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Report

Methods of aggregation and disaggregation

Project results

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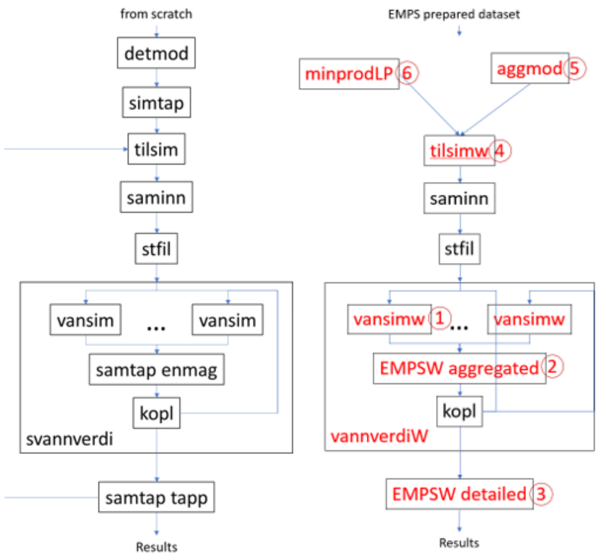

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ABSTRACT
The focus of the project "Methods of aggregation and disaggregation" are improved mathematical methods and computer tools for aggregation and disaggregation of hydro power systems in optimization models. These techniques are necessary for calculating the optimal utilisation of hydropower production in the Nordic power system. Existing methods have been applied for decades and the project re-visited and upgraded these to establish a new model adapted to the analysis of the future electricity market.

The project targets the aggregation techniques of aggregating complex water courses in one or a few equivalent hydro power modules. Moreover, new methods for calculating the best operation strategy for the aggregated hydropower description are tested. However, the tested aggregation and calculation methods did not show their superiority.

For the disaggregation, the project implemented a formal optimisation of the detailed hydropower dispatch, substituting the existing draw down heuristics. This formal optimisation approach provides the opportunity to better assess short-term variability and flexibility in hydro-thermal power systems.

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Summary

The players in the Nordic power market, i.e. producers, transmission system operators and regulators use computer models to plan for the best possible utilization of the power system. The calculation of the optimal operation strategy for the hydro storage in the system is the most important and complicated computation. Emptying the reservoirs may result in curtailment of electricity and too cautious operation may result in unnecessary spillage, which is a loss to the society. The goal for optimal hydropower scheduling is to find operation strategies for all hydro storage in the Nordic system that provides the best utilization and to simulate the consequences of the operation strategy on e.g. prices and reservoir operation for possible futures (inflows, temperatures and wind power production etc.). Moreover, the huge increase in variable, non-controllable renewable production and the stronger coupling to continental Europe results into increasing importance of short-term effects for hydropower scheduling, which cannot be handled properly by the existing computation methods, requiring a re-visit and improving of existing methods.

Utilization of the hydro storage may be formulated as a mathematical optimization problem. The large problem size and complexity requires several simplifications to obtain a solution. One of the most important simplifications is an aggregation of physical hydro storage and plants into an equivalent representation. However, the aggregated hydro model implies more flexibility than the physical system and disaggregation techniques are used to verify that reservoir operation strategies for the aggregated model are feasible for the physical system.

The project "Methods of aggregation and disaggregation" has increased knowledge and improved mathematical methods and computer tools for aggregation and disaggregation of hydro power systems in optimization models. Existing methods has been unchanged for decades and the project re-visited and upgraded these to establish a new model adapted to the analysis of the future electricity market.

The project work started summer 2015 and the focus has been the following activities:

- Literature review and knowledge building related to application of aggregation and disaggregation techniques in hydropower planning outside the Nordic region.

- Knowledge building on existing disaggregation techniques has been combined with finding a new method for disaggregation. It comprised studying parts of the existing disaggregation technique and combining it with formal optimization to give an improved problem solution. The computation time is long and considerable effort has been devoted to testing of methods that can reduce computation time. A prototype model has been main available to the project partners.

- The resulting disaggregation is better at utilizing price variations for pumped storage plants, hydropower in serial watercourses and accounting for non-controllable renewable production. A paper presenting results from an analysis of a future European power system is published in Energies.

- A new Stochastic Dynamic Programming type algorithm was implemented for a general aggregated model structure for strategy calculations where the focus has been on an aggregated two-reservoir model. Testing shows that the new generalized implementation is much more time consuming than existing method. Parallel processing may be utilized to reduce calculation time.

- A Sampling Stochastic Dynamic Programming algorithm (SSDP) was implemented and tested. According to the literature and the properties of the algorithm, SSDP improves on the representation

of correlations in time and space compared to Stochastic Dynamic Programming, e.g. better represent dry/wet years. The SSDP does not give better results than SDP for tested cases.

- An automatic generation of the aggregated model structure to be used in the strategy calculation has been established. This work has been divided into two sub-activities.
    - The first sub-activity addressed a practical method that aggregates one or more river systems into two aggregated parallel reservoirs. This comprises the separation of the physical inflow into storable and non-storable energy inflow. The solution is a formulation of an optimization problem for each river system for each week where the goal is to minimize sum production but fulfill all constraints.
    - In the second sub-activity, a new general aggregation procedure has been implemented and tested. General aggregation means that it can in principle aggregate from any system to any new system that is more aggregate than what it started with in the first place. However, the general procedure is not good enough for practical use in its current form.

Finally, the models developed in the project heavily rely on existing computer algorithms. These algorithms have recently been modernized. Therefore, much effort has been put into upgrading the model to exploit the modern algorithms. A detailed report of the disaggregation method has been distributed to the project members.

1 Introduction

In Scandinavia about 50 % of the electricity is produced by hydropower plants, mainly situated in Norway and Sweden. The Scandinavian hydropower system consists of more than thousand reservoirs, many hundred power plants located in more than 50 river systems. The annual inflow to the reservoirs varies considerably (at least +-/ 20%), which affects the total electricity production and market prices. The hydro reservoirs are used to level out variations caused by inflows, demand and variations in intermittent renewable energy sources like wind and solar power production.

Fundamental power market models are used to forecast how the electricity system will behave and particularly how future electricity prices will be affected by e.g. climate change, the amount of production from new renewables, new transmission infrastructure etc. These fundamental market models describe the whole system based on the installed assets and are formulated as a large scale stochastic dynamic optimization problem. The complexity of this problem is mainly due to the large number of hydropower modules (reservoirs and power plants) and the uncertainties (inflows, wind and solar production, temperatures and thermal production costs). As the storage capacity of the reservoirs ranges from single days to several years a fine time resolution as well as a sufficiently long horizon is needed, which further increases the problem size. The large-scale integration of intermittent renewable energy system, such as wind and solar power puts even more emphasis on short-term variability and effects in hydropower scheduling, requiring a re-visit and improving of existing methods.

1.1 State of the art power market modelling in Scandinavia

Methods to solve this type of stochastic reservoir optimization problems has been known for many decades and evolved with advances in computational power and algorithm development. [1] gives a good overview of the different methods that are applied. Except it does not explicitly mention the Stochastic Dual Dynamic Programming (SDDP) based method [2] that have been in operational use for decades among others in Scandinavia [3] and Brazil [4]. We believe that the SDDP based methods in general are the best and the most applied method for medium to large hydro systems, i.e. system consisting from about 3 to less than 50 reservoirs. Systems consisting of a small number of reservoirs are often solved using stochastic dynamic programming (SDP). The SDP method may include non-linear relations that are difficult to include in SDDP. In Scandinavia the SDDP method is applied to individual river systems ranging from 1 to about 30 reservoirs. If applied to very large systems, the dimensionality of the state space and especially the inflow model makes results poor and hence SDDP is not applicable. Reference [5] describes results from an attempt to apply an SDDP based model at the Norwegian hydro system consisting of about 500 reservoirs.

In Scandinavia a fundamental power market model has been in operational use since the seventies. The model is called EMPS [6, 7] and the objective is to minimize the expected cost of suppling forecasted demand for the planning period that typically is some years ahead. Before the market liberalization in 1991 the model was mainly used to forecast price and exchange of surplus power between different producers within Norway and for exchange with neighbouring countries. After liberalization important applications include spot price forecasting, transmission expansion planning and analysis of security of supply [8].

The EMPS model uses an aggregation/aggregation disaggregation approach and consists of two parts:

- A *strategy* evaluation part computes regional decision tables in the form of expected incremental water values for each of a defined number of aggregate regional subsystems. These calculations are based on the "Water Value Method" which is a variant of SDP, first described by [9]. Instead of storing the future cost in tables, the incremental cost, i.e. water value, is calculated and stored directly. Included in the strategy part is an overlaying hierarchical logic to treat the multi-reservoir aspect of the aggregate problem. Inflows and other weather-related uncertainties are treated as coupled stochastic variables.

- A *simulation* part evaluates optimal operational decisions for a set of scenarios, each defined by different sequences of weather years. Weekly hydro and thermal-based generation are in principle determined via a market clearance process based on the incremental water value tables calculated for each aggregate regional subsystem. Each region's aggregate hydro production is for each week distributed (disaggregated) among available plants using a rule-based reservoir drawdown model containing a detailed description of the hydro system. This ensures that the simulated results are feasible for the physical system.

The EMPS model is today typically run with a five-year planning horizon, 3 hourly time resolution, more than 1000 hydro reservoirs and hundreds of thermal production units that may or may not include linearized unit commitment modelling [10]. The model is run for between 40 and 90 different weather scenarios. With application of parallel processing the computation time for a model run is less than an hour. The model is fast and gives a feasible solution for the very complicated large-scale stochastic optimization problem. Due to the heuristics used for aggregation/disaggregation as well as for the multiarea coordination problem in the strategy part of the model, we know that the results are not necessarily optimal. However, all comparisons done with alternative methods have so far, showed that the EMPS method provide good solutions at extremely low computation times. This is based on comparing EMPS and SDDP based methods when both are applied to systems with a limited number of reservoirs and by comparing with another more formal based optimization model that takes several weeks to run for the large system description [11].

1.2 Challenges for the future Nordic power system

With increasing installation of wind and solar power and the planned increased transmission capacity to central European system, the short-term flexibility of the hydro system is expected to become more valuable, and the hydropower system will more often operate at its limits. It is therefore important to apply planning models that fully optimize the utilization of the hydro system. The EMPS disaggregation heuristic has a known weakness related to the short-term (hour by hour) optimization and utilization of complicated river systems, which is discussed more throughout in chapter 3. The newly developed FanSi model described in [11] and [12] does not have this weakness, but the computation time is too long for many applications. Thus, the objective of the project "Methods of aggregation and disaggregation" is to develop a new "EMPS" type model that addresses known weaknesses and has a much shorter computation time than the FanSi model. Furthermore, a prototype model ProdMarket was developed to test possibilities of applying SDDP in the large-scale hydropower planning problem.

Figure 1 provides an overview of the hydro-thermal power market simulators developed at SINTEF. As it is indicated are all of the models based on the same input data and provide the same types of results. Moreover, some of the models share parts of the implemented methods.

- Within the simulation part, the EMPS solves the weekly hydro-thermal power market problem on an aggregated level and utilises a heuristic based disaggregation method to determine the detailed hydropower dispatch. On contrary, the FanSi, the EMPSW (and ProdMarket) model solve the weekly hydro-thermal power market problem on the detailed level by the formal optimisation using Linear Programming.
- Within the strategy part, the EMPSW use target reservoir heuristics coupled with aggregated water values to make individual water values and solves the weekly hydro-thermal power market problem in a one stage optimisation problem. This calculation of target reservoir and the according water values is similar to EMPS. The FanSi model solves the weekly hydro-thermal power market problem in two stages. The first stage is the weekly hydro-thermal market problem and the second stage is described by a scenario fan comprising of several time steps. Benders cuts from the second stage is used as input to the first stage. ProdMarket applies optimisation problems of individual river systems

based on SDDP, which are coupled to a master problem representing a market clearing based on price-coupling.

- Finally, the computational burden of the EMPSW, which is developed in this project, is lower than the FanSi model, but higher than the EMPS.

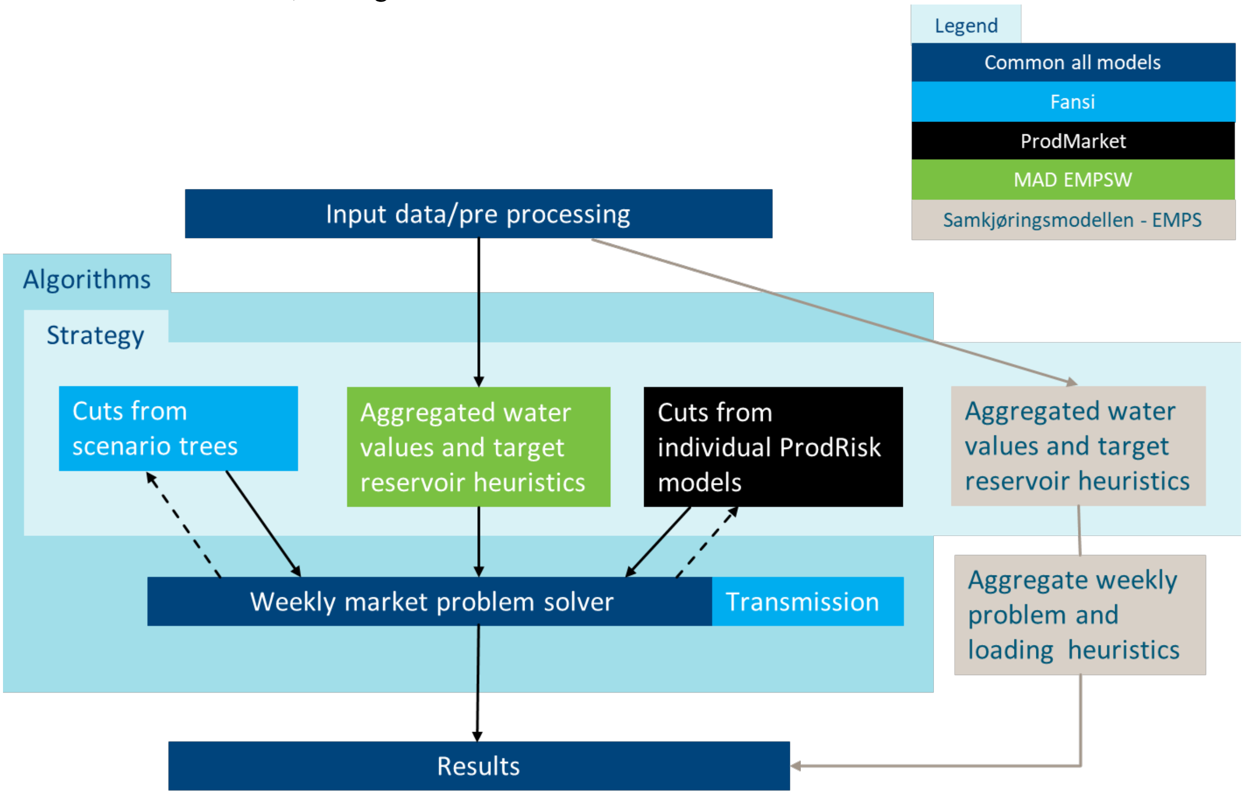

Figure 1: Overview of the different techniques used in the hydro-thermal market simulators developed at SINTEF.

