

# A New Quantile Regression Model to forecast one-day-ahead Value-at-Risk

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# Preface

This study marks the completion of our Master of Science in Financial Economics at the Norwegian University of Science and Technology.

We wish to thank our supervisor Erik Haugom for his helpful comments, suggestions and valuable guidance. We would also like to thank Steinar Veka for his useful help regarding the programming in R. Our master thesis could not have been completed in this manner without their advice and remarks.

Trondheim, June, 2014.

The thesis is in its entirety a joint work between Markus T. Eliassen and Sturla A. Steine.

# Abstract

This master thesis focuses on the problem of forecasting volatility and Value-at-Risk (VaR) in the financial markets. There are numerous methods for calculating VaR. However, research in this area has not currently reached one universally accepted method that can produce good VaR estimates across different data series, and VaR prediction and quality testing is still a very challenging statistical problem.

The thesis has two main purposes, the first is to propose a simple quantile regression model for forecasting one-day-ahead VaR. Our proposed model uses only observable measures of daily, weekly and monthly volatility as input and thus simplifies the optimization compared with many models proposed in the literature. The second is to test our proposed model along with other models found in the literature, and compare them to each other in terms of accuracy. The models' performance is evaluated with both the unconditional (Kupiec, 1995) and conditional (Christoffersen, 1998) coverage tests. Further, we analyze the results and see if any conclusions can be drawn.

In this paper we examine three widely used models to calculate VaR. The models examined are Historical Simulation, RiskMetrics and GARCH(1,1). We compare these approaches with a new quantile regression model, HAR-QREG and our own proposed model, RHAR-QREG. The study is conducted on four different assets, Toyota, Apple, Nike and S&P500, and the conducted data sample ranges from 03.01.2000 – 31.12.2013. We use a rolling window of 1000 days in our study.

When subjected to formal coverage tests for out-of-sample VaR predictions, RHAR-QREG is, overall, more accurate in predicting one-day-ahead VaR compared to the three most popular models used in our study (i.e. Historical Simulation, RiskMetrics and GARCH(1,1)). However, the HAR-QREG model outperforms all of the models, and is found to be a VaR model that can produce good estimates across different datasets. Previous studies argue that the most important return feature to account for when calculating VaR, is volatility clustering. However, our study shows that the most important return characteristic is the distribution of the returns and how well the models capture it.

## Sammendrag

Masteroppgaven ser på problemet knyttet til å predikere volatilitet og Value-at-Risk (VaR) i finansmarkedene. Det finnes i dag uttallige metoder for beregning VaR. Forskning på dette området har imidlertid ikke kommet fram til én universelt akseptert metode som estimerer VaR godt på tvers av dataserier, og VaR-predikering og kvalitetstesting er fortsatt et svært utfordrende statistisk problem.

Masteroppgaven har to hovedformål. For det første, foreslår vi en enkel kvantilregresjonsmodell som kan predikere VaR en dag frem i tid. Vår foreslåtte modell bruker kun observerbare mål på daglig, ukentlig og månedlig volatilitet som input. Modellen forenkler derfor optimalisering sammenlignet med mange andre modeller foreslått i litteraturen. Det andre formålet er å teste vår foreslåtte modell sammen med andre modeller som finnes i litteraturen, og deretter sammenligne dem i forhold til hvor nøyaktig de klarer å predikere VaR en dag frem i tid. Modellenes presisjon blir evaluert ved hjelp av både en ubetinget (Kupiec, 1995) og en betinget (Christoffersen, 1998) dekningsstest. Videre, analyserer vi resultatene og ser om noen konklusjoner kan trekkes.

I studiet ser vi blant annet på tre av de mest benyttede modellene for å predikere VaR. Disse modellene er er Historisk Simulering, RiskMetrics og GARCH(1,1). Vi sammenligner disse modellene med en ny kvantilregresjonsmodell, HAR- QREG, og vår egen foreslåtte modell, RHAR - QREG. Studiet er gjennomført på fire forskjellige aksjeserier, henholdsvis Toyota, Apple, Nike og S&P500. Utvalget strekker seg i tidsrommet 03.01.2000–31.12.2013. Vi bruker et rullerende vindu på 1000 dager i vårt studie.

Når modellene blir evaluert ved hjelp av de nevnte formelle dekningsstestene for out-of-sample VaR prediksjoner, viser resultatene at RHAR-QREG predikerer VaR en dag frem i tid mer nøyaktig enn de tre mest populære modellene som benyttes i vårt studie (Historisk Simulering, RiskMetrics og GARCH(1,1)). RHAR-QREG blir imidlertid utkonkurrert av HAR-QREG, som viser seg å være en god modell da den gir gode estimerer på tvers av dataseriene. Tidligere forskning viser til at volatilitetsklumping er den viktigste egenkapen i avkastningsserier, og som da er det viktigste å ta høyde for når man estimerer VaR. Vårt studie viser imidlertid at det er fordelingen til avkastningsseriene som er viktig å fange opp. Da vil modellene skal være i stand til å predikere VaR, en dag frem i tid, mer presist.

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# 1 Introduction

The purpose of this master thesis is to propose a Value-at-Risk (VaR) prediction model that can easily be implemented by managers, traders and regulators. Further, we compare the performance of various existing models, that can forecast one-day ahead VaR, against our proposed model. According to Jorion (2001), VaR is a measure that represents a prediction of the likely maximum amount that could be lost on a bank's or a trader's portfolio with a certain degree of statistical confidence. In other words, what is the most you can lose on your investment? This is a question that almost every investor asks at some point in time, and VaR tries to provide a reasonable answer.

Many of the traditional models, which have been giving good results in previous studies, require advanced risk management skills along with complex programming, making the analysis suitable for a smaller target market. Therefore, our aim is to propose a simple VaR model based upon quantile regressions. Additionally, since VaR models are known for being data sensitive (Angelidis and Deiannakis, 2006), our study adds value to the research by comparing the traditional models.

We implement different mathematical VaR models to capture the stylized features of the historical returns. These features are crucial to note for the purpose of modeling and forecasting (Brooks, 2008):

- Leptokurtosis: Market returns have distribution with fatter tails than the normal distribution. This gives an excess peak at the mean, which is called kurtosis.
- Volatility clustering/pooling: the tendency for volatility in financial markets to appear in bunches. Volatility exhibits certain patterns and it is not constant over time. Large movements in returns are expected to be followed by further large movements. Thus the economy has cycles with high volatility and low volatility periods.
- Leverage effects: the tendency for volatility to increase more following a large price fall than following a price rise of the same magnitude. In other words, price movements are negatively correlated with the volatility.

We want to study the long and short positions in financial assets. Hence, we want to study the lower and upper tail of the return distributions, which represents risk of loss



on the investment. Losses on investments arise as a combination of two factors: (1) the volatility of the underlying variable and (2) the exposure of it. VaR captures the combined effect of these elements, and estimates the worst-case loss over a period of time, given a particular level of significance (Jorion, 2001). To manage this risk, regulators came up with minimum capital requirements with which all financial institutions have to comply.

The law required that all financial institutions are mandatory by law to set aside money for a buffer in case the money markets would dry out, as seen during the financial crisis<sup>1</sup>. VaR is essential in the calculation of this buffer. In 1988 the Basel Accord guidelines were introduced in order to better control of credit and market risk that financial institutions are facing. The guidelines represent a milestone in the process of risk management, and are used by more than 100 countries worldwide. The guidelines required that the Cook rate<sup>2</sup> should be equal to or above 8 percent. This rate was developed to ensure that all banks had enough capital set aside to reduce the risk of their respective assets. The main criticism of the Cook rate was that all loans banks had were considered to be equally risky, which in fact they were not. The criticism also included the 8 percent rule, which was believed to be arbitrarily determined. The rule moreover failed to capture the diversification effects. On this basis a new framework tool to calculate market risk was developed, the Internal Models Approach. The Internal Models Approach allowed banks to use their own models to calculate VaR, as long as they complied with the official requirements in terms of specific quantitative parameters (Jorion, 2001):

- A time horizon of 10 days, or two calendar weeks.
- A 99% confidence level.
- The observation should include at least one year of historical data and the data should be updated at least once a quarter.

These guidelines are used by most financial institutions (not just banks), and are the foundation for VaR estimations in this study. However, in our study, we use one-day ahead VaR forecasts, in line with several other studies (such as McNeil and Frey (2000), Gencay, Selcuk and Ulugulyagci (2003)).

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<sup>1</sup>If Lehman Brothers had set off an amount equivalent to that which is required by law, they would not have gone bankrupt in 2008.

<sup>2</sup>The Cook-rate is a rate, which calculates the amount of capital a financial institution shall at all times have, relative to the risk-adjusted total assets. Banks in case of unexpected loss use the calculation to determine minimum capital adequacy requirements that must be maintained.

It is especially the tails of the distribution that are of interest when forecasting the return series. Hence, it is interesting to study volatile markets when comparing different VaR models.

The popularity of VaR is due to its simplicity, and the fact that it can convert the likely loss of a portfolio of assets, or a position, to a percentage or a nominal amount. Besides the regulatory framework we also use VaR models to quantify the relationship between risk and return for active traders.

The underlying assets, on which our VaR calculations are based, are Toyota, Apple, Nike and S&P500. We will compare the performance of Historical Simulation (HS), RiskMetrics, GARCH(1,1), HAR-QREG and our proposed model, RHAR-QREG.

When subjected to formal coverage tests for out-of-sample VaR predictions, the RHAR-QREG is, overall, more accurate in predicting one-day-ahead VaR compared to the three most popular models (i.e. Historical Simulation, RiskMetrics and GARCH(1,1)). However, the HAR-QREG model outperforms all of the models, and is found to be a VaR model that can produce good estimates across different datasets.

The paper is organized as follows. Section 2 describes previous studies of the selected models. Section 3 highlights the condition we have set for the study. Section 4 describes the performance criteria. Section 5 provides the features of the chosen data series. Section 6 present how the models are being examined. Section 7 presents our results. Section 8 contains further comments on the results. Section 9 contains the conclusion of our study, while section 10 suggests further research on the topic.

## 2 Literature Review

We will start this section by presenting some general literature on different VaR models. Further, we will take a look at our chosen models, and how they have performed in previous studies.

VaR models have been extensively discussed in literature. The early study by Beder (1995) compared Historical Simulation and Monte Carlo simulation to estimate VaR. The variations of the models were constructed by employing different assumptions with respect to the data samples and/or data correlation. Beder (1995) applied eight different approaches to three hypothetical portfolios. Her findings showed that VaR calculations differ significantly for the same portfolio. In fact, the study shows that the results varied more than 14 times for the same portfolio. Specifically, the sample portfolios demonstrated that the 99% VaR changed significantly based on the time horizon, the underlying data as well as assumptions and applied methodology. Some firms make the mistake of associating VaR under a 99% expectation to the certainty or confidence that the firm will not lose more than the stated amount, more than 1% of the time. However, this study shows that VaR rather provides an expectation of outcomes based on specific assumptions, not a certainty or confidence of outcomes.

Hendricks (1996) compared twelve VaR models to 1000 randomly chosen foreign exchange portfolios. The study shows that the different models tend to produce risk estimates that are similar in average size, but the study also shows substantial differences among the various VaR approaches for the same portfolio on the same date.

