Efficient Modelling and Design Optimization of Large Floating Wind Turbines
NTNU
Norwegian University of Science and Technology

Thesis for the degree of Philosophiae Doctor

Faculty of Engineering
Department of Marine Technology

© John Marius Hegseth

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Abstract

Floating wind turbines (FWTs) are considered a promising solution for wind energy harvesting in deep water, but are currently too expensive to compete with other energy sources. Being a relatively new and immature technology means that there still is a large potential for cost reductions through optimization of the FWT structure. Optimized designs will bring the construction costs of FWTs down and increase profitability, which is currently the major challenge for the industry. Optimized designs can also result in increased reliability, which is an important issue.

A FWT may encounter a large number of different loading situations during its lifetime, including operational, parked, fault, start up, and shutdown events. In each of these situations, various combinations of environmental loads from wind and waves must be considered. Combined with computationally expensive response analyses, this results in a comprehensive design process.

FWTs are also highly multidisciplinary systems, primarily combining the areas of aerodynamics, hydrodynamics, structural dynamics, and control theory. The interactions between the disciplines, and between different components in the system, calls for integrated analysis and design. Due to the complex dynamics, strong couplings, and potentially large number of design variables, identifying optimal design solutions become a difficult task.

The main purpose of this work was to improve the design process for FWTs and thus contribute to reducing the cost of energy. This is addressed through two overall research objectives, which consider i) increased computational efficiency of global design analyses for FWTs, and ii) methods for numerical
design optimization which can help identify cost-effective and reliable design solutions. The main focus was on the support structure and controller for 10 MW spar-type turbines, considering fatigue and ultimate loads.

A linearized aero-hydro-servo-elastic model was shown to yield good results for the fatigue loads in the support structure, where agreement within ± 30 % was achieved for the long-term fatigue damage compared to nonlinear time-domain simulations. Acceptable agreement was also observed for short-term extreme response, especially for the support structure bending moments, which were quite Gaussian also in harsh environmental conditions. The resonant platform pitch response was overestimated by the linear model, especially in near-rated conditions.

A gradient-based optimization approach with analytic derivatives was developed to perform integrated design optimization of the support structure, blade-pitch controller, and mooring system for an elastic 10 MW spar FWT, including the scantling design of the hull, where the goal was to minimize a combination of design costs and rotor speed variation. Different control strategies were compared through integrated design of the controller and support structure, which allowed for identification of optimal control parameters in a lifetime perspective, and fair comparisons between different strategies.

The impact of environmental modelling on the long-term fatigue reliability, and associated design costs, of the support structure was assessed through re-design of the tower and platform. Considering stochastic turbulence intensity, wind-wave misalignment and the wind directional distribution reduced the long-term fatigue damage by approximately two-thirds along the support structure, compared to the base model. Implications of the chosen fatigue safety factor on the trade-offs between CAPEX and OPEX were also assessed.

The methodologies for global response analyses and integrated design optimization developed in the present work have been shown to be suitable for preliminary design of spar FWTs, where they can provide a starting design for later and more detailed design phases. Different modelling and design aspects for cost-effective and reliable solutions have been identified and assessed. The methodologies can be further extended to account for different FWT concepts, additional design parameters, and other load cases, and may help identify novel design solutions.
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Publications

The thesis consists of an introductory part and a collection of papers. The papers that form the basis of the thesis, as well as additional papers and a declaration of authorship, is presented in the following.

List of publications

The following five papers are considered part of this thesis:


P5 John Marius Hegseth, Erin E. Bachynski and Bernt J. Leira (2020). Effect of environmental modelling and inspection strategy on the op-
timal design of floating wind turbines. Submitted to Reliability Engineering and System Safety.

The following paper is not regarded as part of the thesis due to scope:

Declaration of authorship

In P1, P3, P4, and P5, J. M. Hegseth had the main ideas for the papers, developed the numerical models, performed all calculations, and wrote the manuscripts. Profs. E. E. Bachynski, J. R. R. A. Martins and B. J. Leira contributed with valuable suggestions and feedback during the processes. P2 was initiated by C. E. Souza, who also developed the forced oscillation method and wrote the manuscript. J. M. Hegseth derived the analytic expressions for the aerodynamic damping and inertia coefficients, developed the linearized model, and participated in discussions. Prof. E. E. Bachynski provided guidance and feedback.
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Chapter 1

Introduction

This chapter gives an overview of the background and motivation for this work, and states the objectives of the research. A summary of the main contributions to the scientific community, as well as their relations to the research objectives and appended papers, are also presented.

1.1 Motivation and background

The high global demand for energy has led to a large consumption of fossil fuels, with several negative consequences, most notably climate change. The last decades have seen a large increase in the use of wind energy, which more recently has moved offshore. Offshore wind is an attractive energy source due to higher wind speeds with less turbulence, larger areas less susceptible to conflicts with other interests, and with smaller environmental impact than its land-based counterpart.

The majority of energy producing offshore wind turbines installed to date are bottom-fixed, and located in shallow waters. Because suitable sites for such wind parks are limited, it is desirable to develop offshore wind farms in deeper waters. In water depths above 50-60 m, bottom-fixed structures are not economically feasible, and FWTs are expected to be the most cost-effective option (Musial et al. 2006). As 80% of the offshore wind resources in Europe are located at water depths above 60 m (The Carbon Trust 2015), there is a vast potential market for floating wind power, which could become a crucial part of the energy mixture of the future.

The technical potential floating wind capacity across Europe, USA, and Japan is estimated to about 7000 GW, where 4000 GW is located in Europe
Table 1.1: Floating wind farms in Europe with expected commissioning date within 2022 (WindEurope 2019).

<table>
<thead>
<tr>
<th>Country</th>
<th>Wind farm</th>
<th>Floater type</th>
<th>Capacity (MW)</th>
<th>No. of turbines</th>
<th>Water depth (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Portugal</td>
<td>Windfloat Atlantic</td>
<td>Semi-sub</td>
<td>25.0</td>
<td>3</td>
<td>100</td>
</tr>
<tr>
<td>France</td>
<td>EolMed Phase 1</td>
<td>Barge</td>
<td>24.0</td>
<td>4</td>
<td>55</td>
</tr>
<tr>
<td>France</td>
<td>Provence Grand Large</td>
<td>TLP</td>
<td>28.5</td>
<td>3</td>
<td>100</td>
</tr>
<tr>
<td>France</td>
<td>Eoliennes</td>
<td>Semi-sub</td>
<td>30.0</td>
<td>3</td>
<td>65-80</td>
</tr>
<tr>
<td>France</td>
<td>Flottantes de Groix</td>
<td>TLP</td>
<td>28.5</td>
<td>3</td>
<td>60</td>
</tr>
<tr>
<td>UK</td>
<td>Kincardine</td>
<td>Semi-sub</td>
<td>50.0</td>
<td>5</td>
<td>60-80</td>
</tr>
<tr>
<td>Norway</td>
<td>Hywind Tampen</td>
<td>Spar buoy</td>
<td>88.0</td>
<td>11</td>
<td>260-300</td>
</tr>
</tbody>
</table>

(The Carbon Trust 2015). However, moving the energy production to deeper water and larger distances from shore increases the complexity and requires innovative technology. To date, the world’s only floating wind farm is the Hywind Scotland pilot park, which consists of five turbines with an installed capacity of 30 MW. However, more than 30 different FWT concepts have been proposed (Leimeister et al. 2018), and several floating wind farms are planned to start operation in the near future. Seven European farms, listed in Table 1.1, are expected to be commissioned within the next three years, with a total capacity of 274 MW.

The cost of floating wind is currently higher than for bottom-fixed turbines, and large reductions are needed to reach a competitive level. Floating wind turbines have larger construction costs than the bottom-fixed alternative; however, installation costs are expected to be smaller, because the turbines can be assembled at port using onshore cranes instead of expensive jack-up vessels at site (Katsouris and Marina 2016). The current levelized cost of energy (LCOE) is estimated to €180-200/MWh for pre-commercial projects, but large reductions are expected as the technology matures and reaches commercial-scale deployment, and may attain values of €40-60/MWh in 2030 (WindEurope 2018).

Lower LCOE can be achieved in several ways, such as reduction of capital
costs (CAPEX) or operation and maintenance costs (OPEX), increase in the capacity factor, or through lifetime extension. As floating wind is a relatively new and immature technology, there is still a large potential for cost reductions through optimization of the FWT structure. A breakdown of typical CAPEX for floating wind farms is shown in Fig. 1.1, where the turbine and platform are seen to comprise more than 60% of the total costs, suggesting that significant cost reductions in these components may be necessary to reach a sustainable and competitive LCOE. This is also supported by feedback from concept designers, who identify platform size as the most critical technical barrier with respect to cost savings for floating wind farms (The Carbon Trust 2015).

As the technology matures, turbines grow larger, and during the last decade, the average capacity of newly installed offshore wind turbines in Europe has more than doubled, from 3 MW in 2009 to 7.8 MW in 2019 (WindEurope 2019). The turbines are expected to grow further as the technology improves, and could reach 15-20 MW by 2030 (IEA 2019). The increase in turbine size is expected to be the largest driver of cost reduction, because it allows the same amount of energy to be captured with fewer machines, and thus reduces the balance-of-system costs (Smith et al. 2015). In addition, it results in lower relative costs of the substructure, tower and mooring costs, compared to a larger number of turbines with smaller rating (Müller et al. 2019). The increase in size also introduces new challenges and opportunities for design, with innovative turbine and substructure technologies having a large impact on the achieved cost reductions (Chaviaropoulos et al. 2014). It may also help new materials, such as concrete, become more attractive solutions (Müller et al. 2016).

A FWT is subjected to a large number of different loading conditions during its lifetime, involving a highly stochastic environment with irregular loads from wind and waves. According to DNV GL (2018a), the structure shall be designed to sustain all loads during temporary, operating, and damaged conditions, ensure safe operation of the wind turbine, maintain acceptable safety for personnel and environment, and have sufficient durability against deterioration. Because FWT structures are unmanned, the consequences of failure are normally limited to economic losses. Failures may, however, also affect the environment or nearby structures, and FWTs should be designed for an annual target probability of failure of $10^{-4}$ (DNV GL 2018a).

Four different limit states are considered in design (DNV GL 2016):
Ultimate limit state (ULS): Exceedance of maximum load carrying resistance.

Fatigue limit state (FLS): Failure due to cyclic loading.

Accidental limit state (ALS): Failure due to accidental or abnormal loads.

Serviceability limit state (SLS): Exceedance of criteria for normal use or durability.

Offshore wind turbines are highly dynamic systems subjected to static, periodic, stochastic, and transient loads. Combined with compliant structures and a relatively long design lifetime, this often leads to FLS being design driving for several components (Vorpahl et al. 2013). Contrary to the design of conventional offshore structures, which tends to be driven by ultimate loads, this is also the case for the floating platform (Strach-Sonsalla and Muskulus 2016). To give a realistic estimate of the fatigue life of the structure, all relevant environmental conditions throughout the lifetime of the system must be considered, which leads to a comprehensive set of design calculations. This is especially the case if directionality of wind and waves is considered, which is of particular importance for fatigue design (DNV GL 2018a).

In addition to fatigue, the survival of the platform and turbine in extreme
conditions must be ensured. Typical ULS criteria consider overturning, capsizing, yielding, and buckling (DNV GL 2018a). Further constraints may be applied to prevent water on deck, negative air gap, and sloshing inside ballast tanks during storm events (Strach-Sonsalla and Muskulus 2016). Failure caused by accidental loads, or exceedance of ultimate resistance of damaged structures, is covered by ALS criteria (DNV GL 2018a), and is not considered in this thesis.

Lastly, SLS deals with criteria to ensure normal operation of the turbine. It involves limitations on inclinations, accelerations and motions due to sensitive equipment and the power cable (Strach-Sonsalla and Muskulus 2016), in addition to ensuring sufficient hydrostatic stability for efficient power production.

In general, the design criteria can be written

$$S_d \leq R_d,$$  \hspace{1cm} (1.1)

where $S_d$ is the design load effect, and $R_d$ is the design resistance. These are usually found using the partial safety factor method, where the different characteristic load effects and resistances are multiplied by individual load and material factors to obtain their design values. The safety factors reflect the uncertainty in the respective design parameters, as well as the required reliability level.

Probability-based design is an alternative to the partial safety factor method, where the load and resistance variables are described by their probability distributions. The design of the system is then performed to meet a safety level which is expressed by the failure probability:

$$P_F = P[S > R],$$  \hspace{1cm} (1.2)

which should be equal to the annual target failure probability (DNV GL 2018a). In practical applications, the partial safety factor method is by far the most popular, and probability-based design is mainly used to calibrate safety factors, or in design of systems where limited experience is available (DNV GL 2018b).

To evaluate the structure’s margin of safety against the different limit states, an extensive number of design load cases (DLCs) must be assessed. Relevant DLCs for FWTs are defined by DNV GL (2016), and consider eight distinct design situations:

- Power production
Introduction

- Power production plus fault
- Start up
- Normal shutdown
- Emergency shutdown
- Parked turbine
- Parked turbine plus fault
- Transport, installation and repair

Within each of these design situations, a subset of load cases are specified. The subsets consider different wind and wave conditions, fault types, and limit states, and the total number of load cases in a design process may therefore add up to several thousands.

The goal of the design optimization process is to find the design that minimizes or maximizes a given performance measure, such as costs or power production, and at the same time meets the appropriate design criteria. In traditional design optimization, the performance of an initial design is evaluated through analyses or experiments. Based on the results, the designer makes changes to the design, and the process is repeated until a satisfactory design is achieved. This process is typically limited to a few iterations, where changes to the design to a large extent depend on the experience of the designer (Muskulus and Schafhirt 2014).

In contrast, numerical optimization techniques can efficiently explore large parts of the design space to identify improved and possibly non-intuitive solutions, as illustrated in Fig. 1.2. Such techniques have been widely utilized in the automotive and aerospace industries, but have seen limited use in design of wind turbine structures (Muskulus and Schafhirt 2014). Similarly to a conventional design process, the performance of an initial design is evaluated based on results from analyses, and iterations are performed until a satisfactory solution has been achieved. The key differences are that the design changes are made automatically by an optimization algorithm, and that the design process does not end until certain optimality criteria are satisfied. These criteria ensure that the final design not only complies with the relevant constraints, but that it indeed is the solution that minimizes or maximizes the chosen performance measure, at least in that particular neighbourhood of the design space (Martins and Ning 2020).
Figure 1.2: Comparison of main steps in a manual and numerical design optimization process. Adapted from Muskulus and Schafhirt (2014).

Although these techniques enable automation of the design process, they do not remove the need for field-specific expertise from the designer. The outcome of a numerical optimization procedure relies heavily on the formulation of the optimization problem. An appropriate objective function must be chosen, whose value is the only measure of the performance for a given design. Further, relevant constraints should be identified, and proper enforcement strategies must be defined. Finally, a parametrization of the design space is needed, which determines the possibilities for the optimization algorithm to modify the design. The successful application of such methods thus requires profound knowledge about the nature of the system that is considered, and possibly more so than in a traditional design process (Martins and Ning 2020).

As a FWT system experiences important interactions between several different subsystems and disciplines, integrated design is needed to find optimal solutions on the overall system level (Muskulus and Schafhirt 2014). In an
integrated design process, all relevant disciplines are coupled, and the sub-
systems are designed simultaneously. An integrated optimization approach
involves a large number of design variables and load cases, and requires
sophisticated and multidisciplinary numerical tools. Although the overall
system in terms of total cost and energy production is the whole wind farm,
the studies are limited to a single turbine in the present work, and farm-level
effects are not considered.

1.2 Research objectives

The overall aim of the thesis is to improve the efficiency and reliability of
the design process for FWTs, especially catenary-moored spar-type turbines,
and develop methodologies to identify novel and optimized design solutions.
This is addressed by defining two separate research objectives.

During the design process, a numerical model of the FWT system is created
and analysed. The global dynamic response is evaluated using fully coupled
aero-hydro-servo-elastic analyses, considering combined wind-wave loading.
The computational costs associated with such analyses are high, and due to
the variety of design situations and environmental conditions that a FWT
may encounter during its lifetime, a large number of load simulations must
be carried out. There is therefore a need for more efficient analysis tools,
especially during the conceptual and preliminary design phases. While sim-
plified simulation models may not be able to provide accurate response
predictions for all relevant design situations, they should capture important
interactions and response characteristics in the system, and provide consist-
ent design load estimates in comparisons with high-fidelity simulations for
applicable loading conditions.

The first research objective of this thesis is related to reduction of the com-
putational burden associated with global design analyses of FWTs, and is
formulated as:

O1 Suggest and verify methods to increase the computational efficiency
of global design analyses for floating wind turbines.

Due to the complex and multidisciplinary dynamic system, with a large
number of parameters and load cases, the design space cannot be efficiently
explored through traditional design procedures. The important couplings
in the system call for integrated optimization, which further complicates
the design problem. Numerical optimization techniques may help develop
improved and possibly novel solutions, but have seen limited use for FWT structures. Consequently, there is currently no software or framework available for efficient FWT optimization, and there are uncertainties related to the formulation of the optimization problem. Design methodologies with improved accuracy and efficiency, which can help identify innovative and optimized solutions, are expected to result in cost reductions and increased reliability, and can help floating wind technology become commercially feasible.

The second research objective addresses the potential for design improvements through numerical optimization, and is formulated as:

**O2** Develop methodologies for numerical design optimization of floating wind turbine systems, considering ultimate strength and fatigue, and identify possible cost-effective and reliable design solutions.

The two research objectives are connected because contributions related to **O1** can be implemented in, and thus improve the feasibility of, models developed in relation to **O2**. The objectives are achieved using numerical simulations, which are performed for a 10 MW wind turbine rotor mounted on a floating support structure\(^1\). Because a limited amount of available and relevant measurement data from full- or model-scale FWTs exists, results from state-of-the-art numerical simulations are used as benchmark for the models and methods developed over the course of the thesis.

### 1.3 Main contributions

Based on the research objectives defined in Section 1.2, several contributions to the scientific community have been made. The main contributions are further discussed in Chapter 4, and briefly summarized in the following. The relations between research objectives, papers, and contributions are illustrated in Fig. 1.3.

- **C1** A linearized aero-hydro-servo-elastic model for efficient global response analyses of FWTs, extensively verified against state-of-the-art simulations with regards to both fatigue damage and extreme response.

- **C2** Derivation of closed-form expressions for frequency-dependent aerodynamic damping and inertia, and an assessment of their impact on frequency-domain calculations.

\(^1\)The term *support structure* corresponds to the combined tower and platform structure.
**C3** A methodology for integrated design optimization of spar FWTs, considering simultaneous design of platform, tower, controller, and mooring system, using gradient-based optimization with analytic derivatives.

**C4** Evaluation and comparison of different control strategies for FWTs, based on integrated optimization of support structure and blade-pitch controller with realistic design limits.

**C5** Quantification of the impact of environmental modelling on the long-term fatigue reliability and costs of FWT support structures.

**C6** Assessment of trade-off effects between CAPEX and OPEX, and derivation of cost-optimal fatigue safety factors, considering reliability-based inspection planning.

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**Figure 1.3:** Relations between research objectives, papers, and contributions.
Chapter 2

Literature Survey

This chapter summarizes previous work relevant for global response analyses and numerical design optimization of FWT structures. Methods used in state-of-the-art and simplified dynamic simulations are described, some basic optimization theory is given, and specific aspects and challenges related to design optimization of FWTs are discussed.

2.1 Global response analyses

2.1.1 Environmental modelling

FWTs are subjected to stochastic environmental loading, which require a proper statistical description for evaluation of lifetime loads in design. Temporal environmental variations occur at many different time scales, which in engineering applications are separated into short-term and long-term variability. In short-term conditions, the wind and wave processes are considered stationary Gaussian stochastic processes, and are generally assumed to be uncorrelated (DNV GL 2016). Short-term variations in wind speed and wave elevation are commonly given in terms of their power spectral densities, which describe the distribution of energy at different frequencies. For wind, a description of the spatial coherence is also needed. The long-term statistics consider the variation of the statistical properties describing the short-term process, and describe the correlation between wind and waves through a joint probability distribution.

The joint distribution of wind and waves is ideally fitted to measured data from the actual site over a long period of time, and is used to establish scatter diagrams for fatigue calculations, or identify environmental conditions.
with a certain return period. As a minimum, the environmental model should include distributions for the mean wind speed \((U)\) at a reference height, the significant wave height \((H_s)\), and the spectral peak period \((T_p)\). A joint distribution based on measurements from the Northern North Sea was presented by Johannessen et al. (2002), using the marginal distribution of the 1-h mean wind speed at 10 m \((U_{10})\), and conditional distributions of \(H_s\) and \(T_p\):

\[
f_{U_{10},H_s,T_p}(u,h,t) = f_{U_{10}}(u)f_{H_s|U_{10}}(h|u)f_{T_p|U_{10},H_s}(t|u,h). \tag{2.1}
\]

It is also of interest to include directional data in the environmental model. Waves are typically short-crested, which requires a spreading function in addition to the main propagation direction. Also, the wind and waves are in general misaligned, and larger misalignment angles typically occur at smaller wind speeds (Fischer et al. 2011). This is illustrated in Fig. 2.1, where the wind-wave misalignment distribution, based on 60 years of hindcast data from a location in the North Sea, is shown for two wind speed bins. While relative wind-wave directions of more than 90° are quite rare for \(U_{10} > 10\) m/s, larger misalignment angles occur more than 25 % of the time for the lower wind speed bin. The relative wind-wave direction may have a large effect on the response of the structure, and is of particular importance for fatigue design (DNV GL 2018a). While misaligned wind and waves tend to increase the tower fatigue damage for bottom-fixed turbines, the opposite has been observed for FWTs (Barj et al. 2014, Bachynski et al. 2014).

The wind speed in a given point in space consists of longitudinal, lateral,
and vertical components, where the longitudinal component varies around the mean wind speed, and the other two components typically are assumed to have zero mean. Short-term variations in the wind speed are known as turbulence, which are described by so-called turbulence spectra. A measure of the overall level of turbulence is given by the turbulence intensity, which is defined as the ratio between the standard deviation of the wind speed and its mean. In general, the turbulence intensity takes values between 0.1 and 0.4, where higher values occur for lower wind speeds (Manwell et al. 2009). If site-specific data is unavailable, values for the turbulence intensity can be taken from design standards. The IEC Class B normal turbulence model (NTM) (IEC 2005), which is applicable for moderate turbulence levels, is used in the present work.

In addition to temporal variations, the turbulence also varies spatially, which is described by the cross-correlations between fluctuations at points separated laterally and vertically (Burton et al. 2011). The resulting turbulent wind field is assumed to follow Taylor’s frozen turbulence hypothesis (Taylor 1938), where the turbulence is considered as time-invariant eddies moving forward with the mean wind speed. To generate the three-dimensional turbulence box in time domain, several methods are available. A one-dimensional fast Fourier Transform (FFT) can be applied, based on spectra, to each of the three turbulence components independently, together with a coherence function for the spatial cross-correlations. Alternatively, the Mann model can be applied, where correlated components are generated using a three-dimensional FFT (Burton et al. 2011). In the present thesis, the Kaimal spectrum and an exponential coherence function are used to describe the turbulent wind field (IEC 2005).

The mean wind speed also increases with the height above ground. This variation is known as shear, and is commonly modelled using either a logarithmic or power law. The present work uses a power law with exponent 0.14, which is recommended for offshore locations (IEC 2009).

The physics of ocean waves are highly complex; however, for engineering applications, several simplifications can be made while still maintaining reasonable estimates of the wave dynamics. The waves are typically described using linear (Airy) wave theory, which assumes potential (irrotational, inviscid, and incompressible) flow (Faltinsen 1990). The irregular wave elevation process can then be generated from linear superposition of a large number of independent wave components with different frequencies, where the amplitude of each component is found from the wave spectrum.
The wave conditions in a sea state can be separated into two categories: wind seas, which are generated by local wind, and swell, which are waves that have travelled out of the areas where they were generated (DNV GL 2019a). For wind seas, the Pierson–Moskowitz (PM) and JONSWAP spectra are frequently applied (DNV GL 2019a). The JONSWAP spectrum, which was developed for North Sea conditions, is used in the present work.

If the sea state is expected to contain swell components in addition to wind-generated waves, these must also be properly represented (DNV GL 2018a). This can be done through two-peaked spectra, such as the Torsethaugen model (Torsethaugen 1996, Torsethaugen and Haver 2004), which take into account both wind and swell sea. Alternatively, the wind and swell components can be separated and described using two JONSWAP spectra, possibly with different directions, which are superimposed to represent the total wave elevation process (DNV GL 2018a).

### 2.1.2 State-of-the-art simulations

Due to important interactions between aerodynamics, hydrodynamics, structural dynamics, and control dynamics, fully coupled aero-hydro-servo-elastic models are needed to properly simulate the global response of FWTs. Different aspects and interactions considered in such analyses are illustrated in Fig. 2.2. Over the last decade, several fully coupled simulation tools for global analysis of FWTs have been developed, often based on existing codes for either conventional offshore structures or land-based wind turbines. The present thesis uses the SIMA workbench developed by SINTEF Ocean, which couples the RIFLEX and SIMO programs (SINTEF Ocean 2016a;b). A brief summary of common structural, aerodynamic, hydrodynamic, and control capabilities in state-of-the-art analysis tools is given in the following.

The structural dynamics of FWTs are usually modelled using multibody dynamics, modal analysis, finite elements, or a combination of these (Robertson et al. 2014). While modal analysis is computationally efficient, finite elements offer the possibility of geometric and material nonlinearities, and do not require pre-calculation of the system modes. Most finite element models for FWTs are limited to beam elements, which are applicable for long, slender structures such as the tower and blades, whereas the large volume platform typically is considered rigid. The drivetrain is usually modelled using a multibody dynamics approach, where the shaft flexibility often is included through a torsional spring-damper to capture the first torsional mode of the drivetrain (Smilden 2019). The mooring lines are either
represented using nonlinear force-displacement relationships found from a quasi-static analysis, or modelled explicitly using finite elements or lumped masses. While the latter approach is computationally more expensive, it captures the mooring line dynamics, and the effects of mooring line inertia and damping on the platform response. The importance of mooring line dynamics is highly dependent on the considered platform concept and response parameters, but should be included for designs with natural frequencies inside the wave-frequency range and fairleads far away from the platform’s center of mass (Hall et al. 2014b), or if extreme mooring line tension is considered (Azcona et al. 2017).

Various methods exists for calculation of aerodynamic loads on wind turbines. For global response analyses of FWTs, the aerodynamic loads on an operational turbine are commonly computed using blade element momentum (BEM) theory, due to its computational efficiency. Here, the aerodynamic lift and drag forces on the blades are balanced by the change in axial and angular momentum of the air passing through the swept annulus (Burton et al. 2011). However, BEM requires several engineering corrections to give satisfactory results, where the most commonly applied are the Glauert correction for high induction factors, and the Prandtl corrections for tip and hub loss factors. Further, additional corrections are often imposed to account for skewed inflow conditions, dynamic wake, and dynamic stall effects. The generalized dynamic wake (GDW) method is an alternative to BEM, which is based on the potential flow solution of the Laplace equation. GDW inherently includes skewed wake, tip losses, and dynamic wake effects, but is only valid for lightly loaded rotors (high wind speeds), and does not account for wake rotation (Moriarty and Hansen 2005).

Hydrodynamic loads on large volume floating structures are commonly computed from potential flow theory, where the diffraction and radiation loads are found numerically using a panel method. For many structures, the first order solution, which assumes small wave amplitudes and small motions around the equilibrium position, is sufficient to capture the dominant wave loads (Faltinsen 1990). The outputs from the hydrodynamic analysis are frequency-dependent wave excitation, added mass, and wave radiation damping, which are transformed to the time domain before they are applied in the simulations. For structures with natural frequencies outside the wave-frequency range, higher-order wave loads may also be of importance. Second-order loads from potential flow theory, which consist of sum- and difference-frequency terms, can then also be included through so-called quadratic transfer functions (QTFs). To capture important viscous effects,
the quadratic drag term in Morison’s equation is often superimposed to the
potential flow solution in the aero-hydro-servo-elastic simulations.

The majority of modern horizontal axis wind turbines are regulated using variable-speed variable-pitch control. Below the rated wind speed, the generator torque, and consequently the rotational speed of the turbine, is varied to maximize the power output. Above the rated wind speed, the blades are pitched to reduce the structural loads and maintain a constant rotational speed, while the generator torque is kept constant or varied to maintain a constant (rated) power output. While the controllers in below- and above-rated conditions operate independently, they are typically combined in near-rated conditions to limit the tip speed at rated power (Jonkman et al. 2009). For wind turbine analysis tools, control system models should contain, as a minimum, routines for the blade-pitch and generator torque control, in addition to start-up and shut-down procedures (Smilden 2019).

Despite their extensive use in analysis and design, the accuracy of aero-hydro-servo-elastic simulation codes is still somewhat uncertain. As identification of limitations and sources of error in the theoretical models is crucial to reduce safety factors and achieve safe and efficient design solutions, verification and validation of numerical design tools are important. Extensive code-to-code verification studies have been conducted through the OC3, OC4, and OC5 projects (Jonkman and Musial 2010, Popko et al. 2012, Robertson et al. 2014; 2017, Popko et al. 2018; 2019), which have contributed to an understanding of the applicability of different modelling approaches, helped resolve errors in the codes, and identified future improvement needs (Robertson et al. 2014). The recent OC5 project has also considered validation against experiments (Robertson et al. 2017) and full-scale data (Popko et al. 2019).

Limited full-scale measurements from FWTs exist, with very few results being publicly available, and validation studies must therefore to a large degree rely on model-scale experiments. Model tests which consider combined wind-wave response are challenging due to scaling issues. Recently, hybrid testing, where parts of the load picture are calculated numerically and applied on the experimental model using actuators, has shown promise. A real-time hybrid testing method for FWTs has been developed by SINTEF Ocean (Sauder et al. 2016, Bachynski et al. 2016, Berthelsen et al. 2016), where wave basin tests are conducted with aerodynamic loads from simultaneous numerical simulations. A complimentary approach using wind tunnel tests with numerically computed hydrodynamic loads has also been
2.1. Global response analyses

Waves | Platform | Mooring system | Tower | Nacelle | Rotor-nacelle assembly | Rotor | Drive-train | Grid | Control system

Figure 2.2: Aspects of global aero-hydro-servo-elastic analyses for floating wind turbines. Based on a figure presented by Vorpahl et al. (2013).

developed at the Politecnico di Milano (Bayati et al. 2017; 2018).

2.1.3 Linear models

Although FWTs are nonlinear systems, linearized formulations are desirable for several reasons. In addition to the computational efficiency, methods based on linear models are extensively used for analysis of eigenvalues, stability, and control systems, and may provide increased insight in the properties of the underlying system. Because linearity often is an accurate assumption for small displacements, even for nonlinear structures, such models are regarded as suitable for fatigue. Assuming harmonic response, the linear model can be solved very efficiently in the frequency domain. This is also preferable for gradient-based optimization, since the stochastic nature of time-domain results, which depend on the random realization of the environment, complicates the computation of accurate gradients (Zahle
Frequency-domain models have a long history for offshore structures, which is reflected in the early efforts on frequency-domain modelling of FWTs (Wayman et al. 2006, Tracy 2007, Sclavounos et al. 2008, Philippe et al. 2011), where response amplitude operators (RAOs) were established based on diffraction and radiation loads from an external potential flow solver. As hydrodynamic loads from numerical panel codes are computed in the frequency domain, they can be readily applied in such global response models without modification. A drawback with this approach in a design process is that it requires an additional hydrodynamic analysis each time changes are made to the design, which must be run prior to the response analysis, and prevents quick examination of different design solutions. More computationally efficient options include Morison’s equation and MacCamy–Fuchs theory; however, the validity of these methods is limited to simple geometries.

While early studies on frequency-domain methods for FWTs considered only rigid-body modes, Bachynski and Moan (2012) compared a frequency-domain model for a tension-leg platform (TLP) wind turbine with nonlinear time domain simulations, and found that the linear model was insufficient for design calculations, partly due to the rigid modelling of the tower. The effect of tower flexibility on the global response was confirmed by full-scale measurements from the Hywind Demo FWT, where the tower bending moment spectra clearly showed peaks around the first bending mode (Skaare et al. 2015). Kvittem and Moan (2014) used a frequency-domain model, which included the first tower bending mode, to evaluate the fatigue loads for a semi-submersible FWT in stochastic wind-wave conditions, where the shape of the bending mode was found from decay tests in calm air with a coupled time-domain model. A stochastic linearization procedure, considering the two first tower modes, was presented by Kluger et al. (2016), who computed the modes from the eigenvalue problem with an isolated tower.

Although more recent efforts on frequency-domain modelling have included tower modes, platform flexibility has been little explored. As turbines grow larger, support structure flexibility is expected to become more important, and may need to also include the platform to properly capture the correct global behaviour.

Due to the relatively complex aerodynamic calculations, and because the design of the rotor usually is separated from the support structure design, frequency-domain methods have typically focused on decoupling of the rotor dynamics. Early work on FWTs considered only stochastic response due to
waves, while the aerodynamic excitation forces typically were treated as a constant thrust force on the tower top. However, to properly capture the combined wind-wave response, turbulent wind excitation loads are needed, which can be estimated in several ways (Muskulus 2015). A method based on turbulent wind simulations in the time domain, with a fixed rotor and an active controller, was presented by Bachynski (2014), who compared results from linear and nonlinear analyses on a TLP wind turbine. The approach gave reasonably accurate estimates for the low-frequency wind response in certain environmental conditions, but the agreement was highly dependent on the damping in the model.

Calculation of frequency-dependent aerodynamic loads based on time series has some drawbacks. Firstly, the turbulent wind loads are stochastic, meaning that a large number of time-domain simulations may be needed. Secondly, the low-frequency response is sensitive to how the thrust force spectrum is estimated from the time series (Kvittem and Moan 2014). In addition, using pre-calculated aerodynamic loads means that the controller cannot be modified without re-running the time domain simulations. Another approach, which allowed for simultaneous design of the controller, was taken by Sandner et al. (2014), who computed linearized aerodynamic loads on the tower top as a function of wind speed, rotor speed, and blade pitch angle. The formulations were based on a Taylor expansion, using pre-computed look-up tables for the thrust and power coefficients, and a rotor effective wind speed. More sophisticated linear models, involving blade elasticity, unsteady aerodynamics, and hundreds of degrees-of-freedom (DOFs), have previously been developed for wind turbine analysis (Merz 2015, Tibaldi et al. 2016), and more recently, full-system linearization of FWT models based on the OpenFAST software has been presented by Jonkman et al. (2018). The implementation has later been verified against nonlinear simulations from OpenFAST by Johnson et al. (2019).

For support structure models with decoupled rotor dynamics, the change in aerodynamic forces due to the motions of the turbine should also be included. The most important component is the aerodynamic damping, which typically has been represented using a linear dashpot at the tower top, and a number of methods has been applied to calculate an appropriate damping coefficient. Early studies on frequency-domain methods for offshore wind turbines used a value of 4 % of critical based on engineering estimate (van der Tempel 2006). Bachynski (2014) derived a mean wind speed-dependent value from the change in thrust for a small change in wind speed, assuming that the rotor speed and blade pitch angle were
kept unchanged. However, due to interaction with the controller and rotor dynamics, the damping is frequency-dependent, and neglecting these interaction effects leads to non-conservative damping estimates (Jonkman 2008). As damping is most important for the response around resonance, individual values for each mode can be found from decay tests with steady wind and an active controller (Jurado et al. 2018). Model tests have shown that there also is a frequency-dependent inertia (or stiffness) effect from the controller, which results in changes in the pitch natural frequency for the system (Bachynski et al. 2016, Goupee et al. 2017). Souza and Bachynski (2019) computed frequency-dependent inertia and damping coefficients for the NREL 5MW turbine numerically, based on forced harmonic oscillations of the nacelle, and were able to accurately predict pitch decay periods for three different FWTs, using a simplified 2-DOF model with modified inertia and damping matrices.

As linearized models assume small displacements, they are seen as unsuitable for extreme response. However, results from previous studies have shown that both the tower base bending moment (Karimirad and Moan 2011) and global motions (Aggarwal et al. 2017) for spar FWTs can be quite Gaussian in harsh environmental conditions, which suggests that a linear model also may be used to assess the extreme response in early stages of design, or reduce the computational burden by quickly identifying critical load cases. The degree of nonlinearity depends on the considered response parameter, and also varies with location, floater geometry, environmental conditions, and design situations. Therefore, the results obtained with a linear model cannot be generalized to any FWT structure, and such models must be continuously verified against more sophisticated tools during a design process.

2.1.4 Structural reliability

In the design of FWTs, stochastic variables such as turbulence intensity, wind-wave directionality, and fatigue strength are typically replaced by design values, which are obtained from their characteristic values combined with a given safety factor. In order to reduce conservatism and thus the design costs caused by these safety factors, probabilistic analyses can be performed to assess the safety level of the structure through the probability of failure. Such analyses can also be used to evaluate the importance of different model uncertainties on the resulting structural reliability, and possibly calibrate appropriate safety factors, which has been the focus of several studies for bottom-fixed wind turbines. Márquez-Domínguez and Sørensen (2012) calibrated design fatigue factors (DFFs) for a 2.3 MW off-
shore wind turbine, and investigated the effect of inspections on the required DFF values. Horn et al. (2019) quantified the effect of environmental load uncertainties on the fatigue reliability of a 10 MW monopile, and derived corresponding differences in DFFs resulting from the increased model fidelity. Velarde et al. (2020) designed a 10 MW monopile with different DFFs by varying the wall thickness, and used reliability analyses to identify the necessary safety factor to achieve the target probability of failure during a service life of 25 years without inspections.

Structural reliability problems consider the probability of failure for the system, which can be expressed as

\[
P_F = P[g(x) \leq 0] = \int_{g(x) \leq 0} f_X(x) \, dx, \tag{2.2}
\]

where \( X \in \mathbb{R}^n \) is a random vector with joint probability density function (PDF) \( f_X(x) \), which represents the uncertainties in the system. \( g(x) \) is commonly known as the limit state function, where \( g(x) \leq 0 \) denotes the failure domain.

The evaluation of this integral must typically be done numerically, commonly through Monte Carlo simulations or the first- or second-order reliability methods (FORM, SORM) (Madsen et al. 1986). In the widely used and computationally efficient FORM, \( X \) is transformed into a vector of independent standard normal variables \( (U) \), and the limit state function is linearized about the most probable point, which is the point on \( g(u) = 0 \) closest to the origin. The reliability index, \( \beta \), which is the distance from the origin to the most probable point, can be used to calculate the failure probability through the following relation:

\[
P_F = \Phi(-\beta), \tag{2.3}
\]

where \( \Phi \) is the standard normal cumulative distribution function (CDF).

### 2.2 Design optimization

#### 2.2.1 Basic optimization theory

An optimization problem can be formally written as (Nocedal and Wright 2006):

\[
\begin{align*}
\text{minimize} \quad & f(x) \\
\text{with respect to} \quad & x \in \mathbb{R}^n \\
\text{subject to} \quad & \hat{c}_j(x) = 0, \quad j = 1, 2, \ldots, \hat{m} \\
& c_k(x) \geq 0, \quad k = 1, 2, \ldots, m
\end{align*}
\tag{2.4}
\]
where \( f \) is the objective function, \( x \) is a vector of design variables, \( \hat{c} \) is a vector of equality constraints, and \( c \) is a vector of inequality constraints.

