NTNU – Trondheim
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The Net Neutrality Debate

Analysis of economic implications of net neutrality on internet service providers, content providers and internet users

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Abstract

This thesis studies the economic implications of a transition from a neutral to a non-neutral network. A mathematical model with an end to end ecosystem is developed, which includes a backbone internet service provider that provides connectivity for the content providers. The model also includes internet users that pay an access internet service provider for connectivity to interact with the content providers, advertisers that pay the content providers, and access internet service providers that charge the internet users for their connectivity. In the non-neutral regime, the content providers can pay the access internet service provider to access specialized services, a priority lane. The model is solved in a non-cooperative two stage game, and where the backbone and access internet service providers maximize their individual profits. Our analysis shows that the non-neutral unregulated regime always improves social welfare, investment and innovation levels, and lowers the congestion levels. We also show that the backbone internet service provider is always better off in the non-neutral regime, and therefore reduce the benefit of the other players. The result for the other players is generally positive in favor of a non-neutral regime, but much more ambiguous, especially for the access internet service providers. We also show that moderate regulation can increase social welfare. However, even moderate regulation might severely impair individual players, and too much regulation may also impair the non-neutral regime so that the overall social welfare is worse than in the neutral regime.

Sammendrag: Oppgaven studerer de økonomiske konsekvensene ved en overgang fra et nøytralt til et ikke-nøytralt nettverk. Vi utvikler en matematisk modell med et ende-til-ende økosystem, som inkluderer en bakgrunnsleverandør som gir innholdsleverandører tilgang til Internett. Modellen inkluderer også internettbrukere som betaler internettleverandører for tilgang til Internett og dermed innholdsleverandørene, annonsører som betaler innholdsleverandørene, og internettleverandører som får betalt av internettbrukerne. I det ikke-nøytrale regimet kan innholdsleverandørene betale internettleverandørene for å få tilgang til spesialiserte tjenester, en prioritetstilkobling. Modellen løser et ikke-samarbeids to-stegs spill, hvor bakgrunnsleverandøren og internettleverandørene maksimerer sine respektive overskudd. Analysen viser at det ikke-nøytrale og uregulerte regimet alltid forbedrer det sosialøkonomiske overskuddet, investerings- og innovasjonsnivåene, og gir lavere kødannelser i nettverket. Vi viser også at bakgrunnsleverandøren alltid profiterer på det ikke-nøytrale regimet, og derfor reduserer de potensielle fordelene av regimet for de andre aktørene. Overgangen er likevel generelt sett positiv for de andre aktørene, men resultatet er mye mer tvetydig. Dette gjelder særlig for internettleverandørene. Vi viser også at moderat regulering av nettverket kan øke det sosialøkonomiske overskuddet. Men, selv moderat regulering kan gå kraftig utover individuelle aktører, og for mye regulering kan også redusere det sosialøkonomiske overskuddet i et ikke-nøytralt regime slik at overskuddet blir mindre enn i det nøytrale regimet.

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My father endured reading through the first draft, for which I am very grateful.

Thanks again to all who helped me!

A Note to the Reader

A pre study was done on end to end differentiated services, pricing and segmentation of a quality of service enabled network, during the fall of 2013. The pre study focused on segmentation and pricing of differentiated services towards the IUs. While the results proved to be interesting, the work done in this thesis is completely different from that in the pre study. None of the work in the pre study has therefore been carried over to this thesis. Readers interested in the results from the pre study are encouraged to contact the author directly.

This thesis is written by a single student, and the only external aid was provided through meetings with the supervisor and the Telenor research team.

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Cover illustration: Net-neutrality illustration from www.wired.com [41]

Table of Contents

List of Figures and Tables

1 Introduction

The Internet is rapidly evolving, with services demanding ever higher levels of network capacities. This has led to the exploration of alternative ways the access internet service providers can monetize on their position as gatekeepers, in order to finance the high levels of investments. In general what is being explored is the transition from a neutral network to a non-neutral network where the internet service providers can differentiate and prioritize content, and much research has been done to analyze the effects of such a transition [1] [2] [3] [4] [5] [6] [7] [8] [9] [10] [11] [12] [13]. Among these, [1] and [2] are closest to our work. In this context, and throughout the thesis, a neutral network is a network where the internet service providers are not allowed to differentiate content in the network or favor one content provider over another, and the access internet service providers can only charge the internet users once and not the content providers for sending information over the broad band lines to end users. The internet service providers are also not allowed to block internet traffic based on its source, ownership or destination. A non-neutral network is a network where the internet service providers are allowed to differentiate content in the network, and as such prioritize content from one content provider over another. The prioritized lane is called specialized services, and the best effort lane is called internet access services. The internet service providers may also charge both the internet users and the content providers for sending information over the broad band lines to end users. However, the internet service providers are not allowed to block internet traffic.

The articles, [1] - [13], are part in what is referred to as the net neutrality debate. Net neutrality is widely and intensely debated among law and policy makers, as well as the key actors in the ecosystem that are affected by a possible transition to a non-neutral network. The policy makers are afraid that a non-neutral network will threaten the open internet, and impair the traffic that is not being paid for by the content providers. The neutral regime proponents, mostly the content providers, are afraid that a departure from a neutral regime will stifle innovation. The proponents of a non-neutral network, mostly the internet service providers, claim that a non-neutral regime will stimulate innovation by expanding the network infrastructure, help to cope with the increase in internet traffic and that it will secure necessary revenues to maintain and improve networks without charging extra fees on the end-users.

Much important research has been developed in the later years, and especially the late research by Krämer et al. [1], Bourreau et al. [2], Njorge et al. [3] and Economides and Hermalin [4] provide interesting results and thorough insights to many of the possible effects of this transition. Krämer et al. provides a model with heterogeneous content providers, investments from both internet service providers and content providers and multi-homing content providers. Bourreau et al. have further developed a model based on a similar setting to include heterogeneous internet users and competing access internet service providers, providing more crisp and realistic results. They also do a comprehensive analysis on the effect of endogenous regime choice, sabotage and different size of the content providers. While Krämer is more ambiguous towards the non-neutral regime, and shows that the access internet service providers always benefits from a transition, Bourreau et al. are more positive, and show that because of the competition between the access internet service providers they are not certain at all to benefit from a non-neutral regime. Njorge et al. use a different setting from Bourreau et al., however, with the same overall features. They obtain a crisp set of results through a six stage game, and the results mostly agrees with what was found by Bourreau et al. Economides and Hermalin does include heterogeneous content providers and internet users, and investments by the content providers and access internet service providers. However, they do not consider competing access internet service providers and multi-homing content providers. Their results are generally not in favor of a non-neutral regime.

The previous research is largely based on a closed ecosystem, where the content providers have to connect to the same internet service providers as the internet users, or the content provider's connectivity is not included. In reality, the content providers have to connect to the internet, which is modelled in this thesis through a backbone internet service provider. The content providers are dependent on sufficient quality of service from the backbone internet service provider. We assume, because of the much lower costs of backbone network expansion, that the backbone internet service provider delivers sufficient quality of service. We have extended on the work done by Krämer et al. and Bourreau et al. to include the backbone internet service provider, and also include the other features of Bourreau et al., which includes competing internet service providers, investments by the internet service providers and the content providers as measured by innovation, multi-homing content providers and heterogeneous content providers and internet users. The new features in our model are the following

- A backbone internet service provider that provides connectivity to the content providers, and endogenously sets the connectivity price to maximize its own profits
- The opportunity to simulate a weaker market power of the backbone internet service provider
- A regulation scheme on the internet access lane, that forces the access internet service providers to invest in more capacity to obtain sufficient quality of service on their internet access lanes

In addition to confirming propositions of Bourreau et al. in an ecosystem that includes a backbone internet service provider, we also provide new propositions based on the analysis and assumptions of our model

- If the M/M/1 queue assumptions hold, splitting a network into two separate networks can increase the average congestion levels by a factor of two
- A backbone internet service provider always prefers a non-neutral regime, provided he has sufficient quality of service to deliver sufficient quality of service to the specialized service lanes in the non-neutral regime
- The backbone ISP reduce the benefit of the non-neutral regime for the other actors
- Too much regulation of the internet access services' quality of service may reduce the overall social welfare

We perform a numerical experiment where we compare the two regimes, and analyze which regime is favorable over the other in overall social welfare and for the individual actors. Some highlights of our findings are

- The attractiveness of advertising relative to the traffic needed to generate revenues is arguably the most important factor when considering the effects of a neutral versus a non-neutral network because with high and low ratio values the winners and losers of a non-neutral regime is opposite to one another
- Regulating the internet access services lane can reduce the overall social welfare, and significantly reduce the benefit of the internet users when they are dependent on the content providers and high quality of service

This thesis consists of eight parts. Part 1 provides an overview of important definitions and clarifications, an introduction to the net neutrality debate and a description of the problem. Part 2 follows up with a comprehensive survey of previous research on the net neutrality debate and two-sided markets. Part 3 describes the most relevant theory used in this thesis. Part 4 presents mathematical models from four of the latest and most comprehensive papers on the net neutrality debate, and discusses the strengths and weaknesses of the presented models. Part 5 is the core section of this thesis, and contains a presentation of the mathematical models developed and modified for this thesis, solutions to the analytical model, an analytic discussion of the results, and a scenario based numerical experiment with a thorough discussion that compares a neutral and a non-neutral network based on the scenarios in the numerical experiments. Part 6 evaluates the results, and provides a conclusion as well as recommendations for future research. Part 7 contains a full list of references and part 8 contains the appendix, which provides proofs, a thorough derivation of the analytical results and some complimentary numerical results that were used in the analytical discussion in part 5.6.4.

1.1 Definitions and Conceptual Clarifications

The following section shows common expressions and their abbreviations that are frequently used in the net neutrality debate, together with an explanation. Where there is no abbreviation the full expression is used throughout the thesis. However, for most of the thesis the abbreviation will be used when possible.

Expression	Abbreviation	Explanation
Assured forwarding	AF	Assurance of delivery of packet as long as the traffic does not exceed some subscribed rate. [14]
Bandwidth		QoS parameter: Describes the capacity of a link or end-to-end path. Measured in bits per second. [14]
Best Effort	BE	Packet forwarding is performed with the best effort, but without guaranteeing bandwidth, delay bounds etc. [14]
Bits per second	bps	Bandwidth rate describing how much information can be transferred in the network per second. [14]
Content distribution networks	CDN	Networks paid by the big CPs to improve the quality of experience in a BE Internet. They achieve this by building additional infrastructure that bypasses congested routes on the public Internet and by caching frequency downloaded content closer to respective customer class networks. [13]
Content provider	CP	Actor or player in the NN ecosystem that provides content to the IUs through a platform provided by the ISPs. [13]
Delay		QoS parameter: Average delays that packets experience over a specific connection. [14]
Delay variation		QoS parameter: Difference between the bounds of minimum and maximum delay. [14]
Differentiated services	DiffServ/DS	Approach to provide QoS support in large scale networks while avoiding the scalability problems in the IntServ concept. Does this by reducing the flows to a small number of aggregated flows, and admission control and policing are no longer the responsibility of core routers. [14]
Elastic Applications		Application able to adapt to changing QoS parameters and does not fail in that case. Also called Best Effort Applications (e.g. e-mail). [14]
End-to-end	E2E	In the context of this thesis end-to-end refers to the

		principle of treating information from one end of the value chain to the other. Should not be confused with the end-to-end principle of networks.
Inelastic applications		Need strict QoS guarantees. Real-time applications by nature are mostly inelastic, but might have some abilities to adapt to certain QoS parameter changes (e.g. video conferring with changing bandwidth). [14]
Integrated Services	IntServ	Architecture designed to overcome the inability of the Internet to provide guaranteed end-to-end QoS. It is based on the reservation of network and system resources on application parameters. Five parameters (token rate, bucket size, peak rate and minimum and maximum packet sizes) make up an IntServ Traffic Specification. [14]
Interactive Applications		Include human interaction. Typically a human interacts remotely with another end system and expects quick reaction to the performed action (e.g. online gaming). [14]
Internet Protocol	IP	Principal communications protocol in the Internet Protocol suite for relaying datagrams across network boundaries. [15]
Internet Protocol packets	IP packets	Data information to be sent in the network. [15]
Internet Protocol services	IP services	Services provided by data sent through the network. [15]
Internet Service Provider	ISP	Providing access to the Internet for IUs (end-users), and may be viewed as a two-sided market platform under the NNN regime.
Internet user	IU	Internet users, or end-users, are connected to the network by their local access provider (ISP). [13]
Net neutrality	NN	Net neutrality means that broadband service providers charge consumers only once for internet access, do not favor one content provider over another, and do not charge content providers for sending information over broadband lines to end users. [13]
Network Application		Applications for use of network. [14]

Non-interactive Applications		Does not include human interaction (e.g. web browsing, file transfer, chat and multimedia streaming). [14]
Non-net neutrality	NNN	In a NNN regime ISPs understands themselves as two-sided market operators, connecting CPs with the IUs, and can use their last-mile monopoly to charge the CPs extra fees for using their network. [13]
Packet loss rate		QoS parameter: Indicates the number of packets that do not reach the destination in relation to all sent packets. [14]
Premium Service		User negotiates with his ISP a maximum bandwidth for sending packets through the ISP network. Allocates absolute bandwidth for aggregated flows. Packet flow described by the packets source and destination. [15]
Quality of Experience	QoE	The QoE of a service is influenced by three major dimensions: the CPs requirements with respect to QoS, the actual QoS that is delivered by the network and the IUs preferences and expectations about the service experience. [13]
Quality of Service	QoS	A measure of the ability of network and computing systems to provide different levels of services to selected applications and associated network flows. [13]
Queuing		IP Packets in queue in the case of an overload situation [15]
Service Level Agreement	SLA	Contract between an ISP and its customers.
Strict net neutrality		Net neutrality prohibits Internet service providers from speeding up, slowing down or blocking Internet traffic based on its source, ownership or destination. [13]
Termination fee		When CPs are charged extra, just to be able to transmit their data to the access ISPs customers, but without any additional benefits in return. [13]

Table 1: Definitions and conceptual clarifications

1.2 Introduction to the Net Neutrality Debate

The internet is rapidly developing into an essential platform to reach information, entertainment and communication, and the role of network infrastructure owners has shifted to an essential gatekeeper position in the information society. This has leaded both politicians and the public to be concerned about how ISPs are going to monetize access and usage of the networks in the future [13]. Much research has been done to analyze the consequences of different regimes the ISPs monetize on their position as gatekeepers [1] [2] [3] [4] [5] [6] [7] [8] [9] [10] [11] [12] [13]. These articles are part of what is referred to as the net neutrality debate. The debate is currently active in both the EU and the USA [16].

The phrase net neutrality was first coined by Columbia Law Professor Tim Wu in 2003 [7]. The term is used to signify the concept that the Internet is merely a carrier of online content that does not distinguish one website from another. The central idea inherent in this concept is that a «maximally useful public information network aspires to treat all content, sites and platforms equally» [7]. There are several definitions of net neutrality. Two definitions of net neutrality can be read below:

"Net neutrality usually means that broadband service providers charge consumers only once for Internet access, do not favor one content provider over another, and do not charge content providers for sending information over broadband lines to end users." [13]

"NN usually means that broad-band service providers charge consumers only once for Internet access, do not favor one content provider over another, and do not charge content providers for sending information over broadband lines to end users." [7]

There is also a strict definition of net neutrality:

"Net neutrality prohibits Internet service providers from speeding up, slowing down or blocking Internet traffic based on its source, ownership or destination" [13]

The first two definitions refer to that ISPs cannot differentiate among IUs or CPs, and that CPs should be free to send their content to IUs without being charged by the ISPs. The strict net neutrality has much of the same message. However, it also includes blocking of content, which is forced upon the ISPs in some foreign countries. Also the strict definition prohibits ISPs from charging access fees on CPs to reach their end-users.

The techniques necessary to differentiate and manage internet traffic are already implemented in the networks. However, what started the debate was that ISPs have implicitly and overtly signaled that they intend to use these techniques to generate extra revenues. The debate was particularly stimulated in 2005 when the CEO of ATT at the time, Ed Whitacre, stated: "Now what content providers would like to do is use my pipes free, but I ain't going to let them do that because we have spent this capital and we have to have a return on it". Similar statements have been released by major European network operators since then [13].

To understand the reasoning behind the ISPs statements, one has to look at the ISPs as platforms in a two-sided market, which they might be considered economically [18] [19] [20]. The ISPs provide access to the Internet for IUs, who through their connection with the ISP can interact with several CPs. We could say that there are two types of CPs, inelastic and elastic applications. Where elastic applications can adapt to changing QoS, e.g. e-mail, inelastic applications need strict QoS guarantees to deliver sufficient QoE [13] [14]. These inelastic applications usually consume a disproportionate amount of internet traffic. E.g. Netflix amount for nearly 30% of Internet traffic at peak hours [21] (new estimates provide an even higher number [17]). Netflix provides video on demand through online streaming. Their huge demand for traffic is of course not only generated by their services per se, but also by their vast amount of IUs. Thus, an ISP has to provide sufficient access to good enough QoS to the IUs in order for all their users to be satisfied with both the service of the ISP and the experience from using Netflix. If not, they might lose subscribers themselves, or at least many of their customers will not be satisfied. However, with the one-sided pricing most ISPs use today, they cannot charge Netflix explicitly to bear the investment costs that is required to provide a sufficient QoS so that the IUs can enjoy their services. This leads us to the three main arguments of the proponents of NNN:

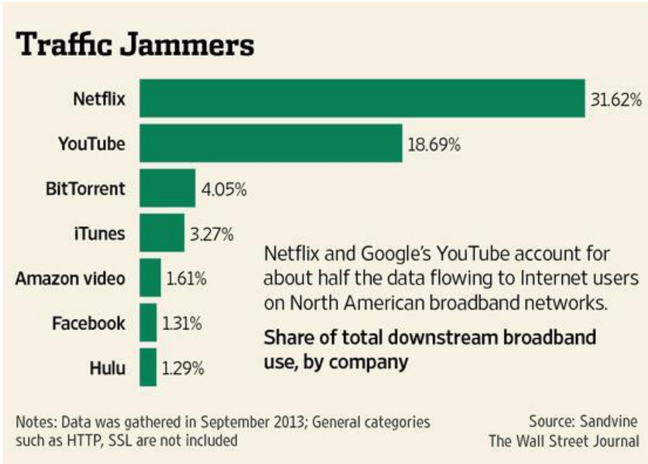

Figure 1: Graph by the Wall Street Journal illustrating the large share of data flow to internet users from Netflix and YouTube [17].

First, the prioritization of bandwidth stimulates innovation because the ISPs can use the money paid for preferential treatment of Internet traffic to pay for the building of network infrastructure that would increase broadband access to more consumers. [16]

Second, video sharing websites, such as YouTube (www.youtube.com), Vimeo (www.vimeo.com), and Vevo (www.vevo.com), take up a lot of bandwidth. According to Cisco, global Internet video traffic was 57 percent of all consumer traffic in 2012. The global Internet video traffic will be 69 percent of all consumer Internet traffic in 2017. This statistic does not include video exchanged through peer-to-peer (P2P) file sharing. Cisco states that the sum of all forms of video traffic, including P2P, will be in the range of 80 to 90 percent of global consumer traffic by 2017. In order to deal with the increased bandwidth requirements, ISPs will need to charge more high-bandwidth websites. [16]

Third, network neutrality decreases the revenues earned by the ISPs. The decreased revenues of the ISPs increase the level of the employment and decrease GDP. Moreover, the decreased revenues of ISPs prevent them from deploying and maintaining networks, and improving them over time. In order to recoup the decrease revenues, the ISPs may charge their customers increased fees. [16]

In short, the main arguments of the NNN proponents are that NNN will stimulate innovation by building our the network infrastructure, it is necessary to cope with the increase in internet traffic and that it secures necessary revenues to maintain and improving networks without charging extra fees on the end-users.

However, there are those who fears that the NNN regime would have negative effects. There are mainly six arguments used by the NN proponents:

First, network neutrality protects the right of freedom of speech. The reason is that network neutrality restricts ISPs from blocking or prioritizing content on the Internet. Countries that have not implemented the principle of network neutrality in their legislation often control or suppress the publishing or accessing of information on the Internet. For example, in China, the government uses a system that does not allow the residents of China to access certain online content. As a result, if an Internet user searches in Google or other search engines for the word "Tibetan independence," "democracy movements," or other blacklisted words, he or she will be redirected to a blank page stating "page cannot be displayed." [16]

Second, ISPs have no right to control data transmitted between end-users. According to this argument, the networks' only function is to move data, not choose which data to move. In this context, Vinton Cerf, a co-inventor of the internet protocol (IP), stated that "the Internet was designed with no gatekeepers over new content or services." [16]

Third, network neutrality does not allow ISPs to restrict content and/or services provided by their competitors. As known, restrictions of competition may lead to increased prices of services and/or goods. For example, in 2009, Deutsche Telekom announced plans to prohibit the use of Skype over iPhones. Such a prohibition will harm the interests of consumers who can otherwise save money on calls by using Skype. [16]

Fourth, the involvement of ISPs in determining what content or services reach consumers will stifle innovators. For instance, if Google can pay ISPs to deliver YouTube videos faster than other sources of Internet video, any startups offering better services than YouTube will have tremendous difficulties enter the online video market. [16]

Fifth, network neutrality preserves the existing Internet standards. The reason is that, at present, the Internet runs on technical standards created by variety of organizations, such as the internet engineering task force (IETF). By using the existing Internet standards, computers, services, and software created by different companies can be integrated together. Without network neutrality, the Internet will be regulated by ISPs under standards chosen by them. [16]

Sixth, network neutrality maintains the end-to-end principle. It "allows nodes of the network to send packets to all other nodes of the network, without requiring intermediate network elements to maintain status information about the transmission". The principle allows people using the Internet to innovate free of any central control. [16]

In short, the main arguments of the NN proponents are that it protects the freedom of speech, is in accordance with "no gatekeepers" principle, protects the interest of consumers with no blocking of content, it preserves the existing internet standards and it maintains the end-to-end principle so that people can use the Internet freely to innovate.

All three arguments posed by the NNN proponents can be tested quantitatively in a mathematical model. The third, fourth and fifth argument of the NN proponents can also be analyzed in a mathematical model. The other arguments of the NN proponents are either technical or principal and will therefore not be analyzed in this thesis.

Regulators have so far responded differently towards NN. In the USA, the Federal Communications Commission (FCC) has adopted a set of rules that should "preserve the Internet's openness and broadband providers' ability to manage and expand their networks". The four core principles are: transparency, no blocking of content, no unreasonable discrimination, and reasonable network management. In Europe, no such rules have been adopted so far. The European Commission is committed to "preserving the open and neutral character of the Internet", it believes that the existing rules on transparency, consumer switching and quality of service are sufficient to ensure competitive outcomes [22]. However, the EU has recently stated that the Internet could be spilt in two, internet access services (IAS) and specialized services (SS). Internet access services should be open without traffic management. On the other hand specialized services allows the operators to manage the networks how they want. The splitting point between these two networks has not yet been decided, and neither has the flexibility in managing the two networks been decided yet. This is to be decided by the European commission this year [23].

Figure 2: FCC (top) and the European (EU) Commission

1.3 Presentation of the Problem Statement

The regulators and policy makers have not yet set the final conditions for a potential NNN regime, where a specialized service lane can be offered to the content providers that are willing to pay for it. The other content providers are left in the internet access lane, which is similar to today's best effort. In order to increase the knowledge of the effects of adopting the NNN regime has on key ecosystem actors, and guide the policy makers in their work on possible regulations, the following problem statement was chosen:

"What effects does leaving the NN regime in favor of the NNN regime involve for key ecosystem actors in an E2E multi actor environment, which scenarios lead to a beneficial outcome for the key actors and overall social welfare, and how does regulating the QoS of the IAS lane in the NNN regime affect the key actors and social welfare?"

1.4 Justification of the Chosen Problem Statement

The problem description given to me by Telenor was the following: "Describe different quantitative methods for analyzing economics effects in E2E multi actor environment for a network operator (ISP), including two-sided pricing models. Furthermore design different scenarios in both network neutrality and non-neutrality settings. Finally analyze economic impact and recommend policy implications for key ecosystem actors using preferred methods." The problem statement of this thesis aims to provide an answer to this problem description, as well as give deeper insights on the effects of regulation.

The EU has largely agreed to allow a separation of the internet into specialized services, and internet access services. However, how this should be implemented, and how flexible it should be, is still to be considered. The problem statement is to provide better insights into the questions the EU commission, FCC and other policy makers have to evaluate before deciding on final guidelines for a possible non-neutral Internet. Also, the problem statement allows for a broad analysis of the implications of a non-neutral internet, which has not yet been fully analyzed in an end to end perspective, which is included in this thesis.

The problem statement could have included a broad analysis on regulation, and look deeper into how regulators could best regulate the network in order to best utilize the benefits and avoid the possible pitfalls of a non-neutral Internet. This is a topic that could also be interesting to investigate further. However, the regulators largest concern is how the QoS of IAS is affected by the non-neutral regime, and therefore this is also chosen as the problem statement in this thesis. A broad analysis of different regulation schemes would also complicate the mathematical model and solutions, which might reduce the quality of the general analysis.

By key actors we consider the main actors in the network ecosystem, the content providers, the backbone ISP, the access ISPs, the internet users and the regulators. The formulation could include a direct analysis of specific actors, like Telenor or Netflix. However, since the insights provided by this thesis aims to aid in a general assessment of adopting the NNN regime, no individual actors are considered. A business analysis that aims to help companies maneuver in the NNN regime to maximize their own profits would benefit from such a formulation and analysis. Single companies should not affect the regulators that have to evaluate the full picture, which is the focus of this thesis.

2 Survey of Previous Research

2.1 Literature on Net Neutrality

The following section contains a survey of previous research on the net neutrality debate. Two articles, [24] and [25], describe the framework of internet interconnections, and how regulators have to take the complexity of the Internet Interconnections into consideration when looking at different regulation policies. Twelve articles, [1] - [12], directly analyze the effects of regulation regimes for the NN debate, will be presented both in tabular form and in a short summary. Some of the findings from these articles will be used explicitly in own sections. As such, they will be presented thoroughly according to the context in which they are being used. An article providing empirical studies [22] is presented, and represents a different approach from the other research papers and the work in this thesis. Lastly, Krämer et al. [13] have written a progress report, and we summarize their most important findings.

2.1.1 Tabular Overview of Previous Research on the NN Debate

This section contains two tables; one that summarizes the framework in which the previous research has analyzed the NN debate, the other table shows the findings in these articles. Both tables were made in MS Excel using Pivot tables.

The columns of the first table include

- **Authors** states who wrote the article
- **Year** states when the article was published
- **Definition NN** is how net neutrality is defined in the article. Some articles consider managing access is defining NN, while other articles define NN as the ability not to prioritize content by offering differentiated classes.
- **Network regime** is how, in the NNN case, the ISP structures the network. It could be by offering tiering services, where there are lanes of different quality, or simply by allowing access to the higher quality lanes or even the network through a termination fee.
- **ISP Comp.** is whether the model captures competition among ISPs. This is important as previous research shows competition for end users are toughened in the NNN regime [2]
- **Invest ISPs** state whether the article consider investments in infrastructure and better quality by the ISPs or not. This is important to analyze long-term effects of regulation policies.
- **Invest CP** states whether the article considers investments or innovations done by CPs, and is again important to analyze long-term effects of regulation.

- **Multih. CPs** is whether end-users connect to several CPs simultaneously.
- **Het. CPs** is whether is whether the CPs are modelled as heterogeneous or not
- **Het. IUs** is whether is whether the IUs are modelled as heterogeneous or not
- **Explicit cong.** is whether congestion is modelled explicitly as a result of demand for packages and investment levels
- **E2E** is whether E2E effects are considered in the model or not

Authors	Year	Definition NN	Network regime	ISP Comp.	Invest ISPs	Invest CPs	Multih. CPs	Het. CPs	Het. IUs	Explicit cong.	E2E
Altman et al.	2011	Acc fee	Ter. fee	No	Yes	Yes	No	No	No	Yes	No
Bourreau et al.	2012	Priority	Tiering	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No
Cañon	2009	Acc fee	Ter. fee	No	Yes	No	No	Yes	Yes	No	No
Cheng et al.	2011	Priority	Tiering	No	Yes	No	No	No	Yes	Yes	No
Choi and Kim	2010	Priority	Tiering	No	Yes	No	No	No	Yes	Yes	Yes
Choi et al.	2011	Priority	Tiering	Yes	No	No	Yes	Yes	Yes	No	No
Economides et al.	2012	Priority	Tiering	No	Yes	Yes	No	Yes	No	Yes	No
Economides et al.	2012	Acc fee	Tiering	Yes	No	Yes	No	Yes	Yes	No	No
Hermalin et al.	2007	Priority	Tiering	No	No	No	No	Yes	Yes	No	No
Krämer et al.	2012	Priority	Tiering	No	Yes	Yes	Yes	Yes	Yes	Yes	No
Musacchio et al.	2009	Acc fee	Tiering	Yes	Yes	Yes	Yes	No	Yes	No	No
Njorge et al.	2012	Acc fee/Prio	Tiering	Yes	Yes	Yes	Yes	Yes	Yes	No	No
This thesis	2014	Priority	Tiering	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Table 2: Features of the mathematical models presented in the previous research, compared to the features of the model in this thesis. This thesis is the only paper with yes in all of the columns.

As can be seen from the overview, only two articles, with the exception of this thesis, use a relatively complete framework, namely Njorge et al. (2012) and Bourreau et al. (2012), which are also some of the newest articles. However, these articles do not consider the effects of an E2E network ecosystem. Njorge et al. does consider connectivity by a CDN, but that is from one of the same ISPs that provide connectivity for the end users, which is rarely the case in international E2E network configurations. This thesis presents the first research what takes this into consideration, in a complete framework. The second table shows a summary of the results found in the different papers. The table reflects a general interpretation of the articles results and argumentation, and does not necessarily represent any personal views the authors may have.

The columns of the second table include

- **NNN** states whether the article in general is positive towards NNN or not. This is usually tied to the overall social welfare. It is important to notice that whether the article is generally in favor of or against NNN is usually dependent on the values of the parameters, and that there usually are values that make either regime profitable.
- **SW** states whether social welfare benefits from NNN or not. Social welfare is in most cases measured as the profit of the ISPs, the CPs and the consumer surplus combined.
- **CP** states whether the CP is better off with NNN or not
- **Backbone ISP** states whether the backbone ISP is better off with NNN or not
- **Access ISP** states whether the access ISPs is better off with NNN or not
- **IU** stated whether the IUs (end-users) are better off with NNN or not
- Where depending is used, the result is dependent on parameter values, and no clear answer can be given based on the general argumentation in the article.
- N/A states that non conclusion could be made from the analysis in the article

Authors	Year	NNN	SW	CP	Backbone ISP	Access ISP	IU
Altman et al.	2011	Depending	N/A	Depending	N/A	Yes	N/A
Bourreau et al.	2012	Yes	Yes	Depending	N/A	Yes	Depending
Cañon	2009	No	Depending	Depending	N/A	Depending	Yes
Cheng et al.	2011	Yes	Yes	No	N/A	Yes	Yes
Choi and Kim	2010	Depending	Depending	Depending	N/A	Depending	Depending
Choi et al.	2011	N/A	Yes	N/A	N/A	Yes	Yes
Economides et al.	2012	No	Depending	No	N/A	Yes	Depending
Economides et al.	2012	Depending	Depending	Depending	N/A	Depending	Depending
Hermalin et al.	2007	Yes	Yes	N/A	N/A	Yes	Yes
Krämer et al.	2012	Depending	Depending	No	N/A	Yes	Yes
Musacchio et al.	2009	Depending	Depending	Depending	N/A	Depending	Depending
Njorge et al.	2012	Yes	Yes	Depending	N/A	Depending	Yes
This thesis	2014	Yes	Yes	Depending	Yes	Depending	Depending

Table 3: Summary of results on profits, consumer surplus and social welfare from the transition to a non-neutral from a neutral regime in the previous research, compared to the results provided by this thesis.

The table reveals a trend that most articles are actually positive towards NNN, especially later research with more comprehensive modelling. However, most articles also point out challenges like "tragedy of the commons" with the ISPs overcharging CPs when the maximize their own profits, ISPs sabotaging BE lanes, hard entry for new CPs or that small CPs might be pushed out of the market. Thus, for regulators, even though NNN should prove beneficial for the society, the articles reveal a need to regulate or monitor some of the business to protect fair competition.

2.1.2 Summaries on Articles Covering the NN Debate

The following section contains short summaries of the articles, their most important findings and suggestions for future research. The summaries are not meant to give a thorough understanding of the results, and any material used explicitly in this thesis will therefore be described independently of this section.

Complexity of Internet Interconnections: Technology, Incentives and Implications for Policy (Faratin, Clark, Gilmore, Bauer, Berger and Lehr) [24]

The article describes the structure and complexity of Internet interconnections. The article describes the different participants in a network system: Autonomous Systems, Prefix, Cone of Prefixes and Default Route. The article also defines tiers for the different ASes, depending on how they interact with each other. There are two ways to interact, transit (A is a customer of X) or peering (exchange of traffic). Peering is what enables ISPs to be in contact with the whole of internet without doing a transit with all networks. However, for peering to work, certain conditions must be met which is regulated by traffic volume, traffic ratio, consistent announcements, marketing considerations or other requirements. The article goes on to describe possible issues and solutions to conflicts, as well as different types of interconnection agreements. The article then describe "Settlement-Free Peering Bargaining", in which different network providers have different willingness to peer (WTP) based on their traffic structure. The article follows up with strategies on when not to peer, and how to create incentives to peer. In general you should avoid to indirectly or directly peering with current or potential customers, and create incentives by lowering performance of the networks that does not want to peer but indirectly are dependent of your network. The article also provides strategies to remain in a current peering policy. Lastly the article provides bilateral negotiations, market failure and entry of content delivery networks. The market failure refers to the fact that there are willingness to pay for better than best effort connectivity, but with no providers able to give such an offer. This enables entry incentives for third party content distribution overlay networks. The article concludes that because of the complexity of the Internet interconnection, regulations have to be very complex in order to reflect reality. The article suggests that regulators should rather focus on the sources of bargaining power and identify anti-competitive opportunism, rather than to impose ex-ante restrictions on the range of bilateral contacts. The article encourage both the industry and the academia to bring "best practice" or "common practice" in interconnection into the light, as this might help reduce bargaining costs, but in a more flexible way than might be achieved vie regulatory constraints. Lastly the article points out that today's "best effort" Internet probably have hindered the emergence of some applications because of lacking QoS, and that some sort of differentiated QoS will be necessary in the future to accommodate the requirements of different types of traffic.

