance analysis of SIS subject to degradation due to proof tests. Reliability Engineering & System Safety. vol. 195 (106702) *-Article II at page Number* 105.

I.4.2 Research Question 2 - Introduction of degradation modeling in qualification of the novel subsea technology

I.4.2.1 Context

In the research article II (at page 105) the framework was developed for the low-demand mode of operation. The framework calculated the probability of failure on demand. In this work, we modeled the degradation of performance of SIS using finite discrete levels. The effect of experiencing a demand situation was not considered in this work. Experiencing a real demand may instantaneously affect the degradation level of SIS. So, it was natural to extend the framework to include the effect of experiencing a demand situation on the assessment of reliability of SIS. This was also strengthened when one of the reviewers from the RESS (for the research article II) raised a query to extend the framework for the high-demand mode of operation. However, it was challenging to find a relevant case study from such a case from the Oil & Gas industry as an actual demand situation is a rare event and has catastrophic outcomes. There was less information available to quantify the effect of demands on the SIS if SIS survived the demand situation [49].

Meanwhile, SUBPRO invited Prof. Dr. Markus Glaser from Aalen University for research exchange. His team has extensively worked on the topic of an all-electric safety system. In one of the presentations, he explained how the all-electric safety system is susceptible to loss of power situations. In the literature review, we found that the all-electric safety system is a relevant case study from the Oil & Gas domain where the phenomena degradation due to demands was relevant.

I.4.2.2 Brief description of scientific problem

The concepts of all-electric control systems have gained much popularity in recent years. These systems are considered as an upgrade of existing electro-hydraulic control systems. Usage of all-electric control systems increases the health of equipment and environmental safety, reduces costs, and increases reliability[113]. In subsea fields, a production tree consists of gate-valves and choke-valves. It mainly controls hydrocarbon production, monitors the well condition, and injects chemicals when required. It also performs the safety function of 'isolating the reservoir' from the environment in case of a shutdown or emergency [11]. The production tree, along with a down-hole safety valve, becomes a safety barrier to the reservoir. All-electric actuation systems use electric springs instead of mechanical springs to activate the movement of valves. Power supply to these springs is generally provided from the topside. In case the power supply from the topside is disrupted, the battery and the battery management system (BMS) integrated at subsea takes over to supply the necessary power to electric springs. Mahler et al. [71] proposed an all-electric architecture for this novel concept of centralized subsea integrated battery and BMS. Failure mode and effect analysis (FMEA) shows that one of the critical high-risk failures occurs when the actuator system provides torque higher than the damage torque of the safety valves[71]. This may cause leakage in the valve, which consequently degrades the performance of the safety valve. This situation is likely to occur whenever the power supply to the electric-spring is disrupted. The event of loss of power supply becomes as a random demand situation for the valves. We can summarize the above discussion in the following manner: 'all-electric safety valves may degrade it's performance due to the experience of random demands situation'. The proposed system's safety capabilities assessment is based on concepts mentioned in the relevant standard IEC 61508[44] for the Oil & Gas industry. These assessments assume perfect restoration after every proof test and no impact of degradation. This is likely to overestimate the safety capabilities of the proposed novel subsea technology. It can lead to wrong decisions in the qualification, i.e., that the conclusion is that the system is sufficiently safe, while in reality, the system will lose its ability after a while, perhaps unnoticed.

We address this research problem by introducing the concept of degradation modeling in the qualification of novel subsea technology, as mentioned above. We will precisely answer the following research questions:

1. Build a time-dependent failure rate model that is a function of the system's current state and the number of demands experienced by the system up to that time.

2. Develop a mathematical framework to model such type of degradation phenomenon, and subsequently to assess the relevant reliability measure (unavailability in this case)

3. Develop analytical formulae to assess the instantaneous unavailability of the system under study when it has experienced a given number of demands with a given maintenance strategy.
4. Develop analytical formulae to calculate the average unavailability over the mission-time when, from the available knowledge about the system, it is known that safety valve will experience a given number of demands.

Related Publications

This question is discussed in detail in the following research article:

- Srivastav, Himanshu; Lundteigen, Mary Ann; Barros, Anne;(2020), Introduction of degradation modeling in qualification of the novelsubsea technology,IEEE Transactions on Reliability - *Article III at page Number* 121.

I.4.3 Research Question 3 - Study of testing and maintenance strategies for redundant final elements in SIS with imperfect detection of degraded state

I.4.3.1 Context

Høyland and Rausand [43] discussed system reliability concepts by extending the methods of unit-level reliability assessment. They pointed out that some failures (termed as 'dormant failures') can only be detected through tests or demands. These failures are under the category of dangerous undetected failures (DU failures). In case of SIS, DU failures are the main contributor to the unavailability of SIS. A SIS with redundancies is a specific application of the concepts of system reliability. Liu [65] discussed the optimal testing strategies for heterogeneously redundant SIS. The work was based on the assumption that there are only binary levels of performances (i.e. 'working', 'failed') for each SIS. Ph.D. scholar with Prof. Lui wanted to extend this analysis for SIS, which experiences degradation. He discussed this idea during one of the intra-RAMS group knowledge sharing seminars. He extended a collaborative opportunity for me as I have worked on modeling the degradation of SIS's performance before. We mutually agreed that the degree of perfect detection of SIS's real state during proof tests has a crucial impact on deciding the system's state, subsequently on condition-based maintenance activity and associated life-cycle cost of maintenance. So, we explored this research challenge with this perspective. My contribution to this research was mainly in formalizing the research question, framing modeling assumptions, developing analytical expression, and discussing the numerical results.

I.4.3.2 Brief description of scientific problem

A Safety-instrumented system(SIS) mainly consist three subsystems: (i) input elements (e.g., sensors), (ii) logic solvers (e.g., programmable electronic solver [PLC]), and (iii) final elements (e.g., safety valves, circuit breakers, alarms) [68]. The main objective of installing a SIS is to perform the associated safety function and bring back equipment/process to the safe state in case of hazardous situations. In the low-demand mode of operation, the final element mostly remains in the dormant state and is vulnerable to degradation mechanisms. Several studies have extended the existing binary state framework to consider the effect of degradation on the performance of SIS [107, 66, 2]. These studies considered the following discrete states 'working', 'degraded', 'failed'. However, these analyses assume that the proof tests reveal the system's true state perfectly. In reality, it may not always be possible to detect the true state perfectly. Since, in many cases, SIS's degradation is not observed directly but determined by the difference between the reference value and the estimated value of the health indicator, while the estimated value is calculated from some relevant process parameters [83, 129]. Zhang et al. [127] discussed that errors in detecting degradation could also come from an inaccurate setting of the reference thresholds. Such imperfectness in state revealing, consequently, weakens the real performance of follow-up actions.

Generally, SIS with a redundant structure is selected to improve its performance. However, the imperfect state revelation during the proof tests could increase the uncertainty of the performance of the SIS with redundancies. Condition-based maintenance strategies and associated life cycle costs may vary significantly concerning the degree to imperfectness of proof tests.

Considering the above discussion, we formulate the following research questions:

1. Modeling and quantifying the imperfectness of state revealing in proof tests and their effects on the performance of redundant final elements

2. Evaluating condition-based maintenance strategies in the contexts where different testing approaches are used.

3. Incorporating and balancing system availability and life cycle costs in seeking testing and maintenance strategies and providing guidance to operational decision-makers

Related Publications

This question is discussed in detail in the following research article:

- Zhang, Aibo; Srivastav, Himanshu; Barros, Anne; Liu,Yiliu(2020), Study of testing and maintenance strategies for redundant final elements in SIS with imperfect detection of degraded state, Reliability Engineering & System Safety- *Article IV at page Number* 139.

I.4.4 Research Question 4 - A Unified Approach for Simultaneous Optimization of Production and Maintenance Schedules

I.4.4.1 Context

I used to get constant feedback during the regular reference group meetings of SUBPRO. It was to extend collaboration with the sub-project 3.8 ‘Control for extending component life’ of the system control group (as shown in figure I.1.1). During the second year of my Ph.D., SUBPRO started an initiative to share knowledge among Ph.D. students by arranging Ph.D. colloquium without involving supervisors. I got the opportunity to share information about my Ph.D. research with the research fellow working with the sub-project 3.8 there. I learned that the system control group wanted to extend typical control problem for the stochastically deteriorating components. It was a tough challenge to be on the same page during the initial phase of this collaboration. The obvious reason was the point of view to look at the problem. Eventually, we grew into the research problem and learned to realize and appreciate the difference between the mindsets of control engineering and RAMS engineering.

My contribution to this research was mainly to modify the typical control problem to accommodate the stochastic nature of components’ degradation. I also supported in the framing of modeling assumptions, discussing the numerical results, and in cross verification of numerical results.

I.4.4.2 Brief description of scientific problem

Subsea production systems have the main objective of extracting hydrocarbon quickly and as much as possible. It generally requires operating these systems very aggressively. This will cause premature wearing of the systems. Subsequently, the maintenance and repair costs will increase. There exists a trade-off between high maintenance and repair costs versus high production profits. In many industries, it is normal to optimize operational decisions and maintenance decisions independent of each other, subject to constraints to ensure the decisions’ feasibility. This generally leads to sub-optimal utilization of the system. Many industries explored the dependencies between operational decisions and maintenance decisions to overcome this issue. Many industries have utilized CBM techniques for maximizing the profitability of operation [2]. The basic idea of CBM is to make maintenance decisions based on the available information about the condition of the system. The maintenance decisions are made in order to maximize the profitability of operations. With the subsea industry, it is a real challenge to monitor the condition of equipment continuously due to the lack of technology or prohibitive cost. Due to the subsea location and environment, there is a requirement of specialized vehicles to perform maintenance and inspection activities. This brings additional complexity to the problem. The existing literature has mainly evaluated the optimum frequency of maintenance, assuming periodic maintenance, although some studies have shown that the aperiodic maintenance schedule may lead to better solutions [56].

Considering the above discussion, we address the following research questions:

1. Develop a framework for integrating deterministic control laws and stochastic degradation models in the presence of various maintenance strategies.
2. What is the optimal way of operating between maintenance, and what is the optimal schedule for the equipment maintenance?
3. Discuss case studies relevant to the Oil & Gas industry in light of the developed method

Related Publications

This question is discussed in detail in the following research articles:

- Verheyleweghen, Adriaen; Srivastav, Himanshu; Barros, Anne; Jäschke, Johannes, (2019), Combined Maintenance Scheduling and Production Optimization. Proceedings of the 29th European Safety and Reliability Conference(ESREL), 22 – 26 September 2019 Hannover, Germany- *Article V at page Number* 155.
- Verheyleweghen, Adriaen; Srivastav, Himanshu; Barros, Anne; Jäschke, Johannes.(2020) A Unified Approach for Simultaneous Optimization of Production and Maintenance Schedules, IEEE Transactions on Reliability- *Article VI at page Number* 165.

I.4.5 Objectives

The main objective of this Ph.D. Project is:

'*to develop systematic frameworks to assess the performance of the subsea system (safety or production) considering the degradation phenomenon it experiences and to support operators or designers in decision making based on the performance assessment*'

Based on the main objective and the research challenges, the more specific objectives are:

1. Literature review on existing methods to assess the performance measure of the EUC. In this thesis main focus is on subsea safety instrumented systems. Subsea production systems are under minor focus.
2. Study factors that may induce degradation phenomenon, determine the methods to quantify the effect of these factors
3. Establish the interaction between external events (such as maintenance) on the degradation process
4. Develop framework to include the effect of degradation to assess of performance of the EUC
5. Address and analyze the inherent optimization problem

The aim in this thesis is contributing to the research challenges to fulfill the specific objectives and therefore the main objective.

I.4.6 Delimitation

This Ph.D. project is limited to the topic of the performance assessment of the subsea system. In the absence of real-time data, we had developed knowledge-based statistical models. In such models, the assumptions become crucial. The assumption for the work carried under this Ph.D. is vetted from experts available within the framework of SUBPRO. The methods and models in this thesis are developed with the objective that it will create a better understanding of the failure mechanism. We have omitted the discussion on model-verification due to no real-time data.

Chapter I.5

Research Methodology and Approach

'Research' as a word has its origin in the Old French language. It consists of two words: *re* and *cerchier*. '*re*' means an intensive force and the word '*cerchier*' means to search. Research is an intensive attempt to search for something new. On the same lines, the online Cambridge Dictionary [19] defines research by 'a detailed study of a subject, especially in order to discover (new) information or reach a (new) understanding'. The central theme of research is to either discover new facts, new concepts, new theories, or new applications; or develop a new understanding of existing concepts, facts, proven theories, or applications. However, it is important to note that there is always an underlying well organized and systematic effort, although the outcome of research sometimes appears coincidental. Creswell [21] defines 'Research is a process of steps used to collect and analyze information to increase our understanding of a topic or issue'.

'*The mystery of human existence lies not in just staying alive, but in finding something to live for*' by Fyodor Dostoyevsky, a famous Russian novelist, from the book named The Brothers Karamazov. The author emphasizes to find purpose in life. Similarly, there should be a purpose of research also. As per the above definitions, it is understood that the purpose of the research is to search for novelty (either on the level of discovery or on the level of understanding) through proper scientific procedures. The term *novelty* is a bit too general and hence, is ambiguous in nature. For different fields and people, it has its own meaning. In today's consumerist society, almost all researches are pretty much well defined at least on the level of purpose. There are specific aims, goals, and deliverable defined even in the proposal stage for applying the grant for research.

The '*why*' is the most important question to answer for me before making the decision to pursue this Ph.D.. After pursuing the Master of Science in Mathematics (with specialization in Statistics) from NTNU, the decision to pursue Ph.D. with faculty of Engineering may seem counter-intuitive. I had a discussion with one of my favorite Prof. Karl Henning Omre from the Dept. of Mathematics about this Ph.D. opportunity. I got the information that this group is good at developing failure distribution. Since, I studied a lot of courses in Master's education about statistics with the assumptions that there always exists a probability distribution. This opportunity seemed to be an opportunity to understand the holistic picture, by complementing my skills on developing a better understanding about probability distributions. Now, after three years while I am writing this thesis, I believe that I made the right choice.

The remaining of this chapter is organized into the following sections: Section I.5.1 discusses the popular ways to classify the research and their applicability to this research; section I.5.2 presents the scientific approach used in performing the research activity, it consists discussion on the scientific method and quality; and section I.5.3 details particular of research design and research method applied.

I.5.1 Classification of research

Depending on the criteria selected, the research may be classified in different ways: Kothari [58] classified research into descriptive research and analytical research based on how it is performed. In descriptive research, research is performed with surveys, polls, and other fact-finding methods. The results of descriptive research make a basis for hypothesis formation on the topic of research. The main attribute of such research is that a variable is not under the control of the researcher. In analytical research, the researcher approaches the problem in analytical ways, such as conducting experiments with controlled variables to test hypotheses or studies the existing knowledge and findings to develop new methods or models based on logical reasoning and analytical thinking. Similarly, research work can also be categorized as between exploratory and confirmatory(also called conclusive) research, quantitative and qualitative research, as well as conceptual and empirical research. Several other classifications may be found in the literature [54, 94, 121].

The Frascati manual (OECD [84]) defines three types of research, namely (i) Basic research, (ii) Applied research, and (iii) Experimental development. *Basic research* refers to experimental or theoretical work undertaken primarily to acquire new knowledge of the underlying foundations of phenomena and observable facts, without any particular application or use in view; *Applied research* refers to an original investigation undertaken to acquire new knowledge. However, it is directed primarily towards a specific, practical aim or objective; *Experimental development* refers to systematic work, drawing on knowledge gained from research and practical experience, and producing additional knowledge, which is directed to producing new products or processes or improving existing products or processes.

The basic research has further classification between *pure basic research* and *oriented basic research*. *Pure basic research* is performed to advance knowledge, without working for long-term economic or social benefits and with no efforts being made to apply the results to practical problems or transfer the results to sectors responsible for its application. *Oriented basic research* is expected to produce a broad base of knowledge likely to form the background to the solution of recognized or expected current or future problems or possibilities.

The research performed in this Ph.D. project falls under the classification of oriented basic research since, in this research, the main focus is to develop frameworks and methods based on current industry challenges and needs, and these may be used for future research. Table I.5.1 summarizes the type of research this Ph.D. falls into.

Research Type	Ph.D. Project
Applied	
Pure Basic	
Oriented Basic	Yes
Experimental development.	
Descriptive	
Analytical	Yes
Exploratory	Yes
Confirmatory	
Quantitative	Yes
Qualitative	
Conceptual	Yes
Empirical	

Table I.5.1: Classification of research for this Ph.D. project

I.5.2 Scientific approach

In RAMS's domain, many scientific studies are related to the development of models, methods, and frameworks. The research performs in this project is funded by Oil & Gas sector companies. This research study aims to provide models, methods that can address practical degradation phenomena experienced by the equipment used in subsea industries. The scientific approach used here consists of three steps (i) selection of scientific method for the development of frameworks, (ii) Evaluation of developed frameworks, and (iii) steps require to assure scientific quality of the results produces under thesis.

I.5.2.1 Scientific Method

The developed frameworks model the degradation phenomena experienced by sub-sea equipment and quantify its effect on the associated decision variables. All frameworks proposed under this thesis are based on the Markov process. The modifications on the Markov process (on case to case basis) are discussed in detail in the papers attached in the thesis. Markov processes are well known and established scientific method in academia.

The usefulness of the developed models in this thesis should be empirically verified(in an ideal situation) by experiments or by collecting real-time data. It is not easy to verify due to the following reasons:

- The event of failure of subsea safety equipment is a rare event. The failure data is not readily available.
- It is very costly and time-consuming to carry out experiments and collect data to confirm the models and modeling results.

I.5.2.2 Model Evaluation

The evaluation and verification of the scientific work and the models must be done by approaches other than empirical or experimental methods. The Model Evaluation Group, launched by the EU in 1992, suggested a model evaluation process in such cases [91]. The group's objective was to improve the culture in which models were developed, particularly by encouraging voluntary model evaluation procedures based on a formalized consensus protocol. The group suggested a model evaluation process consisting of *scientific assessment*, *verification*, *validation*.

The *scientific assessment* should include a comprehensive description of the model, an evaluation of the scientific content, limits of applicability, advantages, and limitations. *Verification* is defined as the process showing that a model has a sound scientific basis, that any assumptions are reasonable, that equations are being solved correctly, and more generally, that the model presented to the user does what the document claims and *validation* as the process of assessing model so that its accuracy and usefulness can be determined. The latter often involves comparison with other models [132].

In this Ph.D. project, the development of frameworks is based on the rational understanding of the subsea equipment's degradation phenomena. The scientific assessment of the proposed frameworks in the thesis relies on the evaluation of assumptions. The assumptions are based on a thorough literature review (both industrial and academic). They have also been assessed by the experts from the industry partners involved in SUBRO. The chosen methods are an improvement of the existing methods suggested by the standards of the domain.

For verification and validation of the framework proposed in this thesis, the industry partners' expert judgments have been utilized. Relevant case-studies from the Oil & Gas sector are presented where these frameworks can be deployed for decision making. The numerical results generated through these frameworks capture the degradation phenomena of the subsea equipment. Hence, they are consistent with the initial objectives and assumptions of these frameworks.

I.5.2.3 Scientific Quality

There is no fixed definition of the term 'scientific quality'. According to the research council of Norway [85], quality research in science consists of *Originality*, *Solidness*, and *Relevance*. Originality is a measure of novelty and innovation. Solidness is a measure of the extent to which the research's statements and conclusions are well supported. Relevance is judged based on the usefulness of the research either for professional development or for society's practical development.

The problems addressed in this thesis stems from collaborations with the industry partners under the framework of SUBPRO or through inter-disciplinary collaborations among NTNU. To the best of our knowledge, the problems addressed are original and are of industrial relevance. The solutions proposed in the thesis are in the form of frameworks. The sensitivity analysis of the frameworks is also performed to show the flexibility and user-friendliness of the solutions.

The quality is also ensured by submitting the research work in peer-reviewed journals and conferences. Scientific papers are updated based on the feedbacks received from the reviewers. The reviewers are considered experts on the subject and their reviews have helped a lot in improving the quality of research. All the scientific work is carried out under the guidance of supervisors and expertise available under the framework of SUBPRO. Their guidance has helped in assuring the scientific quality of the research carried out in this thesis.

I.5.3 Research design and research method

I.5.3.1 Research design

This research project is a sub-project under the framework of SUBPRO, as discussed in chapter I.1. The research performed under this Ph.D. is a combination of integration of external (industry) and internal (other Groups of SUBPRO) forces

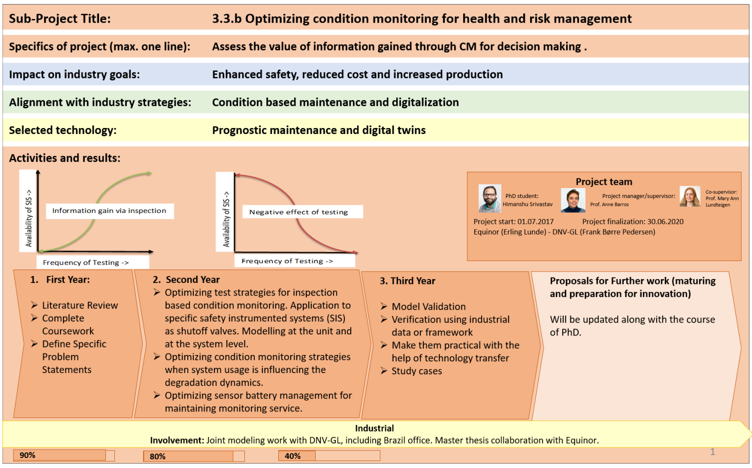


Figure I.5.1: Project Overview

available at SUBPRO. Opinions from industry partners of SUBPRO are regularly sought during review meetings (yearly twice). There is a specific format (termed as *project poster*) to report the progress of the Ph.D. These project posters were the easiest way to communicate the research deliverable and plans for future research activities to industry partners.

A typical project poster consists of three slides (overview, documentation of results and planned activities, and deliverable for next year), as shown in Figure I.5.1, Figure I.5.2, and Figure I.5.3 respectively.

Figure I.5.1 briefly discusses the research objectives, industrial and academic experts involved in the project, selected technology, impact on industry and research schedule, and planned activities. Figure I.5.2 categorizes projects results under three categories: (i) *Academic publications* consist mainly of the research paper published/to be published under this project (ii)*Industrial documentation for SUBPRO partners* consists of master thesis delivered under the collaborations with industry under this Ph.D. project (iii) *Technology transfer* documents the discussion held with industry partners to understand their point of view and concerns. Figure I.5.3 represents the planned activities and deliverables for the next six months.

I.5.3.2 Research method

It is difficult to find an absolute 'research method', which is best for every context. High-quality research requires a documented and logical design of the research project. A research project starts with a research basis and research questions and ends up with the research results. The research methods followed for this Ph.D.

	Current Results	Planned Results
Academic publications	• Paper in ESREL 2019 in collaboration with System Control group with reference to PhD project 3.8 (Control for extending component life) on Combined maintenance scheduling and production optimization -**Published** • Paper in ESREL 2019 in collaboration BRU21 on Time-Dependent Unavailability Assessment of Final Element of Safety Instrumented Systems- An Application of Multi- Phase Markov Process- **Published** • Srivastav Himanshu, Barros Anne, Lundteigen Mary Ann: Journal Paper submitted to RESS in Jan'19: "Modeling framework for performance analysis of SIS subject to degradation due to proof tests" – **Under revision** • Srivastav Himanshu; Gulilherme de Azevedo Vale; Barros Anne, Lundteigen Mary Ann, Pedersen Frank Børre, Hafver Andreas, Oliveira Luiz Fernando: Conference paper submitted to ESREL 2018:"Optimization of Periodic Inspection time of SIS subject to a regular Proof Testing" – **Published**	In collaboration with DNV-GL: • Develop framework for SIS operating in High demand mode • Develop the collaboration further with System control group by extending the work of carried out in ESREL
Industrial documentation for SUBPRO partners (Technical reports, etc.)	• Master thesis by Guilherme de Azevedo Vale (in guidance of DNV-GL) is submitted on comparative study on bad effect of testing for safety system and their modelling. • Master Thesis on battery management for wireless sensors networks is submitted by Sanjay. (in guidance of Equinor) • Master Thesis on condition monitoring and prognosis of remaining life for a heat exchanger is submitted by Bahar. (in guidance of Equinor) • Technical report of Nan Zhang – Researcher hired for procurement of R&D services (3 months: May, June July 2018). Literature review of stochastic dependences in Safety instrumented System. • Presentations at reference group meetings	
Technology transfer	• Close discussion with DNV-GL Oslo and Brazil • Development of simplified approaches to solve equivalent optimization problem. • Joint conference paper.	• An arena for discussion and for challenging different approaches.Numerical studies will be developed to support discussions about other modelling frameworks proposed by the industry (DNV-GL – Shutoff valves and other SIS), to identify complementarities, limitations and validation domains. • An arena for understanding needs from industry for condition monitoring through study case (Equinor – Battery management, some others may come). • Joint publications in international conference and journals. • Development of simulators.

Figure I.5.2: Documentation of results

PLANNED ACTIVITIES AND DELIVERABLES FOR 2019

1. Publications/Reports by the end of 2019 (Most of them will be co-authored with DNV-GL)
- One journal paper about multiphase markov framework to model effect of testing - Finished
- One joint paper with control system group in collaboration with sub-project 3.8 - Finished
- One conference paper challenging the assumptions of assuming constant duration between two inspection times -Finished
- One conference/journal paper about how quality of information will affect the cost of maintenance in case of subsea process -Finished
  conference paper about value of added information through condition monitoring a system with respect to the decision rule -Finished

one paper

2. Technology transfer and gap bridge between existing industry practice and existing research in academia, an active communication with industry partners will be maintained and efforts will be made for join publications, master supervision, simulation tools development. The main actions will be:
- Pursue the specialization project of Sanjay Shah as a Master thesis on Battery management for a network of wireless sensors dedicated to condition monitoring (in collaboration with Equinor). Prediction models for battery lifetime, stochastic optimization for predictive maintenance and inventory planning. - Finished
- Pursue regular working meetings with DNV-GL Oslo and Brasil (in continuation to what has been done in 2018) - Finished
- Establish regular working meetings with Equinor- Rotvoll , in light of the results of Sanjay Shah master thesis. - Finished
- Develop prototypes of simulators in light of outputs from Innovation project – subproject 3.3 - Removed from work plan

3. Integrated project within SUBPRO
- Establish close collaboration with subproject 3.8 (a work has been initiated after PHD colloquium on October 4, 2018) – Finished

4. Integrated project outside SUBPRO (initially not planned)
- Establish collaboration with BRU 21 for similar topic– Finished (Pubished a conference paper in coordination)

Budget for 2019 (1.000 NOK)				
Payroll and indirect expenses	Procurement of R&D services	Equipment	Other operating expenses	Total costs
1 538	-	-	129	3 1 667

Figure I.5.3: Planning of research activities

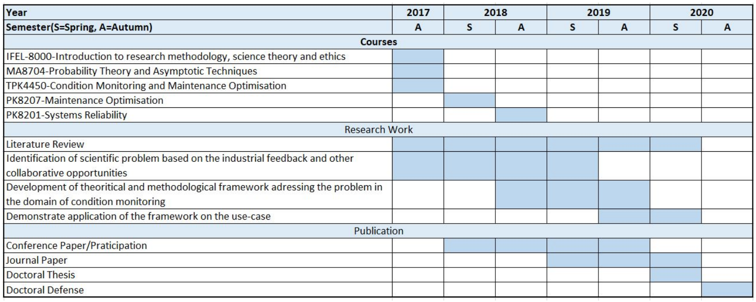


Figure I.5.4: Research Plan and Work Schedule

project consist of the following communicative phases. These are derived from the book 'The essential guide to doing research' [86]. (1) research plan, (2) literature review, (3) model development, and (4) research results.

1. **Research Plan**:

   A research plan is made during the early phase of the Ph.D. to define research challenges and provide a format for further investigation. The typical research plan should answer questions like what is intended to be done; why is the work important; what has already been done; and how should the work be done.

   Figure I.5.4 represents the research plan, including the work schedule. We can see that the initial three semesters were mainly dedicated to learning the required subject knowledge. Under the course work, subjects like condition monitoring, maintenance optimization, system reliability, and probability theory were studied. These subjects helped me to gain in-depth knowledge about the research fields and associated basics. During this period, a significant amount of time is also dedicated to finding relevant research questions and associated literature review. Once the research questions are finalized, the analytical and methodological frameworks are developed to provide the research questions' solutions. In the final stages of Ph.D., all-out efforts are made to publish the research performed. Writing a doctoral thesis and arrange the defense of the doctoral thesis is the final milestone of this Ph.D.

2. **Literature Review**: Karlsson [55] discusses the necessity of careful consideration of existing evidence sources, especially systematic reviews prior to undertaking research. The literature review spanned from the relevant articles published in journals, abstracts, relevant book sections, published reports, and recommended practices by industry.

The general basis for formulating the research challenges in this Ph.D. project is discussions with industry and other departments of NTNU. The main idea behind almost all the research challenges is supporting operators and engineers in better decision-making for the subsea environment. Once the research challenges are established, a thorough literature review was organized with respect to academic and industrial practices.

For the industrial literature review, the online library *OnePetro* [88] was the primary source. *OnePetro* is an online library of technical literature for the oil and gas exploration and production (E&P) industry. It has 21 publishing partners and access to over 200,000 items. The academic literature review is based on research material (articles, papers, and books) available in NTNU's database *Oria*[82]. *Oria* consists of previous studies in the form of master thesis and Ph.D. thesis.

Besides, the professional experience from supervisors and other professors from the department has contributed to the identification and improvement of research challenges.

3. **Model Development**

   Model developments are the next step after the research challenges are finalized, and the necessary information is collected through a literature review. A model only is a simplification of the reality it is designed to represent [1, 90]. The models are, therefore, strongly depend on the assumptions it is based on. There is a famous saying in mathematical modeling '*a model is as good as its assumptions*'. The saying puts the responsibility on the model developer to carefully formulate the modeling assumptions. It is also important to note that there is a trade-off between the generalization of modeling assumptions and computational complexity. Model development is an iterative process until the difference between results delivered and the results proposed(during the modeling assumption stage) are within an acceptable tolerance limit. The level of detail or suitability of a model is restricted by the time, approximation formulas, and software solutions availability. It isn't easy to organize models in order of *correctness*, as different models may be used to analyze the same systems[114]. However, the developers may attribute some order concerning their *usefulness*. One of the most outstanding statisticians of 20$^{\text{th}}$ century, George Box, presented a common aphorism in statistics that '*all models are wrong, but some are useful*'. On the same lines, Stamatelatos et al. [109] claimed that, at best, the model will still only be an approximation. As all models possess limitations in including the natural variability in the real-life system. In all the models, there exists some degree of inherent uncertainty. Standards, guidelines, and internal company policies may often require or recommend specific types of models.

The frameworks proposed under this thesis are based on the models and methods related to this subject and achieve the research objectives.

4. **Research Results**

   Research results should include the application area of the developed models, methods, frameworks, discussion about constraints, and suggestions for new perspectives and ideas for future works. In almost all the papers covered under this thesis, the case study or illustrative example is used to describe the situations explaining research challenges. It is also demonstrated that the developed framework/model us-ability. This thesis's research results are presented to the academic worlds through publication in international journals and proceedings of conferences with peer review processes. To industries, the results were informed through the framework of SUBPRO.

I.5.3.3 Summary

In this chapter, we first classified the research carried out in this research project. Then, we discussed the scientific approach utilized to develop the research ideas. Finally, we presented the research design and research method. It is applied iteratively to develop the research work carried out in this research project. The project's results are delivered in the form of research articles.

Chapter I.6

Main results and future research

We present the the scientific contribution in this chapter by summarizing of the main results and associated discussion from the research articles. All the details about the results and methods are presented in the research articles in part II of the thesis. Specific research questions along-with the objectives of this thesis are mentioned in the Chapter I.4. The purpose of this chapter is to evaluate to what extent these questions have been addressed and solved. Table I.6.1 presents an overview of contribution with respect to research questions of the thesis and main theme of the research questions

Contribution	Research Question	Articles	Main Topic
I	I.4.1	I & II	Degradation in SIS performance due to destructive periodic tests
II	I.4.2	III	Degradation modeling in safety assessment of all-electric actuation system
III	I.4.3	IV	SIS with imperfect detection of degraded state
IV	I.4.4	V & VI	Production optimization and Maintenance Schedules

Table I.6.1: Overview

I.6.1 Contributions I

The first research question is about the degradation in the performance of SIS due to destructive periodic tests. This study aims to provide more realistic estimates of the availability of SIS. A multi-phase Markov process is utilized to describe the natural degradation process of such SIS. The main idea behind utilizing such a model is two-fold: (i) it gives us the freedom to have intermediate performance levels between perfectly working and uniquely failed levels, (ii) the impact of destructive testing can be modeled by altering the transition rate of the degradation process. In this study, we also proposed a time-dependent failure rate model that depends on the current degradation level and the number of tests experienced by the system up to that time.

The detailed problem statement and scientific contribution are discussed in section I.4.1 and research articles I and II respectively. Article I discussed simulation based results whereas articles II developed the analytical framework. Further, we will explain important modeling assumptions and discuss the key results in this section.

I.6.1.1 Key Modeling Assumptions

In this pursuit, we first developed the generic degradation model for SIS. As discussed in section I.3.2.3, multi-state degradation models have very high utility across the industries. We selected a four-state Markov process to represent the degradation process of SIS. This is shown in Figure I.6.1. These states are in the increasing order of degradation, meaning State A represents the minimum or no degradation, whereas State D has degradation beyond acceptable level (failed-state). Dangerously undetectable (DU) failures are considered as the main reason for the unavailability of SIS deployed for subsea. DU failure rate consists of two types of transition rates. One is responsible for progressive degradation (denoted by λ_a), and the other is responsible for sudden failure from any degraded state (denoted by λ_u). Figure I.6.1 shows the state transition diagram. This model gives us time-dependent failure probabilities.

The next step is to identify the factors that affect SIS's degradation process and model the impact of these factors on the degradation process. There were industrial discussions identifying that proof tests are stressful for the mechanical components of SIS and can degrade its performance [87]. Hence, in this study we considered periodic tests as the main factor under study, which may interfere with the degradation process due to its destructive nature. We proposed that the experience of the destructive tests will increase the aging rate of the SIS. For example: if the transition rate responsible for aging is given by λ_0 at the time of the proof test, and if the system is in state A at this time, then the transition rate responsible

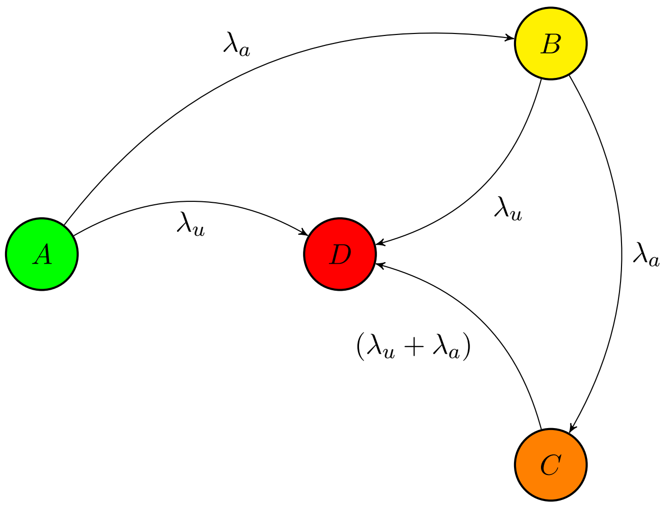


Figure I.6.1: SIS as Markov process

for aging is increased by the factor of α. Similarly, if the system is in state B or state C the multiplicative factors are β, γ respectively. It is important to note that $1 < \alpha < \beta < \gamma$. This ensures that if the system is in a higher degraded state, it will age faster. If the system is found in the failed state, then repair is performed based on the chosen maintenance strategy. There are two maintenance strategies considered:

- AGAN (As-good-as-new): In this maintenance strategy, every time the system is found in the failed state, it is replaced with a new one. This strategy is expensive.
- ABAO (As-bad-as-old): In this maintenance strategy, every time the system is found in the failed state, minimal repaired is performed to make it functioning again. This is the most economical strategy.

I.6.1.2 Key Results

With the above modeling assumptions, we developed a framework that assesses the time-dependent unavailability of SIS. Figure I.6.2 captures the key features of the developed framework. We choose following parameters to develop these plots $\lambda_u = 5 \times 10^{-6}$ hr^{-1},$\lambda_0 = 5 \times 10^{-6}$ hr^{-1},$\tau = 2000$ hr, $\alpha = 1.2, \beta = 1.3, \gamma = 1.5$. It can be argued that the chosen parameters may not belong to realistic parameter space in the domain of SIS. The chosen parameters exhibit important aspects of the model behavior for low numbers of tests.

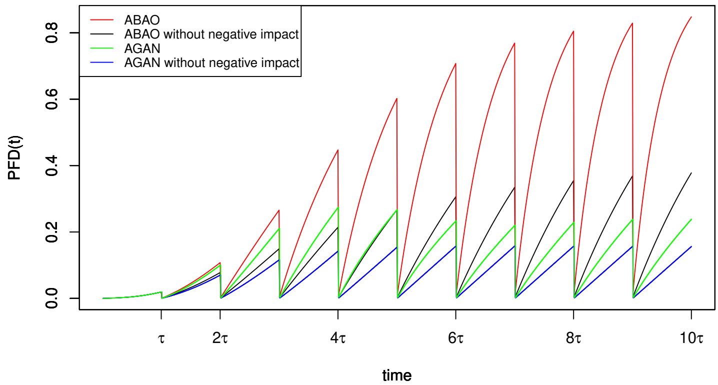


Figure I.6.2: Performance of SIS with time under various testing and maintenance strategies

Figure I.6.2 captures following main phenomena:

- Every time a destructive test is performed, it puts additional stress on the system.
- Stress generated through the previous tests accumulates under the ABAO maintenance strategy.
- The AGAN maintenance strategy with no destructive testing has the lowest unavailability.

Case study

We performed a case study on DHSV based on the developed framework. The purpose of the case study to find the optimum frequency of testing that maximizes the availability of DHSV in given mission time. High frequency of tests will reduce the probability for the SIS to be in an undetected failed state and not to act on demand. On the other side, the stress experienced by the SIS during one test may reduce its probability to perform its safety function for the next period between two tests. Figure I.6.3 explores the optimum frequency of testing for the above case study considering the ABAO maintenance policy. We chose following parameters for this study: Mission Time $= 5$ years, $\lambda_u = \lambda_0 = 5 \times 10^{-6}$ hr^{-1}, $\alpha = 1.01, \beta = 1.03, \gamma = 1.05$.

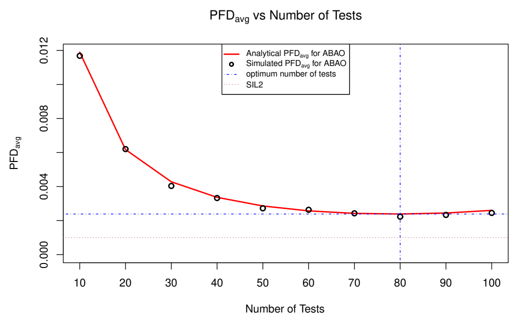


Figure I.6.3: Performance Analysis of DHSV for ABAO maintenance strategy

I.6.2 Contributions II

This research question is about the SIS that are degradable due to the experience of random demands. This study aims to provide more realistic estimates of safety capability of all-electric actuation systems. We used a multi-phase Markov process to model the natural degradation process safety valves used in the novel subsea technology. We proposed time-dependent failure rate model that depends on the current degradation level and the number of demands experienced by the system up to that time. We developed the framework which provides answers to typical questions: (i) instantaneous unavailability of the safety valves when it has experienced a given number of demands with a given maintenance strategy. (ii) estimates average unavailability over the mission-time when it is known that safety valves will experience a given number of demands.

The detailed problem statement and scientific contribution are discussed in section I.4.2 and research article III respectively. Further, we will explain important modeling assumptions and discuss the key results in this section.

I.6.2.1 Key Modeling Assumptions

We utilize the already developed model as a base model for SIS under degradation, as shown in figure I.6.1. Now, the second step is to study the factors which may interfere with the natural degradation process. This study is centered around the all-electric actuation system. Failure mode and effect analysis (FMEA) of such a system shows that one of the critical high-risk failures occurs when the actuator system provides torque higher than the safety valves' damage torque [71]. These safety valves are prone to degradation due to activation caused by power supply interruptions. In this study, we consider the power supply interruptions as factors that may interfere with the safety valves' natural degradation process. We model power supply interruptions as a random demand situation. Destructive periodic tests are considered another factor that can degrade the system's performance.

Modeling of demand situation

We model the arrival of demands with a homogeneous Poisson process (HPP). This is standard practice and followed widely in the available literature [13, 123, 64, 57]. The next step is to model the impact of experiencing a demand on the degradation process of safety valves utilized in all-electric actuation systems. Experiencing a demand situation may affect the safety valve's degradation process one of the following ways:

1. Suppose the experience of the demand causes a strong shock. In this case, the deterioration caused may be to the extent that it can instantaneously change safety valve's degradation level. For example: if safety valve is in state A at the time of demand, then due to demand the state (performance) of safety valve may degrade to one of the states B, C, D; similarly, from state B, it may degrade to states C, D and from state C to state D. This effect is modeled using a matrix transformation. The analyst can choose the degree of safety valve's vulnerability towards the experience of demand situation by tuning the elements of this matrix. .

2. If the deterioration caused by the demand is weak and may not change the degradation level, it will leave residual stress in the safety valve which will increase transition rate responsible for aging. The increment in the transition rate responsible for aging is proportional to the number of demands experienced and the current level of degradation.

   For example: Transition rate of aging increases by the factor of ω_A if the system is in state A just before and just after the demand time. Similarly, for state B and state C, this factor is given by ω_B, ω_C respectively. It is important to note that $1 < \omega_A < \omega_B < \omega_C$. This condition ensure that SIS in higher degraded state will age faster.

3. If system fails due to experience of demand, then we use AGAN and ABAO maintenance policies as described in the section I.6.1.1.

Modeling of periodic tests

Periodic proof can generate additional stress on the safety valves. To incorporate this harmful effect, the existing literature increases the transition rate of ageing by a constant factor [38, 108]. We choose the same way to model the impact of periodic tests on the transition rate responsible for ageing.

I.6.2.2 Key Results

We developed a framework to assesses the time-dependent unavailability of safety valve. It is based on the above modeling assumptions. Figure I.6.4 presents the sensitivity analysis concerning to the number of demands. In this figure, three demands have occurred at $T_1 = 26$ week, $T_2 = 31$ week, $T_3 = 73$ week. We show the evaluation of safety valve's time-dependent unavailability for the first 100 weeks with periodic tests interval (τ) of 20 weeks. Three systems with different vulnerability towards demand experience are considered. We tuned the transformation matrix (as discussed in subsection 1) according to the the vulnerability. System 1, system 2, and system 3 are in the decreasing order of vulnerability towards the demand experience. System 1 has 10% chance to degrade on the occurrence of demand, whereas system 2 has 1% chance to degrade, and system 3 has 0% chance to degrade.