In general, two approaches can be used to solve the optimization problem, namely gradient-free and gradient-based methods (Martins and Ning 2020). Gradient-free methods are relatively easy to combine with existing analysis tools, and do not require differentiability of the simulation outputs. However, as illustrated in Fig. 2.3, these methods scale poorly with the number of design variables, and become infeasible for large-scale optimization problems. The scaling properties are particularly important when expensive analyses are needed to evaluate the objective function or constraints.

Gradient-based methods require derivatives to determine the search direction, which can be acquired in different ways. Numerical gradients are commonly found using either finite-differences or the complex-step method. While finite difference formulas are easier to implement and do not require access to the source code, the complex step approximation is more accurate, as it is not susceptible to subtractive errors (Martins and Ning 2020). Both methods, however, scale linearly with the number of design variables, and thus become prohibitively costly as the problem size increases.

The most preferable way of obtaining gradients is to compute the derivatives analytically. This method requires significant implementation effort, as modification of the specific components of the source code is needed, but offers large improvements in efficiency. Analytic gradients can be computed using either the direct method or the adjoint method, where the preferred approach depends on the problem. The adjoint method, whose cost is independent of the number of design variables, tends to be the most commonly used approach in practice (Gray et al. 2019). This method is further described in Section 3.8.

A function can be either unimodal, meaning that it only has a single minimum, or multimodal, which implies that several minima exist. Unless an optimization problem is known to be convex, it is usually difficult to prove that a global minimum has been found (Nocedal and Wright 2006). However, to prove multimodality, it is sufficient to find two separate local minima. Therefore, Martins (2020) argues that ‘an optimum should be assumed to be the global one until proven otherwise’. This requires that an effort to search for additional minima has been made by performing the optimization with a number of different initial designs.

For certain problems, it may be desirable to minimize several performance
2.2. Design optimization

Figure 2.3: Comparison of required function evaluations until convergence to the optimum using a gradient-free algorithm, finite differences, and analytic derivatives for a multidimensional generalization of the Rosenbrock function (Ning and Petch 2016).

measures. This is known as a multi-objective problem, and may be useful to quantify trade-offs between different parameters, or to produce a number of possible designs when a final decision must be made during a later design stage (Martins and Ning 2020). For these problems, there will no longer be a single optimal solution, but an infinite number of solutions which forms a surface in the objective function space. Multi-objective optimization therefore uses the term Pareto optimality, where a design is considered Pareto optimal if it is impossible to improve the performance in one objective, without worsening the performance in another (Boyd and Vandenberghe 2004).

2.2.2 Offshore wind substructures

Most of the work on numerical design optimization of offshore wind substructures has considered bottom-fixed turbines. The focus has mainly been on jacket structures, because of the complex geometry and large number of potential design variables, which complicates a manual design process, and increases the potential for cost savings by automation. Hydrodynamic shape optimization of floating structures has a long history in the offshore community, using RAOs to identify designs with favourable response to wave loads. An early study on numerical hydrodynamic shape optimization was performed by Clauss and Birk (1996), where various offshore structures were designed to minimize forces or motions, based on a panel
code and a gradient-free optimizer, and in recent years, similar techniques have also been applied on FWT structures (Hall et al. 2013, Steinert et al. 2016, Karimi et al. 2017). Most of the work has considered linear frequency-domain models to limit the computational effort, with an exception of Myhr and Nygaard (2012), who optimized a floating substructure for two extreme load cases using nonlinear time-domain simulations. Due to the computational effort, such optimization studies have rarely included more than two or three design load cases, and the resulting optimized designs may therefore be highly dependent on the chosen environmental conditions. In addition, the applied numerical models may also impact the results. Hall et al. (2013) performed single- and multi-objective hull shape and mooring line optimization for different platform concepts spanning all stability classes, and found that the Pareto front was dominated by relatively complex platforms with several legs and taut mooring lines. The work was later extended by Karimi et al. (2017), using an updated dynamic FWT model and a new optimization algorithm, which significantly affected the derived optimal solutions.

One of the challenges related to design optimization of substructures for floating wind is the parametrization of the geometry. Several platform concepts, with significantly different properties, have been suggested, and there is currently no consensus on the optimal platform type. An illustration of three main classes of FWTs is shown in Fig. 2.4. The derivation of a general formulation that accurately can describe all platform types, including different stability classes, various mooring system configurations, and both single- and multi-hull platforms, possibly connected by braces or pontoons, is a highly involved and difficult task. Consequently, the main properties of the system, including the general shape of the platform, are typically chosen before the optimization problem is formulated, which limits the design space and reduces the chance of obtaining truly novel design solutions. Hall et al. (2014a) tried to overcome this issue by introducing a basis function approach, where the platform design was expressed as a combination of basic geometries with pre-calculated hydrodynamic coefficients. The design space had numerous local minima, and the resulting optimized design was thus highly dependent on the initial starting point. Also, some issues arose when physical geometries was created based on optimal combinations of basis designs, as the resulting hydrodynamic behaviour tended to differ from the superimposed case, due to nonlinearities and interaction effects. While such generalizations can remove the potential bias towards certain concepts and allow for a more direct comparison between different solutions, it remains uncertain whether they are able to fully represent the relevant physics for
2.2. Design optimization

Figure 2.4: Semi-submersible, catenary moored spar, and tension-leg platform wind turbine concepts (Bachynski 2014).

all platform types, or if more concept-specific formulations are needed.

Several studies have considered shape optimization of FWT platforms, but little exists in the literature concerning structural optimization. The hydrodynamic loads on the hull are typically found from a separate hydrodynamic analysis and given as resultant point loads, and complicates a structural optimization procedure based on global response analyses, which requires distributed hydrodynamic loads combined with a formulation for the scantling design criteria. While comprehensive finite element models may be necessary for the detailed structural design of the hull, design criteria based on global forces and moments, such as the formulas provided by DNV GL (2019b), may be used for structural sizing in early phases of design.

2.2.3 Integrated design

Common to previous studies on FWT design optimization is that they consider the design of the platform, mooring system, or both, whereas the tower, rotor, and control system are considered fixed during the optimization. A possible reason is that design of wind turbines and offshore structures, which traditionally have been two completely separated fields, is performed by designers with specific engineering backgrounds. In addition, different companies are responsible for the design of different components. This may lead to partly integrated procedures where loads are exchanged between the support structure designer and the wind turbine manufacturer at a pre-defined interface (Vorpahl et al. 2013). A fully integrated design approach, on the other hand, may be practically impossible in reality, due
to confidentiality issues between the companies. However, because FWTs are multidisciplinary systems, with important interactions between the different subsystems and disciplines, optimizing components separately will in general lead to suboptimal, and in many cases infeasible, solutions on the overall system level (Muskulus and Schafhirt 2014). To find the true optimum, an integrated optimization approach should be applied, where all relevant components are considered simultaneously.

Despite gradient-based optimization being desirable due to its efficiency, a large majority of the work on FWT design optimization has been performed using gradient-free methods. Typical analysis tools for FWTs have not been developed for optimization purposes, which require differentiability, and in the case of analytic gradients, substantial modifications to the source code. One exception is Fylling and Berthelsen (2011), who optimized the hull shape, mooring lines and power cable for a spar FWT using a gradient-based approach with numerical gradients obtained from finite differences. While these methods may be feasible for optimization of smaller parts of the FWT system with a limited amount of design variables and load cases, integrated design optimization considering lifetime loads would result in a large number of input variables and expensive function evaluations, which requires analytic derivatives.

As there are important interactions between the control dynamics and the support structure response, the control system should preferably be integrated in the design optimization procedure. Numerical optimization of a land-based wind turbine controller has earlier been performed by Tibaldi et al. (2014), but has rarely been considered simultaneously to support structure optimization, which typically decouple the rotor dynamics. Sandner et al. (2014) manually derived optimal blade-pitch controller gains for three different spar FWT designs, where the goal was to minimize a weighted combination of rotor speed variation and tower bending. The gains that minimized the cost function was shown to be highly dependent on the platform design. A more automated approach was presented by Lemmer et al. (2017), who optimized four parameters for a FWT substructure to minimize a combination of material cost and damage-equivalent loads, where the blade-pitch controller was tuned in each design iteration using a linear quadratic regulator (LQR). However, the effect of including control system tuning in the substructure optimization loop was not quantified.

Perhaps the most important interaction between the controller and support structure response is the potential introduction of negative damping for the platform motions above rated wind speed (Larsen and Hanson 2007). Sev-
eral methods have been suggested to resolve the issue, such as detuning the controller gains (Larsen and Hanson 2007, Jonkman 2008), or to introduce a feedback term proportional to the pitch velocity (Lackner 2009) or nacelle velocity (van der Veen et al. 2012, Fleming et al. 2014) to manipulate the rotor speed reference. For the latter approach, Fleming et al. (2019) suggested that such controllers should not include the wave-frequency range within their bandwidth, as doing so yielded an increase in the loads around these frequencies. All methods lead to trade-offs between structural loads, rotor speed tracking, and blade-pitch actuator use, and an integrated control and structural design optimization approach is therefore needed to properly evaluate and compare different solutions.

The shapes and natural frequencies of the tower modes, and thus the tower response, are dependent on the platform design. In addition, as turbines grow larger, the flexibility of the platform may become important, which also can affect the response of the tower. In Fig. 2.5, the first bending mode for a 10 MW spar is shown for both a rigid and flexible platform. In addition to the modal shape being clearly affected by the platform flexibility, a 17% decrease in the natural frequency is observed. A similar effect was seen for the OO-Star 10 MW semi-submersible, where the natural frequency of the first tower mode was reduced by 25% when the flexibility of the floater was included (Müller et al. 2018). As the first bending mode of FWT towers typically is located close to the 3P range, an accurate estimation of the natural frequency is important to capture the correct modal excitation forces. Souza and Bachynski (2020) studied the effect of flexible pontoons for a 5 MW TLP wind turbine, and found that the flexible platform resulted in a 12.7% reduction in the weighted 1-h fatigue damage at the tower base, due to the reduced natural frequency of the tower mode, which resulted in less 3P excitation.

Commonly, wind turbine towers are designed with linearly tapered diameter and wall thickness (Jonkman et al. 2009, Bak et al. 2013, Müller et al. 2018), and are thus fully described by the values at the base and top. However, the load picture for a FWT tower alters significantly from a land-based or bottom-fixed offshore tower, with inertial and gravitational loads arising from large platform motions, as well as possible restrictions at the tower base due to compliance with the platform. An integrated design process which considers the actual loads on the FWT system, and consistent reliability levels, may therefore result in cost reductions.

Some effort has been made to perform integrated design optimization of the blades and tower for onshore (Ning and Petch 2016) and bottom-fixed off-
Figure 2.5: First support structure bending mode for a 10 MW spar with flexible and rigid platform, and associated natural periods.
shore (Ashuri et al. 2014) wind turbines, which experience interaction effects due to coupled eigenmodes, increased blade loads from tower shadow effects, and the possibility of tower strike by the blade tips. It is expected that the inclusion of blade design in an integrated optimization process will affect the results also for FWTs. However, design optimization of wind turbine blades is a separate body of literature, which requires high-fidelity aerodynamic and structural formulations to properly capture the blade response, and is therefore considered outside the scope of this thesis.

2.2.4 Formulation of objective and constraints

The success of a numerical optimization process depends heavily on the formulation of objective function and constraints for the design problem. Because the goodness of a design is measured purely through the value of a scalar objective function, it is essential that the applied function reflects a relevant performance measure. The constraints should ideally cover all important limit states, in addition to any restrictions that may be necessary to avoid non-physical or impractical solutions. However, the goal of an optimization process is not necessarily to provide a ready-to-build design, but may also be used to study the sensitivity of different input parameters, or identify design trends that are non-intuitive for a human designer.

A common objective function in design optimization of offshore wind substructures is the structural weight (Pasamontes et al. 2014, Chew et al. 2016, Oest et al. 2017), which is assumed to be closely related to the cost of the design. The weight is a convenient cost parameter in an engineering optimization problem, because it is easily calculated with small uncertainty. However, the weight, or associated material costs, may not be a good measure of the actual cost optimality, and even in the case where only capital costs are considered, manufacturing costs due to e.g. welding of joints should be included (Muskulus and Schafhirt 2014). The manufacturing costs are more uncertain, and are typically expressed as empirically fitted functions of the structural dimensions. Cost models for manufacturing of general steel structures are presented e.g. by Farkas and Jármai (2013); however, their accuracy with respect to FWT structures have not been verified.

Operational costs are rarely considered in design optimization of offshore wind turbines. For conventional offshore structures, the focus has been on inspection planning, where a cost optimal schedule for crack inspection and repair is sought. Madsen et al. (1989) presented an early formulation of an optimization problem which minimized expected costs related to construction, inspection, repair, and failure, where a set of structural design para-
meters, as well as inspection times and qualities, were included as design variables, and a constraint was placed on the reliability index. A small example was solved using gradient-based optimization for the outer problem, combined with FORM for the inner reliability problem. Such approaches may lead to more cost-effective solutions in a lifetime perspective, but increase the complexity of the design problem considerably. In addition, little information exists in the literature regarding costs related to inspection and repair of offshore structural elements, which vary with the inspection technique, location of potential cracks, and extent of the repairs.

An important difference in design optimization of FWTs, compared to conventional offshore structures, is that it is not the cost of the system which should be minimized, but rather the cost of energy. The energy production should therefore be taken into account in the optimization process. Studies, e.g. by Roddier et al. (2017), have suggested that the movement of the platform has a small effect on the total energy production, which is mostly related to the wind farm site. However, the energy quality, which is a measure of how much the energy output fluctuates, is also a relevant parameter. High-quality power, which has low variability in the output, is desirable, and its value is closely related to the rotor speed tracking performance of the system. The power quality may be regarded as an SLS criterion, and thus be constrained based on system specifications, or treated as a performance measure that should be maximized. Because the monetary cost of reduced power quality is difficult to quantify, multi-objective optimization may be needed to take this parameter into account. Ideally, an optimization process should also consider the power quality for the complete wind farm, which may result in different solutions compared to a single turbine.

A commonly constrained response parameter in FWT design optimization is the platform tilt angle, which should be limited due to stability requirements and SLS criteria for the rotor and nacelle assembly (RNA). Such criteria are project-specific, and there is currently no consensus on how the tilt angle should be constrained; both the static and dynamic angle have been considered in the literature, and different values have been applied. The dynamic tilt angle constraint has commonly been expressed using the standard deviation, which is easily obtained from the response spectrum for a frequency-domain analysis. Typical values for the total angle have been 10° (Hall et al. 2013, Karimi et al. 2017, Leimeister et al. 2020) or 15° (Steinert et al. 2016), with possibly different values in operational and survival conditions (Ramachandran et al. 2017).

The constraint on the maximum static tilt angle is mostly related to the
2.2. Design optimization

Figure 2.6: Side view of the wind speed for a spar FWT without tilt (left) and with a tilt angle of 10° (right). The black dots represent the vertical wake center position (Wise and Bachynski 2020).

reduction in projected rotor area and consequently the resulting power production. The use of a static tilt angle has computational benefits, because it can be computed without the need of simulations, using the platform restoring stiffness and the specified rated thrust for the turbine. Common constraint values in the literature are 5° (Matha et al. 2015, Steinert et al. 2016) or 10° (Wayman et al. 2006, Tracy 2007, Hall et al. 2013, Karimi et al. 2017), which, given that the produced energy is proportional to the cosine of the tilt angle to the power three (Matha et al. 2015), result in energy losses of about 1 % and 4.5 %, respectively. Increased stability is likely to translate into larger design costs, and the static tilt angle varies with the mean wind speed. Additionally, Wise and Bachynski (2020) showed that larger tilt reduced the wake loss for a downstream turbine due to vertical wake deflection, as illustrated in Fig. 2.6. Concept-specific assessments of the trade-off with energy loss, also considering farm level effects, are therefore needed to find cost-optimal values for this constraint.

An important consideration in design is to maintain the structural integrity of the power cable, which is designed for relatively small curvatures. The maximum offset of the platform is often constrained in lieu of an explicit dynamic model and design criterion for the power cable. As the relation between offset and power cable loads is somewhat dependent on the design of the specific system, no general offset criterion exists in design guidelines, but limitations are typically expressed as a fraction of the water depth. The maximum offset is also of interest in relation to the tendon connectors for TLP wind turbines, where a maximum offset limitation of 10 % of the water depth has been common industry practice for conventional TLPs (Bachynski 2014).

The offset is to a large degree a function of the mooring system design, which in addition must possess sufficient breaking strength to withstand the design
tension in the ULS and ALS (DNV GL 2018a). A minimum tension should also be maintained at all times, as slack line events may result in high snap loads when the line regains its tension. For drag anchors, which are not designed to take vertical loads, a constraint may also be needed to prevent uplift at the anchors for extreme offsets (Brommundt et al. 2012).

Mooring lines should be designed against fatigue failure; however, as the design life of a mooring line often is shorter than that of the platform and wind turbine, replacement of mooring lines during the design life of the platform is an implicit design assumption (DNV GL 2018a). Consequently, fatigue is not considered a key criterion in mooring system optimization (Benassai et al. 2014). The effect of including a fatigue life constraint for the mooring lines was demonstrated by Fylling and Berthelsen (2011), who found that it resulted in a 20% more expensive mooring system and about 5% increase in spar buoy costs for the optimized system.

A challenge related to constraint formulations for FWTs is to derive expressions based on actual design criteria for fatigue and ultimate loads. Often, constraints are expressed in terms of response parameters which can indicate the level of response, but are only applicable for relative comparison of solutions. However, to achieve cost-effective structures with a consistent reliability level, the constraints should reflect the actual limit state criteria. This requires a large number of simulations, and often, no closed-form solution is available for the design loads. Steinert et al. (2016) attempted to properly express ULS constraints, where the 50-year extreme response in a single parked condition was considered. The constraints were based on the assumption of Gaussian response, which simplifies the estimation of the extreme value distribution considerably. Fatigue damage in the tower and possibly the platform is of concern, but has rarely been assessed directly in design optimization studies, where proxy variables such as tower top displacement due to bending (Sandner et al. 2014, Lemmer et al. 2017) or nacelle acceleration (Hall et al. 2013, Karimi et al. 2017) have been used to assess the support structure loads. Although the large number of required load cases makes optimization for fatigue challenging, it is typically the design-driving limit state for large parts of the support structure, and accurate and consistent fatigue design will thus lead to more effective use of material.

Another reason for constraining the nacelle acceleration has been the assumption of strong correlation with loads on the drivetrain; however, Nejad et al. (2019) showed that the accelerations at the tower top were not a good indicator for either maximum or fatigue loads on the main bearings of the
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As the drivetrain is associated with relatively frequent failures (Nejad et al. 2014), and has been shown to be sensitive to control strategies (Lee et al. 2020) and platform concept (Nejad et al. 2015), it is desirable to have a measure of the drivetrain response in the design optimization process. Global analyses are not appropriate for detailed drivetrain analysis, which typically is performed using de-coupled multibody simulations, but simplified formulations based on global response parameters can be used for qualitative analysis of the gear, shaft, and pitch actuator response with different control system designs (Smilden et al. 2018). Although these methods may be inappropriate to assess the actual fatigue damage in the components, they are applicable for relative comparison between different solutions.

In addition to general requirements, concept-specific constraints may also be needed. For instance, spar platforms are susceptible to Mathieu instability (Haslum and Faltinsen 1999), and TLPs may show ringing responses (Bachynski and Moan 2014). The critical load cases also depend on the system components that are considered. While the largest values for the tower base bending moment, mooring line tension, and global motions of FWTs may occur in operational conditions, the drivetrain and blades are more affected by fault conditions (Bachynski et al. 2013). Insight about the behaviour of the considered system, as well as the specific components that are to be designed, is therefore needed to ensure that feasible solutions are achieved.
Chapter 3

Numerical Models

The results presented in Chapter 4 and the appended papers are based on a linearized numerical model for global response analyses of floating wind turbines, which was developed over the course of the thesis. The main features of the model are summarized in the following. A brief description of the evolution of the model, and the associated publications, is also given.

3.1 Linearized formulation

The coupled dynamics of FWTs are typically analysed using nonlinear time-domain analyses, which capture the complex interactions between aerodynamics, hydrodynamics, structural dynamics and control. For a design procedure which requires several iterations, each involving a large number of load cases, the process becomes computationally prohibitive, and simplified models are therefore desirable.

Using Newton’s second law, the nonlinear equations of motion for the displacement vector \( \mathbf{q} \) is expressed as

\[
\mathbf{M} \ddot{\mathbf{q}} = \mathbf{\tilde{f}}(\dot{\mathbf{q}}, \mathbf{q}, \mathbf{u}),
\]

(3.1)

where \( \mathbf{M} \) is the mass matrix, and \( \mathbf{\tilde{f}} \) contains position- and velocity-dependent terms, in addition to forces due to external inputs (\( \mathbf{u} \)). By defining a state vector, \( \mathbf{x} = [\mathbf{q}^T \dot{\mathbf{q}}^T]^T \), Eq. (3.1) can be written as a set of first order differential equations (Balchen et al. 2003):

\[
\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}).
\]

(3.2)

Considering small perturbations, \( \Delta \), about an operational point defined by
(x₀, u₀), the state and input vectors are written
\[ x = x₀ + Δx, \quad u = u₀ + Δu, \]
while the system in Eq. (3.2) can be linearized by a Taylor series expansion:
\[ \dot{x} ≈ f(x₀, u₀) + \frac{∂f}{∂x} Δx + \frac{∂f}{∂u} Δu. \]
If the operational point is chosen such that \( \dot{x}_0 = f(x₀, u₀) = 0 \), the linearized dynamic system for the differential state and input variables can be written in state-space form as (Chen 2013):
\[ Δ\dot{x} = AΔx + EΔu, \]
where \( A \) is the state matrix, and \( E \) is the input matrix. Given a set of system outputs, \( y \), which are described by \( y = g(x, u) \), a similar linearization can be applied:
\[ Δy = CΔx + DΔu. \]
Here, \( C \) and \( D \) are commonly known as the output matrix and feedthrough matrix, respectively. If the system is time-invariant, the matrices in Eq. (3.5) and Eq. (3.6) are constant (Balchen et al. 2003).

3.2 System overview

An overview of the considered FWT system is shown in Fig. 3.1. The model is valid for single-hull spar platforms with circular cross sections, supporting an arbitrary three-bladed horizontal-axis wind turbine, where the hull is partially filled with solid ballast from the bottom to achieve stability. A catenary mooring system consisting of three lines spread symmetrically about the vertical axis is used for station-keeping, where the so-called crow-feet are neglected, and the lines are assumed to have constant cross-sectional properties and no discrete line components such as buoys or clump weights. Only 2D response is detailed in the following; however, out-of-plane motions have also been included in the present work using the same methodology.

The support structure is modelled as a slender flexible beam with varying cross sectional properties. The model is regarded unsuitable for local blade response, and rotor design is considered outside the scope of the current work. The dynamic response of the blades is assumed to have a small effect on the support structure response, and are therefore considered rigid. Consequently, the RNA is replaced by a point mass and inertia, with resultant aerodynamic loads acting on the tower top. The considered resultant
loads are the thrust force \( F_T \), the tilting moment \( M_T \), and the aerodynamic torque \( Q_A \). In addition, distributed wave excitation forces \( dF_W \) are applied on the hull.

For each wind-wave condition, the linearization point is found from equilibrium when the system is subjected to the mean environmental loads. As the model neglects current and mean wave loads, the wind forces on the rotor and tower are the only load components with non-zero mean. The operational point for a given system design is thus solely a function of the mean wind speed, which determines the references for the rotor speed, collective blade pitch angle, and generator torque, as well as the equilibrium position of the support structure and mooring system.

The model presented here considers three support structure degrees-of-freedom (DOFs), namely surge, pitch, and the first bending mode. For deep-draught spar platforms, the first order wave excitation in heave is small. As a consequence, the heave response is not considered to have a large effect on the structural loads on the system, and is thus not included in the model. However, the natural period can be estimated, and subsequently used in the design process to avoid heave resonance in the wave-frequency range. Another issue relevant for spar platforms is the phenomenon known as Mathieu instability, which can occur when the pitch restoring moment varies harmonically due to large heave motions (Koo et al. 2004). Haslum and Faltinsen (1999) reported that unstable solutions occur for certain ratios between the frequency of the heave motion and the pitch natural frequency, and can thus be prevented assuming that the heave response is dominated by resonant motions.

In addition to the support structure DOFs described above, a rotor speed DOF is included. The inputs to the structural system consist of both control system outputs and disturbances due to wind and wave loads. The control outputs, i.e. the generator torque and the collective blade pitch angle, are described by a separate control system model, which can be combined with the structural model to create a single closed-loop system.

The model considers coupled response of four disciplines: structural dynamics, hydrodynamics, aerodynamics, and control dynamics. The applied formulations for each of the disciplines are detailed in the following subsections.
3.3 Structural dynamics

The linear dynamic response of the support structure is found using generalized coordinates. The total displacements are written as a weighted sum of a set of pre-defined spatial shape functions, \( \psi \), and the accuracy of the formulation thus depends on how well the actual displacement field is captured by the shape functions. If the eigenmodes of the system are chosen as shape functions, the method corresponds to modal superposition. An illustration of the applied shape functions for the present support structure modes is shown in Fig. 3.2.

The total displacement of the support structure at time \( t \) is found by multiplying the shape functions by time-dependent weight functions, \( \chi \), i.e., (Naess and Moan 2013)

\[
w(z, t) = \sum_{k=0}^{n} \psi_k(z)\chi_k(t) = \psi(z)^\top \chi(t).
\]  

The weight functions are assumed harmonic, i.e. \( \chi(t) = \chi_0 e^{i\omega t} \), and are found by solving the generalized equations of motion. Using the principle of virtual work, the generalized system matrices can be established as a combination of continuous and discrete terms. The support structure is modelled as a slender beam undergoing pure bending deformations, and
is assumed to follow Euler–Bernoulli beam theory. Including only terms relevant for the considered FWT support structure, the generalized mass, damping, and stiffness matrices can be expressed as

$$M_{\chi} = \int_{-d}^{l} m(z) \psi(z) \psi^\top(z) dz + \sum_p M_p \psi(z_p) \psi^\top(z_p)$$

$$+ \sum_p I_p \psi,_{z}(z_p) \psi,_{z}^\top(z_p),$$

$$B_{\chi} = \int_{-d}^{l} b(z) \psi(z) \psi^\top(z) dz + \sum_p B_p \psi(z_p) \psi^\top(z_p),$$

and

$$K_{\chi} = \sum_p K_p \psi(z_p) \psi^\top(z_p) + \int_{-d}^{l} EI(z) \psi,_{zz}(z) \psi,_{zz}^\top(z) dz$$

$$+ \int_{-d}^{l} N(z) \psi,_{z}(z) \psi,_{z}^\top(z) dz,$$

respectively, where $m$ and $b$ are distributed masses and dampers, $M_p$, $I_p$, $B_p$, and $K_p$ represent discrete masses, inertias, dampers, and springs, $EI$ is the bending stiffness, $N$ is the axial force, $d$ is the platform draft, and $l$ is the height of the tower top above the still water line (SWL). Here, the mass matrix includes both structural and hydrodynamic mass. In addition to
hydro- and aerodynamic damping, the damping matrix contains structural
damping terms, which are described using stiffness-proportional Rayleigh
damping.

The generalized force vector is written:

\[ \mathbf{F}(t) = \int_{-d}^{l} d\mathbf{F}(z, t) \psi(z) dz + \sum_p \mathbf{F}_p(t) \psi(z_p) + \sum_p \mathbf{Q}_p(t) \psi_z(z_p), \]  

(3.11)

where \( d\mathbf{F} \) are distributed forces, and \( \mathbf{F}_p \) and \( \mathbf{Q}_p \) are discrete forces and moments, respectively. The linear system of equations for the weight functions then becomes

\[ \mathbf{M} \ddot{\mathbf{x}}(t) + \mathbf{B} \dot{\mathbf{x}}(t) + \mathbf{K} \mathbf{x}(t) = \mathbf{F}(t). \]  

(3.12)

To include the controller and power production in the design process, a
model for the rotor and drivetrain is needed. Here, a rigid drivetrain is
assumed, and a single-DOF model is thus applied for the angular rotation
of the shaft. The differential equation is written:

\[ I_D \ddot{\phi} = \mathbf{Q}_A - N_{\text{gear}} \mathbf{Q}_G, \]  

(3.13)

where \( I_D \) is the drivetrain inertia, \( N_{\text{gear}} \) is the gear ratio, \( \mathbf{Q}_G \) is the high-speed shaft (HSS) generator torque, and \( \dot{\phi} \) is the low-speed shaft (LSS) rotational speed.

With the structural DOFs described above, the state vector is defined as

\[ \mathbf{x} = [\chi^\top \dot{\chi}^\top \dot{\phi}]^\top. \]  

(3.14)

The state matrix in Eq. (3.5) can now be derived from the total mass,
damping, and stiffness matrices for the system (Chen 2013):

\[ \mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1} \mathbf{K} & -\mathbf{M}^{-1} \mathbf{B} \end{bmatrix}, \]  

(3.15)

while the input matrix \( \mathbf{E} \) relates the system inputs to the external forces
acting on the states.

The mean configuration of the mooring system, as well as the linearized
mooring stiffness, for a given mean thrust is found using the elastic catenary
equations. For the dynamic response of the mooring lines, the simplified
dynamic model proposed by Larsen and Sandvik (1990), and later extended
by Lie and Sødahl (1993), is utilized. Here, each line is modelled as a single-
DOF spring/damper system, that consider inertia and drag forces due to
the line motion in addition to elastic and geometric stiffness. Given the displacement of the upper end of the line in the tangential direction, \( x(t) \), the dynamic tension in the line can then be calculated from

\[
T_D = k_E [x(t) - u(t)] = c^*|\dot{u}(t)|\dot{u}(t) + k_G u(t) + \omega^2 m^* x(t) \quad (3.16)
\]

where \( u(t) \) is the generalized displacement of the line, found from a quasi-static analysis where the upper end is displaced two times the standard deviation of the platform motion from the equilibrium position (significant motion). The geometric stiffness, \( k_G \), is found as the secant stiffness when the line is moved from equilibrium to the position of significant motion. \( m^* \) and \( c^* \) are the generalized inertia and damping coefficients due to Morison-type hydrodynamic forces, respectively, which are found using the quasi-static line configuration as the shape function. \( k_E \) is the elastic line stiffness. The model is illustrated in Fig. 3.3.

### 3.4 Hydrodynamics

Assuming a circular cross section for the hull, the first order wave excitation forces can be calculated using the analytic expression developed by MacCamy and Fuchs (1954). The force per unit length, \( dF_W \), for a regular wave with unit amplitude is then given by

\[
dF_W(z, \omega) = \frac{4\rho g}{k} \frac{\cosh k(z + h)}{\cosh kh} G e^{i(\omega t - \alpha)}, \quad (3.17)
\]
where $\rho$ is the water density, $g$ is the gravitational acceleration, $k$ is the wave number, $h$ is the water depth, and

$$G = \frac{1}{\sqrt{(J'_1(ka))^2 + (Y'_1(ka))^2}}, \quad \tan \alpha = \frac{J'_1(ka)}{Y'_1(ka)}. \quad (3.18)$$

Here, $J'_n$ and $Y'_n$ are the derivatives of the Bessel function of the first and second kind, respectively, of order $n$, and $a$ is the hull radius. The assumption of a circular cross section also allows for simplification of the radiation forces. The transverse added mass per unit length can be approximated using the analytical 2D coefficient for a circular cylinder in infinite fluid (Newman 1977):

$$a_{11}(z) = \frac{\rho \pi D^2(z)}{4}, \quad (3.19)$$

where $D$ is the hull diameter. For a relatively slender structure, radiation damping will be small and can be neglected, especially at the relevant natural frequencies.

The calculation of the natural period in heave requires an estimate of the added mass in the vertical direction. Here, the vertical added mass is approximated as the value for a 3D circular disc (DNV GL 2017), with the same diameter as the bottom of the platform ($D_b$), which was found to yield good agreement with numerical linear potential flow solutions for several considered spar designs:

$$A_{33} = \frac{\rho D_b^3}{3}. \quad (3.20)$$

The viscous drag forces on the hull are nonlinear, and must therefore be linearized. While viscous wave excitation is small compared to the wind excitation forces and thus not included in the model, viscous damping is important for the low-frequency surge motions. Neglecting the wave particle velocities, the viscous damping due to the velocity of the structure ($\dot{x}$) can be added based on stochastic linearization of the quadratic drag term in Morison’s equation (Borgman 1969):

$$B_{\text{visc}}|\dot{x}|\ddot{x} \approx B_{\text{visc}} \sqrt{\frac{8}{\pi} \sigma_{\dot{x}} \dot{x}} = B_{\text{visc,lin}} \dot{x}, \quad (3.21)$$

where $B_{\text{visc}}$ is the dimensional quadratic damping coefficient, and $B_{\text{visc,lin}}$ is the stochastically linearized coefficient. The standard deviation of the velocity, $\sigma_{\dot{x}}$, depends on the damping forces and vice versa, and must therefore be found using an iteration scheme.
Second order potential flow forces may be important if eigenfrequencies in
the system are located close to the excitation frequencies of the sum- or
difference-frequency loads. However, as studies have shown that these loads
have limited effect on the response for spar-type FWTs (Roald et al. 2013,
Duarte et al. 2014), they are not considered in the model.

3.5 Aerodynamics

Aerodynamic loads in global response analyses of FWTs are typically com-
puted using BEM theory, where induction factors, and subsequently blade
element forces, are found iteratively (Burton et al. 2011). Assuming that
the rotor design does not change during the course of the design process,
induction factors can be pre-calculated and stored in look-up tables, before
they are used in the response analyses. For each blade element, the induc-
tion factors are functions of the blade pitch angle (θ) and tip-speed ratio
(λ), where the latter is defined as

$$\lambda = \frac{\dot{\varphi} R}{v}.$$  \hspace{1cm} (3.22)

Here, $R$ is the rotor radius, and $v$ is the wind speed. In the present model,
the induction factors are computed from the BEM equations following the
procedure presented by Ning (2013), which includes Prandtl hub and tip
loss factors, and a modified Glauert correction as described by Buhl (2005)
for high induction factors. A quasi-static approach is used, meaning that
dynamic wake and dynamic stall effects are neglected. The calculated in-
duction factors at midspan for the DTU 10 MW blade design are shown in
Fig. 3.4.

Following quasi-steady BEM theory, the aerodynamic forces on a blade ele-
ment are nonlinear functions of (relative) wind speed, rotor speed, and
blade pitch angle, which can be found once the induction factors are known.
Linearizing the equations, the normal and tangential blade element forces
relative to the rotor plane can be expressed as:

$$F_n(v, \dot{\varphi}, \theta) \approx F_{n0} + \frac{\partial F_n}{\partial v} \Delta v + \frac{\partial F_n}{\partial \dot{\varphi}} \Delta \dot{\varphi} + \frac{\partial F_n}{\partial \theta} \Delta \theta,$$  \hspace{1cm} (3.23a)

$$F_t(v, \dot{\varphi}, \theta) \approx F_{t0} + \frac{\partial F_t}{\partial v} \Delta v + \frac{\partial F_t}{\partial \dot{\varphi}} \Delta \dot{\varphi} + \frac{\partial F_t}{\partial \theta} \Delta \theta.$$  \hspace{1cm} (3.23b)

The blade root loads of interest, i.e. the flapwise shear force $F_y$, the flapwise
bending moment $M_z$, and the edgewise bending moment $M_y$, are found by
integrating the loads over the length of the blade

\[ F_y = \int_0^R F_n \, dr, \quad (3.24a) \]
\[ M_z = \int_0^R r \, F_n \, dr, \quad (3.24b) \]
\[ M_y = \int_0^R r \, F_t \, dr. \quad (3.24c) \]

Here, the subscripts point to the blade coordinate system, which is defined as in Fig. 3.5. The resultant rotor loads can then be calculated as:
\[ F_T = \sum_{i=1}^{3} F_{y,i}, \] (3.25a)

\[ M_T = \sum_{i=1}^{3} M_{z,i} \cos \phi_i, \] (3.25b)

\[ Q_A = \sum_{i=1}^{3} M_{y,i}. \] (3.25c)

As only the resultant loads on the rotor are of interest in the global response model, it is desirable to simplify the wind field and express it using a single scalar variable. This is achieved by expressing a rotor effective wind speed for each of the resultant rotor loads in Eq. (3.25). The rotor effective wind speed is a spatially constant wind speed which yields identical resultant loads as the full wind field, and can be established from the incoming wind spectrum, the spatial coherence function, and the transfer functions between wind speed and blade element loads from Eq. (3.23). A detailed derivation of the rotor effective wind speed, based on the thesis by Halfpenny (1998), is provided in Appendix B.

In addition to the rotor loads, the aerodynamic drag forces on the tower are taken into account, using a Morison drag formulation. The tower loads are expressed as a combination of a mean force, which is added directly, and a frequency-dependent force, which is found using stochastic linearization. Here, only the excitation forces are considered, meaning that the tower drag forces arising from the movement of the turbine are neglected.

### 3.6 Control system description

The baseline control system determines the changes in collective blade pitch and generator torque based on the rotor speed error, and follows the generic approach for variable-speed variable-pitch wind turbines described by e.g. Jonkman et al. (2009). It consists of two separate and independent regions, namely below and above rated wind speed. Typically, wind turbine controllers also include transition strategies to ensure smooth switchovers between regions, however, for simplicity, this is not included in the model.

In both regions, the rotor speed is low-pass filtered to avoid high-frequency actuation, using a first-order filter:

\[ \ddot{\varphi} + \omega_c \dot{\varphi} = \omega_c \dot{\varphi}, \] (3.26)

where \( \omega_c \) is the corner frequency, and \( \dot{\varphi} \) is the filtered rotor speed. Below rated wind, the blade pitch angle is kept at fine pitch, while the generator...
torque is set proportional to the squared rotor speed, to track the optimal tip-speed ratio and thus optimize the power output:

\[ Q_G = K_g \dot{\varphi}^2, \quad (3.27) \]

where \( K_g \) is the generator torque constant. The change in generator torque for a rotor speed error \( \Delta \dot{\varphi} \) then becomes

\[ \Delta Q_G = \frac{\partial Q_G}{\partial \dot{\varphi}} \Delta \dot{\varphi} = 2K_g \dot{\varphi}_0 \Delta \dot{\varphi}, \quad (3.28) \]

where \( \dot{\varphi}_0 \) is the rotor speed reference. Alternatively, the generator torque can be regulated using a proportional-integral (PI) controller:

\[ \Delta Q_G = k_{p,Q} \Delta \dot{\varphi} + k_{i,Q} \int_0^t \Delta \dot{\varphi} \, d\tau, \quad (3.29) \]

where \( k_{p,Q} \) and \( k_{i,Q} \) are the proportional and integral gains, respectively. Above rated wind speed, the blade pitch angle is found using a gain-scheduled PI controller:

\[ \Delta \theta = \eta_k k_{p,\theta} \Delta \dot{\varphi} + \eta_k k_{i,\theta} \int_0^t \Delta \dot{\varphi} \, d\tau, \quad (3.30) \]

where \( k_{p,\theta} \) and \( k_{i,\theta} \) are the proportional and integral gains, respectively, and \( \eta_k \) is the gain-scheduling parameter. For the torque controller, two common strategies exist for the above-rated regime: (1) constant power or (2) constant torque. The variation in generator torque then becomes

\[ \Delta Q_G = \begin{cases} -\frac{P_r}{N_{\text{gear}} \varphi_r} \Delta \dot{\varphi}, & \text{constant power,} \\ 0, & \text{constant torque,} \end{cases} \quad (3.31) \]

where \( P_r \) and \( \varphi_r \) are the rated power and rotor speed, respectively.

