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## Geometry effect on fracture behavior of V-notched specimens

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### Abstract

Geometric constraint is one of the important issues that affect the fracture behavior of different notched components. Specimens with the same notch geometry can have different fracture strengths due to the geometric constraints around the notch tip. In this paper, the fracture behavior of three different testing specimens made of General Purpose Polystyrene (GPPS) has been studied using an energy-based criterion namely the Average Strain Energy Density (ASED) criterion. According to the formulation of the ASED criterion, all the stress terms around the notch tip were taken into account by considering a volumetric energy-based criterion and the brittle fracture of different GPPS specimens with various geometry constraints were well predicted.

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### 1. Introduction

As one of the important types of material failure, brittle fracture has been widely taken into consideration by researchers, trying to propose precise failure prediction criteria for different brittle and quasi-brittle materials such as rocks, concretes, ceramics and polymers. Dealing with mode I loading condition for a component with symmetric geometry with respect to the notch bisector line which is under to the symmetric loading condition, the crack

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propagation from the notch root is expected to be along the notch bisector line. However, previous researches showed that for the case of specimens with high geometry constraint, crack can deviate from the symmetry line of the specimen. In this case the fracture load is expected to be different from the specimens with lower constraint effect. For specific case of cracked specimens, it has been shown in previous researches that different fracture toughness values can be obtained for an identical material when using various specimens' geometries (Larsson and Carlsson, 1973; Kumar et al., 2011; Ayatollahi et al., 2015, 2016; Moattari and Sattari-Far, 2017; Rashidi Moghaddam et al., 2017). More specifically, Razavi et al. (2018) in a recent publication evaluated the mode I fracture behavior of five different geometries of pre-cracked specimens made of PMMA and three different rocks using an energy-based criterion namely ASED. They reported that for specific categories of materials such as rocks, the effect of geometry constraint is not negligible. Among the studied geometries in their research, application of only the first stress term in Williams' series expansion for fracture prediction of Tapered Double Cantilever Beam (TDCB) specimens made of Harsin marble rock resulted in 47% difference with the ASED results obtained by considering all stress terms in a control volume around the crack tip for the same specimen.

It is worth mentioning that the brittle fracture of notched specimens with negligible geometry constraint is mainly governed by the first singular terms of stress (i.e. notch stress intensity factor, NSIF). However, in the case of components with high geometry constraint, the available fracture criteria which are based on only the singular terms of stress fail to predict the onset of the fracture (Ayatollahi et al., 2015). Some research have been conducted to predict the brittle fracture of notched components by considering both the singular term and the second term of Williams' series which provided better approximation of the fracture behavior, however, they had quite complex formulations compared to the previous models based on NSIF.

#### Nomenclature

$a$	notch length
$E$	elastic modulus
$F$	applied load in finite element model
$F_{ASED}$	theoretical fracture load
$F_{exp}$	experimental fracture load
$h$	height of specimen
$K_{Ic}$	fracture toughness
$r_c$	radius of control volume
$W$	width of specimen
$W_c$	critical SED
$\bar{W}$	average SED
$2\alpha$	notch opening angle
$\nu$	Poisson's ratio
$\sigma_t$	tensile strength
ASTM	American Society of Testing Materials
ASED	Average Strain Energy Density
CT	Compact Tension
DCB	Double Cantilever Beam
GPPS	General Purpose Polystyrene
SED	Strain Energy Density

In this research, we tried to evaluate the fracture behavior of V-notched specimens with different geometry constraints using the well-known ASED criterion proposed by Lazzarin and Zambardi (2001). According to the ASED criterion, brittle fracture occurs when the mean strain energy density (SED) over a circular control volume of radius  $r_c$ , is equal to a critical value,  $W_c$  that is a function of material properties of the material. Application of mean SED in a control volume around the notch tip, considers all terms of Williams' series in resulting in more accurate results. Although successful ability of the ASED criterion has been reported in numerous researches for various loading conditions in different engineering materials (Lazzarin and Berto, 2005; Aliha et al., 2017; Torabi et al., 2018a,b);

Razavi et al., 2018; Razavi and Berto, 2019), the validity of this criterion has not yet been examined for different geometries of notched specimens. Hence, the main aim of the present research is to evaluate mode I brittle fracture in General Purpose Polystyrene (GPPS) testing specimens of three different geometries. For this aim, the experimental behavior of GPPS V-notched specimens are assessed using the ASED criterion. It has been shown that very good agreement exists between the experimental results and the theoretical findings.

