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Volume free strain energy density method for applications to blunt V-notches

Pietro Foti[a]*, Seyed Mohammad Javad Razavi[a], Liviu Marsavina[b], Filippo Berto[a]

[a]*Norwegian University of Science and Technology, MTP Gløshaugen, Richard Birkelands vei 2B, Trondheim 7491, Norway*
[b]*University Politehnica Timisoara, Department Mechanics and Strength of Materials, Timisoara 300222, Romania*

Abstract

This work investigates the application of the strain energy density method to rounded V-notches under mode I loading conditions and to components without stress concentrators through finite element models that do not involve the construction of the control volume in the preprocessing phase of the finite element analysis. The application of the strain energy density method to components without stress concentrators like sharp V-notches requires, according to the conventional procedure of the method, two different numerical simulations; the first simulation defines the point of maximum of the first principal stress along the notch fillet; the second simulation calculates the strain energy density value within the control volume built using the position of the first principal stress maximum evaluated in the first simulation. Several numerical analyses, with changing the notch angle, the notch radius, the control volume radius and the mesh refinement, were carried out to evaluate the error in calculating the strain energy density value through the new procedure presented in the present work whose main advantages are to simplify the method and to decrease the calculation time that, dealing with complex geometries, is reduced by 50% requiring only one simulation instead of the two requested by the conventional procedure

Keywords: Strain Energy Density Method; Finite Element Analysis; Control Volume

Nomenclature

E	Young's modulus

* Corresponding author.
E-mail address: pietro.foti@ntnu.no

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FE	finite elements
r	geometrical parameter in the SED method
R_0	control volume radius
SED	strain energy density
W_C	critical averaged strain energy density
$\Delta\overline{W}$	cyclic averaged strain energy density
Greek	
2α	V-notch opening angle
ρ	notch fitting radius
σ_t	ultimate tensile strength

1. Introduction

Dealing with civil structures and mechanical components, researchers and designers have to deal with the fracture assessment, as well as the fatigue life predictions. Nowadays, the material properties under both static and dynamic conditions are still assessed through global methods that generally lead to an excessive conservative design that is, however, undesirable dealing with mechanical fields that require a lightweight design such as automotive and aircraft engineering.

As regards welded components, the design standards are mainly based on two global approaches due to their simplicity and statistical proof: the nominal stress approach that considers external loads or nominal stresses in the critical cross-section and compares them with the S-N curves that correlate the fatigue strength expressed in various ways, versus the number of cycles; the structural stress approach that considers the stress concentration effects of the component due to the global geometry (A. Hobbacher, 2008; Fricke, 2013; Fricke and Kahl, 2005; Holst et al., 2011; Radaj et al., 2009) and allows the fatigue assessment using the structural stresses with an S-N curve that is independent on the particular type of weld and on the geometry of the component.

The method presented above generally lead to an excessive conservative design and the assessment of a generic mechanical components lacks as a matter of fact a statistical validation being the fatigue strength suggested by the standards based on tests carried out on geometry and conditions rarely encountered in practical applications. A possible alternative to assess the fatigue behavior is given by the local approaches that are able to evaluate with more accuracy the mechanical properties of structural components (Radaj et al., 2006) even if they require an higher expertise to be applied. It is worth to underline that these methods requires the determination of those parameters that have an incisive influence on the component behavior neglecting all the aspects that can be treated in a statistical way in order to avoid complicating, even more, the problem of fatigue assessment.

In this work, we focus on the Strain Energy Density (SED) method that has been validated as a method to investigate both fracture in static condition and fatigue failure (Lazzarin and Zambardi, 2002, 2001).

One of the major drawbacks of this method is that it requires a Finite Elements (FE) model built in order to have a volume, called control volume, centered on the critical point of the components according to the theory of the method that is explained in section 2. It is worth underlining that, in order to apply this method to components without stress concentrators, two different numerical simulations are required making the method less attractive.

