A study of irrational investors in financial markets

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The thesis uses market data to investigate irrational investors in the financial markets. Traditional finance theory states that irrational investors do not influence asset prices. The analysis confirms this statement. The thesis also looks into the survival of irrational investors. The analysis shows that irrational investors will not survive.

Preface

This master thesis is the concluding work of the master in Financial Economics at the department of Economics at Norwegian University of Science and Technology (NTNU).

I am thankful for all the advice I have received from my advisor Egil Matsen. I am also grateful for the help I received with the analysis from Kåre Johansen. This thesis would not have been completed in time without the help of Denise Røsten and all her corrections of my bad grammar.

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1. Introduction

The emergence of technology can be traced back to the spear and arrow. Hunter-gatherers developed these basic tools to enhance the physical labor needed in procuring food. The creation of the machine, such as the tractor, introduced the next stage in the evolution of technology. These complex tools allowed for the substitution of physical labor and permitted humans to exceed physical limitations. This resulted in a tremendous increase in production. Technology, in essence, has always been developed to increase efficiency and productivity.

At this day in age, most things in society are being mediated by automated machines. Unlike their predecessors, these multifaceted machines do not need an operator to control its functions. Instead, automatic algorithms replace human control. One such machine that is used in our daily lives in more ways than one is the computer.

The advances in robotics enable computers to take over human tasks and increase efficiency and productivity by eliminating human weakness. However, human nature in general renders imperfections. There will always be differences in levels of quality and cost. More often than not, humans are considered expensive and lacking in speed and accuracy. To cope with a complex and dynamic world, humans adopt certain survival techniques and short cuts called heuristics to overcome the enormous amounts of information they are bombarded with every day. Our limitations make our decision making flawed.

The financial markets today focus primarily on speed. Investment companies move closer to the physical location of the market place, preferably in the same house, to reduce the time it takes to send and receive information. The majority of the “traffic” is not carried out by human investors, but advanced computer programs that follow a preprogrammed set of rules. As more of the market movements are performed by computer programs, reducing the influence of educated investors, one can assume that the dynamics of the markets will inevitably change.

Norwegian media has focused heavily on the issue of fairness in the market place. Do the computers follow the same rules and regulations as normal investors? Human beings do not possess the same mental processing capacity that computers do. Put simply, computers are faster. They can handle larger amounts of information and have a better problem solving capability. This means that computers can act on information before the traditional investors see the opportunity. Does this mean that computers take advantage of human weakness? This man vs. the machine dilemma is an emerging problem in the Norwegian financial markets. It was not until April 2010 that the OSE[1] decided to upgrade their computer trading system by increasing its speed and consequently claiming the benefits of HFT[2]. The OSE has so far put little regulation on the use of algorithm trading, and it may be too early to clearly predict the effects of this change in the market.

The growth of the internet combined with the use of an electronic trading platform opened up the possibility of online trading of financial products in the financial markets. Well into the 1990's an investor had to contact an investment bank to execute transactions. Now anyone can trade shares from their living room through online investment banks.

Traditional finance theory assumes rational investors (Bodie, Kane & Marcus, 2009). These theories do not account for investors displaying human weakness. One may argue that an educated financial advisor is as rational as an investor can be. They possess expert knowledge of the financial markets and the market mechanisms. In 1985 De Bondt and Thaler published an article called "Does the stock market overreact?" The article looked closely at the reaction pattern of investors to dramatic news and events. Their interest in the combination of "market behavior and the psychology of individual decision making" (De Bondt & Thaler, 1985) was the beginning of modern behavioral finance. With the possibility of online investment banks another group of investors can directly influence the market. These investors may not have deep knowledge about the financial markets. Investors that act on hunches instead of fundamental values are not accounted for in traditional finance theory. Behavioral finance tries to use the knowledge of psychology to see how psychology influences the investors' behavior and subsequently the financial markets (Sewell, 2010).

With an educational background from both psychology and finance it was tempting to fuse these academic disciplines to test some of the hypotheses originating from behavioral finance theory. There has been an increased awareness about algorithm and HFT since strange occurrences has happened in the financial markets. The focus of thesis is not on the potential good or bad consequences stemming from algorithm and HFT; it only looks upon

[1] Oslo Stock Exchange will now be referred to as OSE
[2] High Frequency Trading will now be referred to as HFT

them as ultra-rational investors with a large investment capacity. Rather, it is their influence on the market place dynamics, which is of an interest. This means that the introduction of HFT works as a major increase in the amount of rational investors present in the market.

This thesis attempts to enlighten the role of the irrational agent through quantitative exploration of real world data. This is not done without difficulties. How does one differentiate between rational and irrational investors in publicly available aggregate data? In such volatile times, is it possible to single out the true effects of HFT?

There are two hypotheses that are put to the test:

1. Can irrational investors survive in the financial markets?
2. Do irrational investors have an impact on asset prices?

The first hypothesis looks closer at the survival of the irrational investor. Financial theory states that these investors will "buy high, and sell low" (De Long, Shleifer, Summer & Waldmann, 1991) and in the end run out of the financial strength needed to stay in the markets. HFT increases the number of rational investors that can exploit the mispricing of the irrational investors. The effect should be reflected in the number of transactions performed by the different groups of investors.

The second hypothesis looks closely at the influence of the irrational investors on the financial markets. Financial theory states that irrational investors do not have an impact on asset prices (Friedman, 1953), (Fama, 1965). Our society charges taxes and fees that increases the risk associated with exploiting the mispricing caused by irrational investors, thereby increasing the likelihood of their influence. We also live in a society where financial news can be accessed by anyone. This could lead to irrational investors' unconsciously becoming more rational. Assuming that irrational investors have an influence on asset prices, and that this influence is reduced by the introduction of HFT, it should be revealed in the volatility of OSE.

2. Theory

The theory section of this thesis begins by explaining the arguments of the classical finance theorists. This will then be followed by a detailed presentation of relevant behavioral finance theories. These theories contain the elements that this thesis aims to test and are also the source of the null and alternative hypothesis. Lastly, a short introduction to HFT (HFT) will be given.

2.1 Price formation/ Asset pricing

There are several ways to approach asset pricing. To keep a relatively similar system between the theoretical models, a consumption based asset pricing model will be used in describing the price formation. The model is described in Cochrane (2005, ch.1).

Cochrane (2005) presents a two period model with two types of investors; young and old. When the investors are young they have to decide between consumption now (period t), and consumption later (period t+1). This means that the investor will, in period t, invest the cash that is not used for consumption. The investor will invest in a stock (or asset with uncertain cash flow). This will yield the following base for consumption in period t+1 as the investors will use all assets for consumption:

Eq(1) $$x_{t+1} = p_{t+1} + d_{t+1}$$

Where p_{t+1} represent the price of the stock at time t+1, and d_{t+1} is the dividend payout received. These quantities are unknown at time t, so x_{t+1} is a stochastic variable (Cochrane, 2005).

The motivation behind the investments is for the investors to maximize their expected utility over their life time. Over the two periods their utility function is given by:

Eq(2) $$U(C_t, C_{t+1}) = u(C_t) + \beta E_t\left[u(C_{t+1})\right],$$

β is the investors' subjective discount factor that captures the investors' impatience.

Eq(3) $$\beta = \frac{1}{1+\delta}$$

δ is the time preference of money. It is assumed that people prefer money now rather than later.

The utility function is maximized given the following budget constraints:

Eq(4), Eq(5)
$$C_t = e_t - \xi p_t$$
$$C_{t+1} = e_{t+1} + \xi x_{t+1}$$

e_t, e_{t+1} is exogenous income in *t* and *t+1* (Cochrane, 2005).

The utility function comes with a set of assumptions that creates a frame for our analysis. These assumptions can be seen in figure 1 below:

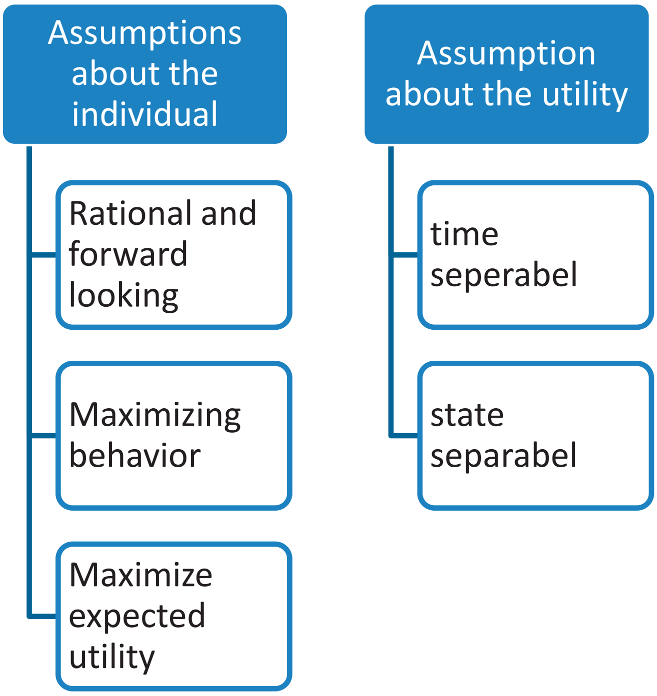

Figure 1: Assumptions for utility function

When the utility function is optimized with regards to ξ it yields the following first order conditions, known as the pricing equation:

Eq(6)
$$p_t = E_t\left[\beta \frac{u'(C_{t+1})}{u'(C_t)} x_{t+1}\right]$$

The pricing equation can be simplified to:

Eq(7) $$p_t = E_t\left[m_{t+1}x_{t+1}\right]$$

Eq(8) $$m_t = \beta\frac{u'(C_{t+1})}{u'(C_t)}$$

Where m_{t+1} is the stochastic discount factor for time t+1.

This pricing formula can be generalized from a two period model by assuming the following:

- power utility function (CRRA)
- lognormal distributed consumption growth
- joint distribution of asset returns and consumption growth is lognormal.

This yields the following pricing equation (Cochrane, 2005):

Eq(9) $$p_t = \frac{E_t[x_{t+1}]}{1+r_t^f} + Cov_t(m_{t+1}, x_{t+1})$$

The asset price is dependent on three factors: a) the risk free interest rate, b) the rational investors' expectation to next year's return, and c) the stochastic discount factor.

The key in this mathematical formula lies in the assumptions that are associated with the utility function. They postulate the existence of only rational investors. A rational investor is an individual who seeks to maximize his or hers expected utility through maximizing his or her consumption with regards to the risk associated with the investment opportunities (Cochrane, 2005).

We can draw lines from the theory of price formation to the theory of random walk (asset price movements). If irrational investors do exist, their faulty views will be eliminated by the rational investors. The rational investors fight with each other for the newest and best information available. New information will automatically be reflected in the stock price. The only powers that can move the stock price are unforeseen shocks and events. This means that the likelihood of a price going up or down is dependent on the probability distribution of the random events. In finance, it has been assumed that the economical shocks that influence the prices are normally distributed with a mean of zero (Fama, 1965)

Eq(10)

$$p_t = \mu + u_t$$
$$u \sim N(0, \sigma_u^2)$$

An investor cannot, consistently, achieve returns that are in excess of the market return. Malkiel (2003) states that, “efficient financial markets do not allow investors to earn above-average returns without accepting above-average risks.” An investor (with a large portfolio) cannot achieve higher returns than the market without taking on additional risk. This theory is in accord with the Efficient Market Hypothesis which looks into the degree of information reflected in the stock price.

2.2 Efficient Market Hypothesis (EMH)

According to Fama (1970) “the primary role of the capital markets is allocation of ownership of the economy’s capital stock”. Fama formed the efficient market hypothesis stating that “a market in which prices always fully reflect all available information is called efficient” (Fama, 1970). The definition, however, of an efficient market is rather vague. It does not specify the exact definitions of the terms “available information” and “fully reflect”.

Instead of a single definition, Fama used Samuelson’s (1965) taxonomy that identifies three forms of market efficiencies: weak-form, semi strong-form, and strong form:

- The weak form hypothesis states that all information that can be derived from historical prices, trading volumes or short interest cannot be used to generate return excess of the market return.
- Semi strong-form hypothesis states that all publicly available information is already reflected in the stock prices.
- Strong- form hypothesis states that all information, public and private, are reflected in the stock prices.

By creating a theory that allows for different states of informational availability, he created a more dynamic theory that covers a wider specter of the world.

These efficiency theories assume that there are no costs associated with the collection of information. In the real world the price of information is connected to its importance, and speed of its delivery. This could create a situation with informational asymmetry, but this is not accounted for in these theories. The simplification makes it easier to use the definitions of EMH in empirical testing. EMH is based on the following assumptions (Shleifer, 2000):

- The investors are independent, rational, profit maximizing individuals
- All information is free but is randomly available
- There are no taxes or transaction costs in the market

2.3 Behavioral finance

As a reaction to the efficient market hypothesis and traditional financial theory, which assume that all investors are rational individuals that invest in a homogenous way, economists influenced by psychological theory began arguing that investors are not rational beings and therefore the financial markets are inefficient. Their arguments against rational investors were founded on the psychological research on heuristics. Heuristics are cognitive methods employed to simplify the world. Heuristics are used to "reduce the complex task of assessing probabilities and predicting values to simpler judgmental operations. In general, these heuristics are quite useful, but sometimes they lead to severe and systematic errors" (Tversky & Kahneman, 1974). These short cuts hinder investors from always behaving rational.

