# Collaborative Fault Diagnosis Decision Fusion Algorithm Based on Improved DS Evidence Theory

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**Abstract.** DS evidence theory has in obtaining a correct diagnosis when confronted with highly conflicting evidence, a collaborative fault diagnosis decision fusion algorithm based on an improved version of DS evidence theory is proposed. The algorithm builds upon the closeness of certain kinds of evidence produced by existing DS evidence theory algorithms. According to the importance of the diagnostic information, weights are assigned to reduce the conflicting information while retaining the important diagnostic information. Simulated example shows that the algorithm could reduce the impact of conflicts in diagnostic information and improve the accuracy of the decision fusion process.

**Keywords:** decision fusion, collaborative fault diagnosis, DS evidence theory, closeness

## 1 Introduction

Collaborative fault diagnosis technology decomposes complex fault diagnosis into multiple sub-fault tasks that are easy to handle and completes the collaborative diagnosis of each sub-fault task with multiple diagnostic resources. The main processes undertaken by the technology include data acquisition, task decomposition, task assignment, and decision fusion. The decision fusion process is particularly responsible for redundancy, conflict and cooperation issues in relation to the diagnostic information of each sub-fault task. This is the focus of this research [1-2].

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Common decision fusion methods include neural networks, Bayesian, fuzzy probability, DS (Dempster Shafer) evidence [3-4]. As the prior knowledge required by DS evidence theory is more intuitive and easier to obtain, DS evidence theory has great advantages for decision fusion. However, in the case of highly conflicting evidence, DS evidence theory can arrive at conclusions that are not consistent with common-sense or even correct. In response to this problem, a number of researchers have proposed improved algorithms. These algorithms can be roughly divided into two categories. One category focuses on potential problems with the fusion rules and recommends modification of the rules (Yager [5], Ali [6] and Cui Jiayu [7]). Although such methods have achieved good fusion results, they destroy the mathematical characteristics of the original fusion rules. The other category focuses on potential problems with the source of the evidence itself. Studies, here, typically recommend modifying the evidence model. There is a particular emphasis placed upon initial pre-processing of the evidence, prior to fusion being conducted using fusion rules (Carlson [8], Sun Quan [9] and Deng Yong [10]).

The above two types of approaches have their own advantages and disadvantages. Although there may be conflicts or inconsistencies between multiple objects, most decision problems relating to individual objects have effective solutions. In that case, it helps to introduce a notion of 'closeness' to indicate the degree of similarity between objects. Some researchers have combined closeness and DS reasoning [11-13], with valuable results. However, most of the above studies focus on decision fusion in situations where there is strong informational conflict and it is difficult to reflect the degree of similarity between objects and attribute weights. This paper therefore proposes, instead, a collaborative fault diagnosis decision fusion algorithm based on improved DS evidence theory. DS evidence theory and closeness analyses are combined for the decision fusion of diagnostic information. This overcomes the errors in fusion results caused by conflicts between diagnostic information, while improving the effectiveness of the decision fusion.

## 2 Collaborative Fault Diagnosis Decision Fusion Model Based on DS Evidence Theory

The description of any problem requires a subjective description of objective problems and the probability of the occurrence of an event is obtained on the basis of subjective and objective analysis. At first, this kind of subjective factor results in a deviation from the objective description of problems. However, as objective problems become more profound and the amount of information increases, subjective understanding is more complete, its knowledge structure is more complete, and subjective judgments are more likely to obtain an accurate representation of probability. Shafer proposed the concept of evidence theory to explain this new approach to probability. DS evidence theory has received the most attention in the fields of decision fusion and expert systems.

In DS evidence theory, $\Theta = \{\theta_1, \theta_1, ..., \theta_n\}$ is a recognition framework, where $\Theta$ is composed of objects that are independent and exclusive. The objects can be a collection of objects that the target recognizes. If the number that can be obtained by throwing a dice is represented by $\Theta$ , the recognition framework can be expressed as $\Theta = \{1,2,3,4,5,6\}$ . The recognition framework is a collection of targets for all situations, which are independent and mutually exclusive, thereby turning abstract problems into mathematical problems.

(1) Basic probability assignment function

The power set $2^n$ of the elements in $\Theta$ represents the possible combination of targets, where any element is called a focal element of $\Theta$. Assuming that $\Theta$ is the recognition framework, the target problem can be represented by the function $m: 2^n \rightarrow [0,1]$ and must satisfy the following:

① The basic probability of an impossible event is 0, i.e. $m(\theta)=0$ .

