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# Optimization and Management of Renewable Natural Resources

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Introduction

**Introduction**

This thesis is a collection of four articles, concerning the optimal management of renewable resources. The resources in question range in generality from generic theoretical natural capital, which also can be interpreted as environmental quality, to red deer in western Norway. The emphasis is on the exploitation of land-based resources; that is, domestic livestock or wildlife. In all papers a well-defined single managing unit is assumed, which is a rational profit maximizer with full information. The planning horizon is infinite, and all models are deterministic. Another common feature of the models presented here is that they all contain more than one state variable. In two of the papers the interaction between man-made and natural capital is studied, while one paper studies the optimal control of a two-species ecosystem and another is a stage structured analysis where the animal stock is divided into several age categories. While all of the articles are predominantly theoretical, three of the papers contain numerical simulations, and two of them with parameters values taken from empirical surveys. The dynamic nature of the models is emphasized, and analytical solutions and interpretation of dynamic processes are provided.

**Chapter 1: On the optimal control of an animal-vegetation ecological system**

This paper studies the optimal management of a grazing system. The animal stock interacts with the vegetation in a predator-prey system, which is controlled by the harvesting of animals, and that is either globally stable or displays perpetually oscillating behavior when unexploited. The underlying assumption is that while vegetation growth follows a logistic growth relationship, growth in the animal stock is density independent. This distinguishes the model from other contributions on optimal pastoral livestock management (e.g. Perrings, 1997). It is shown however, that a simle harvesting rule may lead to stability.

The managing unit maximizes discounted profits over an infinite planning horizon. Income derives from meat production, and costs are associated with stock maintenance only. Both optimal steady states, with comparative statics, and the optimal approach path is characterized analytically. It is shown that an increase in the meat price gives a larger optimal steady stae animal stock. A change in the discount rate, on the other hand, is shown to have an ambiguous impact on the optimal herd size, at least for small discount rates. This leads to the paradoxical situation that a higher discount rate may give a higher current value profit in the steady state. The optimal approach path is shown to exhibit a combination of singular and bang-bang control, where the singular trajectory is a saddle path in state-state space.

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The model is illustrated with a numerical analysis, where it is found that the optimal harvest program, which can be approximated by a constant grazing pressure policy, represents a considerably increase in discounted profit, compared to a constant harvest rate policy. The paper concludes with a discussion of a concave positive stock effect, where the intrinsic value of livestock is taken into account. It is found that this effect leads to a larger herd in optimum, that a price increase now works in the direction of a smaller animal stock, and that the stability properties of the optimal solution and the effect of discounting is not qualitatively different from the baseline model.

**Chapter 2: Optimal exploitation of a renewable resource with capital limitations.**

The second paper studies the optimal management of a renewable resource, with man-made capital requirements. Interaction between natural and man-made capital is treated in the neoclassical growth literature and in the literature on Hartwicks rule, in the context of non-renewable resource management. (Dasgupta and Heal, 1974; Hartwick, 1977; Asheim et.al., 2003). Within that strand of literature, emphasis is on the optimal transition from a resource dependent to a non-resource dependent economy. With a renewable resource, on the other hand, the focus is on optimal steady states where both stocks are co-existing. There has been seminal contributions within the fisheries literature on this issue (in particular, Clark et. al., 1979), but none that we are aware of, where the renewable resource is a domestic animal stock. The differences are important: in domestic livestock management the capital stock does not contribute to the harvesting process, but instead to keeping the animals, in our example through the winter time. Such capital requirements may also be present in e.g. rangeland management and fish farming. We assume not only that the harvesting cost is stock independent, but also that the growth rate of the animal stock is exogenous, and hence assume away any density dependence. The implied linearity of the model, along with pure profit maximization, yields a model that is analytically tractable. Both the optimal steady state and transition paths are analyzed. The paper concludes with a numerical example that is related to sheep farming in Northern Scandinavia.

The comparative static analysis, where both situations where the equilibrium is interior and where the steady is constrained by the maximum annual investment, shows that changes in the meat price and the discount rate have expected effects on the optimal steady state animal and capital stock; a higher meat price leads to larger steady state stocks, while increasing the discount rate yields the opposite effect. For the dynamic workings of the model, the

assumption of irreversibility in both stocks is crucial; live animals cannot be bought at the market and capital cannot be sold. It is found that the approach path towards the optimal steady state is a combination of bang-bang and singular controls, where in general one of the two controls, harvest of the animal stock and investment in produced capital, is interior along the approach path close to the equilibrium. In contrast to the typical fishery model, the capital stock will never overshoot the steady state, but may undershoot the equilibrium when the initial animal stock is small. The animal stock may both undershoot and overshoot the steady state, depending on the initial capital stock.

**Chapter 3: Balancing income and cost in red deer management**

This paper presents a bioeconomic analysis of a red deer population within a Norwegian institutional context. The population is managed by a well-defined managing unit, typically consisting of many landowners cooperating to maximize the present-value income from hunting. Hunting income consists only of profits from selling the meat, net of grazing damage costs, in the baseline scenario, while the effect of including the recreational aspect of hunting is analyzed subsequently. The red deer population is structured in five categories; calves, female and male yearlings, adult females and adult males. The different ways to compose the harvest across age and sex categories and the implied economic benefits are highlighted. A numerical analysis, with parameter values from a Norwegian deer population survey, shows results with respect to the optimal harvest composition, and the effects of parameter changes and constraints.

In the baseline case where the recreational value is excluded, we find that only adult males are harvested extensively. The females and calves are harvested moderately, and calves are harvested only due to the imposed code of conduct restriction whereby the number of calves harvested must be equal to or higher than the number of adult females harvested. There is no harvest of yearlings. This harvest pattern is found to persist when the price of meat goes up and when the grazing damage is ignored by the landowner, although the impact on total population size and economic benefits of these changes are substantial. Relaxing the code of conduct constraint leads to a moderate increase in the harvest rate of adult females, while no calves are harvested. The effect of replacing the optimal harvest composition by a uniform harvest pattern across all categories is shown to be very large, leading to a considerable decrease in net economic benefit.

Without any restrictions on the harvest, and ignoring the recreational aspect of the hunting, we find that the 'biological discounted' values, defined as the ration between the per animal meat value (marginal gain) and the survival rate (marginal loss) for the various categories of animals are instrumental in determining the optimal harvesting composition, without fertility playing any direct role. This seems to be a general result in stage structured models where harvest value is maximized, a result obtained first in the classical Reed (1980) fishery article, see also Tahvonen (2008).

The optimal adjustment path towards the optimal steady state harvest regime is shown to display a typical bang-bang approach where each category is controlled so as to approach the optimal steady state as rapidly as possible. However, when the recreational hunting value is included in the objective function, we find that the optimal harvesting scheme involved oscillating harvest rates. This result is not a new one within the resource management literature (see, e.g. Clark, 1990), and is in this model a consequence of the objective function being no longer concave in the harvest.

**Chapter 4: Sustainable production and the ambiguous effect of discounting**

It is generally acknowledged that a high discount rate implies low levels of man made capital and also a deteriorated environment in the long run. The impact of discounting has been discussed in the sustainable production literature, involving both produced capital and renewable resources, see e.g. by Beltratti et. Al. (1995). Moreover, within the climate change literature, discounting has been heavily debated; a good summary of the different aspects of this debate is provided by Dasgupta (2008). He is, among other thing, concerned about the lack of sensitivity analysis with respect to the impact of discounting in the various integrated assessment models (e.g. Nordhaus, 1994). However, discounting can have surprising effects when more than one state variable is taken into account, something that is not heavily emphasized in this literature.

The aim of this paper is to explore whether discounting can have surprising effects in a theoretical model of production with two inputs; natural and man-made capital. Natural capital can be interpreted as a generic renewable resource, or as environmental quality. The model is deterministic, and growth in population and technology is abstracted from throughout the analysis. The paper is divided into two parts. In the first part, consumption of the produced composite is the only control variable, whereas resource extraction, or

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emissions, is a constant fraction of output. It is shown that either one of the two stock may increse with discounting, for moderate discount rates. Under the assumption that there is a one-to one-relationship between the two stocks across optimal steady states, this implies the existence of paradoxical conumption behavior, where instanteneous stedy state utility increases with discounting for a range of discount rates. In this situation, excess patience leads to overinvestment and too high economic activity in the medium run. The impatioent society may be better off in the long run.

In the second part of the paper, a model is presented where also abatement effort is taken into account as a second control variable. As there is no longer a one-to-one relationship between the two stocks across steady states, the conditions for paradoxical steady states to occur now becomes more difficult to identify. It is shown however, that with the most common production functions, both assets decrease with discounting. However, when utility depends also on environemental quality, there is a possibility that the environmental quality may increase with discounting also with CES production, as the environment is now a consumer good. Letting also emissions into the utility function complicates the picture further, and the relationship between discounting and future becomes increasingly model specific.

**Discussion**

There are several aspects that link together the four papers in this thesis. Obviously, they are all dealing with renewable resource management problems. The three first essays here concern land-based resources; domestic livestock management, semi-domestic herding practice and wildlifde hunting activity. The largest bulk of literature on renewable resource management has been produced within the economics of fisheries, and the managment of terrestrial animal species has not attracted nearly as much attention within the economics literature. Aspects that pertain mainly to terrestrial animal management that are particularly important in the articles presented here include the following. For one, domestic animal stocks, as opposed to wildlife and marine fish stocks, require maintenance. Both land and labour, and to some extent capital structures such as fencing and buildings, are involved in the maintenance process, which all have an opportunity cost. Hence, operating costs are mainly associated with stock size, and not with harvesting in the models presented here. Another property of domestic animal stocks is that growth is not density dependent, something which affects the dynamic workings of the models. Wildlife, on the other hand, is generally density dependent and may not require extensive maintenance, thus being more reminiscent of the

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marine fisheries from a modeling perspective. However, wildlife management may often stand in conflict with alternative land use such as agriculture and other types of human activity. Crop damage, vehicle collisions and other types of nuisance represent possible costs here, that are related to stock size and may, moreover, be of a public nature.

In addition, terrestrial animal species typically have functions aside from being mere providers of meat. The intrinsic value of an animal stock derives from various sources, such as alternative uses of the animals (to confine pests, conserve the cultural landscape, provide means of transport, etc), pure amenity value, leisure activities such as horse riding and wildlife viewing, etc., which are related to the standing stock, and not to harvest. These positive stock effects may be of a privat type, i.e. as a source of social status in the local comminity, but also public, such as the preservation of bidiversity. An early contribution on the topic is the theoretical analysis of 'wealth effects' in a neoclassical growth model by Kurtz (1968). Intrinsic stock values in the context of terrestrial animal resources was introduced by Schulz and Skonhoft (1996). Subsequently, the idea of natural resources having a value of its own has worked its way into the literature. Overviews are provided by Barbier and Swanson (1992), and van Kooten and Bulte (2000).

From a technical perspective, the absence of density dependence among domestic animal stocks introduces linearity into the models, something which may simplify the analysis, but also lead to interesting dynamic developments; both effects of which are represented in the papers included here. Another notable result is that, when operating costs are mainly related to stock size, the size of the steady state animal stock, as well as the amount of inputs required in the operation process, generally increases with the price of meat. The effect of discouting, will generally not be qualitatively different from established insights, however, as an increase in the discount rate gives a more agressive harvesting regime and hence a smaller steady state animal stock. As is shown e.g. in the present thesis however, when several state variable interact this established insight may be subject to revision.

Another common feature of the models presented here are that they are all multi-dimensional. Analytical work on resource problems with several state variables is scarce, for good reason. It is often impossible to obtain insights form mulit-dimentional models that are general, and not specific to the model at hand. Nevertheless, analytical insights are emphasized in these papers, and do to a certain extent appear to have an intuitive economic interpretation. Some

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are surprising, such as the complicated role of discounting, other are more familiar. Dynamic issues become particularly intereseting in multidimentional models, and attempts have been made to analyze optimal approach paths explicitly and to focus on stability problems. Given that multi-dimentional systems are ubiquitous in nature, it is conceivable that these models may have applications beyond the particular cases studied here.

To keep the models tractable however, something must be sacrificed. Of the many aspects that are not treated in this thesis, two seem worth mentioning in particular. First, there is no strategic interaction in none of the models; the only actor present is a single fully informed profit maximizer. This might be reasonable in the case of a single price taking sheep farmer, but for the whole reindeer business in Northern Scandinavia this is clearly an unsatisfactory assumption. Without doubt, strategic behaviour would have to be included to give a realistic picture of the current situation here. Second, the omission of stochastic factors are an obvious neglect in any model of the natural environment. The consequences of increases uncertainty for the optimal management policy would be of great interest in all of the models here, which may be a good starting point for further reserarch.

**References**

Asheim, G., W. Buchholz and C. Withagen 2003. The Hartwick rule: Myths and facts. Environmental and Resource Economics 25: 129-150.

Barbier, E. and T. Swanson, 1992. Economics for the Wilds. Island Press. Washington D.C.

Beltratti, A., G. Chichilniski and G. Heal, 1995. Sustainable growth and the green golden rule. In Golsin, I. and L. A. Wintersd, eds. The Economics of Sustainable Development. Cambridge Univ. Press. Cambridge, New York.

Dasgupta, P., 2009. Discounting climate change. Journal of Risk and Uncertainty 37: 141-169.

Dasgupta, P. S. and G. M. Heal, 1974. The optimal depletion of exhaustible resources. Review of Economic Studies 41: 3–28.

Clark, C., 1990. Mathematical Bioeconomics. Wiley, New York.

Clark, C. W., F. H. Clarke and G. R. Munro 1979. The optimal exploitation of renewable resource stocks: problems of irreversible investment. Econometrica 47: 25-47.

Hartwick, J., 1977. Intergenerational equity and the investing of rents from exhaustible resources. American Economic Review 67: 972–974.

van Kooten G.C. and E.H. Bulte, 2000. The Economics of Nature: Managing Biological Assets. Blackwell, Malden Mass.

Kurz, M., 1968. Optimal economic growth and wealth effects. International Economic Review 9: 348-357.

Nordhaus, W., 1994. Managing the global commons: The economics of climate change. MIT Press. Cambridge, Mass.

Perrings, C. 1997. Economics of ecological resources. Cheltenham, UK. Edward Elgar.

Reed, W. J. 1980. Optimal age-specific harvesting in a nonlinear population model, Biometrics 36: 579–593

Tahvonen, O. 2008. Harvesting an age structured population as biomass_ Does it work? Natural Resource Modelling 21: 525-550.

**Chapter 1**

**On the optimal control of an animal-vegetation ecological system**

**Chapter 2**

**The optimal exploitation of a renewable resource with capital limitations**

**Optimal exploitation of a renewable resource with capital limitations.**

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**Abstract**

A model of interaction between a renewable natural resource with capital limitations, as exemplified by the optimal investment problem of sheep farming in a Nordic context, is analyzed. The model builds on existing studies from the fisheries literature, but the important difference is that while capital is related to harvesting effort in the fisheries, capital attributes to production capacity to keep the animal stock during the winter in our farm model. The paper provides several results where both optimal steady states and the optimal approach paths are characterized analytically. The results are further supported by a numerical example.

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**1.Introduction**

Following the pioneering work of Smith (1968), economic models of renewable resource management have occasionally been extended to include investment in man-made capital. Even though most, if not all, contributions to this strand of literature have been related to fishery management problems, spurred by the seminal contribution of Clark et al. (1979), much of the conclusions obtained here can probably quite easily be extended into the management of other types of wild natural resources, like terrestrial wildlife. In this paper, we look at another type of renewable management problem with capital limitations, namely domestic livestock management. The important difference is that while capital determines the fishing effort in the fishery problem, capital is related to the capacity to keeping animals during the indoors winter season in our farm problem, which is exemplified by sheep farming in a Nordic context.