To avoid the potential problem of negative damping for the platform motions above rated wind speed, a feedback term proportional to the platform pitch velocity or nacelle velocity may also be included, to manipulate the rotor speed reference. Here, a modified rotor speed reference, \( \dot{\varphi}_0' \), as defined by Lackner (2009) is used:

\[ \dot{\varphi}_0' = \dot{\varphi}_0 (1 + k_f \dot{x}_f), \quad (3.32) \]

where \( k_f \) is the velocity feedback gain, and \( \dot{x}_f \) is the nacelle or pitch velocity. An updated expression for the rotor speed error can then be established as

\[ \Delta \dot{\varphi}' = \dot{\varphi} - \dot{\varphi}_0' = \Delta \dot{\varphi} - \dot{\varphi}_0 k_f \dot{x}_f. \quad (3.33) \]

Here, \( \dot{x}_f \) may also be passed through a low-pass filter before it is fed back to the blade-pitch controller.
3.7  Response to stochastic input

After establishing the linearized model, the transfer matrix between system inputs ($u$) and outputs ($y$) is defined using the system matrices in Eq. (3.5) and Eq. (3.6) (Chen 2013):

$$H(\omega) = C (i\omega I - A)^{-1} E + D.$$  (3.34)

The cross spectral density matrix of the response vector $y$ can be calculated from

$$S_y(\omega) = H(\omega)S_u(\omega)H(\omega)^H,$$  (3.35)

where $(\cdot)^H$ denotes the conjugate transpose (Naess and Moan 2013). The response spectra of $y$ are then found along the diagonal of $S_y(\omega)$.

The input vector is expressed as

$$u = \begin{bmatrix} v_{F_T} & v_{M_T} & v_{Q_A} & F_W^T \end{bmatrix}^T,$$  (3.36)

where $v_{F_T}$, $v_{M_T}$ and $v_{Q_A}$ are rotor effective wind speeds for thrust, tilting moment and aerodynamic torque, respectively. The wave load vector, $F_W$, contains the generalized wave excitation force for each support structure DOF. $S_u(\omega)$ is the cross spectral density matrix for the load process, which has the following structure:

$$S_u(\omega) = \begin{bmatrix} S_{\text{wind}}(\omega) & 0 \\ 0 & S_{\text{wave}}(\omega) \end{bmatrix},$$  (3.37)

as the wind and wave processes are assumed to be uncorrelated (DNV GL 2016). Both the wind speed and the wave elevation are assumed to be stationary Gaussian processes, and the response will thus also be a stationary Gaussian process. The $n^{th}$ spectral moment of an arbitrary response parameter $y$, given the variance spectrum $S_y(\omega)$, is defined as

$$m_n = \int_0^\infty \omega^n S_y(\omega) d\omega.$$  (3.38)

The standard deviation of $y$ can then be found from

$$\sigma_y = \sqrt{m_0}.$$  (3.39)

3.7.1 Fatigue damage

To calculate the fatigue damage, formulations for the fatigue resistance and fatigue loading are needed. The fatigue resistance is commonly described
by SN curves, where the relation between the stress range \( S \) and number of cycles to failure \( N \) is given as (DNV GL 2019c):
\[
N = KS^{-m},
\]
(3.40)
where \( K \) and \( m \) are material parameters dependent on the type of fatigue detail. Using the Palmgren-Miner hypothesis of linear accumulation of damage, the fatigue damage in time \( T \) can be expressed as (Naess and Moan 2013):
\[
D = \int_0^\infty \frac{\nu T}{K} s^m f_S(s) ds.
\]
(3.41)
Here, the fatigue loading is described by the cycle rate, \( \nu \), and the PDF of the stress ranges, \( f_S(s) \). If the response process is narrow-banded and Gaussian, the stress ranges can be described by a Rayleigh distribution; however, because the response of FWTs in general is wide-banded, the narrow-band formulation may result in overly conservative damage estimates. For a general wide-banded response, cycle counting methods based on the stress time series are typically applied, with rainflow counting being considered the most accurate for estimation of fatigue damage (Naess and Moan 2013). This method has no closed-form solution in the frequency domain for wide-banded Gaussian response, and given a stress response spectrum, the stress range distribution must thus be estimated either by transformation of the response spectra to the time domain, or directly in the frequency domain using empirical formulae. For the latter approach, various methods have been proposed, where the formulae presented by Dirlik (1985) and Benasciutti and Tovo (2005) have been shown to yield accurate approximations of the rainflow counting method for a variety of spectral shapes and bandwidths (Gao and Moan 2008).

The Dirlik method, which is implemented in the current model, uses the stress response spectrum and empirical factors to fit the PDF of the stress cycles to a combination of an exponential and two Rayleigh distributions, and the accuracy of the method is therefore dependent on how well the rainflow-count can be represented by these distributions. The resulting stress range distribution is given as:
\[
f_S(s) = \frac{G_1}{Q} \exp \left\{ -\frac{Z}{Q} \right\} + \frac{G_2 Z}{R^2} \exp \left\{ -\frac{Z^2}{2R^2} \right\} + G_3 Z \exp \left\{ -\frac{Z^2}{2} \right\}, \quad (3.42)
\]
where \( G_1, G_2, G_3, R, \) and \( Q \) are empirical weight factors found from the spectral moments, and \( Z \) is the normalized stress range:
\[
Z = \frac{S}{2\sqrt{m_0}}. \quad (3.43)
\]
3.7.2 Extreme response

Estimation of extreme values for a stochastic process can be performed using upcrossing analysis, which is based on the assumption that upcrossings of high levels are statistically independent (Naess 1984). Consequently, the number of upcrossings in time $T$ will be Poisson distributed, which, for the response process $y$, gives the following CDF for the extreme value $\Xi$:

$$ F_\Xi(\xi) = \exp \left\{ -\nu^+_\xi \cdot T \right\}, \quad (3.44) $$

where $\nu^+_\xi$ is the mean upcrossing rate of the level $\xi$. Assuming a stationary Gaussian process with zero mean, the CDF can be written (Naess and Moan 2013)

$$ F_\Xi(\xi) = \exp \left\{ -\nu^+_0 \cdot T \exp \left( -\frac{\xi^2}{2\sigma^2_y} \right) \right\}, \quad (3.45) $$

where $\nu^+_0$ is the mean zero-upcrossing rate, which can be found from the zeroth and second order spectral moments:

$$ \nu^+_0 = \frac{1}{2\pi} \sqrt{\frac{m_2}{m_0}}. \quad (3.46) $$

The $p$-fractile of the distribution, i.e. the value which is exceeded with probability $(1 - p)$ during the time $T$, becomes (Naess and Moan 2013):

$$ \xi_p = \sigma \sqrt{2 \ln \left( \frac{\nu^+_0 \cdot T}{\ln(1/p)} \right)}, \quad (3.47) $$

while the most probable maximum value, $\xi_c$, also known as the characteristic largest extreme, can be found approximately from the relation $F_\Xi(\xi_c) = e^{-1}$:

$$ \xi_c = \sigma \sqrt{2 \ln \left( \nu^+_0 \cdot T \right)}. \quad (3.48) $$

3.8 Optimization framework

To enable efficient gradient-based optimization, the FWT model is implemented in the OpenMDAO framework (Gray et al. 2019). OpenMDAO is an open-source framework for multidisciplinary design, analysis and optimization, which contains a large number of solvers and other features making it suitable for large-scale optimization of complex systems. The model is constructed in a modular fashion, where the complex multidisciplinary model
is broken down into smaller units of code (components). The components are gathered in groups that again can be part of other groups, and thus forms a hierarchical structure which can be used to organize the model or define different solver strategies. An overview of the OpenMDAO problem structure is shown in Fig. 3.6.

The partial derivatives of the component outputs with respect to the component inputs can either be calculated numerically using e.g. finite differences, or be provided as analytic functions by the user. Analytic derivatives are desirable due to their computational efficiency and accuracy; however, they require significant implementation effort. In the present work, analytic derivatives are computed for the complete coupled model, which ensures a feasible optimization process even as the number of design variables and load cases become large.

One of the main purposes of the optimization framework is to compute overall model-level derivatives given the component-level derivatives. For a model where all relations are given by explicit functions, the total derivatives can be calculated using the chain rule. However, if implicit relationships between variables exist, more elaborate methods are needed. Given an output of interest, $f$, which is described by the function $f = F(x, y(x))$, where $x$ is the vector of input variables, and $y$ is a vector of implicit variables dependent on $x$, the total derivative can be found from (Martins and Ning 2020):

$$\frac{df}{dx} = \frac{\partial F}{\partial x} + \frac{\partial F}{\partial y} \frac{dy}{dx}. \quad (3.49)$$

The relationship between $y$ and $x$ is defined by the solution of the residual equations $r = R(x, y(x)) = 0$. The total derivative of $r$ with respect to $x$ can be expressed as:

$$\frac{dr}{dx} = \frac{\partial R}{\partial x} + \frac{\partial R}{\partial y} \frac{dy}{dx} = 0. \quad (3.50)$$
It is desirable to calculate $df/dx$ using only partial derivatives, as they do not require solving the residual equations and thus have a lower computational cost. The only term in Eq. (3.49) that is not a partial derivative is $dy/dx$, which from Eq. (3.50) can be expressed as

$$\frac{\partial R}{\partial y} \frac{dy}{dx} = -\frac{\partial R}{\partial x}.$$  \hspace{1cm} (3.51)

Given the partial derivatives of $R$ with respect to $x$ and $y$, $dy/dx$ can then be found by solving the linear system in Eq. (3.51). This is known as the direct (forward) method, where the linear system must be solved once for each model input (Martins and Ning 2020).

Alternatively, $dy/dx$ in Eq. (3.49) can be replaced by the expression in Eq. (3.51):

$$\frac{df}{dx} = \frac{\partial F}{\partial x} - \left[ \frac{\partial F}{\partial y} \left[ \frac{\partial R}{\partial y} \right]^{-1} \frac{\partial R}{\partial x} \right]_{\psi^T}.$$  \hspace{1cm} (3.52)

Here, $\psi$ is known as the adjoint vector, which can be described through the following relation (Martins and Ning 2020):

$$\begin{bmatrix} \frac{\partial R}{\partial y} \end{bmatrix}^T \psi = \begin{bmatrix} \frac{\partial F}{\partial y} \end{bmatrix}^T.$$  \hspace{1cm} (3.53)

Now, $\psi$ can be found by solving the linear system in Eq. (3.53), and used together with Eq. (3.52) to calculate the total derivative. This is known as the adjoint (reverse) method. The linear system in Eq. (3.53) must, contrary to the forward method, be solved once for each model output, and this method therefore offers increased computational efficiency when the number of model inputs is larger than the number of model outputs. OpenMDAO supports both direct and adjoint gradient computations, and the method is chosen based on the specific problem structure.

To illustrate the extent of the analytic gradient implementation, the required calculations are shown for a single model component. The component which calculates the uncoupled heave natural period ($T_{n3}$), given the structural mass of the system ($M$), the added mass in heave ($A_{33}$), and the heave restoring stiffness ($C_{33}$), uses the following relation:

$$T_{n3} = 2\pi \sqrt{\frac{M + A_{33}}{C_{33}}}.$$  \hspace{1cm} (3.54)
The required partial derivatives for the component become:

\[
\frac{\partial T_{n3}}{\partial M} = \frac{\pi}{C_{33}\sqrt{\frac{M+A_{33}}{C_{33}}}}, \quad (3.55a)
\]

\[
\frac{\partial T_{n3}}{\partial A_{33}} = \frac{\pi}{C_{33}\sqrt{\frac{M+A_{33}}{C_{33}}}}, \quad (3.55b)
\]

\[
\frac{\partial T_{n3}}{\partial C_{33}} = -\frac{\pi}{C_{33}}\sqrt{\frac{M+A_{33}}{C_{33}}}. \quad (3.55c)
\]

As both \(M\), \(A_{33}\), and \(C_{33}\) are dependent on the design variables for the optimization problem, the chain rule in Eq. (3.49) is then needed to compute the total derivative of \(T_{n3}\) with respect to an arbitrary design variable \(p\):

\[
\frac{dT_{n3}}{dp} = \frac{\partial T_{n3}}{\partial p} + \frac{\partial T_{n3}}{\partial M} \frac{dM}{dp} + \frac{\partial T_{n3}}{\partial A_{33}} \frac{dA_{33}}{dp} + \frac{\partial T_{n3}}{\partial C_{33}} \frac{dC_{33}}{dp}. \quad (3.56)
\]

If there is an implicit relationship between \(M\), \(A_{33}\), or \(C_{33}\), and \(p\), obtaining the total derivative also requires solving the linear system in Eq. (3.51) or Eq. (3.53).

The component-level derivatives must be provided for all parts of model, and may result in rather involved computations, especially for more advanced components. The complexity and required effort related to the gradient implementation are thus comparable to the development of the analysis model itself, and makes up about 70 % of the total code volume in the present work.

The model layout and data dependencies are illustrated using an extended design structure matrix (XDSM) (Lambe and Martins 2012) in Fig. 3.7. Here, the rounded blue box is the optimizer, green boxes are analysis models, gray parallelograms are data, white parallelograms are fixed input parameters, and grey lines represent data dependencies. The data moves from top to bottom and left to right in the upper triangular part, and from bottom to top and right to left in the lower triangular part.
3.8. Optimization framework

Figure 3.7: XDSM diagram showing the model layout and data dependencies.
3.9 Model development

The linearized FWT model has been developed over the course of the thesis, and the versions applied in the different papers have therefore not been the same. The evolution of the model is briefly summarized in the following.

$V_1$, applied in $P_1$: The structural model for the support structure using generalized coordinates is established, but assumes a rigid platform, while the elasticity of the tower is considered. The hydrodynamic formulations presented in Section 3.4 are implemented, and the quasi-static mooring system response is found from the elastic catenary equations. The rotor DOF is not included, and aerodynamic excitation (thrust and tilting moment) is computed from time-domain simulations with turbulent wind and an active controller. The aerodynamic damping is found as the change in thrust due to a small change in wind speed, assuming no reaction from the rotor or controller. The Dirlik method for fatigue damage in the support structure and extreme response based on upcrossing analysis are implemented.

$V_2$, applied in $P_3$: Building off of $V_1$, a single-DOF model for the rotation of the shaft is implemented, and a flexible platform is included. Linearized aerodynamic loads are derived as functions of wind speed, rotor speed, and blade-pitch angle, using quasi-steady BEM theory and a rotor effective wind speed. Generator-torque and blade-pitch controllers are added to the model. Dynamic mooring line tension is found based on a single-DOF model which consider drag and inertia forces due to the motion of the line. Analytic gradients are calculated for the model, which is implemented in the OpenMDAO framework.

$V_3$, applied in $P_4$: More advanced control strategies for the blade-pitch controller are implemented. The new controllers consider velocity feedback using either the nacelle or the platform pitch velocity, with an optional low-pass filter on the velocity signal.

$V_4$, applied in $P_5$: The model is extended to account for out-of-plane response, and thus allow for wind-wave misalignment.
Chapter 4

Research Findings

This chapter presents the main research findings from the papers included in the thesis, and discusses their relation to the contributions stated in Section 1.3.

4.1 Linearized dynamic analyses

Findings related to the use of linearized dynamic analyses to predict global design loads for FWT structures are presented in the following. First, closed-form expressions for frequency-dependent aerodynamic damping and inertia are derived. Then, verification of the linear model against nonlinear simulations for fatigue and extreme response is presented, and some important aspects and limitations to the model are discussed.

4.1.1 Aerodynamic damping and inertia

The motions of the FWT give rise to changes in the aerodynamic forces on the turbine, which depend on the mean wind speed, motion amplitude, and oscillation frequency. Commonly, frequency-domain analyses of offshore wind turbines have considered aerodynamic damping coefficients which are either constant, or vary only with the mean wind speed. These methods neglect the impact of the controller and rotor dynamics, and consequently overestimate the damping at low oscillation frequencies.

In addition, the forces which arise from the platform motions that are out of phase with the velocity, and thus result in an inertia (or stiffness) effect, are neglected. For FWTs, which typically experience low-frequency resonant motions, frequency-dependent forces should be considered in response
calculations. In the present work, closed-form expressions for frequency-dependent aerodynamic damping and inertia were derived. The key results are presented in the following, while a detailed derivation can be found in P2.

Using linearized aerodynamic forces and a 2-DOF wind turbine model, with the horizontal nacelle displacement \( x \) and the rotation of the shaft \( \varphi \), the following expressions were obtained for the aerodynamic inertia \( a_{aero} \) and damping \( b_{aero} \) coefficients:

\[
a_{aero}(\omega) = -\frac{1}{\omega^2} \text{Re} \left\{ i\omega \frac{\partial F_T}{\partial v} - \left[ i\omega \left( \frac{\partial F_T}{\partial \dot{\varphi}} + k_P \frac{\partial F_T}{\partial \theta} \right) + k_I \frac{\partial F_T}{\partial \theta} \right] \tilde{C}_\varphi(\omega) \right\}, \tag{4.1a}
\]

\[
b_{aero}(\omega) = \frac{1}{\omega} \text{Im} \left\{ i\omega \frac{\partial F_T}{\partial v} - \left[ i\omega \left( \frac{\partial F_T}{\partial \dot{\varphi}} + k_P \frac{\partial F_T}{\partial \theta} \right) + k_I \frac{\partial F_T}{\partial \theta} \right] \tilde{C}_\varphi(\omega) \right\}. \tag{4.1b}
\]

Here, \( F_T \) is the thrust force, \( v \) is the wind speed, \( k_P \) and \( k_I \) are the proportional and integral gains of the blade-pitch controller, and \( \theta \) is the blade-pitch angle. \( \tilde{C}_\varphi \) is the complex transfer function between \( x \) and \( \varphi \), i.e.,

\[
\varphi(\omega) = \tilde{C}_\varphi(\omega)x(\omega). \tag{4.2}
\]

The derived coefficients in Eq. (4.1) were compared to results from numerical simulations using forced oscillations with AeroDyn as described by Souza and Bachynski (2019), as well as constant damping coefficients, which neglect the controller and rotor dynamics. In Fig. 4.1, the coefficients are shown for the low-frequency range, for a mean wind speed of 25 m/s. The overall agreement between the analytical and numerical solutions was good, while the constant damping coefficient was found to significantly overestimate the aerodynamic damping at low frequencies. The magnitude of the aerodynamic inertia is low compared to the total mass of typical FWTs, resulting in negligible consequences for the surge response. Due to the height of the nacelle, however, the contribution to the moment of inertia in pitch is considerable.

To assess the importance of frequency-dependent coefficients, a frequency-domain model of the OO-Star 10 MW semi-submersible, which was part of the the LIFES50+ project (Müller et al. 2018), was created. Results from the frequency-domain model, using different formulations for the aerodynamic damping and inertia, are compared to nonlinear time-domain simulations using SIMA. The response spectra for platform pitch and tower base
4.1. Linearized dynamic analyses

The deviations in response between the frequency-domain models in Fig. 4.2 can be attributed to differences in aerodynamic damping coefficients. The effect of aerodynamic inertia was much less prominent, but can be seen in the calculated platform pitch natural period. Calculated natural periods in pitch, with and without aerodynamic inertia, are shown in Fig. 4.3.

While the constant damping coefficient underestimated the responses, the opposite was observed for the frequency-dependent coefficients. There were also notable differences between the analytical and AeroDyn models, despite the good agreement for the damping coefficient, which suggested that the response was sensitive to resonant motions and consequently the damping level in the model. Although the actual numbers also depend on the considered wind-wave conditions, platform concept, and additional damping in the model, the results demonstrated that the use of constant damping coefficients is non-conservative, and that this can be avoided in a simple manner through the frequency-dependent coefficients in Eq. (4.1).

The deviations in response between the frequency-domain models in Fig. 4.2 can be attributed to differences in aerodynamic damping coefficients. The effect of aerodynamic inertia was much less prominent, but can be seen in the calculated platform pitch natural period. Calculated natural periods in pitch, with and without aerodynamic inertia, are shown in Fig. 4.3.
Research Findings

(a) Platform pitch. (b) Tower base bending moment.

Figure 4.2: Response comparison between SIMA and frequency-domain analyses, 25 m/s mean wind speed.

Figure 4.3: Pitch natural period with and without aerodynamic inertia effect.
An alternative method of achieving frequency-dependent damping and inertia effects in the support structure analysis is to include a description of the rotor and controller dynamics in the support structure model, as described in P3. These effects will then be implicitly included.

### 4.1.2 Long-term fatigue damage

To assess the applicability of the linearized model in design, the degree of nonlinearity in the global response of the support structures is evaluated. Because both the wind speed and the wave elevation in the short term are assumed stationary Gaussian processes, the linear response of the FWT is also stationary and Gaussian. One way to evaluate the nonlinearity of the response is therefore to look at the skewness and (Pearson’s) kurtosis values, which are 0 and 3 for a Gaussian process, respectively. The skewness and kurtosis for the tower bending moment, based on nonlinear time-domain simulations in SIMA, are shown in Fig. 4.4. The results are based on one wind-wave condition for each mean wind speed from 1 to 30 m/s, using a bin size of 1 m/s and six random realizations. The most probable values for $H_s$ and $T_p$ were applied for each mean wind speed, and represent typical fatigue conditions.

For most wind speeds, the response is quite linear, especially above the rated wind speed of 11.4 m/s. Here, the wave loads become increasingly more important relative to the wind loads, which exhibit a larger degree of nonlinearity. Close to the tower top, the response is dominated by aerodynamic loads, and overall more non-Gaussian behaviour is observed. In the rest of the tower, the pattern is more or less identical to the tower base, suggesting that the same load effects are dominating along most of the tower length.

A comparison of weighted 1-h fatigue damage at the tower base, based on both linear and nonlinear analyses, is shown as a function of the mean wind speed in Fig. 4.5, taken from P1. Here, 546 environmental conditions were considered, which corresponded to all conditions within the operational range with probability of occurrence greater than $10^{-4}$ at the considered site. The linear model was, for most wind speeds, able to accurately capture the fatigue response, and the damage estimates based on response spectra using the Dirlik method were found to agree well with inverse Fourier transform combined with rainflow counting. Large errors in the damage estimates were observed at 9 m/s mean wind speed, where the first tower bending mode coincided with the 3P frequency. In such resonance conditions, nonlinear effects become important, and the linear model significantly overestimated
(a) Skewness.

(b) Kurtosis.

Figure 4.4: Skewness and kurtosis values for tower fore-aft bending moment.
4.1. Linearized dynamic analyses

Figure 4.5: 1-h weighted tower base fatigue damage arranged by wind speed.

The results suggest that the linearized model should be used with care if the first bending mode is close to the blade-passing frequency, which is often the case for FWTs. It also highlights the importance of accurate estimation of the tower bending mode and associated natural period, as the modal excitation forces may change rapidly with frequency. This is illustrated for the tower base bending moment response spectrum in Fig. 4.6, based in the linearized model, where the natural frequency of the first bending mode is artificially increased in steps of 0.06 rad/s by modifying the modal stiffness. As the natural frequency approaches the 3P frequency, the resonant response, and consequently the fatigue damage, is significantly increased.

This effect may also be important in the context of the numerical modelling of the platform in the global response analyses. For large turbines, the flexibility of the platform can have a large impact on the tower modes, and should therefore be included to ensure that the correct dynamic behaviour is captured.

4.1.3 Short-term extreme response

Similarly to the skewness and kurtosis values for the fatigue conditions, upcrossing rates were used to assess the linearity of the extreme response in 50-year conditions. For a Gaussian process, the upcrossing rate of level $y$ can be calculated from the process’ standard deviation ($\sigma$) and mean
Figure 4.6: Linearized tower base bending moment response spectrum for $U = 15$ m/s, $H_s = 3$ m, and $T_p = 12$ s, where the natural frequency of the first bending mode is artificially increased in steps of 0.06 rad/s ($\approx 0.01$ Hz) by modifying the modal stiffness. The corresponding increase in calculated 1-h fatigue damage is shown, together with the 3P frequency at rated speed (black line).

For the nonlinear time-domain simulations, a function for the tail of the mean upcrossing rates can be established empirically, as described in P1. The nonlinear and linear average upcrossing rates (AUR) for surge, pitch, and tower base bending moment are shown for two 50-year conditions in 4.7. The plots show the multiplication factor $\kappa$, which is related to the extreme response, $y$, through the relation

$$y = \mu + \kappa \sigma,$$

where $\mu$ and $\sigma$ are the mean and standard deviation of the process.

Very good agreement was obtained with an extreme wind speed in the parked condition, where the response was governed by wave loads. Larger nonlinearities are observed near the rated wind speed, due to the large thrust forces in this condition. This was especially the case for the pitch response. The tower base bending moment, on the other hand, was also quite Gaussian in the operational condition, due to the large influence of inertial forces arising from wave-frequency motions.
4.1. Linearized dynamic analyses

- (a) Surge.
- (b) Pitch.
- (c) Tower base bending moment.

**Figure 4.7:** Upcrossing rates for 11 m/s (left) and 50 m/s (right) mean wind speed.
While these results suggest that a linear model also can be used to evaluate extreme response in early phases of design, especially for the bending moments in the support structure, such a conclusion cannot be drawn in the general case. The accuracy of the linear model depends on the importance of nonlinearities in the system, which will vary with location, floater geometry, and different operational conditions. The agreement with nonlinear analyses also depends on how the system is represented in the time-domain model, and care should be taken to make sure that relevant nonlinearities are included.

4.2 Integrated design optimization

The following subsections present key results and discussion regarding integrated numerical design optimization of FWT systems. The findings are based on the methodology presented in P3, where the tower, spar platform, blade-pitch controller, and mooring system were optimized simultaneously, including the scantling design of the hull, using gradient-based optimization with analytic gradients. The optimization considered buckling and long-term fatigue damage along the hull and tower, maximum offset and pitch angle of the platform during selected extreme conditions, and extreme mooring line tension and anchor uplift.

4.2.1 Multimodality

During the optimization runs performed in the present work, local minima were identified for the tower design. The fatigue damage in the support structure increases significantly if the natural frequency of the first tower bending mode coincides with the blade passing frequency, and the optimizer will therefore try to move the tower mode away from the 3P range. The direction that the mode is moved during the optimization depends on the initial design, as illustrated in Fig. 4.8. Due to the ‘barrier’ created by the blade passing frequency, the optimizer will not be able to move over to the stiff-stiff range if the initial design has a soft-stiff tower, and vice versa. Early studies with the optimization model suggested that no feasible solution existed in the soft-stiff range for the applied design problem, and all subsequent studies were therefore performed with a stiff-stiff initial design. However, this should not be treated as proof that a soft-stiff solution does not exist, as the design analyses were based on a simplified model and a reduced set of environmental conditions. In addition, more advanced control strategies, such as a speed exclusion zone to prevent tower resonance in the below-rated regime, could improve the fatigue behaviour. More detailed studies of potential designs are therefore needed to conclude whether
a feasible soft-stiff tower design exists.

4.2.2 Cost and power quality

The goal of the design optimization was to minimize a combination of system costs and rotor speed variation, where the latter was considered a measure of the (inverse) power quality. The monetary cost of reduced power quality is difficult to quantify, and the terms could therefore not be combined in a straightforward manner. The optimization was therefore performed with different weights on the two sub-objectives, to assess the trade-off effects for the system.

The relative importance of costs and power quality in the objective function primarily affected the blade pitch control system parameters, as shown in Fig. 4.9, where the optimized PI gains for different weight factors are plotted against the resulting normalized rotor speed variation. Increasing the gains results in a faster controller, leading to lower variation in the rotor speed, but increased loads in the support structure.

Within the range of the objective function weights considered in P3, the power quality was found to be much more sensitive to the applied weighting than the system costs, as shown in Fig. 4.10. While a 15 % decrease in rotor speed variation was observed from lowest to highest power quality weighting, the corresponding increase in design costs was only 0.7 %.
As the applied design optimization process considered only stationary conditions, relatively slow controllers were obtained. To ensure proper control action during transient events such as wind gusts, extensions to the presented methodology are needed, which may result in faster controllers with larger negative effects on the support structure costs.

4.2.3 Support structure design

The optimized tower and platform design for a selected combination of objective function weights, based on the integrated approach applied in P3, is shown in Fig. 4.11, together with fatigue and buckling utilization factors. The platform took an hour-glass shape below the wave-zone, which increased the restoring stiffness in pitch and thus improved the low-frequency behaviour. The sharp edges and constant taper angles along the platform were caused by a constraint on the maximum taper angle, which was included to ensure a platform shape that was captured correctly by the numerical model. The fact that this constraint was active for most of the platform length, suggests that reductions in costs could be achieved if the constraint is relaxed. More comprehensive numerical models, and possibly more information about the sensitivity of the platform shape on the manufacturing costs, would be needed in order to relax this constraint.

The tower base diameter was forced to match the top diameter of the platform, which gave rise to conflicting influences. The platform diameter should be low to limit the wave loads, whereas the tower diameter should be large, as this is the most cost-effective way to achieve the required fatigue resistance. The resulting compromise led to an unconventional tower design,
where the diameter increased from the base. The importance of integrated design optimization was demonstrated here, as simultaneous design of the platform and tower is needed to identify the optimal solution near the interface.

**Tower shape**

While typical wind turbine towers are linearly tapered, the integrated optimization procedure employed in the present work suggested a different and somewhat unconventional tower design. The reduction in costs achieved by adopting such a design was quantified by optimizing the tower using three different strategies. In all strategies, 20 years fatigue life and global buckling in selected 50-year conditions were considered, while a $15^\circ$ pitch angle constraint was applied on the platform.

In T1, the tower and platform were optimized in an integrated fashion, as described in previous sections. The same procedure was used in T2, but here, the tower was constrained to a linearly tapered shape. Finally, T3 considered a procedure where only the tower was optimized with a linearly tapered shape using a fixed platform design, taken as the optimal design from T1. The main consequence is then that the tower base diameter is fixed in the optimization, such that it matches the platform. The resulting optimized tower designs are shown in Fig. 4.12.

The resulting normalized tower and platform costs are shown in Table 4.2. The consequence of constraining the tower shape depends on the optimiz-
Figure 4.11: Optimized tower and platform design with fatigue utilization (left) and buckling utilization (right). The wall thickness is scaled by a factor of 40 relative to the diameter for illustration purposes.
Figure 4.12: Optimized tower designs using three different design strategies, with fatigue utilization (left) and buckling utilization (right). The wall thickness is scaled by a factor of 40 relative to the diameter for illustration purposes.
Table 4.2: Normalized platform and tower costs using different design strategies.

<table>
<thead>
<tr>
<th>Design</th>
<th>Tower cost</th>
<th>Platform cost</th>
<th>Total cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>T2</td>
<td>1.00</td>
<td>1.03</td>
<td>1.02</td>
</tr>
<tr>
<td>T3</td>
<td>1.29</td>
<td>1.00</td>
<td>1.11</td>
</tr>
</tbody>
</table>

In the event of an integrated optimization (T2), the tower base diameter is significantly increased to maintain a cost-effective fatigue design, which actually results in a tower with equal costs as the T1 design. However, a consequence of the increased base diameter is an increase in wave loads, which requires a more expensive platform design to satisfy the $15^\circ$ pitch criterion. For the tower optimized with a fixed platform design (T3), a large wall thickness is needed to satisfy the fatigue criterion at the base, which results in an inefficient tower design with almost 30% increase in costs.

From the results in Figs. 4.11 and 4.12, the presented optimization methodology is seen to result in support structure designs which are close to fully utilized with respect to fatigue over a large part of the length, which minimizes the design costs within the applied constraints. However, this affects the fatigue reliability of the total support structure, as discussed in P5. The reliability index after 20 years of operation for support structures optimized with different design fatigue factors (DFFs) is shown in Fig. 4.13(a). Here, the names of the designs indicate the DFF used for the tower, while values twice as high were applied for the platform due to more difficult inspection access. As expected, relatively constant reliability levels are observed along the tower length.

For structures where a single component (i.e. potential fatigue crack location) dominates the failure probability, system effects for the reliability can be neglected. However, the almost constant fatigue reliability along the length of the optimized FWT structures means that system reliability must be considered. The support structure of a spar FWT can be modelled as a series system, where failure in one of the components results in failure of the whole structure.

The importance of system effects on the resulting reliability is illustrated in Fig. 4.13(b), where the system reliability index after 20 years of operation is shown with different number of components included in the system reliability calculations. For systems with less than 12 components, the most
critical ones, based on the results in Fig. 4.13(a), were included.

The importance of system effects depended on the correlation between the components. A common assumption is to use independent fatigue strength in different components, while the loads are taken as fully correlated. This assumption, using typical values from the literature, resulted in relatively low correlation values between the components.

A large reduction in reliability is observed in Fig. 4.13(b) as the number of components increases, due to the consistent failure probabilities and low correlation between the components. Based on the results presented here, the fatigue safety factor needs to be doubled for a system with eight fatigue-critical components to achieve the same reliability after 20 years as a support structure with a single critical component.

These effects must be appropriately considered in design, which can be done in a simple manner by increasing the required DFF along the entire support structure, and optimizing the structural design with consistent probabilities of failure in a large number of components. However, the system effects observed here suggest that more cost-effective designs might be achieved if less steel-intensive components are designed with longer fatigue life (or larger DFF), in order to reduce the system effects. To derive truly optimized fatigue designs for a given safety level, inclusion of system reliability con-
Considerations within the design optimization problem is required, which has not been considered in the present work.

### 4.2.4 Control strategies

Control of FWTs is a complex subject, and several strategies for the blade-pitch controller have been suggested in the literature. All of these control strategies result in trade-offs between structural loads, rotor speed variation, and blade pitch actuator use, which vary with different environmental conditions, and identifying optimal control parameters is a challenging task. Also, due to the strong interactions, simultaneous design of the controller and support structure should be performed to have a fair comparison between different strategies. To properly identify and compare optimal solutions, the integrated control and structural designs should be evaluated over the lifetime of the system, considering actual design limits.

To illustrate the trade-offs between rotor speed variation, blade-pitch actuator use, and structural loads in the tower, the effect of the blade-pitch controller PI gains on the linearized system response is shown in Fig. 4.14 for a given support structure design. To evaluate blade-pitch actuator use, the actuator duty cycle (ADC), defined as the total number of degrees pitched divided by the total simulation time, is used. Here, the ADC values are also normalized by the maximum allowable blade pitch rate for the turbine, which for the DTU 10 MW is equal to 10 deg/s. The contours are shown for the area where the closed-loop system is stable (i.e. no negative damping in any modes). As shown, the optimal solution for an integrated design optimization process depends heavily on the relative importance of the response parameters.

Integrated optimization of the blade-pitch controller and support structure design, considering different control strategies, were investigated in P4. A baseline strategy, using a gain-scheduled PI controller, was considered in addition to three different velocity feedback controllers, as described in Table 4.3. In CS4, the nacelle velocity signal was passed through a first order low-pass filter to remove the wave-frequency components before it was fed back to the blade-pitch controller. On the control side, the considered design variables were the PI gains (CS1-4), the velocity feedback gain from Eq. (3.32) (CS2-4), and the corner frequency of the low-pass filter for the velocity signal (CS4).

In addition to the controller parameters, the design optimization considered the diameter and wall thickness distribution of the tower, with constraints
4.2. Integrated design optimization

(a) Weighted average rotor speed variation.  
(b) Long-term tower base fatigue damage.  
(c) Weighted average ADC.

**Figure 4.14:** Response parameters as functions of PI controller gains.

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>CS1</td>
<td>PI controller</td>
</tr>
<tr>
<td>CS2</td>
<td>PI controller + platform pitch velocity feedback</td>
</tr>
<tr>
<td>CS3</td>
<td>PI controller + nacelle velocity feedback</td>
</tr>
<tr>
<td>CS4</td>
<td>PI controller + nacelle velocity feedback + low-pass filter</td>
</tr>
</tbody>
</table>
Research Findings on long-term fatigue and global buckling. Because the structural design of the platform was found to mostly be driven by hydrostatic buckling loads unaffected by the controller, the study considered only the geometry of the platform through the draft and diameter distribution, while the wall thickness was assumed fixed. This procedure thus neglected the potential cost savings from reduced wall thickness in the uppermost part of the hull, where fatigue loads were more critical.

The optimization aimed to minimize the costs of the support structure, while constraints on the rotor speed standard deviation and actuator duty cycle (ADC) were included to ensure a certain rotor speed tracking and pitch actuator fatigue performance. As appropriate values for these constraints are difficult to quantify, values based on simulations with an onshore DTU 10 MW turbine were used.

The velocity feedback controllers increased the aerodynamic damping and thus reduced the response around the pitch natural frequency. This led to a decrease in fatigue loads, which were the design-driving constraint for the tower. For all four design strategies, a 50-year storm condition with parked turbine was found to be the critical load case for the extreme responses, and the ULS design was therefore not affected by the controller.

The optimized costs of the tower, platform, and tower plus platform for CS2-4 are shown in Fig. 4.15(a), compared to the costs for the optimized CS1. The majority of the cost reductions came from the tower, due to the improved fatigue behaviour, whereas the platform was less affected due to the fixed costs related to the 15° pitch angle constraint. However, because a lighter tower resulted in a lower overall center of gravity, the platform pitch response was somewhat improved, and a small reduction in platform costs of about 2% was also achieved. Because the platform accounted for 70-75% of the total costs for the considered designs, the resulting total cost reduction was approximately 6%.

The rotor speed standard deviations and ADCs, normalized by their maximum allowable values, are shown in Fig. 4.15(b). For each control strategy, there exists a limit where no further reduction in cost can be achieved by increasing the actuator use, as seen from Fig. 4.14. This limit is higher for the velocity feedback controllers than for a controller using only the rotor speed error as input, which caused the ADC constraint to be inactive at the optimum for CS1. For the rotor speed variation, better performance was achieved with CS4 than with the other control strategies. Since there is a trade-off between rotor speed variation and structural loads (and thus costs)
as previously discussed, it is expected that larger cost reductions could be achieved with CS4 if the rotor speed constraint is tightened.

Verification against nonlinear simulations in SIMA showed that the tower base fatigue damage was significantly overestimated for CS1, whereas good agreement was obtained with the other control strategies. The reason for the poor agreement was that the resulting aerodynamic damping for the platform pitch mode was much lower with this control strategy. Consequently, the resonant pitch response became very sensitive to the presence and amount of additional damping in the system, which was either not considered or underpredicted in the linear model. The overestimation of fatigue damage suggests that the cost reductions in Fig. 4.15(a) are highly optimistic, and that CS1 may yield a fatigue design similar to that with a nacelle velocity feedback controller. It also suggests that future optimization studies using the linearized model from the present work should consider more advanced control strategies than CS1, to limit the pitch response error.

