FLEXIBILITY AND COOPERATION:
VIRTUAL LEARNING ENVIRONMENTS IN ONLINE
UNDERGRADUATE MATHEMATICS

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In this paper we investigate the learning environment in the online undergraduate mathematics initiative DELTA. We find that the students work alone and that their most important learning relation is to the teacher. The students need flexibility in respect to when and where they can study, and this affects their ability to have learning relations to other students. Communication on mathematical issues is difficult using computers, and the tools offered by our LMS is insufficient. It seems hard for the students to self organise their online collaboration in mathematics.

ABOUT THE DELTA PROJECT

The DELTA project consists of eight online undergraduate mathematics courses at the Norwegian University of Science and Technology (NTNU). The subjects are the same as taught on campus, but adapted for distant learners. The syllabuses and the exams are the same as in the campus courses. Each course gives 7.5 ECTS credits, and the eight DELTA courses are:

- Basic Calculus I
- Basic Calculus II
- Linear Algebra and Geometry
- Linear Algebra with Applications
- Number Theory
- Geometry
- Probability
- Statistical Methods

The typical DELTA-student is a teacher in upper secondary school who wants to become qualified as a teacher in mathematics. Most of our students are teaching economics or biology, and some of them are mathematics teachers without a formal education in mathematics (60 ECTS credits). The students live in different parts of Norway — some students live more than 1000 km from Trondheim.

The subjects taught in our courses are very traditional undergraduate mathematics courses. We use ICT for communication, flexibility and cooperation, but the use of ICT is not a learning objective in itself. DELTA is based on the use of a learning management system (LMS). The LMS is our most important communication channel, and we use it to publish texts, streamed video lessons and exercises. The students make active use of the discussion groups offered to them inside our LMS [1]. At the Norwegian University of Science and Technology we have a studio for media productions. The media centre produces and streams our video lessons. The students emphasise the video lessons as very instructive and important for their learning.
One of the DELTA project’s main issues is concerning the importance of hand writing in learning and teaching mathematics online. Our LMS is not specially designed for writing mathematics, and the students scan their handwritten hand-ins and deliver via the LMS. The teachers use a pen and a tablet with their computers, and are working directly on screen when correcting and commenting the exercises. The technological problems related to hand writing is still a major problem, precluding the students from effective collaboration online.

Norway Opening Universities (NOU) [2] is a national initiative for change and innovation in Norwegian higher education. NOU supports Norwegian institutions of higher education by funding projects for developing ICT supported flexible learning and distance education courses through an annual application process. In 2006 NOU funded the DELTA project with 500,000 NOK [3].

E-LEARNING AND MATHEMATICS

The challenges in creating an online learning environment might be different when working with mathematics than in other topics. An observation by Mark Guzdial et al. (2002) supports this hypothesis: When introducing a wiki based collaboration tool in undergraduate university teaching, specific resistance was experienced when introducing the technology in mathematics and science classes, this resistance was not seen elsewhere.

There is little research on the specific problems in using e-learning platforms for teaching mathematics, on the contrary distance education in general is relatively well explored (see Andreasen 2003, for an overview). The special situation regarding online mathematics relates, we believe, to several things: For example many of the signs that goes into building mathematical discourse is not available on a standard keyboard, and the way that mathematical communication often is supported by many registers and modalities that are used simultaneously, as writing and drawing various representations on the blackboard, while talking and gesturing (Duval 2006, Rasmussen et al. 2004). Another reason could be that mathematics as an abstract topic rely more on socially negotiated meanings than other topics, and again that this negotiation might be harder to obtain online in mathematics than in other topics. This paper can be seen as a first attempt to investigate the special situation on e-learning and mathematics, and might proceed the development of a specific framework to discuss problems and potentials with elearning and mathematics.

METHOD

The research we describe serves two parallel purposes, to evaluate and further develop the e-learning initiative DELTA and to better understand the specific challenges and potentials in using an online format to teach mathematics. The project has an obligation to perform an evaluation, and the research serves also the purpose of fulfilling this obligation. The research team consists of a teacher and leader of the
educational initiative, and a researcher not, a priori, knowledgeable about the DELTA project.

