Journal of Petroleum Science and Engineering 196 (2021) 107805

Contents lists available at ScienceDirect

Journal of Petroleum Science and Engineering

journal homepage: http://www.elsevier.com/locate/petrol

A material balance model based on mass conservation in the producing layer and gas and water injection in non-communicating gas-cap and aquifer layers

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A B S T R A C T

In this work a material balance model has been developed, expanding the previous work of Borthne (1986), based on: the modified black-oil model, mass-conservation in the producing layer, gas and water injection in non-communicating gas-cap and aquifer layers and volume conservation between the producing, gas-cap and aquifer layers. The model takes as input produced surface quantities of oil (or gas) in time and voidage replacement ratios for gas and water injection and outputs reservoir pressure, associated surface quantities of gas (or oil), fluid saturations and surface volumes of aquifer and gas-cap.

The model is solved by converging the pore volume of the producing layer in each iteration of the Secant method, which is used to converge the oil or gas mass balance of the producing layer. The pore volume of the producing layer is converged by substitution and considers the expansion of gas-cap and aquifer, the injected volumes of gas and (or) water and compressibility of connate water and rock. Injection of gas and (or) water is allocated to a gas-cap or aquifer layer only thus not affecting the saturation and mobility of oil and gas in the producing layer.

The model successfully reproduces the results of the volatile-oil base case of Borthne and a dry gas case with a pot aquifer. Cases considering water and gas injection with different voidage replacement ratios are also simulated and discussed.

1. Introduction

Material balance models are simplified numerical approximations to estimate the evolution of oil and gas reservoirs when undergoing depletion and injection. The model typically assumes that there is one container with, if an oil reservoir, the producing layer, gas-cap and aquifer or, if a gas reservoir, the producing layer and aquifer. The input to the model is usually production and injection profiles, initial surface gas or oil in place, gas-cap and aquifer size, fluid and rock properties and relative permeability curves. The output is profiles in time of reservoir pressure, saturation of oil, gas and water, and incremental produced surface volumes of the associated phase, i.e. gas and water if an oil reservoir or condensate and water if a gas reservoir.

Material balance models are typically derived by applying volume conservation on the container, considering, e.g.: gas cap expansion, gas released from oil or condensate dropout from gas, rock and water expansion, aquifer expansion and influx, water and gas injection. Most material balance models use the traditional black-oil formulation. Some examples are the material balance models of Tarner (1944), Muskat (1945) and Tracy (1955).

Material balance models are typically employed coupled with other models like inflow performance relationships, well and gathering network models, to compute field production profiles. The inflow performance, well and gathering network models are used to compute the rate of wells and production system at a given time and reservoir pressure. Then the rate is integrated to the next simulation step where a new reservoir pressure and flowing gas-oil (or condensate-gas) ratio and water cut are computed with the surface produced volumes and the material balance model.

Material balance models simplify the heterogeneity in the reservoir, neglect well heterogeneity, interference and spatial effects, e.g. saturation variability around the wellbore that impacts the flowing gas-oil (or condensate-gas) ratio and water cut and neglect transients. Therefore, there are often not good approximations to the behavior of real oil and gas reservoirs.

However, in the classroom, material balance models are very useful to introduce students to petroleum engineering and facilitate the learning of fundamentals before progressing to more complex tools like 3D reservoir simulation. Material balance models are also often used to model small reservoirs, dry-gas reservoirs or during early stages of field development and economic evaluation, when information is scarce or unreliable.

In this work the author developed further the material balance model of Borthne (1986). Borthne developed a material balance model based on mass balance of oil and gas (instead of volume conservation, that is

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https://doi.org/10.1016/j.petrol.2020.107805
Received 17 June 2020; Received in revised form 10 August 2020; Accepted 14 August 2020
Available online 21 August 2020
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Nomenclature

A_o	surface volume of oil in producing layer, [Sm^3]
A_g	surface volume of gas in producing layer, [Sm^3]
B_o	oil formation volume factor, [m^3/Sm^3]
B_g	dry gas formation volume factor, [m^3/Sm^3]
B_Q	oil (or gas) formation volume factor, [m^3/Sm^3]
c_f	formation compressibility, [1/bar]
c_w	water compressibility, [1/bar]
G	initial gas in place, [Sm^3]
G_p	cumulative gas production, [Sm^3]
G_{gc}	surface gas volume of gas cap, [Sm^3]
k_{ro}	relative permeability to oil, [fraction]
k_{rg}	relative permeability to gas, [fraction]
N	initial oil in place, [Sm^3]
p	pressure, [bara]
Q	initial oil (or gas) in place, [Sm^3]
R_f	recovery factor [-]
R_s	solution gas/oil ratio in oil, [Sm^3/Sm^3]
r_s	solution oil/gas ratio in gas, [Sm^3/Sm^3]
S_o	oil saturation, [fraction]
S_g	gas saturation, [fraction]
S_w	water saturation, [fraction]
$V_{hc,prod}$	hydrocarbon volume of producing layer, [m^3]
$V_{p,AQ}$	pore volume of aquifer layer, [m^3]
$V_{p,prod}$	pore volume of the producing layer, [m^3]
$V_{p,prod+gc}$	pore volume of the producing layer and gas cap, [m^3]
VRR	Voidage replacement ratio [-]
W	surface volume of aquifer, [Sm^3]
Z	gas deviation factor [-]

Symbols

ΔG_p	incremental gas production, [Sm^3]
ΔG_{inj}	incremental surface volume of gas injected to gas cap, [Sm^3]
ΔV_{inj}	incremental volume injected to gas cap and (or) aquifer [m^3]
ΔN_p	incremental oil production, [Sm^3]
ΔN_{po}	incremental oil production from reservoir oil, [Sm^3]
ΔW_{inj}	incremental surface volume of water injected to aquifer, [Sm^3]
$\rho^*_{\bar{g}}$	ratio of densities of surface gas from reservoir oil and surface gas from reservoir gas, [fraction]
$\rho_{\bar{g}g}$	density of surface gas from reservoir gas, [kg/m^3]
$\rho_{\bar{g}o}$	density of surface gas from reservoir oil, [kg/m^3]
$\rho^*_{\bar{o}}$	ratio of densities of surface oil from reservoir gas and surface oil from reservoir oil, [fraction]
$\rho_{\bar{o}g}$	density of surface oil from reservoir gas, [kg/m^3]
$\rho_{\bar{o}o}$	density of surface oil from reservoir oil, [kg/m^3]
μ	fluid viscosity [cP]