The literature on the management of what may be viewed as two capital stocks, one man-made and the other one biological, is quite small. Clark et al. (1979) emphasized the irreversibility of investment, meaning that man-made capital cannot be sold once having been bought, and they showed how the possible approach paths towards the optimal steady state is greatly affected by this property. Their model is linear in both controls, investment in fishing vessels and harvest of the fish stock, and the approach paths are therefore characterized by a combination of bang-bang and singular controls. Stochastic elements are included in a paper by Charles and Munro (1985), and McKelvey (1985) analyzes open access dynamics in a fishery with man-made capital. Boyce (1995) formulates a similar model to that of Clark et al. (1976), but with non-linear investment costs. He finds, not surprisingly, that the derived optimal approach path is no longer of the bang-bang type. Sandal et al. (2007) extend the literature with a model without any non-negative constraint on investment, but where capital is less valuable when sold than when bought.

In this paper we analyze the optimal investment and harvest, or stocking, decision problem of a sheep farmer. The farmer, assumed to be well-informed and rational, aims to maximize present-value profit generated by meat production. The market price of meat is taken as given, as we consider a single farm, and abstract from both exogenous price fluctuations and (other) stochastic factors such as climatic variations. In addition to the natural capital stock, the animals, the farmer must also hold a certain amount of man-made capital which adheres to the

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familiar mechanisms of investment and depreciation, to keep the animals indoors during the winter season. Man-made capital in this farming system is thus mainly buildings and related equipment which is instrumental in determining farm capacity. We are not aware of other theoretic domestic livestock management models that include man-made capital in addition to animal capital, even though capital theoretical treatments of livestock are frequently found within the resource economics literature, see e.g. Kennedy (1986). Farm models include Jarvis (1974) who formulated a timing problem of cattle grazing, and Skonhoft (2008) who analyzed the optimal stocking problem of Nordic sheep farming. Our model and reasoning builds to some extent on this last paper, but Skonhoft studied a situation with no man-made capital limitations and with different year classes of the animal capital. Different year classes are not included in the present paper. The research problem here is to find the optimal slaughtering and investment policy in such a Nordic farming system, and to characterize both the optimal steady state and approach paths.. In the subsequent analysis, natural and man-made capital will generally be referred to merely as 'animals' and 'capital', respectively.

The rest of the paper is organized as follows. Section 2 describes briefly the Nordic sheep farming system and the model is formulated. Section 3 describes the optimal solution to the model while we in section 4 proceed to analyze the steady states. Having identified the optimal steady states, we analyze in section 5 the approach path and show that it involves a combination of bang-bang and singular controls. Numerical simulations are shown in section 6, while section 7 concludes the paper.

## 2. Model

The following analysis is related to economic and ecological conditions found in Norway, but these also exist in Iceland and Greenland. There are about 2.1 million sheep in Norway during the summer grazing season, divided among some 16,000 family farms. The average farm size is therefore quite small and accounts only for about 130 animals. Sheep farms are located either close to mountain areas and other sparsely populated areas or along the coast, with a means to transport the animals to more distant alpine areas with access to areas of summer grazing land. Such land is typically communally owned and managed. There is a sharp distinction between the summer grazing season and the winter indoors season. While food is abundant during the summer grazing season, housing and indoor feeding is required throughout winter because of snow and harsh weather conditions. The indoors winter season is typically from mid-October to the beginning of May next year. The adult sheep and the

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newborn lambs are then released for outdoors grazing. In September- October slaughtering takes place. In Norway, winter feeding basically consists of hay grown on pastures close to farms, with the addition of concentrate pellets provided by the industry. The main product is meat, which accounts for about 80% of the average farmer's income. The remainder comes from wool, because sheep milk production is virtually nonexistent (Nersten et al. 2003). However, the income from wool is neglected in the following analysis.

We begin with formulating the animal growth equation, given in discrete time, and where additions to the stock occur once a year (in the spring), as does the harvest of animals (in the fall). As our main focus is on the interaction between biological and man-made capital, we do not distinguish between different age classes of animals (but see Skonhoft 2008), but consider a biomass model where 'a sheep is a sheep'. The rate of growth in animal biomass is further assumed constant, as is reasonable with a domestic animal stock facing controlled breeding and maintenance; that is, there is no density dependent growth process. The growth function for animal biomass is thus given as:

(1) $\quad X_{t+1} - X_t = rX_t - H_t,$

where $X_t$ is the animal stock size at time (year) $t$, $H_t$ is harvest and $r > 0$ is the animal stock growth rate, assumed to be constant. A feature of Scandinavian sheep farming is that live animal are generally not traded. Therefore, we do not consider the possibility of restocking and require $H_t \geq 0$.

Man-made capital, also assumed to be homogenous, is used as housing for the animal stock during the winter indoors season. Each year, a constant positive amount of investment is allowed, and a constant fraction of the capital stock depreciates due to wear and tear. The net capital growth is thus given by:

(2) $\quad K_{t+1} - K_t = -\gamma K_t + I_t,$

where $K_t$ is the capital stock and $I_t$ is the accompanying (gross) investment. $\gamma > 0$ is the rate of depreciation, assumed to be fixed.

The revenue of the farmer is made up of income from meat production. With $p > 0$ as the slaughtering price (net of slaughtering costs), the current meat income for the farmer simply reads $pH_t$ and is included as the first term in the profit equation:

(3)　　$\Pi\left(H_t, X_t, K_t, I_t\right) = pH_t - V\left(X_t\right) - Q\left(X_t, K_t\right) - cI_t$.

$p$ is assumed fixed over time and independent of the harvest decision, as explained above (section 1).

We then have the cost side, where we first describe at the operating cost. The operating cost structure differs sharply between the outdoor grazing season and the indoor feeding season. As explained, during the grazing period the sheep may graze on communally owned lands ('commons') or private land. Within the Nordic sheep farming system, such land may be available cost free, or the farmer may pay a fixed annual rent (Austrheim et al. 2008). The variable cost is hence simply assumed to be the indoor season operating cost. These costs, which include labor cost (typically as an opportunity cost), electricity and veterinary costs in addition to fodder, are assumed to determined uniquely by the size of the animal stock, i.e., $V_t = V(X_t)$, and with $V' > 0$, $V'' > 0$, and $V(0) = 0$. The argument for a strictly convex cost function is that fodder production is constrained by the size of the available land; that is, as the stock becomes larger it becomes progressively more costly to provide fodder.

As mentioned, in contrast to what is found in the fisheries literature where capital normally is equivalent to harvesting effort (e.g., Clark et al. 1979), capital in our farm system is housing and related equipment to keep the animal stock during the winter. We assume that there is no absolute constraint on the amount of animals that a given amount of capital can support, so that there is no such thing as 'full' capacity utilization in our farm model. However, as the indoor space per animal diminishes, the operating procedure becomes increasingly cumbersome. We hence include the capacity utilization cost, or congestion cost, function $Q\left(X_t, K_t\right)$ in our current profit equation (3). It increases with the number of animals, for any given amount of capital, such that $Q_x > 0$ and $Q_{xx} > 0$, together with $Q_K < 0$, $Q_{KK} > 0$ and $Q_{KX} < 0$. In addition, we have $Q\left(0, K_t\right) = 0$ when $K_t > 0$ and $\lim_{K_t \to 0} Q(X_t, K_t) = \infty$ for $X_t > 0$. For all positive stock values this function is hence convex in $X_t$ and $K_t$. In the numerical section 6 below we specify this cost function.

The final cost component is the cost of buying new capital equipment. We assume that there is a constraint on the size of investment in each period, due to, say, limited access to credit, such that $I_t \leq I^{\max}$. The cost per unit of investment is fixed and given by $c > 0$, so that the

yearly investment cost reads $cI_t$. An alternative assumption, following e.g. Sandal et al. (2007) , could have been to introduce adjustment costs to limit the amount of investment carried out in each time period. In our model, as in reality, investment is also irreversible; the buildings cannot be sold once having been set up; that is, $I_t \geq 0$.

## 3. Optimal management

The farmer aims to maximize present-value profit subject to the dynamic constraints imposed by the growth equations for animals (1) and capital (2), and the constraints on harvest and investment in each period. We suppose an infinite planning horizon, meaning that we are looking for an optimal steady state. The planning problem of the farmer is then formulated as:

$$max \left\{ \sum_{t=0}^{\infty} \rho^t \left[ pH_t - V(X_t) - Q(X_t, K_t) - cI_t \right] \right\}$$

$$s.t. \quad X_{t+1} - X_t = rX_t - H_t$$

(4)
$$K_{t+1} - K_t = -\gamma K_t + I_t$$

$$0 \leq H_t, \ 0 \leq I_t \leq I^{max}$$

$$X_t, K_t > 0$$

$$X_0, K_0 \text{ given}$$

,

and where $\rho = 1/(1+\delta)$ is the discount factor with $\delta \geq 0$ as the constant discount rate.

The Lagrangean of this problem may be written as:

$$L = \sum_{t=0}^{\infty} \rho^t \left\{ pH_t - cI_t - V(X_t) - Q(X_t, K_t) \right.$$

$$- \rho\lambda_{t+1} \left[ X_{t+1} - (1+r)X_t + H_t \right]$$

$$\left. - \rho\mu_{t+1} \left[ K_{t+1} - (1-\gamma)K_t - I_t \right] \right\},$$

where $\lambda_t$ and $\mu_t$ are the shadow prices of the animal and capital stock, respectively. The necessary conditions for a maximum are:

(5)
$$\frac{\partial L}{\partial H_t} = p - \rho\lambda_{t+1} \leq 0, \ 0 \leq H_t$$

(6)
$$\frac{\partial L}{\partial I_t} = -c + \rho\mu_{t+1} \gtrless 0, \ 0 \leq I_t \leq I^{max}$$

(7)
$$\frac{\partial L}{\partial X_t} = -V' - Q_X + \rho\lambda_{t+1}(1+r) - \lambda_t = 0$$

(8)
$$\frac{\partial L}{\partial K_t} = -Q_K + \rho\mu_{t+1}(1-\gamma) - \mu_t = 0$$

These conditions are also sufficient if the Lagrangean is concave in the states and controls jointly. Since the Lagrangean is linear in the controls, the sufficiency conditions boil down to $L_{XX} = -(V'' + Q_{XX}) \leq 0$, $L_{KK} = -Q_{KK} \leq 0$ and $L_{XX}L_{QQ} - L_{XY}^2 = Q_{KK}(V'' + Q_{XX}) - Q_{KX}^2 \geq 0$, which are satisfied for the given properties of the cost functions. The transversality conditions for the infinite horizon problem must also hold; i.e., $\lim_{t\to\infty} \lambda_t(X_t - X^*) \geq 0$ and $\lim_{t\to\infty} \mu_t(K_t - K^*) \geq 0$ and where the asterisk indicates optimal steady state values.

The interpretation of (5) is that harvest is set to zero whenever the price of meat is lower than the discounted shadow price of the animal stock, and positive otherwise. Similarly, (6) states that there will be positive investment only when the unit investment cost is lower than the discounted shadow price of capital. It can be set to its maximum value, or it can be set to the interior of the control region. This 'singular' control policy is only implicitly defined from the first order conditions. We thus have the following alternatives for harvest policy, letting $H^S$ denote singular harvest:

$$H_t = \begin{cases} \infty & when \quad p > \rho\lambda_{t+1} \quad (impulse\ harvest) \\ H^S & when \quad p = \rho\lambda_{t+1} \\ 0 & when \quad p < \rho\lambda_{t+1} \end{cases}$$

Note that, since there is no upper bound on harvest except from the size of the stock itself, whenever $p > \rho\lambda_{t+1}$, the herd will be reduced immediately (that is, within one time period), to the level where $p = \rho\lambda_{t+1}$, and singular harvest takes over. This is often called impulse control in the optimal control theory literature.

As for investment, we get:

$$I_t = \begin{cases} I^{max} & when \quad c < \rho\mu_{t+1} \\ I^S & when \quad c = \rho\mu_{t+1} \\ 0 & when \quad c > \rho\mu_{t+1} \end{cases}$$

Therefore, investment will be at its maximum or minimum level whenever the per unit investment cost is lower or higher than the shadow price of capital. When the shadow price

reaches the point where it equals the present value of the unit cost of investment, the control will either switch between the two control boundaries – going from maximum to zero investment, or vice versa - or stay at singular investment for some amount of time.

Singular control relationships for both stocks can be further derived from the first order conditions. If singular harvest holds, we have from equation (5) $p = \rho\lambda_{t+1}$, which means that the shadow price of the animal stock is constant, and equalizes $\lambda^* = p / \rho$. When inserted into condition (7), we find the following golden rule condition for the animal stock:

(9) $$(r-\delta)p = V'(X) + Q_X(X,K)$$

Equation (9) therefore describes the relationship between $X$ and $K$ that is consistent with singular harvest. This condition may also be written as

$p = (1/\delta)[pr - V'(X) - Q_X(X,K)]$ indicating that the market revenue from selling one animal should equalize the discounted net benefit from keeping it. Because both $V'$ and $Q_X$ are positive, we must require that the animal growth rate exceeds the discount rate, $r > \delta$, which is a well-known condition for a positive steady state animal stock (see, e.g., Clark 1990). As both $r$ and $\delta$ are constant this must always hold, also outside the steady state.

With singular investment, we have from (6) $c = \rho\mu_{t+1}$, which means that the capital shadow price is constant, $\mu^* = c / \rho$. Inserted into equation (8) gives the golden rule condition for capital:

(10) $$(\gamma + \delta)c = -Q_K(X,K).$$

Equation (10) defines a relationship between $X$ and $K$ that is consistent with singular investment. It may also be written as $c = (1/\delta)[-Q_K(X,K) - \gamma]$ indicating that the unit investment cost should equalize the discounted marginal net benefit from holding capital.

## 4. The steady states

In an interior equilibrium where both $H^* = H^S(K^*, X^*)$ and $I^* = I^S(K^*, X^*)$, the golden rule conditions (9) as well as (10) must hold. But in principle, one, or both, controls may also be set at a boundary at a steady state. From equation (1), however, as long as the rate of animal growth is positive and constant, the harvest rate must be positive in the steady state. Since there is no upper constraint on harvest, the steady state harvest policy must then be singular.

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From equation (2), steady state investment must also be positive, but may be set to its maximum level, where the gross investment in each year equals depreciation, keeping the capital stock at its optimal steady state level given the investment constraint. We therefore have two alternatives for the steady state, and this is stated as:

**Result 1**. There are two steady state alternatives. The first is interior where both controls are singular. In the second harvest is singular while investment is at the maximum level.

We first study the interior steady state in some detail and then discuss the situation where the investment constraint binds. At an interior steady state, the two schedules defined by equations (9) and (10) must intersect. Except from the very special case where the two curves are coinciding, there can be at most a countable number of equilibria. When differentiating (9) and (10) we find $dX / dK = -Q_{XK} / (V'' + Q_{XX}) > 0$ and $dX / dK = -Q_{KK} / Q_{XK} > 0$, respectively. Therefore, both schedules (9) and (10) slope upwards in the $(K, X)$-space, but the curvatures cannot be determined generally without imposing restrictions on third derivatives. This allows for an arbitrary number of intersection points, with a correspondingly arbitrary number of stable and unstable equilibria. However, we find a stable steady state $K^*$ and $X^*$ where the optimal harvest condition (9) intersects the singular investment schedule (10) from above, so that $-Q_{XK} / (V'' + Q_{XX}) > -Q_{KK} / Q_{XK}$ holds at the intersection point. Otherwise, the intersection point is an unstable equilibrium. This holds because a local maximum is found where the Lagrangean is concave around a stationary point which requires that the condition $(V'' + Q_{XX})Q_{KK} - Q_{KX}^2 = \Gamma(X, K) \geq 0$ must be satisfied. Rearranging this expression gives $-Q_{XK} / (V_{XX} + Q_{XX}) > -Q_{KK} / Q_{XK}$ as claimed. When these equilibria are found, the steady state harvest follows from (1) as:

(11) $\qquad H^* = rX^*$,

and the steady state investment from (2) as:

(12) $\qquad I^* = \gamma K^*$.