**Design and research**

In this project we simultaneously attempt to generate general knowledge about the learning environment in online education in mathematics and to evaluate and improve the educational initiative DELTA. In this respect we are conducting design research (DBR Collective, 2003). In our investigation we are focused on obtaining a clearer picture of the learning environment of students enrolled in DELTA and on how various factors shape this environment. Nevertheless we are not, in this investigation, focused on optimizing specific lessons or tasks, even though this would be considered a very valuable side effect.

**Initial interviews and questionnaire**

In order to gain insights into the learning environments of the DELTA students we applied a relatively open approach. First an open interview study was conducted, mainly in order to find relevant themes for a quantitative investigation. The interview guide was developed on the basis of intuition and experience of the involved teacher and researcher and a survey on literature. This open approach is inspired by Strauss and Cobin (1998).

Three informants were initially interviewed by phone. The three were part of a reference group. The interview guide evolved around four main question areas:

- Technological and practical barriers for communication
- What media is used and for what purpose
- Learning together with peers
- The on-campus gatherings

The interviews were transcribed and coded in an open way in order to look for themes. Themes that related to the informants learning environment, and that were of value to the informants (the things they wanted to tell). The teacher’s knowledge was used to support and challenge these findings. The teacher knew some of the students from the gatherings and from contact via the LMS, e-mail and phone, and had an ongoing discussion with them about their needs. He used these ‘meetings’ as a sort of semi-systematic data collection. The teacher’s pre-knowledge, and his need for more information to optimize the learning environment, was used in conjunction with the researcher’s three interviews to find relevant questions for a quantitative investigation. We choose the following six themes as being important for the design of questionnaire:

- the need for flexibility
- the feeling of isolation
- practical barriers for working simultaneously
- the small misconceptions that is only dissolved in face to face situations
• frustration caused by a three day rule [4]
• writing mathematics using computers

Our web-based questionnaire has 32 questions, some are ‘open’, but most of the questions are multiple choice. We got answers from 32 (75 %) of the students.

LEARNING ENVIRONMENTS AND ORGANISATION

The organisation, communicative situation and mutual expectations that are connected to teaching and learning are different in different settings. As relevant to the DELTA project we consider several typical institutional organisations:

• University education in mathematics, for instance as it occurs at NTNU’s on-campus teaching.
• Secondary education, since the main target group for the education are teachers in secondary education.
• Furthermore several distance-learning formats are relevant.

The typical campus university teaching is organised with a combination of lectures and classroom teaching. The social interaction amongst peers are not explicitly supported but due to the easy access to fellow students this allows them to self organise their cooperation. On a schematic level the communication in relation to teaching and learning is centred around the teacher, who communicates to everyone, but equally important is the self organised communication amongst the students. For a deeper description of the mutual (teacher, student) expectations in undergraduate mathematics using the concept of didactical contract see Grønbæk et al. (to appear).

In upper secondary education there is typically a mix between lecture teaching and explicitly organised group work. An important aspect of upper secondary education is that group work is typically facilitated and planned by the teacher, and hence not self organised, as it typically is in undergraduate mathematics.

The classical approach to distance education is more or less to provide educational material such as one or several books, a syllabus and a possibility for evaluating. Here the main interaction is directly between the teacher and a single student (Garrison, 1985). In many approaches to online teaching the role of the teacher is furthermore to moderate and facilitate a discussion between the students. The communicative situation in this case is to focus on the participation of every student in an online learning community (Salmon, 2000).

Having these aspects and examples of learning environment in mind we consider now the situation in DELTA, looking into the student answers to the questionnaire and their comments.
Teacher as a central figure

The introductory interviews gave an impression that there was an almost inherent conflict between flexibility and isolation in DELTA, meaning that obtaining the flexibility needed requires the students to work mostly alone lacking the peer contact that is usually important in university education. In respect to contact (or lack of contact) with the peers the questionnaire shows that it is not very important for the students to study together with peers (figure 1) but slightly more important to be able to contact peers when studying (figure 3). Much more important is the contact with the teachers; 59 % agrees strongly and 41 % agrees, to this importance (figure 4).

It is interesting that the difference between the need for staying in contact with peer-students and with the teacher is so large. Of course the teacher is an important figure...
when studying, but these data shows that in DELTA, the teacher is the most important person for the students.

This is also reflected in the students’ general evaluation of the DELTA courses where the streamed video recordings are described very positively.