Subscripts

e	effective
g	gas
gc	gas cap
k	generic time step "k"
i	initial
o	oil
R	reservoir

typically employed when developing material balance models) and using the modified black oil model. The model is therefore appropriate to model volatile-oil or gas condensate reservoirs. However, the original model of Borthne (1986) did not include, nor discussed the inclusion of gas-cap or aquifer and injection and the treatment of connate water compressibility. To the author's opinion, this is a major drawback and a potential reason why it has not had a more widespread adoption. Most oil and gas field nowadays are developed with injection strategies and there are many reservoirs with gas-cap and aquifer besides the oil (or gas) bearing layer.

Therefore, the present work expands the original model of Borthne to include the presence of gas-cap and aquifer (if an oil reservoir) an aquifer (if a gas reservoir) and to consider injection with an input voidage replacement ratio. Injection of gas and water is allocated to a gas-cap or aquifer layer respectively, and never into the producing layer, thus not affecting the saturation and mobility of oil and gas in the producing layer. Therefore, the gas-cap and aquifer layers are non-communicating[1] with each other or with the producing layer but affect the pore volume of the producing layer during production and injection. This, the author believes, is a suitable approximation to gas and water injection processes used for pressure support.

The model development pursued as main premise to keep to a minimum modifications to the mass balance equations and procedures to model and solve the producing layer presented by Borthne (1986).

The derived model still has the characteristics inherent to material balance models, but it could still be used in the classroom (or elsewhere) to demonstrate a larger variety of scenarios than the original model of Borthne (1986).

2. Model description

A drawing of a mechanical analogue of the material balance model proposed is presented in Fig. 1. The vertical dividers isolate fluids and layers but are free to move up or down in the cavity. The gas-cap, producing and aquifer layers all have the same pressure. Gas Injection is only performed to the gas-cap layer, water injection is only performed to the aquifer layer and production of oil and gas is only performed from the producing layer. The spring at the top represents the rock compressibility that, together with the pressure, dictates the pore volume of all layers.

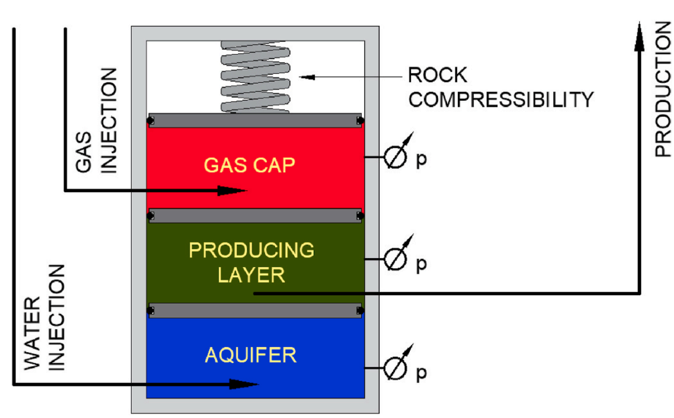

Fig. 1. Drawing of a mechanical analogue of the material balance model proposed.

[1] Here, the term non-communicating means that there is no passage of fluid between compartments

The material balance model of Borthne has been expanded as follows:

- If an oil reservoir, apart from the producing layer (green layer in Fig. 1), a non-communicating gas-gap (red layer in Fig. 1) and aquifer layer (blue layer in Fig. 1) are included.
- If a gas reservoir, apart from the producing layer (green layer in Fig. 1), a non-communicating aquifer layer (blue layer in Fig. 1) is included.
- The aquifer is a pot-aquifer and reacts instantaneously to water injection and expansion.
- It is assumed that the gas-cap has the same connate water saturation as the producing layer.
- It is assumed that the aquifer layer has 100% water saturation.
- There is no transfer of fluids between gas-cap and aquifer layers and the producing layer, thus gas-injection and water injection do not affect the saturation and mobility of oil and gas in the producing layer. However, the gas-cap and aquifer layers affect the pore volume of the producing layer. In Fig. 1, water injection and gas injection cause the vertical dividers to modify the pore volume of the producing layer.
- It is assumed that, if an oil reservoir, no oil is lost due to the movement of the gas-oil contact, or water-oil contact and, if a gas reservoir, no gas is lost due to the movement of the gas-water contact. In fact, if an oil reservoir, the gas-oil contact is the first divider from the top in Fig. 1 and the oil-water contact is the second divider from the top. If a gas reservoir the gas-water contact is the second divider from the top (the first divider in Fig. 1 doesn't exist).
- The pore volume of gas cap, producing and aquifer layer vary when the reservoir is produced and injection is performed.
- Gas and water are injected to the gas-cap and aquifer, respectively.
- Injection quantities are defined with voidage replacement ratios, constant throughout the field's life. The injected volumes of gas and/or water are computed using an input percentage and the volume void left by the oil and gas produced from the producing layer.
- The producing layer does not produce water. The existing water in this layer is connate water and it is assumed to be unmovable.
- The model does not allow for gas, water or oil injection into the producing layer.
- The model does not allow for reservoir pressurization (increasing reservoir pressure).
- The rock in the aquifer, gas-cap and producing layers is compressible and thus the pore volume of each layer depends on the pressure and the rock compressibility.

In this work, the gas-cap and aquifer are modelled as non-communicating between each other and with the producing layer. This is motivated by the fact that, in commercial software (e.g. Petroleum Experts, 2019), the gas, oil and water saturation of the tank are often computed considering aquifer, gas cap and the producing layer. Therefore, if an oil reservoir with a gas cap and aquifer undergoing gas and water injection, the values of gas and water saturation used to estimate relative permeability can be unrealistically high. This could cause prediction of high values of producing gas oil ratio and water cut. Thus, the model discussed in this work is representing the ideal case where the gas cap, aquifer, gas and water injection provide pressure support to the producing layer, but do not introduce external fluids into it.