Equations (9) and (10) and the sufficiency condition can be used to derive some comparative static results about $X^*$ and $K^*$. In a next step, the effects on $H^*$ and $I^*$ follow recursively from equations (11) and (12), respectively. We first look at the effect of a changing meat price and when differentiating (9) and (10) we find $(r - \delta)dp = (V'' + Q_{XX})dX + Q_{KX}dK$ and

$0 = -Q_{KK}dK - Q_{KX}dX$, respectively. Combing these expressions yields the partial price effects $\partial X^* / \partial p = Q_{KK}(r-\delta) / \Gamma(X,K) > 0$ and $\partial K^* / \partial p = -Q_{KX}(r-\delta) / \Gamma(X,K) > 0$. This is stated as:

**Result 2**. An increase in the price of meat will result in a larger stock of animals and man-made capital in an interior optimal steady state.

This result is the opposite of what is found in the standard fishery model (e.g., Clark 1990) where a price increase leads to more aggressive harvest and a lower optimal steady state stock. The reason for the opposite result in our farm model is that costs here are not associated with harvest, but with stock maintenance. With a higher meat price, the farmer thus finds it beneficial to keep a higher stock of both types of capital as the relative maintenance costs decreases. Differentiating (9) and (10) also gives information about the effects of a change in the discount rate. We find positive effects for both stocks, and this is stated as:

**Result 3.** An increase in the discount rate leads to reduced stocks of both animals and capital in the interior optimal steady state.

This result fits conventional economic intuition, but is far from obvious when more than one capital stock is considered and cannot be deduced form the golden rule conditions directly. As shown by e.g. Asheim (2008), paradoxical effects of discounting, such as a positive relationship between discounting and steady state consumption, may result from multi-dimensional models. Also, when there is a trade-off between the two stocks across alternative steady states, something that is typical for predator-prey models, one of the stocks must increase with discounting while the other goes down. However, this does not happen here.

Following the same procedure with respect to the other parameters, all the time assuming that the sufficiency conditions are fulfilled, the other comparative static results can also be computed. All results are reported in Table I where the investment and harvest effects are included as well. An increase in the investments cost or depreciation rate means that it is beneficial for the farmer to reduce the steady state animal and man-made capital stocks, whereas an increase in the growth rate of animals leads to larger optimal stocks of both animals and capital. These results are more or less as expected, and the effects of the

parameters work in the same direction for both stocks. It can also be confirmed that we find the combined effects $\partial X^* / \partial r = \partial X^* / d\gamma - \partial X^* / \partial\delta > 0$, indicating that the negative effect on the animal stock of higher depreciation rate must be smaller than the negative effect of higher discount rate. We also find $\partial K^* / \partial r = \partial K^* / d\gamma - \partial K^* / \partial\delta > 0$, indicating the same different effects on the optimal steady state capital stock.

The effects on the control variables through equations (11) and (12) are quite straightforward, except for the animal growth rate and rate of depreciation which both have direct and indirect effects on the steady state harvest and investment, respectively. For a positive shift in the animal growth rate, the two effects work in the same direction and lead to higher harvest in the steady state, since from (11) $\partial H^* / \partial r = X^* + r\partial X^* / \partial r > 0$. With the depreciation rate, however, the two effects work in the opposite direction as we find $\partial I^* / \partial\gamma = K^* + \gamma\partial K^* / \partial\gamma$ from equation (12). The direct effect is to increase the required amount of investment to maintain a given amount of capital, whereas the indirect effect is to decrease the optimal steady state capital stock. The overall effect is ambiguous with general functional forms.

[Table I about here]

We then consider the other steady state possibility where the investment is no longer singular, and hence the condition

(13) $\quad I^* = I^{max} < I^S$

replaces equation (12) in the interior steady state solution. The steady state capital stock now follows directly through (12) as $K^* = I^{max} / \gamma$ , which inserted into equation (9) yields the steady state animal stock. The amount of capital will now for obvious reasons be below what was found in the interior steady state. Because condition (9) yields a positive relationship between the two stocks, the number of animals will also be below what was found in the interior steady state. The steady state harvest again follows from equation (11) as $H^* = rX^*$, and the number of animals slaughtered will consequently also be below the previous steady state.

The comparative static results are now somewhat different, as indicated with brackets in Table I. When the investment constraint binds, the only factor that affects the steady state

capital stock is the depreciation rate. The sign of the effects on the animal stock are as before, except for the investment cost which now has zero effect.

## 5. Optimal approach paths

We have characterized the two alternatives for an optimal steady state, both the interior solution and the case where the upper investment constraint binds at the optimum. The next task is to study the optimal approach paths. In general, approach paths in multi-dimensional models are often complicated to analyze, as exemplified by the predator-prey model of Mesterton-Gibbons (1996). For a more recent example, see Horan (2005). We find, however, that in our case it is possible to derive an intuitive solution which is easily explained graphically. As indicated above, the optimal trajectories result from a combination of extreme and singular controls. We know that both controls can be singular simultaneously only at an interior steady state, so that one of the control constraints must always bind outside an equilibrium. Whenever the animal stock is above the $H^S$-schedule, it will be harvested down instantaneously (or more precisely, within on time period), until the $H^S$-schedule is reached since $H_t$ is unconstrained from above. Then either i) the system will follow the $H^S$-schedule, with singular harvest for a period of time, or ii) harvest is set to zero, in which case the $H^S$-schedule acts as a switch between extreme controls. Ignoring the case where $H_t > H^S$, which is impossible for any more than one time period, we now consider the various alternative control regimes. The different cases can be best understood with reference to Figure 1 where a situation with a unique interior equilibrium is depicted. The singular control schedules are in accordance with the specific functional forms used in numerical section 6 (Eqs. 9' and 10' below). Four different initial states, labeled A, B, C and D, are shown along with the optimal approach paths originating from them. Notice that initial states A and C have the same value for the animal stock, whereas initial states B and D represent the same capital stock value. These properties are further exploited in the numerical section. Panel a) demonstrates the first three cases, where the upper investment constraint does not bind along the approach path.

Case 1: $H = 0$, $I = I^S$. The only possibility when investment is singular outside of the steady state is that harvest is zero. This happens when, as from an initial situation such as A or B, the initial capital stock is below the steady state level and the system has been controlled to reach the singular investment schedule. The system will then follow the $I^S$-schedule (10) towards the steady state, when the investment constraint does not bind.

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Case 2: $H = H^S$, $I = 0$. From an initial situation such as point D where both stocks are above their interior steady state levels, the animal stock is harvested down until the $H^S$-schedule (9) is reached, and the system moves leftwards along the $H^S$-schedule towards the equilibrium. Note that the $I^S$-schedule plays no role here, and is therefore represented by a dashed line in the figure.

Case 3: $H = 0$, $I = 0$. Here both controls are set to zero, which happens when the state of the system is below both singular control schedules, as at point C. This control regime continues until one of the two singular control schedules is reached, and one of the two above alternatives takes over.

The next two 'intermediate' cases, where the upper investment constraint prevents the system from following the $I^S$-schedule, are shown in panel b) Figure 1. Both these cases depend on the system having reached either the $H^S$-schedule or the $I^S$-schedule below and to the left of the equilibrium. This may happen if the initial states are given by points A or B.

Case 4: $H = 0$, $I = I^{\max}$. When $H^S < 0$, meaning that following the $H^S$-schedule would require restocking of animals, which is omitted in our model, and $I^S > I^{max}$, so that the maximum investment constraint does not allow the system to follow the $I^S$-schedule either, the state of the system will be somewhere below the $H^S$-schedule and above the $I^S$-schedule.

Case 5: $H = H^S$, $I = I^{max}$. This situation arises when the system has reached the $H^S$-schedule (9), either after an initial impulse harvest, or from a situation such as in case 4, but the maximum per period investment is not sufficiently large to detract the system from the $H^S$-schedule. The system will then follow the $H^S$-schedule to the steady state.

The last situation to consider is the alternative steady state where the upper investment constraint is binding. Panel c) Figure 1 demonstrates. The equilibrium can now be found as a point on the $H^S$-schedule, below and to the left of the intersection point, with investment set to its maximum level at every point in time, $I^* = I^{\max} < I^S$. As shown in the figure, the

approach path is along the $H^S$-schedule from both directions in this case. However, for sufficiently low stock values it is still possible to follow the $I^S$-schedule, as the animal stock growth is a constant share of the animal stock size, whereas the maximum investment is assumed to be independent of the size of the existing capital stock.

The different control scenarios can be further characterized by dividing the state-state space into three regions, with three different transitional control regimes. Region I: Above the $H^S$-schedule; impulse harvest, Region II: Below the $H^S$-schedule, above the $I^S$-schedule; $H = 0$, $I = I^{max}$ and Region III: Below both schedules; $H = I = 0$.

As is evident from Figure 1, the singular control schedules act either as switch lines or approach paths, depending on the control constraints. The approach path is identified as a bold line, which in panel a) consists of the part of the $I^S$-schedule (10) that is to the left of the optimal steady state, and the part of the $H^S$-schedule (9) that is to the right of the equilibrium. The upper constraint on investment may also entail that the $H^S$-schedule must be followed even from the left, at least when the equilibrium is sufficiently close. This situation is depicted in panels b) and c) Figure 1.

[Figure 1 about here]

Whenever the initial point is above the singular harvest schedule, a situation exemplified by points A and D in Region I, the stock will be slaughtered down immediately until the $H^s$-schedule is reached. If the state of the system is now above the singular investment schedule, the $H^S$-schedule acts as a switch and harvest is set to zero, as is the case when starting from point A. If not, the rest of the approach path is along the $H^S$-schedule, as with the trajectory from point D. When starting from below both schedules, as from points B and C, the singular approach path is the one of the control schedules that is encountered first, after a period with zero harvest and investment. As indicated in Figure 1a), the approach path is thus the lower one of the two singular control schedules, when feasible. As seen on Figures 1b) and 1c), however, a part of this control path may not be feasible if the upper investment constraint binds. In this case only the leftmost part of the $I^S$-schedule can be followed, while the equilibrium is encountered along the $H^S$-schedule from both directions. The $I^S$-schedule

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may then act partly as a switch between zero and maximum investment, which the trajectory from point B indicates.

By taking the first order difference of eqs. (9) and (10) , and using the growth equations to substitute for $K_{t+1} - K_t$ and $X_{t+1} - X_t$, we can also derive explicit feedback rules for both stocks. Recalling that $H_t = 0$ along the singular investment schedule, singular investment is given by:

$$I^S = \gamma K_t + \frac{V'' + Q_{XK}}{-Q_{KK}} r X_t ,$$

Where the coefficient for $X$ is positive. Singular investment therefore depends positively on depreciation, and also on animal stock growth. A higher stock of animals, and/or a higher animal growth rate means that investment must increase to let capital growth keep pace with growth in the animal stock. Singular harvest is given by:

$$H^S = \begin{cases} r X_t + \dfrac{-Q_{KK}}{Q_{KX}} \gamma K_t & X_t > X^* \\[3mm] r X_t + \dfrac{-Q_{KK}}{Q_{KX}} \left( \gamma K_t - I^{\max} \right) & X_t < X^* \end{cases} ,$$

where the coefficient for $K_t$ is positive, and with a similar interpretation. Note that the singular harvest rule is different depending on whether the system is on the $H^S$-schedule to the left or right of the equilibrium, as investment will be set to $I^{\max}$ or zero in the two situations, respectively.

The next two results regard the monotonicity of the approach paths, and are related to results from the fisheries literature. First observe that the approach path is monotonous with respect to both $X$ and $K$ along the singular control schedules and in Region II (see the discussion above). Also note that a) In Region I there is only impulse harvest and no investment, and b) in Region III the monotonous part of approach path may be encountered on either side of the equilibrium if $K_0 > K^*$ and $X_0 < X^*$. The first of these results is stated as:

**Result 4.** It is never optimal for capital to overshoot its optimal steady state level. However, capital may undershoot the steady state if $X_0 < X^*$.

Proof: Positive investment cannot occur in any of the two regions outside the monotonous part of the approach path. Hence, overshooting is impossible. Undershooting happens if, from Region III with $K_0 > K^*$ and $X_0 < X^*$, the monotonous part of the approach path is reached where $K_t < K^*$.

**Corollary:** It will never be optimal to have excess capacity in the steady state.

This result differs from what is found by Clark et al. (1979), where it is optimal to have excess capacity in the steady state if the depreciation rate is zero. The reason that this does not happen here is that capital plays no role in the harvesting process. Therefore, it is not profitable, or possible, to speed up the approach to the equilibrium by overinvesting, if the initial animal stock is above the equilibrium level. The next result concerns the development of the animal stock:

**Result 5.** The animal stock may either undershoot or overshoot the optimum, depending on the initial situation.

Proof: When $X_0 > X^*$ and $K_0 < K^*$ in Region I, the animal stock will be reduced immediately until the monotonous part of the approach path is reached where $X_t < X^*$, implying undershooting. From Region III with $K_0 > K^*$ and $X_0 < X^*$, overshooting occurs if the monotonous part of the approach path is reached where $X_t > X^*$.

This also contrasts the Clark et al. (1979) model, and subsequent contributions within the fisheries literature. The intuition is that a more profitable rate of capacity utilization can be obtained by temporarily reducing the animal stock below the steady state level if the capital stock is low, and expanding it beyond the steady state level if the capital stock is large. Both situations depend on the fact that the capital stock cannot be adjusted instantaneously in either direction. The next result is stated without proof as:

**Result 6**: If the upper investment constraint is not binding on the approach path, the optimal steady state will almost always be approached with one control set at the interior and the other at zero.

In principle, all control combinations are possible approaches to the equilibrium, but the case where $X_0 > X^*$ and $K_0 = K^*$, so that the equilibrium is reached by a one-time slaughtering down of the animal stock only, and the case where the equilibrium is reached by setting both controls to zero, can both only be satisfied by a fluke. The general approach is along one of the singular control schedules, and ruling out the possibility that $I = I^{\max}$ along the approach path, the non-singular of the controls must be zero.

## 6. Numerical example

To shed some further light on the above analysis, the model is now illustrated numerically[1]. We do not attempt to accurately describe the economic situation of a Nordic sheep farmer, but to demonstrate the workings of the model with reasonably realistic parameter values. First, we specify the functional forms. The congestion cost function is specified as:

$$(11) \qquad Q(X_t, K_t) = \frac{\theta}{2K_t} X_t^2,$$

where $\theta > 0$. It is readily confirmed that this cost function satisfies the properties stated in section 2 above. The operating cost function is next specified as:

$$(12) \qquad V(X_t) = \frac{\eta}{2} X_t^2,$$

with $\eta > 0$. With these functional forms, we find the following expression for the singular harvest and investment schedules:

$$(9') \qquad (r - \delta)p = \left(\eta + \frac{\theta}{K_t}\right) X_t,$$

and

$$(10') \qquad (\gamma + \delta)c = \frac{\theta}{2}\left(\frac{X_t}{K_t}\right)^2.$$

It is easily recognized that both schedules start form the origin and have a positive slope. While the singular investment schedule (Eq. 10') is a straight line, the singular harvest schedule (Eq. 9') yields $X$ as a strictly concave function of $K$, cf. Figure 1 above. They have thus one interior intersection point, provided that the $H^S$-schedule is steeper than the $I^S$-schedule at the origin, which corresponds to a stable equilibrium (see also section 4 above).

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[1] The numerical optimization was performed using the KNITRO for MATLAB solver form Ziena Optimization, with MATLAB release 2011b.

The numerical optimization is performed using the parameter values found in Table II. As indicated, the discount rate, the growth rate for the animals and the indoor feeding cost parameter are taken from Skonhoft (2008) while the depreciation rate is what is used by Statistics Norway for buildings (Statistics Norway 2011). The investment and congestion cost parameters are calibrated for our model such that the number of animals in the steady state should represent a medium sized Norwegian farm. In addition, we assume that the maximum yearly investment is fixed at $I^{\max} = 20$ (m$^2$).