The Historical Simulation (HS) approach has emerged as the most popular method for VaR calculation in the industry. A survey conducted by Perignon and Smith in 2006, shows that 73% of all financial institutions employ HS for predicting VaR. Sharma (2012) has conducted a study where thirty-eight papers were surveyed to understand performance measures for VaR methods and the comparative performance of HS VaR methods (i.e. both unfiltered- and filtered HS). Sharma (2012) shows that the HS method appears to provide superior unconditional coverage in comparison to the simple (variance covariance, Monte Carlo, EWMA) as well as sophisticated GARCH models. However, the study shows that the HS method is not successful when the conditional coverage performance measures are used. However, the filtered HS gives adequate results on the conditional

coverage test. According to Dowd (2005), who summarizes numerous disadvantages of the HS approach, the biggest weakness is the assumptions of IID<sup>3</sup> return series. Consequently, the basic (unfiltered) HS would perform well only if there were no changes in volatility of returns over time. However, from empirical evidence it is known that return series are clearly not independent as they exhibit certain patterns such as volatility clustering. Unfortunately, HS does not take into account such patterns.

JP Morgan was the first to develop comprehensive market risk management methodology based on the VaR concept (Jorion, 1997). They developed the methodology that today is known as RiskMetrics (RiskMetrics Group, 1996). This product has become extremely popular and widespread due to its simplicity and that it performs well at the 5% significance level. McMillan and Kambouroudis (2009) provides a comparison between RiskMetrics and different GARCH models. The paper studies stock index from 31 international markets. The study shows that when forecasting 1% VaR, the RiskMetrics model performs poorly and is the worst performing model amongst all of the models examined in the paper. However, when forecasting the 5% VaR the RiskMetrics model provides a good performance. Hence, the RiskMetrics model appears to perform well when predicting higher VaR measures.

Hansen and Lunde (2001) provide an out-of-sample comparison of 330 different volatility models using daily exchange rate data (DM/\$) and IBM stock prices. The authors use the GARCH(1,1) as a benchmark in the comparison paper. The analysis does not point to a single winner amongst the different volatility models, since there are different models that are best at forecasting the volatility of the two types of assets. However, the best models do not provide a significantly better forecast than the GARCH(1,1) model.

HAR-RV model of Corsi (2009) is an approximating model with long memory and is designed to capture short-, medium- and long-term volatility. Haugom et. al (2014) propose a modified HAR-RV model of Corsi (2009), called HAR-QREG, that predicts the conditional quantile directly. The study compares the HAR-QREG method with four Caviar models of Engle and Manganelli (2004). The four methods are (1) Symmetric Absolute Value, (2) Asymmetric Grinding, (3) Indirect GARCH (1,1) and (4) adaptive. In addition to these, they also compare HAR-QREG with HS and the RiskMetrics method. Finally, the skewed Student t-APARCH model of Ding et al. (1993) is included, which

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<sup>3</sup>Independently and identically distributed.

is known to be a model that works well across different asset (Giot and Laurent, 2003). When HAR-QREG model is subjected to more formal coverage tests on out-of-sample VaR predictions, it turns out that the model compares favorably with more complex models.

Summarizing the presented literature it can be concluded, that there seems to be no ideal VaR model that can produce good estimates across different datasets. It seems that the various VaR models gives both adequate and poor results depending on the characteristics of the different data series, and also on the significance levels chosen. Common sense might suggest that simple models should not produce reliable forecasts and that more advanced models should be used. But the simple HAR-QREG model shows that this is not necessarily true. The question of whether there is a VaR model that can produce sufficiently good estimates for different data series is still unanswered.

### 3 Purpose of the thesis

The thesis has two main purposes, the first is to propose a simple quantile regression model for forecasting one-day-ahead VaR. The second is to test our proposed model along with other models found in the literature, and compare them to each other in terms of accuracy.

Specifically, throughout this study we want to estimate the one-day-ahead VaR with 99% confidence level, as this is in line with the guidelines in the Internal Models Approach. In addition we will include confidence levels at 97.5%, 95% and 90%. This means that the realized losses exceed VaR forecasts in 1, 2.5, 5 and 10 out of 100 times/days, respectively. We want to study the long and short positions in financial assets. Our master thesis compares five different models that are used to estimate the one-day ahead VaR. The models we examine are Historical Simulation, RiskMetrics, GARCH(1,1), HAR-QREG and our proposed model, RHAR-QREG.

We compare the performance of the various models by examining the violation ratio and two formal coverage tests. The unconditional coverage test of Kupiec (1995) assess whether the actual violation rates equal the expected violation rate, while the conditional coverage test of Christoffersen (1998) examines whether the models jointly satisfy both the unconditional property and the independent property (see section 4).

Our conducted data sample ranges from 03.01.2000 – 31.12.2013. This allows us to compare VaR models under extreme conditions, such as the financial crisis in 2008 and the Dot-Com bubble, which had its climax in March 2000. These historical events will provide an additional dimension to our study, as data series in volatile markets shows returns in the tails of the distributions.

## 4 Model Comparison/Backtesting

The most common method to compare the accuracy of different VaR models is the violation ratio method. This method is an important part in evaluating the Internal Models Approach in the Basel Accord. A violation occurs when the actual return of an asset or portfolio at time  $t$  is equal or outside the estimated  $\text{VaR}_{t|t-1}|\Omega_{t-1}(\alpha)$ .

The calculation of the violation ratio for the long position is done by estimating an indicator variable,  $I_t$ , also known as the hit function:

$$I_t(\alpha) = \begin{cases} 1 & \text{if } r_t < \text{VaR}_{t|t-1}|\Omega_{t-1}(\alpha) \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

However, the hit function for the short position is defined as:

$$I_t(\alpha) = \begin{cases} 1 & \text{if } r_t > \text{VaR}_{t|t-1}|\Omega_{t-1}(\alpha) \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

The hit function will give us a series of ones and zeros, and the desired result is when the mean of the hit function,  $E[I_t]$  equals  $\alpha$ .

Most application of VaR are used to control for risk over short time horizons and require conditional VaR estimate that employs information up to time  $t$  to produce VaR for some time period  $t + h$ , where  $h$  is the time horizon of the forecasts. However, the Historical Simulation estimates the unconditional VaR directly (see section 6.2). Note that the value of VaR is dependent on both the VaR approaches and whether the trader has a long (eq. 1) or short (eq. 2) position in the underlying asset.

## 4.1 Kupiec Test

Kupiec (1995) was one of the earliest proposed VaR backtests. The Kupiec test is focusing exclusively on the property of unconditional coverage. Hence, Kupiec (1995) is using a sample of  $T$  observations, and the test statistic measure the proportion of failures (POF), which examines how many times a financial institution's or a trader's estimated VaR is violated over a given time period (i.e. statistically tests if  $I_t = \text{VaR}_{t|t-1}(\alpha)$ ).

Kupiec's (1995) statistic is best defined as a likelihood-ratio (LR) test:

$$LR_{UC/POF} = -2\ln \left( \frac{(1 - \alpha)^{T_0} \alpha^{T_1}}{\left(\frac{T_1}{T}\right) \left[1 - \left(\frac{T_1}{T}\right)\right]^{T_0}} \right) \sim X^2(1), \quad (3)$$

where  $T_0$  and  $T_1$  is the number of zeros and ones in the hit function.  $\alpha$  is VaR's theoretical coverage rate.

The null and the alternative hypothesis of the POF-test is:

$$H_0: E[I_t] = \alpha \quad \text{vs} \quad H_1: E[I_t] \neq \alpha \quad (4)$$

If the value of  $LR_{UC/POF}$ -statistic exceeds the critical value of the  $X^2$  distribution, the null hypothesis will be rejected and the model is regarded as inaccurate, i.e. if the mean of the hit function differs significantly from  $\alpha \cdot 100\%$ . This can also be seen by examining the p-value<sup>4</sup> of the test. We use a 5% significance level when we test the various models. This means that if the p-value is greater than 5%, we can not reject the null hypothesis, and we can conclude that we have an accurate forecasting model.

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<sup>4</sup>P-value is used as an alternative to rejection points to provide the smallest level of significance at which the  $H_0$  would be rejected. The smaller the p-value, the stronger the evidence is in favour of the  $H_1$ .



## 4.2 Christoffersen Test

According to Christoffersen (1998), VaR forecasts are valid if and only if the hit function satisfies the following two properties:

1. The unconditional coverage (UC) property - Kupiec(1995): The probability that the return on a given day exceeding the VaR forecast must be equal to the  $\alpha\%$  coverage rate,

$$\Pr [I_t(\alpha) = 1] = E [I_t(\alpha)] = \alpha \quad (5)$$

The hit function gives a sequence of numbers, e.g.(0, 0, 1, 0, 0, ..., 1), and if the frequency of violations observed over T periods is significantly lower (higher) than the coverage rate then the model used to estimate VaR, overestimates (underestimates) the true level of risk.

2. Independence property: As seen, the UC property places a restriction on how often the violation occurs. The Independence test, on the other hand, places a restriction on how these violations occur. Specifically, two elements of the hit sequence must be independent from each other. In general, a clustering of VaR violations represent a violation of the independence property, which signals a lack of responsiveness in the reported VaR measure. Hence, changing market risk fail to be fully incorporated into the reported VaR measures, which make successive runs of VaR violations more likely (Campbell, 2005). In fact, a model, which does not satisfy the independence property, can lead to clustering of violations (for a given period) even if it has the correct average number of violations. Consequently, there must be no dependence in the hit function, whatever the coverage rate considered (Dumitrescu, 2012).

The relevant test statistic for independence is given by (Kavussanos and Dimitrakopoulos, 2011):

$$LR_{ind} = -2\ln \left[ \frac{\left(1 - \frac{T_1}{T}\right)^{T_0} \left(\frac{T_1}{T}\right)^{T_1}}{\left(1 - \hat{\Pi}_{01}\right)^{T_{00}} \hat{\Pi}_{01}^{T_{01}}} \right] \sim X^2(1), \quad (6)$$

where  $T_{i,j}$ ,  $i, j = 0, 1$  is the number of observations with a  $j$  following and  $i$  in the hit function,  $I_t$ , and  $\hat{\Pi}_{01} = T_{01} / (T_{00} + T_{01})$ .

It is only the hit functions that satisfy both properties that can be considered to be an accurate VaR model, because each property characterizes a different dimension of an accurate VaR model. When property 1. and 2. are simultaneously satisfied, VaR forecasts are said to have a correct conditional coverage. A VaR model that satisfies one property or the other will result in an inaccurate description of the bank's/trader's risk exposure. These two properties are often combined into the single statement, the null hypothesis:

$$I_t(\alpha) \stackrel{\text{i.i.d.}}{\sim} B(\alpha), \quad (7)$$

which means that the hit function,  $I_t(\alpha)$ , is identically and independently distributed as a Beournalli random variable<sup>5</sup> with probability  $\alpha$  (Campbell, 2005). The alternative hypothesis is that the hit function,  $I_t$ , is not i.i.d..

By combining the independence statistic with Kupiec's POF-test we obtain a joint test that examines both properties of a good VaR model, the correct failure rate and independence of violations, i.e. conditional coverage. Hence, we have:

$$LR_{CC} = LR_{UC/POF} + LR_{ind} \sim X^2(2), \quad (8)$$

In this case we have two degrees of freedom due to the fact that we have two separate LR-statistics in the test. If the value of the  $LR_{CC}$ -statistic is lower than the critical value of  $X^2$  distribution, the model passes the test. Hence, a higher value will lead to rejection of the model. Additionally, a p-value of less than 5% concludes that the null hypothesis should be rejected in both tests, which means that the VaR model is inaccurate.

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<sup>5</sup>Beournalli random variable is the probability distribution of a random variable, which takes value 1 with success probability and value 0 with failure probability.

## 5 Data and Descriptive Analysis

In this section we will address the stylized features of the different return series. In the basic descriptive statistics, it is especially important to notice the skewness and kurtosis in terms of the VaR estimations. In the following two sections we will present these features in more detail.