② The sum of the basic probabilities of all the elements in $2^n$ is 1, i.e. $\sum_{A \subseteq \theta} m(A)=1$ .

where, $m$ is the BPA (basic probability assignment) function of $\Theta$, which is also called the basic reliability assignment function. $m(A)$ represents the basic probability assignment function for the target problem, $A$ , i.e. the degree of support for the occurrence of target problem $A$ , rather than the support for the true subset of $A$ . $m(\theta)=0$ indicates that the BPA for an empty set is 0. $\sum_{A \subseteq \theta} m(A)=1$ indicates that each proposition has its own confidence level, but the sum of the confidence in the propositions in the recognition frame is 1.

(2) Confidence function

For any proposition, a confidence function, $Bel(A)$ , is defined as the sum of the basic probabilities corresponding to all subsets, namely:

$$\begin{cases} Bel: 2^{\Theta} \rightarrow [0,1] \\ Bel(A) = \sum_{B \leq A} m(B), A \subseteq \Theta \end{cases} \tag{1}$$

The difference between $m(A)$ and $Bel(A)$ is mainly that $m(A)$ means that the confidence is only assigned to the subset, $A$ , while $Bel(A)$ represents the sum of the confidence relating to all subsets of $A$ .

(3) Likelihood function

The likelihood function represents the degree of trust that $A$ is not false, i.e. the measure of the uncertainty as to whether $A$ is possible, namely:

$$\begin{cases} PI: 2^{\Theta} \rightarrow [0,1] \\ PI(A) = \sum_{B \cap A \neq \phi} m(B), A \subseteq \Theta \end{cases} \tag{2}$$

$PI(A)$ is the sum of BPAs that do not support subsets of $A^c$, i.e. $PI(A) = 1 - Bel(A^c)$.

（4）Fusion Rule

The diagnosis information of each subtask is used for decision fusion, again according to certain rules, until the final result is obtained. Within the same identification framework, there may be several different evidence functions. For example, when there are two pieces of evidence, they can be fused by the fusion rule in DS evidence theory. The fusion formula can be expressed as follows:

$$m(A) = \begin{cases} 0, A=\phi \\ \dfrac{\sum_{A_i \cap B_j = A} m_1(A_i)m_2(B_j)}{1-K}, A \neq \phi \end{cases} \tag{3}$$

where, $K = \sum_{A_i \cap B_j = \phi} m_1(A_i)m_2(B_j)$, with a range of $[0,1]$, is called the conflict factor.

## 3 Decision Fusion Algorithm Based on Improved DS Evidence Theory

It allows us to reduce the impact of conflicting information while retaining more valuable evidence information and obtaining more accurate decision fusion results. Although there is sometimes a conflict between the evidence, there is also varying degrees of closeness. This paper introduces the notion of closeness to indicate the proximity between evidence information and to reflect the degree of conflict between the evidence information, thus improving the accuracy of decision fusion.

Let us assume an identification framework is $\Theta = \{\theta_1, \theta_1, ..., \theta_n\}$. For any proposition, the BPA for obtaining two pieces of evidence is $m_i(\theta_k)$ and $m_j(\theta_k)$, respectively. Then, the closeness of the two pieces of evidence for the proposition is:

$$a_{ij}(k) = \frac{\min(m_i(\theta_k), m_j(\theta_k))}{\max(m_i(\theta_k), m_j(\theta_k))} \tag{4}$$

where, $\min(m_i(\theta_k), m_j(\theta_k))$ is the smaller BPA of the two pieces of evidence and $\max(m_i(\theta_k), m_j(\theta_k))$ is the larger. Therefore, $a_{ij}(k)$ has a value greater than 0 and less than or equal to 1. Given a limiting value, $P$, if $a_{ij}(k) < P$, it means that the two pieces of evidence are not close, which can be expressed as:

$$a_{ij}(k) = \begin{cases} a_{ij}(k), a_{ij}(k) \geq P \\ 0 \quad\quad , a_{ij}(k) < P \end{cases} \tag{5}$$

where, $a_{ij}(k)$ represents the closeness between $m_i(\theta_k)$ and $m_j(\theta_k)$, but does not express the closeness between $i(j)$ and the other $m_n(\theta_k)$ in the recognition framework. The closeness of the propositions between $m_i(\theta_k)$ and the other evidence is $a_{i1}(k), a_{i2}(k), ..., a_{in}(k)$. A matrix can be used to visually express the closeness between the individual evidence for the propositions :

$$A = \begin{bmatrix} 1 & a_{12}(k) & ... & a_{1n}(k) \\ a_{21}(k) & 1 & ... & a_{2n}(k) \\ ... & ... & 1 & ... \\ a_{n1}(k) & a_{n2}(k) & ... & 1 \end{bmatrix} \quad (6)$$

The closeness between the two pieces of evidence in Eq. (6) is 1 and the basic probability assignment function $m_i(\theta_k)$ of a certain piece of evidence $i$ can be obtained by analyzing the closeness between one particular proposition in the matrix, $A$, and other evidence propositions, i.e. $A_{sum}(i) = a_{i1}(k) + a_{i2}(k) + ... + a_{im}(k)$.