We observe that the weaker system (system 1) has higher unavailability when there are frequent demands. This example shows that the framework is flexible enough to handle frequent demands (i.e. two or more demands between two consecutive periodic tests) even if it is highly unlikely in a subsea environment.The following values of parameters are chosen $\lambda_a = .01$ per week;$\lambda_u = .000001$ per week; $\omega_A = 1.03, \omega_B = 1.05, \omega_C = 1.07, \epsilon = 1.01$.

The detailed sensitivity analysis of the framework is discussed in the attached research article III at page 121.

Case study

We assessed the average unavailability of all-electric actuation systems' safety valve for the mission time of five years using the framework developed above. Table I.6.2 shows the effect of the number of demands experienced in a mission time concerning average unavailability. We considered the ABAO maintenance strategy during the periodic testing as it is more economical than the AGAN maintenance strategy. Since system 3 is most vulnerable towards degradation due to demands, the same is reflected in the table where it has the highest unavailability. It is interesting to observe that System2 has only 1% chance of degradation, but over the mission time, the estimate for average unavailability changes significantly from System 3, which has no chance of immediate degradation due to demands. This comparison suggests that even slight vulnerability towards degradation due to demands will change the estimate of average unavailability significantly.

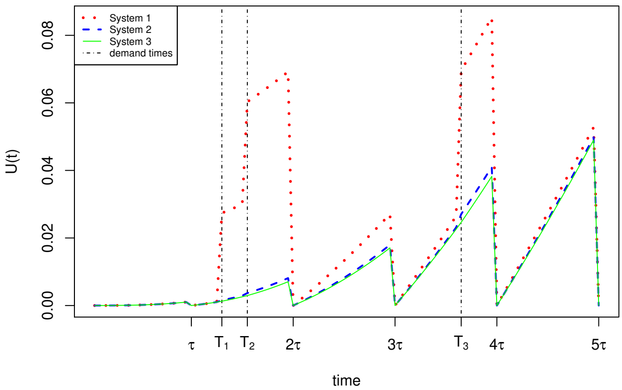


Figure I.6.4: Time dependent Unavailability with three demands

Type of System	Number of demands					
	0	1	2	3	4	5
System 1	1.44E-06	1.94E-04	7.75E-04	1.63E-03	2.76E-03	4.10E-03
System 2	1.44E-06	1.45E-06	7.86E-05	2.29E-04	4.49E-04	7.38E-04
System 3	1.44E-06	1.44E-06	6.39E-06	1.31E-05	2.08E-05	2.91E-05

Table I.6.2: Effect of number of demands on avg Unavailability for use-case

I.6.3 Contribution III

In this research question, we studied the testing and maintenance strategies for redundant final elements in SIS with imperfect detection of degraded state. We evaluated the performance of SIS with redundant final elements. We used a multi-phase Markov process to model the degradation process of each final element. We also developed analytical formulae for the estimation of LCC and relevant reliability measures over a finite time horizon. The proposed method provides support for reliability practitioners of SIS to make an informed decision in choosing particular testing and maintenance strategy.

The detailed problem statement and scientific contribution are discussed in section I.4.3 and research article IV, respectively. Further, we will explain important modeling assumptions and discuss the key results in this section.

I.6.3.1 Key Modeling Assumptions

In this subsection, we first discuss the degradation model utilized to represent the final element of SIS. Then, testing strategies and associated maintenance action are discussed. This research paper mainly considers the use-case of the HIPPS safety system. In HIPPS, two redundant shutdown valves serve as the final elements. These valves are arranged in 1-out-of-2 (1oo2) configuration. In a 1oo2, structure, the SIS continues to perform its dedicated safety function even after one component's failure.

Degradation Model
We consider a Markov process with three states to model the degradation process of each component of the final element. Table I.6.3 describes the states. Figure I.6.5 shows the generic degradation model for each component of the final element whereas, figure I.6.6 shows possible combinations of states on system level for 1oo2 configuration.

Table I.6.3: System state definition

State	status	notation	state description
1	Working	W	System is working as specified
2	Degraded	D	System has a degraded performance but still functioning
3	Failed	F	System has a fault and fails to function

Testing strategies
We considered two types of testing strategies: (i) Simultaneous testing and (ii) Staggered testing. In simultaneous testing both units are tested simultaneously independent of each other. In staggered testing both units are tested sequentially.

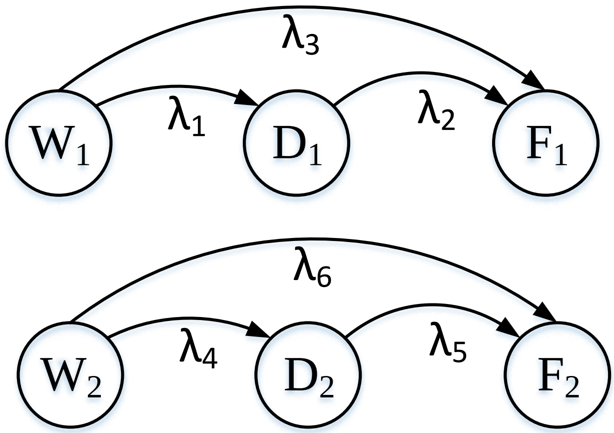


Figure I.6.5: state transition diagrams for (a)1oo1 configuration

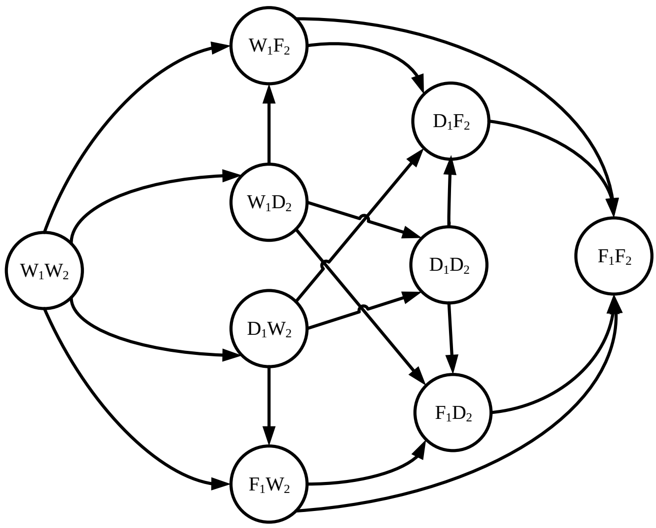


Figure I.6.6: state transition diagrams for 1oo2 configuration

For example: unit 1 is tests at times $= \frac{\tau}{2}, \frac{3\tau}{2}, \frac{5\tau}{2} \cdots$, where as unit 2 is tested at times $= \tau, 2\tau, 3\tau \cdots$. Where τ is periodic testing interval.

The primary purpose of this paper is to study the impact of imperfect detection of degradation during tests. We model the degree to such imperfection in detection by constant factors (given by α_1, α_2 for unit 1 and unit 2 successively). We choose values of α_1, α_2 as in the range $[0, 1]$ to model no detection or full detection of degraded state during tests, respectively.

Follow-up maintenance actions
Based on the information received from the periodic tests about the status of units, we consider the following three types of maintenance action:

- Strategy I: In the simultaneous testing, preventive maintenance (PM) or corrective maintenance (CM) action is taken the unit, which is detected in the degradation state or failed state. Both PM and CM restore the system to the as-good-as-new state.

- Strategy II: In the staggered testing policy, maintenance actions (CM or PM) are performed the tested unit if a degradation state or failed state is detected during the testing.

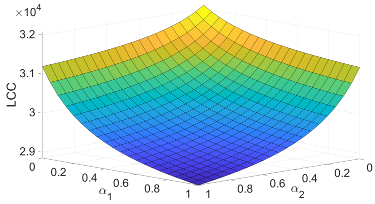
- Strategy III: In the staggered testing, there is an option of performing the opportunistic maintenance. If the tested unit is detected in the failed state:

    1. CM will be performed on the failed unit
    2. Other unit will be replaced independent of its state.

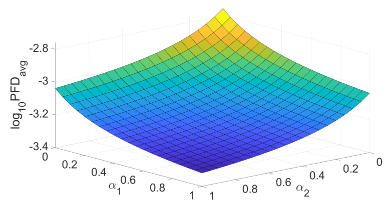
I.6.3.2 Key Results

We developed analytical formulae for time-dependent PFD, average PFD and expected life cycle costing of maintenance for Strategy I, Strategy II and Strategy III. We performed the sensitivity analysis of developed formulae. In this section, we present some key results. Figure I.6.7 presents a sensitivity analysis of expected LCC and $\mathrm{PFD}_{\mathrm{avg}}$ with respect to the degree of imperfection in detection of degraded state of each unit. We considered cost parameters from table I.6.4 and degradation process parameters from table I.6.5 for this analysis. These are selected based on the available literature.

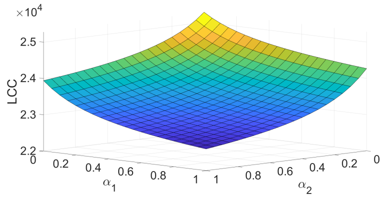
Figure I.6.7(a) shows the sensitivity analysis of LCC for the strategy I. We observe that LCC is maximum for $\alpha_1 = \alpha_2 = 0$ and a minimum for $\alpha_1 = \alpha_2 = 1$. LCC decreases universally with a higher detection factor. The obvious reason



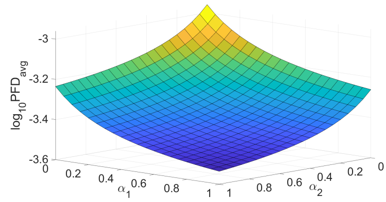
(a) Cumulative cost with strategy I



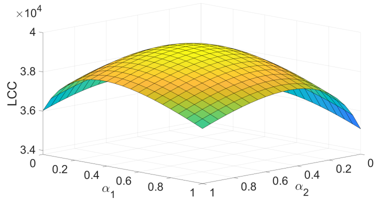
(b) $\mathrm{PFD_{avg}}$ in $(19.5\tau, 20\tau)$ with strategy I



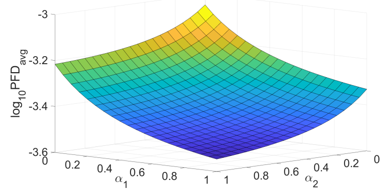
(c) Cumulative cost with strategy II



(d) $\mathrm{PFD_{avg}}$ in $(19.5\tau, 20\tau)$ with strategy II



(e) Cumulative cost with strategy III



(f) $\mathrm{PFD_{avg}}$ in $(19.5\tau, 20\tau)$ with strategy III

Figure I.6.7: Sensitivity of expected LCC and $\mathrm{PFD_{avg}}$ for various maintenance strategies

Table I.6.4: Cost parameters

Parameter	Item	value
C_0	One-time installation cost per unit	600
C_{PT}	test cost per unit	60
C_{PM}	preventive maintenance cost per unit	240
C_{CM}	corrective maintenance (purchase) cost per unit	6940

for such a trend is that increasing the detection of degraded state during tests will increase preventive maintenance instead of corrective maintenance. Since preventive maintenance is less expensive than corrective maintenance, the over all LCC decreases with increasing detection factor. $\mathrm{PFD_{avg}}$ decreases with increasing detection factors. Figure I.6.7(b) shows the corresponding sensitivity analysis of $\mathrm{PFD_{avg}}$ with detection factors. We selected to plot $\mathrm{PFD_{avg}}$ for last phase i.e. $(19.5\tau, 20\tau)$, as $\mathrm{PFD_{avg}}$ has its highest value in the last phase among all phases in the mission time.

Table I.6.5: Degradation parameters

Parameter	value
λ_1	8E-6
λ_2	2E-5
λ_3	4E-6
λ_4	8E-6
λ_5	2E-5
λ_6	4E-6
τ	8760

Similarly, sensitivity analysis for strategy II is shown in figure I.6.7(c) and figure I.6.7(d) show. Figure I.6.7(e) and figure I.6.7(f) represent the sensitivity analysis of LCC and PFD_{avg} for strategy III.

I.6.4 Contribution IV

The fourth research question is about extending degradation modeling techniques to the domain of system control. This study develops a method that integrates the deterministic control laws to the stochastically deteriorating components. The developed method aims to provide support to control system operators to make an informed decision. Control system operators typically decide about the type of maintenance, scheduling inspections, and operational loads scheduling. The developed method quantifies the effect of decisions made by operators in terms of economic indicators. The method provides the optimum values of decision variables, which maximizes the chosen economic indicator. In this study, we considered expected net profit over mission time as the relevant economic indicator, but the method is not limited to this formulation. Some simple modifications in the formulation of a problem can adjust the method for the different economic indicators.

The detailed problem statement and scientific contributions are discussed in section I.4.4 and research articles V and VI, respectively. We divide this section into two subsections. The first subsection explains key modeling assumptions and rationale associated, and the second subsection demonstrates key results.

I.6.4.1 Key Modeling Assumptions

In system control, a typical problem requires optimizing the operational load for maximizing the production. Generally, the objective function of such an optimization problem consists of revenue generated from production and expenses required to perform inspection and maintenance. The objective function is subjected to some constraints such as minimum and maximum availability load, number of interventions to perform maintenance, and system dynamics. The system dynamics do mapping of deterioration in the performance of each component of the control system. Most of the time, system dynamics are represented by a set of differential equations. The introduction of differential equations in the constraints makes the optimization problem a Non-linear problem (NLP). NLPs are solved using off-the-shelf standard non-linear solvers such as IPOPT [98].

In this research collaboration, we extended the framework to include stochastically deteriorating components. For the subsea system, the stochastic nature of the degradation process is two fold: (i) inherent stochastic nature of the process and (ii) insufficient information about current degradation levels. We consider multi-phase Markov process (non-homogeneous) as the degradation model as shown in figure I.6.1 for each component of the system. There are two types of transition rates in the chosen model (i) λ_a is to model the progressive aging of the system and (ii)λ_u

is to model failure due to sudden shock.

After finalizing the degradation model, the next step is to establish the relationship between the input induced operational load and the degradation process of components. The main idea is that higher operational load (u) will push the components to degrade faster. To establish this time-dependent relationship, we used:

$$\lambda_a(t) \propto u(t)$$

It is also important to note that a higher operational load will yield higher production. This means that the revenue generated from production is also proportional to the operation load. We modify the set of equations to include the assumptions mentioned above.

I.6.4.2 Key Results

We developed a method for solving the problem of combined maintenance scheduling and production planning. The method provides solutions to a wide array of problems based on the formulation of the problem. In this research challenge, we applied the method to answer the following types of problems:

- **Maintenance optimization for fixed operational strategy**: In many industries, the operational strategy is fixed, which means that the system performs the same task repeatedly. Then, the intended input-induced loads (u) are constant and are thus not subject to optimization. In such cases, the number of maintenance and maintenance schedules are the main variables in the optimization problem.
- **Joint optimization of operational strategy and number of periodic inspections**: In subsea industries, it is normal to have calendar-based or age-based inspections. In such cases, input induced loads (u) and the number of inspections are optimized to maximize the profit over mission time.
- **Maintenance optimization for fixed number of inspections**: For a fixed number of inspections, the method optimizes the input induced loads (u) and maintenance schedule.

We applied the developed method to study the benefits of optimizing the inspection times and the number of inspections. The study is motivated by the case based on the subsea production system. Figure I.6.8 shows the results of the study. It is observed that periodic inspection generally is sub-optimal. Table I.6.6 consists the paramters selected to generate these results.

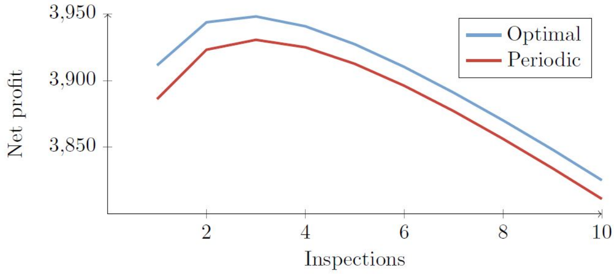


Figure I.6.8: Comparison of net profit for optimal and periodic inspection schedules

Parameters	Description	Value
λ_u	Sudden failure rate	10^{-4} per week
λ_a	Base aging transition rate	10^{-2} per week
d	discounting rate	10^{-2}
c_p	Productivity in each state	$[28, 21, 14, 2.8]^T$
c_m	maintenance cost for AGAN	300
c_i	Inspection cost	30
t_f	Mission time	200 weeks

Table I.6.6: Parameters used for optimization

We also applied the developed method on the more complex case-study of the sub-sea compressor system. This system consists of multiple components, such as two compressors in parallel structure, choke-valve, split-valve, and a separator sub-system. Figure I.6.9 shows a schematic for subsea compressor station. We consider stochastic degradation for each component. We optimized the inspection schedule, input induced load for each compressor, and type of maintenance (corrective or preventive) to maximize the net profit. Figure I.6.10 summarizes the finding from the case study. The parameters selected to generate these results are shown in the table I.6.7.

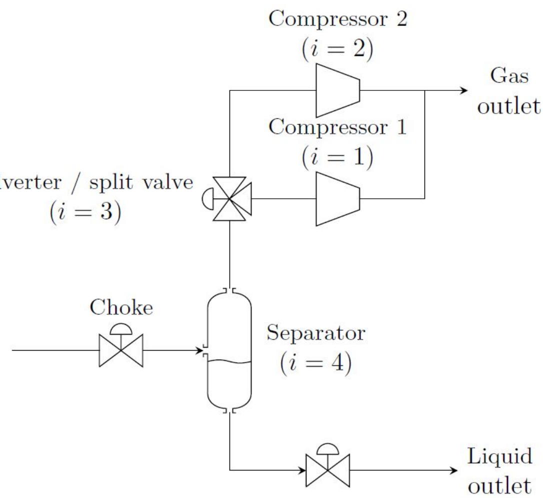


Figure I.6.9: Illustration of the subsea compressor station, with i = $1, \cdots, 4$ indicate the degrading components

Parameters	Description	Value
λ_u	Sudden failure rate for each component	$10^{-4}[2, 2, 1, 0.5]^T$ per week
λ_a	Base aging transition rate for each component	$10^{-2}[1, 1, 0.5, 0.5]^T$ per week
d	discounting rate	10^{-2}
c_p	Productivity in each state	$[10, 5, 5, 0]^T$
c_m	maintenance cost (replacement)	$10^2[5, 5, 1, 2]^T$
c_m	maintenance cost (preventive)	$\frac{10^2}{3}[5, 5, 1, 2]^T$
c_i	Inspection cost	$10^1[1, 1, 1, 1]^T$
t_0	Initial time	0 weeks
t_f	Mission time	200 weeks

Table I.6.7: Parameters used for optimization

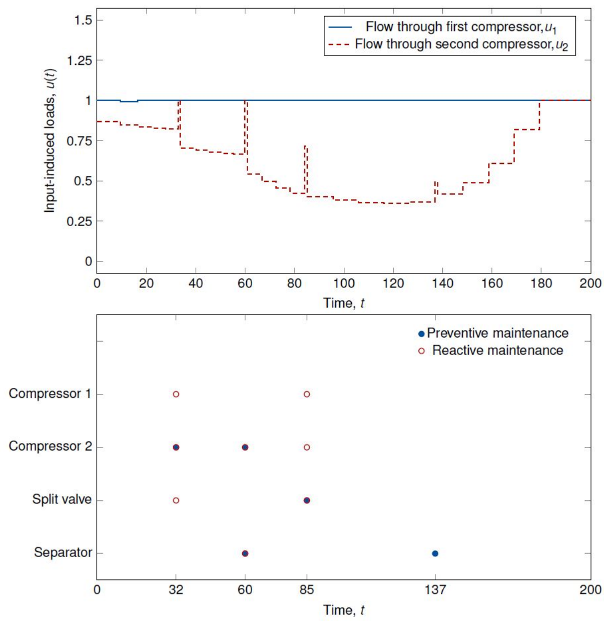


Figure I.6.10: Optimal solution from the optimization of the case study. The upper plot shows the two input-induced loads $u1$ and $u2$, which are the mass flows through the compressors. The bottom plot shows the optimal reactive maintenance times, and the optimal preventive maintenance times.

Chapter I.7

Further considerations and future research

This chapter summarizes the contributions made in this Ph.D. thesis and presents some final considerations. Further, we propose some scientific directions to carry forward the research performed in this thesis.

I.7.1 Conclusion

The research carried out during this Ph.D. provides methods and models to optimize the condition monitoring of subsea equipment. Subsea safety instrumented systems (SIS) are the main focus of this thesis. We first performed a thorough literature review about the methods and techniques that assess the performance of SIS. Under the umbrella of SUBPRO, we had the opportunity to have experts from Oil & Gas industry. We utilized their guidance and expert opinion in formulating the scientific problems that have industrial relevance. Through several discussions with the industry experts, we understood that the industry's standards and current practices lack methods and models that address the real-time degradation phenomena. In this thesis, we focused, particularly on this topic.

We first developed a multi-state Markov process to model the natural degradation process of SIS. This was based on industrial feedback. Then, on case to case basis, we identified the factors that might affect the natural degradation process. We discussed external factors like destructive testing, imperfect maintenance, and experiencing random demands. These factors might interfere with the natural degradation process of SIS. We proposed methods to quantify the effects of such factors on the natural degradation process. We developed frameworks on case to case basis for assessing the time-dependent performance of the SIS. These frameworks account for the interactions among the external factors and the natural degradation process. In general, since we developed the framework to capture the real-time degradation phenomena a SIS goes through, we could perform a more realistic reliability assessment. We also discussed the inherent maintenance optimization problem associated with sub-sea SIS. In subsea SIS, there is a trade-off between information gain about SIS's status from frequent proof testing against the harmful effects it generates on the performance of the SIS performance.

We performed case-studies to showcase the value added by the developed frameworks. In particular, (i) In case of destructive periodic testing, we choose Downhole safety valves (DHSV) as a case study to find the optimum testing frequency that minimizes its unavailability for a finite mission time, (ii) To perform the realistic reliability assessment of components, that are prone to degradation when they experience demands. We choose the safety valves of an all-electric actuation system as a case study.

We extended intra-department collaborations to study the degradation process for subsea SIS with redundancy. In this collaboration, we studied the effect of imperfect inspections (partial detection of degraded state during inspection) on the system's reliability and life cycle costing of different types of condition-based maintenance strategies. We perform a case study on the HIPPS (high-integrity pressure protection system) in this collaboration. Finally, we also got the opportunity to collaborate with a system control group from chemical engineering. In this collaboration, we extended typical control problems for stochastically deteriorating systems. We deployed a multi-state Markov process on the component level to represent the degradation process of components. The developed method optimizes the type of maintenance, time of maintenance, and scheduling of operational loads to maximize the net production profit for a given system. We performed a case study on the subsea compressor station under this collaboration.

I.7.2 Future Research

In this section, we propose future research directions based on the understanding of the limitations of the frameworks developed in this thesis. We discuss them on case to case basis in the research papers attached.

I.7.2.1 Inclusion of other failure modes

The frameworks concerning the reliability assessment of SIS mainly discuss the performance of the final element of SIS. Dangerous undetected (DU) failures are the dominant failure mode for the final elements of SIS. All analysis in this thesis is dedicated to the dominant failure mode. However, to make them more generic, it is required that the developed framework also includes various other failure modes (such as dangerous detected failures and safe failures) into consideration. In the case of SIS with redundant final elements, the framework is based on the assumption that failures of each final element are independent of each other's failure. This assumption needs to be challenged for a more realistic estimation. The inclusion of common cause failures and dependent failures is another direction in which frameworks need to be extended.

I.7.2.2 System level reliability assessment

The frameworks discussed in this thesis only perform a reliability assessment of the final elements of SIS only. Generally, SIS consists of two other subsystems: the sensors subsystem and the logic solver subsystem. These frameworks need to be extended to combine the subsystem level reliability to perform the global level (system level) reliability assessment. The integration of systematic failures in the framework will be necessary to evaluate each subsystem's dependence among the various failure modes. The maintenance optimization in such cases may be an interesting research problem.

I.7.2.3 Investigation about the class of systems across the domain

The work presented in this thesis is dedicated to the subsea systems of the Oil & Gas industry. The key feature for these systems is (i) the degradation in performance of these systems is quantified as finite discrete states, (ii) the transition rate among these degradation levels is affected by external events (such as loads, tests, and type of maintenance). These features are common to the many systems across the industries. Some examples of multi-state degradation models across the domain of power systems and hydro-power systems are discussed in section I.3.2.3. A direction of further research can be to explore the class of systems across the domain where the developed models are applicable as it as or with slight modifications.

I.7.2.4 Model Verification with respect to real-time condition data

The frameworks developed under this thesis are based on identifying the first research problem with industrial interests. The next steps are: hypothesizing about the real-time phenomena that a particular problem is centered around, making applicable assumptions, formulating the frameworks based on these assumptions, and generate and discuss results. Twofold model validation is performed in the thesis (i) All step right from conceptualization of research idea to the results generated through the developed framework are discussed and presented to industry experts, (ii) All the research work carried out in the thesis is submitted and published from peer-reviewed journals and conference. However, we are yet to perform model verification concerning real-time condition data. This verification will justify the number of discrete performance levels, estimation of model parameters from the real-time condition data statistically. The unavailability of real-time data was the main reason behind no model validation. Recently, it seems the Oil & Gas sector is opening up and sharing the data with the research institutes. Now model validation looks like a promising direction for future research.

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Part II

Articles

Chapter II.1

First Paper

Safety and Reliability – Safe Societies in a Changing World – Haugen et al. (Eds)

Optimization of periodic inspection time of sis subject to a regular proof testing

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ABSTRACT: Periodic testing is a method to ascertain the availability of Safety Instrumented Systems (SIS). These systems are generally passive and are activated only on demand. Testing is then required to diagnose their current state and to take the corresponding maintenance action. However, the testing procedure can provoke damage on some units of the SIS (especially the mechanical parts) and the system as a whole becomes more prone to failures. This situation is currently not well covered by standards under the so-called umbrella of imperfect testing. The decision maker must in practice come across to an optimization problem where the objective is to determine the optimal compromise between an accurate diagnostic of the current system state (high tests frequency) and the possible failures or degradation provoked by the testing procedure itself. The commonly used criteria to assess the performance of SIS are all related to the mean downtime of the SIS between two tests. The IEC 61508 provides subsequent analysis for multi-unit SIS when all the units are supposed to follow exponential lifetime distributions. It cannot be applied in this case as some parts of the system have a time varying failure rate which can increase after every test. We propose the use a of Markov process to model the degradation of the mechanical parts upon test and possible preventive maintenance after testing. Since the degradation due to tests is experienced at deterministic dates, we use the modelling framework of multiphase Markov processes to calculate the mean downtime. The paper is focused on explaining the optimization problem between the frequency of testing versus PFD_{avg} and find out the optimum frequency through simulations

1 INTRODUCTION

A Safety Instrumented System (SIS) is often used to detect hazardous events and to mitigate their consequences at facilities and plants that produce or handle hazardous substances, like e.g. hydrocarbon fluids and gases. Due to their criticality, they must obey to regulatory requirements and international standards on safety. IEC 61508 (1998) and related standards (such as IEC 61511 (2002) for the process industry sector) are key in framing the design and operation of SIS. One important requirement mandated by these standards is the need to verify, by quantitative analysis, that the safety performance is adequate in light of risk acceptance criteria. Most safety functions implemented by a SIS, the so-called Safety Instrumented Functions (SIFs), are seldom demanded as the normal operation is managed by a dedicated control system. According to the mentioned IEC standards, the SIFs are classified as operating in the low demand mode.

This means that the SIFs are passive most of the time and are supposed to act only when needed ("on demand"). The reliability of low demand SIFs is measured by the average probability of failure on demand (PFD_{avg}). PFD_{avg} is calculated over a time interval between two proof tests and corresponds to the mean downtime per unit of time between proof tests. The same measure is also used to express the reliability requirement for the each SIF,, but then the associated required value is derived on the basis of a risk analysis (Jin et al. 2012). IEC 61508 suggests four levels of safety integrity levels (SIL), each giving a specified range of PFDavg. For example, a SIF with a SIL 2 requirement must demonstrate that the PFD_{avg} is within $10^{-3}$and 10^{-2}.

The PFD_{avg} can be quantified using different reliability models. These models are based on assumptions and simplifications and in some situations they can lead to different results, depending on the dominating contributing factors. Lowdemand

SIS are periodically tested (proof tests) in order to confirm that they are able to act on demand. Length of intervals between such tests is an important contributor to PFD_{avg}. Normally, it is assumed that the proof tests are perfect, and that the equipment is restored to an to as-good-as-new condition (Shao-Ming et al. 1994). These assumptions imply that the proof tests are carried out in a manner and under conditions which are similar to a real demand, so that all dangerous failure modes,- i.e. failure modes that result in a failure to carry out the SIF, are revealed.The assumptions also imply that no degradation is experienced by the SIS due to the test itself (a non-destructive test). However, in reality, proof tests may not beperfect, and the equipment may degrade from exposures that are applied during the tests. The latter example is also identified by Brissaud et al. (2010). Rausand (2014a) gives one practical example on how the proof test can degrade a Downhole Safety Valve (DHSV) installed in to protect against releases from oil and gas wells. The DHSV is exposed to harsh conditions when operated (due to high pressures drop and in some cases high temperature). A perfect proof test, would imply that the DHSV is closed with full flow from the well (which would be the real demand situation). However, this type of exposure is known to degrade the performance of the DHSV, and the proof test is therefore carried out under non-perfect/imperfect test condtions by closing DHSV with downstream valves already closed. Still, it is interesting to understand better the impact of perfect versus non-perfect/imperfect test conditions. One approach has been suggested by Oliveira et al. (2016), where an additive test-step varying (ATSV) model was elaborated to reflect the increment of the failure rate after each proof test in a blowout preventer (BOP) system. Yet, it is still not clear how to implement the full effect of degradation for the quantification of PFD_{avg}. A review of the modelling framework was performed by Rouvroye & Brombacher (1999) and Bukowski (2005) and both promoted the use of Markov processes when other states than functioning and failed are to be included.

The objective of this paper is to demonstrate the implementation of the Markov process to model the combined effects of degradation due to equipment wear out (aging) and the exposure from the proof test. A simple homogeneous Markov process cannot be used,since the transition rates will change after each proof test. Instead, a multiphase Markov approach is suggested. This method was applied in Strand and Lundteigen (2015) to assess the BOP reliability and also in Innal et al. (2016) to establish new generalized formulas with repair time. Compared to simple Markov processes, multiphase Markov processes allows one to take into account changes of the transition rates at deterministic time points (Wu et al. 2018). The paper is organized as follows: Section 2 provides the problem statement and assumptions. The model is discussed in section 3, within a multiphase Markov framework. Section 4 describes the model implementation in terms of discrete event simulation and Monte Carlo simulations. The last section is devoted to numerical results and the consequent optimization problem.

2 MODELLING FRAMEWORK AND MODEL ASSUMPTIONS

PFD_{avg} is defined as Rausand (2014b):

"..If a demand of safety function of the item occur at a random time in future, the PFD_{avg} is the average probability that the item is not able to react and perform its safety function in response to demand.."

Theoretically, PFD_{avg} value stems from the risk analysis. For practical purposes, it is estimated on the basis of the reliability model of the SIF. In general, an estimator for PFD_{avg} ($\widehat{\mathrm{PFD}_{avg}}$) can be interpreted as long run average value of unavailability, it can be defined as:

$$\widehat{\mathrm{PFD}_{avg}} = \frac{1}{n}\sum_{k=1}^{n}\int_{(k-1)\tau}^{k\tau}\frac{U(t)}{\tau}dt$$

where:

- PFD_{avg} Probability of failure on demand on average
- n = Total number of inspection performed
- τ = Duration between two consecutive inspection
- $U(t)$ = Unavailability of the system at t

Inspection is an integral part of the proof test which reveals about the state of the system at the time of proof test. For all calculations, frequency of inspection is equal to frequency of proof test performed. In this situation $\widehat{\mathrm{PFD}_{avg}}$ is proportion of time on average that the multiphase Markov process spends in the failed state. It is the dangerous failure rate that is considered in the calculation of $\widehat{\mathrm{PFD}_{avg}}$, i.e. the failures that can prevent the SIF from functioning on demand.

The modelling framework to model this problem is described hereafter.

2.1 *Modelling framework*

There are basically two different mindsets for modelling degradation due to equipment wear out (aging) and degradation due to proof test. One mindset is more inherited from Reliability theory: the main idea is to model the degraded unit by a binary random variable moving from working state to failed state and to consider that the transition rate between these two states will increase with time or with the number of tests experienced by the unit. In other words, the unit has a lifetime law with an increasing failure rate which is a function of the number of tests. Another mindset is more applied for people working in the framework of maintenance optimization. The unit is modelled by a random variable with more than two states. The state space can be a discrete finite space, an infinite discrete one or a continuous one. The main idea is that there exists intermediate states between the new one and the failed one. All the intermediate states can be considered as working states but with possibly degraded performances and they are taken as a health indicator of the system. They often correspond to degradation phenomena or symptoms that can be monitored, diagnosed and used as a decision indicator to trigger preventive maintenance actions. The advantage of such models is that

- We can make correspondence between degradation phenomena and the performance of the system (here 1-PFD).
- We can use the intermediate states to optimize and define preventive condition-based maintenance

However, if expert judgments can be relevant enough to define the number and the nature of intermediate states, the law of the sojourn time in every single state may be difficult to estimate. A model relying only on lifetime law and a binary random variable may be then more reasonable.

Most of the existing models that are described in the introduction are inherited from Reliability theory. The calculation of the PFD for SIS is mainly based on binary random variables. In this paper, we want to explore the use of intermediate states in a specific context when the tests have a negative impact on the system condition. We want to investigate such a framework because

- The literature, guidelines and practices related to negative impact of testing should be linked at some point to the identification of some degradation mechanism.
- This seems to be a good way to go ahead and prepare the future for condition-based maintenance and optimal use of condition monitoring.

As a preliminary study, we propose a model with two intermediate states. This number is arbitrarily chosen and we do not investigate any preventive maintenance. We only aim at showing that there is a trade off between the negative effect of tests (pushingthe system randomly into more degraded states) and the added value performing more tests to detect failures earlier.

Equipment wear out is modelled by a finite number of intermediate degraded states between the new state and the failed one. Degradation due to proof test is modelled by an increase of the transition rates between two states at inspection time. In addition, direct transitions are possible from any functioning state to the failed one: they model sudden failures that are not due to wear. Since the unit is passive, all the failures are undetectable without testing, whatever the failure mode is. At last, in order to develop further analytical formulations, we chose a Markovian framework. Because the transition rates are changing at inspection times, we refer it as a Multiphase Markov process. The current paper is only devoted to Monte Carlo simulations in order to demonstrate the relevance of the problem statement and the possible trade off that arises due to the negative effect of testings. Analytical formulations seems to be tractable but are left for further work.

2.2 *Assumptions*

Modelling degradation using Multiphase Markov process, we have used following assumptions:

- In general, we can consider that a SIF equipment is exposed to two types of failures:
  - Dangerous detected (DD) failures, i.e. the dangerous failures revealed by online diagnostics.
  - Dangerous undetected (DU) failures, i.e. the dangerous failures that are not DD and which are to be revealed by regular proof tests.
- For the sake of simplicity to begin with the modelling, we only consider the effect of DU failures in our analysis, since the equipment focused in our study (valves) have no or very limited facilities for diagnostics. However, effect of DD failures,for modeling purposes beyond equipment type in our study, will be considered in the future paper. From now, when we use the term "detected" or "detectable" , it is used to denote DU failures that are revealed by the proof test, in light of the real (non-perfect/imperfect) test conditions.
- DU—failures are of two types: they can be sudden or they can be due to a progressive degradation process named hereafter aging. Sudden failures are modelled by a failure rate λ_u, and aging is modelled by several intermediate states (degradation levels) between new state and failed one, with associated transition rates. Whatever the failure mode is, the system will stay in failed state until the next inspection, and then the system is repaired as per the chosen maintenance policy.

- There are 4 degradation levels: A, B, C, D. These are the states of a Markov process. (A: System working with no degradation, B: System working with degradation of system of level 1, C: System working with degradation of level 2, D: Failed)
- In our model the following instantaneous transitions are possible:
  - System can always jump to next higher state of degradation due to effect of aging.
  - System can always jump to failed state due to sudden DU failures.
  - System can not go to lower degraded state until the maintenance is performed.
- Instantaneous transitions rate for the multiphase Markov process are represented in the Figure 1.
- In the Figures above represents the effect of aging on the system, which changes every time when a proof test is performed on the system. We consider that the proof test has a negative effect on the system condition (shock leading to extra stress) and this negative effect increases the aging transition rates. The modelling of impact of negative effect of testing is done through the following model.

$$\lambda_a(t_0^+) = \begin{cases} 1.01 * \lambda_a(t_0^-) & \text{Current State A} \\ 1.03 * \lambda_a(t_0^-) & \text{Current State B} \\ 1.05 * \lambda_a(t_0^-) & \text{Current State C} \end{cases} \qquad (1)$$

We assume here that a proof test is performed at $t = t_0$ and the current state is the state of the system at $t = t_0$.

- The underlying idea behind this modelling is to show that the negative impact of the proof test increases with the degradation of the system

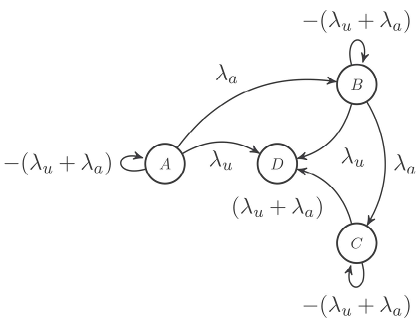

Figure 1. Instantaneous transition rates for the multiphase Markov process.

- Between two consecutive proof tests λ_a and λ_u remains constant.
- When a failure is detected after the proof test, we assume that the mean time to repair the system is negligible.

3 METHODOLOGY

The multiphase Markov process was analyzed using discrete event simulation and exponential distribution for the time spent in each state. System starts in state, degradation time (T_d) and failure time (T_f) are sampled from the exponential distribution of the respective parameters $\lambda_a(t)$ and $\lambda_u(t)$. Then based on the minimum of (T_d, T_f, τ) the next state of the process was chosen. Some specific decisions were made for the modeling:

- If system goes to a failed state (state *D*), the unavailability is calculated by measuring the time spent in the state *D* by the system. On inspection the maintenance action is taken and process is re-initiated.
- If the minimum is τ, then the system stays in the same state for the duration between two consecutive proof tests. Then the inspection is performed and we repeat the process with the increased $\lambda_a(t)$.
- If system goes to more degraded state, then the T'_d, T'_f, are again sampled from the corresponding exponential distributions. Now, the minimum is compared between (T'_d, T'_f, $\tau - T_d$). And the process repeats itself until system goes to failed state. Once the system fails, the unavailability is calculated,the maintenance action is taken and process is re-initiated.

The following maintenance policies were proposed when the system was found to be in the failed state on inspection:

- As-good-as-new (AGAN): System is reset to new state (*A*) and the failure rate of the system is reset to $\lambda_a[i + 1] = \lambda_a[1]$, ie we consider that system is as-good-as-new when we take away the effect of aging after maintenance of the system
- As-bad-as-old (ABAO): On maintenance, the new state of the system is set to *C* and the failure rate of the system is reset to $\lambda_a[i + 1] = \lambda_a[i]$

4 RESULTS AND DISCUSSION

Recall that the PFD_{avg} is the performance measure. Simulations were performed to estimate PFD_{avg} by calculating the average unavailability of the system. The proof test interval (τ) is varied from 3 days to 1 year, where represents the time between two consecutive inspections/proof tests. We considered following values τ of for simulations: τ = (3 days, 6 days, 15 days, 21 days, 1 month, 2 month, 3 month, 4 month, 5 month, 6 month, 7 month, 8 month, 9 month, 10 month, 11 month, 12 month).

Values of parameters like λ_a, λ_u, and mission time are chosen based on industry guidelines on the performance measure. The mission time of the system for the purpose of simulation is chosen to be 5 years. Based on industrial guideline, the impact factor of the proof test is considered as per equation 1. For each value of τ, 500 random realizations were simulated to obtain average unavailability of the system.

Figure 2, shows the estimated value of PFDavg of the system for different values of τ. The borderlines of SIL 1 and SIL 2, showing that the .01 < PFD_{avg} < 0.1 for being within the range of SIL 1 and PFD_{avg} < 0.01 for being in the range of SIL 2. Left side plot in Figure 2 shows that when both λ_a and λ_u are of the order of 10^{-6} per hour, the PFD_{avg} remains within SIL 2 for both AGAN and ABAO maintenance policies for 15 days ≤ τ ≤ 1 year. Right side plot in Figure 2 shows that when λ_a and λ_u are increased to the order of 10^{-5} per hour, the PFD_{avg} increases for both maintenance policies. For AGAN maintenance policy, PFD_{avg} leaves the range of SIL 2 and enters SIL 1 at τ = 15 days and leaves the range of SIL 1 at τ = 6 months. For ABAO maintenance policy the PFD_{avg} leaves range of SIL 1 for $\tau \geq$ 4 months and $\tau \leq$ 15 days and stays within the range of SIL 1 for an optimal proof test interval (15 days < $\tau \leq$ 3 months).

In Figure 2, when the plots pertaining to AGAN maintenance policy are observed, it is found that the information gain through inspection is more significant over the negative effect of testing. This is because with AGAN maintenance policythe system did not carry the history of past tests experienced by the system.

The important conclusion that can be derived from Figure 2 is that when the maintenance policy ABAO is chosen, PFD_{avg} of the system shows a trade off between the negative effect of performing a proof test versus the gain of information by performing the proof test on the system. In other words, when the system undergoes through high frequency of proof tests, the unavailability represented by the PFD_{avg} increases instead of decreasing as it did for AGAN policy. At the same time, when the frequency of proof tests is reduced, the user does not get enough information about the state of the system. Therefore, there exists an optimum frequency of testing which minimizes the value of PFD_{avg} in the Figure 2.

Figure 3 shows the effect, of changing the values of λ_a while keeping the value of λ_u as constant 5×10^{-6} per hour, on the PFD_{avg}. Note that the trade-off between multiplicative negative effect of testing by high frequency of testing versus loss of information by low frequency of testing, is an attribute of ABAO maintenance policy only. Hence, the maintenance policy considered in Figure 3 is ABAO. It is observed from the Figure 3 that the PFD_{avg} remains within the range SIL 2 when the value of $\lambda_a \leq 5 \times 10^{-6}$ per hour for $\tau \in$ [15 days, 5 months]. Plots show that for each value of λ_a, there exists an optimum value of τ for which PFD_{avg} attains a minimum value. It is also observed that the value of PFD_{avg} increases with increasing values of λ_a.