As pointed out also by other researcher (Campagnolo et al., 2020; Fischer et al., 2016; Foti et al., 2020; P. Foti and Berto, 2019; Pietro Foti and Berto, 2019; Zappalorto and Carraro, 2020), alternative procedures that simplify the application of the SED method are possible and it is possible to avoid the construction of the control volume in the FE model without losing the advantages, such as the SED method low sensitivity to the mesh refinement, that makes the method attractive and practical. However, all the researches carried out with the purpose of simplify the method are limited to components with stress concentrator like sharp V-notches neglecting the application of the method to component with complex geometry.

In this work we verified the accuracy of the method applied, through a procedure that does not involve the construction of the control volume in the pre-processing phase of the FE code, dealing with rounded V-notched in mode I and mixed mode loading conditions.

2. Strain Energy Density Method

The SED method is an energetic local approach validated to investigate both fracture in static condition and fatigue failure(Aliha et al., 2017; Berto et al., 2014; Berto and Barati, 2011; Lazzarin et al., 2008b; Lazzarin and Zambardi, 2002, 2001; Razavi et al., 2018; Torabi et al., 2015).

As regard the static condition, the method assumes that the brittle fracture occurs when the local SED, W, averaged over a given control volume, reaches a critical value, that is $\overline{W} = W_C$, that results to be independent on both the local geometry and the loading mode (Lazzarin et al., 2008a; Lazzarin and Zambardi, 2002, 2001). The mean SED critical value can be evaluated for an ideally brittle material under static condition through the conventional ultimate tensile strength,σ_t , and the Young's modulus of the material:

$$\overline{W}_C = \frac{\sigma_t^2}{2E} \tag{1}$$

What stated above represents the basic idea of this method. For more considerations about the analytical frame of this method we remand to (Berto and Lazzarin, 2014; Radaj and Vormwald, 2013).

Dealing with welded joints made of steel or aluminium (Atzori et al., 2006; Berto and Lazzarin, 2011; Lazzarin et al., 2003; Livieri and Lazzarin, 2005), two conditions allow the use of the SED method to assess their fatigue properties in terms of the cyclic average SED, $\Delta\overline{W}$: the brittle nature of the failure and the fact that it happens under the linear elastic regime.

The first validation of the SED method, to assess the fatigue properties of welded joints, involved a study carried out on more than 300 fatigue data with toe failure under different loading modes(Lazzarin et al., 2003). The analysis was later applied to a larger bulk of experimental data involving components with competing failure modes under different loading conditions, providing a final synthesis based on 900 experimental data (Berto and Lazzarin, 2009), where the number of cycles is given as a function of the cyclic average SED.

It is worth underlining some peculiarities of the so-called control volume in which the averaged SED value must be acquired. The control volume has a characteristic length, R_0, that is dependent on the material properties and changes dealing with static and dynamic loadings conditions(Berto and Lazzarin, 2014). Besides, the control volume