[Table II about here]

Table III demonstrates the steady state results. The steady state is interior, as the depreciation is below the investment constraint; $\gamma K^* = 8 < I^{\max}$, and can be found as the solution to equations (9') and (10'). The results with the baseline parameter values are shown in the first column, while the next column shows the results of a 50% increase in the meat price, to 360 (EUR/animal), while all the other parameters are kept at their baseline values. In the last column the discount rate is increased by 50%, to $\delta = 0.06$. In the baseline calculation, the optimal stocking is 120 (animals), the capital stock becomes 200 (m$^2$) while the profit is about 9,800 (EUR). The change in the discount rate has a modest impact on the optimal steady state animal stock level while the effect on the capital stock is somewhat more substantial (see also the comparative static results Table I). The profit is only modestly affected. The slaughter price change, on the other hand, strongly affects the profit which is more than doubled compared to the baseline alternative. The optimal animal stock level is increased by 60%. Note also that capacity utilization is unaffected when the price shifts up. The reason is that the $I^S$-schedule (10') is linear and not affected by the slaughter price. A change in the discount rate, on the other hand, shifts this schedule as well as schedule (9') and hence the rate of capacity utilization changes.

[Table III about here]

The dynamics are demonstrated in Figures 2 and 3 where the panels to the left are for the state variables while the panels to the right depict the corresponding harvest and investment paths. Four different initial situations are considered that correspond roughly to the initial states

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depicted in Figure 1, such that $(K_0, X_0)$ assumes the values $(50,110)$ at point A, $(210,20)$ at point B, $(300,110)$ at point C and $(210,200)$ at point D.

In Figure 2, the approach paths from points A and C are depicted. In both these initial situations the animal stock size is 110 (animals), which is close to the optimum (Table III). The initial capital stock is either far below (A), or well above (C) its steady state level. This figure illustrates the possibilities for the animal stock to over- or undershoot the steady state. If $K_0 = 50$, an immediate harvesting down of the animal stock is followed by a combination of maximum investment and singular harvest until the equilibrium is reached after about 12 years. Since the investment constraint is binding along the approach path, but not in the steady state, the different trajectories corresponds to the ones depicted in panel b) in Figure 1. From $K_0 = 300$, both controls are set to zero and the animal stock grows past its steady state level before the singular harvest schedule is followed, with zero investment.

[Figure 2 about here]

Figure 3 demonstrates the optimal paths when the initial states are found as points B and D. With $K_0 = 210$ in both cases, the initial capital stock is close to the optimum, while the animal stock level is either far below (initial situation B with $X_0 = 20$), or far above (D with $X_0 = 200$), the optimal steady state. In either case, the steady state is reached faster than in Figure 2, as both growth and reduction in the animal stock is faster than for the capital stock. Initial point B entails zero harvest and investment, followed by one period of maximum investment (Region II in Figure 1b) before the equilibrium is encountered. From initial situation D, impulse harvest and depreciation of the capital stock leads to the equilibrium after just two time periods. From point B, the capital stock undershoots the steady state (but may never overshoot, as discussed in section 5).

[Figure 3 about here]

## 7. Concluding remarks

In this paper we have from a theoretical point of view, analyzed the dynamic optimization problem of a profit maximizing farmer who possesses both animals and man-made capital.

The model builds on existing studies from the fisheries literature, but the important difference is that while capital is related to harvesting effort in the fisheries, capital attributes to production capacity to keep the animal stock during the winter in our farm model. The linearity of the model allows an intuitive graphical description that is rare in multi-dimensional optimization problems. Both the steady state and the optimal approach paths have been characterized analytically, and demonstrated by a numerical example related to Scandinavian sheep farming.

The steady state was shown to be either an interior optimum with interior controls, or a constrained optimum with investment set to its maximum value (Result 1). The effects of parameter changes were studied analytically. We found that with a higher meat price the farmer will find it beneficial to increase the stock of animals as well as the amount of capital in the interior steady state (Result 2), while an increase in the discount rate yields opposite effects (Result 3).

As the objective function is linear in both control variables, the approach path is a combination of bang-bang and singular controls, and along the approach path at most one of the controls is singular. The dynamics are different from what is found in the typical fishery models, as in particular there will be a gradual building up of capital, not a one-time impulse investment where the capital stock overshoots the steady state. With capital, only undershooting is possible (Result 4). The animal stock may, on the other hand, both over- and undershoot the optimal steady state (Result 5). In general, one of the controls will be singular along the approach path while the other is set to zero, if the upper investment constraint does not bind (Result 6).

We have focused on a situation with a unique interior equilibrium. However, with different specifications of the cost function there may be several equlibria. With a positive discount rate, the choice of steady state will then in general depend on the initial situation, so that the system is history dependent. The dynamics of such a system will be a rather straightforward generalization of the system analyzed here however, once the optimal steady state is identified. Another possible extension is to include an absolute limit on the number of animals per square meter of housing, typically set by authorities to secure animal health. If this constraint binds along the approach path it will imply maximum investment together with

positive harvest of animals. If the capacity utilization constraint is binding in the steady state, it will imply singular steady state harvest along with maximum investment.

The main contribution of this paper is related to the role of capital which is used here for maintaining the animals and hence plays no role in the harvesting process. In addition, we assume a domestic animal stock where the unit harvest cost is stock independent, and natural growth is density independent and hence also unaffected by stock size. Given that these assumptions also are valid in other types of production involving domestic renewable resources, the model here may have wider applications. Possible examples include other types of livestock management and other areas of modern agricultural production, as well as aquaculture.

**References**

Asheim, G. (2008): Paradoxical consumption behavior when economic activity has environmental effects. Journal of Economic Behavior & Organization 65: 529-546.

Austrheim, G., L.J. Asheim, G. Bjarnason, J. Feilberg, A.M. Fosaa, Ø. Holand, K. Høegh, I.S. Jónsdóttir, B. Magnússon, L.E. Mortensen, A. Mysterud, E. Olsen, A. Skonhoft, G. Steinheim and A.G. Thórhallsdóttir (2008): Sheep grazing in the North Atlantic region: A long term perspective on management, resource economy and ecology. Report Zoology Ser. 2008, 3 Vitenskapsmuseet NTNU.

Boyce, J. R. (1995): Optimal capital accumulation in a fishery: a nonlinear irreversible investment model. Journal of Environmental Economics and Management 28: 324-39.

Kennedy (1986): Dynamic Programming. Applications to Agriculture and Natural Resources. Elsevier Science, London.

Charles, A. T. and G. R. Munro (1985): Irreversible investments and optimal fisheries management: A stocastic analysis. Marine Resource Economics 1: 247-64.

Clark, C. (1990): Mathematical Bioeconomics. Wiley Interscience, New York.

Clark, C., F. Clarke and Munro (1979): The optimal exploitation of renewable resource stocks: Problems of irreversible investments. Econometrica 47: 25-47.

Horan, R. and C. A. Wolf (2005): The economics of managing infectious wildlife disease. American Journal of Agricultural Economics 87: 537-551.

Jarvis, L. (1974): Cattle as capital goods and ranchers as portfolio managers: An application to the argentine cattle sector. Journal of Political Economics 82: 489-520.

McKelvey, R. (1985): Decentralized regulation of a common property renewable resource industry with irreversible investment. Journal of Environmental Economics and Management 12: 287-307.

Mesterton-Gibbons, M. (1996): A technique for finding optimal two-species harvesting policies. Ecological modeling 9: 235-244.

Nersten, N., A. Hegrenes, O. Skjelmo, and K. Stokke (2003): Saueholdet i Norge (Sheep Farming in Norway). Report, Norwegian Agricultural Economic Research, Oslo.

Sandal, L. K., S. I. Steinshamn, A. Hoff (2007): Irreversible investments revisited. Marine Resource Economics 22: 255-266.

Skonhoft, A. (2008): Sheep as capital and farmers as portfolio managers: A bioeconomic model of scandinavian sheep farming. Agricultural Economics 38: 193-200

Smith, V. L. (1968): Economics of production from natural resources. American Economic Review 58: 409-31.

Statistics Norway (2011): Statistical yearbook of Norway. Statistics Norway, Oslo

**Tables and figures**

**Table I: Comparative static results interior steady state with singular harvest and investment. In parentheses the constrained steady state with $I^* = I^{\max}$.**

|  | $p$ | $c$ | $\delta$ | $r$ | $\gamma$ |
|---|---|---|---|---|---|
| $X^*$ | $+\,(+)$ | $-\,(0)$ | $-\,(-)$ | $+\,(+)$ | $-\,(-)$ |
| $K^*$ | $+\,(0)$ | $-\,(0)$ | $-\,(0)$ | $+\,(0)$ | $-\,(-)$ |
| $H^*$ | $+\,(+)$ | $-\,(0)$ | $-\,(-)$ | $+\,(+)$ | $-\,(-)$ |
| $I^*$ | $+\,(0)$ | $-\,(0)$ | $-\,(0)$ | $+\,(0)$ | $\pm\,(0)$ |

**Table II: Baseline parameter values**

| Parameter | Description | Value |
|---|---|---|
| $\delta$ | Discount rate | 0.04 |
| $r$ | Animal growth rate | 0.7 |
| $c$ | Unit investment cost (EUR/m$^{2)}$) | 100 |
| $\eta$ | Feeding cost (EUR/animal$^2$) | 1.1 |
| $\theta$ | Congestion cost (EUR/(animal$^2$/m$^2$)) | 45 |
| $\gamma$ | Depreciation rate | 0.04 |
| $p$ | Meat price (EUR/animal) | 240 |

Sources: $r$, $\eta$ and $p$ based on Skonhoft (2008), $\gamma$ from Statistics Norway (2011), $c$ and $\theta$ calibrated.

**Table III: Steady state results**

|  |  | Baseline | $p$ up 50% | $\delta$ up 50% |
|---|---|---:|---:|---:|
| Animals | $X^*$ | 120 | 192 | 112 |
| Capital (m$^2$) | $K^*$ | 200 | 322 | 168 |
| Capacity utilization (animals/m$^2$) | $X^*/K^*$ | 0.60 | 0.60 | 0.67 |
| Slaughter income (EUR) | $prX^*$ | 20,160 | 48,384 | 18,816 |
| Investment cost (EUR) | $c\gamma K^*$ | 800 | 1288 | 672 |
| Feeding cost (EUR) | $\dfrac{\eta}{2}X^{*2}$ | 7,920 | 20,275 | 6,899 |
| Congestion cost (EUR) | $\dfrac{\theta}{2K}X^{*2}$ | 1,620 | 2,576 | 1,680 |
| Annual profit (EUR) | $\pi^*$ | 9,820 | 24,245 | 9,565 |

**1 a)**



**1 b)**



**1 c)**
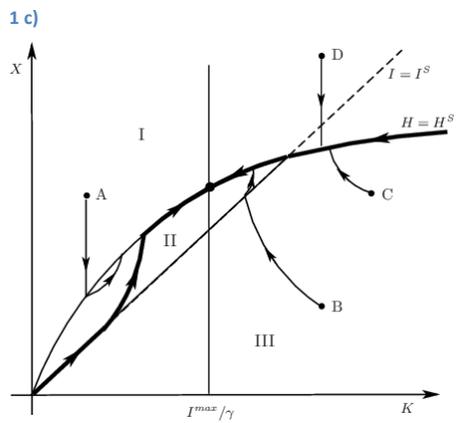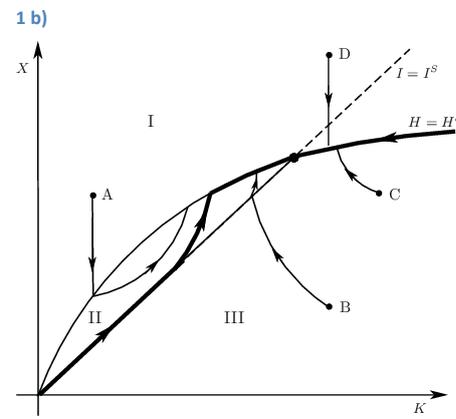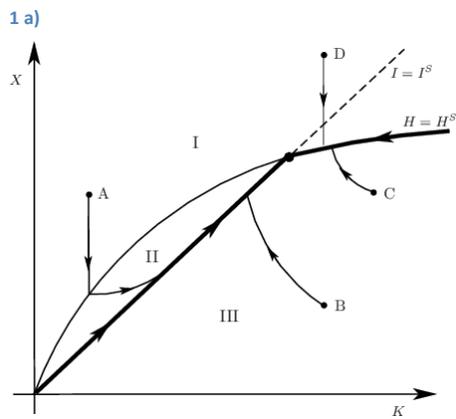


1 a) Both controls singular near the steady state.

1 b): Approach paths with $I^S < I^{\max}$ near the equilibrium, but interior steady state.

1 c): Constrained steady state; $I^* < I^{\max}$.

**Figure 1: Optimal approach paths, unique steady state.**
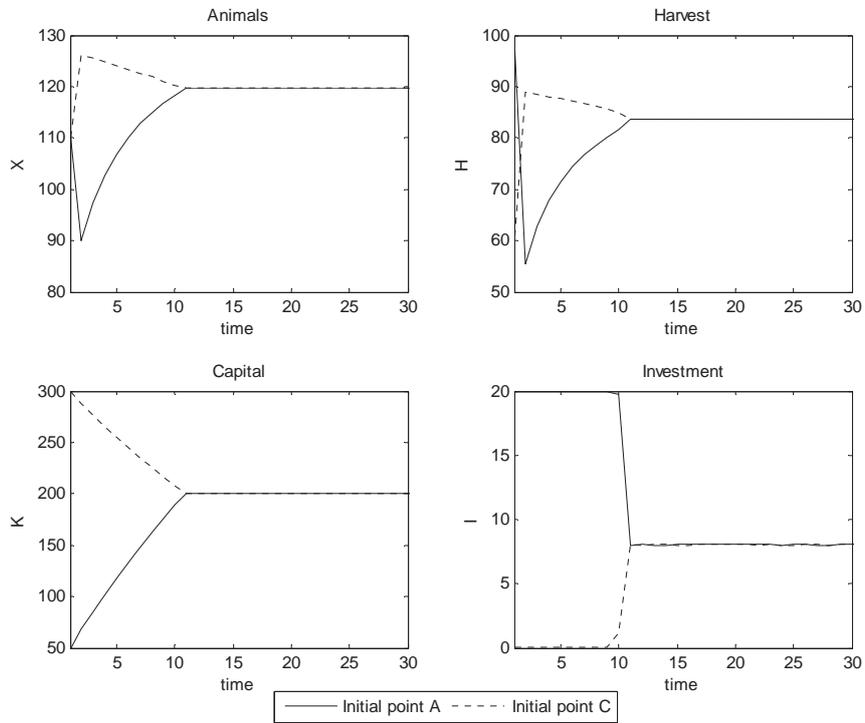
**Figure 2. Optimal approach paths baseline parameter values. Initial situation A ($K_0 = 50$, $X_0 = 110$) and C ($K_0 = 300$, $X_0 = 110$).**
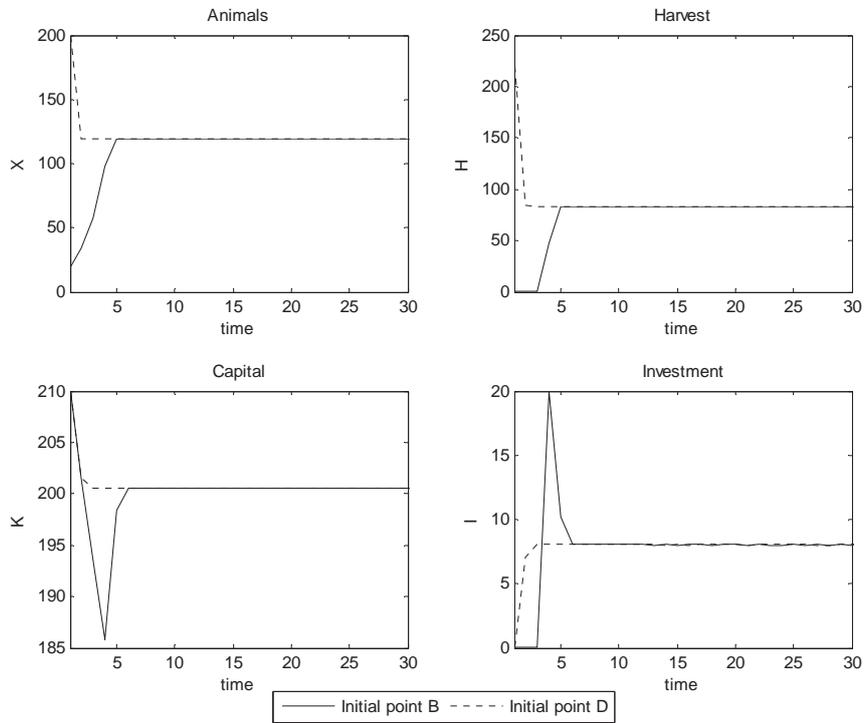
**Figure 3. Optimal approach paths baseline parameter values. Initial situation B** ($K_0 = 210$, $X_0 = 20$) **and D** ($K_0 = 210$, $X_0 = 200$).