Figure 4 shows the effect, of changing the values of the failure rate λ_u, while keeping the value of λ_a constant 5×10^{-6} per hour, on the PFD_{avg}. ABAO maintenance policy is considered for obtaining these plots, using the same arguments as for plots in Figure 3. It is observed from the left side plot in Figure 4 that when λ_u is increased from 10^{-7} per

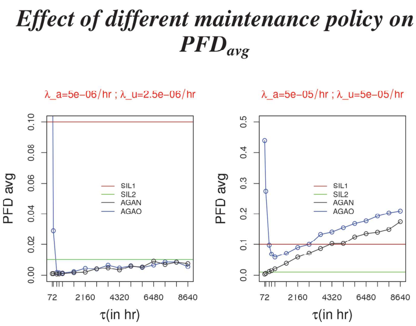

Figure 2. Effect of different maintenance policy on PFD_{avg}.

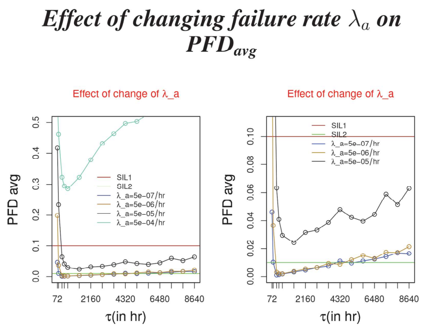

Figure 3. Effect of changing failure rate on PFD_{avg}.

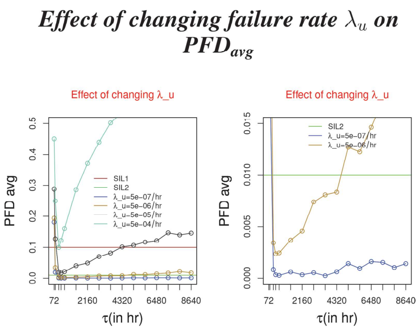

Figure 4. Effect of changing failure rate λ_u on PFD_{avg}.

hour to 10^{-4} per hour the shape of plot of PFD_{avg} changes from a flat convex to a steep convex indicating that PFD_{avg} increases with increase in λ_u.

The optimum time-interval (for performing proof test) which minimizes PFD_{avg} is significantly visible in Figure 4 for higher values of λ_u whereas for lower values of λ_u the curve needs to be zoomed up to observe the optimum time-interval (for performing proof test) as shown in the right side plot of Figure 4.

5 CONCLUSIONS AND IDEAS FOR FUTURE WORK

In AGAN maintenance policy, the technical state of the system is maintained to "as-good-as-new" after regular proof test meaning the system will not aggregate the negative effect of the regular proof test after maintenance. Hence, with AGAN we canmake PFD_{avg} as small as required by increasing the frequency of performing the proof test on the system. But using AGAN maintenance policy may not be economical in most of the practical situations, hence we focus on ABAO maintenance policy in this section.

5.1 *Conclusions*

Our case study showed that in case of ABAO maintenance policy, there are two competitive forces that can increase the PFD_{avg}. The first is the multiplicative negative effect of frequent proof tests, despite the maintenance that is carried out as part of the tests. This force becomes more dominant when the frequency of performing proof test is high. The second is the information obtained about the status of the system by carrying out the proof test. While the second force would like to increase the frequency of performing the proof test to lower PFD_{avg}. The first force would like to decrease the frequency of performing the proof test to obtain the same effect on the PFD_{avg}.

An optimum can be obtained for a regular proof test interval that can be verified against the constraints of the SIL requirement. It is therefore suggested that there exists an optimum frequency for performing the proof test that minimizes the PFD_{avg} of system whenever the following is true:

- The regular proof tests, that involves the inspection of technical state of the system, have some negative effect on the performance of the system due to test conditions and exposures.
- Some dangerous failure modes of the system can only be revealed by the regular proof tests, and not by other means (like e.g. diagnostic testing).
- System is maintained with the ABAO maintenance policy, meaning that the technical state is not "as-good-as-new" after a regular proof test. The ABAO maintenance policy will aggregate the negative effects of regular proof test.

5.2 *Ideas for future work*

The above studies were performed assuming no DD failures and mean time to repair as negligible. It would be an interesting proposition to see the effect of adding DD failures and mean time to repair to the above study. Analytical solutions need to be developed to find out the exact solution of the stochastic differential equation involved in the above situation. Two degraded states were chosen randomly in the above study, the connection between the physical phenomena of the degradation and quantification the degraded states needs to explored. Effect of the predictive maintenance and redundancies on the PFD_{avg} in this situation needs to be studied.

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Chapter II.2

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Modelling framework for performance analysis of SIS subject to degradation due to proof tests

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ABSTRACT

Safety Instrumented Systems (SIS) assure safety of equipment/process by performing the safety functions in demand situations. In low-demand mode of operation, final elements of SIS mostly remain idle and safety performance is measured by probability of failure on demand on average (PFD_{avg}). In this mode, SIS are not continuously monitored but subjected to periodic tests (namely proof tests) to ascertain availability for demand situations. Sometimes, proof tests don't reveal all undetected dangerous failures and may even deteriorate mechanical components by introducing additional stress. To model such degradation phenomena, we propose a framework (based on multiphase Markov process) by adding discrete degraded states between the working and the failed states. The impact of tests is modelled by increasing the transition rates between degraded states. The amplitude increase depends on the current system state at testing time. Then, analytical formulas are developed for the evaluation of the time-dependent PFD under various maintenance policies. Later, a case study on Down hole safety valves (DHSV) is presented to find an optimum test frequency. The optimization problem arises due to the following trade-off: high frequency testing will ensure high availability of DHSV for demand situation, but the stress generated will accelerate degradation to resultant failure.

1. Introduction

Safety-instrumented systems (SIS) are used to detect hazardous events and to mitigate their consequences at facilities and plants that produce or handle hazardous substances, like e.g. hydrocarbon fluids and gases. A typical example of a SIS for subsea oil/gas pipeline is a high intensity pressure protection system(HIPPS). HIPPS prevents the loss by detecting unacceptable levels of pressure and closing dedicated valves to avoid further pressure build-up that may cause pipeline rupture [1]. Due to it's criticality, a SIS must obey to regulatory requirements and international standards on safety. IEC 61508,2016 [2] and related standards (such as IEC 61511,2017 [3] for the process industry sector) are key in framing the design and operation of SIS. One important requirement mandated by these standards is the need to verify, by quantitative analysis, that the safety performance of the SIS is adequate in light of risk acceptance criteria. The underlying idea is to check if the safety functions of a SIS, the so-called safety-instrumented functions (SIFs) are reliable enough.

There is an extensive literature dedicated to quantitative analysis of SIS performances [4] with a large range of methods and models. However, in most of the existing approaches, the physical states of the SIS are reduced to its functional decomposition: the SIS is either available to perform SIF in case of demand (i.e. "OK" state) or unavailable to perform SIF in case of demand (i.e. "KO" state). Even when a physical degradation phenomenon is acknowledged, a binary state model is still considered [5], meaning that all the degraded states are gathered either in a unique "OK" state or in a unique "KO" state. Then, a time dependent failure rate is introduced to model the degradation effect by accelerating the transition from the "OK" state to the "KO" state. The main objective of this paper is to propose a set of models that makes it possible to add degraded states and envision more than two states. By expending the modelling possibilities, we provide the following new insights:

1. Build other forms of failure rates, which can be time dependent but also condition dependent.
2. Implement condition based maintenance policies and optimise inspection strategies.
3. Define other kind of functional decomposition (e.g. degraded mode) and associated performance measures.

The modelling framework is developed in the specific context of low demand mode of operation. In this mode, the mean time between demands is assumed to be greater than one year [6]. This implies that

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final elements of the SIS remain idle most of the time and are activated only in demand situation. Such SIS are periodically submitted to tests (proof tests) in order to confirm that they are able to act on demand. Their performance are then quantified by their mean downtime per unit of time between two proof tests (commonly named average probability of failure on demand, PFD_{avg}). An important challenge raised by the industry is that the proof tests are stressful for the mechanical components of SIS and can degrade its condition [5]. Most of the existing approaches model the test effects by increasing the SIS failure rate at the time of testing. The failure rate is then dependent on the number of tests experienced by the SIS. We propose in this paper to add degraded states and to model the test effects by increasing the transition rates between these intermediate states. Doing so, we can model the situations when the SIS failure rate at time t is dependent on the number of tests experienced before t but also on the SIS condition at the testing times.

The paper is organized as follows: Section 2 describes the problem statement in detail. Section 3 provides state of the art on the existing modelling method. It is intended to show how the proposed framework based on multiphase Markov process adds value to the existing methods for qualitative analysis of SIS. Section 4 develops the modelling framework in light of the modelling assumptions. Section 5 presents numerical results and discussions related to performance analysis of SIS. One shows that the testing period can be challenged to optimize the trade off between the tests benefits (failure detection) and the subsequent degradation for the case study. Concluding remarks on this approach and future ideas are provided in Section 6.

2. Problem statement

A typical SIS consists of three types of subsystems: a set of sensors, a logic-solver, and final elements. Sensors subsystem measures some physical quantity and feeds it to a logic-solver subsystem, then the logic-solver subsystem decides based on the measurement whether the situation is safe or the SIF needs to be activated. The final element subsystem (in most of the cases consists of mechanical parts) performs the safety functions, if required. From now on the term SIS is used for specific subsystem.

2.1. Testing strategies

This paper is focused on SIS operated in the low demand mode. Such SISs remain idle most of the time and act only when needed. They can experience failure modes which can prevent them to act on demand. Such failure modes are classified as “dangerous failure modes” since the SIS is not able to perform its main function. In addition, some of these failure modes can be undetectable by using any on-line monitoring systems and remain hidden until a real demand occurs. They are classified as “dangerous undetected” (DU) failure modes. To avoid such unacceptable situation from the safety point of view, these SIS are periodically tested in order to confirm that they are able to act on demand. The testing strategy is then a key point in the operation of installations under safety constraints.

Mainly three types of tests are performed on a SIS:

- *Proof Test*
  A proof test is a regularly performed test that aims to reveal DU failure modes. A proof test may be defined as full or partial, depending on the planned scope of the test. A full proof test is designed to reveal all DU failures, while a partial proof test is designed to reveal only some selected DU failure modes, typically those where the test can be carried out with only a minor or insignificant impact on the production performance. A partial proof test can never replace a full proof test but can be a useful complement when the motivation is to reduce costs (of testing) or to improve safety [6]. The maintenance policy or operating conditions may influence whether to assume a perfect proof test or imperfect. A proof test is perfect when the following two conditions are met:
  1. All the DU failure modes within the scope of the proof test is revealed
  2. These DU failures must be restored to an as good as new (AGAN) state
  Examples of an imperfect proof is when the maintenance policy is to restore to failed parts to an as good as old (AGAO) state or when DU failures supposed to be revealed by the tests have been overlooked.
  Fig. 1 summarize the above discussion [6]

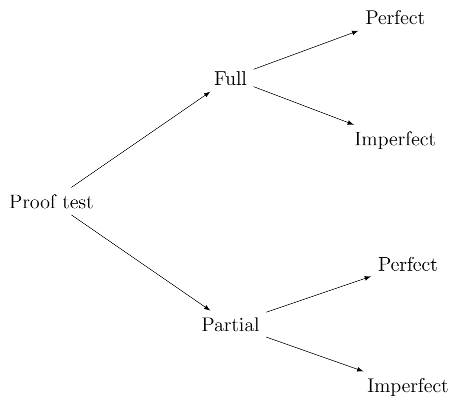


Fig. 1. Classification of proof tests.

- *Function Test*
  Function tests ensure that the SIS performs its safety function properly. Generally, a proof test may be comprised of one or many function tests as failure is always related to non-accomplished functions. For example, the proof testing of valves is split into function test (check of response time) and leakage test (checking if valve keeps tight). Another example is a redundant configuration of SIS, the function test will determine if at least one of the two redundant parts is functioning but it is not sufficient to ensure that the two redundant parts are actually functioning. In situation when SIS only performs one SIF, the function test is regarded as *the* proof test.
- *Diagnostic Test*
  The diagnostic tests are performed in order to detect a specific failure. They are performed rather more frequently than proof tests. Normally, the typical failures detected by diagnostic testing are signal loss, drifted analogue signal, signal out of range, and final element in the wrong position [6].

In this paper, the main focus is on imperfect proof tests and their impact on the SIS.

2.2. Safety levels

The reliability of low demand SIFs is measured by the average probability of failure on demand (PFD_{avg}). PFD_{avg} is calculated over a time interval between two proof tests and corresponds to the mean downtime per unit of time between two proof tests. The same measure is also used to express the reliability requirement for each SIF, but then the associated required value is derived on the basis of a risk analysis (Jin et al. [7]). IEC 61508 suggests four levels of safety integrity levels (SIL), each giving a specified range of PFD_{avg} (see Table 1).

For example, a SIF with a SIL 2 requirement must demonstrate that the PFD_{avg} is within 10^{-3} and 10^{-2}.

Table 1
Safety integrity levels - target failure measures for a safety function operating in low demand mode of operation.

Safety Integrity Level (SIL)	Average probability of a dangerous failure on demand of the safety function (PFD_{avg})
4	$\geq 10^{-5}$ to $< 10^{-4}$
3	$\geq 10^{-4}$ to $< 10^{-3}$
2	$\geq 10^{-3}$ to $< 10^{-2}$
1	$\geq 10^{-2}$ to $< 10^{-1}$

2.3. Degradation phenomena

There are two main reasons for which a SIS can experience deterioration: natural aging and impact of testing.

- *Natural Aging*
  Most of the safety functions of a SIS are implemented by actuators (valves, BOP,...). These actuators are composed of mechanical components. These mechanical parts experience natural deterioration due to the environment and real time demands. This is termed as degradation due to aging.
- *Impact of Testing*
  Perfect proof tests are performed by simulating real demand situation and for which the repair strategy chosen is AGAN. In practice, such tests are not widely implemented and are replaced by imperfect proof tests, meaning that the conditions with which the imperfect proof test is carried out are different from real demand situation. Practical reasons (like safety, high cost, shut-down of equipment/installation) may lead operator to choose imperfect proof tests instead of perfect proof tests. Even if easier to implement in practice, the imperfect proof tests are not fully satisfactory because
  1. They are not able to detect all dangerous failure modes.
  2. They can generate an extra stress on the tested components.

We focus in this paper on the second item, which is about deterioration due to testing. This effect is termed as the **impact of test**. Due to several experiences of impact of tests, some SIS components can become severely deteriorated and this results in SIS performance degradation which eventually leads to failure to perform the associated SIF. From now on the term *test* is used for imperfect proof tests unless otherwise specified.

2.4. Optimization problem

A high frequency of tests will reduce the probability for the SIS to be in an undetected failed state and not to act on demand. On the other side, the stress experienced by the SIS during one test may reduce its probability to perform its safety function for the next period between two tests. Consequently, there is a trade off to optimize between high probability to detect a failure in a short term horizon and high probability to degrade the system after too many tests. The optimal tests frequency strongly depends on the degradation dynamic behaviour.

2.5. Case study

Performance assessment of a subsea safety system namely down hole safety valve (DHSV) is presented as a case study. This equipment is installed as a final element in an oil well (refer Fig. 2). The main safety function of DHSV is to stop flow in the tubing when an uncontrolled flow of crude oil or natural gas occurs. DHSV has two main dangerous failure modes: fail to close on demand and leakage in close position. In the real operation scenario, DHSV shall close against the flowing well and it is called slam-shut closure. In this scenario the valve is exposed to high stresses due to high pressure flow. Rausand [6] states that the DHSV cannot withstand more than a few slam-shut closures without failing. That is why the DHSV is not proof-tested by slam-shut closure (real operation condition). Imperfect proof test is performed by stopping the full flow through it by one or more valves on its downstream side. Then, DHSV is closed against a static well and it is checked for a possible leakage. This practice is widely accepted by oil and gas industry even though it is not complete proof test. Discussions with industry suggest that imperfect proof tests may induce minor degradation in the performance of DHSV. Hence, the optimization problem referred at Subsection 2.4 becomes an interesting problem to solve for this case study. This has already been addressed based on a next even simulation method [8]. However, analytical solution for the optimization problem is still to be explored. In this paper, we present solution of the optimization problem based on the analytical method and compare it with results through simulations.

3. State of the art

There are different approaches to assess the performance of SIS subjected to proof tests. Almost all existing approaches are failure mode analysis techniques with constant or time varying failure rates. IEC

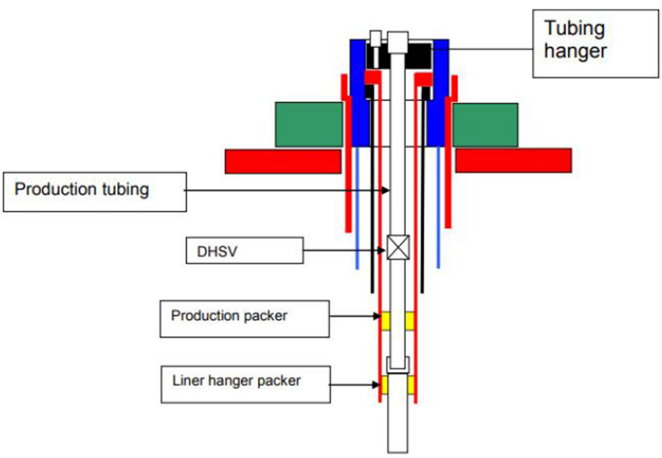


Fig. 2. Position of DHSV in subsea well.

61508 along with others (Rausand [6], Hauge et al. [9]) consider the exponential distribution. For most of the components, this assumption is applicable. However, mechanical components such as valves and pumps may have time varying failure rates when they experience degradation due to aging.

Wu et al. [10] proposes to use Weibull distribution to model the time dependent failure rates for SIS subjected to partial tests. Rogova et al. [11] presents the analytical formulae to assess the performance of redundant SIS with time dependent failure rates. These methods consider that proof tests have no impact on the SIS.

There is less literature available in Oil & Gas sector to assess the performance of a SIS in the presence of test impact. Oliveira et al. [5] proposed a shock degradation mechanism termed as Additive Test-Step Varying (ATSV) model to develop time dependent solutions for PFD. This approach assumes an exponential life time law for the component. In this approach, failure rate increases by a fraction of initial failure rate with each proof test and stays constant between two consecutive proof tests. This framework is applied to assess the performance of BOP components. Later, the ATSV model approach is improved by introducing Multiplicative Step Increasing Model (MTSV) [12]. In this approach, failure rate of the component is multiplied by a constant factor (impact factor of test) at each test.

The nuclear sector is contemporary to the Oil & Gas sector in terms of safety. There exists an extensive literature in nuclear sector on the similar topics. Most of the work is dedicated to performance assessment of safety systems and optimization of some measure of performances. Early research works don't consider the components aging and rely on the assumption of constant failure rate. Some important examples are given hereafter.

Čepin and Martorell [13] developed a framework to evaluate the risk associated with performance in terms of outage (downtime) of safety system under various plant configurations and modes of operations. Vaurio [14] presented a general procedure to optimize the test and maintenance (T&M) intervals for safety system in Nuclear Power plant (NPPs). The optimization is performed to minimize the plant level cost of T&M with constrains on the risk (accident frequency) associated with the downtime. This paper develops analytical formulae for a single component and simple system, but also presents a general procedure to find the optimal interval for more complex systems with multi components. The main assumption in this work is that failure probability of components is linear with T&M interval. In nuclear industry, Probability Safety Assessment (PSA) is an analytical tool to assess the safety of a nuclear power plant. PSA is used to calculate the risk associated with the accidents identified in the study. It mainly uses fault tree and event tree techniques to calculate the probabilities associated with each accident scenario. In PSA, first initiating events that might lead to severe consequences and their occurring frequencies are identified. Then, based on the reliability of the safety system, the failures are arranged in the decreasing order of their probabilities. Then, the sequences of events which will lead to severe accidents are identified. Čepin [15] proposed an optimization algorithm for effective scheduling of safety equipment outage (downtime) due to testing and maintenance which minimizes the risk associated with outage. The work is based on the integration of simulated annealing with probability safety assessment (PSA). Results showed that risk reduction is possible on the plant level with the application of developed algorithm. However, due to less failure data, there is a large uncertainty in the parameters estimates of PSA, that may lead to conclude that the method is best suited for the identification of high-risk schedules rather than finding lowest risk schedules. Čepin and Mavko [16] improved the technical specification regarding surveillance test requirement for safety systems in NPP by optimizing surveillance testing interval on component level, system level and plant level (with minimization of the associated risk). The main limitations of the above referred works is that they are developed with constant failure rates for the components, meaning that the aging on the safety components is not taken into account in the analysis. There has been significant progress on this assumption. Martorell et al. [17] showed that the effective age of the safety component is affected by the surveillance testing, corrective maintenance, time-directed preventive maintenance, time-directed predicted maintenance, overhaul maintenance. Martorell et al. [18] considered that working conditions (both operational and environmental) of NPP have also impact on the ageing of the safety component in addition to surveillance and maintenance activities. Vaurio [19] utilized an extended Weibull hazard rate to assess the time-dependent availability of the ageing of standby units under various testing and repair policies. Based on the proposed model, cost-based optimization is also performed on periodic testing and scheduled maintenances. Martorell et al. [20] adopted a linear ageing model of safety components in simultaneous and multi-criteria optimization of technical specification requirement for testing and maintenance. They used steady state genetic algorithm as a mathematical tool to perform optimization.

The recent literature in the nuclear sector also quantifies the degradation of safety components which undergoes tests and demands. For the assessment of unavailability, both demands and tests are treated in the same way (as far as the degradation-caused is considered). Martorell et al. [21] studied reliability models of safety system considering two failure modes of safety system i.e. (i) demand-caused failures (ii) standby related failures. They developed analytical expression of maximum likelihood parameter estimates for these models. Martorell et al. [22] further extended the reliability model to evaluate the evolution of average unviability of safety systems.

The available literature in both nuclear and Oil & Gas sector considers binary states (i.e. “OK” and “KO”) on component level. Then, based on the case in hand, it improves the life time model associated. The parameters for these models requires real time failure data for meaningful predictions. In the absence of real time data, expert knowledge is used first to a have a sensible guess about the parameters and then to validate the numerical results.

Discussion with industry indicates that it is reasonable to assume more than two states based on the performance of the SIS. These states are categorized as functioning states with degraded performance. For example, partial proof tests of DHSV sometimes detect leakage which is within the acceptable limits. This can be classified as functioning state with degraded performance. In general, an expert opinion can be used to investigate the number of accepted levels of degradation in performance of the SIS. In this situation, traditional methods with life time analysis or failure mode analysis need to be modified significantly.

There exists an extensive literature to model systems with degraded states apart from completely working and failed states. It is often the case for systems like power systems, computer systems. These systems are called multi-state-systems (MSS). Lisnianski and Levitin [23] extend traditional reliability binary methods to develop reliability theory to measure and assess the reliability of MSS. A structure function is a key concept in MSS for the mathematical representation of MSS. It maps every combinations of performance level of components to the performance level of the system. The dimension of the structure function depends on the number of the degraded states of each component and the number of components of the system. The structure function becomes easily very complex due to the various possible combinations of degraded states of the components. To estimate high dimensional MSS structure function, Zaitseva and Levashenko [24] represented the structure function as a Multiple-valued decision diagram. Aubry and Brinzei [25] utilized concepts of graph theory to assess the reliability of MSS. In this approach the structure function of MSS is represented by an ordered graph. A weight is assigned to each node of the graph, based on the performance level of the system corresponding to that node. Then, the reliability is established by a progressive reduction of weighted graph built from the ordered graph. MSS reliability assessment methods are developed for unrepairable systems hence they are not applicable in this case.

For repairable systems [26,27] advocate to use the framework of

Markov processes when other states than functioning and failed have to be included in the modelling. However, the transition rates between the states might change due to the impact of test and the use of a Markov process may not be possible. Hence, in such a situation multiphase Markov approach is utilized. Strand and Lundteigen [28] applied this framework to assess the BOP reliability. Wu et al. [29] also used this approach to analyse performance of subsea blind shear ram preventer subject to testing strategies. We propose to use this framework to model the impact of proof tests in the upcoming sections.

4. Modelling frame-work

This section first presents the relevant modelling assumptions. Formulae for performance analysis of SIS between two consecutive proof tests are then illustrated, considering the proof tests are harmless. Next formulae to model the impact of proof test in the presence of different maintenance strategies are presented, and finally formulae for performance analysis of SIS considering harm full proof tests are developed.

4.1. Modelling assumptions

- There are 4 possible states for the system under study. They are denoted by State A, State B, State C and State D in the increasing order of degradation. State A represents the minimum degradation and State D has degradation beyond acceptable level (failed state).
- DU failures are considered as the main reason for unavailability of SIS. DU failure rate (λ_{DU}) consists of two types of transition rates. One is responsible for progressive aging (λ_a) and the other one is responsible for sudden failure from any degraded state (λ_u). Fig. 3 shows state transition diagram.
- The transitions rates of Markov process can change as a result of impact of proof test.
- Proof tests only reveal whether the system has failed or not but doesn't reveal the current degraded states of the system. Hence, the framework is developed for a such type of condition monitoring.
- Repair time is assumed to be negligible.

4.2. Performance analysis of SIS without impact of test

The Fig. 3 represents the SIS in terms of a Markov process between two consecutive proof tests. We define $\{X_t; t \geq 0\}$: stochastic process which represents the state of the system at time t, $P_t = \{\Pr[X_t = A], \Pr[X_t = B], \Pr[X_t = C], \Pr[X_t = D]\}$: vector represents the probabilities of the process in each state at time t. Then by solving Chapman–Kolmogorov's equation, we can express general solution for P_t as:

$$P_t = P_0 \exp(t\mathcal{A}[\lambda_a]) \tag{1}$$

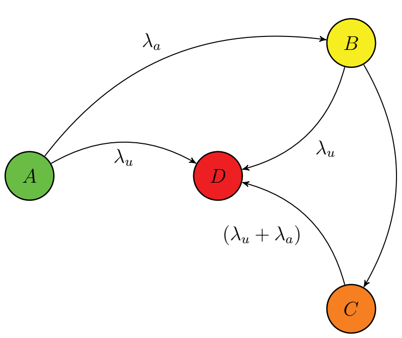


Fig. 3. SIS as Markov process.

where P_0= stands for the initial probabilities vector for the system, and $\mathcal{A}[\lambda_a]$ is defined by the 2.

$$\text{Transition matrix}(\mathcal{A}[\lambda_a]) = \begin{bmatrix} -(\lambda_a+\lambda_u) & \lambda_a & 0 & \lambda_u \\ 0 & -(\lambda_a+\lambda_u) & \lambda_a & \lambda_u \\ 0 & 0 & -(\lambda_a+\lambda_u) & (\lambda_a+\lambda_u) \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{2}$$

In this case the instantaneous PFD (PFD(t)) is given by:

$$\text{PFD}(t) = \Pr[X_t = D] = P_0 \exp(t\mathcal{A}[\lambda_a]) \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{3}$$

If τ is time between two consecutive proof tests then, performance measure of the system is given by:

$$\text{PFD}_{\text{avg}} = \frac{1}{\tau}\int_0^{\tau} \text{PFD}(t)dt = \frac{1}{\tau}\int_0^{\tau} \Pr[X_t = D]dt \tag{4}$$

4.3. Modelling of impact of proof test

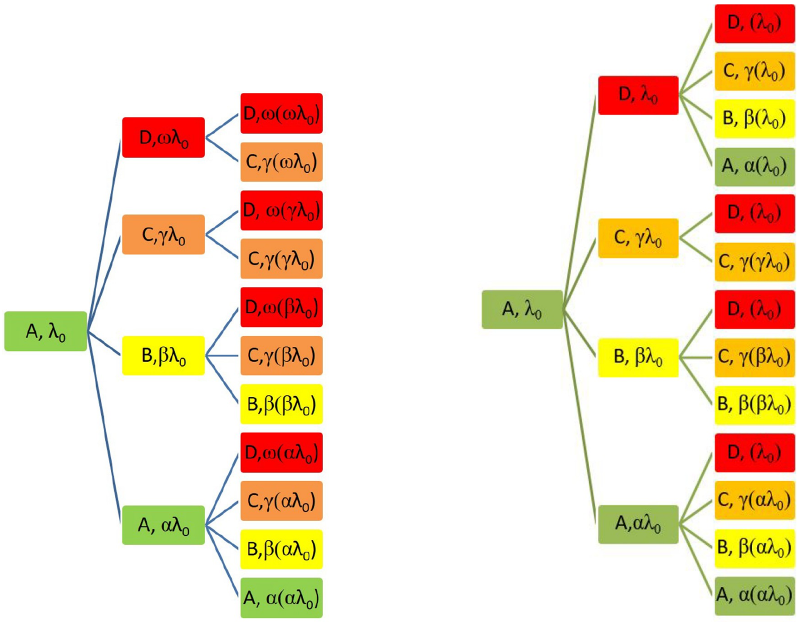
The impact of proof test increases the rate of aging of the system. The transition rate responsible for aging is modelled as a function of both the number of proof tests already experienced and the current degradation level of the system at the proof test time. For example, if the transition rate responsible for aging is given by λ_0 at the time of the proof test, and if the system is in state A at this time, then the transition rate responsible for aging is increased by the factor of α. Similarly if the system is in state B or state C the multiplicative factors are β, γ respectively. It is important to note that $1 < \alpha < \beta < \gamma$. This ensures that if the system is in higher degraded state, it will age faster. If the system is found in failed state, then repair is performed based on the chosen maintenance strategy. There are two maintenance strategies considered in this paper.

- AGAN (As-good-as-new): In this maintenance strategy every time the system is found in failed state, it is replaced with a new one. This strategy is expensive.
- ABAO (As-bad-as-old): In this maintenance strategy every time the system is found in failed state, minimal repaired is performed to make it functioning again. For modelling points of view, the SIS is reset to state C and the transitions rate is the one the system had before failure. This is the most economic strategy.

There are many possible combinations of states and transition rates for aging after a proof test is performed. Fig. 4 shows the combinations after two consecutive proof tests in the presence of AGAN and ABAO maintenance strategy. System starts in the initial condition State A and with the initial transition rate for aging λ_0. In the ABAO maintenance strategy, every time the system is in state D at the time of the proof test, it is reset to state C with aging transition rate increased by the factor ω. In case of AGAN maintenance strategy, the system is reset to state A with aging transition rate (λ_0) after the proof test, if it as failed as testing time.

The dependence on the current degraded state at the time of the proof test introduces stochastic nature in the transition rate pertaining to aging. Hence the transition rate is modelled as a discrete stochastic variable. We define:

α, β, γ, ω: state dependent impact factor of the testing
Λ_n^a: stochastic random variable representing transition rate of aging after nth proof test
S_n: state of the system at the time of (n)th proof test
Initial condition : $\Lambda_0^a = \lambda_0$, $S_0 = A$



(a) ABAO. (b) AGAN.

Fig. 4. Possible combinations of states and transitions rate after proof tests.

- **For ABAO maintenance strategy**

We define impact factors by Eq. (5)

$$\Lambda^a_{(n+1)} = \begin{cases} \alpha\Lambda^a_n & \text{if } S_{n+1} = A \\ \beta\Lambda^a_n & \text{else } S_{n+1} = B \\ \gamma\Lambda^a_n & \text{else } S_{n+1} = C \\ \omega\Lambda^a_n & \text{else } S_{n+1} = D \end{cases} \tag{5}$$

Given the initial condition, the general term is given by λ^i_n for the values of the stochastic random variable Λ^a_n. It can be written in the following manner:

$$\Lambda^a_n = \{\cup\lambda^i_n\} = \{\lambda^1_n, \lambda^2_n, \lambda^3_n, \ldots\}$$

$$\begin{aligned} &\lambda^i_n = \alpha^{x_i}\beta^{y_i}\gamma^{z_i}\omega^{t_i}\lambda_0 \\ &\text{subject to} \\ &x_i + y_i + z_i + t_i = n \\ &x_i, y_i, z_i, t_i \in \mathbb{N} \\ &\text{where} \\ &\mathbb{N} = \{0, 1, 2, 3, \ldots\} \end{aligned} \tag{6}$$

- **For AGAN maintenance strategy**

We define impact factors by Eq. (7)

$$\Lambda^a_{(n+1)} = \begin{cases} \alpha\Lambda^a_n & \text{if } S_{n+1} = A \\ \beta\Lambda^a_n & \text{else } S_{n+1} = B \\ \gamma\Lambda^a_n & \text{else } S_{n+1} = C \\ \lambda_0 & \text{else } S_{n+1} = D \end{cases} \tag{7}$$

Given the initial condition, the general term is given by λ^i_n for the values of the stochastic random variable Λ^a_n. It can be written in the following manner:

$$\Lambda^a_n = \{\cup\lambda^i_n\} = \{\lambda^1_n, \lambda^2_n, \lambda^3_n, \ldots\}$$

$$\begin{aligned} &\lambda^i_n = \alpha^{x_i}\beta^{y_i}\gamma^{z_i}\lambda_0 \\ &\text{subject to} \\ &x_i + y_i + z_i \le n \\ &x_i, y_i, z_i \in \mathbb{N} \\ &\text{where} \\ &\mathbb{N} = \{0, 1, 2, 3, \ldots\} \end{aligned} \tag{8}$$

- In the presence of maintenance strategy, the SIS described as Markov process (refer Fig. 3) exhibits event level properties. They are mentioned at Section A.1. These properties helps to develop the analytical formulae for performance measure of the system.

4.4. Performance analysis of SIS considering the impact of test

Performance indicators are PFD(t) and PFD_{avg} for a SIS operating in low demand mode. It is important to understand that the system is defined by the combination of two stochastic random variables i.e. the state of the system and the transition rate responsible for aging Λ^a at any point of time. Λ^a is a discrete stochastic variable which has a constant value between consecutive tests. It changes instantaneously after the proof test due to impact of the test. With this, we develop analytical formulae for calculating performance indicators. In this regards, system proof test phases are assumed to be $[T_0, T_1]$, $[T_1, T_2]$, $[T_2, T_3]$...$[T_{n-1}, T_n]$, where n represents the number of proof testing phases.

- PFD(t)

$$\begin{aligned}
&\text{For } t\in (T_n, T_{n+1})\\
&\text{PFD}(t) = \text{Probability of failure on demand at time t}\\
&= \Pr[X_t = D]\\
&= \textstyle\sum_{\forall i} \sum_{s\in\{A,B,C,D\}} \Pr[X_t = D|S_n = s;\ \Lambda_n^a = \lambda_n^i]\Pr[S_n = s;\ \Lambda_n^a = \lambda_n^i]\\
&= \textstyle\sum_{\forall i} \{\Pr[X_t = D|S_n = A;\ \Lambda_n^a = \lambda_n^i]\Pr[S_n = A;\ \Lambda_n^a = \lambda_n^i]\\
&\quad + \Pr[X_t = D|S_n = B;\ \Lambda_n^a = \lambda_n^i]\Pr[S_n = B;\ \Lambda_n^a = \lambda_n^i]\\
&\quad + \Pr[X_t = D|S_n = C;\ \Lambda_n^a = \lambda_n^i]\Pr[S_n = C;\ \Lambda_n^a = \lambda_n^i]\\
&\quad + \Pr[X_t = D|S_n = D;\ \Lambda_n^a = \lambda_n^i]\Pr[S_n = D;\ \Lambda_n^a = \lambda_n^i]\}\\
&= \textstyle\sum_{\forall i}\left[\left\{[1,0,0,0]\exp((t-T_n)\mathcal{A}[\lambda_n^i])\begin{bmatrix}0\\0\\0\\1\end{bmatrix}\Pr[\Lambda_n^a=\lambda_n^i;\ S_n=A]\right\}\right.\\
&\quad+\left\{[0,1,0,0]\exp((t-T_n)\mathcal{A}[\lambda_n^i])\begin{bmatrix}0\\0\\0\\1\end{bmatrix}\Pr[\Lambda_n^a=\lambda_n^i;\ S_n=B]\right\}\\
&\quad+\left\{[0,0,1,0]\exp((t-T_n)\mathcal{A}[\lambda_n^i])\begin{bmatrix}0\\0\\0\\1\end{bmatrix}\Pr[\Lambda_n^a=\lambda_n^i;\ S_n=C]\right\}\\
&\quad+\left.\left\{[\mathbb{1}_{\text{AGAN}},0,\mathbb{1}_{\text{ABAO}},0]\exp((t-T_n)\mathcal{A}[\lambda_n^i])\begin{bmatrix}0\\0\\0\\1\end{bmatrix}\Pr[\Lambda_n^a=\lambda_n^i;\ S_n=D]\right\}\right]\\
&\text{Note:}\mathbb{1}_{\text{text}}\text{defined by equation 10}
\end{aligned} \tag{9}$$

$$\mathbb{1}_{\text{text}} = \begin{cases}1 & \text{if “text” is true}\\ 0 & \text{otherwise}\end{cases} \tag{10}$$

It is observed from Eq. (9), PFD(t) depends on the state of the system at the previous proof test (S), transition rate of aging at previous previous proof test(λ_n^i) and type of maintenance strategy chosen. A recursive relation is developed at Section A.2 for AGAN maintenance strategy. For ABAO maintenance strategy, the recursive relation to calculate the associated probabilities is given as following

- $\Pr[\Lambda_n^a = \lambda_n^i;\ S_n = A]$

$$\begin{aligned}
&= \textstyle\sum_{s\in\{A,B,C,D\}}\Pr[\Lambda_n^a=\lambda_n^i;\ S_n=A|\Lambda_{n-1}^a=\frac{1}{\alpha}\lambda_n^i;\ S_{n-1}=s]\Pr\\
&\quad[\Lambda_{n-1}^a=\tfrac{1}{\alpha}\lambda_n^i;\ S_{n-1}=s]\\
&= \Pr[(\Lambda_n^a=\lambda_n^i;\ S_n=A)\cap(\Lambda_{n-1}^a=\tfrac{1}{\alpha}\lambda_n^i;\ S_{n-1}=A)]\\
&\quad(\text{By }\ \textbf{Property 1}\ )\\
&= \Pr[S_n=A|S_{n-1}=A;\ \Lambda_{n-1}^a=\tfrac{1}{\alpha}\lambda_n^i]\Pr[\Lambda_{n-1}^a=\tfrac{1}{\alpha}\lambda_n^i;\ S_{n-1}=A]\\
&(\text{The condition}(x_i=n)\text{must be satisfied for}S_n=A)\\
&= [1,0,0,0]\exp(\tau_n\mathcal{A}[\tfrac{1}{\alpha}\lambda_n^i])\begin{bmatrix}1\\0\\0\\0\end{bmatrix}\Pr[\Lambda_{n-1}^a=\tfrac{1}{\alpha}\lambda_n^i;\ S_{n-1}=A]\mathbb{1}_{(x_i=n)}
\end{aligned} \tag{11}$$

- $\Pr[\Lambda_n^a = \lambda_n^i;\ S_n = B]$

$$\begin{aligned}
&= \textstyle\sum_{s\in\{A,B,C,D\}}\Pr[\Lambda_n^a=\lambda_n^i;\ S_n=B|\Lambda_{n-1}^a=\frac{1}{\beta}\lambda_n^i;\ S_{n-1}=s]\Pr[\Lambda_{n-1}^a=\frac{1}{\beta}\lambda_n^i;\ S_{n-1}=s]\\
&= \Pr[S_n=B|S_{n-1}=A;\ \Lambda_{n-1}^a=\tfrac{1}{\beta}\lambda_n^i]\Pr[\Lambda_{n-1}^a=\tfrac{1}{\beta}\lambda_n^i;\ S_{n-1}=A]\\
&\quad+\Pr[S_n=B|S_{n-1}=B;\ \Lambda_{n-1}^a=\tfrac{1}{\beta}\lambda_n^i]\Pr[\Lambda_{n-1}^a=\tfrac{1}{\beta}\lambda_n^i;\ S_{n-1}=B](\text{By }\ \textbf{Property 2}\ )\\
&= \left[\left\{[1,0,0,0]\exp(\tau_n\mathcal{A}[\tfrac{1}{\beta}\lambda_n^i])\begin{bmatrix}0\\1\\0\\0\end{bmatrix}\Pr[\Lambda_{n-1}^a=\tfrac{1}{\beta}\lambda_n^i;\ S_{n-1}=A]\right\}\right.\\
&\quad+\left.\left\{[0,1,0,0]\exp(\tau_n\mathcal{A}[\tfrac{1}{\beta}\lambda_n^i])\begin{bmatrix}0\\1\\0\\0\end{bmatrix}\Pr[\Lambda_{n-1}^a=\tfrac{1}{\beta}\lambda_n^i;\ S_{n-1}=B]\right\}\right]\mathbb{1}_{(x_i<n;z_i=0;t_i=0)}
\end{aligned} \tag{12}$$

- $\Pr[\Lambda_n^a = \lambda_n^i;\ S_n = C]$

$$\begin{aligned}
&= \textstyle\sum_{s\in\{A,B,C,D\}}\Pr[\Lambda_n^a=\lambda_n^i;\ S_n=C|\Lambda_{n-1}^a=\frac{1}{\gamma}\lambda_n^i;\ S_{n-1}=s]\Pr[\Lambda_{n-1}^a=\frac{1}{\gamma}\lambda_n^i;\ S_{n-1}=s]\\
&(\text{By }\ \textbf{Property 3}\ )\\
&= \Pr[S_n=C|S_{n-1}=A;\ \Lambda_{n-1}^a=\tfrac{1}{\gamma}\lambda_n^i]\Pr[\Lambda_{n-1}^a=\tfrac{1}{\gamma}\lambda_n^i;\ S_{n-1}=A]\\
&\quad+\Pr[S_n=C|S_{n-1}=B;\ \Lambda_{n-1}^a=\tfrac{1}{\gamma}\lambda_n^i]\Pr[\Lambda_{n-1}^a=\tfrac{1}{\gamma}\lambda_n^i;\ S_{n-1}=B]\\
&\quad+\Pr[S_n=C|S_{n-1}=C\cup D;\ \Lambda_{n-1}^a=\tfrac{1}{\gamma}\lambda_n^i]\Pr[\Lambda_{n-1}^a=\tfrac{1}{\gamma}\lambda_n^i;\ S_{n-1}=C\cup D]\\
&= \left[\left\{[1,0,0,0]\exp(\tau_n\mathcal{A}[\tfrac{1}{\gamma}\lambda_n^i])\begin{bmatrix}0\\0\\1\\0\end{bmatrix}\Pr[\Lambda_{n-1}^a=\tfrac{1}{\gamma}\lambda_n^i;\ S_{n-1}=A]\right\}\right.\\
&\quad+\left\{[0,1,0,0]\exp(\tau_n\mathcal{A}[\tfrac{1}{\gamma}\lambda_n^i])\begin{bmatrix}0\\0\\1\\0\end{bmatrix}\Pr[\Lambda_{n-1}^a=\tfrac{1}{\gamma}\lambda_n^i;\ S_{n-1}=B]\right\}\\
&\quad+\left.\left\{[0,0,1,0]\exp(\tau_n\mathcal{A}[\tfrac{1}{\gamma}\lambda_n^i])\begin{bmatrix}0\\0\\1\\0\end{bmatrix}\Pr[\Lambda_{n-1}^a=\tfrac{1}{\gamma}\lambda_n^i;\ S_{n-1}=C\cup D]\right\}\right]\mathbb{1}_{(z_i\neq0)}\\
&\text{Note: (i) Due to ABAO maintenance, system is reset to state C, when it is found in state D.}\\
&\text{(ii) system state C is given by vector [0,0,1,0]}
\end{aligned} \tag{13}$$

- $\Pr[\Lambda_n^a = \lambda_n^i; S_n = D]$

$$= \sum_{s\in\{A,B,C,D\}} \Pr[\Lambda_n^a = \lambda_n^i; S_n = D | \Lambda_{n-1}^a = \tfrac{1}{\omega}\lambda_n^i; S_{n-1} = s] \Pr[\Lambda_{n-1}^a = \tfrac{1}{\omega}\lambda_n^i; S_{n-1} = s]$$

(By **Property 4**)

$$= \Pr[S_n = D | S_{n-1} = A; \Lambda_{n-1}^a = \tfrac{1}{\omega}\lambda_n^i] \Pr[\Lambda_{n-1}^a = \tfrac{1}{\omega}\lambda_n^i; S_{n-1} = A]$$
$$+ \Pr[S_n = D | S_{n-1} = B; \Lambda_{n-1}^a = \tfrac{1}{\omega}\lambda_n^i] \Pr[\Lambda_{n-1}^a = \tfrac{1}{\omega}\lambda_n^i; S_{n-1} = B]$$
$$+ \Pr[S_n = D | S_{n-1} = C \cup D; \Lambda_{n-1}^a = \tfrac{1}{\omega}\lambda_n^i] \Pr[\Lambda_{n-1}^a = \tfrac{1}{\omega}\lambda_n^i; S_{n-1} = C \cup D]$$

$$= \Bigg[\left\{ [1,0,0,0] \exp(\tau_n \mathcal{A}[\tfrac{1}{\omega}\lambda_n^i]) \begin{bmatrix} 0\\0\\0\\1 \end{bmatrix} \Pr[\Lambda_{n-1}^a = \tfrac{1}{\gamma}\lambda_n^i; S_{n-1} = A] \right\}$$
$$+ \left\{ [0,1,0,0] \exp(\tau_n \mathcal{A}[\tfrac{1}{\omega}\lambda_n^i]) \begin{bmatrix} 0\\0\\0\\1 \end{bmatrix} \Pr[\Lambda_{n-1}^a = \tfrac{1}{\omega}\lambda_n^i; S_{n-1} = B] \right\}$$
$$+ \left\{ [0,0,1,0] \exp(\tau_n \mathcal{A}[\tfrac{1}{\omega}\lambda_n^i]) \begin{bmatrix} 0\\0\\0\\1 \end{bmatrix} \Pr[\Lambda_{n-1}^a = \tfrac{1}{\omega}\lambda_n^i; S_{n-1} = C \cup D] \right\} \Bigg] \mathbb{1}_{(z_i \neq 0)}$$

Note: (i) Due to ABAO maintenance, system is reset to state C, when it is found in state D.
(ii) system state C is given by vector [0,0,1,0] (14)

- PFD_{avg}
  With the help of Eqs. (4) and (9), PFD_{avg} for $(n+1)$th testing phase of the system is as follows

$$\text{PFD}_{\text{avg}} = \frac{1}{\tau_{n+1}} \int_{T_n}^{T_{n+1}} \Pr[X_t = D] dt \tag{15}$$

- where $\tau_{n+1} = T_{n+1} - T_n$; τ_{n+1}= time between nth and $(n+1)$th proof test
  Here, the set up is developed to handle generalized situation of non-periodic testing, however it is common in case of SIS to have periodic proof testing. By fixing all the $\tau_i = \tau$, the setup becomes valid for periodic testing also.