The model has two parts,1) mass balance of oil and gas in the hydrocarbon-producing layer, which is taken and further modified from the work of Borthne (1986), and 2) overall volume conservation. The details are described next.

2.1. Mass balance of oil and gas in the producing layer

The oil and gas material balance model proposed, similar to that of Borthne (1986), is based on using the modified black oil model (Whitson and Torp, 1983) and applying mass conservation to oil and gas in the hydrocarbon-bearing reservoir layer (single cell). The resulting equations, one for the oil phase and one for the gas phase, described between two consecutive simulation steps, k and k+1, are listed below:

$$(A_o)_{k+1} - (A_o)_k + (\Delta N_p)_{k+1} = 0 \qquad \text{Eq. 1}$$

$$(A_g)_{k+1} - (A_g)_k + (\Delta G_p)_{k+1} = 0 \qquad \text{Eq. 2}$$

Here, the nomenclature introduced by Whitson and Brulé (1993) is used. A_o and A_g are the current amounts of surface oil and gas in the layer (calculated with Eq. (3) and Eq. (4). $(\Delta N_p)_{k+1}$ and $(\Delta G_p)_{k+1}$ are the incremental quantities of total surface oil and gas respectively produced between steps k and k+1.

$$A_o = \left[\frac{S_o}{B_o} + \frac{(1 - S_w - S_o)\cdot r_s\cdot \rho^*_{\bar{o}}}{B_{gd}}\right]\cdot V_{p,prod} \qquad (3)$$

$$A_g = \left[\frac{S_o\cdot R_s\cdot \rho^*_{\bar{g}}}{B_o} + \frac{(1 - S_w - S_o)}{B_{gd}}\right]\cdot V_{p,prod} \qquad (4)$$

With

$$\rho^*_{\bar{o}} = \frac{\rho_{\bar{o}g}}{\rho_{\bar{o}o}} \qquad (5)$$

$$\rho^*_{\bar{g}} = \frac{\rho_{\bar{g}o}}{\rho_{\bar{g}g}} \qquad (6)$$

where $V_{p,prod}$ is the pore volume of the producing layer. Contrary to this work, in the original development of Borthne, Eq. (1) and Eq. (2) are normalized by the current reservoir bulk rock volume, thus the equation has porosity instead of $V_{p,prod}$.

Other quantities needed to compute the produced amounts of the associated phase (gas if an oil reservoir, or condensate, if a gas reservoir) are:

$$\overline{E}_g = R_s\cdot \rho^*_{\bar{g}} + \frac{k_{rg}\cdot \mu_o\cdot B_o}{k_{ro}\cdot \mu_g\cdot B_{gd}} \qquad \text{Eq. 7}$$

$$\overline{E}_o = 1 + r_s\cdot \rho^*_{\bar{o}}\frac{k_{rg}\cdot \mu_o\cdot B_o}{k_{ro}\cdot \mu_g\cdot B_{gd}} \qquad \text{Eq. 8}$$

$\overline{E}_g$ and $\overline{E}_o$ are calculated as an arithmetic average of $(E_g)_{k+1}$ and $(E_g)_k$ and $(E_o)_{k+1}$ and $(E_o)_k$ respectively.

The procedure (as suggested by Borthne, 1986) to solve for conditions in step "k+1", assuming conditions in step "k" and incremental quantities of surface oil $(\Delta N_p)_{k+1}$ (or gas $(\Delta G_p)_{k+1}$) produced are known, is the following:

If an oil reservoir:

- Assume reservoir pressure in step k+1, calculate PVT properties
- Clear $(S_o)_{k+1}$ from Eq. (1). (Eq. (1.1) in Appendix 1). Calculate mobilities of oil and gas and $\overline{E}_o$ and $\overline{E}_g$
- The accompanying quantity of surface gas produced is calculated by dividing the incremental quantity of surface oil produced by $\overline{E}_o$(giving $(\Delta N_{po})_{k+1}$, incremental surface oil produced from reservoir oil) and then multiplying by $\overline{E}_g$
- Check if the gas mass balance (Eq. (2)) is satisfied. If not, assume another reservoir pressure.

If a gas condensate reservoir:

- Assume reservoir pressure in step k+1, calculate PVT properties
- Clear $(S_o)_{k+1}$ from Eq. (2) (Eq. (1.2) in Appendix 1). Calculate mobilities of oil and gas and $\overline{E}_o$ and $\overline{E}_g$

- The accompanying quantity of surface oil is calculated by dividing the incremental quantity of surface gas produced by $\overline{E}_g$(giving $(\Delta N_{po})_{k+1}$, incremental surface oil produced from reservoir oil) and then multiplying by $\overline{E}_o$
- Check if the oil mass balance (Eq. (1)) is satisfied. If not, assume another reservoir pressure.

In this work, the secant method is used to achieve convergence. In each simulation step, the two initial extreme values for reservoir pressure in step $k+1$ are picked in the following manner:

1. Reservoir pressure from previous simulation step "k"
2. One of 9 values between the current reservoir pressure from previous simulation step "k" and the minimum pressure in the black oil property table. Values are tested sequentially from big to small, picking the one where there is a change of sign of the mass balance (oil or gas, depending if a gas or oil reservoir respectively).

Convergence is achieved when the error in the material balance of the associated phase is lower than 0.0001 Sm^3.