### 5.1 Skewness

The skewness tells us whether the returns are symmetric or not, which in turn tells us if the returns are normally distributed. Specifically, normally distributed data are assumed to have a symmetrical distribution around its mean if it has a skew of 0. A dataset with either a positive or a negative skew therefore deviates from the normal distribution assumptions, which can cause numerical parametric models, such as RiskMetrics and GARCH(1,1), to be less effective. The reason is that these VaR approaches assume that the returns are normally distributed, which can result in an overestimation or underestimation of the VaR value, depending on the skew of the underlying return distribution (Lee, Lee and Lee, 2000). This is graphically shown in Figure 1:

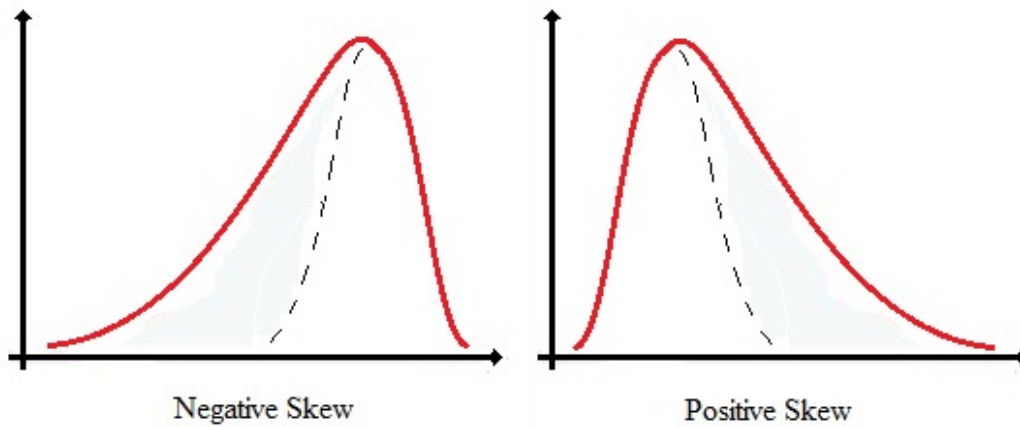


Figure 1: *shows distributions with negative and positive skewness.*

## 5.2 Kurtosis

The kurtosis provides information about how concentrated the returns are around their mean. A high kurtosis means that the returns consist of more extreme values relative to the normal distribution, i.e. the data's variance comes from extreme deviations. According to Lee, Lee, and Lee (2000) a mesokurtic (normal) distribution has a kurtosis equal to 3, and if the return distribution deviates from this number, it can cause problems for the RiskMetrics and GARCH(1,1) models in this study. A kurtosis less than 3 is called platykurtic distribution, while positive excess, kurtosis above 3, is called leptokurtic. (see Figure 2). Regarding the VaR-estimations, a low kurtosis may cause too small VaR values, and vice versa.

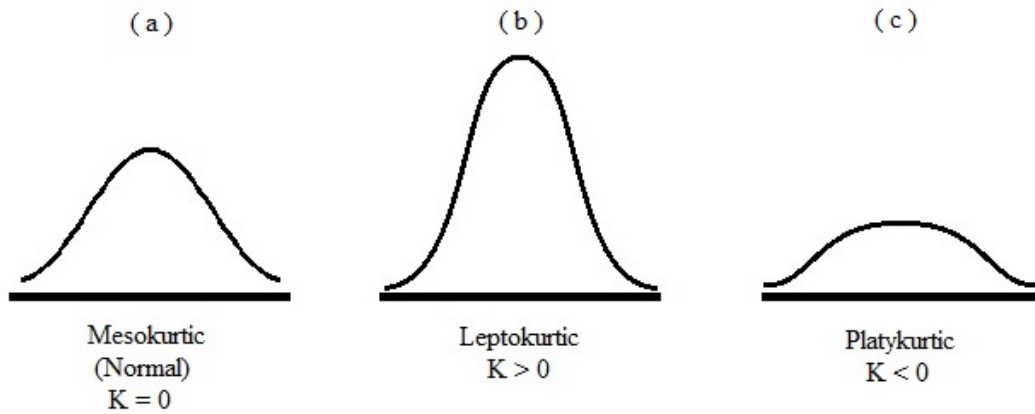


Figure 2: shows (a) normal distribution, (b) leptokurtic distribution and (c) platykurtic distribution.

### 5.3 Data

This study consists of a total of four financial assets, from which the characteristics differ substantially. The four financial assets are:

- Car industry: Toyota for the 03.01.2000–31.12.2013 period.
- Technology: Apple for the 03.01.2000–31.12.2013 period;
- Clothing industry: Nike for the 03.01.2000–31.12.2013 period;
- Index: S&P500 for the 03.01.2000–31.12.2013 period;

In each series we use daily closing prices from the period 03.01.2000 to 31.12.2013, which is obtained from Quandl<sup>6</sup>. For all price series,  $p_t$ , daily continuously compounded returns are defined as:

$$r_t = 100\ln(p_t/p_{t-1}) \quad (9)$$

However, in our proposed model (see section 6.6) we use intraday returns. Therefore, we want to present the differences between the daily close-close and open-close returns.

The open-close return is defined as:

$$R_t = 100\ln(p_{open,t}/p_{close,t}) \quad (10)$$

Descriptive characteristics for the close-close return series are given in Table 1, while descriptive graphs (price, daily close-close returns, density of the daily close-close returns and QQ-plot against the normal distribution) are given in Figures 3 – 6. Table 2 presents the descriptive statistics of the open-close returns, together with the corresponding descriptive graphs, in Figures 7 – 10.

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<sup>6</sup> Quandl - a numeric database: [www.quandl.com](http://www.quandl.com)

## 5.4 Close-close returns

Table 1:

	Toyota	Apple	Nike	S&P500
Count	3520	3520	3520	3520
Mean	0.006 %	0.046 %	0.053 %	0.007 %
Minimum	-18.061 %	-73.125 %	-21.625 %	-9.470 %
Maximum	13.253 %	13.019 %	14.108 %	10.957 %
Standard Deviation	1.822 %	3.361 %	2.047 %	1.315 %
Kurtosis	6.813	142.025	13.563	7.713
Skewness	-0.114	-7.020	-0.686	-0.175
Jarque-Bera	6769	3004543	27497	8715
Jarque-Bera P-value	0.000	0.000	0.000	0.000
Box Ljung	68.889	60.782	85.462	149.201
Box-ljung P-value	0.001	0.004	0.000	0.000
ADF - drift, 2 lag	-35.391	-34.992	-36.516	-36.129

Table 1 shows the main descriptive statistics of the data, along with the Jarque-Bera test, Box-Ljung tests and Augmented Dickey-Fuller. All values are computed using Excel and the packages (*tseries*) and (*fUnitRoots*) in R. Source: Quandl database and authors' calculations.

We see that the mean return is approximately the same for Toyota and S&P500, while Apple and Nike stand out with a significantly higher average returns, respectively 0.046% and 0.053%. Apple and Nike have the greatest volatility, 3.361% and 2.047% respectively. The excess kurtosis is high and exceeds 0<sup>7</sup> by a wide margin in all series. The kurtosis ranges from 6.183 for Toyota to 142.05 for Apple. All of the return series exhibit negative skewness, and Apple stands out with a negative skewness of  $-7.020$ , which for investors can mean a greater chance of extremely negative outcomes.

From Table 1, the Jarque-Bera test statistic and the corresponding p-value test the null hypothesis that the return series is normally distributed<sup>8</sup>. In Table 1 we see that the p-value is 0 for all return series, which indicates that we can reject the null hypothesis of normal distribution.

<sup>7</sup>Note that in Microsoft Excel, a kurtosis of 0 means that the series has no excess kurtosis.

<sup>8</sup>The null hypothesis is that the bias and excess kurtosis is 0. The alternative hypothesis is that at least one of the factors under the null hypothesis is not satisfied.

Ljung Box (Ljung and Box, 1978) Q-statistic is a test for autocorrelation in the return series, i.e. we test if the return series are independently distributed. The null hypothesis is no autocorrelation for a specified number of autocorrelation lags. The number of lags is found by using the *stats* package in R. For all return series we can reject the null hypothesis of no serial correlation. A small p-value is evidence that there is dependence in the return series, and the tests show p-values (almost) equal to zero. We see strong evidence of autocorrelation for all return series, in other words ARCH effects.

A stationary time series is, theoretically, one whose statistical features such as variance, autocorrelation, mean, etc. are all constant over time. This is often not the case for financial time series, which are far from stationary when expressed in their original units of measurements. However, most statistical forecasting is based on the assumption that the return series can be approximately stationary through the use of mathematical transformations, such as the natural logarithm. The ADF-test examines the null hypothesis of random walk,  $y_t I \sim (1)$ , against the alternative hypothesis of stationarity,  $y_t I \sim (0)$ . We see that the test statistics range from  $-34.992$  for Apple to  $-36.516$  for Nike. We see that the test statistic for the return series is sufficiently less than the critical value of  $-3.43$ , and we can thus reject all null hypotheses based on 1% significance level, and conclude that all the series are (approximately) stationary.

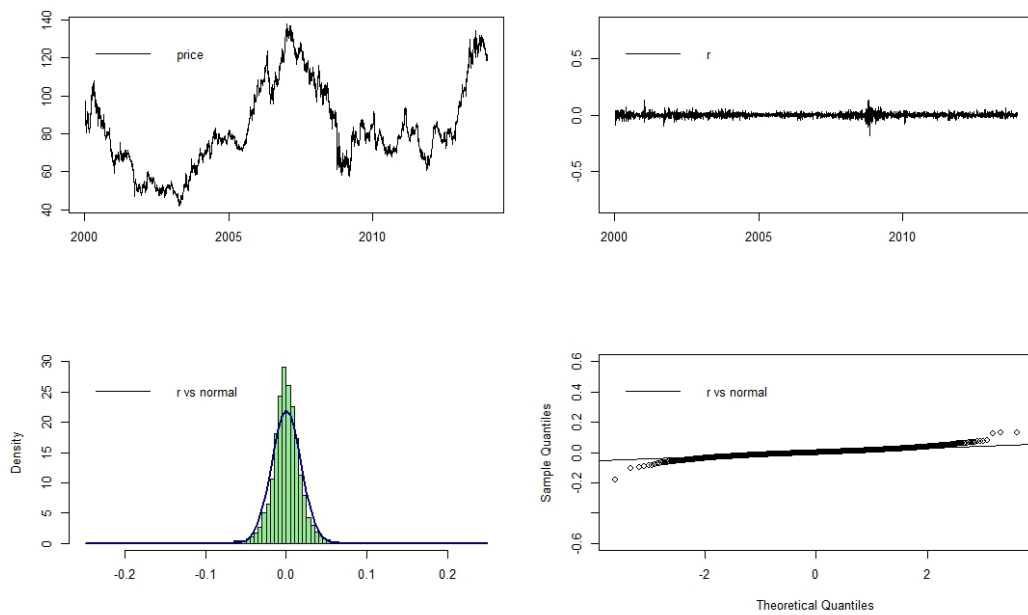


Figure 3: *Toyota cash price in level (cash), daily close-close returns ( $r$ ), daily close-close returns density and QQ-plot against the normal distribution. The time period is 03.01.2000 – 31.12.2013. Source: Quandl database and authors' calculations*

