Optimal Scheduling Policy for Spatio-temporally Dependent Observations using Age-of-Information

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***Abstract*—This paper proposes an optimal scheduling policy for a remote estimation problem, where sensor observations of two spatio-temporally correlated processes are broadcasted to two remote estimators. At each time instant only a single observation can be communicated. For this purpose, a system scheduler determines which sensor measurement is communicated. The scheduler cannot observe measurements, and exploits age-of-information (AoI) to calculate the expected estimation error. We derive an optimal scheduling policy, with AoI as state-variable, that minimizes the average mean squared error for an infinite time horizon. The obtained policy yields a periodic scheduling of the sensor measurements, and we show that the AoI for the process with the largest marginal variance does not exceed one.**

***Index Terms*—Wireless sensor networks, age-of-information, scheduling, spatio-temporal correlation, sequential decision-making**

I. Introduction

Wireless sensor networks (WSN) provide an essential data collecting infrastructure for environmental monitoring and autonomous decision making. In WSN and networked control systems, sensors observe physical processes and communicate measurements to controllers, or remote estimators, that form estimates and track process parameters. Communication resources are often limited, and different communication protocols exist that coordinate network access and transmission instances [1], [2]. Due to a limited number of communication channels and to avoid packet collisions, sensors are allocated dedicated time slots for measurement transmissions. A common task is to design scheduling schemes that assign these time slots to minimize the overall estimation error at the destination. Scheduling problems have been studied under different resource constraints, e.g., limited battery [3], limited packet size [4], or presence of eavesdroppers [5].

Finding optimal scheduling schemes commonly involve optimization problems that are solved using dynamic programming [6], [7]. In [7], authors derive an optimal policy scheme for a system with multiple linear time-invariant sub-systems and a single communication channel. The resulting policy was to schedule the different measurements according to a periodically repeating pattern. For the multiple communication channel case, authors in [8] use deep reinforcement learning to find an optimal policy. In [1], [9], an optimal policy was proposed where the scheduler can observe the sensor measurements.

Previous work regarding scheduling for remote estimation mostly assume that observed processes are independent. Sensor measurements are commonly spatio-temporally correlated, which can be exploited to improve the accuracy at the remote estimators. Papers [3], [10] investigate the optimal transmission frequency for sensors that observe spatio-temporally correlated measurements. In [11], the assumption is that the scheduler can observe the measurements before the scheduling decision. Although such a scheduling strategy may result in a reduced estimation error, it has implications on the privacy and latency of the system.

In this paper, we present an optimal scheduling policy for two sensors that observe spatio-temporally correlated Gaussian processes. The system model is similar to the one in [1], where a scheduler broadcasts measurements to remote estimators with the purpose of improving the overall estimation accuracy. In contrast to [1], [9], [11], we assume that the scheduler is not allowed to observe the measurements, but decides on which measurement to broadcast based on its *age-of-information (AoI)* [12]–[14]. Much of the work related to AoI have revolved around finding optimal system configurations to minimize the average AoI [6], [12]. Recent works have shown that AoI can be used for optimization problems not only concerning the timeliness of a system, but other performance metrics that are non-linear functions of the AoI [15]–[17].

We show that for a WSN with remote estimators and limited channel capacity, the performance can be improved by utilizing spatio-temporal dependency, while having the information at the scheduler restricted to the AoI. We demonstrate that an optimal scheduling strategy takes the form of a periodic sequence and the AoI of the process with the largest variance never exceeds one. We also present a low-complexity recursive numerical method to find an optimal policy.

II. Problem Formulation

We consider a system consisting of two sensors, one scheduler, and two remote estimators as depicted in Fig. 1. Sensor i observes the stochastic process $\theta_i[k] \in \mathbb{R}$ with $\theta_i[k] \sim \mathcal{N}(0, \sigma_i^2)$ at time instant $k \in \mathbb{N}_+$ and $i = 1, 2$. The two processes are correlated over space and time with the cross-covariance given by a positive-definite function [18], [19]

$$\mathbb{E}[\theta_i[k]\theta_j[l]] = \sigma_i \sigma_j \rho_{ij} \rho_t(|k-l|), \quad i, j \in \{1, 2\}, \quad (1)$$

where $\rho_{ij} \in [-1, 1]$ represents the spatial correlation and $\rho_t : \mathbb{R}_+ \to (0, 1]$ is the temporal correlation, which is a strictly decreasing function with $\rho_t(0) = 1$ and $\lim_{|n| \to \infty} \rho_t(|n|) = 0$.