1.3 Report structure

The remainder of the report is structured in the following. A description of the literature review on aggregation and disaggregation techniques is presented in chapter 2. The Sampling Stochastic Dynamic Programming (SSDP) and our experience applying the SSDP method on Norwegian hydropower optimization problems are presented in chapter 3. Chapter 4 focusses on the weaknesses of the existing EMPS/EOPS disaggregation techniques and finally chapter 5 contains the MAD concept describing the main work performed in the project.

Figure 2 illustrates the MAD project concept and compares it to the existing EMPS program structure. The numbered boxes with red fonts on the right are new programs. The aggregation procedure includes programs 1, 2, 4, 5 and 6. While the disaggregation procedure is contained in program 3. Below are introductory descriptions of the programs of the MAD project concept.

1. Vansimw: Stochastic optimisation model for aggregated hydropower
2. EMPSW aggregated: Hydro-thermal power market simulator for aggregate hydropower.
3. EMPSW: Hydro-thermal power market simulator.
4. Tilsimw: Calculation of non-storable and storable energy inflow for aggregated hydropower.
5. Aggmod: Generation of the aggregated model description based on the detailed system data.
6. MinprodLP: Optimization model for calculation of minimum production on detailed level. Minimum production is used in (4) to give non-storable energy inflow to the aggregate model.

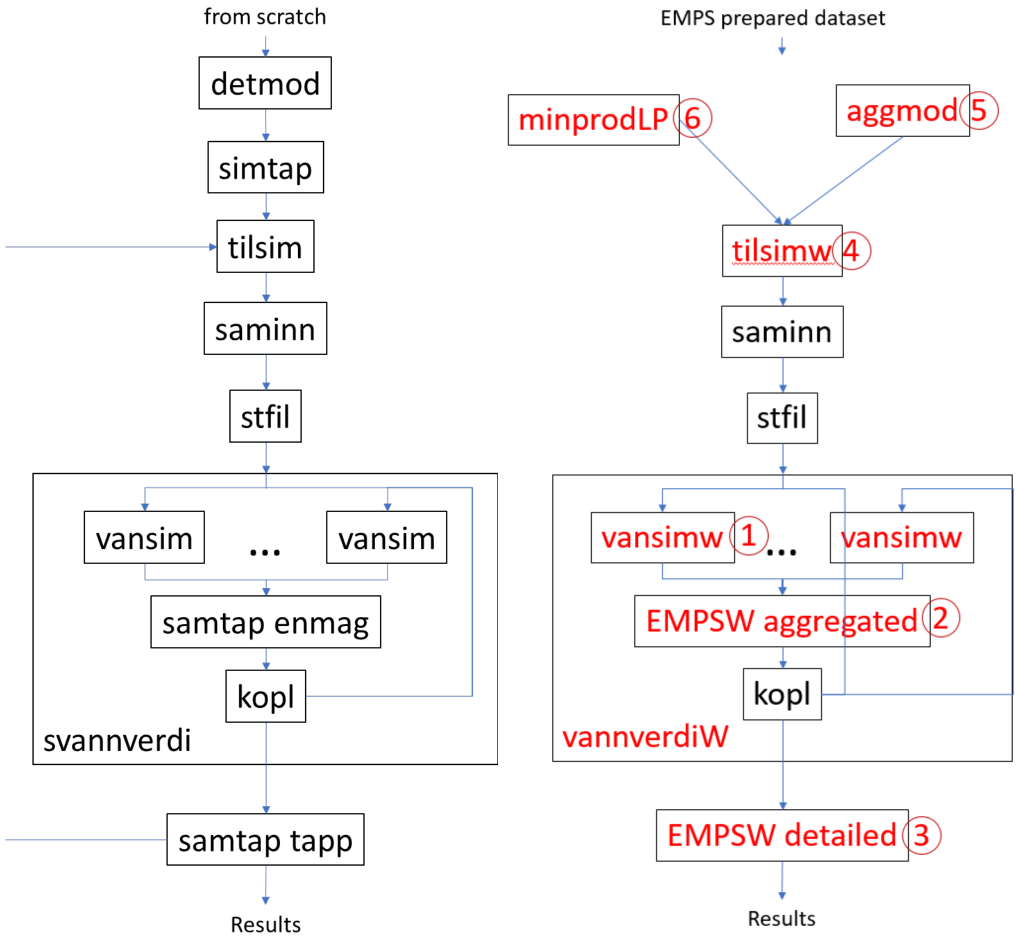

Figure 2: Visualised EMPS and MAD programs run sequence to the left and right, respectively.

EMPS) MAD)

The above figure illustrates the complexity of the existing model framework. Moreover, it shows that changing parts of the solution methods often requires changes in the whole program structure resulting into implementation challenges.

2 Literature review

In the beginning of the MAD project we made a review of the literature with special focus on aggregation and disaggregation techniques and optimization methods for the aggregated model including stochastic inflow modelling. Formal solution methods for the detailed system is not addressed here because it was done when developing the FanSi model [11] and [12]. In this review we focused on simplified methods that can be used for very large systems with reasonable computation time.

Because there is no applicable method that can solve large system problems (i.e. many reservoirs) the usual approach is to aggregate the hydro system into artificial reservoirs and plants that represent the physical system [13], [14] and [15]. The aggregated system is then solved using e.g. SDP or SDDP methodology. In some cases, aggregated results are disaggregated [15] to the physical system depending on the application of model result.

2.1 Aggregation

In reference [16] the multi-reservoir optimization problem is solved using stochastic dynamic programming for aggregated models. For each individual reservoir an aggregated dynamic model consisting of four state variables is solved. The state variables represent the state of all upstream reservoirs, the volume of the focus individual reservoir, the state of all downstream reservoirs and one hydrological state variable (could be snow storage). This method gives separately calculated decisions for the individual reservoirs that combined may not be feasible. A top down correction method that starts with the solution for top individual reservoir and moving to downstream reservoirs is used to find the feasible solution for the multi-reservoir system. The solution for the upper reservoir is always feasible.

[17] extends on the model described in [16] mainly in two ways. The first extension allows for one more state variable to describe the state of the other (not the focus reservoir) reservoirs. The second extension uses principle component analysis to find the states that represent the best experienced distribution of individual reservoirs. In the first paper it was assumed same percentage filling in all reservoirs. The method is tested on a 35 reservoirs system.

Brandao [18] describes the aggregation method used in the Brazilian system. Results from application of the aggregated model are compared with results from a detail model. The comparison is done for one Brazilian river system consisting of three main reservoirs, using deterministic optimization for three hydrological scenarios representing a wet, a medium and a dry year. The paper concludes, the obvious, that a detailed model is better and that development in computer software and algorithms give potential for detailed modelling of the whole Brazilian system. Maceira [19] also describes an aggregation method for the Brazilian system. The detailed physical system is represented by four 4 aggregate models in the NEWAWE model. The paper focuses on the aggregation method, how this is done for hydrological coupled watercourses and details of how controlled and uncontrolled energy inflows are calculated, including correction for head dependencies. The Brazilian methods are very similar to the existing SINTEF approach.

Shayesteh [20] proposes an automatic aggregation that can aggregate to an arbitrary chosen aggregate model structure. The method is made for short-term optimization and verified for deterministic problems. We do not believe that this method could be applied to make aggregate models that are used in long-term stochastic problems.

2.2 Disaggregation

Turgeon [15] presents the optimization method that has been implemented for long-term reservoir management of Hydro Quebec's hydro system. The system consists of 26 large reservoirs and 54 plants in 8 rivers. The method, which is split into three steps apply aggregated models, stochastic dynamic programming and disaggregation methods. In the first step the whole problem is solved by (SDP) for a one-reservoir model of the system. The previous inflow is a state variable in this optimization. The result of this model gives the sum hydro generation. In the second step sum hydro generation is split between river systems. This is done using a two state SDP model with one state representing actual river and one representing the sum of the others. From this optimization, only the marginal water values for each river are used to distribute the sum hydro production from the first step between river systems. In the third and last step sum river production is disaggregated to individual plants. This is formulated as an optimization problem with sum production requirement and an objective function that minimizes overflow and deviation from target reservoirs. The most difficult part is the calculation of target reservoirs. This is done by a heuristic based method where the goal is to have a distribution between reservoirs that maximizes long-term generation.

Valdes [21] also presents an aggregation/disaggregation approach. Aggregation is standard to one reservoir/plant and disaggregation is based on formulation of LP problems with constraints and penalties for overflow etc (details of this is not totally clear). The paper does include a relatively detailed method for inflow discretization and calculation of conditional probabilities for inflow.

Zambelli [22] compares an open-loop deterministic solution approach with the existing Brazilian solution approach based on a chain of models. The paper presents results that show that the deterministic approach is better than existing approach. The existing approach is based on application of SDDP methods for an aggregated representation (for regional aggregated reservoirs) and heuristic for disaggregation. Disaggregation heuristic tries to keep all reservoirs at the same storage levels.

2.3 SDP for multi-reservoir systems

Turgeon [23] compares two methods for solving a "special" multi-reservoir system with all reservoirs and plants in parallel. The first method is called one at a time and solves each reservoir/separately. The second method, which is found to be the best, is based on solving two state SDP problems. One state represents current reservoir and the other state represents the sum of all other reservoirs. The SDP is solved for all reservoirs and a composite future cost function based on all SDP solutions is used to calculate each plants production. Turgeon [15] is an extension and improvement of this method. SINTEF tested a similar method in the eighties [24].

Turgeon [25] presents a method for a simplified representation of a multilag inflow model in a SDP solution approach. Previous inflows are substituted with one hydrological state variable. The benefit is that only one state necessary is used to represent the hydrological state in the optimization. This reduces complexity and computation time.

Serrat [26] compares two different aggregation/disaggregation approaches to optimization of the Rio Grande/Rio Bravo river system. Stochastic Dynamic Programming (SDP) is applied to a one reservoir model and compared with a two reservoir model of the whole system. The two reservoir model is shown to give better results than the one reservoir model. Previous inflows to reservoirs are included as state variables. In both cases aggregated results are disaggregated to find US and Mexicos share of the released water using a nonlinear optimization algorithm.

2.3.1 Sampling stochastic dynamic programming (SSDP)

Kelman [27] proposes a method called Sampling Stochastic Dynamic Programming (SSDP). The method has, as the name suggest, many similarities with the Stochastic Dynamic Programming (SDP). It also means the curse of dimensionality limits the application of SSDP to systems with few states (reservoirs). SSDP differs from SDP in that observed inflow statistics (or scenarios) are used more directly in the calculation than for regular SDP. The main advantage being that inflow correlations in time and space are better represented. In regular SDP, for each time step different discrete inflows are given a probability. In some cases, these inflows are dependent on previous inflows (autocorrelation). In SSDP, the strategy is calculated in backward recursion as in regular SDP but instead of using probabilities for discrete inflows the strategy is calculated for different inflow years in the current time period and conditional probabilities of moving to all other observed inflow years in the following time period. The SSDP version presented in [27] includes a hydrological state variable and therefore also needs transition probabilities for this state variable conditioned on the current inflow. The paper describes a method based on Bayes theorem for the calculation of transition probabilities. The method is applied to a system consisting of one big reservoir, several plants and small reservoirs. Faber [28] applies a variant of the Sampling Stochastic Dual Dynamic Programming method described e.g. in [27] to a one reservoir system. Our SSDP implementation is tested in a simulator type environment. Compared to [27] our SSDP implementation does not include a hydrological state for the stream flow forecast and is therefore simpler. Instead, the actual forecast that was made for each historical year and week is used directly.

Cervellera [29] uses neural networks to approximate the multidimensional future cost function and methods for efficient state space discretization to solve a 10 reservoirs system using SDP. The system consists of 30 states because of a second order autoregressive inflow modelling.
Cote [30] makes a comparison of four different optimization methods for hydropower operation. The methods are applied to the Rio Tinto Alcan hydropower system in Quebec Canada and are tested in a simulator type environment with updated streamflow forecasts. The four tested methods are:

- Deterministic optimization
- Stochastic dynamic programming (SDP)
- Sampling stochastic dynamic programming (SSDP)
- Scenario tree approach (STA)

The hydropower system consists of only three reservoirs (and six plants) which makes it possible to use straight forward implementation of all methods. The paper concludes that the stochastic methods are better than the deterministic method. The deterministic method especially underestimates the risk of spillage. The results also show that the scenario-based methods, i.e. SSDP and STA, where superior to the SDP approach.