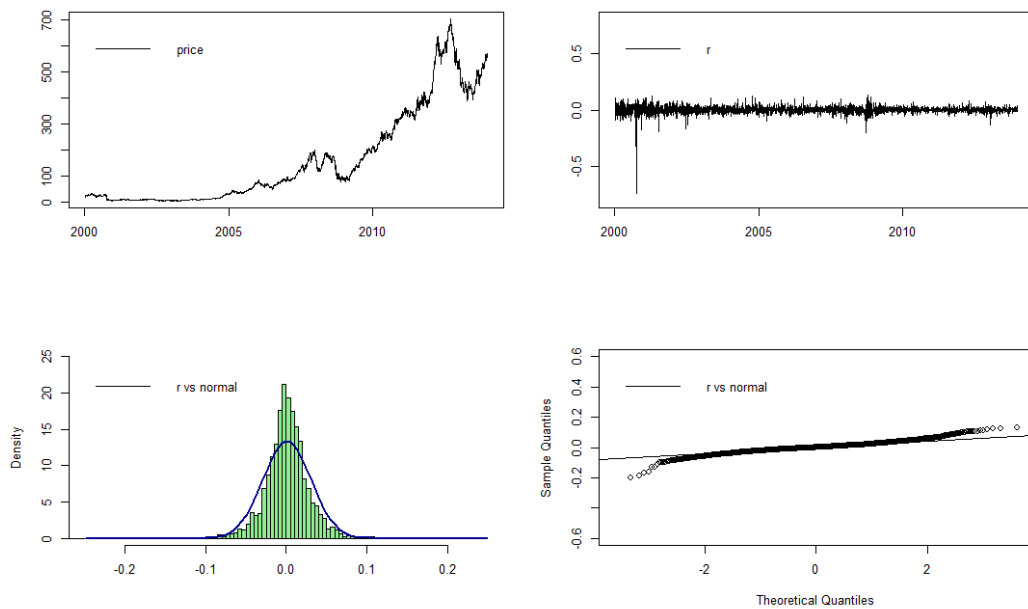


Figure 4: *Apple cash price in level (cash), daily close-close returns ( $r$ ), daily close-close returns density and QQ-plot against the normal distribution. The time period is 03.01.2000 – 31.12.2013. Source: Quandl database and authors' calculations*



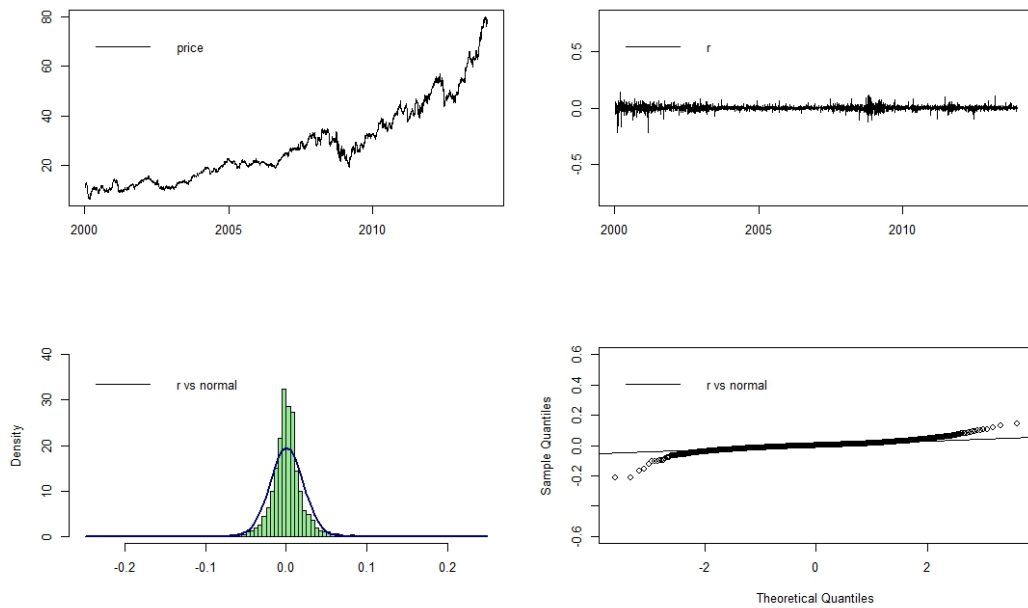


Figure 5: Nike cash price in level (cash), daily close-close returns ( $r$ ), daily close-close returns density and QQ-plot against the normal distribution. The time period is 03.01.2000 – 31.12.2013. Source: Quandl database and authors' calculations

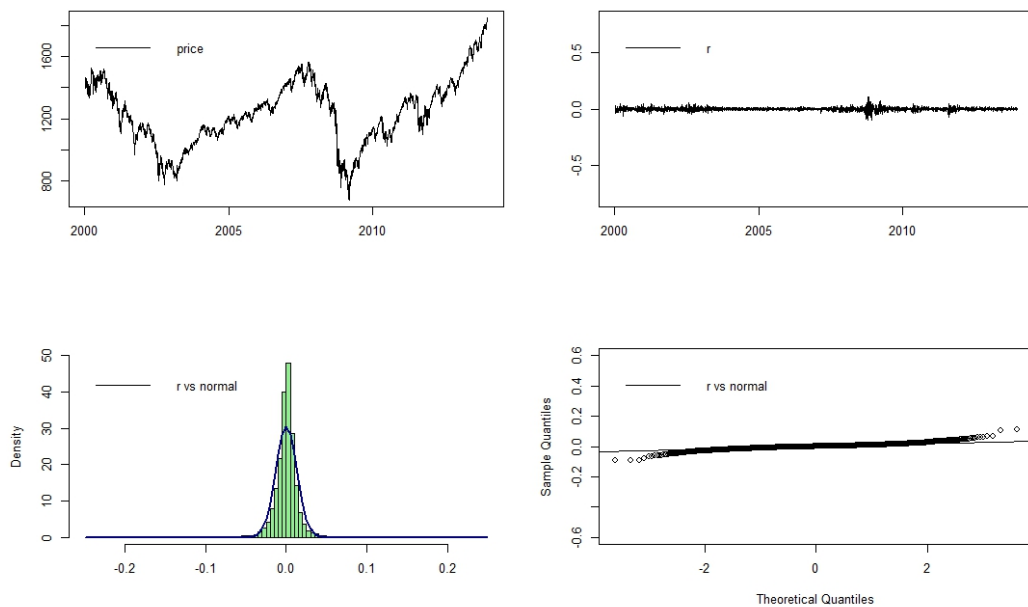


Figure 6: S&P500 cash price in level (cash), daily close-close returns ( $r$ ), daily close-close returns density and QQ-plot against the normal distribution. The time period is 03.01.2000 – 31.12.2013. Source: Quandl database and authors' calculations

The price graphs show that all assets have experienced multiple periods of both bull and bear markets during the sample period. Note that the prices were extremely volatile in late 2000 for Apple and during the financial crisis for Toyota, S&P500 and Nike. A return of  $-73.12\%$  was observed on September 29th, 2000<sup>9</sup>, for Apple as the price fell from 53.50 to 25.75! For Toyota, S&P500 and Nike, the prices were extremely volatile during the recession in the late 2008, with a peak of  $-18.06\%$ ,  $-9.46\%$  and  $-12.59\%$ , respectively. From the figures we notice leverage effects (especially for the Apple stock series), as volatility tends to cluster when the market experience negative shocks, while in periods when the market is bull, the volatility is more stable. In Figure 2, we also see volatility clustering, as we have sub-periods of high volatility, and other periods of low volatility. Volatility clustering leads to an excess kurtosis, which in turn reflects that the volatility is time varying. This is a known feature in financial return series. Volatility clustering creates problems in terms of how to model the data, since we no longer can assume that the series are normally distributed. Both the QQ-plots and the return density figures show the return series against a normally distributed reference. The high peak and the corresponding fat tails means that the distribution is more clustered around the mean, which confirm that the returns are leptokurtic.

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<sup>9</sup>Apple announced that its fourth quarter profit would fall well short of Wall Street forecasts, which spurred a flurry of analyst downgrades.

## 5.5 Open-close returns

Table 2:

	Toyota	Apple	Nike	S&P500
Mean	-0.004 %	0.013 %	-0.088 %	-0.010 %
Minimum	-6.738 %	-12.606 %	-14.351 %	-10.246 %
Maximum	10.481 %	12.789 %	20.990 %	9.127 %
Standard Deviation	1.014 %	2.358 %	1.744 %	1.274 %
Kurtosis	8.501	2.733	11.746	7.258
Skewness	0.427	-0.046	-0.061	0.202

Table 2 shows the main descriptive statistics of the open-close returns.

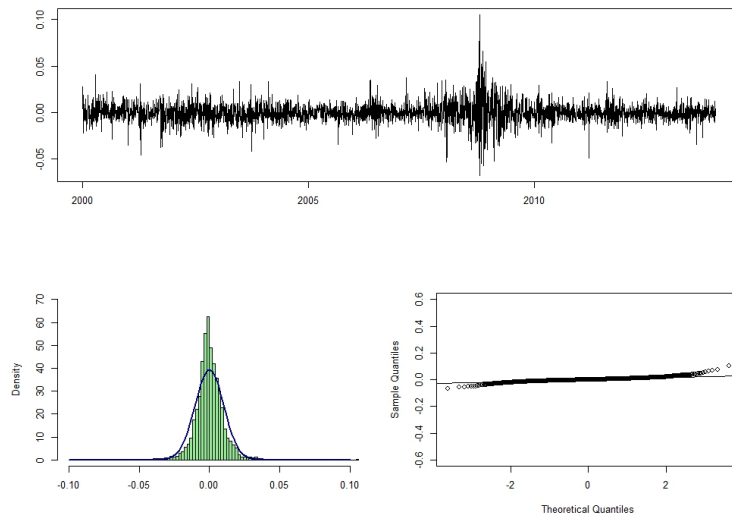


Figure 7: *Toyota daily open-close returns, daily open-close returns density and QQ-plot against the normal distribution. The time period is 03.01.2000 – 31.12.2013. Source: Quandl database and authors' calculations*

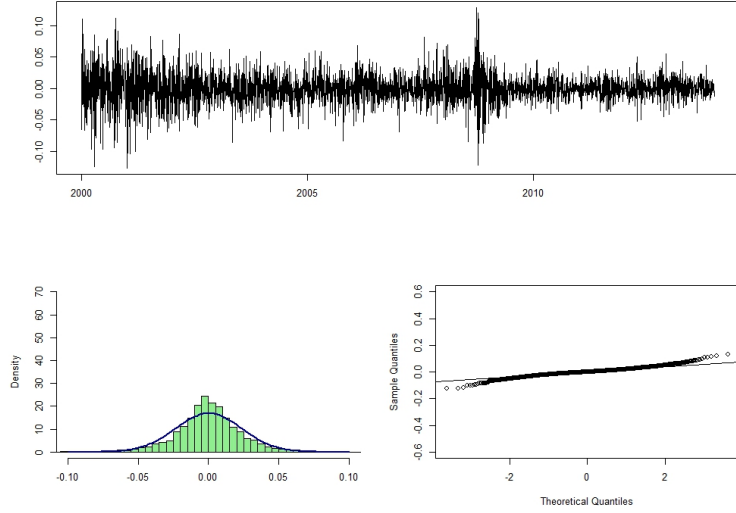


Figure 8: *Apple daily open-close returns, daily open-close returns density and QQ-plot against the normal distribution. The time period is 03.01.2000 – 31.12.2013. Source: Quandl database and authors' calculations*