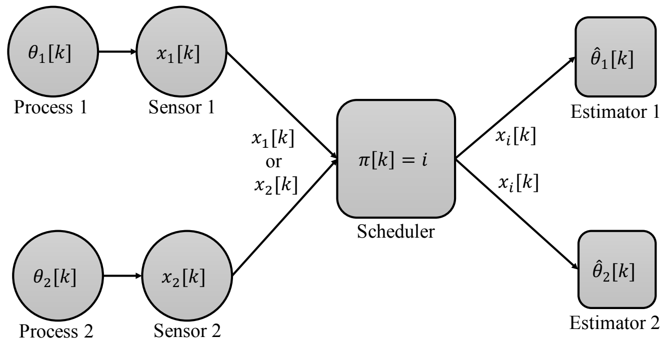

Fig. 1. Schematic of WSN scheduling problem.

At time instant k, Sensor i, $i = 1, 2$ acquires measurement $x_i[k] \in \mathbb{R}$, which is modeled as

$$x_i[k] = \theta_i[k] + w_i[k], \quad k \in \mathbb{N}_+, \tag{2}$$

where $w_i[k] \in \mathbb{R}$ denotes independent identically distributed (iid) measurement noise with distribution $w_i[k] \sim \mathcal{N}(0, \xi^2)$. For each process, $\theta_i[k], i = 1, 2$, there is a corresponding remote estimator that tracks the process and forms an estimate $\hat{\theta}_i[k]$. The estimate $\hat{\theta}_i[k]$ is based on sensor measurements received via the scheduler, see Figure 1.

A. Scheduler

Due to limited channel capacity, the scheduler can broadcast only one sensor measurement at each time instant. The remote estimators can exploit the spatio-temporal correlation to improve their estimation accuracies, irrespective of broadcasted measurement. It is, therefore, the task of the scheduler to choose the sequence of measurements that maximize the overall accuracy given the information at hand.

Let $\pi[k] \in \{1, 2\}$ denote the index of the sensor scheduled at time k. The AoI of Sensor i is denoted by $\Delta_i[k] \in \mathbb{N}_+$, $i = 1, 2$, and defined as the time elapsed between two measurement transmissions, i.e.,

$$\Delta_i[k] = \begin{cases} 0, & \text{if } \pi[k] = i, \\ \Delta_i[k-1] + 1, & \text{if } \pi[k] \neq i. \end{cases} \tag{3}$$

The scheduler is not allowed to observe the measurements $\boldsymbol{x}[k] = [x_1[k], x_2[k]]^{\mathrm{T}}$, but can keep track of previous scheduling decisions. Let γ_k denote the *scheduling strategy* at time k, which provides a mapping from available information set to scheduled measurement index, i.e.,

$$\pi[k] = \gamma_k(\mathcal{I}[k]), \tag{4}$$

where $\mathcal{I}[k] = \{\boldsymbol{\Delta}[0], \boldsymbol{\Delta}[1], \ldots, \boldsymbol{\Delta}[k-1]\}$ is the information set at the scheduler.

We show in next section that the scheduler only needs to store $\mathcal{I}[k] = \{\boldsymbol{\Delta}[k-1]\}$ to decide $\pi[k]$.

B. Remote Estimators

To compute $\hat{\theta}_i[k]$ at time k, Estimator i has access to $\boldsymbol{\Delta}[k] = [\Delta_1[k], \Delta_2[k]]^{\mathrm{T}} \in \mathbb{N}_+^2$ and $\boldsymbol{y}[k] = [y_1[k], y_2[k]]^{\mathrm{T}} \in \mathbb{R}^2$, where

$$y_i[k] = x_i[k - \Delta_i[k]], \quad i = 1, 2. \tag{5}$$

Hence, the minimum mean square error (MMSE) estimate $\hat{\theta}_i[k]$ given $\{\boldsymbol{\Delta}[k], \boldsymbol{y}[k]\}$ is computed as

$$\hat{\theta}_i[k] = \mathbb{E}[\theta_i[k] | \boldsymbol{\Delta}[k], \boldsymbol{y}[k]], \quad i = 1, 2. \tag{6}$$

C. Scheduling Policy

The *scheduling policy* γ is defined as the collection $\gamma = (\gamma_1, \gamma_2, \ldots \gamma_T)$ where T denotes the time horizon. The performance measure of the estimator and scheduling policy is the mean squared error (MSE) of the estimate (6), which is given by

$$J(\gamma, T) = \sum_{k=1}^{T} \sum_{i=1}^{2} \mathbb{E}\Big[(\theta_i[k] - \hat{\theta}_i[k])^2 \Big| \mathcal{I}[k], \pi[k] = \gamma_k(\mathcal{I}[k])\Big], \tag{7}$$

where $\boldsymbol{\Delta}[0] = [1, 0]^{\mathrm{T}}$ without loss of generality.

Our objective is to find an *optimal scheduling policy* γ^* that minimizes the average cost in (7) over an infinite time horizon

$$\min_{\gamma \in \Gamma} \limsup_{T \to \infty} \frac{1}{T} J(\gamma, T), \tag{8}$$

where Γ is the set of all feasible policies based on $\mathcal{I}[k]$. In the following section, we propose a method to solve (8) and obtain an optimal policy.