## 2. Experimental procedure

The geometry of different V-notched specimens used for conducting the fracture tests, namely Compact Tension (CT), and two types of Double Cantilever Beam (i.e. DCB1, DCB2) are schematically shown in Fig. 1. Specimens were cut from a GPPS sheet of 10 mm thick using water jet. Table 1 presents the geometrical dimensions of the test specimens. An initial notch length of  $a/W = 0.5$  was introduced in all the test specimens. The notch opening angle for all of the test specimens was equal to  $30^\circ$ . Fracture tests were conducted in room temperature with a constant displacement rate of 0.1 mm/min and each set of experiments were repeated three times. A series of standard fracture were conducted according to ASTM E1820 to obtain the fracture toughness of GPPS. A fracture toughness of  $K_{Ic} = 35$  was obtained by testing standard pre-cracked CT specimens.

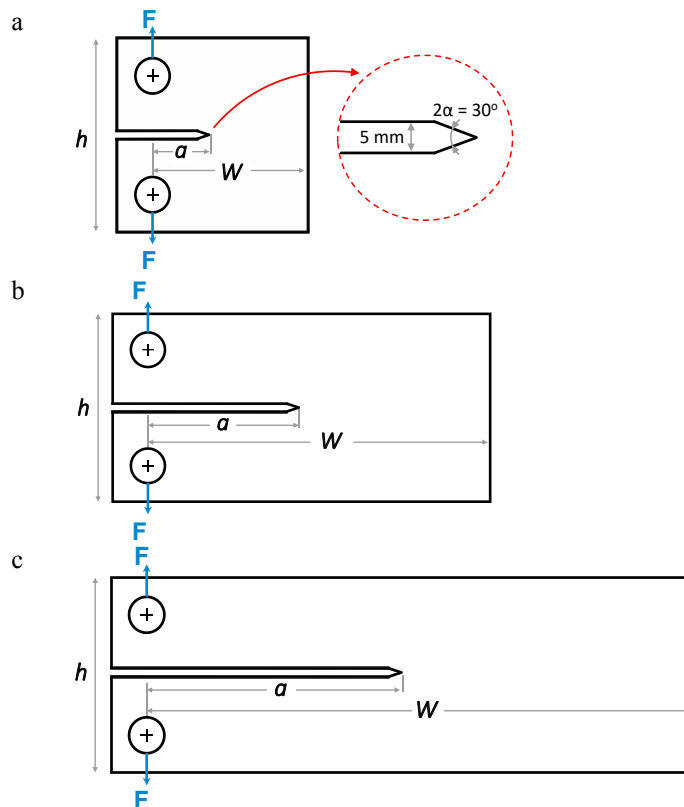


Fig. 1. Schematic view of test specimens; (a) CT, (b) DCB1, and (c) DCB2.

Table 1. The geometrical dimensions of the test specimens (all dimensions in mm).

Specimen type	Width, $W$ (mm)	Height, $h$ (mm)	Notch length, $a$ (mm)	Notch opening angle (degrees)
CT	30	30	15	30
DCB1	90	30	45	30
DCB2	150	30	75	30

### 3. Fracture load prediction using ASED criterion

The ASED criterion was used for fracture load prediction of different test specimens under pure mode I loading condition. According to this criterion, when the mean SED over a control volume,  $\bar{W}$  is equal to a critical SED value,  $W_c$ , the notched component fails. The size of control volume depends on the fracture toughness ( $K_{Ic}$ ) and the ultimate tensile strength ( $\sigma_t$ ) of the materials under static loads. Dealing with V-notched specimens, the control volume is a circle of radius  $r_c$  centered at the notch tip (see Fig. 2). Considering plain-strain conditions for the test specimens,  $r_c$  can be calculated using the following expression

$$r_c = \frac{(1+\nu)(5-8\nu)}{4\pi} \left( \frac{K_{Ic}}{\sigma_t} \right)^2 \quad (1)$$

To avoid any simplifications, the strain energy density values were also directly obtained from the finite element analysis. Two-dimensional linear elastic analyses were performed considering the elastic modulus and Poisson's ratio equal to  $E = 2.9$  GPa and  $\nu = 0.4$ . The model was meshed using iso-parametric 8-node quadrilateral plain strain elements.

