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Test of the Efficient Market Hypothesis by utilizing statistical arbitrage

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Preface

This thesis concludes my Master of Science in Financial Economics at NTNU. I would like to thank my supervisor, Snorre Lindset, for constructive criticism and helpful advice. I would also like to thank Marius Hovde at Sparebank1 SMN Markets for supplying data and being very helpful with every enquiry.

The thesis consists of two parts. The first part gives a review of the Efficient Market Hypothesis and Statistical Arbitrage. The second part tests the theory empirically.

Abstract

The purpose of this paper is to test the efficient market hypothesis. The thesis includes an extensive review of the literature on the Efficient Market Hypothesis and tests if it is possible to earn economic profit by trading on an information set known to all market participants. The trading strategy is based on statistical arbitrage pairs. Pairs trading is a concept based on a co-integrating relationship. Co-integration is the long-term stationary relationship between two asset prices. The pairs in this thesis are untraditionally composed of indices. The test for co-integration uses the price of the ETF for S&P500 and the prices of six other indices from the European continent. I find that there is a stationary long-term relationship between the prices of the respective ETF's of S&P500 and FTSE100 before adjusting for currency. The cointegration test shows a stationary relationship between the two prices and that the spread has mean-reverting properties. A trading strategy based on an out-of-sample period of 85 days does not yield positive profits after adjusting for transaction costs. With USD as the base currency, none of the seven variables co-integrates. In the absence of a co-integrating vector, there is no basis for trading the indices, and the EMH cannot be rejected.

Contents

1. INTRODUCTION

In this thesis, I will test the following hypothesis: Is it possible to make economic profits by trading indices based on an information set θ, which is available to all participants. The weapon of choice is cointegration. In previous research, the test of the efficient market hypothesis has been subject to the joint hypothesis problem. The strength of the cointegration test diminishes this problem. If there is cointegration between two integrated time series, there is a long run relationship between them that implies that one series can be used to explain the other (Granger, 1986). Cointegration tests for a long-term stationary relationship in price series. I investigate whether or not there is a statistical long-term equilibrium between the price of S&P500 and the prices of the European indices. Cointegration and Error Correction Models are tied together through the Granger Representation Theorem (Engle and Granger, 1987). Using this theorem, I model the short-term dynamics and employ a trading strategy on the stationary spread. The out-of-sample test does not yield positive results after adjusting for transaction costs. I find that there is a long-term statistical equilibrium in the prices before correcting for the exchange rate. Testing with a base currency reveals that there are no co-integrating vectors in the system. Both these results are in line with the EMH, as there are no arbitrage opportunities after adjusting for transaction costs. Based on the analysis in this thesis I cannot reject the efficient market hypothesis.

2. DEFINITIONS AND PREVIOUS LITERATURE

2.1 Definitions

In the following section, the efficient market hypothesis and statistical arbitrage are defined.

2.1.1 Definitions of the efficient market hypotheses

Fama (1965) defines three different forms of the EMH. His weak form of the efficient market hypothesis states that all information is fully reflected in previous prices and claims that prices fully reflect the information implicit in the sequence of past prices. The semi-strong form of the hypothesis asserts that prices reflect all relevant information that is publicly available. The strong form of the hypothesis states that all information that is known by any participant is reflected in market prices.

Fama (1970) lists three terms that is sufficient to empower the EMH as a relevant hypothesis. He clearly states that these are not absolute, and that an approximation to these conditions will still yield market efficiency. The first term is the absence of transaction costs. The closest equivalent to no transaction costs are trading liquid stocks or indices. Frequent trading of stocks or indices in an efficient market place makes the bid-ask spreads lower (Alexander, 2008). The second term is perfect and costless information flow. News and financial reports must flow quickly and freely to all market participants. In the age of the internet, the term is almost satisfied. But there is still some investors trading on insider information, at least in regards of equity options (Bradley et al., 2010, Bradley et al., 2012). Insider information is by definition not available to everyone, and thus an example of an imperfect information flow. The third and final term is the agreement about the price implications of information. This idea constitutes that every recipient interprets and understands all the available information in the same way.

In 1978, Michael Jensen wrote "I believe there is no other proposition in economics which has more solid empirical evidence supporting it than the EMH". His definition of the EMH is: "A market is efficient with respect to information set θ_t if it is impossible to make economic profits by trading on the basis of information set θ_t." (Jensen, 1978). Jensen's definition is quite similar to the definition of Burton Malkiel in 1992: "A capital market is said to be efficient if it fully and correctly reflects all relevant information in determining security prices. Formally, the market is said to be efficient with respect to some information set, θ_t, if security prices would be unaffected by revealing that information to all participants.

Moreover, efficiency with respect to an information set, θ_t, implies that it is impossible to make economic profits by trading on the basis of θ_t." (Malkiel, 1992).

2.1.2 Definition of Statistical Arbitrage.

In this thesis, statistical arbitrage is defined as the low risk portfolio composed of two co-integrated asset prices with weights derived from the co-integrating vector. If the tests show a co-integrating relationship, the prices of the two assets will have a long-term stationary relationship. With weights based on the co-integrating vector, the price of the portfolio will be stationary.

2.2 History and Literature review

2.2.1 EMH

The first notable market inefficiency was in Holland during the 1630s. At the time, the tulip was a sign of wealth. So extremely popular that at its price peak one could trade an entire estate for a single tulip. Trading a tulip at the end of the decade would not return more than an onion (Posthumus, 1929).

In 1828 Robert Brown, a Scottish botanist, noticed that grains of pollen in water had a rapid oscillatory random motion when viewed under a microscope. This movement is what later has been called a Brownian motion (Brown, 1828). In 1863 a French stockbroker, Jules Regnault, observed that the price deviation of holding a stock is directly proportional to the square root of time (Regnault, 1863). The first publication regarding a random walk, was made by a British physicist, Lord Rayleigh, in a publication about sound vibrations (Rayleigh, 1880). The first clear concept published on random walk and Brownian motion, was published by a British logician and philosopher in Venn (1888). In "The Stock Markets of London, Paris and New York" published in 1889, George Gibson stated that the value of a commonly known stock was based on the common judgement of available information (Gibson, 1888). A French mathematician, Louis Bachelier, published his PhD thesis, "Théorie de la Spéculation" (1900). In his thesis, he described the statistics of Brownian motion and deduced that the expected value of a speculator is zero. Around the same time, Karl Pearson (1905) introduced the term random walk. Unaware of "Théorie de la Spéculation" , Albert Einstein (1905) developed the equations for Brownian motion in 1905. The first publication that linked finance and Bachelier's thesis was De Montessus' book on probability and its applications in De Montessus (1908). Langevin (1908) developed the stochastic differential equation of Brownian motion. The first publication on leptokurtic distributions of returns was Mitchell (1915). Olivier (1926) delivered unquestionable results, showing that returns are leptokurtic.

Alfred Cowles (founder of Econometric Society, and its journal Econometrica) analysed the performance of investment professionals and concluded that they cannot forecast (Cowles 3rd, 1933). Keynes (1936) famously compared the stock market with a beauty contest, where the participants of the financial markets base their decision on the perception of the other participants' perceptions. Keynes also claimed that most investors' decisions are a result of 'animal spirits'. He previously, in 1923, pointed out that investors in the financial markets are not rewarded for knowing better than the market, but simply for taking on risk (Keynes, 1923).

Slutzky (1937) showed that sums of independent random variables might be the source of cyclic processes. Cowles and Jones published the only paper that found significant market inefficiencies before 1960. They found significant evidence of serial correlation in average time series indices of stock prices (Cowles 3rd and Jones, 1937). In 1944, in a continuation of his forecast publication in 1933, Cowles again reported that investment professionals do not beat the market. Working (1948) showed that in an ideal futures market no forecaster could predict the price.

Friedman (1953) illustrated that due to arbitrage, the EMH holds when the trading strategies of investors are correlated. Kendall (1953) analysed 22 price-series at weekly intervals and concluded that they were random; he also found the time dependence of the empirical variance (non-stationarity). Roberts (1959) demonstrated that the times series of stock prices resembles a random walk. Osborne (1959) was the first to note that the logarithm of common-stock prices follows a Brownian motion. He also found the square root of time rule. This rule states that if the volatility is based on the logarithm of the stochastic returns, where the fluctuations are independent of each other (IID), the volatility can be rescaled by the square root of time.

Larson (1960) found that the central 80% of the distribution resembled the normal distribution, but that the tails are fat. Cowles (1960) wrote "A revision of previous conclusions regarding stock price behaviour". He revisited the results of his 1937 paper, and corrected for the averaging, which had been criticised by Working (1960). In his 1960 paper, Working showed that the use of averages could induce autocorrelations not present in the original time series. The revised Cowles paper of 1960 corrected for averaging and found that there were some temporal dependence.

Independently of Working (1960), Sydney Alexander (1961) published "Price movements in speculative markets: Trends or random walks". Alexander demonstrated that it is possible to induce spurious autocorrelations by averaging. In the same publication, he found leptokurtic distributions, concluded that the random walk model is best fit for the data and conducted one of the first tests for non-linear dependence. Muth (1961) introduced the rational expectations hypothesis in economics. In short, the theory states that the people in the economy make their decisions based on a rational outlook, available information and experiences.

Mandelbrot (1962) defended the statistical law of Pareto, postulating that the law applies to distributions of returns. The same year a paper rejecting the stock market as a random walk, was published by Paul H. Cootner (1962). Osborne (1962) investigated the periodic structure of the Brownian motion in stock prices, and found that the deviations from a simple random walk show a pattern of trade bursts among stocks. Moore (1962) found slightly positive serial correlation for index-prices, and insignificant negative serial correlations for individual stock prices.

Berger and Mandelbrot (1963) tried to connect error clustering in telephone circuits to the financial markets, and argues that if their results were to be applied to the stock market, it may give rise to the Pareto-Levy law of distributions claimed by Mandelbrot in his 1962 paper. Granger and Morgenstern (1963) found that short-run movements are in line with the simple random walk hypothesis, but that long-run movements are not.

Alexander (1964) corrected his 1961 paper, and found that the S&P industrials do not follow a random walk. Steiger (1964) concluded that stock prices do not follow a random walk. The same year, Godfrey, Granger and Morgenstern published "The random walk hypothesis of stock market behaviour"(Godfrey et al., 1964).

Fama (1965) defined an efficient market in the publications "The behaviour of stock market prices" and "Random Walks in stock prices". The analysis of stock market prices concluded that they follow a random walk. Samuelson (1965) used a martingale instead of the random walk in his publication "Proof that properly anticipated prices fluctuate randomly".

Roberts (1967) introduced the concept of efficient market hypothesis and made the distinction between weak and strong form tests, which Fama (1970) elaborated on in his first of three review papers: "Efficient capital markets: A review of theory and empirical work". Fama famously defined an efficient market as a market where prices always fully reflect available information. He also discussed the "joint hypothesis problem". The review was divided into three parts. The first part included weak-form tests, id est. how well past prices or returns predict the future returns. The second part discussed semi-strong-form tests, id est. how quickly the prices adjust to public information announcements. The final part asked whether investors have private information that is not fully reflected in market prices, id est. strong-form tests. These tests assumed that investors are rational. The idea is that if there are irrational investors, their trades will be random and rational arbitrageurs will eliminate their trades so that their effects on prices are zero.

LeRoy (1973) demonstrated that in the presence of risk aversion, there is no theoretical justification of a martingale. Burton G. Malkiel is the man who has popularized the random walk hypothesis through his classic “A Random Walk Down Wall Street” (Malkiel, 1973). The book is now in its 10th edition.

Sanford Grossman (1976) described a model that separated the non-informed from informed traders. The model showed that efficient price systems aggregate information perfectly, leaving the initial price to reflect all available information, and thus reducing the incentive for collecting information. Beja and Hakansson (1977) illustrated that the models and publications of economists are nothing more than a framework for simpler understanding of the financial markets, and have less empirical focus. The authors concluded that the efficiency of a real market is impossible as the market is in constant search of equilibrium.