5. Numerical results and discussions

In this section numerical analysis is performed and some of the interesting plots are presented. These plots are based on the interaction between imperfect testing/perfect testing (impact free testing) and AGAN/ABAO maintenance strategies. Development of time dependent performance for (i) ABAO maintenance strategy in presence of imperfect testing, (ii) ABAO maintenance strategy in presence of perfect testing, (iii) AGAN maintenance strategy in presence of imperfect testing, (iv) AGAN maintenance strategy in presence of perfect testing are shown in red, black, green, blue lines respectively. The objective of these plots is to show that the framework is strong enough to handle real time situation such as age based deterioration process and flexible enough to encompass the user defined inputs/process specific inputs into modelling of the real time phenomena.

5.1. PFD(t) for unrealistic parameters

Fig. 5 shows development of PFD(t) for the system for first 10 testing phases. Following values of parameters are chosen $\lambda_u = 5\times10^{-6}$ h^{-1}, $\lambda_0 = 5\times10^{-6}$ h^{-1}, $\tau = 2000$ h, $\alpha = 1.2$, $\beta = 1.3$, $\gamma = 1.5$. Intuitively, ABAO maintenance strategy with imperfect testing should always have highest PFD(t). This can be verified by the plot in red. The main reasons for this behaviour are:

- stress, generated through the previous tests, accumulates even after the maintenance in ABAO maintenance strategy,
- every time the imperfect tests are performed, it puts additional stress on the system.

Conversely, AGAN maintenance strategy with perfect testing (without impact) always has the lowest value of PFD(t). This can be verified by the plot in blue. For AGAN maintenance strategy, green line is always higher than blue line implying that imperfect testing can not be compensated by using AGAN maintenance strategy for these specific parameters.

It can be argued that the chosen parameters may not belong to realistic parameter space in the domain of SIS. The chosen parameters exhibits important aspects of the model behaviour for law number of

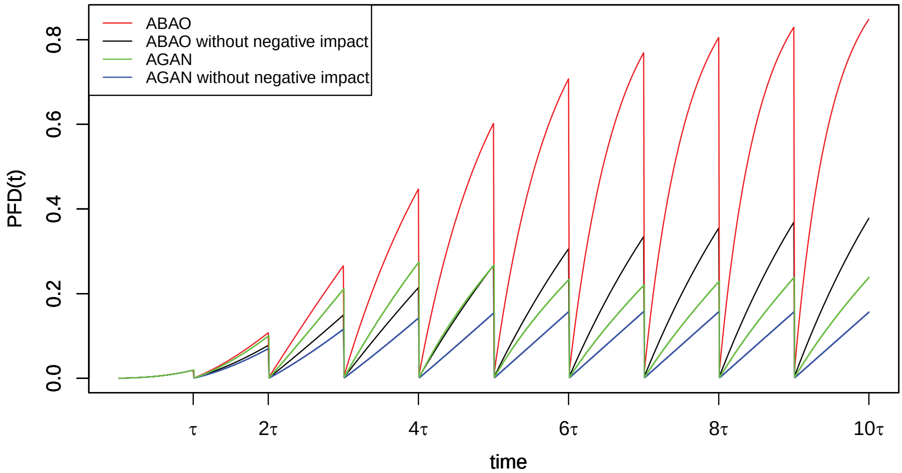


Fig. 5. Performance of SIS with time under various testing and maintenance strategies.

tests and are used to make validation. One of the key aspect exhibited by these plots is that the tests impact increases the aging rate of the system which in turns increases the probability of failure. The realistic parameters will have similar trends but it will take more number of tests to make it obviously visible.

5.2. PFD(t) for the case study

For this case study, values for the parameters are $\lambda_u = 5 \times 10^{-6}$ h^{-1}, $\lambda_0 = 5 \times 10^{-6}$ h^{-1}, $\tau = 1000$ h, $\alpha = 1.01$, $\beta = 1.03$, $\gamma = 1.05$. These parameters are chosen based on the expert knowledge from the industry. Figs. 6 and 7 show the development of PFD(t) over successive 10 testing phases for DHSV, in case of ABAO and AGAN maintenance strategy respectively. It is observed that PFD(t) is not affected by the imperfect testing during initial 10 test phases. One possible reason can be then smaller value tests imact (α, β, γ). With small impact of tests, the stress generated has not become dominant in initial 10 test phases. However, PFD(t) show similar trends as shown in Fig. 5 when test phases are significantly high.

5.3. PFD$_{Avg}$ for case study

It is relevant to study the PFD$_{avg}$, which is the performance measure used in relation to demonstrating SIL. The optimization problem as referred in Subsections 2.4 and 2.5 can be formulated as follows:

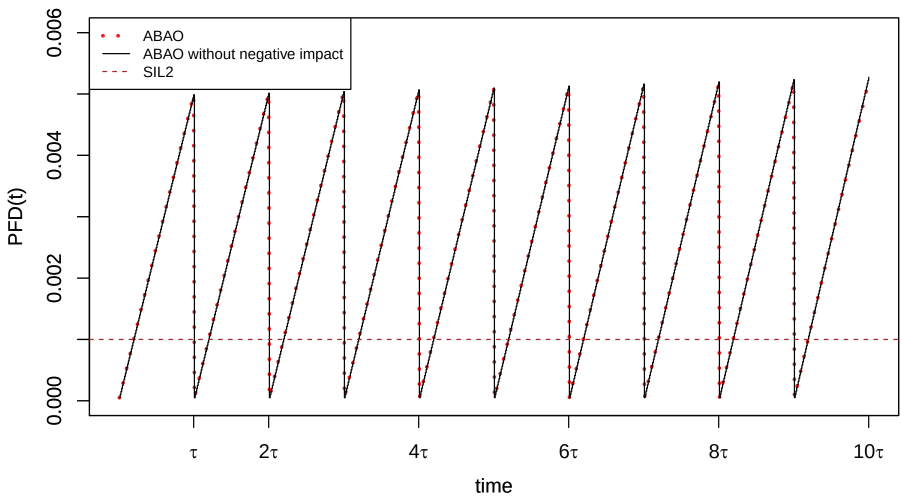


Fig. 6. Performance of DHSV with time under various testing for ABAO maintenance strategy.

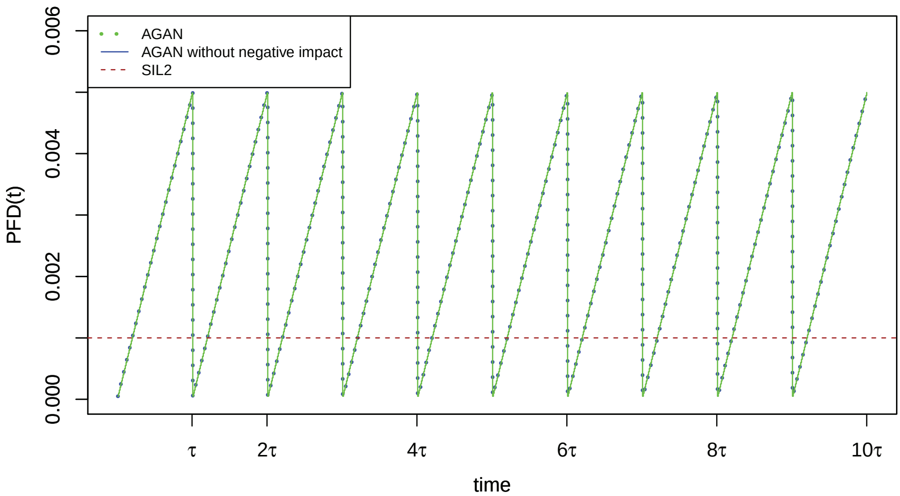


Fig. 7. Performance of DHSV with time under various testing for AGAN maintenance strategy.

$$
\begin{aligned}
N_{\text{opt}} &= \arg\min_{N} \text{PFD}_{\text{avg}}(N)\\
&= \arg\min_{N} \tfrac{1}{L}\textstyle\sum_{n=0}^{N-1}\int_{T_n}^{T_{n+1}} \Pr[X_t = D]dt\\
&\text{(for periodic testing } : L = N\tau,\ T_n = n\tau)\\
&= \arg\min_{N} \tfrac{1}{N\tau}\textstyle\sum_{n=0}^{N-1}\int_{T_n}^{T_{n+1}} \Pr[X_t = D]dt\\
&= \arg\min_{N} \tfrac{1}{N}\textstyle\sum_{n=0}^{N-1}\tfrac{1}{\tau}\int_{n\tau}^{(n+1)\tau} \Pr[X_t = D]dt
\end{aligned}
\tag{16}
$$

N_{opt}= Optimal number of tests which minimizes unavailability of system
N= Total number of proof tests experienced by the system
L= mission time for the system
$\tau = L/N$: Time between two consecutive proof tests
$\Pr[X_t = D]$ is calculated with Eq. (9), with the help of initial conditions
Initial conditions: $L = 5$ years, $\lambda_u = \lambda_0 = 5 \times 10^{-6}$ h^{-1}, $\alpha = 1.01$, $\beta = 1.03$, $\gamma = 1.05$, $\omega = 1$

The following algorithm is used to find solution of optimization problem (16) analytically.

AGAN maintenance policy is expensive as every time system fails, it needs to be replaced with the new system, hence it is not of interest for this case study. Fig. 9 shows variation of PFD_{avg} with number of tests for the above case study considering ABAO maintenance policy. It is observed that PFD_{avg} calculated analytically (using the Algorithm 1) fits well with simulated values.

PFD_{avg} estimated through simulations is taken from the existing literature [8]. Results from simulation are based on the flow chart mentioned at Fig. 8. The fact that the sojourn (waiting) time in each state is exponentially distributed is key concept used in the flow chart. It obtains a random realization down time of system based on the basics of next event simulation. PFD_{avg} is estimated by using the Eq. (17)

$$(\text{PFD}_{\text{avg}})_{\text{est}} = \frac{1}{N_{sim}}\sum_{i=0}^{N_{sim}} \frac{\text{down time}_i}{\text{Mission Time}} \tag{17}$$

where:

$(\text{PFD}_{\text{avg}})_{\text{est}}$= estimated probability of failure on demand on average
N_{sim}= Total number of simulation
down time_i= Down time of system for the ith simulation

Intuitively, PFD_{avg} is high when number of tests are less due to higher uncertainty about the status of system. As the number of proof tests increases, the PFD_{avg} decreases since we get more information (hence less uncertainty) about the status of the system. However, when the number of proof tests increase more than the optimum number = 80 (in this case), the cumulative impact of the test becomes dominant over the information gain. This produces increase in the PFD_{avg} for the system.

This analysis can help industry to ascertain maximum availability of DHSV by observing $\text{PFD}_{\text{avg}}^{\text{min}}$ (for the given initial conditions). Maintenance schedule can be adjusted to achieve maximum availability by observing τ from the analysis.

procedure $N_{\mathrm{OPT}}(\lambda_0, \lambda_u, L, \alpha, \beta, \gamma, \omega)$ ▷ Define initial parameters
initialize: $i \leftarrow 1$; $\mathrm{PFD}_{\mathrm{avg}}^{\min} \leftarrow 1$
Step 1: $\tau \leftarrow L/i$; $S_0 = A$
Step 2: calculate the possible values of $\Lambda_{(i)}^{a}$ by solving equation 6
Step 3: calculate $\Pr[\Lambda_{(i)}^{a}, S_{(i-1)}]$ by equation 11, 12, 13, 14
Step 4: calculate $\mathrm{PFD}_{\mathrm{avg}}(i)$ with the help of equation 15,9
while $\mathrm{PFD}_{\mathrm{avg}}(i) \leq \mathrm{PFD}_{\mathrm{avg}}^{\min}$ **do**
  1. $\mathrm{PFD}_{\mathrm{avg}}^{\min} \leftarrow \mathrm{PFD}_{\mathrm{avg}}(i)$
  2. $i \leftarrow (i+1)$
  3. repeat Step 1,2,3,4
return $\mathrm{PFD}_{\mathrm{avg}}^{\min}, i$ ▷ $N_{\mathrm{opt}} = i$

Algorithm 1. Optimum finder algorithm.

This case study illustrates that in case of harmful imperfect testing there exists a trade off between testing too much versus testing too less. The optimum frequency of testing will ensure maximum availability by balancing the information gain against the impact of testing.

6. Conclusion and future work

In this paper, we have investigated how the performance measure of SISs operating in low-demand mode ($\mathrm{PFD}_{\mathrm{avg}}$) is influenced by imperfect testing in the presence of different maintenance strategy. Two important aspects of imperfect testings are considered while developing the framework:

1. Imperfect testing reveals whether system fails or not. It doesn't reveal the true degraded state of the system.
2. Imperfect testing can generate stress on the mechanical parts of the system resulting in increase of failure probability of the system.

6.1. Conclusion

A new framework is proposed based on the multiphase Markov modelling approach to assess the performance of a SIS subject to imperfect testing. This allows us to model the condition-based impact of testing. It is also illustrated how different maintenance strategies affect the way of modelling the impact of tests. This framework extends the binary state models by considering the intermediate degraded states. The transition rates is modelled as a function of both the condition of the system and the number of proof tests experienced by the system. We can also model constructive control meaning each tests reduces the failure probability of system by selecting impact of tests in the following domain $\alpha \leq 1$, $\beta \leq 1$, $\gamma \leq 1$, $\omega \leq 1$.

However, it requires expert judgment to select initial parameters and the number of degraded states. Hidden degradation of the system introduces stochastic nature in the transition rates, which results in a tree structure of possible combinations for transition rates and system states. As the number of proof tests experienced increases, the number of possibilities increases with power law. This makes the framework computationally extensive and hence time consuming to calculate the analytical performance measure. Table 2 shows a comparison of execution time (in second) for both the methods.

6.2. Future work

Current construction of the framework is dedicated to a SIS operating in low-demand mode. It is natural to extend the framework for a SIS operating in high-demand mode to assess dangerous failure per hour (PFH) as an unconditional failure intensity, mainly by replacing tests by demands. However, some significant modifications may be required in the framework (on case to case basis) to accommodate the demands which are random in nature. The framework is developed considering only DU failures. It needs to be extended to include the effect of other failure modes. It is interesting to find the optimum frequency of testing which minimizes the maintenance cost, with the constrains of availability of system in terms of SIL limits, when more than one failure modes are present. To make the framework more flexible, the assumption of negligible repair time needs to be challenged. The set up is developed for single unit of SIS, however practically there are systems which have redundant final elements to ensure the safety of the process. Hence, it will be useful to develop the framework when the redundancies of SIS are considered. As of now, the

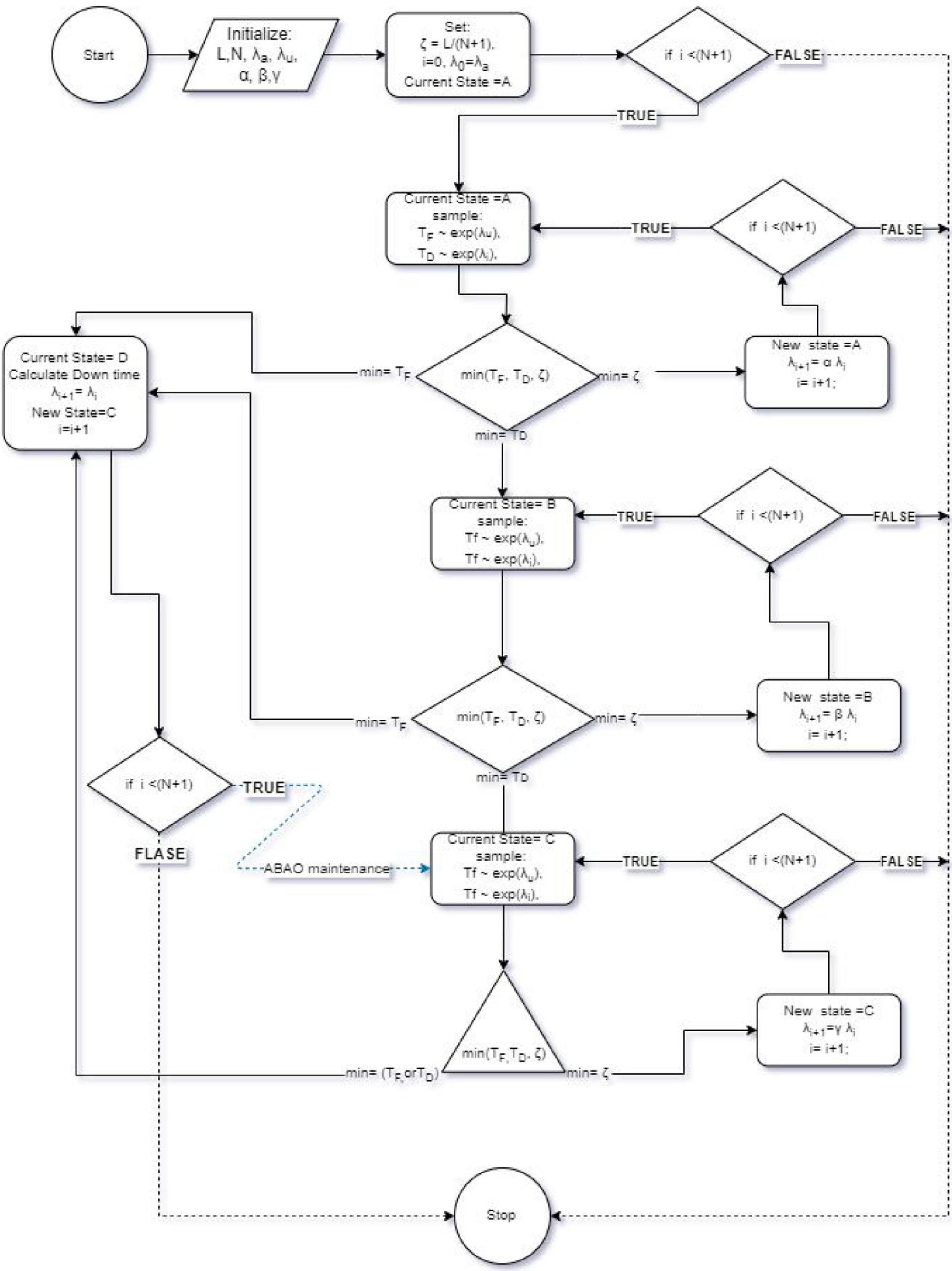


Fig. 8. Flow chart for Monte Carlo simulation to calculate the down time.

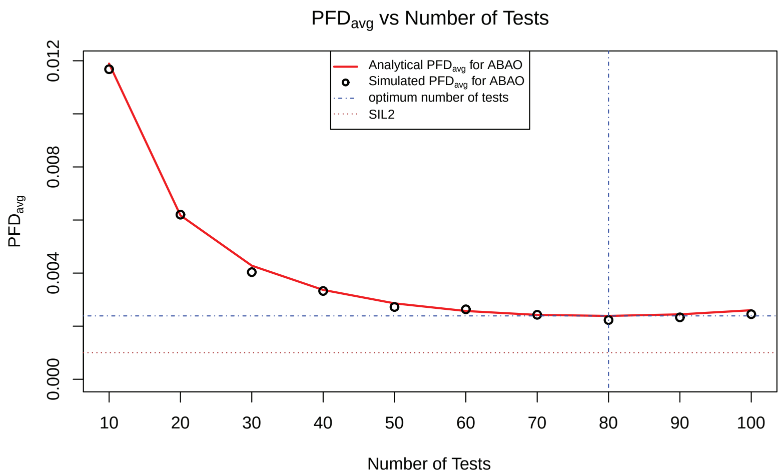


Fig. 9. Performance analysis of DHSV for ABAO maintenance strategy.

Table 2
Processing time (in s) for analytical formulation vs. Monte-carlo simulations.

Number of tests	Analytical formulation processing time	MC Simulation processing time (N_{sim} = 10000)
10	0.28	11.77
20	3.18	12.75
30	23.25	15.66
40	127.95	17.17
50	533.38	19.28
60	1899.16	21.59
70	6052.54	24.81
80	13659.95	26.21

framework only consider corrective maintenance, but since framework provides probabilities at any time in any state, the framework can be used for predictive maintenance also.

It is observed from Table 2 that framework proposed takes a lot of execution time for higher number of tests. It is important to find out efficient ways to implement the algorithm. The main issue with the algorithm is that the number of nodes in the Fig. 4 grow with the power law. For the tree with higher number of proof tests, many nodes have very-very low probabilities (close to 0). These nodes have very small contribution in the calculation of PFD(t). If a suitable threshold on the minimum probability of a node can be imposed, then the number of significant nodes will reduce a lot. This will reduce the execution time for this algorithm.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A

A1. Properties of the process

In ABAO

– **Property 1** :if E_1, E_2, E_3, E_4 are events defined as follows E_1: $[\Lambda_n^a = \alpha\lambda_a]$, E_2: $[S_n = A]$, E_3: $[\Lambda_{n-1}^a = \lambda_a]$, E_4: $[S_{n-1} = A]$, then $E_1 \cap E_2 \cap E_3 \cap E_4 \Leftrightarrow E_2 \cap E_3 \cap E_4$

– **Property 2** :if E_1, E_2, E_3, E_4 are events defined as follows E_1: $[\Lambda_n^a = \beta\lambda_a]$, E_2: $[S_n = B]$, E_3: $[\Lambda_{n-1}^a = \lambda_a]$, E_4: $[S_{n-1} = \{A \cup B\}]$, then $E_1 \cap E_2 \cap E_3 \cap E_4 \Leftrightarrow E_2 \cap E_3 \cap E_4$

– **Property 3** :if E_1, E_2, E_3, E_4 are events defined as follows E_1: $[\Lambda_n^a = \gamma\lambda_a]$, E_2: $[S_n = C]$, E_3: $[\Lambda_{n-1}^a = \lambda_a]$, E_4: $[S_{n-1} = \{A \cup B \cup C \cup D\}]$, then $E_1 \cap E_2 \cap E_3 \cap E_4 \Leftrightarrow E_2 \cap E_3 \cap E_4$.
Transitions from state D is possible due to ABAO maintenance policy.

– **Property 4** :if E_1, E_2, E_3, E_4 are events defined as follows E_1: $[\Lambda_n^a = \omega\lambda_a]$, E_2: $[S_n = D]$, E_3: $[\Lambda_{n-1}^a = \lambda_a]$, E_4: $[S_{n-1} = \{A \cup B \cup C \cup D\}]$, then $E_1 \cap E_2 \cap E_3 \cap E_4 \Leftrightarrow E_2 \cap E_3 \cap E_4$
Transitions from state D is possible due to ABAO maintenance policy.

- In AGAN

– **Property 5** :if E_1, E_2, E_3, E_4 are events defined as follows E_1: $[\Lambda_n^a = \lambda_n^i]$, E_2: $[S_n = A]$, E_3: $[\Lambda_{n-1}^a = \frac{1}{\alpha}\lambda_n^i]$, E_4: $[S_{n-1} = \{A \cup D\}]$, then $E_1 \cap E_2 \cap E_3 \cap E_4 \Leftrightarrow E_2 \cap E_3 \cap E_4$

– **Property 6** :if E_1, E_2, E_3, E_4 are events defined as follows E_1: $[\Lambda_n^a = \lambda_n^i]$, E_2: $[S_n = B]$, E_3: $[\Lambda_{n-1}^a = \frac{1}{\beta}\lambda_n^i]$, E_4: $[S_{n-1} = \{A \cup B \cup D\}]$, then $E_1 \cap E_2 \cap E_3 \cap E_4 \Leftrightarrow E_2 \cap E_3 \cap E_4$

– **Property 7** :if E_1, E_2, E_3, E_4 are events defined as follows E_1: $[\Lambda_n^a = \lambda_n^i]$, E_2: $[S_n = C]$, E_3: $[\Lambda_{n-1}^a = \frac{1}{\gamma}\lambda_n^i]$, E_4: $[S_{n-1} = \{A \cup B \cup C \cup D\}]$, then $E_1 \cap E_2 \cap E_3 \cap E_4 \Leftrightarrow E_2 \cap E_3 \cap E_4$.
– **Property 8** :if E_1, E_2, E_3, E_4 are events defined as follows E_1: $[\Lambda_n^a = \lambda_0]$, E_2: $[S_n = D]$, E_3: $[\Lambda_{n-1}^a = \lambda_{n-1}^i]$, E_4: $[S_{n-1} = \{A \cup B \cup C \cup D\}]$, then $E_1 \cap E_2 \cap E_3 \cap E_4 \Leftrightarrow E_2 \cap E_3 \cap E_4$

A2. Recursive formulae for probability calculation

For AGAN maintenance strategy

$$\begin{aligned}
&\Pr[\Lambda_n^a = \lambda_n^i;\, S_n = A]\\
&= \textstyle\sum_{s\in\{A,B,C,D\}} \Pr[\Lambda_n^a = \lambda_n^i;\, S_n = A | \Lambda_{n-1}^a = \frac{1}{\alpha}\lambda_n^i;\, S_{n-1} = s]\Pr[\Lambda_{n-1}^a = \frac{1}{\alpha}\lambda_n^i;\, S_{n-1} = s]\\
&= \Pr[\Lambda_n^a = \lambda_n^i;\, S_n = A \cap \Lambda_{n-1}^a = \tfrac{1}{\alpha}\lambda_n^i;\, S_{n-1} = A \cup D]\\
&(By\quad \textbf{Property 5}\ )\\
&= \Pr[S_n = A | S_{n-1} = A \cup D;\, \Lambda_{n-1}^a = \tfrac{1}{\alpha}\lambda_n^i]\Pr[\Lambda_{n-1}^a = \tfrac{1}{\alpha}\lambda_n^i;\, S_{n-1} = A \cup D]\\
&= [1,0,0,0]\exp(\tau_n \mathcal{A}[\tfrac{1}{\alpha}\lambda_n^i])\begin{bmatrix}1\\0\\0\\0\end{bmatrix}\Pr[\Lambda_{n-1}^a = \tfrac{1}{\alpha}\lambda_n^i;\, S_{n-1} = A]\mathbb{1}_{(y_i=0\cap z_i=0)}
\end{aligned} \tag{18}$$

$$\begin{aligned}
&\Pr[\Lambda_n^a = \lambda_n^i;\, S_n = B]\\
&= \textstyle\sum_{s\in\{A,B,C,D\}} \Pr[\Lambda_n^a = \lambda_n^i;\, S_n = B | \Lambda_{n-1}^a = \frac{1}{\beta}\lambda_n^i;\, S_{n-1} = s]\Pr[\Lambda_{n-1}^a = \frac{1}{\beta}\lambda_n^i;\, S_{n-1} = s]\\
&= \Pr[\Lambda_n^a = \lambda_n^i;\, S_n = B \cap \Lambda_{n-1}^a = \tfrac{1}{\alpha}\lambda_n^i;\, S_{n-1} = A \cup B \cup D]\\
&(\text{By}\quad \textbf{Property 6}\ )\\
&= \Pr[S_n = B | S_{n-1} = A \cup D;\, \Lambda_{n-1}^a = \tfrac{1}{\beta}\lambda_n^i]\Pr[\Lambda_{n-1}^a = \tfrac{1}{\beta}\lambda_n^i;\, S_{n-1} = A \cup D]\\
&\quad + \Pr[S_n = B | S_{n-1} = B;\, \Lambda_{n-1}^a = \tfrac{1}{\beta}\lambda_n^i]\Pr[\Lambda_{n-1}^a = \tfrac{1}{\beta}\lambda_n^i;\, S_{n-1} = B]\\
&= \left[\left\{[1,0,0,0]\exp(\tau_n \mathcal{A}[\tfrac{1}{\beta}\lambda_n^i])\begin{bmatrix}0\\1\\0\\0\end{bmatrix}\Pr[\Lambda_{n-1}^a = \tfrac{1}{\beta}\lambda_n^i;\, S_{n-1} = A \cup D]\right\}\right.\\
&\quad \left. + \left\{[0,1,0,0]\exp(\tau_n \mathcal{A}[\tfrac{1}{\beta}\lambda_n^i])\begin{bmatrix}0\\1\\0\\0\end{bmatrix}\Pr[\Lambda_{n-1}^a = \tfrac{1}{\beta}\lambda_n^i;\, S_{n-1} = B]\right\}\right]\mathbb{1}_{(y_i\geq 1\cap z_i=0)}
\end{aligned} \tag{19}$$

$$\begin{aligned}
&\Pr[\Lambda_n^a = \lambda_n^i;\, S_n = C]\\
&= \textstyle\sum_{s\in\{A,B,C,D\}} \Pr[\Lambda_n^a = \lambda_n^i;\, S_n = C | \Lambda_{n-1}^a = \frac{1}{\gamma}\lambda_n^i;\, S_{n-1} = s]\Pr[\Lambda_{n-1}^a = \frac{1}{\gamma}\lambda_n^i;\, S_{n-1} = s]\\
&(\text{By}\quad \textbf{Property 7}\ )\\
&= \Pr[S_n = C | S_{n-1} = A \cup D;\, \Lambda_{n-1}^a = \tfrac{1}{\gamma}\lambda_n^i]\Pr[\Lambda_{n-1}^a = \tfrac{1}{\gamma}\lambda_n^i;\, S_{n-1} = A \cup D]\\
&\quad + \Pr[S_n = C | S_{n-1} = B;\, \Lambda_{n-1}^a = \tfrac{1}{\gamma}\lambda_n^i]\Pr[\Lambda_{n-1}^a = \tfrac{1}{\gamma}\lambda_n^i;\, S_{n-1} = B]\\
&\quad + \Pr[S_n = C | S_{n-1} = C;\, \Lambda_{n-1}^a = \tfrac{1}{\gamma}\lambda_n^i]\Pr[\Lambda_{n-1}^a = \tfrac{1}{\gamma}\lambda_n^i;\, S_{n-1} = C]\\
&= \left[\left\{[1,0,0,0]\exp(\tau_n \mathcal{A}[\tfrac{1}{\gamma}\lambda_n^i])\begin{bmatrix}0\\0\\1\\0\end{bmatrix}\Pr[\Lambda_{n-1}^a = \tfrac{1}{\gamma}\lambda_n^i;\, S_{n-1} = A \cup D]\right\}\right.\\
&\quad + \left\{[0,1,0,0]\exp(\tau_n \mathcal{A}[\tfrac{1}{\gamma}\lambda_n^i])\begin{bmatrix}0\\0\\1\\0\end{bmatrix}\Pr[\Lambda_{n-1}^a = \tfrac{1}{\gamma}\lambda_n^i;\, S_{n-1} = B]\right\}\\
&\quad \left. + \left\{[0,0,1,0]\exp(\tau_n \mathcal{A}[\tfrac{1}{\gamma}\lambda_n^i])\begin{bmatrix}0\\0\\1\\0\end{bmatrix}(\Pr[\Lambda_{n-1}^a = \tfrac{1}{\gamma}\lambda_n^i;\, S_{n-1} = C])\right\}\right]\mathbb{1}_{(z_i\neq 0)}
\end{aligned} \tag{20}$$

$\Pr[\Lambda_n^a = \lambda_n^i;\, S_n = D]$

$$= \sum_{s\in\{A,B,C,D\}} \Pr[\Lambda_n^a = \lambda_0;\ S_n = D|\Lambda_{n-1}^a = \lambda_{n-1}^i;\ S_{n-1} = s]\Pr[\Lambda_{n-1}^a = \lambda_{n-1}^i;\ S_{n-1} = s]$$

(By **Property 8**)

$$\begin{aligned}
&= \Pr[S_n = D|S_{n-1} = A\cup D;\ \Lambda_{n-1}^a = \lambda_{n-1}^i]\Pr[\Lambda_{n-1}^a = \lambda_{n-1}^i;\ S_{n-1} = A\cup D]\\
&\quad + \Pr[S_n = D|S_{n-1} = B;\ \Lambda_{n-1}^a = \lambda_{n-1}^i]\Pr[\Lambda_{n-1}^a = \lambda_{n-1}^i;\ S_{n-1} = B]\\
&\quad + \Pr[S_n = D|S_{n-1} = C;\ \Lambda_{n-1}^a = \lambda_{n-1}^i]\Pr[\Lambda_{n-1}^a = \lambda_{n-1}^i;\ S_{n-1} = C]\\
&= \Bigg[\left\{[1,0,0,0]\exp(\tau_n\mathcal{A}[\lambda_{n-1}^i])\begin{bmatrix}0\\0\\0\\1\end{bmatrix}\Pr[\Lambda_{n-1}^a = \lambda_{n-1}^i;\ S_{n-1} = A\cup D]\right\}\\
&\quad + \left\{[0,1,0,0]\exp(\tau_n\mathcal{A}[\lambda_{n-1}^i])\begin{bmatrix}0\\0\\0\\1\end{bmatrix}\Pr[\Lambda_{n-1}^a = \lambda_{n-1}^i;\ S_{n-1} = B]\right\}\\
&\quad + \left\{[0,0,1,0]\exp(\tau_n\mathcal{A}[\lambda_{n-1}^i])\begin{bmatrix}0\\0\\0\\1\end{bmatrix}\Pr[\Lambda_{n-1}^a = \lambda_{n-1}^i;\ S_{n-1} = C]\right\}\Bigg]\mathbb{1}_{(x_i+y_i+z_i<n)}
\end{aligned} \tag{21}$$

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Chapter II.3

Third Paper

This paper is awaiting publication and is not included in NTNU Open

Chapter II.4

Fourth Paper

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Study of testing and maintenance strategies for redundant final elements in SIS with imperfect detection of degraded state

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ABSTRACT

Safety-instrumented systems (SISs) have been widely installed to lower risks of equipment/ process by performing the designed safety functions in cases of demands. Final elements remain dormant mostly in a low demand mode but become vulnerable due to degradation along with time. Tests and maintenances are key activities to prevent the SIS from any failures, including those thank to degradation, to activate upon demands. This paper models the degradation of SIS final elements by considering an intermediate degraded state between the working- and failed states. Sometimes, the actual system states are not distinguished perfectly during proof tests. Such imperfectness in state revealing, consequently, weakens the real performance of follow-up maintenances. The effects of imperfect degradation state revealing are quantified, together with three testing and maintenance strategies for 1-out-of-2 configured SISs. Time-dependent PFD of the system and cumulative life-cycle cost are then estimated in a finite service time. Numerical examples under proposed strategies are presented to provide clues in selection of optimal testing and maintenance strategies for 1oo2 final element in SISs.

1. Introduction

Safety-instrumented systems (SISs) are widely applied in different industries to detect the onset of hazardous event and/or to mitigate their consequences, such as emergency shutdown (ESD) systems on an oil & gas production platform, high pressure protection systems (HIPPSs) in the process industry. Normally, a SIS consists of sensor(s) (e.g. pressure transmitters), logic solver(s) and final element(s) (e.g. shutdown valves) [1,2].

Both ESD and HIPPS are typical SISs operating in a low demand mode, where the activation frequency is less than once per year in general. Some failure modes of final elements will stay hidden until a proof test is executed or an undesired event occurs on the equipment under control (EUC) by the SIS [2]. These hidden failures are called dangerous undetected (DU) failures if they can lead to dangerous events with severe consequences. Redundant structures are often used in SISs to improve the system availability and so to enhance safety. IEC 61508 [3] recommends the average probability of failure on demand (PFD_{avg}) as a measure in the performance evaluation of SISs in the low demand mode.

Some widely used methods have been developed for the calculation of PFD_{avg}, including simplified formulas [1,2,4], fault tree analysis [5–8], Markov methods [9–13], Bayesian methods [14–16], Petri Nets [17–19] and AltaRica modeling [20]. The common for most of these methods is assumed that all elements in a SIS are as-good-as-new after a repair in case a DU is revealed in a proof test. Such an assumption is valid for electronic components with exponentially distributed lifetime, but its validity for mechanical component is in question.

There exists literature in abundance for reliability assessment of units like safety valves under various maintenance strategies such as as-bad-as-old(ABAO) under corrective maintenance or imperfect maintenance under preventive maintenance. The important assumption with these methods is binary state model [21–24].

The final execution elements of SISs, mainly consisted of mechanical components, may not always fail at a constant failure rate. They are rather vulnerable to creeping or other degradation processes [25]. In general, the reliability of a mechanical system decreases as the degradation processes develop [26], which contribute to a time-dependent failure rate. Thus, several dynamic reliability methods with advantage of represent time- and age-dependent performance have been applied to address degradation mechanisms of such mechanical components, e.g. stochastic process [27–29], multi-phase Markov process [9,11,30–32].

For SIS final elements with degradation, Mechri et al. [9] have considered the imprecision on the failure rates of components in

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performance evaluation of the SIS in low demand using fuzzy multi-phase Markov process. Innal et al. [31] have generalized PFD_{avg} formulas by including partial and full periodic tests. Wu et al. [11] have conducted the time dependent unavailability analysis of blind shear ram preventers (BSRPs) by incorporating testing strategies into multi-phase Markov process. Three states for 1oo1 configuration have been considered, including functioning, failed and waiting for repair. Zhang et al. [29] have performed the PFD_{avg} of a 1oo1 configuration subjected to continuous aging degradation process. Different follow-ups based on the system state in proof test are considered. Srivastav et al. [32] have considered the negative effects of proof tests on SIS by adding discrete degraded states between working and failed state.

On the other hand, with the development of sensor technologies, more data about operation conditions and system status can be collected. Numerous parameters such as the lubricant ingredients, vibration signal, thermography picture, corrosion extent and so on can be measured and analyzed for failure prediction and diagnosis [33]. For example, a series of studies have been conducted on choke valve erosion based on the flow coefficient obtained from process parameters [34–37]. The deviation between actual value and reference value is regarded as one useful indicator for choke valve erosion. When the deviation is beyond the acceptable level, the valve is regarded to be failed.

Health indicators are helpful to implement condition-based maintenance on SISs, namely corresponding maintenance actions are conducted based on the observed states. After a proof test on a SIS final element, different following-ups are possible based on the system state of working, degraded or failed. The presence of the degraded state is beyond the scope of binary-state system analysis, and several studies have been conducted on such multi-state systems reliability analysis and maintenance optimization [38–43]. However, the existing literature relies on an assumption that system degradation state revealing is perfect [39,44,45]. This is not always right for SISs because the degradation level of a SIS is not observed directly in many cases but is determined by the difference between a reference value and an estimate value of status, while the estimated value is calculated from some relevant process parameters [34,37]. When the collected data in a proof test, e.g. by sensors, process conditions and media in valve, is imprecise or different from working conditions, these inaccurate measurements will be passed into the physical condition estimation for valves. These unintended errors can be amplified or diminished in calculation of actual status of valves. Errors can also come from inaccurate setting of the threshold between working and degradation [29].

Secondly, existing studies on testing strategies for redundant SISs mainly focus on addressing uncertainty [46] and common cause failures (CCFs) [2,5,47], neglecting degrading units and preventive maintenance policies. In this context of imperfect degradation revealing, it is worth studying to analyze how the degradation of a single unit affects the whole redundant structure under different testing strategies. In addition, the life-cycle cost of an SIS in the designed service time (e.g. 20 years) is more of interest, compared to existing studies focusing on the average long-run cost rate [48,49].

As a response, this paper is aiming to take potential imperfect state revealing into account of state-based SIS assessment, to make a comparison among different testing and maintenance strategies. The specific objectives include:

- Modeling and quantifying the imperfectness of state revealing in proof tests and their effects on the performance of redundant final elements in SISs.
- Evaluating condition-based maintenance strategies in the contexts where different testing approaches are used.
- Incorporating and balancing system availability and life cycle costs in seeking testing and maintenance strategies and providing guidance to operational decision-makers of SISs.

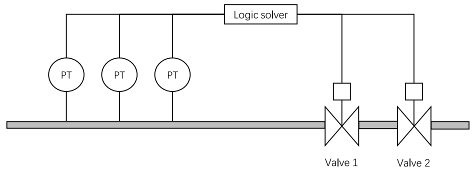


Fig. 1. Example of a HIPPS.