At the onset of fracture, the mean SED,  $\bar{W}$  reaches its critical value,  $W_c$ , which can be calculated by substituting elastic modulus,  $E$  and ultimate strength,  $\sigma_t$  in Eq. (2) (Lazzarin and Zambardi, 2001).

$$W_c = \frac{\sigma_t^2}{2E} \quad (2)$$

According to ASED criterion, the theoretical fracture loads ( $F_{ASED}$ ) can be calculated using the following equation

$$F_{ASED}/F = \sqrt{W_c/\bar{W}} \quad (3)$$

in which,  $F$  is the applied load to the finite element model and  $F_{ASED}$  is the theoretical prediction of fracture load.

### 4. Results and discussion

By substituting the fracture toughness ( $K_{Ic} = 35 \text{ MPa}\sqrt{\text{mm}}$ ) and the tensile strength ( $\sigma_t = 44 \text{ MPa}$ ) of GPPS into Eq. (1), the calculated value of critical radius,  $r_c$  was found to be 0.127 mm. A unit load (i.e.  $F = 1 \text{ N}$ ) was applied to the loading pins of the finite element models to obtain the mean SED in the control volume around the notch tip. The critical strain energy density was equal to  $W_c = 0.334 \text{ mJ/mm}^3$ . The theoretical ASED predictions were obtained using Eq. (3). The mean SED values corresponding to the unit applied load in finite element analyses and the theoretical ASED predictions based on the finite element analysis are presented in Table 2. A comparison between the experimental fracture loads and the ASED predictions based on finite element analysis is presented in Fig. 3.

According to the ASED predictions based on the constant value of  $W_c = 0.334 \text{ mJ/mm}^3$  for different specimen geometries, maximum discrepancies of 1.11, 1.10 and 0.71 were obtained for the fracture load predictions. According to the ASED predictions, it is confirmed that the chosen control volume is capable of considering the geometry effect on the fracture behavior of GPPS notched specimens.

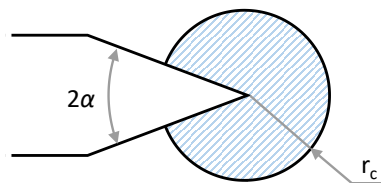


Fig. 2. Schematic view of the control volume around the notch tip.

Table 2. Outline of numerical results of fracture loads.

Specimen type	$F_{exp}$ [N]	$\bar{W}$ [mJ/mm <sup>3</sup> ] × 10 <sup>6</sup>	$F_{ASED}$ [N]	Discrepancy*
CT-1	179			1.11
CT-2	166	10.1	161	1.03
CT-3	159			0.99
DCB1-1	85			0.94
DCB1-2	99	32.4	90	1.10
DCB1-3	92			1.02
DCB2-1	46			0.79
DCB2-2	41	77.8	58	0.71
DCB2-3	51			0.88

\*Discrepancy:  $F_{exp} / F_{ASED}$

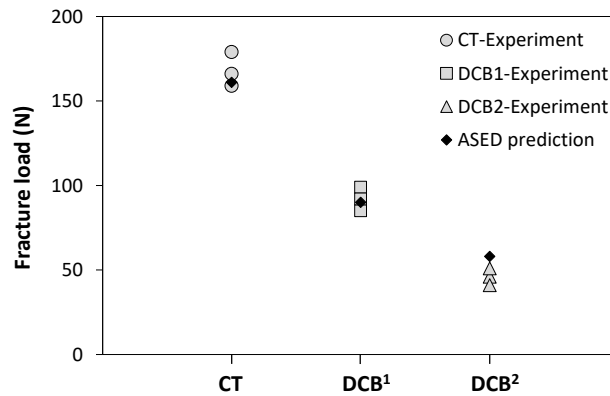


Fig. 3. Comparative results of the experimental fracture loads and ASED predictions.

As mentioned earlier, unlike the two parameter fracture criteria that consider the first and second terms of Williams' stress series, the ASED criterion provides a straightforward methodology to evaluate fracture behaviour of notched components with various geometries. According to the theoretical results of this research, application of ASED criterion provides a good approximation of fracture load for notched components with different geometry constraints. In general, the ASED criterion can be used by engineers and scientists to predict the onset of fracture in complex notched components without requiring costly and time-consuming experiments.

## 5. Conclusions

In this research, the average strain energy density (ASED) criterion was used to predict the fracture load of three different V-notched specimens under mode I. Due to computational complexities of the classical fracture models, it is convenient to use a straightforward methodology for fracture load evaluation, which is capable of taking into account the effect of geometry constraints. The ASED criterion that includes all terms of Williams's series could provide very good predictions for the experimental data obtained from GPPS V-notched specimens of different shapes. The same procedure can be used to estimate the fracture load of other brittle and quasi-brittle materials.

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