Ball (1978) found that there is excess returns after public announcements of earnings. Jensen (1978) defined market efficiency contingent on the information set θ_t . The market is said to be efficient if it is impossible to make a profit by trading based on the information set. Robert E. Lucas Jr (1978) built a theoretical model of rational agents that replicated the conclusions of LeRoy (1973), and demonstrated that the martingale property do not necessarily hold under risk aversion.

Sanford J. Grossman and Joseph E. Stiglitz (1980) pointed out that there must be some degree of disequilibrium in the market prices. Given perfect market efficiency, those who gather information (perform analysis) will not be compensated.

LeRoy and Porter (1981) investigated the implications for asset price dispersion on conventional valuation models. They used a linear vector autoregressive model to create variance bounds. They concluded that the stock market prices have excess volatility. Robert Schiller (1981) performed a test on excess volatility using another approach. The results showed that stock prices move too much in relation to the changes in dividends. Richard Roll (1984) looked at orange juice futures for the US, and the effect of the weather. The results gave clear indications of excess volatility.

Bondt and Thaler (1985) were the first to include elements of psychology in their studies of stock prices. Research in psychology had shown that people overreact to dramatic or unexpected news. The authors investigated whether these results were applicable to the stock market. They formulated an “overreaction hypothesis” based on these psychological elements. The empirical evidence, based on the Center for Research in Security Prices’ (CRSP) monthly return data, were consistent with the overreaction hypothesis. The results contradicted Harry Roberts’ definition of weak form market efficiency. This paper marked the start of behavioural finance.

Fischer Black (1986) introduced the concept of ‘noise traders’, and accredited the functionality and liquidity of financial markets to these traders; who trade on anything other than fundamental information. Lawrence H. Summers (1986) examined the power of statistical tests used to evaluate the efficiency of speculative markets. The research showed that these statistical tests of market efficiency had very low power in discriminating against plausible forms of inefficiency. Fama and French (1986) demonstrated that due to significant negative serial correlations, portfolio returns are predictable in the 3-5 year horizon.

Engle and Granger (1987) published their paper on co-integration and error correction representation, signalizing the start of cointegration.

On Black Monday, October 19, 1987, stock markets around the world crashed. The crash began in Hong Kong, spread west to Europe, then hit the United States causing the largest daily percentage loss in the history of the Dow Jones Industrial Average, -22.61%. The crash started a landslide of academic papers regarding market efficiency. Lo and MacKinlay (1988) rejected the random walk hypothesis for weekly stock market returns by comparing variance estimators derived from data sampled at different frequencies. One weakness of the method in this paper is that the rejection is due to the behaviour of small, illiquid stocks. Poterba and Summers (1988) showed that stock returns have positive autocorrelation in the short run and negative autocorrelation in the long run. Jennifer Conrad and Gautam Kaul (1988) characterized the stochastic behaviour of expected returns on common stock. They found that weekly expected returns are well characterized by a stationary first order AR-process. David Cutler teamed up with Poterba and Summers and published a paper that demonstrated how news do not adequately explain market movements (Cutler et al., 1989). In 1989, Robert Schiller published “Stock Market Volatility”. The book described the human psychology used as the basis for behavioural finance (Shiller, 1989). LeRoy (1989) stated that the previous research on efficient capital markets had taken to lightly on the transition between the intuitive idea of market efficiency and the martingale.

Laffont and Maskin (1990) discussed the efficient market hypothesis in light of insider trading or other forms of imperfect competition, and found that in the case of imperfect competition the EMH may not hold. Lehmann (1990) found reversals in weekly security returns, or “overreactions”, that contradicted the efficient market hypothesis. “The evidence suggests that the “winners” and “losers” one week experience sizeable return reversals the next week in a way that reflects apparent arbitrage profits which persist after corrections for bid-ask spreads and plausible transactions costs”. Jegadeesh (1990) found significant negative first-order serial correlation in monthly stock returns and positive serial correlation for longer lags, the twelve-month serial correlation were particularly strong using a data sample from 1934 to 1987. Kim, Nelson and Startz (1991) compared the stock returns data before and after World War II and concluded that mean reversion is a pre-war phenomenon. They also found evidence that suggested a fundamental change in the stock return process, and accredited the change to the uncertainty of the 1930s and 40s. Fama (1991) published Efficient Markets: II, number two of his three review papers. The paper gave a review of relevant literature regarding efficient markets. He wrote, “Since there are surely positive information and trading costs, the extreme version of the market efficiency hypothesis is surely false”. Instead of weak-form tests (forecasts based on previous returns), the first part now included more general tests for return predictability, with variables like dividend yields and interest rates. The paper also included a section that addresses the joint hypothesis problem.

Malkiel (1992) gave his definition of EMH in ‘Efficient market hypothesis’ published in the New Palgrave Dictionary of Money and Finance. Fama and French (1992) continued the empirical work of Sharpe (1964), Lintner (1965), Mossin (1966) and Black (1972). They demonstrated that market beta, size, book-to-market and earnings-price ratios help explain returns.

Jegadeesh and Titman (1993) published evidence of short term momentum, by enabling trading strategies that bought past winners and sold past losers which realized abnormal returns. In 1995, Robert Haugen published the book The New Finance: The Case Against Efficient Markets. He discussed the overreaction of the market, and how the overreaction may lead to long-term reversals (Haugen, 1995). Chan et al. (1996) examined whether the predictability of future returns from past returns were due to the markets under-reaction to information, in particular to past news of earnings. They found that an earnings surprise caused a drift in the stock price. Market risk, size, and book-to-market effects do not explain the drifts. The results showed that the market responded gradually to new information.

In his third of three reviews, Fama (1998) discussed the EMH against the long term anomalies, and found that the anomalies are chance results, as overreaction to information were about as common as underreacting, and post-event continuation of pre-event abnormal returns were about as frequent as post-event reversal. The publication also directly attacked the methodology of the papers that demonstrated anomalies, and showed that most long-term return anomalies tend to disappear with reasonable changes in technique.

Shleifer (2000) published “Inefficient Markets: An Introduction to Behavioral Finance”. The book discussed the empirical- and theoretical challenges of the EMH, id est. perfect arbitrage and rational investors, it also provided an excellent review of the empirical studies contradicting the EMH. Shiller (2000) published the first edition of Irrational Exuberance. The paper showed that the movement of company earnings or dividends could not sufficiently explain market movements. Based on the findings, the author suggested that the stock prices contains an element of psychology.

Malkiel (2003) defended his book from 1973, and the EMH, by reviewing the attacks of the last decades. Schwert (2003) showed that practitioners implement anomaly research, and thus make the market more efficient.

Timmermann and Granger (2004) discussed the EMH from the perspective of a forecasting pattern in “real time”. They described the constant search for profitable trading strategies as a self-destructing game. The authors suggested that this self-destructing game gives rise to the non-stationarity one still finds in financial markets.

Malkiel (2005) stated that if the market returns are as predictable as the critics of EMH have claimed, then professional investment fund should have outperformed a passive index fund. The paper showed that investment funds did not outperform their index benchmarks.

Wilson and Marashdeh (2007) illustrated that due to co-integration, the markets must be efficient in long-run equilibrium because no arbitrage opportunities exist. They also showed that due to the error correction disequilibrium there were arbitrage opportunities in the short run. The elimination of these arbitrage opportunities means that stock market inefficiency in the short run ensures stock market efficiency in the long-run. Yen and Lee (2008) presented a survey article that gave a chronological review of empirical findings and conclude that the EMH is here to stay.

2.2.2 Statistical Arbitrage

Pairs trading or statistical arbitrage, is a quantitative method of trading used at Wall Street since the 1980's. The concept is to trade divergence between two assets that share common return factors. The trader opens long and short positions simultaneously when the asset prices diverge abnormally, and then close the position when the prices converge (Vidyamurthy, 2004). Hogan et al. (2004) showed that statistical arbitrage avoids the joint hypothesis problem of conventional market efficiency test, mainly because it is not dependent on any equilibrium model. The paper tested momentum and value strategies, adjusted for all relevant transaction costs, and found that these strategies generate a profit.

There are three methods used as statistical arbitrage: The cointegration approach, the distance method and the stochastic method. See Do et al. (2006) for further explanation. In this thesis, I will focus on the cointegration approach. The cointegration analysis will be performed based on the framework provided in Engle and Granger (1987). The reason for the choice of cointegration is that it statistically determines the mean reverting nature of the spread between two assets. Elliott et al. (2005) proposed a mean-reverting Gaussian Markov chain model to model the spread.

Gatev et al. (2006) back-tested a pairs trading strategy with daily data from 1962–2002. Their strategy gave an annualized average return of 11%. These results withstood conservative transaction costs. In an effort to explain the high returns, the authors pointed to the presence of a common return factor. Perlin (2009) performed a similar test on the Brazilian financial market, testing whether daily, weekly or monthly data would yield the highest returns. The study showed that daily returns significantly outperformed the lower frequencies.

Lin et al. (2006) defined pairs trading as "a comparative-value form of statistical arbitrage designed to exploit temporary random departures from equilibrium pricing between two shares". In the 2006 paper, they explained that pairs trading is not riskless. They described how market events and poor statistical modelling might cause losses.

Pole (2007) wrote that the statistical arbitrage strategies from 2003-2005 did poorly, partly driven by the development of trading algorithms, but returned with splendid returns in 2006. The author suggested that the low volatility may have killed the statistical arbitrage profit, but as the algorithms got faster, the profits rose.

Engelberg et al. (2009) showed that the profitability of a pairs trading strategy is at its peak soon after divergence. The paper also showed that idiosyncratic liquidity shocks influenced profitability in a larger scale than idiosyncratic news. Profitability is also influenced by news that affect both parts of the pair, but only when the incorporation of information takes longer in one of the stocks.

Bowen et al. (2010) examined the characteristics of a high frequency pairs trade strategy using price data from FTSE100 from January to December 2007. They find that the profitability is highly sensitive to both transaction costs and the speed of execution.

Binh Do and Robert Faff published a reproduction of Gatev et al. (2006), testing a larger and new sample under the title "Does simple pairs trading still work?" (Do and Faff, 2010). Their results were in line with Pole, and supported the weak form EMH. They speculated that the reduced profit potential were because the trading strategies based on the "Law of one price" do not hold as the common return factors might have changed. The "Law of one" price states that two assets with same expected payoff should have the same price. Do and Faff (2012) examined the effect of transaction costs on pairs trading in the U.S. equity market, from 1963 to 2009. After controlling for commissions, market impact, and short selling fees, they found that pairs trading remains profitable up to 2002, and unprofitable after.

3. THEORY AND METHOD

3.1 Stationarity

Consider a time series (y_1, …, y_T), as the outcome of a draw from a joint probability distribution f(y_1, …, y_T). A strictly stationary process has a distribution independent of time, so that f(y_1, …, y_{t+k})= f(y_{t+m}, …, y_{t+k+m}).

The restriction implies that all moments of the distribution are constant over time. Weak stationarity is less strict, and demands that the first and second order moments of the distribution are constant. The second order moment also includes the auto covariance, so that the covariance between the lags must be constant.

$$\text{cov}(y_t, y_{t+k}) = E[(y_t - \mu_y)(y_{t+k} - \mu_y)] = E[(y_{t+m} - \mu_y)(y_{t+k+m} - \mu_y)]$$

If the restrictions are satisfied, the time series are integrated of order zero, I(0). In the absence of constant second order moments, the series are non-stationary. Without a constant mean or standard deviation, the series will not have a stable equilibrium. Non-stationary series can be made stationary by taking the difference δ times. The series are called integrated of order δ. Non-stationary time series could contain a common time trend, so that one series regressed on the other would seem to be explanatory due to the common trend. If the trend is not included in the regression, this may impose that the covariance between the explanatory variable and the error term is non-zero. This omitted trend may induce invalid answers from the standard hypothesis tests. See appendix on OLS for error term assumptions.