It is the videos that are the backbone of the teaching. A really nice tool that I have used. In addition the discussion group for the subject is good. I have read questions/answers that have helped me.

Furthermore one thing that was revealed by the phone interviews was that the students were very keen on having prompt responses from the teacher when asking questions in the forum. The respondents felt that the teachers would hesitate to answer based on a pedagogical rationale, namely to support students’ interactions, and they were unhappy with that. [4] These data all support the fact that the teacher is a very central person in the learning environment at DELTA.

**Little response to the LMS discussion**

Another thing that we find interesting in the responses is the low number of students that poses questions in the forum and their comments regarding motivation and barriers to pose questions.

Only a few of the students actually use the forum often for asking questions (figure 5). The questions and answers that are posted are read often (several times every week) by almost half of the students (figure 7). Furthermore many of the students weekly, or occasionally, have questions that they would like to raise but choose not to post (figure 8). Several reasons are given for this and of course lack of time and resources to pose the question are important reasons. But among the reasons are also the following (cited from students’ answers to the questionnaire):

I feel I’m falling behind in relation to the expectations.

The questions are often “out-dated” because I am behind schedule ☹.

I sometimes find that my question already has been answered because someone else has posed it. In addition I feel I am lagging behind with the assignments, and concentrate on getting the assignments in rather than doing the voluntary tasks.

This means that some of the students regard the LMS space for discussion as not suitable for “out-dated” problems.
How often do you post questions to the discussion group?

<table>
<thead>
<tr>
<th>Frequency</th>
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<tbody>
<tr>
<td>Several times per week</td>
<td>34%</td>
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<tr>
<td>Weekly</td>
<td>9%</td>
</tr>
<tr>
<td>Now and then</td>
<td>28%</td>
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<tr>
<td>Rarely</td>
<td>28%</td>
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<tr>
<td>Never</td>
<td>0%</td>
</tr>
<tr>
<td>Do not know</td>
<td>0%</td>
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</tbody>
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Figure 5: (Q11) How often do you post questions to the discussion group?

How often do you read other’s questions and answers?

<table>
<thead>
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<td>Now and then</td>
<td>28%</td>
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<tr>
<td>Rarely</td>
<td>9%</td>
</tr>
<tr>
<td>Never</td>
<td>0%</td>
</tr>
<tr>
<td>Do not know</td>
<td>0%</td>
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</tbody>
</table>

Figure 7: (Q13) How often do you read other’s questions and answers?

How often do you answer other students’ questions?

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<thead>
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<td>Now and then</td>
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<td>Rarely</td>
<td>22%</td>
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<tr>
<td>Never</td>
<td>9%</td>
</tr>
<tr>
<td>Do not know</td>
<td>0%</td>
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Figure 6: (Q12) How often do you answer other students’ questions?

How often do you have questions which you really would like answers to, but which you still don’t pose?

<table>
<thead>
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<th>Percentage</th>
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<tr>
<td>Weekly</td>
<td>31%</td>
</tr>
<tr>
<td>Now and then</td>
<td>22%</td>
</tr>
<tr>
<td>Rarely</td>
<td>9%</td>
</tr>
<tr>
<td>Never</td>
<td>0%</td>
</tr>
<tr>
<td>Do not know</td>
<td>0%</td>
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Figure 8: (Q14) How often do you have questions which you really would like answers to, but which you still don’t pose?

Media and mathematics

Writing mathematical signs on a computer is clumsy (Misfeldt, 2006) and this does affect the students’ ability and willingness to contribute to the online forum. For instance one student writes:

a written answer would have bee too complicated and it would not be possible with direct feedback.

In one of the questions, the students explain how they typically communicate with peer students. The most typical way is face to face or via LMS. From the students comments to the questionnaire it seems that the computer is used mainly to send and receive information and given the fact that the students choose to scan their weekly hand-ins is also important because it points to the insufficiency of for instance the e-mail format and our LMS with respect to mathematics (figures 9 and 10).
Figure 9: Excerpt from hand written assignment with student’s question to teacher and a comment from the teacher. The teacher works on screen using a pen tablet for handwriting along with the note utilities in Adobe Acrobat.