**Chapter 3**

**Balancing income and cost in red deer management**

# Balancing income and cost in red deer management

Anders Skonhoft[1][*], Vebjørn Veiberg [2], Asle Gauteplass[1], Jon Olaf Olaussen [3], Erling L. Meisingset [4], Atle Mysterud [5]

## Abstract

This paper presents a bioeconomic analysis of a red deer population within a Norwegian institutional context. This population is managed by a well-defined manager, typically consisting of many landowners operating in a cooperative manner, with the goal of maximizing the present-value hunting related income while taking browsing and grazing damages into account. The red deer population is structured in five categories of animals (calves, female and male yearlings, adult females and adult males). It is shown that differences in the per animal meat values and survival rates ('biological discounted' values) are instrumental in determining the optimal harvest composition. Fertility plays no direct role. It is argued that this is a general result working in stage structured models with harvest values. In the numerical illustration it is shown that the optimal harvest pattern stays quite stable under various parameter changes. It is revealed which parameters and harvest restrictions that is most important. We also show that the current harvest pattern involves too much yearling harvest compared with the economically efficient level.

*Key words:*

Red deer, management, bioeconomic analysis, landowner, cost-benefit, hunting, meat value, crop damage.

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**1.Introduction**

The management of deer have become in focus with recent large increases in population sizes in many areas of both Europe and North America (Gordon et al., 2004). Increased abundance increases potential yields. However, the increasing number of deer-vehicle collisions and their related costs (Langbein et al., 2010), deer damages to forestry and agriculture (Reimoser and Putman, 2010) and their general ecosystem impact (Mysterud, 2006; Reimoser and Putman, 2010) are also important concerns accompanying the population increase. Large herbivores are termed ecological keystone species because of their ability to affect vegetation community patterns and ecosystem functioning (Côté et al., 2004; Hobbs, 1996). Deer management decisions are therefore important issues that generally influence both ecological and economic aspects.

A good example of this development is the Norwegian red deer (*Cervus elaphus*) population, which has increased and spread substantially during the last few decades (Milner et al., 2006). The red deer has become the most numerous wild ungulate in Norway both measured as the number of harvested individuals (Statistics Norway, 2010) as well as number of live animals in the population (Andersen et al., 2010). In 1970 the total number of harvested red deer was about 2,800, and animals were mainly found in scattered and low density populations along the south-western part of the country (Figure 1). In 2009 red deer are found in nearly all municipalities of southern Norway, and the total number of shot animals exceeded 39,000 animals. However, the main populations are still situated in the traditional core areas.

Red deer are extensively utilized as a game species, and the management strategy in Norway has traditionally been oriented towards meat hunting rather than trophy hunting (Milner et al., 2006). Large deer populations both represent a potential source of income for the landowners possessing the hunting rights, as well as a potential cost related to damages from grazing and browsing. The harvest related benefits are typically associated with sale of meat and hunting rights, whereas additional services or products (cabin rental, guiding, etc.) generally represent a modest source of income (Olaussen and Mysterud, Unpublished). The main cost component is normally reduced production from agricultural crops (mostly grass production), both in terms of quality and quantity. In addition, browsing damage to economically important tree species may also be important in some areas. There are also significant costs related to deer-vehicle collisions (Mysterud, 2004). Such costs are both complex to quantify (Bissonette et al., 2008) and paid by the general public.

We apply a five stage population model (calves, female and male yearlings, adult females, and adult males). The main goal is to analyze the basic economic and biologic driving forces behind optimal harvesting strategies given various constraints related to male-female composition, and current code of conduct among hunters regarding to avoid shooting the mother from calves. The theoretical results are illustrated numerically. Since costs related to deer-vehicle collisions are external to landowners, they will not be taken into account here. The landowner, typically consisting of several landowners acting as a single agent through a landowner association (management unit), is assumed to be rational and well-informed aiming to maximize the profit from hunting when accounting for grazing costs, but also taking the recreational aspect of the hunting into account. We also compare the results with the present harvesting pattern, which has been established through cohort analyses (Veiberg et al., 2010). Our analysis may be related to the small, but growing literature, on economic analysis of stage structured fishery and animal models, exemplified by Tahvonen (2009) and Olaussen and Skonhoft (2011). Like these studies a dynamic optimization framework is used, but different from these studies more value components are included in the management problem.

## 2. Methods and material

### 2.1 Population model

In Norway mean dressed body weight (i.e., live weight minus skin, head, viscera, metapodials and bleedable blood) for males and females 2 years and older is usually between 60-125 kg and 50-70 kg, respectively (Solberg et al., 2010, see also Table 1). Due to lack of predators natural mortality rates are generally assumed to be low for all age and sex categories. As mortality throughout summer and fall is regarded to be small, all natural mortality is supposed to take place during the winter and after the hunting season and are assumed density-independent (Loison and Langvatn, 1998). On the other hand, fecundity is affected by female density (Langvatn et al., 2004), while the number of males seems to be of negligible importance for a small and moderately skewed sex composition (Mysterud et al., 2002).

In the present analysis the population at time (year) $t$ is structured in five stages; calves $X_{c,t}$ ( $yr < 1$ ), female yearlings $X_{yf,t}$ ( $1 \le yr < 2$ ), male yearlings $X_{ym,t}$ , adult females $X_{f,t}$ ( $yr \ge 2$ ) and adult males $X_{m,t}$ . The population is measured in spring before calving. All stages may be

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harvested, and where the hunting starts early September and ends mid-November. Neglecting any dispersal in or out of the considered area, the number of calves (recruitment) is first governed by $X_{c,t+1} = r_{yf,t} X_{yf,t} + r_{f,t} X_{f,t}$ , so that $r_{yf,t}$ is the fertility rate of yearlings and $r_{f,t}$ the fertility rate of adults. Unless the number of males becomes extremely low, the male abundance and sex composition has negligible influence on female fertility rates (Mysterud et al., 2002). We therefore assume that only the total number of females, yearlings and adults, affect female fertility rates. Moreover, as female yearlings and adult females compete over the same resources, we assume that the fertility rates for both categories are affected only by the total number of females present. We then have $r_{yf,t} = r_{yf}(X_{yf,t} + X_{f,t})$ , with $r_{yf}(0) > 0$ and $\partial r_{yf} / \partial (X_{yf,t} + X_{f,t}) = r'_{yf} < 0$ for yearlings, and $r_{f,t} = r_f(X_{yf,t} + X_{f,t})$ , with $r_f(0) > 0$ and $r'_f < 0$, for adults. The density dependent recruitment function then yields:

$$X_{c,t+1} = r_{yf}(X_{yf,t} + X_{f,t}) X_{yf,t} + r_f(X_{f,t} + X_{yf,t}) X_{f,t}. \tag{1}$$

The number of female and male yearlings follows next as:

$$X_{yf,t+1} = \psi s_c (1 - h_{c,t}) X_{c,t} \tag{2}$$

and

$$X_{ym,t+1} = (1 - \psi) s_c (1 - h_{c,t}) X_{c,t} \tag{3}$$

where $s_c$ is the fixed calf natural survival rate (fraction) and $\psi$ is the proportion of female calves. $0 \le h_{c,t} \le 1$ is the calf harvesting rate. Finally, the abundance of adult females and males become:

$$X_{f,t+1} = s_{yf}(1 - h_{yf,t}) X_{yf,t} + s_f(1 - h_{f,t}) X_{f,t} \tag{4}$$

and

$$X_{m,t+1} = s_{ym}(1 - h_{ym,t}) X_{ym,t} + s_m(1 - h_{m,t}) X_{m,t} , \tag{5}$$

respectively. $s_f$ and $s_m$ are the fixed natural survival rate of adult females and males, and where $s_f \ge s_m$. $s_{yf}$ and $s_{ym}$ are the yearling survival rates, also with $s_{yf} \ge s_{ym}$ (Table 1). $0 \le h_{yf,t} \le 1$ and $0 \le h_{ym,t} \le 1$ are the harvesting rates of female and male yearlings, respectively, while $0 \le h_{f,t} \le 1$ and $0 \le h_{m,t} \le 1$ are the harvesting rates of the two adult stages.

Equation (2) and (3) together yield:

$$X_{ym,t} = [(1 - \psi) / \psi] X_{yf,t}. \tag{6}$$

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Therefore, with $\psi \approx 0.5$ the number of female and male yearlings will be similar at every point in time. Using equation (6) to eliminate $X_{ym,t}$ from equation (5), we find the abundance of adult males as:

$$X_{m,t+1} = s_{ym}(1-h_{ym,t})[(1-\psi)/\psi]X_{yf,t} + s_m(1-h_{m,t})X_{m,t} \tag{7}$$

Equations (1), (2), (4) and (7) now constitute a reduced form model in four stages and five harvesting rates, and where all equations are first order difference equations. We employ this system of equations when studying exploitation below.


## 2.2 Landowner benefits and costs

As indicated, management units regularly consist of several landowners. Each landowner contributes to the total area (the counting area) with the part of their property that satisfies given criteria defined in regulations. On the basis of the total counting area, the municipality then allows harvest of a given number of deer. Within the management unit, decisions related to e.g. distribution of hunting licenses, costs and income are made by majority of votes. In the voting process all landowners either represent equal votes or their influence is differentiated through weighted voting on the basis of their contribution to the common counting area. The traditional landowner management goal has been directed towards maximizing the population productivity in order to produce meat, but also with a strong recreational component (Andersen et al., 2010; Milner et al., 2006). The hunters have typically been local people, the landowners and their families and friends, and the hunting has had some flavor of subsistence hunting (Andersen et al., 2011; Olaussen and Mysterud, 2012). In what follows, we think of the hunters as the local people enjoying the meat value and the recreational value of the hunting. The utility derived from the meat value is assumed to be linear in the hunt, while the recreational value is linear in the hunting rates. As will be seen, this difference has a substantial impact on the solution of the model.

We start to look at the hunting where the number of animals removed through hunting in year $t$ is defined as $H_{i,t} = h_{i,t}X_{i,t}$ ($i = c, yf, ym, f, m$). The value (price) per kg harvested meat $\hat{p}$ is assumed to be independent of number of animals hunted, population density and animal category, as well as assumed to be fixed over time. The current gross meat value is accordingly $\hat{p}(w_c h_{c,t}X_{c,t} + w_{yf}h_{yf,t}X_{yf,t} + w_{ym}h_{ym,t}X_{ym,t} + w_f h_{f,t}X_{f,t} + w_m h_{m,t}X_{m,t})$ and where $w_c < w_{yf} \leq w_{ym} < w_f < w_m$ are the (average) dressed weights (kg per animal) of the different

categories of animals (Table 1). For every animal shot the landowner pays a fixed fee $q_i$ (NOK per animal) to the municipality. This fee is typically differentiated between calves and older animals (Table 2). For landowners the yearly net benefit from hunting measured as meat value is then defined as:

$$V_t = p_c h_{c,t} X_{c,t} + p_{yf} h_{yf,t} X_{yf,t} + p_{ym} h_{ym,t} X_{ym,t} + p_f h_{f,t} X_{,t} + p_m h_{m,t} X_{m,t},\qquad (8)$$

and where $p_i = (\hat{p}w_i - q_i)$ ($i = c, yf, ym, f, m$) is the net per animal meat value (NOK per animal).

In addition to the meat value, the hunters also obtain a recreational value. Studies from elk hunting in US (Aiken and Pullis La Rouche, 2001) and from Sweden (Boman et al., 2011) indicate that time spent on hunting is valuable. In what follows, only the hunting time is considered as an indicator for recreational value and hence all other valuable aspects of the hunting, except of the meat value, are lumped into this indicator. When using a production function approach and where the production (harvesting) function is assumed to be of the standard Schaefer type with perfect selectivity (e.g., Clark, 1990), the number of hunted animals of category $i$ becomes $H_{i,t} = \theta_i N_{i,t} X_{i,t}$ ($i = c, yf, ym, f, m$). $N_{i,t}$ hence indicates the hunting effort, measured in number of hunting days, and $\theta_i$ is the catchability (productivity) coefficient (1/hunting days). The value will typically differ somewhat between the various categories of animals, but probably not too much. In what follows, it is assumed to be identical. Therefore, with $z$ as the recreational value (NOK per hunting day), the recreation value writes $B_t = z(N_{c,t} + N_{yf,t} + N_{ym,t} + N_{f,t} + N_{m,t})$, or

$$B_t = (z/\theta)(h_{c,t} + h_{yf,t} + h_{ym,t} + h_{f,t} + h_{m,t})\qquad (9)$$

when substituting for the harvest functions. The total yearly hunting benefit is hence described as $(V_t + B_t)$.

The underlying causes explaining the occurrence and extent of deer damages are complex and not fully understood, but crop type and population density are assumed to be the most relevant factors (Reimoser and Putman, 2010). In a descriptive study from one of the core hunting areas in Norway Thorvaldsen et al. (2010) found that crop damages on agricultural land was more important than damages to commercial tree species. Only crop damage is considered here, and as it takes place during the spring, summer and early fall, it is the pre-harvest population size that is relevant. When assuming that the agriculture damage cost functions are

linear, with unit values as $d_c < d_{yf} \le d_{ym} < d_f < d_m$ (Table 2), these costs may thus be written as:

$$D_t = d_c X_{c,t} + d_{yf} X_{yf,t} + d_{ym} X_{ym,t} + d_f X_{f,t} + d_m X_{m,t}. \tag{10}$$

Therefore, the landowner yearly net benefit is described by:

$$R_t = V_t + B_t - D_t \tag{11}$$

## 2.3 Optimal management

### 2.3.1 Hunting meat value benefit only

We first study the situation where the landowner maximizes the hunting benefit meat value only. With $\rho = 1/(1+\delta)$ as the discount factor with $\delta \ge 0$ as the (yearly) fixed discount rate, the goal is thus to find harvest rates that maximizes the present-value $\max\limits_{h_{c,t},h_{yf,t},h_{ym,t},h_{f,t},h_{m,t}} \sum\limits_{t=0}^{t=\infty} \rho^t V_t$ subject to the biological growth equations (1), (2), (4) and (7). In addition, a female – calf harvest restriction is included (Olaussen and Skonhoft, 2011) due to the common code of conduct among hunters to avoid shooting the mother from calves:

$$h_{f,t} X_{f,t} \le h_{c,t} X_{c,t}. \tag{12}$$

A restriction on the number of harvested males is also incorporated to avoid a too skewed sex ratio:

$$h_{m,t} \le \overline{h}_m \text{ and } h_{ym,t} \le \overline{h}_m. \tag{13}$$

In a Supplement section the optimal control conditions are stated and the basic driving forces steering the harvest composition are revealed. We now discuss these driving forces, first by disregarding the code of conduct constraints (12) and (13).