The Growing Complexity of Internet Interconnection (Faratin, Clark, Bauer, Lehr, Gilmore and Berger, 2008) [25]

The article is an abbreviated version of [24]. However, some elaboration of different alternative interconnections is presented. The early internet grew on the assumption that the ISPs were homogeneous in size, and thus in costs. This makes it beneficial for the large Tier 1 ISPs to peer. However, following the recent development of the Internet with bandwidth heave content providers, the Autonomous Systems' (AS) are now more heterogeneous. They can be divided into "eyeball" heavy networks, which provide connectivity to users, and "content" heavy networks that provide connectivity to the content providers. The "eyeball" networks connects to many users that send relatively little traffic to the "content" networks, that on the other hand send loads of traffic back to the "eyeball" networks. This makes for unfair peering, as the "eyeball" networks have to carry much cost that they do not get compensated for directly. The "eyeball" networks thus sometimes refuse to peer with "content" network; this could be because of several reasons: Switching costs are now high for consumers, so the "eyeball" networks may perceive that they have increased bargaining power. The "eyeball" networks believe that the natural direction of value flow is towards them. Especially since the content providers are so dependent on advertising to users to generate revenues. The last-mile networks of the broadband "eyeball" networks are more capital intensive, which makes for a cost recovery challenge.

Some alternative peering and transit is therefore discussed, paid peering and partial transit. Paid peering is also called "settlement based peering", and is similar to "settlement free peering" except that the traffic is no longer exchanged without payment. This is the best solution for heterogeneous ASes that want to peer. Partial transit is where a provider sells access to and/or from a subset of the Internet prefixes to another network. E.g. a network that only buy the ability to send traffic but not receive can work as a content network but not like an eyeball network.

Network Neutrality and Congestion Sensitive Content Providers: Implication for Content Variety, Broadband Investment, and Regulation (Krämer and Wiewiorra, 2012) [1]

The article answers and analyses many of the arguments in the net neutrality debate, welfare enhancement, capacity expansion, BE unable to provide necessary QoS. The article finds results that do not agree with previous research on capacity expansion. The reason is that the article also considers the effect of new CP entrants, and competition between ISPs, that force them to invest in higher capacities. The analysis also considers the effect of bad QoS, and thus capture the disutility that might be the effect from congestion with traditional BE networks. The article also analyses different strategic options for policy makers, like minimum quality standards or strategic degradation. The first may be effective but inefficient concerning investments, while the latter may decrease the overall welfare as fewer CPs enter the network. All in all, except for the capacity expansion, the article agree with much earlier research in that in general the CPs are worse off while the overall welfare in general is higher. However for very congestion sensitive CPs QoS tiring might put them better off because QoS tiring can allocate congestion to congestion insensitive CPs. The article concludes that QoS tiering is the preferred regime for ISPs, and that prohibiting this can eventually be harmful to content variety, broadband investment, and welfare.

Net Neutrality with Competing Internet Platforms (Bourreau, Kourandi and Valleti, 2013) [2]

The article studies the effect of a net neutrality regulation on capacity investment in the market for Internet access, and on innovation in the market for content. The article finds that under discrimination investments in broadband capacity and content innovation are both higher than under net neutrality, and that the total welfare increases. Also, the end users benefit from less congestion. However, the discriminatory regime is not always beneficial to the ISPs as it can intensify competition for subscribers. Though welfare-enhancing, the discriminatory regime has some undesirable effects, as the discriminatory regime hurts the small CPs more than the large ones. The article also considers sabotage by the ISPs on the CPs, and shows that it can only emerge, with adverse effects, under discrimination. Under discrimination each ISP benefits from degrading the quality of the non-priority lane in order to extract higher profits from the priority lane. This effect should be considered by policy makers, and if possible be regulated while allowing the welfare-improving discriminatory regime. This could be done my monitoring the traffic quality.

Investment in two-sided markets and the net neutrality debate (Njorge, Ozdaglar, Stier-Moses and Weintraub, 2012) [3]

The article develops a game theoretic model, where the participants make decisions in a specified order, to investigate the net neutrality debate. The article summarizes the research done on the NN debate, and explicitly shows the limitations of the other articles that this article covers. The paper claims to be the first to analyze the NN debate in a model that simultaneously considers competition between ISPs, ISP investment decisions, CPs and consumers heterogeneity and CPs endogenous participation decisions as a proxy for content innovation. The model has a high quality platform, and a low quality platform that offer their services. The model is solved theoretically and exhaustively through numerical simulation. In general ISPs investments are larger in the non-neutral regime, because the ISPs can extract surplus from the CPs. The results in the article suggests that net neutrality regulation could possibly be an inapt policy to increase value in the Internet, because it could limit the investment incentives of network providers, which results in decreased CPs and consumer utility directly through low QoS and indirectly through bottleneck effects. The results are in general, and the article also provides tables to illustrate which actors prefer what under different circumstances. E.g. the low quality platform always prefers non-neutrality. However the high quality platform prefer (in the numerical solution) non-neutrality when the CPs are very heterogeneous, and neutrality when they are not (theoretically the high quality platform always prefer neutrality). This is because with heterogeneous CPs the platforms can use their monopoly power more effectively charging higher prices to CPs that enroll and appropriating the higher revenues that CPs earn in this regime. The results also suggest that a non-neutrality policy could potentially reduce the CPs participation and innovation. The article suggests that future research could implement transaction costs in the non-neutral regime, to let the CPs quality be determined endogenously and finally to model congestion more explicitly.

The economics of network neutrality (Economides and Hermalin, 2012) [4]

The article analyzes the private and social implications of two-sided pricing by ISPSs when the network is congested and more traffic implies greater delays. The article analyses under the assumption that the ISP can price discriminate only the CP side. The article argues that even though network neutrality might not generally be welfare maximizing, it might be welfare maximizing within second degree price discrimination, where the content providers are induced to play their part in the set. In particular, if the elasticity of content demand with respect to transmission time does not increase with households' time sensitivity for the content, then network neutrality is welfare maximizing within the set of feasible schema. The article features a complex and abstract mathematical model, which provides some direct insights. E.g. that it is impossible to increase total welfare by excluding content, and that it could be welfare maximizing to hold back inelastic content in favor of elastic content, because the IUs will adapt to the situation. Overall the article argues in favor on NN. If that elasticity is invariant with households' time sensitivity for the content, then network neutrality is welfare superior to all schema. (This is of course not realistic).

The mathematical model in the article has a number of simplifying assumptions. However, the simplifications are relaxed individually to prove that the major standings are reliable also under relaxation and thus a more realistic scenario. Other issues to be explored are an analysis of what happens when the ISP is engaging in price discrimination on both sides of the market.

Network Non-neutrality Debate: An Economic Analysis (Altman, Legout and Xu, 2004) [5]

The article considers an environment with one ISP and one CP and analyzes a two-sided pricing model with two separate analyses, where the first considers revenue generation of CPs through subscriptions, and the latter through advertising. The analysis has three important features, the relative price sensitivity, the CP's revenue model and the QoS provided by the ISP. The analysis shows that under certain conditions it is beneficiary for the ISP to charge a side-payment. With the subscription model, the relative price sensitivity determines whether the ISP should charge the side payment from the CP or not. With the advertisement model, the charge of the side payment depends on the ability of the CP's investment to attract demand. Under some circumstances it is also beneficial for both when the side payment is charged by the ISP, due to better QoS delivered to end users, which increase demand.

Regulation Effects on Investment Decisions in Two-Sided Market Industries: The Net Neutrality Debate (Carlos Cañon, 2009) [6]

The paper discusses the impact of regulation on a platform's pricing scheme, on investment decisions, on network users' decision to join the network and on welfare. The article studies three scenarios, a profit maximizing platform, a profit maximizing platform that cannot charge content providers and a profit maximizing platform that cannot charge end users. Concerning investments the article concludes that a profit maximizing platform with no regulations will invest less. However, if regulators care about the mass of network users that join in, the results are a bit more two-sided: The profit maximizing platform will exclude more content providers and end-users than the profit maximizing platform forced to either charge content providers or end users a zero access fee. On the other hand, if network effects are such that content providers trade surplus exceeds enough end-users trade surplus, the profit maximizing platform will exclude more content providers but less end users than when forced to charge either group zero for access. The welfare is maximized if the profit maximizing platform is forced to charge the content providers a zero access fee.

The Debate on Net Neutrality: A Policy Perspective (Cheng, Bandyopadhyay and Guo, 2011) [7]

The article discusses two major issues, who are the gainers and losers when abandoning NN, and if broad band providers will have a greater incentive to expand their capacity without NN. The model is based on a simplistic model with one ISP and two CPs, Y and G have inverse proportional market share, and a uniformed distributed demand of end users for the ISP. The Social welfare and consumer surplus is also considered when the results are analyzed. Concerning the first issue, the ISP is ambiguously better off without NN. Social welfare either increases or remains unchanged depending on the parameter values. Likewise, consumer surplus either increase or remain unchanged. The CPs are usually worse off under NNN (non-net neutrality), except when the content provider fee has the same surplus as under NN. Regarding expanding capacity, it is shown that, except under one condition, the optimal capacity is higher under NN than with NNN. The scenario when capacity expansion is higher for NNN is equivalent to when the revenue rates, r_G and r_Y, is such that r_G is a certain ratio hither than r_Y. G is the content provider that attracts the best customers for its advertisers, and therefore can collect higher advertising fees.

Net neutrality and investment incentives (Choi and Kim, 2010) [8]

The article analyzes the effects of net neutrality regulation on investment incentives for ISPs and CPs, and their implications for social welfare. The article use a simple model based on queuing theory to capture the congestion in the network. The article shows that the ISPs incentives to invest in a multitiered network vs. a nondiscriminatory network under net neutrality regulation depends on a potential tradeoff between the two sides of the market: the network access fee from end users and the revenue from content providers through the potential trade of the first priority in delivery. The article also finds that the relationship between the net neutrality regulation and investment incentives is subtle in the case of CPs incentives to invest and social welfare. For future research the article emphasize the simplifying assumptions of the model regarding pricing strategies of several players, and it should be further analyzed. Research on two-sided markets show what the equilibrium depends crucially on the pricing scheme, which makes this an important extension. Another important extension would be competition between the content providers. Content providers can perform different types of investments, e.g. firm-specific investments or investments that have spillover effects. This is also suggested as an extension for future research.

Net Neutrality and Internet Interconnection (Choi, Jeon and Kim, 2011) [9]

The article analyzes competition between interconnected networks when content is heterogeneous in terms of its sensitivity to deliver quality. The article considers, in a two-sided market, under a neutral and non-neutral regime how packet delivery can take place. In the neutral regime, all packets are delivered with the same quality, and under the non-neutrality regime ISPs are allowed to offer multiple lanes with different delivery quality levels. Cooperation is needed to assure quality in an interconnected network, and the article provides a framework of two-sided markets in which ISPs compete with each other to serve as platforms that connect CPs and end consumers.

The article shows that, under a non-neutral regime, ISPs agree on charges that enable them to behave as monopoly bottlenecks towards the CPs, and that the competition in the consumer side of the market is intensified. However, under a neutral regime, ISPs make losses from the CPs side of the market and the consumer plays the role as the competitive bottleneck. Also the competition to attract consumers is softened. The model in the article is static in nature, and there are no dynamic investment incentives facing ISPs and CPs. This could be included in future research.

Network neutrality on the Internet: A two-sided market analysis (Economides and Tåg, 2012) [10]

The article considers two sided pricing in a monopoly and duopoly scenario (ISPs), with several CPs and customers. The demand from CPs and users are both dependent on the expected demand of the other, because more users attract more CPs, and more CPs attracts more users. The model does not consider QoS, and discuss the net neutrality debate looking at prices and demand only. This is a major simplification. The article concludes that under different circumstances, the total surplus might be higher or lower, depending on parameter values, under non-net neutrality. The customers however are usually better off, together with the ISP. However the CPs will be worse off with a two sided pricing system. The article suggest further analysis on network neutrality implications on innovation, price discrimination and two part tariffs to consumers and content providers. The results in the article rely heavily on the fact that ISPs cannot extract the entire surplus from content providers and customers.

The Economics of Product-Line Restrictions with an Application to the Network Neutrality Debate (Hermalin and Katz, 2006) [11]

The article examines the effects of product-line restrictions, such as those called for by some proponents of net neutrality regulation. The regulation being analyzed in the article is towards product lines for the consumer, and two-sided pricing of the CPs and the end-users. The article finds that by restricting a monopoly supplier to a single product, the follow effects apply: consumers who would otherwise have consumed a low-quality variant are excluded from the market, consumers in the middle range of the market consume a higher and more efficient quality, and consumers at the top of the market consume a lower and less efficient quality. The total surplus might rise or fall. A duopoly model is also analyzed, and the article finds that a single-product restriction always reduces welfare. The welfare is reduced because of the loss of variety as a result from the regulation.

A Two-Sided Market Analysis of Provider Investment Incentives with an Application to the Net-Neutrality Issue (Musacchio, Schwartz and Walrand, 2009) [12]

The article address whether local ISPs should be allowed to charge content providers, who derive advertising revenue, for the right to access end-users. The article compares two-sided pricing with one-sided pricing. When the ratio between parameters characterizing advertising rate and end-user price sensitivity is either high or low, two-sided pricing is favorable. When the ratio is high, such that the content providers revenue from advertising is large and ISPs revenue from end users low, the ISPs incentives to invest are suboptimal unless they can extract some extra revenues from the content providers through two-sided pricing. However, when the ratio is very small the ISPs have to pay the content providers in order to get the CPs to invest adequately. This whether ISPs should charge CPs through two-sided pricing is a question on how the revenue is the value chain is divided between CPs and ISPs. When the ratio is in the intermediate range, both ISPs and CPs have adequate incentives to invest. An interesting aspect is the effect of when the number of ISPs is large. Each ISP only sees the benefit of increasing its own prices, but not the externalities the community suffers from the raised prices. Thus all ISPs will overprice the CPs, which is analogous to the tragedy of the commons. The model in the article is limited due to a fixed amount of network providers, no heterogeneity among the providers or net users, full commitment to the declared prices and the content provider's price on advertisers is not a variable. Future studies could take these limitations into account, in order to capture a more realistic image of the industry structure.

Net Neutrality, Foreclosure and the Fast Lane An empirical study of the UK (Laura Nurski, 2012) [22]

The article analyzes the NN debate using empirical data, as the first and, to our knowledge, the only article to do so. The empirical analysis is based on data from the UK. The paper investigates the inceptives for an ISP to break net neutrality empirically. The empirical data is used in a two-stage consumer demand model of differentiated online content providers and downstream ISPs. The demand estimates are then used to explore two counterfactual simulations of breaking net neutrality, with focus on YouTube in particular. There is one analysis where a fast lane is offered and another where the ISP can foreclosure through quality degradation. The results in the article are preliminary. They reveal that a fast lane increases consumers' surplus, industry revenues and advertising revenues. However, foreclosure reduce the foreclosing ISP's revenues from selling broadband more than it can recuperate through advertising on online content, and is therefore an unlikely scenario. The article also shows a summary empirical data: market share among the players, competition in the local exchanges, statistics on households, share of households that consume different type of only content by speed, and product characteristics. The author is going to expand the preliminary model to allow for more realistic substitution patterns as well as estimate complementary or substitutability between content providers. Future empirical research should look at the long run effects of net neutrality regulation on investment in the broadband network as well as on innovation among content providers.

Net neutrality: A progress report (Krämer, Wiewiorra and Weinhardt, 2012) [13]

The article is a summary of the work done on net neutrality. The article provides a definition of net neutrality, and a non-net neutrality framework to categorize the different NNN scenarios along two dimensions, pricing regime and network regime. The pricing regime refers to whether the pricing is one-sided or two-sided. The network regime refers to the way the network is managed, either through capacity only, managed network or quality of service.

		Pricing regime	
		One-sided	Two-sided
Network regime	Quality of Service	**User tiering** (IUs choose priority class)	**Content and service provider tiering** (CPs and/or IUs choose priority class)
	Managed network	**Status quo** (Best effort network with traffic engineering and/or managed services)	**Termination fee** (Additional fee for CPs to terminate traffic at access ISP)
	Capacity only	**Strict net neutrality** (No discrimination based on source, destination or content)	N/A

Table 4: Setup of network and pricing regimes in one- and two-sided markets in the NN debate

The conclusion is that today's network is still not NN per definition. The article move on to discuss a strict NN model, the Termination fee model, the CP tiering model and the User tiering model by summing up the most important findings from the research done on the subject. The strict NN model could lead to congestion at peak hours, as ISPs would not be allowed to manage congestion. The counter play would be overprovisioning of network capacity, and overall the ISPs revenue would be reduced because business models that rely on managed services could not be provided anymore. Either the consumer price would increase, or the rate of investments in network infrastructure be reduced. In the Termination fee model, ISPs use the last mile monopoly to charge the CPs additional fees for terminating their traffic to the installed customer base. This would only be a financial burden for the CPs, without any immediate reward. The main concern of NN proponents concerning this model is that the additional termination fee causes CPs to cease or to be discouraged from ever offering their services. It is therefore argued that two-sided pricing reduces innovation of the internet. The research done on the area is divided on whether this would benefit society or not. The CP tiering model is a response to the fact that many services would be much better off if they were allowed to pay for priority, e.g. Skype. The most relevant scenario in this model is that only the CP that opts for better quality pays and the other CPs receive best effort. The general tone of CP tiering is that this practice should be welfare enhancing, and especially if there exists competition between access ISPs. UI tiering is a one sided model where users can pay for better quality. Many of the arguments opted by the NN proponents do not hold with this model, because the CPs are not charged with any fees. However, even though some ISPs already do some IU tiering, they hesitate to do it on a big scale. This is probably due to a different psychological perspective on fairness from IUs vs. CPs, where IUs might negative get negative emotions to a proposed tiering pricing scheme.

The article continues by drawing policy conclusion. The debate on net neutrality rests on two fundamental assumptions, the internet traffic will increase to such a high level that we get a severe congestion problem, and the ISPs claim that they cannot bear the costs for the necessary network infrastructure investments without tapping additional revenue streams. The survey in the article shows that deviations from the NN are generally welfare enhancing if the appropriate remedies are applied. The article also suggests a flow chart that could guide policy makers through the potential threats that are associated with the different NNN scenarios. For each threat, a remedy is suggested that can potentially deal with it. The chart is simplified to a binary decision process, and in reality the conclusion might be finer grained. The article further discusses different regulation policies. Lastly the article consider neutrality in the internet ecosystem, and argue the debate may only be onset of a larger debate on neutrality in the Internet ecosystem, due to the fact that the Internet ecosystem is affected by competition up and down the internet value chain. There are also other participants that should be included in the debate, namely content and services, the devices that connect users to the network and the content delivery networks.

As a conclusion the paper state that while there are some consensus on the NN debate (e.g. allowing reasonable network management), there is still considerable disagreement and topics that should be researched further: The effect of competition between ISPs on IUs, empirical papers to reject some of the assumptions driving the analytical models, and finally to extend the debate to other participants on the Internet.

2.2 Literature on Two-sided Markets

The net neutrality debate in many ways centers around the understanding of two-sided markets, because debate is about whether an ISP should operate as a platform in a two-sided market or not. Because of network externalities and a complicated ecosystem it is not straight forward to deduce the effects of a transition based on general research on two-sided markets. Still, the previous research provides important insights on modelling two-sided markets, strategies and general assumptions. The following section contains short summaries of relevant literature on two-sided markets. Three of the articles contain mathematical models, [18] - [20], two are strategic, [26] and [27], and the last descriptive, [28].

2.2.1 Summaries on Articles Covering Two-sided Markets

Competition in two-sided markets (Mark Armstrong, 2006) [18]

The article discus two-sided pricing with three different models: a monopoly platform, a model of competing platforms where agents join a single platform and a model of competitive bottlenecks where one group joins all platforms. The article provides necessary and sufficient condition for a market sharing equilibrium to exist, and postulate formulas to calculate welfare maximizing and profit maximizing formulas. The article also discusses different pricing schemes, e.g. two-part tariffs or uniform prices. The models are put up with examples from real life, e.g. Supermarkets where the customers enter for free and the shops pay a sum to the supermarket (this is a competitive bottleneck). The models are analyzed and the effects of two sided pricing are discussed for the different scenarios.

Two-Sided Markets: An Overview (Rochet and Tirole, 2004) [19]

The article is very similar to the progress report by the same authors, [20], which was published two years later. Much of the results in that article is based on the research in this article, and the conclusions the same. However, this article is a bit more comprehensive in describing the different types of two sided markets, that can be interaction between two end users, but end-users can also connect to the platform through intermediaries or "service providers". There can also be "on us" or "on net" interactions. If the buyers and sellers interact through the same platform, it is said to be "on us". An example could be when the user and retail shop have a transaction through a credit card using the same bank. If for example a telephone operator may serve both the caller and the callee, and the backbone serve the website and the web user; the traffic is said to be "on net". There may also be multiple non-interconnected platforms, for example when a seller wants to interact with as many buyers as possible through different platforms, or video game developers that create games for multiple platforms. They are then said to be "multi-homing". The article also provides a definition of two-sided markets, and a list of a platforms motivation to charging membership fees, in contrast to usage based fees only. The article provides the same mathematical model as in [20], and list several weaknesses to the model; that side i only cares about the number of users on the other side and no other factors such as quality, that the model excludes same side externalities, that the model does not consider users who might have ex ante private information about their future per-transaction benefit and that the model involves simultaneous courting of buyers and sellers. The article lastly discusses how platforms can regulate the interactions between the users, to encourage positive externalities and discourage negative ones.

Two-sided markets: a progress report (Rochet and Tirole, 2006) [20]

The article is a comprehensive analysis of two sided markets, defining two side markets and necessary and sufficient conditions for a market to be two sided. The article also provides a general mathematical model to analyze two-sided markets. Factors that make a market two-sided include transaction costs among end-users, platform-imposed constraints on pricing between end-users and membership fixed costs or fees. A market is defined as two-sided if the volume is dependent of the price structure (that is if the total price of the two agents are constant) the market is said to be two sided. A necessary condition for a market to be two-sided is the break-down of the Coase theorem; if the outcome of negotiations between two informed agents with established tradable goods and no transaction costs are not Pareto efficient. In other words the optimal allocation of resources cannot be negotiated between the two agents when they maximize their own surplus, and the platform has to set prices in order to maximize the surplus as both sides benefit from externalities by the other side. The article also discusses an extended model that allows for payments between end users with both symmetric and asymmetric information. With symmetric information Coasian bargaining between end-users calls for a pass-through of variable costs by the platform to the end users. However, with asymmetric information the platform should subsidize the transactions between end users. Concerning pricing the article concludes that it is tied to the Lerner formula (price elasticity), however because of the two-sidedness the cost must be replaced with opportunity costs.

Strategies for Two-Sided Markets (Eisenmann, Parker and Alstyne, 2006) [26]

The article discusses different strategies to win the battle in a two-sided market. The article considers different scenarios and examples from real life business. The article identifies three challenges in two sided markets: Pricing the platform, winner-take-all dynamics and the threat of envelopment. Concerning pricing it is important to price and substitute the right side, dependent of their characteristics and costs of serving them and externality effects. The winner-take-all dynamics refer to situations when multi-homing is expensive for at least one side. For example DVD, which took the whole market as it is costly for both producers and consumers to support more than one platform. The threat of envelopment refers to the situation when one platform develops so that it also supports another platform, and can use bundle synergies to outcompete the smaller platform. The article provides different strategies companies can apply depending on which situation they are in.

The Economics of Two-Sided Markets (Mark Ryasman, 2009) [27]

The article describes what a two-sided market is, and discuss the two-sided market through the newspaper and media industry, payment card industry and operating systems industry. The article discusses strategies in two sided markets, and divides the strategies into pricing and openness. Where pricing focus on finding the right price for the two agents interacting through the platform. Openness refers to the choice of being one- or multi-sided, or whether to be compatible or incompatible with competitors. Sometimes it can be advantageous to be one-sided to begin with, in order to attract the valuable side before charging the other side. Openness can also be exemplified with Apple that produces both its own hardware and software with no third party producers. Microsoft on the other hand allows anybody to manufacture computers for their operating system, lowering the bar to use their product. Based on the openness discussion the article concludes that it is better to discuss two sided "strategies" rather than two-sided "markets", as whether the market is one-, two or multi-sided depends on the strategy of the companies operating in the environment. The article also discuss other strategies, such as innovation, and public policies such as antitrust and regulations. The article asks for more work to be done on how open one should be, and how many sides of the market to allow.

Everything you wanted to know about Two-Sided Markets (Evans and Passell, 2003) [28]

The article describes what two-sided markets is, and provide some examples of two-sided markets, e.g. dating services and credit cards. The article further describes the economics of two-sided markets, and how the number and quality of one side attracts the other, which the platform can reap high revenues from. The article further describes profit maximization, entry costs and balancing demand in two-sided markets. Lastly the article discusses regulation and how two-sided markets often attract competition authorities, and the challenges of establishing a working two-sided pricing model. Usually the companies have to invest a few years before a profit can be reaped. When the model works, one side of the market usually bears most of the costs. This creates a challenge for the authorities as they have to distinguish between actions that would undermine competition in one-sided markets, yet serve long-term interests of consumers in two-sided markets. Two-sided markets also generally encourage novel partnerships arrangements among competitors, to increase competition on one side of the market, and reduce the competition on the other. An example are banks collaborating with VISA and MasterCard, reducing competition on the retail side by collaborating with a mutual platform, however increasing the competition for customers as there are more banks able to provide an offer. The article suggests that regulators have to question whether the restraint of competition is justified by efficiency, and if regulation in the context of interest group pressure work better than without regulation.

Examples of two-sided markets:

Figure 3: The operating system Microsoft Windows charges consumers and subsidizes developers with free developer tools [26]

Figure 4: Credit cards are usually free to use for the customers, but the shops pay for the credit card's services [27]

Figure 5: Shopping malls usually allow customers to enter for free, and the retailers pay a sum to the shopping mall [26]

Figure 6: Video game consoles subsidizes consumers and charges the developers [26]

3 Relevant Theory

3.1 Economics and Game Theory

3.1.1 Microeconomics

The demand curve shows how much of a good consumers are willing to buy as the price per unit changes. The supply curve shows the quantity of a goods that producers are willing to sell at a given price. The supply curve and demand curve can be put together. Where the two curves intersect there is an equilibrium, which also is called market clearing price and quantity. The market mechanism is the tendency in a free market for the price to change until the market clears [29].

The supply function is dependent on the market situation. In an oligopoly with several suppliers, it might be stepwise, reaching a certain threshold enables more supplies to enter the market. Under perfect competition, where no participants are large enough to have market power to set the price, the supply is independent of price, and the price is equal the variable costs of the suppliers. In general, every participant in the market are "price takers". The opposite is true with a monopoly, where there is only one supplier that has all the market power. The market power is dependent on the price elasticity of demand. The higher the price elasticity, the less is the market power. In the case of a monopoly, the supply is based on the variable costs of the single supplier that will offer the supply which maximizes the profit under the current demand [29].

The demand function can be estimated empirically [29] or analytically using utility functions. The consumer net surplus can be estimated as the expected utility of the purchase less the expected price to obtain that utility. As long as the net utility is ≥ 0, the customer will be interested in the purchase [30]. The customer's expected net surplus can be expressed as follows:

$$ES_i(x,p) = EU_i(x) - E[p] \qquad (1)$$

Where ES_i is the expected net surplus for customer type i, EU_i is the expected utility for customer type i as a function of x, where x is the evaluation of the offer by the customer. E[p] is the price the customer expects to pay for the service. Setting $ES_i = 0$ and rearranging for x as a function of P yields a function of the fraction of the total market of customer type i that is indifferent to the offer. This fraction can be used to calculate the amount of total demand described by customer type i [30]. This way a market is divided into several parts where the users are considered to be homogeneous, which of course is a simplification of reality, however the resolution can be set dependent of the need of the analysis, and thus might work quite well for analytical purposes.

To find the optimal price for a supplier one has to know the demand and cost function of the supplier. Once that is known, calculus can be used to find the optimal levels. One way of doing this is by expressing both revenue and cost as a function of price [29]:

$$P(p) = p * D(p) - C * D(p) \tag{2}$$

Here p is the price, D(p) is the demand as a function of price, and C is the marginal cost of production. C does not have to be constant, and thus the variable cost function does not have to be linear. Regardless, the process of finding the optimal price and quantity are the same. The next step is to take the derivative of P with respect to p:

$$\frac{dP(p)}{dp} = D(p) + p * \frac{dD(p)}{dp} - C * \frac{dD(p)}{dp} \tag{3}$$

By setting the derivative equal to zero and rearranging for p, one can find an expression for the optimal price to maximize profits. Another way of arranging the equation after setting the derivative equal to zero is:

$$D(p) + p * \frac{dD(p)}{dp} = C * \frac{dD(p)}{dp} \tag{4}$$

This equation says that profits are maximized when the marginal revenue (MR) on the left side equals the marginal cost (MC) on the right side [29].

3.1.2 Nash Equilibrium, and Subgame Perfect Nash Equilibriums and Backward Induction

A Nash equilibrium is a solution concept in a game theoretic analysis of a non-cooperative game involving two or more players. Each player is assumed to know the equilibrium strategies of the other players. Formally, a Nash equilibrium can be stated as:

A Nash equilibrium is an action profile a* with the property that no player i can do better by choosing an action different from a^*_i, given that every other player j adheres to a^*_j." [31] [32]

A subgame is a subset of any game that includes an initial node, which has to be independent from any information set, and all its successor nodes [33]. The figure below shows how a subgame works in extensive form.

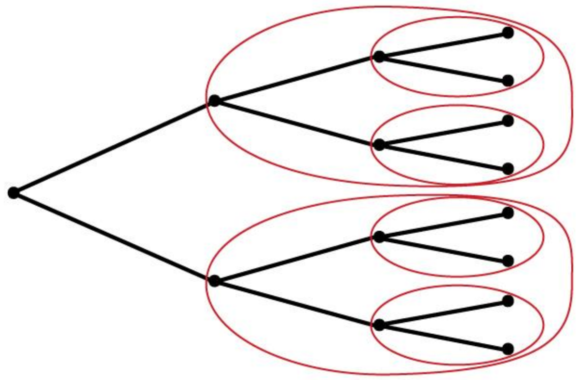

Figure 7: Illustration of a game and its subgames, which are marked by red circles. As seen by the figure, a subgame can contain several subgames of its own [33].

To determine a subgame perfect equilibrium one can apply the method of backward induction. In backward induction one first considers the last actions of the game and determines which actions the final player should take in each possible circumstance to maximize his/hers own utility. One assumes that all players make the choice that maximizes their own utility. After the last move has been determined, the second move is determined, and so on. The process continues until the first mover is reached. When the optimum decision of the first mover is decided based on the optimal strategies of the later stages, the optimal strategies of the later stages automatically falls into place. The strategies that remain are the set of all subgame perfect equilibria for finite-horizon extensive games of perfect information. [31]

3.1.3 Cournot and Bertrand Conditions of Competition

With a demand function and knowledge of the variable costs from the provider, it is possible to calculate the optimal price to maximize profits. If there is only one provider, this will be the market clearing price. However, with several providers a game theoretic analysis based on the results of maximizing profits of the individual supplier is needed to find the market clearing price and corresponding Nash equilibrium. This can be done under a Cournot Model where the suppliers compete on quantity, or a Bertrand Model where the suppliers compete on price. Both models analyze the competition to find Nash equilibrium [29].

3.1.4 Tragedy of the Commons

When a resource is freely accessible, the users of the resource become locked in to a pattern of behavior where all seek to take more of it [34]. That the resources are freely accessible is important, as the agents do not feel the consequences they inflict on the whole population by their own actions. Thus the resource is used inefficiently. This is a common problem for network services under flat-rate pricing, as the customers are not forced to compensate for excess use of the bandwidth available to all users.

3.2 Optimization

3.2.1 Non-linear Optimization and Convex Analysis

This theory is applied in section 0 to find the optimal solutions to the simplified mathematical model developed in this thesis, which are presented in section 5.4.

Definition 9.5: A mathematical program is a convex problem if the objective function is a concave in a maximization program, or convex in a minimization program, and the feasible region defined by the constraints is a convex set [35].

Definition 9.2:

The Hessian **H** is a matrix consisting of all partial second order derivatives to the function f(**x**) and can be expressed as [35]:

$$\boldsymbol{H}(x_1, \dots, x_n) = \begin{bmatrix} \frac{\partial f}{\partial x_1^2} & \frac{\partial f}{\partial x_1 \partial x_2} & \cdots & \frac{\partial f}{\partial x_1 \partial x_n} \\ \frac{\partial f}{\partial x_2 \partial x_1} & \frac{\partial f}{\partial x_2^2} & \cdots & \frac{\partial f}{\partial x_2 \partial x_n} \\ \vdots & \vdots & & \vdots \\ \frac{\partial f}{\partial x_n \partial x_1} & \frac{\partial f}{\partial x_n \partial x_2} & \cdots & \frac{\partial f}{\partial x_n^2} \end{bmatrix} \quad (5)$$

Definition 9.6

The quadratic matrix **H** is positive definite (positive semi-definite) if $\mathbf{d}^T\mathbf{H}\mathbf{d} > 0$ for all vectors $\mathbf{d} \neq \mathbf{0}$ ($\mathbf{d}^T\mathbf{H}\mathbf{d} \geq 0$, for all d) [35].

This definition makes us able to formulate the following theorem:

Theorem 9.5:

Suppose the function f(**x**) is twice differentiable defined on a convex set X. Then we have [35]:

- f(**x**) is convex a function on X if the Hessian matrix **H** is positive semi-definite for all **x** ϵ X.

- f(**x**) is a strict convex function on X if the Hessian matrix **H** is positive definite for all **x** ϵ X.
- f(**x**) is a concave function on X if the Hessian matrix **H** is negative semi-definite for all **x** ϵ X.
- f(**x**) is a strict concave function on X if the Hessian matrix **H** is negative definite for all **x** ϵ X.

An alternative approach to check if the matrix is positive or negative definite is to examine all minor determinants to **H.** If h_1 is the first minor determinant, h_2 the second minor determinant and $h_n = \mathbf{H}$ is the last minor determinant, we can state the following theorem

Theorem 9.7

H is positive definite if and only if

$$\det h_1 > 0, \det h_2 > 0, \det h_3 > 0, \dots, \det \boldsymbol{H} > 0 \tag{6}$$

H is negative definite if and only if

$$\det h_1 < 0, \det h_2 > 0, \det h_3 < 0, \dots \tag{7}$$

3.2.2 M/M/1 Queuing Theory

This theory is applied in section 5.2 and 5.3 to define the mathematical models, and in section 0 to solve the simplified mathematical model.

The M/M/1 queuing system has been widely used by scholars in operations research to study congestion problems and priority pricing. The setup is well known to be a very good approximation for the arrival process in real systems, in which the number of customers is sufficiently large so that the impact of single customers on the performance of the system is very small, and all customers' decisions to use the system are independent of other users'. [8]

The first and second M in the M/M/1 setup means that the distribution of arrivals and the distribution if service are exponentially distributed, respectively. The "1" means that there is one service station, which represents the single network in our setup. If λ is the average arrivals per time unit, and μ is the average service per time unit, the following representation of an M/M/1 queue can be made [36]

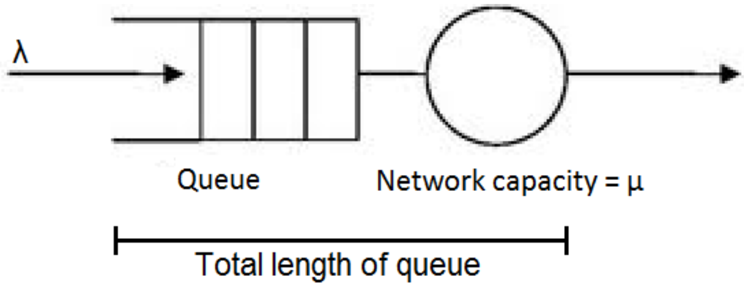

Figure 8: schematic representation of the M/M/1 queue with capacity μ and average arrivals per time unit μ

The state transition diagram shows the relation between each stage of the queue, where there is a probability of reaching the next stage of the queue depending on the average arrival rate and capacity or service time.