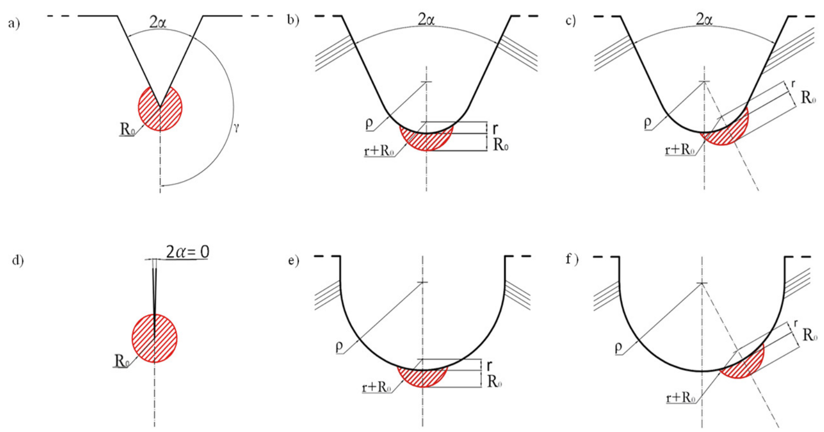

Figure 1: Control volume for: a) Sharp V-notch; b) blunt V-notch under mode I loading ($r = \rho \cdot (\pi - 2\alpha)/(2\pi - 2\alpha)$); c) blunt V-notch under mixed mode loading ($r = \rho \cdot (\pi - 2\alpha)/(2\pi - 2\alpha)$); c) Crack; d) U-notch under mode I loading ($r = \rho / 2$); e) U-notch under mixed mode loading ($r = \rho / 2$)

shape varies with the local geometry keeping the characteristic length R_0 constant and its position moves along the notch fillet depending on the loading conditions.

As regards cracks and sharp V-notches in plane problems both in mode I and mixed mode loading, the control volume is a sector-shaped cylinder of radius equal to the characteristic length R_0 with its axis along the notch tip line as shown in Figure 1 a) for V-notch and in Figure 1 d) for cracks.

As regards blunt notches (Berto et al., 2012, 2007; Gómez et al., 2008, 2007; Lazzarin et al., 2013, 2004, 2003; Lazzarin and Berto, 2005a, 2005b; Yosibash et al., 2004), the control volume has a crescent shape, with R_0 being its maximum depth inside the component. In this case, the control volume is delimited by two arches defined by the intersection between the component and a circle of radius $r + R_0$, which centre is not always located on the notch bisector, but also on a line rigidly rotated with respect to it and centred on that point where the SED reaches its maximum value, following, essentially, the mode I dominance concept. The centre of this circle is located between the notch edge and the notch-fitting radius centre, at a distance r from the notch edge as shown in Figure 1 b) and c) for blunt V-notches and in Figure 1 e) and f) for U-notches. The need to know the position of the maximum of first principal stresses field, in order to locate the control volume, leads to two different numerical simulations to apply the SED method to components that do not present stress concentrators like sharp V-notches or cracks.

3. Finite Element Analysis

Two different details are taken into account for this study. The first detail, shown in Figure 2 a) and used to assess the possibility to use the SED method according to the volume free procedure for blunt V-notches under mode I loading condition, is a flat, double V-notched, specimen while the second detail, shown in Figure 2 b) and used to verified the application of the procedure in mixed mode loading condition, is a component made by two plates, perpendicular to each other, with a blunt V-notch at the connection.