III. Optimal Scheduling Policy

As seen in (3), the AoI at k $\boldsymbol{\Delta}[k]$, is completely determined by the scheduled sensor index $\pi[k]$, and the AoI at $k-1$, $\boldsymbol{\Delta}[k-1]$. Given the scheduling policy γ and the initial AoI, $\boldsymbol{\Delta}[0]$, it is possible to determine the AoI at any time $k \in [1, T]$. We can, therefore, express the cost function as

$$J(\gamma, T) = \sum_{k=1}^{T} \sum_{i=1}^{2} \mathbb{E}\Big[(\theta_i[k] - \hat{\theta}_i[k])^2 \Big| \boldsymbol{\Delta}^{\gamma}[k]\Big], \tag{9}$$

where $\boldsymbol{\Delta}^{\gamma}[k]$ is the AoI at time k generated by policy γ. To evaluate (9), we need an expression for the MSE in (6) as a function of $\boldsymbol{\Delta}[k]$.

The process vector $\boldsymbol{\theta}[k] = [\theta_1[k], \theta_2[k]]^{\mathrm{T}}$ follows a bivariate Gaussian distribution $\boldsymbol{\theta}[k] \sim \mathcal{N}(\mathbf{0}, \mathbf{C}_{\theta\theta})$ with covariance matrix $\mathbf{C}_{\theta\theta}$. Likewise, substituting (2) in (5), we see that the observation vector $\boldsymbol{y}[k] = [y_1[k], y_2[k]]^{\mathrm{T}}$ follows $\boldsymbol{y}[k] \sim \mathcal{N}(\mathbf{0}, \mathbf{C}_{yy}[k])$. The covariance matrix $\mathbf{C}_{yy}[k] = \mathbb{E}[\boldsymbol{y}[k]\boldsymbol{y}[k]^{\mathrm{T}}]$ can be calculated using expressions (1)–(5), and is given by

$$[\mathbf{C}_{yy}[k]]_{i,j} = \sigma_i \sigma_j \rho_{ij} \rho_t(\Delta_{ij}[k]) + \xi^2 \delta(i - j), \quad i, j \in \{1, 2\} \tag{10}$$

where $\Delta_{ij}[k] = |\Delta_i[k] - \Delta_j[k]| \in \mathbb{N}_+$ is the AoI difference between the two processes, and $\delta(\cdot)$ is the Dirac delta function.

The estimate $\hat{\theta}_i[k], i \in \{1,2\}$ is the linear MMSE estimate given in [20]. The estimate $\hat{\boldsymbol{\theta}}[k] = [\hat{\theta}_1[k], \hat{\theta}_2[k]]^{\mathrm{T}}$ in (6), given as a function of $\boldsymbol{\Delta}[k]$ and $\boldsymbol{y}[k]$, becomes

$$\hat{\boldsymbol{\theta}}[k] = \mathbb{E}[\boldsymbol{\theta}[k] | \boldsymbol{\Delta}[k], \boldsymbol{y}[k]] = \mathbf{C}_{\theta y}[k]\mathbf{C}_{yy}^{-1}[k]\boldsymbol{y}[k], \tag{11}$$

where $\mathbf{C}_{\theta y}[k] \in \mathbb{R}^{2\times 2}$ is the cross-covariance between $\boldsymbol{y}[k]$ and $\boldsymbol{\theta}[k]$ given as

$$[\mathbf{C}_{\theta y}[k]]_{i,j} = \sigma_i\sigma_j\rho_{ij}\rho_t(\Delta_j[k]), \quad i,j \in \{1,2\}. \tag{12}$$

The MSE can now be expressed as a function of $\boldsymbol{\Delta}[k]$, i.e.,

$$\begin{aligned} E(\Delta_1[k], \Delta_2[k]) &= \sum_{i=1}^{2} \mathbb{E}\Big[(\theta_i[k] - \hat{\theta}_i[k])^2 \Big| \boldsymbol{\Delta}[k]\Big] \\ &= \operatorname{tr}\Big(\mathbf{C}_{\theta\theta} - \mathbf{C}_{\theta y}[k]\mathbf{C}_{yy}^{-1}[k]\mathbf{C}_{\theta y}^{\mathrm{T}}[k]\Big), \end{aligned} \tag{13}$$

where $\operatorname{tr}(\cdot)$ denotes the trace of its argument matrix. We will herein refer to $E(\Delta_1[k], \Delta_2[k])$ as the *error function*. By substituting (10) and (12) in (13), $E(\Delta_1[k], \Delta_2[k])$ can be simplified to

$$\begin{aligned} &E(\Delta_1[k], \Delta_2[k]) = (\sigma_1^2 + \sigma_2^2) \\ &+ \beta[k]\Big(2(\sigma_1\sigma_2\rho_{12})^2\rho_t(\Delta_1[k])\rho_t(\Delta_2[k])\rho_t(\Delta_{12}[k])(\sigma_1^2+\sigma_2^2) \\ &- \rho_t^2(\Delta_1[k])(\sigma_2^2+\xi^2)((\sigma_1\sigma_2\rho_{12})^2+\sigma_1^4) \\ &- \rho_t^2(\Delta_2[k])(\sigma_1^2+\xi^2)((\sigma_1\sigma_2\rho_{12})^2+\sigma_2^4)\Big), \end{aligned} \tag{14}$$

with $\beta[k] = \Big((\sigma_1^2+\xi^2)(\sigma_2^2+\xi^2) - (\sigma_1\sigma_2\rho_{12})^2\rho_t^2(\Delta_{12}[k])\Big)^{-1}$.