2.2. Overall volume conservation

The change in hydrocarbon volume of the producing layer ($V_{hc,prod}$) between steps "k+1" and "k" is estimated with Eq. (9) (based on Pinczewski, 2001) that considers (right side of the equation, from left to right):

- Expansion (or shrinkage) of the gas in the gas-cap
- Expansion of the connate water and rock in the pore volume of the producing layer and gas cap ($V_{p,prod+gc}$)
- Expansion of the rock in the pore volume of the aquifer ($V_{p,AQ}$)
- Expansion (or shrinkage) of the water in the aquifer
- Injection to the gas-cap, aquifer or both.

$$\begin{aligned}(V_{hc,prod})_k-(V_{hc,prod})_{k+1}=&\left[(B_{gd,gc})_{k+1}-(B_{gd,gc})_k\right]\cdot(G_{gc})_k\\&+(c_f+(S_w)_k\cdot c_w)\cdot(V_{p,prod+gc})_k\cdot\left[(p_R)_k-(p_R)_{k+1}\right]\\&+c_f\cdot(V_{p,AQ})_k\cdot\left[(p_R)_k-(p_R)_{k+1}\right]\\&+\left[(B_w)_{k+1}-(B_w)_k\right]\cdot(W)_k+(\Delta V_{inj})_{k+1}\end{aligned} \quad \text{Eq. 9}$$

Eq. (9) can be written alternatively using the pore volume of the producing layer:

$$\begin{aligned}(V_{p,prod})_k\cdot(1-(S_w)_k)-(V_{p,prod})_{k+1}\cdot(1-(S_w)_{k+1})=&\left[(B_{gd,gc})_{k+1}-(B_{gd,gc})_k\right]\cdot(G_{gc})_k\\&+(c_f+(S_w)_k\cdot c_w)\cdot(V_{p,prod+gc})_k\cdot\left[(p_R)_k-(p_R)_{k+1}\right]\\&+c_f\cdot(V_{p,AQ})_k\cdot\left[(p_R)_k-(p_R)_{k+1}\right]\\&+\left[(B_w)_{k+1}-(B_w)_k\right]\cdot(W)_k+(\Delta V_{inj})_{k+1}\end{aligned} \quad \text{Eq. 10}$$

The initial (i) pore volume of the producing and gas cap layers and assuming that the producing layer is undersaturated oil or gas (Q):

$$(V_{p,prod+gc})_i=\frac{1}{(1-S_{wc})}\left[Q\cdot(B_Q)_i+(G_{gc})_i\cdot(B_{gd,gc})_i\right] \quad (11)$$

The initial pore volume of the producing layer:

$$(V_{p,prod})_i=\frac{Q\cdot(B_Q)_i}{(1-S_{wc})} \quad (12)$$

The initial pore volume of the aquifer:

$$(V_{p,AQ})_i=W_i\cdot(B_w)_i \quad \text{Eq. 13}$$

The local volume amounts of injected gas and (or) water are estimated using a voidage replacement ratio and the quantities of surface oil and gas produced between steps "k" and "k+1":

$$\begin{aligned}(\Delta V_{inj})_{k+1}=V_{RR}\cdot\left(\frac{1}{1-\overline{R}_s\cdot\overline{r}_s}\right)\cdot&\left[(\Delta G_p)_{k+1}\cdot\overline{B}_g-(\Delta N_p)_{k+1}\cdot\overline{R}_s\cdot\overline{B}_g\right.\\&\left.-(\Delta G_p)_{k+1}\cdot\overline{B}_o\cdot\overline{r}_s-(\Delta N_p)_{k+1}\cdot\overline{B}_o\right]\end{aligned} \quad \text{Eq. 14}$$

Where, from left to right, the first term is reservoir gas that becomes surface gas, the second is reservoir gas that becomes surface oil, the third is reservoir oil that becomes surface gas and the last is reservoir oil that becomes surface oil. Black oil properties are an average of properties evaluated at k and k+1 (this is what the bar on top of the property symbol indicates).

Since it is necessary to keep track in each step of the total pore volume of the reservoir and the surface volumes of gas cap and aquifer, the following equations are used:

$$(V_{p,prod+gc})_{k+1}=(V_{p,prod+gc})_k\cdot\left\{1-c_f\cdot\left[(p_R)_k-(p_R)_{k+1}\right]\right\} \quad (15)$$

$$(V_{p,AQ})_{k+1}=(V_{p,AQ})_k\cdot\left\{1-c_f\cdot\left[(p_R)_k-(p_R)_{k+1}\right]\right\} \quad (16)$$

$$(G_{gc})_{k+1}=(G_{gc})_k+(\Delta G_{inj})_{k+1} \quad (17)$$

$$(W)_{k+1}=(W)_k+(\Delta W_{inj})_{k+1} \quad (18)$$

$$(S_w)_{k+1}=(S_w)_k\cdot\frac{\left\{1+c_w\cdot\left[(p_R)_k-(p_R)_{k+1}\right]\right\}}{\left\{1-c_f\cdot\left[(p_R)_k-(p_R)_{k+1}\right]\right\}} \quad (19)$$

The local volumetric amounts of injected gas or water (Eq. (14)) are converted to surface conditions by using the gas (or water) formation volume factor of the gas-cap or aquifer at step "k+1".

$$(\Delta G_{inj})_{k+1}=\frac{(\Delta V_{inj})_{k+1}}{(B_{gd,gc})_{k+1}} \quad (20)$$

$$(\Delta W_{inj})_{k+1}=\frac{(\Delta V_{inj})_{k+1}}{(B_w)_{k+1}} \quad (21)$$

2.3. Solving procedure

The complete model is solved by converging the pore volume of the producing layer in each iteration of the Secant method (used to converge the oil or gas mass balance of the producing layer). The procedure

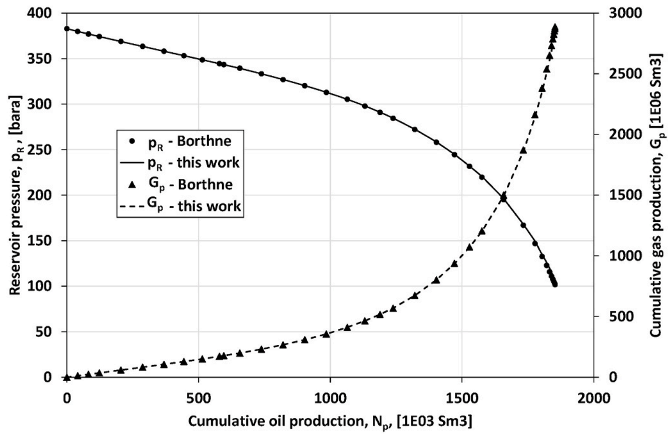

Fig. 2. Reservoir pressure (p_R) and cumulative gas production (G_P) versus cumulative oil production (N_p) presented by Borthne (1986, dots) and current work (lines).