For both sexes, we find that simultaneous harvest of both yearlings and adults does not satisfy the optimality criteria. Next, it can be shown that positive female yearling harvest together with zero adult female harvest implies $p_{yf} / s_{yf} > p_f / s_f$ ; that is, the 'biological discounted' female yearling hunting value (in NOK per animal) exceeds the 'biological discounted' adult female hunting value. When the adult category is more valuable than the yearling category, $p_f > p_{yf}$, and as long as the survival rates $s_{yf}$ and $s_y$ do not differ too much (Tables 1 and 2, numerical section), this inequality represents a contradiction. In the same manner, we also find that positive male yearling harvest together with zero adult male harvest implies

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$p_{ym} / s_{ym} > p_m / s_m$ which also represent a contradiction within the range of realistic parameter values. Therefore, positive female yearling harvest and zero female adult harvesting and positive male yearling harvest and zero adult male harvesting are not possible options when the code of conduct constraints (12) and (13) are not binding and when the harvest recreation value is small and hence neglected.

Since, the 'biological discounted' value of the calf category is lower than for the yearlings, $p_c / s_c < p_i / s_i$ ($i = yf, ym$) (see Table 1 and 2), we may also suspect that calf harvest together with zero adult harvest will not represent an optimal harvesting policy when the code of conduct constraint (12) is not binding. We find that zero yearling harvest together with positive harvest of one, or both, of the adult categories do not contradict the optimality conditions. The reason for this outcome is simply that the per animal values are higher for the adults than for yearlings while the survival rates do not differ too much.

Based on the optimality conditions it is difficult to say something definite about the harvest composition among the adults. However, if both adult females and males are harvested in the unbinding case, this result may change when the restrictions (12) and (13) are included. First suppose that restriction (12) is included, while we still neglect restriction (13). If females are harvested, the code of conduct restriction (12) indicates that at least the same number of calves must be harvested. Since the restriction is unbinding if more calves than females are harvested, we already know from the unbinding case that no calves should be harvested. Therefore, harvesting more calves than females represents a contradiction both in the unbinding as well as the binding case. Hence, the restriction must bind with the same positive number of calves and females harvested, or zero harvest of both. When constraint (12) binds, it will certainly also influence the various possibilities of yearling harvest together with adult female and male harvest. We may hence find that yearling harvest together with male harvest occurs in the optimal solution. When restriction (13) is included and binds, $h_{m,t} = h_{ym,t} = \overline{h}_m$, this will generally also influence the various optimal harvest options. Both yearling female, calf and adult female harvest may be affected due to less yearling and adult male harvest. The intuition is straightforward as e.g., calf harvest is an indirect way of controlling the yearling category when the yearling male harvest is restricted. Hence, even if calf harvest is not an optimal option in the unrestricted case, it may very well be optimal in the restricted case.

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Summing up, we have demonstrated that differences in the 'biological discounted' value of various animal categories are the basic driving forces behind the optimal harvest composition. Fertility plays no direct role. However, certain modifying and complicating factors related to the common code of conduct constraint (12) and male harvest constraint (13) are present. These constraints also complicates our understanding of how the harvest composition and the populations may evolve over time; that is, the dynamic properties of the model. However, because of the strong degree of linearity in the model together with density dependent regulation through the recruitment function, we may suspect that oscillations will not be present and that the harvesting composition will approach a stable equilibrium.

### 2.3.2 Crop damage taken into account

We then look at the problem when also the damage costs are taken into account, but the recreational hunting value is still not included. Therefore, the problem now is to find the harvesting strategy that $\max\limits_{h_{c,t},h_{yf,t},h_{ym,t},h_{f,t},h_{m,t}} \sum\limits_{t=0}^{t=\infty} \rho^t (V_t - D_t)$, again subject to the biological growth equations (1), (2), (4) and (7) and the code of conduct constraints (12) and (13).

Because the damage in year $t$ takes place *before* the harvest (see Eq. 10), the first order necessary control conditions will be identical to the above problem of maximizing the hunting benefit only (Supplement section). However, we may suspect that the harvesting pressure will be higher when damage costs are included (see e.g., Skonhoft and Olaussen, 2005)..

### 2.3.3 Recreational value included

When taking the recreational value of hunting into account, the current net benefit is defined by equation (11), and the problem is hence to $\max\limits_{h_{c,t},h_{yf,t},h_{ym,t},h_{f,t},h_{m,t}} \sum\limits_{t=0}^{t=\infty} \rho^t (V_t + B_t - D_t)$, subject to the same constraints as above. The solution to this problem gives the 'overall' optimal solution for the landowners. In this case, not only differences in per animal values and survival rates ('biological discounted' values) are instrumental in determining the harvest composition, but also the demographic composition of the population (see Supplement section). We may also suspect that the harvesting composition is more prone to changes over time because the strong degree of linearity in the model is no longer present. Generally, we find the dynamics now to be strikingly different. See Results section.

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## 3. Results

### 3.1 Data and specific functional forms

The model is now illustrated numerically for an area located in the county Sogn and Fjordane on the south western coast of Norway. This county is the core area for red deer hunting in Norway. The fertility rate (see Methods and material section) is expressed as a sigmoid function predicting increased degree of density dependence with higher population densities (Langvatn et al., 2004). The function reads:

$$r_{i,t} = r_i \left( X_{yf,t} + X_{f,t} \right) = \frac{\tilde{r}_i}{1 + \left[ \left( X_{yf,t} + X_{f,t} \right) \big/ K \right]^{b_i}} \; ; \; (i = yf, f) \tag{14}$$

with $\hat{r}_i > 0$ as the intrinsic growth rate (maximum number of calves per female), $K > 0$ as the female 'carrying capacity' and $b_i > 0$ as a compensation parameter. Equation (14) then implies the recruitment function (1) of the Shepherd type. Table 1 gives the baseline parameter values for this function as well as the other biological parameter values. The parameter $K$ is scaled for our study area and is 4,000 (animals). See Table 1.

The baseline economic parameter values are given in Table 2. The value of the catchability coefficient $\theta = 0.0001$ (1/hunting days) is calibrated based on reported hunting productivity. The per day recreational hunting value $z = 50$ (NOK/day) is taken from a study from the actual area (Thorvaldsen et al., 2010). The crop damage cost parameters are based on Olaussen and Mysterud (2012) while the meat value $\hat{p}$ is assumed similar among all categories of animals. We assume a zero discount rate in the baseline scenario. As known (e.g., Clark, 1990), this means that the (possible) steady state of the dynamic optimization problem coincides with the solution to the problem of maximizing current net benefit in biological equilibrium. The numerical results are first presented when only meat value and damage costs are included. Then we present results when also the recreation value of the hunting is taken into account.

### 3.2 Results when recreation value is not included

We first present the basic dynamic harvesting results[2]. As already indicated, because of the strong degree of linearity in the model together with density dependent regulation through the recruitment function, the model solution approaches a stable equilibrium. Because the profit function is linear in harvest, economic theory for one-stage models (or biomass models)

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[2] The dynamic optimization was performed with the KNITRO for MATLAB solver from Ziena Optimization, with MATLAB release 2011b.

suggest that harvest should be adjusted such as to lead the populations to their optimal steady states as rapid as possible – if no constraints are binding (for proof of the Most Rapid Approach Path theorem, see Spence and Starrett, 1975). Figure 2 demonstrates the optimal harvest rate dynamics with the baseline parameter values, and when both meat value and grazing damage are included (Case 1 in Table 3 below). The results here seem to partly confirm the most rapid approach principle also in our linear multi-stage model. Starting with populations equal to 300 for all categories, calves and adult females are not harvested until they have grown to their optimal population sizes. Adult males, on the other hand, are harvested at the maximum rate even though the population is below its optimal steady state. This occurs because constraint (13) binds for adult males in the steady state, so postponing harvest until the optimal population size is reached would lead to overshooting. We also studied the effect of changing the discount rate on the time path and steady state in this Case 1. We find, not surprisingly, that increasing the discount rate results in progressively smaller populations while the dynamics do not change qualitatively. The effects of other initial population values were examined as well without changing the qualitative structure of the dynamics. Neither was other steady state values found ('ergodic' dynamics).

Tables 3 and 4 report the detailed steady state results for different configurations of parameter values and constraints. Table 3 shows the optimal harvest rates and population sizes for all stages. Table 4 presents the economic results. From Table 3 we see that 20 % of the calves are harvested in Case 1, because the female-calf harvesting constraint (12) is binding, and the harvest rate of adult females is 0.15. The adult male harvest constraint (13) also binds with $h_m = 0.55$. It is not beneficial with harvest of yearlings because the 'biologically discounted' values are lower for yearlings than for adults. The adult male/female sex ratio becomes $740 / 2,158 \approx 0.34$. In this case, as well as in the other cases, we find identical numbers of yearling males and yearling females because of equal sex ratio in the recruitment, $\psi = 0.5$ (see Methods and material section and Table 1). From Table 4, we find the total yearly meat value to be 4,071 (1,000 NOK), with grazing damage costs being 1,843. Net average yearly income per hunted animal is 2.102 (1,000 NOK).

Case 2 describes what happens when the meat value is increased with 50 % while all other parameter values are kept constant. This leads to a slight adjustment in the harvest ratios for calves and adult females, but probably more important, to higher standing populations (Table

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3). This result contrasts standard biomass harvesting theory (e.g., Clark, 1990). Obviously, the net economic value per animal both when hunted and when alive (as 'capital') increases when the meat value shifts up. For a given harvest activity, a price increase leads to an equal increase in income. When costs are unchanged, the increase in net benefit will, however, always be higher. In addition, the harvest activity will be adjusted to take further advantage of the price change. Following the 50 % price increase, we hence find a doubling of total net benefit (Table 4).

In Case 3 the grazing damage $D_t$ in equation (10) is ignored; that is, the landowners optimize without taking the grazing damage into account. Optimal population sizes are affected as the cost of keeping the population, as viewed by the landowner, is reduced. This leads to a change in harvest pattern and population size that works in the same direction as the price increase. The implied economic effect, on the other hand, goes in the opposite direction as the *actual* grazing damage will be larger, and amounts to a 10 % decrease in total net benefit (Table 4).

The female-calf constraint (12) is excluded in Case 4. As expected, there is no harvest of calves. The male harvesting constraint still binds with $h_m = 0.55$, and the adult female harvest rate increases to $h_f = 0.21$. Population sizes increase moderately, except for adult females. Total net benefit increases with about 14 %, from 2,228 (1,000 NOK) to 2,537, and there is a significant increase in the net benefit per hunted animal as the average weight of a hunted animal is higher. Therefore, the cost of the code of conduct constraint (12) is quite significant.

The contours in Figure 3 indicate the steady state net economic benefit $(V - D)$ for different combinations of $h_f$ and $h_m$, and demonstrates the sensitivity of the net economic benefit to changes in the adult harvesting rates, when all the other harvesting rates are kept constant at their optimal levels. The left panel is for the baseline Case 1, while the right panel is for Case 4 (i.e., without the cow-calf constraint). When the female-calf harvesting constraint is included as in Case 1, the maximum possible harvest ratio of adult females is around 0.15, as a higher harvest ratio implies a violation of constraint (12) when the harvest rate of calves is kept constant. In Case 4 however, the maximum sustainable harvest rate of adult females is around 0.4, which is equal to the recruitment rate of adult females when no calves or female yearlings are harvested.

From the left panel it is seen that combinations of the male harvesting rates within the range 0.40 – 0.55 and female rates within the range 0.15 – 0.10 yields a net benefit of about 2,000 (1,000 NOK) which implies an approximate 10 % reduction in net benefit compared to the optimal steady state (see also Table 4). This indicates that the profit loss is modest within a quite wide range of harvesting combinations. In the right, panel, where the code of conduct constraint (12) is omitted, net benefit is even less sensitive to adjustments in the adult harvest ratios. The inclusion of the zero profit contour line indicates when profit turns negative because of high grazing damage, and occurs when harvest rates become low and populations become large. In Case 4, net benefits decrease to around -7 mill. NOK for zero harvest of adults (while still keeping other harvest rates constant at their optimal levels). This dramatic impact on profit is to a large extent due to grazing damage being linearly related to population size, an assumption that may be unrealistic for large populations.

### 3.3 Uniform harvesting pattern

One interesting aspect of stage structured harvesting is to what extent the complexity involved in this kind of management practice pays its way through a significantly higher economic benefit yield than a *uniform* harvest rate across all stages. Such uniform hunting pattern implies an exploitation scheme where the present value benefit is maximized under the same biological constraints as above (but not constraints 12 and 13), but with the additional constraint of equal harvest rates across *all* age classes imposed. Results from this uniform harvesting scheme are reported as Case 5 in Table 3 and 4. As indicated here, uniform harvest implies a highly inefficient exploitation pattern, where the net benefit is reduced with about 50 % compared to the baseline Case 1.

### 3.4 Also including the recreational aspect

So far, the hunting recreational value has not been included in the numerical analysis. As mentioned above (Optimal management section) this adjustment complicates the solution considerably. The long term optimal solution may no longer be a steady state, but perpetual oscillations between more profitable, but unsustainable, outcomes. Figure 4 shows that, under the same assumptions as in Case 1, and with a recreational value of $z = 50$ (NOK/day) included, oscillating harvest rates for the calves and the female categories emerge. The harvest rates of both adult and yearling males become constant and equal to their upper constraint levels, whereas the harvest rates of calves, yearling and adult females oscillates between zero and 0.45, 0.30 and 1, respectively.

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The pulse cycle lengths seem to be quite similar for the different stages. However, the periods of hunting are not synchronized and where the top of the cycles occurs earlier for the adult than the yearlings and the calves. As indicated above (Optimal management section), more complex dynamics may evolve when the strong degree of linearity, or concavity, in the model is no longer present. Once the non-concavity is introduced by the recreational value, the possibility for oscillations depends on combinations of all parameters in the model. Generally, it seems that this non-concavity effect is stronger for smaller populations, so that everything that leads to reduced populations will increase the possibility of oscillations, once the recreational value is incorporated into the analysis. This includes higher discount rates, higher grazing damage, lower meat price and also the magnitude of the recreational value itself. All these factors may also influence the synchronizing of the pulse hunting. For baseline parameter values, pulse harvesting occurs only if the recreational parameter $z$ value exceeds a certain critical value ($z = 25$).

## 4. Discussion and conclusion

In this paper we have analyzed the cost and benefit of a red deer population within a five stage model with density-dependent fertility and density independent mortality. This population is managed by a well-defined manager with the goal of maximizing the present-value hunting related income while taking grazing damages into account. The different ways to compose the harvest across age and sex categories and the implied economic benefits are highlighted. Without any restrictions on the harvest, and ignoring the recreation aspect of the hunting, we find that the 'biological discounted' values for the various categories of animals are instrumental in determining the optimal harvesting composition. The difference in the 'biological discounted' value between two animal categories is made up of the net per animal meat value difference as the marginal gain while the difference in natural mortality rate represents the marginal loss. The importance of these factors seems to be a general result in stage structured models where harvest value is maximized. See e.g., the moose harvest model in Olaussen and Skonhoft (2011), the fishery analysis in Skonhoft et al. (2012) and the classical Reed (1980) fishery article.

The Results section illustrates numerically the predictions from the theoretical model. In the baseline case (Case 1) where the meat value and the grazing damage, but not the recreational value, are included, we find that all stages except yearlings are harvested. The females and

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calves are harvested only moderately, and calves are harvested due to the imposed code of conduct restriction (12). We also find that the same pattern persists when the meat price goes up (Case 2) and when grazing damage costs are ignored (Case 3), but the total population becomes substantially higher in both cases. However, the implied economic effect differs between the two cases. When the code of conduct constraint (12) is excluded (Case 4), we find that no calves are harvested, and the harvest of adult females increased to 21 %. Net benefit increased by 14 %. The landowner net benefit is also calculated when the optimal selective harvesting patterns were replaced by 'optimal' uniform harvest rates (Case 5).