Consider the following regression:

$$y_t = \mu + \phi y_{t-1} + u_t$$

If 0 <ϕ < 1, the process described by the equation is an autoregressive process. If ϕ = 1 the process is a random walk. If ϕ=0 the process is white noise, id est. independent and identically distributed (IID) with zero mean and constant variance. For the EMH to hold, one would expect prices to be random walks and returns to be white noise. A time series with a stochastic trend can be modelled as a random walk with drift.

$$y_t = \gamma_1 + y_{t-1} + u_t \ , y_0 = \gamma_0 \ , u_t \sim IID(0, \sigma_u^2)$$

The equation is composed of a drift term, γ_1, and a random walk term, id est. the lagged value of the dependent variable (where the coefficient is one) and an error term that is white noise. If $y_0 = \gamma_0$ then recursive iterations give the equation,

$$y_t = \gamma_0 + \gamma_1 t + \sum_{i=1}^{t} u_i$$

where the error-term has a non-stationary variance $\sigma_u^2 t$. In comparison, a deterministic trend is on the form

$$y_t = \gamma_0 + \gamma_1 t + e_t \ , \ e_t \sim IID(0, \sigma_e^2)$$

The difference between a deterministic trend and a stochastic trend is the error term. Using the stochastic trend, one can re-write the expression so that the error term of the stochastic trend equation follows an AR-process,

$$y_t = \gamma_0 + \gamma_1 t + v_t$$

$$v_t = \delta v_{t-1} + \varepsilon_t$$

ε_t is a stationary process with zero mean. If $\delta = 1$, then the error term is a non-stationary random walk.

Subtracting δy_{t-1} from the re-written equation yields:

$$y_t = (1-\delta)\gamma_0 + (1-\delta)\gamma_1 t + \delta y_{t-1} + \varepsilon_t$$
$$y_t = \theta_0 + \theta_1 t + \delta y_{t-1} + \varepsilon_t$$

To test for non-stationarity one can either test $(\theta_1, \delta) = (0,1)$ by an F-test or $\delta = 1$ with a t-test. This test is what is known as a Dickey-Fuller test (Dickey and Fuller, 1979). In this thesis however, I use the augmented Dickey-Fuller (ADF) test. The reason for the use of ADF is that the ADF includes an extra term that captures any autocorrelation in the error term, so that ε_t is white noise. It does so by adding lagged values of the dependent variable to capture autocorrelation in the process (Said and Dickey, 1984). In addition, the equation below shows the ADF on difference form,

$$\Delta y_t = \theta_0 + \theta_1 t + (\delta - 1) y_{t-1} + \varepsilon_t + \sum_{j=1}^{k} \rho_j \Delta y_{t-j}$$

The H_0, H_0: (1- δ) = 0, is that the time series are non-stationary. The null hypothesis of standard interference tests are based on stationary time series. Because the H_0 of the ADF is based on a non-stationary time series, the assumptions regarding typical t-tests are not met. The instability of the mean or the variance of the non-stationary time series prevents the use of typical t-values when performing a t-test on H_0. The ADF-test employs simulated, stricter values. OxMetrics gives a 5% t-adf as -2.86 and a 1% t-adf as -3.44.

3.2 Cointegration

3.2.1 Engle-Granger method

The test for co-integration is based on Engle and Granger (1987). The test is a two-step process. Step one is an OLS regression of the two non-stationary variables with the hypothetic stationary relationship. Step two is performed on the residuals of the regression in step one. If the coefficient of the first regression is significant and the residuals are stationary, there is a co-integrating relationship between the two variables. Engle and Yoo (1987) show that the traditional critical values do not suffice when testing the stationarity of the residuals. The critical value for two variables and sample size bigger than 200 is 3.25 on the 5%-level and 3.75 for the 1% level (Engle and Yoo, 1987). Given two time series, x and y generated from the model

$$y_t - \beta x_t = u_{1t} \ , \ u_{1t} = u_{1t-1} + e_{1t} \ , \ e_{1t} \sim IID(0, \sigma_{e_1})$$
$$y_t - \alpha x_t = u_{2t} \ , \ u_{2t} = \rho u_{2t-1} + e_{2t} \ , \ |\rho| < 1 \ , \ e_{2t} \sim IID(0, \sigma_{e_2})$$

Imposing α ≠ β, the first equation is I(1) and the second is I(0). The second equation contains a linear stationary combination of the two series, both of which are I(1) (can be shown in reduced form). Engle and Granger (1987) prove that the alpha from the regression in the second equation is not only consistent, but super-consistent. See appendix OLS and super-consistency.

As the error term follows an autoregressive process it is possible to model the short term dynamics as

$$y_t - \alpha x_t = u_{2t} \vee u_{2t} = \rho u_{2t-1} + e_{2t} \ , \ |\rho| < 1$$

The difference of the residuals are expressed as

$$\Delta u_{2t} = (\rho - 1) u_{2t-1} + e_{2t} \ , \ |\rho| < 1 \ \vee \Delta u_{2t} = \Delta y_t - \alpha \Delta x_t$$

with

$$u_{2t-1} = (y - \alpha x)_{t-1}$$

Combining these equations gives the Equilibrium Correction Model (EqCM) or Error Correction Model (ECM),

$$\Delta y_t = \alpha \Delta x_t - (1-\rho)(y - \alpha x)_{t-1} + e_{2t}$$

in the EqCM/ECM all variables are I(0) and e_{2t} is white noise. The representation of a co-integrated relationship as an ECM is known as the Granger representation theorem (Engle and Granger, 1987).

There are two problems with Engle Granger. The first is when N>2, then the test will be influenced by the choice of dependent variable. Second, Engle Granger only test for one co-integrating vector. These problems can be avoided by using the cointegration test known as the Johansen method (Johansen, 1988).The Johansen method seeks the linear combination that is most stationary. The Engle-Granger two-step method seeks the stationary linear combination that has the minimum variance.

3.2.2 Johansen method

The Johansen method generalizes the argument from the Dickey-Fuller Unit root test to a system with n integrated variables. Consider a first order vector autoregressive process (VAR), written on matrix form:

$$\mathbf{X}_t = \mathbf{A} + \mathbf{B}\mathbf{X}_{t-1} + \varepsilon_t,$$

where

$$\mathbf{X} = \begin{pmatrix} X_{1t} \\ \vdots \\ X_{nt} \end{pmatrix}, \mathbf{A} = \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_n \end{pmatrix}, \mathbf{B} = \begin{pmatrix} \beta_{11} & \cdots & \beta_{1n} \\ \vdots & \ddots & \vdots \\ \beta_{n1} & \cdots & \beta_{nn} \end{pmatrix}$$

By subtracting $\mathbf{X}_{t-1}$ from both sides, I obtain the Dickey Fuller equivalent,

$$\Delta \mathbf{X}_t = \mathbf{A} + \Pi \mathbf{X}_{t-1} + \varepsilon_t,$$

where Π=**B-I**, and I is the (n x n) identity matrix. To remove any autocorrelation in the residuals, one adds an appropriate number of lags of the dependent variable,

$$\Delta \mathbf{X}_t = \mathbf{A} + \Pi \mathbf{X}_{t-1} + \Gamma_1 \Delta \mathbf{X}_{t-1} + \ldots + \Gamma_q \Delta \mathbf{X}_{t-q} + \varepsilon_t ,$$

if all the variables (X_1, … ,X_n) are integrated the difference on the left-hand side is stationary. The stationary left-hand side requires the right-hand side to represent a stationary process, id est. Π$\mathbf{X}_{t-1}$ must be stationary. This condition has no implications for the relationship between the variables (X_1, … ,X_n) if the rank of the Π matrix is zero. However, if the rank of the Π matrix is r, with r>0, then when $\mathbf{X}_{t-1}$ is stationary, there are r independent linear relations between the variables (X_1,..,X_n) id est. the variables are co-integrated. Hence, the test for cointegration is a test of the rank of the Π matrix, and the rank of the matrix is the number of co-integrating vectors (Alexander, 2008).

Johansen and Juselius (1990) suggests a trace test to test for non-zero eigenvalues. The H_0 is that r ≤ Rank and the H_1 is that r>Rank (Johansen and Juselius, 1990). The test static is given by:

$$T_r = -T \sum_{i=R+1}^{n} \ln(1-\lambda_i) \quad , \quad 1 > \lambda_1 > \ldots > \lambda_n > 0$$

Where T is the sample size, R is the hypothetical rank one tests for and λ are the eigenvalues of the matrix Π.

4. EMPIRICAL ANALYSIS

4.1 Variables and data

Marius Hovde, in Sparebank 1 SMN Markets, has supplied the data

4.1.1 Indices

The analysis in this thesis uses indices to minimize the return influences by the proven return anomalies in previous literature, so that the test of EMH does not included any single stock anomalies. The choice of indices are based on liquidity and geographic focus. Appendix Unit root tests of indices show the results from a unit root test performed on the indices. All the indices are I(1).

Table 1 – Overview of ETFs

The first column shows the ticker of the index/ETF and the second column shows the underlying index of the ETF. The third column shows the geographic region of the index. The last column denotes which currency the index is traded in.

Ticker	Index	Geographic region	Currency
SPY	S&P500	USA	USD
EZU	MSCI EMU	European Economic and Monetary Union	USD
FTSEMIIB	FTSEMIB	Italy	EUR
SXXP	Stoxx 600	Europe	EUR
IBEX	IBEX	Spain	EUR
ISF LN	FTSE100	United Kingdom	GBP
UKX	FTSE100	United Kingdom	GBP
EFA	MSCI EAFE	Europe, Australasia and the Far East	USD

4.2 Cointegration

The notation L before a variable implies that the time series of the variable has been log-transformed, id est. it is the natural logarithm of the variable. The notation DL means that the variable is on log-difference form, id est. the t-1 lagged natural logarithm of the variable has been subtracted from the natural logarithm of the variable at time t, giving the log-returns.

4.2.1 Engle-Granger

The Engle Granger two-step procedure is applied on the raw price series and then I log-transform the price series that are co-integrated with S&P500. I then use the log-transformed prices to perform a log-log OLS-regression. The coefficient from the level regression can be interpreted as number of shares in the co-integrating index. Typically, the co-integrating vector (1,-α) represents one share in S&P500 and α shares in the respective variable. The log-log coefficient requires another interpretation as a log-log coefficient returns the elasticity of the dependent variable with respect to a change in the explanatory variable, see section 5.1.

Table 2 – Engle-Granger OLS

The table shows the Engle-Granger OLS-regression with the ETF for S&P500 as the dependent variable and the respective variables as explanatory variables one at a time.

$$SPY_t = \hat{\alpha}_0 + \hat{\alpha}\, EZU_t + \hat{u}_t$$
$$SPY_t = \hat{\alpha}_0 + \hat{\alpha}\, FTSEMIIB_t + \hat{u}_t$$
$$SPY_t = \hat{\alpha}_0 + \hat{\alpha}\, SXXP_t + \hat{u}_t$$
$$SPY_t = \hat{\alpha}_0 + \hat{\alpha}\, IBEX_t + \hat{u}_t$$
$$SPY_t = \hat{\alpha}_0 + \hat{\alpha}\, UKX_t + \hat{u}_t$$
$$SPY_t = \hat{\alpha}_0 + \hat{\alpha}\, ISF_t + \hat{u}_t$$
$$SPY_t = \hat{\alpha}_0 + \hat{\alpha}\, EFA_t + \hat{u}_t$$

The first row gives the name of every dependent variable. The third row shows the coefficient known as the Engle-Granger alpha. The estimation sample is 31.03.2004 – 31.12.2013. The residuals from the OLS are saved and then tested for stationarity with the Augmented Dickey-Fuller test.

$$\Delta \hat{u}_t = \hat{\theta} + (\delta - 1) y_{t-1} + \varepsilon_t + \sum_{j=1}^{2} \Delta \hat{u}_{t-1}$$

The critical value for two variables and sample size bigger than 200 is 3.25 on the 5%-level and 3.75 for the 1% level (Engle and Yoo, 1987). The results in the second lowest row show that there is cointegration between the ETFs of S&P500 and FTSE100, but no cointegration between the other indices. The residuals of FTSE100 are stationary, whilst the other indices yield non-stationary residuals and therefore the EG OLS is considered spurious.

