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Brittle fracture prediction of key-hole notched specimens by means of J-integral expression

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Abstract

The main goal of the present research is to check if J-integral criterion is capable of predicting the onset of brittle fracture in key-hole notched isostatic poly-granular graphite plates. In this way, this article provides the prediction of fracture loads in several key-hole notched rectangular graphite specimens with five different notch tip radii subjected to pure mode I loading condition, which has been previously reported in the literature by Lazzarin et al. (2013). The fracture load predictions for five dimensional cases were obtained through the J-integral calculations. It is revealed that the J-integral criterion is capable of predicting well the fracture loads of graphite key-hole notched specimens, independent of the size of key-hole notch.

Keywords: Brittle fracture; J-integral; Key-hole notch; Mode I loading.

1. Introduction

Some notch features are not original, meaning that they are resulted from applying a repairing method to remove crack and damage emanating from the notch tip or border. For instance, if a small crack initiates from the tip of a U-notch, a usual repairing method is to remove the crack by drilling a hole having the radius equal to the crack length. After the repairing process, a new notch called the key-hole notch is created. Recently, a large number of experimental

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results dealing with brittle fracture of key-hole notched specimens have been published in the literature (Lazzarin et al., 2013; Majidi et al., 2018). The purpose of the present research is to study the brittle fracture of the key-hole notched specimens reported by Lazzarin et al. (2013) under tension loading by using the J-integral fracture criterion.

2. Experimental results reported in literature

Lazzarin et al. (2013) have recently published a research paper in which a series of experiments on the key-hole notched rectangular specimens have been performed to assess experimentally the brittle fracture in key-hole notches under tension loading. The material utilized in the experiments has been a type of isostatic poly-granular graphite. Details of their experimental program can be found the published research by Lazzarin et al. (2013). Fig. 1 shows the tested specimen during pure mode I fracture test. The mechanical properties of the tested graphite are listed in Table 1. Four different values of notch tip radius have been investigated. The experimentally obtained fracture loads for the tested key-hole notched graphite plates are presented in Table 2.

Table 1. Some of the properties of the tested graphite material (Lazzarin et al., 2013).

Material property	Graphite
Elastic modulus (MPa)	8050
Poisson's ratio	0.2
Ultimate tensile strength (MPa)	46
Plane strain fracture toughness (MPa.m$^{0.5}$)	1.06

Table 2. Summary of the experimental results reported by Lazzarin et al. (2013) for the key-hole notched graphite specimens.

ρ (mm)	P_1 (N)	P_2 (N)	P_3 (N)	$P_{av.}$ (N)
0.25	3768	4032	4100	3967
0.5	4069	3916	4193	4060
1	4200	3758	4035	3998
2	5285	4789	4827	4967
4	4889	4992	4848	4910

3. J-integral criterion

Up to date, The J-integral criterion has been widely utilized for brittle fracture prediction in a number of materials containing notch or crack defects (Matvienko 1994; Majidi et al. 2019a; Torabi et al. 2019a), similarly to other well-known approaches such as averaged strain energy density (ASED), cohesive zone model (CZM), finite fracture mechanics (FFM), the theory of critical distances (TCD) (Aliha et al. 2017; Gomez et al. 2008; Majidi et al. 2019b; Ayatollahi et al. 2016; Torabi et al., 2019b; Carpinteri et al. 2008; Razavi et al., 2018). J-integral equation can generally be proposed as follows (Rice, 1968):

$$J_k = \int_{\varphi} (W n_k - T_i \frac{\partial u_i}{\partial x_k} ds) \qquad (k = 1, 2) \tag{1}$$

The parameters n_k, u_i, T_i, W, and J_k in Eq. 1 are the unit normal vector to the specified contour φ, the components of the displacement and traction vectors, strain energy density and the value of J-integral, respectively. In order to calculate J-integral for key-hole notched specimens subjected to pure mode I loading, the inner arc of the specified control volume, which previously utilized in previous researches dealing with ASED criterion (Razavi et al. 2017; Torabi et al. 2018a,b), should be considered. The crescent-shaped control volume is resulted between the two arcs with different curvatures. Fig. 1 depicts that the specified control volume is located at the notch tip for mode I loading conditions. Yosibash et al. (2004) has derived Eq. 2 for evaluating the critical radius of the control volume under plane-strain conditions.

$$R_C = \frac{(1+\nu)(5-8\nu)}{4\pi}\left(\frac{K_{IC}}{\sigma_u}\right)^2 \tag{2}$$