2.4 Comments from the literature review

Aggregation and disaggregation methods are in operational use in large hydro systems such as the Brazilian system and the Hydro Quebec system. Stochastic Dynamic Programming (SDP) is normally used to solve the aggregated model, SDDP is used in Brazil. SINTEF has also implemented and tested a SDDP based model called Samplan that was applied to aggregate hydro models in each price area [31,32]. The results from Samplan was not satisfactory and the model has not been further developed since 2005.

Some other observations:

- Aggregated models usually consist of one plant and one reservoir, except [1] and [20] and applications in Brazil.
- Literature about disaggregation methods are rare, specific and difficult to understand all the details of.
- There is an increasing amount of literature describing methods/applications based within the field of computational intelligence. Both [1] and [33] give overviews. The methods could be categorized as follows:
    - Genetic based optimization algorithms. These are heuristic based optimization methods where new solutions to be investigated are generated by replicating evolution processes in nature. The methods find near optimum solutions.
    - Unsupervised learning, i.e. learning from "training data". The method requires input/output pairs for training.
    - Reinforced learning. The method has many similarities with dynamic programming and does not require input/output pairs. The algorithm learns how to obtain a defined goal. [34] describes how reinforced learning could be applied to reservoir operation. The paper includes an example of application to a one reservoir system.

  SINTEF has so far not any experience with application of these methods to the hydro optimization problem.

There is no attempt anywhere to model a system with close to similar size, defined by the number of reservoirs, as the Nordic system. This is because the other physical systems consist of fewer reservoirs and/or because the systems can be represented good enough with a reduced number of storage. E.g. in the system operated by BC Hydro in Canada, which is also like the Norwegian almost entirely based on hydro, more than 90 % of the storage capacity can be represented by two physically decoupled reservoirs.

The review identified the Sampling Stochastic Dynamic Programming method as an interesting alternative to SDP and the water value method. The main advantage being the more explicit utilization of the inflow scenarios and consequently improved handling of prolonged periods with very little or very much inflow. We therefore did some more investigation of this method which is described in chapter 3.

3 Sampling Stochastic Dynamic Programming (SSDP)

3.1 Introduction

Stochastic dynamic programming, (SDP) is a method for solving long-term hydropower scheduling problems. SDP is based on Bellman's principle, making it possible to reformulate the problem from a multi-stage stochastic problem to several smaller one-stage problems. This is one of the main advantages of applying SDP on hydropower scheduling problems, as it allows for complex problems to be solved within reasonable computational time and resources. A disadvantage is that the application of the method is limited to maximum 3-4 state variables (reservoirs, snow storage, previous inflows). Another possible disadvantage of using SDP on hydropower scheduling problems is the statistical representation of inflow. Uncertainty in inflow is a key factor in hydropower scheduling and using a statistical representation may poorly preserve inflow characteristics, leading to underestimation of extreme weather years. This challenge is addressed by the sampling stochastic dynamic programming (SSDP) method, aiming to better preserve inflow characteristics through more direct use of historical inflow years in the algorithm. The SSDP method is presented in [27] and has further shown promising results in [28,30]. In the latter study four optimization algorithms were compared finding that methods based on scenarios, such as SSDP, are superior to methods based on probability distributions, such as SDP.

The objective of our work was to apply the SSDP method to typical Norwegian hydropower optimization problems and compare the results with a similar, operational implementation of the water value method. The hypothesis was that using SSDP for water value calculation, rather than SDP, will better preserve inflow and give a better representation of extreme weather years. This should give an improved operation of the hydro resources, lower operations costs and less curtailment.

3.2 Methodology

The main structure of the SSDP method is similar to SDP. As in SDP, the problem is solved step by step from the last stage to the first, finding the optimal decisions in each stage given the initial state of the system. An expected future cost function is used in the decision problem to include the future cost of a decision, given uncertainty about the future. This function is updated between each stage based on the optimal solution found in the previous stage. Optimal solution in each stage is found by minimizing total cost as given in equation (1), i.e. minimizing the resulting cost in the current stage and the expected future cost. The differences between SSDP and SDP lays in how uncertainty is represented and how the expected future cost function is calculated. This is previously described in [28,30]. The formulation used in this study is based on [30], with the main differences being that the problem is formulated with a cost minimizing objective and that historical inflow is used to calculate the transition probabilities. To preserve statistical characteristics of inflow, uncertainty is represented directly by use of historical data as scenarios instead of a probability distribution. Transition probabilities describe the probability of transitioning between scenarios from one stage to the next. In addition, scenario specific costs functions are used to accumulate the costs backwards along each scenario path. These cost functions represent the actual cost of the made decisions given that the specified scenario is realized. The functions are updated in each stage with the realized cost of the found optimal solution (which is made under uncertainty) in each scenario, as given in equation (2). An expected future cost function is used to include uncertainty in the decision problem. The expected cost is a function of the system state given a known scenario in the current stage and uncertainty about scenario realizations in future stages. As given in equation (3), the expected future cost functions are calculated using the transition probabilities and the scenario specific costs functions in the following stage.

$$f_t(S_t) \;=\; \min_{R_t} \{C_t(S_t,Q_t,R_t) + \; E_{j|i}[f_{t+1}(S_{t+1},j)]\,\} \tag{1}$$

$$f_t(S_t,i) = C_t(S_t,Q_t,R_t) + f_{t+1}(S_{t+1},i) \tag{2}$$

$$E_{j|i}[f_{t+1}(S_{t+1},j)] = \sum_{\mathrm{j}\in M} P_t(j|i)\;\; [f_{t+1}(S_{t+1},j)] \tag{3}$$

Notation	Definition
$\boldsymbol{t \in 1,\dots,T}$	Discretization of time stages, where T is the last stage
$\boldsymbol{i,j,k \in M}$	Inflow scenarios
$\boldsymbol{S_t}$	State of the system in stage *t*
$\boldsymbol{R_t}$	Release decision in stage *t*
$\boldsymbol{Q_t}$	Inflow in stage *t*
$\boldsymbol{C_t(S_t,Q_t,R_t)}$	Cost in stage t given decision R_t, inflow Q_t and system state S_t
$\boldsymbol{f_t(S_t,i)}$	Scenario specific cost given state S_t and scenario i in stage *t*
$\boldsymbol{E_{j\|i}[f_{t+1}(S_{t+1},j)]}$	Expected cost of state S_{t+1} and scenario *j* in stage *t+1* conditioned on scenario *i* in stage *t*
$\boldsymbol{P_t(j\|i)}$	Probability of transitioning to scenario *j* in stage *t+1* conditioned on scenario *i* in stage *t*

Transition Probabilities. Different methods can be used to calculate the transition probabilities depending on the available data. The method used in this study is based on Bayes Theorem and the work of [28,30], but has been limited to the use of historical inflow data. Since inflow usually have strong seasonal variations, the data has been normalized to weaken the seasonal effect, using equation (4). The transition probabilities are calculated using the probability density functions (pdfs), as described in equation (6). The pdfs were found from the conditional probability distributions of the model given by equation (5). First a prediction of inflow in stage *t* was calculated by regressing the inflow in stage *t* on the inflow in stage *t+1* using a least squares polynomial fit of first degree. Then the conditional normal distribution of the random inflow in stage *t* was found, assuming the prediction of inflow in stage *t* as mean and the standard error of the estimate as standard deviation. Knowing the probability density functions, the transition probabilities can be found using equation (6).

$$z_t^i = \frac{\left(Q_t^i - \overline{Q_t}\right)}{\sigma_t} \tag{4}$$

$$p[z_t|z_{t+1}^j] \sim N(\,\hat{z}_t\big(z_{t+1}^j\,\big), \sigma_e)) \tag{5}$$

$$P_t(\mathrm{j}|\mathrm{i}) = P_t\big[z_{t+1}^j\,|z_t^i\big] = \frac{p\big[z_t^i|z_{t+1}^j\big]\,p^j}{\sum_{k\in M} p[z_t^i|z_{t+1}^k\,]p^k} \tag{6}$$

Notation	Definition
$\boldsymbol{z_t^i}$	Normalized weekly inflow
$\boldsymbol{\overline{Q_t}}$	Mean inflow in period *t* given all scenarios
$\boldsymbol{\sigma_t}$	Standard deviation of inflow in stage *t*
$\boldsymbol{P\left[z_{t+1}^j \middle\| z_t^i\right]}$	Probability of inflow, z_{t+1}^j , in stage *t+1* given the inflow in stage *t*
$\boldsymbol{p[z_t\|z_{t+1}^j]}$	Pdf of inflow, z_t, in stage *t* given inflow, z_{t+1}^j , the following week
$\boldsymbol{P^j}$	Unconditional probability of inflow scenario *j*
$\boldsymbol{\hat{z}_t\left(z_{t+1}^j\right)}$	Prediction of inflow in stage *t,* given the inflow in stage *t+1*
$\boldsymbol{\sigma_e}$	Standard deviation of random inflow *(*standard error of the estimate)

3.3 Implementation

The SSDP model was implemented based on an existing SDP model [35]. The decision problem solved in each iteration is identically formulated for the SDP model and the SSDP model. Input data include thermal production units, an aggregated hydro system description including inflows, and firm and price dependent load. The model calculates the water values given all states, scenarios and stages using the SSDP or SDP algorithm. The resulting water value table is then taken as input to a simulator which simulates optimal operation for the system for a set of inflow scenarios. The simulation is done on the same inflow scenarios used in the optimization problem calculating the strategy (water values). The results from the simulator gives expected operational cost of the given strategy.

3.4 Testing

3.4.1 Test cases

The models have been tested on three aggregated representations of different regions of the Norwegian hydro system. The representations are taken from datasets in operational use and include inflow, reservoir size and maximum discharge, as listed in **Table 1**. To represent the stochasticity in the system, 83 historical inflow years have been used. Firm and price dependent demand have been generated for each region. Demand and availability of thermal power production have been scaled to simulate cases with different flexibility in the system, referred to as slack, base and tight cases. The tight case has a very high curtailment probability.

Table 1. Characteristics of the hydro in the Norwegian regions.

	Reservoir size [GWh]	Max discharge [GWh/week]	Average inflow [GWh/yr]	Max inflow [GWh/yr]	Min inflow [GWh/yr]
Reg 1	9 361	716	19 089	22 863	11 316
Reg 2	33 330	2015	47 796	76 726	28 113
Reg 3	11 650	550	13 862	20 657	8 021

3.4.2 Results

The SSDP and SDP models have been tested on the described cases for a one-year time horizon (52 weeks). The resulting power price, reservoir operation, production and operation costs are compared for the two models. We observe only minor differences between the solutions for the slack and base cases for Reg 1, 2 and 3.

Table 2. Operational cost in the SSDP and SDP solutions for the slack and base cases

	Slack Reg 1	Slack Reg 2	Slack Reg 3	Base Reg 1	Base Reg 2
SDP [mill. €]	-34.4	-72.5	-26.8	-9.7	-13.4
SSDP [mill. €]	-35.1	-73.3	-26.8	-9.5	-12.5

No model consistently performs better than the other. The economic results for the slack and base case runs are given in **Table 2**. In addition, the models were tested on a more pressed system, the Reg 1 tight case. In this case we observe larger differences in the resulting strategies, as illustrated in **Figure 3**. The SSDP model keeps a higher reservoir filling, reducing the amount of curtailment and more than halving the associated cost. However, this also more than doubles the flooding in the system. Assessing the economic results, the

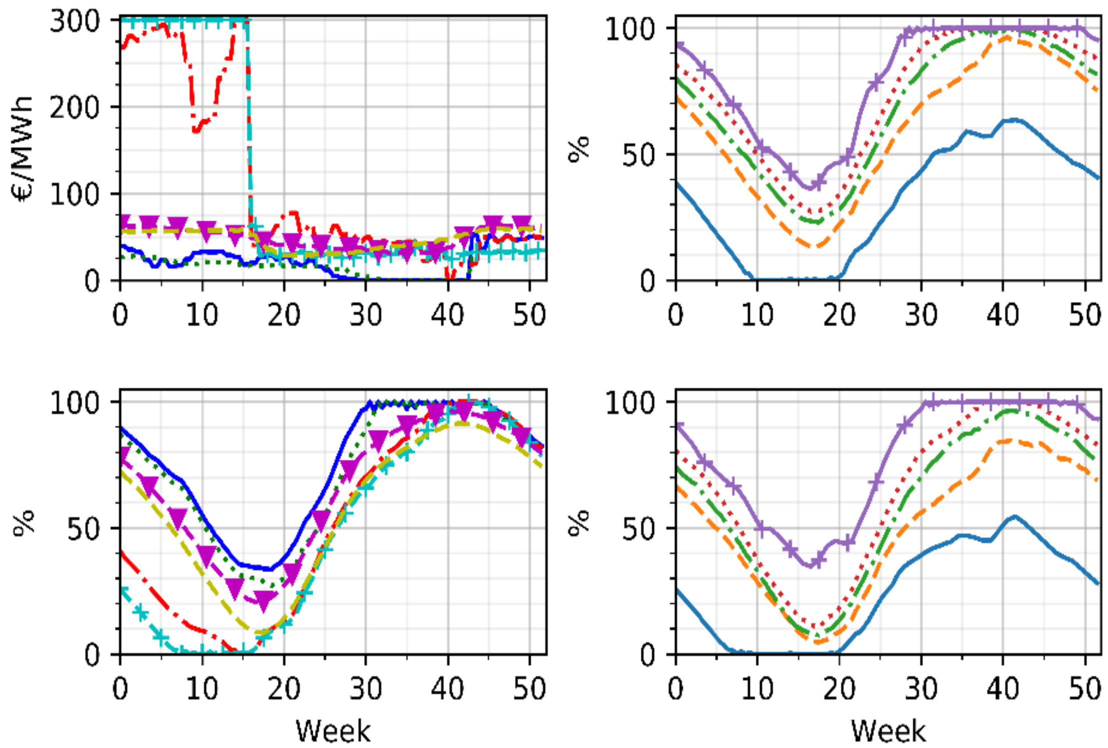

Figure 3: Results from the Reg 1 tight case, SSDP and SDP solution. The top left plot shows the power price for two extreme scenarios and the average. The bottom left plot shows the reservoir filling for the same scenarios. The plots to the right show the reservoir filling in percentiles (top plot: SSDP, bottom: SDP). The reservoir filling is higher in the SSDP solution.