	EZU	FTSEMIIB	SXXP	IBEX	UKX	ISF	EFA
Constant	88.2263	119.271	41.7855	105.645	-19.9837	-21.1913	-21.1913
Coefficient	1.02927	0.000325	0.305135	0.002129	0.026579	0.267055	0.267055
T-value (Coefficient)	27.2	7.29	55.1	12.2	112	111	48.1
ADF tests on residuals from the previous regression (T=2545, Constant; 5%=-2.86 1%=-3.44)							
D-Lag	t-adf	t-adf	t-adf	t-adf	t-adf	t-adf*	t-adf
2	0.7893	-0.0415	0.7871	0.2458	-1.841	-1.697	1.221
1	0.6042	-0.1923	0.3139	0.1049	-2.295	-2.204	0.9868
0	0.4206	-0.4942	-0.899	-0.3195	-3.898**	-3.853**	0.9717

**Significant in relation to the 1% critical values supplied by OxMetrics

The level regressions show that there is a significant co-integrating relationship between S&P500 and FTSE100 (UKX and ISF). Although all the t-values for the coefficients from the Engle Granger regression is significant on the 5% significance level for all variables, the corresponding residuals are not stationary. The non-stationary residuals indicate that the Engle Granger regression is spurious.

In the further analysis, the only interesting variable is the ETF for FTSE100, id est. ISF LN. Because the price of one unit ISF is higher than the price of SPY, I estimate the Engle-Granger regression again, this time with ISF as the dependent variable. In addition to the level regression, I also perform the regression analysis on the log-transformed data.

Table 3 – Engle-Granger OLS with FTSE100 as dependent variable

The table shows the Engle-Granger OLS regression with the ETF of FTSE100 as the dependent variable on the ETF of S&P500. In the upper part of the table, the price series are not transformed. The level-level regression equation is given as,

$$ISF_t = \hat{\alpha}_0 + \hat{\alpha}\, SPY_t + \hat{u}_t$$

The lower part contains the same regression on the log-transformed time series, id est., the log-log regression,

$$\ln(ISF_t) = \hat{\alpha}_0 + \hat{\alpha}\ln(SPY_t) + \hat{u}_t$$

The residuals from the EG OLS is tested for a unit root with the ADF-test.

$$\Delta \hat{u}_t = \hat{\theta} + (\delta - 1) y_{t-1} + \varepsilon_t + \sum_{j=1}^{2} \Delta \hat{u}_{t-1}$$

The results from the ADF is in the right part of the table.

LEVEL

Engle-Granger			ADF	
	Coefficient	t-value	D-lag	t-adf
Constant	160.737	44.5	2	-2.76
SPY	3.10742	111	1	-3.13*
			0	-4.51**

LOG

Engle-Granger			ADF	
	Coefficient	t-value	D-lag	t-adf
Constant	2.80564	92.6	2	-3.133*
ln(SPY)	0.725675	116	1	-3.513**
			0	-5.006**

**Significant in relation to the 1% critical values supplied by OxMetrics

The critical value for two variables and sample size bigger than 200 is 3.25 on the 5%-level and 3.75 for the 1% level (Engle and Yoo, 1987). Hence, the results in the last column show that the residuals are stationary and that there is co-integration between the two ETFs. The results show that the OLS regression is sensitive to the ordering of the variables. The OLS minimizes the squared sum of the residuals of the dependent variable to find the optimal fit. The problem may be eliminated by using a model that uses the residuals from both the dependent and independent variable such as Total Least Squares. In this thesis, I will try to circumvent the problem by using the more robust regression, id est. ISF LN as the dependent variable. The fact that the coefficients are not the inverse of each other could imply a problem with the hedge ratio, but as long as the coefficients are used consistently, it should be possible to avoid the problem.

4.2.2 Error Correction Model

The error correction model is defined in Engle and Granger (1987) and is given by

$$\Delta y_t = \beta_1 \Delta x_t + \gamma_1 Z_{t-1} + e_{1t}$$
$$\Delta x_t = \beta_2 \Delta y_t + \gamma_2 Z_{t-1} + e_{2t}$$

Table 4 – Error correction model

The table gives the Error Correction Model, id est. the results from the regressions shown above. DL ISF is Δy from the equation above, and is the log-difference of ISF, id est. the log-returns of ISF. DL SPY is Δx, and is the log-difference of SPY. Z is the spread given by the residuals of the Engle-Granger regression.

DL ISF	Coefficient	t-value	DL SPY	Coefficient	t-value
Constant	0.05303	5.14	Constant	-0.0334	-3.07
DL SPY	0.56087	36.9	DL ISF	0.62213	36.9
Z_1	-0.0189	-5.17	Z_1	0.01195	3.08

The Engle-Granger regression returns a positive alpha. The name error correction describes the short-term adjustment back to the equilibrium. The coefficients of Z, found in the last row of table 4, are interpreted as the speed of adjustment back to the equilibrium. The ECM will only include an error correction mechanism if $\gamma_1<0$ and $\gamma_2>0$ given a positive alpha. The size of γ_1 and γ_2 gives the speed of adjustment. In our case they are -0.019 and 0.012. These small coefficients signal a low speed of adjustment. As the model corrects to the equilibrium, it should be applicable for trading.

To check if there is any one way Granger flows, the two equations from the ECM is re-written. Given the ECM-equations,

$$\begin{aligned}\Delta y_t &= \beta_1 \Delta x_t + \gamma_1 Z_{t-1} + e_{1t} \\ \Delta x_t &= \beta_2 \Delta y_t + \gamma_2 Z_{t-1} + e_{2t}\end{aligned},$$

then Y and X can be expressed using only predetermined lags:

$$y_t = (1 + \frac{(\beta_1\gamma_2 + \gamma_1)}{(1-\beta_1\beta_2)})y_{t-1} - \alpha\frac{(\beta_1\gamma_2 + \gamma_1)}{(1-\beta_1\beta_2)}x_{t-1} + e_t$$

Inserting the expression for Y into the orignal expression for x yields

$$x_t = (\beta_2\frac{(\beta_1\gamma_2 + \gamma_1)}{(1-\beta_1\beta_2)} + \gamma_2)y_{t-1} + (1 - (\alpha\beta_2\frac{(\beta_1\gamma_2 + \gamma_1)}{(1-\beta_1\beta_2)} + \alpha\gamma_2))x_{t-1} + e_{2t}$$

Table 5 – Predetermined values from ECM

The table shows the transformed ECM containing only predetermined variables on the right-hand side. The regression investigates whether the other variable has significant explanatory power. The results show that the log prices of the ETF for FTSE100 are explained partly by previous log prices of SPY, the ETF for the S&P500, but not the other way around.

ln(ISF)	Coefficient	t-value	ln(SPY)	Coefficient	t-value
ln(ISF)_1	0.98129	216	ln(SPY)_1	0.99781	263
Constant	0.05838	4.02	Constant	0.00879	0.574
ln(SPY)_1	0.01239	3.44	ln(ISF)_1	0.00032	0.09476

Estimating y_t and x_t by ordinary least squares illustrates an interesting point. Namely that the regression that explains the price of ISF (left side of table) is statistically significant for every coefficient, but the regression with SPY as the dependent variable only contain one significant coefficient. Recall the definition of Granger causality; X Granger Causes Y if lagged values of x increase the precision of current and future predictions. The Granger causality indicates that SPY determines the equilibrium and the equilibrium is adjusted trough ISF, id est. the error correction term and the influence SPY has on ISF.

To verify the findings I test the first differences of ln(ISF) and ln(SPY). Table 6 shows the results from the regression:

$$\begin{aligned}\Delta y_t &= \alpha_1 + \beta_{11}\Delta x_{t-1} + \beta_{12}\Delta y_{t-1} + \gamma_1(y_{t-1} - \alpha x_{t-1}) + u_{1t} \\ \Delta x_t &= \alpha_2 + \beta_{21}\Delta x_{t-1} + \beta_{22}\Delta y_{t-1} + \gamma_2(y_{t-1} - \alpha x_{t-1}) + u_{2t}\end{aligned},$$

where y is ln(ISF) and X is ln(SPY). The formal test for Granger causality will then be:

ln(SPY) Granger causes ln(ISF) if the H_0 is rejected.

$$H_0^{\ln(SPY)} : \beta_{11} = \gamma_1 = 0$$

ln(ISF) Granger causes ln(SPY):

$$H_0^{\ln(ISF)} : \beta_{22} = \gamma_2 = 0$$

Table 6 – Granger causality test

The table shows the results from the Granger causality test, where respectively the first difference of the log prices of the ETF for FTSE100 and S&P500 are dependent variables. The variables are regressed on lagged values and the lagged spread provided by the Engle-Granger regression.

	Dln(ISF)			Dln(SPY)	
	Coefficient	t-value		Coefficient	t-value
Dln(ISF)_1	-0.30253	-13.1	Dln(SPY)_1	-0.09745	-3.97
Constant	0.03283	2.72	Constant	0.001892	0.14
Dln(SPY)_1	0.40646	18.5	Dln(ISF)_1	0.011015	0.428
z_1	-0.01166	-2.71	z_1	-0.0006	-0.125

The results show that the return on SPY are not explained by the lagged return of ISF LN or the spread and that the simultaneous $H_0^{\ln(SPY)}$ is rejected so that SPY Granger Cause ln(ISF). $H_0^{\ln(ISF)}$ is not rejected, stating that ln(ISF) does not Granger cause ln(SPY).

4.2.3 USD as base currency

Table 7 – Johansen cointegration test

The table shows the results from the Johansen cointegration test. The variables included are EFA, EZU, FTSEMIB, IBEX, ISF, SPY, and SXXP in the base currency USD. The first column show the different H_0 and describes the number of co-integrating equations (CE). The third column show the trace statistic and the forth column show the critical value (5%).

Unrestricted Cointegration Rank Test (Trace)

No. of CE(s)	Eigenvalue	Statistic	Critical Value	Prob.**
None	0.017311	117.0119	125.6154	0.1483
At most 1	0.011017	72.64032	95.75366	0.6303
At most 2	0.007078	44.48993	69.81889	0.8467
At most 3	0.004992	26.43960	47.85613	0.8746
At most 4	0.003004	13.72191	29.79707	0.8559
At most 5	0.002033	6.077962	15.49471	0.6863
At most 6	0.000357	0.906304	3.841466	0.3411

The results show that there are no co-integrating equations in the system using a 5%-significance level after adjusting for currency. If the market capitalization of the indices are similar, these results rejects a purchasing power parity-test (PPP). However, Purchasing Power Parity is not the focus of this thesis. The focus for this thesis is the efficient market hypothesis. The fact that the indices corrected for currency are not co-integrated implies that the price of the indices evolve randomly. These results support the efficient market hypothesis.

