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THE SILENCE OF THE LAMBS

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Sheepmodell1006

The silence of the lambs

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Abstract
A model analyzing the economics of sheep farming is formulated. The basic idea is simple. Sheep are capital and they are held by farmers as long as their capital value exceeds their slaughter, or meat, value. The farmers are therefore portfolio managers aiming to find the optimal combination of different categories of animals and the yields are compared with the yields from other assets. The model is formulated within a Northern Scandinavian economic and biological setting with a crucial distinction between the outdoors grazing season and the indoors season, and with adult sheep and lambs being different categories. In the first step, the management problem is analyzed with only the meat income of the farmers taken into account. In the next step, income from wool production is considered as well. The analysis provides several results that differ from standard harvesting theory.

Keywords: natural resource modeling, sheep farming

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1. Introduction

In this paper, a model analyzing the economics of sheep farming is formulated. The basic idea is simple. Sheep are capital and farmers hold them as long as their capital value exceeds their slaughter, or meat, value. The farmers are therefore portfolio managers aiming to find the optimal combination of different categories of animals and the yields are compared with the yields from other assets. This problem has, therefore, similarities with the archetypical renewable natural resource problem (see, e.g., Clark 1990). However, whereas the standard fishery (or wildlife) problem is formulated in a biomass framework (‘a fish is a fish’), the different *age categories* of the sheep asset are central in the following analysis. The study is carried out with a crucial distinction between the outdoors grazing season and the indoors winter season, which is the typical situation found in Northern Scandinavia as well as in other places in Europe (e.g., mountain areas in France and Spain) and elsewhere. However, the analysis is essentially related to the economic and biological setting found in Northern Scandinavia and Norway[1].

There are about 20,000 sheep farms in Norway. These are family farms, and there are around two million animals during the outdoors grazing season. The average farm size is therefore quite modest and most of the farms are located in mountain areas and other sparsely populated areas; there are also some sheep farms along the coast. The main product is meat, which accounts for about 80% of the average farmer’s income. The rest comes from wool, as sheep milk production is nonexistent. Housing and indoor feeding is required throughout the winter because of snow and harsh weather conditions. The lambs are born during late winter to early spring, and in early spring the animals usually graze on fenced land. When the weather conditions allow, the sheep are released into rough grazing areas in the valleys and mountains, which are typically communally owned (‘commons’). The outdoors grazing season ends around late September to the middle of October. The animals are then gathered, the wool may be cut and slaughtering takes place. During the summer rough grazing period, the flocks may be vulnerable to large predators and to sickness and other injuries. Aunsmo et al. (1998), Nersten et al. (2003) and Dyrmundsson (2005) provide more details.

[1] France is an important sheep producing country, but, in contrast to Northern Scandinavia (see below), milk that is processed to cheese is the main product. Milk production is also the most significant product in Mediterranean countries such as Spain, but meat production from lambs is also important. Wool production is of most importance in Sweden as well as in Finland (Aunsmo et al. 1998).

Figure 1 about here

Within this farming system, the farmers face several investment decision problems. One problem is to find the optimal size of a farm; that is, the capacity to keep animals indoors during the winter season. Another problem is the so-called replacement problem, i.e., to find the optimal categories, or year classes, of adult females, as fertility (as well as mortality) varies over the life cycle. A third problem is, for a given farm capacity, to find the capacity utilization that gives the optimal number of animals to be fed and kept indoors during the winter season. A corollary of this problem is to find the optimal number of lambs to be slaughtered before the winter season. The main content of this third problem can be studied by considering just two categories, or stages, of the sheep population, i.e., lambs and adults. This investment problem is analyzed in the following few pages, and because only two categories of animals are included, it is possible to solve the problem within a simple optimal control framework. The analysis is at the farm level, where the farmer aims to maximize present-value profit.

There is extensive literature on the economics of livestock management (see, e.g., Kennedy 1986 and Jarvis 1974), but most of this literature has little relevance for a farming system with a distinct seasonal subdivision between the winter indoors season and the outdoors grazing. The problem of the typical cow-calf operator in the western United States, however, has some similarities with the Scandinavian sheep farming system, but the problem here is typically to determine the length of the grazing season, in addition to determine the stocking level (see, e.g., Huffaker and Wilen 1991). In contrast to this, the length of the grazing season is fixed in our model. There are several papers that analyze the replacement problem and consider the different categories of the adult sheep (see, e.g., Avramita et al. 1981). Typically, these models are large and detailed linear programming-type models. Fisher (2001) is an example of a detailed linear programming model that analyzes the economics of what are called a spring lambing system, a winter lambing system and an accelerated lambing system in Canada. In the following analysis, the spring lambing scheme is taken for granted. It is also assumed that the outdoors grazing conditions represent no constraint on the size of the flock and on the growth of the animals; this problem is taken up in an accompanying paper. On the other hand, as already indicated, winter farm capacity is assumed fixed. Relaxing this assumption is also analysed in the accompanying paper.