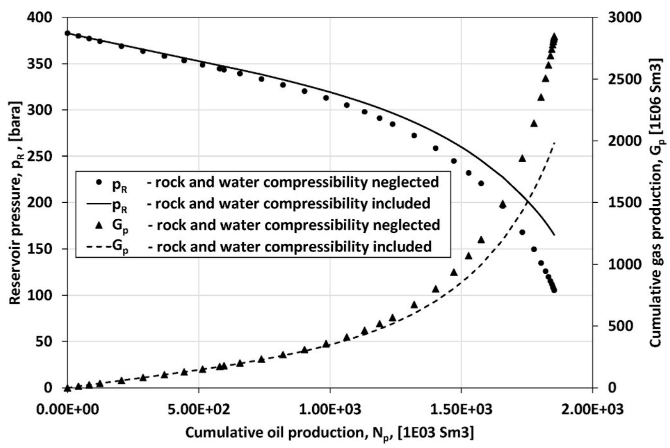

Fig. 3. Reservoir pressure (p_R) and cumulative gas production (G_P) versus cumulative oil production (N_p) calculated with the model neglecting (markers) and including (lines) connate water and formation compressibility.

consists of:

1. At start, assume $(V_{p,prod})_{k+1} = (V_{p,prod})_k$
2. Reservoir pressure value is given by the iteration of the Secant method
3. Compute the injected quantities of gas and water with Eq. (14)
4. Compute the pore volume of the producing layer with Eq. (10)
5. If the value obtained in 4 is not equal to the previous within a tolerance (in this work 0.0001 m^3) this new value of pore volume of the producing layer found is used in step 3. If the values are equal, then one proceeds with the Secant method.

Table 1
Cases simulated within study Case 2.

Case nr.	Gas cap volume G_{gc} [Sm^3]	Aquifer volume W [Sm^3]	$V_{rr,gc}$ [-]	$V_{rr,aq}$ [-]
2.1	0	1.46E+07	0	0
2.2	0	1.46E+07	0	0.8
2.3	2.84E+09	0	0	0
2.4	2.84E+09	0	0.8	0
2.5	2.84E+09	1.46E+07	0	0
2.6	2.84E+09	1.46E+07	0.4	0.4

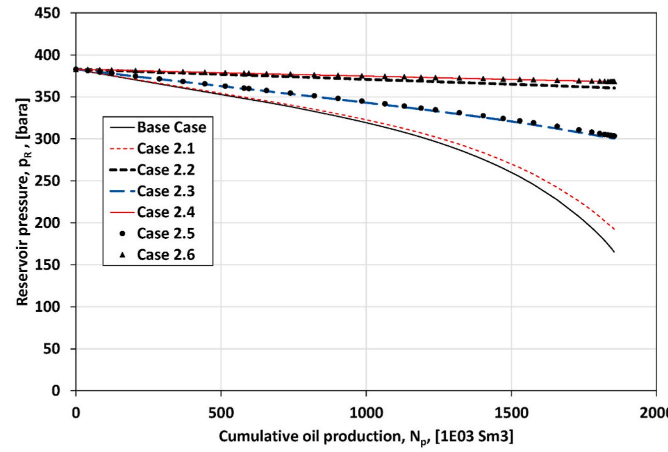

Fig. 4. Reservoir pressure (p_R) versus cumulative oil production (N_p) obtained for Case 1 and Cases 2.1–2.8.

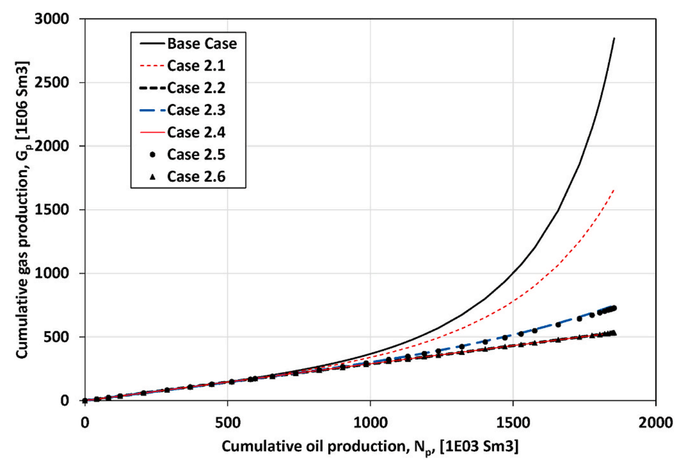

Fig. 5. Cumulative gas production (G_P) versus cumulative oil production (N_p) obtained for Case 1 and Cases 2.1–2.8.

3. Study cases and results

3.1. *Case 1 Volatile-oil reservoir of Borthne*

Borthne (1986) presented, solved and discussed a study case of a volatile-oil field. The details are given in Appendix 2. Values of reservoir pressure and cumulative gas production presented by Borthne and computed using the model proposed are shown in Fig. 2, neglecting compressibility of connate water and rock. The average and maximum relative deviation are 1.8 and 5.4% for reservoir pressure and 2.0 and 4.9% for cumulative gas production.

Fig. 3 shows reservoir pressure and cumulative gas production output by the model when the compressibility of connate water and rock is enabled. The decline of reservoir pressure and the increase in cumulative gas production is much less when compared against the case where compressibility is neglected.

3.2. *Case 2 Volatile-oil reservoir case of Borthne with gas-cap, aquifer and undergoing gas and water injection*

The cases shown in Table 1 were simulated, using several combinations of gas cap and aquifer surface volumes and voidage replacement factors for injection in gas cap, aquifer or both. All cases considered formation and connate water compressibility.