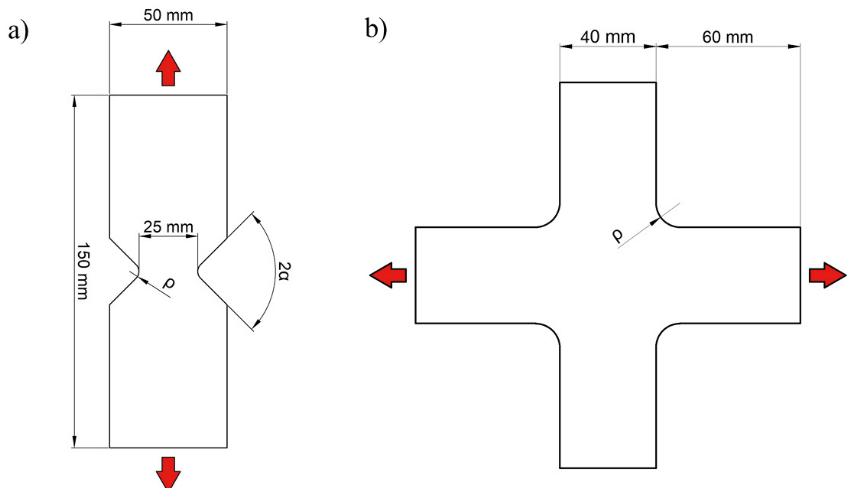

Figure 2: Schematic representation of details geometry

As regards the first detail, six different notch opening angles, 2α, are considered, $[30°; 45°; 60°, 90°; 120°; 135°]$ while the notch fitting radius, ρ, assumes five different values, $[0.1; 0.5; 2; 4; 8]\ mm$. As regards the SED evaluation for the first detail, this was carried out considering five different values of the control volume radius $[0.2; 0.4; 0.6; 0.8; 1]\ mm$, six different mesh refinements, defined as a fraction of R_0, are considered for each set of the geometrical parameters listed above: $[4; 8; 10; 12; 16;\ 20]$. As regards the second detail, the notch fitting radius, ρ, assumes five different values, $[1; 2; 4; 6; 8]\ mm$. As regards the SED evaluation for the second detail, this was carried out considering seven different values of the control volume radius $[0.14; 0.28; 0.42; 0.56; 0.70; 0.84; 0.98]\ mm$, six different mesh refinements, defined as a fraction of R_0, are considered for each set of the geometrical parameters listed above: $[4; 8; 10; 12; 16;\ 20]$.

As regard the first detail, two different models, shown in Figure 3, are taken into account. The first model, identified as model 1A and shown in Figure 3 a), is represented by a conventional FE model built to evaluate the SED value with the construction of the control volume while the second model, identified as model 1B and shown in Figure 3 b), has instead a completely free mesh pattern with only a refinement in the notch tip in order to estimate the error in the evaluation of the SED considering the easiest way possible to build the model.

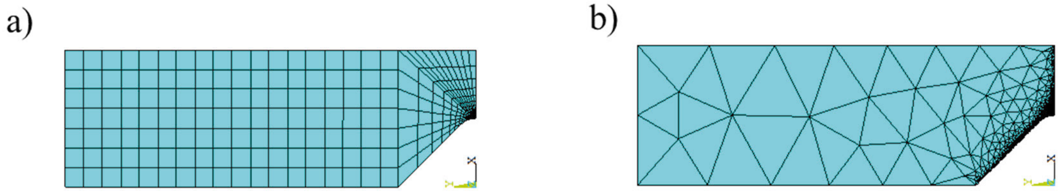

Figure 3: FE models for: a) mapped mesh pattern with control volume; b) free mesh pattern without control volume

As regard the second detail, three different models, shown in Figure 4, are taken into account. The first model, identified as model 2A and shown in Figure 4 a), is built in order to have a mapped mesh for the geometry considered but without the construction of the control volume. The second model, identified as model 2B and shown in Figure 4 b), has instead a completely free mesh pattern with only a refinement along the notch fitting curve in order to estimate the error in the evaluation of the SED considering the easiest way possible to build the model. The second model, identified as model 2C and shown in Figure 4 c), is represented by the conventional model built to evaluate the SED value with the construction of the control volume; in this case, since the control volume is no longer centred along the notch bisector, the position of the maximum first principal stress comes from the model 2A, Figure 4 a).

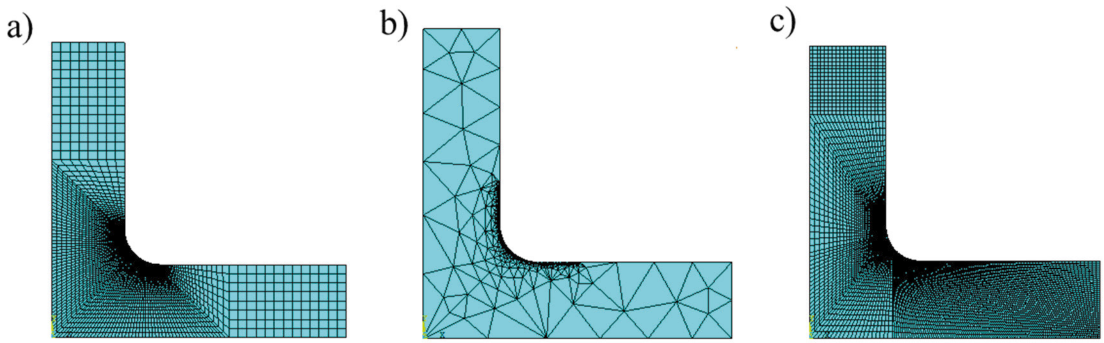

Figure 4: FE models for: a) mapped mesh pattern without control volume; b) mapped mesh pattern with control volume; c) free mesh pattern without control volume