Figure 9: Representation of the state transition diagram of an M/M/1 queue

The system represents a birth-and-death process, which can be formulated as a Markowitz chain. By solving the system of equations, we can obtain the following solutions

In a neutral network, where all packets are treated equally without and priority classes, each customer has the expected waiting time, w, of [8]

$$w = \frac{1}{\mu - \lambda} \quad (8)$$

If there is a differentiated network with two priority classes, where λ_1 is the total amount of traffic sent with the first priority, the expected waiting time for consumers waiting for packets in the priority lane is [8]

$$w_1 = \frac{1}{\mu - \lambda_1} \quad (9)$$

The customers who requests content without the first priority faces the expected waiting time, w_2, of

$$w_2 = \frac{1}{\mu - \lambda} w_1 = \frac{1}{\mu - \lambda} \frac{1}{\mu - \lambda_1} \tag{10}$$

Based on standard results of queuing theory, we can infer that the customer will face a higher waiting cost by requesting the non-prioritized content instead of the prioritized one, that is [8]

$$Property\ 1: w_2 > w > w_1 \tag{11}$$

We define β as the share of the total traffic on the priority. Based on standard results of queuing theory we can obtain the following relation between the average congestion level, w, and the priority and non-priority lane w_1 and w_2, respectively [2]

$$Property\ 2: w = \beta w_2 + (1 - \beta) w_2 \tag{12}$$

3.3 Two-sided Markets

In a two-sided market there are two groups of agents that interact through a platform. The agents could either single-home or multi-home. Were single-home means that the agent only interact with one platform, and multi-home means that the agents interacts with multiple platforms. This provides for three different setups, both sides single- or multi-home, or one side single-homes and the other is multi-homing. [18] More examples of different two-sided markets can be seen in the article by Rochet and Tirole (2004, 2006) [19] [20].

Examples of such markets, as described above, are the credit cards where the credit card companies connect shops with customers by enabling the customers to pay for goods in the shop, dating industry that connects men and women through a dating platform, gaming consoles that connects video game developers with players, operating systems connecting developers with end users, media industry that connects advertisers with viewers/readers and shopping malls that attracts customers for the shops inside the shopping mall [18] [19] [20] [26].

Two-sided markets are especially interesting when cross-group externalities are present, so the benefit enjoyed by a member of one group depends on how well the platform does in attracting custom from the other group [18]. The value of the platform to any given user largely depends on the number of users on the platform's other side. Value grows as the platform matches demand from both sides. Because of these effects, successful platforms enjoy increasing returns to scale, and better margins. As a result, mature two-sided industries are usually dominated by a handful of large platforms [26].

In general, there is a difference between the valuations of the two groups, where one of the groups is considered to be more valuable than the other. The value side, also called the “subsidy side” are users who, when attracted in volume, are highly valuated by the “money side” [26]. This means that the money-side is willing to pay to reach the value side. Thus the platform usually subsidies the value side in order to attract enough users so that the money side will pay handsomely to reach them [18] [26]. Examples are credit cards that are practically free to use for the customers, while the shops pay the credit card companies to use their services. The customers are the value side, and it is important for them to be able to use a credit card in order to make a transaction with the shops. Thus, the shops are willing to pay the credit card companies in order to use their services to be able to make transactions with customers. Sometimes, the value of the subsidy side might be so high that platforms are willing to pay to make the users interact with their platforms. E.g. ladies night at night-clubs, where girls get free drinks as they are considered more valuable than the men, whom ultimately have to pay for the fun.

Mark Armstrong (2006) presents in an analysis on competition of two-sided markets a model of competitive bottlenecks, where one group single-home, and the other interacts with all platforms, that is multi-homes. This model is what closest represents the market in which the NN debate takes place under the NNN regime. CPs want to reach all the customers, who usually interact with a single ISP. Currently there is little differentiation of CPs through the ISPs, except for a few examples [16] [21]. Thus, since the CPs connect with all the ISPs with similar quality, there is little reason for the IUs to multi-home [18]. This assumption is important as it simplifies the solution process. In a world where niche ISPs provide superior quality for a certain type of CPs, and the CPs only connect through the niche ISPs, it is likely that some ISPs would multi-home if the services of those CPs are deemed important.

3.3.1 Mathematical Model of Competitive Bottlenecks

Mark Armstrong (2006) [18] has developed a mathematical model of competitive bottlenecks. The model is used a foundation I several research papers on the net neutrality debate, and is also the basis for the model provided in this thesis. In the model, group 1 is single-homing, while group 2 wishes to deal with each platform, which is a form of multi-homing. Implicit in the model is the idea that group 2 puts more weight on the network benefit of being in contact with the widest population of group-1 consumers that it does on the costs of dealing with more than one platform. Since group wishes to deal with all the platforms, there is no competition between platforms to attract group-2 customers. [18] This setting is similar to the setting in the net neutrality debate, where the IUs are single-homing and the CPs are multi-homing with respect to the access ISPs.

i	Platform i
j	Platform j
1	Group-1 single-homes
2	Group 2 multi-homes (heterogeneous)
n_1^i	Number of group-1 agents on platform i
n_2^i	Number of group-2 agents on platform i
u_1^i	Utility of platform i's group-1 users
$\hat{u}_1^i$	Utility offered to group-1 agents by platform i
$\hat{n}_1^i$	Group-1 agents attracted by utility $\hat{u}_1^i$
$\hat{n}_2^i$	Optimal number of group-2 agents
p_1^i	What platform i charges group-1 to join the platform
p_2^i	Fixed fee for group 2 to join platform i

Table 5: Indexes, parameters and variables of the mathematical model of competitive bottlenecks [18]

The number of group-2 agents prepared to pay a fixed fee to join platform i is denoted

$$n_2^i = ø^i(n_1^i, p_2^i) \qquad (13)$$

The function $ø^i$ is decreasing in p_2^i and increasing in n_1^i. A group-2 agent's decision to join one platform does not depend on whether she chooses to join the rival platform. The revenue of platform i from group-2 when it has n_1^i group-1 agents, sets group-2 price is defined as

$$R^i(n_1^i, ø^i(n_1^i, p_2^i)) \equiv p_2^i ø^i(n_1^i, p_2^i) \qquad (14)$$

A platform i's group-1 utility u_1^i is given by

$$u_1^i = U^i\left(n_2^i\right) - p_1^i \qquad (15)$$

The function U^i might be decreasing, for instance when newspaper readers find advertisements to be a nuisance. If a group-1 agent's utility is u_1^i with platform i, suppose the platform will attract

$$n_1^i = \Phi^i(u_1^i, u_1^j) \qquad (16)$$

Where Φ^i is increasing in the first argument and decreasing in the second. If platform i's total cost of serving the two sides is denoted $C^i(n_1^i,n_2^i)$, its profit is

$$\pi^i = p_1^i n_1^i + R^i(n_1^i, n_2^i) - C^i(n_1^i, n_2^i) \tag{17}$$

The number of group-2 agents on each platform in equilibrium is derived as a function of the equilibrium market shares for group-1. Platform i offers utility $\hat{u}_1^i$ to its group-1 agents, and attracts a number $\hat{n}_1^i$ of such agents (given the function Φ^i). Then the platform must be maximizing its profits given this group-1 utility $\hat{u}_1^i$. Consider varying p_1^i and n_2^i so that utility $\hat{u}_1^i = U^i(n_2^i) - p_1^i$ is constant. Writing $p_1^i = U^i(n_2^i) - \hat{u}_1^i$ in (17) means that profit is

$$\pi^i = \hat{n}_1^i\left[U^i(n_2^i) - \hat{u}_1^i\right] + R^i(\hat{n}_1^i, n_2^i) - C^i(\hat{n}_1^i, n_2^i) \tag{18}$$

Given $\hat{n}_1^i$, platform i will choose to serve a number $\hat{n}_2^i$ of group-2 agents, where $\hat{n}_2^i$ maximizes

$$n_1^i U^i(\cdot) + R^i(\hat{n}_1^i, \cdot) - C^i(\hat{n}_1^i, \cdot) \tag{19}$$

The equilibrium price to group-2 is $\hat{p}_2^i$, where this satisfies

$$\hat{n}_2^i = ø^i(\hat{n}_1^i, \hat{p}_2^i) \tag{20}$$

For a given $\hat{n}_1^i$, notice that (19) measures the total surplus of platform i and its group-1 agents as the number of group-2 agents is varied. Therefore, the number of group-2 agents is chosen to maximize the joint interests of the platform and its group-1 agents, and the interests of group-2 are ignored. In general, this implies that there is a market failure, and there are a suboptimal number of group-2 agents on each platform for a given distribution of group-1 agents. [18]

If $V^i(n_1^i, n_2^i)$ is the gross group-2 surplus on platform I, and there are no externalities within the set of group-2 agents, this surplus function differentiates to give the inverse demand function, so that

$$\frac{\partial}{\partial n_2^i} V^i(n_1^i, n_2^i) \equiv \frac{R^i(n_1^i, n_2^i)}{n_2^i} \tag{21}$$

The right hand expression is the price paid by group-2 agents. This formula is only valid when there are no intragroup externalities that could make a group-2 agent better off should there be fewer group-2 agents. The market failure can be seen by looking at the total surplus on platform i, which is maximized by choosing n_2^i to maximize

$$n_1^i U^i(\cdot) + V^i\left(\hat{n}_1^i,\cdot\right) - C^i(\hat{n}_1^i,\cdot) \qquad (22)$$

Since $V^i() - R^i$, group-2's net aggregate surplus, is increasing in n_2^i, the maximum of (22) is greater than the maximum of (19), and there are too few group-2 agents serviced in equilibrium. This is summarized in proposition 4 [18]:

"In the competitive bottleneck model, in any equilibrium the number of group-2 agents on a platform is chosen to maximize the joint surplus of the platform and its group-1agents, and the interests of group 2 are ignored. Unless there are externalities within the set of group-2 agents, there are too few group-2 agents on each platform given the distribution of group-1 agents on each platform. [18]"

4 Mathematical Models from Previous Research

The following section contains descriptions of some of the latest mathematical models developed in the previous research on the NN debate. The first model by Krämer et al. includes a quite sophisticated model with explicit congestion. However, the model does not consider competing ISPs. This is included in the next model by Bourreau et al. The last two models use a different structure than Krämer et al. and Bourreau et al., and illustrate some of the different approaches when analyzing the NN debate.

4.1 Congestion Sensitive Model by Krämer and Wiewiorra

Krämer and Wiewiorra [1] developed a mathematical model to analyze the effects of a QoS tiering regime in which the ISP charges for prioritization on a non-discriminatory basis. The model considers investments by the ISP and CPs, multi-homing CPs and heterogeneous CPs and IUs. The model specifically addresses congestion sensitivity and effects of re-congestion after investments, inter-class externality and endogenous entry by CPs.

$F(\theta)$: [0,1]	Distribution of continuum of CPs
$\tilde{\theta}$	CP that is indifferent between choosing the priority and BE class under NNN
$\bar{\theta}$	Congestion sensitivity characterizing indifference between staying active or not
θ	Individual congestion sensitivity of the CPs
$\beta = 1 - \frac{F(\tilde{\theta})}{F(\bar{\theta})}$	Share of CPs choosing the priority class under a QoS tiering regime
r	Average revenue-per-click for CPs
$\bar{\eta}$	Share of internet customers in equilibrium
$b > 0$	Base utility for IUs to join the internet
$v > 0$	Marginal utility for IUs by adding an extra CP
	IUs marginal disutility because of congestion
$\iota > 0$	Average traffic from each customer
λ	Average rate at which IUs aggregate content requests arrive at the
$\Lambda = \lambda\bar{\eta}F(\bar{\theta})$	ISP's network
$c(\mu)$	The costs on capacity expansion
w	The CPs perceived average level of network congestion
w_Q	Average congestion level for IUs in NNN regime ($=\beta w_{Q1} + (1-\beta) w_{Q2}$)
p	Price of priority transmission class for CPs

a	Internet access fee charged by the ISP
μ	Average rate at which service requests are handled (transmission quality)

Table 6: Indexes, parameters and variables of the mathematical model by Krämer and Wiewiorra [1]

If (1-θw) is the click through rate of a CP, the CPs profit under net neutrality is

$$\Gamma_N(\theta) = \begin{cases} (1-\theta w_N)\lambda\bar{\eta}r, & if\ active \\ 0, & if\ otherwise, \end{cases} \quad (23)$$

Under NN all CPs receive the same level of congestion w_N. In the QoS tiering regime, however, CPs can opt for priority transmission class with w_{Q1} ¡ w_N at a price of p per click. On the other hand, the CPs that remain in the BE class receive a higher congestion level $w_{Q2} > w_N$.

$$\Gamma_N(\theta) = \begin{cases} (1-\theta w_{Q2})\lambda\bar{\eta}r, & if\ active\ in\ BE\ class \\ (1-\theta w_{Q1})\lambda\bar{\eta}r - \lambda\bar{\eta}p, & if\ active\ in\ priority\ class \\ 0, & if\ otherwise, \end{cases} \quad (24)$$

The utility of IUs are formally

$$U = \begin{cases} (b + v\bar{\theta} - \iota w - a, & if\ connected \\ 0, & if\ otherwise, \end{cases} \quad (25)$$

The network congestion is measured through Internet consumers' average waiting time following a content request. The model use an M/M/1 queuing model to fix ideas on the relationship between average waiting time, network traffic, and capacity. The average expected waiting time under NN is

$$w_N = \frac{1}{\mu - \Lambda} \quad (26)$$

It is assumed that μ > Λ. Under the NNN regime the congestion is expressed for the priority lane and BE lane accordingly

$$w_{Q1} = \frac{1}{\mu - \beta\Lambda}, w_{Q2} = \frac{\mu}{\mu - \Lambda} w_{Q1} \quad (27)$$

The ISP controls the two-sided Internet market, over which it has a terminating monopoly. Under an NN regime the ISPs profit is

$$\Pi_N = \bar{\eta}a - c(\mu) \qquad (28)$$

Under an NNN regime, the ISP can also chose the strategic variable p. The profit function becomes

$$\Pi_N = \bar{\eta}a + \beta\Lambda p - c(\mu) \qquad (29)$$

The previous investment decision in transmission capacity is considered to be sunk in all regimes, and is therefore not necessary to be considered in the short run maximization.

4.2 Competing Internet Platforms by Bourreau et al.

Bourreau et al. [2] have developed a mathematical model to compare the effects of competition between ISPs under a NNN and NN regime. While the ISPs mechanically benefit from NNN in a monopoly, it is less clear that switching to an NNN regime would benefit competing ISPs. The model also considers investments by ISPs and CPs, multi-homing CPs and heterogeneous CPs and IUs. The setting is quite similar to that of Krämer et al. However, this model also includes competing ISPs. The notation used in the article is presented in the following table.

$i \in \{A, B\}$	Index of the two horizontally-differentiated competing ISPs
$h \in [0, \infty)$	Continuum of CPs with mass 1
λ	Constant number of visits per user, which is the same for all web-sites
a	Per click advertising revenue
R	IUs utility of connecting to as ISP
v	IUs preference for product variety supplied by the CPs
d	Parameter for the IUs preference for the speed of the connection
t	The standard Hotelling unit transporting cost
w_i	Congestion on ISP i's network
x_i	Number of end-users subscribing to ISP i
$\bar{h}_i$	Marginal CP which is indifferent between connecting to ISP I and not connecting
$\tilde{h}_i$	Marginal CP which is indifferent towards priority lane and non-priority lane

p_i	Subscript for IUs to connect to ISP i's network
μ_i	ISP i's network capacity investments

Table 7: Indexes, parameters and variables of the mathematical model by Bourreau et al. [2]

With a click trough rate on ISP i for CP h of (1-hw_i), the advertising revenue for CP h is

$$a\lambda x_i(1 - hw_i) \tag{30}$$

Under NN (N), the profit of CP h is

$$\Pi_h^N = \begin{cases} a\lambda x_A^N(1 - hw_A^N) + a\lambda x_B^N(1 - hw_B^N) & , if\ it\ connects\ to\ both\ ISPs \\ a\lambda x_i^N(1 - hw_i^N) & , if\ it\ connects\ only\ to\ ISP\ i \\ 0 & , otherwise \end{cases} \tag{31}$$

Under NN, all CPs are active at ISP I are treated equally and face the same average level of congestion w_i^N. However, under the NNN (D), a CP may choose to pay a fixed fee to ISP i to benefit from a priority (P) lane where the congestion is lower. The profit for CP h under NNN is given by

$$\Pi_h^N = \begin{cases} a\lambda x_A^D(1 - hw_A^P) - f_A + a\lambda x_B^D(1 - hw_B^P) - f_B & if\ priority\ at\ both\ ISPs \\ a\lambda x_i^D\left(1 - hw_i^P\right) - f_i + a\lambda x_j^D(1 - hw_j^{NP}) & if\ priority\ only\ at\ ISP\ i \\ a\lambda x_A^D(1 - hw_A^{NP}) + a\lambda x_B^D(1 - hw_B^{NP}) & if\ non - priority\ at\ both\ ISPs \\ a\lambda x_i^D\left(1 - hw_i^P\right) - f_i & if\ priority\ at\ ISPi, no\ entry\ at\ ISP\ j \\ a\lambda x_i^D\left(1 - hw_i^{NP}\right) & if\ non - priotity\ at\ ISP\ i, no\ entry\ at\ ISP\ j \\ 0 & otherwise \end{cases} \tag{32}$$

In the NNN regime the CP that connects to ISP choose either to pay for access to the priority lane or to use the non-priority (BE) lane for free.

The two ISPs are located at the extremities of a linear city of length one, with ISP A located at point 0 and ISP B located at point 1. The investment costs C(μ_i) is increasing and convex in μ_i ($C' > 0$ and $C'' > 0$). Under NN the profit function of ISP i is

$$\Pi_i^N = p_i^N x_i^N - C(\mu_i^N) \tag{33}$$

In the NNN regime, the ISPs can also charge a fixed fee, f_i, to the CPs that opt for the priority lane. The ISP profit function in the NNN regime is

$$\Pi_i^D = p_i^D x_i^D + (\bar{h}_i^D - \tilde{h}_i) f_i - C(\mu_i^D) \tag{34}$$

Due to capacity constraints the IUs might suffer from congestion. Congestion is measured by the waiting time for IUs when they request content from CPs. The M/M/1 queue model is used to determinate the average level of congestion as a function of network capacity and traffic. Under the NN regime, the average level of congestion for ISP i is

$$\mathrm{w}_i^N = \frac{1}{\mu_i^N - \bar{h}_i^N \lambda x_i^N} \tag{35}$$

Under the NNN regime, each ISP sorts CPs into two traffic lanes, the priority lane and the non-priority lane. The congestion for the priority lane (P) operated by ISP i is given by

$$\mathrm{w}_i^P = \frac{1}{\mu_i^D - (\bar{h}_i^D - \tilde{h}_i) \lambda x_i^D} \tag{36}$$

The congestion for the non-priority lane (NP) is given by

$$\mathrm{w}_i^{NP} = \frac{\mu_i^D}{\mu_i^D - \bar{h}_i^D \lambda x_i^D} w_i^P \tag{37}$$

The formula implies that congestion is always higher in the non-priority lane. If $b_i = 1 - \tilde{h}_i / h_i^D$ is the share of CPs that buy priority from ISP i, note that the average congestion under the NNN regime satisfies

$$\mathrm{w}_i^D = b_1 w_i^P + (1 - b_i) w_i^{NP} = \frac{1}{\mu_i^D - \bar{h}_i^D \lambda x_i^D} \tag{38}$$

So if the volume and capacity are the same under the NN and NNN regime, the average congestion will be the same ($w_i^N = w_i^D$). This is a well-known property of the M/M/1 queuing model.

There is a unit mass of IUs uniformly distributed along the unit interval. Each IU subscribes only to one ISP (single-homes). Under NN, a user located at x_j on the unit interval and who subscribes to ISP A, obtains utility

$$U_j = R + v\bar{h}_A^N + \frac{d}{w_A^N} - p_A^N - tx_j \tag{39}$$

A similar expression can be obtained for ISP B. It is assumed that R is sufficiently high so that the market is covered in equilibrium in both regimes. Under the NNN regime, the end user located at x_j obtains utility

$$U_j = R + v\bar{h}_A^D + \frac{d}{w_A^D} - p_A^D - tx_j \tag{40}$$

It is also assumed that the IUs value content sufficiently compared to the disutility they suffer from congestion. In particular, in a symmetric equilibrium, it must be that

$$v > \mathrm{d}\lambda/2 \tag{41}$$

To see why, put (38) into (40)

$$U_j = R + (v - d\lambda x_j^D)\bar{h}_A^N + d\mu_i^D - p_A^D - tx_j \tag{42}$$

For consumers to value the CPs, the bracket has to be positive. Therefore, $v - d\lambda x_j^D > 0$, which implies what is stated above as in an equilibrium solution $x_i^D = 1/2$.

In the NN regime, the model is solved in a two stage game with the following set-up

1. The two ISPs choose their capacities μ_A^N and μ_B^N, and set the subscription fees to the end users p_A^N and p_B^N.
2. The CPs choose which ISP(s) to connect to (if any), and the end users choose which ISP to subscribe to.

The model is solved backwards to find the symmetric subgame perfect equilibrium (SPE).

In the NNN regime, each ISP offers a priority lane and a non-priority lane to CPs. The CPs that opt for priority at ISP i pay a fixed fee f_i, whereas the non-priority lane is offered for free. The two-stage game is modified accordingly:

1. The two ISPs choose their capacities μ_A^D and μ_B^D, set their subscription fees to the end users, p_A^D and p_B^D, as well as the fees for their priority lanes f_A and f_B.
2. The CPs choose which ISP(s) to connect to (if any) and whether to pay for priority, and the IUs choose which ISP to subscribe to.

4.3 Investments and Competition model by Njorge et al.

Njorge et al. [3] developed a mathematical model to analyze the NNN regime in a two-sided market that includes ISP competition, ISPs investments, CPs investments, Multihome CP (whether IUs connects to several CPs simultaneously) and heterogeneity among both CPs and IUs. The model is based on work by Shaked and Sutton (1982), Rochet and Tirole (2003), Gabszewicz and Wauthy (2004), Roson (2005), Parker and Alstyne (2005) and Armstrong (2006).

NN model

The model considers two platforms (or ISPs), a continuum of users and CPs of unit mass.

$z \in [\alpha,\beta]$	Indexes of the two platforms (ISPs)
$j \in [0,1]$	Index over CPs
i	Index of consumers (IUs)
$f \in [0,1]$	Mass of consumers
y_{basic}	Minimum threshold that ISPs are enforced to provide to all CPs by regulation
γ_j	The quality of CP j. Uniformly distributed random variable, independent and identically distributed across the population of CPs
a	Support variable of γ_j, $[\bar{\gamma}\text{-a}, \bar{\gamma}\text{+a}]$ and $0 < a < \bar{\gamma}$
k_α, k_β	Value of extra services (e-mail, virus scans) provided by the ISPs, defined as random variables with the same distributions as those of γ_j
θ_i	Consumer heterogeneity, uniformly distributed on the interval [0,f]
R	Reservation utility of joining a platform
$y_z \in R_+$	The QoS chosen by platform z. We assume $y_\alpha \geq y_\beta \geq y_{basic}$
$ø:[0,f]\text{->}\{\alpha,\beta\}$ $\hat{ø}:[0,1]\text{->}\{\alpha,\beta\}$	Connection decisions that map the space of consumers and CPs, respectively, to the set of platforms
r_α, r_β	Aggregation of mappings representing the masses of CPs that join each platform
q_α, q_β	Aggregation of mappings representing the masses of IUs that join

	each platform
p_z	Connection fee to consumers on platform z
w_z	Fixed fee for a CP to connect to a platform z
v_j	Utility of a CP, defined as its profit

Table 8: Indexes, parameters and variables of the mathematical model by Njorge et al. [3]

A consumer i on a platform ø(i) connecting to a CP j on platform $\hat{ø}(j)$ receives utility

$$u_{ij}\left(y_{ø(i)}, y_{\hat{ø}(j)}, \gamma_j, k_{ø(i)}, r_{\hat{ø}(j)}\right) = \min\{y_{ø(i)}, y_{\hat{ø}(j)}\}\left(\frac{\gamma_j}{r_{\hat{ø}(j)}} + k_{ø(i)}\right) \quad (43)$$

The formula multiplies the quality of the network transmission, given by the worst of the two platforms, by the value of the content plus additional services offered by the platform. To compute the value of the content, the quality of the CP j is divided by the mass of CPs that connect to the same platform to incorporate congestion effects. The congestion is not considered on the consumer side, which is a simplification. [3]

Each consumer connects to a single platform, but has access to all content because of interconnection. In particular, a consumer i on platform ø(i) connects to all CPs subscribed to either platform since $u_{ij} \geq 0$ for all j [3]. The overall utility perceived by IU consumer i that joins platform ø(i) is

$$F_i\left(y_{ø(i)}, y_{ø(-i)}, \bar{\gamma}, a, r_\alpha, r_\beta\right) = \int_0^1 E\left[u_{ij}\left(y_{ø(i)}, y_{\hat{ø}(j)}, \gamma_j, k_{ø(i)}, r_{\hat{ø}(j)}\right)\right] dj \quad (44)$$

Here ø(-i) denotes the other platform, and the expectation is taken over the random parameters such as γ_j and $k_{ø(i)}$. [3]

Platform z charges consumers a connection fee of p_z. Consumers have reservation utility of R and consumers' preferences are heterogeneous, which is represented by the parameter θ_i. The utility of a consumer i connecting to platform ø(i) is given by

$$U_i\left(ø(i)\right) = max\left\{R + \theta_i F_i\left(y_{ø(i)}, y_{ø(-i)}, \bar{\gamma}, a, r_\alpha, r_\beta\right) - p_{ø(i)}, 0\right\} \quad (45)$$

Consumers join the platform that yields the highest utility, provided it is positive. [3]

If CPs connect to a platform z ϵ [α, β], they pay a fixed connection fee w_z and make revenue by selling advertising and showing it to consumers. The utility v_j of a CP j is defined to be its profit

$$v_j = V_j(\gamma_j, y_\alpha, y_\beta, q_\alpha, q_\beta) - w_{\hat{\emptyset}(j)} \quad (46)$$

Where V_j is its gross revenue, given by

$$V_j(\gamma_j, y_\alpha, y_\beta, q_\alpha, q_\beta) = \begin{cases} g(\gamma_j, y_\alpha)q_\alpha + g(\gamma_j, y_\beta)q_\beta, & if\ \hat{\emptyset}(j) = \alpha \\ g(\gamma_j, y_\beta)q_\alpha + g(\gamma_j, y_\beta)q_\beta, & if\ \hat{\emptyset}(j) = \beta \end{cases} \quad (47)$$

Here $g(\gamma_j, y_{\hat{\emptyset}(j)})$ is a function that represents as prices. It is increasing in both parameters: Ad prices are high when content quality is good because it is easier to attract advisers. In addition, consumers have a better experience with high-quality platforms and, therefore, they spend more time in these sites which increase the advertisers brand exposure. Note that is CP j join the higher quality platform, it is able to charge a higher ad price for connection arising from consumers on that platform. If a CP join the lower quality platform its ad prices are the same across the two platforms because when a customer and CP connect to different platforms, the QoS is given by the works of them. [3]

The platforms pay for their quality investment, which modelled by an increasing and convex investment cost $I(y_z)$ to achieve a QoS of y_z. This results in decreasing returns to investment. The investment function is differentiable, and $I(0) = 0$. The payoff π_z experiences by platform z is given by

$$\pi_z = p_z q_z + w_z r_z - I(y_z) \quad (48)$$

Timing:

1. Quality Investment Decisions: Platforms simultaneously choose QoS y_α and y_β
2. CP Pricing Decisions: Platforms simultaneously choose fees w_α and w_β
3. CP Connection Decisions: CPs decide which platform to join
4. Consumer Pricing Decisions: Platforms simultaneously choose prices p_α and p_β
5. Consumer Connection Decisions: Consumer decide which platform to join
6. Consumer Consumption Decisions: Consumers decide which CPs to get service from

The timing of the extensive game is predicated on the view that investments adjust more slowly than prices. The former is viewed as a medium to long-term decision whereas the latter is a shorter term decision. Thus investments are the first stage of the game. The prices of CPs are set before IUs to reflect the longer horizon of contracts between CPs and ISPs, as opposed to those of IUs and ISPs. The game is solved by considering a subgame perfect Nash equilibrium (SPE), focusing on optimal actions/decisions along the equilibrium paths. The game can be solved with backward induction. [3]

NNN model

The model is equal to that in the neutral regime, the NN model, except for one important difference: a CP that joins and pay fees to a platform enjoys the enhanced quality level y_z from that platform. However, if the CPs does not directly connect to that platform, it only enjoys the basic quality level u_{basic} ($\leq y_\alpha$) from that provider. In this sense, there are priority lanes in the non-neutral model. When y_{basic} is said to be zero, it effectively means that a CP has to pay the platform to reach the customers (else the IUs utility could be zero connecting to the CP). That way it is possible to analyze the NN regime through both a tiering and termination fee regime. [3]

In the NNN-model, the utility of the different CP connection decisions are a bit different due to the implicit access fee and tiering

$$v_j(\gamma_j, y_\alpha, y_\beta, q_\alpha, q_\beta) = \begin{cases} g(\gamma_j, y_\alpha)q_\alpha - w_\alpha, & if\ \hat{\emptyset}(j) = \alpha \\ g(\gamma_j, y_\beta)q_\beta - w_\beta, & if\ \hat{\emptyset}(j) = \beta \\ g(\gamma_j, y_\beta)q_\alpha + g(\gamma_j, y_\beta)q_\beta - w_\alpha - w_\beta, & if\ \hat{\emptyset}(j) = both \end{cases} \quad (49)$$

(49) being the three alternative buying options for the CPs.

4.4 Congestion Sensitive Model by Economides and Hermalin [4]

Economides and Hermalin (2012) developed a mathematical model to analyze the effects of NNN and NN that explicitly considers congestion, and does not take the amount of traffic (e.g. number of packets) sent by a given content provider as fixed. The amount of content purchased by IUs can vary, which means that expansion in bandwidth does not necessarily increase speed because larger bandwidth will attract more traffic. This effect is also observed in physical highways, when adding lanes does not always significantly reduce commute times. The model also consider differentiation in provision of quality (transmission speed), which is important for their result that, for a fixed amount of bandwidth, a case can be made that welfare is greater under neutrality than under multiple tiers. The model also considers investments by ISPs and CPs, and heterogeneous IUs and CPs. [4]

$\theta \in [\underline{\theta}, \overline{\theta}) \subseteq R_+$	Continuum of content providers of measure one
$\Theta \in [\underline{\theta}, \overline{\theta})$	Set of content providers
j ϵ [1,J]	Set of subbandwidths
B	Bandwidth of a pipe from CPs to IUs (capacity for B units of content / unit time)
B_j	Sub-bandwidth (where $\Sigma B_j = B$)
$F: [\underline{\theta}, \overline{\theta}) \rightarrow [0,1]$	Distribution of θ, where F'() exists and is positive for all $\theta \in (\underline{\theta}, \overline{\theta})$
F(Θ)	Proportion of application-provider types that are in set Θ
X(θ)	Units of content send by content provider θ
$t(\Theta)$	$t(\Theta) \equiv \frac{\int_\Theta X(\theta)dF(\theta)}{B_\Theta}$, The time necessary to send all of the content of the CPs in Θ, and a measure of congestion faced by CPs in Θ.
τ(θ)	Deliver time for CP θ. If θ is in Θ, then τ(θ) = t(Θ).
α(τ(θ),θ)	Adjustment factor on some indication of the value households place on content
y	Numéraire good in households utility
c	Marginal cost of content production and transmission
x	Unit of content
p	Price of content from CPs
q	Advertising rate CPs
s	Payment to ISP
π	The "equilibrium gross profit factor"
σ	The "equilibrium consumer surplus factor"
η	Hook-up fee for IUs connecting to ISP

Table 9: Indexes, parameters and variables of the mathematical model by Economides and Hermalin [4]

A household's marginal utility from the x^{th} unit of content from content provider θ is taken to be

$$m\left(\frac{x}{\alpha(\tau(\theta),\theta)}\right) \quad (50)$$

Where the adjustment factor, α(τ(θ),θ), reflects the congestion in transmission, τ(θ), some indication of the value the household assigns that content, and how much the household cares about delay or congestion vis-à-vis that content. A household's (IUs) overall utility is

$$U = y + \int_{\underline{\theta}}^{\overline{\theta}} \left(\int_0^{x(\theta)} m\left(\frac{x}{\alpha(\tau(\theta),\theta)} \right) dx \right) dF(\theta) \qquad (51)$$

The utility function assumes that all households are homogeneous in their preferences, and that they have additive separability across different content. The marginal utility m() is assumed to be twice differentiable and decreasing, and never too convex, specifically

$$zm''(z) + m'(z) < 0 \qquad (52)$$

For all z ϵ_+. It is further assumed that households prefer faster content delivery to slower content delivery, all else being equal, that is $\tau > \tau'$ implies $\alpha(\tau,\theta) < \alpha(\tau',\theta)$ for all θ. Also, households view content from higher-θ CPs to be more time sensitive than from lower-θ CPs, that is for all $\theta > \theta'$ and all $\tau > \tau'$,

$$\alpha(\tau,\theta) - \alpha(\tau',\theta) < \alpha(\tau,\theta') - \alpha(\tau',\theta') \qquad (53)$$

It is further assumed that consumption of the content providers' good plus any hook-up fee paid the ISP never consumes a household's entire income. This and the assumption of additively separable and quasi-linear utility mean that each household acquires the amount of the θ^{th} CPs product that equates marginal utility to marginal cost. A household's demand is thus

$$x(p,\theta) = \alpha(\tau(\theta),\theta)m^{-1}(p) \equiv \alpha(\tau(\theta),\theta)\omega(p) \qquad (54)$$

A CPs profit is

$$(q + p - c)x(p,\theta) - s \qquad (55)$$

It is here assumed that the advertising rate and content cost are common across content providers. Assumption (52) implies that $\omega()$ is log concave. To rule out infinite consumption it is assumed that $\lim_{p \downarrow c-q} \omega(p) < \infty$. These assumptions are sufficient for a CPs pricing problem,

$$\max_{p}(q+p-c)x(p,\theta)-s \tag{56}$$

to have a unique and finite solution. The fee paid to the ISP, s, and the content delivered, θ, is treated as constants and is thus not need to be evaluated in order to maximize (56), which could be solved by

$$\max_{p}(q+p-c)\omega(p) \tag{57}$$

The solution is, for further reference, defined as

$$\pi=(q+p^{*}-c)\omega(p^{*}) \tag{58}$$

and π is referred to as the "equilibrium gross profit factor". The household (consumer) surplus from trade with CP θ is

$$\int_{p^{*}}^{\infty}x(p,\theta)dp=\alpha(\tau(\theta),\theta)\int_{p^{*}}^{\infty}\omega(p)dp\equiv\alpha(\tau(\theta),\theta)\sigma \tag{59}$$

The total welfare, the sum of CPs and household's profits, is

$$W=\int_{\underline{\theta}}^{\overline{\theta}}(\pi+\sigma)\alpha(\tau(\theta),\theta)dF(\theta) \tag{60}$$

Since the IUs are homogeneous, the ISP can capture the entire consumer surplus. Thus the hook-up fee posed on the IUs by the ISP is

$$\eta=\sigma\int_{\underline{\theta}}^{\overline{\theta}}\alpha(\tau(\theta),\theta)dF(\theta) \tag{61}$$

If the ISP can discriminate continuously across content providers it will impose the scheme that maximizes expected virtual surplus subject to τ()'s being non-decreasing and the bandwidth constraint. The ISP seeks to maximize

$$\sigma \int_{\underline{\theta}}^{\overline{\theta}} \alpha(\tau(\theta),\theta) dF(\theta) + \int_{\underline{\theta}}^{\overline{\theta}} \left(\pi\alpha(\tau(\theta),\theta) - \pi \frac{1-F(\theta)}{f(\theta)} \frac{\partial \alpha(\tau(\theta),\theta)}{\partial \theta} \right) f(\theta) d\theta \qquad (62)$$

The first term is to maximize the income from IUs, while the second term is the profit captured from the CPs. The model can also be modified to take investments in capacity by the ISP into account.