Results are plotted in Fig. 4 and Fig. 5 that depict reservoir pressure

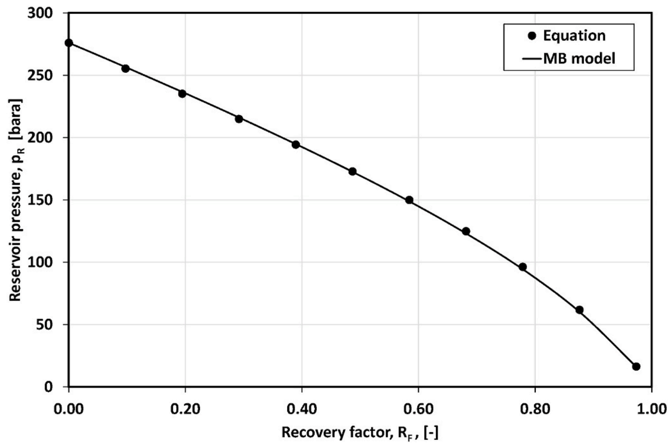

Fig. 6. Reservoir pressure (p_R) versus recovery factor (R_F) obtained for with the material balance model proposed and the analytical equation.

versus oil cumulative production and gas cumulative production versus oil cumulative production respectively. The decline in reservoir pressure with oil cumulative production is less pronounced when a higher voidage replacement ratio is employed and when there is a gas cap present. Similarly, the increase in cumulative gas production is less pronounced when employing a higher voidage replacement ratio and when there is a gas cap present.

Cases 2.2, 2.4 and 2.6 are undergoing injection with the same voidage replacement ratio, (80% void replacement with water, gas or both respectively) but have aquifer, gas-cap or both, respectively. These cases exhibit a similar behavior of reservoir pressure and cumulative gas production versus cumulative oil production. Case 2.2, that has an aquifer, shows a decline in reservoir pressure slightly more pronounced than cases 2.4 and 2.6 that have a gas-cap. This indicates that the presence of the gas-cap and aquifer have little influence in the decline and that injection is dominating the process.

Cases 2.3 and 2.5 have gas-cap and gas-cap and aquifer respectively, with no injection. These cases exhibit a similar behavior of reservoir pressure and cumulative gas production versus cumulative oil production. This indicates that the gas-cap has the most influence in the decline and the effect of the presence of the aquifer is very modest. This is somewhat expected because the compressibility of the gas is much larger than the compressibility of the water.

3.3. *Case 3 Dry gas reservoir with pot aquifer*

A dry gas reservoir with a pot-aquifer was simulated using the proposed model. The data of the case is provided in Appendix 3. Results were compared against the output of an analytical equation (details presented in appendix 4). The simulated behavior or reservoir pressure versus recovery factor predicted by the model and the analytical equation is presented in Fig. 6. The model results had an average relative deviation of 1% with respect to the results of the analytical equation and a maximum of 2.5%.

4. Remarks

Borthne (1986) validated his model against the model by Tarner (1944), a dry gas model, and a commercial, fully implicit, black oil, three-dimensional reservoir simulator (Exploration Consultants Limited, 1984). Two models were created in the three-dimensional simulator, one with a single gridblock and one with a one-dimensional radial mesh with 20 gridblocks. The study case simulated by Borthne (except when using the dry gas model) was the volatile oil field studied in this work (Cases 1 and 2).

Borthne (1986) reports a good match between the results of his model and Tarner, the dry gas model and the monoblock three-dimensional simulator. The match was fair for the multi-block three-dimensional simulator.

The model developed in the present work reproduced with an acceptable accuracy the results of the base case of Borthne. Additionally, it reproduced with an acceptable accuracy an analytical model of a case of a dry gas reservoir with a pot aquifer. The later model is derived and solved using an approach significantly different than the one presented in this work. These comparisons indicate that the proposed model should be correct as far as these tests shown. However, these studies by no means constitute a comprehensive and complete validation required for a broader-scope code verification, which is outside of the scope of this work.

Points of interest for future work are:

- Testing and validation of other cases (e.g. gas condensate reservoirs)
- Allow for reservoir re-pressurization
- Model coning from the aquifer and gas-cap layers to the producing layer
- Allow variation of the voidage replacement ratio in time
- Incorporate other aquifer models

5. Conclusions

The material balance model of Borthne was successfully expanded to include the presence of gas-cap and pot-aquifer (if an oil reservoir) an aquifer (if a gas reservoir) and to consider injection with an input voidage replacement ratio. Also, it considers formation compressibility in the producing layer, gas cap layer and aquifer and the expansion of connate water. The model was developed modifying as little as possible the mass balance equations and procedures to model and solve the producing layer presented by Borthne (1984).

The model reproduces with an acceptable accuracy the original results of the base case presented by Borthne (1986) that was validated extensively against other models and a commercial simulator. The model also reproduces with an acceptable accuracy the results of a dry gas case with pot-aquifer calculated with an analytical equation.

The model predicts logic behavior of reservoir pressure and cumulative gas production versus cumulative oil production computed for the base case of Borthne when a gas cap, aquifer and injection in gas cap and aquifer are considered.

The derived model could be used in the classroom (or elsewhere) to demonstrate a larger variety of scenarios than the original model of Borthne (1986).

CRediT authorship contribution statement

Milan Stanko: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Project administration, Resources, Software, Supervision, Validation, Visualization, Writing - original draft, Writing - review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix

A executable version of the material balance model is available here:https://drive.google.com/open?id=1ep_S5txpeh1RXC_WqlCHXkAca-WWTpiv.

Instructions of use:

- This program is compatible with Windows operating systems only.
- Download and extract the folder to your computer
- Open and modify the Excel file (input.xlsx) to provide model input. Save and close the file afterwards
- Run the executable file (main.exe).
- After some time, a pop-up window will appear after some time indicating if the run is successful or not.
- Results are printed to the text file "results.dat". Use any text editor to open and inspect this file.