As regards the models 1A and 2B, the SED value is acquired following the conventional procedure of the SED method while, for the models 1B, 2A and 2C , the SED value is acquired through a selection of the elements using a polar coordinate system centred along the segment that links the point of maximum of the first principal stress and the center of the connection fillet at a distance r , assessed as stated in section 2, from the component surface. The result of such a selection is shown in Figure 5 that reports the elements selection carried out for model 2A (Figure 5a)) and for model 2B (Figure 5 b)) and the mesh patter inside the control volume as regards the model 2C (Figure 5 a)).

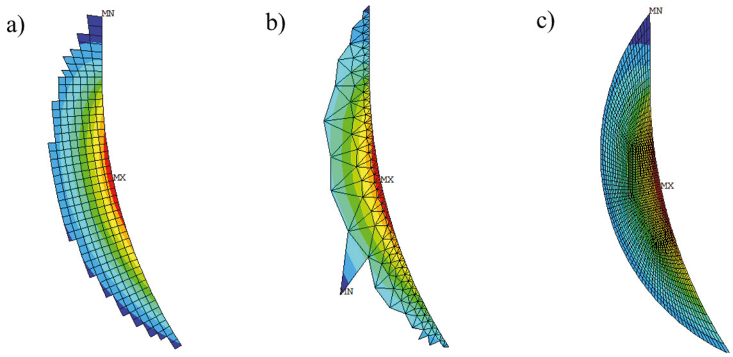

Figure 5: Control volume and SED contour plot for: a) Model 2A; b) Model 2B; c) Model 2C

4. Results and Discussions

The purpose of the present work is to evaluate the error in achieving the SED value for components without stress concentrator, such as sharp V-notches, considering only one numerical simulation carried out though a model with a free mesh pattern built imposing only a refinement near the notch fitting curve.

For the model 1B, the reference case corresponds to the value acquired with the numerical analysis carried out with the most refined mesh pattern through the model 1A that corresponds to the conventional procedure utilised to estimate the SED value, while as regards the model 2A and 2B, the reference value corresponds to the value acquired with the numerical analysis carried out with a very refined mesh pattern through the model 2C.

The Table 1 reports, for the models 1B, 2A and 2B, the mean value, μ, the standard deviation, σ, the maximum and the minimum error achieved for all the possible set of parameters listed in section 3 with changing the mesh refinement.

Table 1: SED error statistic for the models considered

Model	$R_0/Mesh\ Size$	μ	σ	Err max %	Err min %
1B	4	0.29	1.46	7.91	-3.70
	8	0.05	0.84	3.97	-3.14
	10	-0.02	0.88	5.05	-4.66
	12	0.10	0.79	7.68	-1.65
	16	0.04	0.72	5.49	-2.08
	20	0.10	0.94	8.32	-2.01
2A	4	0.43	0.96	1.52	-3.66
	8	0.03	0.32	0.71	-1.19
	10	-0.10	0.52	1.69	-2.39
	12	-0.03	0.40	1.66	-1.27
	16	-0.07	0.35	1.33	-1.37
	20	-0.02	0.17	0.59	-0.51
2B	4	-0.81	4.52	17.56	-8.17
	8	0.47	3.48	14.26	-8.20
	10	-0.11	2.25	8.07	-6.21
	12	0.05	2.79	4.81	-5.10
	16	0.21	1.57	5.68	-4.91
	20	0.25	1.69	6.05	-4.98

The data acquired show that an acceptable estimation of the SED value is possible through the volume free methodology as regards blunt V-notches under mode I and mixed mode loading condition.