The rest of the paper is structured as follows. In the next section, the biological model is formulated and the conditions for equilibrium harvesting, or slaughtering, are found. The revenue and cost functions are introduced in section three, and the portfolio management problem of the individual farmer is formulated and solved in section four. The next section studies how changing economic and biological conditions may affect stock composition and slaughtering. The base model is extended to include wool production and predation in section six and section seven provides a numerical illustration.

2. The Biological Model

The biological model is formulated in a time-discrete manner with a seasonal subdivision between the outdoors grazing period (spring, summer and fall) and indoors feeding period (winter) (Figure 1). The sheep population is structured (e.g., Caswell 2001) as adult females, and young females and males, henceforth called lambs. The lambs are recruited in late winter to early spring, just before the grazing season starts. Lambs not slaughtered enter the adult population after the slaughtering period (i.e., September–October). All male lambs are assumed slaughtered since only very few (or none when artificial insemination is practiced) are kept for breeding. Therefore, only female adults are considered. Fertility is fixed. Natural mortality differs between adults and lambs and is fixed and independent of the number of animals as well. All natural mortality is assumed to occur during the grazing season. Demographic data on sheep are available in Mysterud et al. (2002).

When stochastic variations in biology and environment are ignored, the number of adult females in year $(t+1)$ just after slaughtering is made up of the previous year's adults surviving natural mortality and not slaughtered and the female lambs surviving natural mortality and not slaughtered (see Figure 1). This may be written as $X_{1,t+1} = X_{1,t}(1-m_1)(1-h_{1,t}) + X_{0,t}(1-m_0)(1-h_{0,t})$, where $X_{0,t}$ is the number of female lambs, m_1 and m_0 are the mortality fractions of adult females and lambs, respectively, and $h_{1,t}$ and $h_{0,t}$ are the fractions slaughtered[2]. With the fecundity rate f (lambs per adult female) and the same number of male and female lambs being recruited, $X_{0,t} = 0.5fX_{1,t}$ yields the number of female lambs. Therefore, the adult female population growth is:

[2] New animals from outside may be added, but this possibility is ignored in the present exposition.

(1) $X_{1,t+1} = X_{1,t}(1-m_1)(1-h_{1,t}) + 0.5 f X_{1,t}(1-m_0)(1-h_{0,t})$.

In a linear growth model such as this, it is well known that, with no removal of animals, i.e., $h_{1,t} = h_{0,t} = 0$, the population will either grow without bounds or die out (e.g., Caswell 2001)[3]. However, with slaughtering, there are infinite combinations of harvesting fractions that may sustain a stable population. Such steady-state harvesting rates are found when $X_{1,t+1} = X_{1,t} > 0$, and may be written as:

(2) $h_1 = 1 - \frac{1-0.5f(1-m_0)(1-h_0)}{(1-m_1)}$.

Equation (2) describes a downward sloping line in the (h_0, h_1) plane, and a constant population can hence be sustained with either a 'low' h_0 and a 'high' h_1 or the opposite. Harvesting combinations outside this line mean a shrinking population, whereas combinations inside yield growth. Condition (2) intersects with the h_1 -axis at $[1-[1-0.5f(1-m_0)]/(1-m_1)]$, which may be above or below one (see also Figure 2). Therefore, the highest adult-harvesting rate compatible with the steady state is $\min\{1,[1-[1-0.5f(1-m_0)]/(1-m_1)]\}$. For all realistic parameter values, it will be below one (see numerical section), and only this situation is considered (but see section four). Equation (2) intersects with the h_0 -axis at $[1-m_1/0.5f(1-m_0)]<1$ and is the highest lamb-harvesting rate compatible with the steady state. Not surprisingly, these maximum values increase with higher fertility and lower mortality.