SSDP strategy has an overall cost of 95 mill. € compared to an overall cost of 84 mill. € in the SDP solution. The results indicate that the SSDP model evaluate the consequence of low inflow extreme years as more severe than the consequence of spillage in high inflow extreme years and reflects this in the strategy (water values).

The reduced curtailment in the SSDP solution for the high price (dry) year gives lower power price in periods, e.g. for week 10 the price in the SDP solution is the curtailment price, while the SSDP solution has a price below 200 €/MWh. Furthermore, for the low-price (wet) year we observe that the simulated price in the SSDP solution is kept higher closer to the periods with spillage. This indicates that the marginal value of the water is higher in the SSDP solution even for scenarios with a large risk of spillage. This is also shown in Figure **4** where we see that the marginal water value of the low- price scenario is higher than for the SDP solution up to about 80% reservoir filling. Furthermore, we observe that the marginal water value in the dry scenario is higher than the marginal value of the wet scenario.

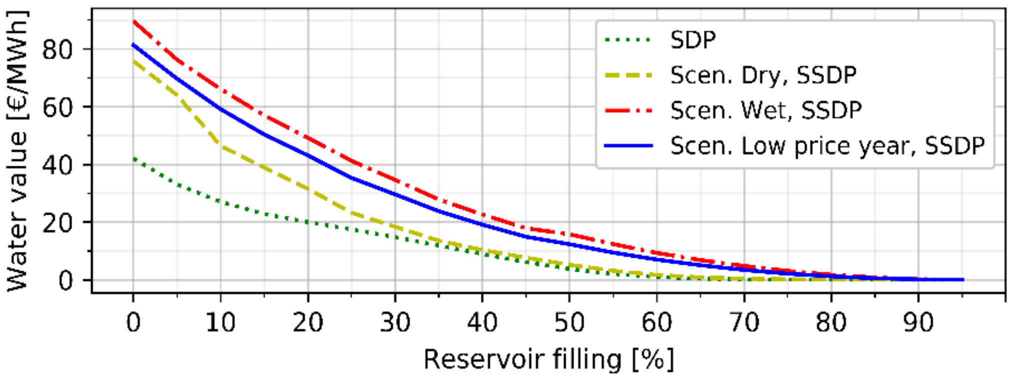

Figure 4: Marginal value of water for the SSDP and SDP solution in week 20 as a function of reservoir filling. For the SSDP solution are the scenarios with highest (wet) and lowest (dry) inflow in week 20 plotted as well as the marginal value of water in the low price scenario.

3.5 Conclusion SSDP

We have implemented a test model for long term hydropower scheduling using Sampling Stochastic Dynamic Programming (SSDP). The model was tested on cases based on real aggregated hydro representations from operational datasets and compared to results from a SDP-implementation similar to the one in operational use in Norway. The simulated results from the tight case show the expected response from the SSDP model, with a more careful operation for dry years giving less curtailment. The analyses do not show one method to consistently perform better than the other, and we do not see the same promising potential of the SSDP method as previous studies. This could possibly be explained with differences in the statistical properties of the inflow series compared to the mentioned references. In our study aggregated energy inflow series where used.

Taken that the SSDP model use information of the current inflow scenario to calculate the transition probabilities it was expected that the SSDP model should perform significantly better than the SDP model without autocorrelation. Considering this the SSDP model is expected to overall perform worse than an SDP model with autocorrelation. Therefore, we did not go further with the SSDP implementation, especially because we in EMPS work on aggregate models and use calibration to fine tune the strategy.

A natural extension of the SSDP implementation, which is also included in the literature, would be to include snow storage information. Further work on the SSDP algorithm could for example be done through Master studies at NTNU. A first step could be to test with a physical Norwegian hydro system.

4 Existing EMPS disaggregation weaknesses

4.1 Introduction

This section discusses some aspects of the aggregation/disaggregation technique used in standard EMPS and shows some examples of how this works for a few Swedish water courses. This work was motivated by questions by one of the model users and project participants regarding why simulated maximum production from these water courses was much less than the sum of the individual plant capacities and also less than observed production at a given stage. The question relates directly on- to the main tasks of MAD project.

The example is from one area in the model that includes Lule- og Skellefteelven. In total, the system in this area includes 35 modules. To simplify testing and to make it easier to manipulate inputs we made a EOPS dataset of this system. The disaggregation and feedback to the aggregate model in EMPS is the same as in EOPS, therefore it should not make any difference that we are using EOPS for testing purposes.

The river optimization is done using EOPS with exogenously given prices. The percentiles (0, 25 50, 75, 100) and the average for weekly market prices are shown in Figure 5 from week 50 to week 104. For testing, the prices in week 50 are set almost three times as high as the average for the other weeks. The model is run with four load periods within a week. The prices in load period 4, which has the lowest price, are on average a bit higher than 50 % of prices in load period 1.

The total installed capacity for the hydro system is about 5300 MW, which corresponds to a weekly maximum production of 890 GWh. Load period 1 consists of 25 aggregated hours and give a potential maximum weekly production of 132.5 GWh for that period only. This is to explain the results that are taken directly from Kurvetegn. The initial reservoir filling in week 50 is set to 70 % in all reservoirs.

The price in week 50 is set equally high for all load periods, hence it should be optimal to produce close the physical maximum.

Figure 6 shows the percentiles (0, 25 50, 75, 100) and the average weekly production from the EOPS model. The maximum weekly production is far below the installed capacity. The only exception is for the 100 percentile which shows a production of 899 MW for week 81. In week 81 for the 100 percentile, all reservoirs are at their maximum. This results in high head and explains why simulated production is higher than installed capacity which refers to nominal head. Figure 7 shows the same results for the first load period.

To sum up, the EOPS model only gives production close to maximum when all reservoirs are full and there are high inflows. A separate test with 100 % reservoir filling in week 50 does not significantly change simulated maximum production, but 100 % filling in week 80 does. Thus, high prices alone do not give maximum production, even if prices are very high.

It is not straight forward to evaluate what is the correct physical maximum production for given initial conditions for complicated water courses as the two ones included in this example. There are several factors to consider:

- Head dependencies, i.e. the installed capacities refer to a given head.
- Different discharge capacities for hydropower plants in series. Is it possible for all plants to produce at maximum even in cases with little local inflows? Is there enough water? Is bypassing needed? What is the loss of the bypass compared to its benefit?
- Plant efficiencies. The plant efficiency is decreasing with increasing discharge (over nominal production).

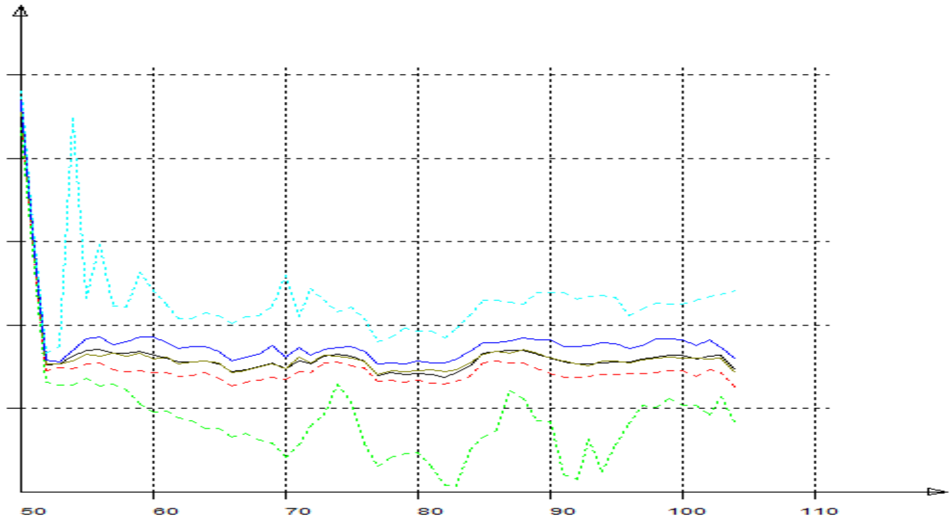

Figure 5: Assumed market prices for week 50 to week 104.

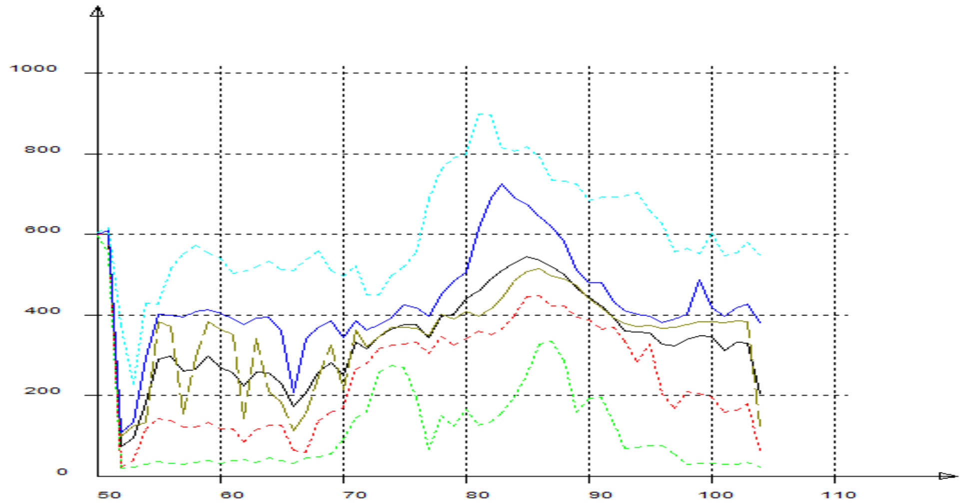

Figure 6: EOPS, sum weekly production (GWh) (installed capacity corresponds to 890 GWh).

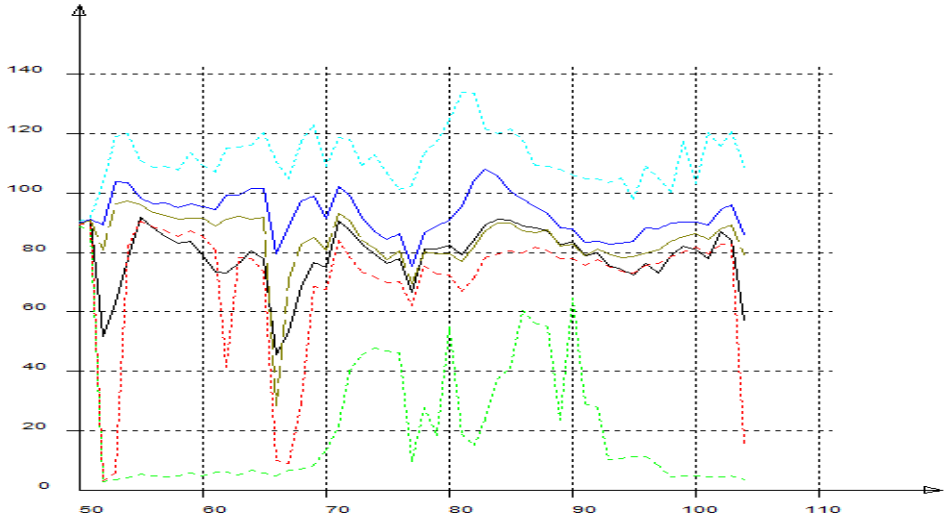

Figure 7: EOPS, simulated sum production (GWh) in load period 1. (max. production =132.5 GWh)

An alternative to EOPS is to use the SDDP based ProdRisk model. ProdRisk is based on formal optimization for a detailed physical description. We have applied the ProdRisk model to the same case. The simulated production from ProdRisk is shown in Figure 8 and Figure 9.

Results from ProdRisk shows higher production in week 50 when prices are high and higher production throughout the year in the highest price period. ProdRisk simulates the system using 29 sequential time steps within a week based on the same four accumulated load periods as used in EOPS. If we disregard time delays and other physical properties, normally not include in SINTEFs long-term models, ProdRisk should give the best estimate of the "real" maximum production.

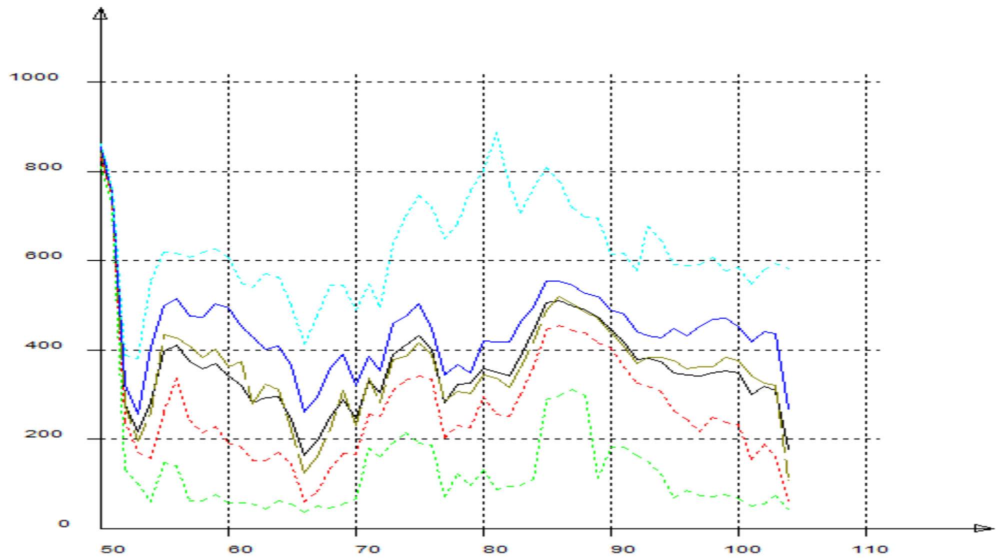

Figure 8: ProdRisk, sum weekly production (GWh) (installed capacity corresponds to 890 GWh).