5. TRADING STRATEGY

5.1 Design

This section includes the intuitive reasoning behind the trading strategy. The trading strategy is based on the co-integrating relationship of S&P500 and FTSE100. The two indices are traded in USD and GBP, respectively. Because the co-integrating relationship disappears when we use a base currency, the trading strategy will include a higher transaction cost due to the exchange of GBP for USD. I assume that the investor performing the trade is a hedge fund or similar with power to reduce the transaction costs. Regarding transaction costs, Marius Hovde said the following: "Transaction costs varies from instrument to instrument. The major players will be able to trade in the spread and escape this item. When I look at the SPY, traded live now as we speak , this trades at 188.17 / 188.18 and it looks like one point spread is pretty standard here, and it's practically negligible. One might argue that in the turbulent period (financial crisis, etc.) then the spread could be substantially higher. ISF LN is now 682.6 / 683.0 (…), but when you have several hundred millions to shop for, then you will surely be able to get down towards to zero here as well".

If there is cointegration between the asset prices, the residuals from the OLS are stationary. Id est. the spread is stationary. As the spread is stationary, the spread will return to its equilibrium value. The strategy is based on two threshold values: The equilibrium value +/- delta multiplied with the standard deviation of the spread return estimated from the in-sample estimation. Delta is an arbitrary number. The strategy opens a long position when the spread is below the lower threshold. It opens a short position when the spread is above the upper threshold. The threshold, λ, is expressed mathematically as

$$\lambda_U = \mu_Z + \delta\sigma_Z \ \vee \ \lambda_L = \mu_Z - \delta\sigma_Z$$

Whenever the spread is in between the two thresholds, the strategy yields a signal equal to the lagged signal.

The quotes used are average bid-ask closing prices, but the bid-ask spread will be incorporated by subtracting a percentage whenever there is a change of position. The bid-ask spread of the ETFs based on S&P500 and FTSE100 are minimal. The cost of short-selling will include a cost of borrowing ETFs to short. This cost will be included in the model by subtracting a percentage.

In Gatev et al. (2006) the authors construct a capital neutral portfolio. A capital neutral portfolio is constructed by using the proceeds from short selling to cover the long position. I will not construct capital neutral portfolios in this thesis.

The trading strategy consists of three different approaches. The first approach uses the untransformed price series and the corresponding co-integrating vector as the portfolio weights. This strategy is called Level Unit, because the co-integrating vector can be interpreted as number of shares. The two other approaches use the log-transformed price series. The first of the strategies that use log-transformed price series, is called Log Unit, and use the co-integrating vector as portfolio weights. The last strategy is called Log Relative Weight in Capital, or Log RWC. Lin et al. (2006) constructs a market neutral portfolio by using the cointegration coefficient as a hedge ratio. The interpretation of the Engle-Granger alpha as a hedge ratio is dependent on the price series used in the regression. If the regression uses raw, untransformed prices, the coefficient can be interpreted as a number of share. If the regression uses log-transformed data, the coefficient should be interpreted as relative weights in capital. The third strategy uses the weights given in the equation below. Assume the regression

$$\ln(ISF_t) = \hat{\alpha} + \hat{\alpha}_{EG} \ln(SPY_t) + \hat{u}_t \text{ ,}$$

then the weights for a market neutral portfolio are defined as

$$w_{ISFUK} = \frac{P_t^{ISF}}{P_t^{ISF} + \hat{\alpha}_{EG} P_t^{ISF}} \text{ ,}$$

with the corresponding weight of SPY

$$w_{SPY} = \frac{\hat{\alpha}_{EG} P_t^{ISF}}{P_t^{ISF} + \hat{\alpha}_{EG} P_t^{ISF}}$$

These weights are necessary for a market neutral portfolio if the prices are log-transformed.

To evaluate the trading strategies I will use standard deviation and return, and some risk-adjusted measures like the Sharpe ratio of Sharpe (1966) and an approximation to the Treynor index portrayed in Treynor (1965). The Sharpe ratio (SR) is expressed as,

$$SR = \frac{R_p - R_f}{\sigma_{R_p}}$$

Where R_p is the return of the portfolio, σ_{Rp} is the standard deviation of portfolio returns and R_f is the risk-free rate. The Treynor index differs from the Sharpe ratio in the use of risk adjustor. The treynor index uses the portfolio return beta (defined below). If the investor is fully diversified, she will only care about systematic risk (or market risk) and the Treynor index will be a suitable measurement. Beta is expressed as,

$$\beta_{R_p} = \frac{Cov(R_m, R_p)}{Var(R_m)}$$

So that the treynor index is,

$$Treynor = \frac{R_p - R_f}{\beta_{R_p}}$$

My approximation of the Treynor index is determined by the use of R_m. Both FTSE100 and S&P500 are suitable for use as the market return, but because the FTSE100 is used as the dependent variable in the regressions I will only use FTSE100 as the market return.

5.1.1 Level Unit

When modelling the level spread, the co-integrating vector is interpreted as number of shares. Do et al. (2006) discusses the long term level difference of pairs, and argues that the long-term difference will not be constant, but that it will increase as the stock prices go up and decrease as they go down. Given the spread from the Engle-Granger regression

$$z_t^{Level} = SPY_t - \hat{\beta} ISF_t = \hat{\mu} + \varepsilon_t$$

The prices at time t+1 can be written as:

$$SPY_{t+1} = SPY_t(1 + r_{t+1}^{SPY}) \vee ISF_{t+1} = ISF_t(1 + r_{t+1}^{ISF})$$

The spread at time t+1 is then:

$$z_{t+1}^{Level} = SPY_t(1 + r_{t+1}^{SPY}) - \hat{\alpha}\, ISF_t(1 + r_{t+1}^{ISF}) = z_t^{Level} + SPY_t r_{t+1}^{SPY} - \hat{\beta}\, ISF_t r_{t+1}^{ISF}$$

From the spread at time t+1 it becomes clear that the last difference must be equal to zero if the spread should remain constant. In this thesis, I estimate the expected value of the spread based on the historical data.

5.1.2 Log Unit

When the price of the portfolio consisting of one unit ISF LN and α unit of SPY is above the threshold value, I short the portfolio and vice versa.

5.1.3 Relative weight in capital (Log RWC)

The portfolio consisting of weights defined by the relative weight in capital applies the same strategy, id est. when the price of the portfolio is above the threshold value the portfolio is shorted, and vice versa. The use of rebalancing weights will incur daily transaction costs. However, the fluctuations in the in-sample estimation are minimal. Therefore, the cost of rebalancing is included in the transaction cost term, rather than a daily percentage.

5.2 In-Sample Estimation

Table 8 – Trading strategies (In-sample)

The table shows the standard deviation (σ_Z) of the returns of the spreads and indices, the value index (VI) without transaction costs (TC), the Sharpe ratio (SR), the Treynor index and the beta of the returns. The risk-free rate is based on the YTM of a 10-year US government bond quoted at 01.04.2004. The YTM is 3.8786% annually. The Sharpe ratios in the fourth column to the right are only positive for the level unit strategy and the buy and hold strategy for the S&P500. The sharpe ratio is negative for the two log strategies. The equilibrium value is given in the second column.

	EV	σZ	VI TC=0%	SR	Treynor (ISF)	Beta R_{ISF}
Level Unit	159.2596	3.44 %	2.045303	1.19	0.570141	1.021233
Log Unit	2.803661	0.99 %	1.402321	-0.43	-0.15809	0.384178
Log RWC	10.22022	0.36 %	1.143798	-6.34	-2.31913	0.137663
SPY	-	1.26 %	1.494405	2.49	0.02622	1.195616
ISF LN	-	1.20 %	1.379768	-6.94	-0.08329	1

The SR of the long position in SPY beats all the trading strategies. The risk adjusted return of ISF is lower than of all the other trading strategies. The level unit has the highest standard deviation, and the second highest Sharpe ratio. An undiversified risk-averse investor would prefer a long position in SPY instead of the level unit strategy. The Treynor index of the level unit strategy is higher than the Treynor index for the long position in SPY. A well-diversified investor might prefer the level unit strategy, as the systematic risk (given by beta) is lower. The other strategies has lower standard deviations and lower returns than SPY. If one assume the returns to be a predictor of what to come, then the choice of strategy will be contingent on investor preferences. The low risk strategy of Log relative weight in capital (RWC) in row number 3 does not exceed the risk free rate, but has the lowest standard deviation of all the strategies. The large and negative SR for the log RWC strategy is due to the low standard deviation. The log RWC strategy has the lowest beta in regards to FTSE100, and may therefore be seen as a market neutral portfolio, or a hedge. If the hedging had been perfect, the Sharpe ratio would have been zero. The fact that the strategies do not yield a higher risk adjusted return than SPY may be because the Sharpe ratio does not incorporate the inherent hedge that the spread-based position has.

5.3 Out-of-sample test

The out-of-sample test is based on 85 trading days, from 01.01.2014 to 29.04.2014. The out-of-sample uses the equilibrium and threshold values estimated by the in-sample model.

Table 9 – Trading strategies (Out-of-sample)

The table shows the standard deviation (σ_Z) of the returns of the spreads and indices, the value index (VI) with and without transaction costs (TC), the corresponding Sharpe ratio (SR), the Treynor index and the beta of the returns based on the portfolio with transaction costs. The estimation period is from 01.01.2014 to 29.04.2014. The risk-free rate is based on the YTM of a 1 month US government bond quoted at 01.01.2014. The YTM is 1.01% annually. The value index includes a transaction cost. The cost is set to 5% pr. Position change for the cointegration-based strategies and 1% for the long positions in the indices

	σ_z	VI TC=0%	SR	VI TC=5%	SR	Treynor	Beta
Level Unit	3.01 %	0.9906	-0.15	0.9406	-0.74	-0.0221	2.83
Log unit	0.55 %	0.9992	-0.28	0.9492	-3.56	0.0955	-0.57
Log RWC	0.21 %	1.0001	-0.60	0.9501	-6.73	-0.2455	0.22
SPY	0.90 %	1.0164	1.74	1.0064	0.41	0.0056	0.55
ISF LN	0.69 %	1.0162	1.91	1.0062	0.42	0.0029	1.00

None of the cointegration-based strategies yields positive returns after adjusting for transaction costs. The level unit strategy that performed well in the in-sample test gives approximately the same standard deviation, but with far lower return. The log RWC is still close to market neutral, but has the most negative SR index after transaction costs. This highly negative SR is because the return of the portfolio is lower than the risk-free, and has a very low (0.21%) standard deviation. The log unit strategy returns a negative beta. I consider the negative beta to be a coincidence since the beta of the strategy is not negative in the in-sample estimation. The negative beta makes the Treynor index of the Log unit strategy to the highest. The worst SR from the in-sample estimation was ISF, but now the ISF has the highest SR. The fact that the SR is "unusual", in addition to the negative beta of the Log unit strategy, might indicate that the out-of-sample period is an atypical period. The ECM gave a γ_1 and γ_2, that were are quite low. The low coefficients from the ECM tells us that mean reversion will take some time, and the convergence may take longer than 85 days.

6. DISCUSSION

One interesting finding of this thesis is that S&P500 and FTSE100 are co-integrated. A co-integrating relationship between the two indices means that there is statistically significant stationary long-term equilibrium. Modelling the short-term dynamics shows that the spread based on the co-integrating vector has mean reverting properties. The ECM shows that the mean reverting will take some time. This mean-reversion is not captured in the 85-day out-of-sample test. I find that SPY Granger Causes ISF LN, meaning that the returns of SPY help describe the returns of ISF LN better than just the lagged returns of ISF LN. The one-way granger causality may be due to the different opening hours, and that the indices are open for trade at the same time only a couple of hours a day. In spite of the mean-reverting process proven by the ECM, the trading strategies did not yield positive results. The fact that there is a short term disequilibrium may be the disequilibrium mentioned by Grossman and Stiglitz (1980). As mentioned in the literature review, Grossman and Stiglitz argues that there must be some degree of disequilibrium in the market prices. If there is perfect efficiency in the markets, those who gather information will not receive compensation.