Figure 2 about here

3. Revenue and Costs

At this stage, we are neglecting any income from wool production, and the sale of meat is the only revenue component. Because slaughtering takes place after natural mortality, the number of adult animals removed is $H_{1,t} = X_{1,t}(1-m_1)h_{1,t}$. The number of slaughtered female lambs is

[3] It is easily recognized that the population will die out if $m_1 > 0.5f(1-m_0)$. Therefore, with removal of animals, the demographic parameters must be scaled such that $f > m_1/0.5(1-m_0)$.

$HF_{0,t} = 0.5 fX_{1,t}(1-m_0)h_{0,t}$, and the entire male lamb subpopulation is removed, $HM_{0,t} = 0.5 fX_{1,t}(1-m_0)$. With p_1 and p_0 as the net (net of slaughtering costs) adult and lamb slaughtering prices (Euro per animal), respectively, and assumed to be constant over time and independent of the number of animals supplied at the farm level, the meat income for year t reads:

(3) $Q_t = p_1 X_{1,t}(1-m_1)h_{1,t} + p_0 0.5 fX_{1,t}(1-m_0)(h_{0,t}+1)$.

The cost structure differs sharply between the outdoors grazing season and the indoors feeding season, and generally the indoors costs are much higher. The length of the indoors season is strictly steered by climate conditions and is therefore exogeneously given. Throughout this analysis, it is assumed that farm capacity is fixed (section one)[4]. Therefore, the costs of buildings, machinery and so forth are constant, and given by γ . The indoor season variable costs include labor cost (typically as an opportunity cost), electricity, and veterinarian costs, in addition to fodder and vary with the given length of the indoors season. The variable costs increase with the size of the winter population and, as the capacity constraint is approached, these costs may increase steeply. This is approximated by a convex function, and the winter total cost function is specified as $CW_t = \gamma + (\beta/2)X_{1,t}^2$.

As indicated, during the grazing period the sheep essentially graze on communally owned lands (commons). In Norway, such land is always available cost free. There may be some transportation and maintenance costs, which altogether are assumed to be linearly related to the size of the grazing flock and become $CS_t = \alpha(X_{1,t} + fX_{1,t})$ when measured before natural mortality. Therefore, when ignoring discounting within the year, the yearly cost is:

(4) $C_t = \gamma + \alpha(1+f)X_{1,t} + (\beta/2)X_{1,t}^2$.

The difference to the standard resource economic model (e.g. Clark 1990) is apparent. While the costs are typically decreasing in the stock size in the standard model (due to lower unit

[4] The problem of also allowing for physical capital accumulation and changing farm capacity is progressively more difficult to analyse because one has to account for irreversibility (see the pioneering work of Clark, Clarke and Munro 1979 in a fishery context). As mentioned, this problem is taken up in an accompanying paper.

harvesting costs), they are increasing in the present model. However, we often find the similar cost structure in models of terrestrial wildlife resources (e.g., Swanson 1994, Skonhoft 1999).

4. The Optimal Program

It is assumed that the farmer aims to maximize present-value profit over an infinite time horizon, $\sum_{t=0}^{\infty}\rho^t[Q_t - C_t]$, where $\rho = 1/(1+\delta)$ is the discount factor with $\delta \geq 0$ as the (yearly) discount rent[5]. The current-value Hamiltonian of this problem reads

$$\Psi = p_1 X_{1,t}(1-m_1)h_{1,t} + p_0 0.5 f X_{1,t}(1-m_0)(h_{0,t}+1) - [\gamma + \alpha(1+f)X_{1,t} + (\beta/2)X_{1,t}^2]$$
$$+\rho\lambda_{t+1}[X_{1,t}(1-m_1)(1-h_{1,t}) + 0.5 f X_{1,t}(1-m_0)(1-h_{0,t}) - X_{1,t}]$$, where $\lambda_t > 0$ is the resource shadow price. The first-order conditions with $X_{i,t} > 0$ and the adult-harvesting fraction below one (but see below) yield:

(5) $\partial\Psi/\partial h_{1,t} = p_1 - \rho\lambda_{t+1} \leq 0\,;\; h_1 \geq 0$

(6) $\partial\Psi/\partial h_{0,t} = p_0 - \rho\lambda_{t+1} \leq 0\,;\; h_0 \geq 0$

and

(7) $$-\partial\Psi/\partial X_t = -p_1(1-m_1)h_{1,t} - p_0 0.5 f(1-m_0)(h_{0,t}+1) + \alpha(1+f) + \beta X_{1,t}$$
$$-\rho\lambda_{t+1}[(1-m_1)(1-h_{1,t}) + 0.5 f(1-m_0)(1-h_{0,t}) - 1] = \rho\lambda_{t+1} - \lambda_t\,.$$