Pieces of evidence $i$ and $j$ have the same closeness to the same proposition, $\theta_k$, i.e. $a_{ij}(k)$ equals $a_{ji}(k)$, which satisfies the symmetry of matrix $A$. As the values in matrix $A$ are always positive, the rules of linear algebra dictate that matrix $A$ must have eigenvalues $\lambda(\lambda > 0)$ and corresponding eigenvectors $R$, namely:

$$AR = \lambda R(\lambda > 0) \quad (7)$$

For some piece of evidence, the closeness of the proposition reflects the credibility of the evidence associated with the proposition. Thus, the weight of the evidence for a proposition can be expressed by its closeness. The weight of a proposition, $w_i(\theta_k)$, can be expressed as follows:

$$w_i(\theta_k) = c_1 a_{i1}(\theta_k) + c_2 a_{i2}(\theta_k) + ... + c_n a_{in}(\theta_k), (i = 1, 2, ..., n) \quad (8)$$

$w_i(\theta_k)$ can be obtained from Eq. (8), but should also satisfy:

$$\sum_{i=1}^{n} w_i(\theta_k) = 1 \quad (9)$$

The following matrix representation can be obtained through further simplification:

$$W = AC \quad (10)$$

where, $W = [w_1(\theta_k), w_2(\theta_k), ..., w_n(\theta_k)]^T$ and $C = [c_1, c_2, ..., c_n]^T$. According to $AR = \lambda R$ $(\lambda > 0)$ in Eq. (7), Eq. (10) can be linearly transformed, namely:

$$W = \lambda P \quad (11)$$

The matrix, $P$, in Eq. (11) includes $p_1(k), p_2(k), ..., p_n(k)$. Subsequently, the weight of the proposition can be obtained according to the following solution matrix:

$$W_i(\theta_k) = \frac{p_i(k)}{p_1(k) + p_2(k) + ... + p_n(k)} \quad (12)$$

The weight, $w_i(\theta_k)$, of each probabilistic BPA function, $m_i(\theta_k)$, for the proposition can be calculated in turn, according to Eq. (12). To solve $w_i(\theta_k)$,

$p_1(k), p_2(k),..., p_n(k)$ must be obtained first. For the matrix, $P$, it can be obtained by multiple transformations using the rules of linear algebra. In this paper, we use a membership function with a normal distribution:

$$\varphi(x) = e^{-(\frac{x_i(k)-a}{b})^2}, (a > 0, b > 0) \tag{13}$$

The function, $x_i(k)$, in Eq. (13) can represent the BPA function, $m_i(\theta_k)$, of the proposition, where $a$ is the mean and $b$ is the variance. The above formula can be substituted into Eq. (13):

$$p_i(k) = e^{-(\frac{x_i(k)-a}{b})^2}, (a > 0, b > 0) \tag{14}$$

where, $a = \bar{x} = \frac{\sum_{i=1}^{n} x_i(k)}{n}, b = s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})}$.

$p_i(k)$ can be obtained and substituted into Eq. (12). Subsequently, the weight, $w_i(\theta_k)$, of the evidence for the proposition can be obtained. The BPA is then recalculated and, finally, the new BPA can be used to carry out the decision fusion.

## 4 Numerical Simulation Analysis

Assuming that a recognition framework, $\Theta = \{\theta_1, \theta_1, \theta_3\}$, is used to indicate three possible causes of an equipment fault. DS evidence theory-based decision fusion can now be performed on the newly generated BPA functions. This can be compared with the outcomes of traditional DS theory, Yager's method [5], Sun Quan's method [9], and Deng Yong's method [10]. The comparison results are shown in Table 1, 2 and 3.