The other index prices do not have a statistical significant long-term equilibrium with S&P500. The results are in line with the EMH seeing as S&P500 cannot be used to predict the returns of the other indices. The long-term equilibrium cannot be directly linked to any of the existing asset pricing models (CAPM, APT, FF etc.), which has previously been used to test the EMH. But, if one test the definition "A market is efficient with respect to information set θ_t if it is impossible to make economic profits by trading on the basis of information set θ_t." then the cointegration approach is a great tool for testing the EMH, as the information set is known for everybody and easy accessible. Because of the resemblance to previous studies that have implemented similar strategies on single stock pairs, and returned positive returns, it might seem as though Schwert (2003) has a valid point as he describes how the practitioners implement the anomalies found by researchers. The fact that practitioners implement the research gives credibility to Bowen et al. (2010), which showed that the speed of trading is crucial. Timmermann and Granger (2004) described the constant search for profitable trading strategies as a self-destructing game. This self-destructing game could be one of the reasons for the negative results in the out-of-sample test.

The analysis of this thesis has its drawbacks, apart from the currency risk faced in the trading strategy, the stationary relationship may be altered as the composition of stocks in the index changes. In addition, an improved modelling of the threshold values could maybe enhance the profitability of the trading strategies.

7. CONCLUSION

The test of the efficient market hypothesis shows that it is not possible to make economic profits trading based on the information set, θ. The interesting part of the empirical analysis is that although there is a stationary short-term disequilibrium between the asset prices, the trading of the ETFs based on the spread is not profitable. These unprofitable trades may also point in favour of the semi-strong EMH, and is in line with the results of Grossman and Stiglitz (1980). The analysis in this thesis supports the EMH.

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APPENDICES

Appendix: OLS-coefficients and the superconsistent EG-OLS coefficient.

Given the model

$$y_i = \beta_0 + \beta_1 x_1 + u_i$$

The model does not capture the underlying process exactly, and the stochastic error term is noted u. If one estimates the model

$$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$$

Then the observable equivalent to the error term is defined as $\hat{u}_i$, and is called the residuals. The OLS rests on the assumptions that the error term follows a normal distribution with expectation zero and standard deviation σ, the covariance between the explanatory variable and the error term is zero and that the auto covariance of the error term is zero. Under these assumptions, the OLS-estimators follows a normal distribution.

The ordinary least squares minimizes the squared sum of residuals by choosing the OLS-estimators, β_0 and β_1

$$\min_{\hat{\beta}_0,\hat{\beta}_1} SSR^2 = \min_{\hat{\beta}_0,\hat{\beta}_1} \sum_{i=1}^{n} (\hat{u}_i)^2 = \min_{\hat{\beta}_0,\hat{\beta}_1} \sum_{i=1}^{n} (y_i - \hat{y})^2 = \min_{\hat{\beta}_0,\hat{\beta}_1} \sum_{i=1}^{n} (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)^2$$

The first order condition,

$$FOC: \frac{\partial SSR^2}{\partial \hat{\beta}_0} = 2\sum_{i=1}^{n} (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)(-1) = 0$$

Yields the expression for the constant, β_0, by multiplying the FOC with 1/2n,

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

To find an expression for the explanatory coefficient β_1, insert the expression for β_0 in the original minimizing problem. Then minimize with respect to β_1,

$$\min_{\hat{\beta}_1} \sum_{i=1}^{n} (y_i - \bar{y} - \hat{\beta}_1 (x_i - \bar{x})^2$$

$$FOC: 2\sum_{i=1}^{n} \left[(y_i - \bar{y}) - \hat{\beta}_1 (x_i - \bar{x}) \right] (-1)(x_i - \bar{x}) = 0$$

$$\sum_{i=1}^{n} (y_i - \bar{y})(x_i - \bar{x}) - \hat{\beta}_1 \sum_{i=1}^{n} (x_i - \bar{x})^2 = 0$$

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n} (y_i - \bar{y})(x_i - \bar{x})}{\sum_{i=1}^{n} (x_i - \bar{x})^2} (*)$$

Given the regression,

$$y_i = \beta_0 + \beta_1 x_i + u_i$$

And,

$$\bar{y} = \beta_0 + \beta_1 \bar{x} + \bar{u}$$

Then,

$$y_i - \bar{y} = \beta_1 (x_i - \bar{x}) + (u_i - \bar{u})$$

Inserting the last expression into (*) yields the following expression for the OLS-estimator,

$$\hat{\beta}_1 = \beta_1 + \frac{\sum_{i=1}^{n} (u_i - \bar{u})(x_i - \bar{x})}{\sum_{i=1}^{n} (x_i - \bar{x})^2} = \beta_1 + \frac{\sum_{i=1}^{n} u_i (x_i - \bar{x})}{\sum_{i=1}^{n} (x_i - \bar{x})^2} (**)$$

Consider the difference between the estimator and the parameter. Consistency requires that the probability of the difference being less than an arbitrary small number is zero, when number of observations goes towards infinity.

$$\lim_{n \to \infty} P(\left| \hat{\beta} - \beta \right| < \varepsilon) = 0$$

Where ε is an arbitrary small number. The estimator is consistent if the probability limit of the estimator equals the parameter,

$$p\lim \hat{\beta} = \beta$$

To investigate if (**) is consistent, I will expand the denominator and the numerator with 1/n and then take the probability limit of both sides.

$$p\lim \hat{\beta} = \beta_1 + \frac{p\lim \frac{1}{n}\sum_{i=1}^{n}(u_i - \bar{u})(x_i - \bar{x})}{p\lim \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

When n approaches infinity, the expression becomes

$$p\lim \hat{\beta} = \beta_1 + \frac{Cov(u_i, x_i)}{Var(x_i)}$$

In other words, if the covariance between the error term and the explanatory variable is zero, as earlier assumed, then the estimator is consistent. In the Engle-Granger regression both y and x in the above model is non-stationary, while u is a stationary AR-process. If we consider the probability limit of (**), then the numerator will explode as it contains a non-stationary process. However, the denominator contains a squared non-stationary process, and will thus dominate the numerator so violently that it does not matter if the error term is independent of the explanatory variable.

Appendix: Unit Root tests

Unit-root tests of the variables included in the analysis.

ADF tests (T=2542, Constant+Trend; 5%=-3.41 1%=-3.97)

3 lags captures the relevant autocorrelation that might be present.

SPY US

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob	AIC	F-prob
2	-0.8067	0.99882	1.412	-2.628	0.0086	0.6918	
1	-0.9354	0.99863	1.413	-4.303	0	0.6937	0.0086
0	-1.151	0.99831	1.418			0.7002	0

EZU US

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob	AIC	F-prob
2	-1.752	0.99759	0.6214	-0.916	0.3597	-0.9495	
1	-1.778	0.99755	0.6214	-4.805	0	-0.9499	0.3597
0	-1.921	0.99735	0.6241			-0.9417	0

FTSEMIB

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob	AIC	F-prob
2	-1.873	0.99779	337.6	-0.5334	0.5938	11.65	
1	-1.884	0.99777	337.6	-1.653	0.0985	11.65	0.5938
0	-1.918	0.99773	337.7			11.65	0.2216

SXXP In

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob	AIC	F-prob
2	-1.479	0.99811	3.158	-1.342	0.1799	2.302	
1	-1.522	0.99805	3.158	-1.366	0.1721	2.301	0.1799
0	-1.567	0.99799	3.159			2.301	0.1601

IBEX In

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob	AIC	F-prob
2	-2.056	0.99715	148.5	-1.991	0.0466	10	
1	-2.108	0.99708	148.6	-0.5226	0.6013	10	0.0466

0	-2.123	0.99706	148.6			10	0.1204

UKX Ind

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob	AIC	F-prob
2	-2.162	0.99593	62.01	-1.711	0.0871	8.256	
1	-2.241	0.99578	62.03	-2.554	0.0107	8.257	0.0871
0	-2.363	0.99555	62.1			8.259	0.0089

ISF LN:

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob	AIC	F-prob
2	-2.157	0.99589	6.279	-2.548	0.0109	3.676	
1	-2.272	0.99567	6.285	-2.917	0.0036	3.678	0.0109
0	-2.412	0.9954	6.295			3.681	0.0006

EFA US

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob	AIC	F-prob
2	-1.739	0.99735	0.8122	-1.857	0.0635	-0.414	
1	-1.8	0.99725	0.8126	-5.635	0	-0.4135	0.0635
0	-2.001	0.99693	0.8175			-0.4018	0

LSPY: A

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob	AIC	F-prob
2	-1.223	0.99806 0	0.01262	-3.718	0.0002	-8.743	
1	-1.39	0.99779 0	0.01265	-4.56	0	-8.739	0.0002
0	-1.606	0.99744 0	0.0127			-8.731	0

LEZU: A

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob	AIC	F-prob
2	-1.962	0.99683 0	0.01788	-1.333	0.1826	-8.047	
1	-2.008	0.99676 0	0.01788	-4.627	0	-8.047	0.1826
0	-2.178	0.99647 0	0.01795			-8.039	0

LFTSEMI

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob	AIC	F-prob
2	-2.083	0.99689 0	0.01541	-1.381	0.1675	-8.344	
1	-2.127	0.99682 0	0.01541	-0.2377	0.8121	-8.344	0.1675
0	-2.136	0.99681 0	0.01541			-8.344	0.375

LSXXP:

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob	AIC	F-prob
2	-1.59	0.99780 0	0.01218	-2.331	0.0198	-8.815	
1	-1.671	0.99768 0	0.01219	-0.5995	0.5489	-8.813	0.0198
0	-1.694	0.99765 0	0.01218			-8.814	0.0554

LIBEX:

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob	AIC	F-prob
2	-2.198	0.99670 0	0.015	-2.405	0.0162	-8.397	
1	-2.27	0.99659 0	0.01502	0.7816	0.4345	-8.395	0.0162
0	-2.248	0.99663 0	0.01502			-8.396	0.041

LUKX: A

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob	AIC	F-prob
2	-2.233	0.99570 0	0.01199	-2.642	0.0083	-8.845	
1	-2.353	0.99547 0	0.01201	-2.298	0.0217	-8.843	0.0083
0	-2.463	0.99526 0	0.01202			-8.842	0.0022

LISFUK:

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob	AIC	F-prob
2	-2.204	0.99572 0	0.01205	-3.61	0.0003	-8.836	
1	-2.366	0.99540 0	0.01207	-2.604	0.0093	-8.832	0.0003
0	-2.492	0.99516 0	0.01209			-8.83	0.0001

Appendix: Engle-Granger regression before correcting for base currency

The appendix shows the Engle-Granger regression with the ETF for S&P500 as dependent variable. The residuals from the EG-regression is then tested with an Augmented Dickey Fuller test. S&P500 and the dependent variable has a co-integrating relationship if the residuals are stationary, id est the t-adf for D-lag Zero is higher than the critical value. Significant values are marked with * on the 5% level and ** on the 1%. The table shows that there are no co-integrating relationships when the regression is performed with only one dependent variable.