Next we present properties of the error function $E(\Delta_1[k], \Delta_2[k])$ and how the MSE evolves for a given scheduling strategy and policy. This will aid the derivation of an optimal policy γ^*.

A. Properties of the Error Function

For every time instant k, one of the two sensors are scheduled and the AoI, $\boldsymbol{\Delta}[k]$, evolves as

$$[\Delta_1[k], \Delta_2[k]]^{\mathrm{T}} = \begin{cases} [0, \Delta_2[k-1]+1]^{\mathrm{T}}, & \text{if } \pi[k]=1, \\ [\Delta_1[k-1]+1, 0]^{\mathrm{T}}, & \text{if } \pi[k]=2. \end{cases} \tag{15}$$

As there is no constraint on scheduling frequency in (8), the AoI $\Delta_i[k]$, $i=1,2$, can grow unbounded as $k\to\infty$, i.e., one of the sensors is never scheduled. However, the error $E(\Delta_1[k], \Delta_2[k])$ is bounded when either $\Delta_1[k]$ or $\Delta_2[k]$ tends to ∞. This is because the correlation approaches zero in (14) when AoI increases. We define the error boundaries as

$$\begin{aligned} E_1^{\infty} &= \lim_{\Delta_1[k]\to\infty} E(\Delta_1[k], 0), \\ E_2^{\infty} &= \lim_{\Delta_2[k]\to\infty} E(0, \Delta_2[k]). \end{aligned} \tag{16}$$

The upper limits E_2^{∞} and E_1^{∞} are obtained by setting $\rho_t(\Delta_2[k]) = \rho_t(\Delta_{12}[k]) = 0$ and $\rho_t(\Delta_1[k]) = \rho_t(\Delta_{12}[k]) = 0$, respectively in (14). For $\sigma_1 \geq \sigma_2$, the upper limit E_2^{∞} is the lowest average MSE the system can attain when there is no temporal correlation in (1), i.e., $\rho_t(|k-l|)=0$ for all $|k-l|>0$. The objective in (8) would then be minimized by always scheduling Sensor 1.

If $\sigma_1 \geq \sigma_2$, the error function has the following properties:

$$E(0,\Delta_2[k]) \leq E(\Delta_1[k], 0), \quad \forall\Delta_2[k] \leq \Delta_1[k] \tag{17a}$$
$$E(0,\Delta_2[k]) \leq E(0, \Delta_2[k]+\epsilon) \leq E_2^{\infty}, \quad \epsilon \in \mathbb{N}_+ \tag{17b}$$
$$E(\Delta_1[k],0) \leq E(\Delta_1[k]+\epsilon, 0) \leq E_1^{\infty}, \quad \epsilon \in \mathbb{N}_+. \tag{17c}$$

Inequality (17a) shows that the error is always greater or equal for Sensor 1 than for Sensor 2 with respect to AoI. Inequalities (17b) and (17c), imply that the error is monotonically increasing with AoI but bounded by $E_i^{\infty}, i=1,2$.

In the next section we show that an optimal policy γ^* results in a periodic scheduling sequence for an infinite scheduling horizon.

B. Optimal Scheduling Policy

For $\sigma_1 \geq \sigma_2$, the properties in (17) imply that scheduling Sensor 2 more frequently than Sensor 1 results in a higher average MSE over time. Expressions in (17) give that when $\pi[k-1]=2$, the error at time k will be smaller if Sensor 1 is scheduled, i.e., $\pi[k]=1$, instead of Sensor 2 because

$$E(0,1) < E(\Delta_1[k-1]+1, 0), \quad \forall\Delta_1[k-1] \in \mathbb{N}_+. \tag{18}$$

We define a scheduling strategy γ_k^m, $k \in \mathbb{N}_+$ as

$$\pi[k] = \gamma_k^m(\mathcal{I}[k]) = \begin{cases} 1, & \text{if } \Delta_2[k-1]+1 < m, \\ 2, & \text{if } \Delta_2[k-1]+1 \geq m, \end{cases} \tag{19}$$

where $m \in \mathbb{N}_+$ and $m \geq 2$. Let γ^m be a policy $\gamma^m = (\gamma_1^m, \gamma_2^m, \ldots, \gamma_T^m)$. For a finite m, the policy γ^m gives that Sensor 2 will be scheduled at every mth time instant. For $m=\infty$, only Sensor 1 is consistently scheduled. The policy will, therefore, render a periodic scheduling sequence with period m for a finite m and with period 1 if $m=\infty$.