4.3 Environmental modelling

Due to lack of site-specific data, or to limit the computational effort during the design process, conservative assumptions about the environmental loads are often used. The impact of different environmental modelling uncertainties on the fatigue reliability and associated design costs was assessed in P5, using an environmental model based on hindcast data from a realistic floating wind park site. In addition to the base case, four different environmental models were considered:

- Stochastic turbulence intensity described by the Weibull distribution from IEC (2005), rather than the 90% design value.
- Inclusion of the wind directional distribution.

Figure 4.15: Objective and constraint function values for optimized designs.
- Inclusion of the relative wind-wave direction distribution.

- A two-peak wave spectrum to account for swell.

The resulting equivalent fatigue factors with the different environmental models, based on Monte Carlo simulations on the linearized FWT model, are presented in Fig. 4.16(a). Here, the equivalent fatigue factor is defined as the maximum expected 20-year fatigue damage around the circumference, normalized by the base case results. The factors are shown for the fatigue-critical parts of the support structure, which consisted of the tower and upper part of the platform. For depths below approximately 10 m, shell buckling became the design driving constraint, and the differences in fatigue loads did not affect the design process.

The effect of turbulence and wind-wave misalignment varied significantly along the length of the structure due to differences in the relative importance of wind and wave response. The tower top response, which was almost exclusively governed by wind loads, was much more affected by the turbulence modelling than the tower base, and vice versa for the wind-wave misalignment. This also resulted in some location-dependent effects when the wind directional distribution was included. The inclusion of swell through a two-peaked wave spectrum had small impact on the fatigue reliability for the considered support structure.

Applying all the aforementioned environmental model uncertainties resulted in an almost constant reduction in fatigue damage of about 65% along the length of the tower and upper part of the platform. This indicates that an additional safety factor on fatigue of about three is implicitly applied when these model uncertainties are neglected in the design process for such FWT structures.

Reductions in support structure costs with the different environmental models were quantified through re-design of the support structure, using a numerical optimization scheme as described in the previous sections. To account for the environmental modelling effects in the design optimization procedure, the equivalent fatigue factors from Fig. 4.16(a) were included in the applied fatigue safety factors along the tower and platform. The resulting cost reductions are shown in Fig. 4.16(b), where total cost reductions in the order of 5-11% is observed. These numbers are only indicative, and depend on the platform concept, considered design constraints, and the metocean conditions at the actual wind park site.
4.4 Inspection schedule

This section considers derivation of cost-optimal DFFs, including costs related to construction and inspections during the operational lifetime. The required fatigue crack inspection interval for a FWT depends on the applied DFF, which results in trade-offs between CAPEX and OPEX. To minimize the cost of energy, the DFF resulting in the lowest lifetime costs should be applied. This issue was investigated in \textit{P5}, based on probabilistic fracture mechanics and reliability updating using Bayes theorem. The study was performed with the three different set of DFFs described in Section 4.2.3. To evaluate lifetime costs of the considered designs, necessary inspection intervals with DFF1 and DFF2 to achieve the same reliability as DFF3 without inspections at the end of the service life were identified. The inspections were assumed to occur with fixed intervals for each design.

Figure 4.17(a) shows the accumulated reliability index over the service life of the system, assuming inspections with no detection of cracks every two and five years for DFF1 and DFF2, respectively. This corresponds to a total of nine and three inspections during the lifetime of the structure, which resulted in reliability indices within 5\% of DFF3 without inspections after 20 years.

Based on the results in Fig. 4.17(a), trade-offs between CAPEX and OPEX were derived. In Fig. 4.17(b), the lifetime costs are shown for different
(a) System reliability index during service life, including the effect of inspections.
(b) Combined design and inspection costs comparison, $r=0.05$.

**Figure 4.17:** Reliability and total costs with different DFFs and inspection schedules.

Values of the inspection costs, assuming a real interest rate of 5%. In this particular case, DFF1 was the most cost-effective solution for costs per inspection lower than 210 000 € while DFF3 was the cost-optimal design for costs of more than 650 000 € per inspection. This corresponded to about 2% and 6% of the support structure costs with DFF1, respectively.

In the present work, higher DFFs were achieved by increasing the structural dimensions. The fatigue life could also be extended by reducing the SCF, which may be a more cost-effective option. However, the cost of improved fabrication tolerances or detail geometry is highly uncertain, and SCF reduction was therefore not considered.
Chapter 5

Conclusions and Recommendations for Future Work

This chapter summarizes the contributions of this thesis to the research community. Suggestions for future studies are also made, based on limitations identified in the current work.

5.1 Conclusions

This thesis has sought to improve the design process for FWTs through the development of efficient methods for global dynamic response analyses, and numerical design optimization techniques. The main focus has been on the support structure for 10 MW spar-type turbines, considering fatigue and ultimate loads. Some attention has also been given to the blade-pitch controller and mooring system, particularly related to coupling effects with the support structure.

A linearized aero-hydro-servo-elastic model with four structural DOFs was shown to yield acceptable accuracy for the support structure response with co-directional wind and waves. Nonlinear effects resulted in poor performance of the linearized model when the tower bending natural frequency was close to the 3P range. Some overestimation was also observed for the resonant platform pitch response, which caused the model to be conservative in general compared to nonlinear analyses. Analytical frequency-dependent aerodynamic damping and inertia coefficients were derived, which removed
the underestimation of the low-frequency response observed with constant damping coefficients.

A gradient-based optimization approach with analytic derivatives was used to perform integrated design optimization of the support structure, blade-pitch controller, and mooring system for a 10 MW spar FWT, including the scantling design of the hull, where the goal was to minimize the system costs and rotor speed variation. An hour-glass shape was found to be favourable for the platform below the wave zone, as it increased the effective rotational stiffness of the system and improved the low-frequency behaviour in pitch. The inclusion of structural hull design ensured a realistic distribution of material and proper penalization of designs with large drafts. Due to compliance with the platform, the tower took an unconventional shape, where the diameter increased from the base. Integrated optimization is needed to identify the optimal solution near the interface.

The optimized support structure was fully utilized with respect to fatigue along most of the tower length, as well as in the uppermost part of the platform. Below approximately 10 m below the SWL, shell buckling became design-driving due to the large hydrostatic pressure. The consistently high fatigue utilization in the upper part of the structure led to important system effects for the total fatigue reliability. By considering the support structure as a series system, an additional fatigue safety factor of two was needed for a system with eight fatigue-critical welds, to achieve the same reliability after 20 years as a single critical component. System effects may thus be important for cost-optimal designs with a given target safety level.

Trade-offs between rotor speed tracking, blade-pitch actuator use, and structural loads make controller design a challenging task, and due to important interaction effects, simultaneous design of the controller and support structure should be preferably be performed. The blade-pitch controller affected the fatigue loads in the tower and upper part of the platform due to damping of the resonant pitch response; however, because the critical extreme responses occurred in either below-rated or parked conditions, no effect was observed on the ULS design. Different control strategies were compared through integrated design optimization with constraints on rotor speed variation and blade-pitch actuator use. While appropriate values for these constraints are difficult to quantify, the methodology allowed for identification of optimal control parameters in a lifetime perspective, and fair comparison between different solutions.

Uncertainties in environmental input parameters used in global design ana-
yses are often chosen based on conservative assumptions. The impact of these assumptions on the long-term fatigue reliability and associated costs of the support structure was assessed using Monte Carlo simulations and re-design of the tower and platform based on derived fatigue factors. Wind-wave misalignment, wind directionality, and stochastic turbulence intensity were all shown to have non-negligible effect on the long-term fatigue damage. The inclusion of all three resulted in an almost constant reduction in fatigue along the support structure length of about two-thirds, suggesting that an additional safety factor of three is inherent in the fatigue design when these effects are neglected.

5.2 Recommendations for future work

Based on the studies conducted in P1-5, the following suggestions are made for future work:

- **Alternative platform concepts:**
  The developed numerical design optimization methodology is only applicable for spar platforms, which imposes serious limitations on the design space for the floater. A more general description of the platform geometry would allow the optimizer to explore other parts of the design space, and possibly identify non-intuitive and more cost-effective solutions. This requires a numerical panel method for the hydrodynamic loads, with the support of gradient calculations. For structural optimization of a general platform shape, a more sophisticated finite element model, using e.g. shell elements, may be needed.

- **Nonlinear and transient events:**
  The linearized dynamic model developed in the present work cannot be used to analyse transient or highly nonlinear events. Because such conditions may be design-driving for certain parts of the system, it is recommended to focus on computationally efficient methods to include them in the design optimization process.

- **Improved cost models:**
  The optimal design from a numerical optimization process depends on the cost models. While material costs can be calculated in a straightforward manner, there are large uncertainties related to manufacturing, installation, maintenance, and decommissioning costs. If more reliable engineering cost models can be established, based on the actual lifetime costs of the system, this may have a large effect on the resulting cost-optimal designs.
Conclusions and Recommendations for Future Work

- **Mooring system and power cable representation:**
  A more detailed description of the mooring system, including bridle lines, varying cross sections, and discrete components such as clump weights and buoys, would enable more complete and realistic optimization of the mooring design. Together with wind and wave directionality, such procedures could also consider the layout of the mooring system. A model describing the power cable dynamics would allow for simultaneous design optimization of the cable, including explicit constraints on the curvature.

- **Blade and drivetrain response:**
  A fully integrated FWT design optimization procedure must also consider simultaneous design of the rotor. This requires more detailed structural and aerodynamic models to accurately capture the blade and drivetrain response, as well as appropriate cost models and constraints for these subsystems.

- **Reliability-based design optimization:**
  FWTs are subjected to a stochastic environment, and uncertainties in loads and response are typically considered through partial safety factors. The design conservatism could be reduced through the application of probabilistic design, where the structure is optimized to minimize the cost for a given target probability of failure.

- **Farm-level design optimization:**
  Similarly to an integrated design optimization approach on the turbine level, design optimization considering farm-level effects could affect and improve the overall system behaviour. Important considerations include wake effects, total wind farm costs, system reliability effects, and possibly shared mooring lines or anchors.


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35th International Conference on Ocean, Offshore and Arctic Engineering (OMAE2016), Busan, South Korea, 2016.


SINTEF Ocean. RIFLEX user guide, 2016a.

SINTEF Ocean. SIMO user guide, 2016b.


Appendix A

Appended Papers
Paper 1

A semi-analytical frequency domain model for efficient design evaluation of spar floating wind turbines

John Marius Hegseth and Erin E. Bachynski

Marine Structures, 2019
A semi-analytical frequency domain model for efficient design evaluation of spar floating wind turbines

John Marius Hegseth*, Erin E. Bachynski
Department of Marine Technology, NTNU, 7491 Trondheim, Norway

ABSTRACT

A linear model for efficient design evaluation of spar floating wind turbines is presented, and verified against a nonlinear time domain model with regards to long-term fatigue and short-term extreme response for two different spar designs. The model uses generalized displacements and a semi-analytical approach to establish the equations of motion for the system, which are solved in the frequency domain. The results show agreement within ±30% for the long-term fatigue considering operational conditions, however, the linear fatigue damage estimates are sensitive to the accuracy of the estimated natural frequency of the first bending mode. The results also suggest that a small number of environmental conditions can be simulated with a nonlinear time domain model to verify and possibly tune the linear model, which then can be used to run the full long-term analysis. Short-term extreme tower base bending moments and surge and pitch motions are observed to be nearly Gaussian above cut-out wind speed, as the response is dominated by wave forces. Consequently, the linear model is able to accurately capture the upcrossing rates, which are used to calculate the characteristic largest extreme response. For an operational case near rated wind speed, the response is somewhat non-Gaussian, which gives larger discrepancies between the linear and nonlinear models. However, due to large mean values in this condition, the total error in the extreme response is reduced, and reasonable agreement is achieved.

1. Introduction

The offshore wind industry has had significant growth over the last decade, and as a majority of the global wind resources are located in deeper waters, there has recently been an increased interest in floating wind turbines (FWTs). For floating wind farms to be economically feasible, cost-effective and reliable designs are needed. Design optimization of FWTs is a complex task, and due to interactions between aerodynamics, hydrodynamics, structural dynamics and control, coupled nonlinear time domain (TD) simulations are usually applied. As numerous load cases need to be analysed, the design process becomes computationally very expensive. It is therefore desirable to use simplified models, especially in preliminary design [1]. Frequency domain (FD) models provide an efficient way of performing dynamic simulations and are frequently used for floating structures in the offshore industry, usually in fatigue analyses [2]. Although less common in the offshore wind industry, design standards state that frequency domain analyses also can be used to calculate the fatigue loads for FWTs, if validation against time domain analyses or experiments is performed [3].

Several studies have investigated the applicability of FD models for FWTs. Typically, these models have considered first order hydrodynamic loads from potential theory and steady aerodynamic loads, and have been used to calculate response amplitude operators (RAOs) for the rigid body motions of the platform [4–7].

* Corresponding author.
E-mail address: john.m.hegseth@ntnu.no (J.M. Hegseth).

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Although useful information about the system may be found using only rigid body motions, the elasticity of the tower has been shown to significantly affect the global response of FWTs. Bachynski and Moan [8] compared a linear FD model with three rigid body modes to a nonlinear TD model for different tension leg platform wind turbine designs in both wave-only and combined wind-wave conditions. The study was later expanded to include turbulent wind excitation and aerodynamic damping [9], and concluded that the tower base bending moment response was expressed as a dynamic amplification factor. The model was found to perform reasonably well in the 13 considered load cases, however, the fatigue damage was underestimated by up to 60% due to its exponential nature. A somewhat different approach was applied by Kluger et al. [11], who used statistical linearization to develop a FD model for the OC3 spar wind turbine [12]. The two first elastic tower modes were found for a fixed foundation and included in the analysis, and equivalent fatigue stress (EFS) at the tower base due to wave excitation and steady wind was calculated for 11 different environmental conditions. The results were compared to values from TD simulations reported by Matha [13], where the FD analysis was found to underestimate the total EFS by 12%.

Although earlier work has assessed the fatigue damage based on FD analyses for FWTs, these calculations have only been performed for a limited number of environmental conditions. Thus, little information exists regarding the accuracy of such models for a full long-term analysis. In addition, the design process must also consider the extreme response of the system, which can be a comprehensive task. Results from previous studies have shown that both the tower base bending moment and global motions [14] for spar FWTs can be quite Gaussian in harsh environmental conditions, which suggests that a linear model may also be used to assess the extreme response in early stages of design. For a spar FWT, in addition to the ultimate stresses in the structure, the extreme surge and pitch response of the platform may be of interest, as these motions are important for loads on the mooring system and nacelle components, respectively.

In the present work, a semi-analytical FD approach is presented, where generalized degrees-of-freedom (DOFs) are used to describe the dynamic behaviour of the FWT. The system is linearized, and due to the simple geometry of the hull, closed-form expressions may be used for the hydrodynamic loads with good accuracy. This removes the need for a separate numerical hydrodynamic analysis, which significantly increases the computational efficiency. The aerodynamic loads are found numerically from TD simulations, and the forces are superimposed to find the total response. The approach is then compared to fully coupled nonlinear TD simulations for two different 10 MW spar FWT designs, considering long-term fatigue damage at the tower base, as well as short-term
extreme values for the tower base bending moment and surge and pitch motions of the platform in selected conditions along the 50-year environmental contour. The linear FD model presented in the present work is not applicable for detailed analyses, but may complement state-of-the-art tools in early stages of design. The model can help the designer get a quick overview of the response of the system, and give indications of how the system responds to changes in the design parameters. It may also be used to identify critical load cases, which then can be analysed using coupled nonlinear TD analyses.

2. System description

2.1. Platform designs

The two spar buoys considered in the current work are shown in Fig. 1. The designs are based on the OC3-Hywind design [12] and consist of two cylinders with different diameters, connected by a linearly tapered section. The hull extends to a height of 10 m above still water level (SWL), where it is fixed to the tower. Spar 1 has the same draft as the original OC3 spar of 120 m, but with increased diameters to provide enough buoyancy to support a 10 MW wind turbine, and to match the tower base diameter. The large draft of the spar provides good stability and hydrodynamic performance, however, it limits the use to deep water, and complicates construction and towing to site. The draft of the second design (spar 2) was therefore reduced by 25%, which makes it more suitable for intermediate water depths. In addition to the draft reduction, the lower hull diameter was increased by 25% compared to spar 1, to achieve a sufficient amount of buoyancy.

The two designs are presented in Table 1, where values for the vertical centre of gravity (CoG) and moment of inertia are calculated by assuming that the hulls are partially filled with concrete ballast to achieve the correct draft, using a ballast density of 2600 kg/m³, while a constant wall thickness of 6 cm is assumed for the steel hull. The water depth for both platforms is 320 m.

![Fig. 1. Spar designs considered in the present work. Spar 1 (left) and spar 2 (right).](image_url)

Table 1
Platform properties.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Spar 1</th>
<th>Spar 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Draft (m)</td>
<td>120.0</td>
<td>90.0</td>
</tr>
<tr>
<td>Elevation to tower base above SWL (m)</td>
<td>10.0</td>
<td>10.0</td>
</tr>
<tr>
<td>Depth to top of taper below SWL (m)</td>
<td>4.0</td>
<td>4.0</td>
</tr>
<tr>
<td>Depth to bottom of taper below SWL (m)</td>
<td>12.0</td>
<td>12.0</td>
</tr>
<tr>
<td>Diameter above taper (m)</td>
<td>8.3</td>
<td>8.3</td>
</tr>
<tr>
<td>Diameter below taper (m)</td>
<td>12.0</td>
<td>15.0</td>
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<tr>
<td>Mass including ballast (kg)</td>
<td>1.18E+7</td>
<td>1.33E+7</td>
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<tr>
<td>Displacement (m³)</td>
<td>1.31E+4</td>
<td>1.49E+4</td>
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<tr>
<td>Moment of inertia about CoG (kgm²)</td>
<td>6.53E+9</td>
<td>3.42E+9</td>
</tr>
<tr>
<td>Vertical CoG below SWL (m)</td>
<td>94.7</td>
<td>72.3</td>
</tr>
<tr>
<td>Vertical CoB below SWL (m)</td>
<td>62.0</td>
<td>47.8</td>
</tr>
</tbody>
</table>
2.2. DTU 10MW wind turbine with modified tower

The spar buoys support the DTU 10 MW reference wind turbine [16], with the tower shortened by 10 m to achieve the same hub height as the onshore turbine at 119 m above SWL. The tower has a linearly tapered outer diameter and consists of ten sections with constant wall thickness in each section. In the modified tower, each section is shortened by 1 m, while the top and bottom diameters are kept unchanged. The key characteristics of the turbine are listed in Table 2.

The natural frequencies of the tower change when the turbine is placed on a floating substructure, and preliminary calculations showed that the natural frequency of the first bending mode for spar 2 was very close to the blade passing frequency at rated rotor speed. The wall thickness in all sections of the tower was therefore increased by 50% for spar 2, which moved the natural frequency above the 3P range.

2.3. Mooring system

The mooring system, described in Table 3, consists of three catenary lines spread symmetrically about the vertical axis. As in the OC3 project, a simplification is made in that the delta lines are removed, and lines with constant properties are instead used all the way up to the fairleads. A rotational spring is added to the model to ensure that the yaw stiffness from the mooring system is included. The fairleads are placed at a depth equal to the total CoG of the system including the wind turbine, in order to limit the coupling between surge and pitch motion. This results in a slightly stiffer mooring system for spar 2.

3. Time domain model

The fully coupled nonlinear aero-hydro-servo-elastic analyses are carried out in the time domain using the simulation workbench SIMA developed by SINTEF Ocean, which couples two computer codes: Riflex, a finite element solver developed for flexible beam elements; and SIMO, which calculates large volume hydrodynamic loads [17,18]. The spar buoys are modelled as six-DOF rigid bodies with first order wave forces found numerically from potential flow theory using WAMIT [19], combined with viscous forces from the drag term in Morison’s equation, which are integrated up to the instantaneous wave elevation. Second order potential flow forces are not considered in the model, as studies have shown that these loads have limited effect on the response for spar-type FWTs [20,21].

Bar elements with only axial stiffness are used to model the mooring lines, together with hydrodynamic loads from Morison’s equation. The model thus includes the nonlinear restoring forces from the mooring system, as well as the dynamic behaviour of the mooring lines. The tower and blades are modelled using flexible beam elements.

The aerodynamic loads are calculated using blade element/momentum (BEM) theory, including Glauert correction, Prandtl hub and tip loss factors, dynamic stall, dynamic wake, skewed inflow and tower shadow. The code has previously been verified for FWTs [22,23]. As wind drag forces on the tower may become important at high wind speeds, this effect is included for the extreme response conditions using a drag force formulation with a drag coefficient of 0.7.

An external control system written in Java is used to modify the generator torque and blade pitch. In order to avoid pitch motion

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Spar 1</th>
<th>Spar 2</th>
</tr>
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<tbody>
<tr>
<td>Radius to anchors (m)</td>
<td>855.2</td>
<td>855.2</td>
</tr>
<tr>
<td>Unstretched mooring line length (m)</td>
<td>902.2</td>
<td>902.2</td>
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<tr>
<td>Equivalent mooring line mass density (kg/m)</td>
<td>155.4</td>
<td>155.4</td>
</tr>
<tr>
<td>Equivalent mooring line axial stiffness (MN)</td>
<td>3.84E-8</td>
<td>3.84E-8</td>
</tr>
<tr>
<td>Fairlead depth below SWL (m)</td>
<td>77.2</td>
<td>56.3</td>
</tr>
<tr>
<td>Yaw spring stiffness (Nm/rad)</td>
<td>1.48E+8</td>
<td>1.48E+8</td>
</tr>
</tbody>
</table>
instability above rated wind speed due to negative feedback \[24,25\], the proportional and integral gains for the blade pitch are modified from the original controller. With the resulting gains, the controller has a natural frequency of 0.13 rad/s and a damping ratio of 0.7.

4. Frequency domain model

In the frequency domain model, a linear representation of the system is created as shown in Fig. 2. For simplicity, only the response in the xz-plane is considered in the present study. The platform is considered rigid, while the tower is modelled as a slender flexible beam. The rotor and nacelle assembly (RNA) is replaced by a point mass and inertia at the top of the tower, with resultant wind loads (thrust force, rotor pitching moment, and aerodynamic damping) acting on the tower top. The mooring system is represented by a linear spring at the position of the fairleads. As the structure is statically determinate, internal loads in the hull can be found by considering equilibrium between external, inertial, damping, restoring and internal reaction forces. In the present work, the tower base bending moment is the only internal load which is considered (see also Appendix A), but the method can easily be applied to other components.

Generalized displacements are used in combination with the principle of virtual work to establish the dynamic equations of motion \[26\], which are solved in the FD. In this procedure, the total response is described by a weighted combination of an arbitrary number of shape functions, \( \psi \). The accuracy of the formulation thus depends on how well the actual displacement field is captured by the shape functions, which typically are chosen to represent the most important eigenmodes of the system.

Three generalized DOFs are included in the model, namely surge, pitch, and the first bending mode, as illustrated in Fig. 3. Orthogonal eigenmodes can be found numerically from the solution of the eigenvalue problem for a finite element (FE) model of the linearized system. However, as the shapes of the rigid body modes are known a priori, they are instead chosen as analytical functions for simplicity. In addition, to have global motions that are consistent with the TD model, the pitch mode is chosen as a rotation about the SWL. Using non-orthogonal modes introduces off-diagonal terms in the system matrices, however, these coupling effects are readily taken into account in the formulation.

The actual shape of the bending mode, on the other hand, is not known, and is therefore found from the eigenvalue solution. A third-order spline function is then fitted to the nodal displacements to have continuous expressions for the first and second order derivatives. The FE model used to solve the eigenvalue problem is based on the linearized system in Fig. 2 and does not include the blades, however, added mass on the hull is included using strip theory.
4.1. Load formulation

In order to predict the dynamic behaviour of the system, the inertia, damping, restoring and excitation forces on the structure need to be determined. Due to the simple and relatively slender geometry of the hull, the hydrodynamic loads on the floater can be simplified without significant loss of accuracy. The transverse added mass per unit length is approximated using the analytical expression for a 2D circular cylinder with diameter $D$ in infinite fluid [27]:

$$a_{11}(z) = \frac{\rho \pi D^2}{4} z$$

(1)

The linear wave excitation forces are taken from MacCamy-Fuchs theory [28]. The force per unit length, $dF_w$, for a regular wave with unit amplitude is given by

$$dF_w(z, \omega) = \frac{4\pi g}{k} \frac{\cosh k(z + h)}{\cosh kh} \frac{\cosh(\omega t - z)}{k} G e^{i(\omega t - z)}$$

(2)

Fig. 3. DOFs considered in the analysis.

Fig. 4. Aerodynamic damping ratio for the first fore-aft mode of the land-based DTU 10 MW turbine as a function of wind speed.
where
\[ G = \frac{1}{\sqrt{(J'_1(ka))^2 + (Y'_1(ka))^2}}, \quad \tan \alpha = \frac{J'_1(ka)}{Y'_1(ka)}. \] (3)

For the aerodynamic loads, the BEM equations usually have to be solved in the TD [29], and there is no straight-forward way to establish linearized frequency-dependent wind forces. In the present study, the turbulent thrust force \( F_r \) and rotor moment \( M_r \) for different wind speeds are extracted from TD simulations on a fixed rotor in SIMA. The resulting time series are transformed to the FD using fast Fourier transform (FFT), and normalized with the incoming wind spectrum to create force transfer functions. In addition, the so-called aerodynamic damping, which arises due to the change in thrust force as a result of the nacelle’s velocity, needs to be explicitly calculated, which can be done in various ways [29,30]. Here, the linearized damping is estimated following the procedure described by Bachynski [9]. Time domain simulations with a range of constant wind speeds are run on a fixed rotor, and the damping values are found as the change in thrust force for small variations in wind speed while the blade pitch and rotor speed are kept fixed:
\[ B_{\text{aero}} = -\frac{dF_r}{dU}. \] (4)

The resulting aerodynamic damping values are shown as a function of mean wind speed in Fig. 4, presented as critical damping ratio for the first fore-aft tower mode of the original land-based DTU 10 MW turbine. The linearized aerodynamic damping applies to all generalized modes with nonzero deflection at the tower top, and the coefficient \( B_{\text{aero}} \) is varied with the mean wind speed. Damping caused by pure rotation of the rotor is not considered. It is assumed that any changes in aerodynamic forces that arise due to the motions of the FWT are captured by the aerodynamic damping term, which means that variations in control system outputs caused by platform motions are not considered in the model.

Rayleigh damping is used to model structural damping in the tower, with a stiffness-proportional coefficient \( \beta = 0.007 \) for both designs. For a given \( \beta \), the damping ratio is proportional to the natural frequency, and using a constant value will thus reward stiffer designs, such as spar 2, without any physical reason. In an actual design process, the coefficient should therefore be continuously updated to keep the damping ratio constant. However, as the scope of the current work is not to perform design calculations, but rather to compare different modelling techniques, a constant coefficient is used for simplicity. The applied coefficient corresponds to a damping ratio of 0.9% and 1.2% at the first bending natural frequency for spar 1 and spar 2, respectively.

Viscous damping, which is important for the low-frequency surge response, is added based on stochastic linearization of the quadratic drag term in Morison’s equation [31]:
\[ \frac{1}{2} \rho C_d D(z) u(x) = \frac{1}{2} \rho C_d D(z) \frac{S_\zeta}{\pi} \sigma_{\zeta}(z) \alpha = b_{\text{visc}}(z) u(x) \] (5)
with a drag coefficient \( C_d = 0.7 \). The standard deviation of the velocity, \( \sigma_{\zeta}(z) \), is found using an iteration scheme. Viscous wave excitation was found to be small compared to the wind excitation forces, and is therefore not included in the model. Wave radiation damping is also neglected. The aerodynamic drag forces on the tower, which only are included in the extreme response conditions, are taken into account in the linear model as a combination of a mean force, which is added directly, and a frequency-dependent force, which is found using stochastic linearization. Here, only the excitation forces are considered, meaning that the tower drag forces arising from the movement of the turbine are neglected.

The analytical solution for quasi-static horizontal tension in elastic catenary lines can be found in e.g. Faltinsen [32]. In order to find the linear mooring stiffness used in the model, \( K_t \), this equation is differentiated numerically. The stiffness is calculated for the zero offset position of the hull in all environmental conditions, as the offset was found to have little effect on the results. However, updated stiffness values based on the actual mean position of the platform can easily be included. Mooring line excitation, damping and inertia are neglected in the model.

The hydrostatic restoring in pitch, which is applied as a rotational spring at the SWL, is found from
\[ K_r = \rho g V z_B - M g z_B + \rho g I_{\text{yp}}. \] (6)

4.2. Establishing the equations of motion

Using the vector containing the shape functions and the terms derived in the previous section, the generalized mass (including added mass), damping and stiffness matrices can be established [26]:
\[ M = \int_0^{z_{\text{top}}} m(z) \psi^T(z) \psi(z) dz + \int_{-L}^0 [m(z) + a_1(z)] \psi^T(z) \psi(z) dz + M_{\text{RNA}} \psi^T(z_{\text{top}}) \psi(z_{\text{top}}) + I_{\text{RNA}} \frac{\psi^T(z_{\text{top}}) \psi(z_{\text{top}})}{z_{\text{top}}}, \] (7)
\[ B = \beta \int_{-T_{\text{TB}}}^{z_{\text{top}}} \frac{E l(z) \psi^T(z) \psi(z) dz + \int_{-L}^0 b_{\text{visc}}(z) \psi^T(z) \psi(z) dz + B_{\text{aero}} \psi^T(z_{\text{top}}) \psi(z_{\text{top}})}{z_{\text{top}}}, \] (8)
where \( N(z) \) is the axial force in the tower. The generalized wind and wave load vectors are found from

\[
\mathbf{F}_W(z) = \int \mathbf{T}(z) \psi(z) dz + \int N(z) \mathbf{u}_w(z) dz + \mathbf{K}_w \psi(z) \psi^T(z) \mathbf{0},
\]

\[
\mathbf{F}_W(z) = \int \mathbf{T}(z) \psi(z) dz.
\]

4.3. Frequency domain response

The transfer functions relating modal response to wave and wind input, \( H_{\mathbf{X}}(\omega) \) and \( H_{\mathbf{U}}(\omega) \) respectively, are defined as

\[
H_{\mathbf{X}}(\omega) = \mathbf{H}_{\mathbf{X}}(\omega) \mathbf{F}_W(\omega), \quad H_{\mathbf{U}}(\omega) = \mathbf{H}_{\mathbf{U}}(\omega) \mathbf{F}_W(\omega)
\]

where

\[
\mathbf{H}_{\mathbf{X}}(\omega) = [-\omega^2 \mathbf{M} + i \omega \mathbf{B} + \mathbf{K}]^{-1}
\]

is the frequency response function matrix. Transfer functions between tower base bending moment and wind/wave input (\( H_{\mathbf{M}}(\omega) \) and \( H_{\mathbf{U}}(\omega) \)) are found by considering equilibrium between external, inertial, damping and internal forces. The complete equations may be found in Appendix A. As both the wind speed and wave elevation are considered to be stationary Gaussian processes within the short-term duration, the linear response will also be a stationary Gaussian process. The responses to wind and wave input are assumed to be independent; that is, there is no interaction between the responses at different frequencies. This is an assumption inherent in the linearization, and may not be equally applicable for all FWTs.

The response spectrum for an arbitrary response parameter \( \xi \) is then found by superimposing the wind and wave responses:

\[
\mathbf{S}_\xi(\omega) = |H_{\mathbf{X}}(\omega)|^2 S^\xi(\omega) + |H_{\mathbf{U}}(\omega)|^2 S^\xi(\omega),
\]

while the variance is given as

\[
\sigma^2(\xi) = \int_0^\infty S(\omega) d\omega.
\]

5. Dynamic simulations

5.1. Environmental conditions

The long-term fatigue assessment uses the joint probability distribution of metocean parameters given in Johannessen et al. [33]. Probability density functions (PDFs) for the parameters are shown in Fig. 5. IEC 61400-3 [34] prescribes a minimum bin size of 2 m/s for the wind speed, 0.5 m for wave heights, and 0.5 s for wave periods. Kvittem and Moan [35] performed a long-term fatigue analysis of a semi-submersible FWT, where damage sensitivities with regards to simulation length, bin size and number of samples were addressed. Based on their findings, the bin sizes for wave heights and wave periods in the present work are increased to 1 m and 1 s, respectively, and each condition is simulated using six 1-h realizations after removal of transients.

Long-term fatigue analyses used in design of FWTs should cover all relevant conditions over the lifetime of the structure, including operational, fault, idling, and survival conditions. The fatigue analysis presented in this study is limited to operational cases, which means that the mean wind speed is varied between 4 and 25 m/s, and the turbine is assumed to be operating normally. A total of 546 environmental conditions (ECs) are thus considered in the analysis, which corresponds to all ECs within 4–25 m/s with probability of occurrence greater than 10^{-4}. It is worth noting that, although not employed in the current fatigue comparison, the FD model also is applicable for wind speeds below cut-in or above cut-out, as long as only steady-state conditions are considered.

Three-dimensional turbulent wind time series are generated using TurbSim [36], assuming a Kaimal spectrum for IEC61400-1 class B turbines and a normal turbulence model. A power law profile with exponent 0.14 is used to model the vertical wind shear [34]. The same wind files are used both directly in the TD simulations, and to establish the wind thrust and rotor moment transfer functions used in the FD calculations, which are derived by taking the average values over the six seeds. The calculated transfer functions for each individual wind time series, as well as the average curves, are shown in Fig. 6 for 11 m/s mean wind speed, to illustrate the degree of variability between the realizations. Wave time series are generated using JONSWAP spectra with a peakedness parameter of 3.3. Co-directional waves and wind travelling in the positive x-direction are applied in all simulations, and no current is considered in the analysis.

Four ECs along the 50-year contour surface are selected to compare the short-term extreme response, as described in Table 4. In EC 1, the mean wind speed is close to the rated speed of the turbine, which gives the maximum thrust force on the rotor. EC 2 considers an operational condition near cut-out, while ECs 3 and 4 represent wind speeds above cut-out, where the turbine is parked.
and the blades are feathered. As for the fatigue analysis, 1-h simulations (excluding transients) are used in the extreme response comparison, but with 20 different random seeds for each condition. No fault conditions are considered, however, previous studies on blade pitch faults, grid loss and shutdown have shown that such events primarily affect the loads on the blades and shaft, and that the tower base fore-aft bending moment and global motions tend to be smaller than during fault-free conditions for spar-type FWTs [37,38].

5.2. Fatigue damage assessment

The fore-aft axial stress at the outer radius of the tower is calculated from

\[
\sigma_x = \frac{N}{A} + \frac{M_y}{I} r.
\]  

(16)

Stress variations due to fluctuations in the axial force are found to be negligible compared to the moment-induced stress.

![Fig. 5](image-url). Marginal distribution of 1-h wind speed, and conditional distributions of wave height and peak period. Based on Johannessen et al. [33].

![Fig. 6](image-url). Calculated transfer functions for aerodynamic excitation loads at 11 m/s mean wind speed. Six different wind realizations (grey) and average values (black).

Table 4

<table>
<thead>
<tr>
<th>Condition</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean wind speed at hub height, ( U ) (m/s)</td>
<td>11.0</td>
<td>25.0</td>
<td>36.0</td>
<td>50.0</td>
</tr>
<tr>
<td>Significant wave height, ( H_s ) (m)</td>
<td>6.9</td>
<td>10.3</td>
<td>12.9</td>
<td>15.1</td>
</tr>
<tr>
<td>Spectral peak period, ( T_p ) (s)</td>
<td>10.0</td>
<td>12.5</td>
<td>14.0</td>
<td>16.0</td>
</tr>
<tr>
<td>Turbulence intensity at hub height, ( I ) (−)</td>
<td>0.18</td>
<td>0.14</td>
<td>0.13</td>
<td>0.12</td>
</tr>
</tbody>
</table>
variations, and are thus not included in the fatigue calculations. In the TD, rainflow counting is used together with SN curves and the Miner-Palmgren rule to estimate the fatigue damage, which is taken as the average value over the six realizations for each EC. In the FD, two different methods are considered:

1. Inverse Fourier transform (IFT)
2. Dirlik method

In the first method, 1-h stress time series are generated by performing IFT on the response spectrum with random phase angles. The fatigue damage is then calculated using rainflow counting and the Miner-Palmgren rule. The phase angles introduce randomness to the damage estimates, and the method is therefore performed with six different random seeds, as for the TD model. The average value is then used in the fatigue comparison.

The Dirlik method [39] uses the stress response spectrum and empirical factors to fit the PDF of the stress cycles to a combination of an exponential and two Rayleigh distributions, and the accuracy of the method is therefore dependent on how well the rainflow-count can be represented by these distributions. The expression for the fatigue damage during $T$ seconds becomes [26]:

$$D_{DK} = \frac{\nu_{ref}^T}{K}(2\sigma)^n\left(G_1Q^n\Gamma(1 + m) + (\sqrt{2})^n(G_2R^n + G_1)\Gamma\left(1 + \frac{m}{2}\right)\right)\left(\frac{t}{t_{ref}}\right)^{mk}$$

(17)

where

$$G_1 = \frac{2(x_m - \alpha_1^2)}{1 + \alpha_1^2}, \quad G_2 = \frac{1 - \alpha_1 - G_1 + G_1^2}{1 - R}, \quad G_3 = 1 - G_1 - G_2, \quad Q = \frac{1.25(\alpha_2 - G_3 - G_2R)}{G_1}, \quad R = \frac{\alpha_2 - x_m - G_1^2}{1 - \alpha_2 - G_1 + G_1^2}$$

(18)

and

$$\alpha_2 = \frac{m_2}{\sqrt{m_0m_4}}, \quad x_m = \frac{m_1}{m_0} \left(\frac{m_2}{m_4}\right)^{\frac{1}{2}}, \quad \nu_{ref} = \frac{1}{2\pi} \sqrt{\frac{m_4}{m_2}}.$$

(19)

Here, $m_n$ is the $n$th spectral moment, defined as:

$$m_n = \int_0^\infty \omega^n S(\omega) d\omega.$$

(20)

The Dirlik method has previously been shown to give accurate results over a wide range of bandwidths for a stationary Gaussian process [40], and the closed-form expression makes it well-suited for efficient design optimization. However, the accuracy of the method deteriorates if the fatigue loads are dominated by a few frequency components [41].

The SN curve used for the tower base is the D curve in air taken from DNV-RP-C203 [42]. The curve is bilinear, i.e. the material parameters, and thus also the slope of the curve, change at a certain stress range. For the rainflow procedure, this is easily taken into account, however, the Dirlik method only allows for single-slope curves. Based on initial calculations, the Dirlik damage estimation is performed with material parameters valid for stress ranges above 52.63 MPa (fewer than $10^7$ cycles).