As regards the model 1B a good estimation of the SED value is possible with a mesh size of $1/8$ of R_0 that leads to a mean error of 0% with a standard deviation of 1% and a maximum error of 4%. It is worth underlining that a trend has been noticed between the error achieved and the notch opening angle; the error in evaluating the SED value decreases with increasing the notch opening angle. As regards the model 2A that, unlike the other models considered, has a mapped mesh pattern without the construction of the control volume in the pre-processing phase a good estimation of the SED value is possible with a mesh size of $1/4$ of R_0 that leads to a mean error of 0.5% with a standard deviation of 1% and a maximum error of 4%. As regards the model 2B a good estimation of the SED value is possible with a mesh size of $1/8$ of R_0 that leads to a mean error of 0.5% with a standard deviation of 4%. In this case the maximum and minimum error reported seems to be too high in absolute value but it is worth underlining that they are related to a bad element selection due to the mesh pattern, as shown in Figure 6. A check to the shape of the selection to be sure that it does not deviate excessively from the theoretical control volume shape will avoid errors of this magnitude.

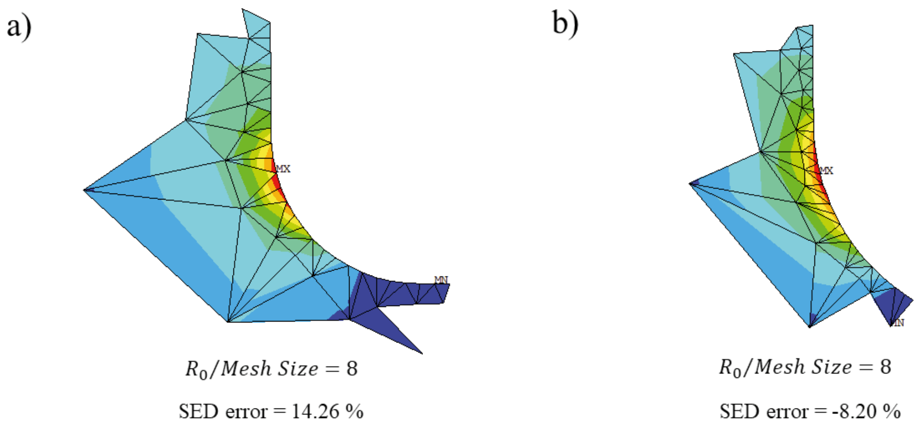

Figure 6: Elements selections for model 2B deviating excessively from the control volume theoretical shape

5. Conclusions

The present work investigates, through FE simulations, the possibility to apply the volume free procedure to acquire the SED value with regards to component with blunt V-notches under mode I loading condition and to component without stress concentrators under mixed mode loading condition considering several sets of the geometrical parameters describing the details taken into account.

As regards the detail subjected to mode I loading condition the conclusions are the following:

- The error in evaluating the SED value decreases with increasing the notch opening angle and increasing the mesh refinement.
- A mesh size of $1/8$ of R_0 leads to an acceptable evaluation of the SED value with a standard deviation of 1% and a maximum absolute error of 4%.

As regards the detail subjected to mixed mode loading condition the conclusions are the following:

- A mapped mesh that uses 8 node elements without control volume and with a mesh size of $1/4$ of R_0 leads to a very good evaluation of the SED value with a standard deviation of 1% and a maximum error of 4%
- A free mesh realized imposing a mesh size along the notch fitting curve of $1/8$ of R_0 leads to an acceptable evaluation of the SED value with a standard deviation of 3.5 % and a maximum error of 14.5%.

Besides, it is important to check that the shape of the selection does not deviate excessively from the theoretical control volume shape in order to avoid having higher value of the error in estimating the SED value.

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