We define the *average error* $\bar{E}_{\gamma^m}$ as

$$\bar{E}_{\gamma^m} = \limsup_{T\to\infty} \frac{1}{T} J(\gamma^m, T), \tag{20}$$

which represents the average error per time instant for policy γ^m over an infinite time-horizon. Since γ^m is periodic, $\bar{E}_{\gamma^m}$ will converge to the average error per decision over its period and $\bar{E}_{\gamma^m}$ in (20) reduces to

$$\bar{E}_{\gamma^m} = \begin{cases} \dfrac{\sum_{i=1}^{m-1} E(0,i) + E(1,0)}{m}, & \text{if } 2 \leq m < \infty, \\ E_2^{\infty}, & \text{if } m=\infty. \end{cases} \tag{21}$$

Theorem 1. *If $\sigma_1 \geq \sigma_2$, policy γ^{m^*} with m^* given by*

$$m^* = \operatorname*{arg\,min}_{m\geq 2} \quad \bar{E}_{\gamma^m} \tag{22}$$

is an optimal scheduling policy.

Proof. For $\sigma_1 \geq \sigma_2$, the properties in (17) and inequality in (18) imply that for an optimal policy γ^*, Sensor 1 is always scheduled at a time instant k if $\Delta_1[k-1]=1$. Let us, therefore, define a scheduling policy $\gamma^{\boldsymbol{m}}$ that satisfies this condition.

Let $\gamma^{\boldsymbol{m}}$ be a policy expressed as

$$\gamma^{\boldsymbol{m}} = (\underbrace{\gamma_1^{m_1}, \ldots, \gamma_{m_1}^{m_1}}_{m_1}, \underbrace{\gamma_{m_1+1}^{m_2}, \ldots, \gamma_{m_1+m_2}^{m_2}}_{m_2}, \ldots, \gamma_T^{m_N}), \quad (23)$$

where $\boldsymbol{m} = [m_1, m_2, \ldots, m_N]^{\mathrm{T}}$ and $m_j \in [2, 3, \ldots, \infty)$, $j = 1, 2, \ldots, N$ and $N \leq T$. For example, if $N \geq 2$, the policy would generate a scheduling sequence that follows

$$(\pi[1], \pi[2], \ldots,) = (\underbrace{1, 1, \ldots, 1, 2}_{m_1}, \underbrace{1, 1, \ldots, 1, 2}_{m_2}).$$

There exist a combination $\boldsymbol{m}^*$ such that $\gamma^{\boldsymbol{m}^*}$ is an optimal scheduling policy γ^* that minimizes (8)

$$\limsup_{T\to\infty} \frac{1}{T} J(\gamma^{\boldsymbol{m}^*}, T) = \min_{\gamma\in\Gamma} \limsup_{T\to\infty} \frac{1}{T} J(\gamma, T). \quad (24)$$

To find an optimal $\boldsymbol{m}^*$ in (24), we first define the average error as

$$\bar{E}_{\gamma^{\boldsymbol{m}}} = \limsup_{T\to\infty} \frac{1}{T} J(\gamma^{\boldsymbol{m}}, T), \quad (25)$$

and minimize (25) to get

$$\boldsymbol{m}^* = \arg\min_{\boldsymbol{m}} \bar{E}_{\gamma^{\boldsymbol{m}}}. \quad (26)$$

The average error $\bar{E}_{\gamma^{\boldsymbol{m}}}$ can be calculated as a time-weighted average of each m_j, $j = 1, ..., N$, which represents choosing scheduling strategy $\gamma_k^{m_j}$ over a time interval of length m_j. Expression $\bar{E}_{\gamma^{m_j}}$ given in (21), equals the average error per time instant for using strategy $\gamma_k^{m_j}$ over a time interval of length m_j. The optimization problem in (26) can be recast as

$$\min_{\{\alpha_j \geq 0\}_{j=1}^N} \sum_{i=1}^N \alpha_j \bar{E}_{\gamma^{m_j}}, \quad \text{s.t.} \sum_{i=1}^N \alpha_j = 1, \quad (27)$$

where $\alpha_j = \frac{m_j}{\sum_{j=1}^N m_j}$ and $\bar{E}_{\gamma^{m_j}}, \forall m_j$, is given in (21). The optimal solution for (27) is given by

$$\alpha_j^* = \begin{cases} 1, & \text{for } j = \arg\min_{\{m_i\}_{i=1}^N} \bar{E}_{\gamma^{m_i}}, \\ 0, & \text{for all other } j, \end{cases} \quad (28)$$

which shows that for any combination $\boldsymbol{m}$ the average error is lower bounded by

$$\min_{\{m_j\}_{j=1}^N} \bar{E}_{\gamma^{m_j}} \leq \bar{E}_{\gamma^{\boldsymbol{m}}}. \quad (29)$$