4.5 Strengths and Weaknesses of the Mathematical Models from Previous Research

In this section a discussion of the four presented mathematical models will be provided. First, we will discuss the strengths and weaknesses of the mathematical models, before our findings are summarized in Table 10. Lastly we will describe the strengths that are important to include in the analysis, and the weaknesses that are treated in this thesis and why they are important to include in the analysis.

Krämer et al. and Bourreau et al. have many similarities in their model settings. Both models explicitly formulate congestion through the M/M/1 model, which is a well proven model to analyze the effects of network capacities and congestion [8]. They also have an explicit formulation of all the players which simplifies the process of analyzing results and making extensions. They also provide a holistic perspective on the CPs and IUs rather than analyzing individual players. This makes it possible to do a traceable overall assessment of the effect of the NN- and NNN regimes. Both models also have heterogeneous CPs and IUs. However, Krämer use a monopoly setting for the ISPs. The argument is that most ISPs operate under monopoly conditions in the US. However, this is not true in all regions, e.g. do most IUs have several ISPs to choose from in Norway. Thus, the conclusions based on the monopoly setting are not valid in many regions. Krämer et al.'s formulation of the IUs utility does not allow explicit formulation of the solutions. However, Bourreau et al. solved this by modelling good QoS as a utility, instead of congestion as a disutility, which makes it possible to do more sophisticated analytical analysis based on the explicit solutions. This is a major advantage over Krämer et al.'s model. Bourreau et al. does also consider competition among the ISPs. Both Krämer et al. and Bourreau et al. only have two stages in their set up. While this greatly simplifies the solving process, it is not completely realistic, as e.g. investments are usually made a long time prior to sale. Thus, it is often possible to adapt price levels based on the conditions at the time of sale. On the other hand, the decision to make investments is much more difficult as it directly affects how the other decisions can be made, and has to be made based on reliable forecast or analyses. Njorge et al. has solved this in their sophisticated six stage model. However, their model is very complex and much more difficult to analyze than Krämer et al. and Bourreau et al.'s. All three models only consider CP revenues from advertising. This makes for a much easier analysis. However, many of the services that are likely to opt for a priority lane, e.g. Netflix, are subscription services. Subscription services can directly extract money from the IUs, and are thus positioned differently than advertising CPs to benefit from a NNN regime. Neither Krämer et al. nor Bourreau et al. formulate congestion sensitive IUs, rather they react to the average level of congestion. This is in favor of the NN regime, as the NNN regime more efficiently allocated congestion [1] [2]. This effect is captured directly by an increase in CPs revenues. However, the effect is not captured by the IUs, whom obviously would also be congestion sensitive by the same argument as the CPs are congestion sensitive. The reason why this is omitted is to simplify the solution process.

Njorge et al. have developed a sophisticated model that provides very crisp results. However, congestion, which is a very important factor in the NN debate, is not included directly in their model. They do include a form of E2E by requiring the CPs to connect to either of the ISPs, and that the overall QoS is dependent on the worst link in the value chain. However, the Internet is international, which means that rarely the CPs connect to the same ISP, or CDN, as the IUs. Even in the same regions, there are many devoted CDNs that connect the CPs to the backbone of the Internet. Thus, the analysis becomes more of a closed ecosystem than the international ecosystem where multiple ISPs work together through transit and peering. Also, the CDNs investments costs are usually much lower than that of the access ISPs [24], which implies that the bottleneck of the network is usually at the eyeball and not at the backbone of the Internet.

Economides and Hermalin does provide an analysis of CPs that offer both advertising and subscription services. The model does also include variable costs of the CPs, which is more realistic than only modelling revenues. However, their model has an implicit solution that complicates possible extensions. The model also considers a network where the capacity is separated, which can be proven to be less effective than a singular network with a high capacity. Analytically it is a nice feature, however, should provide sub-optimal results concerning network management.

	Krämer et al.	Bourreau et al.	Njorge et al.	Economides et al.
Strengths	• Long and short term analysis • Heterogeneous CPs and IUs • Explicit congestion	• Long and short term analysis • Heterogeneous CPs and IUs • Explicit solutions • Competing ISPs • Explicit congestion	• Long and short term • Competing ISPs • Many stages • Different revenue for CPs based on quality of CDN	• Variable CP costs • Both subscription and advertising revenues • Explicit capacity allocation • Explicit congestion
Weaknesses	• No ISP competition • No variable costs • Constant visits/IU/CP • Only advertising revenues • No transaction cost • Few stages • IUs not congestion sensitive • No E2E assessment	• No variable costs • Constant visits/IU/CP • Only advertising revenues • No transaction cost • Few stages • IUs not congestion sensitive • No E2E assessment	• No variable costs • ISPs also CDN, no real E2E assessment • Only advertising revenues • No explicit model of congestion • No transaction cost • IUs not congestion sensitive	• No ISP competition • Homogeneous IUs • Abstract model • No transaction cost • IUs not congestion sensitive • Separated network • No E2E assessment

Table 10: Summary of strengths and weaknesses of the mathematical models from previous research

All models have heterogeneous CPs, which is very important to avoid the ISPs to extract all the surplus profits of the CPs in the NNN regime, and get a good assessment on the effect of CPs innovation levels. All but Economides and Hermalin also include heterogeneous IUs. Regulators and policy makers are very careful not to harm IUs. When the IUs are homogenous, the ISP can extract all available utility from the IUs, leaving no consumer surplus. Thus, no good analysis on the effect a transition to the NNN regime has on the IUs consumer surplus can be made. Heterogeneous IUs is therefore a very important feature to include in the model.

All models include investments in capacity by the ISPs and investments in content by the CPs. These are all important features of analyzing the NN debate. First, because all wealth stems from the capacity in the network and the content the CPs send through it. Secondly, because the arguments posed by either side both focus on the effects of investments from both the ISPs and the CPs. Thus, to provide insights into the arguments posed by the two sides of the debate, it has to be directly included into the model.

All models consider congestion, and all but Njorge et al. consider explicit congestion explicitly. This is important accurately analyze the effect of extra investments from lower congestion levels, but also from re-congestion as the CPs innovate and increase their volume of content.

None of the mathematical models consider E2E. The Internet is global and CPs connects to access ISPs all over the world. They do connect to the Internet, but not necessarily through the same ISPs as the IUs. Rather, they connect through a CDN, which connects the CPs to the backbone of the Internet. This is included in our model through the backbone ISP, and is important because as the CPs pay the access ISPs for higher QoS and possibly offer more content to the IUs, the backbone ISP will also be affected. How the backbone ISP reacts in a NNN regime will again affect the outcome for the other actors. If this is not included some actors might believe they are winning from the NNN regime, while they actually are losing because the backbone is eating away their profits.

None of the models include transaction costs. However, there will be transaction costs related to the adoption of an NNN regime. New SLAs have to be made, systems of monitoring to prevent moral hazard would have to be developed by the CPs and probably to the interest of IUs as well, and an administration would probably have to be set up to manage the new service. Due to the added complexity of this feature it is not included in this thesis. Congestion sensitive IUs are also not included by any of the models. However, this is not that important as the main effect would be a higher benefit of the NNN regime for the IUs [1] [2]. This analysis is therefore conservative regarding the IUs benefit of an NNN regime, and thus the analysis and results provided by this thesis is on the safe side for policy makers and regulators.

5 Mathematical Models, Numerical Experiments and Discussion

In this section we will first, in section 5.1, describe the NN debate ecosystem, the actors involved, how they play, and how they are implemented in the general model. Then, in section 5.2, we develop a general mathematical model based on the NN debate ecosystem, and partly the previous research as presented in this thesis. In section 5.3 we develop a simplified mathematical model, which can be solved analytically. This model is based on the research by Kramer et al. and Bourreau et al. We extend on their work by including a backbone ISP and a regulating mechanism on the IAS lane. The solutions to the simplified mathematical model are presented in section 5.4. Section 5.5 provides a discussion of the solutions, as well as an analysis and discussion of the analytical solutions where the two regimes, NN and NNN, are compared. We further compare the two regimes in section 0 with several numerical experiments. Lastly, in section 5.7, we summarize the strengths and weaknesses of this model compared to the previous research.

5.1 The Net Neutrality Debate Ecosystem

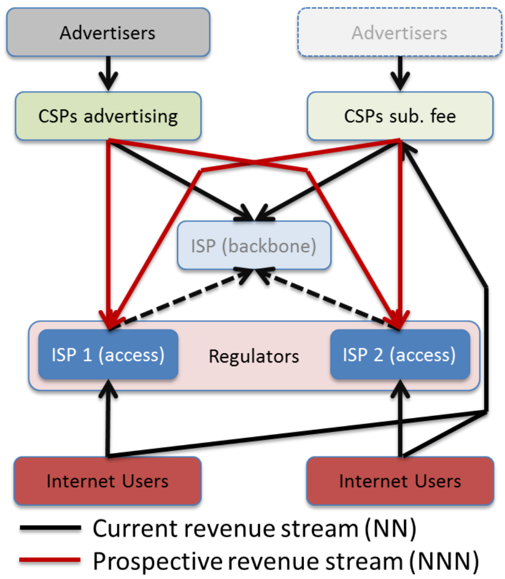

Figure 10: The ecosystem of the NN debate

There are mainly six participants in the NN debate ecosystem, the advertisers, the CPs, the access and backbone ISPs, the IUs and the regulators [13] [16]. The advertisers pay the CPs to show adds, and they usually pay a proportional amount to the number of visitors, clicks per visitor and their click-through rate. The CPs either make money from the advertisers, or by subscription fees. Either way, they are dependent on having users using their content. Some CPs obtain revenues from both advertisements and subscriptions, e.g. common for e-newspapers like VG.no. However, this will not be considered in the model because indirectly it is analyzed by separating the business into two business units, advertising and subscribing business (like VG and VG+). This analysis can then be used to see the pure effects NN and NNN has on CP business that advertise and CP business from subscription fees. The CPs pay an ISP backbone connectivity provider to get access to the internet, and the access ISPs have to connect to the Internet through the backbone either with a transit or peering agreement [13] [25]. If there is no explicit agreement on assured services between the backbone and the ISPs, the connectivity between the CPs and the IUs can obviously not be assured, which is the case in a BE Internet. It is assumed that a settlement free peering between the access and backbone ISPs ensures sufficient QoS. However, for SS to be effective, the CPs have to purchase similar services from the backbone as that from the access ISPs. That is, the QoS of the backbone ISP have to be as high as or better than that of the access ISPs. This is actually a likely outcome as eyeball networks are more capital intensive than backbone networks [24]. The IUs pay the access ISPs to connect to the Internet and through that platform use the content of the CPs. In this respect, the ISPs (access) could under an NNN regime act as a platform in a two-sided market, and charge both the IUs and the CPs for connecting the two agents. Regulators (FCC, EU etc.) may put restrictions from which the ISPs have to operate in compliance with. Lastly it could be worth noticing that the CPs that connect advertisers with IUs could represent a platform in a two-sided market, connecting the advertisers with the IUs. However, analyzing this is beyond both the scope and focus of this thesis, and therefore the effect on advertisers will not be analyzed.

Figure 11: From top left: Example of IUs, access ISP (Telenor) and CPs

The advertisers main decision is how much to pay per click, which usually is set by the CPs. This decision will be treated exogenously, and the revenue per click is thus considered to be constant. The CPs have more strategic decisions. They can chose business model, whether they should get revenue from advertising or subscriptions. In a NNN regime they may also decide whether to pay for SS, which could be important for inelastic CPs. The CPs are heterogeneous in respect to the demand for connectivity and sensitivity towards congestion. The sensitivity for congestion is proportional to the amount of traffic the CPs generate. The CPs will have to multi home in order to reach all consumers with SS. However, the access ISPs cannot block the CPs from delivering content through IAS. In this respect, the ISPs have a monopoly for offering SS to reach their respective IUs. The CPs have to connect to the backbone of the Internet, regardless of NN or NNN, and the backbone ISP have to provide a high enough QoS in order for .

Both access and backbone ISPs have to decide how much to invest in capacity and prices for providing connectivity to the IUs and CPs, respectively. This is normally a fixed fee for the IUs connectivity, and thus not usage based. In an NNN regime, the access ISPs may also decide to offer Specialized Services (SS), with a higher QoS, that the CPs have to pay for. This could be a fixed fee, or a usage based fee and thus price discriminate based on the CPs aggregated traffic. The access ISPs also have to set the QoS of SS, which in turn may affect the QoS of IAS since the networks are not separated in closed networks. The networks could be separated. However, this would increase the overall average congestion:

Proposition 1: If the M/M/1 queue assumptions are valid, splitting the networks into two independent networks will increase the overall average congestion by a factor of two.

The proof, **(171)**-**(174)**, can be seen in the appendix. Therefore, the network should be open, and a protocol for priority of some packages should be applied rather than physically separating the information flows. This is also how it is implemented in this model.

The IUs decide if they want connectivity, and if so from which ISP as they are single-homing. Then they decide which CPs they want to use. The IUs are sensitive to congestion, and would thus value a higher QoS to a lower QoS, as long as the price does not outweigh the benefit from higher QoS. This is also true for the ISPs and CPs, as congestion might result in lower revenues for both the ISPs and the CPs should the IUs not be interested in their service because of congestion. The IUs are heterogeneous with respect to demand for content through subscription CPs and preferences towards the ISPs.

Lastly, the regulators may put restrictions on the IAS lane, and thus either force the ISPs to invest in more capacity, or restrict the QoS of SS.

Decision and timing under NN	***Decisions and timing under NNN***
0. Exogenous parameters set 1. ISPs make investments in capacity 2. Backbone ISPs set the price for CPs 3. CPs choose whether to participate or not 4. ISP set the IUs price to access the Internet 5. IUs choose which ISP to connect to 6. IUs interact with the CPs	0. Exogenous parameters set, including regulators restrictions 1. ISPs make investments in capacity 2. Backbone ISPs set the price for CPs 3. Access ISPs set the priority fees and the QoS of SS 4. CPs choose whether they should participate, and if so whether they want SS or IAS through either or both ISPs 5. ISP set the IUs price to access the Internet 6. IUs choose which ISP to connect to 7. IUs interact with the CPs

Table 11: The general decision procedure and timing of the NN debate ecosystem

5.2 The General Mathematical Model

The general mathematical model is developed based on previous research presented in this thesis. Especially the research by Njorge et al, Economides and Hermalin, Krämer and Wiewiorra, Bourreau et al., and Gaivoronski and the Telenor research team influence the formulation. However, some modifications are made in order to incorporate a more accurate ecosystem and description of the agents, and include the perspective of E2E. Specifically the model has split the CPs into advertising and subscribing CPs, a backbone ISP is included, which the CPs have to pay both for connectivity and SS, congestion sensitive IUs and heterogeneous IUs towards the subscription CPs, and different regulating mechanisms. The work by Marc Armstrong (2006) on two sided markets has laid the foundation for modelling the two sided market. This model will follow a game theoretic approach, with symmetric information. This makes it easier to solve, but it might be a less accurate representation of the real world.

First the notation in the mathematical model will be presented. The mathematical formulation of the ecosystem will then presented together with reasoning behind the formulations.

Notation

$j_A \in [0,\rightarrow)$	Continuum of CPs that earn revenues from advertising
$j_S \in [0,\rightarrow)$	Continuum of CPs that earn revenues from subscription fees
$i \in \{a, b, c\}$	Access ISP a and b, and backbone ISP c
$x \in [0,1]$	Unit mass of IUs, modelled as residents in a linear city between 0 and 1
$L \in \{N,D\}$	Superscript to separate between a neutral (N) regime or a non-neutral, differentiation, regime (D)
SS	Specialized Services (usually as superscript)
IA	Internet Access Services (usually as superscript)
$Z \in \{A,S\}$	Sub-index to separate between Advertising CPs (A) and Subscription CPs (S)
r	NPV of revenue per click from advertising per click for CPs
κ_{jA}	Click traffic per IU for CP j_A
λ_{jZ}	Package traffic per click per time unit per IU for CP j_A or j_S
h_{jZ}	Sensitivity towards congestion for CP j_A or j_S
c_{jZ}	Operating/marginal costs for CP j_A or j_S
N	Number of subscription CPs competing with each other
$C^V(\bar{J}_{Ai}^L, \bar{J}_{Si}^L, x_i^L)$	Variable costs for the ISPs of providing connectivity
$C^I(\mu_i^L)$	Investments costs of the ISPs from investing in capacity
Y	IUs numéraire utility of being provided connectivity

m_A	Marginal utility per content from advertising CPs for IUs
t	Standard Hotelling unit cost of transportation
Ψ, Ψ_1, Ψ_2	Constant set by regulators to prevent impairment of the QoS on IAS
M	Big M, relaxation constant to change restriction type from regulators
v_{jS}	Subscription fee for CP j_S towards the IUs
η_{ij_S}	Fraction of demand from ISP i for CP j_S that purchase the CPs offer
Λ	Total aggregated traffic per time unit
μ_i^L	Capacity as measured by packages (traffic) per time unit
w	Measurement of congestion as average waiting time per packet
β_i	Share of CPs in SS for ISP i
f_i	Fee per traffic unit for connecting to SS on ISP a or b, or to the backbone ISP c
u_{xi}^A	Utility for IU x from interacting with advertising CPs on ISP i
u_{xi}^S	Utility for IU x from interacting with subscription CPs on ISP i
p_i^L	Price for connectivity at ISP i for the IUs

Table 12: Indexes, parameters and variables of the mathematical model from this thesis

The CPs

The utility of the CPs is chosen to be their profit from serving the IUs. There are two types of CPs, advertising and subscription CPs. Both types of CPs are sensitive towards congestion. It is assumed that subscription CPs are more sensitive to congestion as the IUs pay for the content and thus should expect to get what they pay for. Also many subscription CPs are traffic heavy, such as video games and movie streaming services. That is in general $h_{j_S} > h_{j_A}$.

Advertising CPs

The advertising CPs generate revenues that are dependent on the number of visitors and their click through rate. There are no internal externalities among the advertising CPs, except that more CPs mean more congestion, which lowers the revenues for all CPs (as well as the ISPs).The average number of clicks per site per IU, κ_{jA} is dependent on the quality of the CP. If the congestion sensitivity is h_{jA}, the congestion w_i, the click through rate $\left(1 - h_{j_A} w_a^N\right)$, the advertising revenues, r, the share of customers, x_i, f_c the unit cost for traffic by connecting to the backbone ISP and the cost function, c_{jA}, the profit of a CP in the NN scenario can be modelled as

$$\Pi_{j_A}^{N} = \begin{cases} r\kappa_{j_A} \sum_{i\epsilon\{a,b\}} x_i^N \left(1 - h_{j_A} w_{j_A i}^N\right) - \lambda_{j_A}\kappa_{j_A} \sum_{i\epsilon\{a,b\}} f_c^N x_i^D - c_{j_A} \\ 0 \end{cases} \quad if \quad \begin{matrix} 1 \\ 2 \end{matrix} \qquad (63)$$

Where the two options are

1. The CP enters the market
2. The CP does not enter the market

In the neutral regime (N) the CPs can only choose to enter the market. However, the CPs cannot choose which customers to send its content to. The revenues from advertising will never be negative. However, due to costs of operating the service, c_{jA}, some CPs will choose not to enter the market.

The QoS cannot be higher than the weakest link in the chain, which means that the QoS experienced by the CPs users is the following

$$w_{j_A i}^N \geq w_i^N, w_{j_A i}^N \geq w_c^N, i\epsilon\{a,b\} \qquad (64)$$

In the differentiation scheme (D) the CPs can choose whether to pay for SS or not, or whether they want to compete in the market or not. If f_i is the price per traffic unit on SS charged by the ISPs, and λ_{jA} is the traffic per click per IU, the profit functions depending on their choices can be described as follows

$$\Pi_{j_A}^{D} = \begin{cases} r\kappa_{j_A} \sum_{i\epsilon\{a,b\}} x_i^D \left(1 - h_{j_A} w_{j_A i}^{DSS}\right) - \lambda_{j_A}\kappa_{j_A} \sum_{i\epsilon\{a,b\}} (f_c^{DSS} + f_c + f_i) x_i^D - c_{j_A} \\ r\kappa_{j_A} \sum_{i\epsilon\{i,-i\}} x_i^D \left(1 - h_{j_A} w_{j_A i}^{DSS/IA}\right) - (f_c^{DSS} + f_c^D + f_i)\lambda_{j_A}\kappa_{j_A} x_i^D - f_c^D \lambda_{j_A}\kappa_{j_A} x_{-i}^D - c_{j_A} \\ r\kappa_{j_A} \sum_{i\epsilon\{a,b\}} x_i^D \left(1 - h_{j_A} w_{j_A i}^{DIA}\right) - \lambda_{j_A}\kappa_{j_A} \sum_{i\epsilon\{a,b\}} f_c^D x_i^D c_{j_A} \\ 0 \end{cases} \quad if \quad \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} \qquad (65)$$

where the four options are

1. The CP connects to SS from both access ISPs and backbone ISP
2. The CP connects to SS on access ISP i, and IAS on ISP -i, and both SS and IAS from backbone ISP
3. The CP connects to IAS on both access ISPs and the backbone ISP
4. The CP does not choose to enter the market

This is a simplification, as the CP might also choose to go SS on access ISPs and IAS on the backbone ISP, provided the general QoS of the backbone is higher than of the access ISP. However, to simplify the analysis it is assumed that the CPs either connect to SS on both backbone and access ISPs, or none. Because this analysis will be made in the special case of symmetric access ISPs, option 2 will not be considered as the CPs will always choose both or neither access ISPs when evaluation SS cs. IAS.

Immediately it might look as all CPs will favor SS, as the cost of connecting to SS is proportional to the amount of traffic generated by the ISP, which is almost the same as with revenues. However, the benefit from joining SS depends on the difference in click through rate IAS vs. SS. Also, if the sensitivity towards congestion, h_{jA} is small, the difference between SS and IAS will not matter much for the CPs revenues, and it might not choose to enter SS.

Again, the QoS cannot be higher than the weakest link in the chain, which means that the QoS in the four alternatives are the following

$$\begin{matrix} w_{j_A i}^{DSS} \geq w_i^{DSS}, w_{j_A i}^{DSS} \geq w_c^{DSS} \\ w_{j_A i}^{DSS} \geq w_i^{DSS}, w_{j_A i}^{DSS} \geq w_c^{DSS}, w_{j_A -i}^{DIA} \geq w_{-i}^{DIA}, w_{j_A -i}^{DIA} \geq w_c^{DIA} \\ w_{j_A i}^{DIA} \geq w_i^{DIA}, w_{j_A i}^{DIA} \geq w_c^{DIA} \\ 0 \end{matrix} \quad if \quad \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix}, i \epsilon \{a, b\} \tag{66}$$

Subscription CPs

To model the subscription CPs we assume Bertrand competition with differentiated goods, as there is a limit to how much content the IUs can consume, and thus the subscription CPs have to compete on price. In this respect, we see the IUs time and ability to consume content as a scarce resource. The IUs obtain the following utility from interacting with a subscription CP j_S

$$u_{xj_S}^{N/D} = \iota_{j_S} \lambda_{j_S} \left(1 - h_{j_S} w^{N/D}\right) - v_{j_S}^{N/D} - \frac{1}{N} \sum_{i \neq j_S} \left(\lambda_i \left(1 - h_i w^{N/D}\right) - v_i^{N/D}\right) \tag{67}$$

Here, ι_{J_S} is the fraction of IUs that are indifferent to the offer, which is multiplied with the utility offer made by CP j_S. The second term is disutility from the price set by the CP j_S, the third term is disutility from the opportunity IUs have to connect to competing CPs. If we assume that all the CPs have the same utility offer $\lambda_{j_S}\left(1 - h_{j_S} w^{N/D}\right)$: By setting the utility equal to zero we can obtain an expression for the fraction of demand that is generated by the selected price and QoS, we obtain

$$\eta_{j_S} = 1 - \iota_{j_S} = \frac{1}{N} - \frac{v_{j_S}^{N/D}}{\lambda_{j_S}\left(1 - h_{j_S} w^{N/D}\right)} + \frac{\sum_{i \neq j_S} v_i^{N/D}}{\lambda_{j_S}\left(1 - h_{j_S} w^{N/D}\right) N} \tag{68}$$

Lemma 1: The optimal price under Bertrand competition, when the competing CPs are homogeneous in demand functions and marginal costs, but heterogeneous in IUs preferences, and the CPs are homogenous, for CP j_S is

$$v_{j_S}^{N/D} = \frac{\lambda_{j_S}\left(1 - h_{j_S} w_{j_S i}^{D/N}\right) + Nc}{N+1} \quad (69)$$

The proof of Lemma 1 can be seen in the Appendix in section 8.1, **(176)-(181)**. It should be noted that marginal cost c is equal to ($c_{js} + f_c\lambda_{js}$) in the NN regime, and ($c_{js} + f_c\lambda_{js} + f_i\lambda_{js}$) is the NNN regime if the CP is active in SS, as the CP then has to pay extra for sending packages in the priority lane.

The formula (69) implies that price goes up as the QoS gets better, but goes down with increasing competition. When the competition is infinite (perfect competition), the price is as expected equal to the marginal cost, c. The assumption that the CPs are homogenous is obviously only valid for small N, as the continuum de facto are defined as heterogeneous. However, we assume in the model that neighboring CPs can be modelled as homogenous. This also implies that the price of the competing CPs will be equal.

Lemma 2: The fraction of demand for a single CP under Bertrand competition, when the competing CPs are homogeneous in demand functions and marginal costs, but heterogeneous in IUs preferences, and the CPs are homogenous, for CP j_S is

$$\eta_{ij_S} = 1 - \iota_{j_S} = \frac{1}{N+1}\left(1 - \frac{c}{\lambda_{j_S}\left(1 - h_{j_S} w_{j_S i}^{N/D}\right)}\right) \quad (70)$$

The proof of Lemma 2 can be seen in the Appendix in section 8.1, (182) -(188). We see that the QoS will increase both demand and price, and thus should improve the revenues of CPs. It is also worth noticing that more competition, a higher N, decrease demand and the service price for the single CP, which will decrease its revenues, which is not surprising. However, overall demand increases significantly with higher levels of competition

Lemma 3: Assuming Bertrand competition, homogeneous CPs in demand functions and marginal costs, but heterogeneous in IUs preferences, the total fraction of demand covered is equal to

$$\eta_T = \frac{N}{N+1}\left(1 - \frac{c}{\lambda_{j_S}\left(1 - h_{j_S} w^{N/D}\right)}\right) \quad (71)$$

Lemma 3 is obtained by multiplying the demand of a single CP, (70), with N. Lemma 1 and Lemma 3 combined implies that more intense competition, a higher N, means significantly higher consumer surplus and social welfare from consumers interacting with the CPs because the price is lower and total demand covered higher. Thus the level of competition is important for the overall consumer surplus in the model, as a higher level of competition will move surplus from the CPs to the IUs.

With the price and fraction of demand served, we can now formulate the profit functions of the subscription CPs. The profit in the NN regime is

$$\Pi_{js}^{N} = \left(v_{js}^{N} - c_{js}\right) \sum_{i\epsilon\{a,b\}} \eta_{ijs}^{N} x_i^{N} \tag{72}$$

The revenues are thus dependent on the price the CPs set, and the number of IUs that connects, which is dependent on price and the QoS. As for the advertising CPs, the QoS cannot be higher than the weakest link in the network

$$w_{jsi}^{N} \geq w_i^{N}, w_{jsi}^{N} \geq w_c^{N} i\epsilon\{a,b\} \tag{73}$$

In an NNN regime, the CPs can choose to connect to SS, or stay in IAS for free. The profit depending on the their choices is then

$$\Pi_{js}^{D} = \begin{cases} \left(v_{js}^{D} - c_{js}\right) \sum_{i\epsilon\{a,b\}} \eta_{ijs}^{D} x_i^{D} - \lambda_{js} \sum_{i\epsilon\{a,b\}} f_i \eta_{ijs}^{D} x_i^{D} \\ \left(v_{js}^{D} - c_{js}\right) \sum_{i\epsilon\{i,-i\}} \eta_{ijs}^{D} x_i^{D} - \lambda_{js} f_i \eta_{ijs}^{D} x_i^{D} & \quad if \quad \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} \\ \left(v_{js}^{D} - c_{js}\right) \sum_{i\epsilon\{a,b\}} \eta_{ijs}^{D} x_i^{D} \\ 0 \end{cases} \tag{74}$$

where the four options are

1. The CP connects to SS from both access ISPs and backbone ISP
2. The CP connects to SS on access ISP i, and IAS on ISP -i, and both SS and IAS from backbone ISP
3. The CP connects to IAS on both access ISPs and the backbone ISP
4. The CP does not choose to enter the market

What applies for the scenarios regarding the advertising CPs, also applies for the subscriber CPs. That is only option 1, 3 and 4 are possible outcomes in the symmetrical analysis.

The optimal price formula (69) will not be valid in scenario 2, SS on one ISP and IAS on the other, or in general where the QoS varies on different lanes. However, this analysis will be in a special symmetrical case where the ISPs are homogenous, and thus a CP will either connect to SS on both or neither of the ISPs, and the QoS will be equal on both access lanes. Therefore, the optimal price formula is valid throughout the whole analysis. In a non-symmetrical case, the optimal price would have to be calculated from maximizing the revenues from IUs at both ISPs simultaneously, where IUs through ISP i have SS and IUs through ISP –i only have IAS.

Again, the QoS is dependent on both the access and backbone ISP, and cannot be higher than the weakest link in the chain, which means that the QoS in the four alternatives are the following

$$\begin{matrix} w_{jsi}^{DSS} \geq w_i^{DSS}, w_{jsi}^{DSS} \geq w_c^{DSS} \\ w_{jsi}^{DSS} \geq w_i^{DSS}, w_{jsi}^{DSS} \geq w_c^{DSS}, w_{js-i}^{DIA} \geq w_{-i}^{DIA}, w_{js-i}^{DIA} \geq w_c^{DIA} \\ w_{jsi}^{DIA} \geq w_i^{DIA}, w_{jsi}^{DIA} \geq w_c^{DIA} \\ 0 \end{matrix} \quad if \quad \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix}, i \epsilon \{a, b\} \qquad (75)$$

The access ISPs

There are two ISPs modelled as in a linear city between 0 and 1, where each ISP is on one of the extremes. In the NN regime, the ISPs make their income by providing connectivity to the IUs. They have to pay to serve the users of their network, which can be seen as a variable cost, and invest in broadband capacity to serve the traffic demand generated from the CPs and IUs. Under net neutrality the ISP profit is

$$\Pi_i^N = p_i^N x_i^N - C^V(\bar{j}_{Ai}^N, \bar{j}_{Si}^N, x_i^N) - C^I(\mu_i^N) \qquad (76)$$

The function C^V represents variable operating costs (e.g. customer service, electricity and maintenance), while C^I are the costs of investing in more capacity.

In the NNN regime, the ISP can also charge a fee f_i, which is a fee per package (traffic) the CPs have to pay in order to enter the BE lane. The profit in the NNN regime for ISP i is then

$$\Pi_i^D = p_i^D x_i^D + f_i \Lambda_i^{DSS} - C^V(\bar{j}_{Ai}^D, j_{Si}^D, x_i^D) - C^I(\mu_i^D) \qquad (77)$$

The function Λ_i^{DSS} () is dependent on how many CPs that choose SS and their need to send content, and is equal to

$$\Lambda_i^{DSS} = \int_{\bar{J}_{Ai}^D}^{\bar{J}_{Ai}^D} \lambda_{j_A} \kappa_{j_A} x_i^D dj_A + \int_{\bar{J}_{Si}^D}^{\bar{J}_{Si}^D} \lambda_{j_S} \eta_{ij_S}^D x_{ij_S}^D dj_S \qquad (78)$$

In the NNN scenario, the ISPs have to set the price of connectivity, p_i, the basic price/package in the priority lane, f_i, and the amount to invest, μ_i, in order to maximize their profit function Π_i^D. The ISPs can also set the QoS of SS and IAS according to the discussion in the section presenting congestion. It is also assumed that the ISPs have a base capacity, μ_i^0, which means that the ISPs does not really have to invest in extra capacity to serve demand, especially if the initial capacity is high and demand for traffic is low.

The backbone ISP

To reduce complexity, and since the focus is on the access ISPs, the backbone ISPs are modelled as one player. The CPs only have to connect to one backbone ISP. To avoid the effects of monopoly pricing the mark-up is reduced through the competition parameter, φ, which simulates competition. Since the model does not differ from traffic going up and down the network, and that it obviously has to be in balance between the access ISPs and the backbone ISPs within this ecosystem, a settlement free peering is assumed. Thus, the ISPs in this model are considered to be Tier 1 ISPs, which means that the ISPs do not purchase any transit service, peers with all the other ISPs and pay no settlement on their peering agreement [24]. In reality there would also be many smaller ISPs that have to pay larger ISPs for access to the Internet through a transit agreement.

The backbone ISP invests in a capacity, μ_c, to serve the CPs, and obtains income from providing connectivity to the CPs, which pay a price, f_c, per traffic. The backbone ISP also has a variable cost C^V dependent on the amount of CPs that connect to the backbone ISP, and C^I dependent on how much capacity the ISP invest in. The profit of the backbone ISP in the NN regime becomes

$$\Pi_c^N = f_c^N \Lambda_c^N - C^V(\bar{J}_A^N, \bar{J}_S^N) - C^I(\mu_c^N) \qquad (79)$$

The function C^V represents variable operating costs (e.g. customer service, electricity and maintenance), while C^I are the costs of investing in more capacity.