Appendix 1. Additional equations

Oil saturation derived from Eq. (1)

$$(S_o)_{k+1} = \left[\frac{(A_o)_k - (\Delta N_p)_{k+1} - \left[\frac{V_{p,prod}\cdot(1-S_w)\cdot r_s\cdot\gamma_{\bar{o}}^{*}}{B_{gd}}\right]_{k+1}}{\left[V_{p,prod}\cdot\left(\frac{1}{B_o}-\frac{r_s\cdot\gamma_{\bar{o}}^{*}}{B_{gd}}\right)\right]_{k+1}}\right] \tag{1.1}$$

Oil saturation derived from Eq. (2)

$$(S_o)_{k+1} = \left[\frac{(A_g)_k - (\Delta G_p)_{k+1} - \left[\frac{V_{p,prod}\cdot(1-S_w)}{B_{gd}}\right]_{k+1}}{\left[V_{p,prod}\cdot\left(\frac{R_s\cdot\gamma_{\bar{g}}^{*}}{B_o}-\frac{1}{B_{gd}}\right)\right]_{k+1}}\right] \tag{1.2}$$

Appendix 2. Data of the volatile-oil reservoir case of Borthne

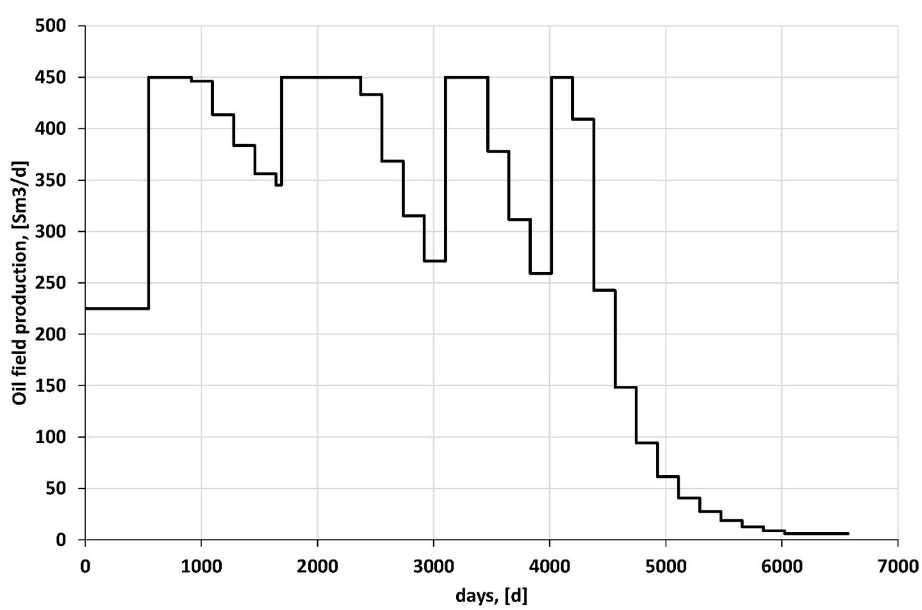

Fig. 2.1. Production profile.

Table 2.1
Reservoir properties

N [1E06 Sm^3]	16.03
P_{Ri} [bara]	382.91
S_{wc} [-]	0.30
Φ [-]	0.25
Rock compressibility [1/bar]	6E-05
Water compressibility [1/bar]	4E-05

Table 2.2
Relative permeability tables

S_o S_g [-]	k_{ro} [-]	k_{rg} [-]
0.0000	1.0000E-10	0.0000E+00
0.0149	4.3600E-09	1.2890E-03
0.0298	1.3950E-07	3.2190E-03
0.0447	1.0600E-06	5.8720E-03
0.0596	4.4660E-06	9.3190E-03
0.0745	1.3630E-05	1.3622E-02
0.0894	3.3910E-05	1.8835E-02
0.1043	7.3260E-05	2.5004E-02
0.1192	1.4290E-04	3.2167E-02
0.1340	2.5740E-04	4.0353E-02
0.1489	4.3600E-04	4.9587E-02
0.1638	7.0200E-04	5.9886E-02
0.1787	1.0850E-03	7.1258E-02
0.1936	1.6190E-03	8.3710E-02
0.2085	2.3450E-03	9.7241E-02
0.2234	3.3110E-03	1.1184E-01
0.2383	4.5720E-03	1.2751E-01
0.2532	6.1910E-03	1.4423E-01
0.2681	8.2390E-03	1.6197E-01
0.2830	1.0796E-02	1.8072E-01
0.2979	1.3953E-02	2.0046E-01
0.3128	1.7808E-02	2.2115E-01
0.3277	2.2471E-02	2.4276E-01
0.3426	2.8064E-02	2.6527E-01
0.3575	3.4719E-02	2.8865E-01
0.3723	4.2580E-02	3.1285E-01
0.3872	5.1806E-02	3.3785E-01
0.4021	6.2565E-02	3.6360E-01
0.4170	7.5041E-02	3.9008E-01
0.4319	8.9433E-02	4.1725E-01
0.4468	1.0595E-01	4.4508E-01
0.4617	1.2483E-01	4.7353E-01
0.4766	1.4631E-01	5.0259E-01
0.4915	1.7064E-01	5.3221E-01
0.5064	1.9811E-01	5.6237E-01
0.5213	2.2901E-01	5.9306E-01
0.5362	2.6365E-01	6.2426E-01
0.5511	3.0236E-01	6.5594E-01
0.5660	3.4548E-01	6.8810E-01
0.5809	3.9340E-01	7.2074E-01
0.5957	4.4649E-01	7.5385E-01
0.6106	5.0516E-01	7.8743E-01
0.6255	5.6985E-01	8.2150E-01
0.6404	6.4099E-01	8.5608E-01
0.6553	7.1907E-01	8.9119E-01
0.6702	8.0459E-01	9.2685E-01
0.6851	8.9805E-01	9.6310E-01
0.7000	1.0000E+00	1.0000E+00

Table 2.3
Black Oil Table.

p	B_o	B_g	R_s	r_s	μ_o	μ_g	$\rho_{\bar{o}g}$ $\rho_{\bar{o}o}$	$\rho_{\bar{g}o}$ $\rho_{\bar{g}g}$
[bara]	[m^3/Sm^3]	[m^3/Sm^3]	[Sm^3/Sm^3]	[Sm^3/Sm^3]	[cp]	[cp]	[kg/m^3]	[kg/m^3]
97.54	1.188	0.012463	47.5	0.000034	0.9244	0.01569	793.43	0.8209
132.01	1.239	0.009145	66	5.45E-05	0.7611	0.01721	793.07	0.8221
166.49	1.294	0.007327	85.9	8.44E-05	0.6274	0.01914	792.94	0.8231
200.96	1.355	0.006221	107.8	0.000121	0.5181	0.02142	793.03	0.8240
235.44	1.422	0.005498	132.2	0.000163	0.428	0.02395	793.23	0.8246
269.91	1.499	0.005001	159.9	0.000209	0.3535	0.02662	793.30	0.8250
304.38	1.589	0.004644	192	0.000257	0.2914	0.02939	792.94	0.8252
338.86	1.696	0.004382	230.2	0.000307	0.2394	0.03225	791.85	0.8254
365.4	1.795	0.004226	265.5	0.00035	0.2051	0.03456	789.87	0.8259
382.91	1.872	0.004139	292.6	0.000373	0.1846	0.03596	787.46	0.8271