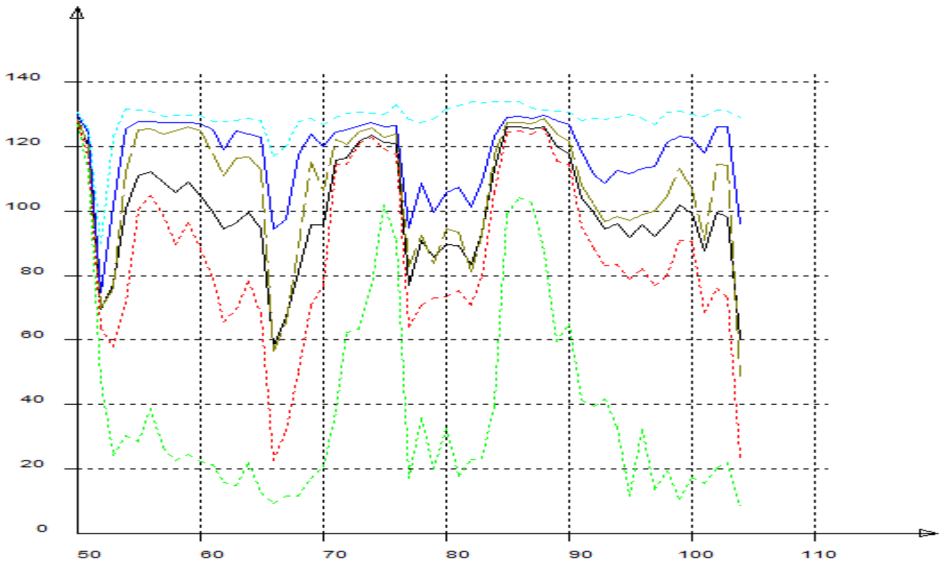

Figure 9: ProdRisk, simulated sum production (GWh) in load period 1. (max. production =132.5 GWh)

Figure 10 and Figure 11 show percentiles for the simulated sum of bypass and overflow from ProdRisk and EOPS, respectively. The main difference is that ProdRisk is using bypassing in week 50 to achieve the high production. In the ProdRisk simulation overflow/bypass is possible at a small cost. If these costs are increased ProdRisk also simulate less maximum production. This has been verified by testing.

The simulated sum overflow/bypass in ProdRisk is less than EOPS for all other weeks than week 50. Thus, it seems that it is possible to produce close to physical maximum for a few hours without bypassing, but it is not possible to produce at maximum for all hours in the week without bypassing. I.e. by using the small reservoirs in the river system for daily storage it is possible to produce much more than simulated by EOPS/EMPS.

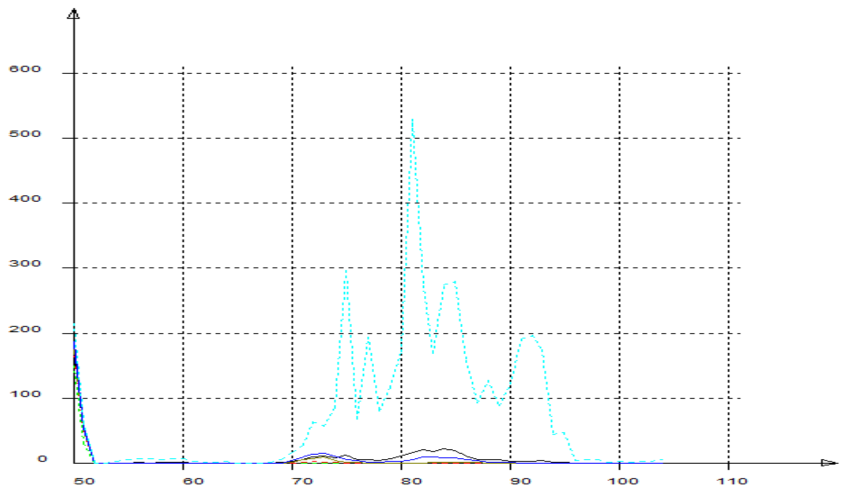

Figure 10: ProdRisk simulated sum bypass and overflow (weekly average 8.36 GWh)

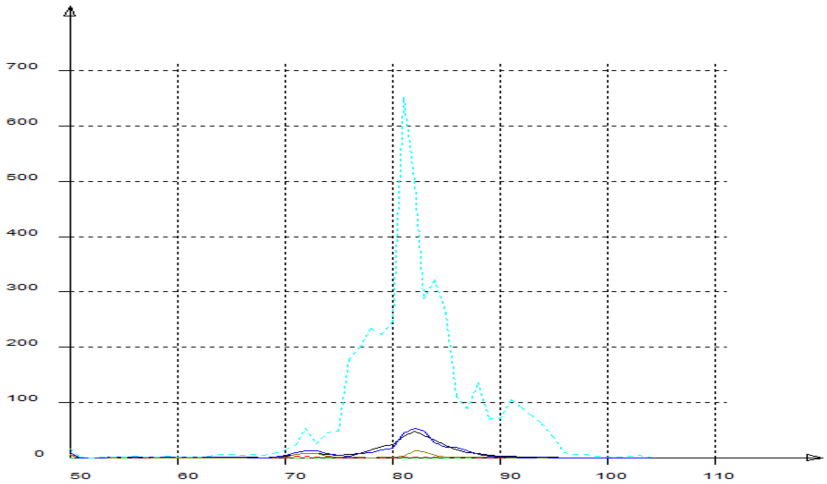

Figure 11: EOPS simulated sum bypass and overflow (weekly average 6.48 GWh)

The main principles of the interaction between the aggregated model and the detailed model are as follows:

The aggregated model is used in the market clearing process, i.e. it is directly part of the price calculation. Sum hydro production from the market clearing is sent to the disaggregation procedure that tries to load this production and update the aggregated model. The following values are updated through this feedback/iteration procedure:

- Efficiency for different segments (giving the relation between energy from aggregated storage and energy on to the bus bar).
- Segment capacities (sum of all segments is at least equal to installed capacity)
- Segment cost. This represents the marginal production cost of using a given segment.

Regulated and non-regulated energy inflow is also updated, but this has not been focused here, because it is not supposed to be related to the stated problem.

Segment efficiency is representing the marginal plant efficiency given by individual PQ descriptions, losses caused by bypassing and head effects. Segment cost is representing non-optimal distribution of end reservoir filling relative to the optimal distribution given by the discharge strategy.

4.2 Disaggregation method and load periods - simple example

We illustrate issues discussed in the previous section. Figure 12 shows a simple artificial system used to test the EMPS/EOPS model disaggregation method. We test the production with significant price variation between load periods. The basic idea behind the example is that plant 2 is 4 times as large (MW production) as plant 1 and that the local reservoir connected to plant 2 only can store enough water for 12 hours full production, assuming that plant 1 is producing at maximum at the same time. If Plant 1 is producing at maximum for the whole week, it should be possible to produce at maximum in plant 2 for half of the week. We assume constant plant efficiencies and no head effects.

We define two accumulated load periods of 84 hours. Prices in the first load period are twice as high as the price in the second load period, approximately. Optimal utilisation is maximum production for the whole week in plant 1 and maximum production in the high price period for plant 2.

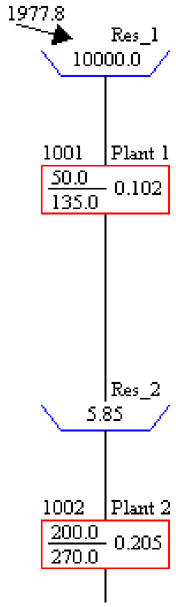

Figure 12: Simple test system.

Figure 13 shows the simulated production from EOPS in plant 2 for the high price period. The plant seldom produces more than 100 MW. The model does not see the potential production of 200 MW because the model does not include sequential time resolution or reservoir balance constraints.

Figure 14 shows the simulated production from ProdRisk in plant 2 for the high price period. ProdRisk used sequential time resolution where 12 high price hours are followed by 12 low price hours for each day of the week. Plant 2 produces much more often at 200 MW.

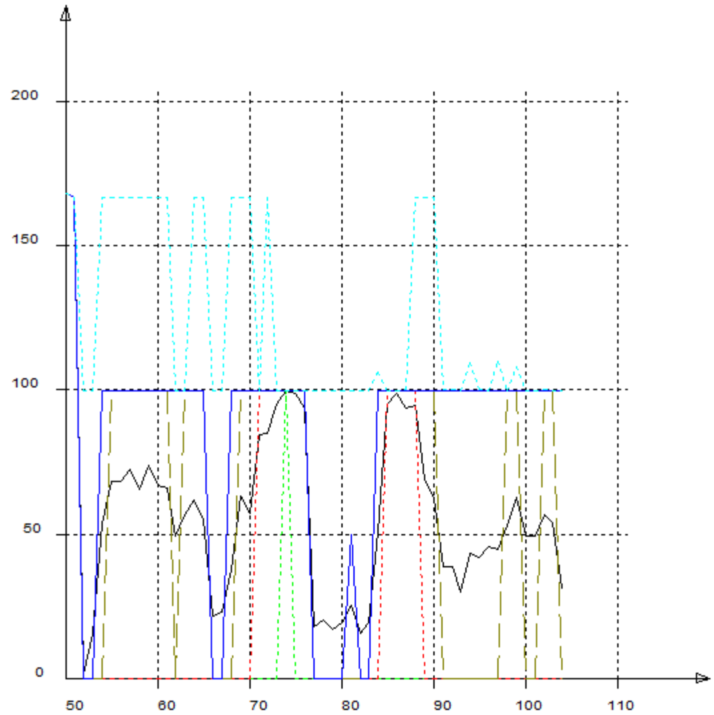

Figure 13: EOPS: Percentiles for simulated production (MW) in plant 2 in the high price period.

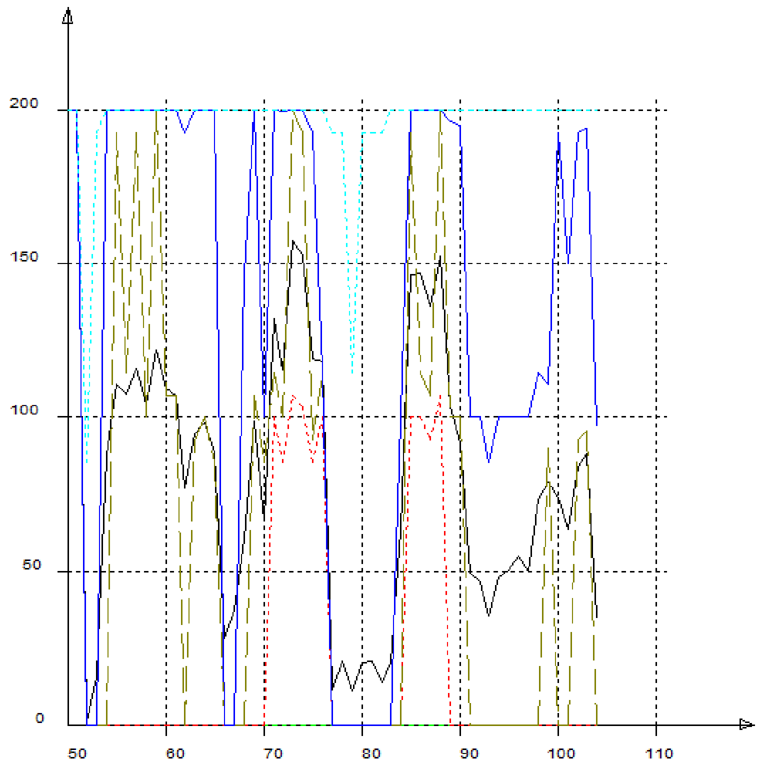

Figure 14: ProdRisk: Percentiles for simulated production (MW) in plant 2 in the high price periods.

The market clearing in EOPS and EMPS is done for an aggregate model that is built iteratively based on signals from the loading of the detailed model. The intra week storage in buffer reservoirs is not considered. Table 3 and Table 4 present the final aggregate model in week 50 for the first simulated inflow year for two different initial reservoir fillings. They show segment cost, efficiency and accumulated production capacity for the segments. The relative efficiency is reduced to 66.7 % for production above a specific production capacity of 155 MW and 151.4 MW. The efficiency reduction occurs because if plant 2 is producing more than 100 MW bypassing from reservoir 1 is needed, according to the loading heuristics. When bypassing, water from reservoir 1 is only used with 2/3 efficiency. The accumulated production capacity is larger than 150 MW because the initial stored water in reservoir 2 is used before bypassing is need. If this water is evenly distributed over the whole week, 155 MW can be produced without bypassing. Table 3 and Table 4 show that the accumulated production is reduced, from 155 MW to 151.4 MW when the initial storage is reduced. The segment cost reflects deviation from optimal target reservoirs given by the heuristics.

Table 3: Aggregate model in week 50, initial storage 70% in both reservoirs.

Segment number	Accumulated production capacity (MW)	Cost	Relative efficiency
1	155	0	1.0
2	250	-0.2	0.67

Table 4: Aggregate model in week 50, initial storage 20% in both reservoirs.

Segment number	Accumulated production capacity (MW)	Cost	Efficiency
1	151.4	0	1.0
2	250	-0.2	0.67

4.3 Conclusions- existing disaggregation methodology

The above examples illustrate some of the properties of the disaggregation methodology. The information from the disaggregation is fed back to the aggregate model and used in the market clearing. It shows some deficiencies for maximal production of small reservoirs in serial river systems in peak hours. This is a model weakness, especially for analyses of future systems where the hydro system is expected to balance more short-term variations caused by wind and solar power.