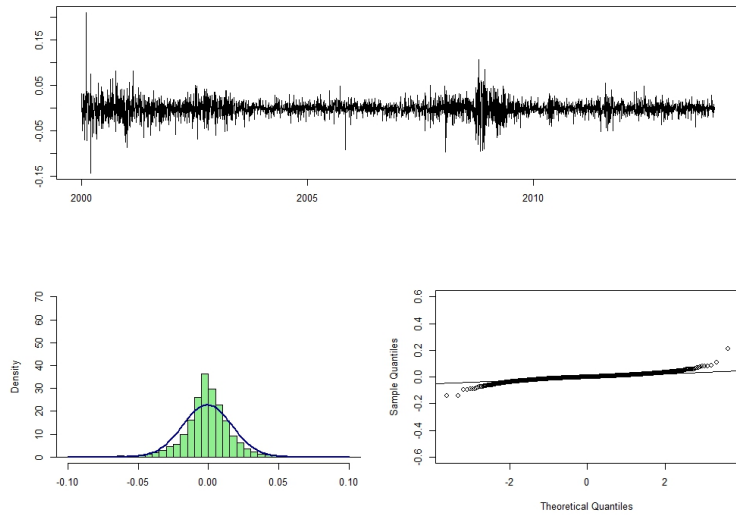


Figure 9: *Nike daily open-close returns, daily open-close returns density and QQ-plot against the normal distribution. The time period is 03.01.2000 – 31.12.2013. Source: Quandl database and authors' calculations*

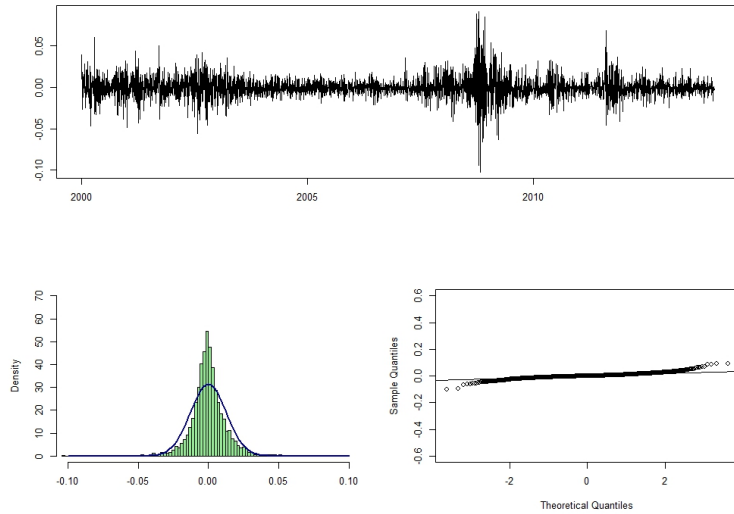


Figure 10: *S&P500 daily open-close returns, daily open-close returns density and QQ-plot against the normal distribution. The time period is 03.01.2000 – 31.12.2013. Source: Quandl database and authors' calculations*

From Table 2, we see that the open-close return series are significantly less volatile compared to close-close returns in Table 1. Specifically, comparing the minimum returns in Table 1 and 2, we see that these are considerably lower in Table 1. The standard deviations are lower for all of the returns series in Table 2 compared to Table 1. We observe positive skewness for Toyota and S&P500 and negative skewness for Apple and Nike, whereas in Table 1 all the returns series have negative skewness. Figures 7 – 10 show the descriptive graphs of the open-close return series. The figures highlight the differences of the stylized features when using open-close returns compared to the close-close returns.

Summarizing the presented descriptive statistics it can be concluded, that the distribution of the open-close returns is different from the close-close returns. Consequently, the open-close returns will produce different VaR forecasts compared to using close-close returns. Common sense suggests that the close-close returns will capture more of the information of the historical volatility, since it captures all the information within a 24 hours time span, i.e. from the closing price at day $_{t-1}$  to the closing price at day $_t$ , whereas the open-close captures the information during each trading day. However, we are using the open-close returns as proxies only when estimating the short- and medium volatility component, which may be beneficial (see section 6.6 for further explanation).

## 6 Methodology

In this section we will explain VaR, Quantile Regression, In-/Out-of-sample and our chosen models in more detail.

### 6.1 Value-at-Risk, Quantile Regression and In-/Out-of-sample

#### Value-at-Risk

According to Alexander (2008), VaR can be defined as the loss, in present value terms, that we are  $100 \cdot \alpha\%$  confident will not be exceeded if the portfolio is held static over a certain period of time (however, it might be better to interpret the VaR value as an expectation of outcomes, as discussed under section 2). Specifically, if we let  $\{r\}_{t=1}^T$  denote a time series of portfolio returns and  $\alpha \in (0, 1)$  represents the probability that a forecasted value of the portfolio returns will be exceeded each period, the challenge is to find  $VaR_t$  such that  $Pr[r_t < VaR_{t|t-1} | \Omega_{t-1}] = \alpha$ , which is defined as the conditional VaR.  $\Omega_{t-1}$  represents the information set available at time  $t - 1$ .

The RiskMetrics and GARCH(1,1) estimates the conditional variance and assume normally distributed return series,  $r_t$ , and according to Kavussanos and Dimitrakopoulos (2011), we can then forecast the conditional VaR as follows:

$$VaR_{\alpha,t+1|t} = \hat{\mu}_{t+1} - F^{-1}(\alpha)\hat{\sigma}_{t+1}, \quad (11)$$

where  $F^{-1}$  is the number of standard deviations that corresponds to the selected confidence level  $(1 - \alpha)$  from the assumed distribution (e.g. normal distribution), and  $\hat{\mu}_{t+1}$  and  $\hat{\sigma}_{t+1}$ , denotes the forecasted location and scale parameters, respectively. Since we are predicting one-day-ahead VaR (short time horizon) in our study, Figlewski (1997) states that we can assume that the sample mean ( $\hat{\mu}_{t+1}$ ) in equation (11) is zero.

The alternative models for forecasting  $\hat{\sigma}_{t+1}$  (RiskMetrics and GARCH(1,1)) and hence VaR, are presented in the following. Note that the Historical Simulation, HAR-QREG and RHAR-QREG estimate VaR directly.

## Quantile Regression

VaR is simply put a particular percentile of future returns, conditional on current information, and therefore, a quantile regression seems to be an obvious choice for VaR forecasting. In the general case, if  $Y_t$  is the dependent variable and  $X_{1,t}$ ,  $X_{2,t}$  are the independent variables, the quantile regression is given by:

$$Y_t = \omega_q + \beta_1 X_{1,t} + \beta_2 X_{2,t} + \varepsilon_t \quad (12)$$

Where  $\varepsilon_t$  has an unspecified distribution function. The conditional  $q^{th}$  quantile,  $0 < q < 1$ , is defined as any solution to the minimization problem (Koenker and Bassett, 1978 and Haugom et. al., 2014):

$$\min_{\omega, \beta_1, \beta_2} \sum_{t=1}^T (q - 1_{Y_t \leq \omega + \beta_1 X_{1,t} + \beta_2 X_{2,t}})(Y_t - (\omega + \beta_1 X_{1,t} + \beta_2 X_{2,t})), \quad (13)$$

where

$$1_{Y_t \leq \omega + \beta_1 X_{1,t} + \beta_2 X_{2,t}} = \begin{cases} 1 & \text{if } Y_t \leq \beta_1 X_{1,t} + \beta_2 X_{2,t} \\ 0 & \text{otherwise} \end{cases} \quad (14)$$

The quantile regression method explicitly allows you to model all relevant quantiles of the distribution of the dependent variable. Because VaR simply is a particular conditional quantile of future returns, the conditional quantile function can be expressed as follows:

$$VaR_{q,t+1}|X_{1,t}, X_{2,t} = \widehat{\omega}_q + \widehat{\beta}_{1,q} X_{1,t} + \widehat{\beta}_{2,q} X_{2,t} + \varepsilon_{t_q}|X_{1,t}, X_{2,t} \quad (15)$$

A unique set of regression parameters  $\beta_{1,q}$  and  $\beta_{2,q}$  can be obtained for each quantile of interest and the whole conditional distribution can be modeled or forecasted. In our study we are concerned with the upper and lower tails of the return distribution.

## In-/Out-of-sample

Out-of-sample forecasts simulates an actual risk management setting. We move our present back in time and use the out-of-sample as an unknown future, yet measurable. We test how precise the models are when predicting VaR for the out-of sample period, using the hit function. This gives us a violation if the return in the out-of-sample period exceeds the prediction. A perfect result is when the mean of the hit function, consisting of ones and zeros, is equal to the significance level used in the prediction.

The size of the in- and out-of-sample can vary, but as the title suggests we have chosen a one-day out-of-sample approach. In practice we do this by using the interval defined as time  $t - 1000$  to time  $t$ , as the in-sample data, to estimate the out-of-sample  $t + 1$ .

## 6.2 Historical Simulation

This non-parametric approach does not make use of conditioning information. Hence the approach estimates the unconditional VaR. the HS method involves creating a database consisting of the daily returns based on closing prices over a period of time. We then use a rolling window of 1000 days, and find the upper (and lower) quantiles of the rolling distributions, which per definition is the one-day-ahead unconditional VaR (Hull and White, 1998)., Mathematically defined as (Kavussanos and Dimitrakopoulos, 2010):

$$VaR_{\alpha,t+1} = Q^{\alpha}(\{r_t\}_{t=1}^n), \quad (16)$$

where  $Q^{\alpha}$  denotes the  $\alpha$ -quantile and  $\{r_t\}_{t=1}^n$  is the series of returns from 1 to  $n$ , where  $n$  represents the rolling window. We use a rolling window of 1000 days/observations, and a  $VaR_{0.05,t+1}$  is simply the 50<sup>th</sup> lowest observation in the window, with returns sorted from low to high:

$$r_1 < r_2 < \dots < r_{n-1} < r_n$$

where  $n = T$  is used to denote an order not based on time.



### 6.3 RiskMetrics

The RiskMetrics model is based on the unrealistic assumption of normally distributed returns, and completely ignores the presence of fat tails in the probability distribution, which is one of the most important features of financial data. However, the RiskMetrics model incorporates another important feature, volatility clustering. RiskMetrics capture the phenomenon of volatility clustering by choosing a particular autoregressive moving average process to model price process, which is known as the exponentially weighted moving average (EWMA).

RiskMetrics exploits a restricted Integrated GARCH (IGARCH) filter for returns, with a zero constant ( $\omega = 0$ ), and predetermined parameters  $\alpha_1 = 0.06$  and  $\beta = (1 - \alpha_1) = 0.94$ . The value of  $\beta = 0.94$  produces the best backtesting results, which implies a high degree of persistence in the variance (RiskMetrics Group, 1996).

The estimator for the conditional variance is given by:

$$\sigma_{t+1}^2 = \alpha_1 r_t^2 + \beta \sigma_t^2 \quad (17)$$

The notation  $\sigma_{t+1}$  (square root of  $\sigma_{t+1}^2$ ) emphasizes that the volatility estimated on a given day ( $t$ ) is actually used as a predictor for the volatility of the next day ( $t + 1$ ).  $VaR_{\alpha,t+1|t}$  can then be calculated (under the assumption of normal distribution) by multiplying  $\sigma_t$  with  $F^{-1}$ , as shown in equation (11).

### 6.4 GARCH (1,1)

The AutoRegressive Conditional Heteroscedasticity ARCH (Engle, 1982) and Generalized AutoRegressive Conditional Heteroscedasticity GARCH (Bollerslev, 1986) can capture the time varying volatility, like volatility clustering and serial correlation, which is common in most financial return series. The GARCH model captures these features by allowing the variance of the returns to be conditional on previous values of the returns, defined as:

$$\sigma_t^2 = Var (r_t | r_{t-1}, r_{t-2}, r_{t-3} \dots) \quad (18)$$

Hence, a GARCH model can be used as an appropriate tool when forecasting VaR. A GARCH(p,q) forecasting model is defined as:

$$\sigma_{t+1}^2 = \omega + \sum_{i=0}^{q-1} \alpha_i r_{t-i}^2 + \sum_{j=0}^{p-1} \beta_j \sigma_{t-j}^2 \quad (19)$$

The estimated parameters  $\omega$ ,  $\alpha_i$  and  $\beta_j$  must satisfy the non-negativity of the conditional variance, i.e.  $0 < \omega$ ,  $0 \leq \alpha_1$  and  $0 \leq \beta_1$ , which means that  $\alpha_1 + \beta_1 < 1$ . See Bollerslev (1986) and Nelson and Cao (1992) for details on the non-negativity and stationarity conditions of the GARCH process.