Data for Case 2

Table 2.4
Black Oil Table of Gas Cap and Aquifer

p [bara]	B_w [m^3/Sm^3]	B_{g_gc} [m^3/Sm^3]
97.54	1.037451	0.011762
132.01	1.035889	0.008464
166.49	1.034325	0.006651
200.96	1.032763	0.005557
235.44	1.031200	0.004852
269.91	1.029637	0.004372
304.38	1.028075	0.004028
338.86	1.026512	0.003771
365.4	1.025309	0.003613
382.91	1.024515	0.003524

Appendix 3. Data of Dry gas reservoir case with pot aquifer

Table 3.1
Reservoir parameters

Connate water saturation	0.25
G [1E06 Sm^3]	270,000
Pore volume of producing layer [1E06 Sm^3]	1620
Gas specific gravity,	0.55
Initial reservoir pressure [bara]	276
Reservoir temperature [°C]	92
Water compressibility [bar^{-1}]	3.70E-5
Aquifer surface volume [1E06 Sm^3]	7794
Aquifer pore volume [1E06 m^3]	8100
Rock compressibility [bar^{-1}]	2.27E-4

Table 3.2
Black Oil Table

p [bara]	Z [-]	B_g [m^3/Sm^3]	B_w [m^3/Sm^3]
1.0	0.999	1.282937	1.050000
2.5	0.998	0.512533	1.049942
5.0	0.996	0.255736	1.049845
10.0	0.992	0.127347	1.049650
15.0	0.988	0.084558	1.049456
20.0	0.984	0.063170	1.049262
25.0	0.980	0.050342	1.049068
30.0	0.977	0.041795	1.048873
35.0	0.973	0.035693	1.048679
40.0	0.969	0.031120	1.048485
45.0	0.966	0.027567	1.048291
50.0	0.963	0.024727	1.048096
55.0	0.960	0.022406	1.047902
60.0	0.957	0.020475	1.047708
65.0	0.954	0.018843	1.047514
70.0	0.951	0.017446	1.047319
75.0	0.948	0.016238	1.047125
80.0	0.946	0.015183	1.046931
85.0	0.944	0.014255	1.046737
90.0	0.941	0.013431	1.046542
95.0	0.939	0.012696	1.046348
100.0	0.937	0.012037	1.046154
105.0	0.936	0.011442	1.045960
110.0	0.934	0.010903	1.045765
115.0	0.933	0.010412	1.045571
120.0	0.931	0.009964	1.045377
125.0	0.930	0.009554	1.045183
130.0	0.929	0.009176	1.044988
135.0	0.928	0.008828	1.044794
140.0	0.928	0.008507	1.044600
145.0	0.927	0.008209	1.044406
150.0	0.927	0.007932	1.044211
155.0	0.926	0.007675	1.044017

(*continued on next page*)

Table 3.2 (*continued*)

p [bara]	Z [-]	B_g [m^3/Sm^3]	B_w [m^3/Sm^3]
160.0	0.926	0.007435	1.043823
165.0	0.927	0.007210	1.043629
170.0	0.927	0.007001	1.043434
175.0	0.927	0.006804	1.043240
180.0	0.928	0.006619	1.043046
185.0	0.929	0.006446	1.042852
190.0	0.930	0.006282	1.042657
195.0	0.931	0.006128	1.042463
200.0	0.932	0.005983	1.042269
205.0	0.933	0.005846	1.042075
210.0	0.935	0.005716	1.041880
215.0	0.937	0.005593	1.041686
220.0	0.938	0.005477	1.041492
225.0	0.940	0.005366	1.041298
230.0	0.942	0.005261	1.041103
235.0	0.945	0.005161	1.040909
240.0	0.947	0.005066	1.040715
245.0	0.949	0.004976	1.040521
250.0	0.952	0.004890	1.040326
255.0	0.955	0.004807	1.040132
260.0	0.958	0.004729	1.039938
265.0	0.960	0.004654	1.039744
270.0	0.964	0.004582	1.039549
275.0	0.967	0.004513	1.039355
276.0	0.967	0.004500	1.039316

Appendix 4. Analytical equation for material balance of dry gas reservoir and pot aquifer

The material balance equation for dry gas with a pot aquifer is:

$$\frac{p_R}{Z_R}\cdot\left(1-\overline{c}_e\cdot\left(p_{R,i}-p_R\right)\right)=\frac{p_{R,i}}{Z_{R,i}}\cdot\left(1-\frac{G_p}{G}\right) \tag{4.1}$$

with $\overline{c}_e$, the effective compressibility:

$$\overline{c}_e = \left[\frac{\overline{c}_f+\overline{c}_w\cdot S_{wc}+\left(\frac{V_{p,AQ}}{V_{p,prod}}\right)\cdot\left(\overline{c}_f+\overline{c}_w\right)}{1-S_{wc}}\right] \qquad \text{Eq. 4.2}$$

Expanding Eq. (4.1) gives.

$$\overline{c}_e\cdot {p_R}^2+p_R\cdot\left(1-\overline{c}_e\cdot p_{Ri}\right)-\frac{p_{R,i}}{Z_{R,i}}\cdot Z_R\cdot\left(1-\frac{G_p}{G}\right)=0 \tag{4.3}$$

Which is a second-degree polynomial function on p_R, that can be solved with the quadratic equation. The gas deviation factor Z has been calculated with the correlation of Hall and Yarborough (1973).

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