Since $\boldsymbol{m}$ belongs to $\boldsymbol{m} \in [2, 3, \ldots, \infty)^N$, we have

$$\min_{m\geq 2} \bar{E}_{\gamma^m} = \min_{\boldsymbol{m}} \bar{E}_{\gamma^{\boldsymbol{m}}}, \quad (30)$$

and that $\boldsymbol{m}^* = [m^*, m^*, \ldots, m^*]^{\mathrm{T}}$ is the solution to the optimization in (24) with m^* given by

$$m^* = \arg\min_{m\geq 2} \quad \bar{E}_{\gamma^m}.$$ □

Lemma 1. *The scheduling strategy $\gamma_k^{m^*}$ at time $k \in \mathbb{N}_+$ for an optimal policy γ^{m^*} only depends on $\mathcal{I}[k] = \{\boldsymbol{\Delta}[k-1]\}$ and is independent of $\{\boldsymbol{\Delta}[0], \boldsymbol{\Delta}[1], \ldots, \boldsymbol{\Delta}[k-2]\}$.*

Proof. Theorem 1 gives that an optimal policy γ^* is γ^{m^*}. As seen in (19), only the information $\mathcal{I}[k] = \{\boldsymbol{\Delta}[k-1]\}$ is needed for scheduling strategy $\gamma_k^{m^*}$ at time $k \in \mathbb{N}_+$. □

Lemma 1 implies that the scheduler only needs to store the most recent AoI vector, $\boldsymbol{\Delta}[k-1]$, i.e., we have $\mathcal{I}[k] = \{\boldsymbol{\Delta}[k-1]\}$. Now that we have an optimal policy γ^{m^*}, for $\sigma_1 \geq \sigma_2$, we need to determine the optimal period m^*. The next result shows how an optimal m^* can be obtained.

Theorem 2. *For $\sigma_1 \geq \sigma_2$, the optimal m^* is $m^* = \infty$ if*

$$\sum_{i=1}^{\infty} \left(E_2^{\infty} - E(0,i)\right) \leq E(1,0) - E_2^{\infty}, \quad (31)$$

else, if inequality (31) is not satisfied, the optimal m^* is finite and given by

$$m^* = \inf\left\{m \geq 2 \middle| \frac{\sum_{i=1}^{m-1} E(0,i) + E(1,0)}{m} \leq E(0,m)\right\}. \quad (32)$$

Proof. We prove that $\bar{E}_{\gamma^{m^*}}$ is a minimum point, i.e., $\bar{E}_{\gamma^{m^*}} \leq \bar{E}_{\gamma^{m^*+l}}$, for $m^* \geq 2$, $l \in \mathbb{N}$ and $2 - m^* \leq l < \infty$.

We start with proving $\bar{E}_{\gamma^{m^*}} \leq \bar{E}_{\gamma^{m^*+l}}$ for $2 - m^* \leq l \leq 0$. If $m^* \geq 3$, we use (21) to express $\bar{E}_{\gamma^{m^*-1}}$ in terms of $\bar{E}_{\gamma^{m^*}}$ as

$$\bar{E}_{\gamma^{m^*-1}} = \frac{m^* \bar{E}_{\gamma^{m^*}} - E(0, m^*-1)}{m^*-1}, \quad (33)$$

and we can re-write the inequality $\bar{E}_{\gamma^{m^*}} \leq \bar{E}_{\gamma^{m^*-1}}$ as

$$E(0, m^*-1) \leq \bar{E}_{\gamma^{m^*}}. \quad (34)$$

If $m^* \geq 4$, we again use (21) to express $\bar{E}_{\gamma^{m^*-2}}$ in terms of $\bar{E}_{\gamma^{m^*}}$ as

$$\bar{E}_{\gamma^{m^*-2}} = \frac{m^* \bar{E}_{\gamma^{m^*}} - E(0, m^*-1) - E(0, m^*-2)}{m^*-2}, \quad (35)$$

and the inequality $\bar{E}_{\gamma^{m^*-1}} \leq \bar{E}_{\gamma^{m^*-2}}$ becomes

$$E(0, m^*-1) + (m^*-1)E(0, m^*-2) \leq m^* \bar{E}_{\gamma^{m^*}}, \quad (36)$$

which is satisfied due to (34) and $E(0, m^*-2) \leq E(0, m^*-1)$ from (17b). By repeating the process for $2 - m^* \leq l \leq 0$ we find that

$$\bar{E}_{\gamma^{m^*}} \leq \bar{E}_{\gamma^{m^*-l}} \leq \cdots \leq \bar{E}_{\gamma^2}. \quad (37)$$