5.3. Short-term extreme response

Several methods have been used to estimate the short-term extreme response of FWTs, such as the Gumbel method and Weibull tail method for global motions [15,43], and the Winterstein method for the tower base bending moment [14]. The present study uses the average upcrossing rate (AUR) method, see e.g. Naess and Gaidai [44], which previously has been shown to perform well for a bottom-fixed offshore wind turbine under combined wind and wave loading [45]. The AUR method is based on the assumption that upcrossings of high levels are statistically independent, which means that the number of upcrossings during time $T$ will be Poisson distributed. The cumulative distribution function (CDF) of the extreme value $Y$ may thus be written as

$$F_Y(y) = \exp\{-\nu^*(y) T\},$$

(21)

where $Y$ can represent an arbitrary response parameter, such as tower base bending moment, global motions of the platform, or mooring line tension. $\nu^*(y)$ is the mean upcrossing rate of level $y$, which can be estimated empirically from $j$ simulated time series of length $T_0$:

$$\nu^*(y) = \frac{1}{jT_0} \sum_{i=1}^j n_i^+(y; T_0)$$

(22)

where $n_i^+(y; T_0)$ is the number of upcrossings in time series $i$. Assuming that the appropriate asymptotic extreme value distribution for the considered response is the Gumbel distribution, the tail of the mean upcrossing rate may be written as

$$\nu^*(y) = q(y) \exp\{-a(y - b)^c\}, \quad y \geq y_0,$$

(23)

where $q(y)$ is slowly varying and can be approximated as a constant. The parameters $a, b, c$ and $q$ are then determined by fitting Eq. (23) to the empirical data, as described in detail by Saha and Naess [46]. This is done by minimizing the mean square error function:
\[ F(a, b, c, q) = \sum_{i=1}^{N} w_i |\ln \hat{v}^*(y) - \ln q + a(y_i - b)|^2 \]

using the Levenberg-Marquardt least-squares optimization algorithm. \(w_i\) is a weight factor which puts more emphasis on the more reliable data points, and is found from the estimated 95% confidence interval.

The AUR method is also applicable for the linear model, where an analytical expression is available. For a stationary Gaussian process with zero mean, the upcrossing rate can be written as \([26]\):

\[ \nu^*(y) = \nu^*(0) \exp \left( -\frac{y^2}{2\sigma^2} \right), \]

where \(\nu^*(0)\) is the mean zero-upcrossing rate, which can be found from the zeroth and second order spectral moments:

\[ \nu^*(0) = \frac{1}{2\pi} \sqrt{\frac{m_2}{m_0}}. \]

The characteristic largest extreme response \(y_{th}\), i.e. the most probable maximum value in one hour, is used to compare the TD and FD models. The value is found approximately from the following relation:

\[ F_T(y_{th}) = e^{-1}. \]

6. Results

6.1. Natural periods

The natural periods of the linear system are estimated and compared to results from decay tests performed with the TD model in still water, as shown in Table 5. The decay tests are performed by releasing the platform from an offset position in the considered DOF, and the natural period is then found from the decaying motion. For the first bending mode, the hull is released together with the tower top, such that the coupling between platform pitch and tower bending is included.

The FD model slightly overestimates the natural periods, which for the rigid modes is found to mainly be a consequence of the simplified added mass formulation. The reason why the first bending natural frequency is better approximated for spar 1 than spar 2 can be understood by examining the eigenmodes. In Fig. 7, the bending mode from the linearized eigenvalue solution is compared to the mode shape found from a decay test in TD, where a bandpass filter around the natural frequency is used to extract the shape including one of the blades. Due to the increased stiffness of the tower in spar 2, the coupled blade-tower mode changes, and the blades undergo a larger amount of bending. The simplification of replacing the RNA by a point mass and inertia is thus less accurate than for spar 1, where the blades behave more like a rigid body in the first tower bending mode. The shape of the tower is, however, seemingly not affected by the blade behaviour, and both designs show good agreement between the decay tests and the eigenvalue solutions.

6.2. Fatigue damage

6.2.1. Linear and nonlinear estimates

Weighted 1-h fatigue damage is shown as a function of wind speed in Fig. 8. The linear predictions are seen to agree well with the TD results for spar 1, especially at wind speeds from 11 m/s and higher. For spar 2, the linear model is seen to overpredict the fatigue damage quite consistently by about 25–35%. The reasons for this overprediction are further explored in Section 6.2.2. The good agreement for spar 1 can be explained by the fact that the tower base bending moment is close to Gaussian in most of the considered environmental conditions, in particular at higher wind speeds, which are associated with larger waves. The response in these conditions is dominated by wave forces and by resonant motions, which tend to be Gaussian even for non-Gaussian loads \([47]\).

A notable difference between the linear and nonlinear results for spar 1 is seen at 9 m/s, where the 3P frequency is very close to the natural frequency of the first bending mode, which leads to resonance and thus a significant increase in fatigue damage for the FD model. In TD, the resonance has a rather limited effect on the damage estimates, possibly due to nonlinearities in the model.

With a few exceptions, the IFT and Dirlik results agree well for all conditions in both designs. Compared to the TD model, the error in total fatigue damage is 10.6% (IFT) and 7.1% (Dirlik) for spar 1, and 30.4% (IFT) and 30.9% (Dirlik) for spar 2.

Table 5
Natural periods.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Spar 1</th>
<th>Spar 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TD</td>
<td>FD</td>
</tr>
<tr>
<td>Surge (s)</td>
<td>120.55</td>
<td>122.22</td>
</tr>
<tr>
<td>Pitch (s)</td>
<td>35.48</td>
<td>36.39</td>
</tr>
<tr>
<td>1st bending (s)</td>
<td>2.41</td>
<td>2.42</td>
</tr>
</tbody>
</table>
Scatter plots showing the calculated fatigue damage for each environmental condition in TD and FD are presented in Fig. 9, where the Dirlik results are used for the linear model. The solid black line indicates perfect agreement, while conservative FD predictions are found below the line.

The accuracy of the linear model is to a large extent insensitive to the amount of fatigue damage in the individual conditions, and the data follow a fairly straight line, with most points located within ±10% of the line of best fit for both designs. For spar 1, this line, when neglecting the 9 m/s conditions, is very close to the diagonal. The 9 m/s results, although being highly overestimated by the linear model, are also seen to be reasonably consistent, as the ratio between the linear and nonlinear estimates is more or less constant with increasing damage.

For spar 2, all points are located below the 1:1 line, and as for spar 1, the conditions with the largest discrepancies between linear and nonlinear predictions are in general found at lower wind speeds (5–9 m/s). These wind speeds are associated with low sea states, and little excitation of the bending mode. The tower base bending moment is thus dominated by the low-frequency wind response, which is less accurately predicted by the linear model than the wave- and bending-frequency response. The low-frequency response in the linear model is also somewhat sensitive to how the thrust force spectrum is estimated, as reported by Kvittem and Moan [10]. It should be noted that these conditions are associated with relatively small bending moments, and their contribution to the total fatigue damage over the lifetime of the structure is thus not significant.

The limited scatter in the data for both designs suggest that it is possible to simulate a small number of environmental conditions in the TD to verify and, if necessary, calibrate a linear model which can be used to perform the full long-term analysis. The verification cases should cover the entire wind speed range, as resonance effects may have a large impact on the accuracy of the model. In addition, a 1 m/s interval for the wind speed bins may be needed to correctly capture the response around possible resonance wind speeds [35].

### 6.2.2. Sensitivity to errors in natural frequency

From the tower base stress spectra in Fig. 10, the reason for the general overprediction in fatigue damage by the linear model for spar 2 is obvious: the response in the region of the first bending mode is captured quite well by the linear model for spar 1, but is significantly overestimated for spar 2. The wave-frequency response is accurately estimated for both designs, whereas the low-frequency response is underestimated at the pitch resonance frequency, mainly due to interaction between platform motion and the wind turbine controller in the TD model. Due to the limited number of cycles, however, the low-frequency response has little influence on the fatigue damage in the tower compared to the wave- and bending-frequency responses.

The overestimation of the response around the natural frequency of the first bending mode originates mainly from the small error in estimated natural period for the bending mode discussed in Section 6.1, as the wind excitation loads vary significantly with frequency in this region. In Fig. 11, the natural frequencies found from FD and TD are shown together with the transfer functions for thrust force and rotor moment at 21 m/s mean wind speed. From the figure, one clearly sees that the loads are reduced when the
The eigenmode in tower bending depends on wind speed, due to increased blade stiffness caused by centrifugal forces and pitching of the blades, as well as changes in the mean platform pitch angle. This is illustrated in Fig. 13, where mode shapes from decay tests are shown for calm air and 21 m/s uniform wind. As a consequence, the natural frequency of the bending mode is slightly increased by 2–3% at higher wind speeds. In addition, the mode shape derivative at the tower top is reduced by up to 20%.

From Fig. 11, an increase in the natural frequency of the bending mode is seen to result in higher wind loads for spar 1. However, this effect is counteracted by the decrease in the tower top derivative, which reduces the generalized wind load caused by the rotor moment (see Eq. (10)). For spar 2, on the other hand, both effects reduce the wind excitation loads for the bending mode, and cause the tuned linear model to somewhat overestimate the fatigue damage at higher wind speeds, as observed in Fig. 12.

Regarding the bending-frequency response, it should also be mentioned that, even though no validation has been performed, the natural frequency for spar 2 is increased. The effect is most prominent for the rotor moment, which is about 25% lower at the TD natural frequency than at the natural frequency found from the linearized eigenvalue solution.

To quantify the importance of this error on the fatigue damage results, the natural period of the bending mode is shifted from 1.85 s to 1.79 s in the linear model by adding an artificial stiffness term, and the fatigue analysis is rerun. The results, presented in Fig. 12, show a large improvement at most wind speeds, and the total fatigue damage error is reduced by approximately 50%. This highlights the importance of having accurate estimates for the natural frequency and shape of the first bending mode, which is dependent on how well the RNA can be approximated as a rigid body. In an optimization process, the accuracy of the model simplification may vary as changes are made to the design, however, this can be taken into account by a straightforward calibration of the natural frequency in the linear model.

FIG. 8. 1-h weighted fatigue damage arranged by wind speed.

(a) Spar 1.

(b) Spar 2.
Fig. 9. Time and frequency domain fatigue damage for individual ECs.

(a) Spar 1.

(b) Spar 2.

Fig. 10. Tower base stress response spectrum for $U = 21$ m/s, $H_s = 7.5$ m, $T_p = 14.5$ s.

(a) Spar 1

(b) Spar 2

Fig. 10. Tower base stress response spectrum for $U = 21$ m/s, $H_s = 7.5$ m, $T_p = 14.5$ s.
present study assumes that the behaviour of the physical system is described correctly by the TD model. Full-scale measurements from the Hywind Demo FWT [48] have shown that the tower base bending moment response spectra contain energy around the first tower bending mode and that the numerical models thus capture real effects, however, the actual level of the response is sensitive to the modelling of the system, and may therefore not be correctly represented in the simulations.

6.3. Extreme value prediction

The short-term extreme response of the system in the four 50-year environmental conditions is calculated for the tower base bending moment, as well as for the surge and pitch motions of the platform. The results are presented in the following subsections.

6.3.1. Tower base bending moment

Response statistics for the tower base bending moment in the 50-year conditions are listed in Table 6. For a Gaussian process, the skewness is equal to zero, while the Pearson’s kurtosis is equal to 3. The skewness and kurtosis values for the tower base bending moment suggest that the responses in EC 2–4 are very close to Gaussian, which is expected, as the bending moments in conditions above cut-out are completely dominated by wave loading, whereas the response in operational conditions with high wind speeds is governed by a combination of wave forces and resonance. The response in EC 1 exhibits some non-Gaussian behaviour due to the large aerodynamic thrust near rated wind speed, however, the degree of nonlinearity is small. In this context, it should also be emphasized that the TD model only includes first order wave loads, with the exception of viscous forces arising from the drag term in Morison’s equation.

The standard deviations are accurately predicted by the linear model in all conditions, with relative errors between $-2.9\%$ and

---

**Fig. 11.** Wind excitation transfer functions around first bending natural frequency, $U = 21$ m/s.

**Fig. 12.** 1-h weighted fatigue damage arranged by wind speed, spar 2 with tuned natural frequency for first bending mode.
The mean moments are somewhat less accurate, with the largest errors of 5.5% (spar 1) and 7.8% (spar 2) occurring in EC 1, near rated wind speed, where also the largest mean values are found.

Empirical upcrossing rates are plotted together with optimized curve fits and Gaussian upcrossing rates based on the FD results in Figs. 14 and 15. The plots show the multiplication factor $\kappa$, which is related to the extreme response, $y$, through the relation

$$y = \mu + \kappa \sigma,$$

where $\mu$ and $\sigma$ are the mean and standard deviation of the process. The characteristic largest 1-h extreme for a given environmental condition, which is used in the comparison, corresponds to an upcrossing rate of $2.78 \times 10^{-4}$.

The Gaussian upcrossing rates are seen to be conservative for EC 1, due to the slightly nonlinear behaviour which becomes more prominent as the bending moments increase. In the parked conditions, on the other hand, the response is linear also at lower upcrossing rates, and the empirical data are well-described by the linear model.

For spar 1, the FD solution is seen to follow the optimized curve fit closely in EC 2, however, larger discrepancies are observed for spar 2. The fact that the shape of the fitted curve is captured quite well, suggests that the error is related to the overprediction of the bending response discussed in the previous section. As the bending mode is located at a relatively high frequency, the response level has significant influence on the second order spectral moment. Consequently, the mean zero-upcrossing rate (see Eq. (26)) is overestimated by 29% compared to the TD model, which causes a right shift in the linear upcrossing rate curve as observed in Fig. 15b.

Table 6

<table>
<thead>
<tr>
<th>Model</th>
<th>Statistical quantity $^a$</th>
<th>Condition 1</th>
<th>Condition 2</th>
<th>Condition 3</th>
<th>Condition 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time domain</td>
<td>Mean (MNm)</td>
<td>247.2</td>
<td>123.0</td>
<td>50.3</td>
<td>105.2</td>
</tr>
<tr>
<td></td>
<td>Std.dev. (MNm)</td>
<td>89.1</td>
<td>101.0</td>
<td>106.5</td>
<td>118.8</td>
</tr>
<tr>
<td>Skewness</td>
<td>−0.20</td>
<td>−0.03</td>
<td>−0.01</td>
<td>0.01</td>
<td>−0.30</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>3.03</td>
<td>3.00</td>
<td>3.00</td>
<td>3.01</td>
<td>3.13</td>
</tr>
<tr>
<td>Frequency domain</td>
<td>Mean (MNm)</td>
<td>260.8</td>
<td>127.8</td>
<td>52.3</td>
<td>101.3</td>
</tr>
<tr>
<td></td>
<td>Std.dev. (MNm)</td>
<td>89.4</td>
<td>100.8</td>
<td>106.3</td>
<td>115.1</td>
</tr>
</tbody>
</table>

$^a$ Time domain results are averaged over 20 seeds.

Figure 13. Normalized bending mode shapes in calm air and for $U = 21$ m/s. Natural periods found from decay tests are 2.41 s (calm air) and 2.35 s (21 m/s) for spar 1, and 1.79 s (calm air) and 1.75 s (21 m/s) for spar 2.
The calculated 1-h extreme bending moments are listed in Table 7. The linear estimates give reasonably good approximations of the extreme response, particularly for EC 2–4, as indicated by the upcrossing results. Due to the high mean value caused by the thrust force, the largest extreme response is found in EC 1. However, this may not be the case in general, and previous studies comparing short-term extremes in selected operational and survival conditions have shown that the largest tower base bending moments for a spar FWT can occur in storm conditions with extreme wind speeds \[49,50\].

The critical load cases may vary with floater design and turbine size, as it alters the ratio between wind and wave loads, and for long-term extremes, the probability of the conditions must be taken into account. The most critical operational condition is also not necessarily associated with maximum thrust force on the turbine. In a recent study by Sultania and Manuel \[51\], the largest 50-year tower base bending moment for a 5MW spar FWT, considering only operational cases, was found for a mean wind speed of 21.7 m/s. This suggests that a wider range of environmental conditions should be considered in an actual design process.

6.3.2. Global motions

Tables 8 and 9 list extreme response statistics for surge and pitch, respectively. The global motions show similar behaviour to the tower base bending moments, but as a larger part of the response is located in the low-frequency range, there is less agreement between the TD and FD results for the standard deviations. The values are in general found to be underestimated by the linear model, with errors up to about 20% for both surge and pitch.

As for the bending moments, the upcrossing rates for the global motions (Appendix B) are accurately described by the Gaussian curves in EC 2–4, while the nonlinear behaviour in EC 1 results in conservative predictions by the linear model. For both spar designs, some slight nonlinearities are also seen at higher wind speeds, which causes the linear model to underestimate $\kappa$ at lower upcrossing rates. The discrepancies are likely caused by the aerodynamic drag forces on the tower, which, although they mostly affect the mean position of the FWT, also become more important for the dynamic response as the wind speed increases. The good agreement for the surge motions in the parked conditions suggests that the viscous hydrodynamic forces on the hull contribute little to the motion response.

The resulting characteristic largest 1-h extreme values are listed in Tables 10 and 11. As expected, the most severe global motions are also found in EC 1. The pitch response, which shows the most pronounced non-Gaussian behaviour near rated wind speed, has the

![Fig. 14. Tower base bending moment upcrossing rates for spar 1.](image-url)
least accurate linear predictions in this condition. Large discrepancies are also seen in the most extreme wave condition (EC 4), mostly due to underestimation of the standard deviations in FD. For the surge motions, the extremes found from FD generally agree well with the TD simulations. Most notable are the results in EC 1, especially for spar 1, where the relative error between FD and TD is only $-1\%$. However, this result is somewhat coincidental, as it is a consequence of the error in $\kappa$ being almost exactly balanced out by the errors in $\mu$ and $\sigma$. Nonetheless, the results suggest that the extreme surge motions may be predicted with roughly the same accuracy as the tower base bending moments, despite larger discrepancies in the standard deviations, as the mean values account for a larger share of the total extreme response.

The accuracy of the linear model depends on the importance of nonlinearities in the system, which will vary with location, floater geometry, and different operational conditions. Therefore, the results presented here cannot be generalized to any FWT, and the FD model should be continuously verified against state-of-the-art tools during a design process. The agreement with nonlinear analyses also depends on how the system is represented in the TD model, and care should be taken to make sure that relevant nonlinearities are included.

Table 7
Characteristic largest extreme tower base bending moment $y_{l_{\text{tr}}}$, 50-year environmental conditions.

<table>
<thead>
<tr>
<th>Model</th>
<th>Spar 1</th>
<th>Spar 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Condition</td>
<td>Condition</td>
</tr>
<tr>
<td>Time domain (MNm)</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Frequency domain (MNm)</td>
<td>534.2</td>
<td>480.6</td>
</tr>
<tr>
<td>Relative error (%)</td>
<td>7.8</td>
<td>2.3</td>
</tr>
</tbody>
</table>

Fig. 15. Tower base bending moment upcrossing rates for spar 2.
Table 8
Platform surge response statistics, 50-year environmental conditions.

<table>
<thead>
<tr>
<th>Model</th>
<th>Statistical quantity</th>
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<th>Spar 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Time domain</td>
<td>Mean (m)</td>
<td>27.3</td>
<td>13.8</td>
</tr>
<tr>
<td></td>
<td>Std.dev. (m)</td>
<td>6.9</td>
<td>2.4</td>
</tr>
<tr>
<td></td>
<td>Skewness</td>
<td>−0.14</td>
<td>−0.02</td>
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<tr>
<td></td>
<td>Kurtosis</td>
<td>2.49</td>
<td>2.90</td>
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<td>Frequency domain</td>
<td>Mean (m)</td>
<td>28.6</td>
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</tr>
<tr>
<td></td>
<td>Std.dev. (m)</td>
<td>5.4</td>
<td>2.3</td>
</tr>
</tbody>
</table>

* Time domain results are averaged over 20 seeds.

Table 9
Platform pitch response statistics, 50-year environmental conditions.

<table>
<thead>
<tr>
<th>Model</th>
<th>Statistical quantity</th>
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</tr>
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<tr>
<td></td>
<td></td>
<td>1</td>
<td>2</td>
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<tr>
<td>Time domain</td>
<td>Mean (deg)</td>
<td>6.7</td>
<td>3.4</td>
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<tr>
<td></td>
<td>Std.dev. (deg)</td>
<td>1.8</td>
<td>1.0</td>
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<tr>
<td></td>
<td>Skewness</td>
<td>−0.62</td>
<td>0.03</td>
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<td>Kurtosis</td>
<td>3.37</td>
<td>2.90</td>
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<tr>
<td>Frequency domain</td>
<td>Mean (deg)</td>
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<tr>
<td></td>
<td>Std.dev. (deg)</td>
<td>1.7</td>
<td>1.0</td>
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</table>

* Time domain results are averaged over 20 seeds.

Table 10
Characteristic largest extreme surge response $y_{1,s}$, 50-year environmental conditions.

<table>
<thead>
<tr>
<th>Model</th>
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</thead>
<tbody>
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</tr>
<tr>
<td>Time domain</td>
<td>44.2</td>
<td>21.4</td>
</tr>
<tr>
<td>Frequency domain</td>
<td>43.8</td>
<td>21.4</td>
</tr>
<tr>
<td>Relative error (%)</td>
<td>−1.0</td>
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</tbody>
</table>

Table 11
Characteristic largest extreme pitch response $y_{1,p}$, 50-year environmental conditions.

<table>
<thead>
<tr>
<th>Model</th>
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<td>Frequency domain</td>
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<td>6.6</td>
</tr>
<tr>
<td>Relative error (%)</td>
<td>14.8</td>
<td>−0.9</td>
</tr>
</tbody>
</table>

7. Conclusions

A semi-analytical FD model has been developed, which allows for efficient preliminary design optimization of spar FWTs. The model has been verified against a fully coupled nonlinear TD model for two different 10 MW spar designs, with regards to fatigue damage in the tower base and short-term extreme response in both operational and survival conditions. The fatigue results show that the FD model is accurate in most environmental conditions, especially for spar 1, where the long-term fatigue damage is
overestimated by 7%. For spar 2, larger discrepancies are observed, and the linear model overpredicts the damage by approximately 30%. This is found to primarily be caused by errors in the response around the first bending mode, which is highly sensitive to the estimated natural frequency. The accuracy depends on how well the RNA can be approximated as a rigid body for the first bending mode, however, tuning the frequency by adding an artificial stiffness term can be performed in the case where the approximation leads to unacceptable errors.

For 9 m/s mean wind speed, the 3P frequency of the rotor coincides with the natural frequency of the first bending mode for spar 1, which leads to increased damage values and causes the linear model to significantly overpredict the response. The sensitivity with regards to wind speed also suggests that a finer discretization of the wind bins should be applied in a design process, to accurately capture the response around possible resonances. The accuracy of the linear model is found to be fairly independent of wave height and wave period, also for the wind speed which induces 3P resonance with the first tower bending frequency. This indicates that a small number of environmental conditions, which cover the full wind speed range, can be simulated with a TD model to verify and possibly tune the linear model, which then can be used to carry out the full long-term analysis.

The extreme response, both with regards to tower base bending moment and rigid body motions in surge and pitch, is seen to be nearly Gaussian near cut-out and in parked conditions, as it is dominated by wave forces. This results in good agreement for the linear model in these conditions. The largest extremes values are found in EC 1, near rated wind speed, which indicates that a wider range of environmental conditions may be important for the extreme response. The large aerodynamic forces in this condition also introduce some non-Gaussian behaviour, which causes the linear model to overestimate the multiplication factors $\kappa$. However, as the mean values in this condition are relatively large, the inaccuracies in the total extreme response are reduced, and the total error in $\gamma_{th}$ is within 16% for all considered responses in this condition. Regarding the standard deviations, good agreement between the linear and nonlinear models is obtained for the tower base bending moments, while larger discrepancies are seen for surge and pitch. The reason for this is that the global motions are more dependent on the low-frequency (wind-induced) response, which is less accurately captured by the linear model than the wave- and bending-frequency response. The TD model in the present study only includes first order wave forces, and could thus be improved by also considering higher order loads, which may be important for certain FWT concepts. The model should also be validated through comparisons with physical measurements.

The developed FD model is shown to be well-suited for preliminary design. It gives reasonable agreement with a fully coupled nonlinear TD model, both with regards to fatigue and extreme response, and the semi-analytical approach provides an efficient way to explore the design space. In addition, the closed-form expressions offer the possibility of using analytic gradients in a computer-aided optimization process. However, the efficiency of the model is partly due to the simple geometry of the platform, and the formulations are thus not valid for more complex platform designs, where more comprehensive numerical calculations may be necessary. Also, the approach is only applicable for steady-state conditions, and transient events, which also must be considered, cannot be analysed. Nonetheless, it can serve as a useful complement to more sophisticated models in the design process of spar-type FWTs, and significantly reduce the needed computational effort.

A Transfer functions

A.1 Wave elevation to tower base bending moment

The transfer function from wave elevation to tower base bending moment is given in Eq. (A.1). The equation considers bending moments due to inertial forces in the tower and RNA, aerodynamic damping forces, and gravitational forces.

\[
H_{TM}(\omega) = \int_{z_{TB}}^{z_{op}} m(z)\chi_{\omega}(\omega)\psi(z)(z - z_{TB})dz - M_{RNA}\chi_{\omega}(\omega)\psi(z_{op})(z_{op} - z_{TB})
- \chi_{RNA}\chi_{\omega}(\omega)\psi(z_{op}) - B_{\text{aero}}\chi_{\omega}(\omega)\psi(z_{op})(z_{op} - z_{TB})
+ \int_{z_{TB}}^{z_{op}} m(z)g\chi_{\omega}(\omega)\psi(z) - \psi(z_{TB})]dz + M_{RNA}g\chi_{\omega}(\omega)[\psi(z_{op}) - \psi(z_{TB})]
\]

(A.1)

A.2 Wind speed to tower base bending moment

The transfer function from wind speed to tower base bending moment is given in Eq. (A.2). The equation considers bending moments due to inertial forces in the tower and RNA, aerodynamic damping forces, gravitational forces, and wind excitation forces.

\[
H_{TM}(\omega) = \int_{z_{TB}}^{z_{op}} m(z)\chi_{\omega}(\omega)\psi(z)(z - z_{TB})dz - M_{RNA}\chi_{\omega}(\omega)\psi(z_{op})(z_{op} - z_{TB})
- \chi_{RNA}\chi_{\omega}(\omega)\psi(z_{op}) - B_{\text{aero}}\chi_{\omega}(\omega)\psi(z_{op})(z_{op} - z_{TB})
+ \int_{z_{TB}}^{z_{op}} m(z)g\chi_{\omega}(\omega)\psi(z) - \psi(z_{TB})]dz + M_{RNA}g\chi_{\omega}(\omega)[\psi(z_{op}) - \psi(z_{TB})]
+ F_{\omega}(\omega)(z_{op} - z_{TB}) + M_{\omega}(\omega)
\]

(A.2)
B Upcrossing rates for global motions

B.1 Surge

Figure B.1. Platform surge upcrossing rates for spar 1.
Figure B.2. Platform surge upcrossing rates for spar 2.
B.2 Pitch

![Figure B.3. Platform pitch upcrossing rates for spar 1.](image)

(a) 11 m/s  
(b) 25 m/s  
(c) 36 m/s  
(d) 50 m/s
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Frequency-dependent aerodynamic damping and inertia in linearized dynamic analysis of floating wind turbines

Carlos Eduardo S. Souza, John Marius Hegseth and Erin E. Bachynski

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Frequency-Dependent Aerodynamic Damping and Inertia in Linearized Dynamic Analysis of Floating Wind Turbines

Carlos Eduardo S Souza, John Marius Hegseth and Erin E Bachynski
Department of Marine Technology, Norwegian University of Science and Technology, 7491 Trondheim, Norway
E-mail: carlos.souza@ntnu.no

Abstract. In frequency-domain (FD) models of floating wind turbines (FWTs), it is common to regard the interaction between nacelle motions and thrust by means of a constant aerodynamic damping coefficient. This approach is effective at higher motions frequencies, but does not consider interactions between nacelle motions and the blade pitch control system. As a result, the motions and loads at frequencies closer to the controller bandwidth may be underpredicted. A remedy for this problem is to include the linearized thrust expression in the FD model, such that the dynamic effects related to control are considered. In this paper, these dynamic effects are related to frequency-dependent damping and inertia terms. Expressions for damping and inertia coefficients are obtained with two different methods, and then included in the FD model. The resulting responses are compared to those obtained with the constant damping coefficient method, and also with coupled time-domain simulations of a semi-submersible 10 MW FWT. The better performance of the FD model with frequency-dependent inertia and damping coefficients encourages the adoption of the linearized thrust approach for representing the interaction between nacelle motions, thrust and control system.

1. Introduction
Frequency-domain (FD) methods can be helpful in the design of floating wind turbines (FWTs), providing responses for a large number of loading conditions with relatively low computational time. Previous work has indicated that the response of FWTs in moderate environmental conditions may be estimated reasonably well using FD models [1, 2, 3]. However, their reliability depends on an accurate prediction of loads and interactions with linearized models.

An especially interesting interaction takes place between nacelle motions and the rotor thrust. Fluctuations in the nacelle's horizontal velocity provoke changes in the flow through the rotor, leading to oscillations in the rotor speed and thrust. An aerodynamic damping effect results from this interaction, and is normally included in FD models by means of a constant aerodynamic damping coefficient. This coefficient may be obtained from the thrust derivative w.r.t. the relative wind speed [1, 3, 4], or by means of decay simulations of the FWT under different incident wind speeds [2].

This method is convenient to implement due to its relative simplicity, and is normally adopted in combination with the frequency-dependent thrust obtained for a fixed wind turbine, installed on the top of the tower. A disadvantage of this approach is that the interactions of the nacelle...
motions with the blade pitch control system are not considered, an effect that is known to reduce the damping or even render it negative [5, 6, 7], besides introducing an inertia effect [8]. As a consequence, the damping effect can be significantly overpredicted at frequencies close to the controller bandwidth, if the constant coefficient is used.

Control system effects can be included in an FD model by means of linearization of the thrust and torque. The rotor speed is then included as an additional degree of freedom, and the blade-pitch controller can be written in terms of the system states [9].

The objective of the present work is to relate the linearized thrust equations to the above-mentioned inertia and damping effects, providing a visualization of how those effects vary with wind speed and frequency for a given control system.

The aerodynamic inertia and damping coefficients are calculated both from the linearized thrust equations and from simulations where the nacelle is forced to oscillate with different frequencies, under uniform wind. The obtained coefficients are then included in an FD model of a semi-submersible 10 MW FWT. The responses predicted with the different methods for obtaining the coefficients are compared to coupled time-domain simulations, where the thrust is calculated with the blade element momentum (BEM) theory.

Section 2 presents the frequency domain model for the FWT; Section 3 introduces the thrust linearization procedure and provides formulations for the frequency-dependent aerodynamic inertia and damping coefficients; the time-domain simulations are explained in Section 4, and the results are presented in Section 5; final discussions and conclusions are provided in Section 6.

2. Frequency-domain analysis of FWT

The frequency domain model used in the present work consists of three degrees-of-freedom (DOFs), namely surge, heave, and pitch. The potential-theory hydrodynamic model is the same as the one adopted in the time-domain simulations, presented in Section 4. Viscous excitation is neglected, but viscous damping on the platform is added using stochastic linearization of the drag term in Morison’s equation, where the standard deviation of the velocity along the length of the columns and pontoons is found using an iteration scheme.

The stiffness matrix includes both hydrostatic restoring and linearized mooring forces. The mooring stiffness matrix is a function of the mean offset, based on static equilibrium between mooring forces and mean thrust.

Transfer functions from wind speed to thrust force, \( F_{UT}(\omega) \), are obtained for each mean wind speed as the squared root of the ratio between the spectrum of thrust time series, obtained from simulations with turbulent wind on a fixed turbine, and the incoming wind spectrum. The wind force vector is then found as

\[
F_U(\omega) = \begin{bmatrix} F_{UT}(\omega) & 0 & z_{hub}F_{UT}(\omega) \end{bmatrix}^T,
\]

where \( z_{hub} \) is the hub height above still water level (SWL). Transfer functions from wave and wind input to platform response, \( H_{\xi X}(\omega) \) and \( H_{UX}(\omega) \) respectively, are expressed as

\[
H_{\xi X}(\omega) = H_{FX}(\omega)F_{\xi}(\omega), \quad H_{UX}(\omega) = H_{FX}(\omega)F_U(\omega),
\]

where

\[
H_{FX}(\omega) = \left[ -\omega^2(M + A(\omega)) + i\omega B(\omega) + K \right]^{-1}
\]

is the frequency response function matrix. After solving for the platform motions, the tower base bending moment response is found by considering dynamic equilibrium.

Using the wind and wave spectra, the response spectrum for an arbitrary response parameter \( \xi \) is then expressed as

\[
S_\xi(\omega) = |H_{\xi \xi}(\omega)|^2S_\xi(\omega) + |H_{U \xi}(\omega)|^2S_U(\omega).
\]
3. Aerodynamic damping and inertia effects
The FD model described in Section 2 neglects the variations in the thrust caused by the FWT motions. The interactions between thrust and FWT motions are often represented by a constant damping coefficient, which is added to the matrix $B$ in equation (3). This approach neglects the controller action. When the controller is included, its effect results in frequency-dependent damping and inertia coefficients, as shown further below.

3.1. Thrust linearization without controller
A common way to calculate the aerodynamic damping in FD models [1, 4, 3] is to linearize the thrust with respect to the relative wind speed at hub height, $v$, at different mean incoming wind speeds, $U_w$, considering no change in the blade pitch angle or rotor speed. Using a first order Taylor series expansion, the thrust is then expressed as

$$T = T_0 + \frac{\partial T}{\partial v} \Delta v.$$  

(5)

Let the nacelle dynamics be represented by an 1-DOF, 1st-order system:

$$m\ddot{x} + c\dot{x} + kx = T.$$  

(6)

Uniform, non-turbulent wind is assumed for now. Recalling that $v = U_w - \dot{x}$, it is noted that $\Delta v = v - U_w = -\dot{x}$. Replacing (5) in (6), the damping effect becomes clear:

$$m\ddot{x} + \left(c + \frac{\partial T}{\partial v}\right) \dot{x} + kx = T_0,$$  

(7)

and the coefficient, $b_{aer}$, can be expressed as

$$b_{aer} = \frac{\partial T}{\partial v}.$$  

(8)

The damping effect estimated with this method considers the change in the steady-state thrust for a small perturbation in the uniform wind speed seen by the rotor, while neglecting the effect of the control system and rotor dynamics. Different coefficients are obtained for each incident wind velocity, but they are constant with respect to the nacelle frequency of oscillation. This method is relatively simple to use and normally provides satisfactory results when the turbine oscillates in a frequency range distant from the controller bandwidth. For lower frequencies of oscillation, interaction with the control system takes place and the damping coefficient dependence on frequency becomes more important.

3.2. Thrust linearization including controller
A more comprehensive method, which captures effects from the wind turbine controller, may be used by including the rotor speed DOF and control system in the FD model, and also linearizing the thrust with respect to rotor speed, $\Omega$, and blade pitch angle, $\beta$:

$$T = T_0 + T_v \Delta v + T_\Omega \Delta \Omega + T_\beta \Delta \beta,$$  

(9)

where the index indicates partial derivative w.r.t. the indicated variable, i.e., $A_B = \frac{\partial A}{\partial B}$. In order to find the damping coefficient, Equation (9) must be written in terms of $\dot{x}$ only. First, the aerodynamic torque, $Q$, is also given in its linearized version:

$$Q = Q_0 + Q_v \Delta v + Q_\Omega \Delta \Omega + Q_\beta \Delta \beta,$$  

(10)
In addition, the rotor dynamics are given by

$$I_d\ddot{\Omega} = Q - N_gE,$$  \hspace{1cm} (11)

where $I_d$ is the drivetrain inertia, $N_g$ is the gear ratio and $E$ is the electrical (generator) torque, here assumed as constant above rated wind speed. In equilibrium, $Q_0 = N_gE$, so replacing (10) in (11):

$$I_d\ddot{\Omega} = Q_v \Delta v + Q_\Omega \Delta \Omega + Q_\beta \Delta \beta.$$ \hspace{1cm} (12)

It is assumed that a PI controller commands the blade pitch angle, i.e.,

$$\Delta \beta = K_p \Delta \Omega + K_i \int \Delta \Omega \, dt,$$ \hspace{1cm} (13)

where $K_p$ and $K_i$ are the proportional and integral controller gains. Equations (9) and (12) are now rewritten replacing $\Delta \beta$ as given in (13), and noting that $\Delta \Omega = \dot{\phi}$, where $\phi$ is the rotor azimuth angle:

$$T = T_0 - T_v \dot{x} + (T_\Omega + K_p T_\beta) \dot{\phi} + K_i T_\beta \phi,$$ \hspace{1cm} (14)

$$I_d \ddot{x} - (Q_\Omega + K_p Q_\beta) \dot{x} - K_i Q_\beta x = -Q_v \dot{x}.$$ \hspace{1cm} (15)

Assuming harmonic oscillation, Equation (15) can be written in the frequency domain, and the following transfer function between $x(\omega)$ and $\phi(\omega)$ is obtained:

$$\phi(\omega) = \frac{i\omega Q_v}{I_d \omega^2 + (Q_\Omega + K_p Q_\beta) i\omega + K_i Q_\beta} x(\omega) = C(\omega) x(\omega).$$  \hspace{1cm} (16)

The nacelle dynamics (Equation 6) are now written in terms of $T$ as given in (14), and also expressed in the frequency domain, with $\phi(\omega)$ as given in (16):

$$\begin{cases} 
-m\omega^2 + i\omega c + k + i\omega T_v - [(T_\Omega + K_p T_\beta) i\omega + K_i T_\beta] C(\omega) \\
-\omega^2 a_{aer}(\omega) + i\omega b_{aer}(\omega)
\end{cases} \quad x(\omega) = T_0.$$ \hspace{1cm} (17)

Based on the assumption that $T$ responds harmonically to harmonic oscillations of the nacelle, the thrust can be written as a combination of terms proportional to the nacelle acceleration and velocity [8]. This assumption is further discussed in Section 3.3, but an immediate consequence is the definition of frequency-dependent, aerodynamic inertia and damping coefficients:

$$a_{aer}(\omega) = -\frac{1}{\omega^2} \text{Re} \left\{ i\omega T_v - [(T_\Omega + K_p T_\beta) i\omega + K_i T_\beta] C(\omega) \right\}$$ \hspace{1cm} (18)

$$b_{aer}(\omega) = \frac{1}{\omega} \text{Im} \left\{ i\omega T_v - [(T_\Omega + K_p T_\beta) i\omega + K_i T_\beta] C(\omega) \right\}$$ \hspace{1cm} (19)

3.3. Forced oscillations

The thrust and torque derivatives in Section 3 can be calculated analytically, based on the BEM equations, or numerically, using e.g. the central differences method. Both techniques assume that the thrust and torque respond immediately to changes in $v$, $\Omega$ and $\beta$, while in reality the time constants associated with rotor dynamics, aerodynamics, and control system, can be relevant when the nacelle moves at frequencies close to the controller bandwidth.