A GARCH(1,1) model is defined as (Jorion, 2001):

$$\sigma_t^2 = \omega + \alpha_1 r_{t-1}^2 + \beta_1 \sigma_{t-1}^2, \quad (20)$$

where,  $\sigma_{t-1}^2$  is the lagged conditional variance and  $r_{t-1}^2$  is the lagged squared returns. Note that the  $\alpha_1$  denotes how fast the variance reacts to shocks in the squared returns. In other words, a large  $\alpha_1$  indicates that the shock almost instantly will be reflected in the variance forecast for the next period, while small values of the coefficient forecasts a smoother transition in the future variance pattern. Values of  $\beta_1$  represent how much yesterdays variance estimation weights into the forecast.

The  $\omega$ ,  $\alpha$  and  $\beta$  is estimated numerically using Maximum Likelihood Estimation (MLE), defined as (Jorion, 2001):

$$\max F(\sigma_t) = \sum_{t=1}^T \left( \ln \left( \frac{1}{\sqrt{2\pi}\sigma_t} \right) - \frac{r_t^2}{2\sigma_t^2} \right), \quad (21)$$

where  $\omega$ ,  $\alpha$  and  $\beta$  gives the values of the  $\sigma_t^2$ . We maximize this function by adjusting  $\omega$ ,  $\alpha$  and  $\beta$  (with their non-negativity constraints), which in turn gives us the optimal values of  $\omega$ ,  $\alpha$  and  $\beta$ . We use a rolling window size equal to 1000 days, and the optimization takes into account each and every observation within this window. Further, the  $\omega$ ,  $\alpha$  and  $\beta$  are adjusted by the MLE.

The actual VaR value is calculated by multiplying the square root of the one-day-ahead variance (i.e. standard deviation) with  $F^{-1}$ , as shown in equation (11)

## 6.5 HAR-QREG

The HAR-QREG model predicts the conditional quantiles directly by using quantile regressions. We have used `quantreg` package in R to estimate the quantile regressions. Haugom et. al. (2014) define  $r_t$  as the daily logged return at day  $t$ , and the daily, weekly and monthly backward-looking volatility as:

$$\sigma_{day,t} = \sqrt{r_t^2}$$

$$\sigma_{week,t} = \sqrt{\frac{1}{5} (r_{t-4}^2 + r_{t-3}^2 + \dots + r_t^2)}$$

$$\sigma_{month,t} = \sqrt{\frac{1}{20} (r_{t-19}^2 + r_{t-18}^2 + \dots + r_t^2)}$$

The HeterogenousAutoregressive – Quantile Regression Model (HAR-QREG) is then defined as:

$$r_{qt+1} = \omega_q + \beta_{1,q}\sigma_{day,t} + \beta_{2,q}\sigma_{week,t} + \beta_{3,q}\sigma_{month,t}, \quad (22)$$

where  $r_{qt+1} = VaR_{q,t+1}$  is the conditional quantile of the day-ahead return, and  $\omega_q$ ,  $\beta_{1,q}$ ,  $\beta_{2,q}$ , and  $\beta_{3,q}$ , are parameters which estimate the constant term, daily, weekly and monthly historical volatility, respectively.

## 6.6 RHAR-QREG

We have chosen to modify the HAR-QREG model, by using squared intraday return as a proxy when estimating the realized short- and medium-term volatility components. The RHAR-QREG will incorporate the same features as the HAR-QREG, but by using intraday volatility on the short and medium term components, we believe it will capture the historical volatility even better. The other models are constructed with the data of closing prices, which might neglect the important intraday information of the price

movements, and in return this might lead to loss of information and efficiency. For example, if yesterday's closing price equals to today's closing price, the price return will be zero, even though the price variation during the today might be turbulent. However, the return based models that use only closing prices cannot capture this. The problem of getting zeros can also be found when using open-close returns, but all our data series consist of more zeros in the close-close returns than in the open-close returns. Hence our hypothesis is to capture more of the medium- and short-term variation. We have therefore incorporated the intraday return by using the daily opening and closing prices of the financial assets, defined as:

$$R_t = 100\ln(p_{open,t}/p_{close,t}) \quad (23)$$

Further we use  $R_t$  to define the daily and weekly historical volatility components:

$$\sigma_{day,t} = \sqrt{R_t^2}$$

$$\sigma_{week,t} = \sqrt{\frac{1}{5} (R_{t-4}^2 + R_{t-3}^2 + \dots + R_t^2)}$$

Since all traders are concerned with the long-term volatility, the long-term volatility component will have a strong effect on the conditional return quantiles. In Haugom et. al (2014) the monthly component is substantially significant. As mentioned, the open-close returns may also be equal to zero, but by implementing the close-close return as a proxy on the long-term volatility component we will still capture some of the variation in the returns. Hence, we have chosen to use the daily closing prices on the monthly component, rather than intraday return. We suspect that the long-term component will capture more of the historical volatility when using closing prices, and that the short- and medium-term component is better estimated when the intraday return is incorporated, because of their short time horizon. The monthly component will be defined in the same way as in Haugom et. al (2014):

$$\sigma_{month,t} = \sqrt{\frac{1}{20} (r_{t-19}^2 + r_{t-18}^2 + \dots + r_t^2)},$$

where  $r_t$  is the estimated daily return constructed with the data of closing prices. Our proposed Remodeled HeterogenousAutoregressive – Quantile Regression Model (RHAR-QREG) model can then be defined as:

$$r_{q_{t+1}} = \omega_q + \beta_{1,q}\sigma_{day,t} + \beta_{2,q}\sigma_{week,t} + \beta_{3,q}\sigma_{month,t} \quad (24)$$

where  $r_{q_{t+1}} = VaR_{q,t+1}$  is the conditional quantile of the day-ahead return, and  $\omega_q$ ,  $\beta_{1,q}$ ,  $\beta_{2,q}$ , and  $\beta_{3,q}$ , are parameters which estimate the constant term, daily, weekly and monthly historical volatility, respectively.

## 7 Results

We compare the accuracy of the VaR forecasts from the RHAR-QREG method with Historical Simulation, RiskMetrics, GARCH(1,1) and HAR-QREG. The estimations are performed using R (R Core Team, 2013).

In all cases we obtain VaR using a rolling window of 1000 days to estimate the parameters and then predict one-day-ahead, out-of-sample with each method. The choice of using 1000 days is based on the findings of Alexander and Sheedy (2008).

In table 2 we present the failure rates for eight VaR-levels (1%, 2.5%, 5%, 10%), long and short positions, respectively. We present the results for two different coverage tests to assess the accuracy and independence of the provided VaR forecasts. These are the unconditional coverage test of Kupiec (1995), and the conditional coverage test of Christoffersen (1998), which were explained earlier in the thesis. We use a significance level of 5% in both tests.

The results from these coverage tests are presented in Table 3:

Table 3: Results

<b>Toyota</b>																	
Model	VaR for long positions								VaR for short positions				Passes				
	1.00 %	2.50 %	5.00 %	10.00 %	90.00 %	95.00 %	97.50 %	99.00 %	95.00 %	97.50 %	99.00 %						
Historical Simulation	1.43 %	‡	2.86 %	‡‡	4.57 %	‡‡	8.87 %	†	90.58 %	†	95.15 %	†	97.57 %	†	98.85 %	‡‡	68.75 %
RiskMetrics	1.39 %	‡‡	2.63 %	‡‡	4.93 %	‡‡	8.99 %	‡‡	90.25 %	‡‡	94.19 %	‡‡	96.38 %		98.09 %		75.00 %
GARCH(1,1)	1.19 %	‡‡	2.35 %	‡‡	4.02 %	‡‡	7.99 %		91.41 %	‡‡	95.23 %	‡‡	97.14 %	‡‡	98.57 %	‡	81.25 %
HAR-QREG	1.19 %	‡‡	2.35 %	‡‡	4.77 %	‡‡	9.46 %	‡‡	89.98 %	‡‡	95.23 %	‡‡	97.50 %	‡‡	98.81 %	‡‡	100.00 %
RHAR-QREG	1.51 %		2.94 %	‡‡	5.45 %	‡‡	9.34 %	‡‡	89.70 %	‡‡	94.63 %	‡‡	96.70 %	‡	98.17 %		81.25 %
<b>S&amp;P500</b>																	
Model	VaR for long positions								VaR for short positions				Passes				
	1.00 %	2.50 %	5.00 %	10.00 %	90.00 %	95.00 %	97.50 %	99.00 %	95.00 %	97.50 %	99.00 %						
Historical Simulation	1.67 %		3.06 %	†	5.16 %	†	8.33 %		91.35 %		95.40 %	‡‡	97.42 %	‡‡	98.69 %	‡‡	50.00 %
RiskMetrics	1.96 %		3.86 %		5.88 %	‡	10.49 %	‡‡	90.22 %	†	94.88 %	‡‡	97.10 %	‡‡	98.66 %	‡‡	62.50 %
GARCH(1,1)	1.90 %		3.49 %		5.48 %	‡‡	9.05 %	‡‡	90.87 %		95.71 %	‡‡	97.74 %	‡‡	99.17 %	‡‡	62.50 %
HAR-QREG	1.47 %	‡	2.90 %	‡‡	5.04 %	‡‡	9.29 %	‡‡	89.56 %	†	94.68 %	‡‡	97.34 %	‡‡	98.89 %	‡‡	87.50 %
RHAR-QREG	1.55 %		2.54 %	‡‡	5.16 %	‡‡	9.33 %	‡‡	89.60 %	†	94.80 %	‡‡	97.38 %	‡‡	98.65 %	‡‡	81.25 %
<b>Apple</b>																	
Model	VaR for long positions								VaR for short positions				Passes				
	1.00 %	2.50 %	5.00 %	10.00 %	90.00 %	95.00 %	97.50 %	99.00 %	95.00 %	97.50 %	99.00 %						
Historical Simulation	0.95 %	‡‡	1.87 %		4.30 %	†	8.31 %		92.12 %		96.30 %		97.85 %	‡‡	99.16 %	‡‡	43.80 %
RiskMetrics	1.43 %	‡‡	2.35 %		4.34 %		8.00 %		89.81 %	‡‡	94.51 %	‡‡	96.50 %	‡‡	97.81 %		50.00 %
GARCH(1,1)	1.35 %	†	2.07 %	†	3.34 %		6.80 %		91.09 %	†	95.19 %	†	97.06 %	†	98.37 %		31.25 %
HAR-QREG	0.99 %	‡‡	2.47 %	†	4.69 %	‡‡	9.67 %	‡‡	90.69 %	‡‡	95.27 %	‡‡	97.45 %	‡‡	98.93 %	‡‡	93.75 %
RHAR-QREG	1.19 %	‡‡	2.59 %	†	4.81 %	†	9.75 %	‡‡	90.37 %	‡‡	95.19 %	‡‡	97.41 %	‡‡	98.81 %	‡‡	87.50 %
<b>Nike</b>																	
Model	VaR for long positions								VaR for short positions				Passes				
	1.00 %	2.50 %	5.00 %	10.00 %	90.00 %	95.00 %	97.50 %	99.00 %	95.00 %	97.50 %	99.00 %						
Historical Simulation	1.39 %	†	2.78 %	†	4.97 %	†	9.11 %	†	91.09 %	†	95.31 %	†	97.26 %	‡‡	98.97 %	‡‡	62.50 %
RiskMetrics	1.35 %	‡‡	2.55 %	‡‡	4.18 %	†	7.40 %		91.33 %	‡	94.91 %	‡‡	96.82 %	‡	98.09 %		62.50 %
GARCH(1,1)	1.15 %	‡‡	1.91 %	†	3.66 %		6.72 %		92.33 %		95.63 %	‡‡	97.46 %	‡‡	98.57 %	‡‡	56.25 %
HAR-QREG	1.03 %	‡‡	3.10 %	‡‡	5.13 %	‡‡	10.06 %	‡‡	90.38 %	‡‡	95.11 %	‡‡	97.57 %	‡‡	98.77 %	‡‡	100.00 %
RHAR-QREG	1.15 %	‡‡	2.66 %	‡‡	5.17 %	‡‡	9.86 %	†	90.10 %	‡‡	94.99 %	‡‡	97.57 %	‡‡	99.05 %	‡‡	93.75 %

Table 3: Failure rates for the three financial series. † indicates that the model passes the unconditional coverage test of Kupiec (1995) at the 5% significance level, and ‡ indicates that the model passes the conditional coverage test of Christoffersen (1998) at the 5% significance level. We have used the quantileVaR package by Steinar Veka(2014) in R. The column labeled ‘Passes’ shows what fraction the model passes the two tests at all VaR-levels. Best possible score is  $16/16 = 100\%$  (eight VaR-levels times two tests). The Sample period is January 3.2000 to December 31.2013. The results are based on out-of-sample forecasts using a 1000-days rolling window.