The remainder of this paper is organized as follows: Section 2 illustrates the characteristics of final elements in SIS, as well as the testing and maintenance strategies; Section 3 investigates the calculation of system PFD_{avg} and cumulative life-cycle cost given the certain assumptions; Section 4 conducts a numerical example to present the system performance and cumulative cost with state revealing coverage under different test and maintenance strategies and discusses the pros and cons of different strategies; Concluding remarks are given in Section 5.

2. System description

2.1. Structure and operations of a SIS

As mentioned, a typical SIS consists of sensor(s), logic solver(s) and final element(s). Without losing generality, a high pressure protection system (HIPPS) in oil & gas industry is used to study SIS operations and tests here, whose architecture is shown in Fig. 1. Two redundant shutdown valves (Valve 1 and 2), serving as the final elements in HIPPS, are installed on the same pipeline to stop the flow and relieve pressure in case the downstream pressure is too high. When one of two valves cannot be activated, the process, namely EUC, is still safe if the other valve works. Such kind of configuration is called as 1-out-of -2 (1oo2), which can improve system availability and so to enhance safety to some extent.

The performance measure of valves in HIPPS is expressed by an average probability that the item will not be able to perform its required safety function if the demand occurs, and it is denoted as Probability of Failure on Demand (PFD_{avg}) [2]. IEC 61508[3] specifies the requirement into four safety integrity levels (SILs), with SIL1 being the least reliable and SIL4 being the most reliable. To fulfill the requirements of a SIL, the SIS in low demand mode must have a PFD_{avg} in the corresponding interval.

Given the inevitable degradation mechanisms in valves, the actual performance of a mechanically final element always degrades along with time. Through the life-cycle of valves, at least three distinguishable states can be defined which are linked with the physical condition of system. (See Table 1.)

2.2. Proof test and maintenance strategies

Proof tests address the necessary functional safety requirements of SIS, including functions such as response time and leakage class of safety valves, with reflecting real conditions as accurately as possible. During a test it is possible to check the actual performance of valves, e.g. fully open/closed, the time to perform safety function and leakage rate in closed position. These kind of information can be employed as indirect indicators which provide us an opportunity to prognostics the valve condition [50].

In the designed phase of SISs, the final elements, such as valves, are allocated a target value with acceptable deviation to meet the specified performance requirement, e.g. leakage rate and closing time. When the leakage rate or closing time exceeds the acceptable deviation, as a safety barrier, the valve will not meet the performance requirements for risk mitigating of EUC. The corresponding failure modes are called 'leakage (through the valve) in a closed position (LCP)' and 'closing too

Table 1
System state definition.

State	Status	Notation	State description
1	Working	W	System is working as specified
2	Degraded	D	System has a degraded performance but still functioning
3	Failed	F	System has a fault and fails to function

slowly', respectively. In most cases, it is not possible to observe such kind of failure without activating the valve, so these failures are DU failures. When DU failure presents, the SIS will be into a fault state as losing the corresponding pre-designed safety function.

LCP failure mode is mainly caused by erosion on the gate or the seat [2]. Referring to the existing studies of erosion in valves, a series of work have been conducted on selection of performance indicator. A potential erosion indicator is the difference value between the calculated result from collected information and a reference value from vendor data sheet. Complied to the performance requirement of SIS, when the difference is too big, the valve is said to be failed (in a fault state).

Considering state classification and the updated status indicator after a proof test, the condition-based maintenance can be adopted to improve system performance: (1) no action if the difference value is quite small, it means the system is the working condition; (2) preventive maintenance (PM) is executed if the difference value is quite big but still within the required range, in this case, the performance is not satisfying even though is still kind of working; (3) corrective maintenance (CM) if the difference value exceeds the required range, namely, a DU is found (with respect to this particular function).

3. SIS modeling and performance analysis

This part firstly presents the relevant modeling assumptions. Markov chain is one approach quoted in IEC 61511 [51] for reliability assessment of SIS. When using Markov chains, it is possible to make a dynamic analysis of the system in each test interval. The state of the tested units are observed and known through periodic proof test, which implies the inapplicability of the classical Markov chain. Thus, the probability that the SIS sojourns in a certain state is known or partially known in each proof test. The proof test and its follow-up maintenance reallocate the distribution of system states from the modeling perspective, and create a new phase in the Markov chain for latter phase. Thus, a multi-phase Markov process is used to model the performance of SIS.

3.1. Assumptions

For unavailability and maintenance analysis, the following assumptions are needed as most of the existing literature:

- DU failures of units follow the exponential distribution;
- All units are repairable and repair time is negligible;
- Proof tests are executed periodically to check system performance and independently for units.
- Both preventive and corrective maintenance once conducted are perfect to make the objective as-good-as-new (AGAN).
- Common cause failures (CCFs) are excluded, with the purpose to illustrate the effects of α_i in a single unit on the redundant structure apparently.

In this study, proof tests are imperfect in revealing degraded states with a revealing probability or testing coverage α_i for unit i. When identifying failed states, tests are perfect.

3.2. Performance analysis

Considering the discrete states assumption, a system can be in $r+1$ distinct states with a state space $\{1,\ldots,r+1\}$. We define the stochastic process $\{X(t), t \geqslant 0\}$ to represent the system state at time t. Vector $\mathbf{P}(t) = [\mathbf{P}_1(t), \mathbf{P}_2(t), \ldots, \mathbf{P}_{r+1}(t)]$ stands for the probabilities of the process in each state at time t. The system is always in one of states, so that the sum of state probabilities should be equal to 1 at any time. A generic mathematical notion of a Markov model is

$$\frac{d\mathbf{P}(t)}{dt} = \mathbf{QP}(t) \tag{1}$$

where $\mathbf{Q}$ is the Markov transition matrix containing all transition rates (assumed to be constant in each phase). Considering the periodic proof tests, the overall life cycle of system could be modeled by multi-phase Markov process, the i testing intervals are denoted as $[0, \mathrm{T}_1], [\mathrm{T}_1, \mathrm{T}_2], \ldots, [\mathrm{T}_{(i-1)}, \mathrm{T}_i]$, accompanying with Markov transition matrix $\mathbf{Q}_i$ and $\mathbf{M}_i$ to represent the transition rates and probability matrix of different states after a testing/repair action in the ith test phase, respectively. To accompany the set of equations, a set of initial state probabilities $\mathbf{P}(t=0) = \mathbf{P}_0$ is also required. Then by solving Chapman–Kolmogorov's equation, we can calculate system state probabilities at time t in first test phase $[0, \mathrm{T}_1]$.

$$\mathbf{P}(t) = \mathbf{P}_0 \cdot exp(\mathbf{Q}_1 \cdot t) \tag{2}$$

If the time immediately before a test (pretest) at time T_1 is indicated as T_1^- and immediately after a test (post-test) as T_1^+, the effect of test and maintenance actions at time T_1 can be described as

$$\mathbf{P}(\mathrm{T}_1^+) = \mathbf{P}(\mathrm{T}_1^-) \cdot \mathbf{M}_1 \tag{3}$$

where $\mathbf{M}_1$ represents the probability matrix of different states after a testing and repair action. $\mathbf{P}(\mathrm{T}_1^+)$ stands for the state probabilities at time T_1. So, the system state probabilities at time t in second phase can be calculated as:

$$\begin{aligned}\mathbf{P}(t) &= \mathbf{P}(\mathrm{T}_1^+) \cdot \exp(\mathbf{Q}_2 \cdot (\mathrm{t} - \mathrm{T}_1)) \\ &= \mathbf{P}(\mathrm{T}_1^-) \cdot \mathbf{M}_1 \cdot \exp(\mathbf{Q}_2 \cdot (\mathrm{t} - \mathrm{T}_1)) \\ &= \mathbf{P}_0 \cdot exp(\mathbf{Q}_1 \cdot \mathrm{T}_1) \cdot \mathbf{M}_1 \cdot \exp(\mathbf{Q}_2 \cdot (\mathrm{t} - \mathrm{T}_1))\end{aligned} \tag{4}$$

Therefore, we can have $\mathbf{P}(\mathrm{T}_2^-)$

$$\begin{aligned}\mathbf{P}(\mathrm{T}_2^-) &= \mathbf{P}(\mathrm{T}_1^+) \cdot \exp(\mathbf{Q}_2 \cdot (\mathrm{T}_2 - \mathrm{T}_1)) \\ &= \mathbf{P}_0 \cdot exp(\mathbf{Q}_1 \cdot \mathrm{T}_1) \cdot \mathbf{M}_1 \cdot \exp(\mathbf{Q}_2 \cdot (\mathrm{T}_2 - \mathrm{T}_1))\end{aligned} \tag{5}$$

Similarly, $\mathbf{P}(\mathrm{T}_{(i-1)}^-)$ could be calculated as

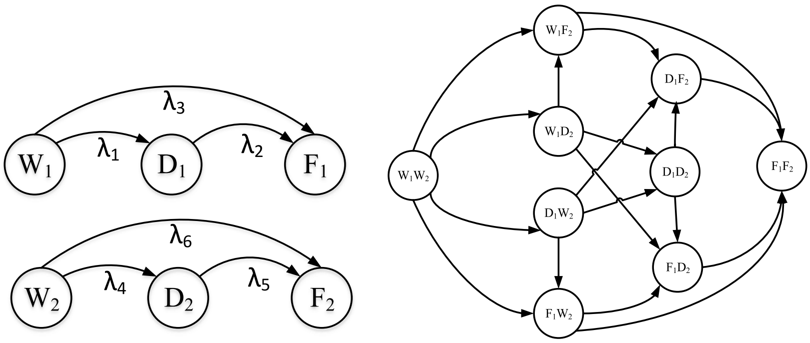
$$\begin{aligned}\mathbf{P}(\mathrm{T}_{(i-1)}^-) &= \mathbf{P}(\mathrm{T}_{i-2}^+) \cdot \exp(\mathbf{Q}_{i-1} \cdot (\mathrm{T}_{i-2} - \mathrm{T}_{i-1})) \\ &= \mathbf{P}_0 \prod_{n=1}^{i-2} (exp(\mathbf{Q}_n \cdot (\mathrm{T}_n - \mathrm{T}_{n-1})) \cdot \mathbf{M}_n) \cdot \exp(\mathbf{Q}_i \cdot (\mathrm{T}_{i-1} - \mathrm{T}_{i-2}))\end{aligned} \tag{6}$$

Then if t is in the i testing phase $[\mathrm{T}_{(i-1)}, \mathrm{T}_i]$, we can have $\mathbf{P}(t)$

$$\begin{aligned}\mathbf{P}(t) &= \mathbf{P}(\mathrm{T}_{i-1}^-) \cdot \mathbf{M}_{i-1} \cdot \exp(\mathbf{Q}_i \cdot (\mathrm{t} - \mathrm{T}_{i-1})) \\ &= \mathbf{P}_0 \prod_{n=1}^{i-1} (exp(\mathbf{Q}_n \cdot (\mathrm{T}_n - \mathrm{T}_{n-1})) \cdot \mathbf{M}_n) \cdot \exp(\mathbf{Q}_i \cdot (\mathrm{t} - \mathrm{T}_{i-1}))\end{aligned} \tag{7}$$

For a 1oo1 configuration, the system will not be functional in the failed state, and the instantaneous PFD(t) in each testing phase is given by

$$\mathrm{PFD}(t) = \Pr(X(t) = F) = \mathbf{P}(t) \cdot [0, 0, 1]^{\mathbf{T}} \tag{8}$$



(a) (b)

Fig. 2. State transition diagrams for (a) 1oo1 configuration and (b) 1oo2 configuration.

Meanwhile, for a 1oo2 configuration, the system will not be functional when both of two units are in the failed states, then the instantaneous PFD(t) is given by

$$\mathrm{PFD}(t) = \Pr(X(t) = \mathrm{FF}) = \mathbf{P}(t) \cdot [0,0,0,0,0,0,0,0,1]^{\mathrm{T}} \tag{9}$$

Then performance measure of system, $\mathrm{PFD}^{i}_{\mathrm{avg}}$, in ith testing phase is given by

$$\mathrm{PFD}_{\mathrm{avg}}{}^{i} = \frac{1}{T_i - T_{i-1}} \int_{T_{i-1}}^{T_i} \mathrm{PFD}(t)dt \tag{10}$$

3.3. Modeling for proof tests and maintenances

In this paper, each unit in a 1oo2 configuration is assumed to have three states, including working, degraded and failed. The transition diagram for 1oo1 and 1oo2 configuration is shown in Fig. 2, the corresponding transition matrix is $\mathbf{Q}$ as shown in Appendix B.

As assumptions in Section 3.1, proof tests are perfect in revealing failed states, but imperfect in revealing degraded states. To quantify such imperfectness, a coverage indicator α is defined as the conditional probability that a degraded state will be detected by the proof test, given that degradation has occurred when initiating the proof test.

$$\alpha = \Pr(\text{Degradation is detected in a proof test } | \text{Degradation has occurred}) \tag{11}$$

The parameter α does not affect the transition matrix and diagram as the unrevealed degraded state is physically in degraded. Since the maintenance actions are based on the detected state of system, the imperfectness in revealing of degraded state should be taken into matrix which upon testing and maintenance actions.

3.3.1. Testing strategies

Two different testing strategies for a redundant structure of SIS final element will be investigated here, include:

- Simultaneous testing: Two units are tested at (almost) same time with a fixed interval τ. The ith proof test is executed at time $t_i = i\tau, (i = 1, 2, \ldots)$, and independently for two units.
- Staggered testing: Two units are tested at different times with a constant test interval. Here, we assume that unit 1 is tested at time $t_{2j-1} = (2j-1)\times\tau/2$ and unit 2 at time $t_{2j} = (2j)\times\tau/2$, $(j = 1, 2, \ldots)$, since $\tau/2$ has been identified as the optimal interval [52].

3.4. Follow-up maintenance strategies

Considering the aforementioned testing strategies, several optional maintenance strategies are proposed for 1oo2 configuration:

- Strategy I: Under the simultaneous testing policy, the tests for two units are two separate processes. A PM or CM action will be executed if any unit is found in the degraded or failed state in test. Both PM and CM actions are perfect and make units as-good-as-new.
- Strategy II: Under the staggered testing policy, repair actions are only executed on the tested unit. A PM or CM will be executed when the tested unit is in degraded or failed state, respectively. Since no information of another unit is collected during the testing, then no repair is executed on the untested unit.
- Strategy III: Opportunistic maintenance with perfect action under the staggered testing policy. The maintenance policy is described as follows: 1. PM will be executed for tested degraded unit and perform CM if the tested unit fails. 2. At the moment of CM, this opportunity is taken to perform a replacement action on the other unit no matter the actual state is.

3.5. Life-cycle cost

Life-cycle cost for final elements in SISs mainly consists of purchase, installation, maintenance and disposal, while almost three-quarters of total cost goes for maintenance while one fifth goes for purchase [53]. The huge proportion for maintenance cost represents an opportunity for cost reduction.

The acknowledged maintenance criteria is to optimize certain parameter with renewal theorem. Differ from usual production systems, most SISs are designed with finite service time and thus the steady-state criteria is not applicable [29]. Therefore, the life-cycle cost of SISs could be estimated by the sum of expected cost after each proof test.

To quantify the life-cycle cost, several cost items related maintenance and testing actions are defined as: C_0, C_{PT}, C_{PM}, C_{CM} represents one-time installation cost per unit, proof test cost per unit, preventive maintenance cost and corrective maintenance cost (purchase) per unit, respectively.

The expected maintenance cost after ith test (EC_i) should equal to the sum of proof test cost (EC_{PT}), expected PM cost (EC_{PM}) and CM cost (EC_{CM}) in ith test interval, where expected cost depends on the system state probability and corresponding maintenance actions.

$$EC_i = EC_{PT} + EC_{\mathrm{PM}} + EC_{\mathrm{CM}} \tag{12}$$

Considering the imperfectness of revealing degraded state, the expected maintenance cost should be linked with parameter α, for 1oo1 configuration after the first test,

$$\begin{aligned} EC_{\mathrm{PM}} &= \mathbf{P}_2(\tau^-)\cdot \mathrm{C}_{\mathrm{PM}} = \mathbf{P}_2(\tau^+)\cdot \alpha \cdot \mathrm{C}_{\mathrm{PM}} \\ EC_{\mathrm{CM}} &= \mathbf{P}_3(\tau^-)\cdot \mathrm{C}_{\mathrm{CM}} = \mathbf{P}_3(\tau^+)\cdot \mathrm{C}_{\mathrm{CM}} \end{aligned} \tag{13}$$

Then the expected maintenance cost EC_1 for 1oo1 configuration SIS after first test can be expressed as following,

$$EC_1 = C_{PT} + \mathbf{P}((\tau)^+)\cdot \begin{pmatrix} 0 \\ \alpha \cdot C_{\mathrm{PM}} \\ C_{\mathrm{CM}} \end{pmatrix} \tag{14}$$

Afterwards, the total expected life-cycle cost (LCC) for 1oo1 configured SIS in n test intervals can be estimated as

$$\mathrm{LCC} = C_0 + \sum_{i=1}^{n} EC_i \tag{15}$$

Similarly, the expected maintenance cost for 1oo2 configuration after single proof test with Strategy I can be estimated as Eq. (16),

$$EC_i = 2C_{PT} + \mathbf{P}((i\tau)^+) \cdot \begin{pmatrix} 0 \\ \alpha_2 \cdot C_{\mathrm{PM}} \\ C_{\mathrm{CM}} \\ \alpha_1 \cdot C_{\mathrm{PM}} \\ \alpha_1\cdot(1-\alpha_2)\cdot C_{\mathrm{PM}} + \alpha_1\cdot(1-\alpha_2)\cdot C_{\mathrm{PM}} + 2\cdot\alpha_1\cdot\alpha_2\cdot C_{\mathrm{PM}} \\ \alpha_1\cdot(C_{\mathrm{PM}}+C_{\mathrm{CM}}) + (1-\alpha_1)\cdot C_{\mathrm{CM}} \\ C_{\mathrm{CM}} \\ \alpha_2\cdot(C_{\mathrm{PM}}+C_{\mathrm{CM}}) + (1-\alpha_2)\cdot C_{\mathrm{CM}} \\ 2C_{\mathrm{CM}} \end{pmatrix} \tag{16}$$

the total expected life-cycle cost (LCC) for 1oo2 configured SIS with Strategy I in n test intervals can be estimated as

$$\mathrm{LCC} = 2\cdot C_0 + \sum_{i=1}^{n} EC_i \tag{17}$$

For Strategy II, unit 1 is tested at time $t_{2j-1} = (2j-1)\times\tau/2$ and unit 2 at time $t_{2j} = (2j)\times\tau/2$, $(j = 1,2,\ldots)$, the expected cost after single test can be estimated by Eq. (18).

$$\begin{aligned} EC_{2j-1} &= C_{PT} \\ &\quad + \mathbf{P}(((2j-1)\cdot\tau/2)^+) \\ &\quad \cdot \left(0,0,0,\alpha_1\cdot C_{\mathrm{PM}},\alpha_1\cdot C_{\mathrm{PM}},\alpha_1\cdot C_{\mathrm{PM}},C_{\mathrm{CM}},C_{\mathrm{CM}},C_{\mathrm{CM}}\right)^{\mathbf{T}} \\ EC_{2j} &= C_{PT} \\ &\quad + \mathbf{P}(((2j)\cdot\tau/2)^+) \\ &\quad \cdot \left(0,\alpha_2\cdot C_{\mathrm{PM}},C_{\mathrm{CM}},0,\alpha_2\cdot C_{\mathrm{PM}},C_{\mathrm{CM}},0,\alpha_2\cdot C_{\mathrm{PM}},C_{\mathrm{CM}}\right)^{\mathbf{T}} \end{aligned} \tag{18}$$

Similarly, for Strategy III, the expected cost after each test can be estimated by Eq. (19).

$$\begin{aligned} EC_{2j-1} &= C_{PT} \\ &\quad + \mathbf{P}(((2j-1)\cdot\tau/2)^+) \\ &\quad \cdot \left(0,0,0,\alpha_1\cdot C_{\mathrm{PM}},\alpha_1\cdot C_{\mathrm{PM}},\alpha_1\cdot C_{\mathrm{PM}},2C_{\mathrm{CM}},2C_{\mathrm{CM}},2C_{\mathrm{CM}}\right)^{\mathbf{T}} \\ EC_{2j} &= C_{PT} \\ &\quad + \mathbf{P}(((2j)\cdot\tau/2)^+) \\ &\quad \cdot \left(0,\alpha_2\cdot C_{\mathrm{PM}},2\cdot C_{\mathrm{CM}},0,\alpha_2\cdot C_{\mathrm{PM}},2\cdot C_{\mathrm{CM}},0,\alpha_2\cdot C_{\mathrm{PM}},2\cdot C_{\mathrm{CM}}\right)^{\mathbf{T}} \end{aligned} \tag{19}$$

Using Eq. (17), the total expected LCC for 1oo2 configuration under Strategy I in a finite lifetime can be estimated by summing up the expected cost from Eq. (16). Similar equations could be conducted for Strategy II and Strategy III by summing up results from Eqs. (18) and (19), respectively.

Table 2
Parameter value.

Parameter	value
λ_1	8E−6
λ_2	2E−5
λ_3	4E−6
λ_4	8E−6
λ_5	2E−5
λ_6	4E−6
τ	8760

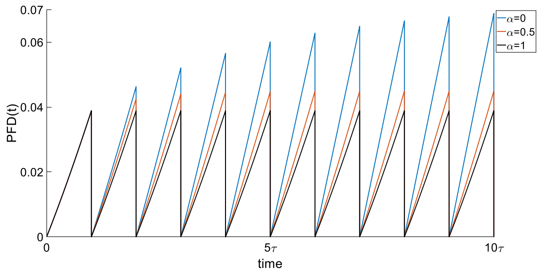


Fig. 3. PFD(t) of 1oo1 configuration.

4. Numerical example

To illustrate the proposed model and maintenance strategies, a numerical example is conducted here. Assumed parameters for transition rates in the example are listed in Table 2.

4.1. Effect of α on the performance of a 1oo1 configuration

To investigate the effect of imperfectness in revealing degraded state α on the 1oo1 configuration, a perfect PM or CM will be executed if the system is manifested in degraded or failed state in proof tests. The effect of coverage α of proof test in revealing degraded state is shown in Fig. 3.

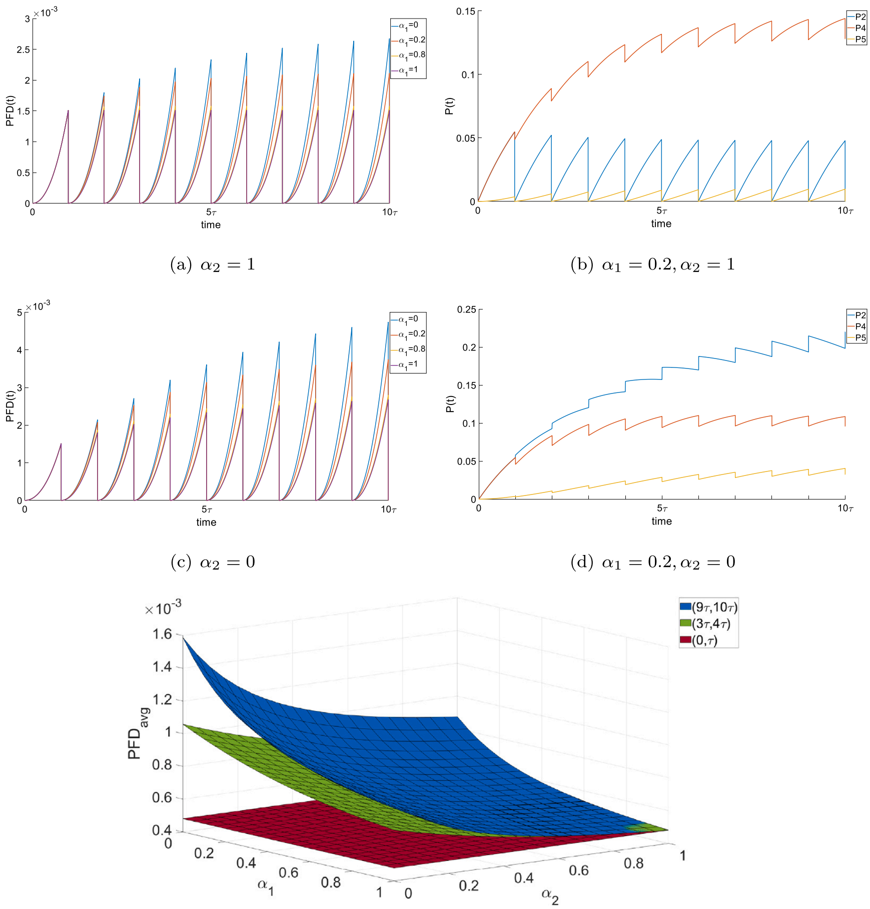
It is easy to notice that the testing coverage α has an obvious effect on system PFD(t). In the first test phase $(0,\tau)$, system PFD(t) is overlapped when $\alpha = 0, 0.5, 1$, thanks to the same initial state probability $\mathrm{P}(t) = [1,0,0]$ at $t = 0$. When $\alpha = 1$, the proof testings are perfect in revealing degraded states and failed state, the element will reach a stable and lowest tendency since the initial state is $\mathrm{P}(t) = [1,0,0]$ in each test phase. When $\alpha < 1$, the system is still possible in the degraded state after perfect PM or CM, and then the initial state of the system in each phase is $\mathrm{P}(t) = [1-\alpha\mathrm{P}_2(t^-), \alpha\mathrm{P}_2(t^-), 0]$. Consequently, system PFD(t) is increasing with time under imperfect testing as $\alpha = 0$ and $\alpha = 0.5$ in each test phase as shown in Fig. 3. When $\alpha = 0$, the system PFD(t) reaches the highest value in same test phase.

4.2. Effect of α on the performance of a 1oo2 configuration

Performance of a 1oo2 configuration is analyzed according to the proposed testing and maintenance strategies respectively.

4.2.1. Simultaneous testing with maintenance strategy I

For strategy I, given the imperfect revealing coverage on degraded state for two units, undoubtedly, the observed state probabilities will not be equal to the actual physical ones when $\alpha_i < 1$. According to assumptions in Section 3.1, test and repair time is assumed to be negligible. The instantaneous state transition process at time $i\tau, i = 1,2,\ldots$ with revealing coverage α_1 and α_2 on degraded state for selected states are shown in Table 3. The whole matrix regarding test and repair is shown as $\mathbf{M}$ in Appendix B.



(a) $\alpha_2 = 1$ (b) $\alpha_1 = 0.2, \alpha_2 = 1$

(c) $\alpha_2 = 0$ (d) $\alpha_1 = 0.2, \alpha_2 = 0$

(e) $\mathrm{PFD_{avg}}$ in 1st, 4th and 10th test phase

Fig. 4. PFD(t) and selected state probabilities of 1oo2 configuration under strategy I.

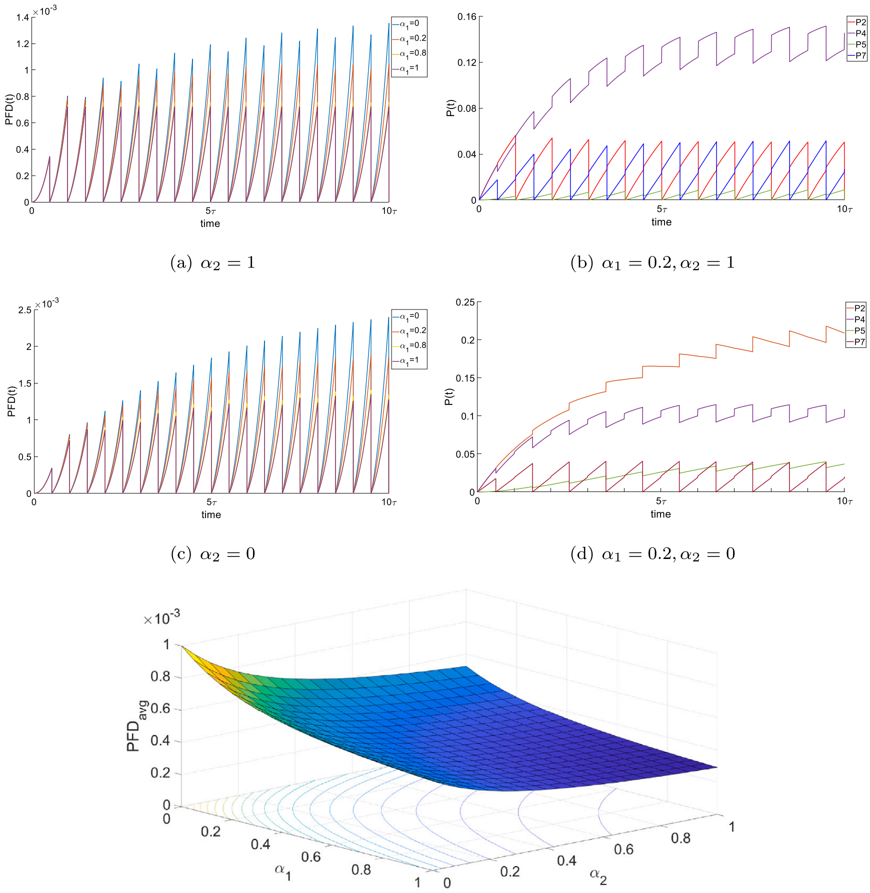
Table 3
Instantaneous state transition at test time $i\tau$ with strategy I.

Physical at $i\tau^-$	After test	After repair	Physical at $i\tau^+$
F_1D_2	α_2 F_1D_2 $1-\alpha_2$ F_1W_2	α_2 W_1W_2 $1-\alpha_2$ W_1W_2	α_2 W_1W_2 $1-\alpha_2 W_1D_2$
D_1D_2	$\alpha_1\alpha_2$ D_1D_2 $\alpha_1(1-\alpha_2)$ D_1W_2 $(1-\alpha_1)\alpha_2$ W_1D_2 $(1-\alpha_1)(1-\alpha_2)$ W_1W_2	$\alpha_1\alpha_2$ W_1W_2 $\alpha_1(1-\alpha_2)$ W_1W_2 $(1-\alpha_1)\alpha_2$ W_1W_2 –	$\alpha_1\alpha_2$ W_1W_2 $\alpha_1(1-\alpha_2)W_1D_2$ $(1-\alpha_1)\alpha_2D_1W_2$ $(1-\alpha_1)(1-\alpha_2)$ D_1D_2
D_1F_2	α_1 D_1F_2 $1-\alpha_1$ W_1F_2	α_1 W_1W_2 $1-\alpha_1$ W_1W_2	α_1 W_1W_2 $1-\alpha_1D_1W_2$

System PFD(t) and selected state probabilities of 1oo2 configuration with strategy I are shown in Fig. 4.

System PFD(t) is increasing under strategy I with the set parameters in Table 2 when $\alpha_i < 1$, meaning that system unavailability is increasing in each testing phase. In Fig. 4(a), the test coverage of revealing degraded state α_1 for unit 1 has a more evident effect on PFD(t) with time when $\alpha_2 = 1$. When α_1 closes to 1, PFD(t) has a slowing decrease with α_1 in each test interval. System PFD(t) with $\alpha_1 = 0.8$ is almost overlapping with that of $\alpha_1 = 1$. Selected state probabilities with $\alpha_1 = 0.2, \alpha_2 = 1$ is shown are 4(b). When $\alpha_2 = 1$, the degraded state of unit 2 will be revealed perfectly after each test. Then the state probabilities for state 2 (W_1D_2) and 5 (D_1D_2) will decrease to 0 at the beginning of each test phase. Meanwhile, the state probability of state 4 (D_1W_2) should theoretically equal to 0. But, given the imperfect revealing coverage for unit 1, the state probability $P_4(i\tau^-)$ decreases at each test point ($P_4(i\tau^-) < P_4(i\tau^+)$) with overall increases ($P_4(i\tau^-) < P_4((i+1)\tau^-)$) instead, which comes from the partly imperfect repair of state 5 (D_1D_2) and 6 (D_1F_2) as shown in Table 3.

Similar as system PFD(t) tendency in Fig. 4(a), PFD(t) in Fig. 4(c) is also increasing along with time. In each test phase, PFD(t) monotonically increases in each test phase and reaches a maximum at $i\tau^+, i = 1, 2, \ldots$. PFD($t$) decreases slowly with a higher α_1. State probabilities $P_2(t), P_4(t)$ and $P_5(t)$ in Fig. 4(d) show different tendencies compared to Fig. 4(b). Since $\alpha_2 = 0$, no degraded state for unit 2 is revealed in proof tests. For state 2 (W_1D_2), $P_2(i\tau^+) > P_2(i\tau^-)$, the increment comes from the partly repair of state 5 (D_1D_2) and 6 (D_1F_2) as described in Table 3. $P_5(i\tau^-)$ will be divided into four possible states 5(D_1D_2), 4(D_1W_2), 2(W_1D_2)and 1(W_1W_2) with portions 0,0.2,0,0.8, respectively. When the system is in $P_5(i\tau^-)$, it has 20% of probability to be repaired, and the probability of being skipped is 80%.



(a) $\alpha_2 = 1$

(b) $\alpha_1 = 0.2, \alpha_2 = 1$

(c) $\alpha_2 = 0$

(d) $\alpha_1 = 0.2, \alpha_2 = 0$

(e) $\mathrm{PFD_{avg}}$ in $(9.5\tau, 10\tau)$

Fig. 5. PFD(t) and selected state probabilities of 1oo2 configuration under strategy II.

System PFD_{avg} with α_1 and α_2 in selected test phases is shown in Fig. 4(e). In first test phase (0, τ), PFD_{avg} shows a flat surface with the value of 4.81×10^{-4} for independent on α_1 and α_2. It means that the system performance in first phase is only depending on the initial state vector and the length of test. It is reasonable to conclude that system PFD_{avg} is increasing with time, since showing a highest value for 10th with an intermediate and lowest value for 4th and 1st test phase in Fig. 4(e), respectively. Meanwhile, it is not difficult to notice that PFD_{avg} reaches a minimum value when $\alpha_1 = \alpha_2 = 1$ and a maximum value when $\alpha_1 = \alpha_2 = 0$ with up to 1.59×10^{-3} for 10th and 1.06×10^{-3} in 4th test phase. This finding also provide clues to take system PFD_{avg} in final test phase as a reference in the whole life-cycle in the further discussions.

4.2.2. Staggered testing with maintenance strategy II

The point of testing for unit 1 is shifted with a time $\tau/2$ compared to the unit 2. And unit 1 is tested at $t_{2j-1} = (2j-1) \times \tau/2$ and unit 2 at time $t_{2j} = (2j) \times \tau/2$, $(j = 1, 2, \ldots)$. System PFD(t) of 1oo2 configuration with strategy II is shown in Fig. 5. In the first testing phase, system PFD(t) has no relation with either α_1 or α_2 thanks to the same initial state probability $\mathbf{P}_0$.

As mentioned in Section 3.4, the staggered testing procedure introduces two separate matrices, which are shown in Appendix B, $\mathbf{M}_{U_1}$ is valid after a test of unit 1 and $\mathbf{M}_{U_2}$ is valid after a test of unit 2. When $\alpha_2 = 1$, in Fig. 5(a), system PFD(t) increases with a lower value of α_1 in each testing phase. Several system states, e.g. state 4(D_1W_2), state 5(D_1D_2) and state 6(D_1F_2) will still be hidden and not be repaired during the testing of unit 1 when $\alpha_1 \neq 0$. Because of the alternation and imperfect coverage, these hidden states after testing of unit 1 contribute to a fluctuating PFD(t) in the consecutive testing phase of unit 2. Similar tendencies are demonstrated in Fig. 5(c) with $\alpha_2 = 0$.

Selected state probabilities with $\alpha_1 = 0.2, \alpha_2 = 1$ are shown in Fig. 5(b). For example, state probability $P_4(t)$ for state 4 (D_1W_2) decreases instantly after testing of unit 1 because of the imperfect coverage α_1 but jumps to a higher value given the repair of state 5 (D_1D_2) and state 6 (D_1F_2) after testing of unit 2. Similarly, compared to Fig. 5(b), the lower increment magnitude of $P_4(t)$ in Fig. 5(d) comes from the repair of state 6 (D_1F_2) since no state 5 (D_1D_2) is revealed with $\alpha_2 = 0$ in tests of unit 2.

It is worth noting that there are two specific cases: (1) $\alpha_1 = 0, \alpha_2 = 0$ (2) $\alpha_1 = 1, \alpha_2 = 1$.

(1) When $\alpha_1 = 0, \alpha_2 = 0$, it means that even the physical state of unit has shifted from working to degraded state, but no degraded states for either unit 1 or unit 2 are revealed in tests. Consequently, no PM will be executed. Therefore, system PFD(t) reaches a maximum value in each

Table 4
Different transition rates for unit 2.

Parameter	Value			
	Unit 21	Unit 22	Unit 23	Unit 24
λ_4	0.5 × 8E−6	8E−6	2 × 8E−6	3 × 8E−6
λ_5	0.5 × 2E−5	2E−5	2 × 2E−5	3 × 2E−5
λ_6	0.5 × 4E−6	4E−6	2 × 4E−6	3 × 4E-6

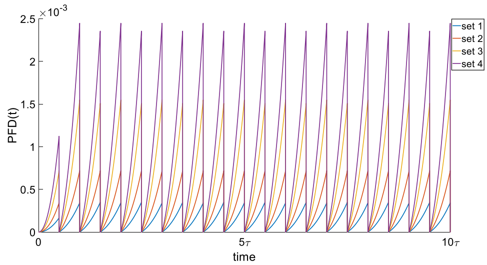


Fig. 6. PFD(t) of 1oo2 configuration under strategy II.

test phase, as shown in Fig. 5(c). This finding is also demonstrated by the maximum value of system PFD$_{\text{avg}}$ in $(9.5\tau, 10\tau)$ after test of unit 1 at time 9.5τ in Fig. 5(e). Meanwhile, PFD$_{\text{avg}}$ increases with a higher magnitude when either α_1 or α_2 is closing to 0.

(2) When $\alpha_1 = 1, \alpha_2 = 1$, it means that degraded state of unit 1 and unit 2 will be perfectly revealed in the tests. Corresponding repair actions are taken, system PFD(t) reaches a stable tendency and minimum value after few phases since two units are assumed identical with same transition rates.

To demonstrate the effect of transition rates, a brief study is conducted here. The transition rates for unit 1 keep the same values as in Table 2. Four optional unit 2 for 1oo2 configuration, which marked as Unit 21, 22, 23 and 24, are listed in Table 4 with different transition rates. For the simplification in the following, symbol 'set i' is employed to stand for the 1oo2 configuration with unit 1 and unit 2i.

The calculation result of PFD(t) for the 1oo2 configuration under strategy II with nonidentical units are shown in Fig. 6. It is obvious that system PFD(t) increases with higher values of transition rates for unit 2. Given the unequal transition rates for two units, system PFD(t) fluctuates when $\alpha_1 = \alpha_2 = 1$ with the test of unit 1 and 2 except a stable tendency for set 2.

4.2.3. *Staggered testing with maintenance strategy III*

The main difference between strategy II and strategy III is an additional replace action on the untested unit. It is easy to infer that system PFD$_{\text{avg}}$ will be to some extent lower with strategy III compared to strategy II. Similarly as strategy II, the staggered testing procedure introduces two separate matrices, which are shown in Appendix B, $\mathbf{M}_{U_1}$ is valid after a test of unit 1 and $\mathbf{M}_{U_2}$ is valid after a test of unit 2.

System PFD$_{\text{avg}}$ results with parameters from Table 2 under two strategies are shown in Fig. 7.

When $\alpha_1 = \alpha_2 = 1$, in Fig. 7(a), system PFD$_{\text{avg}}$ reaches a constant value 2.91×10^{-4} with strategy II and a lower value with strategy III, at 2.84×10^{-4}, representing 2.45% decrease.

When PFD$_{\text{avg}}$ if $\alpha_1 = \alpha_2 = 0$, only failed unit will be restored to working state. In Fig. 7(b), it is obvious that system PFD$_{\text{avg}}$ keeps increasing with time with strategy II and III. Strategy III has a more evident advantage along with time on PFD$_{\text{avg}}$.

The main shortcoming of strategy III is the abuse of restoring the untested unit, which consequently will contribute to a increasing maintenance cost. Therefore, the upcoming consideration is how to balance the decreased PFD$_{\text{avg}}$ and economic loss.

Table 5
Parameter value regarding maintenance and test items.

Parameter	Item	value
C_0	One-time installation cost per unit	600
C_{PT}	test cost per unit	60
C_{PM}	preventive maintenance cost per unit	240
C_{CM}	corrective maintenance (purchase) cost per unit	6940

4.2.4. PFD$_{\text{avg}}$ *Comparisons among proposed strategies*

For strategy I with $\alpha_1 = \alpha_2 = 1$, either degraded or failed state will be repaired. The system state probabilities will be same as initial vector $\mathbf{P}_0$, which leads to a stable performance of system in each test phase. As proved in previous sections, system will have a lower PFD$_{\text{avg}}$ with $\alpha_1 = \alpha_2 = 1$ in same strategy. When α_1 and α_2 take same values, staggered test (strategy II and III) can lead to a better system performance than simultaneous test (strategy I).

For $\alpha_1 = \alpha_2 = 1$, in Fig. 8(a), system PFD$_{\text{avg}}$ under strategy II and III is up to 60.6% and 59.2% of that under strategy I, respectively. In $(9.5\tau, 10\tau)$, the corresponding value is 63.1% and 54.4% for $\alpha_1 = \alpha_2 = 0$. It is worth mentioning that, in Fig. 8(b), system performance meet SIL 3 with $\alpha_1 = \alpha_2 = 0.5$ under any of proposed maintenance strategy.

To quantify the differences for PFD$_{\text{avg}}$ under proposed strategies, an indicator k_{ji} is proposed here as following,

$$k_{ji} = \frac{\text{PFD}_{\text{avg}} \text{ with strategy } j}{\text{PFD}_{\text{avg}} \text{ with strategy } i} \tag{20}$$

In Figs. 8(c) and 8(d), indicator k_{21} and k_{31} fluctuates with time thanks to the unstable performance for 1oo2 configuration in the early stage when $\alpha_1 = \alpha_2 = 0$, meanwhile, fluctuations of k_{21} and k_{31} decreases gradually along with time.

From Fig. 8(c), the indicator k_{21} gradually reaches a constant value under the specified value of α_1 and α_2 after around 10τ. The overall of effects of strategy II can be approximated estimated in the range of $(0.6, 0.65)$ of strategy I. To infer from these findings that indicator k_{21} has quite weak relation with the value of α_1 and α_2 when the service time is quite long.

However, the indicator k_{31} shows a non-identical tendency in Fig. 8(d). PFD$_{\text{avg}}$ of strategy III mainly located in the range of $(0.5, 0.6)$ with that of strategy I. Imprecision of revealing coverage in tests shows a more obvious effect on PFD$_{\text{avg}}$ when α_1 and α_2 is less than 0.5. For example, k_{31} equals to 0.513 for $\alpha_1 = \alpha_2 = 0$ at 20τ, while 0.589 and 0.592 for $\alpha_1 = \alpha_2 = 0.5$ and $\alpha_1 = \alpha_2 = 1$, respectively.