Traditional financial theory that is based on the EMH and the assumption of rational investor cannot satisfyingly explain all of the actual market movements (De Bondt & Thaler, 1984). De Bondt and Thaler (1984) looked closer at the overreaction of stock markets. They found that prior "looser" stocks had a tendency to outperform prior "winner" stocks and therefore proving a degree of predictability in the market. Other market movements that cannot be explained are irrational bubbles, sudden market crashes, and the equity premium puzzle (EPP) among others. The EPP stems from the equity premium (excess return of a risky asset over a riskless) found in the US stock markets. This premium is greater than what "can be rationalized in the context of the neoclassical paradigm of financial economics" (Mehra, 2003). Behavioral finance theorists argue that including human imperfections in the financial models will make the models better describe the observed phenomena.

Because of its psychological roots behavioral finance theorists tend to focus on psychological shortcuts that remove our rational being. Most critics say that behavioral finance is a study of anomalies rather than a theory of the markets as a whole and that this is their shortcoming. The following section will present two behavioral finance theories which will form the basis of this thesis' two research questions.

2.4 Limits of Arbitrage Model

The limits of Arbitrage model looks into a situation where rational agents cannot eliminate all of the influence that irrational agents have on asset prices. The mispricing caused by the irrational agent creates added risk to the rational agent thereby reducing their willingness to exploit the mispricing (Matsen, 2011). In this model, the assumption of the rational homogenous agent is relaxed. These agents would normally guarantee that the financial markets are efficient (market fully reflects all available information).

Even if there is a situation where irrational agents are present in the market, one can still have efficient markets if:

- the irrational agents act in a random manner $I \sim N(0,\sigma^2)$ so that their irrationality is cancelled out by other irrational agents, or
- the irrational agents deviate in a systematic way, which leads to a mispricing of the asset that the rational investors can eliminate with opposite position. This eliminates the irrational agents influence on the asset price.
  (Matsen, 2011)

If the markets are efficient it means that the irrational agents do not influence the asset price, and therefore do not influence the market volatility.

Limits of arbitrage argue that these irrational (outside investors) agents can influence the market price of an asset through the unwillingness of rational (arbitrageurs) investors to take on the added risk created by the irrational agents. (Matsen, 2011).

The limits of arbitrage model consist of rational and irrational agents. The rational agents trade on fundamental values whereas the irrational agents trade on noise. Black (1986) labels noise as information without any informational value (without real fundamental value). In financial markets a rational agent will face the fundamental risk that stems from the uncertainty about future fundamental values. However, they also face the possibility that the noise traders' influence can get stronger making their positions worse off. This risk arises from the possibility that the already mispriced asset will become even more mispriced at a later time; since most rational investors have short investment horizon they may not be able to wait for the mispricing to disappear. This added risk reduces the rational agents' thirst of exploiting the irrational agents' mispricing (Gromb & Vayanos, 2010).

The mathematical model for cross asset arbitrage was introduced by Gromb and Vayanos (2010).

2.4.1 Cross asset arbitrage (Gromb & Vayanos, 2010)

The cross asset arbitrage model has two foundations on which it explains the dynamics of the financial market. Limits of arbitrage are normally considered as the first "building block needed to explain anomalies. The other building block is demand shocks experienced by investors other than arbitrageurs" (Gromb & Vayanos, 2010). The demand shocks move asset prices away from fundamental values. Limits of arbitrage prohibit the arbitrageurs to correct the mispricing.

The model consists of two types of investors. The first type of investor is the traditional rational investor. They are competitive, risk averse and utility maximizing. Gromb and Vayanos referred to these investors as the arbitrageurs. The second type of investor is called the outside investor. These agents are the irrational investors in this model. Their irrationality is represented by an inelastic demand (u) for the risky asset A. u is the demand shock that moves asset prices away from fundamental values (Gromb & Vayanos, 2010).

It is a two period model that has two risky assets (A and B). These assets have payoff d_A and d_B. $\overline{d_A}, \overline{d_B}, \sigma_A, \sigma_B, \rho$ represents the mean payoff for A and B, standard deviation for A and B and the correlation between the payoff of A and B. Since arbitrageurs are rational their payoff is equal to the mean payoff $d_B = \overline{d_B}$. We assume that d_A and d_B are jointly normal distributed to simplify the calculations.

The model describes a shock driven economy. The economy only consists of the arbitrageurs and outsiders so their total demand makes up the total demand of the economy. To simplify the model Gromb and Vayanos (2010) normalized the demand for asset A to zero. In a model such as this the actual demand in number of units is not interesting. It is the effect of a demand shock that is of an interest. By setting the net demand to zero the model will clearly display these effects, and be simpler mathematically. In the instance of a demand shock ($u \neq 0$) asset prices will move away from fundamental values and arbitrageurs will try to exploit the mispricing (Gromb & Vayanos, 2010).

In period one, the arbitrageurs have to choose their investment in asset A (X_A) and in asset B (X_B) to maximize their expected utility:

Eq(11) $$-E_1\left[\exp(-\alpha W2)\right]$$

This function is subject to the budget constraint:

Eq(12) $$W_2 = W_1 + x_A(d_A - p_A) + x_B(d_B - p_B)$$

Where α = risk aversion of arbitrageurs. By inserting the budget constraint into the formula for expected utility, using the assumption of normality, and the assumption of $p_B = \overline{d_B}$. Optimizing $-e^{-X}$ will be equivalent to maximizing the mean variance objective function, where one tries to minimize the outcome:

Eq(13) $$x_A(d_A - p_A) - \frac{\alpha}{2}(x_A^2\sigma_A^2 + x_B^2\sigma_B^2 + 2x_A x_B \sigma_A \sigma_B \rho)$$

This gives the following optimal investments in asset A and B (Gromb & Vayanos, 2010):

Eq(14) $$x_A = \frac{\overline{d_A} - p_A}{\alpha\sigma_A^2(1-\rho^2)}$$

Eq(15) $$x_B = -\frac{\rho(\overline{d_A} - p_A)}{\alpha\sigma_A\sigma_B(1-\rho^2)}$$

If we take into account that asset A is in zero net supply:

Eq(16) $$x_A + u = 0 .$$

We get the following equilibrium price for asset A:

Eq(17) $$p_A = \overline{d_A} + \alpha\sigma_A^2(1-\rho^2)u$$

As we can see from the equation, the price of asset A, p_A, will increase when there is a demand from outside investors. This excess demand will drive the price above fundamental value. The price of asset A will be higher:

- the higher the risk aversion of the arbitrageurs, α
- the higher the volatility of asset A, σ_A^2
- the lower the correlation between asset A and B (due to the poor hedging possibility), ρ
- the higher the demand from outside investors, u

There are two types of risk associated with asset A. First there is fundamental risk which stems from the uncertainty of the assets future value (dividend stream). The second source of uncertainty is called non fundamental risk. In the model Gromb and Vayanos (2010) a third period (period 0) where $\overline{d_A}, \overline{d_B}, u$ are stochastic is introduced. The asset price depends "on the realization of u" (Gromb & Vayanos, 2010), which is unknown and stochastic. From the pricing formula we can find the equation for the non fundamental risk in period 0 created by the outside investors (Gromb & Vayanos, 2010). By taking the variance of the price of asset A, then taking the square root of the variance (Matsen, 2011):

$$
\begin{aligned}
VAR(P_A) &= VAR(\overline{d_A} + \alpha\sigma_A^2(1-\rho^2)u) \\
VAR(P_A) &= \sigma_A^2 + \alpha^2\sigma_A^4(1-\rho^2)^2\sigma_u^2 \\
VAR(P_A) &= \sigma_A^2(1+\alpha^2\sigma_A^2(1-\rho^2)^2\sigma_u^2)
\end{aligned}
$$

Taking the square root of the final expression above leads to equation for non fundamental risk.

Eq(18) $$\sigma_{pA} = \sigma_A\sqrt{1+\alpha^2\sigma_A^2(1-\rho^2)^2\sigma_u^2}$$

The presence of outside investors will increase the volatility of asset A through $\alpha^2\sigma_A^2(1-\rho^2)^2\sigma_u^2$. When there is high uncertainty in the market, either represented through the variance of asset A or the demand shock *u*, it leads to a larger mispricing of the asset. Alpha represents the level of risk aversion inherent in the arbitrageur. This coefficient reports how accepting an investor is to uncertainty. A high value of the coefficient means that the investor is not willing to take on a lot of risk. The investor will then be less willing to exploit arbitrage opportunities, thereby letting the outside investors influence the asset price which leads to increased volatility in the price of the asset. This will lead to a situation where the outside investors can permanently influence the price and volatility of the asset. The non fundamental risk can be reduced through hedging opportunities if asset A or asset B is highly correlated.

2.5 Noise Trader Model

The second model that is introduced is called the noise trader model. The theory stated in this section is taken from the article “Noise trader risk in financial markets” (1990) by De Long, Shleifer, Summers, and Waldmann.

As stated earlier the “unpredictability of noise traders’ beliefs creates a risk in the price of the asset that” (De Long, Shleifer, Summers & Waldmann, 1990) will deter the rational investors from seeking to exploit the mispricing. This means that according to noise trader theory irrational investors have a permanent impact on the price of an asset.

The theory uses an overlapping generation model. There are two types of investors in model: the rational investors and the irrational investors (called noise traders). The economy lasts forever, but the individual agent will only live for two periods. The initial investment will occur in the first period, and the wealth will be consumed in the second period. There are two assets present, one risk free and one risky. The risk free asset pays an interest rate r, is in elastic supply and has a price of 1. The risky asset pays a dividend of d, is in inelastic supply normalized to one unit, and has an unknown price p_t. In this model d=r. The total demands for the risky asset is given by the demand from the rational investors λ^R , and the demand from the irrational investors λ^I (De Long, Shleifer, Summers & Waldmann, 1990). Since the rational and the irrational investors make up the entire population of investors it is easier to work with normalized values. This means that the total amount of investors equal 1 (100%). The rational investors will make up $1-\mu$ of the population, and the irrational investors will make up μ .

The model assumes that the rational investors are risk averse with short investment horizons. The irrational investors have faulty expectations about future asset prices. This mispricing (ρ_t) is comprehensive for all noise trade and is normally distributed with a mean of ρ^* and variance of σ_ρ^2 (De Long, Shleifer, Summers & Waldmann, 1990)

$$\rho_t \sim N(\rho^*, \sigma_\rho^2) .$$

ρ_t is the "expectational error in period t regarding the price in period t+1" (Matsen, 2011). " ρ^* is a measure for the average "bullishness" of the noise traders" (De Long, Shleifer, Summers & Waldmann, 1990). σ_ρ^2 represents the variance of the "misperceptions of the expected return per unit of the risky asset" (De Long, Shleifer, Summers & Waldmann, 1990). The noise traders will therefore optimize their investments based on faulty expectations.

Both investors utility is represented through a constant absolute risk aversion (CARA) function:

Eq(20) $$U = -e^{-(2\gamma)w} .$$

W is the initial wealth of the investor and γ is the coefficient of absolute risk aversion (De Long, Shleifer, Summers & Waldmann, 1990). The investors will maximize their expected utility

Eq(21) $$E(U) = E_t\left[-e^{-(2\gamma)w}\right]$$

"With normally distributed returns to holding a unit of the risky asset, maximizing the" (De Long, Shleifer, Summers & Waldmann, 1990) utility is the same as maximizing the mean variance objective function:

Eq(22) $$E(U_{t+1}) = E_t\left[w_{t+1}\right] - \gamma \operatorname{var}_t(w_{t+1})$$

2.5.1 Demand functions

Wealth in time t+1 is the following for the investors:

Eq(23/24) $$\begin{aligned} w_{t+1}^R &= (w_t - \lambda_t^R p_t)(1+r) + \lambda_t^R (p_{t+1} + r) \\ w_{t+1}^I &= (w_t - \lambda_t^I p_t)(1+r) + \lambda_t^I (p_{t+1} + r) \end{aligned}$$

Where w_t is the initial wealth at time t, p_t is price of the risky asset at time t, and p_{t+1} is price of the risky asset at time t+1.