An alternative method to find the $a_{aer}(\omega)$ and $b_{aer}(\omega)$ coefficients is through simulations of forced nacelle oscillations, with aerodynamics calculated nonlinearly and under influence of
the control system. When the turbine oscillates harmonically, the thrust also oscillates nearly harmonically, but with a phase relative to the nacelle velocity. This can be seen in Figure 1, where the DTU 10 MW wind turbine is forced to oscillate with period 100.0 s and under constant incident wind speed of 13.0 m/s. The thrust can then be assumed to be composed of a mean plus an oscillating component:

$$T = T_0 + T_{osc}(U_w, \omega).$$

(20)

Writing the nacelle velocity \(\dot{x}\) as

$$\dot{x} = v_0 \cos(\omega t),$$

(21)

the oscillating part of the thrust can then be assumed to be given by [8]

$$T_{osc} = f_0 v_0 \cos(\omega t + \alpha)$$

$$= f_0 \left[ v_0 \cos(\omega t) \cos(\alpha) - v_0 \sin(\omega t) \sin(\alpha) \right]$$

$$= f_0 \cos(\alpha) \dot{x} + \frac{f_0 \sin(\alpha)}{\omega} \ddot{x}.$$  

(22)

The amplifying factor \(f_0\) and the phase \(\alpha\) between nacelle velocity and thrust depend on the dynamic effects mentioned above, and are not straightforward to determine analytically. However, for a given turbine and control system they vary with the incident wind speed and frequency of oscillation, while motion amplitude does not seem to have a significant influence. They can therefore be obtained from forced oscillations covering the ranges of interest for both parameters. Fast Fourier transforms (FFTs) of the nacelle velocity and thrust are calculated, and \(f_0\) and \(\alpha\) can be directly obtained from the ratio between amplitudes or phases of both FFTs at the period of interest. Formulations for the aerodynamic inertia and damping coefficients are then directly determined from equation (23):

$$a_{aer}(U_w, \omega) = -\frac{f_0 \sin(\alpha)}{\omega}, \quad b_{aer}(U_w, \omega) = -f_0 \cos(\alpha),$$

(23)

3.4. Coefficients calculation and comparison
The aerodynamic inertia and damping coefficients were obtained both from the linearized thrust and the forced oscillations method. For the former case, the thrust and torque derivatives were
obtained from the linearized BEM equations. For the latter, forced oscillations of the DTU 10 MW wind turbine were performed with a simulator which obtains aerodynamic loads from AeroDyn [10], updates the blade pitch angle based on the same controller strategy presented in Section 4, and updates the rotor speed based on the rotor dynamics as in Equation (11). Oscillation periods varied from 20.0 s to 130.0 s, and uniform wind with the three velocities in Table 3 was considered. The BEM formulation with dynamic stall is adopted for the AeroDyn calculations.

The coefficients obtained with both approaches are shown in Figures 2 and 3. In spite of the oscillatory character of the curves obtained with the oscillation method, the agreement between both methods looks satisfactory, especially for higher wind velocities. The inertia effect is of the order of 1% of the mass of typical FWTs, resulting in negligible consequences for the surge dynamics. The contribution to the moment of inertia in pitch is however considerable, due to the nacelle height. Important changes in the pitch natural period can then result, as shown in [8]. The constant damping coefficients as obtained in equation (5) are also plotted in Fig. 3, showing that the discrepancy w.r.t. the frequency-dependent coefficients is larger for lower frequencies and higher wind velocities.

3.5. The aerodynamic inertia and damping effects on the FWT dynamics

It was already shown that the fluctuations in the aerodynamic thrust resulting from the nacelle motions may be treated as frequency-dependent inertia and damping effects. The derivation assumed non-turbulent wind, under the hypothesis that the fluctuations in the apparent wind flow in the rotor due to turbulence can be decoupled from those caused by the nacelle motions.

Under the same assumption, the fluctuations in the thrust due to nacelle oscillations can now be added to the FD model presented in Section 2. This is done by noting that, from Equations (22) and (23), the oscillating component of the thrust can be written as

\[ T_{osc} = -a_{aer,\omega} \ddot{x} - b_{aer} \dot{x}. \]  

(24)

The following aerodynamic and damping matrices, \( A_{aer} \) and \( B_{aer} \), are then defined:

\[
A_{aer}(\omega) = \begin{bmatrix}
a_{aer}(\omega) & 0 & a_{aer}(\omega)z_{hub} \\
0 & 0 & 0 \\
a_{aer}(\omega)z_{hub} & 0 & a_{aer}(\omega)z_{hub}^2
\end{bmatrix}, \quad B_{aer}(\omega) = \begin{bmatrix}
b_{aer}(\omega) & 0 & b_{aer}(\omega)z_{hub} \\
0 & 0 & 0 \\
b_{aer}(\omega)z_{hub} & 0 & b_{aer}(\omega)z_{hub}^2
\end{bmatrix}.
\]  

(25)

\( A_{aer} \) and \( B_{aer} \) are now summed to \( A \) and \( B \) in equation (3), which includes the effect of nacelle motion in the FD analysis.

4. Simulations

The platform considered is the OO-Star 10 MW [11], a semi-submersible concept designed for the LIFES50+ project. The main properties are summarized in Table 1. The potential-theory hydrodynamic loads are generated with WADAM, and viscosity is added in form of the Morison drag term. The platform is installed at a water depth of 130.0 m, and the mooring system consists on a simplified 3-catenary line arrangement, which differs from the original of taut lines with clump weights.

The DTU 10 MW turbine (Table 2) is installed at the top of the tower, at a height of 118.4 m. The blade-pitch controller corresponds to the PI formulation of Equation (13), i.e., the power error contribution of the original controller [12] is not included. The controller gains are tuned such that its bandwidth is below the pitch natural period, and a gain scheduling strategy corrects the gains according to the current blade pitch angle.

The time-domain simulations were performed with SIMA, a workbench which allows for coupled analyses between floating bodies and slender elements [14, 15]. The platform, nacelle
and hub are modeled as rigid bodies, while the blades, tower and mooring lines are modeled as flexible structures, with finite elements. The aerodynamics are based on the BEM theory, with dynamic stall and dynamic wake effects based on Øye’s models [16]. Hub and tip losses are modeled with the Prandtl factor, and Glauert’s correction is applied for high induction factors. The environmental conditions are listed in Table 3. Turbulent wind was generated with TurbSim [17], using the IEC Kaimal spectrum with turbulence characteristic B (moderate). A JONSWAP
Figure 3: Aerodynamic damping coefficients, calculated with the linearized approach (both constant and frequency-dependent) or obtained from the forced oscillations method.

![Graphs showing aerodynamic damping coefficients for different wind speeds](image)

(a) $U_w = 13.0$ m/s.  
(b) $U_w = 19.0$ m/s.  
(c) $U_w = 25.0$ m/s.

Table 2: Main properties of the DTU 10MW wind turbine [13]. The controller gains refer to $\beta = 0^\circ$.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotor radius, $R$ (m)</td>
<td>89.2</td>
</tr>
<tr>
<td>Rated rotor speed, $\Omega_0$ (rad/s)</td>
<td>1.0</td>
</tr>
<tr>
<td>Drivetrain inertia, $I_d$ (kg.m$^2$)</td>
<td>$1.6 \times 10^8$</td>
</tr>
<tr>
<td>Gear ratio, $N_g$ (-)</td>
<td>50</td>
</tr>
<tr>
<td>Rotor-nacelle assemble mass, $m_{RNA}$ (kg)</td>
<td>$6.7 \times 10^5$</td>
</tr>
<tr>
<td>Proportional controller gain, $K_p$ (s)</td>
<td>0.1794</td>
</tr>
<tr>
<td>Integral controller gain, $K_i$ (-)</td>
<td>0.0165</td>
</tr>
</tbody>
</table>

wave spectrum with a $\gamma$ factor of 3.3 was used to describe the sea state. The simulations lasted for one hour, which showed to provide a sufficient number of LF oscillations for the analysis. Only one realization of each condition was considered.
Table 3: Environmental conditions.

<table>
<thead>
<tr>
<th></th>
<th>(U_w) (m/s)</th>
<th>(H_s) (m)</th>
<th>(T_p) (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>13.0</td>
<td>2.7</td>
<td>10.3</td>
</tr>
<tr>
<td>C2</td>
<td>19.0</td>
<td>4.0</td>
<td>11.1</td>
</tr>
<tr>
<td>C3</td>
<td>25.0</td>
<td>5.8</td>
<td>12.1</td>
</tr>
</tbody>
</table>

5. Results

Figure 4 shows the frequency-domain pitch response for the three conditions in Table 3, obtained with time-domain simulations in SIMA; and with the FD model from Section 2, using the different approaches for including the aerodynamic damping and inertia effects, as presented in Section 3. When the constant damping coefficient from Equation (8) is adopted, the method is referred to as \textit{FD constant}; with the frequency-dependent inertia and damping coefficients as given by Equations (18) and (19), the approach is named \textit{FD calculated}; and when the coefficients are obtained from the forced harmonic oscillations, as in Equation (23), the \textit{FD forced} identifier is adopted.

The curves indicate that the adoption of a constant damping coefficient overestimates the damping effect at lower frequencies, resulting in significantly underpredicted resonant response for frequencies below 0.05 Hz. When frequency-dependent coefficients are adopted, however, the situation is the opposite: the damping effect is underestimated, resulting in higher response for lower frequencies. Using the coefficients based on the forced oscillations leads to improved results in comparison with those obtained from the linearized thrust equations, in terms of the pitch motion standard deviation (Table 4). All three methods perform equivalently in the wave-frequency range (0.05-0.15 Hz), where the response is underpredicted due to the absence of viscous excitation in the FD model.

Similar conclusions can be obtained regarding the tower-base bending moment (Figure 5). The most remarkable difference in comparison with the plots for the pitch response is for \(U_w = 13.0\) m/s (Figure 5a), which shows a persistent underprediction for all the three FD models for frequencies above 0.04 Hz. The errors in predicted tower base bending moment, shown in Table 4, are in general lower than for the pitch motion, since wave-frequency and 3p loads also influence the tower-base bending moment, and are less affected by the damping dependence of frequency. The accuracy is satisfactory for calculated and, especially, forced methods.

Figure 6 compares the low-frequency responses in pitch, for \(U_w = 19.0\) m/s, when the inertia effect is included or disregarded. The FD model with damping coefficient obtained from forced oscillations is adopted, but only the blue curve includes the inertia coefficient – resulting in a peak period about 2.5 s longer than when the inertia effect is not considered.

6. Conclusion

A method was developed to illustrate the importance of including the interaction between the nacelle motions and thrust in the frequency-domain representation of a FWT. The thrust fluctuations resulting from nacelle motions can be interpreted as frequency-dependent inertia and damping effects, which challenges the traditional approach of adopting a constant aerodynamic damping coefficient.

Expressions were derived for the respective coefficients based both on a linearized expression for the thrust and on forced oscillations of a wind turbine in the time domain. The obtained coefficients were included in a 3 DOF FD model of the OOstar semi-submersible FWT. Comparisons were made between the models using a constant aerodynamic damping coefficient, the frequency-dependent inertia and damping coefficients obtained in two different ways, and
Figure 4: Frequency-domain platform pitch response – comparison between SIMA simulations and frequency-domain analyses with different approaches for calculating the aerodynamic inertia and damping.

In any case, a better agreement of the LF response should still be pursued. Other linearization
<table>
<thead>
<tr>
<th>Condition</th>
<th>FD constant (%)</th>
<th>FD calculated (%)</th>
<th>FD forced (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>-0.2</td>
<td>+23.7</td>
<td>+21.8</td>
</tr>
<tr>
<td>C2</td>
<td>-24.8</td>
<td>+38.0</td>
<td>+20.0</td>
</tr>
<tr>
<td>C3</td>
<td>-29.6</td>
<td>+55.0</td>
<td>+30.6</td>
</tr>
</tbody>
</table>

Table 4: Error in pitch standard deviation compared to time domain simulations.

Figure 5: Frequency-domain tower base bending moment – comparison between SIMA simulations and frequency-domain analyses with different approaches for calculating the aerodynamic inertia and damping.

methods could be attempted, trying to preserve effects other than those obtained with the partial derivatives of thrust and torque w.r.t. relative wind velocity, rotor speed and blade-pitch angle.
<table>
<thead>
<tr>
<th>Condition</th>
<th>FD constant (%)</th>
<th>FD calculated (%)</th>
<th>FD forced (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>-7.1</td>
<td>+13.0</td>
<td>+10.7</td>
</tr>
<tr>
<td>C2</td>
<td>-29.8</td>
<td>+15.1</td>
<td>+2.9</td>
</tr>
<tr>
<td>C3</td>
<td>-33.0</td>
<td>+19.2</td>
<td>+5.3</td>
</tr>
</tbody>
</table>

Table 5: Error in tower base bending moment standard deviation compared to time domain simulations.

Figure 6: Pitch response estimated in the frequency domain for $U_w = 19.0$ m/s, with aerodynamic damping estimated using the forced oscillations method and with/without considering the aerodynamic inertia effect.

References


Paper 3

Integrated design optimization of spar floating wind turbines
John Marius Hegseth, Erin E. Bachynski and Joaquim R. R. A. Martins
Marine Structures, 2020
Integrated design optimization of spar floating wind turbines

John Marius Hegseth,*, Erin E. Bachynski a, Joaquim R.R.A. Martins b

a Department of Marine Technology, NTNU, 7491, Trondheim, Norway
b Department of Aerospace Engineering, University of Michigan, Ann Arbor, MI, 48109, USA

A linearized aero-hydro-servo-elastic floating wind turbine model is presented and used to perform integrated design optimization of the platform, tower, mooring system, and blade-pitch controller for a 10 MW spar floating wind turbine. Optimal design solutions are found using gradient-based optimization with analytic derivatives, considering both fatigue and extreme response constraints, where the objective function is a weighted combination of system cost and power quality. Optimization results show that local minima exist both in the soft-stiff and stiff-stiff range for the first tower bending mode and that a stiff-stiff tower design is needed to reach a solution that satisfies the fatigue constraints. The optimized platform has a relatively small diameter in the wave zone to limit the wave loads on the structure and an hourglass shape far below the waterline. The shape increases the restoring moment and natural frequency in pitch, which leads to improved behaviour in the low-frequency range. The importance of integrated optimization is shown in the solutions for the tower and blade-pitch control system, which are clearly affected by the simultaneous design of the platform. State-of-the-art nonlinear time-domain analyses show that the linearized model is conservative in general, but reasonably accurate in capturing trends, suggesting that the presented methodology is suitable for preliminary integrated design calculations.

1. Introduction

Floating wind turbines (FWTs) are complex multidisciplinary systems, which makes design optimization a challenging and time consuming task. Different components, such as the wind turbine control system, tower, platform, and mooring system, are typically designed separately in a sequential manner; however, due to strong coupling between the disciplines, this often leads to suboptimal solutions at the overall system level. It is therefore desirable to use an integrated optimization approach where the components are designed simultaneously while considering all relevant disciplines. Multidisciplinary design optimization (MDO) techniques can be used to solve such problems [1].

A large body of literature exists on the topic of FWT design, often using parametric studies to assess the behaviour of the system. Tracy [2] performed simplified frequency-domain analyses to evaluate a large variety of FWT designs. Bachynski and Moan [3] used spreadsheet calculations together with fully coupled time-domain simulations to analyse different design parameters for five tension leg platform (TLP) wind turbines. Gilloteaux and Bozonnat [4] examined the response of cylinder-shaped FWTs with different geometries.

An early study on hydrodynamic shape optimization of offshore structures using computer-aided optimization was carried out by

* Corresponding author.
E-mail address: john.m.hegseth@ntnu.no (J.M. Hegseth).

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Clauss and Birk [5], and more recently, these techniques have also been applied in the design of FWT systems. Brommundt et al. [6] optimized the catenary mooring system for a semi-submersible FWT using frequency-domain analyses. The model included the directionality of the environmental loads and thus also considered the layout of the mooring lines. Hall et al. [7] performed single- and multi-objective hull shape and mooring line optimization for different platform concepts spanning all stability classes and found that the Pareto front was dominated by relatively complex platforms with several legs and taut mooring lines. The work was later extended by Karimi et al. [8], using an updated dynamic model and a new optimization algorithm, which significantly affected the derived optimal solutions. Myhr and Nygaard [9] used time-domain simulations and two extreme load cases to optimize the structural design of a tension leg buoy and found that the wave forces could be significantly reduced if the cylindrical platform section in the wave zone was replaced with a space frame. Muskulus and Schahirt [10] offers a comprehensive review of wind turbine support structure optimization.

Despite gradient-based optimization being desirable due to its efficiency, especially for large design spaces [10], most of the work on FWT design optimization has been performed using gradient-free methods. This is likely due to the difficult task of computing accurate gradients. One exception is Fylling and Berthelsen [11], who optimized the hull shape, mooring lines and power cable for a spar FWT using a gradient-based approach with numerical gradients obtained using finite differences.

The aforementioned studies consider the design of the platform, mooring system, or both, but the tower, rotor, and control system are considered fixed during the optimization. Because the design process must consider the energy production and not only the cost of the FWT system, the control system should preferably be integrated in the optimization procedure [12]. Numerical optimization of a land-based wind turbine controller has been performed by Tibaldi et al. [13], but little exists in the literature concerning FWTs. A first step in this direction was made by Lemmer et al. [14], who optimized four main dimensions of a three-column semi-submersible FWT, where the controller was automatically tuned in each design iteration using a linear quadratic regulator (LQR). The optimization sought to minimize a combination of material costs and damage-equivalent loads in the tower for seven operational conditions, with a constraint on the static pitch angle at rated thrust.

A limitation of previous work on FWT design optimization is that the large-volume platform is considered rigid in the design analyses. In addition, the tower design is typically not considered in the optimization process. Since turbines are getting larger, the flexibility of the platform is becoming increasingly important and may need to be considered to obtain the correct global behaviour of the system. The elasticity of the platform affects both the shape and the natural frequency of the structure’s eigenmodes, which may influence the tower response.

To address these issues, the current work develops an integrated optimization process that considers the tower and platform design simultaneously, and captures the coupling effects between the two subsystems. A flexible hull, combined with distributed hydrodynamic forces, also enables the structural design of the platform to be integrated in the global optimization loop. A computationally efficient low-fidelity model is used to analyse the linearised aero-hydro-servo-elastic dynamics of a 10 MW spar FWT in the frequency domain. The model is implemented in the OpenMDAO framework [15], which enables efficient gradient-based optimization using analytic gradients. The spar platform, tower, mooring system, and control system are then optimized in an integrated fashion, considering both fatigue and extreme response constraints, where a weighted combination of system cost and power quality is employed as the objective function. Finally, the results are verified against nonlinear time-domain analyses. The rotor design is not modified during the optimization, since the model is considered unsuitable for local blade response. The design optimization of wind turbine rotors is considered to be outside the scope of the current work, but has been considered separately [16–18].

Since the presented design optimization methodology is based on a simplified dynamic model, its main purpose is to help designers in the preliminary design phase. Due to the computational efficiency of the model, it can also be used to quickly get insight into the system behaviour, and to study the effect of changes in system parameters on the dynamic response and resulting design solutions. Detailed design would require more accurate nonlinear analyses incurring a much greater computational cost.

The paper is organized as follows. The linear FWT model used to perform the integrated design analyses is presented in Section 2. In Section 3, the optimization problem is summarized and the design variables, constraints, and objective function are described in detail. The results from the optimization study are presented and discussed in Section 4, and the optimized design solutions are verified using a high-fidelity simulation tool. Finally, the findings are summarized in Section 5.

2. FWT model

The coupled dynamics of FWTs are typically analysed using nonlinear time-domain analyses, which capture the complex interactions between aerodynamics, hydrodynamics, structural dynamics, and control. For a design optimization procedure that requires several iterations to converge, each involving a large number of load cases, the process becomes computationally prohibitive. Simplified models are therefore needed, especially in the conceptual design phase, where large parts of the design space are explored to identify possible solutions.

The present design optimization study considers a spar buoy that supports the DTU 10 MW reference wind turbine [19] at a water depth of 320 m. The steel hull is partially filled with concrete ballast to achieve the correct draft, using a ballast density of 2600 kg/m³. The interface with the tower is located 10 m above the still water line (SWL), while the hub height is 119 m above the SWL. A catenary mooring system consisting of three lines spread symmetrically about the vertical axis is used for station-keeping. For simplicity, the so-called crowfeet (or delta lines) are removed, and lines with constant cross-sectional properties are used. The yaw motions of the platform are not included in the linearized model.

Only the response in the x-z-plane is considered in the current work, and co-directional waves and wind travelling in the positive x-direction are applied in all simulations. An overview of the FWT system, including the mooring system layout, is shown in Fig. 1.
2.1. Linearized system formulation

The system is linearized to obtain the equations of motion, which are solved in the frequency domain. The model is expressed in state-space form, and consists of a structural part and a control system part, which are connected to obtain the complete closed-loop aero-hydro-servo-elastic model in Section 2.4. The linearized system considers perturbations in the state and input variables, \( x \) and \( u \) respectively, about the operational point:

\[
x = x_0 + \Delta x, \quad u = u_0 + \Delta u.
\]

The dynamic equations of motion are then expressed as

\[
\Delta \dot{x} = A \Delta x + B \Delta u,
\]

where \( A \) is the state matrix, and \( B \) is the input matrix. In the rest of this work, the \( \Delta \) sign is omitted for brevity.

2.2. Structural model

2.2.1. Platform and turbine

The structural model for the platform and turbine considers four degrees-of-freedom (DOFs), as shown in Fig. 2. The equations of motion for the three support structure DOFs (surge, pitch, and the first bending mode of the tower and platform) are derived by using generalized displacements similar to Hegseth and Bachynski [20], but using a flexible hull. Hydrodynamic excitation loads are found from MacCamy–Fuchs theory, while the added mass is based on analytical 2D coefficients. Radiation damping is neglected, while viscous damping is computed using a stochastic linearization of the drag term in Morison’s equation. The three support structure DOFs are gathered in the vector \( \xi = [s_1, \xi_3, \xi_7]^T \). The differential equation for the rotor speed, \( \dot{\phi} \), considers only rigid body dynamics and thus neglects the torsional DOF of the shaft:

\[
I_o \dot{\phi} = Q_A - N_{gear} Q_G.
\]

Here, \( I_o \) is the rotor and drivetrain inertia, \( Q_A \) is the aerodynamic torque, \( N_{gear} \) is the gear ratio, and \( Q_G \) is the generator torque.

![Diagram of the FWT system](image)

Fig. 1. Overview of the FWT system.
For each wind-wave condition, the linearization point for the system is found from static equilibrium when the system is subjected to the mean environmental loads. In addition to the mean aerodynamic loads on the rotor, the static component of the aerodynamic quadratic drag force on the tower is included.

Although the linearized model presented here may yield reasonably accurate estimates for the global response of the tower, platform and mooring lines, it is not considered suitable for blade design, which typically involves nonlinear analyses in the time domain. One exception is Merz [21], who performed conceptual blade design for a stall-regulated offshore wind turbine using linear frequency-domain analyses. Because modern wind turbine blades are made of composite materials, and because they are more sensitive to local aerodynamic effects than the support structure and mooring system, including the blade design would require significant enhancements to the aerodynamic and structural models presented here, which is outside the scope of the current work. In addition, previous studies on FWTs have indicated that transient fault conditions, which cannot be analyzed with the presented methodology, may be critical for extreme blade loads [22,23]. The blade response is therefore not considered in the model; instead, the aerodynamic forces are applied as resultant loads on the tower top.

With the structural DOFs described above, the state vector is defined as

\[
\mathbf{x} = \begin{bmatrix} \xi \\ \zeta \\ \phi \end{bmatrix}.
\]

(4)

The inputs to the structural system consist of both control system outputs and disturbances due to environmental loads, which are separated in two different vectors \(\mathbf{u}_w\) and \(\mathbf{u}_d\). The control input vector is written as

\[
\mathbf{u}_w = \begin{bmatrix} Q_t \\ \theta \end{bmatrix}.
\]

(5)

where \(\theta\) is the collective blade pitch angle, which is found from the control system model, together with the generator torque. The current implementation neglects the dynamics of the generator and pitch actuator systems, and the generator torque and blade pitch are thus instantaneously set to their reference values at each time step.

The wind speed varies over the rotor disk; however, because only the resultant aerodynamic loads at the tower top are of interest in the analysis, it is desirable to express the wind input using a single scalar variable. This is achieved by expressing a rotor effective wind speed for each of the three resultant forces in Fig. 1. The rotor effective wind speed is a spatially constant wind speed that yields the same resultant forces on the rotor as the full wind field and may therefore be considered as a weighted average of the wind speed seen by the different blade elements. The disturbance vector is expressed as

\[
\mathbf{u}_d = \begin{bmatrix} v_{f_i} \\ v_{m_i} \\ v_{Q_i} \\ F_{W,1} \\ F_{W,5} \\ F_{W,7} \end{bmatrix}^T,
\]

(6)

where \(v_{f_i}, v_{m_i}\) and \(v_{Q_i}\) are the effective wind speeds for thrust, tilting moment and aerodynamic torque, respectively, which are derived in Section 2.5.2. \(F_{W,a}\) represents the generalized wave excitation force in support structure DOF \(n\), found from MacCamy–Fuchs theory. The resulting system is written as:
\[
\begin{align*}
\dot{x}_i &= A_s x_i + B_{sc} u_{sc} + B_{sd} u_{sd}, \\
y_i &= C_s x_i,
\end{align*}
\]
where the state matrix is found from the total mass \((M, \text{including hydrodynamic added mass})\), stiffness \((K)\), and damping \((D)\) matrices for the system
\[
A_s = \begin{bmatrix}
0 & -M^{-1} K & I \\
-M^{-1} & -M^{-1} D
\end{bmatrix}
\]
(8)
The input matrices \(B_{sc}\) and \(B_{sd}\) relate control inputs and disturbances to forces on the structural DOFs, while the output matrix for the structural model, \(C_s\), is chosen such that the rotor speed is the single output to match the control system input:
\[
C_s = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1].
\]
(9)

2.2.2. Mooring system

The response of the mooring lines is calculated using the simplified dynamic frequency-domain model described by Larsen and Sandvik [24] and later extended by Lie and Sødahl [25]. Here, the lines are modelled as a single DOF system, as shown in Fig. 3.

The dynamic tension in the line can then be derived as
\[
T_D = k_E [x(t) - u(t)] = c^* \dot{u}(t) |\dot{u}(t)| + k_G u(t) + \omega^2 m^* x(t),
\]
(10)
where \(u(t)\) is the generalized displacement of the line, found from a quasi-static analysis where the upper end is displaced two times the standard deviation of the platform motion from the equilibrium position (significant motion). The geometric stiffness \((k_G)\) is found as the secant stiffness when the line is moved from equilibrium to the position of significant motion. The generalized inertia coefficient \((m^*)\) and damping coefficient \((c^*)\) are found using the quasi-static line configuration as the shape function. The linearized damping coefficient \((c_L)\) is computed using the stochastic linearization
\[
c^* \dot{u}(t) |\dot{u}(t)| \approx c^* \sqrt{\frac{8}{\pi}} \sigma_{\dot{u}} \ddot{u}(t) = c_L \ddot{u}(t),
\]
(11)
where \(\sigma_{\dot{u}}\) denotes the standard deviation of \(\dot{u}\). Because Eq. (10) can be considered to be a variant of Morison’s equation, non-Gaussian extremes for the dynamic mooring line tension can be found from the normalized maxima distribution of the Morison force, which consists of a Rayleigh distribution for low probability levels and an exponential distribution for high probability levels [25]:
\[
F_Y(y) = \begin{cases} 
1 - \exp \left[ - \left( 3k^2 + 1 \right) \frac{y^2}{2} \right], & 0 \leq y \leq y_0 \\
1 - \exp \left[ - \sqrt{\frac{3k^2 + 1}{2k}} \left( y - \frac{y_0}{2} \right) \right], & y \geq y_0,
\end{cases}
\]
(12)
where \(y = T_D / \sigma_{\dot{u}(t)}\) is the normalized tension, and
\[
y_0 = \frac{1}{2k\sqrt{3k^2 + 1}}
\]
(13)
The parameter \(k\) is a measure of the relative importance of drag and inertia forces.

![Fig. 3. Simplified dynamic mooring line model as illustrated by Lie and Sødahl [25].](image)
2.3. Control system model

The baseline linear control system follows the generic approach for variable-speed pitch-controlled wind turbines [26]. The approach uses two independent controllers: a generator-torque controller for below-rated wind speeds and a collective blade-pitch controller for above-rated wind speeds. Typically, these types of controllers also include transition strategies to ensure smooth switchovers between regions; however, for simplicity, this is not included in the current implementation. In both control regimes, the control system is described with the following first-order system

\[
\dot{x} = Ax + Bu, 
\]

(14a)

\[
y = Cx, 
\]

(14b)

where the rotor speed is the only input, and the outputs are the generator torque and the blade pitch angle:

\[
u = [\dot{\phi}], \quad y = \begin{bmatrix} Q_c \\ \theta \end{bmatrix}.
\]

(15)

First, the rotor speed is filtered through a first-order low-pass filter with corner frequency \(\omega_p\) to avoid high-frequency excitation of the control system

\[
A_s = \begin{bmatrix} 0 & 1 \\ 0 & -\omega_p \end{bmatrix}, \quad B_s = \begin{bmatrix} 0 \\ \omega_p \end{bmatrix}, \quad x_s = \begin{bmatrix} \dot{\phi}_p \\ \phi_p \end{bmatrix},
\]

(16)

where \(\dot{\phi}_p\) is the low-pass filtered rotor speed. Below the rated wind speed, the generator torque is set to be proportional to the square of the rotor speed to balance the aerodynamic torque and thus maintain the optimal tip-speed ratio, which maximizes the power output. Above the rated wind speed, the generator torque is kept constant at the rated torque, and a PI controller is used to modify the collective blade pitch angle:

\[
C_s = \begin{bmatrix} 0 & 2K \dot{\phi}_p \\ 0 & 0 \end{bmatrix}
\]

(below rated),

(17)

\[
C_s = \begin{bmatrix} 0 & 0 \\ \eta_k k_c & \eta_k k_p \end{bmatrix}
\]

(above rated).

Here, \(K\) is the generator torque constant, \(\eta_k\) is the gain-scheduling parameter, while \(k_c\) and \(k_p\) are the integral and proportional gains of the PI controller, respectively.

2.4. Closed-loop system

The structural and control system models can be written as a single closed-loop system by setting \(y_s = u_s\) and \(u_{sc} = y_c\):

\[
x = Ax + Bu_{sc},
\]

(18a)

\[
y = Cx,
\]

(18b)

where

\[
A = \begin{bmatrix} A_s & B_s C_s \\ B C_s & A_s \end{bmatrix}, \quad B = \begin{bmatrix} B_{sd} \\ 0 \end{bmatrix}, \quad x = \begin{bmatrix} x_s \\ x_c \end{bmatrix},
\]

(19)

and \(C\) is set to the identity matrix. The closed-loop system is then transformed to the frequency domain. The transfer matrix between disturbances and outputs for a frequency \(\omega\) is defined as:

\[
H(\omega) = C(j\omega I - A)^{-1}B.
\]

(20)

The cross-spectral density matrix of the response vector \(y\) can be calculated from

\[
S_y(\omega) = H(\omega)S_u(\omega)H(\omega)^H,
\]

(21)

where \((\cdot)^H\) denotes the conjugate transpose [27]. The variance spectra of \(y\) are then found along the diagonal of \(S_y(\omega)\).

The cross spectral density matrix \((S_u(\omega))\) is used for the load process and has the following structure

\[
S_u(\omega) = \begin{bmatrix} S_{wind}(\omega) & 0 \\ 0 & S_{wave}(\omega) \end{bmatrix}
\]

(22)

The off-diagonal terms above are zero because the wind and wave processes are assumed to be uncorrelated [28]. The resulting response spectra are applied, together with dynamic equilibrium, to find the bending moment response along the
tower and platform. The fatigue damage is then calculated at selected locations using the Dirlik method [29].

The extreme response of the support structure is computed using the average upcrossing rate (AUR) method [30]. Assuming that the response process is stationary and Gaussian with zero mean, the cumulative distribution function (CDF) of the extreme value \( Y \) for an arbitrary response parameter can be written as [27].

\[
F_Y(y) = \exp \left\{ - \nu^+(0) T \exp \left( - \frac{y^2}{2\sigma^2} \right) \right\},
\]

where \( \nu^+(0) \) is the mean zero-upcrossing rate, which can be found from the zeroth- and second-order spectral moments:

\[
\nu^+(0) = \frac{1}{2\pi} \sqrt{\frac{m_2}{m_0}}.
\]  

The \( n \)th spectral moment is defined as:

\[
m_n = \int_0^\infty \omega^n S(\omega) d\omega.
\]  

The most probable maximum value in 1 h, \( y_{1h} \), is used in the design constraints related to extreme response. The value is approximated using:

\[
F_Y(y_{1h}) = e^{-1}.
\]

The mean value of the response process is then added to \( y_{1h} \) to obtain the total extreme value. We have used this methodology to compute fatigue damage in the tower base, as well as extreme surge, pitch, and tower base bending moment response, for two 10 MW spar FWT designs in previous work [20]. We found this methodology to yield good agreement within \( \pm 30\% \) for long-term fatigue and \( \pm 15\% \) for short-term extremes when compared to nonlinear time-domain simulations.

2.5. Aerodynamic forces

2.5.1. Linearized BEM theory

The aerodynamic forces on the blades are computed using blade-element momentum (BEM) theory [31,32]. A quasi-steady formulation is used, meaning that dynamic wake and dynamic stall effects are neglected. The Glauert correction and hub and tip loss factors are included in the model. The normal and tangential inflow velocities for a blade section at radius \( r \) are defined as:

\[
V_n = v (1 - a_n),
\]

\[
V_t = \phi r (1 + a_t),
\]

where \( v \) is the relative wind speed, and \( a_n \) and \( a_t \) are the normal and tangential induction factors, respectively. In the analyses, induction factors for the different blade sections are found from pre-calculated lookup tables as functions of blade pitch angle and tip speed ratio. The inflow velocity \( W \) and inflow angle \( \varphi \) seen by the blade element are then expressed as

\[
W = \sqrt{V_n^2 + V_t^2},
\]

\[
\varphi = \tan^{-1} \frac{V_n}{V_t}.
\]

The angle of attack is the difference between the inflow angle and the element's combined pitch and twist angle:

\[
\alpha = \varphi - (\theta + \beta).
\]

Here, \( \beta \) is the twist angle. The lift and drag forces can then be computed using tabulated lift and drag coefficients

\[
L = \frac{1}{2} \rho c W^2 C_L(\alpha),
\]

\[
D = \frac{1}{2} \rho c W^2 C_D(\alpha).
\]

The normal and tangential forces relative to the rotor plane are found from a coordinate system transformation

\[
F_x = L \cos \varphi + D \sin \varphi
\]

\[
F_y = L \sin \varphi - D \cos \varphi.
\]

The blade root loads of interest, i.e., the flapwise shear force \( F_y \), the flapwise bending moment \( M_x \), and the edgewise bending moment \( M_y \), are found by integrating the loads over the length of the blade.
\[ F_z = \int_0^R F_z(r) \, dr, \quad (32a) \]
\[ M_z = \int_0^R r F_z(r) \, dr, \quad (32b) \]
\[ M_y = \int_0^R r F_y(r) \, dr. \quad (32c) \]

As seen in Eqs. (27)-(32), the aerodynamic forces are nonlinear functions of wind speed, rotor speed, and blade pitch angle. To obtain linear forces that can be applied in the simplified FWT model, the blade root forces are linearized using a first order Taylor expansion:

\[ F_z \approx F_{z0} + F_{z1} \Delta v + F_{z2} \Delta \phi + F_{z3} \Delta \theta. \quad (33a) \]
\[ M_z \approx M_{z0} + M_{z1} \Delta v + M_{z2} \Delta \phi + M_{z3} \Delta \theta, \quad (33b) \]
\[ M_y \approx M_{y0} + M_{y1} \Delta v + M_{y2} \Delta \phi + M_{y3} \Delta \theta. \quad (33c) \]

where the subscripts after the commas denote partial derivative with respect to the variable, i.e.,
\[ A_{ij} = \frac{\partial A_i}{\partial y}. \quad (34) \]

2.5.2. Rotor effective wind speed

A rotor effective wind speed is used to describe the incoming wind field. The effective wind speed is dependent on the type of resultant load (thrust force, tilting moment, or aerodynamic torque), as well as the mean wind speed.

As shown by Halfpenny [33], the (single sided) cross-spectral density of the rotationally sampled wind speed for two blade elements \( j \) and \( k \) can be written as:

\[ S_{\psi\psi}(\omega) = \sum_{n=-\infty}^{\infty} e^{i\omega n} K_n^{(0)}(n \phi) S_{\nu}(\omega - n \phi), \quad (35) \]

where \( S_{\nu}(\omega) \) is the incoming wind spectrum, \( \psi \) is the azimuth angle between the elements, and \( K_n \) is the \( n \)th Fourier coefficient of the coherence function \( \gamma \), which is Fourier expanded in the rotor plane:

\[ K_n^{(0)}(\omega) = \frac{1}{\pi} \int_0^\pi \gamma(\omega, d, \phi) \cos(n\phi) \, d\phi. \quad (36) \]

The coherence function for two points with separation distance \( d \) and frequency \( f \) (in Hz) is taken from IEC [34] and expressed as

\[ \gamma(f, d) = \exp \left\{ -12 \left[ \left( \frac{f d}{U_{hub}} \right)^2 + \left( \frac{0.12d}{L_c} \right)^2 \right]^{0.5} \right\}, \quad (37) \]

where \( U_{hub} \) is the mean hub-height wind speed and \( L_c \) is a coherence scale parameter.

The forces on each blade element can then be found from the rotationally sampled wind speed and weight factors relating wind speed to forces on the element. These weight factors are the partial derivatives of the blade element forces in Eq. (31) with respect to wind speed. When summing the forces on the blade elements to obtain the total tower top loads, cancellation of harmonics results in peaks at multiples of three of the rotor frequency (i.e., 3P, 6P, 9P, etc.) for the resultant force spectra. In the current work, harmonics higher than 3P are neglected. Tower shadow and wind shear are not included.

The spectra for the effective wind speeds are calculated from the resultant force spectra and the total weight factors over the rotor:

\[ S_{\psi j}(\omega) = \frac{S_{\psi z}(\omega)}{3 F_{z0}}, \quad (38a) \]
\[ S_{\psi k}(\omega) = \frac{S_{\psi z}(\omega)}{3 F_{z0}}, \quad (38b) \]
\[ S_{\psi 0}(\omega) = \frac{S_{\psi z}(\omega)}{3 F_{z0}}. \quad (38c) \]

The wind load submatrix in Eq. (22), \( S_{\text{wind}}(\omega) \), can then be found simply by observing which harmonics of the wind are shared by
the different load components. As shown by Halfpenny [33], the thrust force and aerodynamic torque on the turbine are perfectly correlated, since they share all harmonics, and completely uncorrelated from the tilting moment. This is because they have no harmonics in common.

2.5.3. Resultant aerodynamic forces
Following the derivation of weight factors in the previous section, the relation between derivatives of the blade root forces and resultant forces on the rotor becomes evident:

\[ F_{y,r} = 3 F_{y,r}, \]  
\[ M_{y,r} = \frac{3}{2} M_{y,r}. \]  

Fig. 4. Comparison of spectra for resultant aerodynamic loads on the tower top. Mean wind speeds of 7 m/s (left) and 15 m/s (right).
\[ Q_{av} = 3 \, M_{sv}. \]  

(39c)

The same relations hold for the derivatives with respect to the rotor speed and blade pitch angle; however, a change in rotor speed or collective blade pitch angle affects all parts of the rotor plane equally, and perturbations in these variables therefore do not contribute to the tilting moment. The total resultant aerodynamic forces are thus expressed as:

\[ F_T \approx F_{T0} + F_{T,\Delta v} + F_{T,\Delta \phi} + F_{T,\Delta \theta}, \]  

(40a)

\[ M_T \approx M_{T0} + M_{T,\Delta v}, \]  

(40b)

\[ Q_4 \approx Q_{w0} + Q_{w,\Delta v} + Q_{w,\Delta \phi} + Q_{w,\Delta \theta}. \]  

(40c)

2.5.4. Verification

To verify the linearized aerodynamic formulation, the resultant tower top loads are compared to results from nonlinear time-domain simulations performed using the SIMA software developed by SINTEF Ocean, which couples the RIFLEX [35] and SIMO [36] programs. Here, the blades are modelled using nonlinear beam elements, and the aerodynamic loads on the blades are computed using BEM theory with a Glauert correction, hub and tip loss factors, dynamic wake, dynamic stall, and tower shadow. The three-dimensional turbulent wind field is created using TurbSim [37], where the vertical wind shear is modelled by the power law with an exponent of 0.14 [38].