By means of Eq. 2, the critical value of the SED for the isostatic graphite considered in the present investigation is found to be W_c= 0.13 MJ/m^3, whereas the radius of the control volume is found to be equal to 0.17 mm. Since the x axis coincides with the notch bisector line (as can be seen in Fig. 1), the value of J_2 becomes zero. Also, the arc ACB in Fig. 1 is considered as an integration path. Because the traction-free conditions exist on the arc ACB, the second term of J-integral equation vanishes. Therefore, the J-integral equation for pure mode I loading conditions can be expressed as follows:

$$J = \int_{\varphi} W dy = W_1(y_2 - y_1) + W_2(y_3 - y_2) + W_3(y_4 - y_3) + \ldots + W_{n-1}(y_n - y_{n-1}) \tag{3}$$

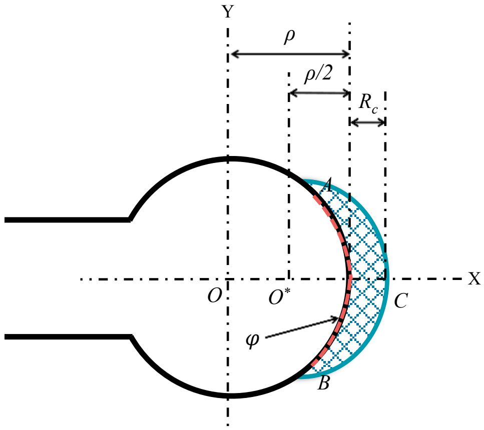

Fig. 1. Material-dependent control volume for a U-notch under mode I loading.

Therefore, one can simply and conveniently calculate the values of J-integral and critical J-integral for the tested key-hole notched specimens by using J-integral criterion. For this purpose, SED distribution along the specified contour path is essentially needed which can be evaluated by means of numerical analysis in finite element (FE) software. In this way, a FE model is created for each U-notched specimen and analysed to determine SED distribution along the contour path defined at the notch vicinity. Moreover, the values of the parameters J_{eq}, arc (ACB), and J_{cr} are presented in Table 3.

According to J-integral criterion, brittle failure occurs when the value of J-integral over a specified contour path along the inner arc of the specified crescent-shaped control volume which embraces the notch border reaches the critical value of J-integral, called J_{cr} (Barati and Berto, 2011). Also, it has been previously expressed by Barati and Berto (2011) that the critical value of J-integral (J_{cr}) could be linked to the critical SED (W_{cr}) by considering the inner arc of the control volume as the specified contour path (see the arc ACB in Fig. 1). So, the following expression can be utilized as presented previously by Barati and Berto (2011):

$$J_{cr} \cong W_{cr} \times arc(ACB) \tag{4}$$

Now, the parameter W_{cr} should also be calculated. In this way, the value of critical SED can be simply evaluated according to Beltrami's expression:

$$W_{cr} = \frac{(\sigma_u)^2}{2E} \tag{5}$$

Finally, it can be stated that for evaluating fracture load of a key-hole notched brittle component loaded under pure mode I condition, one should first evaluate the value of the J-integral for an arbitrary load (e.g. 1 N) by using Eq. 6 and after that, the load is increased till J_{eq} attains $J_{cr/EMC}$. So, the critical load can simply be evaluated by using the following expression:

$$\frac{P_{cr}}{P_{applied}} = \sqrt{\frac{J_{cr}}{J_{eq}}} \quad (6)$$

4. Results and discussion

Pure mode I fracture was investigated theoretically in key-hole notched specimens. The experimental fracture loads were predicted by means of the J-integral criterion. The fracture load values obtained for the key-hole-notched graphite plates are provided in Table 1 for different notch tip radii. It is evident from Table 3 that the fracture load increases as the notch tip radius increases from 0.25 to 4 mm. This trend of the experimental data is in a good consistency with those of the theoretical predictions obtained from J-integral criterion. An average discrepancy of around 6% was obtained from the theoretical predictions revealing the applicability of the J-integral criterion for the studied material and notch geometry.

Table 3. Summary of the numerical results with theoretical prediction for the key-hole notched graphite specimens under pure mode I loading.

ρ (mm)	$P_{av.}$ (N)	J_{eq} (N/mm)	arc (ACB) (mm)	J_{cr} (N/mm)	$P_{J\text{-integral}}$ (N)	Dis (%)
0.25	3967	0.132	0.8564	0.1113	3647	8.1
0.5	4060	0.148	0.9947	0.1293	3789	6.7
1	3998	0.148	1.2825	0.1667	4249	6.3
2	4967	0.234	1.7305	0.2250	4871	1.9
4	4910	0.267	2.3892	0.3106	5301	8.0
Average discrepancy						**6.2%**

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