This will give the rational agent a demand for the risky asset equal (Matsen, 2011):

Eq(25/26/27)

$$\begin{aligned}
&E_t^R\left[w_{t+1}\right]=(w_t-\lambda_t^R p_t)(1+r)+\lambda_t^R(E_t^R\left[p_{t+1}\right]+r) \\
&Var_t^R(w_{t+1})=(\lambda_t^R)^2 Var_t(p_{t+1}) \\
&\lambda_t^R=\frac{r+E_t^R\left[p_{t+1}\right]-(1+r)p_t}{2\gamma Var_t(p_{t+1})}
\end{aligned}$$

The irrational agents demand for the risky asset (Matsen, 2011):

Eq(28/29/30)

$$\begin{aligned}
&E_t^I\left[w_{t+1}\right]=(w_t-\lambda_t^I p_t)(1+r)+\lambda_t^I(E_t^I\left[p_{t+1}\right]+r)\Leftrightarrow \\
&E_t^I\left[w_{t+1}\right]=(w_t-\lambda_t^I p_t)(1+r)+\lambda_t^I(E_t^R\left[p_{t+1}\right]+\rho_t+r) \\
&Var_t^I(w_{t+1})=(\lambda_t^I)^2 Var_t(p_{t+1}) \\
&\lambda_t^I=\frac{r+E_t^R\left[p_{t+1}\right]-(1+r)p_t}{2\gamma Var_t(p_{t+1})}+\frac{\rho_t}{2\gamma Var_t(p_{t+1})}=\lambda_t^R+\frac{\rho_t}{2\gamma Var_t(p_{t+1})}
\end{aligned}$$

The irrational agents' demand is equal to the demand of the rational agents plus the inherent misperception of the expected return of the risky asset. In this model the investors can have a negative demand, which means they are allowed to take short positions in the assets (De Long, Shleifer, Summers & Waldmann, 1990). When the irrational agents are "bullish" they will have a higher demand for the risky asset then the rational investors; therein driving the price of the risky asset above its fundamental values. The opposite is true when the irrational investors are "bearish".

2.5.2 Equilibrium price

In equilibrium, the demand must equal the supply. Earlier the supply of the risky asset was normalized to one unit. This means that the total demand of the rational investors and the total demand of the irrational investor must sum to one:

Eq(31)

$$\mu\lambda_t^I+(1-\mu)\lambda_t^R=1$$

This means that the price of the risky asset is given by:

Eq(32)

$$p_t=\frac{1}{1+r}\left[r+E_t^R\left[p_{t+1}\right]-2\gamma Var_t(p_{t+1})+\mu\rho^*\right]$$

De long et al. focused on a world with steady state equilibriums in their paper. In mathematical terms one can rewrite the conditional expectation on the price in t+1 to the unconditional expectation of price in t, because they will be identical. Therefore the pricing equation in steady state will be:

Eq(33)
$$p_t = 1 + \frac{\mu(\rho_t - \rho^*)}{1+r} + \frac{\mu\rho^*}{r} - \frac{2\gamma\mu^2\sigma_\rho^2}{r(1+r)^2}$$

The price of the risky asset will be driven above or below fundamental values depending on the “bullishness” of the irrational investors. In this analysis it is assumed that the noise trader risk is systematic (Matsen, 2011). That means that noise traders are bullish and bearish as a group. When these investors are bullish they will drive the prices higher, yet they will continue to invest in the asset. If they are bearish they will drive the prices down below fundamental values and keep selling. This partly fits with Friedman’s (1953) opinion that noise traders cannot survive in the financial markets. He claimed that these investors buy when prices are high, and sell when the prices are low and will therefore lose all their wealth. In essence this means that all irrational agents that enter the financial markets will exit the markets when all their wealth is consumed. In the long run there would be no irrational agents left in the market.

2.6 HFT

In the 80’s investment companies began using electronic trading platforms. Along with the enhancement of computer technology came steady improvements in the speed, capacity and accuracy of the trading platforms and its software. Some of the benefits with HFT are that computers have a higher processing capacity than humans; they do not get sick, and have faster reaction (Aldridge, 2010). Along with with reduced brokerage fees this savings opportunity is tempting. However, HFT cannot only be cost saving it also has to create profit.

HFT is based on a complex set of algorithms that dictate what the computer is allowed to do. The algorithms are the rules, and the information it receives from the financial markets are the input. With all the data at hand the computer uses its vast capacity to find investment opportunities (Aldridge, 2010). “Trading software incorporates optimal execution algorithms for achieving the best execution price within a given time interval through timing of trades, decisions on market aggressiveness, and sizing orders into optimal lots" (Aldridge, 2010).

The computers do not use fundamental values to find investment opportunities. Instead, they follow the principles of EMH which claim that the information is reflected in the market price. With all the information available the computers look for irregularities in the market that may be profitable. Therefore the HFT computers will only need to look at the market values of the assets to pick up market wide or company specific changes. The information appears faster in the data then it does through the traditional news channels. Therefore the computer will have an informational advantage compared to human investors ("HFT," 2010). "At the heart of HFT is a simple idea that properly programmed computers are better traders than humans" (Aldridge, 2010).

For HFT to be of any interest "two requirements must be met: the ability to quickly move in and out of positions and sufficient market volatility to ensure that changes in prices exceed transaction costs" (Aldridge, 2010). In Norway one such market is the OSE (equity market). It has the market liquidity that is needed, though it is a small market, and in April 2010 it upgraded its computer platform making it fast enough to benefit HFT.

HFT software has different trading strategies but they all break down to exploiting small opportunities through informational advantage. "High-frequency trading opportunities range from microsecond price moves allowing a trader to benefit from market-making trades, to several minute-long strategies that trade on momentum forecasted by microstructure theories, to several-hour-long market moves" (Aldridge, 2010).

3. Methodology

3.1 Data collection

This thesis takes a closer look into the dynamic world of financial markets with a behavioral finance frame of mind. It is crucial to be able to differentiate between rational institutional investors and irrational private investors. It is assumed that investors that influence the markets through an investment bank consisting of financial advisors with the relevant education within finance (or similar) and working in an environment where they share and possess expertise knowledge of the financial markets are rational investors. On the other hand, it is assumed that investors that influence the financial markets directly through internet investment banks without consulting financial advisors are irrational investors. In this definition of irrational investors there may be individuals who are rational that are grouped in with the irrational investors; but hopefully there will only be a few. The rational investor will receive advice based on fundamental values, where as the irrational investors will trade on information found in the media. This information may have aspects of fundamental value, but it is very likely that they are opinion based views and therefore considered noise. The Norwegian equity markets can be accessed through traditional investment banking contacts, but also directly through internet (investment) banking. Most finance institutions offer services to both of these customer groups making it difficult to differentiate between rational and irrational investors in the publicly available cumulative data.

3.1.1 Data for the first hypothesis

There were attempts to obtain differentiated data from the investment banks themselves during the data collection part of this thesis. However, due to the secret nature of this industry, none were willing to disclose detailed information. As such, publicly available information from the OSE was used. In order to obtain an in-depth look at the difference between rational and irrational investors, two samples representing the two groups were created. Theoretically, a rational investor has been defined, among others, as a rational being with optimizing behavior and utility maximizing. This definition acts as a basis for the slightly more superficial definition used in this thesis. As a sample for the rational investors, investment banks that do not offer internet banking are used. In this scenario, the customer must contact trained, rational, financial advisors before making an investment. In this situation an investor cannot participate in the market without taking the advice of an expert. This makes them the rational sample. The second sample consists of the investment banks that only offer internet transactions. The customers of such investments banks do not have contact with any of the banks' employees before making an investment. They may invest on hunches or news on the internet, but there is no guarantee that fundamental values are the basis for their investment. This makes them the irrational sample. There is a chance that rational investors are grouped into the irrational sample group. This can influence the analysis, but it is assumed that they would be in a minority and would not significantly influence the analysis. The sample groups are:

- Rational investors:
    - SEB
    - Arctic
    - Carnegie

- Irrational investors:
    - Skandiabanken
    - Nordnet
    - Netfunds

This thesis looks upon the number of transactions performed by these banks to see if irrational investors can survive in the market place. Another choice could have been to look at turnover, but that would have made the analysis complicated due to the interconnection between turnover and the overall economy. Even though the economy changes, the number of transactions may not alter as much as the average size (turnover) of the transactions thereby giving a more accurate measure of the number of participants in the market. The two samples used account for roughly thirty percent of the transactions at the OSE (Appendix A):

Market share of Transactions (Average 2008-2012)

Skandiabanken	1,61	SEB	8,51
Nordnet	8,69	Artic	1,98
Netfonds	3,6	Carnegie	2,4
Total Irrational	13,90	Total Rational	12,89

Table 1: The average market share (transactions) for the sample groups from 2008-2012

thereby, hopefully giving a good representation of the market as a whole. One negative aspect of this data sample is the length of the sample period. The data is monthly (number of transactions per month), and goes back less than five years. Therefore, there will only be a small sample prior to the computer upgrade and an even smaller one afterwards. The sample may be too short to display the effects of HFT. Another problem may be that some or all of these rational investment banks may themselves use HFT and therefore not represent a sample of traditional rational investors. This thesis attempts to determine whether or not traditional investors would be influenced or unaffected by HFT, but they may have adopted the practice.

3.1.2 Data for the second hypothesis

To investigate the second research question, a sample to find the market volatility was needed. As there are almost 11 years of market data available, the daily adjusted value of OSE to calculate the historical market volatility was used. The historical volatility was determined by first calculating the continuous return in the adjusted (data corrected for dividends and splits) value of OSE through:

Eq(34) $$\ln\left(\frac{V_t}{V_{t-1}}\right)$$

Then the continuous return value (data over 30 days) to calculate the volatility over a thirty day period was used:

Eq(35) $$SD(V_t : V_{t-30}) * \sqrt{365}$$

In the formula above the volatility is scaled to annualized historical volatility by multiplying with the square root of 365. It does not have to be the number of days in the year. It could also have been scaled by number of trading days per year.

The historical volatility for visual inspection was utilized. The data was acquired at yahoo.finance.com. The volatility was calculated over a thirty day period. The period from 2008-today has been subject to an extremely volatile macro economy.

In the analysis, some of the effects of the most extreme market movements have been controlled by introducing a variable of the VIX index. The VIX index is the volatility index of Chicago Board Options Exchange. It is considered a good measure of the market volatility, or market fear ("The CBOE volatility," 2009). Since the economical unrest is a global matter more than a regional Norwegian matter, the volatility that is captured in the VIX can be a good proxy for the unrest that influences the Norwegian markets. The data of the VIX index was acquired from yahoo finance.

It is important to note that there is no guarantee that HFT was introduced directly after the computer upgrade. It is more likely that companies began developing and introducing the software over a time period after April 2010. This will make it more difficult to find clear results in either of my hypotheses, but we may be able to see the beginning of a trend.

3.2 Testing

3.2.1 Testing of hypothesis 1

In hypothesis one, evidence that could prove or disprove whether irrational investors can survive in the financial markets is sought after. Number of transactions per month measures the activity/health of the institution. According to traditional finance theory an investor who trades on noise will lose their money to rational investors and cannot survive in the market place. Over time they will lose all their wealth and exit the market. This leads to the following hypothesis:

- H_0 : irrational investors can survive in the market place
- H_1 : irrational investors cannot survive in the market place

I run a test with total amount of irrational transactions as a Y variable, and a dummy variable for the transactions performed after April 2010 in order to determine whether there has been a significant decline of irrational investors. Oxmetrics6 is the statistical program used where PcGive is chosen. Under categories, time series data is selected, and single equation modeling using PcGive. The software then aids me in selecting a model by choosing automatic model selection. PcGive then runs several models picking the model that fits my data sample the best.

Under the same hypothesis I investigate the effects of the upgrade on the rational sample. Rational investors trading on fundamental values will not change their investment philosophy despite the irrational investors being present or not. However, one can argue that the noise traders' behavior influence how rational investors act since rational investors try to exploit the mispricing caused by the noise traders. My view is that the rational investors will base their investment decisions on the fundamental values of the assets regardless of whether noise traders are present or not. Therefore, I would not expect a significant change in the number of transactions performed by the rational investors. However, it is important to note that the rational investors may have begun using HFT. My definition of rational investor emphasizes that it is investment banks that are rational. Internationally, it is these institutions that are using HFT. Therefore, it is not unlikely that some of the investment banks in the rational sample have engaged in the practice. If this is the case, an increase in the number of transactions performed by the rational sample may be seen.

- H_2 : rational investors are not affected by the upgrade
- H_3 : rational investors are affected by the upgrade

On both of my tests I look for changes in the coefficients.

The last test under hypothesis 1 looks at the total number of transactions in the market as endogenous variable (Y). As explanatory variables I have total transactions for both irrational and rational and dummy variables (for transactions after April 2010) for both irrational and rational investors. This test is performed to understand the market rather than to test a hypothesis based on a theoretical framework. It is natural to assume that all parties involved (the rational and irrational samples) will have a positive impact on the total amount of transactions. In other words, an increase in the transaction amount for the irrational or rational will lead to an increase of the total number of transactions.

3.2.2 Testing of Hypothesis 2

Hypothesis 2 attempts to determine whether or not the volatility of the OSE has changed after the computer upgrade in April 2010. "It is unlikely in the context of financial time series that the variance of the errors will be constant over time" (Brooks, 2008). It is more likely that the variance will be influenced by its preceding values. This is one of several features inherent in the volatility of financial markets that promotes a non linear model. The other elements are leptokurtosis (fat tail distributions in asset returns), volatility clustering (tendency for volatility to appear in groups), and leverage effects (tendency for volatility to increase more after negative shocks). Financial data needs a model that allows it to "follow different processes at different points in time" (Brooks, 2008).