Therefore, one of the important goals of the MAD project has been to replacement or improve the disaggregation methodology. In the end we decided to replace the disaggregation heuristic with a formal optimization problem while still utilizing parts of the disaggregation methodology.

5 MAD concept

This chapter describes the new model concept that we have worked towards in the project after the initial competence building activities. The concept includes two major improvements compared to the existing EMPS model:

- A new disaggregation methodology. This is represented with a new model called EMPSW
- A new aggregated model structure and a new optimization algorithm applicable to the new aggregate structure called Vansimw

The new disaggregation methodology addresses the weaknesses discussed in chapter 4. Figure 15 shows the EMPS run sequence to the left and the MAD run sequence to the right. The MAD concept requires one run of the EMPS run sequence to prepare the dataset. The concepts are similar, e.g. there is still aggregation and disaggregation. The differences are related to the properties and methods used to solve each individual task in the figure. The main differences can be summarized by the following in relation to Figure 15.

7. Stochastic Dynamic Programming is used to calculate the optimal strategy for the aggregate model. Linear Programming is used to solve the one stage problem within the dynamic programming loop. The new model is called Vansimw and is intended to replace the old water value method that is made specially for one storage problem with weekly time resolution.
8. A new market simulator for aggregate representation of hydro. Needed because aggregate hydro is represented by more than one storage and the marginal value of hydro storage is represented differently in systems with more than one storage.
9. The new disaggregation model (EMPSW) is the hydro-thermal power market simulator and replaces the EMPS draw-down model.
10. Calculation of non-controlled and controlled inflow. Replaces Tilsim in EMPS model.
11. Generation of the aggregated model description based on the detailed system data. The aggregated model data include storage capacities (GWh), time dependent constraints on storage, production capacities etc.
12. A new optimization model that calculates minimum production for each plant in the system. Minimum production is used in (4) to give non-controlled inflow to the new aggregate model. The model replaces what is called bounded simulation in simtap of the EMPS model.

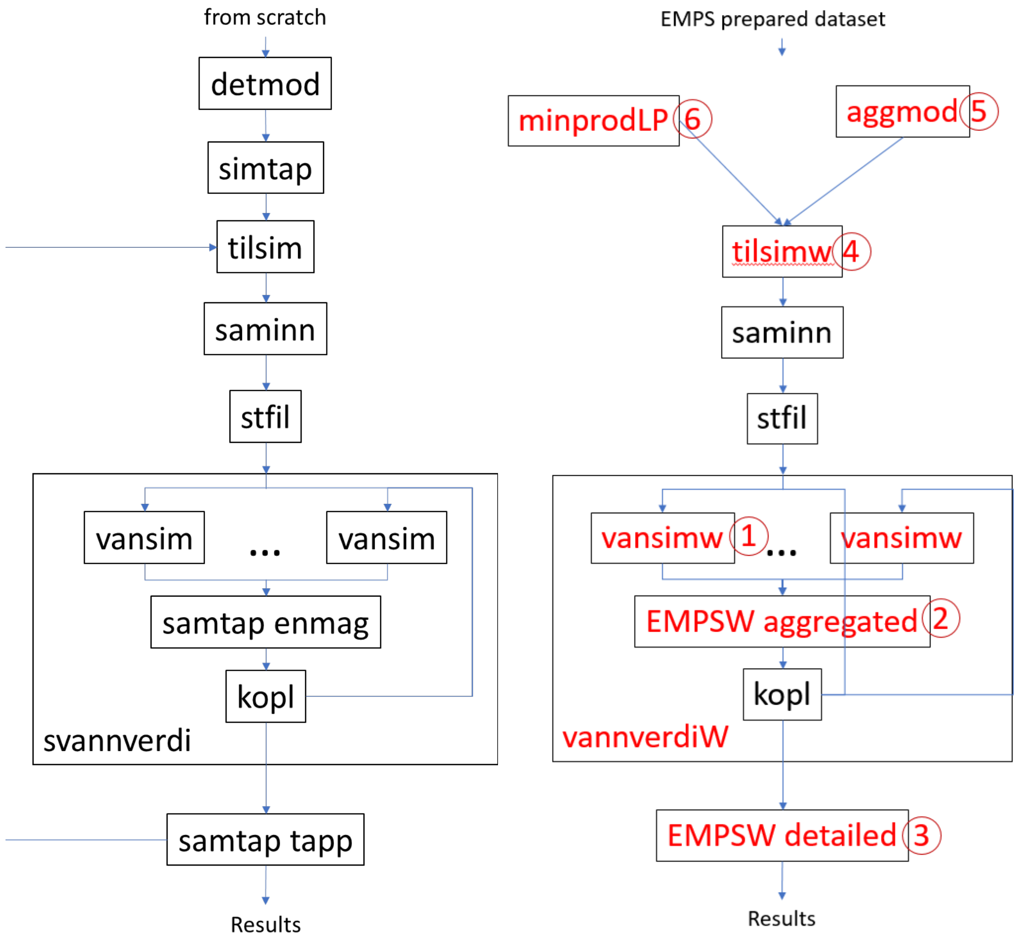

Figure 15: Visualised EMPS and MAD programs run sequence to the left and right, respectively.

EMPS) MAD)

5.1 EMPSW – The hydro-thermal power market simulator

The EMPSW model is the new hydro-thermal power market simulator. EMPSW uses formal optimization to solve the weekly market clearing problem with a detailed representation of each hydro plant included. Individual end of the week water values are used as input to this optimization. These water values are based on the aggregated reservoirs water values and using parts of EMPS disaggregation heuristic for individualisation.

EMPSW is the most important result from the MAD project. Only a short description of the EMPSW model is included here and the model is described more detailed in [36]. The description here focuses on the methodology used for individualisation of aggregated water values and new types of constraints that was included into the weekly market clearing problem. These constraints were implemented because simulation results show that simulated market prices for today's electricity system varies less within the week than can be observed in the market. Price variation is also less than what we get from the EMPS model. The weekly market clearing problem in EMPSW is formulated as a Linear Programming (LP) problem and includes a detailed description of the hydro system in addition to the standard description (i.e. the same as in EMPS) of all other market options. The time resolution for the weekly problem can be hourly. Because formal optimization is used, complicated river systems are utilized better which again decreases price variation. The new constraints discussed in section 5.2.2-5.2.5 are real constraints that reduces flexibility and makes short-term price variation more in line with observations.

5.1.1 Aggregated water values

The aggregate area water values are calculated for 51 discrete values of the reservoir volume for each stage, i.e. end of the week. These water values are used in EMPSW to specify water values by the end of the week for each individual reservoir.

This is done by adding the following term to the weekly LP-problem:

$$\text{Max } [\ldots + \sum_{i=1}^{N} \sum_{j=1}^{J} c_{i,j} x_{i,j} \;],$$

where

$$c_{i,j} = -W_j E_i,$$

and

$$X_i = \sum_{j=1}^{J} x_{i,j}.$$

W_j - [øre/kWh] is the aggregated water value for segment j.
j - is the index for discrete values of storage,
J - is the number of discrete water values (51).
E_i - is the energy conversion factor to sea [kWh/Mm^3] for hydro module j.
$x_{i,j}$ - is a model variable representing segment j in storage i [Mm3].
N - The number of hydro modules that are included in the particular aggregate model.
X_i - Calculated storage by the end of the week

The simplest individualisation method is to give all $x_{i,j}$ variables upper bounds corresponding to 2% of the reservoir size of reservoir j, the aggregated water value would then be distributed evenly to all individual reservoirs independent of the properties of the particular reservoirs. Overflow risk and regulation flexibility would not matter. This is too simple and would give poor individual water values.

Therefore, we have utilized the target reservoir calculations in the exiting EMPS disaggregation heuristics to improve on the above simple individualisation method. Formally this is done by adjusting the upper bounds of each $x_{i,j}$ based on the individual characteristics of the storage. More on this in the next chapters. The lower bound is zero for all $x_{i,j}$. and the sum j of all upper bounds give the maximum storage capacity for storage i.

5.1.1.1 Target reservoir calculations

The disaggregation heuristics of the EMPS/EOPS model include the target reservoir calculation and the loading procedure of the different plants[37]. In the new EMPSW we utilise the target reservoir calculations coupled with aggregated water values to make individual water values for all hydropower plants. The aggregated water values are obtained from EMPS.

The target reservoir calculation has two different objectives depending on the season, the filling season or the depletion season. In the filling season, the expected inflow is greater than the expected discharge. In the depletion season the expected discharge is greater than the expected inflow. The objective in the filling season is to minimise reservoir spillage, while the objective in the depletion season is to avoid emptying the reservoirs and thereby maintain the production capacity of all hydropower plants if possible. These objectives are the basis for the target reservoirs calculations.

The filling season

The strategy is to minimise reservoir spillage. To minimise spillage the relative damping is held constant for all reservoirs according to equation **(8)**

$$D = \frac{M_{\max} - M}{M_{\max}} \cdot R\,, \tag{8}$$

where

D = Relative damping,
M_{max} [Mm3] = Maximum reservoir volume,
M [Mm3] = Current reservoir volume,
R = Degree of regulation.

The relative damping is an estimate for risk of spillage. The degree of regulation is defined by

$$R = \frac{M_{\max}}{T}\,, \tag{9}$$

where

T [Mm3/Year] is the expected annual inflow to the reservoir.

The strategy in the filling season is to keep the relative damping at an equal level for all reservoirs, which gives a target reservoir volume M^i_{target} for each time step

$$\frac{M^i_{\max} - M^i_{target}}{M^i_{\max}} \cdot R^i \text{ equal for all reservoirs,} \tag{10}$$

while equation **(11)** is fulfilled

$$\sum_{i=1}^{N} M^i_{target} \cdot E^i_k = E_{tot}\,, \tag{11}$$

where

i = reservoir index,
M^i_{target} [Mm3] = target reservoir,
N = number of reservoirs,

E_k^i [kWh/m3] = energy conversion factor for the reservoir, calculated to sea level,

E_{tot} [GWh] = stored energy of the aggregate reservoir.

For a system with N reservoirs equation 10 give N-1 equations and together with equation 11 this give N equations that can be solved to give target reservoirs. I.e. if the aggregated storage is equal to E_{tot}, the solution of the N equations gives how this energy should be distributed according to the heuristics. There are some additional technicalities in the solution of these equations that involves ensuring that all target reservoirs are within specified constraints.

The depletion season

The strategy in the depletion season is twofold; to avoid emptying any reservoir too early and at the same time end the depletion season with the same relative damping.

To fulfil these two competing strategies a tapered reservoir volume trajectory is used for all reservoirs. The reservoir volume trajectory is tapered down from full reservoir at the beginning of the depletion season to an end of the depletion season target reservoir volume, as shown in Figure 16.

The end of the depletion season target reservoir volume is calculated as follows:

- First the lowest probable storage volume for the aggregate model by the end of the depletion season is calculated. This calculation assumes low inflow and high market prices.
- The aggregate storage volume by the end of the depletion season target reservoirs are distributed with equal damping as described in the section about the depletion season.

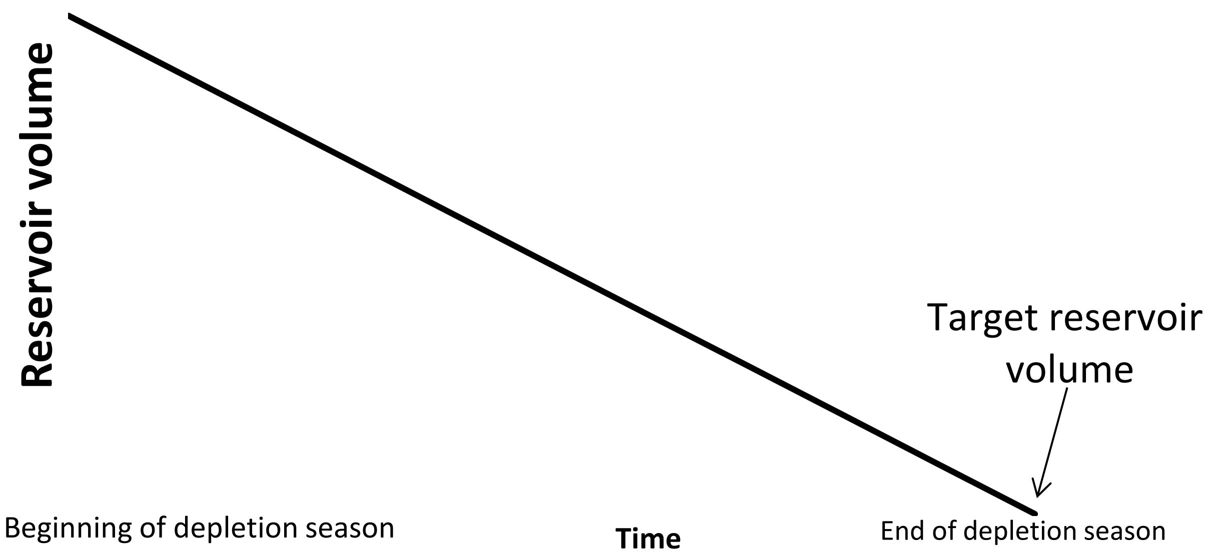

Figure 16: Tapered reservoir volume trajectory in the depletion season. Tapered from full reservoir volume to an end of the depletion season target reservoir volume.

In the depletion season the target reservoirs can then be calculated using the same equations as in the filling season except that maximum storage is substituted with tapered volume in equation (11):

$$\frac{M_{Tr}^{i} - M_{target}^{i}}{M_{\max}^{i}} \qquad (12)$$

where

M_{Tr} [Mm3]= tapered reservoir volume trajectory.

By following the tapered reservoir volume trajectory, as illustrated in Figure 16, one can avoid emptying the reservoirs too early. It's important to stress that the tapered trajectory is only a reference trajectory for each time step. It's the relation between the water value and the market price that ultimately controls the reservoir volume and any deviation from the trajectory.