The interpretation of control condition (5) is that adult slaughtering should take place up to the point where the marginal meat income is equal to, or below, the cost of reduced growth in stock numbers, evaluated at the shadow price. It holds as an equation when the removal of this subpopulation is optimal at the steady state. The lamb control condition (6) has the same interpretation. Equation (7) is the portfolio condition, which essentially states that the number of adult females should be maintained so that the natural growth equals the shadow price

[5] The realism of present-value maximizing as a management goal may be questioned for various reasons. However, it can be shown that maximizing current profit under the condition of equilibrium harvesting, $X_{1,t+1} = X_{1,t}$, yields the same solution as the steady state of present-value maximization for a zero discount rent, $\delta = 0$ (see the main text below). Therefore, the steady-state flock size for a zero discount rent is similar to the flock size when current-value profit is maximized for a stable population.

growth, adjusted for the discount factor. With the assumption that the steady state is reachable from the initial value $X_{1,0}$, the dynamics will typically be of the Most Rapid Approach Path (*MRAP*) as the Hamiltonian is linear in the controls and hence will involve a 'bang-bang' control. Accordingly, if the stock is above that of the steady state, it should be slaughtered down the first year. In the opposite situation with too few animals, slaughtering should be postponed until the steady state is reached.

At the steady state, the shadow price is fixed through the control conditions. However, these conditions cannot be generally satisfied simultaneously as equations. It is a 'knife-edge' harvesting problem and, except when prices are equal, only one of the categories should be harvested. Therefore, we have to distinguish between the three cases: i) if $p_0 > p_1$, lamb-only slaughtering is optimal; ii) if $p_0 < p_1$, adult-only slaughtering is optimal, and iii) if $p_0 = p_1$, both categories should be slaughtered. The argument for case i) is simply that if steady state lamb-only slaughtering is beneficial, condition (6) should hold as equality and condition (5) as an inequality. It is easily recognized that this demands $p_0 > p_1$. The argument for case ii) follows in a similar manner. This is stated as:

Result 1. For $p_0 \neq p_1$, *only one-stage slaughtering is optimal at the steady state.*

A corollary to Result 1 and the biological equilibrium condition (2) is:

Result 2. *When* $p_0 \neq p_1$, *slaughtering should take place at the highest level compatible with the steady state.*

Therefore, when the lamb price is above that of the adult price, which fits reality and is considered as the main case (cf. numerical section), the case i) steady-state harvesting rates are:

(8) $h_1^* = 0$

and

(9) $h_0^* = 1 - \dfrac{m_1}{0.5 f(1 - m_0)}$.

Note that the optimal harvest rates are independent of the steady state number of animals, which is due to the linear structure of the population model. The accompanying shadow price is $\lambda^* = p_0 / \rho = p_0(1+\delta)$. When evaluating the portfolio condition (7) at the steady state, replacing the shadow price and doing some small rearrangements, we next find the golden rule condition as:

$$(10) \quad (1+\delta) = f(1-m_0) + (1-m_1) - \frac{\alpha(1+f) + \beta X_1}{p_0}.$$

This condition states that the internal rate of return, comprising the natural growth rate adjusted for the cost–price ratio, should be equal to the external rate $(1+\delta)$, and Eq. (10) alone determines the unique solution of the steady-steady number of adult animals X_1^*. Note that biological as well as economic parameters influence X_1^* while only biological parameters determine the size of the steady state harvest rate.

In the opposite case ii) of $p_0 < p_1$, the steady-state harvesting rates are $h_1^* = [1-[1-0.5f(1-m_0)]/(1-m_1)] < 1$ and $h_0^* = 0$, and the golden rule condition reads $(1+\delta) = \frac{(p_0 + p_1)}{p_1} 0.5 f(1-m_0) + (1-m_1) - \frac{\alpha(1+f) + \beta X_1^*}{p_1}$. Hence, the harvesting rate is still not included in the golden rule condition, but both prices influence the internal rate of return in this adult-only harvesting case[6]. The lamb harvesting price influences the optimal flock size when there is no lamb slaughtering because of the biologically 'indirect' nature of this stage; lambs this year are slaughtered as adults next year.