In Oxmetric6, one can test the null hypothesis that the variance of the errors is constant (homoscedastic) as they are in the classical linear models (Brooks, 2008). If we reject the null hypothesis and find that the variance is not constant (heteroscedastic) normal regression models will not suffice. This means that the volatility data is best described by a GARCH (1, 1) model. (GARCH stands for generalized autoregressive conditionally heteroscedastic.)

In this test the hypothesis would be:

- H_5 : the market variance is not influenced by the computer upgrade
- H_6 : the market variance is influenced by the computer upgrade

I used a dummy variable (volatility data after April 2010) to test for any effects that may have come from the computer upgrade. In an effort to control for the market volatility I have included a variable for the VIX index. The macroeconomic turbulence that has been moving through the financial markets since 2008 does not have its origin in Norway. Norway has only been affected because of the interconnected economies. Therefore, it is not unlikely to assume that the VIX index, which is considered to measure the volatility in the US market, can model the external volatility influencing the Norwegian financial markets.

4. Tests and models used

There are a large variety of diagnostic tests and statistical models that can be used in this analysis. The next section provides a brief overview of the tests and model that I will be utilizing. The specification tests that I use look at aspects that are associated with time series analysis, and ordinary least squares analysis.

4.1 ARCH – Heteroscedasticity

The arch test looks for "ARCH effects" in the residuals. With ARCH effects I mean that the value of the error term is influenced by the squared value of the preceding error terms (Wooldridge, 2009). To test for ARCH effects one can run a normal regression:

Eq(36) $$y_t = \beta_1 + \beta_2 x_{2t} + \beta_3 x_{3t} + u_t$$

Save the residual $\hat{u}_t$, than square them and run a regression on q of its own lags:

Eq(37) $$u_t^2 = \gamma_0 + \gamma_1 u_{t-1}^2 + \gamma_2 u_{t-2}^2 + ... + \gamma_q u_{t-q}^2$$

The test statistic is defined as TR^2. This is the number of observations multiplied with the statistic of the goodness of fit of the model for the previous regression (Brooks, 2008). The statistic has chi square distribution $\chi^2(q)$. The null hypotheses states that the gamma values (except γ_0) equals zero, which means that the variance of the error terms are constant. The alternative hypothesis states that at least one of the gamma values is not equal to zero, so the variance of the error terms is time varying.

4.2 Normality test/ Asymptotic test

The normality test assumes that the error term is independent of the explanatory variable. It tests to see if the data is well modeled by the normal distribution. The null hypothesis will state that the data has a normal distribution. The alternative hypothesis will state that the data does not have a normal distribution. Its difference is measured in the data's kurtosis and skewness. The asymptotic test examines the same hypothesis, but with a slightly transformed test statistic (PcGive). OxMetrics reports the test statistic which is a χ^2 distribution.

It is also possible to perform a visual analysis by plotting the data's density function against the probability density function of the normal curve.

4.3 Autocorrelation (ACF - Correlogram)

The autocorrelation function visually describes the correlation between values of a variable and its preceding values. This means that r_t (correlation coefficient) is the correlation between variables x_t and x_{t-1}. The correlogram shows whether shocks introduced to the system will fade or if they will persist. If shocks persist in the data it is a non stationary process. This instability will make it difficult to estimate or forecast future values.

4.4 GARCH (1, 1)

The GARCH (1, 1) model is given by:

Eq(38) $$y_t = \beta_0 + D_t upgrade + \gamma VIX + u_t$$

Eq(39) $$u_t \sim N(0, \sigma_t^2)$$

Eq(40) $$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta\sigma_{t-1}^2 + \delta D_t upgrade + \gamma VIX$$

This "model allows the conditional variance to be dependent upon" (Brooks, 2009) its previous lags. This means that the conditional variance is influenced by the error term of the previous period ($\alpha_1 u_{t-1}^2$), in addition to its own previous value ($\beta\sigma_{t-1}^2$), and a long term average (α_0) (Brooks, 2009). Because of the non linear nature of the model it uses maximum likelihood estimation instead of ordinary least squares. Maximum likelihood will find the parameter values that optimize the equation.

5. Analyses

5.1 Hypothesis one

The first hypothesis investigates Freidman's (1953) statement of the survival of noise traders. Friedman claimed that noise traders (irrational investors) would die out because they will buy when the prices are high, and sell when the prices are low. The rational investors will take advantage of their mispricing. If I were to estimate what would happen based on this theory, I would expect that the amount of irrational investors to decline after April 2010, but that the amount of rational investors stay the same or increase. As the irrational investors are losing money they will soon run out of funds and motivation to invest. The rational investors will take advantage of the irrational investors, but they are not dependant on their survival. HFT is another form of rational investing, but I do not know if my sample group contains companies that actively pursue this form of trading. If they do not use HFT I would expect their level of transactions to remain, relatively, unchanged by the event of April 2010. If they use HFT, I would expect their level of transactions to increase significantly.

Looking at the data we can see from figure 2 that the irrational investor group has had a declining market share since early 2009. This is before the computer upgrades that lead to HFT. It is therefore difficult to conclude anything specific from a visual analysis. The rational investors have had a stable market share all through the time series. Figure 2, on the next page, shows a declining market share for the irrational investors, but that does not mean that the total number of transactions is declining. If the total amount of transactions in the market increases drastically, the irrational investors may still have an increasing amount of transactions.

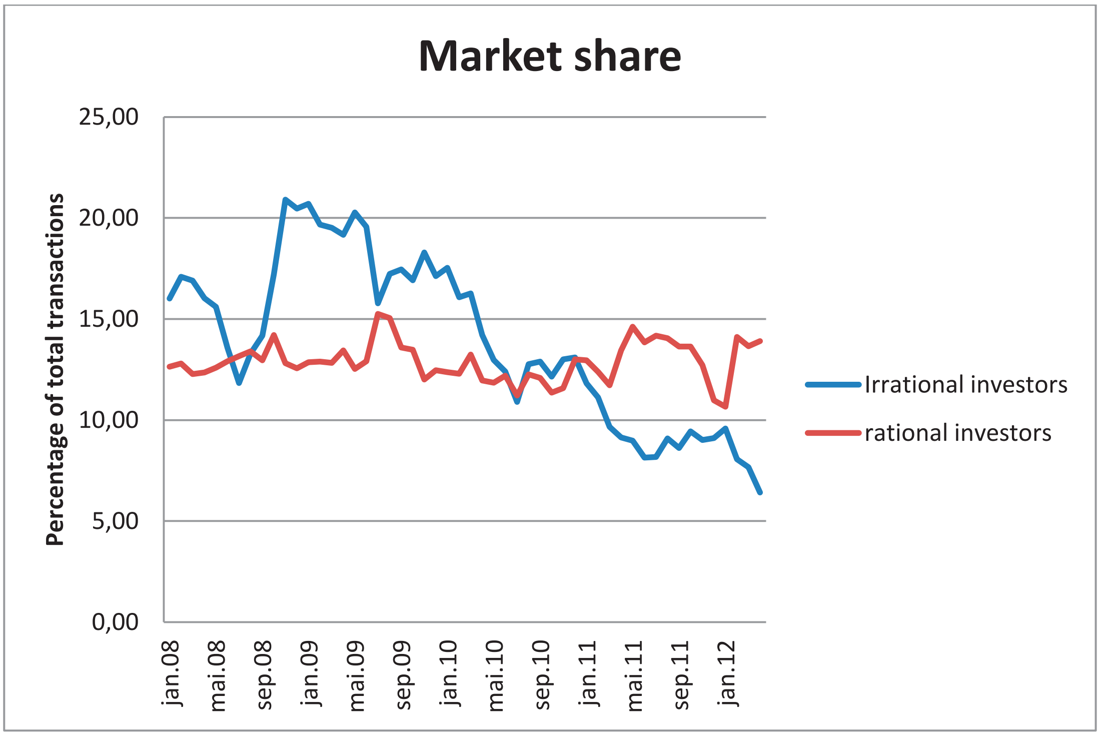

Figure 2: The samples share of the total amount of transactions (in percent)

If we look at the actual number of transactions the image changes only slightly.

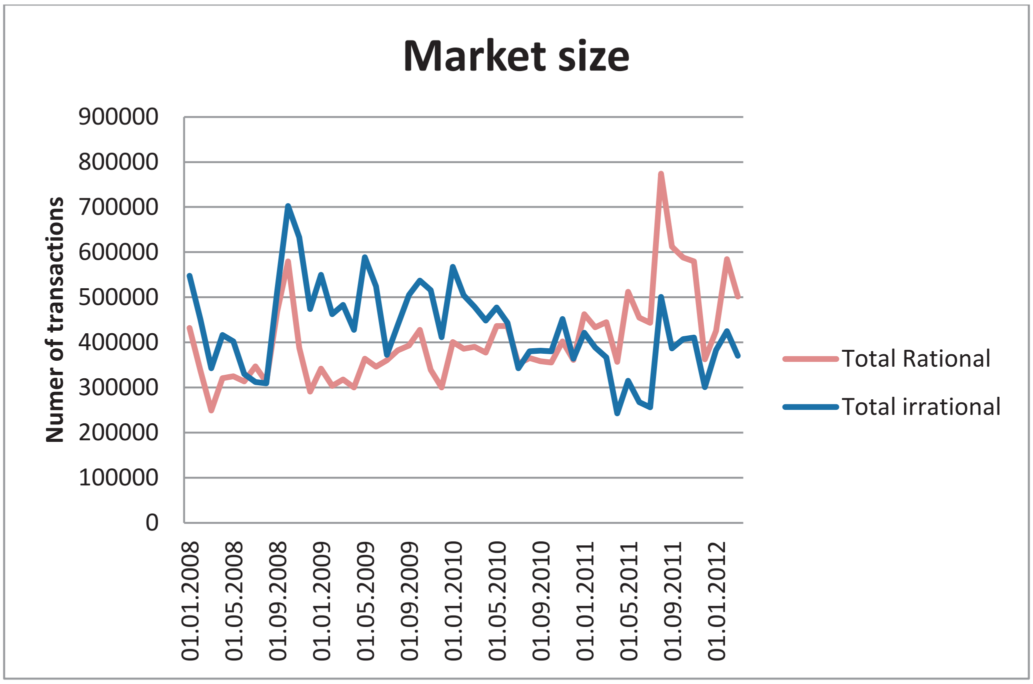

Figure 3: The total amount of transactions of the two sample groups

Figure 3 shows the actual amount of transactions the two sample groups have performed between January 2008 and April 2012. The irrational sample clearly performed more transactions than the rational investors up to April 2010. From April 2010 till January 2011 it was a period of analogous transaction amounts, but from then on the rational investors have performed more transactions than the irrational. These graphs do not prove or disprove anything, but they give a visual confirmation of my expectations. To check whether the changes have been significant I must apply statistical measures. The null hypothesis states that there is no significant difference in the data after April 2010. The alternative hypothesis states that the data after April 2010 is significantly different than from before April 2010.

Before selecting a model a closer look at the data sample is needed to outline its underlying characteristics.

First I look at the autocorrelation functions to see if there are signs of autocorrelation. I look at the total sample and both sub sample to see if the sample groups display different tendencies then the market as a whole. This is done because classical finance theories claim that irrational investors behave in a different manner than rational investors. If this is true, this should be displayed in the characteristics of the data.

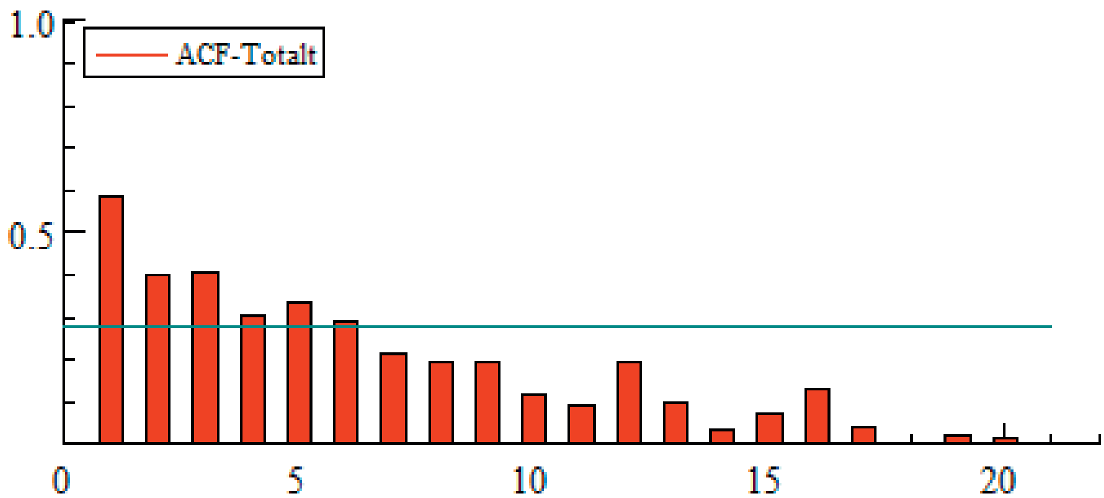

Figure 4: Correlogram showing the ACF for the entire sample

The total sample data (number of transactions at OSE) is clearly auto correlated. Figure 4 shows a dying autocorrelation function. If a shock is introduced to the model it will eventually die out. It is interesting that the data does not decline smoothly. The irregular occurrence of the”spikes” in the ACF speaks against seasonality. It may be caused by the volatile nature of the financial markets and the macro economy.