	Coefficient	Std.Error	t-value	t-prob	Part.R^2
Constant	88.2263	1.498	58.9	0	0.5769
EZU US Equity	1.02927	0.0379	27.2	0	0.2248

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob	AIC	F-prob
2	0.7893	1.0008	0.9203	-3.114	0.0019	-0.1645	
1	0.6042	1.0006	0.9219	-3.14	0.0017	-0.1615	0.0019
0	0.4206	1.0004	0.9235			-0.1584	0.0001

	Coefficient	Std.Error	t-value	t-prob	Part.R^2
Constant	119.271	1.229	97	0	0.7873
FTSEMIB Index	0.000325	4.46E-05	7.29	0	0.0205

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob	AIC	F-prob
2	-0.04157	0.99994	1.351	-2.712	0.0067	0.6028	
1	-0.1923	0.99975	1.352	-5.394	0	0.6049	0.0067
0	-0.4942	0.99934	1.36			0.6155	0

	Coefficient	Std.Error	t-value	t-prob	Part.R^2
Constant	41.7855	1.585	26.4	0	0.2147
SXXP Index	0.305135	0.005541	55.1	0	0.5439

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob	AIC	F-prob
2	0.7871	1.0012	1.037	-7.741	0	0.0734	
1	0.3139	1.0005	1.049	-20.26	0	0.09595	0

0	-0.899	0.99854	1.13			0.245	0

	Coefficient	Std.Error	t-value	t-prob	Part.R^2
Constant	105.645	1.849	57.1	0	0.5621
IBEX Index	0.002129	0.000174	12.2	0	0.0555

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob	AIC	F-prob
2	0.2458	1.0003	1.255	-2.508	0.0122	0.4559	
1	0.1049	1.0001	1.256	-7.554	0	0.4576	0.0122
0	-0.3195	0.9996	1.27			0.479	0

	Coefficient	Std.Error	t-value	t-prob	Part.R^2
Constant	-19.9837	1.324	-15.1	0	0.0822
UKX Index	0.026579	0.000236	112	0	0.8325

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob	AIC	F-prob
2	-1.841	0.99443	1.271	-7.459	0	0.4813	
1	-2.295	0.99299	1.285	-21.55	0	0.5022	0
0	-3.898**	0.98711	1.397			0.6694	0

	Coefficient	Std.Error	t-value	t-prob	Part.R^2
Constant	-21.1913	1.348	-15.7	0	0.0886
ISF LN	0.267055	0.002398	111	0	0.8299

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob	AIC	F-prob
2	-1.697	0.99475	1.307	-7.858	0	0.5377	
1	-2.204	0.99312	1.323	-21.13	0	0.561	0
0	-3.853**	0.98702	1.434			0.7222	0

	Coefficient	Std.Error	t-value	t-prob	Part.R^2
Constant	49.7294	1.648	30.2	0	0.2638
EFA US Equity	1.33443	0.02773	48.1	0	0.4766

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob	AIC	F-prob
2	1.221	1.0011	0.658	-3.665	0.0003	-0.8355	
1	0.9868	1.0009	0.6596	-0.273	0.7849	-0.831	0.0003
0	0.9717	1.0009	0.6595			-0.8318	0.0012

Appendix EG Regression with USD as base currency.

The appendix shows the Engle-Granger regression with the ETF for S&P500 as dependent variable. The residuals from the EG-regression is then tested with an Augmented Dickey Fuller test. S&P500 and the dependent variable has a co-integrating relationship if the residuals are stationary, id est the t-adf for D-lag Zero is higher than the critical value. Significant values are marked with * on the 5% level and ** on the 1%. The table shows that there are no co-integrating relationships when the regression is performed with only one dependent variable.

ADF tests (T=2542, Constant; 5%=-2.86 1%=-3.44)

	Coefficient	Std.Error	t-value	t-prob	part.R^2
Constant	118.798	1.23	96.6	0	0.7857
FTSEMIB USD	0.000282	3.67E-05	7.69	0	0.0227

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob	AIC	F-prob
2	-0.06775	0.99991	1.348	-2.707	0.0068	0.5988	
1	-0.2164	0.99971	1.35	-5.475	0	0.6009	0.0068
0	-0.5195	0.99931	1.357			0.6118	0

	Coefficient	Std.Error	t-value	t-prob	Part.R^2
Constant	38.915	1.497	26	0	0.2099
SXXP USD	0.258737	0.004294	60.3	0	0.5881

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob	AIC	F-prob
2	-0.4434	0.99928	1.078	-6.831	0	0.1512	
1	-0.7475	0.99877	1.087	-19.58	0	0.1686	0
0	-1.692	0.99703	1.166			0.3084	0

	Coefficient	Std.Error	t-value	t-prob	Part.R^2
Constant	102.512	1.868	54.9	0	0.5421
IBEX USD	0.001997	0.000145	13.8	0	0.0698

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob	AIC	F-prob
2	0.1441	1.0002	1.24	-2.452	0.0143	0.4322	

1	0.01252	1	1.241	-8.085	0	0.4338	0.0143
0	-0.4218	0.99947	1.257			0.4584	0

	Coefficient	Std.Error	t-value	t-prob	Part.R^2
Constant	41.7032	1.505	27.7	0	0.2319
UKX USD	0.008979	0.000155	58.1	0	0.5702

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob	AIC	F-prob
2	0.6583	1.001	1.072	-6.946	0	0.14	
1	0.2214	1.0004	1.082	-20.17	0	0.158	0
0	-1.031	0.99823	1.165			0.3059	0

	Coefficient	Std.Error	t-value	t-prob	Part.R^2
Constant	42.6887	1.523	28	0	0.2359
ISF USD	0.088442	0.001559	56.7	0	0.5586

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob	AIC	F-prob
2	0.7219	1.0011	1.087	-6.988	0	0.1686	
1	0.2768	1.0004	1.097	-19.79	0	0.1869	0
0	-0.9676	0.99834	1.179			0.3296	0

	Coefficient	Std.Error	t-value	t-prob	Part.R^2
Constant	49.7294	1.648	30.2	0	0.2638
EFA US Equity	1.33443	0.02773	48.1	0	0.4766

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob	AIC	F-prob
2	1.221	1.0011	0.658	-3.665	0.0003	-0.8355	
1	0.9868	1.0009	0.6596	-0.273	0.7849	-0.831	0.0003
0	0.9717	1.0009	0.6595			-0.8318	0.0012

	Coefficient	Std.Error	t-value	t-prob	Part.R^2
Constant	88.2263	1.498	58.9	0	0.5769

EZU US Equity	1.02927	0.0379	27.2	0	0.2248

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob	AIC	F-prob
2	0.7893	1.0008	0.9203	-3.114	0.0019	-0.1645	
1	0.6042	1.0006	0.9219	-3.14	0.0017	-0.1615	0.0019
0	0.4206	1.0004	0.9235			-0.1584	0.0001

Appendix: The Johansen test tables

The following table is a test provided by eViews. The test determines if there is a co-integrating relationship between the included variables. The test uses information criteria to the test for co-integrating relationships in five ways, all with no or different trends or intercepts.
Testing with all variables included, except UKX.

The table includes the results from a test perfomed by eviews. The test determines via information criteria whether the series should be modelled with a trend and/or a intercept.

Sample: 3/31/2004 12/31/2013

Included observations: 2540

Series: EFA_USD EZU_USD FTSEMIB_USD IBEX_USD ISF_USD SPY_USD SXXP_USD

Lags interval: 1 to 4

Selected (0.05 level*) Number of Cointegrating Relations by Model

Data Trend:	None	None	Linear	Linear	Quadratic
Test Type	No Intercept	Intercept	Intercept	Intercept	Intercept
	No Trend	No Trend	No Trend	Trend	Trend
Trace	0	0	0	0	0
Max-Eig	0	0	0	0	0

*Critical values based on MacKinnon-Haug-Michelis (1999)

Information Criteria by Rank and Model

Data Trend:	None	None	Linear	Linear	Quadratic
Rank or	No Intercept	Intercept	Intercept	Intercept	Intercept
No. of CEs	No Trend	No Trend	No Trend	Trend	Trend
Log Likelihood by Rank (rows) and Model (columns)					
0	-49534.98	-49534.98	-49529.66	-49529.66	-49524.22
1	-49516.22	-49514.02	-49508.78	-49508.04	-49502.64
2	-49505.89	-49499.40	-49494.73	-49493.16	-49487.90
3	-49498.64	-49489.23	-49485.18	-49480.82	-49476.09

4	-49491.45	-49482.03	-49478.82	-49471.28	-49467.46
5	-49487.60	-49477.81	-49474.82	-49465.21	-49462.44
6	-49485.13	-49474.02	-49472.16	-49461.23	-49459.69
7	-49485.04	-49471.56	-49471.56	-49458.70	-49458.70

Akaike Information Criteria by Rank (rows) and Model (columns)

0	39.15825	39.15825	39.15958	39.15958	39.16081
1	39.15450	39.15356*	39.15416	39.15436	39.15483
2	39.15739	39.15386	39.15412	39.15446	39.15425
3	39.16271	39.15767	39.15762	39.15655	39.15598
4	39.16807	39.16380	39.16364	39.16085	39.16021
5	39.17606	39.17229	39.17151	39.16788	39.16728
6	39.18514	39.18112	39.18044	39.17656	39.17614
7	39.19609	39.19099	39.19099	39.18637	39.18637

Schwarz Criteria by Rank (rows) and Model (columns)

0	39.60889*	39.60889*	39.62631	39.62631	39.64363
1	39.63733	39.63868	39.65308	39.65559	39.66985
2	39.67241	39.67347	39.68523	39.69017	39.70145
3	39.70991	39.71177	39.72092	39.72675	39.73537
4	39.74746	39.75239	39.75912	39.76554	39.77179
5	39.78764	39.79537	39.79919	39.80705	39.81105
6	39.82891	39.83868	39.84031	39.85022	39.85209
7	39.87205	39.88304	39.88304	39.89452	39.89452

The previous table shows that the cointegration test should be performed with a linear deterministic trend. The following table employs the Johansen cointegration test based on a linear deterministic trend.

Sample (adjusted): 4/06/2004 12/31/2013

Included observations: 2541 after adjustments

Trend assumption: Linear deterministic trend

Series: EFA_USD EZU_USD FTSEMIB_USD IBEX_USD ISF_USD SPY_USD SXXP_USD

Lags interval (in first differences): 1 to 3

Unrestricted Cointegration Rank Test (Trace)

Hypothesized		Trace	0.05	
No. of CE(s)	Eigenvalue	Statistic	Critical Value	Prob.**
None	0.017311	117.0119	125.6154	0.1483
At most 1	0.011017	72.64032	95.75366	0.6303
At most 2	0.007078	44.48993	69.81889	0.8467
At most 3	0.004992	26.43960	47.85613	0.8746
At most 4	0.003004	13.72191	29.79707	0.8559
At most 5	0.002033	6.077962	15.49471	0.6863
At most 6	0.000357	0.906304	3.841466	0.3411

Trace test indicates no cointegration at the 0.05 level

* denotes rejection of the hypothesis at the 0.05 level

**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized		Max-Eigen	0.05	
No. of CE(s)	Eigenvalue	Statistic	Critical Value	Prob.**
None	0.017311	44.37161	46.23142	0.0782
At most 1	0.011017	28.15039	40.07757	0.5511
At most 2	0.007078	18.05034	33.87687	0.8751
At most 3	0.004992	12.71769	27.58434	0.9000
At most 4	0.003004	7.643948	21.13162	0.9242
At most 5	0.002033	5.171658	14.26460	0.7200
At most 6	0.000357	0.906304	3.841466	0.3411

Max-eigenvalue test indicates no cointegration at the 0.05 level

* denotes rejection of the hypothesis at the 0.05 level

**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegrating Coefficients (normalized by b'*S11*b=I):

EFA_USD	EZU_USD	FTSEMIB_USD	IBEX_USD	ISF_USD	SPY_USD	SXXP_USD
-0.806115	0.475294	-0.000405	-0.000366	0.031864	-0.200906	0.099872
-0.275541	-0.518278	5.29E-05	0.001770	0.026936	0.102520	-0.052134
-0.049624	0.004152	3.58E-05	-0.000223	-0.013141	-0.024600	0.053460
-0.686233	0.509439	1.25E-05	0.000405	0.006857	0.107219	-0.021597
0.161073	-0.154736	9.61E-05	0.000434	-0.004315	0.031596	-0.015749
-0.019929	0.070223	-0.000115	0.000471	-0.013305	-0.015409	0.027798
0.399565	-0.498699	0.000127	0.000185	0.010438	-0.057510	-0.041853