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[6] If the biological conditions are such that $h_1^* = 1$ (which, as mentioned, is unrealistic due to the actual biological parameter values), the lamb-harvesting rate compatible with the steady state follows from equation (2) as $h_0^* = [1 - 1/0.5 f(1-m_0)]$. In this special version of case ii), the complementary slack conditions change and the above control condition (5) reads $p_1 - \rho\lambda > 0$ at the steady state. Moreover, as lambs are slaughtered as well, (6) holds as an equation at the steady state, $p_0 - \rho\lambda = 0$. It is recognized that these conditions are consistent with $p_0 < p_1$.

Case iii), with $p_0 = p_1 = p$, means that both categories should be harvested at the steady state, $h_0^* > 0$ and $h_1^* > 0$. After some rearrangements, it can be shown that the portfolio condition is just as the above equation (10), except that p replaces p_0 . As the control conditions (5) and (6) now yield the same information, there is one degree of freedom in the system of equations and unknowns. Accordingly, either the lamb- or the adult-harvesting rate must be given exogenously. Hence, all harvesting rates along the equilibrium harvesting schedule (2) are equally beneficial for the farmer.

5. Changes in the Economic and Biological Environment

The above analysis demonstrates that the meat prices alone determine the harvesting decision, whereas biological conditions only determine the size of the harvesting rates. On the other hand, biological as well as economic factors determine the optimal adult flock size to be kept during the winter, and hence the optimal outdoors grazing lamb population. When considering the main case i) and rewriting the golden rule condition (10), the adult steady-state flock size becomes:

(10´) $$X_1^* = \frac{p_0[f(1-m_0)+(1-m_1)-(1+\delta)]-\alpha(1+f)}{\beta} .$$

Not surprisingly, it is observed that permanent higher mortality rates mean fewer animals. The fertility rate effect is, on the other hand, positive when the lamb marginal harvesting income dominates the marginal outdoors season cost; that is, when $p_0(1-m_0) > \alpha$. This must hold to secure a positive steady state profit. Higher fertility also means a higher harvesting rate (Eq. 9), and it can easily be shown that the equilibrium profit $(Q^* - C^*) = p_0 0.5 f X_1^*(1-m_0)(h_0^* + 1) - [\gamma + \alpha(1+f)X_1^* + (\beta/2)X_1^{*2}]$ increases as well. This fits intuitive reasoning as higher fertility is to be considered as a cost-free gift of Mother Nature.

More costly farming, either during the indoors feeding season, β , or the grazing season, α , yields fewer animals. The discount rent has the standard negative stock effect as well, whereas condition (10´) clearly indicates a positive price effect, $\partial X_1^* / \partial p_0 > 0$. This is stated as:

Result 3. *A higher slaughter price yields more animals.*

This result contrasts with what is found in the standard harvesting model (Clark 1990). The reason is, however, straightforward as there is no stock-dependent harvesting, or slaughtering, cost. Therefore, a higher p_0 simply means that it becomes relatively less expensive to keep animals as both α / p_0 and β / p_0 decrease (see also Swanson 1994 and Skonhoft 1999). With more adult animals, there will also be more grazing lambs, $X_0^* = 0.5 f X_1^*$. The optimal number of removed animals (cf. the above section three), consisting of lambs only (female and male), is $HF_0^* + HM_0^* = 0.5 f X_1^* (1 - m_0)(h_0^* + 1) = X_1^* [f(1 - m_0) - m_1]$, and slaughtering increases with a higher harvesting price as well.

6. Extending the Basic Model

Income from wool production has so far been ignored. However, wool income can be significant for some farmers and, as indicated, it contributes about 20% of the total sheep farm income in Northern Scandinavia (e.g., Aunsmo et al. 1998). Following today's practice, the farmer may choose whether to shear the sheep once or twice a year. If the fleece is shorn once, this will be in spring and for adults only. Therefore, the lambs that survive natural mortality and slaughtering, are not shorn before they are one-year old. In the other case of shearing two times a year, there is an additional shearing just before slaughtering. The last scheme is considered here, as this is the most common practice (Aunsmo et al. 1998). The yearly wool yield is then written as $W_t = q[\sigma_s X_{1,t} + \sigma_a (1 - m_1) X_{1,t} + \tau f X_{1,t} (1 - m_0)]$, where q is the net (net of shearing costs) wool price (euro per tonne wool), σ_s and σ_a are the (average) per unit adult spring and autumn outputs (tonne per animal), respectively, and τ is the per lamb output. This may be simplified to:

(11) $W_t = q\theta X_{1,t}$,

where θ is the demographic and seasonally adjusted per unit output coefficient. Accordingly, adding wool implies joint production, meat and wool, of the fixed coefficient type.