Table 3 presents the result for the four financial assets. The first row shows the results for the Historical Simulation, followed by the RiskMetrics, GARCH(1,1), HAR-QREG and RHAR-QREG. The RHAR-QREG outperforms all the models across the different series, except on the Toyota series where the model is on par with GARCH(1,1). However, the HAR-QREG model delivers extraordinarily good results and outperforms all the models.

The Toyota stock series gives the overall best results. Both the RHAR-QREG and GARCH(1,1) models passes 81.25% against the RiskMetrics (75.00%) and Historical Simulation (68.75%). The HAR-QREG outperforms the other models with its 100% score.

For the *S&P500* series, the RHAR-QREG model passes 81.25% of the coverage tests and is outperformed by the HAR-QREG model, with a score of 87.5%. The Historical Simulation method, RiskMetrics and GARCH(1,1) models pass 50.00%, 62.50% and 62.50%, respectively.

The results are similar for the Apple series. The RHAR-QREG delivers a good performance and outperforms the Historical Simulation, RiskMetrics and GARCH(1,1). The RHAR-QREG passes 87.5% in the coverage tests for this series, while the HAR-QREG model passes 93.75%. GARCH(1,1) has its worst performance and passes 31.25%. Historical Simulation (43.80%) and RiskMetrics (50.00%) again performs poorly relative to the quantile models.

For the Nike stock series, the RHAR-QREG has its best performance with a score of 93.75% on the coverage tests. Yet again, it is outperformed by the HAR-QREG's which passes 100% of the tests. Still, the RHAR-QREG model outperforms the Historical Simulation (62.50%), RiskMetrics (62.50%) and GARCH(1,1) (56.25%).

RHAR-QREG's total success rate, i.e. across the four stocks and the eight theoretical significance levels, is equal to 85.94%. The Historical Simulation, RiskMetrics, GARCH(1,1) and HAR-QREG have total success rates equal to 56.25%, 62.50%, 57.81% and 95.31%, respectively.

Note, the two most popular methods, Historical Simulation and RiskMetrics, perform substantially worse than the RHAR-QREG model for all the series.



## 8 Comments

On the basis of these estimation outputs and the estimation methods discussed in the previous sections, we briefly summarize the advantages and disadvantages of the chosen models.

### 8.1 Historical Simulation

The HS method performs worst in our study. Overall, the HS approach performs best at higher confidence levels and especially the upper tail (i.e. short positions at 97.5% and 99%), where the approach performs adequately in terms of the Kupiec (1995) and Christoffersen (1998) tests. The reason for this may be that the historical window size is better suited for these confidence levels as it considers the extreme values that fall out of the normal distribution. On the lower tail (i.e. long trading positions) and with lower confidence levels, the HS method generally produces too few violations and the violations comes in clusters, which gives more rejections from the unconditional and conditional coverage tests, relative to the best models.

The HS method has no assumptions regarding the distribution of the return series of financial assets, but fail to capture the time-varying volatility (e.g. volatility clustering) due to its assumption of i.i.d return series. Specifically, the HS method captures the fatter tails of the return series, but the results show the importance of the model also capturing the time-varying volatility in the return series when forecasting one-day-ahead VaR.

This approach looks at the latest 1000 daily returns when calculating VaR (i.e. a rolling window of 1000 days, in line with the other models examined). A rolling window of 1000 days makes the approach react slowly to new information and changes in the daily returns. If we had chosen a shorter interval, we would have increased the impact of new observations, which in turn would have put a higher focus on recent market conditions, which might improve forecasts.

Generally, the historical simulation model is considered to perform well with return distributions that are leptokurtic, but this study shows that when the leptokurtosis is extreme, the method cannot capture this. The bad performance on the Apple stock series supports this argument.

Based on our study, the HS method with a rolling window of 1000 days can be recommended when estimating VaR at high confidence levels for traders with short trading positions, and on return distributions that are not too leptokurtic. However, it seems that the rolling window size has to be chosen with respect on the confidence level and the kurtosis.

## 8.2 RiskMetrics

The RiskMetrics scores overall good at the 5% significance level. This is expected due to the fact that the model is designed estimating VaR on the 5% significance level. D. Nelson (2010) shows that even though the RiskMetrics model is considered to be a misspecified model, it still estimates volatility reasonably accurately. The RiskMetrics sets the confidence level at 95%, and the prescription to obtain this 5% quantile is to multiply the volatility estimate by  $-1,65$  (long position) or  $1.65$  (short position), assuming the returns are conditionally normally distributed. It is very often found that despite the presence of fat tails (leptokurtic distribution), for many distributions the 5% quantile is roughly  $-1,65$  (or  $1.65$ ) times the standard deviation. Thus, the RiskMetrics at a 95% confidence level will perform well (S. Pafka and I. Kondor, 2001).

However, our study shows that for higher confidence levels (e.g. 99%) the effect of fat tails becomes much stronger, and therefore VaR is underestimated (i.e. too many returns exceed the VaR predictions) if one assumes normal distribution.

RiskMetrics has its best performance on the Toyota series. Based on the descriptive analysis in section 5, we found that the return series for Toyota exhibits characteristics most similar to a normally distributed series. Hence, the model's assumption of normality is reflected in the adequate performance score.

The RiskMetrics approach has its worst performance on the Apple stock series, and this may be due to the fact that the distribution exhibits significant leptokurtosis compared to the other series. Consequently, the model cannot capture the presence of fat tails due to the model's assumptions of normality.

### 8.3 GARCH(1,1)

The model building approach, like GARCH(1,1) captures the time-varying volatility, but the disadvantage is that the model assumes normal distributed returns. The GARCH(1,1) model has been giving adequate results, except on the Apple stock series. It seems that it fails to capture more extreme price movements, which is a highlighted feature in the Apple series (see Data and Descriptive Analysis). Not surprising, due to the fact that the approach assumes normality. Consequently, this is a common disadvantage to a standard GARCH model. The GARCH(1,1) model is not able to capture the asymmetries of the volatility with respect to the sign, since we are squaring the lagged error term (i.e. we are losing its sign). In other words, the standard GARCH model fails to capture leverage effects, and it seems that this defect has led to a poor performance for the Apple series. The threshold GARCH (TGARCH), by Rabemananjara and Zakoian (1993), may be a consideration for future volatility models, as it can capture the asymmetry in the volatility.

Not surprisingly, the GARCH(1,1) performs best on the Toyota stock series, and is in fact on par with the RHAR-QREG model. As explained in the comments of RiskMetrics, this is due to the fact that the Toyota return series exhibit properties that are most similar to a normal distribution.

Since the Apple series exhibit the characteristics least compatible with the normal distribution, we also suspect that the assumption of normality has caused the maximum likelihood estimation (MLE) to generate values  $\omega$ ,  $\alpha$  and  $\beta$  that are off their optimum. It seems that the model captures the volatility clustering, but fails to capture the distribution of the series. Hence, this may have caused the GARCH(1,1) to estimate poorly on both coverage tests relative to the better models. A future consideration could be to work on adjusting the MLE function in R to better fit the asset returns in order to increase the performance of the GARCH(1,1) model.

## 8.4 HAR-QREG

HAR-QREG delivers good results with a 100% score in two of four data series. According to Haugom et. al (2014), the advantage of the HAR-QREG is that it can capture all shapes of the conditional return distribution and isolate the effect from short-, medium-, and long-term volatility. To exemplify, if the conditional day-ahead distribution of a return series is skewed to either side, this will be captured in either the constant term or the slope-coefficients for “similar” quantiles in the two tails. Additionally, quantile regression estimates are more robust against outliers (i.e. fat tails) in the response measurements (Koenker and Roger, 2005). The model has thus no assumptions regarding the distribution.

The model exhibits the important feature of capturing the heterogeneous effects occurring from different traders’ investments horizons, for both the lower and upper tails, i.e. long and short positions. The heterogeneous effects will be revealed through the parameters in the model, i.e the short-, medium-, and long-term volatility components.

## 8.5 RHAR-QREG

The RHAR-QREG model delivers good results, as it captures many of the features mentioned for the HAR-QREG model. Still, it is outperformed by the HAR-QREG. The main reason seems to be that the violations in the RHAR-QREG's hit function comes more in clusters compared to HAR-QREG's hit function (i.e. the HAR-QREG performs better in the conditional coverage test). In other words, changing market risk fail to be fully incorporated into RHAR-QREG's VaR measures, even though RHAR-QREG has the correct average number of violations. HAR-QREG incorporates yesterday's return in the short-, medium and long-term volatility, which may be a critical factor to capture changing market risks. It appears that HAR-QREG captures more of the information of the historical volatility, since it captures all the information within a 24 hours time span, i.e. from the closing price at day  $t-1$  to the closing price at day  $t$ , whereas the RHAR-QREG only capture the information during each trading day (se Figure 11). Our hypothesis of capturing more of the historical volatility using intraday return on the short-, and medium-term volatility component was rejected.

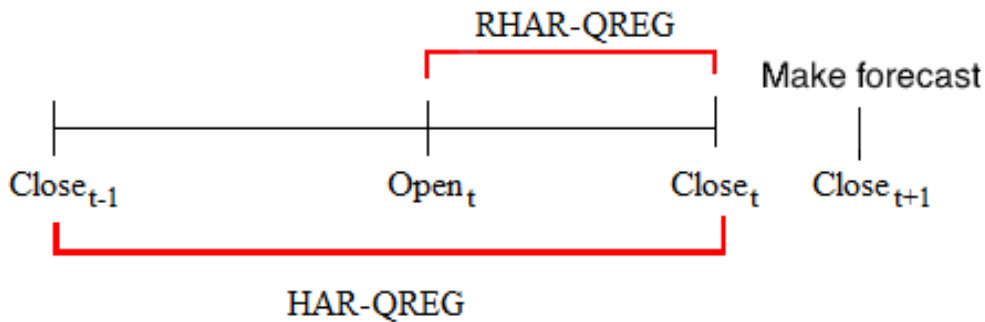


Figure 11: Shows the differences between RHAR-QREG and HAR-QREG in terms of capturing the historical volatility/information.