The total traffic, $\Lambda_c^{D/N}$ is equal to all the traffic sent from the CPs to the IUs on both access ISPs

$$\Lambda_c^L = \int_0^{\bar{j}_A^L} \lambda_{j_A} \kappa_{j_A} (x_a^L + x_b^L) dj_A + \int_0^{\bar{j}_{Si}^D} \lambda_{j_S} (\eta_{aj_S}^L x_{aj_S}^L + \eta_{bj_S}^L x_{bj_S}^L) dj_S \tag{80}$$

In the NNN regime, the ISP can also charge a fee f_c^{DSS}, which is a fee per package (traffic) the CPs have to pay in order to enter the SS lane in addition to the fee f_c^D for access to the network. The profit in the NNN regime for ISP i is then

$$\Pi_c^D = +f_c^D \Lambda_c^D + f_c^{DSS} \Lambda_c^{DSS} - C^V(\bar{J}_A^D, j_S^D) - C^I(\mu_c^D) \tag{81}$$

Where the traffic on the SS lane equals

$$\Lambda_c^{DSS} = \int_{\tilde{j}_{Ai}^D}^{\bar{j}_A^D} \lambda_{j_A} \kappa_{j_A} (x_a^D + x_b^D) dj_A + \int_{\tilde{j}_{Si}^D}^{\bar{j}_{Si}^D} \lambda_{j_S} (\eta_{aj_S}^D x_{aj_S}^D + \eta_{bj_S}^D x_{bj_S}^D) dj_S \tag{82}$$

The regulators

The regulators are concerned about how SS will affect IAS, and thus might be interested in putting restrictions on the QoS of IAS. If Ψ is a constant set by the regulators, the restriction posed upon the access ISPs from the regulators can be modelled differently based on type of restriction. One alternative is

$$w_i^{DIA} \leq \Psi \tag{83}$$

Where Ψ > 0. This type of restriction requires an absolute minimum QoS, and therefore poses requirements on the minimum capacity. Also, this restriction does not adapt to traffic, which means that in some scenarios the QoS of IAS might be higher than SS, which obviously will not be tolerated by the CPs that pay for SS. Also, during times of extreme traffic this restriction might be impossible to comply altogether. However, this restriction is easy to monitor which makes is a practical, and thus realistic, alternative. Another alternative is

$$w_i^{DIA} \leq \Psi w_i^{DSS} \tag{84}$$

Where Ψ >1. The type of restriction controls how much better the QoS can be on SS relative to IAS. It is important to remember that lower w (higher QoS) is better. This type of restriction does not set any restriction on the amount of capacity that is built. Also this restriction adapts to different levels of congestion, which makes it possibly to comply regardless of the amount of traffic. Therefore, this type of restriction would probably be a better regulation mechanism. However, much more complicated to monitor and implement in practice. Another alternative is a combination of the two

$$\begin{gathered} w_i^{DIA} \leq \Psi_1 + My \\ w_i^{DSS} \geq \Psi_1 y \\ w_i^{DIA} \leq \Psi_2 w_i^{DSS} + M(1-y) \\ y \in \{0,1\} \end{gathered} \tag{85}$$

Here M is a number big enough to relax either the first or third line of the restriction. This restriction requires a minimum QoS of the IAS. However, when the traffic is so high that the QoS of SS will be worse than Ψ_1, that is IAS becomes better than SS, the alternative restriction is a relative QoS requirement between IAS and SS. Thus under normal operation y = 0, and a minimum QoS of IAS is assured. However, during periods with peak levels of traffic, the requirement can be relative so that the QoS of SS is never lower than the QoS of IAS. However, the QoS of IAS and SS might be equal. The benefit of this restriction is the ability to have strict requirements, while still being flexible concerning variability in traffic. However, this type of restriction does also require a minimum invested capacity. In reality, there is already a built in minimum level of invested capacity, which is controlled by the marked. In order to be competitive, an ISP always has to provide equal QoS relative to price to its competitors. Therefore, the minimum QoS requirement in (83) and (85) might be considered to be redundant, and the relative QoS restriction (84) most suitable for the regulators and ISPs.

The IUs

The IUs get utility from the advertising CPs and by subscribing to CPs. The utility from advertising *CPs* is dependent on the marginal utility from receiving the content, m_A, and the total content received, $\lambda_{jA}\kappa_A$ less a factor determined by the congestion, w, and congestion sensitivity, h_{jA}. The utility from content received by Advertising CPs can thus be modelled as

$$u_{xi}^{AN} = m_A \int_0^{\bar{J}_{Ai}^N} \lambda_{j_A} \kappa_{j_A} \left(1 - h_{j_A} w_{j_A i}^N\right) dj_A \tag{86}$$

In the NN regime, and

$$u_{xi}^{AD} = m_A\left(\int_0^{\tilde{j}_{Ai}^{D}} \lambda_{j_A}\kappa_{j_A}\left(1-h_{j_A}w_{j_Ai}^{DIA}\right)dj_A + \int_{\tilde{j}_{Ai}^{D}}^{\bar{j}_{Ai}^{D}} \lambda_{j_A}\kappa_{j_A}\left(1-h_{j_A}w_{j_Ai}^{DSS}\right)dj_A\right) \quad (87)$$

in the NNN regime. The utility is dependent on the amount of content received from the CP, and the CP's congestion sensitivity. This means that very congestion sensitive CPs may in certain scenarios provide less utility than CPs that provide little content, but are not sensitive to congestion. This will be dependent on the QoS provided by the ISP and distribution of CPs.

The utility from the subscription *CPs* is the consumer surplus from the demand that is served by the subscribing CPs. It is the obtained by taking the integral of the demand function (68), with (69) substituted as the competitors price, from the clearing price to the maximum obtainable price. Then we take the integral over all the subscribing CPs. In the NN regime

$$u_{xi}^{SN} = \int_o^{\bar{j}_{Si}^{N}}\int_{v_{clear}^{N}}^{v_{max}^{N}}\left(\frac{2\lambda_{j_S}\left(1-h_{j_S}w_{j_Si}^{N}\right)+(N-1)c}{\lambda_{j_S}\left(1-h_{j_S}w_{j_Si}^{N}\right)(N+1)} - \frac{v_{j_S}^{N}}{\lambda_{j_S}\left(1-h_{j_S}w_{j_Si}^{N}\right)}\right)dv_{j_S}^{N}dj_S \quad (88)$$

is the consumer surplus. In the NNN regime we have to split SS and IAS

$$\begin{aligned} u_{xi}^{SD} = &\int_0^{\tilde{j}_{Si}^{D}}\int_{v_{clear}^{D}}^{v_{max}^{D}}\left(\frac{2\lambda_{j_S}\left(1-h_{j_S}w_{j_Si}^{DIA}\right)+(N-1)c}{\lambda_{j_S}\left(1-h_{j_S}w_{j_Si}^{DIA}\right)(N+1)} - \frac{v_{j_S}^{D}}{\lambda_{j_S}\left(1-h_{j_S}w_{j_Si}^{DIA}\right)}\right)dv_{j_S}^{N}dj_S \\ &+ \int_{\tilde{j}_{Si}^{D}}^{\bar{j}_{Si}^{D}}\int_{v_{clear}^{D}}^{v_{max}^{D}}\left(\frac{2\lambda_{j_S}\left(1-h_{j_S}w_{j_Si}^{DSS}\right)+(N-1)c}{\lambda_{j_S}\left(1-h_{j_S}w_{j_Si}^{DSS}\right)(N+1)} - \frac{v_{j_S}^{D}}{\lambda_{j_S}\left(1-h_{j_S}w_{j_Si}^{DSS}\right)}\right)dv_{j_S}^{D}dj_S \end{aligned} \quad (89)$$

The IUs also obtain a numéraire utility from connecting to an ISP, Y. However, loose utility p_i from the price charged by the CP, and the possible utility offered by the other ISP. This is modelled through the Hotelling transfer cost, t. The utility of IU x in the NN regime from connecting to ISP i is thus the sum of utility obtained from connecting to the ISP, connecting to advertising and subscribing CPs and the disutility of price and opportunity to connect to a competing ISP

$$U_{ix}^{N} = Y + \mathrm{u}_{xi}^{AN} + \mathrm{u}_{xi}^{SN} - p_i^N - tx_i^N \tag{90}$$

And in the differentiation, NNN, regime

$$U_{ix}^{D} = Y + \mathrm{u}_{xi}^{AD} + \mathrm{u}_{xi}^{SD} - p_i^D - tx_i^D \tag{91}$$

Congestion

To model the congestion in the system, the M/M/1 model is adopted. This model has been widely used in operations research by many scholars to study congestion problems and priority pricing [8]. The M/M/1 model is also widely used in previous research in the NN debate [2] [8] [1]. The reason for choosing this system is because it captures the effect of capacity being a scarce resource, and it fits a system where the arrival of customers is so large that the impact of a single customer on the performance of the system is very small [8].

In the NN regime, all packages are treated equally according to BE. If μ_i^{NT} is the total capacity of ISP i in the NN regime and Λ_i the total traffic, the average congestion for ISP i is

$$w_i^N = \frac{1}{\mu_i^{NT} - \Lambda_i^N} \tag{92}$$

which is equal for all packages, and where the total traffic Λ_i^L in both regimes, $L \in \{N,D\}$, is equal to

$$\Lambda_i^L = \int_0^{\bar{J}_{Ai}^L} \lambda_{j_A} \kappa_{j_A} x_i^L dj_A + \int_0^{\bar{J}_{Si}^L} \lambda_{j_S} \eta_{ij_S}^L x_i^L dj_S, \mathrm{i} \in \{\mathrm{a}, \mathrm{b}\} \tag{93}$$

, and

$$\mu_i^{LT} = \mu_i^0 + \mu_i^L, \forall \mathrm{i} \tag{94}$$

For the backbone ISP, the value of x is 1 because all traffic has to go through the backbone ISP.

In the NNN regime the access ISPs can set a higher standard in the SS lane, which can be no higher than

$$w_i^{DSS} \geq \frac{1}{\mu_i^{DT} - \beta_i \Lambda_i^D}, \forall i \qquad (95)$$

Here the β_i is the share of total CPs that pay for SS on ISP i, which can be modelled as

$$\beta_i = \frac{\Lambda_i^{DSS}}{\Lambda_i^D}, \forall i \qquad (96)$$

Where Λ_i^{DSS} can be calculated from equation (78) or (82). Equation (95) says that SS is restricted to that it can be no better than if all the packaged in SS are sent directly first in line. However, the QoS can be SS if chosen to be so by the ISP. Depending on the QoS, w^{DS}_i, set by the ISP, the QoS of IAS can be no better than

$$w_i^{DIA} \geq \frac{w_i^D - \beta_i w_i^{DSS}}{(1 - \beta_i)}, \forall i \qquad (97)$$

Here w^D_i is the average congestion in the whole network if ISP i, which equals

$$w_i^D = \frac{1}{\mu_i^{DT} - \Lambda_i^D}, \forall i \qquad (98)$$

The formulation implies that the ISPs can degrade the quality of IAS should it want to, and if it is allowed by the policy makers. However, it is worth noticing that degrading the quality of IAS does not improve the QoS of SS, or at least not above the maximum QoS as illustrated by (95). If the ISP sets the average QoS equal to the best possible according to invested capacity, the QoS of the IAS is

$$w_i^{DIA} = \frac{\mu_i^{DT}}{\mu_i^{DT} - \Lambda_i^D} w_i^{DSS}, \forall i \qquad (99)$$

This is obviously higher than the QoS of SS. The QoS levels have the property that $w^{DSS} \leq w^D \leq w^{DIA}$ as long as the QoS of SS is equal or better than the systems total QoS. It would not make sense for the ISP to set the QoS lower (or equal) to the system's total QoS, as no rational CP would be willing to pay for that.

5.3 Simplified Mathematical Model that Allows Analytical Treatment

In this section the general model has been simplified in order to make it easier to solve analytically, and thus bear much resemblance to the work by Krämer et al. and Bourreau et al. However, this model still includes the aspect of E2E by including a backbone ISP, and the aspect of regulation on the IAS lane. Both are important extensions to better understand the effects of a transition to an NNN regime. First, the notation will be presented, followed by the equations of the mathematical model.

Notation

$y \in [0,\rightarrow)]$	Continuum of CPs
$i \in \{a, b, c\}$	Access ISP a and b, and backbone ISP c
$x \in [0,1]$	Unit mass of IUs, modelled as residents in a linear city between 0 and 1
$L \in \{N,D\}$	Superscript to separate between a neutral (N) regime or a non-neutral, differentiation, regime (D)
SS	Specialized Services (superscript)
IA	Internet Access Services (superscript)
r	Advertising revenue per advertising traffic unit per IU
λ	Package traffic unit per IU for CPs
h	Sensitivity towards congestion for CPs
$C^V(x_i^L)$	Variable costs for the access ISPs of providing connectivity to IUs
$C_c^V(\bar{y}_i^L)$	Variable costs for the backbone ISP of providing connectivity to the CPs
$C^I(\mu_i^L)$	Investments costs of the CPs from investing in capacity
Ψ	Minimum QoS of IAS set by regulators
R	IUs numéraire utility of being provided connectivity
u_C	Utility per CPs for IUs
u_w	QoS utility for IUs
t	Standard Hotelling unit cost of transportation
ω	Market power of backbone ISP
μ_i	Capacity as measured by package traffic per time unit
w	Measurement of congestion as average waiting time per packet
β_i	Share of CPs in SS for ISP i
f_i^{DSS}	Connection fee to SS on access ISP a or b
f_c^{DSS}	Connection fee to SS on the backbone ISP
f_c	Connectivity price for CPs on backbone ISP
p_i	Price for IUs connectivity for IUs at ISP a and b

Table 13: Indexes, parameters and variables of the simplified mathematical model

Decision and timing under NN	*Decisions and timing under NNN*

0. Exogenous parameters set
1. Backbone and access ISPs decisions
  a. IUs connectivity price (access only)
  b. Investment levels (access only)
  c. Connectivity fee CPs (backbone only)
2. CPs and ISPs decisions
  a. CPs choose which access ISPs to connect to
  b. IUs choose which platform, a or b, to connect to
	0. Exogenous parameters set, including regulators restrictions
	1. Backbone and access ISPs decisions
	  a. IUs connectivity price (access only)
	  b. Investment levels (access only)
	  c. Connectivity fee CPs (backbone only)
	  d. Connectivity fee for SS (both)
	2. CPs and ISPs decisions
	  a. CPs choose which access ISPs to connect to, and either SS or IAS
	  b. IUs choose which platform, a or b, to connect to

Table 14: Timing and decisions in the NN and NNN regime. Decisions under the same number are taken simultaneously, and the players have the same information.

The CPs

NN regime

$$\Pi_y^N = \begin{cases} \sum_{i\epsilon\{a,b\}} \left[r\lambda x_i^N\left(1 - h y_i^N w_i^N\right) - f_c^N\right] & if \quad \textit{it enters the market} \\ 0 & \textit{otherwise} \end{cases} \qquad (100)$$

NNN regime

$$\Pi_y^D = \begin{cases} \sum_{i\epsilon\{a,b\}} \left[r\lambda x_i^D\left(1 - h y_i^D w_i^{DSS}\right) - f_c^D - f_i^{DSS} - f_c^{DSS}\right] & 1 \\ \sum_{i\epsilon\{a,b\}} \left[r\lambda x_i^D\left(1 - h y_i^D w_i^{DSS/IA}\right) - f_c^D\right] - f_i^{DSS} - f_c^{DSS} & if \quad 2 \\ \sum_{i\epsilon\{a,b\}} \left[r\lambda x_i^D\left(1 - h y_i^D w_i^{DIA}\right) - f_c^D\right] & 3 \\ 0 & 4 \end{cases} \qquad (101)$$

where the four options are

1. The CP connects to SS from both access ISPs and backbone ISP
2. The CP connects to SS on access ISP i, and IAS on ISP -i, and both SS and IAS from backbone ISP
3. The CP connects to IAS on both access ISPs and the backbone ISP
4. The CP does not choose to enter the market

The access ISPs

NN regime

$$\Pi_i^N = p_i^N x_i^N - C^V\left(x_i^N\right) - C^I\left(\mu_i^N\right), \mathrm{i}\epsilon\{\mathrm{a,b}\} \tag{102}$$

NNN regime

$$\Pi_i^D = p_i^D x_i^D + f_i^{DSS}\left(\bar{y}_i^D - \tilde{y}_i^D\right) - C^V\left(x_i^D\right) - C^I\left(\mu_i^D\right), \mathrm{i}\epsilon\{\mathrm{a,b}\} \tag{103}$$

The backbone ISP

NN regime

$$\Pi_c^N = \sum_{i\epsilon\{a,b\}} \left[f_c^N \bar{y}_i^N - C_c^V(\bar{y}^N)\right] \tag{104}$$

NNN regime

$$\Pi_c^D = \sum_{i\epsilon\{a,b\}} \left[f_c^D \bar{y}_i^D + f_c^{DSS}\left(\bar{y}_i^D - \tilde{y}_i^D\right) - C_c^V(\bar{y}^D)\right] \tag{105}$$

Regulators

$$w_i^{DIA} \leq \Psi \tag{106}$$

The IUs

NN regime

$$U_{ix}^N = R + u_C \bar{y}_i^N + \frac{u_w}{w_i^N} - p_i^N - t x_i^N \tag{107}$$

NNN regime

$$U_{ix}^D = R + u_C \bar{y}_i^D + \frac{u_w}{w_i^D} - p_i^D - t x_i^D \tag{108}$$

Congestion

$$\Lambda_i^L = \int_0^{\bar{y}_i^L} \lambda x_i^L dy_i^L = \lambda x_i^L \bar{y}_i^L, \text{i}\epsilon\{\text{a, b}\}, \forall \text{L} \tag{109}$$

$$\Lambda_i^{DSS} = \int_{\tilde{y}_i^D}^{\bar{y}_i^D} \lambda x_i^D dy_i^D = \lambda x_i^D \left(\bar{y}_i^D - \tilde{y}_i^D\right), \text{i}\epsilon\{\text{a, b}\} \tag{110}$$

$$\beta_i = \frac{\Lambda_i^{DSS}}{\Lambda_i^D} = 1 - \frac{\tilde{y}_i^D}{\bar{y}_i^D}, \forall \text{i} \tag{111}$$

NN regime

$$w_i^N = \frac{1}{\mu_i^N - \Lambda_i^N} = \frac{1}{\mu_i^N - \lambda x_i^N \bar{y}_i^N}, \text{i}\epsilon\{\text{a, b}\} \tag{112}$$

NNN regime

$$w_i^D = \frac{1}{\mu_i^D - \Lambda_i^D} = \frac{1}{\mu_i^D - \lambda x_i^D \bar{y}_i^D} = \beta_i w_i^{DSS} + (1 - \beta_i) w_i^{DIA}, \text{i}\epsilon\{\text{a, b}\} \tag{113}$$

$$w_i^{DSS} = \frac{1}{\mu_i^D - \beta_i \Lambda_i^D} = \frac{1}{\mu_i^D - \lambda x_i^D \left(\bar{y}_i^D - \tilde{y}_i^D\right)}, \text{i}\epsilon\{\text{a, b}\} \tag{114}$$

$$w_i^{DIA} = \frac{\mu_i^D}{\mu_i^D - \Lambda_i^D} w_i^{DSS} = \frac{\mu_i^D}{\mu_i^D - \lambda x_i^D \bar{y}_i^D} w_i^{DSS}, \text{i}\epsilon\{\text{a, b}\} \tag{115}$$

Some necessary conditions

For IUs to value the presence of CPs in the symmetric equilibrium, the following must apply

$$u_C > \frac{u_w \lambda}{2} \tag{116}$$

This can be seen by inserting (112) into (107) or (113) into (108)

$$U_{ix}^D = R + \left(u_C - u_w \lambda x_i^D\right) \bar{y}_i^D + u_w \mu_i^D - p_i^D - t x_i^D \tag{117}$$

The value in the bracket must be positive.

From the solutions of $\bar{y}$ and f_c

$$\frac{2\mu^N(r\lambda - 2f_c^N) - r\lambda^2 + 2\lambda f_c^N}{2r\lambda} < h \quad (118)$$

$$h > \frac{2C_c^V - r\lambda}{2r} \quad (119)$$

This model is allows analytical solutions, and the results are presented in the next section.

5.4 Solutions to the Simplified Mathematical Model

The following section contains the solutions to the simplified model, as well as solutions from a reduced version of the simplified model where the backbone SS-lane has been removed. This is to obtain explicit solutions, whereas the model including the backbone ISP SS fee can only be solved implicitly. This section only provides the solutions, and readers interested in how the solutions were derived from the mathematical model can find the process of deriving the solutions, as well as the second order conditions for optimality, in the appendix section 0. The next paragraphs will briefly summarize the assumptions that were made to derive the solutions, before the solutions are presented.

The model is solved under the assumption of perfect information. This is not realistic, as strategic and financial information is usually closely guarded secret by most corporations. In a realistic scenario one would expect the large ISPs to have more information than the other players, because they directly interact with all players in the ecosystem. Not just all the types of players, like the CPs that actually also deal with all the types of players, but with literary all the players, as one ISP might do business with all CPs, but a CP usually does not do business with many other CPs. It might be that ISPs could take advantage of this asymmetry. However, to reduce complexity of the analysis this will not be considered in this model.

Another simplification is that ISP a and b are symmetrical. The analysis is meant to analyze the effect between the types of players, and not effects within a type of players. E.g. a large Tier 1 ISP could potentially squeeze out the smaller ISPs in one of the regimes. Such an analysis would require a much more complicated solution process, which is beyond the scope of this thesis. The effect of competing access ISPs is still included, which is important when analyzing effects between the types of players to avoid unrealistic high margins of the access ISPs.

When solving the model we consider its subgame perfect Nash equilibrium (SPE), where we focus on optimal decisions along the equilibrium paths [2] [31]. To solve the game, we used backward induction. The timing in the model as already been set, and thus when we start at the last stage, it is assumed that all previous decisions have been made and the information is available to all players.

The cost and investment functions were assumed to be the following when deriving the solutions

- $C^V(x_i^N) = C^V x_i^{D/N}$
- $C_c^V(\bar{y}_i^N) = C_c^V y_i^{D/N}$
- $C^I(\mu_i^N) = \frac{C^I}{2}\mu_i^{D/N^2}$ (same as Bourreau et al. [2])

NN regime

Proposition 2: In Net Neutrality, the ISPs investments, the IUs connection fee, the CPs connectivity fee, the participation level of the CPs and the average congestion are given by

$$\mu^N = \frac{1}{C^I}\frac{u_C(r\lambda - 2f_c^N) + u_w\lambda rh}{\lambda(r\lambda - 2f_c^N + 2rh)} \tag{120}$$

$$p^N = 4\mu^N\left(\frac{r\lambda(u_C(r\lambda - 4f_c^N) + r\lambda u_w h) + 4u_C f_c^N(f_c^N - rh)}{\lambda(r\lambda - 2f_c^N + 2rh)^2}\right) + C^V + t \tag{121}$$

$$f_c^N = \left(\frac{2rh + r\lambda - \sqrt{rh(4rh + 2r\lambda - 4C_c^V)}}{2}\right)\frac{1}{\omega} \tag{122}$$

$$\bar{y}^N = \frac{2\mu^N(r\lambda - 2f_c^N)}{\lambda(r\lambda - 2f_c^N + 2rh)} \tag{123}$$

$$w^N = \frac{r\lambda - 2f_c^N + 2rh}{2\mu^N rh} \tag{124}$$

Proposition 3: The total profit of CPs, the access and backbone ISPs, consumer surplus and social welfare in Net Neutrality is given by

$$\Sigma\Pi_{CSP}^N = \frac{\mu^N(r\lambda - 2f_c^N)^2}{\lambda(r\lambda - 2f_c^N + 2rh)} \tag{125}$$

$$\Sigma\Pi_{ISP_{back}}^{N} = \frac{4\mu^N(f_c^N - C_c^V)(r\lambda - 2f_c^N)}{\lambda(r\lambda - 2f_c^N + 2rh)} \quad (126)$$

$$\Sigma\Pi_{ISP_{access}}^{N} = 4\mu^N\left(\frac{r\lambda(u_c(r\lambda - 4f_c^N) + r\lambda u_w h) + 4u_c f_c^N(f_c^N - rh)}{\lambda(r\lambda - 2f_c^N + 2rh)^2}\right) + t - C^I {\mu^N}^2 \quad (127)$$

$$CS_{IU}^{N} = R + 2\mu^N\left(\frac{u_C(r\lambda - 2f_c^N) + u_w r\lambda h}{\lambda(r\lambda - 2f_c^N + 2rh)}\right) - 4\mu^N\left(\frac{r\lambda(u_C(r\lambda - 4f_c^N) + r\lambda u_w h) + 4u_c f_c^N(f_c^N - rh)}{\lambda(r\lambda - 2f_c^N + 2rh)^2}\right) - C^V - t\frac{5}{4} \quad (128)$$

$$W^N = \Sigma\Pi_{CSP}^{N} + CS_{IU}^{N} + \Sigma\Pi_{ISP_{access}}^{N} + \Sigma\Pi_{ISP_{back}}^{N} = R + (r\lambda - 2C_c^V)\bar{y}^N - \frac{r\lambda h {\bar{y}^N}^2 w^N}{2} + u_C\bar{y}^N + \frac{u_w}{w^N} - t\frac{1}{4} - C^V - C^I{\mu^N}^2 \quad (129)$$

The proof of Proposition 2and Proposition 3 can be seen in the appendix in section 8.2.1, along with the second order optimum conditions in section 8.2.4.

NNN regime

Proposition 4: In Non-Net Neutrality, the ISPs investments, the IUs connection fee, the CPs connectivity fee, the SS lane connectivity fees, the participation level of the CPs and the average congestion levels are given by

$$\mu^D = \frac{1}{C^I}\left(\frac{u_C(r\lambda - 2f_c^D) + u_w r\lambda h}{\lambda(r\lambda - 2f_c^D + 2rh)} + f_i^{DSS}\frac{(2r\lambda - 4f_c^D)(r\lambda - 2f_c^D - 2f_i^{DSS} - 2f_c^{DSS}) - 8rh(f_i^{DSS} + f_c^{DSS})}{\lambda(r\lambda - 2f_c^D - 2f_i^{DSS} - 2f_c^{DSS})(r\lambda - 2f_c^D + 2rh)}\right) \quad (130)$$

$$\mu^D \geq \frac{1}{\Psi}\frac{(r\lambda - 2f_c^D + 2rh)^2}{4(rh)^2(1 + f_i^{DSS} + f_c^{DSS})} \quad (131)$$

$$p^D = 4\mu^D\left(\frac{r\lambda(u_C(r\lambda-4f_c^D)+r\lambda u_w h)+4u_C f_c^D(f_c^D-rh)}{\lambda(r\lambda-2f_c^D+2rh)^2}\right)-f_i^{DSS}Y^D+C^V+t \qquad (132)$$

where

$$Y^D = \mu^D\left(\frac{(16rhf_c^D-(2r\lambda-4f_c^D)^2)(r\lambda-2f_c^D-2f)^2}{\lambda\left((r\lambda-2f_c^D-2f_i^{DSS}-2f_c^{DSS})(r\lambda-2f_c^D+2rh)\right)^2} - \frac{16rhf\left(4(f_c^D+f_i^{DSS}+f_c^{DSS})(rh-f_c^D)-r\lambda(3r\lambda-8f_c^D-4f_i^{DSS}-4f_c^{DSS}+4rh)\right)}{\lambda\left((r\lambda-2f_c^D-2f_i^{DSS}-2f_c^{DSS})(r\lambda-2f_c^D+2rh)\right)^2}\right) \qquad (133)$$

$$\begin{aligned} f_c^D:\ & 2[(r\lambda-2f_c^D)(r\lambda-2f_c^D+2rh) \\ & - 4rh(f_c^D+f_c^{DSS} \\ & - C_c^V)][(r\lambda-2f_c^D)(r\lambda-2f_c^D-2f_i^{DSS}-2f_c^{DSS})]^2 \\ & - 16f_c^{DSS}[rh(f_i^{DSS}+f_c^{DSS})(2r\lambda-4f_c^D-2f_i^{DSS}-2f_c^{DSS})(r\lambda \\ & - 2f_c^D)(r\lambda-2f_c^D+2rh) \\ & - 2(rh)^2(f_i^{DSS}+f_c^{DSS})(r\lambda-2f_c^D)(r\lambda-2f_c^D-2f_i^{DSS} \\ & - 2f_c^{DSS})] = 0 \end{aligned} \qquad (134)$$

$$\begin{aligned} f_i^{DSS}:\ & (r\lambda-2f_c^D)(r\lambda-2f_c^D-2f_i^{DSS}-2f_c^{DSS})^2 \\ & - rh(f_i^{DSS}+f_c^{DSS})(4r\lambda-8f_c^D-8f_i^{DSS}-8f_c^{DSS}) \\ & - rh(4r\lambda-8f_c^D)f_i^{DSS} = 0 \end{aligned} \qquad (135)$$

$$\begin{aligned} f_c^{DSS}:\ & (r\lambda-2f_c^D)(r\lambda-2f_c^D-2f_i^{DSS}-2f_c^{DSS})^2 \\ & - rh(f_i^{DSS}+f_c^{DSS})(4r\lambda-8f_c^D-8f_i^{DSS}-8f_c^{DSS}) \\ & - rh(4r\lambda-8f_c^D)f_c^{DSS} = 0 \end{aligned} \qquad (136)$$

$$\bar{y}^D = \frac{2\mu^D(r\lambda-2f_c^D)}{\lambda(r\lambda-2f_c^D+2rh)} \qquad (137)$$

$$\tilde{y}^D = \frac{4rh(f_i^{DSS}+f_c^{DSS})}{(r\lambda-2f_c^D)(r\lambda-2f_c^D-2f_i^{DSS}-2f_c^{DSS})}\bar{y}^D \qquad (138)$$

$$w^D = \frac{r\lambda - 2f_c^D + 2rh}{2\mu^D rh} \qquad (139)$$

$$w^{DSS} = \frac{(r\lambda - 2f_c^D - 2f_i^{DSS} - 2f_c^{DSS})(r\lambda - 2f_c^D + 2rh)}{2rh\mu^D(r\lambda - 2f_c^D)} \qquad (140)$$

$$w^{DIA} = \frac{(r\lambda - 2f_c^D - 2f_i^{DSS} - 2f_c^{DSS})(r\lambda - 2f_c^D + 2rh)^2}{4(rh)^2\mu^D(r\lambda - 2f_c^D)} \qquad (141)$$

If regulators set a minimum QoS on IAS, the following restriction is added

$$\mu^D \geq \frac{1}{\Psi}\frac{(r\lambda - 2f_c^D - 2f_i^{DSS} - 2f_c^{DSS})(r\lambda - 2f_c^D + 2rh)^2}{4(rh)^2(r\lambda - 2f_c^D)} \qquad (142)$$

The proof of these results can be seen in the appendix, in section 8.2.2.

As one can readily verify by observation, solving the equations numerically involves a complex procedure of solving for the implicit solutions given by (134)-(136). By solving this system of equations, the investment level given by (131) can be calculated, and from that all the other solutions are then obtained. So without these solutions, the others cannot be calculated. Numerical experimentation in Matlab 2012a using "fsolve" was not able to solve the system of equations. Therefore, a reduced version of the model without the backbone SS price, f_c^{DSS}, was used to derive direct numerical results. It is possible to provide a logic explanation to why this is a reasonable assumption to make. Including the backbone SS price would make more sense if the QoS of the backbone also was dependent on the investment levels of the backbone. This means that the overall QoS is equal to the weakest link, as explained in the general mathematical model. Since this feature is not implemented, it makes more sense to analyze the situation where the CPs have to pay for connectivity to the backbone provider only, and assume that the QoS provided by the backbone is always sufficient. This is not an unrealistic assumption, as the cost of building network for the backbone ISPs are much lower than that of the access (eyeball) ISP [24]. By removing the backbone SS price, f_c^{DSS}, the following results were obtained:

Proposition 5: In Non-Net Neutrality, the ISPs investments, the IUs connection fee, the CPs connectivity fee, the SS connectivity fee, the participation level of the CPs and the congestion levels are given by

$$\mu^D = \frac{1}{C^I}\left(\frac{u_C(r\lambda - 2f_c^D) + u_w r\lambda h}{\lambda(r\lambda - 2f_c^D + 2rh)} + f_i^{DSS}\frac{(2r\lambda - 4f_c^D)(r\lambda - 2f_c^D - 2f_i^{DSS}) - 8rhf_i^{DSS}}{\lambda(r\lambda - 2f_c^D - 2f_i^{DSS})(r\lambda - 2f_c^D + 2rh)}\right) \tag{143}$$

$$\mu^D \geq \frac{1}{\Psi}\frac{(r\lambda - 2f_c^D + 2rh)^2}{4(rh)^2(1 + f_i^{DSS})} \tag{144}$$

$$p^D = 4\mu^D\left(\frac{r\lambda(u_C(r\lambda - 4f_c^D) + r\lambda u_w h) + 4u_C f_c^D(f_c^D - rh)}{\lambda(r\lambda - 2f_c^D + 2rh)^2}\right) - f_i^{DSS}Y^D + C^V + t \tag{145}$$

where

$$Y^D = \mu^D\left(\frac{(16rhf_c^D - (2r\lambda - 4f_c^D)^2)(r\lambda - 2f_c^D - 2f_i^{DSS})^2}{\lambda\left((r\lambda - 2f_c^D - 2f_i^{DSS})(r\lambda - 2f_c^D + 2rh)\right)^2} - \frac{16rhf_i^{DSS}\left(4(f_c^D + f_i^{DSS})(rh - f_c^D) - r\lambda(3r\lambda - 8f_c^D - 4f_i^{DSS} + 4rh)\right)}{\lambda\left((r\lambda - 2f_c^D - 2f_i^{DSS})(r\lambda - 2f_c^D + 2rh)\right)^2}\right) \tag{146}$$

$$f_c^D = \left(\frac{2rh + r\lambda - \sqrt{rh(4rh + 2r\lambda - 4C_c^V)}}{2}\right)\frac{1}{\omega} \tag{147}$$

$$f_i^{DSS} = \frac{\left(r\lambda - 2f_c^D + 2rh - \sqrt{2rh(r\lambda - 2f_c^D + 2rh)}\right)(r\lambda - 2f_c^D)}{2(r\lambda - 2f_c^D + 2rh)} \tag{148}$$

$$\bar{y}^D = \frac{2\mu^D(r\lambda - 2f_c^D)}{\lambda(r\lambda - 2f_c^D + 2rh)} \tag{149}$$

$$\tilde{y}^D = \frac{4rhf_i^{DSS}}{(r\lambda - 2f_c^D)(r\lambda - 2f_c^D - 2f_i^{DSS})}\bar{y}^D \tag{150}$$

$$w^D = \frac{r\lambda - 2f_c^D + 2rh}{2\mu^D rh} \tag{151}$$

$$w^{DSS} = \frac{(r\lambda - 2f_c^D - 2f_i^{DSS})(r\lambda - 2f_c^D + 2rh)}{2rh\mu^D(r\lambda - 2f_c^D)} \tag{152}$$

$$w^{DIA} = \frac{(r\lambda - 2f_c^D - 2f_i^{DSS})(r\lambda - 2f_c^D + 2rh)^2}{4(rh)^2\mu^D(r\lambda - 2f_c^D)} \tag{153}$$

The results from the analytic NNN regime model, along with the results from the NN regime, will be used in the following sections in the discussion of analytical results and the numerical experiments. In the regulator scenario, the following restriction is added

$$\mu^{DT} \geq \frac{1}{\Psi}\frac{(r\lambda - 2f_c^D - 2f_i^{DSS})(r\lambda - 2f_c^D + 2rh)^2}{4(rh)^2(r\lambda - 2f_c^D)} \tag{154}$$

This is equal to that the QoS of IAS has to be better or equal to the average QoS in NN.