With W_t as part of the farm income, a stock effect is added to the harvesting decision and represents a similarity to the so-called 'wealth effect' in models of optimal growth (see the classic Kurz 1968 article). When maximizing present-value profit including wool income, $\sum_{t=0}^{\infty} \rho^t [Q_t - C_t + W_t]$, under the biological constraint (1), it follows directly that the control conditions (5) and (6) stay unchanged. This is stated as:

Result 4. *Including a stock value leaves the harvesting decision unchanged, and the harvesting fractions stay unchanged as well.*

Because adding a stock value changes the relative valuation between keeping an asset and selling it, this result also contrasts standard harvesting theory and intuitive reasoning. It holds irrespective of the profitability of the wool production, q, whether the per lamb wool output coefficient is 'low' or 'high' and whether there is a price difference between adult and lamb wool (which is not considered here). On the other hand, the portfolio condition (7) changes as it is extended with the marginal wool income term ('wealth effect'). In the lamb-only harvesting case i), it can be verified that the steady-state adult flock size now becomes:

$$(12) \quad X_1^* = \frac{p_0[f(1-m_0)+(1-m_1)-(1+\delta)]+q\theta-\alpha(1+f)}{\beta}.$$

The effect $\partial X_1^* / \partial q > 0$ is easily recognized. As a corollary of this effect and $\partial h_0^* / \partial q = 0$, the number of animals removed increases, $\partial (HF_0^* + HM_0^*) / \partial q > 0$. Hence, in contrast to the expected result that the farmer should sell less of an asset that becomes more valuable, the farmer sells more. This is stated as:

Result 5. *Including a stock value leads to more animals being slaughtered.*

The model may also be extended to include predation. During the grazing period, sheep in Northern Scandinavia are vulnerable to predation from four big predators: bears (*Ursus arctos*), wolverines (*Gulo gulo*), wolves (*Canis lupus*) and lynxes (*Lynx lynx*). Although the total loss is modest (yearly, less than 1% of the sheep population is reported lost), farmers in some few areas may be seriously affected. With r_0 and r_1 as the fixed lamb and adult predation rates, respectively, and assuming pure additively to natural mortality (which fits reality, see Environment Department 2003), the population growth equation (1) changes to[7]:

[7] The assumption of fixed predation rates reflects the situation of a constant predation pressure through time, e.g., a fixed number of wolves through time. As the predation loss increases linearly with sheep density,

(13) $X_{1,t+1} = X_{1,t}(1-m_1)(1-r_1)(1-h_{1,t}) + 0.5fX_{1,t}(1-m_0)(1-r_0)(1-h_{0,t})$.

Accordingly, the new harvesting equilibrium schedule shifts down relative to the previous condition (2).

With predation, the meat income of the farmer also changes and reads $\tilde{Q}_t = p_1 X_{1,t}(1-m_1)(1-r_1)h_{1,t} + p_0 0.5 fX_{1,t}(1-m_0)(1-r_0)(h_{0,t}+1)$. When the present-value profit is maximized under the new biological growth equation (13), we find the same control conditions as above, i.e., (5) and (6) hold irrespective of the relative predation pressure on adults and lambs. However, the harvesting fraction reduces, and the lamb-only harvesting case i) now yields $h_0^* = \{1-[1-(1-m_1)(1-r_1)]/0.5f(1-m_0)(1-r_0)\}$. Therefore, predation on lambs, but also on adults, reduces the harvesting rate. This is stated as:
Result 6. *Predation leaves the slaughtering decision unchanged. The optimal harvesting fraction reduces.*

When the wool income term again is ignored, the steady-state female adult flock size becomes $X_1^* = (1/\beta)\{p_0[f(1-m_0)(1-r_0)+(1-m_1)(1-r_1)-(1+\delta)]-\alpha(1+f)\}$. Not surprisingly, the flock size is reduced compared with the no-predation situation, $\partial X_1^* / \partial r_i < 0$ ($i = 0,1$). Consequently, the number of animals slaughtered and the meat income are reduced as well.