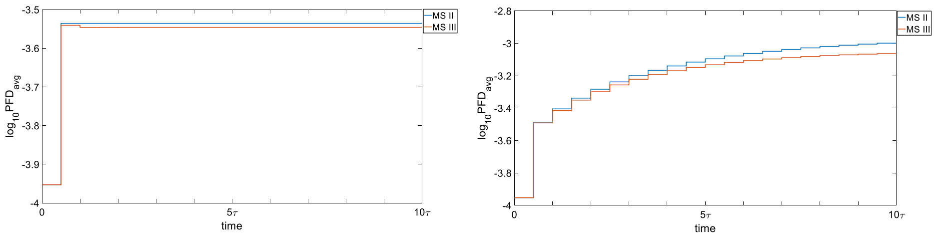
Fig. 8(e) depicts the differences between strategy II and III regarding imprecision revealing coverage α_1 and α_2 in tests. It demonstrates that system has a better performance under strategy III than strategy II as the indicator $k_{32} < 1$, which complies to the findings in Fig. 8(a) and Fig. 8(b). Similar as k_{31} in Fig. 8(d), indicator k_{32} shifts from 0.817 to 0.962 when α_1 and α_2 from 0 to 0.5 at 20τ, while only from 0.962 to 0.976 when α_1 and α_2 from 0.5 to 1. In the long run, strategy III results in an optimistic system performance compared to strategy I and II when the test coverage is quite low.

To conclude, for system PFD$_{\text{avg}}$, staggered test could lead to a better system performance that simultaneous test when the state revealing coverage α_i takes same value. Meanwhile, strategy III is ahead of strategy II to some extent, which is strongly linked with parameter α_i.

4.2.5. *Life-cycle cost*

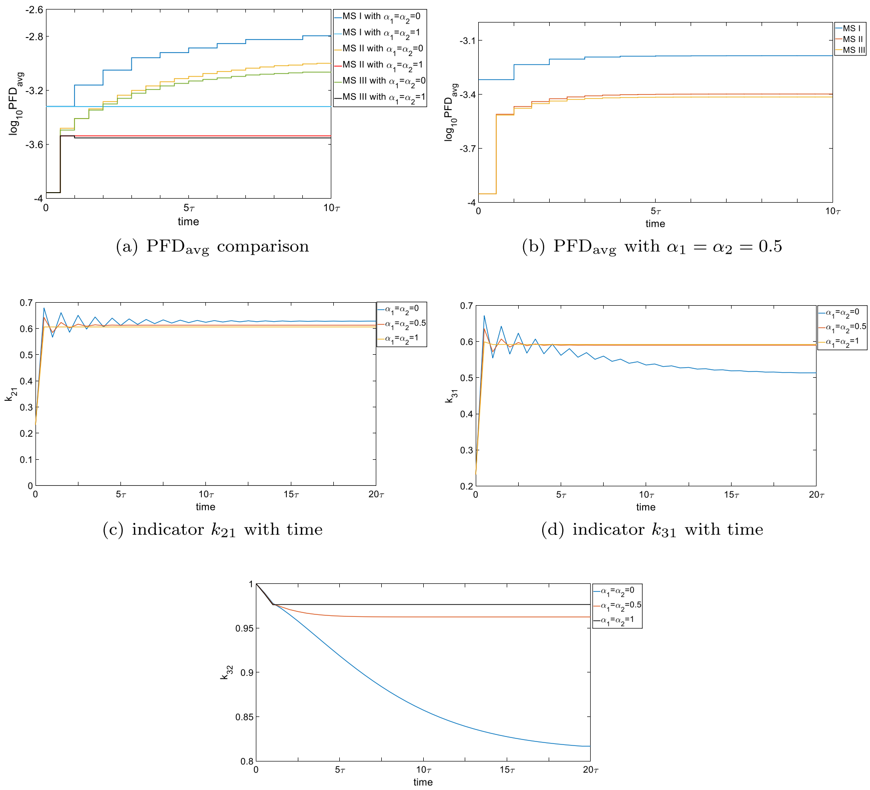
Life-cycle cost items and corresponding values are partly adopted from [47]. Maintenance cost parameters and values are presented in the following Table 5. Based on the finding in Section 4.2, system PFD$_{\text{avg}}$ in final test phase is used as a reference of system performance in the whole life-cycle.

Cumulative maintenance cost for 1oo2 configuration in 20τ with different strategies are depicted in Fig. 9.



(a) $\alpha_1 = \alpha_2 = 1$ (b) $\alpha_1 = \alpha_2 = 0$

Fig. 7. System PFD_{avg} comparison between strategy II and strategy III.



(a) PFD_{avg} comparison (b) PFD_{avg} with $\alpha_1 = \alpha_2 = 0.5$

(c) indicator k_{21} with time (d) indicator k_{31} with time

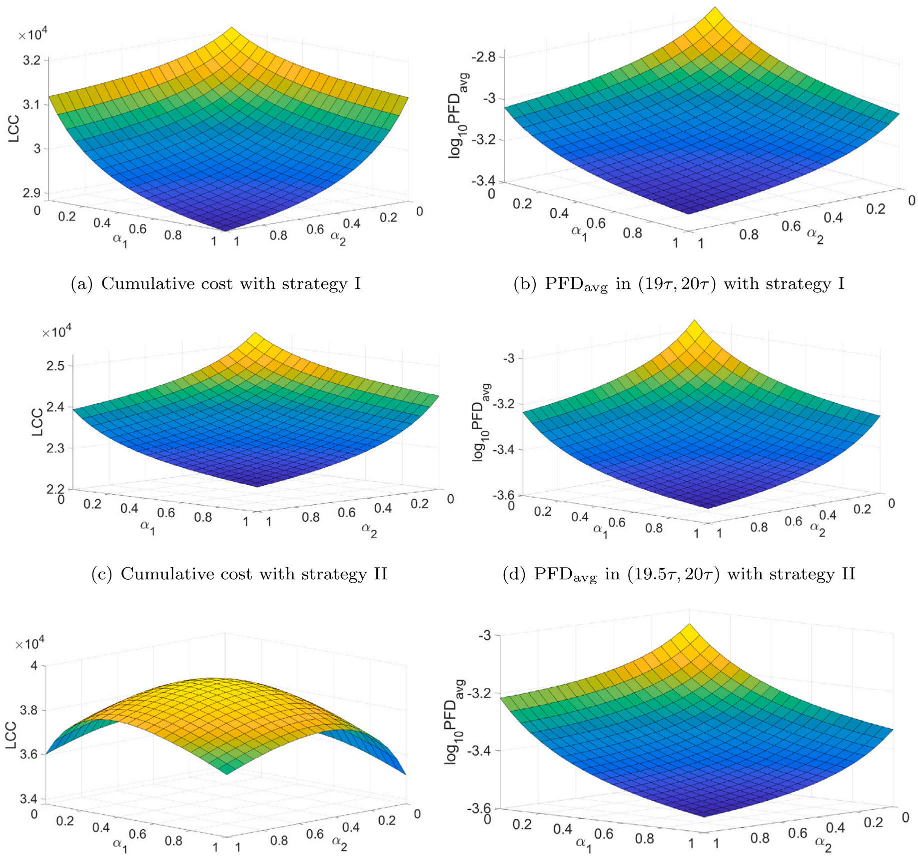
(e) indicator k_{32} with time

Fig. 8. Summary of system PFD_{avg} based on proposed strategies.

In Fig. 9(a), it is obvious that cumulative maintenance cost reaches a maximum value with $\alpha_1 = \alpha_2 = 0$ and a minimum value when $\alpha_1 = \alpha_2 = 1$. Cumulative maintenance cost decreases universally with a higher state revealing probability α_i. When the revealing probability is quite low, the SIS will be remained at the degraded state after proof test. The hidden degraded state will gradually develop to failed state, which will contribute an expensive CM cost compared to PM. This finding is demonstrated by the tendency of PFD_{avg} in $(19\tau, 20\tau)$ in Fig. 9(b). System performance in $(19\tau, 20\tau)$ locates in SIL2 with quite low revealing test coverage, while in SIL3 with a better revealing coverage.

LCC with coverage α_i under strategy II in Fig. 9(c) shows a similar tendency but a lower value than that under strategy I in Fig. 9(a). Considering different test sequences of units 1 and 2, $\mathbf{P}(i\tau^+)$ will redistribute after the prior test and maintenance. The redistribution of state probabilities contributes to the phenomena that LCC is asymmetry about $\alpha_1 = \alpha_2$ given the certain testing sequences of unit 1 and 2, similar result also can be drawn for strategy III in Fig. 9(e).

Distinguished from those by strategies I and II, LCC under strategy III reaches a minimum value when $\alpha_1 = \alpha_2 = 0$, namely, CM would only be executed when an item fails. When $\alpha_i \neq 0$, an additional CM on untested unit will be executed along with the PM for tested unit. Consequently, this maintenance action contributes to a higher life-cycle cost. Given $\mathbf{P}(i\tau^+)$ is time-dependent and α_i-dependent, the whole LCC in 20τ is not a monotonic with α_i. In fact LCC increases with α_i and reaches a peak, subsequently, decreases slightly. When revealing



(a) Cumulative cost with strategy I

(b) $\mathrm{PFD}_{\mathrm{avg}}$ in $(19\tau, 20\tau)$ with strategy I

(c) Cumulative cost with strategy II

(d) $\mathrm{PFD}_{\mathrm{avg}}$ in $(19.5\tau, 20\tau)$ with strategy II

(e) Cumulative cost with strategy III

(f) $\mathrm{PFD}_{\mathrm{avg}}$ in $(19.5\tau, 20\tau)$ with strategyIII

Fig. 9. Cumulative maintenance cost in 20τ.

coverage α_i is quite low, less PMs will be taken, but which could lead to higher possibility of CM. PM cost contributes to an increment in accumulation with coverage α_i at first. When the efficiency of proof tests on degraded state is higher, PM increases and potential CM cost decreases as well. Decrement of potential CM contributes to a decline accumulative cost with higher coverage α_i.

Another potential doubt here is that PM cost is far less than CM (purchase) with values in Table 5. Therefore, a further calculation is conducted here with $C_{\mathrm{PM}} = 2400$. $\mathrm{PFD}_{\mathrm{avg}}$ should be independent with the value of C_{PM}. The accumulative LCC in 20 years with different strategies is shown in Fig. 10.

It is obvious that each strategy has a higher cost with an expensive PM cost than previous results in Fig. 9. Inconsistent with the result in Fig. 9(a), LCC under strategy I has a minimum value when $\alpha_1 = \alpha_2 = 0$ and a maximum value when $\alpha_1 = \alpha_2 = 1$. It implies that the cumulative PM cost takes a higher proportion in life-cycle. For strategy II, LCC increases with α_i and reaches a peak, subsequently, decreases slightly, which is similar as the result with strategy III in Fig. 9(e). When it comes to strategy III, thanks to the opportunistic replacement of untested unit when maintenance action is executed on tested unit, the tendency of accumulative cost should be consistent with Fig. 9(e).

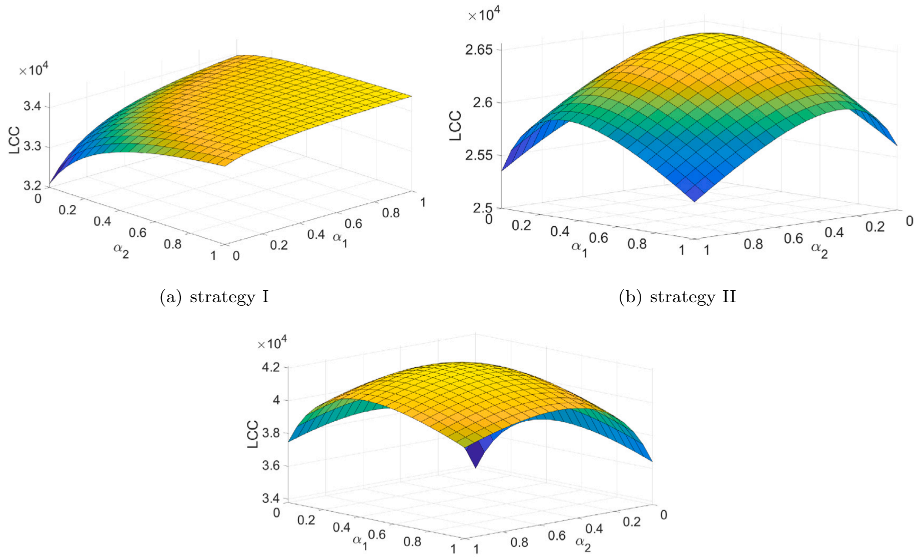
Combined the results from Figs. 9 and 10, generally, from the aspect of LCC, it is easy to conclude that strategy III > strategy I >strategy II in 20τ. But when the PM cost is quite high, the LCC in 20τ have an obvious increment, namely, the maintenance actions also need to be considered carefully. As for $\mathrm{PFD}_{\mathrm{avg}}$, from the result in Figs. 9(b), 9(d) and 9(f), system performance with staggered test is universally better than simultaneous test. System with simultaneous test in $(19\tau, 20\tau)$ is within SIL2 and SIL3. For strategy II, except the extreme low revealing coverage of degraded state ($\alpha_1 < 0.2$ and $\alpha_2 < 0.2$), system performance mainly in SIL3. Namely, strategy II contributes to a better system performance than strategy I. Compared to strategy II, system $\mathrm{PFD}_{\mathrm{avg}}$ in $(19.5\tau, 20\tau)$ complies to SIL3 totally with strategy III.

Table 6
Comparisons among proposed maintenance strategies.

Strategy	$\mathrm{PFD}_{\mathrm{avg}}$	LCC
Strategy I	Poor	Medium
Strategy II	Medium	Low
Strategy III	Good	High

The universal pros and cons of proposed maintenance strategies without taking the values of revealing coverage α_i into consideration are listed in Table 6.

In reality, following the previous findings, if the α_i quite high ($\alpha_i > 0.5$), from Fig. 9, $\mathrm{PFD}_{\mathrm{avg}}$ under each maintenance strategy is within SIL3. Therefore, LCC should be prioritized to reduce unnecessary economic loss. That is, the proposed strategy II is the optimal option.



(a) strategy I

(b) strategy II

(c) strategy III

Fig. 10. Cumulative maintenance cost in 20τ with an expensive PM cost.

On the contrary, if the α_i quite low ($\alpha_i < 0.5$), not all system SIL complies to SIL3, PFD_{avg} is in the higher priority when it comes to select optimal test and maintenance strategy.

Meanwhile, it is obvious to conclude from Figs. 8 and 9 that the proposed strategy III can lead to the highest LCC and optimum PFD_{avg} regardless of the value of α_i. Nevertheless, in terms of PFD_{avg}, it has slight improvement compared to strategy II especially when α_i quite high ($\alpha_i > 0.5$). The high LCC is the definite disadvantage of the proposed strategy III.

Given that the inevitable degradation phenomena in mechanical elements, it is needed to study how dynamic monitoring can be better utilized. An indicator reflecting the working condition and system status could provide clues for maintenance actions. When a PM is implemented (parameter $\alpha_i > 0$ in this paper), the system performance is better, but LCC is higher. A systematic testing and maintenance policy for the SIS with coordinating the trade-off between PFD_{avg} and LCC should be carefully considered in the designed phase.

5. Concluding remarks

This paper has presented a state-based approach for performance analysis of redundant final elements in SIS subject to imperfect degradation state revealing. The system performance is calculated based on a multi-phase Markov process. Estimation methods for maintenance cost in a finite time regarding imperfect state revealing have been proposed.

A numerical example is given to illustrate the usefulness of the proposed strategies. Based on the assumption, for a 1oo2 configuration, we found that staggered tests can contribute to a better system performance compared to simultaneous tests. From the aspect of LCC, strategy III > strategy I > strategy II in 20τ. Through the proposed method and discussions, a systematic consideration in incorporating system availability and life cycle cost need to be conducted, for reliability practitioners of SISs, when choose testing and maintenance strategy in the overall life-cycle for redundant final element.

This paper focuses on the comparisons among three proposed testing and maintenance strategies for 1oo2 SIS subject to imperfect state revealing. However, several limitations have been remained here in terms of testing and maintenance for SISs, e.g. partial test, common cause failures (CCFs), time-dependent degradation state revealing probability and imperfect maintenance etc. Another point here is about the estimation of potential economic loss of EUC due to the testing and maintenance of SISs.

For further studies, it would be interesting to extend and apply this model to realistic issues of SISs with risk-based EUC cost involved.

CRediT authorship contribution statement

Aibo Zhang: Visualization, Methodology, Software, Investigation. **Himanshu Srivastav:** Visualization, Methodology. **Anne Barros:** Methodology, Writing - review & editing, Supervision. **Yiliu Liu:** Conceptualization, Validation, Writing - review & editing, Supervision.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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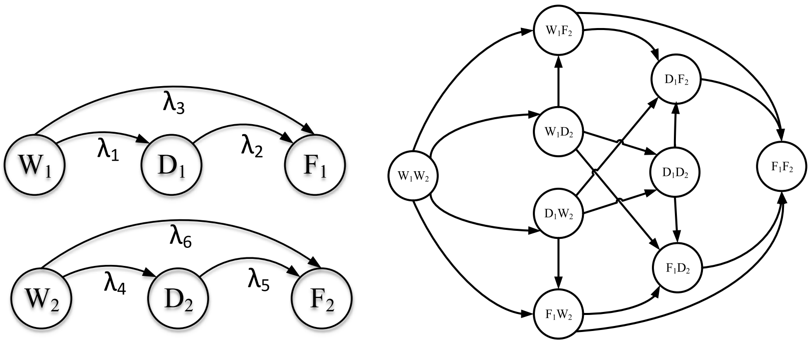
Appendix A. Possible states for 1oo2 configuration

See Table A.1 and Fig. A.1.



(a) (b)

Fig. A.1. State transition diagrams for (a) 1oo1 configuration and (b) 1oo2 configuration.

Table A.1
Possible states for 1oo2 configuration.

State	Notation
1	W_1W_2
2	W_1D_2
3	W_1F_2
4	D_1W_2
5	D_1D_2
6	D_1F_2
7	F_1W_2
8	F_1D_2
9	F_1F_2

Appendix B. Matrices mentioned in this paper

There are 3 possible states for each single unit under study. They are denoted by State W (working), State D (degraded) and State F (failed).

Transition rate matrix $\mathbf{Q}_{U_1}$ and $\mathbf{Q}_{U_2}$ for unit 1 and 2:

$$\mathbf{Q}_{U_1}=\begin{array}{c} \\ W_1 \\ D_1 \\ F_1 \end{array}\begin{array}{c}\begin{array}{ccc} W_1 & D_1 & F_1 \end{array}\\ \begin{pmatrix} -(\lambda_1+\lambda_3) & \lambda_1 & \lambda_3 \\ & -\lambda_2 & \lambda_2 \\ & & \end{pmatrix}\end{array} \quad \mathbf{Q}_{U_2}=\begin{array}{c} \\ W_2 \\ D_2 \\ F_2 \end{array}\begin{array}{c}\begin{array}{ccc} W_2 & D_2 & F_2 \end{array}\\ \begin{pmatrix} -(\lambda_4+\lambda_6) & \lambda_4 & \lambda_6 \\ & -\lambda_5 & \lambda_5 \\ & & \end{pmatrix}\end{array}$$

Transition rate matrix $\mathbf{Q}$ for 1oo2 configuration

$$\mathbf{Q}=\begin{array}{c} \\ 1\\2\\3\\4\\5\\6\\7\\8\\9 \end{array}\begin{array}{c}\begin{array}{ccccccccc} 1&2&3&4&5&6&7&8&9 \end{array}\\ \begin{pmatrix} -\Sigma & \lambda_4 & \lambda_6 & \lambda_1 & & & \lambda_3 & & \\ & -\Sigma & \lambda_5 & & \lambda_1 & & & \lambda_3 & \\ & & -\Sigma & & & \lambda_1 & & & \lambda_3 \\ & & & -\Sigma & \lambda_4 & \lambda_6 & \lambda_2 & & \\ & & & & -\Sigma & \lambda_5 & & \lambda_2 & \\ & & & & & -\Sigma & & & \lambda_2 \\ & & & & & & -\Sigma & \lambda_4 & \lambda_6 \\ & & & & & & & -\Sigma & \lambda_5 \\ & & & & & & & & -\Sigma \end{pmatrix}\end{array}$$

The coverage indicator α_i is defined as the conditional probability that a degraded state will be detected by the proof test of unit i, given that degradation has occurred when initiating the proof test.

α_i = Pr(Degradation is detected in a proof test |Degradation has occurred)

$\mathbf{M}$ represents the probability matrix of different states after a testing and repair action.

$\mathbf{M}_{U_1}$ represents the probability matrix of different states after a testing and repair action of unit 1.

$\mathbf{M}_{U_2}$ represents the probability matrix of different states after a testing and repair action of unit 2.

Matrix $\mathbf{M}$ for simultaneous testing with testing coverage α_i and maintenance strategy I

$$\mathbf{M}=\begin{array}{c} \\ 1\\2\\3\\4\\5\\6\\7\\8\\9 \end{array}\begin{array}{c}\begin{array}{ccccccccc} 1&2&3&4&5&6&7&8&9 \end{array}\\ \begin{pmatrix} 1 & & & & & & & & \\ \alpha_2 & 1-\alpha_2 & & & & & & & \\ 1 & & & & & & & & \\ \alpha_1 & & & 1-\alpha_1 & & & & & \\ \alpha_1\alpha_2 & (1-\alpha_2)\alpha_1 & & (1-\alpha_1)\alpha_2 & (1-\alpha_1)(1-\alpha_2) & & & & \\ \alpha_1 & & & 1-\alpha_1 & & & & & \\ 1 & & & & & & & & \\ \alpha_2 & 1-\alpha_2 & & & & & & & \\ 1 & & & & & & & & \end{pmatrix}\end{array}$$

Matrix $\mathbf{M}$ for staggered testing with testing coverage α_i and maintenance strategy II

$$\mathbf{M}_{U_1}=\begin{array}{c} \\ 1\\2\\3\\4\\5\\6\\7\\8\\9 \end{array}\begin{array}{c}\begin{array}{ccccccccc} 1&2&3&4&5&6&7&8&9 \end{array}\\ \begin{pmatrix} 1 & & & & & & & & \\ & 1 & & & & & & & \\ & & 1 & & & & & & \\ \alpha_1 & & & 1-\alpha_1 & & & & & \\ & \alpha_1 & & & 1-\alpha_1 & & & & \\ & & \alpha_1 & & & 1-\alpha_1 & & & \\ 1 & & & & & & & & \\ & 1 & & & & & & & \\ & & 1 & & & & & & \end{pmatrix}\end{array}$$

$$\mathbf{M}_{U_2}=\begin{array}{c} \\ 1\\2\\3\\4\\5\\6\\7\\8\\9 \end{array}\begin{array}{c}\begin{array}{ccccccccc} 1&2&3&4&5&6&7&8&9 \end{array}\\ \begin{pmatrix} 1 & & & & & & & & \\ \alpha_2 & 1-\alpha_2 & & & & & & & \\ 1 & & & & & & & & \\ & & & 1 & & & & & \\ & & & \alpha_2 & 1-\alpha_2 & & & & \\ & & & 1 & & & & & \\ & & & & & & 1 & & \\ & & & & & & \alpha_2 & 1-\alpha_2 & \\ & & & & & & 1 & & \end{pmatrix}\end{array}$$

Matrix **M** for staggered testing with testing coverage α_i and maintenance strategy III

$$\mathbf{M}_{U_1} = \begin{array}{c|ccccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \hline 1 & 1 & & & & & & & & \\ 2 & & 1 & & & & & & & \\ 3 & & & 1 & & & & & & \\ 4 & \alpha_1 & & & 1-\alpha_1 & & & & & \\ 5 & & \alpha_1 & & & 1-\alpha_1 & & & & \\ 6 & & & \alpha_1 & & & 1-\alpha_1 & & & \\ 7 & 1 & & & & & & & & \\ 8 & 1 & & & & & & & & \\ 9 & 1 & & & & & & & & \end{array}$$

$$\mathbf{M}_{U_2} = \begin{array}{c|ccccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \hline 1 & 1 & & & & & & & & \\ 2 & \alpha_2 & 1-\alpha_2 & & & & & & & \\ 3 & 1 & & & & & & & & \\ 4 & & & & 1 & & & & & \\ 5 & & & & \alpha_2 & 1-\alpha_2 & & & & \\ 6 & 1 & & & & & & & & \\ 7 & & & & & & & 1 & & \\ 8 & & & & & & & \alpha_2 & 1-\alpha_2 & \\ 9 & 1 & & & & & & & & \end{array}$$

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Chapter II.5

Fifth Paper

Combined Maintenance Scheduling and Production Optimization

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Optimal operation of complex production and processing plants is important, but challenging to achieve in practice. The reason for this is that decisions in different disciplines, such as design, operations and maintenance, are made independently of each other. This can lead to a large degree of conservativeness. In this paper, we present a unified approach for maintenance- and production planning, which reduces the conservativeness and leads to more economical operation. We model the system using differential equations and then formulate the problem of optimal operation as a numerical optimization problem. The problem is a mathematical program with equilibrium constraints (MPEC), which we solve using off-the-shelf optimization software. Some model approximations were made to make the system numerically tractable. We demonstrate the method on a subsea-inspired case example.

Keywords: Reliability modeling, maintenance scheduling, production optimization

1. Introduction

For certain classes of production systems there exist a trade-off between producing as much as possible, and prematurely wearing the system out. For example, in subsea oil and gas production systems the revenue is directly related with the amount of produced hydrocarbons, while a too high production rate may lead to fast system degradation, with expensive maintenance operations and possible production loss due to downtime.

From an economical point of view, there exists an optimal trade-off between the maintenance cost, the inspection cost and the operational profit. Moreover, when the system has degraded, the operator needs to make a decision relating to what degree the system should be restored. Using commonly employed Monte Carlo methods to obtain the optimal production profile and the optimal maintenance schedule, represents a significant challenge due to the sheer amount of possible scenarios that need to be explored. Numerical optimization seems like an attractive alternative to Monte Carlo methods due to the potential for faster convergence to an optimal solution.

In this paper we propose an integrated approach to operate the system optimally, that is, we propose to integrate the decisions on 1) the system load (how much to produce), 2) when to perform a maintenance operation, and 3) to what degree the system should be restored, in a unified framework.

To demonstrate our approach, we model a subsea oil and gas production system using a four-state Markov chain, where state A represents the new, healthy system, states B and C represent progressively degraded systems, and state D represents the failed, inoperable system. Arrival at the failed state D can be caused by unexpected sudden failure, or due to progressive degradation. The time dependent transition rates are a function of the input usage, thus yielding a multiphase Markov decision process. The production system model is described by a non-linear differential-algebraic equation system (DAE).

Optimal production and operation planning is defined as the case when the sum of the expected value of the revenue minus the inspection cost and

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the maintenance cost, is maximized. As decision variables in the optimization problem we assume the inspection times and the input profile. By inputs, we here mean the operating mode that the plant is running at. For example, the inputs of a compressor could be its throughput and frequency.

At each inspection, all systems found in the failed state D are restored to state A (if we follow the as-good-as-new (AGAN) policy). If the system is not found in state D, the system does not reveal whether its true state is A, B or C, and no maintenance is performed. Consequently, the model becomes a switched differential algebraic model, and the optimization problem can be formulated as a non-smooth, non-linear program. We solve this problem using state of the art methods for non-smooth optimization.

Authors of previously published work on the topic of combined maintenance and production planning typically formulate the problem as a lot-allocation problem (Iravani and Duenyas, 2002; Fitouhi and Nourelfath, 2012; Wolter and Helber, 2016). This often results in a mixed-integer (non-)linear program (MI(N)LP). Our proposed method is different as we do not consider "lots" of products, but rather the case where production can be adjusted in a continuous fashion. We also avoid the use of integer variables by formulating the problem with complementary constraints instead.

The remainder of the paper is structured as follows; in Section 2 we show how a continuous differential equation can be derived for the case of a degrading system with discrete inspection- and maintenance times. In Section 3 it is shown how the derived model can be used in the context of optimization, where the aim is to manipulate inputs and the maintenance times to minimize some objective function. In Section 4, the method is demonstrated on the subsea case example. Finally, concluding remarks and thoughts on future work are given in Section 5.

2. Modeling the degrading system

Given a four-state Markov process as shown in Fig. 1. Let $\boldsymbol{\mu}(t) = [\mu_A(t)\ \mu_B(t)\ \mu_C(t)\ \mu_D(t)]^\mathsf{T}$ denote the probabilities for being in any state at time t. Assuming time-invariant transition rates λ_a and λ_u between the states, the change in probabilities between two inspections is given by

$$\frac{d\boldsymbol{\mu}}{dt} = \boldsymbol{\Lambda}\cdot\boldsymbol{\mu}(t) \tag{1}$$
$$= (\boldsymbol{\Lambda}_a + \boldsymbol{\Lambda}_u)\cdot\boldsymbol{\mu}(t) \tag{2}$$

where

$$\boldsymbol{\Lambda}_a = \begin{bmatrix} -\lambda_a & 0 & 0 & 0 \\ \lambda_a & -\lambda_a & 0 & 0 \\ 0 & \lambda_a & -\lambda_a & 0 \\ 0 & 0 & \lambda_a & 0 \end{bmatrix} \tag{3}$$

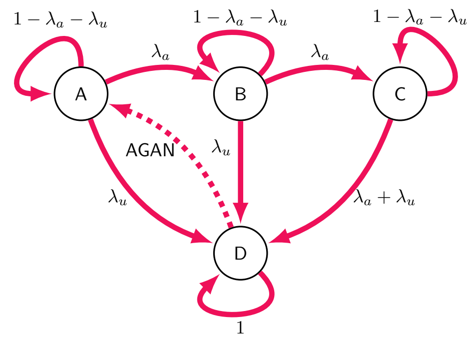

Fig. 1. Markov chain for a system with four discrete degradation states

and

$$\boldsymbol{\Lambda}_u = \begin{bmatrix} -\lambda_u & 0 & 0 & 0 \\ 0 & -\lambda_u & 0 & 0 \\ 0 & 0 & -\lambda_u & 0 \\ \lambda_u & \lambda_u & \lambda_u & 0 \end{bmatrix}. \tag{4}$$

In the above expressions, $\boldsymbol{\Lambda}$ is known as the transition matrix. $\boldsymbol{\Lambda}$ is decomposed into $\boldsymbol{\Lambda}_a$, which is describing the transitions due to aging, and $\boldsymbol{\Lambda}_u$, which is describing the transitions due to unforeseen failures.

Integrating Eq. (1) gives

$$\boldsymbol{\mu}(t) = \exp(\boldsymbol{\Lambda}\cdot(t-t_0))\boldsymbol{\mu}(t_0) \tag{5}$$

2.1. *Probabilities between two inspections*

Upon inspection of the system at time t_1, we reveal if the system is broken down (in state D), or not (either in state A, B or C). This leads to two different cases, depending on the outcome:

Case I:
Upon inspection, we find the system is in state D. Thus, we restore the system by performing maintenance (without time lag). Assuming perfect repairs according to the AGAN policy, the new initial conditions t_1 are

$$\boldsymbol{\mu}^+_{\text{Case I}}(t_1) = [1\ 0\ 0\ 0]^\mathsf{T} \tag{6}$$

and

$$\boldsymbol{\mu}_{\text{Case I}}(t > t_1) = \exp\left(\boldsymbol{\Lambda}\cdot(t-t_1)\right)\boldsymbol{\mu}^+_{\text{Case I}}(t_1) \tag{7}$$

Note that we use the notation $\boldsymbol{\mu}^+(t_1)$ to indicate the right-handed limit of $\boldsymbol{\mu}(t_1)$, i.e. directly after the inspection at t_1, and $\boldsymbol{\mu}^-$ to indicate the left-handed limit of $\boldsymbol{\mu}$, i.e. directly before the inspection at t_1. Because $\boldsymbol{\mu}$ is discontinuous at t_1, these two limits will generally not be equal.

Case II:
Upon inspection, we find that the system is not in state D. However, we do not know if the system

is in state A, B or C. The new initial conditions at t_1 are:

$$\boldsymbol{\mu}^{+}_{\text{Case II}}(t_1) = \begin{bmatrix} \frac{\mu^{-}_{A}(t_1)}{1-\mu^{-}_{D}(t_1)} & \frac{\mu^{-}_{B}(t_1)}{1-\mu^{-}_{D}(t_1)} & \frac{\mu^{-}_{C}(t_1)}{1-\mu^{-}_{D}(t_1)} & 0 \end{bmatrix}^{\mathsf{T}} \tag{8}$$

and

$$\boldsymbol{\mu}_{\text{Case II}}(t > t_1) = \exp\left(\boldsymbol{\Lambda} \cdot (t - t_1)\right) \boldsymbol{\mu}^{+}_{\text{Case II}}(t_1) \tag{9}$$

Expressing the probabilities from the perspective of t_0
However, we can not know ahead of time which of the two cases will be observed in the future (non-anticipativity). We must therefore forecast the probabilities into the future by taking the weighted average of both cases.

$$\begin{aligned} \boldsymbol{\mu}^{+}(t_1) &= \boldsymbol{\mu}^{+}_{\text{Case I}}(t_1) \cdot \mu^{-}_{D}(t_1) && (10)\\ &\quad + \boldsymbol{\mu}^{+}_{\text{Case II}}(t_1) \cdot \left(1 - \mu^{-}_{D}(t_1)\right) && (11)\\ &= \begin{bmatrix} \mu^{-}_{A}(t_1) + \mu^{-}_{D}(t_1) \\ \mu^{-}_{B}(t_1) \\ \mu^{-}_{C}(t_1) \\ 0 \end{bmatrix} && (12) \end{aligned}$$

Or, in matrix notation:

$$\begin{aligned} \boldsymbol{\mu}^{+}(t_1) &= \boldsymbol{M} \cdot \exp\left(\boldsymbol{\Lambda}_0 \left(t_1 - t_0\right)\right) \cdot \boldsymbol{\mu}(t_0) && (13)\\ &= \boldsymbol{M} \cdot \boldsymbol{\mu}^{-}(t_1) && (14) \end{aligned}$$

where we further decompose $\boldsymbol{M}$ as

$$\boldsymbol{M} = (\boldsymbol{I} + \boldsymbol{R}\boldsymbol{S}^{\mathsf{T}}) \tag{15}$$

where $\boldsymbol{I}$ is the identity matrix, $\boldsymbol{R}$ is the repair matrix, and $\boldsymbol{S}$ is a selection matrix. The selection matrix chooses the failed state D.

$$\boldsymbol{S} = [0\ 0\ 0\ 1]^{\mathsf{T}} \tag{16}$$

For AGAN repairs, we have

$$\boldsymbol{R} = [1\ 0\ 0\ {-1}]^{\mathsf{T}}\,. \tag{17}$$

In Section 2.2, we will come back to why the decomposition of $\boldsymbol{M}$ into $\boldsymbol{R}$ and $\boldsymbol{S}$ is useful.

The evolution of the probabilities for case 1 and 2, and the weighted average of the two cases is illustrated in Figure 2.

Generalization
Using the general expression for the probabilities between two maintenance stops from Eq. (5), we can express the probabilities at any time t as the piece-wise function

$$\boldsymbol{\mu}(t) = \begin{cases} \exp\left(\boldsymbol{\Lambda} \cdot (t - t_0)\right) \cdot \boldsymbol{\mu}^{+}(t_0) & \text{if} \quad t < t_1 \\ \exp\left(\boldsymbol{\Lambda} \cdot (t - t_1)\right) \cdot \boldsymbol{\mu}^{+}(t_1) & \text{if} \quad t > t_1 \end{cases} \tag{18}$$

where

$$\boldsymbol{\mu}^{+}(t_0) = \boldsymbol{\mu}(t_0) = \boldsymbol{\mu}_0 = [1\ 0\ 0\ 0]^{\mathsf{T}} \tag{19}$$

is the specified initial condition.

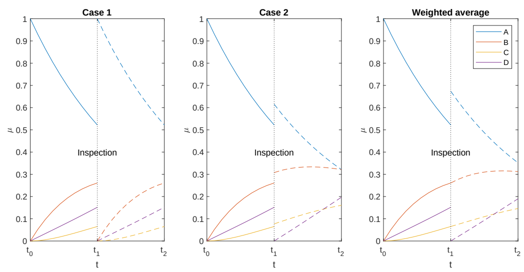

Fig. 2. Illustration of evolution of the probabilities μ before and after inspection. Left: system is found to be in the failed state D upon inspection, and is repaired without time lag. Middle: system is found to be in working order upon inspection, and no repairs are performed. Right: weighted average of the two previous cases.

Taking it one step further, we can have an arbitrary amount of inspections and maintenances, k, between two times t_0 and t_f and express the probabilities as

$$\boldsymbol{\mu}(t) = \begin{cases} \exp\left(\boldsymbol{\Lambda} \cdot (t - t_0)\right) \cdot \boldsymbol{\mu}^{+}(t_0) \\ \quad \text{if} \quad t_0 < t < t_1 \\ \exp\left(\boldsymbol{\Lambda} \cdot (t - t_1)\right) \cdot \boldsymbol{\mu}^{+}(t_1) \\ \quad \text{if} \quad t_1 < t < t_2 \\ \cdots \\ \exp\left(\boldsymbol{\Lambda} \cdot (t - t_k)\right) \cdot \boldsymbol{\mu}^{+}(t_k) \\ \quad \text{if} \quad t_k < t < t_f \end{cases} \tag{20}$$

where

$$\begin{aligned} \boldsymbol{\mu}^{+}(t_i) &= \boldsymbol{M} \cdot \boldsymbol{\mu}^{-}(t_i) && (21)\\ \boldsymbol{\mu}^{-}(t_i) &= \exp\left(\boldsymbol{\Lambda} \cdot (t_i - t_{i-1})\right) \cdot \boldsymbol{\mu}^{+}(t_{i-1}) && (22) \end{aligned}$$

2.2. *Differentiating to get the differential model*

Equation (1) described the evolution of the state $\boldsymbol{\mu}$ between two inspection times. However, as we have shown, we can express the state at any given time by the piece-wise model from equation (20). If we differentiate (20), we get

$$\frac{d\boldsymbol{\mu}}{dt} = \boldsymbol{\Lambda} \cdot \boldsymbol{\mu}(t) + \boldsymbol{R}\boldsymbol{S}^{\mathsf{T}}\boldsymbol{\mu}(t) \cdot \left. \sum_{i=1}^{k} \delta(t - t_i) \right) \tag{23}$$

where δ is the Dirac delta function.

Let us now introduce the variable

$$\boldsymbol{r}(t) = \boldsymbol{S}^{\mathsf{T}}\boldsymbol{\mu}(t) \cdot \left. \sum_{i=1}^{k} \delta(t - t_i) \right) \tag{24}$$

to obtain the form

$$\frac{d\boldsymbol{\mu}}{dt} = \boldsymbol{\Lambda} \cdot \boldsymbol{\mu}(t) + \boldsymbol{R} \cdot \boldsymbol{r}(t). \tag{25}$$

We choose to work with this form of the reliability model as it allows us to distinguish between the degradation of the system due to aging and unforeseen failures (first term of Eq. (25)), and the maintenance of the system (second term of Eq. (25)). Our aim is to do numerical optimization of the maintenance times, which can be achieved by optimizing the breakpoints t_i of the function $\boldsymbol{r}(t)$. More on this in Section 3.

Furthermore, we can easily change the maintenance strategy from as-good-as-new (AGAN) to as-bad-as-old (ABAO) by changing $\boldsymbol{R}$

$$\boldsymbol{R}_{\text{AGAN}} = [1\ 0\ 0\ -1]^{\intercal} \tag{26}$$

$$\boldsymbol{R}_{\text{ABAO}} = [0\ 0\ 1\ -1]^{\intercal} \tag{27}$$

An illustration of how $\boldsymbol{\mu}(t)$ changes as a function of $\boldsymbol{r}(t)$ is shown in Fig. 3. Note that $\boldsymbol{r}(t)$ is a sum of Dirac functions. In addition, we show the integral $\int_0^{t_f} \boldsymbol{r}(t)$, which is proportional to the maintenance cost.

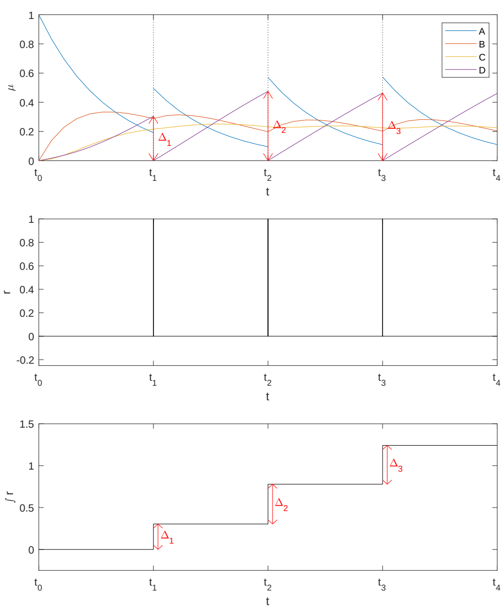

Fig. 3. Illustration of how $\boldsymbol{r}(t)$ influences $\boldsymbol{\mu}(t)$. The cumulative maintenance cost is proportional to the integral of $\boldsymbol{r}$, shown in the bottom plot, while the inspection cost is proportional to number of spikes (three, in this case)

2.3. *Modeling the effect of inputs*

The second contribution of this paper is the inclusion of the effect of inputs $\boldsymbol{u}(t)$, which influence the degradation rate of the system. Thus by changing $\boldsymbol{u}(t)$, we can actively steer the rate of degradation of the system. This is very useful, as it allows us to optimize the performance of the system by co-optimizing $\boldsymbol{u}(t)$ and $\boldsymbol{r}(t)$. We model this behavior by letting $\boldsymbol{\Lambda}_a$ be a function of the inputs and time, instead of a constant matrix like before. $\boldsymbol{\Lambda}_u$ remains constant, as we assume that unexpected failures cannot be influenced by changing the inputs. The differential model now is

$$\frac{d\boldsymbol{\mu}}{dt} = (\boldsymbol{\Lambda}_a(\boldsymbol{u}, t) + \boldsymbol{\Lambda}_u) \cdot \boldsymbol{\mu}(t) + \boldsymbol{R} \cdot \boldsymbol{r}(t). \tag{28}$$

Note that if $\boldsymbol{\Lambda}_a(\boldsymbol{u}, t)$ is a piece-wise constant function, meaning we can write it as

$$\boldsymbol{\Lambda}_a(\boldsymbol{u}, t) = \begin{cases} \boldsymbol{\Lambda}_{a,1} & \text{if} \quad t_0 < t < t_1 \\ \boldsymbol{\Lambda}_{a,2} & \text{if} \quad t_1 < t < t_2 \\ \dots & \\ \boldsymbol{\Lambda}_{a,k} & \text{if} \quad t_k < t < t_f \end{cases}, \tag{29}$$

the system becomes a Multiphase Markov process. However, we do not require this assumption, and are free to choose whatever form of $\boldsymbol{\Lambda}_a(\boldsymbol{u}, t)$ we need.