Proposition 6: The total profit of CPs, the backbone and access ISPs, consumer surplus and social welfare in Non-Net Neutrality is given by

$$\begin{aligned}\Sigma\Pi_{CSP}^{D} &= \frac{2\mu^D(r\lambda - 2f_c^D)^2}{\lambda(r\lambda - 2f_c^D + 2rh)} - \frac{8rh\mu^D f_i^{DSS^2}}{\lambda(r\lambda - 2f_c^D)(r\lambda - 2f_c^D - 2f_i^{DSS})} \\ &\quad - \frac{\mu^D\left(\left((r\lambda - 2f_c^D)(r\lambda - 2f_c^D - 2f_i^{DSS})\right)^2 - 16(rh)^2 f_i^{DSS^2}\right)}{\lambda(r\lambda - 2f_c^D + 2rh)(r\lambda - 2f_c^D)(r\lambda - 2f_c^D - 2f_i^{DSS})} \\ &\quad - \frac{4\mu^D f_i^{DSS}\left((r\lambda - 2f_c^D)(r\lambda - 2f_c^D - 2f_i^{DSS}) - 4rhf_i^{DSS}\right)}{\lambda(r\lambda - 2f_c^D + 2rh)(r\lambda - 2f_c^D - 2f_i^{DSS})}\end{aligned} \tag{155}$$

$$\Sigma\Pi_{ISP_{back}}^{D} = \frac{4\mu^D(f_c^D - C_c^V)(r\lambda - 2f_c^D)}{\lambda(r\lambda - 2f_c^D + 2rh)} \tag{156}$$

$$\begin{aligned}&\Sigma\Pi_{ISP_{access}}^{D} \\ &= 4\mu^D\left(\frac{r\lambda(u_C(r\lambda - 4f_c^D) + r\lambda u_w h) + 4u_C f_c^D(f_c^D - rh)}{\lambda(r\lambda - 2f_c^D + 2rh)^2}\right) + t \\ &+ f_i^{DSS}\left[2\left(1 - \frac{4rh(f_i^{DSS} + f_c^{DSS})}{(r\lambda - 2f_c^D)(r\lambda - 2f_c^D - 2f_i^{DSS} - 2f_c^{DSS})}\right)\frac{2\mu^D(r\lambda - 2f_c^D)}{\lambda(r\lambda - 2f_c^D + 2rh)} \right. \\ &\left. - Y^D\right] - C^I {\mu^D}^2\end{aligned} \tag{157}$$

$$CS_{IU}^{D} = R + 2\mu^{D}\left(\frac{u_c(r\lambda - 2f_c^{D}) + u_w r\lambda h}{\lambda(r\lambda - 2f_c^{D} + 2rh)}\right) - 4\mu^{D}\left(\frac{r\lambda(u_C(r\lambda - 4f_c^{D}) + r\lambda u_w h) + 4u_C f_c^{D}(f_c^{D} - rh)}{\lambda(r\lambda - 2f_c^{D} + 2rh)^2}\right) + f_i^{DSS}Y^{D} - C^{V} - t\frac{5}{4} \quad (158)$$

$$W^{D} = \Sigma\Pi_{CSP}^{D} + CS_{IU}^{D} + \Sigma\Pi_{ISP_{access}}^{D} + \Sigma\Pi_{ISP_{back}}^{D} = R + \frac{2\mu^{D}(r\lambda - 2f_c^{D})(r\lambda - 2C_c^{V})}{\lambda(r\lambda - 2f_c^{D} + 2rh)} - \frac{\mu^{D}\left((r\lambda - 2f_c^{D} - 2f_i^{DSS})^2(r\lambda - 2f_c^{D}) + 8rhf_i^{DSS^2}\right)}{\lambda(r\lambda - 2f_c^{D} + 2rh)(r\lambda - 2f_c^{D} - 2f_i^{DSS})} + 2\mu^{D}\frac{u_c(r\lambda - 2f_c^{D}) + u_w r\lambda h}{\lambda(r\lambda - 2f_c^{D} + 2rh)} - t\frac{1}{4} - C^{V} - C^{I}\mu^{D^2} \quad (159)$$

The proof of Proposition 5 and Proposition 6 can be seen in the appendix in section 8.2.3, along with the second order optimum conditions in section 8.2.4.

5.5 Discussion of the Analytical Results

This section will provide a general discussion of the equilibrium properties of Proposition 2, Proposition 3, Proposition 5 and Proposition 6, and a discussion that compares the two regimes analytically. By an analytic comparison of the two regimes we will derive the following propositions:

- Proposition 7: The investments levels are always higher in the NNN regime, than in the NN regime. That is $\mu^{D} > \mu^{N}$.
- Proposition 8: The innovation level is always higher in the NNN regime, than in the NN regime. That is $\bar{y}^{D} > \bar{y}^{N}$.
- Proposition 9: The average congestion is always lower in the NNN regime than in the NN regime. That is $w^{D} < w^{N}$.
- Proposition 10: The congestion level of SS in the NNN regime is always lower than the average congestion in the NN regime. That is $w^{DSS} < w^{N}$.
- Proposition 11: The backbone ISP always prefers the NNN regime.
- Proposition 12: The presence of a backbone ISP always reduces the benefit of NNN of the other players.
- Proposition 13: Overall social welfare is always higher in the NNN regime, than in the NN regime.

- Proposition 14: Some regulation might increase social welfare, however, the winners and losers of regulation is ambiguous.

Proofs of the propositions are provided in the discussion of section 5.5.3.

5.5.1 NN Equilibrium Properties, a Discussion of Proposition 2 and Proposition 3

In the standard Hotelling setting the equilibrium price would be $C^V + t$ [18]. However, because of the presence of externalities in the model settings, the equilibrium price for the IUs is higher. Increasing the price, p, decreases demand, which can be observed directly by the utility function of the IUs. However, reducing demand also improve the perceived QoS because of less congestion. This increases utility from QoS as well as the number of CPs, which also increases the utility of the IUs and thus demand. The total effect on demand from increasing the price is negative, but less negative than in the standard setting. This enables the ISPs to claim a price above the standard Hotelling setting. The effect increases proportionally with the investment levels, because the IUs directly have to finance the access ISPs investments. This result was also obtained by Bourreau et al [2]. Bourreau et al. also point out that in the degenerate case of no traffic ($\lambda \rightarrow 0$) or investment in capacity ($\mu \rightarrow 0$), the result converges to the standard Hotelling setting ($p \rightarrow C^V + t$). This implies that there would be no active CPs, and thus this result is only valid as long as the numéraire utility, R, is high enough so that the IUs would connect. This degenerate case is obviously of no interest to analyze as it implies there are no marked for Internet services and it follows that the numéraire utility, R, would also be zero, so there are no market for Internet access. However, there are two other factors in this model that affects the price, the backbone ISP and the congestion sensitivity, h. When the congestion sensitivity approaches infinity, ($h \rightarrow \infty$), the market converges to the degenerate case of no active CPs, and the price converges to the standard Hotelling setting. Because this increases the competition for IUs, as there are fewer CPs, the IUs are actually better off as the CPs sensitivity to congestion, h, goes up. As the backbone's market power increases, less CPs are able to join the market, which lowers the need for investments, and thus the price, p, is also lower. This is also because the IUs utility from CPs is lower, so the ISPs cannot claim as much value from the IUs as when there are many CPs.

In contrast to the model by Bourreau et al. the revenues per advertisement, r, also affects the access ISPs and IUs. This is because of the E2E effect, where the backbone ISP is able to extract much of the surplus from the CPs, and thus affect the number of CPs that participates which affects the price and investment levels by the ISP, which again affects the IUs. However, as can be seen in the numerical experiment, the effect is still small and mostly affect the backbone ISP and the CPs, which are able to claim significantly higher profits when r increases.

By increasing the demand for traffic, λ, the investment levels starts to increase, but then decrease as congestion effects take over and the cost of investing is higher than the extra value the ISPs can claim from the IUs. The IUs are always worse off when λ increase, because they have to pay for the extra investments, while the CPs and backbone ISP earn from the extra advertising revenues, and the access ISPs are able to claim a higher price because of the extra CPs joining the network. This result should not be compared to scenario where the demand for traffic increases because the packages themselves are bigger. This result would require the model to separate click traffic and traffic through the network. The intuition behind this result is that by increasing λ we increase the IUs need to interact with the CPs. However, since they only gain utility from the number of CPs, they do not obtain extra benefit, but have to pay for their extra demand. This obviously makes the IUs worse off.

Increasing the variable costs for the access ISPs does not affect the performance of the CPs and backbone ISP, and neither does it affect the profit of the ISPs. This is because the operational costs are linked to the number of IUs served by the ISP only, and thus the ISP compensates 1:1 by increasing the price an equal amount to the increase in variable costs, C^V. The IUs are therefore the only losers when the variable costs of the access ISPs go up. However, increasing the variable costs of the backbone ISP affects the whole system, because the backbone's variable cost affects the CPs connectivity price, f_c, which in turn affects the number of CPs that join the market. The number of CPs affects the whole system. The backbone ISP by far lose the most on increased costs, because the backbone ISP cannot compensate the full amount from the CPs as the access ISP can with the IUs. The CPs also lose, although not as much as the backbone ISP, and even less when the market power of the backbone ISP is less than monopolistic. If the IUs or access ISP loose or gain in this scenario is ambiguous. If the IUs are very dependent on the CPs, e.g. u_C and u_w are high, they pay more than they gain on higher investments, so they gain surplus as the cost of the backbone increase, because of less investments, and the ISPs loose. If the IUs does not value the CPs that much, the access ISP cannot gain as much value from their investments from the IUs, and thus the IUs are worse off when the variable costs of the backbone ISP goes up, because there is less need to make as much investments.

The connectivity price, f_c, is only dependent on the exogenous parameters, and is not affected by the access ISPs decisions. In contrast, the price set by the backbone ISP affects everything the access ISPs do. The intuition is that the backbone cannot control the size of the pie, only its share of the pie, which is what it tries to maximize. The access ISPs, on the other hand, can control both the size of the pie and their share, and therefore tries to maximize the product of their share and size of the total profit pool. This product is dependent on the backbone ISP connectivity price, f_c.

The cost of network investments, C^I, is arguably the most important value driver in the system, because all the players are dependent on the network capacity to make profits. Both ISPs and CPs earn higher profits by decreasing the investment costs. Increasing network capacity means more CPs, which again increase the revenues of the backbone, which does not have to do anything. However, in equilibrium the total investments costs are actually higher when the cost of investing goes down. This can be seen by direct substitution of μ^N in to $C^I(\mu^N)$. This extra cost is extracted from the IUs, which actually loose surplus by lowering the network investment costs. The effect is dampened when the IUs utility for CPs and congestion, u_C and u_w, go down. However, the overall effect is still a loss for the IUs.

Lastly, the IUs preference for QoS or CPs in general make all players, except the IUs, better off. The intuition is that the more highly IUs value CPs and QoS, the more they are willing to pay the ISPs to make higher investments. All players, except the IUs, take advantage of this. The system is therefore in favor of the platforms and CPs, as the IUs are not able to draw benefits from being more dependent on the CPs and their services. The congestion levels also decrease as u_C and u_w go up, while the number of CPs goes up. This is because an increase in the IUs utility leads to higher investment levels.

5.5.2 NNN Equilibrium Properties, a Discussion of Proposition 5 and Proposition 6

Generally, the equilibrium properties in the NNN regime follow much of the same logic as in the NN regime. However, there are some differences. As in the NN regime, the price, p, does not generally equal the standard Hotelling setting, $p = C^V + t$. The price might be both higher and lower depending on the parameter values, as the ISPs are now able to also extract revenues from the CPs. When r increases, the access ISPs are able to extract more revenues from the CPs, which makes the IUs more valuable. The access ISPs might even, in extreme scenarios, be willing to pay the IUs in order to be attractive for the CPs. This is not an option in the NN regime. Increasing λ always make the price increase, as the increase in investment costs are higher than the ISPs ability to extract revenues from SS, so the IUs have to pay for the extra capacity.

The priority fee, f_i^{DSS}, is always positive in the equilibrium solution. The fee generally increases with r and λ. The intuition is that as advertising becomes more valuable, the access ISPs can extract more revenues from the CPs.

The CPs connectivity fee, f_c, is the same in both regimes. This is because the backbone ISP can only maximize its share of the profit pool, and not the size of the pool. Thus, the backbone cannot directly gain any extra profits in the NNN regime by its own decisions. However, it may earn higher profits indirectly through an increase in network capacity investments.

The average congestion level, w^D, has the same logic as in the NN regime. However, the IAS and SS congestion levels, w^{DIA} and w^{DSS}, follows a different logic. Generally we see that $w^D < w^{DIA}$, and $w^{DSS} < w^D$. However, in the case where $f_i^{DSS} = 0$, $w^{DSS} = w^D$, while w^{DIA} is always higher than w^D. This is because when $f_i^{DSS} = 0$, all CPs connects to the SS lane, so it obviously follows that $w^{DSS} = w^D$, since these would be the same. In practice there would be no traffic on the IAS lane, so congestion would be zero. However, theoretically the congestion level is higher, as would have been experienced if a single package were sent on the IAS lane, as it then would be put at the back of the line, behind all the other packages. However, this degenerate case will not happen as f_i^{DSS} always is > 0 in the equilibrium solution.

5.5.3 Net Neutrality vs. Non-Net Neutrality, a General Discussion Comparing the Two Regimes

The main question in this thesis is whether NN or NNN is preferable, which we will partly try to answer in this section through an analytical analysis. However, the main conclusions will be drawn in the numerical experiments in the next section, as the model is too complex to be fully analyzed analytically. First we will discuss the investment and innovation levels. Then we will discuss the effect on price levels and congestion. Lastly the profits, consumer surplus and social welfare will be briefly discussed.

Effects on investments and innovation

In this section we compare the investments in capacity by the access ISPs and the number of CPs that enter the market.

Proposition 7: The investments levels are always higher in the NNN regime, than in the NN regime. That is $\mu^D > \mu^N$.

Proof: Since we know that $f_c^N = f_c^D$, we can obtain the following result by utilizing the results obtained in Proposition 2 equation (120) and Proposition 5 equation (143)

$$\frac{\mu^D}{\mu^N} - 1 = f_i^{DSS} 2 \frac{(r\lambda - 2f_c^D)(r\lambda - 2f_c^D - 2f_i^{DSS}) - 4rhf_i^{DSS}}{(u_C(r\lambda - 2f_c^N) + u_w\lambda rh)(r\lambda - 2f_c^D - 2f_i^{DSS})} \qquad (160)$$

This result is always positive in equilibrium, even when the backbone ISP has monopoly power. This means that the investments will always be higher in the NNN regime, than in the NN regime. □

Proposition 8: The innovation level is always higher in the NNN regime, than in the NN regime. That is $\bar{y}^D > \bar{y}^N$.

Proof: Since we know that $f_c^N = f_c^D$, we can obtain the following result by utilizing the results obtained in Proposition 2 equation (123) and Proposition 5 equation (149)

$$\frac{\bar{y}^D}{\bar{y}^N} - 1 = \frac{\mu^D}{\mu^N} - 1 \qquad (161)$$

The proof follows from Proposition 7. □

This result coincides with Bourreau et al [2].

Bourreau et al also find that there might be a scenario where all CPs that were active in the NN regime, are in IAS in the NNN regime, and that only a portion of the extra CPs enter SS. This is very unlikely in our analysis because of the congestion sensitivity, which keeps the number of CPs that enter down. However, it is possible when the IUs utility for CPs and QOS, u_C and u_w, are low.

Effect on price levels and congestion

Whether the IUs connectivity price will be higher or lower in the NNN regime is ambiguous. We can obtain the following result by utilizing the fact that $f_c^N = f_c^D$, and the results obtained in Proposition 2 equation (121) and Proposition 5 equation (145)

$$p^D - p^N = 4\left(\frac{r\lambda\left(u_C\left(r\lambda - 4f_c^{D/N}\right) + r\lambda u_w h\right) + 4u_C f_c^{D/N}\left(f_c^{D/N} - rh\right)}{\lambda\left(r\lambda - 2f_c^{D/N} + 2rh\right)^2}\right)(\mu^D - \mu^N) - f_i^{DSS} Y^D \qquad (162)$$

It is not clear whether the NNN regime will make the price higher or lower. Generally, the effect of extra investments increases the price. However, the ISPs ability to extract revenues from the CPs lowers the price. How well the access ISPs can extract revenues from the CPs is therefore essential for whether p^D is higher or lower than p^N.

Proposition 9: The average congestion is always lower in the NNN regime than in the NN regime. That is $w^D < w^N$.

Proof: Since we know that $f_c^N = f_c^D$, we can obtain the following result by utilizing the results obtained in Proposition 2 equation (124) and Proposition 5 equation (151)

$$\frac{w^N}{w^D} - 1 = \frac{\mu^D}{\mu^N} - 1 \qquad (163)$$

It follows from Proposition 7 that the average congestion level in the NN regime is always higher than the average congestion in the NNN regime, so $w^D < w^N$. □

This is also in correspondence with the results by Bourreau et al [2].

It follows that the SS lane is always lower than the NN-lane, because of the M/M/1 queuing system property $w^{DSS} < w^D < w^{IAS}$. Formally, based on the previous discussion and the properties of the M/M/1 queue model, we can write

Proposition 10: The congestion level of SS in the NNN regime is always lower than the average congestion in the NN regime. That is $w^{DSS} < w^N$.

Proof: The proof follows directly from the M/M/1 Property 1, (11), and Proposition 9. □

The IAS lane in the NNN regime might be either higher or lower than the average congestion in the NN regime, so the results are ambiguous. Since we know that $f_c^N = f_c^D$, we can obtain the following result by utilizing the results obtained in Proposition 2 equation (124) and Proposition 5 equation (153)

$$\frac{w^N}{w^{DIA}} = \frac{\mu^D}{\mu^N} \frac{2rh(r\lambda - 2f_c^D)}{\left(r\lambda - 2f_c^D - 2f_i^{DSS}\right)(r\lambda - 2f_c^D + 2rh)} \qquad (164)$$

We already know that $\mu^D/\mu^N > 1$. However, the value of the last fractional varies depending on the value of r and λ. As the numerical experiments will reveal, there are scenarios with a high r/λ ratio where also the congestion level of IAS is lower than the average congestion in NN. Also, when the IUs utility for CPs and congestion is low, w^{DIA} is lower than w^N because the revenues from the CPs are so important that they manage to increase the investments sufficiently in the NNN regime to lower the congestion for all CPs.

Effect on profits, consumer surplus and social welfare

The access ISPs

By utilizing the results obtained in Proposition 3 and Proposition 6, subtracting (127) from (157), we obtain

$$\Sigma\Pi^{D}_{ISP_{access}} - \Sigma\Pi^{N}_{ISP_{access}} = \left(4\frac{r\lambda\left(u_c\left(r\lambda - 4f_c^{D/N}\right) + r\lambda u_w h\right) + 4u_c f_c^{D/N}\left(f_c^{D/N} - rh\right)}{\lambda\left(r\lambda - 2f_c^{D/N} + 2rh\right)^2}\right)\left(\mu^D - \mu^N\right) + f_i^{DSS}\left[4\mu^D\frac{(r\lambda - 2f_c^D)(r\lambda - 2f_c^D - 2f_i^{DSS}) - 4rh(f_i^{DSS} + f_c^{DSS})}{\lambda(r\lambda - 2f_c^D + 2rh)(r\lambda - 2f_c^D - 2f_i^{DSS})} - Y^D\right] - C^I\left({\mu^D}^2 - {\mu^N}^2\right) \quad (165)$$

We know that the investments are always higher in the NNN regime, so the ISP has higher costs making investments. The first parenthesis, which is equal to the externality price effect on the IUs, is equal in both regimes, but since the increase in investments costs are quadratic, and the increase in the externality price effect is linear, the externality effect is not enough to pay for the extra investments that is required in NNN. Whether the NN- or NNN regime is beneficial for the access ISPs is dependent on the access ISPs ability to extract extra revenues from the CPs, compared to the rebate they offer the IUs. Thus, NNN is not always beneficial for the access ISPs, as will be confirmed in the numeric analysis. This result is also proven analytically by Bourreau et al [2]. In comparison, Krämer et al. have shown that the NNN regime is always beneficial for the ISPs in a monopoly setting [1].

The backbone ISP

Proposition 11: The backbone ISP always prefers the NNN regime.

Proof: By utilizing the results obtained in Proposition 3 and Proposition 6, subtracting (126) from (156), we obtain

$$\Sigma\Pi^{D}_{ISP_{back}} - \Sigma\Pi^{N}_{ISP_{back}} = \frac{4\left(f_c^{D/N} - C_c^V\right)\left(r\lambda - 2f_c^{D/N}\right)}{\lambda\left(r\lambda - 2f_c^{D/N} + 2rh\right)}\left(\mu^D - \mu^N\right) \quad (166)$$

It follows from Proposition 7 that the backbone ISPs profits are always higher in the NNN regime, than in the NN regime. □

The intuition is that this result applies as long as the backbone ISPs already provide sufficient QoS, and that it is the QoS of the access ISPs that are holding the perceived QoS back. This means that the backbone does not have to make any investments, and the profits increase in proportion to the extra investments done by the access ISPs. From this we can conclude the following.

Proposition 12: The presence of a backbone ISP always reduces the benefit of NNN of the other players.

Proof: Since the backbone does not create any value in the model, and always benefits from the NNN regime, the other players must lose benefit relative to a system with no backbone ISP effects because some of their benefit is taken by the backbone ISP. This proposition does only hold if the backbone ISP is not the bottleneck in the system, and can make investments that affects the other players. □

It is obvious from a practical standpoint that a real network cannot operate without a backbone ISP, and if the backbone ISP can make larger investments from adopting the NNN regime, and these investments are crucial for the performance of the system, the backbone might increase the benefit for other players. However, this is not possible to analyze in this model, as it is assumed that the backbone already has sufficient QoS, and thus operates as a free rider when the access ISPs adopts the NNN regime.

The CPs

By utilizing the results obtained in Proposition 3 and Proposition 6, subtracting (125) from (155), we obtain

$$\Sigma\Pi_{CSP}^{D} - \Sigma\Pi_{CSP}^{N}$$
$$= \frac{(2\mu^D - \mu^N)\left(r\lambda - 2f_c^{D/N}\right)^2}{\lambda\left(r\lambda - 2f_c^{D/N} + 2rh\right)} - \frac{8rh\mu^D {f_i^{DSS}}^2}{\lambda\left(r\lambda - 2f_c^{D/N}\right)\left(r\lambda - 2f_c^{D/N} - 2f_i^{DSS}\right)}$$
$$- \frac{\mu^D\left(\left(\left(r\lambda - 2f_c^{D/N}\right)\left(r\lambda - 2f_c^{D/N} - 2f_i^{DSS}\right)\right)^2 - 16(rh)^2 {f_i^{DSS}}^2\right)}{\lambda\left(r\lambda - 2f_c^{D/N} + 2rh\right)\left(r\lambda - 2f_c^{D/N}\right)\left(r\lambda - 2f_c^{D/N} - 2f_i^{DSS}\right)} \quad (167)$$
$$- \frac{4\mu^D f_i^{DSS}\left(\left(r\lambda - 2f_c^{D/N}\right)\left(r\lambda - 2f_c^{D/N} - 2f_i^{DSS}\right) - 4rhf_i^{DSS}\right)}{\lambda\left(r\lambda - 2f_c^{D/N} + 2rh\right)\left(r\lambda - 2f_c^{D/N} - 2f_i^{DSS}\right)}$$

No clear analysis can be drawn from this result. We know that the total number of CPs will be greater in the NNN regime. However, the margins of the CPs might be lower, so the aggregate profit is less even though there are more active CPs. The numerical experiments will later show that, although the CPs are usually better off, there are scenarios where the aggregate profit is less in the NNN regime, than in the NN regime.

The IUs

By utilizing the results obtained in Proposition 3 and Proposition 6, subtracting (128) from (158), we obtain

$$CS_{IU}^{D} - CS_{IU}^{N} = CS_{IU}^{D} = 2(\mu^{D} - \mu^{D})\left(\frac{u_c\left(r\lambda - 2f_c^{D/N}\right) + u_w r\lambda h}{\lambda\left(r\lambda - 2f_c^{D/N} + 2rh\right)}\right) - 4(\mu^{D} - \mu^{N})\left(\frac{r\lambda\left(u_C\left(r\lambda - 4f_c^{D/N}\right) + r\lambda u_w h\right) + 4u_C f_c^{D/N}\left(f_c^{D/N} - rh\right)}{\lambda\left(r\lambda - 2f_c^{D/N} + 2rh\right)^2}\right) + f_i^{DSS} Y^{D} \quad (168)$$

From Proposition 7 it follows that the IUs will always obtain more utility from CPs and congestion in the NNN regime. However, depending on how the payment for the extra capacity is split between the IUs and the access ISPs. The numerical experiments will reveal that the results on the consumer surplus are ambiguous. However, the IUs are usually better off in the NNN regime because of both higher utility from CPs and congestion, and a lower price from the access ISPs.

Total welfare

Proposition 13: Overall social welfare is always higher in the NNN regime, than in the NN regime.

Proof: By utilizing the results obtained in Proposition 3 and Proposition 6, subtracting (129) from (159), we obtain

$$W^{D} - W^{N} = (\mu^{D} - \mu^{N})\left[\frac{2\left(r\lambda - 2f_c^{D}\right)\left(r\lambda - 2C_c^{V}\right) + 2u_c\left(r\lambda - 2f_c^{D}\right) + 2u_w r\lambda h - \left(r\lambda - 2f_c^{N}\right)^2}{\lambda\left(r\lambda - 2f_c^{D} + 2rh\right)}\right] + \mu^{D}\frac{2f_i^{DSS}\left(\left(r\lambda - 2f_c^{D}\right)^2 - 2f_i^{DSS}\left(r\lambda - 2f_c^{N} + 2rh\right)\right)}{\lambda\left(r\lambda - 2f_c^{D} + 2rh\right)\left(r\lambda - 2f_c^{D} - 2f_i^{DSS}\right)} - C^{I}\left({\mu^{D}}^2 - {\mu^{N}}^2\right) \quad (169)$$

By observation, one can establish that both the two first factors are always positive in the equilibrium solution. And because the investments are reverse proportional to the basic cost unit, C^I, the sum of the two terms are higher than the difference in investment costs. □

The second factor is interesting, because it shows that even if μ^D - μ^N = 0, the social welfare is still higher in NNN because of better allocation. This means that the short run effects of NNN, that is before any investments are made, should also increase the social welfare because of better allocation. The first factor is increasing with (μ^D − μ^N), which we know is always positive because of Proposition 7. Even though the investments costs are higher in the NNN regime, they always increase less than the gross increase in social welfare. This is because the levels of investment, μ^D, that optimizes social welfare is always higher than the levels that optimize the access ISPs profits. The intuition is that since the access ISP cannot obtain all the profits by increasing the investments, it will always under invest according to the maximizing criteria of the social welfare. However, by the definition of the maximum criteria of the access ISP, the access ISP will not invest so that the marginal cost of investments is higher than the marginal increase in revenues, so an increase in investments will increase the revenues for the ISP. However, that increase is always less than the total increase in surplus and revenues, so it follows that the social welfare is always higher in NNN. Except for the backbone ISP, which always prefers NNN, it is ambiguous which players that benefit from NNN. However, the social welfare is always higher than in NN.

Proposition 14: Some regulation might increase social welfare, however, the winners and losers of regulation is ambiguous.

Proof: If μ^{D^*} is the access ISPs optimal investment levels in equilibrium, by deriving equation (159) from Proposition 6 with respect to μ^D, we obtain

$$\left.\frac{\mathrm{d}W^D(\mu^D)}{\mathrm{d}\mu^D}\right|_{\mu^D=\mu^{D^*}}$$
$$= \frac{\left(r\lambda - 2f_c^D\right)\left(r\lambda + 2f_c^D + 2f_i^{DSS} - 4C_c^V - 4f_i^{DSS}\left(r\lambda - 2f_c^D - 2f_i^{DSS}\right)\right)}{\lambda\left(r\lambda - 2f_c^D + 2rh\right)} \quad (170)$$
$$+ \frac{8rhf_i^{DSS^2}}{\lambda\left(r\lambda - 2f_c^D + 2rh\right)\left(r\lambda - 2f_c^D - 2f_i^{DSS}\right)} > 0$$

We already know from (165), (167) and (168) that it is ambiguous whether the individual players, except the backbone ISP, will benefit from the NNN regime. Therefore, higher investments, as might be enforced from regulation, does not necessary mean that the individual actors are better off, even though the social welfare increases. Equation (171) does only state that the overall social welfare is improved when the investments are enforced to be higher than the access ISPs optimum levels. Too much regulation can force the investment levels out of the social optimum levels, so that the social optimum is worse off than with no or little regulation. □

Because the individual players benefit from regulation is ambiguous, whether regulation is favorable or not depends on the objective of the regulators. If regulators want to maximize the IUs consumer surplus, there are scenarios where the IUs market power is such that they have to pay for the increase investments. Also, if the regulation is too strict, the social welfare might lose on the fact that the total investments are higher than the social welfare optimum levels. Last, but not least, this discussion has not considered the fact that an access ISP will only operate in the long run as long as it is profitable. Thus, if regulating the IAS lane makes the access ISPs unprofitable, they might not choose to operate even though it maximizes the social welfare. Obviously, the social welfare is not maximized if no access ISPs choose to operate, as no investments then would be made.

Since regulating the IAS lane's QoS might force the investment levels of the access ISPs out from optimum levels, one would expect the access ISPs to always be worse off from regulation. However, because of the competition between the access ISPs there are scenarios, as we will see in the numerical experiments, where the competition makes the ISPs under invest. Thus some regulation might actually increase the profit of the ISPs, as it forces the ISPs to reach the optimum monopoly levels of investments. This result is ambiguous and might go in either direction depending on the effect of competition between the access ISPs.

Our results coincide with those of Bourreau et al. in their Propositions 3, 4, 5 and 6 [2] . However, due to the introduction of the backbone ISP, the benefit of the NNN regime is less than that of Bourreau et al. Still, our results are in agreement with theirs. Also, we have shown that the backbone ISP always prefers the NNN regime, and because of this the potential benefit of the other players in the NNN regime is reduced, or if the actor is losing from the NNN regime the downside is strengthens. Also, we have shown that some regulation might improve social welfare. However, too much regulation might reduce the overall social welfare, and even some regulation might damage individual players.

Figure 12: Investments in network capacity is always higher in the NNN regime

Figure 13: The volume of content from the CPs is always higher in the NNN regime

Figure 14: The average congestion is always lower in the NNN regime

Figure 15: The backbone ISP always prefers the NNN regime

Figure 16: Social welfare is always higher in the NNN regime

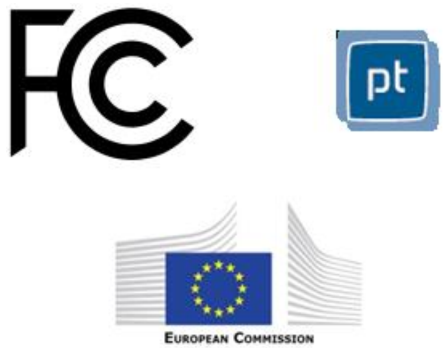

Figure 17: Some regulation can increase social welfare, but risk hurting the IUs, CPs or access ISPs

5.6 Numerical Experiments

This section will provide a numerical analysis, where the purpose is to clarify under which circumstances the different regimes are preferred for the different actors, and how changing parameters affect the results provided in the model. The numerical experiments will also confirm all the propositions from the analytic discussion based on the analytic solutions. Lastly, we will also do a numerical experiment on the effects of regulation. To achieve this, a scenario based sensitivity analysis will be conducted. By scenarios we mean analyzing the effects on a transition from the NN regime to the NNN regime where all parameters are kept constant, except those analyzed in the scenario, and parameters that have to change in order to satisfy the conditions for the model, (116) - (119).

The following scenarios will be analyzed and presented in this section:

1. Varying attractiveness of online advertising
2. Varying level of interactive and live services
3. Varying operational performance of ISPs
4. Varying effectiveness of building infrastructure
5. IUs becomes more or less dependent on online services
6. Regional differences in backbone ISPs market power
7. A regulated network
   a. Base scenario
   b. High and low IU dependence for CPs and QoS
   c. Low and high r/λ ratio

The scenarios are meant to analyze different regional effects, as well as how the use and performance criteria of the Internet might change in the future, will affect a possible transition to the NNN regime. The analysis is purely synthetic, and should thus not be compared to empirical numbers or results from the real world. However, the relations between the numbers are supposed to represent a real scenario, and the different scenarios analyses effects that might occur in real life depending on local demographics and future development of the Internet. A few highlights from the results of the analysis are

- The attractiveness of advertising is a crucial factor to consider, and in extreme scenarios either the IUs and CPs benefit, and the access ISPs lose, when the attractiveness of advertising is low, or opposite when the attractiveness of advertising is high
- Regulation might severely reduce, and even revert, the benefit of the NNN regime for IUs when they are heavily dependent on the CPs and high QoS
- Regulating the IAS lane is ineffective when the IUs are less dependent on CPs and QoS, or when the attractiveness of advertising is high

The next section will contain a description on how the model was implemented, followed by a section describing the scenarios in more detail. The main results of the numerical analysis will then be presented together with a short discussion of each scenario. Lastly, a general discussion on NN vs NNN and regulation will be provided.

5.6.1 Implementation

The implementation is based on the explicit results of the analytic mathematical model, equations (120) - (124) and (143) - (153). This model can be solved directly, and does not require any algorithms to obtain the solutions. A standard computer can easily obtain good results within seconds using either a spreadsheet or a mathematical programming language like Matlab.

The results presented in this thesis was calculated on a standard laptop, ASUS UX32VD Ultrabook with a dual core processor (i7-3517U) running at 1.9GHz and 10GB RAM. The software in use was Microsoft Excel 2010. Excel was chosen because of its ability to rapidly perform sensitivity analyses using Data Tables. This is effective as long as the results can be calculated directly, which is the case with this model. After implementation, results can be calculated immediately, with an instant response to parameter changes.

5.6.2 Description of Scenarios

This section will present a more detailed description of the scenarios. Each scenario will be described with a text providing the context for the analysis, and a description of the sensitivity parameter(s) in the analysis.

Base scenario

The base scenario is meant to be a realistic representation of today's situation. The parameters for the base scenario are presented in Table 17 together with the results in the base scenario. The base scenario is set to provide a realistic split of welfare between the players, reduce the volume of CPs so the total volume of content, $\bar{y}^N$, is short of 100% in the NN regime, as well as highlight the fact that in an advertising model a lot of traffic must be generated in order to generate a reasonable amount of revenues. This is why the r/λ ratio is so low, as seen by the overview in Table 17.

Sensitivity scenarios

The number in the following list states the name of the scenario, the following letter, a, describes the parameter that is being changed in the analysis, and the last letter, b, explains the relevance and context of the scenario.