Above, it is tacitly assumed that the farmer receives no economic compensation for the predation loss. However, compensation is normally paid by the State (Environment Department 2003). If natural mortality is assumed to take place before predation (the opposite will not change the results qualitatively), the number of adults and lambs lost through predation is $R_{1,t} = X_{1,t}(1-m_1)r_1$ and $R_{0,t} = fX_{1,t}(1-m_0)r_0$, respectively. With $k_i > 0$ ($i = 0,1$) as the per unit compensation value, the yearly compensation is $K_t = k_1 X_{1,t}(1-m_1)r_1 + k_0 fX_{1,t}(1-m_0)r_0$, which may be simplified to:

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assuming constant predation rates (see main text below), the ecological interaction is consistent with the famous Lotka–Volterra predator–prey model (see, e.g., Clark 1990).

(14) $K_t = \phi X_{1,t}$,

where ϕ is the demographic adjusted per adult compensation value.

Current profit is now $(\tilde{Q}_t - C_t + K_t)$ (when wool income is ignored), and present-value profit maximization yields the same control conditions as above. Therefore, when considering the lamb-only harvesting case i) with $p_0 > p_1$, we find the same harvesting fraction h_0^* as under predation without compensation, but lower than that without predation. The optimal steady-state population size changes to:

$$(15) \quad X_1^* = \frac{p_0[f(1-m_0)(1-r_0) + (1-m_1)(1-r_1) - (1+\delta)] + \phi - \alpha(1+f)}{\beta}.$$

Predation with compensation leads to more animals than without predation (Eq. 10´) if $\{p_0[f(1-m_0)(1-r_0) + (1-m_1)(1-r_1)] + \phi\} > p_0[f(1-m_0) + (1-m_1)]$. This inequality may also be written as $(\phi / p_0) > [f(1-m_0)r_0 + (1-m_1)r_1]$. In principle, the farmer should be exactly compensated (Environment Department 2003)[8]. With $k_i = p_i\,(i = 0,1)$, the adjusted per adult compensation value is $\phi = p_1(1-m_1)r_1 + p_0 f(1-m_0)r_0$. The above condition reduces then to $p_1 > p_0$, which is violated under the lamb-only harvesting scheme. Therefore, even if the farmer is fully compensated, it is optimal to keep a smaller flock size and slaughter fewer animals than to farm without predation.

More interesting, however, is that the profit will be reduced as well. The reason is that predation imposes an additional constraint on the slaughtering decision of the profit-maximizing farmer. With predation, the farmer must ‘harvest’ twice a year and the first harvest, i.e., the predation, takes place with predation rates fixed by Mother Nature. So, even when the price per animal of this ‘harvest’ is the same as under the regular slaughtering, the profit will decrease compared with the situation of no such constraint. The numerical simulations confirm this reasoning. This is stated as:

[8] However, in reality, the amount of compensation may differ because of various reasons, such as cheating and overestimating the loss.

Result 7. *Predation with full compensation yields a smaller flock and fewer animals slaughtered than without predation. The profit reduces as well.*

7. Numerical Illustration

To shed some further light on the above analysis, the model is illustrated numerically. The intention is to mirror only some qualitative aspects of the Northern Scandinavian sheep-farming practice. The Appendix gives the data used in the simulations and Table 1 reports the results of the lamb-only harvesting case i).

Table 1 about here

Without wool production and predation, the optimal lamb-only harvesting fraction is 0.93 (Result 1 and Result 2) and the optimal adult steady-state flock size is 119 animals. If the lamb price shifts up 25%, the adult flock size increases to 155 animals (Result 3) and the number of lambs slaughtered increases from 160 to 208. With wool production and the baseline lamb meat price, but still no predation, both the adult flock size and number of animals slaughtered increase (Result 5) and the harvesting fraction stays unchanged (Result 4). Predation with full compensation, but without wool production, gives a small reduction in the number of adult animals compared with the no wool and no predation scenario. The harvesting fraction is more affected, as it reduces to 0.88 (Result 6). The profit drops, albeit slightly, irrespective of the full compensation scheme, from 8,218 to 8,149 (Result 7).

8. Concluding Remarks

The paper has analyzed the economics of sheep farming at the farm level in a Northern Scandinavian context with a crucial distinction between the outdoors grazing season and the winter indoors feeding season. Meat production is the basic product. Farm capacity is assumed given and the problem is to find the optimal number of animals to be kept indoors during the winter and the number of animals to be slaughtered before the winter season. In this two-stage model of lambs and adult females, it is demonstrated that the harvesting decision is determined by economic factors alone and, for the given price structure, lamb-only harvesting is the best strategy. On the other hand, the lamb-harvesting fraction is determined by biological factors alone, whereas the optimal flock size is determined jointly by biological and economic factors. The reason for this sharp distinction between the effects of economic and biological forces is the lack of any density-dependent factors regulating sheep population growth. Including wool income in addition to meat value, and including predation during the grazing season, leaves this structure more or less unchanged. Several results contrasting the standard natural resource harvesting theory are provided.