3. Formulating the optimization problem

In order to find the optimal combined production and maintenance strategy, we first formulate an optimization problem in terms of an objective function and constraints. [a]

$$\min_{\boldsymbol{u},\boldsymbol{r}} \quad \int_{t_0}^{t_f} \Big(-f(t, \boldsymbol{\mu}, \boldsymbol{u}) \Big) dt + f_i(t, \boldsymbol{\mu}, \boldsymbol{r}) + f_m(t, \boldsymbol{\mu}, \boldsymbol{r}) \tag{30a}$$

$$\text{s.t.} \quad \frac{d\boldsymbol{\mu}}{dt} = \boldsymbol{\Lambda}(t, \boldsymbol{u}) \cdot \boldsymbol{\mu}(t) + \boldsymbol{R} \cdot \boldsymbol{r}(t) \tag{30b}$$

$$\boldsymbol{r}(t) = \boldsymbol{S}^{\intercal} \boldsymbol{\mu}(t) \cdot \left(\sum_{i=1}^{k} \delta(t - t_i) \right) \tag{30c}$$

$$0 \le \boldsymbol{\mu} \le 1 \tag{30d}$$

$$\sum_{i \in \{A,B,C,D\}} \mu_i = 1 \tag{30e}$$

$$0 \le \boldsymbol{r} \le \infty \tag{30f}$$

$$\boldsymbol{u}_{\min} \le \boldsymbol{u} \le \boldsymbol{u}_{\max} \tag{30g}$$

In the above optimization problem, f denotes some economical objective which is to be maximized (typically profit or production), f_i denotes

[a] Note that the constraint in Eq. (30e) is implied by Eq. (30b), but we include it for completeness.

the inspection cost, which is typically proportional to the number of inspections k,

$$f_i(t, \boldsymbol{\mu}, \boldsymbol{u}) \propto k, \tag{31}$$

and f_m denotes the maintenance cost, which is assumed to be proportional to the integral of $\boldsymbol{r}(t)$

$$f_m(t, \boldsymbol{\mu}, \boldsymbol{u}) \propto \int_{t_0}^{t_f} \boldsymbol{r}(t)dt. \tag{32}$$

3.1. *Problem re-formulation for numerical optimization*

The optimization problem from Eqs. (30a)-(30g) can be solved in a multitude of ways. A common approach is to approximate the dynamic problem by a static non-linear programming (NLP) problem through the use of so-called direct methods, where direct multiple shooting and direct collocation are common approaches (Biegler, 2010). In this work, we use the direct collocation approach.

One issue with (30) is in $\boldsymbol{r}(t)$. Since it is the summation of Dirac functions, it is unbounded as shown in Eq. (30f). An alternative to using the formulation adopted in this paper is to formulate the problem as a mixed integer problem, as was done in Ashayeri et al. (1996). Due to the nonlinear nature of the problem, we have to solve a mixed-integer non-linear programming (MINLP) problem, as done in e.g. And and Grossmann (1998); Georgiadis and Papageorgiou (2000). These MINLP problems are however known to be very difficult to solve in the general case, despite recent progress in algorithmic development.

Instead, we approximate $\boldsymbol{r}(t)$ using Boxcar functions as

$$\boldsymbol{r}(t) \approx \tilde{\boldsymbol{r}}(t) \tag{33}$$

$$\tilde{\boldsymbol{r}}(t) = \sum_{i=1}^{k} \text{Boxcar}(t) \tag{34}$$

$$= \sum_{i=1}^{k} h_i \Big(\text{H}(t - t_i) - \text{H}(t - t_i - \epsilon_i) \Big) \tag{35}$$

where H is the Heaviside function, h_i is the height and ϵ_i is the width of each "box".

An illustration of this approximation is shown in the middle plot in Fig. 4. Note that the approximation for $\boldsymbol{\mu}$ is good if ϵ is sufficiently small, and that the approximation gives the same cumulative maintenance cost $\int \tilde{\boldsymbol{r}}(t)dt$. Furthermore, we observe that $\boldsymbol{\mu}$ is now continuous (although nonsmooth), which makes the optimization problem easier to solve.

Another numerical issue is posed by the inspection cost from Eq. (31). In the original

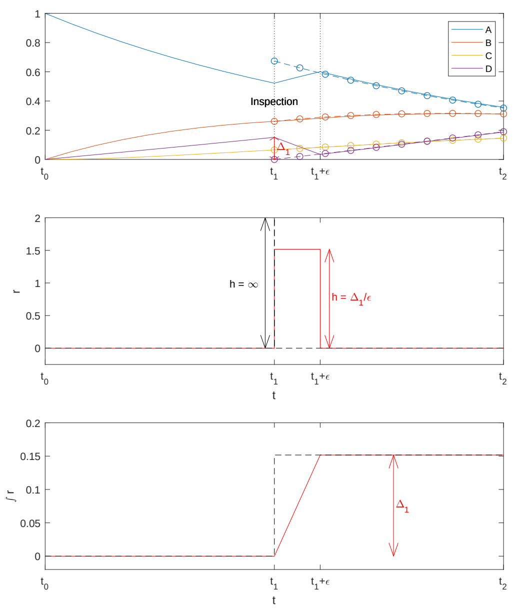

Fig. 4. Illustration of how $\boldsymbol{r}(t)$ (dashed line, circles) can be approximated by $\tilde{\boldsymbol{r}}(t)$ (solid line) to obtain a continuous $\boldsymbol{\mu}$.

formulation with Dirac functions, one might be tempted to find k as

$$k = \int_{t_0}^{t_f} \Big(\sum_{i=1}^{k} \delta(t - t_i) \Big) dt \tag{36}$$

$$= \int_{t_0}^{t_f} \Big(\big(\boldsymbol{S}^\mathsf{T} \boldsymbol{\mu}(t) \big)^{-1} \cdot \boldsymbol{r}(t) \Big) dt, \tag{37}$$

but for this to work, we must assert that

$$\mu_D(t_i) = \mu_D^-(t_i) \tag{38}$$

to avoid division by zero. Such a condition might be difficult to impose numerically.

Instead, we propose to solve the problem by introducing the additional variable $\boldsymbol{y}$, which we use to formulate additional constraints:

$$0 \leq (1 - \boldsymbol{y}) \perp \tilde{\boldsymbol{r}} \geq 0 \tag{39}$$

$$0 \leq \boldsymbol{y} \leq 1 \tag{40}$$

Here, the $\perp$ operator indicates complementary, i.e. we require that at all times either $\tilde{\boldsymbol{r}}$ or $(1 - \boldsymbol{y})$ or both are zero. The inspection cost can then be written as

$$f_i(t, \boldsymbol{\mu}, \boldsymbol{u}) \propto \frac{\boldsymbol{y}}{\epsilon} \tag{41}$$

In order to minimize the cumulative inspection cost, $\boldsymbol{y}(t)$ will be a function that is either at its lower bound (zero) when no inspection is performed, or at its upper bound (one) when inspection is performed. By integrating we get

$$k = \int_{t_0}^{t_f} \Big(\sum_{i=1}^{k} \delta(t - t_i) \Big) dt \approx \int_{t_0}^{t_f} \frac{\boldsymbol{y}}{\epsilon} dt \tag{42}$$

4. Case study

As a case example, we consider a system inspired by subsea oil and gas production. Subsea technology is key to satisfying the energy demands of tomorrow, due to the intermittent nature of renewables and the continued need for petroleum products also in a green society. Reliability is a major issue for subsea installations, as maintenance interventions are very costly. Consequently, it is important to optimize both production from the subsea installation, as well as the maintenance interventions.

Assume that production from the subsea installation actively degrades critical components such as pumps, valves and heat exchangers. The transition rates can therefore be assumed to be proportional to the inputs $\boldsymbol{u}$ that we apply to the system. In our case, $\boldsymbol{u}(t)$ represents the production rate of oil and gas. A higher production rate will give more immediate profit, but also increased degradation.

Poor instrumentation and a lot of measurement uncertainty mean that a system may not be properly diagnosed to have failed without inspection. An example of this could be the failure of a single well going to a manifold with several other wells. The failure of the single well may be masked by the large variability in production of the other wells. Well tests (which can be thought of as inspections) are thus required to reveal the state of the single well.

4.1. *Objective function*

The objective is to maximize the average production from the well over the lifetime of the field, while simultaneously minimizing the inspection and maintenance costs. This economical objective can be written as

$$\min_{\boldsymbol{u}(t), \tilde{\boldsymbol{r}}(t)} \int_{0}^{t_f} \Big(\frac{-f + f_m + f_i}{(1+d)^t} \Big) dt \tag{43a}$$

where

$$f(t) = \boldsymbol{c}_p^{\mathsf{T}} \cdot \boldsymbol{\mu}(t) \cdot \boldsymbol{u}(t) \tag{43b}$$

$$f_m(t) = \boldsymbol{c}_m^{\mathsf{T}} \cdot \tilde{\boldsymbol{r}}(t) \tag{43c}$$

$$f_i(t) = \boldsymbol{c}_i^{\mathsf{T}} \cdot \boldsymbol{y}(t). \tag{43d}$$

Here, $\boldsymbol{c}_p$ is the productivity in each state, $\boldsymbol{c}_m$ is the maintenance cost, $\boldsymbol{c}_i$ is the inspection cost. The entire economic objective is discounted by a factor d to reflect the decreasing value of future income streams compared to present income streams. Note that the objective is non-linear due to the bi-linear term in $f(t)$.

4.2. *Constraints*

The objective function is optimized subject to the following constraints

$$s.t. \quad \frac{d\boldsymbol{\mu}}{dl} = \boldsymbol{\Lambda}(t, \boldsymbol{u})\boldsymbol{\mu}(t) + \boldsymbol{R}\tilde{\boldsymbol{r}}(t) \tag{43e}$$

$$\boldsymbol{\Lambda}(t, \boldsymbol{u}) = \boldsymbol{\Lambda_a} \cdot \boldsymbol{u}(t) + \boldsymbol{\Lambda}_u \tag{43f}$$

$$\boldsymbol{\mu}(0) = \boldsymbol{\mu}_0 \tag{43g}$$

$$0.1 \leq \boldsymbol{u} \leq 1.0 \tag{43h}$$

$$0 \leq \tilde{\boldsymbol{r}} \leq \frac{1}{\epsilon_{\min}} \tag{43i}$$

$$\epsilon_{\min} \leq \boldsymbol{\epsilon} \leq \epsilon_{\max} \tag{43j}$$

$$0 \leq \boldsymbol{\mu} \leq 1 \tag{43k}$$

$$0 \leq (1 - \boldsymbol{y}) \perp \tilde{\boldsymbol{r}} \geq 0 \tag{43l}$$

$$0 \leq \boldsymbol{y} \leq 1 \tag{43m}$$

Note that the transition matrix $\boldsymbol{\Lambda}(t, \boldsymbol{u})$ is linear in $\boldsymbol{u}$. Since we require $\boldsymbol{u}(t)$ to be a piece-wise constant function, $\boldsymbol{\Lambda}(t, \boldsymbol{u})$ is also a piece-wise function. Consequently we are dealing with a Multiphase Markov process, as discussed in Section 2.3.

The parameters for the problem are summarized in Table 1.

Table 1. Parameters used for the optimization

Parameter	Description	Value
λ_u	Sudden failure transition rate	10^{-4}
λ_a	Base aging transition rate	10^{-2}
d	Discount rate	.001
$\boldsymbol{c}_p$	Productivities in each state	$\begin{bmatrix}28 & 21 & 14 & 2.8\end{bmatrix}^{\mathsf{T}}$
$\boldsymbol{c}_m$	Maintenance cost	300
$\boldsymbol{c}_i$	Inspection cost	30
t_f	Final time	200 weeks

4.3. *NLP formulation*

Multiple approaches exist to solve dynamic problems like (43), but we choose to use orthogonal collocation on finite elements. The original dynamic problem is reformulated as an NLP, which can be solved using standard non-linear optimization algorithms. We will not go into details about how to discretize the problem, see e.g. Biegler (2010) for a summary.

The resulting NLP is implemented in MATLAB using Casadi 3.4.1 (Andersson et al., 2018). The interior-point solver IPOPT 3.12.3 (Wächter and Biegler, 2006) is used to solve the optimization problem.

4.4. *Solution strategy*

Our problem is non-convex and local solvers such as IPOPT will consequently only find local solutions. In other words, we cannot guarantee global optimality of the solution. In order to ensure global optimality, global solvers such as BARON (Sahinidis, 2017) have to be used. Global solvers come with some drawbacks, such as being computationally intractable for large problems.

To remedy this, we use a multi-start approach where the problem is repeatedly re-optimized with different initial guesses. After a certain number of optimizations (1000 in this case), the best local minimum is returned. Fewer than 1000 optimization runs could suffice, but as they are computationally cheap, we choose to run 1000 to ensure that a good solution is obtained.

4.5. *Solution*

The optimized production and maintenance strategy is shown in Fig. 5. As can be seen, the optimal strategy is to operate at maximum $\boldsymbol{u}$ all the time (a typical property of almost-linear optimization problems), while inspections / maintenance is performed at $t = 88$, 127 and 160 weeks. The objective function value is 3924 M$. Note that the first inspection is quite late. The reason for this is that the probabilities of being in the degraded states are initially low. As a result, performing inspections and maintenance at an early stage in the race is sub-optimal.

In an actual implementation, one would re-optimize the problem upon obtaining new information about the system state (such as after an inspection). This is known as model-predictive control (MPC) or rolling horizon optimization (Biegler, 2010). However, the optimization problem itself remains the same with only the initial conditions (43g) changing. We therefore chose to skip the closed-loop results in the interest of time and space.

5. Conclusions and future work

In this paper, we have introduced a method for simultaneous production- and maintenance optimization. The problem was motivated by a Markov-chain representation of a degrading production system, which would have been difficult to optimize using the traditional Monte Carlo based approach. We reformulated the problem as a algebraic-differential equation system, which we solved using a non-linear optimization approach. While not all problems can be solved like this, we showed how for the specific problem at hand, the problem could be cast into a form which could be solved using off-the-shelf solvers. The concept of input-dependent transition rates can easily be included in the framework. Some approximations were introduced to make the problem numerically tractable. The method was demonstrated on a case example inspired by subsea oil and gas production.

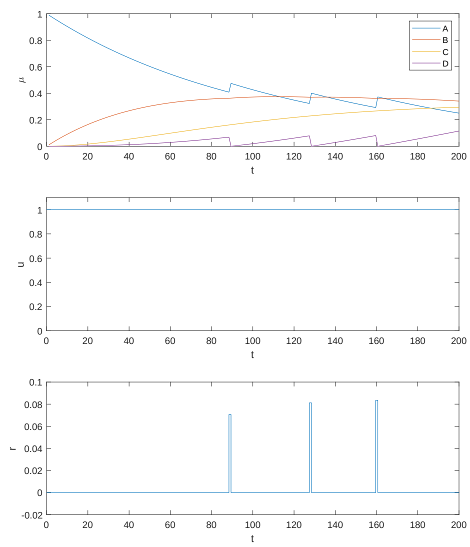

Fig. 5. Optimal solution of the case example

Possible future research directions include:

- Detailed comparison to Monte Carlo-based methods for optimization.
- Inclusion of more maintenance strategies by modification of $\boldsymbol{R}$ and optimization of the trade-off between the different maintenance strategies.
- A distributionally robust problem formulation to safeguard the solution against uncertainties in the transition rates.
- A multi-step approach to include the value of future information in the open-loop optimization problem.
- Looking at the case where maintenance is not instantaneous, i.e. when there is lag-time.
- Analysis of the losed-loop performance.
- A more complex case study with multiple simultaneously degrading units

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Chapter II.6

Sixth Paper

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Simultaneous Optimization of Production and Maintenance Schedules

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***Abstract*—Many systems in process industry experience degradation which is dependent on the usage of the system. At the same time, the systems generate profit when utilized, which typically increases the more the system is used. Often, this leads to a trade-off between maintenance costs and the profit associated with the utilization of the system. How to optimally operate the system in order to achieve a desired trade-off is a little-explored topic in literature. Common practice in many industries is to optimize operational decisions and maintenance decisions more or less independently of each other, subject to constraints to ensure feasibility of the obtained decisions. This leads to sub-optimal utilization of the system. In this paper, we propose a framework for formulating optimization problems which can be solved to obtain operational strategies which simultaneously optimize the trade-off between production profit and maintenance costs. We show how the discrete decisions pertaining to maintenance times can be approximated by a continuous model and solved using off-the-shelf optimization solvers. Because a global solution cannot be guaranteed using our proposed method, we employ a heuristic multi-start approach to find a near-global solution. We demonstrate the method on a case study inspired by subsea oil and gas production.**

***Index Terms*—Maintenance optimization, maintenance modeling, production optimization, degrading systems**

I. Introduction

In this paper, we propose a unified approach for simultaneous optimization of the production or the system load, and maintenance schedules. Within the process systems engineering community, the development of operational strategies that maximize some economic objective of a chemical production plant, subject to various constraints, is an important research question. How to optimally adjust operational degrees of freedom to achieve higher throughput, while still ensuring that the products are on-spec, is addressed on several time-scales and levels of system complexity, all the way from operation of single components to plant-wide operation [1].

Since plants like these inevitably degrade over time, maintenance interventions need to be performed. The optimization of maintenance schedules is also a much-explored research topic. Especially the field of condition-based maintenance (CBM) has attracted a lot of attention in recent years, both from academia and from industrial practitioners [2]. The main idea behind CBM is to monitor the degradation of the plant, so that maintenance can be performed when necessary according to some safety or economic criterion. This is turn leads to more efficient use of money, time and other resources by avoiding premature (preventive) maintenance. However, in certain industries, continuous process monitoring may not be possible due to lack of technology or prohibitive cost. One such example is subsea oil and gas production, where usually real-time equipment monitoring is currently not available due to lack of qualified instrumentation. Another thing that further complicates the issue is the fact that inspections are very costly, due to the need for special inspection remotely operated vehicles (ROVs). Depending on the location of the subsea installation, inspection might also be impossible to perform for long periods of time due to weather conditions. In such cases, there might be significant economical benefits to finding an optimum inspection and maintenance schedule in a rigorous manner.

In this work, we will therefore focus on such cases where continuous monitoring is not possible, and there is significant benefit to finding an optimal maintenance schedule. For these kind of applications, literature has traditionally focused on finding the optimum frequency of inspections for a periodic (equidistant in time) inspection schedule [3]. However, as we shall discuss later in this paper, periodic maintenance schedules are not optimal in general. One reason for the focus on period schedules in literature is that the tools commonly employed for finding these schedules are not well suited for the aperiodic case.

A major problem associated with maintenance optimization is that the problems tend to be stochastic. Since degradation of process plants is typically not deterministic (either due to inherent stochasticity of the process, or insufficient information), failure times and consequently maintenance costs are usually stochastic in nature. Analytical expressions for the distributions may exist, but could be difficult to express. Consequently, a popular approach to obtain the distributions is by repeated simulation of the system by the use of sampling (Monte Carlo) methods. Once the distribution has been obtained, we can use it to evaluate the maintenance schedule according to some criteria, such as the expected cost or the value at risk.

During optimization of this criteria, we may need to evaluate the cost function multiple times. Every single one of these function evaluations may be very computationally expensive, due to the need for a large number of Monte Carlo samples when the dimensionality of the problem is large. If the aim is to find optimal non-periodic maintenance schedules, i.e. if maintenance can occur at any point in time and not just at fixed times (maintenance time is a continuous variable),

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the search space becomes infinitely large and many function evaluations are necessary for the optimization algorithm to converge. Extensions of traditional/textbook Monte Carlo algorithms have been developed to somewhat reduce the amount of samples necessary for each evaluation of the cost function by choosing "smart" sample points. Unfortunately, this only partially mitigates the problem, and evaluations of the cost function through the use of sampling methods tends to be computationally expensive for large problem sizes.

As an alternative to sample-based methods for the evaluation of the cost, we may use numerical integration, which is generally computationally efficient if the functions are sufficiently smooth and convex. An underlying assumption is that we are able to express the evolution of the probabilities as a differential equation. However, it is not necessary that a closed-form solution of the integral exists. The advantage of expressing the cost as a numerical integration is that we can subsequently use theory from the field of dynamic optimization to find the optimal maintenance schedule. Algorithms for numerical optimization can efficiently solve certain large-scale decision making problems, with hundreds of thousands or even millions of variables [4]. Sampling-based methods for evaluating the cost would be infeasible for these large-scale applications due to the sheer amount of cost function evaluations required.

In this paper, we propose a unified approach for simultaneous optimization of the production and maintenance schedules. Note that because the problem is stochastic, we optimize a statistic of the probability density function. Here, we chose to optimize the expected economical profit of the system by assigning an economic value to each of the discrete degradation states. Note also that the obtained strategies are only optimal in the expected sense, and not optimal for any one given realization of the uncertainties. We start by using a differential model to describe the degradation of the system and resetting the initial conditions each time maintenance is performed. We then show how this piece-wise, discontinuous model can be approximated by a non-smooth, continuous model which is more suitable for optimization.

There exists a body of papers dedicated to combined production and maintenance optimization. Many of them formulate the problem as an optimal-control problem. In [5], a stochastic optimization model is developed, in which production, subcontracting and preventive maintenance are considered simultaneously. The Hamilton-Jacobi-Bellman (HJB) optimality conditions are formulated for the optimal control problem, and a discretized version of the problem is solved using a simulation-based approach. In [7], the problem of combined preventive maintenance and the repair/replacement policy of a failure-prone manufacturing system is addressed. The optimal policies, which minimize the average cost over an infinite planning horizon, are calculated numerically.

Numerical approaches to combined production optimization and maintenance scheduling were previously addressed in works by e.g. [8, 9, 10, 11, 12]. Typically, the problem is formulated as a lot-allocation problem, meaning that the operation is split into different modes or lots of products, the sequence of which is to be optimized. Consequently, the problems end up being mixed-integer (non-) linear programs (MI(N)LP). Generally, MINLPs are very difficult to solve, with solution times being orders of magnitude larger than for NLP problems of comparable size and complexity. In general, solution time of MINLPs grow exponentially with the number of integer variables [13]. Although algorithms have been developed to deal with mixed-integer problems, we will avoid the use of integer variables (because this leads to much more difficult optimization problems) by reformulating the problem.

In this work, we reformulate the problem using complementary constraints, such that it can be solved using off-the-shelf NLP solvers to obtain local solutions efficiently [13]. Furthermore, we consider the case where operation of the equipment can be adjusted in a continuous fashion, which is also a largely un-explored topic in reliability literature.

Several methods for optimization of non-smooth problems exist in literature. Among the most popular methods are subgradients methods and bundle methods. See e.g. [14] for an in-depth discussion on the topic and the different methods. In our case, the non-smoothness stems from the introduction of complementary constraints. Using problem reformulations for mathematical programs with complementarity constraints (MPCCs) from [15], the resulting problem can be solved using of-the-shelf standard non-linear solvers such as IPOPT [16]. One disadvantage of using local solvers is that only locally optimal solutions can be guaranteed. Depending on the non-convexity of the problem, the local solutions may be significantly worse we use a multi-start approach to converge to a solution close to the global optimum in this paper. A subsea case example is used to demonstrate the proposed method.

The remainder of the paper is structured as follows; in Section III.1, we use a small toy example to show how a non-periodic maintenance schedule can result in higher expected net profit than a periodic maintenance schedule, motivating the rest of the paper. Then in Section II we show how to derive the general continuous differential model. In Section III-A we show how to formulate an optimization problem to optimize the net profit of a system by adjusting its inspections and operation. Approximations to make the problem numerically tractable are shown in Section IV. The case study illustrating our method is presented in Section V. Finally, concluding remarks are given in Section VI.

II. MODELING FRAMEWORK FOR DEGRADING SYSTEMS

A. Modelling degrading systems

By "degradation" of process systems, we mean the evolution of a health state or reliability state in time. The evolution in time can be modeled either discretely or continuously. In reliability engineering, continuous-time, discrete-state models are the most common [17, 18]. Consequently, in this work, we limit ourselves to study systems whose degradation can not be determined with arbitrary fidelity, but whose degree of degradation can be divided into n_x distinguishable, discrete degradation levels. Let $X(t)$ denote the state of the system at time t. The state space is the set containing all possible states of the system, and is denoted by

$$\mathcal{X} = \{1, 2, ..., n_x\} \tag{1}$$

The transition between these discrete degradation levels is stochastic in nature, and happens at randomly distributed times. Let us denote by $\boldsymbol{x}(t) : \mathbb{R} \rightarrow \mathbb{R}^{n_x}$ a vector of probabilities

$$\boldsymbol{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_{n_x}(t) \end{bmatrix}, \tag{2}$$

where we define

$$x_i(t) = \Pr(X(t) = i) \tag{3}$$

as the probability that the system is found in that particular degradation state i at time t, such that

$$0 \leq \quad x_i(t) \quad \leq 1 \tag{4}$$

$$\sum_{i=1}^{n_x} \quad x_i(t) \quad = 1. \tag{5}$$

In the following sections, we sometimes simplify notation by not indicating the time dependency of $\boldsymbol{x}$:

$$\boldsymbol{x} = \boldsymbol{x}(t). \tag{6}$$

In the general case the evolution of the probabilities is given by the following ordinary differential equation (ODE)

$$\frac{d\boldsymbol{x}}{dt} \quad = \quad f(\boldsymbol{x}, t), \tag{7a}$$

f is known as the state equation, and is assumed to be piecewise continuous in time and locally Lipschitz, meaning that at a point $\boldsymbol{x}_1$, there exists a neighborhood around $(\boldsymbol{x}_1, t)$, $N(\boldsymbol{x}_1, r) = \{\boldsymbol{x} \in \mathbb{R}^n | \|\boldsymbol{x} - \boldsymbol{x}_1\| < r\}$ within which f satisfies the Lipschitz condition.

$$\|f(\boldsymbol{x}_1, t) - f(\boldsymbol{x}_2, t)\| \leq L\|\boldsymbol{x}_1 - \boldsymbol{x}_2\| \tag{8}$$

where $L > 0$. If there exists a connected open subspace D in which all points are locally Lipschitz, then f is said to be locally Lipschitz in D.

Remark II.1

Requiring this condition for f guarantees that the solution of the ODE (7) *does not only exist, but is also unique. Uniqueness and existence of the solution is necessary to guarantee causal determinism. In other words, if we have perfect information about the system and the input-induced loads acting on it, we can predict the future state probabilities. If we were not able to predict the future of the system, we would be unable to optimize decisions influencing the future of the system, as we shall do later.*

Example II.1 (Application to CTMC)

One method for describing the dependability or availability of a system is with continuous time Markov chains (CTMCs). The transition time between the discrete degradation states is random, and the transition rates between two states are Markovian, meaning that they only depend on the current state of the system. For the case where the transition rates between the states are constant over time, the evolution of the distribution of states over time in a CTMC can be written mathematically as

$$\frac{d\boldsymbol{x}}{dt} \quad = \quad \boldsymbol{A}\boldsymbol{x} \tag{9}$$

where $\boldsymbol{A}$ is known as the transition rate matrix. In this case, the ODE (7) is a linear system. Alternatively, in some textbooks on CTMCs, the above equation is written as

$$\frac{d\boldsymbol{x}^{\intercal}}{dt} \quad = \quad \boldsymbol{x}^{\intercal}\boldsymbol{A}^{\intercal}, \tag{10}$$

where $\boldsymbol{x}^{\intercal}$ indicates the transpose of $\boldsymbol{x}$. We prefer to use the former notation, where $\boldsymbol{x}$ is a column vector, and will continue to use this notation throughout the rest of this article.

B. Modeling the input-induced loads

As alluded to in the introduction, there are many examples of processes where the rate of degradation depends on the operation of the system. For example, [19] reports that the degradation of airplane gas turbine engines evolves exponentially, but the rate of degradation is impacted by the operating mode of the airplane, including parameters such as altitude, ambient temperature and throttle lever angle.

In marine vessels, propeller cavitation and fatigue due to cyclic loading are two phenomena which negatively influence the efficiency of a vessel propulsion system [20]. These degradation phenomena are influenced by the operating mode of the vessel, so that higher vessel speeds lead to faster degradation.

Let us denote by $\boldsymbol{u}(t) : \mathbb{R} \rightarrow \mathbb{R}^{n_u}$ the input-induced load on (or simply inputs to) the system. For the sake of readability, we sometimes simplify the notation in the following sections by not indicating the time dependency of $\boldsymbol{u}$

$$\boldsymbol{u} = \boldsymbol{u}(t) \tag{11}$$

We assume that the state equation $f(\boldsymbol{x}, \boldsymbol{u}, t)$ is a function of these input-induced loads. Consequently, by changing $\boldsymbol{u}$, we can actively steer the evolution of the system degradation in a desired direction.

If f is Lipschitz in $\boldsymbol{x}, \boldsymbol{u}, t$, then the solution $\boldsymbol{x}$ exists and is unique [21].

C. Modeling repairs

In the context of this work, let us by "repairs" refer to the action of resetting the state of the system, i.e. shifting probability from one discrete health state to another. In practice, the probability of being in the degraded or failed state will be reduced and the probability for being in a healthy state will increase correspondingly, i.e. the system will be repaired. Note that since we are dealing with probabilistic systems with multiple discrete health states, the expectancy of being in the fully repaired state is not necessarily 1 after a maintenance intervention, depending on the chosen maintenance strategy.

Repairs are assumed to be instantaneous, consequently they cause a discontinuity in the system state. Inspection refers to the action of inspecting the system or a component in order to assess which discrete state it is in. Inspections are planned in advance and occur at deterministic dates.

If the system is found to be failed upon inspection, replacement, i.e. reactive repair, is necessary to restore the system to an operational state. If the system is found to be degraded, preventive maintenance could be performed to restore the system to a less degraded state and reduce probability of a system failure.

We denote by $\boldsymbol{x}^-(T_1)$ the left-handed limit of $\boldsymbol{x}$ when t approaches T_1, meaning $\boldsymbol{x}$ at the moment immediately before the inspection at time T_1. Conversely, $\boldsymbol{x}^+(T_1)$ denotes the resetted probabilities, i.e. after inspections and maintenance. For inspection and maintenance at time T_1, the system probabilities before and after the inspection and maintenance intervention are

$$\begin{aligned}\boldsymbol{x}^-(T_1) &= \lim_{t\to T_1^-} \boldsymbol{x}(t) = \int_{t_0}^{t} f(\boldsymbol{x},\boldsymbol{u},\tau)d\tau \\ \boldsymbol{x}^+(T_1) &= \lim_{t\to T_1^+} \boldsymbol{x}(t) = \boldsymbol{M}\boldsymbol{x}^-(T_1),\end{aligned} \tag{12}$$

where t_0 is the starting time and $\boldsymbol{M} \in \mathbb{R}^{(n_x,n_x)}$ is a square repair (maintenance) matrix. Since f is Lipschitz, the limit $\boldsymbol{x}^-(T_1)$ exists and is unique.

For k inspections at times $\boldsymbol{T} = \begin{bmatrix}T_1, ..., T_k\end{bmatrix}^\intercal$, where

$$t_0 < T_i < T_{i+1} < t_f \quad \forall i \in \{1, 2, ..., k-1\},$$

where t_f is a (fixed) final time / horizon length, the system probabilities can be expressed at any time t by the following piecewise model

$$\boldsymbol{x}(t) = \begin{cases} \int_{t_0}^{T_1} f(\boldsymbol{x},\boldsymbol{u},t)dt, & \text{if} \quad t_0 \le t < T_1 \\ \int_{T_1}^{T_2} f(\boldsymbol{x},\boldsymbol{u},t)dt, & \text{if} \quad T_1 \le t < T_2 \\ \vdots & \\ \int_{T_k}^{t_f} f(\boldsymbol{x},\boldsymbol{u},t)dt, & \text{if} \quad T_k \le t \le t_f \end{cases} \tag{13}$$

where

$$\boldsymbol{x}^-(T_i) = \int_{T_{i-1}}^{T_i} f(\boldsymbol{x},\boldsymbol{u},t)dt \tag{14}$$

$$\boldsymbol{x}^+(T_i) = \boldsymbol{M}(T_i)\cdot \boldsymbol{x}^-(T_i). \tag{15}$$

Here $\boldsymbol{x}^+(T_i)$ is used as the initial condition at time T_i for the time interval $T_i \le t < T_{i+1}$, and $\boldsymbol{x}(t_0)$ is some known initial condition for the probabilities at the known starting time t_0.

Again, because f is Lipschitz, all the pieces of the piecewise $\boldsymbol{x}$ are well defined. $\boldsymbol{M}(t) : \mathbb{R}^1 \to \mathbb{R}^{(n_x,n_x)}$ is a repair function. This allows different maintenance policies to be used at different points in time.

We can differentiate the above model to obtain

$$\frac{d\boldsymbol{x}}{dt} = f(\boldsymbol{x},\boldsymbol{u},t) + \boldsymbol{R}\boldsymbol{r}(t) \tag{16}$$

where

$$\boldsymbol{r}(t) = \boldsymbol{S}^\intercal \boldsymbol{x}^-(t)\cdot\left(\sum_{i=1}^{k}\delta(t-T_i)\right). \tag{17}$$

Here δ is the Dirac delta function, and

$$\boldsymbol{M}(t) = (\boldsymbol{I} + \boldsymbol{R}\boldsymbol{S}(t)^\intercal). \tag{18}$$

where $\boldsymbol{I} \in \mathbb{R}^{(n_x,n_x)}$ is the identity matrix. $\boldsymbol{R} \in \mathbb{R}^{(n_r,n_x)}$ is the maintenance and repair policy matrix. Finally, $\boldsymbol{S} : \mathbb{R}^1 \to \mathbb{R}^{(n_r,n_x)}$ is a selection function, which selects the origin states, i.e. which correspond to negative entries in $\boldsymbol{R}$.

Example II.2 (Structure of $\boldsymbol{R}$ and $\boldsymbol{S}$)

For example, consider a four state system, $\mathcal{X} = \{1, 2, 3, 4\}$, with corresponding probabilities $\boldsymbol{x} = \begin{bmatrix}x_1 & x_2 & x_3 & x_4\end{bmatrix}^\intercal$, where x_1 corresponds to a completely new system, and x_4 corresponds to the failed system. For such a four state system, an As-Good-As-New (AGAN) maintenance policy corresponds to the following $\boldsymbol{R}$ matrix

$$\boldsymbol{R}_{\text{AGAN}} = \begin{bmatrix}1\\0\\0\\-1\end{bmatrix}. \tag{19}$$

which resets the system state to the first state (as good as new) by repairing the system if it is found in state four. If the system is found in either state two or state three, it is not repaired. Thus, in our stochastic modelling framework, the probabilities for being in these two states do not change when a maintenance intervention is performed.

The corresponding selection function $\boldsymbol{S}(t)$ that selects which state to reset, is

$$\boldsymbol{S}_{\text{AGAN}}(t) = \begin{bmatrix}0\\0\\0\\1\end{bmatrix}. \tag{20}$$

Similarly, if the third state corresponds to a barely functioning system, the As-Bad-As-Old (ABAO) maintenance policy can be expressed as

$$\boldsymbol{R}_{\text{ABAO}} = \begin{bmatrix}0\\0\\1\\-1\end{bmatrix}. \tag{21}$$

Since the origin state is the same as for the AGAN policy, the selection functions $\boldsymbol{S}_{\text{AGAN}}(t)$ and $\boldsymbol{S}_{\text{ABAO}}(t)$ are identical. Note that in these two previous examples, the selection function is static and time independent. If our maintenance strategies involves having different maintenance policies applied at different times, this can be expressed as

$$\boldsymbol{R}_{\text{mix}} = \begin{bmatrix}1 & 0\\0 & 0\\0 & 1\\-1 & -1\end{bmatrix}. \tag{22}$$

In the case where two instances of maintenance are to be performed, one at T_1 and one at $T_2 > T_1$, we may use

$$\boldsymbol{S}_{\text{mix}}(t) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{T_2-t}{T_2-T_1} & \frac{T_1-t}{T_1-T_2} \end{bmatrix} \tag{23}$$

to indicate that the AGAN maintenance is performed at T_1 and the ABAO maintenance is performed at T_2. More sophisticated maintenance policies can be obtained by adjusting the $\boldsymbol{R}$ and $\boldsymbol{S}$ matrices continuously.

III. Joint optimization of production and maintenance times

A. General problem description

The aim of this paper is to find a strategy that jointly optimizes the maintenance schedule and the operation of the plant. In terms of the model introduced in the previous section, this implies finding the optimal system load $\boldsymbol{u}$, and the times $\boldsymbol{T}$ at which to perform maintenance, given repair policies $\boldsymbol{S}$ and $\boldsymbol{R}$. Optimizing the performance of the system is a matter of optimizing the trade-off between the expected monetary benefit of its function, and the expected monetary loss of maintenance, as discussed in the introduction. This trade-off may be described mathematically in terms of a weighted economical objective, which is to be maximized using the available degrees of freedom.

In the following sections, a number of assumptions are made to solve this problem:

1) The system degradation is not monitored in real-time. The system must be inspected to observe in order to determine in which state it currently is.
2) Repairs and replacements are assumed to be instantaneous.
3) The resulting optimization problem is solved initially at the start of the planning horizon to get a production and maintenance schedule which is optimal in some statistical sense. The problem is re-optimized regularly to adjust the production and maintenance schedule according to observed plant behavior. This re-optimization should be done as often as possible in order to make sure that the production and maintenance schedule is always up to date and accounting for the current system conditions.

B. Mathematical problem formulation

A general version of the optimization problem can be formulated as

$$\begin{aligned}
\max_{\boldsymbol{u},k,\boldsymbol{T}} \quad & \sum_{i=1}^{k-1} \int_{T_i}^{T_{i+1}} \Big(\phi(t,\boldsymbol{x},\boldsymbol{u})\Big) dt \\
& + \int_{t_0}^{T_1} \Big(\phi(t,\boldsymbol{x},\boldsymbol{u})\Big) dt \\
& + \int_{T_k}^{t_f} \Big(\phi(t,\boldsymbol{x},\boldsymbol{u})\Big) dt \\
& -\phi_I(t,k) - \phi_M(t,\boldsymbol{r}) && \text{(24a)} \\
\text{s.t.} \quad & \frac{d\boldsymbol{x}}{dt} = f(t,\boldsymbol{u},\boldsymbol{x}) + \boldsymbol{R}\boldsymbol{r} && \text{(24b)} \\
& \boldsymbol{r}(t) = \boldsymbol{S}^{\mathsf{T}}\boldsymbol{x}^{-}(t) \cdot \left(\sum_{i=1}^{k} \delta(t-T_i)\right) && \text{(24c)} \\
& \boldsymbol{0} \le \boldsymbol{x} \le \boldsymbol{1} && \text{(24d)} \\
& \boldsymbol{1}^{\mathsf{T}}\boldsymbol{x} = 1 && \text{(24e)} \\
& \boldsymbol{u}_{\min} \le \boldsymbol{u} \le \boldsymbol{u}_{\max} && \text{(24f)}
\end{aligned}$$

here we use $\boldsymbol{0}$ and $\boldsymbol{1}$ to denote vectors of appropriate dimensions with all zero or one entries, respectively.

The objective function comprises three terms, as described below.

1) ϕ is the economical profit rate generated by the production, which is to be maximized
2) ϕ_I is the inspection cost, which is proportional to the number of inspections k

$$\phi_I(t,k) \propto k, \tag{25}$$

3) ϕ_M is the maintenance cost, which is proportional to the integral of $\boldsymbol{r}$ (maintenance costs only have to be paid if maintenance occurs, i.e. it is dependent on the outcome of the inspection)

$$\phi_M(t,\boldsymbol{r}) \propto \int_{t_0}^{t_f} \boldsymbol{r}(t)dt. \tag{26}$$

All three terms are functions of t since they either depend on $\boldsymbol{x}(t)$, or because one might want to discount these terms to reflect the time value of money.

The constraints of the optimization problem comprise bound constraints on the state variables and the intended input-induced loads, and any additional model equations that describe the system. These model equations could describe aerodynamic properties of an aircraft system, or mass and energy conservation in a chemical production system.

Remark III.1 (Some special cases of the problem formulation from (24)**)**

The problem formulation from (24) *describes a general case. However, in some circumstances, we may alter the problem formulation slightly to obtain a special problem formulation.*

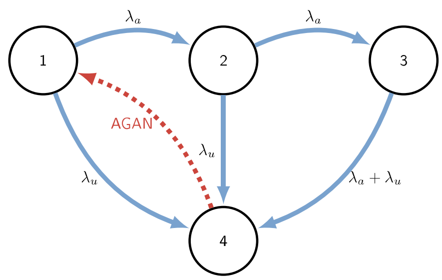

Fig. 1. Illustration of a typical Markov chain

Examples of such problem formulations include, but are not limited to:

- ***Special case 1: Maintenance optimization for fixed operational strategy***
  If the operational strategy is fixed, as is the case in many industries where a system performs the same task over and over, then the intended input-induced loads $\boldsymbol{u}$ are constant and are thus not subject to optimization. The degrees of freedom are k and $\boldsymbol{T}$.
- ***Special case 2: Joint optimization of operational strategy and number of periodic inspections*** *If the maintenance strategy is fixed, e.g. we have to perform maintenance according to a clock-based or age-based schedule, the inspection times T are no longer subject to optimization, and can be reformulated as constraints to the optimization problem. The degrees of freedom are $\boldsymbol{u}$ and k*
- ***Special case 3: Maintenance optimization for fixed number of inspections***
  If the number of inspections is fixed, k ceases to be a decision variable in the optimization problem. The degrees of freedom are $\boldsymbol{u}$ and $\boldsymbol{T}$

These special cases are numerically easier to solve than the general case. For example, fixing the number of inspections removes the discrete nature of the problem, and we are left with an ordinary NLP instead of an MINLP or an MPCC. In this work, we focus on the latter case.

Example III.1 (What is the benefit of optimizing the inspection times and the number of inspections?)

To illustrate the proposed method, consider the following example. A production system has four degradation states labeled 1, 2, 3 and 4, where state 4 is the failed state, and states 1 - 3 are progressively degraded states. An illustration of the Markov chain is shown in Fig. 1 The transition rates are assumed constant, meaning that we can use the following model to describe the probabilities

$$\frac{d\boldsymbol{x}}{dt} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{R}\boldsymbol{r} \tag{27}$$

$\boldsymbol{A}$ is the constant transition rate matrix

$$\boldsymbol{A} = \begin{bmatrix} -0.0101 & 0 & 0 & 0 \\ 0.0100 & -0.0101 & 0 & 0 \\ 0 & 0.0100 & -0.0101 & 0 \\ 0.0001 & 0.0001 & 0.0101 & 0 \end{bmatrix} \tag{28}$$

and

$$\boldsymbol{R} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} \tag{29}$$

such that the system, if found in the failed state 4, when inspected, will be repaired to state 1.

In this case, we have made the transition rates independent of inputs. The inputs $\boldsymbol{u}$ do not enter in the problem formulation and are thus omitted (Special case 1).