Target reservoirs may also be corrected based on the utilisation time of each plant. Utilisation time may reduce the target reservoirs calculated from the basic strategy both in the filling season and depletion season, if it is too high. We will not comment more on this here.

5.1.1.2 Artefact of the weekly valuation of water

The target reservoir heuristics changes instantaneously when moving from one season to the next. Consequently, there can be significant changes in the individualized water values when transitioning to a new season. This may also result in rather large jumps in the simulated power price at the transition points (typically week 18 and 40). The transition points are specified by the model user. To avoid the observed consequence of instantaneous changes in strategy, a smoothing of the individualized waters values is introduced. The transition period between seasons is extended from one to several weeks. The abrupt change in the valuation of water at the transition point is then smoothened out over several weeks. Figure 17 shows an example of simulated average area power prices before and after introduction of the smoothing method.

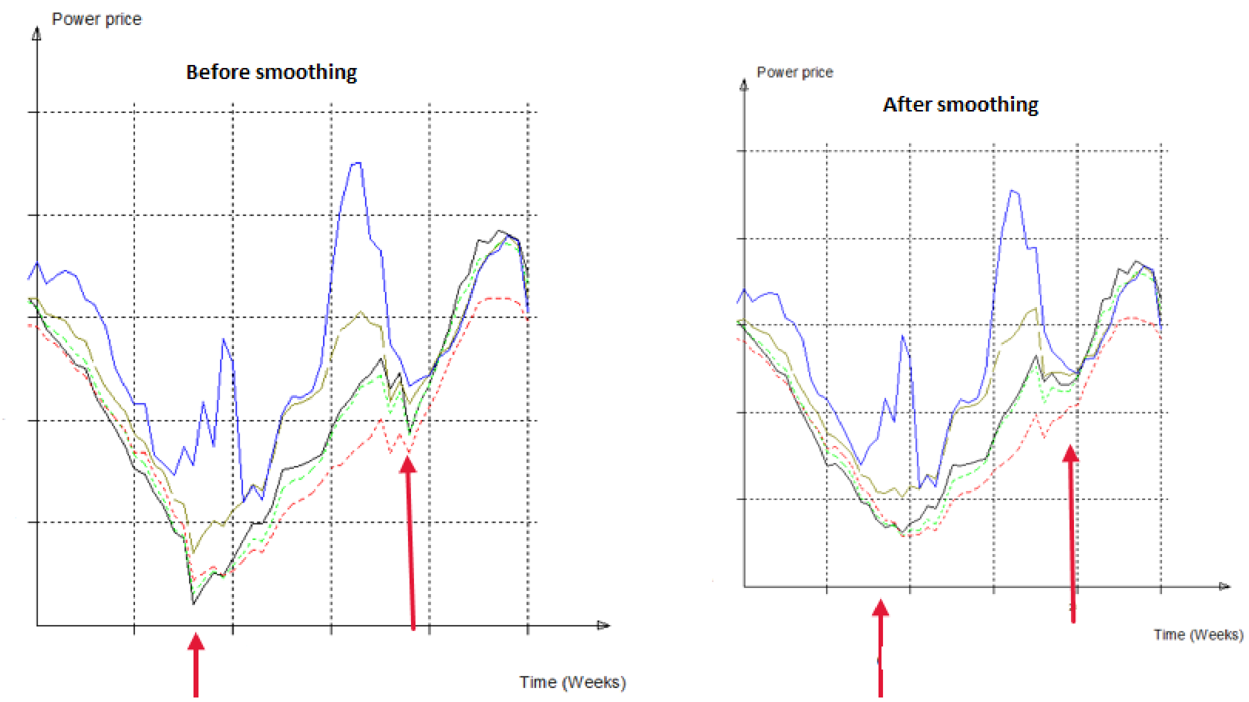

Figure 17: Area mean power prices from 5 different areas. The results are obtained from the EMPSW before and after smoothing was applied.

5.1.2 Time delay on water flow

Time delay on water flow is the time it takes for water to travel from one plant to the next. EMPSW may be used with a time resolution of 1 hour and time delay on water flow can make a significant impact on the operation. In a long river string, it may not be possible to produce at maximum for all plants at the same time because of time delays and small local storage capacity.

Time delay on water flow is handled in the hydro balance constraints of the weekly LP-problem. The LP-problem contains hydro balance constraint for each time period. This allows for inclusion of time delay on water flow where the water released from an upstream reservoir in one period arrives at the next plant in a later load period. In standard modelling released water is immediately available in for the next plant.

When time delay is modelled there will for all time periods be water in transit to its destination point. In the EMPSW the LP-problems are formulated and solved per week, which means that at the end of the week, some flow of water is in transit and will arrive the next week. If transit water is not given a value the model will produce more than optimal in the last time periods because the value of transit water is not seen by the model. Therefore, as an approximate solution, water in transit is set to be equal to the water value in the last time period for the previous week for each reservoir. The water values for the current week is not known when the optimization problem is formulated.

Simulation results show that modelling time delays on water increases intra-week power price variation.

5.1.3 Discharge ramping constraint

Discharge ramping constraints restricts the *change* in water discharge from a hydropower station between two consecutive load periods. Such constraints may be caused by environmental considerations, but may also be used to emulate technical limitations (e.g. startup limits)

In the EMPSW discharge ramping constraints are modelled as follows:

$$b \geq q(t-1) - q(t) \geq -b, \qquad \textbf{(13)}$$

where

- b is the specified ramping constraint limit [m^3/s]
- q is model discharge variable [m^3/s]
- t is time period

Ramping constraints increases LP-problem size and computation time considerably. The EMPSW handles load periods with different time lengths.

The model can deviate from the ramping constraints in certain cases. Deviation from constraints is penalised by costs in the calculation. For example, in a case where the discharge must be ramped down fast to avoid breaching the minimum reservoir level constraint, the model will allow a deviation from the ramping constraint or from the minimum reservoir level constraint based on the deviation penalty cost.

When including discharge ramping constraints in the simulations, the intra-week power price variation is increased.

5.1.4 Transmission line ramping constraints

Ramping constraints on transmission lines restricts the *change* in exchange between areas over a period.

Ramping constraints are handled per load period. The allowed variation in exchange E (in MW) between load periods is

$$b \geq E(t-1) - E(t) \geq -b, \qquad \textbf{(14)}$$

where b is the specified ramping constraint, t is the present load period and t-1 is the previous load period. Thus, the ramping constraints are always relative to two consecutive load periods. The ramping constraints are added to the LP-problem, expanding the LP-problem size. The constraints must be specified by the user.

The model can deviate from the ramping constraints. Deviation from constraints is penalised by costs in the calculation. For example, in a case where the exchange must be ramped down fast due to maintenance, the model will allow a deviation from the ramping constraint. The penalty cost can be specified by the user.

When including transmission line ramping constraints in the simulations, the effect on intra-week power price variations is area dependent. Areas with low flexibility have the highest intra-week power price sensitivity to transmission line ramping constraints.

5.1.5 Reducing LP-problem size

The objective function of the LP-problem is optimized subjected to a set of constraints. One way to reduce calculation time is to reduce the number of constraints. When building the LP-problem in the current EMPSW, the hydro balance constraints are included for all time periods within the week. However, some reservoirs are unlikely to breach their maximum or minimum constraints within the week and can therefore be represented with one hydro balance equation representing the whole week. Before building the LP-problem one can calculate which reservoirs need hydro balance constraints per load period and which reservoirs need only one weekly constraint. Typically, large reservoirs will for many weeks not reach its limits and can be omitted. This way the LP-problem size is reduced.

The LP-problem size reduction implementation was implemented, but the reduction in computation time was not consistent. This is because the reduction method gives changes in LP problems structure between many weeks and therefore not allow the same efficient utilization of "warm start". No attempt was performed to make a consistent reduced LP-problem size for all weeks.

5.1.6 Summary

The method for individualization of aggregate water values and utilization in the new EMPS has been presented. The individualization method resulted in an artificial jump in simulated power price at the season transitions. A transitional period was introduced to smoothen out the immediate shift of water values and this improved the results significantly.

The implementation of hydropower production ramping constraints and time delay on water flow increased the intra-week variations on power prices. The impact on intra-week power price variations is area dependent. All new constraints reduce as expected the simulated socio-economic surplus from the model. The effect of including ramping constraints on transmission lines on power prices is area dependent, each area will have varying sensitivity to transmission line ramping constraints.

5.2 VANSIMW – The stochastic optimisation model for aggregated hydropower

The new stochastic optimization model for the aggregate model is named Vansimw. Vansimw solves a problem where the area hydropower is represented by two aggregated parallel reservoirs with corresponding hydropower plants. In Vansim the area hydropower is represented by one aggregate hydro reservoir and the problem is solve using a special made algorithm that is difficult to expand to more the one reservoir or more time periods within the week. The Vansim water value calculation has therefore been replaced by Stochastic Dynamic Programming formulation where a Linear Programming algorithm is used to solve the one stage transition problem for a given value of the uncertain variables. The new formulation gives more modelling flexibility, e.g. the possibility to use sequential time periods within the week, but the implementation is made for a given number of storage, in our case two. Linear Programming is chosen for the one stage problem because it is by far the quickest solution method for this generalized problem.

5.2.1 Representation of future costs

In textbook implementations of Stochastic Dynamic Programming algorithms, the future cost function $\boldsymbol{\alpha}$ is calculated and stored as a function of time (t) for discrete values of the state (x), in our case storage level.

With two storage x becomes two dimensional. The future cost in a given time step t depend on the volume of both storage.

Because we are using Linear Programming to solve the one stage decision problem within the Stochastic Dynamic Programming (SDP) formulation the future cost function must be linear and concave function of both state variables. To account for the cost functions interdependency on both state variables we chose to use Benders cut for this representation. The solution that we ended up implementing therefore has very much in common with the Stochastic Dual Dynamic Programming formulation:

- One stage problem formulated as an LP problem
- Future costs are described by Benders cuts.
- The Benders cuts are calculated in a backward procedure for discrete value of the state variables

but there are some differences

- There is no forward and backward iteration procedure because the discrete states for all time periods are predefined as in regular SDP.

The planning period is extended automatically to be long enough so that the initial guess of the end value function does not affect the results for the planning period.

The model minimises the expected value of operation dependent costs within imposed constraints by building and solving weekly LP-problems within an SDP framework.

The weekly one stage LP-problem for a given outcome of the uncertainties can be described by the following:

$$Minimise\ \{\boldsymbol{c}^T\boldsymbol{x} + \boldsymbol{\alpha}\}, \tag{15}$$

where $\boldsymbol{c}^T$ is the transposed costs vector, $\boldsymbol{x}$ is a vector of decision variables including reservoir storage and $\boldsymbol{\alpha}$ is the expected future cost at the end of the week.

The objective function is subject to the constraints:

$$\boldsymbol{Ax} \geq \boldsymbol{b}, \tag{16}$$

$$\boldsymbol{x} > \boldsymbol{0}, \tag{17}$$

where $\boldsymbol{b}$ is vector of constants, e.g. inflows. $\boldsymbol{A}$ is coefficients matrix given by power and reservoir balance constraints, reservoir constraints and discharge constraints etc. In addition, we have the cut constraints that represent the future costs function:

$$\boldsymbol{\alpha}(\boldsymbol{x}) \geq \boldsymbol{\alpha_0} + \boldsymbol{\mu_0}(\boldsymbol{x} - \boldsymbol{x_0}), \tag{18}$$

where $\boldsymbol{\alpha_0}$ is the future costs for reservoir level $\boldsymbol{x_0}$, $\boldsymbol{\mu_0}$ is the water values for discrete reservoir levels $\boldsymbol{x_0}$ and $\boldsymbol{x}$ is the two-dimensional vector of reservoir levels.

The predefined discretization level for the two-reservoir states are user defined. The one stage problem is solved for each discrete combination of state variables for each time step for each outcome of the uncertainties. Computation time therefore increases quadratically with the discretization level.

5.2.2 Experiences and results

The new stochastic optimisation model Vansimw has been tested on a two-storage system that purposely are made equal, with the intention of getting a symmetrical future income as function of the two state variables. The characteristics of the hydropower plants and reservoirs are listed in Table 5.

Table 5: Characteristics of the hydropower plants and reservoirs of the SINTEF dataset 1.

SINTEF dataset 1	**Hydropower characteristics**	
Reservoir no.	1	2
Non-storable inflow	373 Mm3	373 Mm3
Storable inflow	523 Mm3	523 Mm3
Minimum discharge	11.4 m3/s	11.4 m3/s
Maximum reservoir storage	706 Mm3	706 Mm3
Installed capacity	145 MW	145 MW
Maximum discharge	40 m3/s	40 m3/s

The volume of the two reservoirs are discretised in 25 levels from V=0 Mm^3 to V=706 Mm^3. For each timestep the number of states is 25*25 = 625. The calculation is performed with weekly timesteps for 52 weeks. Stochastic inflows are lumped into 7 discrete outcomes with given probabilities, similar to standard water value calculations. Based on inflow outcomes and corresponding probabilities the expected future income can be calculated for all timesteps and states (reservoir volume). The firm power, thermal production, price elastic demand etc are modelled as in the standard water value calculations. The dataset consist of 4 time periods within the week and includes seven thermal units.

The expected future income as a function of the reservoir volumes for week 1 is shown in Figure 18. The results are obtained by running Vansimw on SINTEF dataset 1. The expected future income function should be symmetrical on each side of the horizontal plane extending from the points $(V_1, V_2) = (0,0)$ to $(V_1, V_2) = (706,706)$ Mm^3, due to the equality of the two hydropower plants and reservoirs. The figure indeed shows this symmetry.