The model's main results replicate today's practice where farmers basically slaughter and sell lamb meat (Aunsmo et al. 1998). Within our model, the only factor explaining this practice is that lamb meat is more valuable than adult meat. Therefore, a crucial part of the farmer's portfolio management problem boils down to a very simple decision rule. On the other hand, how much capital to hold is progressively more difficult to answer, as economic as well as biological factors determine the optimal population size. The baseline numerical example indicates a lamb-harvesting fraction of 0.93. This is above today's practice, which averages about 0.75 in Norway (Aunsmo et al. 1998). The low adult mortality assumption (0.05) explains most of this difference. Thus, if adult mortality is increased somewhat due to today's practice of slaughtering old females because of high natural mortality and low fertility, our result would be more in accordance with reality. However, to find such age-specific harvesting rates is beyond the scope of the present analysis as it is a part of the replacement investment problem (see the introductory section).

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Appendix

The Appendix reports the base-line data used in the simulations. The biological parameter values are based on Mysterud et al. (2002) and Aunsmo et al. (1998). Aunsmo and Nersten et al. (2003) provide economic data, but some of the key parameters are calibrated to ensure realistic stock size. The variable marginal winter cost is crucial here. All prices are in 2003 value.

Table A1 about here

Table 1

Steady-state different value categories included. Lamb only harvesting case ($p_0 > p_1$) . X_1^* number adult animals, h_0^* female lamb harvesting fraction, $(HF_0^* + HM_0^*)$ total lamb slaughtering and π^* profit (in Euro).

	X_1^*	h_0^*	$HF_0^* + HM_0^*$	π^*
No wool production. No predation. Baseline price p_0	119	0.93	160	8,218
No wool production. No predation. 25% price increase p_0	155	0.93	208	13,876
Wool production, No predation. Baseline price p_0	138	0.93	181	10,503
Predation with full compensation. No wool production. Baseline price p_0	118	0.88	147	8,149

Table note: Fixed costs neglected.

Table A1
Baseline values prices and costs, ecological parameters and other parameters

Parameter	Parameter description	Value
m_0	-Natural mortality fraction lambs	0.09
m_1	-Natural mortality fraction adult	0.05
f	-Fertility rate	1.53 (lamb per adult)
r_0	-Predation fraction lambs	0.05
r_1	-Predation fraction adult	0.03
p_0	-Lamb slaughter price	120.0 (Euro per animal)
p_1	-Adult slaughter price	100.0 (Euro per animal)
α	-Fixed marginal cost summer grazing	10.5 (Euro per animal)
β	-Variable marginal cost winter	1.1 (Euro per animal)
q	-Wool price	4,300 (Euro per tonne)
θ	-Adjusted wool output coefficient	0.005 (tonne per animal)
δ	-Discount rent	0.03

Figure 1
Seasonal subdivision Northern Scandinavian sheep farming

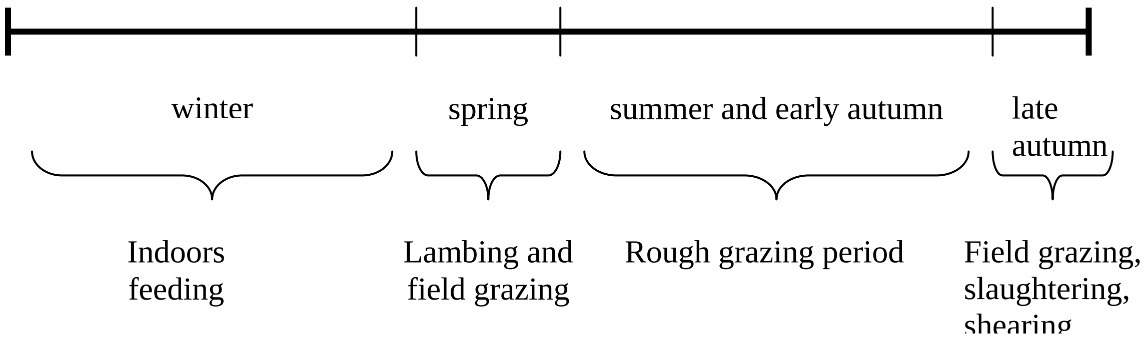

Figure 2
Steady-state harvesting relationship Eq. (2) (no predation)