Unrestricted Adjustment Coefficients (alpha):

D(EFA_USD)	0.017093	0.008312	0.051472	-0.009003	-0.021369	0.009657	-0.003771
D(EZU_USD)	0.014270	0.011688	0.042472	-0.013633	-0.011994	0.002458	-0.002305
D(FTSEMIB_USD)	-16.89271	-0.069832	24.81940	-2.495155	-11.63698	-3.748310	0.697195
D(IBEX_USD)	-4.301999	-5.434888	10.28191	-2.071032	-4.183419	-2.485268	-0.887754
D(ISF_USD)	-0.551908	-0.170965	0.811270	-0.142480	-0.141790	0.157918	-0.040636
D(SPY_USD)	0.024634	0.016591	0.054853	-0.047191	-0.048841	0.017828	-0.005757
D(SXXP_USD)	-0.217541	0.012339	0.209321	-0.024790	-0.097446	-0.012524	-0.022340

1 Cointegrating Equation(s): Log likelihood -49561.43

Normalized cointegrating coefficients (standard error in parentheses)

EFA_USD	EZU_USD	FTSEMIB_USD	IBEX_USD	ISF_USD	SPY_USD	SXXP_USD
1.000000	-0.589611	0.000503	0.000454	-0.039528	0.249227	-0.123893
	(0.13829)	(6.1E-05)	(0.00037)	(0.00711)	(0.04790)	(0.02270)

Adjustment coefficients (standard error in parentheses)

D(EFA_USD)	-0.013779
	(0.01296)
D(EZU_USD)	-0.011504
	(0.00993)
D(FTSEMIB_USD)	13.61746
	(6.38208)
D(IBEX_USD)	3.467904
	(2.81016)
D(ISF_USD)	0.444901
	(0.19047)
D(SPY_USD)	-0.019858
	(0.02257)
D(SXXP_USD)	0.175363
	(0.05935)

2 Cointegrating Equation(s): Log likelihood -49547.36

Normalized cointegrating coefficients (standard error in parentheses)

EFA_USD	EZU_USD	FTSEMIB_USD	IBEX_USD	ISF_USD	SPY_USD	SXXP_USD
1.000000	0.000000	0.000337	-0.001187	-0.053424	0.100952	-0.049170
		(6.7E-05)	(0.00024)	(0.00570)	(0.05405)	(0.02281)
0.000000	1.000000	-0.000281	-0.002785	-0.023569	-0.251480	0.126733
		(8.9E-05)	(0.00032)	(0.00757)	(0.07178)	(0.03028)

Adjustment coefficients (standard error in parentheses)

D(EFA_USD)	-0.016069	0.003817
	(0.01370)	(0.01131)
D(EZU_USD)	-0.014724	0.000725
	(0.01049)	(0.00866)
D(FTSEMIB_USD)	13.63670	-7.992815
	(6.74461)	(5.56744)
D(IBEX_USD)	4.965437	0.772069
	(2.96835)	(2.45027)
D(ISF_USD)	0.492009	-0.173711
	(0.20127)	(0.16614)
D(SPY_USD)	-0.024429	0.003110
	(0.02385)	(0.01969)
D(SXXP_USD)	0.171963	-0.109791
	(0.06273)	(0.05178)

3 Cointegrating Equation(s): Log likelihood -49538.33

Normalized cointegrating coefficients (standard error in parentheses)

EFA_USD	EZU_USD	FTSEMIB_USD	IBEX_USD	ISF_USD	SPY_USD	SXXP_USD
1.000000	0.000000	0.000000	0.000510	0.044993	0.217260	-0.365820
			(0.00146)	(0.03532)	(0.19129)	(0.09999)
0.000000	1.000000	0.000000	-0.004201	-0.105720	-0.348564	0.391048
			(0.00125)	(0.03027)	(0.16394)	(0.08569)
0.000000	0.000000	1.000000	-5.039770	-292.2420	-345.3670	940.2666
			(4.30621)	(104.421)	(565.520)	(295.592)

Adjustment coefficients (standard error in parentheses)

D(EFA_USD)	-0.018623	0.004030	-4.64E-06

	(0.01369)	(0.01128)	(6.6E-06)
D(EZU_USD)	-0.016832	0.000902	-3.64E-06
	(0.01048)	(0.00864)	(5.0E-06)
D(FTSEMIB_USD)	12.40506	-7.889772	0.007728
	(6.74285)	(5.55667)	(0.00324)
D(IBEX_USD)	4.455204	0.814756	0.001823
	(2.96824)	(2.44608)	(0.00143)
D(ISF_USD)	0.451750	-0.170343	0.000244
	(0.20114)	(0.16576)	(9.7E-05)
D(SPY_USD)	-0.027151	0.003337	-7.14E-06
	(0.02387)	(0.01967)	(1.1E-05)
D(SXXP_USD)	0.161576	-0.108922	9.63E-05
	(0.06273)	(0.05170)	(3.0E-05)

4 Cointegrating Equation(s): Log likelihood -49531.97

Normalized cointegrating coefficients (standard error in parentheses)

EFA_USD	EZU_USD	FTSEMIB_USD	IBEX_USD	ISF_USD	SPY_USD	SXXP_USD
1.000000	0.000000	0.000000	0.000000	0.028577	0.141737	-0.282470
				(0.02485)	(0.09213)	(0.07318)
0.000000	1.000000	0.000000	0.000000	0.029530	0.273676	-0.295678
				(0.02620)	(0.09712)	(0.07714)
0.000000	0.000000	1.000000	0.000000	-129.9959	401.0710	116.4724
				(33.1452)	(122.872)	(97.5955)
0.000000	0.000000	0.000000	1.000000	32.19315	148.1095	-163.4587
				(12.1060)	(44.8779)	(35.6458)

Adjustment coefficients (standard error in parentheses)

D(EFA_USD)	-0.012446	-0.000556	-4.75E-06	-6.69E-06
	(0.01757)	(0.01393)	(6.6E-06)	(3.0E-05)

D(EZU_USD)	-0.007476	-0.006044	-3.81E-06	4.52E-07
	(0.01345)	(0.01066)	(5.0E-06)	(2.3E-05)
D(FTSEMIB_USD)	14.11731	-9.160900	0.007697	-0.000488
	(8.65245)	(6.86137)	(0.00324)	(0.01475)
D(IBEX_USD)	5.876414	-0.240307	0.001798	-0.011181
	(3.80867)	(3.02026)	(0.00143)	(0.00649)
D(ISF_USD)	0.549525	-0.242928	0.000242	-0.000339
	(0.25809)	(0.20466)	(9.7E-05)	(0.00044)
D(SPY_USD)	0.005233	-0.020704	-7.73E-06	-1.10E-05
	(0.03062)	(0.02428)	(1.1E-05)	(5.2E-05)
D(SXXP_USD)	0.178588	-0.121551	9.60E-05	4.48E-05
	(0.08050)	(0.06383)	(3.0E-05)	(0.00014)

5 Cointegrating Equation(s): Log likelihood -49528.15

Normalized cointegrating coefficients (standard error in parentheses)

EFA_USD	EZU_USD	FTSEMIB_USD	IBEX_USD	ISF_USD	SPY_USD	SXXP_USD
1.000000	0.000000	0.000000	0.000000	0.000000	-0.111910	-0.067902
					(0.14674)	(0.04938)
0.000000	1.000000	0.000000	0.000000	0.000000	0.011568	-0.073953
					(0.15266)	(0.05137)
0.000000	0.000000	1.000000	0.000000	0.000000	1554.893	-859.5836
					(638.104)	(214.745)
0.000000	0.000000	0.000000	1.000000	0.000000	-137.6317	78.25910
					(154.960)	(52.1495)
0.000000	0.000000	0.000000	0.000000	1.000000	8.875837	-7.508360
					(5.05550)	(1.70136)

Adjustment coefficients (standard error in parentheses)

D(EFA_USD)	-0.015887	0.002750	-6.81E-06	-1.60E-05	0.000123

	(0.01775)	(0.01415)	(6.8E-06)	(3.1E-05)	(0.00071)
D(EZU_USD)	-0.009408	-0.004188	-4.96E-06	-4.75E-06	0.000170
	(0.01359)	(0.01083)	(5.2E-06)	(2.4E-05)	(0.00055)
D(FTSEMIB_USD)	12.24291	-7.360241	0.006579	-0.005534	-0.833190
	(8.74179)	(6.96645)	(0.00333)	(0.01513)	(0.35137)
D(IBEX_USD)	5.202576	0.407018	0.001396	-0.012994	-0.414733
	(3.84854)	(3.06696)	(0.00147)	(0.00666)	(0.15469)
D(ISF_USD)	0.526686	-0.220988	0.000228	-0.000401	-0.033217
	(0.26085)	(0.20787)	(9.9E-05)	(0.00045)	(0.01048)
D(SPY_USD)	-0.002634	-0.013146	-1.24E-05	-3.22E-05	0.000398
	(0.03093)	(0.02465)	(1.2E-05)	(5.4E-05)	(0.00124)
D(SXXP_USD)	0.162892	-0.106472	8.66E-05	2.50E-06	-0.009100
	(0.08134)	(0.06482)	(3.1E-05)	(0.00014)	(0.00327)

6 Cointegrating Equation(s): Log likelihood -49525.56

Normalized cointegrating coefficients (standard error in parentheses)

EFA_USD	EZU_USD	FTSEMIB_USD	IBEX_USD	ISF_USD	SPY_USD	SXXP_USD
1.000000	0.000000	0.000000	0.000000	0.000000	0.000000	-0.134400
						(0.01944)
0.000000	1.000000	0.000000	0.000000	0.000000	0.000000	-0.067079
						(0.03431)
0.000000	0.000000	1.000000	0.000000	0.000000	0.000000	64.35220
						(83.8669)
0.000000	0.000000	0.000000	1.000000	0.000000	0.000000	-3.523235
						(15.0602)
0.000000	0.000000	0.000000	0.000000	1.000000	0.000000	-2.234234
						(0.30070)
0.000000	0.000000	0.000000	0.000000	0.000000	1.000000	-0.594212
						(0.13906)

Adjustment coefficients (standard error in parentheses)

D(EFA_USD)	-0.016080	0.003429	-7.92E-06	-1.14E-05	-5.86E-06	-0.005637
	(0.01776)	(0.01419)	(7.0E-06)	(3.2E-05)	(0.00074)	(0.00406)
D(EZU_USD)	-0.009457	-0.004015	-5.25E-06	-3.59E-06	0.000137	-0.004592
	(0.01359)	(0.01086)	(5.4E-06)	(2.4E-05)	(0.00057)	(0.00311)
D(FTSEMIB_USD)	12.31760	-7.623459	0.007010	-0.007297	-0.783320	2.198657
	(8.74281)	(6.98818)	(0.00345)	(0.01558)	(0.36673)	(2.00126)
D(IBEX_USD)	5.252104	0.232495	0.001681	-0.014164	-0.381668	-0.261768
	(3.84878)	(3.07635)	(0.00152)	(0.00686)	(0.16144)	(0.88100)
D(ISF_USD)	0.523539	-0.209898	0.000210	-0.000327	-0.035318	0.051206
	(0.26087)	(0.20851)	(0.00010)	(0.00046)	(0.01094)	(0.05971)
D(SPY_USD)	-0.002989	-0.011894	-1.45E-05	-2.38E-05	0.000161	-0.011475
	(0.03093)	(0.02472)	(1.2E-05)	(5.5E-05)	(0.00130)	(0.00708)
D(SXXP_USD)	0.163141	-0.107352	8.80E-05	-3.39E-06	-0.008933	0.034277
	(0.08135)	(0.06502)	(3.2E-05)	(0.00014)	(0.00341)	(0.01862)