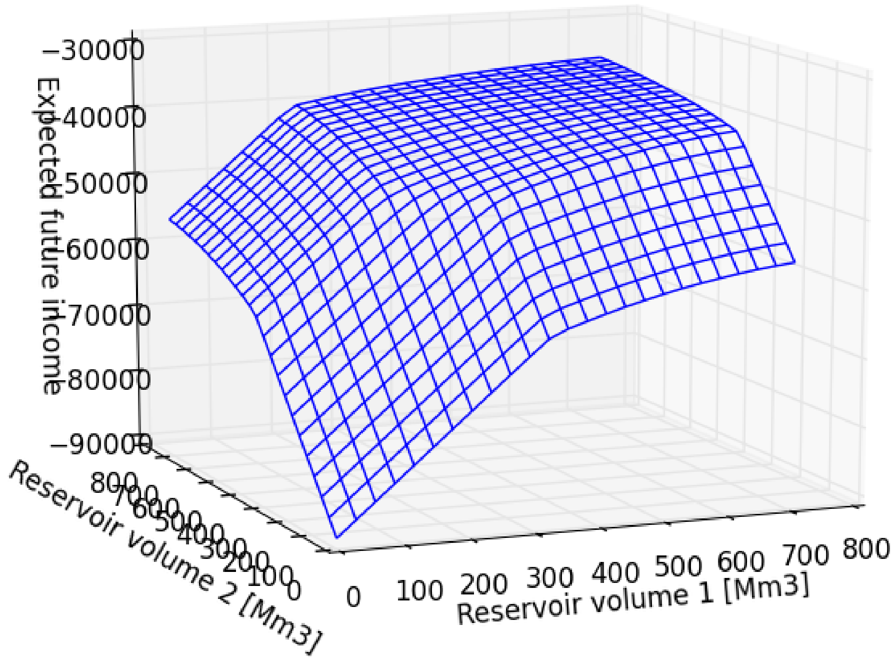

Figure 18: Expected future income as a function of the reservoir volumes for week 1.

In Table 6 the characteristics of a modified test dataset (dataset 2) is listed. The only differences to the previous are the maximum reservoir storage. The volume of the two reservoirs are discretised in 25 points each from $(V_1, V_2) = (0,0)$ to $(V_1, V_2) = (1059,353)$ Mm^3. For each timestep the number of states is 25*25 = 625. The calculation is performed with weekly timesteps for 52 weeks. The number of weighted inflow scenarios is 7. Based on these inflow scenarios the expected future income is calculated for all timesteps and states (reservoir volume).

Table 6: Characteristics of the hydropower plants and reservoirs of the SINTEF dataset 2.

SINTEF dataset 2	**Hydropower characteristics**	
Reservoir no.	1	2
Non-storable inflow	373 Mm3	373 Mm3
Storable inflow	523 Mm3	523 Mm3
Minimum discharge	11.4 m3/s	11.4 m3/s
Maximum reservoir storage	1059 Mm3	353 Mm3

Installed capacity	145 MW	145 MW
Maximum discharge	40 m3/s	40 m3/s

The slope of the expected future income function represents the water value. The water values of reservoir 1 is expected to be higher than the water values of reservoir 2, since reservoir 1 has a higher degree of regulation.

Figure 19 shows the expected future income as a function of reservoir volume for week 1. The slope of the future income is indeed higher when moving in the direction of higher volume of reservoir 1 than reservoir 2. Meaning that the water values are higher in reservoir 1 than 2. The expected future income should be increasing with increasing reservoir volume.

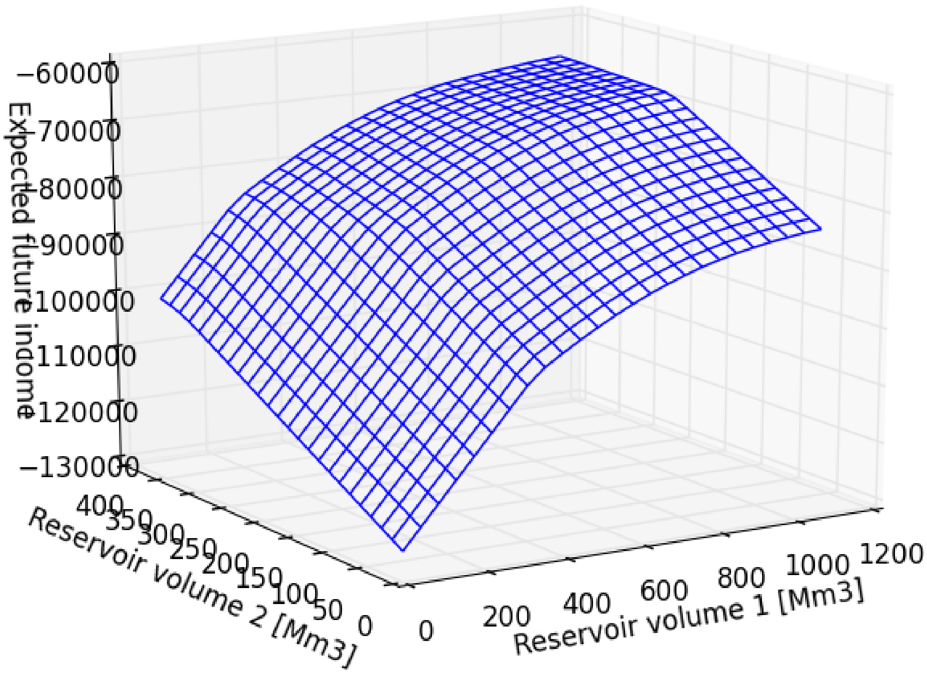

Figure 19: Expected future income as a function of the reservoir volumes for week 1.

The main disadvantage of the new implementation is a very high increase in calculation time. However, there is significant potential for parallelisation. The project did not allow for implementation of parallelisation due to time constraints.

The calculation time along with the LP-problem size and number of LP-problems solved before convergence when running dataset 1 is listed in Table 7. The calculation time is long. The number of solved LP-problems is given by multiplying the number of iterations, weeks, inflow outcomes and system states (9*52*7*625 ~2

M). In our dataset some of the inflow outcomes have zero probability and we end up solving approximately 1.23 M problems.

Table 7: Calculation time, the number of solved LP-problems before converging and the LP-problem size for SINTEF dataset 1.

LP-problem size		Solved LP-problems	Calculation time
Variables	Constraints	1.23 Million	**10 minutes**
80	~25*25 = 625		

The calculation time for the standard water value calculation for same dataset, but with only one aggregated reservoir, is 1 second. It cannot be compared directly, since the system in Vansim is simpler, but it illustrates the need for reduction of calculation time. Parallel processing is one method to reduce calculation time.

It is discussable whether it is useful to use a specially made SDP for the new aggregate model structure. It was a good experience to implement, especially because it was implemented by a person without any experience of this type of calculations, but the two SDDP based models that SINTEF have developed can probably solve the problem equally good with some adaptations.

- Samplan was developed to solve the same problem as solved by EMPS with a one reservoir aggregate hydro model in each price area. The model has not been developed since 2005 and is not in use.
- The ProdRisk model is used operationally by many utilities for long and medium-term hydro scheduling. The model is continuously development.

The Samplan model uses almost the same data and solves a problem similar to the two-storage aggregate model, but the model has not been developed for many years and therefore uses older data structures.

In general, non-linear relations can be represented when using SDP and cost functions for discrete state variables. The future cost as function of the state variables do not need to be a concave function of the states. But because we need to use LP for the one stage problem, the cost function must be concave to avoid use of binary variables and one of the SDP's advantages is lost. The SDDP models are more general and applicable to aggregate models with more reservoirs because of computation time issues, curse of dimensionality is avoided. In short, sampling and iterations replaces predefined discretization.

5.3 Aggregated structure and selection criteria (Aggmod)

The new aggregated structure represents the detailed hydropower of the area by two aggregated parallel reservoirs with corresponding hydropower plants, see Figure 20. The advantage of having two aggregated plants/reservoirs is the possibility to differentiate between different reservoir characteristics.

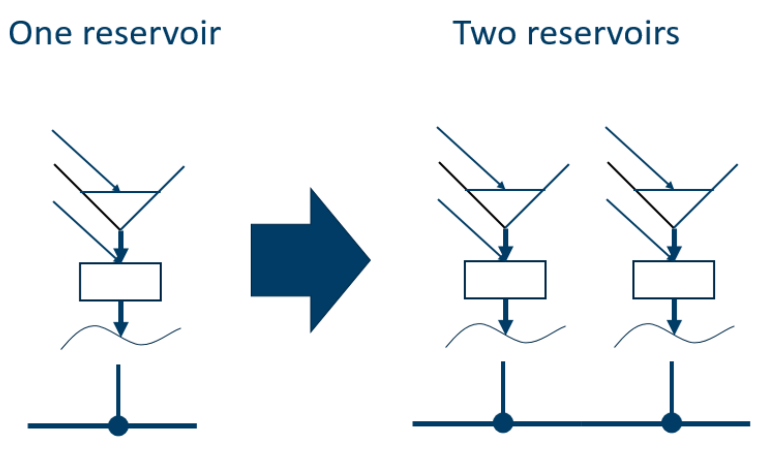

Figure 20: The old and the new aggregated structure.

A hydropower plant is selected and placed in one of the two aggregated hydropower reservoirs/plants based on plant/reservoir characteristics. The selection is based on the characteristics called degree of regulation:

$$R = \frac{V}{I},$$

where V is the reservoir maximum volume and I is the average annual inflow volume. First the area mean regulation degree, μ_R,is calculated.
The selection criteria are:

$$R \geq \mu_R, aggregated\ reservoir\ no.1.$$
$$R < \mu_R, aggregated\ reservoir\ no.2.$$

The selection criteria are very simple and should be revisited at a later stage. The plan was to write an ascii-file with details of all reservoirs/plants and to which aggregated reservoir they belong. In this file the user should be able to manually choose reservoirs/plants for aggregation.

Based on these results the aggregated reservoirs/plants can be calculated. The calculation is at the current stage only possible for one reservoir/plant, but can with some work be expanded to two reservoirs/plants. The aggregated model is described on *.detd-files which is the description used for the detailed hydro power plants in the EMPS/EMPSW, giving the advantages:

- Possible to expand to multiple reservoirs per area, both parallel and series reservoirs.
- Possible to run simulation (EMPSW) with the new aggregated structure in the search for convergent water values.
- Ascii file, human readable.

5.4 Calculation of aggregated energy inflow

To give as good result as possible the total inflow to the aggregate model is separated into storable and non-storable inflow. The energy inflows are in standard EMPS calculated by the program Tilsim based from the results of a simulation with the detailed drawdown model. The inflow calculation is done for one aggregated reservoir/plant.

A new energy inflow calculation is necessary for the new two reservoir aggregate model because we found that the method used for similar calculations in today's EMPS/EOPS model was not easily adaptable to a general aggregate model structure or to the new two storage model. Existing method is based on simulation and heuristics.

The implemented calculation method consists of two parts.

5.4.1 Non-storable inflow

First the non-storable energy inflow must be calculated. To give as good aggregate representation as possible of the detailed physical system, the non-storable energy inflow should include more than just the sum of physical non-storable inflow. It should also include production due to minimum discharge and/or bypass constraints, forced production to avoid overflow and energy used for pumping to avoid overflow. These results cannot cannot be obtained from the EMPSW results and therefore must be calculated by a special made algorithm.

In this algorithm the non-storable inflow is calculated by building weekly LP-problems including all constraints where the objective is to production while fulfilling all constraints. The value of water in all reservoirs by the end of each week has small positive vale. The optimisation is performed for each area individually to reduce the LP-problem size and calculation time. Detailed simulation results must be available for the whole simulation period, because for each week the reservoir levels are used to set the state of the system. The resulting minimum production for each plant is summed up to give the aggregated non-storable energy inflow.

5.4.2 Storable inflow

The storable energy inflow can be calculated, in the same way as in standard EMPS, when then non-storable energy inflow is known, following the equation below.

	Storable energy inflow =
	Sum production (including purchased load factor contracts)
-	non-storable inflow
-	energy used for pumping
+	increase in sum reservoir volume(or – reduction of reservoir volume)

The sum production, energy used for pumping and change in sum reservoir volume on the detailed level is obtained from MinprodLP. The aggregated storable and non-storable energy inflow are stored as time series.

6 Status new model concept

The project "Methods of aggregation and disaggregation" provides two main results, the prototype of a hydropower scheduling model EMPSW and the testing of new aggregation and disaggregation methods compared to the existing modelling framework.

EMPSW is well tested prototype that is delivered to all project participant ready to be used on operational data sets, including the new version 10 data structures and formats. EMPSW solves the main challenge with the existing EMPS model regarding short-term optimization of complicated river systems and handling of pumped storage plants. Based on first analyses [ref Ingeborg Energies] with the model it can be observed that EMPSW model is better suited than EMPS for analyses of future systems with more intermittent power production as wind and solar production that need balancing by pumped storage or batteries. However, the computation time for EMPSW is longer than for the EMPS model, but shorter than the formal optimisation model FanSi [11] and [12].

The remainder of the new model concept, which comprises the new aggregation procedure including inflow calculations, new aggregate model structures, optimal strategy for new aggregate model structure (VansimW), market simulation with new aggregate models. These parts are all implemented but are not robust and general enough for practical use or testing by the users. Based on experiences from testing, especially concerning calculation times, Vansimw needs both more refinement and parallel processing before it can used in practice.

The existing stochastic optimisation model for aggregated hydropower included in the EMPS called Vansim is computationally very fast, as Vansim is custom made for a predefined aggregated model structure with one aggregated reservoir and one aggregated power plant. The project showed that adapting Vansim to a new aggregated model structure is rather challenging. Furthermore, it is unlikely that an implementation using formal optimisation methods as Linear Programming can match the low computation time of Vansim. However, there is a large potential for parallelisation with the strategy calculation.

The EMPS model is suitable for use on today's power system, but not the future power system with a higher share of intermittent renewable energy sources. The detailed drawdown of the EMPS model is not adapted to intermittent intra-week power production [38]. Finally, more effort is necessary to compare the performance of the EMPS, EMPSW and FanSi models and find the right balance between model performance and calculation time.

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