1. **Varying attractiveness of online advertising**
   a. Parameter to be evaluated: The ratio r/λ will vary to test the effects of varying performance of the online advertising business model. A high ratio means the model creates a lot of welfare relative to the investment requirements. The opposite may still generate high revenues; however, to achieve this, much higher levels of investments are required, as much more traffic is transferred through the network. The available revenues with no congestion, $r^*\lambda$, will be constant throughout the analysis.
   b. The wealth of the CPs comes from online advertising. The attractiveness of online advertising in comparison to other medias is thus very important for the CPs performance and the performance of other parties in the ecosystem. As services become more complex, it does not necessary mean that the CPs can earn more money. However, it does put a higher stress on the network capacity which affects the performance of the network, and thus the performance of the whole ecosystem. An example of this is by releasing 4k video streaming due to competition between video services, which requires a high bandwidth, but does not necessary provide more revenues from advertising.
2. **Varying level of interactive and live services**
   a. Parameter to be evaluated: Increasing and decreasing the congestion sensitivity, h
   b. Many services in the future will likely involve more live action and interaction with the IUs, and would thus probably be more congestion sensitive. This will likely enforce higher demands on network capacity
3. **Varying operational performance of ISPs**
   a. Parameter to be evaluated: Increasing and decreasing variable costs, C^V and C_c^V
   b. Simple analysis to see the effect of decreasing performance by the backbone and access ISPs cost structure. As their operational costs increase, some other players are likely to pay much, or most, of the extra costs to provide sufficient service
4. **Varying effectiveness of building infrastructure**
   a. Parameter to be evaluated: Increasing and decreasing investment costs, C^I
   b. This might vary from region to region. Low investment costs means that there is likely to be a higher network capacity available, and more wealth should be generated for the ecosystem

5. **IUs becomes more or less dependent on online services**
    a. Parameter to be evaluated: Increasing/decreasing IUs CSP and congestion utility, u_C and u_w
    b. When we become more and more dependent on the Internet, we are also likely to be willing to pay more for the services, which is illustrated by increasing the utility for the number of CPs and low congestion levels
6. **Regional differences in backbone ISPs market power**
    a. Parameter to be evaluated: The backbone ISP market power (ω) will vary from a high value, which corresponds to a low backbone ISP market power, to 1, representing full monopoly power
    b. The market power of the backbone ISP might vary in different regions, which means that there might be local differences in the share of profit. The backbone can directly harvest the surplus of the CPs, and also reap benefits from SS, without paying for the capacity expansion. This gives the backbone ISP a unique position where it can reap the benefits of the access ISPs investments without having to pay for the extra investments
7. **A regulated network: Base scenario, high IU utility levels and low IU utility levels, low r/λ and high r/λ**
    a. Parameter to be evaluated: The Minimum QoS req. (Ψ) will be set very strict, and relaxed until the equilibrium solution investment levels are obtained. Regulation is varied from 0 to 100%, where 0% means that the access ISPs can do as the like, and 100 % means that the investments must be so that $w^{DIA} \leq \Psi = w^N$.
    b. The analysis is made to see the effects by regulators restricting the QoS on IAS, which could be a likely outcome of the NN debate. Regulating policies will limit the flexibility of access ISPs to manage their network, or make investments, which is the problem being analyzed in this scenario. Five scenarios are analyzed under regulation: the base scenario, high u_C and u_w, and low u_C and u_w, and the effects of regulation when r/λ is low and high. Table 15 shows the values of the parameters used in the analysis.

Scenario	Parameter changed	Low value	High value
Base scenario	None	-	-
IU dependence	u_C and u_w	5,26 and $6,37 \cdot 10^{-2}$	41,25 and 0,5
Advertising attractiveness	r/λ	$1,81 \cdot 10^{-4}$	$1,81 \cdot 10^{-2}$

Table 15: Values changed to perform sensitivity analysis of the effects from regulation the NNN regime

5.6.3 Presentation and Discussion of the Results

This section contains graphs showing the main results from the scenario analysis, followed by a short discussion of each scenario. Some additional results, used in the discussion, are presented in the appendix section 8.3. Since this is a synthetic analysis, where the purpose is to see the relative effect of NN vs. NNN, all results have been normalized with basis on the NN solutions. The table below illustrates the normalization process that was used to present the results and compare the two regimes.

Investments and innovation levels. Preferably N > 0	Price and congestion levels. Preferably N < 0 or as low as possible	Profits, consumer surplus and social welfare. Preferably N > 0
$N_{\mu} = \frac{\mu^D - \mu^N}{\mu^N}$	$N_p = \frac{p^D - p^N}{p^N}$	$N_{CSP} = \frac{\Sigma\Pi_{CSP}^D - \Sigma\Pi_{CSP}^N}{\Sigma\Pi_{CSP}^N}$
$N_{\bar{y}} = \frac{\bar{y}^D - \bar{y}^N}{\bar{y}^N}$	$N_{f_c^D} = \frac{f_c^D - f_c^N}{f_c^D}$	$N_{ISP_{back}} = \frac{\Sigma\Pi_{ISP_{back}}^D - \Sigma\Pi_{ISP_{back}}^N}{\Sigma\Pi_{ISP_{back}}^N}$
$N_{\tilde{y}} = \frac{\tilde{y}^D}{\bar{y}^N}$	$N_{f_i^{DSS}} = \frac{f_i^{DSS}}{p^N}$	$N_{ISP_{acc}} = \frac{\Sigma\Pi_{ISP_{acc}}^D - \Sigma\Pi_{ISP_{acc}}^N}{\Sigma\Pi_{ISP_{acc}}^N}$
	$N_{w^D} = \frac{w^D - w^N}{w^N}$	$N_{CS_{IU}} = \frac{CS_{IU}^D - CS_{IU}^N}{CS_{IU}^N}$
	$N_{w^{DIA}} = \frac{w^{DIA} - w^N}{w^N}$	$N_W = \frac{W^D - W^N}{W^N}$
	$N_{w^D} = \frac{w^{DSS} - w^N}{w^N}$	

Table 16: The normalization process in the numerical experiments to compare the results of the NNN regime to the NN regime. See Table 17 for a description of their representation in the scenario analysis' graphs and diagrams

First the base scenario will be presented, followed by graphs with results from the seven scenarios. The results will be presented by graphs showing the relative development from NN to NNN of profits and welfare, price and congestion levels, and investment and CPs participation levels in the different scenarios. The base scenario shows the parameters that are the basis for the whole scenario analysis. Each scenario analysis will show how the sensitivity variables are changed in the scenario.

Base scenario

Parameters	Value
Revenue click (r)	0,2
Traffic per IU (λ)	110
Congestion sensitivity (h)	20
Variable cost access ISP (C^V)	0,1
Variable cost backbone ISP (C_c^V)	0,1
Investment cost coeff. (C^I)	0,001
Minimum QoS req. (Ψ)	-
Numéraire utility (R)	15
Utility CSP (u_C)	8,25
Utility congestion (u_w)	0,1
Hotelling unit cost (t)	1
Backbone ISP market power (ω)	6

Variable	Description
Investment	μ, network investments
y_bar	CP volume (innovation)
y_tilde	SS indifference level
p	IUs connectivity price
fiDSS	SS lane fee for CPs
wD	Average congestion
wIAS	IAS congestion
wDSS	SS congestion
fc	CPs connectivity fee

Table 17: Parameter setting in the base scenario, and explanation of the variable names in the scenario analysis representing the relative change

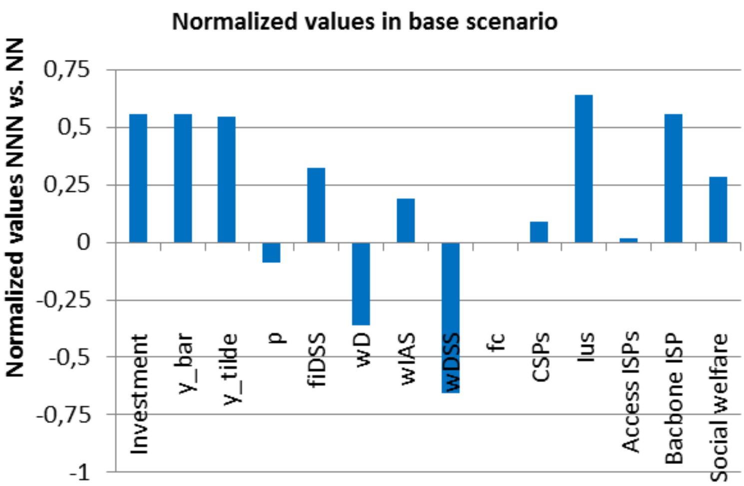

Figure 18: Normalized results from Base scenario, comparing NNN against the basic values of NN

The base scenario has an even spread of absolute wealth in the NN regime, and all the players benefit from the NNN regime. This means that if there is a downward trend of benefit in a sensitivity scenario, an actor that is worse off in the base scenario might be tipped over sooner to the side where the NNN regime is not favorable. The most dramatic parameter to change the result from the base scenario is presented in the first sensitivity analysis, in scenario 1: Varying attractiveness of online advertising, which will be presented in the first scenario analysis.

Varying attractiveness of online advertising

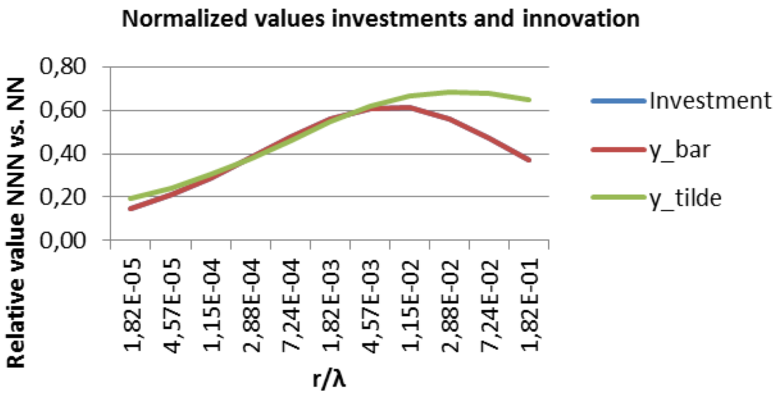

Figure 19: Normalized values of investments and innovation in the attractiveness of online advertising scenario

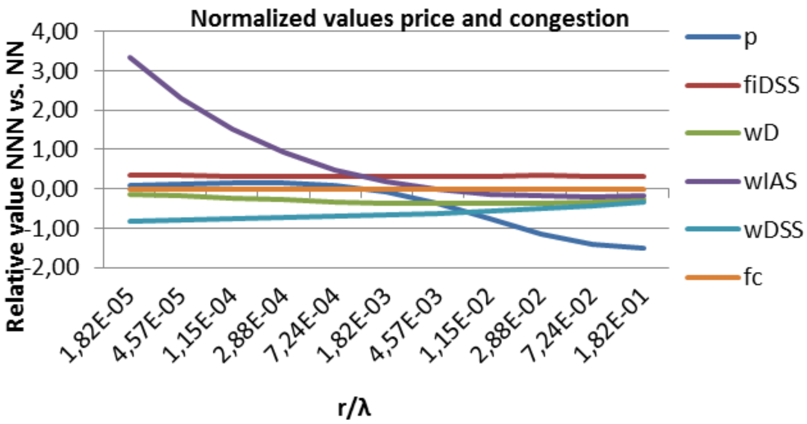

Figure 20: Normalized values of price and congestion in the attractiveness of online advertising scenario

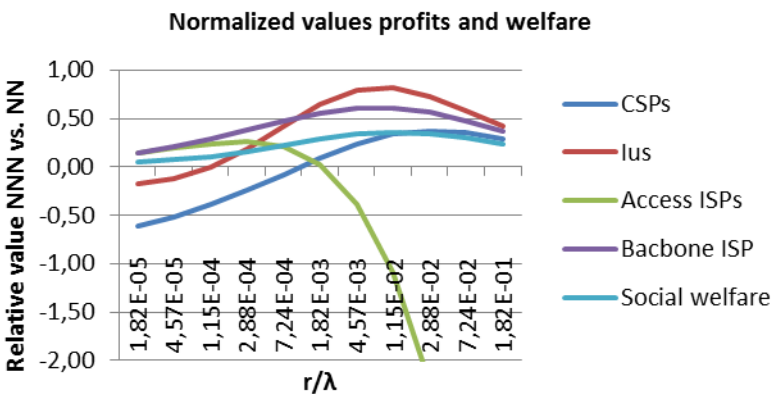

Figure 21: Normalized values of profits and welfare in the attractiveness of online advertising scenario

In the first scenario, with varying attractiveness of advertising scenario, the r/λ goes from very small, to relatively very large. However, the total potential earnings from advertising, $r*\lambda$, are the same throughout the whole analysis. At a low ratio, the benefit of NN is relatively small. This is because there is a tremendous amount of traffic that has to be sent in order to generate revenues for the CPs, which makes the whole ecosystem inefficient. The CPs and IUs are also worse off with a low r/λ ratio. This is because the demand for traffic relative to the ability to generate revenues is so high, that almost all CPs have to join the SS line in order to get sufficient QoS. The result is that the IAS line becomes severely impaired, and that the SS-line is not good enough to compensate for the SS fee the CPs have to pay. Because there is such a high requirement to make large investments, but relatively little revenues to be extracted from the CPs, the IUs have to pay a higher price to make up for the larger investments. Even though the IUs get a higher utility from more CPs and better average congestion, it is not enough to make up for the extra price they have to pay in the NNN regime. At a high ratio, all players, except the access ISPs, profits from the NNN regime. This is because the IUs are now so valuable, because their utility for CPs and QoS are lower, that the ISPs have to pay them to use their platforms, and the revenues they generate from the SS-lane is not enough to compensate. Concerning impairment of the IAS-line it should be noted that, when the r/λ-ratio is high, adopting the NNN regime actually improves all congestion levels and makes the congestion level of IAS better than the average congestion in the NN regime. Lastly, the relative difference between the two regimes seems to peak at some point for all the players. This is because in the extremes, when there is a very high r/λ-ratio, the access ISPs are under-investing because of their inability to extract sufficient revenues from the system. As a result, the total investments are smaller, which also makes the relative difference between NNN and NN smaller. When the ratio is very small, the CPs simply do not generate enough revenues compared to investment requirements so that, even though beneficial, the transition from NN to NNN does not make much difference. By observation of the absolute values, neither extreme seems likely because the spread of wealth in the NN regime is huge in both extremes, favoring the access ISPs when the ratio is low and the CPs and IUs when the ratio is high. Thus a middle scenario is probably more likely where the spread of wealth between the players is not that large, albeit still in the region where there could be a biased spread of wealth favoring one more players.

Figure 22: The attractiveness of online advertising is important for the individual players' benefit of the NNN regime.

Varying level of interactive and live services

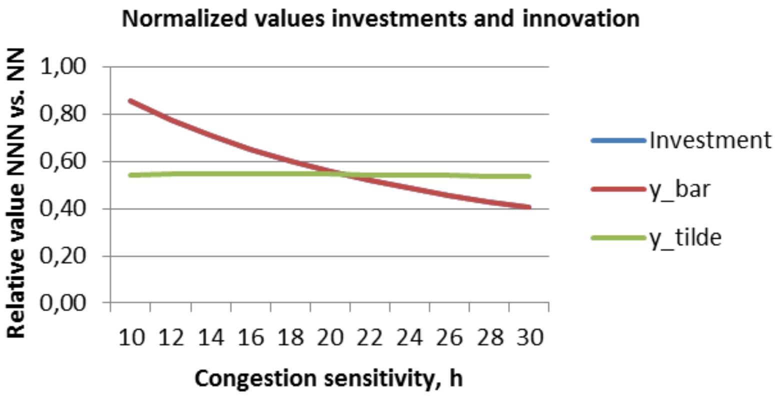

Figure 23: Normalized values of investments and innovation in the interactive and live services scenario

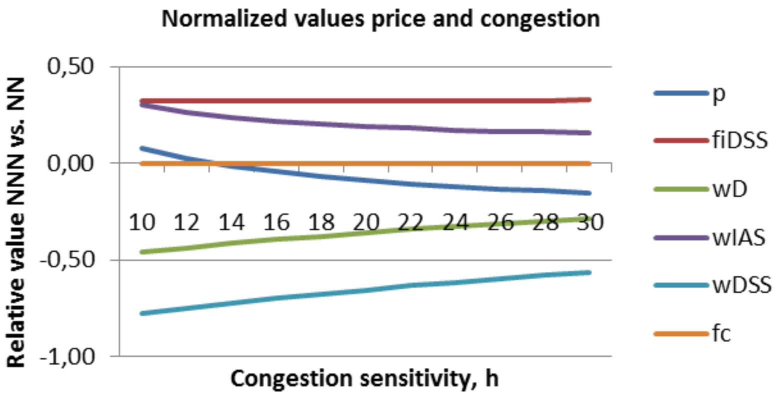

Figure 24: Normalized values of price and congestion in the interactive and live services scenario

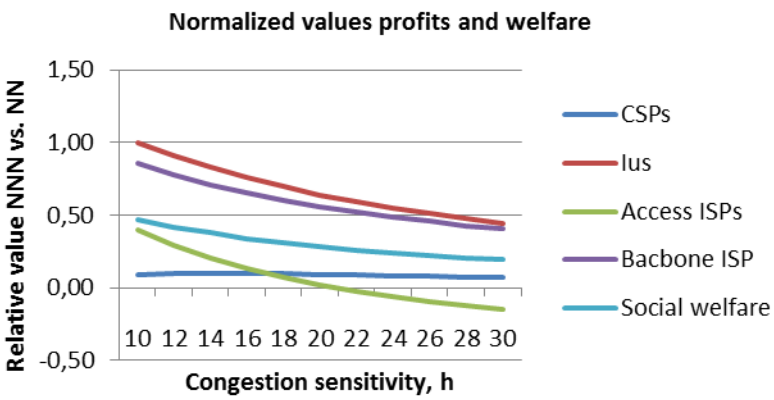

Figure 25: Normalized values of profits and welfare in the interactive and live services scenario

In the second scenario, where the sensitivity of interactive and live services is tested, the congestion level goes from relatively small to relatively large. When the congestion levels are small, all players benefit from the NNN regime. However as the congestion levels increase, the benefits go down and the access ISP is actually worse off as the congestion levels are high. Relatively the benefits for all players decrease. The reason for this is because the high congestion level makes the whole system less effective, which also makes it relatively harder to benefit from the NNN regime. Especially for the ISP that has to make relatively much higher investments compared to how many CPs are in the market. Since there are fewer CPs the access ISPs has a worse value proposition for the IUs, and can thus not charge as high prices. However, they cannot either extract as much revenues from the CPs as they generate less revenues per CP, and the total number of CPs is lower. If the backbone CPs market power is very low, this effect is much smaller, so that the NNN regime is still profitable under high congestion sensitivity. However, since the backbone ISP is only maximizing its own share, it is much harder for the access ISP to extract enough revenues from the CPs, so that it is worse off in the much more competitive NNN regime than in NN. The NNN regime does provide one important improvement, an increase in investments and innovation. As the CPs become more congestion sensitive, fewer are able to join the market and the overall investments are decreasing in the NN regime as the congestion sensitivity goes up. The same effect is observed in the NNN regime. However, adopting the NNN regime is a good way to actually increase both the investment levels and innovation levels, to higher levels than what they would even be when congestion sensitivity is very low in the NN regime. This shows how the NNN regime might actually benefit the CPs: Even though their conditions for competition are worsened, their ability to fund investments makes more CPs able to join the market in an NNN regime!

Figure 26: The level of interactive and live services, like Skype, reduce the relative benefit of the NNN regime. However, as proposed the NNN regime can contribute to maintain the overall investment levels

Varying operational performance of the access ISP and backbone ISPs

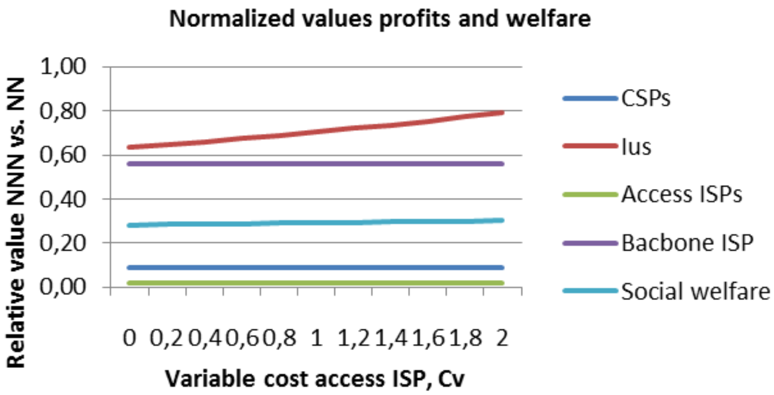

Figure 27: Normalized values of profits and welfare in the operational performance of the access ISPs scenario

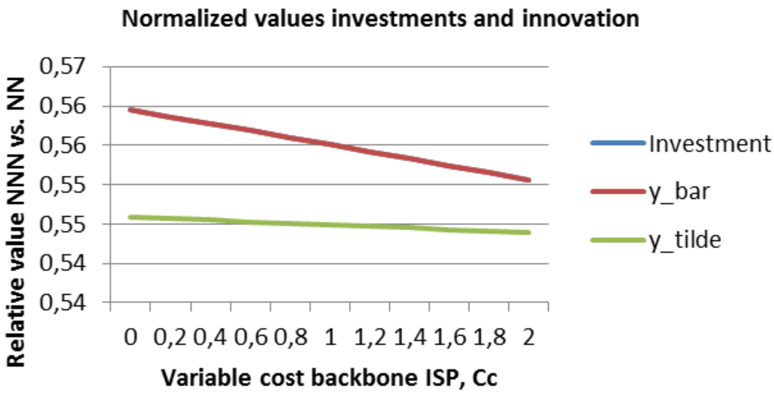

Figure 28: Normalized values of investments and innovation in the operational performance of the backbone ISP scenario

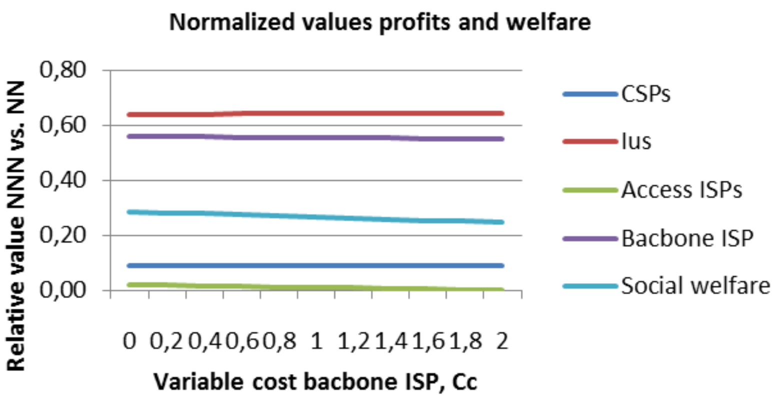

Figure 29: Normalized values of profits and welfare in the operational performance of the backbone ISP scenario

In the third scenario, where first the performance level of the access ISPs is tested, the variable costs of the access ISP go from relatively low to relatively high. The results are trivial. The extra costs are extracted directly from the IUs, because the variable costs of the access ISPs are only connected to the number of IUs on the platform. Thus the consumer surplus goes down in proportion to the variable costs of the access ISPs. However, the absolute benefit of the NNN regime is equal to all players throughout the whole scenario. Since the consumer surplus of the IUs is relatively smaller when the operational costs of the access ISPs are high, they benefit relatively more from the NNN regime. The profits of the CPs, and backbone and access ISPs are unchanged in the whole scenario. The result would probably have been different if the access ISPs variable costs also were linked to the CPs, as this most likely would decrease the number of CPs in the system, which would affect all the players. Since the total number of IUs is the same, and not affected by price, the other players are not affected in this scenario.

Figure 30: The variable costs of the backbone ISP affects the whole ecosystem.

The results are different when the variable cost of the backbone ISP goes from relatively small to relatively large. In contrast to the previous test, changing the variable costs of the backbone ISP affects the whole system. This is because the variable costs of the backbone ISP is linked to the number of CPs, which affects all the players. In absolute values, the backbone ISP's profits are reduced the most. However, the relative difference between NN and NNN is almost unchanged. This is because the relative difference of the backbone only depends on the investment levels, which relatively does not change much. No players lose relatively much benefits of NNN by increasing the variable costs, which illustrates how this effect is spread much more evenly among the players than in the previous scenario. So the backbone ISP affects the whole system, and not just its own direct business relations. However, the access ISPs are clearly the players that benefit relatively much less from the NNN regime when the variable costs of the backbone is high, than the others. This is because the total effects for all values are small. In the NN regime, the only effect for the ISPs is a minor decrease in price and investments. However, in the NNN regime, in addition to the decrease in price and investments, the SS connectivity fee and the number of CPs also decrease, which makes the total effect larger. Thus the access ISPs are less efficient in the NNN regime than when the variable costs of the backbone ISP is low, and it follows that the relative benefit of NNN is also smaller.

Varying effectiveness of building infrastructure

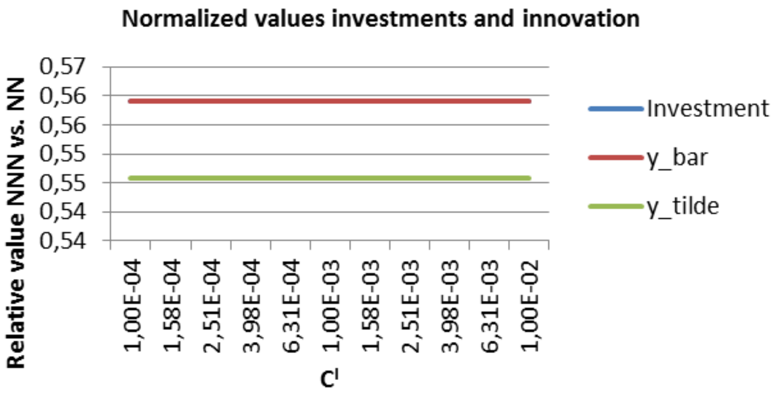

Figure 31: Normalized values of investments and innovation in the effectiveness of building infrastructure scenario

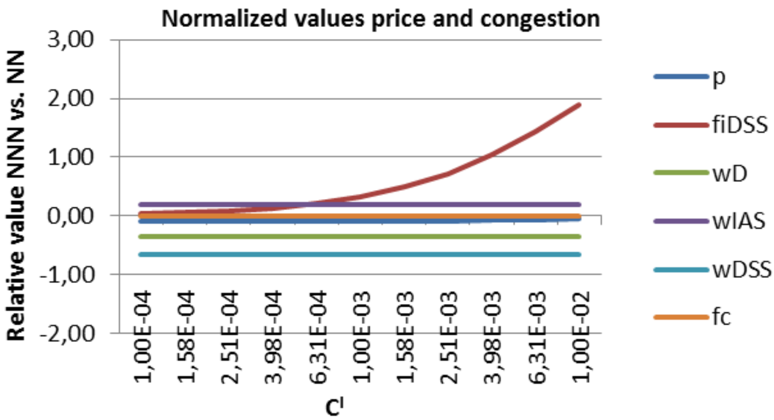

Figure 32: Normalized values of price and congestion in the effectiveness of building infrastructure scenario

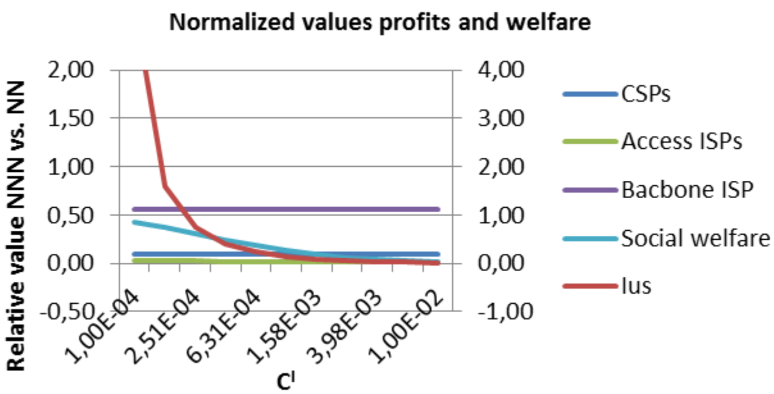

Figure 33: Normalized values of profits and welfare in the effectiveness of building infrastructure scenario. The secondary axis is for the IUs relative effect.

In the fourth scenario, where the effectiveness of building infrastructure is tested, the base unit for investment costs, C^I, goes from a reduction by a factor of 10, to an increase by a factor of 10. The results show that the absolute benefit of NNN is inverse proportional to the base investment cost, C^I. This is already known from the analytic discussion. It is of more interest that relatively the benefits of NNN only changes for the IUs and the ISPs. This is because the connectivity price and SS fee is independent of the investment levels, so they are unchanged in the whole scenario. The CPs and backbone ISPs profits are thus only dependent on the investment levels. Since all other parameters are equal, the relative difference in investment levels are the same, and thus the relative profits for the backbone ISP and the CPs are the same. The total investment costs are actually much higher when C^I is low, because the convexity of the investment costs. As can be seen both numerically and by studying the formulas, in the optimum solution the product of $C^I*\mu$ is constant, but the cost is a squared function depending on μ, so the total investment costs are 10 times as high when C^I is one tenth of the original unit. This also makes the relative benefit of sharing investment costs with the CPs much higher, which is why the IUs and access ISPs are relatively much better off in the NNN regime when C^I is low. The reason why f_i^{DSS} is relatively much larger when C^I is low is because it is unchanged by the investment levels, and thus also C^I. However, the IUs price is much lower when C^I is high because of the overall much lower investment costs, as previously discussed.

Figure 34: Fiber optic cables have revolutionized the telecommunication industry and the way we transfer information, providing better and more efficient network infrastructure.

IUs becomes more or less dependent on online services

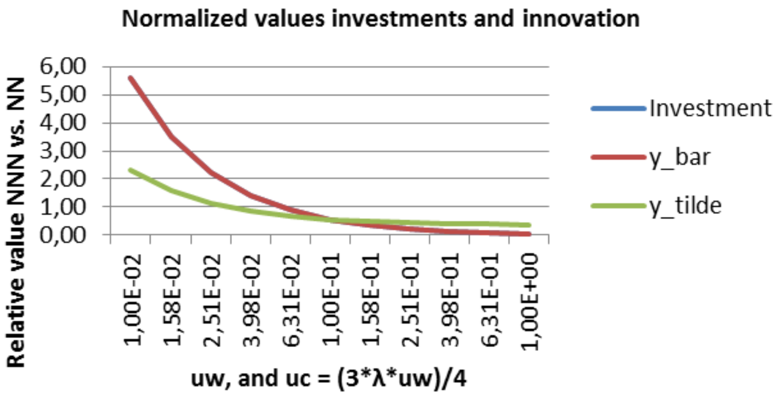

Figure 35: Normalized values of investments and innovation in the IUs dependence on online services scenario

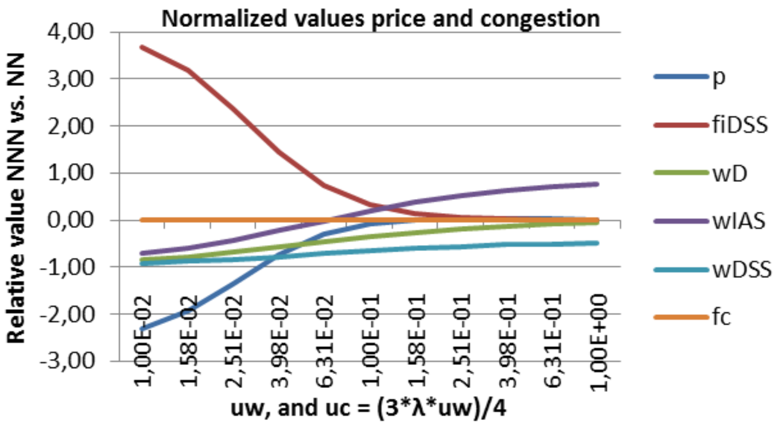

Figure 36: Normalized values of price and congestion in the IUs dependence on online services scenario

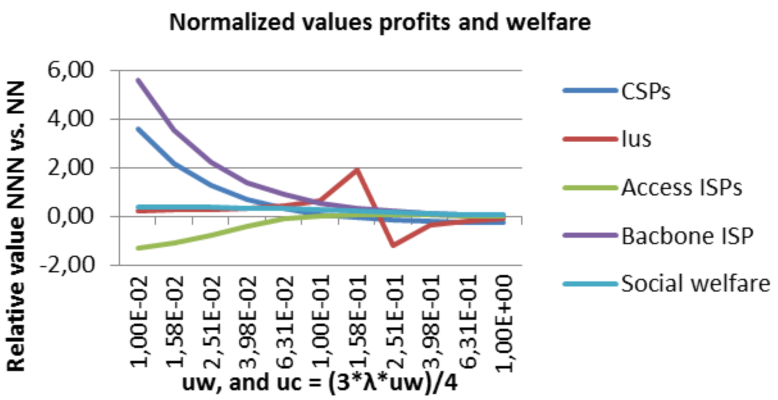

Figure 37: Normalized values of profits and welfare in the IUs dependence on online services scenario

In the fifth scenario, where the IUs dependence on CPs and QoS is tested, u_w is varied from 0,01 to 1, and u_C from 0,825 – 82,5 (u_C is, as in all scenarios, dependent on u_w and λ). The graphs clearly shows that the relative benefit of NNN is decreasing for the CPs and the backbone ISP, as the IUs becomes more and more dependent on CPs and low QoS, while the benefit of NNN is increasing for the access ISPs and the IUs. The break in the IUs graph is because the utility of the IUs become negative (this can be adjusted by increasing R) as they become so dependent that they bear most of the investment costs. Thus after the break, the graph shows that NNN makes the IU utility relatively less negative in the NNN regime, so the IUs are actually increasingly better off in the NNN regime when they are more dependent on CPs and QoS. The reason why the backbone ISP and the CPs get relatively less benefits from the NNN regime with high IU utility is because when the utility for CPs and QoS is high, the IUs are already paying for huge investments in infrastructure in the NNN regime, so there is not that much to gain by adopting the NNN regime for the CPs and the backbone ISP. In absolute value, they are much better off in both regimes when the IUs utility is high, as one would expect, but the relative difference is low. In contrast, when the IUs utility is low, they are not willing to pay for as much investments. As a result, the CPs can obtain a huge benefit by paying the access ISPs for higher investments, as the total investments then are relatively much higher in the NNN regime than NN. The IUs always benefit from the NNN regime in this scenario. However, the access ISPs actually loses by adopting the NNN regime when the IUs utility is low. This is because the intensified competition in the NNN regime forces the access ISPs to over-invest. They cannot reclaim enough value from neither the CPs nor the IUs to compensate, in contrast to when the IUs utility is high, when the access ISPs can make the IUs pay for the extra investments. The relative increase in social welfare is much higher when the IUs utility is low, than when their utility is high. The intuition behind this result is straight forward. If there is a one-sided market, and the side being charged is much more willing to pay than the other side, which is most valuable for the system, the benefit of adopting a two-sided market is small. However, if the side being charged is the most valuable, and is not willing to pay compared to the other side, the benefit of adopting a two-sided market is high.

Figure 38: Internet is rapidly becoming a more important part of our lives, and we carry it with us almost everywhere with our phones.