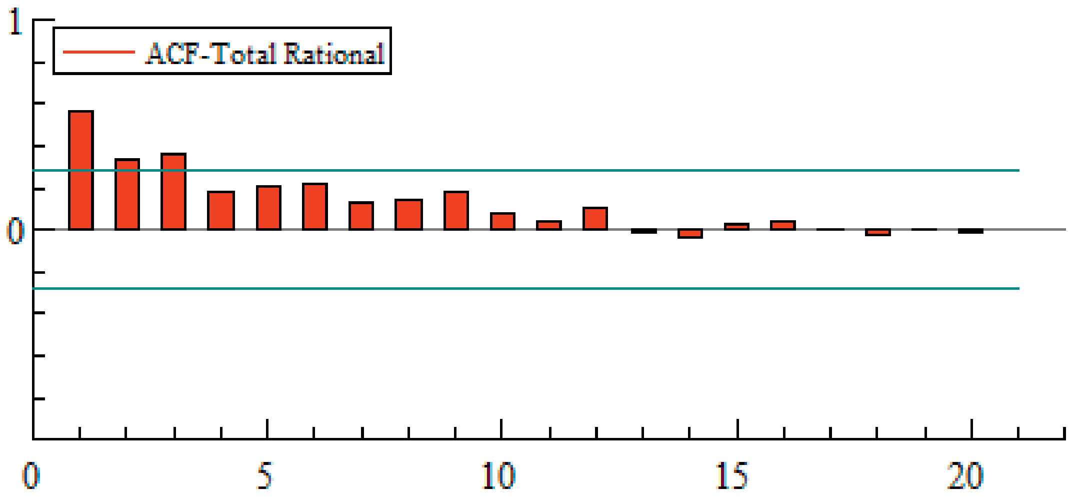

Figure 5: Correlogram showing the ACF for the rational sample

Figure 5 displays the ACF for the rational sample. The movement of the ACF for the rational sample is similar to the movements of the ACF of the total sample, showing clear autocorrelation. Figure 7 displays the ACF for the irrational sample group. This ACF is clearly different than the previous ones.

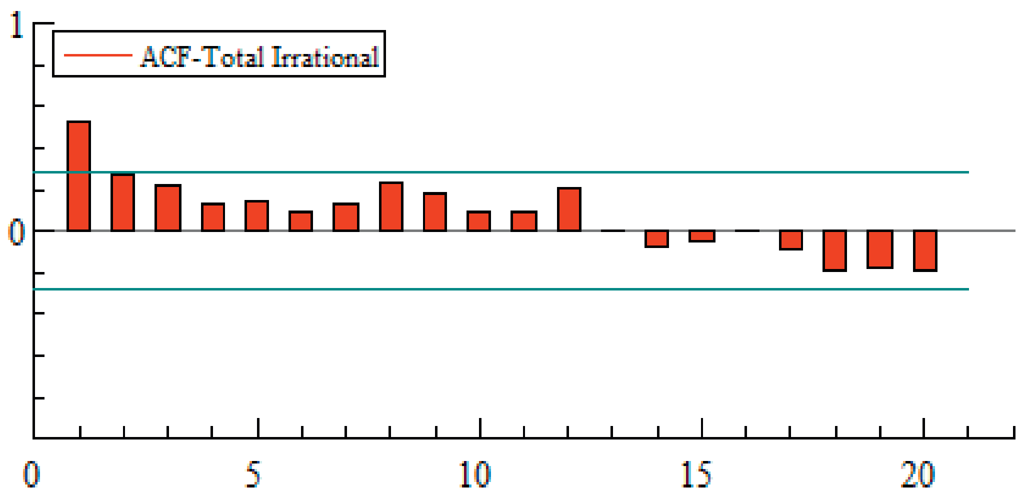

Figure 6: Correlogram showing the ACF for the irrational sample

The ACF is declining, but has high peaks around periods 8, 9, and 12. After period 14 it becomes negative, and then increases in negativity. Due to a limited data sample it is unclear how the ACF behaves over a longer period of time. From the sample it is difficult to tell whether a shock would disappear or not. It is clear that the ACF for the irrational investors are different than the ones for the market and the rational investors. The rational and total sample clearly shows that shock will die out, where as the irrational ACF do not show clear signs of shocks dying out. This could mean that irrational investors have a harder time adapting to shocks. It is difficult to specify how their behavior changes. It seems that the effects of a shock lingers on, which could mean that they overreact and therefore prolong the shock themselves.

Next I look at the probability density distribution of the sample data. In these graphs the actual sample is compared to a normal distribution curve. This can give a good illustration of whether the data is well modeled by a normal distribution. Classical linear regression assumes that the error terms are normally distributed. If the data shows clear signs of skewness or kurtosis it can create difficulties with the analysis (Wooldridge, 2009).

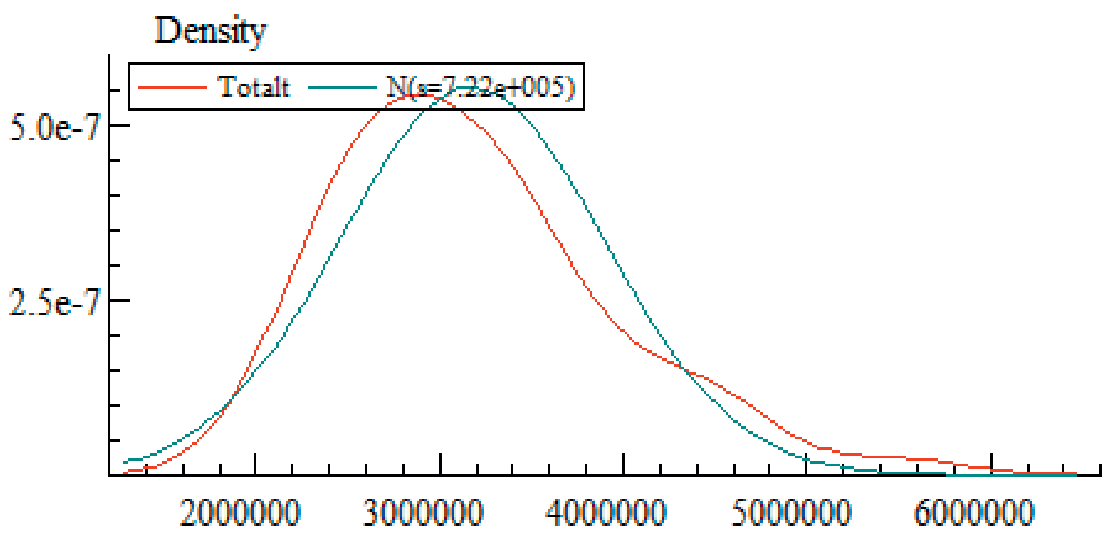

Figure 7: The distribution of the total sample compared to the normal curve (skewness & kurtosis)

Figure 7 show that the total sample has a positive skewness. This means that the distribution has a thicker tale on the right hand side. This demonstrates that there is a higher probability of achieving higher values in the real world then it is in a normal distribution.

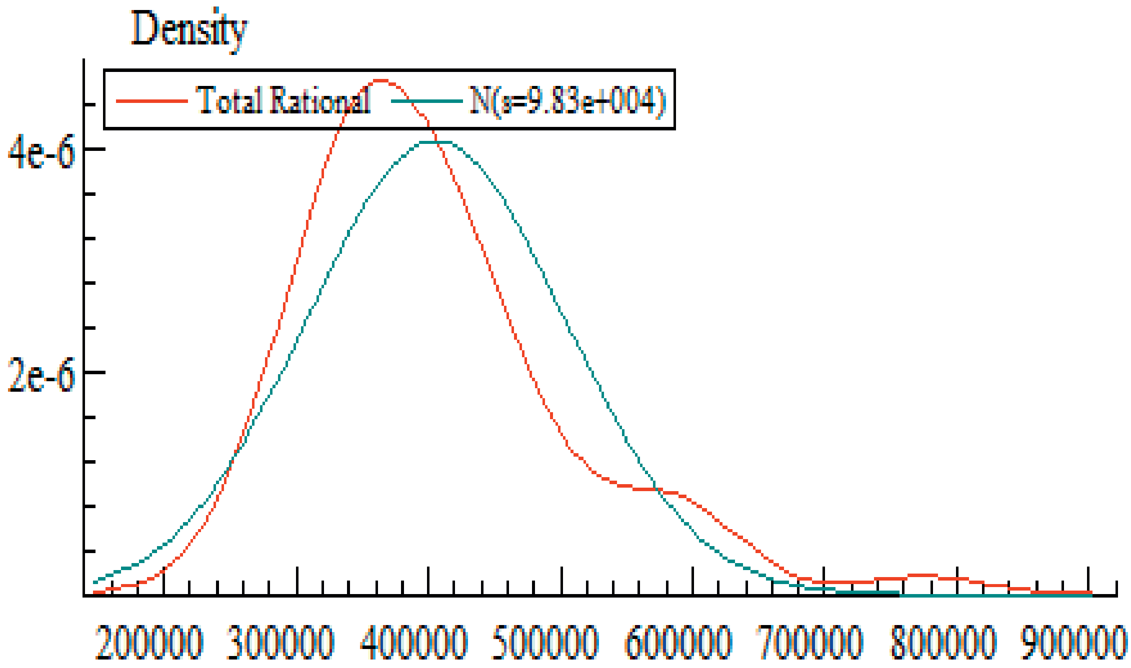

Figure 8: The distribution of the rational sample compared to the normal curve (skewness & kurtosis)

Figure 8 show that the rational sample has similar traits as the total sample. The only difference is a larger kurtosis.

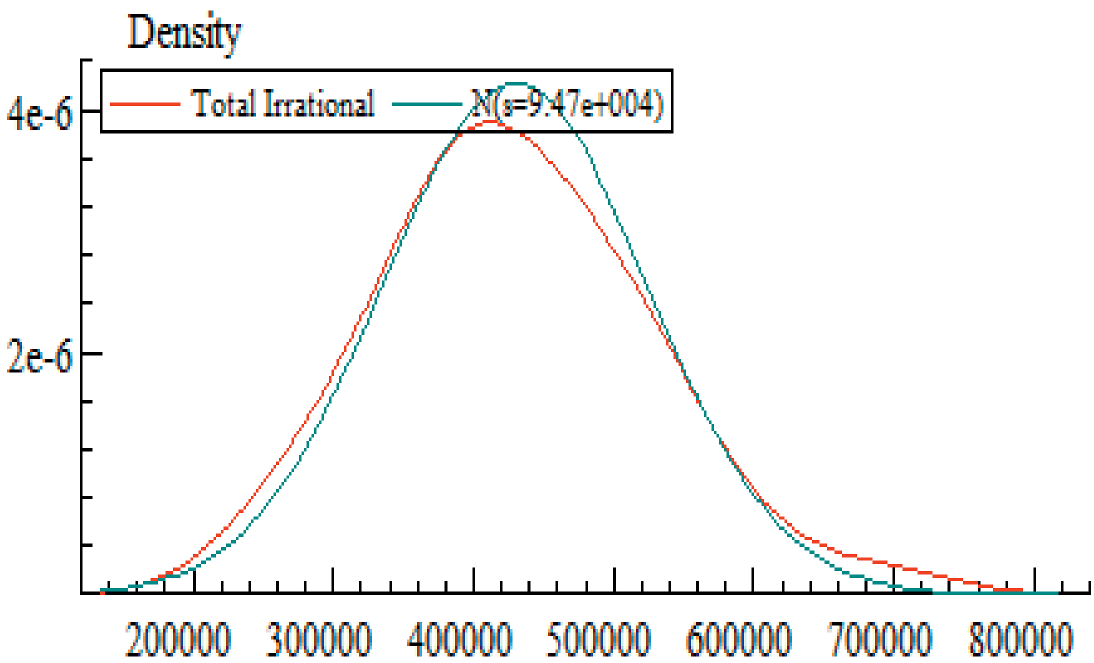

Figure 9: The distribution of the irrational sample compared to the normal curve (skewness & kurtosis)

Again, it is the irrational sample group that differs. The irrational investors have a density function that is very similar to the normal distribution. It is interesting to see the distinction of the irrational sample. This could mean that the irrational investors act in a more random manner than their rational counterparties. As the rational investors and the market as a whole have similar characteristics it is natural to assume that the market is predominately rational.

Correlation matrix:	Totalt Sample	Total Rational	Total Irrational	Total rational/bin	Total irrational/bin
Totalt Sample	1.0000	0.94531	0.11265	0.75939	0.68300
Total Rational	0.94531	1.0000	0.12914	0.69541	0.56812
Total Irrational	0.11265	0.12914	1.0000	-0.42504	0.38895
Total rational/bin	0.75939	0.69541	-0.42504	1.0000	0.96089
Total irrational/bin	0.68300	0.56812	-0.38895	0.96089	1.0000

Table 2: Correlation matrix for the sample groups, and the whole population

From table 2 it can be seen that there is high correlation between the rational sample and the total sample group. The irrational sample has a low correlation with both the total population and the rational sample group. The rational sample and the total population have an almost perfect correlation. In financial theory, it is assumed that the markets are rational. This data seems to support that notion. It is very interesting to see that the total irrational sample is almost unrelated to the rational sample. This could support the notion that noise traders act on information without any fundamental value. It may lead support to the notion that they act in a random manner.