Fig. 4 shows the comparison for a fixed turbine with the control system active for mean wind speeds of 7 and 15 m/s. The results show overall good agreement, with the exception of frequencies close to the 3P frequency, where the linear model overestimates the loads. This discrepancy may be due to the assumption of constant rotor speed in the effective wind speed formulation, which causes the linear model to overestimate the response of the 1st bending mode (and thus the fatigue damage in the structure) if the natural frequency is close to the blade passing frequency. However, this is not expected to significantly affect the results from the optimization, because it is highly unlikely that the optimizer will converge to a design solution with the first bending frequency within the 3P range.

2.6. Environmental conditions

The fatigue design should consider all relevant environmental conditions (ECs) over the lifetime of the structure; however, to limit the computational effort, 30 ECs are used to evaluate the long-term fatigue performance in the present work. The mean wind speeds range from 1 to 30 m/s with 1 m/s steps. For each mean wind speed, the most probable values for the significant wave height and spectral peak period are used. These values, as well as the probability associated with each condition, are found from the joint probability distribution presented by Johannessen et al. [39], and shown in Fig. 5. The probabilities are normalized to achieve a total probability of unity, i.e.,

\[ \sum_{i=1}^{N_{EC}} p_i = 1, \]  

(41)

where \( N_{EC} \) is the number of ECs considered in the analysis, and \( p_i \) is the probability of condition \( i \).

![Fig. 5. Significant wave heights, spectral peak periods, and normalized probabilities associated with mean hub height wind speeds applied in the fatigue calculations.](image)
The three ECs described in Table 1, which are located on the 50-year contour surface, are selected to evaluate the extreme response. In EC 1, the mean wind speed is close to the rated speed of the turbine, which yields the maximum thrust force on the rotor. EC 2 considers an operational condition with above-rated wind speed, while EC 3 represents a wind speed above cut-out, where the turbine is parked and the blades are feathered.

For all ECs, a Kaimal spectrum for IEC class B turbines and a normal turbulence model (NTM) [34] are used to describe the incoming wind.

3. Optimization problem

The present optimization study considers the design of four different subsystems simultaneously:

1. Spar platform
2. Wind turbine tower
3. Mooring system
4. Blade-pitch control system

The formulation of the optimization problem is described in the following subsections.

3.1. Framework

The FWT model is implemented in OpenMDAO [15], which is an open-source framework for multidisciplinary design, analysis and optimization. A modular approach is utilized, where smaller units of code (components) are connected to form the larger and more complex multidisciplinary model. Inputs, outputs, and partial derivatives for each component are given to the framework, which handles the data transfer between different parts of the code, performs the analysis, and calculates the total system derivatives needed by the optimizer [40]. To analyze a large number of load cases in an efficient manner, the code is parallelized using MPI.

The design is optimized using a gradient-based approach where analytic derivatives are computed for the coupled model. Analytic gradient computation requires significant implementation effort, but offers increased computational efficiency and accuracy compared to finite-difference approximations. The total derivatives are computed using the adjoint method, whose cost is independent of the number of design variables [41]. With the current model, which has 80 inputs and 18 outputs, the run time for the gradient computation is roughly 4× the analysis time, where a single wind-wave condition runs in approximately 5 s on a standard laptop. The model layout and data dependencies are illustrated using an extended design structure matrix (XDSM) [42] in Fig. B.1 in Appendix B.

The SNOPT algorithm [43] is used to solve the optimization problem. SNOPT uses a sequential quadratic programming (SQP) approach to solve nonlinear constrained optimization problems, and is connected to OpenMDAO using the pyOptSparse interface [44].

Because gradient-based methods have the problem of potentially getting stuck in local minima if the objective function is multimodal, the optimization is run starting from several initial design points.

3.2. Objective

Ideally, a FWT design optimization procedure should try to minimize the levelized cost of energy (LCOE), which is the lifetime costs of the system divided by the total energy production. However, because the LCOE depends on numerous unknown parameters that cannot be included properly in an engineering optimization model, simpler objectives are needed. A common choice is the weight or the associated material costs of the system, but cost functions that do not take into account production cost components, such as welding, are considered unrealistic [10]. In addition, a measure of the power production performance should be included. The amount of produced energy will mostly depend on the wind farm site and is unlikely to vary much with the design of the FWT system. Another relevant performance measure is the quality of the power delivered by the turbine, which describes how much the energy output fluctuates. High-quality power, which has low variability in the output, is desirable, and its value for a given design is proportional to the variation in rotor speed. The objective function used in the present work is thus a weighted combination of two parameters: the system costs, \( C_{\text{total}} \), which cover material and manufacturing, and the weighted average of the rotor speed standard deviation, \( \sigma_{\phi} \), which is a measure of the power quality:

\[
f = w_1 C_{\text{total}} + w_2 \sigma_{\phi},
\]

(42)

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Environmental conditions for extreme response calculations.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Condition</td>
<td>1</td>
</tr>
<tr>
<td>Mean wind speed at hub height, ( U ) (m/s)</td>
<td>11.0</td>
</tr>
<tr>
<td>Significant wave height, ( H_s ) (m)</td>
<td>7.5</td>
</tr>
<tr>
<td>Spectral peak period, ( T_p ) (s)</td>
<td>12.0</td>
</tr>
<tr>
<td>Turbulence intensity at hub height, ( I ) (–)</td>
<td>0.18</td>
</tr>
</tbody>
</table>
The relative importance is controlled by the weight factors \( w_1 \) and \( w_2 \), where \( w_1 + w_2 = 1 \); \( w_1, w_2 \in [0, 1] \). Because the monetary cost of reduced power quality is difficult to quantify, the optimization is run with different values for the weight factors, to assess the trade-off between the two sub-objectives.

The total cost of the system is

\[
C_{\text{total}} = C_{\text{spar}} + C_{\text{tower}} + C_{\text{moor}},
\]

(43)

where \( C_{\text{spar}} \), \( C_{\text{tower}} \) and \( C_{\text{moor}} \) are the costs of the platform, tower, and mooring system, respectively. The platform and tower costs take into consideration both material and manufacturing, using cost models from Farkas and Jármai \[45\]. Costs related to installation, maintenance, and decommissioning are not included. The cost of the platform (and similarly for the tower) is expressed as

\[
C_{\text{spar}} = k_m M_{\text{spar}} + k_f \sum_i T_i,
\]

(44)

where \( k_m \) is the steel cost factor, \( M_{\text{spar}} \) is the steel mass of the hull, and \( k_f \) is the fabrication cost per unit time. \( T_i \) is the time spent at fabrication stage \( i \), expressed as a function of the geometry. The manufacturing costs include forming of plate elements into shells, assembly, welding, and painting. The ratio between the material and fabrication cost factors, \( k_m/k_f \), depends on the labour cost, which varies with the fabrication site. In the present work a ratio of 1.0 is assumed, which is a typical value for West European labour \[45\]. Based on numbers from Teillant et al. \[46\], the cost of structural steel is about 80 times the cost of concrete ballast (per kg). The cost of ballast is therefore neglected in the current work.

Many different estimates for the cost of steel for floating support structures can be found in the literature, spanning from 1.0 €/kg (only material \[47\]) to 4.5 €/kg (including manufacturing and assembly \[14\]). Here, the steel cost factor, \( k_m \), is assumed to have a value of 2.7 €/kg, which is approximately the same as the value used by Fylling and Berthelsen \[11\]. The mooring line costs are expressed as a function of weight, with a price of 3–4.5 €/kg, depending on the grade \[48\]. In the present work, steel grade R4 with an associated cost of 3.5 €/kg is assumed, which results in mooring line costs that are about 5–10% of the support structure costs. The relative importance of mooring and support structure costs, which is somewhat uncertain, may affect the optimization results due to trade-off effects; however, preliminary analyses indicate that the system is fairly insensitive to the cost distribution.

The weighted average of the rotor speed standard deviation, used in Eq. (42), is computed by summing the values from each of the 30 FLS conditions described in Section 2.6, weighted by the probability of the condition:

\[
\sigma_{(\epsilon)} = \sum_{i=1}^{N_{\epsilon}} p_i \sigma_{(\epsilon) i},
\]

(45)

where \( \sigma_{(\epsilon) i} \) is the rotor speed standard deviation for condition \( i \).

---

Fig. 6. Scantling design parameters.
3.3. Design variables

3.3.1. Spar platform

In the design process, the hull is discretized using ten sections in the vertical direction. The length of each section, as well as the diameter and wall thickness at each connecting node are included as design variables in the optimization. The diameter and wall thickness are assumed to vary linearly within each element. The number of elements that must be welded together during production is assumed to be equal to the number of hull sections, and is thus independent of the section lengths.

In addition, the study considers the scantling design of the hull. The spar platform is assumed to be equipped with circumferential T-ring stiffeners, which are described by five variables: the thickness and length of the web and flange, and the distance between the stiffeners. This is illustrated in Fig. 6. The ring stiffener variables are assumed to be constant within each section of the hull, and a material density of 7850 kg/m$^3$ is used for all components.

3.3.2. Tower

Similarly to the platform, the tower is parameterized as ten sections with linearly interpolated cross sectional properties. The section lengths are kept fixed during the optimization to achieve the same hub height as the original DTU 10 MW turbine at 119 m above SWL. Here, each section has a length of 10.5 m, except for the uppermost section, which has a length of 11.13 m. The design variables are the diameter and wall thickness for each section, and the inner structure in the tower is not considered. Because secondary structures in the tower are not modelled, the material density is increased to 8500 kg/m$^3$ to achieve a more realistic tower mass [19].

3.3.3. Mooring system

The mooring system optimization includes four design variables: the mooring line diameter, the depth of the fairleads below the SWL, the total length of the lines, and the horizontal distance between fairlead and anchor. Because the analyses only consider the 2D response of the turbine with co-directional wind and waves, the layout of the mooring system in the horizontal plane is not considered in the study.

The mooring line diameter affects both the mass and stiffness properties of the lines. The equations for the submerged weight and axial stiffness, $w_{\text{moor}}$ and $EA_{\text{moor}}$ respectively, are taken from Barltrop [49]:

$$w_{\text{moor}} = 0.1875 \, D_{\text{moor}}^2 \quad \text{[N/m]}$$

(46)

and

$$EA_{\text{moor}} = 90000 \, D_{\text{moor}}^3 \quad \text{[N]}. \quad (47)$$

where $D_{\text{moor}}$ is expressed in mm.

| Table 2 |
|-----------------|------------|-----------------|-----------------|
| Description | Symbol | Number of variables | Units |
| Spar diameter | $D_{\text{spar}}$ | 11 | m |
| Spar wall thickness | $t_{\text{spar}}$ | 11 | m |
| Spar length | $l_{\text{spar}}$ | 10 | m |
| Ring stiffener flange thickness | $t_{\text{stiff}}$ | 4 | m |
| Ring stiffener web thickness | $t_{\text{w stiff}}$ | 4 | m |
| Ring stiffener flange width | $b_{\text{stiff}}$ | 4 | m |
| Ring stiffener web height | $h_{\text{stiff}}$ | 4 | m |
| Ring stiffener distance | $l_{\text{stiff}}$ | 4 | m |
| Tower diameter | $D_{\text{tower}}$ | 11 | m |
| Tower wall thickness | $t_{\text{tower}}$ | 11 | m |
| Mooring line diameter | $D_{\text{moor}}$ | 1 | m |
| Fairlead depth | $r_{\text{moor}}$ | 1 | m |
| Mooring line length | $l_{\text{moor}}$ | 1 | m |
| Horizontal distance from fairlead to anchor | $x_{\text{moor}}$ | 1 | m |
| Proportional gain of blade-pitch controller | $k_p$ | 1 | rad/s/rad |
| Integral gain of blade-pitch controller | $k_i$ | 1 | rad/rad |
3.3.4. Control system

The control system optimization considers the proportional and integral gains of the blade-pitch PI controller. Although more advanced control strategies are not employed in the present work, they could be implemented with their associated parameters as design variables within the present framework, as long as the system can be reasonably accurately represented by a linear control model. A relevant example is the well-known “negative damping problem” related to the blade-pitch controller above rated wind speed, which can induce instabilities in the platform motions of the system [50]. A suggested way to resolve this issue is to use a feedback term proportional to the nacelle velocity [51] or platform pitch velocity [52] to manipulate the rotor speed set point. The velocity feedback gain could then be incorporated as a design variable in the optimization process.

3.3.5. Summary

Concatenating the design variables for each of the subsystems described in the previous sections would result in a total of 110 design variables. To reduce the required computational effort, the number of design variables is decreased by introducing B-splines for the ring stiffener parameters. B-splines use a pre-defined number of control points, which we set to \( n_p = 4 \), to produce smooth distributions for the parameters. The remaining support structure design variable vectors are kept at their original sizes, because we found this to result in better optimization convergence. Thus, the optimization problem was reduced to the 80 design variables detailed in Table 2.

3.4. Constraints

The present optimization study includes constraints related both to fatigue (FLS) and ultimate (ULS) limit states for the system. In addition, a number of restrictions are added to avoid non-physical or infeasible design solutions, as detailed below. These may not be stated explicitly in design standards, but are chosen here based on common practice or engineering judgment.

3.4.1. Spar platform

The fatigue damage in the hull is calculated at each node using SN curves and the Dirlik method. The D curve with cathodic protection developed by DNV GL [53] is applied together with a design fatigue factor (DFF) of 3.0. This DFF is higher than the minimum required value given by DNV GL [54], but corresponds to the DFF for a normal safety class in the previous DNV standard [55]. The lifetime of the FWT system is chosen to be 20 years, and the fatigue design constraint that needs to be satisfied is thus expressed as

\[
D_{tot} \leq \frac{1.0}{\text{DFF}},
\]

where \( D_{tot} \) is the total fatigue damage in 20 years:

\[
D_{tot} = N_{30} \sum_{i=1}^{N_{30}} p_i D_i.
\]

Here, \( N_{30} \) is the number of short term conditions in 20 years, and \( D_i \) is the fatigue damage in condition \( i \).

Hull buckling is assessed using design criteria stated by DNV GL [56], where the following buckling modes are considered:

1. Shell buckling: Buckling of shell plating between ring stiffeners
2. Panel ring buckling: Buckling of shell plating including ring stiffeners
3. Column buckling: Buckling of the cylinder as a column

For shells, the stability requirement is given by

\[
\sigma_{\text{S3}} \leq f_{\text{stl}},
\]

where \( \sigma_{\text{S3}} \) is the design equivalent von Mises stress, and \( f_{\text{stl}} \) is the design shell buckling strength. To avoid panel ring buckling, several requirements must be met. The cross sectional area of the ring frame, \( A_{\text{stl}} \), must satisfy

\[
A_{\text{stl}} \geq \left( \frac{2}{Z_i} + 0.06 \right) I_{\text{eff}} / L_{\text{eff}} ,
\]

where \( Z_i \) is a curvature parameter. Also, the effective moment of inertia of the ring frame, \( I_{\text{stl}} \), must satisfy

\[
I_{\text{stl}} \geq I_x + I_y + I_z.
\]

where the terms on the right hand side are the minimum required moments of inertia for cylindrical shells subjected to axial, bending, or both (\( I_x \)), torsion, shear, or both (\( I_y \)), and external hydrostatic pressure (\( I_z \)). Because the ballast inside the platform is assumed to be solid, the horizontal pressure from the ballast, which counteracts the external pressure, is not accounted for in the buckling calculations [54]. To avoid local buckling of the ring frame, the web height is limited to
\[ h_{\text{bult}} \leq 1.35 \cdot r_{w,\text{still}} \sqrt{\frac{E}{f_r}} \]  

(53)

The torsional buckling of the ring frame need not be considered if

\[ h_{\text{bult}} \geq \frac{7h_{\text{bult}}}{\sqrt{10 + \frac{\gamma}{2}} \cdot E \cdot f_r} \]  

(54)

where \( E \) is the Young’s modulus, and \( r \) is the mid-plane shell radius. The yield stress, \( f_r \), is set to 355 MPa. To avoid assessing column buckling, the following equation should be satisfied

\[ \left( \frac{kL_c}{L_c} \right)^2 \leq 2.5 \cdot \frac{E}{f_r} \]  

(55)

where \( k \) is the effective length factor (set to 1.0), \( L_c \) is the total length of the cylinder, and \( i_c \) is the radius of gyration of the cylinder section. For the hull buckling, a load factor \( t_f = 1.35 \) is applied on the environmental loads \([54]\), while the material factor, \( t_M \), is given by the following equation \([54]\):

\[ t_M = \begin{cases} 
1.10, & 0.80 + 0.60 \cdot t_s, \\
1.40, & 0.5 \leq t_s \leq 1.0 \\
1.0, & t_s > 1.0.
\end{cases} \]  

(56)

where \( t_s \) is the reduced shell slenderness defined in DNV GL \([56]\).

The buoyancy forces should be equal to the total weight of the system, which is achieved by filling the inside of the platform with concrete ballast from the bottom until equilibrium is reached. In addition, the platform is limited to have a maximum pitch angle of no more than 15° in any of the extreme EEs. A maximum offset requirement of 32 m, which corresponds to 10% of the water depth, is also applied to avoid breakage of the power cable. These constraints are based on common industry practice and should be verified before they are used in an actual design process.

Another potentially constrained response parameter is the nacelle acceleration, which has been frequently considered in design optimization of FWTs, either as a constraint \([11,57]\) or as part of the objective function \([7,8]\). The rationale behind setting a limit on the maximum acceleration at the nacelle is typically related to reduced loads on the drivetrain; however, a recent study by Nejad et al. \([58]\) found that the accelerations at the tower top were not a good indicator for either maximum or fatigue loads on the main bearings of the gearbox. This parameter is therefore not considered in the present work.

The heave response is not calculated in the model and is therefore not constrained, but the natural period in heave is limited to minimum 25 s to avoid the possibility of heave resonance in the wave frequency range. Here, the added mass in heave is approximated as the value for a 3D circular disc provided by DNV GL \([59]\), with the same diameter as the bottom of the platform.

Another issue relevant for spar platforms is the phenomenon known as Mathieu instability, which can occur when the pitch restoring moment varies harmonically due to large heave motions \([60]\). Haslum and Faltsen \([61]\) reported that unstable solutions occur when the ratio between the heave motion and the pitch natural frequency is 0.5, 1.0, 1.5, or 2.0. Mathieu instability is avoided in the current work by placing a constraint on the ratio between the natural periods in heave and pitch, to prevent it from reaching a value close to one of the critical ratios.

The presented model is only strictly valid for hull sections with vertical walls. However, the diameter varies with the platform nodes, and abrupt changes in geometry are unwanted due to fatigue performance. Therefore, the sections are conical in reality. To avoid platform shapes where the physics are not captured correctly by the model, a maximum taper angle of 10° is applied as a constraint for each section.

To limit the computational effort, constraints that must be satisfied along the length of the platform, such as the fatigue and buckling constraints, are aggregated using Kreisselmeier–Steinhauser (KS) functions \([62,63]\). This is also done for constraints that must be satisfied in each of the 50-year conditions.

### 3.4.2. Tower

The fatigue damage in the tower is assessed in the same way as described for the platform, but with the D curve in air and DFF = 2.0 \([55]\).

Tower buckling is assessed using Eurocode 3 \([64]\), assuming that the tower is stiffened between each section to reduce the buckling length. For each tower section, the following equation must be satisfied

\[ \sigma_t \leq \sigma_c \cdot \frac{1}{t_M t_f} \]  

(57)

where \( \sigma_t \) is the axial stress on the outer radius of the tower, \( \sigma_c \) is the critical buckling stress, and \( t_f \) and \( t_M \) are load and material factors, respectively. The factors applied in the current work are \( t_M = 1.1 \) \([65]\) and \( t_f = 1.35 \) \([54]\). To ensure a smooth transition between the platform and tower, the tower base diameter and wall thickness are set equal to the diameter and wall thickness at the platform top. As for the hull, KS functions are used to aggregate the constraints.
3.4.3. **Mooring system**

Two constraints are applied to the mooring system: the maximum mooring line tension should not exceed the breaking strength of the chain, and the tension at the anchor should be purely horizontal. The first constraint is expressed as \[ T_d \leq 0.95 S_{\text{min}}, \] where \( S_{\text{min}} \) is the minimum breaking strength and \( T_d \) is the design tension, \[ T_d = \gamma_{\text{mean}} T_c,\text{mean} + \gamma_{\text{dyn}} T_c,\text{dyn}. \]

Here, \( T_c,\text{mean} \) and \( T_c,\text{dyn} \) are the characteristic mean tension and the characteristic dynamic tension in the 50-year condition, respectively, and \( \gamma_{\text{mean}} = 1.3 \) and \( \gamma_{\text{dyn}} = 1.75 \) are the load factors for the normal safety class [54]. The minimum breaking strength is computed using \[ S_{\text{min}} = c D^2_{\text{moor}} (44 - 0.08 D_{\text{moor}}), \] where \( c \) is a constant dependent on the mooring chain grade [66].

The second constraint is based on the assumption that the anchors are not designed to take vertical forces, which requires the mooring lines to have sufficient length to avoid uplift at the anchors in all relevant ULS conditions [67]. The constraint limits the maximum experienced offset at the fairleads to be less than the critical value, which is found from an iteration scheme where the offset is increased until the effective length of the mooring line equals the total line length.

3.4.4. **Control system**

For certain combinations of the PI controller gains, the FWT system may become unstable. This is avoided by adding a requirement that the closed-loop poles of the system should have negative real parts, which ensures stability [68]. Analyzing the poles of the linearized system is not an exact method for determination of stability; Sandner et al. [69] found that a spar FWT with gains that gave poles in the right half plane still showed reasonable performance in nonlinear simulations. However, this analysis provides a starting point for a more detailed design of the control system.

4. **Results**

The results of the optimization study, presented in the following subsections, are divided into six parts. First, we discuss the multimodality of the design space, and identify local minima. Trade-off effects between the two sub-objectives in Eq. (42) are then assessed by running the optimization problem stated in Section 3 with different weight factors. Subsequently, a single combination of weight factors is selected, and the resulting optimized design of the support structure, mooring system, and controller is described in detail. Finally, the design is verified using nonlinear time domain simulations.

4.1. **Soft-stiff and stiff-stiff tower design**

The fatigue damage in the support structure increases significantly if the natural frequency of the first tower bending mode coincides with the blade passing frequency, and the optimizer will therefore try to move the tower mode away from the 3P range. The direction that the mode is moved during the optimization will depend on the initial design, as illustrated in Fig. 7. Due to the “barrier” created by the blade passing frequency, the optimizer will not be able to move over to the stiff-stiff range if the initial design has a soft-stiff tower, and vice versa.

Initial studies performed in this work showed that the tower mode had to be placed in the stiff-stiff range for the optimization problem to converge to a feasible solution, because the optimizer was unable to find a design that satisfied the fatigue constraints in the
soft-stiff range. Although these results suggest that it is challenging to design such a FWT system with a soft-stiff tower, it should not be treated as proof that a solution does not exist. The design analyses are based on a simplified model and a reduced set of ECs for the long-term fatigue calculations, which may give inaccurate results for certain combinations of parameters. In addition, more advanced control strategies, such as a speed exclusion zone to prevent tower resonance in the below-rated regime, could improve the fatigue behaviour. More detailed studies of potential designs are therefore needed to conclude whether a feasible soft-stiff tower design is possible.

4.2. Trade-offs between system costs and power quality

To assess the trade-offs between costs and power quality for the system, the optimization is performed with different values for the weight factors in Eq. (42). Here, the rotor speed standard deviation is normalized by the value achieved with the land-based DTU 10 MW wind turbine and the original controller [70], found from nonlinear time domain analyses using SIMA. The land-based weighted average rotor speed standard deviation is found to be 0.043 rad/s. Because the present work considers a floating system with a simplified controller, the rotor speed variations are expected to be larger than the land-based reference value.

The results, shown in Fig. 8, suggest that a significant reduction in rotor speed variation can be achieved with relatively small increases in the total costs. However, these results are dependent on the environmental conditions that drives the extreme response. The reduction in rotor speed variation is primarily achieved by modifying the blade-pitch controller gains, but for both platform motions and buckling loads in the support structure, the below-rated 50-year condition (EC 1) yields the largest response. Therefore, the blade-pitch controller does not affect the extreme response of the system, and the trade-off effects are limited to the response in the above-rated fatigue conditions, where a faster controller (and thus better rotor speed tracking) leads to an increase in the fatigue loads and consequently to an increase in material costs.

In the following sections, the results are presented for the case shown with a black circle in Fig. 8, which corresponds to \( w_1 = 7 \times 10^{-8} \) and \( w_2 = (1 - 7 \times 10^{-8}) \) in Eq. (42).

4.3. Platform and tower

The optimized tower and platform design, including fatigue and buckling utilization, is shown in Fig. 9. For the hull, the value for the most critical buckling mode is shown.

The probable reason for the relatively small platform diameter in the wave zone is to reduce the wave loads on the structure. Because a smaller diameter increases the axial stress for a given bending moment, and because the fatigue loads in this part of the structure are large, a large wall thickness is needed in this area to satisfy the FLS design criteria. Fatigue is the critical failure mode down to a depth of approximately 10 m, beyond which buckling due to hydrostatic pressure becomes the constraint driving the design.

Below the wave zone, the hull exhibits an hourglass shape, with maximum and minimum values for the diameter approximately 32 m and 56 m below the SWL, respectively. The sharp edges located where the diameter reaches its maximum and minimum and the constant gradient between them is a result of the maximum taper angle constraint being active for most of the platform sections. The constraint is a consequence of the simplified hydrodynamic and structural formulations, and its value is based solely on engineering judgment. The results indicate that the performance of the platform can be further improved if this constraint is relaxed, which might require a more comprehensive numerical model and possibly more information about the effect of the platform shape on the manufacturing costs.

The hourglass shape serves several purposes that are beneficial for the platform dynamics. The large diameter just below the wave

![Fig. 8. Trade-offs between rotor speed variation and system costs in the multiobjective optimization.](image)
Fig. 9. Optimized tower and platform design with fatigue utilization (left) and buckling utilization (right). The wall thickness is scaled by a factor of 40 relative to the diameter for visualization purposes.
zone raises the centre of buoyancy (CoB), and because spar platforms achieve most of their restoring moment in roll and pitch from the distance between the CoB and the total centre of gravity (CoG), this effectively increases the rotational stiffness of the system.

The increase in diameter near the bottom of the spar can be explained by two factors. First, because the hull is filled with concrete ballast from the bottom until equilibrium between weight and buoyancy is reached, a consequence of the increased diameter is that a larger share of the required ballast can be placed near the bottom, which lowers the platform CoG. Second, the increased surface area at the bottom increases the added mass and thus the natural period in heave, which is constrained from below to avoid resonance in the wave-frequency range. One effect of increasing the restoring moment is that the natural period in pitch is reduced. At higher frequencies, there is less energy in the wind (less turbulence) and thus less excitation around the natural frequency as well.

The lower draft compared to other designs, such as the 5 MW OC3-Hywind design [71], is advantageous from a material cost point of view, because the high hydrostatic pressure at large depths requires thick-walled sections. Although costs related to installation are not a part of the objective in the current optimization scheme, a shallower draft is also favourable with respect to assembly and towing to site, which is one of the challenges related to spar buoys.

Both the maximum platform pitch constraint of 15° and the maximum offset constraint of 32 m in the 50-year conditions are active for the optimized design. The critical condition for both constraints is EC 1, where the mean wind speed is close to rated. The static pitch angle at rated thrust for the optimized design is 7.7°. The key characteristics for the optimized spar platform are listed in Table 3, while natural periods for the system in calm air are listed in Table 4. Distributed properties for the optimized platform and tower are given in Tables 10 and 11 in Appendix A.

Surprisingly, the diameter of the tower increases from the base and reaches its maximum at a height of 20.5 m above the SWL. This is uncommon for wind turbine towers, which typically are widest at the bottom and taper gradually towards the nacelle. The reason for this unconventional design is related to the integrated optimization approach, which takes into account the cost and performance of the tower and platform simultaneously. In general, within the limits set by the buckling constraints, the most cost-effective way to achieve the required fatigue performance for a given tower section is to increase the diameter and decrease the wall thickness. However, at the tower base, the diameter also has to match the diameter at the top of the platform, which should be reduced to limit the wave loads on the structure. The result of these conflicting influences is that the optimized tower starts with a smaller-than-optimal diameter at the base and consequently larger-than-optimal wall thickness (from the point of view of an isolated tower), before the diameter is increased and the wall thickness is significantly reduced.

The spar platform has clearly separated areas where the design is driven by either fatigue or ultimate loads because of the large bending moments in the wave zone and high hydrostatic pressure in the lower parts. Unlike the spar platform, both tower constraints are dominated by bending moments. Consequently, the optimizer is able to find a tower design where both the FLS and ULS constraints are close to the limit along the whole length. Table 5 lists the main parameters of the optimized tower, where the elevations to the tower base and tower top are fixed during the optimization.

In Fig. 10, the optimized tower design is compared to the tower of the OO-Star Wind Floater Semi 10 MW, which was part of the LIFES50+ project [72]. The comparison shows that the present design in general has a larger diameter and a smaller wall thickness than the OO-Star tower. Comparing the overall tower properties, the mass of the tower optimized in the present work is approximately 24% lower than the OO-Star tower, despite using a slightly larger effective mass density (8500 vs. 8243 kg/m³). The natural frequency of the first fore-aft bending mode (with flexible substructure) is 0.8 Hz and 0.59 Hz for the optimized tower and the OO-Star tower, respectively, which places them both well within the stiff-stiff range. One drawback with the large diameter in the tower presented here is that tower shadow effects, which are not considered in the model, become more prominent. This may in turn lead to increased loads on the blades and increased 3P excitation on the tower, which could complicate the design process.

The optimized scantling design parameters are shown in Fig. 11. All five variables follow the same pattern, where a reduced distance between stiffeners results in smaller required stiffener size. In general, the most critical buckling mode driving the ring stiffener design is the buckling of the shell plating.

4.4. Control system

The optimized control system parameters for different weight factors are plotted against the resulting power quality in Fig. 12. Increasing the gains results in a faster controller, leading to lower variation in the rotor speed.

To assess the importance of integrating the controller in the FWT design loop, the optimization procedure is repeated with the control parameters $k_p$ and $k_i$ fixed at their initial values, as shown in Table 6. The values are those obtained by Hegseth and Bachynski [20], where a simple detuning procedure was used to reduce the bandwidth of the original DTU 10 MW controller [70] and thus place

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Draft</td>
<td>80.4</td>
<td>m</td>
</tr>
<tr>
<td>Mass including ballast</td>
<td>$1.57 \times 10^7$</td>
<td>kg</td>
</tr>
<tr>
<td>Displacement</td>
<td>$1.72 \times 10^4$</td>
<td>m³</td>
</tr>
<tr>
<td>Moment of inertia about CoG</td>
<td>$4.66 \times 10^9$</td>
<td>kgm²</td>
</tr>
<tr>
<td>Vertical CoG below SWL</td>
<td>66.0</td>
<td>m</td>
</tr>
<tr>
<td>Vertical CoB below SWL</td>
<td>42.9</td>
<td>m</td>
</tr>
</tbody>
</table>
Table 4
Natural periods for the optimized FWT system.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surge</td>
<td>144.7</td>
<td>s</td>
</tr>
<tr>
<td>Pitch</td>
<td>34.4</td>
<td>s</td>
</tr>
<tr>
<td>Heave</td>
<td>25.0</td>
<td>s</td>
</tr>
<tr>
<td>1st bending</td>
<td>1.25</td>
<td>s</td>
</tr>
</tbody>
</table>

Table 5
Optimized tower properties.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elevation to tower base above SWL</td>
<td>10.0</td>
<td>m</td>
</tr>
<tr>
<td>Elevation to tower top above SWL</td>
<td>115.63</td>
<td>m</td>
</tr>
<tr>
<td>Total mass</td>
<td>$9.5 \times 10^5$</td>
<td>kg</td>
</tr>
<tr>
<td>Vertical CoG above SWL</td>
<td>49.1</td>
<td>m</td>
</tr>
<tr>
<td>Moment of inertia about CoG</td>
<td>$7.46 \times 10^8$</td>
<td>kgm$^2$</td>
</tr>
</tbody>
</table>

Fig. 10. Comparison of optimized tower design with LIFESS0+ OO-Star Wind Floater.

it below the platform pitch natural frequency, while the damping ratio was kept constant.

The cost and power quality for the optimized solutions with and without the controller (for the same objective function weights) are shown in Table 7. Using the initial detuned controller results in both slightly higher costs, due to larger fatigue loads in the tower, and larger rotor speed variation. Although the actual numbers depend on the values used for the gains and thus are only valid for this particular example, it shows that neglecting the controller in the structural design process may lead to more expensive solutions with lesser performance.
Fig. 11. Optimized ring stiffener parameters. The black dots show a possible stiffener layout.

Fig. 12. Optimized blade-pitch controller gains plotted against resulting rotor speed variation.

### Table 6
Fixed control system parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_p$</td>
<td>0.1794</td>
<td>rad/s/rad</td>
</tr>
<tr>
<td>$k_i$</td>
<td>0.0165</td>
<td>rad/rad</td>
</tr>
</tbody>
</table>

### Table 7
Objective function values with and without controller included in the optimization.

<table>
<thead>
<tr>
<th></th>
<th>System cost [$10^6$ €]</th>
<th>Normalized rotor speed std.dev. [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>With controller</td>
<td>12.17</td>
<td>1.65</td>
</tr>
<tr>
<td>Without controller</td>
<td>12.19</td>
<td>1.82</td>
</tr>
<tr>
<td>Difference</td>
<td>0.2%</td>
<td>10.9%</td>
</tr>
</tbody>
</table>
4.5. Mooring system

The optimized mooring system parameters are listed in Table 8. The most critical ULS condition for both mooring system constraints is found to be EC 1, as for the maximum offset and pitch angle constraints in Section 4.3. This is due to the large thrust forces in this condition. The optimized solution consists of relatively long mooring lines to meet the requirement of no uplift at the anchors. The line length and thus the material costs of the system, could be significantly reduced if vertically loaded anchors were used; however, these types of anchors are more expensive to install [73]. More information about the total costs of the mooring system, including installation and anchoring, would be needed to identify the most cost-effective solution.

Based on the results presented here, the significance of designing the mooring system simultaneously with the platform is not as clear as for the tower or control system. Although the mass of the platform and tower affect the natural period in surge and subsequently the level of excitation around resonance, the surge response is mostly governed by the rotor and mooring system design and is to a large degree independent from the design of the support structure. The most important coupling effects between the support structure and mooring system are related to the location of the fairleads, which influences the surge-pitch coupling and thus the pitch response of the platform. The fairlead position also affects the fatigue damage in the hull because the mooring forces contribute to the internal loads in the platform. Another coupling effect is that low-frequency mooring line dynamics, which is dependent on the platform motions, will induce damping for the low-frequency platform response [74]. However, this coupling effect is not considered in the present work.

The delta connections are omitted from the mooring system in the present model, and the yaw response of the platform is neglected in the analysis. Since one of the main tasks of the mooring system for a spar FWT is to limit yaw motions [75], the optimized mooring design presented here may have to be modified to obtain satisfactory performance for the platform response. In addition, common mooring line components that can improve the system behaviour, such as buoys or clump weights, can not be modelled with the current methodology and are not considered.

4.6. Design verification

The proposed design optimization approach is verified by analysing initial and optimized designs using nonlinear aero-hydro-elastic time-domain simulations in SIMA. Here, the tower, hull, and blades are modelled using nonlinear beam elements. Hydrodynamic loads in the horizontal direction are applied on the platform using MacCamy–Fuchs theory, together with viscous forces from the drag term in Morison’s equation. In heave, the hydrodynamic loads are limited to buoyancy forces integrated up to the instantaneous free surface and added mass on the lower end of the platform, meaning that Froude–Krylov and diffraction forces are neglected. Since the heave response is not considered in the linearized model, and the first order wave excitation in heave is small, this is deemed acceptable for the present work.

Bar elements with only axial stiffness are used to model the mooring lines, and hydrodynamic loads on the lines are computed using Morison’s equation. A rotational spring is added to the model to ensure that the yaw stiffness from the mooring system is included, using a spring stiffness of 1.48 × 10^6 Nm/rad. The aerodynamic loads on the rotor are computed using BEM theory as described in Section 2.5.4, and viscous drag forces on the tower are included.

The fatigue performance of the tower and hull is assessed by analysing the 30 FLS conditions using six 1-h realizations. Rainflow counting and the Miner–Palmgren rule are used to estimate the fatigue damage, which is taken as the average value over the six seeds. The extreme response, i.e., the most probable maximum value in 1 h in the 50-year conditions, is found using the AUR method, where the function describing the upcrossing rate is fitted to the empirical data [76]. 1-h simulations with 20 different random seeds are used for each 50-year condition.

The initial design is based on the “Spar 1” design obtained by Hegseth and Bachynski [28], but with 60% increase in wall thickness and 40% increase in diameter along the hull and tower to place the first tower mode in the stiff-stiff range. This FWT system has not been through a proper design process and does not represent a realistic solution. The purpose of including it in the verification is not to use it as a benchmark for the optimized design, but rather to show that the presented methodology is able to capture real trends.

Fatigue and buckling utilization factors from the nonlinear time-domain analyses are shown in Fig. 13 and Fig. 14 for the initial and optimal designs, respectively. For the initial design, the fatigue damage values are overestimated by the linear model in the wave zone and lower parts of the tower, where the largest utilization factors are found. This is primarily due to limitations in the Dirlik calculations in the current work, which assumes a single-slope SN curve valid for high-cycle fatigue. Therefore, this assumption becomes less accurate for designs with large damage values, where there is also a significant contribution from low-cycle fatigue. The buckling utilization factors agree very well, suggesting that the extreme response in this condition is relatively linear.

For the optimized designs, the fatigue damage and buckling utilization factors in the tower agree reasonably well with the results from the linearized model, which is usually conservative. This is primarily due to the overestimation of the platform pitch response in the linear model, which become more prominent in extreme conditions. Because the pitch response for the initial design is in general much smaller than for the optimized solution and thus contributes less to the bending moments in the structure, this effect is not seen for the initial design results in Fig. 13.