When the recreational hunting value is included in the objective function, we find that the optimal harvesting scheme involved oscillating harvest rates. This result is not a new one within the resource management literature; it has been shown to exist in various models (e.g.,Wirl, 1995), and in particular when there is imperfect selectivity in the harvest (Tahvonen, 2009).

Our results may be compared to the current red deer harvesting practices. The present harvesting regimes of red deer in Norway varies a great deal as a consequence of variation in local population density, demography and red deer related conflicts. On the other hand, a general pattern is that calves and yearlings are rather extensively harvested. This is also so for calves in our numerical illustration. As the traditional management practice in general has been oriented towards stimulating population increase, males have also been subject to a more extensive harvest compared to females. This male-biased harvest has resulted in female-biased populations with a high growth potential. Based on extensive data series from culled animals, population size and demographic composition have been reconstructed through cohort analyses for three regions in Norway (Veiberg et al., 2010). The adult sex ratio varied between 3-3.7 females per male. The harvest rates for adult females were rather moderate and stable through all of their lifespan (0.13-0.16). In contrast, the harvest rates of adult males were significantly higher (0.40-0.49) and varied considerably with age. The empirical harvest rates for adult females are quite in line with the steady state alternatives described for Case 1-3. These three cases also represent the most realistic harvest scenarios given the present code of conduct and the Norwegian hunting tradition. The experienced adult sex ratio was also quite equal to the steady state sex ratios in Case 1-3. A striking difference between the current hunting practice and all the steady state alternatives, apart from Case 5, is that yearling harvest represents a significant proportion of the total red deer harvest. On the national level

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yearlings in general represent 30 % of the total red deer harvest (Statistics Norway, 2010), and the sex specific harvest rates were found to vary between 0.16-0.26 for yearling females and 0.40-0.49 for yearling males (Veiberg et al., 2010).

This indicates that there is a clear scope for improving the economics of the present hunting practice by lowering the yearling harvest. The current harvesting strategy is consolidated through a mixture of economic, ecological and evolutionary factors in addition to cultural history and former management aims lingering after the aim has been achieved (Milner et al., 2011). The current practice with a relatively large proportion of both yearlings and calves in the harvest in both Sweden and Norway for both red deer and moose was introduced many decades ago when population growth was a main management goal. There is, however, an increasing awareness of red deer as an economic resource (Olaussen and Mysterud, 2012) which may stimulate a future approach to a more economically optimized harvesting strategy as presented here.

**Literature cited**

Aiken, R., and G. Pullis La Rouche. 2001. Net economic values for wildlife-related recreation in 2001. Addendum to the 2001national survey of fishing, hunting and wildlife-associated recreation, U.S. Fish and Wildlife Service.

Andersen, O., A. Mysterud, V. Veiberg, E. L. Meisingset, and L. Wold, C. 2011. Jakten på hjortejegeren - en spørreundersøkelse blant norske hjorteviltjegere med fokus på framtidig utøvelse av hjortejakt, Norsk institutt for naturforskning, Trondheim.

Andersen, R., E. Lund, E. J. Solberg and B.-E. Sæther. 2010. Ungulates and their management in Norway. In: M. Apollonio, R. Andersen and R. Putman (eds.) European ungulates and their management in the 21$^{st}$ century. p 14-36. Cambridge University Press.

Bissonette, J. A., C. A. Kassar, and L. J. Cook. 2008. Assessment of costs associated with deer-vehicle collisions: human death and injury, vehicle damage, and deer loss. Human-Wildlife Conflicts 2: 17-27.

Boman, M., L. Mattsson, G. Ericsson, and B. Kriström. 2011. Moose hunting values in Sweden now and two decades ago: The swedish hunters revisited. Environmental and Resource Economics 50: 515-530.

Clark, C. 1990. Mathematical bioeconomics. New York; John Wiley and Sons Inc.

Côté, S. D., T. P. Rooney, J. P. Tremblay, C. Dussault, and D. M. Waller. 2004. Ecological impacts of deer overabundance. Annual Review of Ecology Evolution and Systematics 35: 113-147.

Gordon, I. J., A. J. Hester, and M. Festa-Bianchet. 2004. The management of wild large herbivores to meet economic, conservation and environmental objectives. Journal of Applied Ecology 41: 1021-1031.

Hobbs, N. T. 1996. Modification of ecosystems by ungulates. Journal of Wildlife Management 60: 695-713.

Langbein, J., R. Putman, and B. Pokorny. 2010. Traffic collisions involving deer and other ungulates in Europe and available measures for mitigation. In: R. Putman, M. Apollonio and R. Andersen (eds.) Ungulate management in Europe: Problems and practices. p 215-259. Cambridge University Press.

Langvatn, R., and A. Loison. 1999. Consequences of harvesting on age structure, sex ratio and population dynamics of red deer Cervus elaphus in central Norway. Wildlife Biology 5: 213-223.

Langvatn, R., A. Mysterud, N. C. Stenseth, and N. G. Yoccoz. 2004. Timing and synchrony of ovulation in red deer constrained by short northern summers. American Naturalist 163: 763-772.

Loison, A., and R. Langvatn. 1998. Short- and long-term effects of winter and spring weather on growth and survival of red deer in Norway. Oecologia 116: 489-500.

Milner, J. M. et al. 2006. Temporal and spatial development of red deer harvesting in Europe: biological and cultural factors. Journal of Applied Ecology 43: 721-734.

Milner, J.M. , C. Bonenfant and A. Mysterud 2011. Hunting Bambi-evaluating the basis for selective harvesting of juveniles. European Journal of Wildlife Research 57: 565-574.

Mysterud, A. 2004. Temporal variation in the number of car-killed red deer Cervus elaphus in Norway. Wildlife Biology 10: 203-211.

Mysterud, A. 2006. The concept of overgrazing and its role in management of large herbivores. Wildlife Biology 12: 129-141.

Mysterud, A., T. Coulson, and N. C. Stenseth. 2002. The role of males in the dynamics of ungulate populations. Journal of Animal Ecology 71: 907-915.

Nilsen, E. B. et al. 2005. Moose harvesting strategies in the presence of wolves. Journal of Applied Ecology 42: 389-399.

Olaussen, J. O. and A. Skonhoft. 2011. A cost-benefit analysis of moose harvest in Scandinavia. A stage strucctured approach. Resource ad Eneregy Economics 33: 589-611

Olaussen, J. O. and A. Mysterud. 2012. Red deer hunting - commercialising versus availability. European journal of wildlife Research 58: 597-607.

Reed, W. J. 1980. Optimum age-specific harvesting in a nonlinear population model. Biometrics 36: 579-593.

Reimoser, F., and R. Putman. 2010. Impacts of wild ungulates on vegetation: costs and benefits. In: R. Putman, M. Apollonio and R. Andersen (eds.) Ungulate management in Europe: Problems and practices. p 144-191. Cambridge University Press.

Skonhoft, A., and J. O. Olaussen. 2005. Managing a migratory species that is both a value and a pest. Land Economics 81: 34-50.

Solberg, E. J. et al. 2010. Hjortevilt 2009 - Årsrapport fra Overvåkingsprogrammet for hjortevilt, NINA, Trondheim.

Spence, M., and D. Starrett. 1975. Most rapid approach paths in accumulation problems. International Economic Review 16: 388-403.

Tahvonen, O. 2009. Economics of harvesting age-structured fish populations.
J.Environ.Econ.Manage. 58: 281-299.

Thorvaldsen, P. et al. 2010. Kostar hjorten meir enn han smakar? Del 1: Berekning av kostnad
og nytteverdi av hjort i Eikås storvald i Jølster (In Norwegian), Bioforsk.

Veiberg, V., E. B. Nilsen, and M. Ueno. 2010. Framtidig forvaltning av norske
hjortebestandar - utfordringar knytt til bestandstettleik og demografi, NINA,
Trondheim.

Wirl, F. 1995. The cyclical exploitation of renewable resource stocks may be optimal.
J.Environ.Econ.Manage. 29: 252-261.

**Table 1.** Baseline biological parameter values.

| Parameters | Description | Value | Reference/source |
|---|---|---|---|
| $\tilde{r}_{yf}$ | maximum specific growth rate yearlings | 0.6 | Langvatn et al. (2004) |
| $\tilde{r}_f$ | maximum specific growth rate adults | 0.96 | Langvatn et al. (2004) |
| $K$ | carrying capacity females | 4000 (animals) | Calibrated |
| $b_{yf}$ | density compensation parameter yearlings | 2 | Nilsen et al. (2005) |
| $b_f$ | density compensation parameter adults | 2 | Nilsen et al. (2005) |
| $w_c$ | mean weight calves | 25.5 (kg) | Solberg et al. (2010) |
| $w_{yf}$ | mean weight female yearlings | 43.1 (kg) | Solberg et al. (2010) |
| $w_{ym}$ | mean weight male yearlings | 49.4 (kg) | Solberg et al. (2010) |
| $w_f$ | mean weight females | 57.3 (kg) | Solberg et al. (2010) |
| $w_m$ | mean weight male | 77.6 (kg) | Solberg et al. (2010) |
| $s_c$ | survival rate calves | 0.75 | Langvatn and Loison (1999) |
| $s_{yf}$ | survival rate female yearlings | 0.92 | Langvatn and Loison (1999) |
| $s_{ym}$ | survival rate male yearlings | 0.87 | Langvatn and Loison (1999) |
| $s_f$ | survival rate adult females | 0.93 | Langvatn and Loison (1999) |
| $s_m$ | survival rate adult males | 0.93 | Langvatn and Loison (1999) |
| $\psi$ | calf sex ratio | 0.5 | |

**Table 2.** Baseline economic parameter values.

| Parameters | Description | Value | Reference/source |
|---|---|---|---|
| $\hat{p}$ | meat price | 75 (NOK/kg) | Olaussen and Mysterud (2012) |
| $q_c$ | hunting fee calf | 210 (NOK/animal) | Directorate for Nature Management |
| $q_{yf}$ | hunting fee yearling female | 350 (NOK/animal) | Directorate for Nature Management |
| $q_{ym}$ | hunting fee yearling male | 350 (NOK/animal) | Directorate for Nature Management |
| $q_f$ | hunting fee female | 350 (NOK/animal) | Directorate for Nature Management |
| $q_m$ | hunting fee male | 350 (NOK/animal) | Directorate for Nature Management |
| $\theta$ | catchability coefficient | 0.0001 (1/hunting day) | Calibrated |
| $d_c$ | crop damage cost calves | 175 (NOK/animal) | Olaussen and Mysterud (2012) |
| $d_{yf}$ | crop damage cost yearling female | 295 (NOK/animal) | Olaussen and Mysterud (2012) |
| $d_{ym}$ | crop damage cost yearling male | 338 (NOK/animal) | Olaussen and Mysterud (2012) |
| $d_f$ | crop damage cost adult female | 393 (NOK/animal) | Olaussen and Mysterud (2012) |
| $d_m$ | crop damage cost adult male | 532 (NOK/animal) | Olaussen and Mysterud (2012) |
| $z$ | recreational value | 50 (NOK/day) | Thorvaldsen et al. (2010) |
| $\delta$ | discount rate | 0 | |

**Table 3.** Steady state harvest rates and populations. Recreational value not included. $h_c$, $h_{yf}$, $h_{ym}$, $h_f$, and $h_m$ are harvest rates for calves, female yearlings, male yearlings, adult females and adult males, respectively. $H$ is total harvest (in number of animals) and $X_c$, $X_{yf}$, $X_{ym}$, $X_f$, and $X_m$ are population sizes (in number of animals) of calves, female yearlings, male yearlings, adult females and adult males, respectively. $X$ is total population size.

|          | Case 1 | Case 2 | Case 3 | Case 4 | Case 5 |
|----------|--------|--------|--------|--------|--------|
| $h_c$    | 0.20   | 0.19   | 0.18   | 0      | 0.18   |
| $h_{yf}$ | 0      | 0      | 0      | 0      | 0.18   |
| $h_{ym}$ | 0      | 0      | 0      | 0      | 0.18   |
| $h_f$    | 0.15   | 0.13   | 0.11   | 0.21   | 0.18   |
| $h_m$    | 0.55   | 0.55   | 0.55   | 0.55   | 0.18   |
| $H$      | 1,060  | 1,088  | 1,096  | 951    | 732    |
| $X_c$    | 1,651  | 1,717  | 1,799  | 1,632  | 1,165  |
| $X_{yf}$ | 494    | 520    | 555    | 612    | 360    |
| $X_{ym}$ | 494    | 520    | 555    | 612    | 360    |
| $X_f$    | 2,158  | 2,449  | 3,042  | 2,097  | 1,165  |
| $X_m$    | 740    | 778    | 830    | 916    | 1,102  |
| $X$      | 5,532  | 5,984  | 6,780  | 5,869  | 4,151  |

Case 1: Baseline parameter values. Maximizing meat value minus grazing damage. Both constraints (12) and (13) included, and (13) with $\bar{h}_{ym} = \bar{h}_m = 0.55$.

Case 2: As Case 1, but meat price increased by 50 %

Case 3: As Case 1, but maximizing meat value only

Case 4: As Case 1, but excluding the female – calf constraint (12)

Case 5: Uniform harvest pattern. Maximizing meat value minus grazing damage

**Table 4.** *Steady state landowner yearly cost and benefit (in 1,000 NOK).*

|  | Case 1 | Case 2 | Case 3 | Case 4 | Case 5 |
|---|---|---|---|---|---|
| Meat value *(V)* | 4,071 | 6,477 | 4,303 | 4,521 | 2,620 |
| Crop damage *(D)* | 1,843 | 2,006 | 2,303 | 1,984 | 1,476 |
| Net benefit *(R=V-D)* | 2,228 | 4,472 | 2,000 | 2,537 | 1,144 |
| Net benefit per total hunted animal *(R/H)* | 2.102 | 4.108 | 1.825 | 2.667 | 1.563 |
| Net benefit per total living animal *(R/X)* | 0.403 | 0.747 | 0.295 | 0.432 | 0.276 |

Case 1: Baseline parameter values. Maximizing meat value minus grazing damage. Both constraints (12) and (13) included, and (13) with $\bar{h}_{ym} = \bar{h}_m = 0.55$.

Case 2: As Case 1, but meat price increased by 50 %
Case 3: As Case 1, but maximizing meat value only
Case 4: As Case 1, but excluding the female – calf constraint (12)
Case 5: Uniform harvest pattern. Maximizing meat value minus grazing damage

**Figure 1.** Number of harvested red deer per 10 km$^2$ in Norwegian municipalities in 1970 (left) and 2009 (right). The figure illustrates both the change in population density and geographical distribution. Source: Statistics Norway.

**Figure 2**. Dynamic harvest pattern Case 1. Harvesting rates $h$ and time (years). Harvest rates of both male and female yearlings are zero all the time.

**Figure 3**. Contour plots showing how different levels of landowner net benefit, meat hunting value minus grazing damage $(V - D)$ (1,000NOK) relates to variation in harvest rates of adult males ($h_m$) and adult females ($h_f$). The left and right panel illustrates the conditions described in Case 1 and Case 4 respectively.The optimal steady state is identified with dots in both panels. The numbers along the level curves indicate net economic benefit (in 1,000 NOK), and the distance between two adjacent curves is 1 mill. NOK. Note the different scaling of the two panels.