The data has several characteristics that make it difficult to choose an appropriate model. Oxmetrics automatic model selection was used to choose the model.

Modeling Total Irrational by OLS

The estimation sample is: 1 - 51

	Coefficient	Std.Error	t-value	t-prob
Constant	462134	1,66E+04	27,80	0
Total irrational/bin	-0,191408	0,06477	-2,96	0,0048

AR 1-2 test:	F(2,47) =	3,7970 [0,0296]*	sigma	89006,7
ARCH 1-1 test:	F(1,49)	3,7820 [0,0576]	R^2	0,151285
Normality test:	Chi^2(2)	0,77121 [0,6800]	Adj,R^2	0,133965
Hetero test:	F(2,48) =	1,0177 [0,3691]	no, of bservations	51
Hetero-X test:	F(2,48) =	1,0177 [0,3691]	mean(Y)	429628
RESET23 test:	F(2,47)	14,236[0,0000]**	RSS	3,8818771+011
Hetero test:	F(2,48) =	1,0177 [0,3691]	F(1,49) =	8,734 [0,005]
Hetero-X test:	F(2,48)	1,0177 [0,3691]	log-likelihood	-652,566
RESET23 test:	F(2,47)	14,236[0,0000]**	no, of parameters	2
			se(Y)	95643,4

Table 3: Test scores of irrational investors with dummy

Table 3 shows the result of the regression of the irrational investors. I try to determine whether there has been a significant change in the behavior of the irrational investors after April 2010. The table reports the test scores of the variables and the results of several tests on the data. I will comment on the most important findings of these tests. The t-probability test score shows that both the constant and the dummy variable are significant. The dummy coefficient tells us that the irrational agents significantly reduced the number of transactions after April 2010. The test also shows that the data for the irrational sample is normally distributed. The normality test has a p-value of 0, 68 which means we will keep the null hypothesis of normal distribution. The ARCH test shows slight (not significant on a 5% level) heteroscedasticity but that is to be expected since the number of transactions is influenced by the economical and financial situation.

Modeling Total Rational by OLS

The estimation sample is: 1 - 51

	Coeff	Std.Error	t-value	t-prob	Part.R^2
Constant	346541	1,34E+04	26	0	0,9322
Totrat/bin	0,284927	0,04206	6,77	0	0,4836

sigma	72047,1	RSS	2,54E+1	AR 1-2 test: F(2,47)	5,5435[0,0069
R^2	0,483598	F(1,49) =	45,89[0,0	ARCH 1-1 test: F(1,49)	0,625[0,4325
Adj,R^2	0,473059	log-likelihood	-641,785	Normality:Chi^2(2)	10,149[0,0063
				Hetero test:	5,1714
no, of obs	51	no, of paramet	2	F(2,48)	0,0092]
				Hetero-X test:	5,1714
mean(Y)	405771	se(Y)	99251,1	F(2,48)	0,0092]
				RESET23 test:	28,778
				F(2,47)	0,0000]

Table 4: Test scores of rational investors with dummy

Table 4 shows the test of the rational sample and the effects of a dummy variable for the transactions after April 2010. There is a significant change for the rational sample after the introduction of the computer upgrade. There is an increase in the number of transactions after the upgrade. The normality test shows that the data is not normally distributed.

The last test that I performed under hypothesis one looks at the relationship between the sample groups and the total market. The total transactions are the endogenous variable. The total rational and total irrational plus their dummy variables are the explanatory variables.

Modelling Totalt by OLS					
The estimation sample is: 1 - 51					
	Coefficient	Std.Error	t-value	t-prob	Part.R^2
Constant	272610.	1.615e+005	1,69	0.0982	0.0583
Total irrational	0.831344	0.5220	1,59	0.1181	0.0523
Total rational	5.84846	0.7309	8.00	0.0000	0.5819
Total rational/bin	-0.406711	0.8058	-0.505	0.6162	0.0055
Total irrational/bin	1.51108	0.7358	2,05	0.0457	0.0840
sigma	189836	RSS	1.65773932e+012		
R^2	0.937602	F(4,46)	172.8 [0,000]		
Adj.R^2	0,932176	log-likelihood	-689,584		
no. Of obs	51	no. Of param.	5		
mean(Totalt)	3.175e+006	se(Totalt)	728934		
AR 1-2	F(2,44)	7,0387 [0,0022]			
ARCH 1-1	F(1,49)	4,9132 [0,0313]			
Normality	Chi^2(2)	7,0387 [0,2353]			
Hetero	F(8,42)	1,4646 [0,1992]			
Chow	F(14,32)	3,1744 [0,0034]			

Table 5: The analysis of the total market with the sample groups as explanatory variables

From the analysis displayed above it is clear that neither the total irrational sample nor the total rational dummy is significant in explaining the total number of transactions. This would mean that these variables have moved in a random manner through their sample periods, and cannot explain the overall movements of the transaction markets.

EQ(2) Modelling Totalt by OLS					
The estimation sample is: 1 - 51					
	Coefficient	Std.Error	t-value	t-prob	Part.R^2
Total irrational	1,42985	0,2733	5,23	0	0,3632
Total rational	5,79017	0,3496	16,60	0	0,851
Total irrational/bin	1,19408	0,2062	5,79	0	0,4113
sigma	193247	RSS	1,79E+12		
log-likelihood	-691,578				
no. Of obs.	51	no. Of. Par.	3		
mean(Totalt)	3,18E+06	se(Totalt)	728934		
AR 1-2	F(2,46)	6,5176	[0,0032]		
ARCH 1-1	F(1,49)	2,3739	[0,1298]		
Normality	Chi^2(2)	2,7445	[0,2535]		
Hetero	F(6,44)	0,9193	[0,4903]		
Hetero-X	F(8,42)	1,6062	[0,1521]		
RESET23	F(2,46)	2,1945	[0,1229]		

Table 6: The adjusted analysis of the total market with the sample groups as explanatory variables

In the final analysis, displayed in table 6, OxMetrics has removed the rational dummy. The three remaining variables are significant in explaining the total number of transactions in the market. If the markets are rational it would be logical to assume that the samples of the rational investors are significant in explaining the market. Since the total sample of rational investors is significant, but the dummy variable is not, could mean that the rational investors have not been influenced by the introduction of HFT and have not changed their behavior dramatically after April 2010.

This section will outline the conclusion of hypothesis 1. It is clear from the first analysis that the irrational investors have been significantly less active after the computer upgrade. This could mean that the introduction of HFT has a negative effect on the irrational investors. It is difficult to draw a definite conclusion because of the economical turbulence that is reflected in the data samples. However, if it was the macroeconomic conditions that caused the relationship it is curious that the rational investors significantly increased their activity level after the computer upgrade since it was expected that the rational investors would remain unaffected by it. The increase could be a consequence of the adoption of HFT by any of the three rational investors. It is difficult to confirm this because most companies do not proclaim their involvement with HFT. It is important to note that the two tests involving the rational dummy led to two different conclusions. This could mean that the data is borderline significant, which would make it difficult to draw conclusions. With the limited time series data that I have available it seems that the irrational investors are being squeezed out of the financial markets. This is just an interpretation of the data up to this point, how it will evolve is another matter.

5.2 Hypothesis two

In the second hypothesis I attempt to discover whether irrational investors can survive in the market place. If irrational investors have an influence on the asset price they will increase the market volatility. If the introduction of more arbitrageurs (rational investors) through HFT reduces the market volatility it can be proof of the irrational investors influence. I run a test to see if there have been any significant changes to the volatility after the introduction of the new computer system in April 2010. The sample period has been very volatile, this should have (theoretically) reduced the willingness of rational investors to exploit the mispricing, and increased the irrational investors influence on the asset price.

The variable in this analysis is the volatility of the stock market. Volatility is known to be serially correlated (Brooks, 2008). Figure 10 shows the daily percentage change in total value of OSE. This figure displays periods of both high and low levels of change in the market.

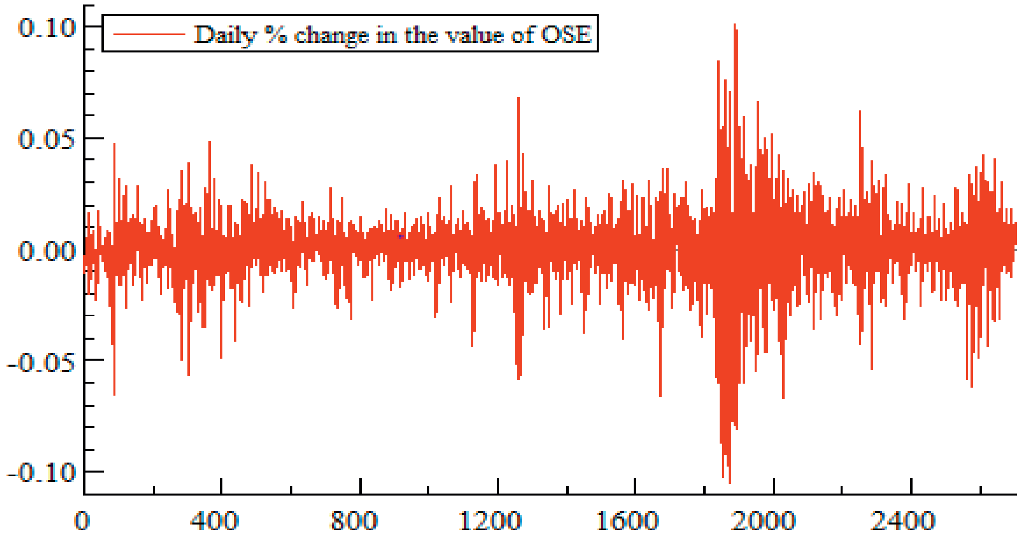

Figure 10: Daily percentage change in the value of OSE (2001-2012)

Figure 11 shows a similar graph, but it displays the first difference of the actual volatility of the OSE. As we can see from both of the graphs below, periods of high volatility tends to follow periods of high volatility, and periods of low levels of volatility also tends to follow each other.

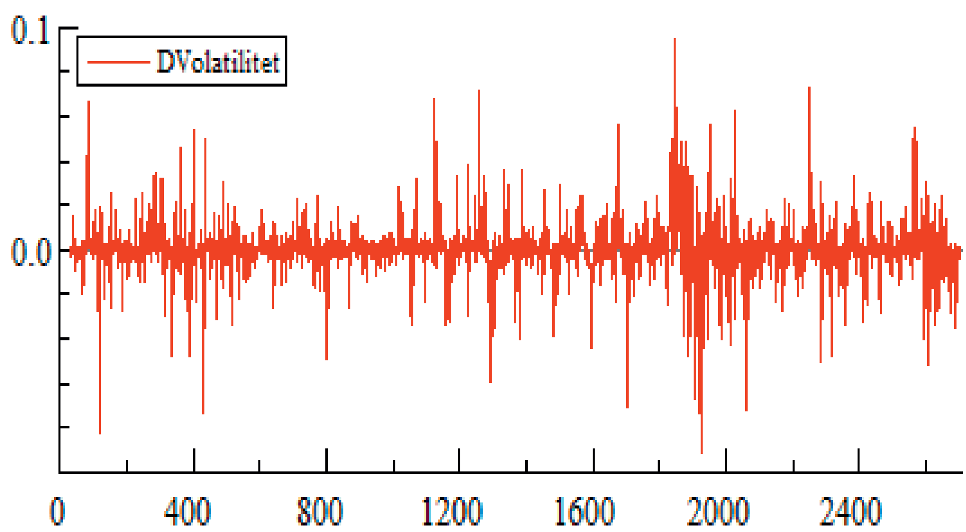

Figure 11: First difference of the volatility of OSE

Visual analysis shows that periods with large changes tend to group together. This is called volatility clustering and is common in financial data (Brooks, 2008). There are four-five periods of large and frequent changes in the sample. Both graphs can be seen as evidence that the data set is serially correlated.