Next, we show that $\bar{E}_{\gamma^{m^*}} \leq \bar{E}_{\gamma^{m^*+l}}$ for $l \geq 0$. Given expression (21), we can express $\bar{E}_{\gamma^{m^*+1}}$ in terms of $\bar{E}_{\gamma^{m^*}}$ as

$$\bar{E}_{\gamma^{m^*+1}} = \frac{m^* \bar{E}_{\gamma^{m^*}} + E(0, m^*)}{m^*+1}, \quad (38)$$

and we can re-write the inequality $\bar{E}_{\gamma^{m^*}} \leq \bar{E}_{\gamma^{m^*+1}}$ as

$$\bar{E}_{\gamma^{m^*}} \leq E(0, m^*). \quad (39)$$

We then express $\bar{E}_{\gamma_{m^*+2}}$ in terms of $\bar{E}_{\gamma_{m^*}}$ as

$$\bar{E}_{\gamma_{m^*+2}} = \frac{m^* \bar{E}_{\gamma_{m^*}} + E(0, m^*) + E(0, m^*+1)}{m^*+2}, \tag{40}$$

and the inequality $\bar{E}_{\gamma_{m^*+1}} \leq \bar{E}_{\gamma_{m^*+2}}$ becomes

$$m^* \bar{E}_{\gamma_{m^*}} + E(0, m^*) \leq (m^*+1) E(0, m^*+1). \tag{41}$$

The inequality in (41) satisfied due to (39) and $E(0, m^*) \leq E(0, m^*+1)$ from (17b). If we continue the previous steps, we find that

$$\bar{E}_{\gamma_{m^*}} \leq \bar{E}_{\gamma_{m^*+1}} \leq \cdots \leq \bar{E}_{\gamma_\infty}. \tag{42}$$

The inequalities in (37) and (42) demonstrate that $\bar{E}_{\gamma_{m^*}}$ is a minimum point of the average error, i.e.,

$$\bar{E}_{\gamma_{m^*}} \leq \bar{E}_{\gamma_{m^*+l}}, \quad 2 - m^* \leq l < \infty. \tag{43}$$

The inequalities in (34) and (39) also gives that $\bar{E}_{\gamma_{m^*}}$ is bounded by

$$E(0, m^*-1) \leq \bar{E}_{\gamma_{m^*}} \leq E(0, m^*),$$

which together with (17b) and the definition of $\bar{E}_{\gamma_{m^*}}$ in (21), shows that the optimal m^* is found at

$$m^* = \inf \left\{ m \geq 2 \middle| \frac{\sum_{i=1}^{m-1} E(0,i) + E(1,0)}{m} \leq E(0,m) \right\}.$$

This proves expression (32) in Theorem 2.

If $m^* = \infty$ is optimal, (21) and (37) give

$$\bar{E}_{\gamma_\infty} = \frac{m E_2^\infty}{m} \leq \bar{E}_{\gamma_m} = \frac{\sum_{i=1}^{m-1} E(0,i) + E(1,0)}{m}, \ \forall m \geq 2, \tag{44}$$

which can be re-written as

$$\sum_{i=1}^{m-1} \left(E_2^\infty - E(0,i) \right) \leq E(1,0) - E_2^\infty, \quad \forall m \geq 2. \tag{45}$$

Due to (17b), the sum of the left-hand side of (45) increases with m, which gives

$$\sum_{i=1}^{\infty} \left(E_2^\infty - E(0,i) \right) \leq E(1,0) - E_2^\infty \tag{46}$$

if $m^* = \infty$ else, inequality (46) is not satisfied and m^* is finite. This proves inequality (31) in Theorem 2. □

Corollary 1. *For $\sigma_1 = \sigma_2$, the optimal m^* equals $m^* = 2$.*

Proof. For $\sigma_1 = \sigma_2$, we find in (14) and (17) that the error function is lower bounded by

$$E(0,1) = E(1,0) \leq E(\Delta_1[k], 0) \leq E(0, \Delta_2[k]),$$

where $\Delta_1[k], \Delta_2[k] \in \mathbb{N}_+, \forall k$. By choosing $m^* = 2$, the AoI $\mathbf{\Delta}[k] \in \{(0,1),(1,0)\}, \forall k \in \mathbb{N}_+$, and the average error equals the lower boundary, i.e., $\bar{E}_{\gamma_2} = E(1,0) = E(0,1)$. □

Algorithm 1 presents a recursive method, based on Theorem 2, for finding the optimal m^*.