1+(x/(4x))^2= 1 + x^2 - 2x(1/(4x) + 1/(4x)^2 = (x+1/(4x))^2 the square root of this then becomes x+1/(4x)

Figure 10: Example from the calculus discursion in our LMS. This is hard both to write and to read, and with a suitable editor the student could have written this in standard mathematical notation as:

\[
1 + \left( x - \frac{1}{4x} \right)^2 = 1 + x^2 - 2x \left( \frac{1}{4x} \right) + \frac{1}{(4x)^2}
\]

\[
= x^2 + \frac{1}{2} + \frac{1}{(4x)^2} = \left( x + \frac{1}{4x} \right)^2 \text{ and}
\]

\[
\sqrt{\left( x + \frac{1}{4x} \right)^2} = \left( x + \frac{1}{4x} \right)
\]

ANALYSIS

The interviews left us with the impression that there is an inherent conflict between flexibility and cooperation/isolation when working with mathematics in an online environment. And the quantitative data shows that both the concerns, to obtain flexibility and to avoid isolation, are considered important by the students. The data also shows that many of the students mainly work alone (figure 2).

The students consider the contact with the teacher as more important than the contact with peer-students. In that sense the teacher is the central person having “learning relations” with each individual student. There can be several explanations for this teacher centric learning environment. The subject mathematics is different and in a sense more authoritative than for instance social sciences. This means that the online discussion format (Salmon, 2000) does not really apply to mathematics education (Guzdial et al., 2002). This does not mean, however, that the students cannot benefit from collaborating on their work, but it does mean that the discussion based format, as it is described in (Salmon, 2000) might be insufficient for supporting mathematical learning processes.

The culture around undergraduate mathematics education as it exists in campus settings can also play a detrimental role in the learning environment at DELTA. From the questionnaire we see a tendency not to pose questions in the online forum because the question is related to topics and tasks posed several weeks ago. This is actually the most typical reason that was stated in the comments to the question of why the students did not participate in the online forum. But it really does not matter that you are behind schedule, for you to pose questions. There is a big difference between
posing a question in an online forum and asking questions in an auditorium where all the attention of hundreds of students is focused on you and important teaching time is used. But the reason that the students give leaves us with the impression that they feel as if they are wasting other people’s time in posing “old” questions. It is typical in campus teaching that collaboration between peer students happens automatically without the teacher organising it. The lack of day to day contact between peer students might be reason enough for the teacher to be more explicit in organising group work and collaboration, as it is also suggested in (Salmon, 2000).

The learning environment is also greatly influenced by the communicative difficulties that mathematical representations pose to online environments. It might be an extra barrier for using the online forum that it cannot handle mathematical formalism. It seems reasonable to assume that the students’ difficulties in establishing well functioning collaborative relations online is partly due to their difficulties in communicating mathematically online. There is a large difference between the teachers’ ability to communicate with the students through video-lectures and the students’ online communicative abilities. This can be part of the explanation as to why the teacher seems very central in the learning environment at DELTA. By using the video lectures the teacher is placed in a very special position, performing well produced instructional sequences rather than participating in an equal dialogue.

CONCLUSION

This paper has investigated the learning environment in the online educational initiative DELTA. We have seen that the students engaged in DELTA mainly work alone and that their most important learning relation is to the teacher. We also see a number of reasons for this. The students in general need a lot of flexibility in respect to when and where they can study, and this affects their ability to have learning relations to other students. Mathematics is an authoritative discipline and, to some extent the teacher is governing the truth. Furthermore communication on mathematical issues is difficult using computers. And the teacher (giving video lectures) has better ways to overcome these barriers than the students.

Looking ahead three changes in the organisation of DELTA could be considered. Firstly the teachers might support the students’ collaboration more explicitly, for instance by asking for cooperatively authored assignments. To trust the students to self organise their collaboration is insufficient in the online format. Partly because the students do not meet, and partly because they have trouble communicating using the computer. Secondly, it might be worthwhile to consider introducing online meeting programs as for instance Marratech [5] to support both teacher-student and student-student communication. And finally we have to find discussion software with a better editor for mathematical writing.
NOTES

1. We use the LMS It's Learning (http://www.itsolutions.no)

2. http://nou.no

3. Approx. 60,000 EUR

4. This turned out to originate from a misunderstanding. We do not go into a detailed analysis on the frustration caused by the three day rule in this paper.


REFERENCES


Garrison D.R.: 1985, ‘Three generations of technological innovation in distance education’, *Distance Education* 6, 235-241