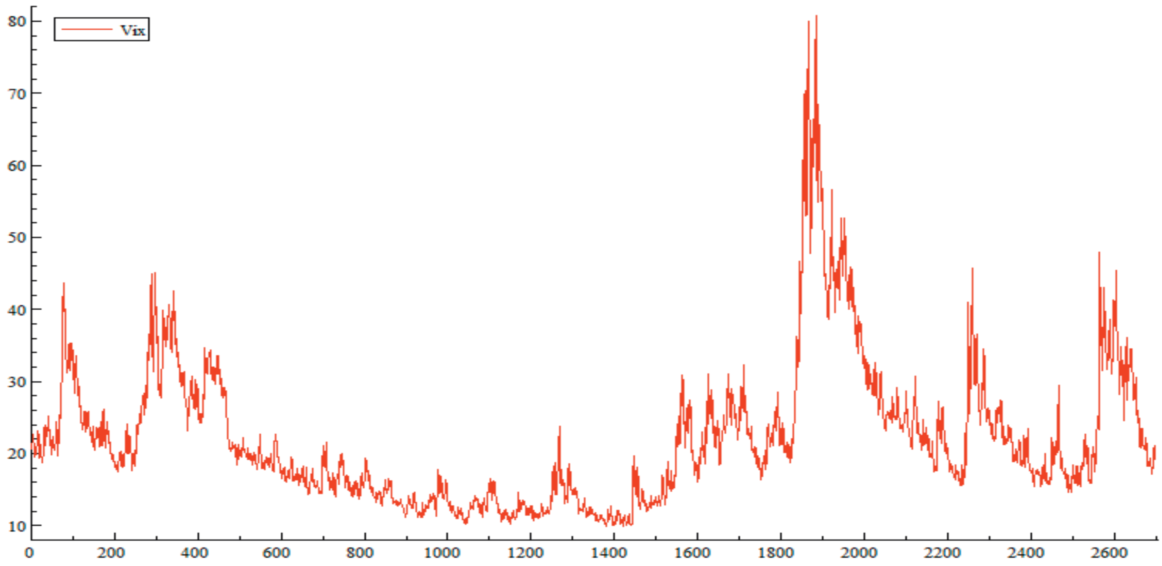

Figure 12: VIX index distribution

The control variable for the economical unrest in the market (VIX) displays a similar pattern as the OSE. There are five periods with high levels of unrest. This could mean that the incorporation of the VIX index may be able to control for the macroeconomic noise influencing OSE.

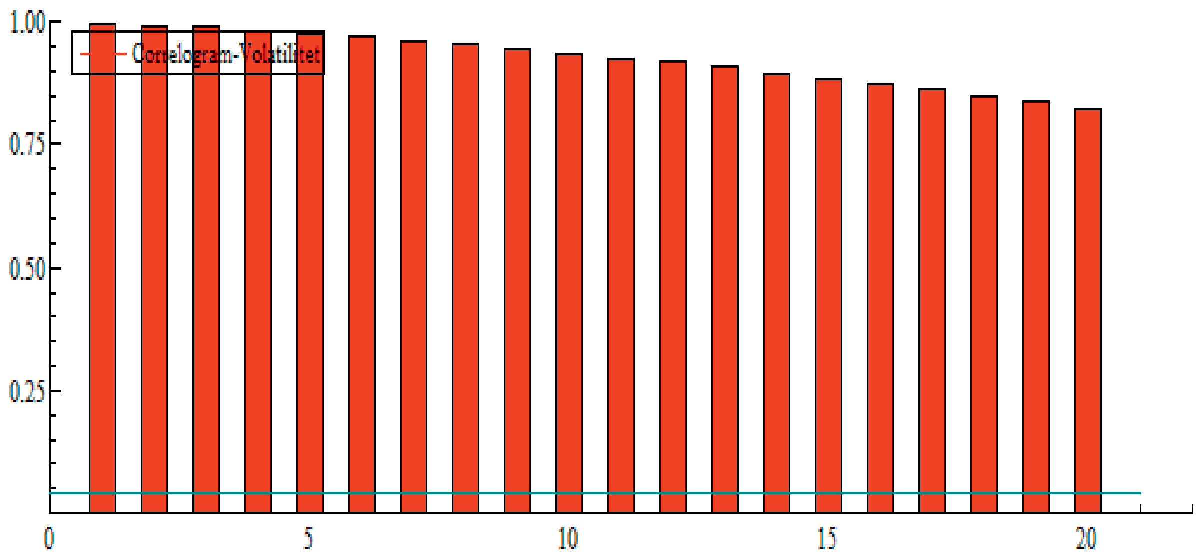

Figure 13: Correlogram showing the Auto correlation function for the volatility

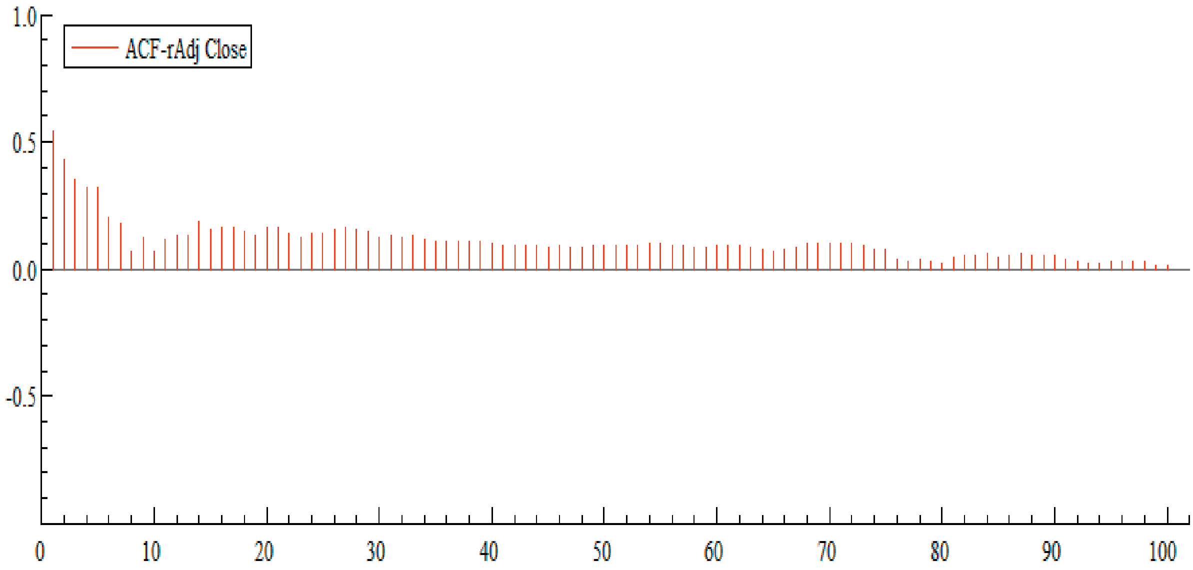

Figure 14: Correlogram showing the Auto correlation function for the residual of the adjusted closing price of OSE

The auto correlation function (figure 13) shows that the volatility process will adjust to shocks very slowly. This means that the volatility data seems significantly auto correlated. In classical regression analysis, autocorrelation creates problems because it breaches an assumption for ordinary least squares regression. If auto correlation is present in the data it means that the error terms in the GARCH analysis will be correlated. The correlation in the error terms leads to an underestimating of the standard errors which leads to overestimated t-scores (Brooks, 2008). Figure 14 shows the correlogram for the residuals of the adjusted closing price. This ACF shows that shocks introduced to the system will disappear, but it will take a long time for the effects of the shock to die out.

Looking at the historical volatility data it is natural to expect that the density function of the historical volatility to be of a non normal form. Volatility tend to appear in clusters and be serially correlated which makes it natural with a non normal density distribution. Figure 15 shows the density function of the historical volatility compared to the normal density function.

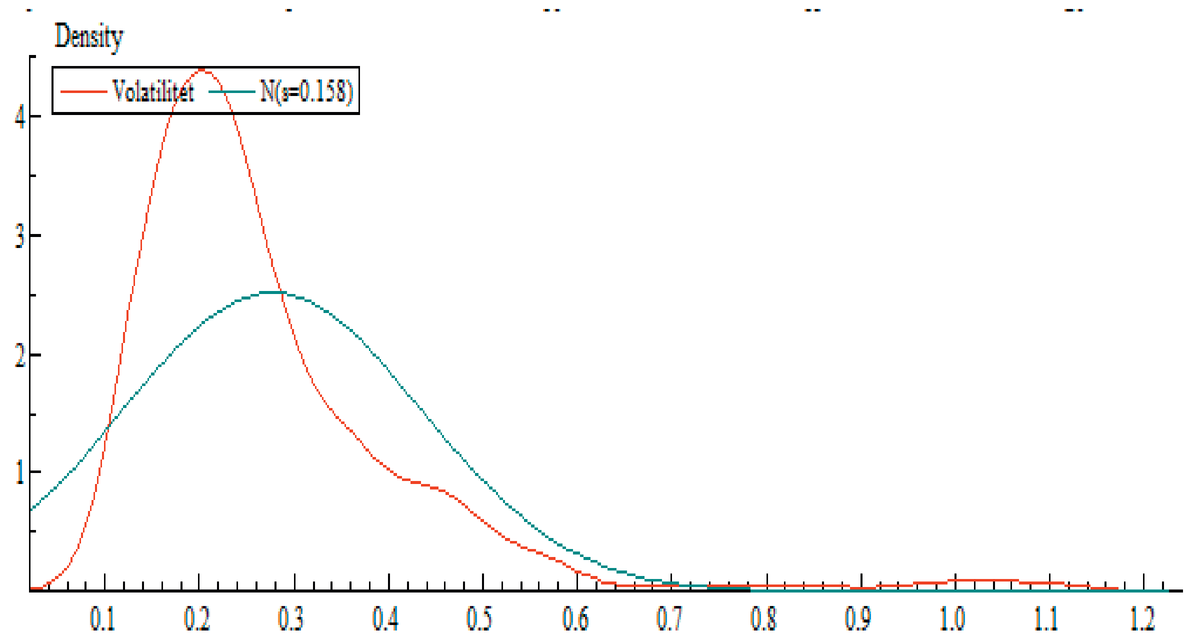

Figure 15: Shows the kurtosis and skewness of the volatility data's probability distribution

The kurtosis describes how the data's probability distribution creates a peak, instead of the traditional bell shaped normal curve. Figure 15 shows a significant kurtosis in the data sample. This means that the residuals from the linear regression are not normally distributed. If they are not normally distributed there are certain tests that cannot be utilized on the data. It also shows significant positive skewness.

Visual inspections are always prone to fault. A closer look at the descriptive statistics in Oxmetrics gave the following information:

Observations	2700
Mean	298.85
Std.Devn.	119.30
Skewness	-0.0060737
Excess Kurtosis	-1.3235
Minimum	98.570
Maximum	524.37
Asymptotic test: Chi^2(2)	197.07 [0.0000]
Normality test: Chi^2(2)	372.65 [0.0000]
ARCH 1-2 test: F(2,2691)	18.470 [0.0000]

Table 7: Descriptive statistics for the OSE adjusted closing value

In the normality test and the asymptotic test the null hypothesis states that the data samples error terms are normally distributed, where as the alternative hypothesis states that the data samples error terms are not normally distributed. From this analysis we can clearly see in the asymptotic test (p-value = 0), and the normality test (p-value = 0) that the data does not possess normally distributed error terms. The ARCH test is a test for autocorrelation. The test shows that the error terms has a non linear pattern (p-value = 0). A non linear pattern means that the values of the error terms are influenced by its preceding values.

The characteristics that the data displays are normal when dealing with financial time series. To analyze my data further I use a model that can handle the non linear character inherent in the data. One such model is the GARCH (1, 1).

The GARCH analysis examines whether the volatility of OSE has changed significantly after April 2010. The test yielded the following results:

```
VOL( 1) Modelling Adj Close by restricted GARCH(1,1)
        The dataset is: C:\Users\Denise\Desktop\OsL2.xls
        The estimation sample is: 1 - 2699

                 Coefficient  Std.Error  robust-SE  t-value  t-prob
Constant      X      370.135     0.7125      1.237     299.   0.000
Bin price     H   0.00947744    0.03159    0.03325    0.285   0.776
Vix           H      1.33453     0.3451     0.3586     3.72   0.000
alpha_0       H  6.29899e-007
alpha_1       H     0.933139    0.06266    0.05565     16.8   0.000
beta_1        H    0.0639083    0.05403    0.05611     1.14   0.255

log-likelihood    -15505.7845  HMSE                  0.243921
mean(h_t)             19299.6  var(h_t)          4.26608e+008
no. of observations      2699  no. of parameters            6
AIC.T               31023.569  AIC                  11.494468
mean(Adj Close)       298.802  var(Adj Close)         14233.4
alpha(1)+beta(1)     0.997048  alpha_i+beta_i>=0, alpha(1)+beta(1)<1
```

Table 8: Modeling volatility with a GARCH (1, 1) model

Table 8 displays the results from the GARCH analysis investigating the volatility of OSE. Alpha 0 is a constant. The variable Bin price is the dummy variable for the adjusted price level to see if the prices have significantly changed after April 2010. The VIX variable is the control variable that will control for the macroeconomic noise that has been significant in the sample period. Alpha 1 is the coefficient for the first order autocorrelation, and beta is the coefficient of the first order auto regression. The table also shows other statistics relating to descriptive statistics and model fit. The analysis shows that the adjusted prices are not significantly different after the computer upgrade. The control variable for the macro economy is significant (p-value = 0) in describing the price movements. It is surprising that the volatility is not significantly dependant on its lagged values (p=0,089). With volatility clustering present in the data I would have expected this parameter to be significant. The effect of the lagged shock is significant. It is surprising that the lagged values of the volatility is not significant, but that could be an effect of the unique macroeconomic situation. Since 2008 the economy has been thrown back and forth from constant economical shocks. These frequent and significant shocks may have caused such movements in the volatility that the previous values became less important predictors. This could mean that the volatility over this period has been shock driven (mainly influenced by u). The financial markets have been extremely volatile in the sample period. This makes it impossible to determine whether the relationship is a normal phenomenon or caused by the extreme conditions.