**Table 1** Comparison of fusion results for two diagnostic nodes

| $m_1 m_2$ | $m_{12}(\theta_1)$ | $m_{12}(\theta_2)$ | $m_{12}(\theta_3)$ | $m_{12}(\varphi)$ |
|---|---|---|---|---|
| DS Evidence Theory | 0.0 | 0.01 | 0.99 | 0.0 |
| Yager's Method | 0.0 | 0.0001 | 0.0099 | 0.99 |
| Sun Quan's Method | 0.18 | 0.004 | 0.194 | 0.622 |
| Deng Yong's method | 0.18 | 0.004 | 0.194 | 0.622 |
| Method in this paper | 0.1859 | 0.0044 | 0.0392 | 0.7715 |

**Table 2** Comparison of fusion results for three diagnostic nodes

| $m_1 m_2 m_3$ | $m_{123}(\theta_1)$ | $m_{123}(\theta_2)$ | $m_{123}(\theta_3)$ | $m_{123}(\varphi)$ |
|---|---|---|---|---|
| DS Evidence Theory | 0.0 | 0.0 | 0.0 | 1.0 |
| Yager's Method | 0.0 | 0.0 | 0.00099 | 0.999 |
| Sun Quan's Method | 0.321 | 0.003 | 0.188 | 0.488 |
| Deng Yong's method | 0.3594 | 0.0038 | 0.2103 | 0.4255 |
| Method in this paper | 0.4686 | 0.0027 | 0.0518 | 0.4769 |

**Table 3** Comparison of fusion results for four diagnostic nodes

| $m_1 m_2 m_3 m_4$ | $m_{1234}(\theta_1)$ | $m_{1234}(\theta_2)$ | $m_{1234}(\theta_3)$ | $m_{1234}(\varphi)$ |
|---|---|---|---|---|

| | | | | |
|---|---|---|---|---|
| DS Evidence Theory | 0.0 | 0.0 | 0.0 | 1.0 |
| Yager's Method | 0.0 | 0.0 | 0.00099 | 0.999 |
| Sun Quan's Method | 0.42 | 0.003 | 0.181 | 0.369 |
| Deng Yong's method | 0.4557 | 0.0033 | 0.1967 | 0.3442 |
| Method in this paper | 0.6481 | 0.0017 | 0.0511 | 0.2991 |

It can be seen from Table 3 that the accuracy of the decision fusion is greatly improved when compared with traditional DS evidence theory and Yager's method. When compared with Sun Quan's method, the improvement in the accuracy of the decision fusion is about 22% and it is about 19% higher than Deng Yong's method. Thus, the proposed algorithm improves DS accuracy to a significant degree.

Fig. 2 shows the change in the degree of support for the three faults in the identification framework $\Theta = \{\theta_1, \theta_1, \theta_3\}$ according to the amount of evidence, with $\varphi$ indicating the degree of support.

It can be seen from Fig. 2 that traditional DS evidence theory always assigns a degree of support of $0$ to $\theta_1$ when dealing with conflicting information. This is obviously not consistent with common-sense, indicating that traditional DS evidence theory is not able to deliver a correct decision fusion result in the face of highly conflicting evidence. If Yager's method is compared with traditional DS evidence theory, it simply discards the conflicting information when it appears. In other words, it is assigned to $m(\varphi)$ and no other processing is performed, so the correct decision fusion result cannot be obtained. Sun Quan's method and Deng Yong's method obtain a decision fusion result, but the accuracy is low and a large proportion of evidence is assigned to $m(\varphi)$. By contrast, the algorithm proposed in this paper effectively overcomes the problem of conflicting evidence. The larger the number of diagnosis nodes, the greater the degree of support for fault $\theta_1$, according to the algorithm proposed in this paper. In comparison to other algorithms, it effectively overcomes the disruption of decision fusion caused by conflicting information and can converge rapidly.

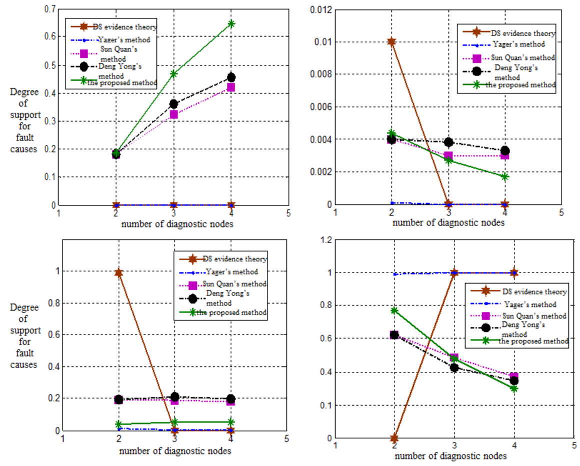


**Fig. 2.** Degree of support for the faults $\theta_1, \theta_2, \theta_3$ and $\varphi$

# 5 Conclusions

In this paper, the collaborative fault diagnosis decision fusion model based on DS evidence theory has been discussed. Its limitations have been noted and an improved DS evidence theory-based algorithm has been proposed. An example simulating use of the algorithm was used to verify the performance of the algorithm in comparison to other related algorithms. The results have shown that the improved algorithm performs better and can obtain more accurate decision results.


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