The goal is to find the maximum expected net profit by optimizing the k inspection times $\boldsymbol{T}$. The initial time is $t_0 = 0$ and the final mission time is $t_f = 200$ weeks. The objective terms are

$$\phi(t, \boldsymbol{x}) = \boldsymbol{c}^\intercal \boldsymbol{x} \tag{30}$$

$$\phi_I(t, k) = c_I \cdot k \tag{31}$$

$$\phi_M(t, \boldsymbol{r}) = \int_{t_0}^{t_f} \boldsymbol{c}_M^\intercal \boldsymbol{r}(t) dt \tag{32}$$

with

$$\boldsymbol{c} = \begin{bmatrix} 27.8 \\ 20.8 \\ 13.9 \\ 2.8 \end{bmatrix} \tag{33}$$

$$c_I = 30 \tag{34}$$

$$\boldsymbol{c}_M = \begin{bmatrix} 300 \end{bmatrix}. \tag{35}$$

Further, fixing the number of inspections to $k = k^*$, the problem may then be written as

$$\max_{\boldsymbol{T}} \quad \sum_{i=1}^{k^*-1} \left(\int_{T_i}^{T_{i+1}} \left(\boldsymbol{c}^\intercal \boldsymbol{x} \right) dt \right) + \int_{t_0}^{T_1} \left(\boldsymbol{c}^\intercal \boldsymbol{x} \right) dt + \int_{T_k}^{t_f} \left(\boldsymbol{c}^\intercal \boldsymbol{x} \right) dt - c_I \cdot k^* - \int_{t_0}^{t_f} \boldsymbol{c}_M^\intercal \boldsymbol{r}(t) dt \tag{36a}$$

$$\text{s.t.} \quad \frac{d\boldsymbol{x}}{dt} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{R}\boldsymbol{r} \tag{36b}$$

$$\boldsymbol{r}(t) = \boldsymbol{S}^\intercal \boldsymbol{x}^-(t) \cdot \left(\sum_{i=1}^{k} \delta(t - T_i) \right) \tag{36c}$$

$$\boldsymbol{0} \le \boldsymbol{x} \le \boldsymbol{1} \tag{36d}$$

$$\boldsymbol{1}^\intercal \boldsymbol{x} = 1 \tag{36e}$$

Solving the optimization problem Eq. (24) repeatedly for different k^*, we obtain the optimal maintenance schedules shown in Table I. Note that the decision variables in this optimization problem are only the inspection times $\boldsymbol{T}$.

For reference, Table II also shows the objective function values for a periodic (equidistant in time) maintenance schedule, which is prevalent in industry today. As expected, the time between inspections becomes shorter and shorter in the optimal maintenance schedule, as opposed to in the periodic maintenance schedule, where the time between inspections is constant.

Figure 2 shows the comparison of the net profit from the two approaches, and shows that the optimization approach always yields higher net profits, which motivates the use for

TABLE I
OPTIMIZED MAINTENANCE SCHEDULE FOR THE MOTIVATING EXAMPLE IN SECTION III.1

Inspections k^*	Net profit	Inspections times $\boldsymbol{T}$					
1	3913.9	128.398					
2	3946.0	103.228	149.136				
3	3950.3	88.616	127.258	159.178			
4	3942.9	78.984	112.972	140.713	165.239		
5	3929.4	72.169	102.931	127.776	149.563	169.356	
6	3912.4	67.136	95.549	118.281	138.070	155.928	172.369

TABLE II
PERIODIC MAINTENANCE SCHEDULE FOR THE MOTIVATING EXAMPLE IN SECTION III.1

Inspections k	Net profit	Inspections times $\boldsymbol{T}$					
1	3886.1	100.000					
2	3923.1	66.667	133.333				
3	3930.5	50.000	100.000	150.000			
4	3924.9	40.000	80.000	120.000	160.000		
5	3912.5	33.333	66.667	100.000	133.333	166.667	
6	3896.1	28.571	57.143	85.714	114.286	142.857	171.429

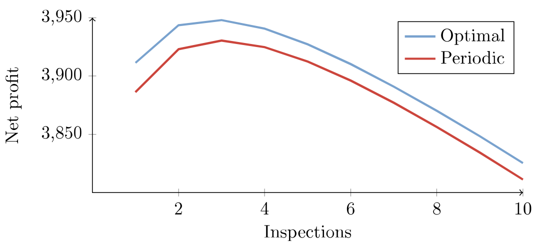

Fig. 2. Comparison of net profit for optimal and periodic inspection schedules

an optimization-based method for determining the inspection times, as proposed in this paper.

IV. CONTINUOUS APPROXIMATION OF THE DISCONTINUOUS OPTIMIZATION PROBLEM

A. Continuous approximation of repairs $\boldsymbol{r}(t)$

Before proceeding, let us recall that the aim of this paper is to find an optimization-based method with which we can determine the optimal operational strategy in terms of production, inspections and maintenance. However, the expression for the evolution of the probabilities in Eq. (16) contains the Dirac delta function. Due to its discontinuous nature, this is problematic for large-scale numerical optimization, since algorithms for solving large-scale numerical optimization problems typically require that the problem contains only differentiable functions. A possible solution is to reformulate the problem as a mixed integer problem, as was done in e.g. [22, 23, 24], instead of using Dirac delta functions. Unfortunately, MINLP problems are known to be difficult to solve for large-scale problems, despite recent progress in algorithmic development [25].

Instead of using a mixed-integer formulation to solve the problem, we chose to approximate $\boldsymbol{r}$ by a sum of Boxcar functions instead:

$$\boldsymbol{r}(t) \quad \approx \quad \tilde{\boldsymbol{r}}(t) \tag{37}$$

where

$$\begin{aligned} \tilde{r}_i(t) &= \sum_{j=1}^{k} \mathrm{Boxcar}_{i,j}(t) \\ &= \sum_{j=1}^{k} h_{i,j}\Big(\mathrm{H}(t-T_j) - \mathrm{H}(t-T_j-\epsilon)\Big) \\ &\quad \forall i \in \{1, ..., n_r\}. \end{aligned} \tag{38}$$

Here, H is the Heaviside function, $h_{i,j}$ is the height and ϵ is the width of boxcar i, j. By choosing a nonzero ϵ we ensure that $\tilde{\boldsymbol{r}}(t)$ is bounded and Lipschitz continuous, and consequently that $\boldsymbol{x}(t)$ is continuous (although still non-smooth).

An illustration of the approximation of the Dirac functions with boxcar functions is shown in Fig. 3. Observe that the approximation for $\boldsymbol{x}(t)$ is good if ϵ is sufficiently small. ϵ should be chosen as small as possible in order to minimize the approximation error, but big enough to avoid numerical problems with the chosen solver. In practice, we have found that choosing $\epsilon \approx \frac{1}{1000}(t_f - t_0)$ yielded satisfactory performance for our applications.

B. Smooth problem re-formulation for numerical optimization with off-the-shelf NLP solvers

The problem in Eq. (24) is discrete in nature due to the inclusion of the integer variable k, therefore making it difficult to solve without the use of MINLP solvers as explained in Section IV-A, or by fixing the number of inspections k, as illustrated in Example III.1.

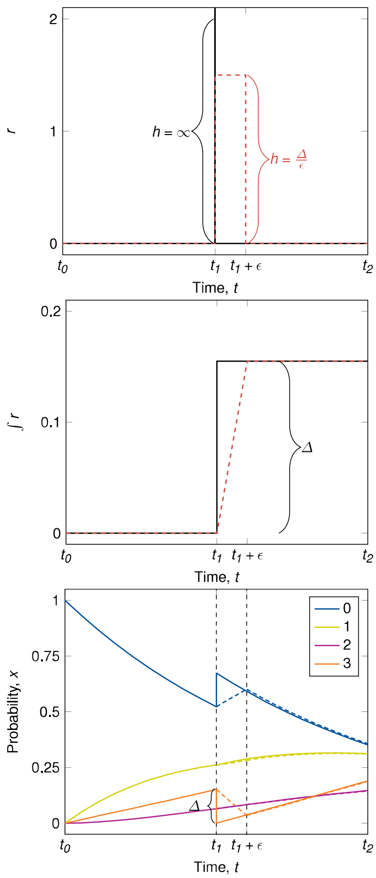

Fig. 3. Illustration of how $\boldsymbol{r}(t)$ (solid, black line) can be approximated. The top plot shows $\boldsymbol{r}(t)$ and the Boxcar approximation $\tilde{\boldsymbol{r}}(t)$ (dashed, red line). The bottom plot shows the original discontinuous states $\boldsymbol{x}$ (solid lines), and the states due to the approximation (dashed), which are continuous.

In order to avoid both of these solution alternatives, we include the simplifications introduced in the previous sections. However, we still need to reformulate the inspection and maintenance costs, which contain the integer variable k. In Eq. (25), the inspection cost is said to be proportional to the number of inspections, k. However, since we reformulated the optimization problem, k is no longer explicitly included. Instead, it is "baked" into $\tilde{\boldsymbol{r}}(t)$. In order to ensure that the reformulated optimization problem still approximates the original discrete problem, we introduce a variable $\boldsymbol{z}(t)$ which is complementary to $\tilde{\boldsymbol{r}}$

$$\mathbf{0} \leq (\mathbf{1} - \boldsymbol{z}) \perp \tilde{\boldsymbol{r}} \geq \mathbf{0} \tag{39a}$$

$$\mathbf{0} \leq \boldsymbol{z} \leq \mathbf{1} \tag{39b}$$

Here, the $\perp$ operator indicates complementary, i.e. we require that at all times either $\tilde{\boldsymbol{r}}$ or the term $(\mathbf{1} - \boldsymbol{z})$ or both to have all zero entries. Now we can formulate the number of inspections as

$$\phi_I(t, \boldsymbol{z}) \propto k \approx \int_{t_0}^{t_f} \frac{1}{\epsilon} \boldsymbol{z}(t) dt, \tag{40}$$

where ϵ is the width of the Boxcar in the approximation $\tilde{\boldsymbol{r}}(t)$. A graphical illustration of this is shown in Fig. 3

Due to the introduction of the complementary constraints into the optimization problem, the resulting problem is classified as a mathematical program with complementary/equilibrium constraints (MPCC / MPEC).

As a result, we reformulate optimization problem from Eq. (24) as

$$
\begin{aligned}
\max_{\boldsymbol{u}, \tilde{\boldsymbol{r}}, \epsilon} \quad & \int_{t_0}^{t_f} \Big(\phi(t, \boldsymbol{x}, \boldsymbol{u})\Big) dt \\
& -\phi_I(t, \boldsymbol{z}) - \phi_M(t, \tilde{\boldsymbol{r}}) && (41a) \\
\text{s.t.} \quad & \frac{d\boldsymbol{x}}{dt} = f(t, \boldsymbol{u}, \boldsymbol{x}) + \boldsymbol{R}\tilde{\boldsymbol{r}} && (41b) \\
& \mathbf{0} \leq \boldsymbol{x} \leq \mathbf{1} && (41c) \\
& \mathbf{1}^{\mathsf{T}} \boldsymbol{x} = 1 && (41d) \\
& \mathbf{0} \leq \tilde{\boldsymbol{r}} \leq \frac{1}{\epsilon_{\min}} \mathbf{1} && (41e) \\
& \epsilon_{\min} \leq \epsilon \leq \epsilon_{\max} && (41f) \\
& \boldsymbol{u}_{\min} \leq \boldsymbol{u} \leq \boldsymbol{u}_{\max} && (41g) \\
& \mathbf{0} \leq (\mathbf{1} - \boldsymbol{z}) \perp \tilde{\boldsymbol{r}} \geq \mathbf{0} && (41h) \\
& \mathbf{0} \leq \boldsymbol{z} \leq \mathbf{1} && (41i) \\
& \phi_I(t, \boldsymbol{z}) = \int_{t_0}^{t_f} \frac{\boldsymbol{z}(t)}{\epsilon} dt && (41j)
\end{aligned}
$$

The optimization problem from Eqs. (41a)-(41j) is discretized using direct collocation [15] and solved using IPOPT [16].

Remark IV.1 (Regarding the complementary constraints) *Due to the non-convex nature of the functions in the optimization problem, standard solvers for nonlinear programs, such as IPOPT [16], will only provide locally optimal solutions. To explore more of the feasible space and hopefully converge to a near-global optimum, we solve the problem using a multi-start approach. Here we initiate the optimization algorithm at multiple different initial points. Each optimization run yields a local solution, and the best local solution is chosen. Of course a globally optimal solution cannot be guaranteed, but our experience is that there are many solutions with an objection similar to the the globally optimal value, which are only marginally worse and are acceptable from an engineering perspective.*

Another measure that we have taken is to reformulate the MPEC. Broadly speaking, these reformulations can be categorized as either relaxation methods or penalty methods. In relaxation methods, the complementary conditions are temporarily relaxed. Upon repeated solving of the relaxed problem with gradual tightening of the complementary constraints, the solution of the relaxed problem converges to the true solution.

Similarly, penalty methods remove the complementary condition from the constraints and penalize them in the objective instead. Gradual increase of the penalty weight means that the complementary condition is satisfied upon convergence. Using a relaxation and hybrid method, (41) *with the added complementarity constraints* (39) *can be reformulated as*

$$\max_{\boldsymbol{u},\tilde{\boldsymbol{r}},\boldsymbol{\epsilon}} \quad \int_{t_0}^{t_f} \Big(\phi(t,\boldsymbol{x},\boldsymbol{u})\Big)dt$$

$$-\phi_I(t,\boldsymbol{z}) - \phi_M(t,\tilde{\boldsymbol{r}}) \tag{42a}$$

$$-\boldsymbol{\gamma}(l)\cdot \mathrm{vec}\left((\mathbf{1}-\boldsymbol{z})\otimes\tilde{\boldsymbol{r}}\right) \tag{42b}$$

$$\text{s.t.} \quad \frac{d\boldsymbol{x}}{dt} = f(t,\boldsymbol{u},\boldsymbol{x}) + \boldsymbol{R}\tilde{\boldsymbol{r}} \tag{42c}$$

$$\mathbf{0} \le \boldsymbol{x} \le \mathbf{1} \tag{42d}$$

$$\mathbf{1}^{\mathsf{T}}\boldsymbol{x} = 1 \tag{42e}$$

$$\mathbf{0} \le \tilde{\boldsymbol{r}} \le \frac{1}{\epsilon_{\min}}\mathbf{1} \tag{42f}$$

$$\epsilon_{\min} \le \epsilon \le \epsilon_{\max} \tag{42g}$$

$$\boldsymbol{u}_{\min} \le \boldsymbol{u} \le \boldsymbol{u}_{\max} \tag{42h}$$

$$\mathbf{0} \le \boldsymbol{z} \le \mathbf{1} \tag{42i}$$

$$\mathrm{vec}\left(\tilde{\boldsymbol{r}}\otimes(\mathbf{1}-\boldsymbol{z})\right) \ge \boldsymbol{\xi}(l) \tag{42j}$$

Here, $(\cdot)$ *denotes the inner product and* $(\otimes)$ *denotes the outer product. This problem is solved sequentially* l *times such that*

$$\lim_{l\to\infty}, \boldsymbol{\xi}\to 0, \boldsymbol{\gamma}\to\infty$$

The MPEC solution will be approached asymptotically [13].

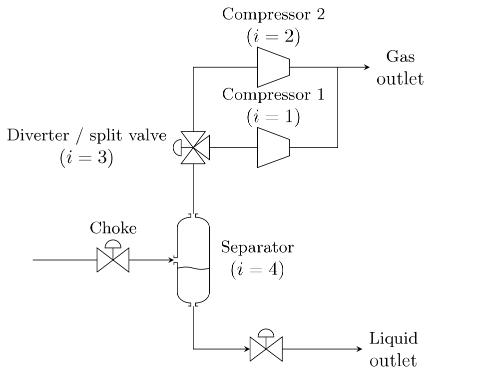

Fig. 4. Illustration of the subsea compressor station that is studied in this case example, with $i = 1...4$ indicate the degrading components.

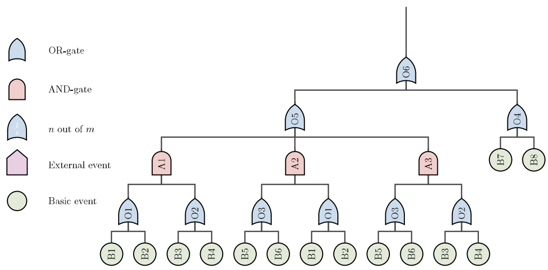

Fig. 5. Fault tree for the subsea compressor station. The gates and basic events are described in Tab. III

V. Case Example: Operation & Maintenance Planning for a Subsea Oil and Gas Separation System

To illustrate our proposed method, we apply it to the case study of an subsea oil and gas separation plant. We will solve the problem using the approximations introduced in Section IV-B.

A. Motivation and general description

An illustration of the plant we are studying in this case example is shown in Fig. 4.

It is a subsea gas compression station with two compression trains (referred to as units 1 and 2) in parallel. These units are dry gas compressors, meaning that the wet gas from the reservoir must be cleaned of water and oil droplets in a scrubber-type separator (referred to as unit 4) before entering the compressors. If liquid droplets enter the compressors, this will lead to rapid degradation. The parallel configuration was chosen for the sake of flexibility. Failure of one of the units will only lead to partial failure of the whole system, since the system can continue production. Each compressor alone is able to provide the handle the full load, albeit with the consequence of faster degradation.

A single separator was deemed sufficient, since an additional, second separator unit would significantly increase complexity, cost and footprint of the overall system. Since the separator operates statically without moving parts, its reliability is acceptably high to be in a series configuration with the compressors. A fault tree analysis was performed for the system, resulting in the simplified fault tree shown in Fig. 5. A comprehensive root cause analysis is outside the scope of this paper, so we instead use the encompassing events "failure from load-induced wear" for the compressors and "actuator failure" for the separator.

B. Modeling the component degradation

Let us start by modelling the degradation of each of the four components of the system. The degradation of component i is described in terms of its probabilities. For each of the four components, the probabilities of being in any of the four degradation states are denoted by

$$\boldsymbol{x_i} = \begin{bmatrix} x_{i,1} & x_{i,2} & x_{i,3} & x_{i,4} \end{bmatrix}^{\mathsf{T}} \tag{43}$$

such that

$$\mathbf{0} \le \boldsymbol{x_i} \le \mathbf{1} \tag{44}$$

$$\mathbf{1}^{\mathsf{T}}\boldsymbol{x_i} = 1 \tag{45}$$

and

$$\frac{d\boldsymbol{x_i}}{dt} = f_i(\boldsymbol{x},\boldsymbol{u},t). \tag{46}$$

TABLE III
DESCRIPTION OF LABELS OF EVENTS AND LOGIC GATES IN FAULT TREE IN FIG. 5

Label	Description
O_6	Failure of the entire system
O_5	Failure of the compression subsystem
O_4	Failure of the separation subsystem
A_1	Failure of compressor 1 and compressor 2
A_2	Failure of compressor 1 and split valve
A_3	Failure of compressor 2 and split valve
O_1	Failure of compressor 1
O_2	Failure of compressor 2
O_3	Failure of split valve
B_1	Sensor failure, loss of communication or other unforeseen failure
B_2	Compressor failure due to load induced wear
B_3	Sensor failure, loss of communication or other unforeseen failure
B_4	Compressor failure due to load induced wear
B_5	Sensor failure, loss of communication or other unforeseen failure
B_6	Insufficient actuation due to load induced wear of split valve
B_7	Sensor failure, loss of communication or other unforeseen failure
B_8	Insufficient actuation due to load induced wear of separator valve

where

$$\boldsymbol{x} = \begin{bmatrix} \boldsymbol{x_1} & \boldsymbol{x_2} & \boldsymbol{x_3} & \boldsymbol{x_4} \end{bmatrix}^\intercal \tag{47}$$

f_i is split into two parts:

1) f_a, which is the contribution due to aging, which is dependent on the inputs $\boldsymbol{u}$
2) f_u is the sudden failure due to shock damage, which is independent from the inputs $\boldsymbol{u}$.

The basic events B_1, B_3, B_5 and B_7 from the fault tree in Fig. 5 correspond to unexpected shock failure of a component. These events are assumed to be independent of the operating mode, and are expressed through f_u

$$f_{u,i}(\boldsymbol{x}, t) = \begin{bmatrix} -\lambda_{u,i} & 0 & 0 & 0 \\ 0 & -\lambda_{u,i} & 0 & 0 \\ 0 & 0 & -\lambda_{u,i} & 0 \\ \lambda_{u,i} & \lambda_{u,i} & \lambda_{u,i} & 0 \end{bmatrix} \boldsymbol{x_i} \tag{48}$$

The basic events B_2, B_4, B_6 and B_8 are linked to aging of the components. We model the contribution from aging as

$$f_{a,i}(\boldsymbol{x}, \boldsymbol{u}, t) = \begin{bmatrix} -\lambda_{a,i} & 0 & 0 & 0 \\ \lambda_{a,i} & -\lambda_{a,i} & 0 & 0 \\ 0 & \lambda_{a,i} & -\lambda_{a,i} & 0 \\ 0 & 0 & \lambda_{a,i} & 0 \end{bmatrix} \boldsymbol{x_i} \tag{49}$$

where the degradation rates of each of the four components is assumed to be proportional to the load applied to it

$$\begin{aligned}
\lambda_{a,comp1} &= \tilde{\lambda}_{a,comp1} \cdot (u_1^2 + u_1) && (50)\\
\lambda_{a,comp2} &= \tilde{\lambda}_{a,comp2} \cdot (u_2^2 + u_2) && (51)\\
\lambda_{a,split} &= \tilde{\lambda}_{a,split} \cdot (u_1 \cdot (1 - x_{comp1,N}) \\
&+ u_2 \cdot (1 - x_{comp2,N})) && (52)\\
\lambda_{a,sep} &= \tilde{\lambda}_{a,sep} \cdot (u_1 \cdot (1 - x_{comp1,N}) && (53)\\
&+ u_2 \cdot (1 - x_{comp2,N})), && (54)
\end{aligned}$$

Note that the transition rates of the split and the separator depend on the reliability of the two compressors. The reason for this is that if one of the compressors fails, the consequence is that the overall load on the system is reduced since no flow goes through the failed compressor (safety shut-off systems prevent this from happening). In reality, an operator may observe that the overall flow from the system is reduced, and may react to this by increasing the set-points to the compressors in order to keep the total flow constant. However, this manual intervention is not included in the optimization framework.

Furthermore, we note that the transition rates for the compressors are quadratic functions of the flows through the compressor. Due to reduced compressor efficiencies at higher flow rates, compressors have to be run harder to satisfy the desired pressure increase. Some literature also suggest that the degradation of compressors is quadratic with the usage of the compressor [26].

At each inspection time, there are two options available for maintenance interventions:

1) If the component is found in the failed state, then it is replaced by a new component (as-good-as-new)
2) If the component is found in the most degraded state, then preventive maintenance may be performed to restore the component. Due to imperfect repairs, the health component will end up as-good-as-new 80% of the time, and partially degraded 20% of the time.

C. Expressing the system degradation

The condition of the overall system can be formulated as algebraic relationships of the component reliability probabilities $\boldsymbol{x_i}$. We are interested in the following four system probabilities

$$\boldsymbol{y} = \begin{bmatrix} y_1 & y_2 & y_3 & y_4 \end{bmatrix}^\intercal \tag{55}$$

where, $y_1, ..., y_4$ denote progressively deteriorated condition of the overall system. Specifically,

1) No failed units, system is working at full capacity
$$y_1 = \mathbb{P}(O_1{}^c \cap O_2{}^c \cap O_3{}^c \cap O_4{}^c) \tag{56}$$
Here, $\mathbb{P}$ indicates the probability operator, O_i refers to the corresponding events from Table III, c indicates the complement operator, and $\cap$ is the intersection operator.
2) Either compressor 1 has failed or compressor 2 has failed, but split valve and separator are still operational. The system is running at reduced capacity.
$$y_2 = \mathbb{P}((O_1 \oplus O_2) \cap O_3{}^c \cap O_4{}^c) \tag{57}$$
where $\oplus$ is the exclusive OR operator.
3) The split valve has failed, but the separator and both compressors are still operational. The split ratio can no longer be adjusted. System is running at reduced capacity.
$$y_3 = \mathbb{P}(O_1{}^c \cap O_2{}^c \cap O_3 \cap O_4{}^c) \tag{58}$$
4) System has failed. No production.
$$y_4 = \mathbb{P}(O_6) \tag{59}$$

Note that logically, the assertions

$$\mathbf{0} \leq \boldsymbol{y} \leq \mathbf{1} \tag{60}$$
$$\mathbf{1}^\mathsf{T}\boldsymbol{y} = 1 \tag{61}$$

must hold because the system must be in one of these states. Either the system has no failed components, some non-critical failed components, or the overall system has failed. To get expressions for $\boldsymbol{y}(t)$, we consider at the fault tree and derive the expressions for the probabilities.

First, we express

$$\begin{aligned}\mathbb{P}(A_1) &= \mathbb{P}(O_1 \cap O_2)\\ &= \mathbb{P}(O_1)\mathbb{P}(O_2)\\ &= x_{1,4}x_{2,4} \end{aligned}\tag{62}$$
$$\begin{aligned}\mathbb{P}(A_2) &= \mathbb{P}(O_1 \cap O_3)\\ &= \mathbb{P}(O_1)\mathbb{P}(O_3)\\ &= x_{1,4}x_{3,4} \end{aligned}\tag{63}$$
$$\begin{aligned}\mathbb{P}(A_3) &= \mathbb{P}(O_2 \cap O_3)\\ &= \mathbb{P}(O_2)\mathbb{P}(O_3)\\ &= x_{2,4}x_{3,4} \end{aligned}\tag{64}$$

We can now express the probability of a failure of the compression subsystem as

$$\begin{aligned}\mathbb{P}(O_5) &= \mathbb{P}(A_1 \cup A_2 \cup A_3)\\ &= \mathbb{P}(A_1) + \mathbb{P}(A_2) + \mathbb{P}(A_3) - \mathbb{P}(A_1 \cap A_2)\\ &\quad - \mathbb{P}(A_1 \cap A_3) - \mathbb{P}(A_2 \cap A_3) + \mathbb{P}(A_1 \cap A_2 \cap A_3)\\ &= x_{1,4}x_{2,4} + x_{1,4}x_{3,4} + x_{2,4}x_{3,4}\\ &\quad - x_{1,4}x_{2,4}x_{3,4} - x_{1,4}x_{2,4}x_{3,4}\\ &\quad - x_{1,4}x_{2,4}x_{3,4} + x_{1,4}x_{2,4}x_{3,4}\\ &= x_{1,4}x_{2,4} + x_{1,4}x_{3,4} + x_{2,4}x_{3,4} - 2x_{1,4}x_{2,4}x_{3,4} \end{aligned}\tag{65}$$

Consequently, the probability of system failure can be expressed as

$$\begin{aligned}\mathbb{P}(O_6) &= \mathbb{P}(O_4 \cup O_5)\\ &= \mathbb{P}(O_4) + \mathbb{P}(O_5) - \mathbb{P}(O_5 \cap O_4)\\ &= \mathbb{P}(O_4) + \mathbb{P}(O_5) - \mathbb{P}(O_4)\mathbb{P}(O_5)\\ &= x_{4,4} + (1 - x_{4,4})\,(x_{1,4}x_{2,4} + x_{1,4}x_{3,4}\\ &\quad + x_{2,4}x_{3,4} - 2x_{1,4}x_{2,4}x_{3,4})\\ &= y_4 \end{aligned}\tag{66}$$

Similarly for the other three system probabilities

$$\begin{aligned}y_1 &= \mathbb{P}({O_1}^c \cap {O_2}^c \cap {O_3}^c \cap {O_4}^c)\\ &= (1 - x_{1,4}) \cdot (1 - x_{2,4}) \cdot (1 - x_{3,4}) \cdot (1 - x_{4,4}) \end{aligned}\tag{67}$$
$$\begin{aligned}y_2 &= \mathbb{P}((O_1 \oplus O_2) \cap {O_3}^c \cap {O_4}^c)\\ &= \mathbb{P}(((O_1 \cup O_2) - (O_1 \cap O_2)) \cap {O_3}^c \cap {O_4}^c)\\ &= (x_{1,4} + x_{2,4} - 2x_{1,4}x_{2,4}) \cdot (1 - x_{3,4}) \cdot (1 - x_{4,4}) \end{aligned}\tag{68}$$
$$\begin{aligned}y_3 &= \mathbb{P}({O_1}^c \cap {O_2}^c \cap O_3 \cap {O_4}^c)\\ &= (1 - x_{1,4}) \cdot (1 - x_{2,4}) \cdot x_{3,4} \cdot (1 - x_{4,4}) \end{aligned}\tag{69}$$

Finally, we check that the sum of the four system probabilities is

$$\begin{aligned}\sum_{i=1}^{4} y_i &= y_1 + y_2 + y_3 + y_4\\ &= (1 - x_{1,4}) \cdot (1 - x_{2,4}) \cdot (1 - x_{3,4}) \cdot (1 - x_{1,4})\\ &\quad + (x_{1,4} + x_{2,4} - x_{1,4}x_{2,4}) \cdot (1 - x_{3,4}) \cdot (1 - x_{4,4})\\ &\quad + (1 - x_{1,4}) \cdot (1 - x_{2,4}) \cdot x_{3,4} \cdot (1 - x_{4,4})\\ &\quad + x_{4,4} + (1 - x_{4,4})\,(x_{1,4}x_{2,4}\\ &\quad + x_{1,4}x_{3,4} + x_{2,4}x_{3,4} - 2x_{1,4}x_{2,4}x_{3,4})\\ &= 1 \end{aligned}\tag{70}$$

as expected.

Now that we have the expressions for the four system probabilities $\boldsymbol{y}$ as a function of the component probabilities $\boldsymbol{x}$, we may formulate the following optimization problem.

D. Economic objective

The objective is to maximize the throughput through the compressor station. Simultaneously, we want to minimize the inspection and maintenance costs. The economical objective is the sum of discounted cash flows.

$$\min_{\boldsymbol{u}(t),\tilde{\boldsymbol{r}}(t)} \quad \int_0^{t_f} \Big(\frac{-\phi + \phi_I + \phi_M}{(1+d)^t}\Big)dt \tag{71a}$$

where

- The first term ϕ is expected the production, which is proportional to the system probabilities $\boldsymbol{y}(t)$ via the weighing vector / productivity vector $\boldsymbol{c}_p$.

$$\phi(t, \boldsymbol{y}, \boldsymbol{u}) = \big(\boldsymbol{c}_p^\mathsf{T}\boldsymbol{y}\big) \cdot (\mathbf{1}^\mathsf{T}\boldsymbol{u}) \tag{71b}$$

  $c_{p,i}$ indicates how well the system is able to perform its desired task in the corresponding state i.
- The second term ϕ_I is the inspection cost, which is paid every time an inspection is performed. It is proportional to $\boldsymbol{z}(t)$, the variable introduced to count the number of inspections.

$$\phi_I(t, \boldsymbol{z}) = \boldsymbol{c}_I^\mathsf{T}\boldsymbol{z} \tag{71c}$$

- The third and last term ϕ_M is the maintenance cost, which is proportional to $\tilde{\boldsymbol{r}}(t)$, i.e. it is proportional to the probability of being in the undesired state at the inspection time.

$$\phi_M(t, \tilde{\boldsymbol{r}}) = \boldsymbol{c}_M^\mathsf{T}\tilde{\boldsymbol{r}} \tag{71d}$$

  As mentioned previously, there are two maintenance options:
  1) Corrective replacement
  2) Preventive (imperfect) repairs

$$\boldsymbol{c}_M = \begin{bmatrix}\boldsymbol{c}_{M,corr}\\ \boldsymbol{c}_{M,prev}\end{bmatrix} \tag{71e}$$

- The entire objective is discounted by the factor d. The discount factor is chosen according to the economics and the time scale of the optimization.

The decision variables are the maintenance and inspection times, as well as the maintenance type (corrective replacement or preventive repairs)

$$\tilde{\boldsymbol{r}}(t) = \begin{bmatrix} \tilde{\boldsymbol{r}}_{corr}(t) \\ \tilde{\boldsymbol{r}}_{prev}(t) \end{bmatrix} = \begin{bmatrix} \tilde{r}_{corr,1}(t) \\ \tilde{r}_{corr,2}(t) \\ \tilde{r}_{corr,3}(t) \\ \tilde{r}_{corr,4}(t) \\ \tilde{r}_{prev,1}(t) \\ \tilde{r}_{prev,2}(t) \\ \tilde{r}_{prev,3}(t) \\ \tilde{r}_{prev,4}(t) \end{bmatrix} \tag{71f}$$

and the inputs $\boldsymbol{u}$. The maintenance matrix $\boldsymbol{R}$ is

$$\boldsymbol{R} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0.8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0.8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0.8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0.8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{71g}$$

E. Constraints

The optimization problem outlined in the previous section is solved subject to the following constraints

$$\begin{aligned} s.t. \quad & \frac{d\boldsymbol{x}}{dt} = f(\boldsymbol{x}, \boldsymbol{u}, t) && (71h) \\ & \boldsymbol{y}(t) = \boldsymbol{g}(\boldsymbol{x}, t) && (71i) \\ & \boldsymbol{x}(0) = \boldsymbol{x}_0 && (71j) \\ & \boldsymbol{u}_{\min} \le \boldsymbol{u} \le \boldsymbol{u}_{\max} && (71k) \\ & \boldsymbol{1}^{\mathsf{T}}\boldsymbol{u} \le \boldsymbol{u}_{\max} && (71l) \\ & \boldsymbol{0} \le \tilde{\boldsymbol{r}} \le \frac{1}{\epsilon_{\min}}\boldsymbol{1} && (71m) \\ & \epsilon_{\min} \le \epsilon \le \epsilon_{\max} && (71n) \\ & \boldsymbol{0} \le \boldsymbol{x} \le \boldsymbol{1} && (71o) \\ & \boldsymbol{0} \le \boldsymbol{z} \le \boldsymbol{1} && (71p) \\ & \boldsymbol{0} \le \boldsymbol{y} \le \boldsymbol{1} && (71q) \\ & \boldsymbol{0} \le (\boldsymbol{1} - \boldsymbol{z}) \perp \tilde{\boldsymbol{r}} \ge \boldsymbol{0} && (71r) \end{aligned}$$

where

$$\boldsymbol{x} = \begin{bmatrix} \boldsymbol{x_1} & \boldsymbol{x_2} & \boldsymbol{x_3} & \boldsymbol{x_4} \end{bmatrix}^{\mathsf{T}} \tag{72}$$

and

$$f(\boldsymbol{x}, \boldsymbol{u}, t) = diag(\begin{bmatrix} f_{u,1} \\ f_{u,2} \\ f_{u,3} \\ f_{u,4} \end{bmatrix}) + diag(\begin{bmatrix} f_{a,1} \\ f_{a,2} \\ f_{a,3} \\ f_{a,4} \end{bmatrix}) + \boldsymbol{R}\tilde{\boldsymbol{r}} \tag{73}$$

TABLE IV
PARAMETERS USED FOR THE OPTIMIZATION

Parameter	Description	Value
λ_u	Sudden failure transition rate	$10^{-4} \cdot \begin{bmatrix} 2.0 & 2.0 & 1.0 & 0.5 \end{bmatrix}^{\mathsf{T}}$
$\tilde{\lambda}_a$	Base aging transition rate	$10^{-2} \cdot \begin{bmatrix} 1.0 & 1.0 & 0.5 & 0.5 \end{bmatrix}^{\mathsf{T}}$
d	Discount rate	0.001
$\boldsymbol{c}_p$	Productivities in each state	$\begin{bmatrix} 10 & 5 & 5 & 0 \end{bmatrix}^{\mathsf{T}}$
$\boldsymbol{c}_m$	Maintenance cost (replacement)	$10^2 \cdot \begin{bmatrix} 5.0 & 5.0 & 1.0 & 2.0 \end{bmatrix}^{\mathsf{T}}$
$\boldsymbol{c}_m$	Maintenance cost (preventive)	$\frac{10^2}{3} \cdot \begin{bmatrix} 5.0 & 5.0 & 1.0 & 2.0 \end{bmatrix}^{\mathsf{T}}$
$\boldsymbol{c}_i$	Inspection cost	$10^1 \cdot \begin{bmatrix} 1.0 & 1.0 & 1.0 & 1.0 \end{bmatrix}^{\mathsf{T}}$
t_0	Initial time	0 weeks
t_f	Final time	200 weeks

Eqs. (71h)-(71i) describe the system model. Eq. (71j) is the initial condition for the states. Eqs. (71k)-(71q) are variable bounds. Finally, the last constraint in Eq. (71r) are the complementarity constraints for the introduced counter variable $\boldsymbol{z}$ that is used to keep track of the number of inspections k.

F. Simulation results

The model described in Section V was discretized using direct orthogonal collocation and implemented in MATLAB and CasADi 3.4.1 [27]. IPOPT 3.12.3 [16], an interior point solver for NLPs, was used to solve the problem. The parameter values for the problem are given in Tab. IV.

The optimal system probabilities obtained from the optimization are shown in Fig. 6. The corresponding decision variables ($\boldsymbol{u}(t)$ and $\tilde{\boldsymbol{r}}(t)$) are shown in Fig. 7.

We observe that the optimal solution obtained for this case study is quite complex. The planned maintenance times are non-periodically distributed, and not all components are maintained at every inspection, as this would lead to higher maintenance costs. The optimal input profile shows that one compressor is used at maximum capacity the entire time, whereas the other is throttled to run at less than maximum speed for some time to improve the overall economic performance by lowering the probability of system failure and thus the expected maintenance cost. The input-induced loads increase temporarily at the times when inspections are performed, but these are only numerical artifacts and do not alter the solution significantly.

Initially, the system is to be inspected and maintained quite often, as the net present value formulation implies a preference for high availability early on. At $T_1 = 32$, $T_2 = 60$, $T_3 = 85$ and $T_4 = 137$ weeks, inspections are performed on all four components. However, various equipment is maintained more often than others. For example, compressor 2 is scheduled to be maintained reactively the first two maintenance interventions, and preventively the first three interventions. The separator, on the other hand, is only scheduled to be repaired reactively on the second and fourth intervention, and preventively at the second inspection. Following the final

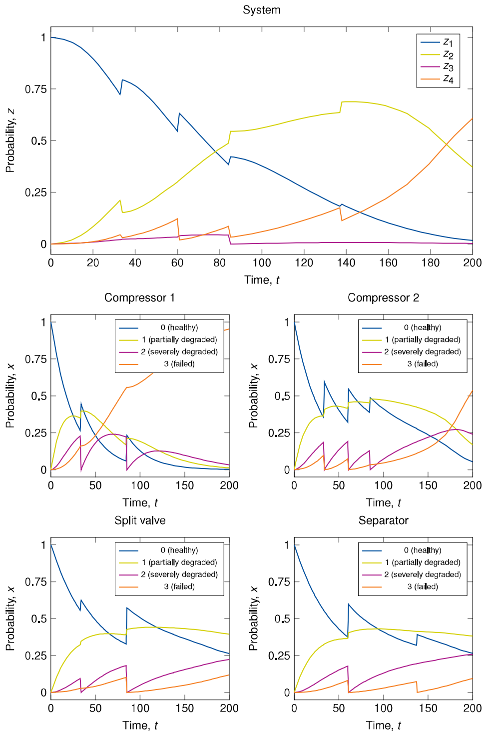

Fig. 6. Optimal solution obtained from the optimization of the case study. The subplot on top shows the system states, i.e. the four composite events described in Section V-C. The four smaller subplots each show the probabilities x_i of being in the four health states for compressor 1, compressor 2, the split valve and the separator, respectively (counting left to right, top to bottom).

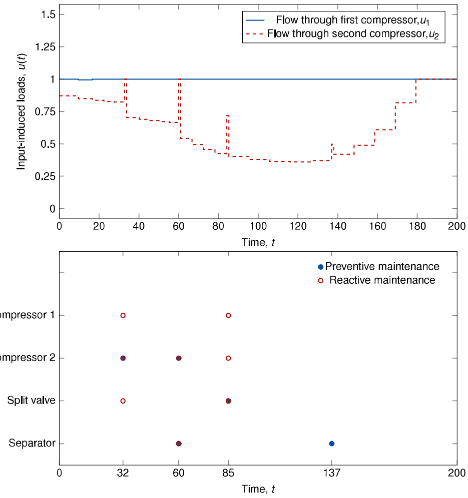

Fig. 7. Optimal solution from the optimization of the case study. The upper plot shows the two input-induced loads u_1 and u_2, which are the mass flows through the compressors. The bottom plot shows the optimal reactive maintenance times, and the optimal preventive maintenance times.

inspection and maintenance intervention, the system is allowed to degrade and fail, as it is of no economic benefit to keep the availability of the system high when the intended mission time of the system has been reached.

It should be noted that the solution obtained here is only a local solution, as discussed earlier. We performed multi-start optimization until the improvement in the objective function value did not improve significantly for a couple of iterations, at which point it was assumed that a near-global solution was found. At that point, our method had managed to improve the initial guess, a periodic schedule with four inspections and both compressors running at maximum, by about 10%, which we deem to be a significant improvement for engineering applications.

We also checked to make sure that the particular solution has a mirror solution which is equally good. This was confirmed by testing the strategy where the opposite compressor was used at max capacity while throttling the other. This strategy was found to be equally good. Since the compressors are identical, this makes sense. If the compressors would have been different, this would no longer have been the case.

VI. Conclusion

We have presented a method for solving the problem of combined maintenance scheduling and production planning for arbitrary degradation models and arbitrary maintenance strategies. The obtained maintenance schedule is non-periodic, and may include multiple different maintenance actions, if included in the model. We derived a general differential model and showed how the resulting optimization problem can be approximated to yield a continuous, although non-smooth, optimization problem that can be solved using standard, off-the-shelf tools for numerical dynamic optimization. Non-convexity of the problem required a multi-start approach to obtain a reasonable, near-global solution. Complementary constraints were handled using a relaxation and penalty method.

Finally, we illustrated the method on a small illustrative example and a more complex case example from the subsea oil and gas industry. For the case study, the method managed to produce a non-periodic and non-intuitive inspection and maintenance schedule, which was found to be better than a simple periodic inspection and production schedule.

Future work within this area should focus on improving the numerical aspects of the problem formulation. Due to non-convex nature of the problem, convergence can be quite slow. A more sophisticated multi-start approach, in combination with a more sophisticated method for handling the complementary constraints, could potentially improve performance. It is known that interior-point solvers such as IPOPT are not well suited to solve MPCCs without reformulations, as the solution has no interior [15]. Active-set methods such as the one in CONOPT [28], which implements a sequential quadratic programming (SQP) method for solving the NLP, may converge quicker due to the efficient detection of active sets and handling of dependent constraints. On the other hand, IPOPT is efficient at solving large-scale optimization problems with many inequality constraints, which is the case when using a direct collocation method to reformulate the dynamic optimization problem, as we did in this work. A comparison of different NLP solver strategies would be interesting. Furthermore, a comparison with global MINLP solvers such as BARON [25] would also be highly interesting to see the whether the MPCC reformulation is advantageous over solving solving the MINLP directly.

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