If I assume that there is nothing wrong with the data or the analysis it would mean that the introduction of HFT has had no effect on the volatility of the stock market. There are certain difficulties associated with the data sample. The biggest problem is the amount of macroeconomic noise that has influenced the markets since 2008. There have been periods of extreme volatility. I believe I have been able to remove most of the influence of these factors from my analysis by adding the VIX as a control variable, but this period of time has been unique so the results may have been different with longer data samples.

Another issue is that I have assumed that HFT robots act rationally. In the sample period, there is evidence of at least one trading robot acting irrationally in a case where two investors were charged with market manipulation. The robot observed movements in stocks with low liquidity and treated it as a stock where the investor interest increased dramatically. The investors triggered the robots interest and then reversed their position for a profit. In this instance, the computer was taken advantage of because of its simple trading behavior (Steinsland and Dahl, 2012). If enough robots act in an irrational manner it can remove the foundation that this thesis is built upon.

6. Conclusion

It is not easy to distinguish between irrational and rational investors in aggregated data. In my first hypothesis I looked into Friedman's (1953) claim of the survival of irrational investors. The analysis shows that the numbers of transactions performed by the irrational investors were significantly reduced after April 2010. This is not evidence proving that the irrational investors will die out. It only demonstrates that the irrational investors have been less active. It is therefore acceptable to conclude that the introduction of HFT has had a negative effect on irrational investors which could lead to their extinction (if the trend continues).

The second hypothesis investigated the power of irrational investors' influence on asset prices. The analysis showed that the volatility of OSE had not changed significantly after April 2010 when controlled for the macroeconomic volatility. This supports the notion that irrational investors do not have an influence on the asset price. Aspects of the sample data from hypothesis 1 illustrated that the irrational sample was well modeled by the normal distribution. These results could support the idea of random behavior among irrational investors therein nullifying their own price effect.

It is interesting to see from the data under hypothesis 1 that the irrational sample clearly showed different characteristics than the rational sample and the market data. This could be seen as evidence for a rational market and that the irrational investors act in a random manner. It would be interesting to perform deeper enquiries in these topics. However, detailed information would need to be obtained from the investment banks on their rational and irrational investors. At the moment there is too much information that is withheld to fully understand the relationship between irrational and rational investors.

To conclude H_0 is rejected, and H_1 is accepted. Irrational investors cannot survive in the financial markets. H_2 is rejected, and H_3 is accepted. Rational investors had an increase in activity after April 2010 which could be explained by their adoption of HFT. H_5 is accepted. The market volatility is unaffected by the introduction of HFT proving that irrational investors do not have an influence on asset prices.

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8. Appendix A

Dataset used in hypothesis 1, acquired from OSE.no.

	Transaction (Market share)								
	Skandiabanken	Nordnet	Netfonds	Total irrational	SEB	Artic	Carnegie	Totalt rational	Total sample size
jan.08	1,61	9,46	4,95	16,01	7,65	1,63	3,35	12,64	28,65
feb.08	1,74	10,11	5,23	17,08	7,43	1,88	3,48	12,79	29,87
mar.08	1,86	9,98	5,06	16,89	7,54	1,65	3,09	12,28	29,17
apr.08	1,57	9,66	4,79	16,03	7,27	1,89	3,19	12,36	28,39
mai.08	1,72	9,51	4,37	15,60	8,17	1,83	2,59	12,59	28,19
jun.08	1,16	8,80	3,61	13,57	8,38	2,08	2,45	12,91	26,47
jul.08	1,07	7,40	3,37	11,83	9,16	1,74	2,26	13,16	24,99
aug.08	1,12	8,46	3,71	13,29	8,93	2,29	2,19	13,41	26,69
sep.08	1,27	9,35	3,56	14,18	9,19	1,87	1,88	12,95	27,13
okt.08	1,98	10,81	4,42	17,21	10,29	1,95	1,97	14,21	31,42
nov.08	2,30	13,36	5,24	20,90	9,51	1,81	1,49	12,81	33,71
des.08	2,36	12,96	5,15	20,47	8,83	2,18	1,55	12,56	33,03
jan.09	2,26	13,01	5,44	20,71	8,68	2,44	1,75	12,87	33,57
feb.09	2,14	12,64	4,89	19,68	9,01	2,06	1,81	12,89	32,57
mar.09	1,99	12,72	4,80	19,52	9,08	2,16	1,58	12,83	32,35
apr.09	2,12	12,23	4,81	19,16	8,67	2,22	2,55	13,45	32,61
mai.09	2,33	12,95	5,01	20,28	8,14	2,12	2,27	12,53	32,81
jun.09	2,08	12,74	4,74	19,56	8,58	1,99	2,34	12,90	32,46
jul.09	1,91	9,94	3,92	15,78	11,20	1,94	2,12	15,26	31,03
aug.09	1,85	11,21	4,17	17,24	10,30	2,25	2,49	15,04	32,28
sep.09	1,83	11,66	3,97	17,46	8,45	2,58	2,56	13,59	31,05
okt.09	1,88	11,02	4,03	16,92	8,55	2,45	2,48	13,48	30,40
nov.09	1,94	11,97	4,39	18,29	7,83	2,18	2,00	12,01	30,30
des.09	1,86	10,96	4,31	17,12	8,28	2,21	1,98	12,47	29,60
jan.10	1,89	11,44	4,21	17,54	7,57	2,39	2,41	12,37	29,91
feb.10	1,58	10,91	3,59	16,08	7,96	2,31	2,02	12,30	28,37
mar.10	1,94	10,50	3,82	16,27	8,51	2,13	2,60	13,24	29,51
apr.10	1,85	8,97	3,39	14,21	7,46	2,08	2,43	11,97	26,17
mai.10	1,51	8,46	3,00	12,97	8,26	1,68	1,92	11,85	24,82
jun.10	1,42	8,09	2,89	12,40	8,08	1,79	2,34	12,20	24,60
jul.10	1,37	6,90	2,64	10,90	8,22	1,13	1,87	11,22	22,12
aug.10	1,48	8,13	3,15	12,76	8,14	2,23	1,88	12,26	25,02
sep.10	1,53	8,19	3,17	12,89	7,69	2,11	2,30	12,09	24,98
okt.10	1,52	7,70	2,94	12,15	7,25	1,97	2,14	11,36	23,52
nov.10	1,93	7,89	3,19	13,00	7,10	2,06	2,41	11,58	24,58

des.10	2,03	7,72	3,35	13,10	7,92	1,74	3,36	13,01	26,11
jan.11	1,82	7,02	2,97	11,81	7,60	2,13	3,23	12,96	24,77
feb.11	1,81	6,46	2,84	11,11	7,80	1,62	2,98	12,39	23,50
mar.11	1,40	5,75	2,52	9,67	7,67	1,30	2,76	11,72	21,39
apr.11	1,27	5,36	2,51	9,14	8,64	1,84	2,96	13,44	22,58
mai.11	1,22	5,37	2,40	8,98	9,97	1,74	2,92	14,62	23,60
jun.11	1,07	4,98	2,10	8,14	10,33	1,05	2,45	13,84	21,98
jul.11	1,17	4,92	2,08	8,17	10,40	1,30	2,47	14,17	22,35
aug.11	1,33	5,24	2,53	9,09	10,44	1,31	2,29	14,05	23,14
sep.11	1,05	5,20	2,37	8,61	9,95	1,50	2,18	13,64	22,25
okt.11	1,26	5,46	2,72	9,44	9,35	1,58	2,70	13,63	23,07
nov.11	1,17	5,40	2,45	9,02	8,26	1,56	2,90	12,73	21,75
des.11	1,33	5,05	2,74	9,11	7,58	1,55	1,85	10,98	20,10
jan.12	1,38	5,25	2,95	9,58	7,00	1,69	1,96	10,66	20,23
feb.12	0,98	4,50	2,59	8,07	8,10	3,61	2,41	14,12	22,19
mar.12	0,80	4,56	2,31	7,67	7,77	3,22	2,67	13,66	21,33
apr.12	0,72	3,79	1,91	6,42	8,17	3,03	2,71	13,90	20,32
Average	1,61	8,69	3,60	13,91	8,51	1,98	2,40	12,88	26,79

Total number of transactions

	Skandia-banken	Nordnet	Netfonds	Total Irrational	SEB	Artic	Carnegie	Total Rational	Totalt
01.01.2008	54977	323424	169306	547707	261779	55703	114663	432145	**3420128**
01.02.2008	45851	267008	138159	451018	196195	49629	91874	337698	**2640068**
01.03.2008	37614	202334	102573	342521	152842	33516	62561	248919	**2027454**
01.04.2008	40776	250936	124465	416177	188877	49076	82893	320846	**2596456**
01.05.2008	44388	245289	112801	402478	210764	47181	66852	324797	**2580320**
01.06.2008	28134	213831	87744	329709	203710	50498	59439	313647	**2430182**
01.07.2008	28104	194926	88726	311756	241501	45945	59482	346928	**2635336**
01.08.2008	25966	196896	86194	309056	207670	53180	51027	311877	**2326288**
01.09.2008	46099	338079	128810	512988	332502	67792	68152	468446	**3617330**
01.10.2008	80757	441222	180433	702412	419960	79411	80413	579784	**4080656**
01.11.2008	69730	404865	158619	633214	287954	54921	45190	388065	**3029418**
01.12.2008	54784	300190	119254	474228	204608	50471	35960	291039	**2316652**
01.01.2009	60005	345428	144322	549755	230334	64786	46548	341668	**2655038**
01.02.2009	50437	297331	115027	462795	212007	48545	42676	303228	**2352040**
01.03.2009	49346	315060	118774	483180	224881	53551	39243	317675	**2475938**
01.04.2009	47409	272892	107273	427574	193587	49650	56846	300083	**2231724**
01.05.2009	67548	376033	145354	588935	236225	61571	66048	363844	**2903688**
01.06.2009	55627	341553	127049	524229	229843	53268	62778	345889	**2680314**
01.07.2009	45198	234711	92637	372546	264486	45773	49953	360212	**2361152**
01.08.2009	47035	285119	106087	438241	261989	57139	63356	382484	**2542692**

01.09.2009	52887	337136	114906	504929	244416	74550	74076	393042	**2891904**
01.10.2009	59513	349602	127805	536920	271401	77661	78612	427674	**3173402**
01.11.2009	54624	337718	123760	516102	220964	61612	56342	338918	**2821546**
01.12.2009	44617	263191	103596	411404	198917	53150	47573	299640	**2402442**
01.01.2010	61230	370331	136203	567764	245113	77447	77932	400492	**3237370**
01.02.2010	49485	342532	112562	504579	249947	72566	63396	385909	**3138698**
01.03.2010	57278	309254	112622	479154	250508	62876	76613	389997	**2945076**
01.04.2010	58324	283005	107034	448363	235388	65524	76691	377603	**3155794**
01.05.2010	55636	311212	110270	477118	303739	61682	70687	436108	**3678908**
01.06.2010	50820	288951	103381	443152	288599	64068	83508	436175	**3573882**
01.07.2010	42976	216843	82934	342753	258559	35387	58789	352735	**3143806**
01.08.2010	43971	242320	93934	380225	242595	66569	56167	365331	**2979824**
01.09.2010	45435	242623	93795	381853	227748	62391	68135	358274	**2962702**
01.10.2010	47416	240598	91886	379900	226597	61686	66878	355161	**3125922**
01.11.2010	66872	273980	110852	451704	246676	71722	83804	402202	**3473608**
01.12.2010	56409	214116	93070	363595	219710	48236	93111	361057	**2775240**
01.01.2011	64976	250277	106035	421288	270946	76095	115313	462354	**3566860**
01.02.2011	63273	225982	99240	388495	272602	56483	104012	433097	**3495988**
01.03.2011	53019	217940	95593	366552	290870	49206	104548	444624	**3792216**
01.04.2011	33758	142300	66516	242574	229294	48940	78579	356813	**2654650**
01.05.2011	42787	188068	83971	314826	349314	60886	102345	512545	**3505172**
01.06.2011	35039	163533	68930	267502	339308	34594	80620	454522	**3284260**
01.07.2011	36730	154057	64973	255760	325286	40813	77380	443479	**3128618**
01.08.2011	73237	288578	139217	501032	575044	72416	126334	773794	**5508920**
01.09.2011	46967	233296	106146	386409	446627	67467	97917	612011	**4487308**
01.10.2011	54444	235507	117437	407388	403674	68057	116653	588384	**4316530**
01.11.2011	53457	245988	111388	410833	376392	71208	132107	579707	**4555214**
01.12.2011	43703	166546	90278	300527	249897	51198	61112	362207	**3297630**
01.01.2012	55292	209571	117846	382709	279802	67676	78187	425665	**3994950**
01.02.2012	62673	243271	119099	425043	386437	93961	103880	584278	**4670294**
01.03.2012	53061	213907	103068	370036	328693	75288	97282	501263	**4283174**