Algorithm 1 Finding optimal m^*

if Inequality (31) is satisfied **then**
  set $m^* = \infty$
else
  Initialize $m = 2$
  while $\bar{E}_{\gamma_m} > E(0,m)$ **do**
    $\bar{E}_{\gamma_{m+1}} = \dfrac{m\bar{E}_{\gamma_m} + E(0,m)}{m+1}$
    $m = m+1$
  end while
  set $m^* = m$
end if

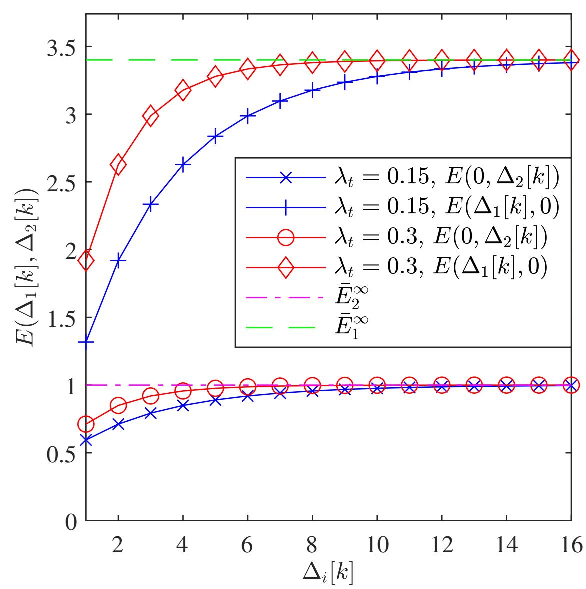

Fig. 2. Error function versus $\Delta_i[k]$, $i = 1, 2$, for $\sigma_1 = 2, \sigma_2 = 1, \rho_{12} = -0.5$ and $\xi = 0.5$.

IV. Numerical Example

We consider a system with statistical parameters $\sigma_1 = 2$, $\sigma_1 = 1$, $\rho_{12} = -0.5$, and $\xi = 0.5$. For the temporal correlation ρ_t in (1), we use $\rho_t(\Delta) = e^{-\lambda_t \Delta}$, $\Delta \in \mathbb{N}_+$, where $\lambda_t \in \mathbb{R}, \lambda_t > 0$ [18].

Figure 2 shows $E(0, \Delta_2[k])$ and $E(\Delta_1[k], 0)$ versus the AoI, $\Delta_i[k]$, $i = 1, 2$, for $\lambda_t = 0.15$ and $\lambda_t = 0.3$. The functions converge to the error boundaries $E_2^\infty = 1$ and $E_1^\infty = 3.4$, given by (16), as $\Delta_i[k]$ increases. For a larger decay constant λ_t, the MSE converges faster for both $E(0, \Delta_2[k])$ and $E(\Delta_1[k], 0)$.

Figure 3 shows the average error $\bar{E}_{\gamma_m}$ versus m for $\lambda_t = (0.05, 0.15, 0.2, 0.3)$, with m^* respectively being $4, 5, 6$ and ∞. Solid lines depicts theoretical values obtained from (21) and markers show Monte Carlo simulated estimates of $\bar{E}_{\gamma_m}$, based on simulating 100 sequences with $T = 300$ per m. We see that the simulations matches the theory. The optimal m^* and the average error $\bar{E}_{\gamma_m^*}$ grows for an increasing λ_t since

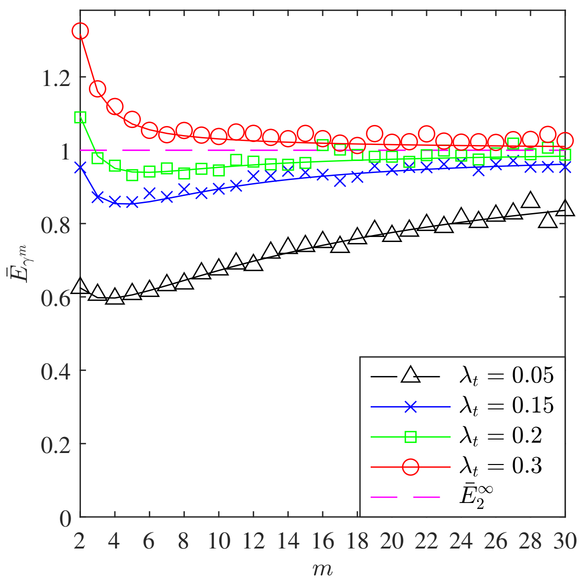

Fig. 3. Average error $\bar{E}_{\gamma^m}$ versus m for system parameters $\sigma_1 = 2, \sigma_2 = 1, \rho_{12} = -0.5$ and $\xi = 0.5$. Solid lines shows results derived from theory and markers show simulation results.

the temporal correlation decreases. For all $\lambda_t \geq 0.25$, we have the optimal $m^* = \infty$ due to (32).

V. Conclusion

This paper studied a scheduling problem for two sensors that observe two spatio-temporally dependent stochastic processes. A remote estimator tracks each process by forming an estimate based on sensor measurements transmitted by the scheduler that cannot read the measurement and can transmit data from only one sensor at each time. We derived an optimal scheduling policy using the age-of-information as a state-variable that achieves the minimum average MSE over time. We proved that the optimal scheduling policy has a periodic structure, and presented a recursive numerical method to find the optimal policy. For the proposed policy, the AoI of the process with the largest variance never exceeds one.

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