**Figure 4.** Dynamic harvest pattern when recreational value is included. $z = 50$ (NOK/hunting day). Other parameter values and constraints as in Case 1. Harvesting rates and time (years). Harvest rates of both adult males and yearlings are kept constant at 0.55.
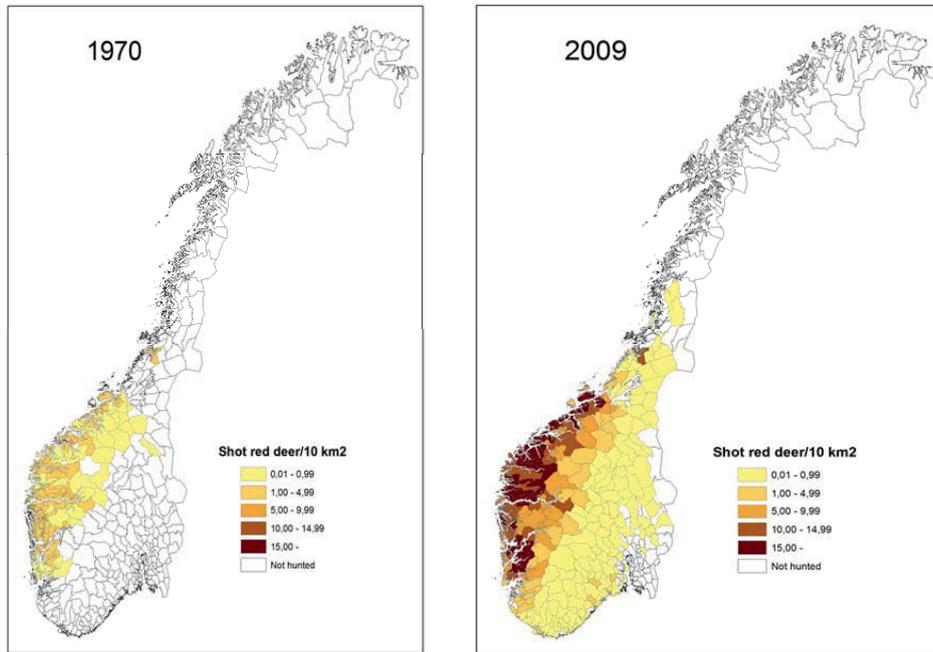
**Figure 1**
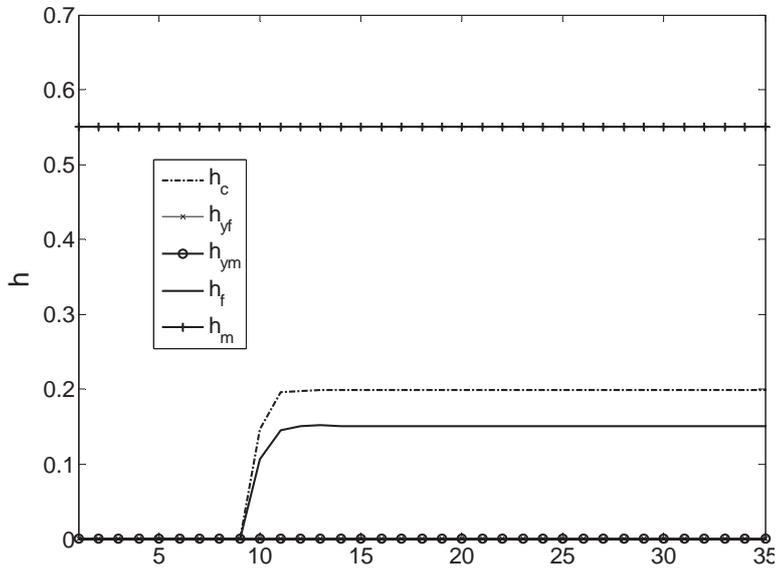
**Figure 2**
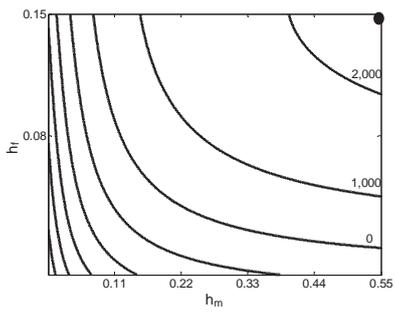


Case 1                                            Case 4

**Figure 3**

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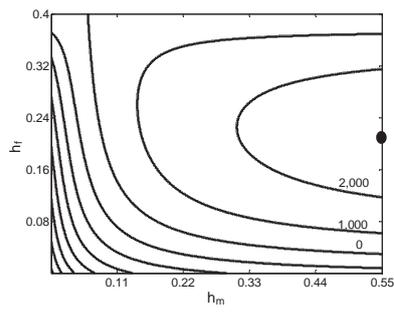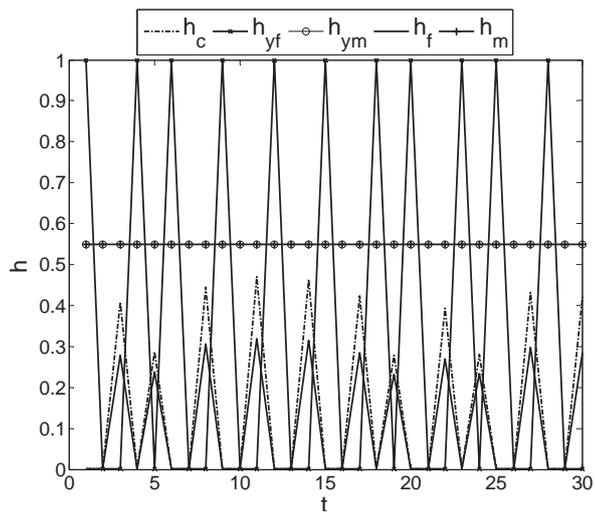
**Figure 4**

**Supplement section**

The Lagrangian of the problem of maximizing the present value benefit when only meat hunting value is included given the biological constraints plus constraints (12) and (13) may be written as

$$L = \sum_{t=0}^{\infty} \rho^t \Big\{ \ p_c h_{c,t} X_{c,t} + \big[ p_{yf} h_{yf,t} + p_{ym} h_{ym,t}(1-\psi)/\psi \big] X_{yf,t} + p_f h_{f,t} X_{,t} + p_m h_{m,t} X_{m,t}$$

$$- \rho \gamma_{t+1} \Big[ X_{c,t+1} - r_{yf}\left( X_{yf,t} + X_{f,t} \right) X_{yf,t} - r_f \left( X_{f,t} + X_{yf,t} \right) X_{f,t} \Big]$$

$$- \rho \eta_{t+1} \Big[ X_{yf,t+1} - \psi s_c \left( 1 - h_{c,t} \right) X_{c,t} \Big]$$

$$- \rho \lambda_{t+1} \Big[ X_{f,t+1} - s_{yf}\left( 1 - h_{yf,t} \right) X_{yf,t} - s_f \left( 1 - h_{f,t} \right) X_{f,t} \Big]$$

$$- \rho \mu_{t+1} \Big[ X_{m,t+1} - [s_{ym}(1-h_{ym,t})(1-\psi)/\psi] X_{yf,t} - s_m(1-h_{m,t}) X_{m,t} \Big]$$

$$- \rho \omega_{t+1}\left( h_{f,t} X_{f,t} - h_{c,t} X_{c,t} \right) - \rho \zeta_{t+1}\left( h_{m,t} - \overline{h}_m \right) - \rho \chi_{t+1}\left( h_{ym,t} - \overline{h}_m \right) \Big\}$$

$\gamma_t > 0$, $\eta_t > 0$, $\lambda_t > 0$, $\mu_t > 0$ are the shadow prices of the calf population constraint (1), female yearling constraint (2), adult female constraint (4) and adult male population constraint (7), respectively. $\omega_t \geq 0$ is the shadow price of the female – calf harvesting restriction (12), and $\zeta_t \geq 0$ and $\chi_t > 0$ are the shadow prices of the adult male and male yearlings harvesting restrictions (13). Notice that the population constraint shadow prices, in contrast to the two harvesting constraints (12) and (13), always will be strictly positive because these constraints always bind (Kuhn-Tucker theorem).

The first-order necessary control conditions of this maximizing problem are stated with the actual complementary slackness conditions and where the possibility for keeping each of the stages unexploited is considered while harvesting whole subpopulations are not considered as a possible option. These control conditions with $X_{i,t} > 0$ ($i = c, yf, ym, f, m$) read then:

(A1) $\quad \dfrac{\partial L}{\partial h_{c,t}} = X_{c,t}\left( p_c - \rho \eta_{t+1}\psi s_c + \rho \omega_{t+1} \right) \leq 0 \ ; \ 0 \leq h_{c,t} < 1$,

(A2) $\quad \dfrac{\partial L}{\partial h_{yf,t}} = X_{yf,t}\left( p_{yf} - \rho \lambda_{t+1} s_{yf} \right) \leq 0 \ ; \ 0 \leq h_{yf,t} < 1$,

(A3) $\quad \dfrac{\partial L}{\partial h_{ym,t}} = X_{ym,t}\left( p_{ym} - \rho \mu_{t+1} s_{ym} \right) \leq 0 \ ; \ 0 \leq h_{ym,t} < 1$,

(A4) $\quad \dfrac{\partial L}{\partial h_{f,t}} = X_{f,t}\left( p_f - \rho \lambda_{t+1} s_f - \rho \omega_{t+1} \right) \leq 0; \ 0 \leq h_{f,t} < 1$

and

(A5)   $\dfrac{\partial L}{\partial h_{m,t}} = X_{m,t}(p_m - \rho\mu_{t+1}s_m - \rho\xi_{t+1} / X_{m,t}) \le 0$  ; $0 \le h_{m,t} < 1$.

The portfolio conditions $\partial L / \partial X_{i,t} = 0$ ( $i = c, yf, f, m$) are rather messy and are not stated here.

Condition (A1) says that calf harvest should take up to the point where marginal harvest value is equal to, or below, the cost in term of reduced yearling growth of both categories evaluated at their shadow prices while also taking the discounting into account. The female - calf harvesting restriction shadow price is also included in this condition. When this condition holds as an inequality, the marginal benefit is below its marginal cost and harvesting of this stage is thus not profitable, $h_{c,t} = 0$. In a similar manner, condition (A2) and (A3) indicate that harvesting of the yearling female (male) category should take place up to the point where the marginal benefit is equal to, or below, the cost in terms of reduced population of adult females (males). The female and male adult conditions (A4) and (A5), respectively, can be given a similar interpretations, but the shadow cost of the female - calf harvesting constraint is included in the female condition (A4) while the shadow cost of the male harvesting rate is included in condition (A5).

These first order control conditions together the portfolio conditions and the biological constraints comprise a complex dynamic system. It is therefore difficult, if not impossible, to assess the dynamics analytically. However, based on the control conditions, it is possible to draw some important conclusions about the harvesting composition. To see this, we look away from the constraints (12) and (13). The above conditions are then simplified to:

(A1')   $\dfrac{\partial L}{\partial h_{c,t}} = X_{c,t}(p_c - \rho\eta_{t+1}\psi s_c) \le 0$ ; $0 \le h_{c,t} < 1$,

(A2')   $\dfrac{\partial L}{\partial h_{yf,t}} = X_{yf,t}(p_{yf} - \rho\lambda_{t+1}s_{yf}) \le 0$ ; $0 \le h_{yf,t} < 1$,

(A3')   $\dfrac{\partial L}{\partial h_{ym,t}} = X_{yf,t}(p_{ym} - \rho\mu_{t+1}s_{ym}) \le 0$ ; $0 \le h_{ym,t} < 1$,

(A4')   $\dfrac{\partial L}{\partial h_{f,t}} = X_{f,t}(p_f - \rho\lambda_{t+1}s_f) \le 0$; $0 \le h_{f,t} < 1$

and

(A5')     $\dfrac{\partial L}{\partial h_{m,t}} = X_{m,t}(p_m - \rho\mu_{t+1}s_m) \le 0$  ; $0 \le h_{m,t} < 1$.

From these conditions it is straightforward to find that harvesting female yearlings and females at the same time cannot represent a possible optimal policy. To see this, assume first that female yearling harvest is optimal. (A2') (with $X_{yf,t} > 0$) holds then as $p_{yf} = \rho\lambda_{t+1}s_{yf}$. In a similar manner, assume next that female harvest is optimal. With $X_{f,t} > 0$, (A4') then reads $p_f = \rho\lambda_{t+1}s_f$. Combining these two equation yields then $p_{yf} / s_{yf} = p_f / s_f$. As only parameters are included here, this equation holds only by accident, Therefore, harvesting both these two stages at the same time cannot represent an optimal solution when the constraints (12) and (13) are not binding and when the harvest recreation value is not included. When combining (A3') and (A5') we can draw a similar type of conclusion; that is, harvesting male yearlings and males at the same time is not in accordance with the optimality conditions. In the main text (section 5.1) more discussions based on (A1') – (A5') are found. For example, harvesting female yearlings together with zero adult female harvesting yields $p_{yf} / s_{yf} > p_f / s_f$, which represents a contradiction due to the structure of the parameter values.

When including grazing damage, $D_t = d_c X_{c,t} + d_{fy} X_{fy,t} + d_{my} X_{my,t} + d_f X_{f,t} + d_m X_{m,t}$, but still not including the recreational hunting value, the landowner problem is to $\displaystyle\max_{h_{c,t},h_{fy,t},h_{my,t},h_{f,t},h_{m,t}} \sum_{t=0}^{t=\infty} \rho^t (V_t - D_t)$, again subject to the biological growth equations (1), (2), (4) and (7) together with constraints (12) and (13). It is easily recognized that the control conditions of this problem will be similar to that of (A1) – (A5) because the harvest rates are not includet in the $D_t$ expression; that is, the damage occurs before the hunting.

When the recreational value also is included, the problem is to maximize

$$\sum_{t=0}^{\infty} \rho^t (V_t - D_t + B_t) = \sum_{t=0}^{\infty} \rho^t \Big\{ \; p_c h_{c,t} X_{c,t} + \big[ p_{yf} h_{yf,t} + p_{ym} h_{ym,t}(1-\psi)/\psi \big] X_{yf,t} + p_f h_{f,t} X_{,t} + p_m h_{m,t} X_{m,t}$$
$$- (d_c X_{c,t} + d_{fy} X_{fy,t} + d_{my} X_{my,t} + d_f X_{f,t} + d_m X_{m,t}) + (z/\theta)\big( h_{c,t} + h_{yf,t} + h_{ym,t} + h_{f,t} + h_{m,t} \big) \Big\}$$

to the same constraints as above.

The first-order necessary control conditions with $X_{i,t} > 0$ ($i = c, yf, ym, f, m$) read now:

(A1'') $\quad \dfrac{\partial L}{\partial h_{c,t}} = X_{c,t}\left(p_c + z/\theta X_{c,t} - \rho\eta_{t+1}\psi s_c + \rho\omega_{t+1}\right) \le 0 \ ; \ 0 \le h_{c,t} < 1,$

(A2'') $\quad \dfrac{\partial L}{\partial h_{yf,t}} = X_{yf,t}\left(p_{yf} + z/\theta X_{yf,t} - \rho\lambda_{t+1}s_{yf}\right) \le 0 \ ; \ 0 \le h_{yf,t} < 1,$

(A3'') $\quad \dfrac{\partial L}{\partial h_{ym,t}} = X_{ym,t}\left(p_{ym} + z\psi/(1-\psi)\theta X_{yf,t} - \rho\mu_{t+1}s_{ym}\right) \le 0 \ ; \ 0 \le h_{ym,t} < 1,$

(A4'') $\quad \dfrac{\partial L}{\partial h_{f,t}} = X_{f,t}\left(p_f + z/\theta X_{f,t} - \rho\lambda_{t+1}s_f - \rho\omega_{t+1}\right) \le 0 ; 0 \le h_{f,t} < 1$

and

(A5'') $\quad \dfrac{\partial L}{\partial h_{m,t}} = X_{m,t}\left(p_m + z/\theta X_{m,t} - \rho\mu_{t+1}s_m - \rho\xi_{t+1}/X_{m,t}\right) \le 0 \ ; 0 \le h_{m,t} < 1.$

The important difference is now that the population sizes are included in these control conditions.

**Chapter 4**

**Sustainable production and the ambiguous effect of discounting**

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