## 9 Conclusion

In this master thesis, we examine VaR models relevant for stock traders who have long and short trading positions in stock markets. Our time horizon is short-term as we focused on the one-day-ahead market risk. In an out-of-sample study, covering car industry (Toyota), technology (Apple), clothing industry (Nike), and the index (S&P500), we assessed the performance of the Historical Simulation, RiskMetrics, GARCH(1,1), HAR-QREG and RHAR-QREG. While the HAR-QREG performs best in all cases, the RHAR-QREG model nevertheless delivered good results. An important feature of the quantile-based models is that it simplifies the optimization compared to other models. Hence, managers and regulators can easily implement RHAR-QREG (and HAR-QREG) in a spreadsheet. Another important feature is that HAR-QREG and RHAR-QREG has no assumption regarding the distribution of the return series.

In general, we see that all the models examined in this study, perform better at the upper tail of the return distribution. We suspect that there are several factors that contribute to these findings: We see that all the close-close return series exhibit negative skewness. This means that the peak of the distribution is tilted to the right relative to the normal distribution (see Figure 1). Combined with a leptokurtic distribution, this means that there are more positive than negative returns. Hence, we have more observations on the upper tail (i.e. between zero and maximum return) of the return distribution, which makes it easier for the models to forecast the one-day-ahead VaR. The leverage effects also causes the volatility to be more stable when the market is bull (i.e. positive returns), which may be an important factor.

Our findings also show that RiskMetrics and GARCH(1,1) give good results, when the return series exhibits normally distributed patterns. Not surprising, since this is the key assumption in the two models. According to previous studies, they argue that the most important return feature to account for when calculating VaR, is volatility clustering. In our opinion, and according to our study, it seems that the most important return characteristic is the distribution of the returns and how well the models capture it.

## 10 Further Research

Several extensions to our study can be considered. It would be interesting to conduct a study on other assets (i.e. other stocks and/or commodities) to see if the conclusions drawn would be the same. Other assets can have characteristics that fit our model better leading to better results. Additionally, it would have been interesting to examine suitable window sizes for the calculation of both Historical Simulation and the RHAR-QREG (and HAR-QREG) model. The window size is the only factor that influences the VaR prediction on the Historical Simulation approach, but also for the RHAR-QREG and HAR-QREG.

Instead of using only the open-close volatility estimator in the RHAR-QREG, it could be an alternative to use the High-Low-Open-Close volatility estimator introduced by Garman and Klass (1980). This volatility estimator, which is built upon the Parkinson (1980) estimator, incorporates both the high and low and opening and closing historical prices when estimating variance and hence volatility.

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## 11 Appendix

Figure 12: *R-Coding for Historical Simulation with rolling window.*

```
HS<-function(x, pval){
  #x represents the return-series
  #pval the quantile in question, e.g 0,05 for the 5% quantile
  N<-length(x)
  y=rep(NA,N-1000)
  for(i in 1:length(y))
  {
    y[i]<-quantile(x[i:(i+999)], pval)
  }
  test<-ifelse((y>x[1001:length(x)]),1,0)
  return(mean(test))
  #the violation ratio is returned
}
```

Figure 13: *R-Coding for RiskMetrics.*

```
RM<-function(x) #x is the return-series
{
  T<-length(x)
  mu=mean(x)
  l d<-0.94
  sigRM<-rep(NA, (T))
  sigRM[2]<-(1-l d)*(x[1]^2)+l d*(x[2] )^2
  for(i in 3:T)
  {
    sigRM[i]<-l d*sigRM[i-1]+(1-l d)*(x[i-1] )^2
  }
  B<-na.omit(sigRM)
}
```

Figure 14: *R-Coding for GARCH(1,1) with rolling window.*

```

garchv = function(x, start = 1000, roll = 5, n.ahead = 1, spec = NULL, cluster = NULL)
{
  # x : is a series of closing prices
  # start : is the number of periods which are used for the first rolling
  # estimation window
  # roll : the number of periods to roll the 1-ahead forecast before
  # re-estimating the model parameters
  # n.ahead : the forecast horizon
  # spec : the univariate GARCH model specification (UGARCHspec object)
  # cluster : a pre-created cluster object from the parallel package
  require(rugarch)
  if (!is.xts(x))
    stop('\nx must be an xts object')
  # create continuous return series
  r = na.omit(diff(log(x), 1))
  n = nrow(r)
  if (n <= start)
    stop('\nstart cannot be greater than length of x')
  # the ending points of the estimation window
  s = seq(start + roll, n, by = roll)
  m = length(s)
  # the rolling forecast length
  out.sample = rep(roll, m)
  # adjustment to include all the datapoints from the end
  if (s[m] < n) {
    s = c(s, n)
    m = length(s)
    out.sample = c(out.sample, s[m] - s[m - 1])
  }
  # the default specification if none is provided is set to an eGARCH-std
  # model
  if (is.null(spec))
    spec = ugarchspec(variance.model = list(model = 'fGARCH', garchOrder=c(1,1), submodel='GARCH'), distribution = 'std')
  if (is.null(cluster)) {
    clusterEvalQ(cluster, library(rugarch))
    clusterEvalQ(cluster, library(xts))
    clusterExport(cluster, varlist = c('r', 's', 'roll', 'spec', 'out.sample'),
                  envir = environment())
    tmp = clusterApply(cl = cluster, 1:m, fun = function(i) {
      fit = ugarchfit(spec, r[1:s[i], ], out.sample = out.sample[i], solver = 'hybrid')
      f = ugarchforecast(fit, n.ahead = 1, n.roll = out.sample[i] - 1)
      y = sigma(f)
      # use xts for indexing the forecasts
      y = xts(as.numeric(y), tail(time(r[1:s[i], ]), out.sample[i]))
      return(y)
    })
  } else {
    tmp = lapply(1:m, FUN = function(i) {
      fit = ugarchfit(spec, r[1:s[i], ], out.sample = out.sample[i], solver = 'hybrid')
      f = ugarchforecast(fit, n.ahead = 1, n.roll = out.sample[i] - 1)
      y = sigma(f)
      # use xts for indexing the forecasts
      y = xts(as.numeric(y), tail(time(r[1:s[i], ]), out.sample[i]))
      return(y)
    })
  }
  # re-assemble the forecasts
  forc = tmp[[1]]
  for (i in 2:m) {
    forc = rbind(forc, tmp[[i]])
  }
  colnames(forc) = 'v'
  return(forc)
}

```

Figure 15: *R-Coding for HAR-QREG. Supplied by Erik Haugom and Steinar Veka.*

```

har_qreq <- function(x, pval){
  n <- length(x)
  nn <- n - 19
  y <- x[-(1:20)]
  xx <- abs(x)
  x1 <- xx[-c(1:19)]
  x2 <- mavg(xx[-(1:15)], 5)
  x3 <- mavg(xx, 20)

  fit <- rq(y~x1[-nn]+x2[-nn]+x3[-nn], tau=pval)

  sum(coefficients(fit)*c(1, x1[nn], x2[nn], x3[nn]))
}

```

Figure 16: *R-Coding for RHAR-QREG.*

```

rhar_qreg <- function(x,z, pval){
  n <- length(x)
  T <- z[2:length(z)]
  nn <- n - 19
  y <- x[-(1:20)]
  xx <- abs(T)
  x1 <- xx[-c(1:19)]
  x2 <- mavg(xx[-(1:15)], 5)
  x3 <- mavg(xx, 20)

  fit <- rq(y~x1[-nn]+x2[-nn]+x3[-nn], tau=pval)

  sum(coefficients(fit)*c(1, x1[nn], x2[nn], x3[nn]))
}

```

Figure 17: *R-Coding for the Unconditional (Kupiec (1995) and Conditional (Christoffersen (1998)) coverage tests supplied by Steinar Veka.*

```

Christo <- function(prob, y, var){
  n <- length(y)
  I <- (y<var)
  n1 <- sum(I)
  n0 <- n-n1
  n00 <- 0; n01 <- 0; n10 <- 0; n11 <- 0;

  for(i in 2:length(y)){
    if(I[i-1]==0 & I[i]==0)n00 <- n00+1
    else if(I[i-1]==0 & I[i]==1)n01 <- n01+1
    else if(I[i-1]==1 & I[i]==0)n10 <- n10+1
    else n11 <- n11+1
  }

  pi01 <- ifelse((n00+n01)>0, n01/(n00+n01), 0)
  pi11 <- ifelse((n10+n11)>0, n11/(n10+n11), 0)
  pi2 <- (n01+n11)/(n00+n10+n01+n11)

  t.uncond <- -2*( n0*log(1-prob) + n1*log(prob) - ( n0*log(1-(n1/n)) +n1*log(n1/n) ) )

  t.indep <- -2*log(
    ( (1-pi2)^(n00+n10)/2) / (1-pi01)^n00 ) * ( (1-pi2)^(n00+n10)/2) / pi01^n01 ) *
    ( ( pi2^(n01+n11)/2) / (1-pi11)^n10 ) * ( pi2^(n01+n11)/2) / pi11^n11 ) )
  ) #Factorized for avoiding to low values for OS

  if(is.na(t.indep)) #If to low values for OS
  t.indep <- -2*( (n00+n10)*log(1-pi2)+(n01+n11)*log(pi2) - ( n00*log(1-pi01) + n01*log(pi01) +
    n10*log(1-pi11) + n11*log(pi11) ) )
  #Does not handle n00=0 or n11=0 and is therefore not the default formula for t.indep

  t.cond <- t.uncond+t.indep

  list(t.uncond = t.uncond, t.indep=t.indep, t.cond=t.cond,
    p.uncond = pchisq(t.uncond, 1, lower.tail=FALSE),
    p.indep = pchisq(t.indep, 1, lower.tail=FALSE),
    p.cond = pchisq(t.cond, 2, lower.tail=FALSE) )
}

```

Figure 18: R-Coding for example VaR estimation. Here Riskmetrics.

```

var.RM<-function(x,pval){
  #x=return-series, pval=significance level, e.g for 5% 1,64(one-sided)
  RMx<-RM(x)
  sdx<-sqrt(RMx)
  varlow<-pval*sdx
  varup<-pval*sdx
  N<-length(x)
  x.1<-x[2:N]
  low<-ifelse((varlow>x.1),1,0)
  up<-ifelse((varup<x.1),1,0)
  VaRnedre=mean(low)
  VaRovre=mean(up)
  return(data.frame(VaRnedre, VaRovre))
}

```

Figure 19: VaR illustration, (a) is the significance interval, (b) is the critical value ( $\alpha$ ) e.g. 5% and the confidence level  $\rho$  e.g. 95%, (c) is the confidence interval.

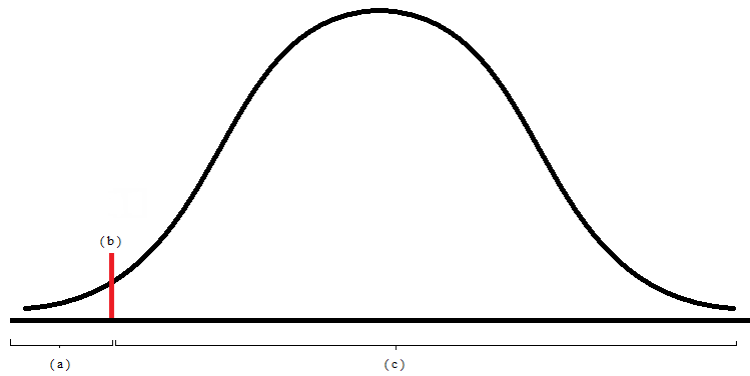


Figure 20: In- and Out-Of-Sample illustration