There is a notable difference between the linear and nonlinear models for the fatigue damage at the tower top in Fig. 14, where the accumulated damage based on time-domain simulations in SIMA is about twice as high. We believe this to be a consequence of somewhat non-Gaussian bending moment response at the tower top, which causes the accuracy of the Dirlik method to deteriorate. For the rest of the tower, the response is close to Gaussian, which earlier has been shown to result in good agreement between the Dirlik method and rainflow counting for spar FWTs [28].
Table 8
Optimized mooring system parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_{tow}$</td>
<td>0.081</td>
<td>m</td>
</tr>
<tr>
<td>$x_{tow}$</td>
<td>-10.7</td>
<td>m</td>
</tr>
<tr>
<td>$L_{tow}$</td>
<td>1433.0</td>
<td>m</td>
</tr>
<tr>
<td>$x_{tow}$</td>
<td>1497.2</td>
<td>m</td>
</tr>
</tbody>
</table>

Fig. 13. Comparison of fatigue and buckling utilization factors, initial design.

Almost perfect agreement is observed for the hull buckling, despite the overprediction of the bending moments by the linear model. The reason for this is that the hydrostatic pressure, which is easily obtained without error, is the most important load component for buckling of the shell plating in the hull.

The extreme values for the offset, pitch angle, and mooring tension are listed in Table 9. The linearized model overestimates the platform motions, mainly because the aerodynamic loads are linearized and the mooring line damping is neglected. The mooring tension, on the other hand, is underestimated by the linear model, which also predicts a 4% decrease in the extreme value from initial to optimized design that is not observed in the nonlinear model.

5. Conclusions

A linearized aero-hydro-servo-elastic FWT model has been developed and used to optimize the design of the platform, tower, mooring system, and blade-pitch controller for a 10 MW spar FWT. The goal of the optimization is to minimize a weighted combination of the system costs and rotor speed variation, which is used as a measure of the power quality.

The results of the optimization depend on the initial tower design because a local minimum exist in both the soft-stiff and stiff-stiff range for the first bending mode. Initial analyses suggest that feasible soft-stiff tower designs may be challenging to achieve for these types of FWT systems, since no solution that satisfied the FLS constraints was identified by the optimizer. However, including more advanced control strategies could improve the fatigue behaviour in the soft-stiff range, and potentially result in a different global minimum for the problem.
Table 9
Comparison of extreme response constraints.

<table>
<thead>
<tr>
<th>Model</th>
<th>Initial design</th>
<th>Optimized design</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Surge</td>
<td>Pitch</td>
</tr>
<tr>
<td></td>
<td>35.1 m</td>
<td>7.0 deg</td>
</tr>
<tr>
<td>Linear</td>
<td>29.4 m</td>
<td>5.9 deg</td>
</tr>
<tr>
<td>Nonlinear</td>
<td>32.0 m</td>
<td>15.0 deg</td>
</tr>
<tr>
<td></td>
<td>27.1 m</td>
<td>11.3 deg</td>
</tr>
</tbody>
</table>

The platform adopts an hourglass shape below the wave zone, where the diameter is kept small to limit the wave loads on the structure. The hourglass shape maximizes the distance between the CoB and the CoG, which increases the restoring moment and natural frequency in pitch and consequently improves the behaviour in the low-frequency range for the platform pitch response. The large bottom diameter also increases the added mass in heave, which helps place the natural frequency outside the wave-frequency range.

The importance of integrated design is evident in the optimized solutions for the tower and blade-pitch control system, where the interaction with the platform clearly affects the design. For the tower, the integrated optimization results in an unconventional design where the diameter increases from the tower base, because the base diameter is constrained to match the diameter at the upper end of the platform. Varying the weight factors in the multi-objective optimization primarily affected the control system variables, which governed the trade-offs between rotor speed variation and fatigue damage in the support structure (and thus the material costs). Optimizing with fixed gains from a simple detuning procedure resulted in 0.2% higher cost and 10.5% larger rotor speed standard deviation compared to the integrated solution.

The loads on the drivetrain are not considered in the present optimization study because there is a lack of response parameters that can serve as proxies for the drivetrain performance with the current methodology. If such parameters are identified, additional terms may be included in a multi-objective optimization approach to penalize designs with unfavourable drivetrain response. As discussed in Section 3.2, finding appropriate weights for the different sub-objectives is challenging, and sensitivity studies would be needed to assess trade-offs between the variables. This would in turn increase the computational effort.

The use of linear analyses in the frequency domain imposes important limitations on the computations, since nonlinear and
transient effects are not captured. The accuracy of the model, and thus its applicability in design optimization studies, is therefore dependent on the importance of these effects. Also, the proposed methodology is only applicable for spar platforms, which limits the choices of the optimizer. A more general description of the platform geometry, which requires more comprehensive models for the hydrodynamics and structural dynamics, would allow the optimization to explore other parts of the design space and possibly identify more cost-effective solutions.

The feasibility of the optimized design is verified using state-of-the-art simulations, which show that the simplified model gives reasonably accurate predictions for both fatigue and extreme response. In general, the linearized model is conservative, especially for the platform pitch motions, which tended to be more nonlinear than the internal loads in the support structure. The main reason for the overprediction is likely to be the linearization of the aerodynamic loads.

The proposed methodology is shown to be suitable for preliminary design, where it can provide a starting design for later and more detailed design phases. The integration of several subsystems in a single design optimization process clearly affects the results and improves the overall system behaviour. The methodology can be extended to account for additional concepts, parameters, and load cases, which may help identify novel design solutions.

Declaration of competing interest

☐ The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgements

Financial support from the Research Council of Norway (NFR) through project number 274827 - “Green Energy at Sea” is gratefully acknowledged. The first author is grateful to John Jasa for helpful discussions regarding the OpenMDAO implementation.

A. Distributed Properties for the Optimized Platform and Tower

| Table 10 | Distributed platform properties. |
| --- | --- | --- |
| Elevation above SWL [m] | Outer diameter [m] | Wall thickness [m] |
| 80.427 | 21.365 | 0.084 |
| 75.064 | 19.505 | 0.075 |
| 68.117 | 17.055 | 0.064 |
| 62.249 | 15.003 | 0.056 |
| 56.242 | 12.940 | 0.051 |
| 50.677 | 14.070 | 0.054 |
| 44.312 | 16.314 | 0.061 |
| 31.584 | 20.681 | 0.068 |
| 18.829 | 16.212 | 0.060 |
| 8.741 | 12.784 | 0.085 |
| 10.000 | 12.660 | 0.061 |

| Table 11 | Distributed tower properties. |
| --- | --- | --- |
| Elevation above SWL [m] | Outer diameter [m] | Wall thickness [m] |
| 10.000 | 12.660 | 0.061 |
| 20.500 | 14.377 | 0.037 |
| 31.000 | 14.096 | 0.033 |
| 41.500 | 13.827 | 0.030 |
| 52.000 | 12.742 | 0.029 |
| 62.500 | 11.422 | 0.029 |
| 73.000 | 10.473 | 0.027 |
| 83.500 | 9.892 | 0.024 |
| 94.000 | 8.904 | 0.021 |
| 104.500 | 7.789 | 0.018 |
| 115.630 | 6.774 | 0.015 |
B. XDSM Diagram

Fig. B.1. XDSM diagram showing the model layout and data dependencies. Rounded blue box is the optimizer, green boxes are analysis models, gray parallelograms are data, white parallelograms are fixed input parameters, and gray lines represent data dependencies. The data moves from top to bottom and left to right in the upper triangular part, and from bottom to top and right to left in the lower triangular part. See Lambe and Martins [42] for more details on how to interpret this figure.

References


Design optimization of spar floating wind turbines considering different control strategies

John Marius Hegseth, Erin E. Bachynski and Joaquim R. R. A. Martins

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Design Optimization of Spar Floating Wind Turbines
Considering Different Control Strategies

John Marius Hegseth¹, Erin E. Bachynski¹, Joaquim R. R. A. Martins²

¹ Department of Marine Technology, NTNU, 7491 Trondheim, Norway
² Department of Aerospace Engineering, University of Michigan, Ann Arbor, MI 48109, USA
E-mail: john.m.hegseth@ntnu.no

Abstract. One of the challenges related to the design of floating wind turbines (FWTs) is the strong
interactions between the controller and the support structure, which may result in an unstable system.
Several control strategies have been proposed to improve the dynamic behaviour, all of which result in
trade-offs between structural loads, rotor speed variation, and blade pitch actuator use, which makes
controller design a challenging task. Due to the interactions, simultaneous design of the controller and
support structure should be performed to properly identify and compare different solutions. In the present
work, integrated design optimization of the blade-pitch controller and support structure is performed for a
10 MW spar FWT, considering four different control strategies, to evaluate the effect of the controller on
the structural design and associated costs. The introduction of velocity feedback control reduces the platform
pitch response and consequently the fatigue loads in the tower, which leads to a decrease in the tower
costs compared to a simple PI controller. Low-pass filtering of the nacelle velocity signal to remove the
wave-frequency components results in reduced rotor speed variation, but offers only small improvements
in costs, likely due to the limited wave-frequency response for the considered designs. Comparisons with
nonlinear time-domain simulations show that the linearized model is able to capture trends with acceptable
accuracy, but that significant overpredictions may occur for the platform pitch response.

1. Introduction
For floating wind turbines (FWTs), the performance of the control system generally depends on the
support structure design and vice versa. A well-known interaction between the blade pitch controller and
platform motions is the introduction of negative damping above rated wind speed [1]. While detuning
the controller gains such that the bandwidth is reduced achieves stability, this results in poorer rotor
speed tracking performance [1, 2]. Several alternative methods have also been suggested to resolve the
issue, such as introducing a feedback term proportional to the pitch velocity [3] or nacelle velocity [4, 5]
to manipulate the generator speed reference. For these types of feedback control, Fleming et al. [6]
also suggested to remove the wave-frequency components from the velocity signal, as this reduced the tower
loads. All of these control strategies result in trade-offs between structural loads, rotor speed variation,
and blade pitch actuator use, which vary with different environmental conditions. Therefore, identifying
optimal control parameters is a challenging task.

Due to the strong interactions, simultaneous design of the controller and support structure should be
performed to have a fair comparison between different solutions. A step toward simultaneous design was
made by Lemmer et al. [7], who optimized the main dimensions of a three-column semi-submersible
FWT. They minimized a combination of material costs and damage-equivalent loads in the tower for
seven operational conditions, with a constraint on the static pitch angle at rated thrust. The controller
was tuned at each design iteration using a linear quadratic regulator (LQR) approach. To properly
identify and compare optimal solutions, the integrated control and structural designs should, however,
be evaluated over the lifetime of the system, considering actual design limits. The purpose of the present
study is to perform integrated design optimization of the blade-pitch controller and support structure for
a 10 MW FWT, considering long-term fatigue damage and extreme response constraints, to evaluate the
effect of different control strategies on the structural design and associated costs.

2. Linearized FWT dynamics

2.1. FWT definition

The present study performs the design optimization of a spar buoy that supports the DTU 10 MW reference wind turbine [8] at a water depth of 320 m. The steel hull is partially filled with concrete ballast to achieve the correct draft, using a ballast density of 2600 kg/m$^3$. The interface with the tower is located 10 m above the still water line (SWL), while the hub height is 119 m above the SWL. A catenary mooring system consisting of three lines spread symmetrically about the vertical axis is used for station-keeping. Only the response in the $xz$-plane is considered in the current work, and co-directional waves and wind travelling in the positive $x$-direction are applied in all simulations. An overview of the FWT system is shown in Fig. 1.

![Figure 1: Overview of the FWT system.](image)

2.2. Linearized equations of motion

The system is linearized and expressed in state-space form, and consists of a structural part and a control system part, which are connected to obtain the complete closed-loop aero-hydro-servo-elastic model as described by Hegseth et al. [9]. For each wind-wave condition, the operational point is found from static equilibrium when the system is subjected to the mean wind loads. The linearized system considers perturbations in the state and input variables, $x$ and $u$ respectively, about the operational point:

$$x = x_0 + \Delta x, \quad u = u_0 + \Delta u. \quad (1)$$

The dynamic equations of motion are then expressed as

$$\Delta \dot{x} = A \Delta x + B \Delta u, \quad (2)$$

where $A$ is the state matrix and $B$ is the input matrix.

2.3. Structural model

The structural model for the platform and turbine considers four degrees-of-freedom (DOFs), namely surge, pitch, first tower/platform bending mode, and rotor speed. The equations of motions for the former three are found from generalized displacements similar to Hegseth and Bachynski [10], but using a flexible hull to ensure a correct natural frequency for the first bending mode, while a rigid drivetrain and rotor is assumed for the rotor dynamics. The structural state vector is thus written as

$$\mathbf{x}_s = \begin{bmatrix} \xi_1 & \xi_5 & \xi_7 & \dot{\xi}_1 & \dot{\xi}_5 & \dot{\xi}_7 & \dot{\varphi} \end{bmatrix}^T, \quad (3)$$

where $\xi_n$ represents generalized support structure DOF $n$, and $\dot{\varphi}$ is the rotor speed.

Hydrodynamic excitation loads on the hull are described by MacCamy–Fuchs theory, and transverse added mass is based on analytical 2D coefficients. Radiation damping is neglected, while viscous damping is found from stochastic linearization of the drag term in Morison’s equation.

Wind loads on the rotor are derived from linearized BEM theory with the incoming wind field described by the Kaimal spectrum and an exponential coherence function for the longitudinal wind
velocity component [11]. The blades are considered rigid in the model, and the aerodynamic forces on the rotor are applied as resultant loads at the tower top. In addition, the static component of the aerodynamic quadratic drag force on the tower is included.

The inputs to the structural system consist of both control system outputs and disturbances due to environmental loads. The control input vector contains the references for the generator torque \( Q_G \) and the collective blade pitch angle \( \theta \), and is defined as

\[
u_{sc} = [Q_G \quad \theta]^	op.
\]

The disturbance vector contains rotor-effective wind speeds for thrust, tilting moment and aerodynamic torque, and generalized wave excitation forces for each support structure DOF, i.e.

\[
u_{sd} = [v_{F,T} \quad v_{M,T} \quad v_{Q,A} \quad F_{W,1} \quad F_{W,5} \quad F_{W,7}]^	op.
\]

2.4. Controller description

The baseline linear control system consists of a generator-torque controller and a collective blade-pitch controller, which work independently in below-rated and above-rated wind speeds, respectively. Below rated wind speed, the generator torque is set to be proportional to the square of the rotor speed to maintain the optimal tip-speed ratio. Above rated wind speed, the generator torque is kept constant, and four different strategies are considered for the blade-pitch controller:

- CS1: Gain-scheduled PI controller
- CS2: Gain-scheduled PI controller + platform pitch velocity feedback
- CS3: Gain-scheduled PI controller + nacelle velocity feedback
- CS4: Gain-scheduled PI controller + nacelle velocity feedback + low-pass filter

For feedback control using platform pitch or nacelle velocity, we use the modified rotor speed reference, \( \dot{\varphi}_0' \), defined by Lackner [3]:

\[
\dot{\varphi}_0' = \dot{\varphi}_0(1 + k_f\dot{x}_f)
\]

where \( \dot{\varphi}_0 \) is the nominal rotor speed reference, \( k_f \) is the velocity feedback gain, and \( \dot{x}_f \) is either the platform pitch velocity or the nacelle velocity. An updated expression for the rotor speed error can then be established as

\[
\Delta\dot{\varphi}' = \dot{\varphi} - \dot{\varphi}_0' = \Delta\dot{\varphi} - \dot{\varphi}_0k_f\dot{x}_f,
\]

where \( \Delta\dot{\varphi} \) is the nominal rotor speed error. In CS4, the nacelle velocity signal is passed through a first order low-pass filter to remove the wave-frequency components before it is fed back to the blade-pitch controller.

2.5. Response calculations

The structural and control system models are written as a single closed-loop system, which is solved in the frequency domain. Dynamic force equilibrium is then used together with the response spectra to calculate the bending moment response along the tower.

The fatigue damage is calculated at selected locations in the tower using the Dirlik method [12], while the extreme response of the support structure is found using the AUR method, where the most probable maximum value in one hour is used in the design constraints. The model has earlier been verified against fully coupled nonlinear time domain simulations in SIMA [9].

The rotor speed tracking performance is evaluated using the weighted average of the rotor speed standard deviation, which is found by summing the values from each short-term condition that is considered, weighted by their associated probabilities:

\[
\sigma(\dot{\varphi}) = \sum_{i=1}^{N_{EC}} p_i \sigma(\dot{\varphi}),.\]
Here, $N_{EC}$ is the number of short-term conditions, $\sigma(\dot{\phi}_i)$ is the rotor speed standard deviation in condition $i$, and $p_i$ is the probability of the condition.

To evaluate blade-pitch actuator fatigue, the actuator duty cycle (ADC) was defined in Kendall et al. [13] as the total number of degrees pitched divided by the total simulation time. Although ADC cannot be used as an absolute measure of the actuator fatigue damage, it is suitable for comparison of different control strategies. The normalized ADC, which is used in the current work, is defined by Bottasso et al. [14] as

$$\text{ADC}_i = \frac{1}{\dot{\theta}_{max}} \int_0^T \left| \frac{\dot{\theta}_i(t)}{\dot{\theta}_{max}} \right| dt,$$

where $\dot{\theta}_i$ is the blade pitch rate in condition $i$, $T$ is the total simulation time, and $\dot{\theta}_{max}$ is the maximum allowable blade pitch rate, which for the DTU 10 MW is equal to 10 deg/s.

If the process is ergodic, the ADC can be expressed using the expected value:

$$\text{ADC}_i = \frac{1}{\dot{\theta}_{max}} E \left[ \left| \dot{\theta}_i(t) \right| \right].$$

Assuming that the blade pitch rate is Gaussian, the expected value can be calculated as

$$E \left[ \left| \dot{\theta}_i(t) \right| \right] = \sqrt{\frac{2}{\pi}} \sigma(\dot{\theta}_i).$$

For the linearized model, the ADC can thus be derived from the blade pitch rate standard deviation. The weighted average ADC used in the design optimization is then found similarly as in Eq. (8).

2.6. Environmental conditions

Fifteen different ECs are used to evaluate the long-term fatigue performance in the present work. The conditions span mean wind speeds from 1-30 m/s with 2 m/s step, and for each mean wind speed, the most probable values for the significant wave height and spectral peak period are used. These values, as well as the associated probabilities of occurrence, are found from the joint probability distribution derived by Johannessen et al. [15], and shown in Fig. 2.

![Figure 2: Significant wave heights, spectral peak periods, and normalized probabilities applied in the fatigue calculations.](image)

Three ECs, described in Table 1, along the 50-year contour surface, are selected to evaluate the extreme response. EC 1 and 2 represent operational wind speeds above rated, while EC 3 considers an extreme storm condition, where the turbine is parked with feathered blades. For all ECs, the IEC Class B normal turbulence model (NTM) [11] is used for the incoming wind, while the waves are described by a JONSWAP spectrum with a peakedness parameter of 3.3.

3. Optimization problem

The FWT model is implemented in OpenMDAO [16], which is an open-source framework for multidisciplinary design, analysis, and optimization. The design is optimized using a gradient-based approach, where the derivatives of the model are computed analytically using coupled adjoints. The optimization problem is solved using the SNOPT algorithm [17], which uses a sequential quadratic programming (SQP) approach, through the pyOptSparse Python interface [18].

3.1. Objective function

The objective function used in the present work is the combined cost of the platform and tower, $C_{spar}$ and $C_{tower}$ respectively:

$$f = C_{spar} + C_{tower}.$$
Table 1: Environmental conditions for extreme response calculations.

<table>
<thead>
<tr>
<th>Condition</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean wind speed at hub height, $U$ (m/s)</td>
<td>13.0</td>
<td>21.0</td>
<td>50.0</td>
</tr>
<tr>
<td>Significant wave height, $H_s$ (m)</td>
<td>8.1</td>
<td>9.9</td>
<td>15.1</td>
</tr>
<tr>
<td>Spectral peak period, $T_p$ (s)</td>
<td>14.0</td>
<td>15.0</td>
<td>16.0</td>
</tr>
<tr>
<td>Turbulence intensity at hub height, $I$ (-)</td>
<td>0.17</td>
<td>0.14</td>
<td>0.12</td>
</tr>
</tbody>
</table>

The costs consider both material and manufacturing, using the cost models developed by Farkas and Jármai [19]. Costs related to installation, maintenance, and decommissioning are not included. The cost of the platform (and similarly of the tower) is expressed as

$$C_{spar} = k_m M_{spar} + k_f \sum_i T_i,$$

(13)

where $k_m$ is the steel cost factor, $M_{spar}$ is the steel mass of the hull, and $k_f$ is the fabrication cost per unit time. $T_i$ is the time spent at fabrication stage $i$, expressed as a function of the geometry. The steel cost factor, $k_m$, is assumed to have a value of 2.7 €/kg, while the ratio between the material and fabrication cost factors, $k_m/k_f$, is set to 1.0, which is a typical value for West European labour [19]. The cost of the concrete ballast is neglected in the current work.

3.2. Design variables

Both the platform and tower are discretized into ten sections along the length. For the tower, the diameter and wall thickness at the nodes connecting the sections are set as design variables. The length of the tower sections is kept fixed during the optimization, to maintain the original hub height.

The structural design of the spar platform is primarily governed by buckling loads, which for most parts of the hull are dominated by the hydrostatic pressure. The structural design of the platform is therefore not considered in the study, and only the diameter at the platform nodes and the length of each section are included in the design optimization. The wall thickness is expressed as a function of depth, based on the optimized structural design from Hegseth et al. [9]. This ensures a proper mass distribution, and penalizes designs with large drafts and consequently high external pressure loads, which require increased use of material. The mooring system design is kept fixed during the optimization.

For the control system, the optimization considers the proportional ($k_p$) and integral ($k_i$) gains for the blade-pitch controller, as well as the velocity feedback gain ($k_f$) for the pitch and nacelle velocity feedback control. For CS4, the corner frequency of the nacelle velocity low-pass filter ($\omega_f$) is also included.

3.3. Constraints

The fatigue damage at each tower node is evaluated using an SN curve approach, where the D curve in air from DNV-RP-C203 [20] is applied together with a design fatigue factor (DFF) of 2.0 [21], and the lifetime of the FWT system is chosen to be 20 years. The fatigue design constraints are thus expressed as

$$D_{tot} = N_{20} \sum_{i=1}^{N_{EC}} p_i D_i \leq \frac{1.0}{DFF},$$

(14)

where $D_{tot}$ is the total fatigue damage in 20 years, $N_{20}$ is the number of short term conditions in 20 years, and $D_i$ is the fatigue damage in condition $i$.

Tower buckling is assessed using Eurocode 3 [22], assuming that the tower is stiffened between each section to reduce the buckling length. To ensure a smooth transition between the platform and tower, the tower base diameter is set to be equal to the diameter at the platform top. Both fatigue and buckling constraints are aggregated using Kreisselmeier–Steinhauser (KS) functions [23].
The maximum platform pitch angle in the considered 50-year conditions is limited to 15°. Although the heave response is not included in the model, heave resonance in the wave frequency range is avoided by placing a lower limit of 25 s on the heave natural period. The added mass in heave is approximated as the value for a 3D circular disc with the same diameter as the platform bottom [24].

The presented model is valid strictly for hull sections with vertical walls, and a maximum taper angle of 10° is therefore applied as a constraint for each section of the platform, to avoid shapes where the physics are not captured correctly. Offset constraints are not considered, as the surge response is mostly governed by the rotor and mooring system design.

Appropriate upper limits for the rotor speed variation and blade-pitch actuator use are difficult to quantify. The constraints are therefore based on values taken from the land-based DTU 10 MW wind turbine with the original controller [25], where the weighted average rotor speed standard deviation and ADC are found from nonlinear time domain analyses using the simulation tool SIMA. Initial analyses found that the rotor speed variation obtained with the land-based turbine was unrealistic for the floating system with the simplified controllers considered in the present work. To enlarge the feasible region of the design space, the constraints for both the rotor speed variation and the ADC are scaled by a factor of 1.5 compared to the land-based values, as shown in Table 2.

4. Results

4.1. Optimized designs

The optimized support structure design for CS1 is illustrated in Fig. 3. The hourglass shape taken by the platform below the wave zone increases the distance between the center of buoyancy and the center of gravity, which leads to increased pitch restoring stiffness, while the relatively large diameter at the bottom results in larger added mass and consequently longer natural period in heave. For the upper part of the platform and intersection with the tower, the optimizer finds a balance between a small diameter, which is desirable with regards to hydrodynamic loads, and a large diameter, which (together with a small wall thickness) is the most cost-effective way to achieve the required fatigue life.

The optimized tower diameter and wall thickness distributions for the different control strategies are plotted in Fig. 4. All four solutions follow the same trends, and the effect of velocity feedback control is most visible for the wall thickness, where the values for CS2-4 are approximately 20 % lower than for the simple PI controller (CS1) along most of the tower length. The reduced wall thickness is enabled by a decrease in the fatigue loads for these controllers, which is the design-driving constraint for the tower. For all four control strategies, the 15° pitch angle constraint is also active; however, as the 50-year storm condition with parked turbine (EC3) is found to be the critical load case for extreme response, this constraint is not affected by the controller.

The improved fatigue performance can be understood by examining the response spectra for the optimized designs. In Fig. 5a, the tower base bending moment spectra are shown for a mean wind speed of 15 m/s. Large differences are seen around the pitch natural frequency at 0.15 rad/s, where the velocity feedback controllers increase the aerodynamic damping and thus reduce the response. The low-pass filtering of the nacelle velocity removes the wave-frequency range from the signal, which results in a higher optimal velocity feedback gain and nearly eliminates the tower base bending moment response.
Table 2: Land-based and applied constraint values for the rotor speed variation and blade-pitch actuator use.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Land-based value</th>
<th>Constraint value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(\dot{\varphi})$ [rad/s]</td>
<td>4.22E-2</td>
<td>6.33E-2</td>
</tr>
<tr>
<td>ADC [-]</td>
<td>5.10E-3</td>
<td>7.65E-3</td>
</tr>
</tbody>
</table>

Figure 4: Tower diameter and wall thickness for optimized designs.

arising from resonant pitch motions. The wave-frequency bending moments, on the other hand, is unaffected by the control strategy. The blade pitch spectra in Fig. 5b show how the larger PI gains in the velocity feedback controllers result in overall increased actuator use, with the exception of the pitch natural frequency, as well as the wave-frequency band for CS4.

The optimized costs of the tower, platform, and tower plus platform for CS2-4 are shown in Fig. 6a, compared to the optimized costs for CS1. The majority of the cost reductions come from the tower, due to the improved fatigue behaviour, whereas the platform is less affected due to the fixed costs related to buckling resistance and the 15° pitch angle constraint. However, because a lighter tower results in a lower overall center of gravity, the platform pitch response is somewhat improved, and a small reduction in platform costs of about 2 % is also observed. Because the platform accounts for 70-75 % of the total costs for the considered designs, the resulting total cost reduction is approximately 6 %.

The resulting rotor speed standard deviations and ADCs, normalized by their maximum allowable values from Table 2, are shown in Fig. 6b. For each control strategy, there exists a limit where no further reduction in cost can be achieved by increasing the actuator use. This limit is higher for the velocity feedback controllers than for a controller using only the rotor speed error as input, which causes the ADC constraint to be inactive at the optimum for CS1. A larger ADC may result in higher probability of fatigue failure for the actuator bearings, and therefore more detailed design considering the lifetime of the system should be performed to determine appropriate values for this constraint.

For the rotor speed variation, better performance is achieved with CS4 than with the other control strategies. Since there is a trade-off between rotor speed variation and structural loads (and thus costs) as previously discussed, it is expected that larger cost reductions can be achieved with CS4 if the rotor speed constraint is tightened.

Some limitations to this work should be noted. The standard deviation in steady-state conditions is used as the only measure of the rotor speed tracking performance, and extreme rotor speed excursions due to gusts are not considered. The effect of the controllers on surge motions, drivetrain response, mooring line tension, or blade response has also not been studied. In addition, the performance of the FWT system could likely be further improved by also adding individual pitch control or modifications to the torque controller, which have not been examined in the present work.
4.2. Sequential versus multidisciplinary optimization

In the presented methodology, the platform, tower, and blade-pitch controller are optimized simultaneously. This approach, commonly known as multidisciplinary design optimization (MDO) [26], is preferred for coupled systems, where a sequential optimization process in general leads to suboptimal solutions on the overall system level [27]. To assess the importance of integrated design, results using MDO are compared to a sequential optimization for CS4, where the controller and support structure are optimized separately. For the sequential approach, the control parameters are first optimized to minimize the rotor speed variation, with constraints on system stability and ADC. All support structure parameters are kept constant and structural constraints are not considered. The support structure is then optimized for minimum cost, keeping the controller parameters fixed at their optimized values from the previous step. This procedure is repeated until the system has converged. Since this method results in different objective functions for the two sequential problems, a multi-objective approach is utilized for the MDO study, where a combination of costs and rotor speed variation is minimized with different relative weighting.

Figure 7 shows the MDO Pareto front together with the optimal solution from the sequential approach. The results confirm that improved designs can be achieved with integrated optimization; however, the differences are small. For the same rotor speed variation, using MDO results in
approximately 0.5 % reduction in the costs of the tower, or 0.2 % in total costs. Although small coupling effects are seen for the considered optimization problem, larger differences are expected in cases where the controller has a greater effect on the structural response.

4.3. Verification

The optimized controller designs are verified through fully coupled nonlinear time-domain simulations using SIMA, where two different ECs above rated wind speed are simulated with each control strategy for a specified support structure design. Comparisons with the linearized model for different response parameters, weighted by the probability of each condition, are shown in Fig. 8.

The linearized model is seen to mostly follow the trends observed in the nonlinear simulations, but some errors are present. The largest errors are observed for the tower base fatigue damage, which is significantly overestimated for CS1, whereas good agreement is obtained with the other control strategies. The reason for the poor agreement is that the aerodynamic (and thus the overall) damping for the platform pitch mode is much lower with this control strategy. Consequently, the resonant pitch response becomes very sensitive to the presence and amount of additional damping in the system, which is either not considered or underpredicted in the linear model. This disagreement was also observed for a linearized model of the 10 MW OO-Star semisubmersible in Souza et al. [28], which used a control strategy similar to CS1. The overestimation of fatigue damage means that the optimized tower design for CS1 is more conservative than for the other control strategies, suggesting that the cost reductions in Fig. 6a are highly optimistic, and that CS1 may yield a fatigue design similar to that with a nacelle velocity feedback controller. It also suggests that future optimization studies using the linearized model should consider more advanced control strategies than CS1, to limit the pitch response error.

In addition, the rotor speed standard deviation is consistently underestimated by about 20 % for all four control strategies. This disagreement could be taken into consideration in the optimization process by adjusting the constraint value.

5. Concluding remarks

The design of the platform, tower, and blade-pitch control system for a 10 MW spar FWT was optimized simultaneously using a linearized aero-hydro-servo-elastic model and gradient-based optimization with...
analytic derivatives. The goal has been to minimize the material and manufacturing costs of the support structure, with constraints on tower fatigue damage and buckling, extreme platform pitch motions, rotor speed variation, and blade-pitch actuator use, considering four different strategies for the blade-pitch controller.

The effect of the controller on the structural response was limited to the fatigue damage in the tower, since the storm condition with parked turbine was found to govern the extreme responses of the system considered here. The reduction in tower loads for the velocity feedback controllers compared to the simple PI control system was a consequence of lower platform pitch response, which led to a reduction in the wall thickness required to satisfy the long-term fatigue damage constraint. Consequently, the tower costs were reduced, and also the platform costs due to better dynamic performance. Although low-pass filtering of the nacelle velocity signal did not offer significant cost reductions, this control strategy also saw a reduction in rotor speed variation, since the constraint was inactive at the optimum. It is also expected that the effect of this filter will be more prominent for FWT concepts with larger wave-frequency response.

Comparisons with nonlinear time-domain simulations showed that the linearized model in general is able to capture trends with acceptable accuracy, but that the platform pitch response can be significantly overpredicted for designs with low aerodynamic damping if contributions from other sources of damping are small. This was the case for CS1, which indicates that the cost reductions achieved for CS2-4 are considerably overestimated. For the velocity feedback controllers, which increases the amount of aerodynamic damping induced by the control system, this problem was not observed, and good agreement was achieved.

The presented approach is useful for conceptual FWT design, where it can be used to quickly explore the design space before resorting to higher fidelity tools for detailed subsystem analysis and design. The model captures important interactions between the controller and support structure, and enables assessment of trade-off effects in a lifetime perspective. Further, the methodology can be extended to account for additional design parameters and load cases, which may help identify novel design solutions.

References

Paper 5

Effect of environmental modelling and inspection strategy on the optimal design of floating wind turbines

John Marius Hegseth, Erin E. Bachynski and Bernt J. Leira

Submitted to: Reliability Engineering and System Safety, 2020

This article is awaiting submission and is therefore not included
Appendix B

Formulation of Rotor Effective Wind Speed

As shown by Halfpenny (1998), the (single sided) cross spectral density of the rotationally sampled wind speed for two blade elements $j$ and $k$ may be written:

$$S_{\psi}^{(jk)}(\omega) = \sum_{n=-\infty}^{\infty} e^{\iota n \psi} K_n^{(jk)}(|\omega - n\dot{\phi}|) S_U(|\omega - n\dot{\phi}|),$$  \hspace{1cm} (B.1)

where $S_U(\omega)$ is the incoming wind spectrum, $\psi$ is the azimuth angle between the elements, and $K_n$ is the $n^{th}$ Fourier coefficient of the coherence function $\gamma$, which is Fourier expanded in the rotor plane:

$$K_n^{(jk)}(\omega) = \frac{1}{\pi} \int_{0}^{\pi} \gamma(\omega, d_{jk}) \cos(n\theta) \, d\theta.$$  \hspace{1cm} (B.2)

The coherence function for two points with separation distance $d$ and frequency $f$ in Hz is taken from IEC (2005) and expressed as

$$\gamma(f, d) = \exp \left( -12 \left[ \left( \frac{fd}{U_{\text{hub}}} \right)^2 + \left( \frac{0.12d}{L_c} \right)^2 \right]^{0.5} \right),$$  \hspace{1cm} (B.3)

where $U_{\text{hub}}$ is the mean hub-height wind speed and $L_c$ is a coherence scale parameter.

Using Eq. (B.1), the wind speed seen by each blade element can be found, and used together with aerodynamic derivatives to calculate blade root
Formulation of Rotor Effective Wind Speed

forces and subsequently resultant rotor loads. For a three-bladed turbine with \( n_b \) blade elements on each blade, \( S_v(\omega) \) is a \( 3n_b \times 3n_b \) matrix with the following structure:

\[
S_v(\omega) = \begin{bmatrix}
S_{v,11}(\omega) & S_{v,12}(\omega) & S_{v,13}(\omega) \\
S_{v,21}(\omega) & S_{v,22}(\omega) & S_{v,23}(\omega) \\
S_{v,31}(\omega) & S_{v,32}(\omega) & S_{v,33}(\omega)
\end{bmatrix}
\]

However, if only the resultant loads are of interest, and the blades are identical, the calculations can be simplified by defining a single ‘effective’ blade, and the cross spectral density matrix for the effective blade can be found from the auto-spectral density for one of the blades in Eq. (B.4). For the thrust force and aerodynamic torque, which is the sum of the blade root shear force \( F_y \) and edgewise bending moment \( M_y \) respectively, the same formulation can be used:

\[
\bar{S}_v(\omega) = 3^2 \sum_{n=-\infty}^{\infty} \bar{S}_{v,n}(\omega), \quad (B.5)
\]

where

\[
\bar{S}_{v,n}(\omega) = \begin{cases}
K_n(|\omega - n\dot{\phi}|) S_U(|\omega - n\dot{\phi}|), & n \in \{\ldots, -6, -3, 0, 3, 6, \ldots\} \\
0, & \text{otherwise}
\end{cases}
\]

and \( K_n \) only considers elements on the same blade. For the tilting moment, the effective blade formulation is somewhat different, as the azimuth angle of the blade affects the contribution from the blade root moment on the resultant load. The contribution from the flapwise blade root moment on the rotor tilting moment is proportional to \( \cos(\dot{\phi}t) \), and the cross spectral density matrix thus becomes:

\[
\bar{S}_v(\omega) = \left( \frac{3}{2} \right)^2 \sum_{n=-\infty}^{\infty} \bar{S}_{v,n}(\omega), \quad (B.7)
\]

where

\[
\bar{S}_{v,n}(\omega) = \begin{cases}
K_n(|\omega - (n + 1)\dot{\phi}|) S_U(|\omega - (n + 1)\dot{\phi}|), & n \in \{\ldots, -7, -4, -1, 2, 5, \ldots\} \\
K_n(|\omega - (n - 1)\dot{\phi}|) S_U(|\omega - (n - 1)\dot{\phi}|), & n \in \{\ldots, -5, -2, 1, 4, 7, \ldots\} \\
0, & \text{otherwise}
\end{cases}
\]

(B.8)
As seen from the non-zero terms in Eqs. (B.6) and (B.8), the cancellation of harmonics with a three-bladed turbine results in peaks at multiples of three of the rotor frequency (i.e. 3P, 6P, 9P, etc.) for the resultant loads. The tower top load spectra can be found by pre- and post-multiplying $\bar{S}_v(\omega)$ with the weight factors for each element, i.e. factors that relate wind speed to the aerodynamic loads on the element:

$$S_{F_T}(\omega) = \mathbf{W}_{F_T}^T \bar{S}_v(\omega) \mathbf{W}_{F_T}$$  \hspace{1cm} (B.9a)
$$S_{M_T}(\omega) = \mathbf{W}_{M_T}^T \bar{S}_v(\omega) \mathbf{W}_{M_T}$$  \hspace{1cm} (B.9b)
$$S_{Q_A}(\omega) = \mathbf{W}_{Q_A}^T \bar{S}_v(\omega) \mathbf{W}_{Q_A}.$$  \hspace{1cm} (B.9c)

Here,

$$\mathbf{W}_{F_T} = \begin{bmatrix} F_{n,v}(r_1)\Delta r \\ F_{n,v}(r_2)\Delta r \\ F_{n,v}(r_N)\Delta r \end{bmatrix}, \quad \mathbf{W}_{M_T} = \begin{bmatrix} r_1 F_{n,v}(r_1)\Delta r \\ r_2 F_{n,v}(r_2)\Delta r \\ r_N F_{n,v}(r_N)\Delta r \end{bmatrix}, \quad \mathbf{W}_{Q_A} = \begin{bmatrix} r_1 F_{t,v}(r_1)\Delta r \\ r_2 F_{t,v}(r_2)\Delta r \\ r_N F_{t,v}(r_N)\Delta r \end{bmatrix}.$$  \hspace{1cm} (B.10)

The spectra for the effective wind speeds can be calculated from the tower top load spectra and the total weight factors over the rotor:

$$S_{v_{F_T}}(\omega) = \frac{S_{F_T}(\omega)}{(3 F_{y,v})^2}$$  \hspace{1cm} (B.11a)
$$S_{v_{M_T}}(\omega) = \frac{S_{M_T}(\omega)}{\left(\frac{3}{2} M_{z,v}\right)^2}$$  \hspace{1cm} (B.11b)
$$S_{v_{Q_A}}(\omega) = \frac{S_{Q_A}(\omega)}{(3 M_{y,v})^2}.$$  \hspace{1cm} (B.11c)

The cross spectral density matrix for the wind loads can be found by observing which harmonics of the wind are shared by the different load components. As shown by Halfpenny (1998), the thrust force and aerodynamic torque on the turbine are perfectly correlated, as they share all harmonics, and completely uncorrelated from the tilting moment, as they have no harmonics in common.
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