Regional differences in backbone ISP's market power

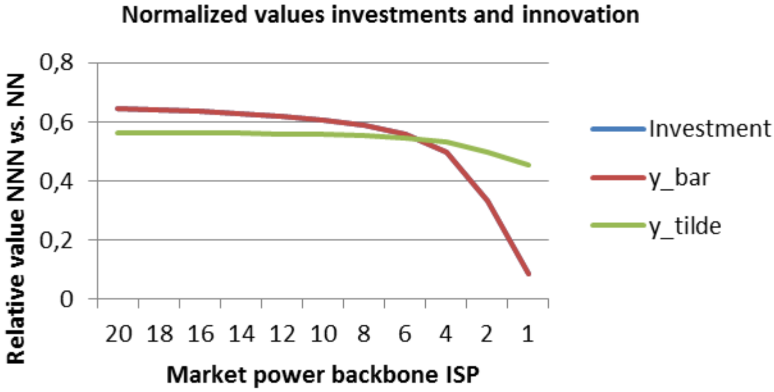

Figure 39: Normalized values of investments and innovation in the backbone ISPs market power scenario

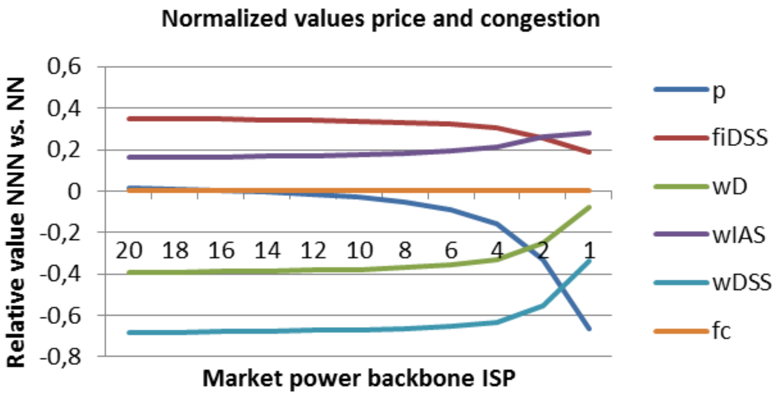

Figure 40: Normalized values of price and congestion in the backbone ISPs market power scenario

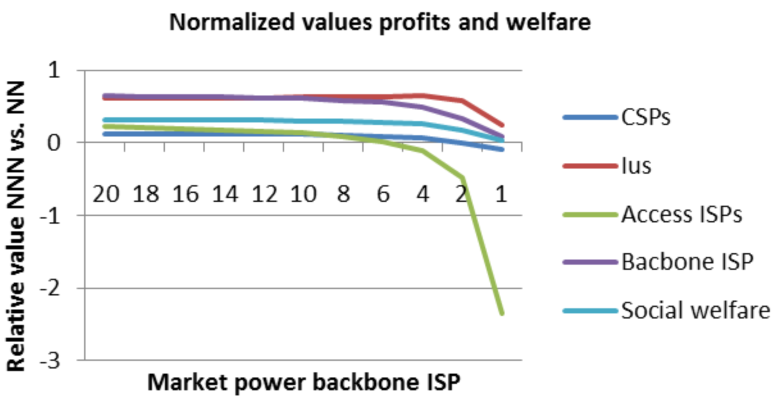

Figure 41: Normalized values of profits and welfare in the backbone ISPs market power scenario

In the sixth scenario, the effect of varying the backbone ISP's market power is tested, where the backbone goes from almost perfect competition to full monopoly. Since the backbone ISP has a major influence on the CPs profit, and thus their participation, which in turn affects all the players, varying the backbone market power affects the whole system, as can be seen by the results. In absolute terms, the backbone and IUs are actually profiting from the increase in backbone ISP market power, while the CPs and access ISPs loses. This is because the price of the IUs decrease relatively more than the loss in utility from CPs. The utility from congestion is higher because of the relatively lower amount of CPs compared to investments. As the market power of the backbone goes up, the margins of the CPs naturally go down, so they are obviously worse off. In relative terms, all players gain less benefits of adopting the NNN regime when the backbone ISP's market power is high. This is because the backbone decreases the margins of the CPs, so the access ISPs cannot extract as much revenues from the CPs. The result is that the relative change in investments are very small when the backbone has a monopoly, compared to when there is a high level of competition. We already know that the investments in network capacity, μ, is the most important value creator, so it follows that when the relative increase in investments are small, the relative increase in profits and welfare is also smaller. The access ISPs actually end up losing on adopting the NNN regime as the backbone ISP becomes more powerful. This is due to the competitiveness of the NNN regime. The access ISPs absolute change of p is lower when the backbone ISP has monopoly. However, the relative change is extremely high. Thus, the access ISPs lose a relatively large amount of revenues from substituting the IUs, more than it manages to gain from the SS lane's connectivity fee. One important aspect of this analysis is that the backbone ISP does not generate any value to the ecosystem in this model. The foundation for creating value is made by the access ISPs investments, and the ability these investments make for the IUs to interact with the CPs. However, in the real world the backbone ISP is obviously an important value creator, as the backbone is a fundamental part of the Internet and has to make investments on their own. Thus, the results should not be interpreted to mean that the backbone ISP is stealing value from the ecosystem, but rather that the backbone's ability to claim value from the system is weakening the benefit if the NNN regime for the other players. Also, a model where investments by the backbone is included, and vital for the overall performance, might even provide a totally different result where it is important that the backbone claims enough value to make sufficient investments. Therefore, the key takeaway from this analysis is that the NNN regime can still be favorable for all players, even though a large share of the value created is captured by the backbone, which might be a necessary and real life scenario.

A regulated network – base scenario

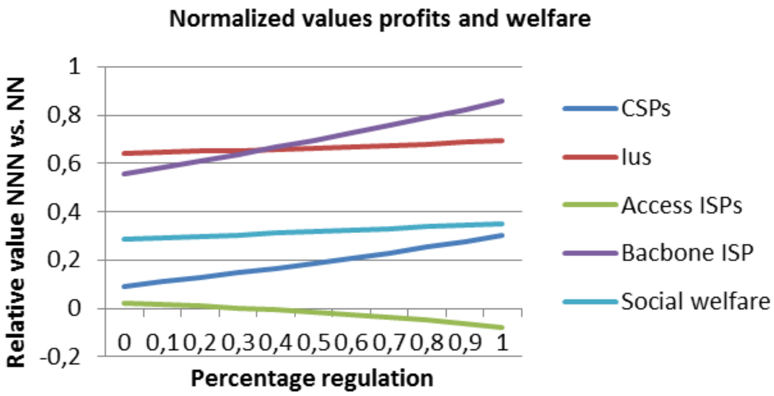

Figure 42: Normalized values of profits and welfare in the regulated network – base scenario

The seventh scenario, where the effects of regulation is tested, consists of three tests; one test with the base scenario assumptions, another case where the IUs dependence on CPs and QoS is low, and one where it is high. In the basic scenario all players, except the access ISP, benefits from regulation. This is not surprising, as we already know the investment levels that maximizes social welfare is higher than the investment levels that optimize the ISPs investment levels. This result is in correspondence with Proposition 14. When the access ISP is forced to over-invest in the NNN regime, the relative benefit is naturally smaller. And regulation might actually force the access ISPs to lose by adopting the NNN regime. This is because regulation only forces the access ISP out of its optimum solution in the NNN regime, so the result in the NN regime is not affected. When the regulation is strict enough, the effect of regulation might reduce the optimum of the access ISPs sufficiently to make the regime unprofitable for the access ISPs. From a social standpoint, regulation in this scenario is preferable.

A regulated network – high and low IU utility on CPs and QoS

The results are quite different when regulation is tested under high and low IU dependence on CPs and QoS. When their dependencies low, the congestion of IAS is equal to or better in the NNN regime than the average congestion level in the NN regime. This, regulation is redundant. However, when the IUs are very dependent on CPs and QoS they take on much higher investment costs, and so the benefits of NNN are very high to begin with for the IUs as in accordance with the fifth scenario. Since the optimal investment levels are not much higher in NNN than NN, regulation forces the investments to be much higher than both the optimum of the access ISPs, and even the social optimum. The result is that the IUs have to pay for most of the extra investments posed by the regulators, and the price is higher than the benefit of extra IUs and lower congestion. Thus the IUs are actually worse off by regulation, as they are forced to pay for most of the extra investments. The effect is so great that 100% regulation actually makes the social welfare worse in the NNN regime, than in the NN regime. So regulation is not optimal from a social standpoint when the IUs are very dependent on CPs and QoS.

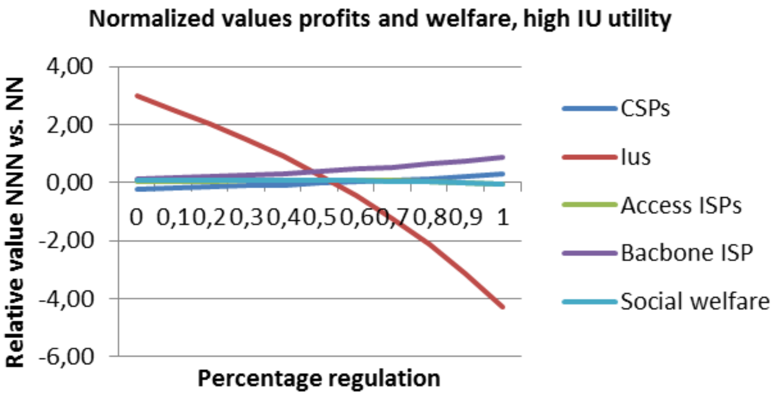

Figure 43: Normalized values of profits and welfare in the regulated network – high IU utility on CPs and QoS scenario

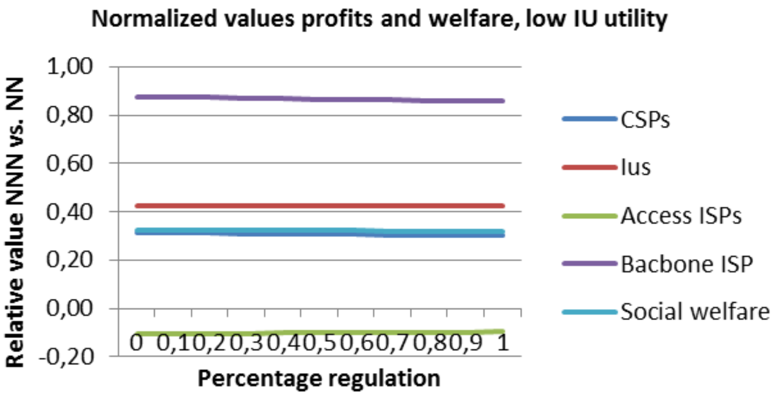

Figure 44: Normalized values of profits and welfare in the regulated network– low IU utility on CPs and QoS scenario

A regulated network – low and high r/λ

Because the attractiveness of advertising (the r/λ ratio) is arguably the most important parameter to evaluate the benefit of adopting the NNN regime, a numerical experiment considering regulation was also performed with a lower and higher ratio than the base scenario. When the ratio is high, the congestion level of IAS is already better in the NNN regime than in the NN regime, so regulation is redundant. However, when the r/λ ratio is low regulation might have positive effects. To begin with, the CPs profits are greatly reduced in the NNN regime. However, because the price for the SS lane, f_i^{DSS}, is unaffected by investments the larger investments means relatively much more profits for the CPs. At about 60% regulation, the CPs are relatively better off in the NNN regime. The access ISPs have a minor increase while the IUs are actually slightly worse off, but this is only marginally. The overall social welfare does also increase to a certain point. However, as the regulation enforces too high investments, the access ISP starts to lose significantly from regulation, until the combined effects of the IUs and access ISPs means that the social welfare is reduced. The backbone ISP is naturally better off the more regulation, because its profits increase proportionally to investments. The implication of this analysis is very important, as it implies that some moderate regulation may actually widen the gap of the positive range in the r/λ scenario. Meaning that the probability of a real life scenario where the NNN regime is favorable, and can be accepted by all players, is increased.

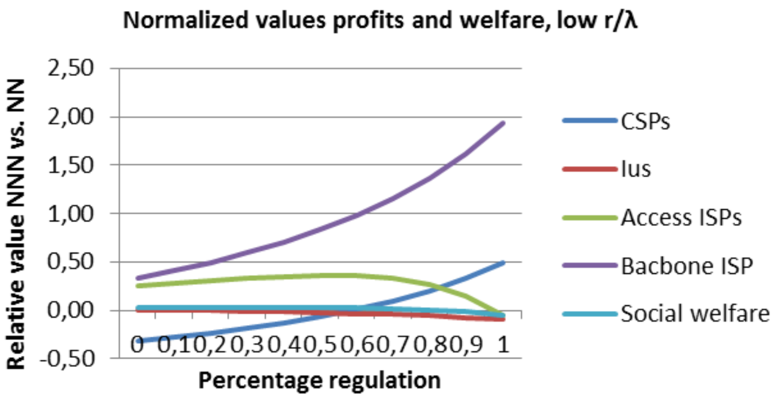

Figure 45: Normalized values of profits and welfare in the regulated network– low r/λ scenario

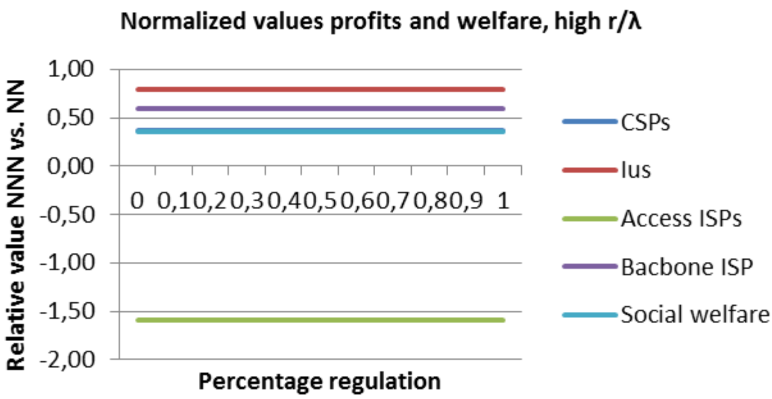

Figure 46: Normalized values of profits and welfare in the regulated network– high r/λ scenario

5.6.4 General Discussion on NNN vs. NN

The following table summarizes the results from the scenario analyses, showing whether the actors benefit or lose by the adoption of the NNN regime under the corresponding parameter values.

Player surplus\Scenario	Low r/λ	High r/λ	Low h	High h	Low C^V	High C^V	Low C^V_c	High C^V_c	Low C^I	High C^I	Low u_C, u_w	High u_C, u_w	High ω	Low ω
CPs	-	+	+	+	+	+	+	+	+	+	+	-	+	-
Backbone ISP	+	+	+	+	+	+	+	+	+	+	+	+	+	+
Access ISPs	+	-	+	-	+	+	+	+	+	+	-	+	+	-
IUs	-	+	+	+	+	+	+	+	+	+	+	+	+	+
Social welfare	+	+	+	+	+	+	+	+	+	+	+	+	+	+

Table 18: Summary of profits, consumer surplus and social welfare from the numerical experiments. See section 5.3 page 74 for an explanation of the different parameters.

By studying the table is might seem obvious that an overall assessment should conclude that adopting the NNN regime is clearly the favorable choice. However, this results stems from the fact that all scenarios are based on the base scenario, where all players have a positive benefit from the NNN regime. The general development in the scenarios should therefore be guiding instead of absolute values. That is, e.g. if the IUs are in fact becoming more and more dependent on CPs and QoS, the development goes towards a general benefit of an NNN regime, and such a regime could help protect the IUs interests. Also, this table illustrates which parameters are important, as those parameters that change surplus from positive to negative clearly has a larger impact than the ones that do now. By observation, and from this table, the r/λ ratio is arguably the most important parameter. However, both congestion sensitivity, h, the IUs utilities, u_C and u_w, and the backbone ISPs market power is also parameters that could severely change the outcome of adopting the NNN regime. Actors and policy makers should therefore possess some knowledge on which of these scenarios is representing their situation, as it might have huge consequences for the final outcome.

In all scenarios the social welfare increases by adopting an unregulated NNN regime. The access ISPs generally argues that the NNN regime will generate higher investments, and we have shown this to always be true. Still, there are scenarios where one could argue that the practical outcome of NNN is not favorable. Even if the social welfare improves, the profit or surplus of the individual players might be worse in NNN, and even negative in some extreme scenarios. Obviously, if a player obtains negative profit or utility, it will not participate at all, which makes the whole system break down. So no scenario with negative profits will prevail in the NNN regime. The absolute profit of the CPs, the backbone ISP and the IUs is always positive. However, the access ISPs might obtain a negative profit by adopting the NNN regime. In general, the increased competitiveness of the NNN regime enforces much higher investments. Whenever these investments are higher than the extra revenues obtained by the access ISPs in the NNN regime, they actually risk obtaining a negative profit, which would put them out of the market. This is true when the r/λ ratio is high, the IUs utility for CPs and QoS, u_C and u_w, is low, and when the market power of the backbone ISP is approaching monopoly power. Bourreau et al. [2] has shown that even though the ISPs do not always benefit from adopting the NNN regime, they are locked in a prisoner's dilemma game, and would thus always choose to adopt the regime. Therefore, in some extreme scenarios, the NNN regime might make the access ISPs compete themselves out of the market.

However, even in scenarios where the absolute profits and consumer surplus is positive, the NNN regime might not be considered favorable if some players are suffering to a large extent to the benefit of others. In general, all players, except the access ISPs, are better off in the NNN regime. The exception is in the scenario with varying r/λ ratio. This scenario is arguably the most important to analyze as it, in either extremes, split whom is benefitting by the adoption of the NNN regime in two. When the ratio is low the CPs and IUs do not benefit from the NNN regime. As the r/λ ratio increase, the IUs quickly benefit from the NNN regime. However, the CPs do not benefit, before the ratio is significantly higher. The CPs are also marginally worse off when the IUs are heavily dependent on their services and QoS. The last scenario where the NNN regime might not be favorable for the CPs is when the backbone ISP's market power is large. Although the range where the CPs benefit from the NNN regime is larger than that where the CPs do not benefit, the results show there might be some truth to the arguments posed by the CPs, that they would be worse off in the NNN regime.

The benefits of adopting the NNN regime are much more ambiguous for the access ISPs than the other players. In general the Access ISPs are relatively worse off in the NNN regime when the r/λ is high, the CPs congestion sensitivity, h, is high, the IUs utility for CPs and QoS, u_C and u_w, is low and the backbone ISP's market power is high. There are far more scenarios where the access ISPs are worse off than the other players. Since the access ISPs generally argue pro the NNN regime, one must assume they consider the real life scenario to be in their favor. One important aspect is the assumption of complete information. This is not true in the real world, where one would expect the access ISPs to have more information than the other players, because the access ISPs directly do business with all the other players and should therefore be able to make a better map of the competitive landscape.

The backbone ISP was included in this analysis to provide an E2E assessment of the effects of adopting the NNN regime. The analysis shows that the backbone ISP always benefits from the NNN regime. Since the backbone ISP does not create any value on this model, it implies that the backbone ISP benefitting on the cost of other players. This is quite obvious as all players benefit increases as the backbone ISPs market power goes down. Also, the r/λ ratio range where all players benefit increases severely as the market power of the backbone ISP is weakened. This aspect of the analysis provides two important insights for potential regulators and key actors. First of all the analysis confirms that the NNN regime is still the socially optimal regime in the E2E ecosystem. Secondly, this also shows that the presence of a backbone ISP severely impairs the performance of the ecosystem, and also the net benefit of adopting the NNN regime is impaired and much more ambiguous for individual players. In practice this means that where the previous analysis with no E2E assessment provided a positive result for the key actors, this analysis shows that the presence of a backbone ISP might be enough to make the benefit of the NNN regime negative for certain players in some scenarios.

Concerting the arguments posed by either camp, as presented in the introduction to the net neutrality debate chapter, the results shows that there are some truths in both of the sides' arguments. As posed by the proponents of the NNN regime, it does lead to higher investments, and lead to relatively much higher investments compared to the extra prices forced on the IUs. In many scenarios the IUs are even substituted in the NNN regime. The results thus confirm the arguments posed by the NNN regime proponents. The NN-proponents, however, have other arguments, which are to a larger extent more conceptual than directly result oriented. Most of these principal arguments are in agreement with the work already done by the EU-commission, and the results from this analysis are not in contradiction to the principle of an open E2E Internet. However, the fear of that the NNN regime will stifle innovation is proven not to be true; measured in volume, the level if innovation is always higher in the NNN regime than in the NN regime.

Generally, the unregulated NNN regime is preferable to the NN regime. This is because it generally increases profits and welfare, is in most scenarios favorable by all players, or does not provide any major changes (except in the extreme scenarios). Also, as seen by the numerical analysis, the NNN regime can prove to be a major benefit for the IUs as they become more dependent on the CPs, and also counter the effect of less investments when the CPs becomes more sensitive to congestion, maintaining and even increasing the investment levels of today. However, it should be emphasized that a clear understanding of where we are on the r/λ scale, and the implications of this result, should be provided before a final decision is made. As this is, as seen by these numerical experiments, arguably the most important parameter to understand who will benefit, and who might possibly not benefit, by the adoption of the NNN regime.

5.6.5 General Discussion on Regulation

The numerical experiments confirm that regulation might increase the social welfare and thus the overall benefits of obtaining the NNN regime. However, when the IUs are very dependent on CPs and QoS, too much regulation might reduce the social welfare, and greatly reduce the social surplus of the IUs. When the IUs are not that dependent, or the r/λ ratio is high, regulation is redundant. However, some minor regulation might be beneficial. As seen by the previous analytic and numerical discussion, some regulation should always lead to an increase in social welfare. Also, some regulation could lead to a wider r/λ range where the NNN regime is favorable, or at least acceptable, for all players. Too much regulation can on the other hand be very dangerous. It can lead to a loss of social surplus, and severely damage the surplus of individual players. Therefore, based on the previous discussion, too much regulation is not recommended as the outcome in uncertain and might severely impair especially the IUs that are sensitive to an increase in the access ISPs investments. As seen by Proposition 1, neither should the policy makers force the ISPs to separate the networks into two separable networks for SS and IAS. This is ineffective, and, assuming the M/M/1 assumptions are valid, will increase the average congestion levels by a factor of two. This will make investments in network capacity much less effective, and should therefore be avoided to maximize the performance of the NN ecosystem.

5.7 Strengths and Weaknesses Compared to Previous Research

The presented model is based on the research by Krämer et al. and Bourreau et al., and have adopted the modifications and strengths posed by Bourreau et al., alas, also the weaknesses of no transaction costs or subscription CPs. However, this model does provide the E2E perspective by including the backbone ISP, which represents the effect of CDN and their agreements with other ISPs to connect the CPs to the backbone of the Internet. This has enabled us to prove that there are parts of the ecosystem, that has not yet been analyzed, that always benefit on the NNN regime. It has also enabled us to show the effect of smaller margins of the other actors in the ecosystem when the backbone ISP is introduced. The result was Proposition 11 and Proposition 12, which are not similar to any other propositions from previous research. Our model does also include variable costs of the ISPs, which have enabled us to show how much more sensitive the system is to variable costs involving the CPs. Even though this model is more complicated due to the presence of the backbone ISP, it is still possible to provide explicit analytical solutions and thus apply a direct analysis from which we have derived our propositions.

Concerning our game theoretical approach, with our two-stage game, the model is not as sophisticated as Njorge et al, whom have managed to solve a six stage game. Neither does the model consider subscription CPs, as done by Economides and Hermalin. Both are valuable contributions to the analysis of the NN debate ecosystem. In contrast to their research, however, we were able to provide explicit results and analysis.

6 Recommendations to the Enterprise, Evaluations and Conclusion

Section 6.1 will summarize our most important findings into recommendations to Telenor and how they could maneuver in the NN debate. Section 6.2 will evaluate the results compared to the problem statement, and section 6.3 will provide a conclusion to this thesis. Section 6.4 will propose recommendations for future research on the NN debate.

6.1 Recommendations to the Enterprise Based on the Analysis and Evaluation of the Results

Based on the results in this thesis, the most important recommendations to Telenor are

- General recommendations towards adopting an NNN regime
    - Analyze the attractiveness of advertising and be sure that it is not high enough to make the competitiveness of the NNN regime reduce the overall profit of the enterprise from introducing the SS lane.
    - Be certain of your competitive position in the value chain, and that the backbone ISPs will not eat away the benefits of an NNN regime. In general this should not be a problem if the backbone ISPs do not have a monopoly.
    - Be sure that the IUs are dependent enough on CPs and QoS so that they are still willing to pay when the CPs are charged in the SS lane
- General recommendations to persuade policy makers to allow the NNN regime
    - It is very important to show that the attractiveness of advertising is sufficient enough for the IUs and CPs to benefit from the NNN regime.
    - Focus on the following arguments, as proven in section 5.5.3 and 5.6.3
        - The overall social welfare always improves in the unregulated NNN regime
        - Investments, Innovation and average QoS always improves in the NNN regime
        - Assuming sufficient QoS in the backbone of the Internet, the access ISPs investments is the value creator of the ecosystem. Higher investments means more value to the whole ecosystem, and the access ISP will only claim a fraction of the extra value by itself

        - Even though the benefit for individual players is ambiguous, all players benefit around the medium ranges in the scenarios
        - As the IUs become more and more dependent on the CPs and QoS, they increasingly benefit more from the NNN regime
        - The analysis confirms the three arguments proposed by the NNN-proponents
- General recommendations to persuade policy makers to allow an unregulated and flexible network
    - Focus on the following arguments, as proven in section 5.5.3 and 5.6.3
        - Splitting the network in two makes the network much less effective, doubling the overall congestion levels
        - Regulation might severely impair individual players, and especially the IUs are exposed when they are willing to pay more for investments than they gain in utility
        - Too much regulation might reduce social welfare, and is therefore potentially dangerous
        - Even though some regulation usually increases the social welfare, individual actors might be severely impaired. Even some regulation can therefore be dangerous if one aims to protect the interest of individual actors
    - Bourreau et al. [2] have shown that the access ISPs have no incentives to sabotage unless the IAS lane unless the attractiveness of advertising is sufficiently high. However, in when the attractiveness is sufficiently high, as shown in this thesis, the NNN regime is not beneficial to begin with, so given that the access ISPs prefer an NNN regime, sabotaging the IAS lane is not likely assuming the access ISPs only interests are to maximize their own profits

The results of this thesis are dependent on the assumption that the QoS of the backbone is sufficient so that no extra investments by the backbone ISP are needed in order to manage the extra traffic in the NNN regime. This is not unlikely, as the eyeball network is much more expensive and harder to build than the backbone network [24]. However, this is likely to vary from region to region, and where the QoS of the backbone is not sufficient they either have to accept a lower QoS than possible, or invest which will probably lead to an increase in prices for the CPs that use them.

The results are also dependent on the assumption of settlement free peering. On average this should be true as the access ISPs and backbone ISPs are equally dependent on each other. However, the difference between individual players can be substantial, meaning that many non-tier 1 ISPs have to pay extra for transit or paid peering in order to obtain the necessary increase in QoS in a NNN regime. This analysis is in connection with the different actors, and thus differences within the type of actors is not part of this thesis. Intuitively the overall result should be the same. However, the smaller ISPs that have to pay extra for transit or peering are likely to benefit less from the NNN regime. On the other hand, ISPs that can demand payment for transit or peering would likely be better off in a model where this factor is included.

This model assumes that the SS lane is prioritized and that the packages are put first in line, while the IAS packages are next in line to the SS lane packages. There are other ways to manage the internet, which could alter the result of this analysis. However, it is not possible to make the QoS of the SS lane any better than what is assumed in this analysis, and thus, unless the ISPs sabotage, the QoS of IAS cannot be worse either. Regarding the concerns of the NN proponents, this analysis could be seen as a worst case scenario for the IAS lane, as long as the ISPs do not sabotage. Bourreau et al. have shown in a similar setting, but without E2E, that sabotaging IAS might be preferred in the NNN regime when the advertising rate, r, is sufficiently high [2]. Intuitively this is in agreement with our analysis since in this scenario the access ISP are worse off in the NNN regime because not enough CPs pay for the huge investment requirements.

Lastly, these results assume complete information, which means that the ISPs can set their prices with full knowledge of demand. That all players have complete information is most likely not realistic, and incomplete information generally in favor of the buyers when the sellers set the price, as is the case in this model [37]. The outcome in this setting, however, is not certain. Generally this means that in a real setting the IUs and CPs, the buyers, should be even better off than the sellers, as the backbone and access ISPs would not be able to extract their full potential margins because of the uncertainty. However, we could also expect this effect to be reduced by the fact that the ISPs do business with most of actors and individual identities within the type of actors and should thus be able to provide fairly good estimates to compensate for the lack of full information. Therefore, the results in this thesis should not be regarded as unreliable, but rather slightly optimistic on the access ISPs behalf.

6.2 Evaluation of the Results Compared to the Problem Statement

The problem statement of this thesis was "*What effects does leaving the NN regime in favor of the NNN regime involve for key ecosystem actors in an E2E multi actor environment, which scenarios lead to a beneficial outcome for the key actors and overall social welfare, and how does regulating the QoS of the IAS lane in the NNN regime affect the key actors and social welfare?*"

This thesis does provide insights on the relative benefit of the NNN regime compared to the NN regime in an E2E multi actor environment, and we have also shown which scenarios are beneficial for the individual actors. Lastly, we have made an assessment of regulating the IAS lane as measured by a minimum QoS requirement relative to the average QoS in the NN regime. The analysis also provides general recommendations concerning the effects of regulation, and how policy makers could use regulation to improve the relative benefit of the NNN regime for all actors and maximize social welfare.

6.3 Conclusion

We have summarized the current situation and findings in the NN debate. We have formulated a general model to analyze the effect of adopting the NNN regime, and developed a simplified model, which is possible to solve analytically in order to obtain solutions that can be used to analyze the effects of adopting the NNN regime in an E2E multi actor environment. The analysis is based on the simplified model.

We model the following situation: Under the net neutrality regime, the CPs earn revenues from advertisers who pay proportionally to the IUs click-through rate. The CPs also have to connect to a backbone ISP in order to connect to the Internet. The access ISPs connects the backbone of the Internet to the IUs, who pays the access ISPs for connectivity. In the NNN regime, the ISPs can offer specialized services (SS), similar to a priority lane, which put all the packages in specialized services first in line. The CPs can pay a fee to access SS. Those who do not pay have to connect through internet access services, which is similar to the neutral best effort internet.

We have solved the model analytically and done numerical experiments to compare the effect of adopting the NNN regime versus the NN regime.

We show that in the unregulated NNN regime the investments are always higher since they are able to be partly compensated for their investments by the additional revenues in the SS lane. The consequence is that the innovation levels measured in volume is higher and the average congestion levels are lower. We have shown that the backbone ISP always prefers the NNN regime, and also reduce the benefit of the NNN regime for the other actors in the ecosystem. We also show that the social welfare is always higher in the unregulated NNN regime. However, the individual player's benefits are more ambiguous, with the exception of the backbone ISP that always benefits from the NNN regime. Lastly, we have shown that some regulation on the IAS lane's QoS can improve the overall social welfare.

Our numerical experiments show the importance of understanding how the attractiveness of advertising (the r/λ ratio) is in the ecosystem, and that extreme values of this ratio severely separated the benefiters and losers of the NNN regime on two. When the ratio is low, the CPs and IUs are worse off while the access ISP is better off. The opposite is true when the ratio is high. Regulators and policy makers should therefore be wary of this ratio. We also show that the NNN regime might enforce higher investments as the CPs becomes more congestion sensitive, and that it greatly benefits the IUs as they become more dependent of the CPs and QoS. This is because the NNN regime provides considerably higher investments and innovation levels compared to the price the IUs are willing to pay, when the CPs also pay for investments in the NNN regime. We show that a flexible network is favorable, and that splitting the network in two will double the average congestion levels. Lastly, we show that some regulation on the IAS lane could be positive, and lead to an overall higher social welfare as well as a widened r/λ ratio range where all players are better off in the NNN regime. However, regulation may also have some unwanted effects where especially the IUs can be severely damaged as they are forced to pay for the extra investments that might be required by regulation. Too much regulation can also lead to a worsened overall social welfare in the NNN regime, in contrast to the unregulated NNN regime where it is always positive.

We do confirm the arguments posed by the NNN regime proponents. In our model the conceptual arguments posed by the NN-proponents are already in accordance with most of our model settings, as there is no blocking of any content. And, our findings conclude that there should not be any fear that the NNN regime will stifle innovation as measured in volume of content from the CPs.

The main contribution to the NN debate from this thesis is the introduction of the backbone ISP. We show that the NNN regime is still favorable in the E2E perspective. However, we also show that, under our assumptions, the backbone ISP is always better off in the NNN regime. This means that the backbone is eating off the other player's benefits, which makes their benefits of the NNN regime more ambiguous. However, the overall assessment is still in favor of the NNN regime because of the increase in social welfare, investments and innovation, an improved average QoS and the ability to greatly benefit the IUs as they become more dependent on the Internet. This is because an NNN regime protects the interests of the IUs in such a scenario, as the regime prevents the IUs from paying more for investments than they gain from the extra utility from more volume of CPs content and a higher QoS. The Internet is rapidly becoming a more important part of everybody's lives, so a scenario where we are sufficiently dependent on CPs and QoS to pay more than our gained utility is not unlikely. Maybe, in the future, the NNN regime can protect us, the IUs, from that.

6.4 Recommendations for Future Research

To further gain insights to the effects of the individual players, future researches could try to solve the model with a subdivided set of actors, e.g. small and large CPs, high and low quality CPs, small and large ISPs and heavy and light IUs. The algebraic challenges to do this are substantial, however conceptually it should be possible to follow the steps, as in section 0, to solve such a model.

Lastly, this thesis provides a model to analyze the effect the NNN regime has on subscription CPs. These CPs can directly price the IUs from their services, and would therefore probably behave differently than the advertising CPs, which most of the current research is based on.

Two major simplifications in this model are the assumption of asymmetric information and the lack of transaction costs. Asymmetric information would generally lead to better conditions for the buyers, and it would be interesting to see how this applies on our setting. Also, one must assume that there will be transaction costs in connection with introducing and selling the SS-lane. If these costs are substantial, the overall social benefit might not always improve as suggested by this thesis.

There are no internal externalities among the CPs, which is a simplifying assumption. Since the total number of IUs is constant, it is likely that the number of clicks per CSP would go down as the number of CPs increase. The effect would decrease the effect of extra income from innovation, however, also reduce the re-congestion effect of innovation. Whether this benefits the CPs or not depends whether the increased click-through rate compensates for the decrease in revenues from the IUs per volume of CSP content.

Lastly, the model in this analysis is conceptual, and the numerical experiments use synthetic data. Future researchers could try to conduct an analysis based on empirical data sets. Also, the analysis in this thesis is based on average numbers. Regulators and NN-proponents fear that introducing the SS-lane will impair the IAS lane. On average, with the model settings in this thesis, this is shown not to be true. However, the traffic levels of IAS and SS may depart far from average levels during specific times of the day. Thus, a simulation of specific traffic patterns on the SS-lane and IAS-lane could further benefit the insights on the practical effects of the NNN regime.

The first extension mentioned, separating the different actors into different quality and sizes, should not provide significant technical difficulties, but some algebraic challenges will be added because of the increased complexity. The other four extensions, however, will likely be challenging to implement and obtain good solutions. The empirical studies and simulations would require a different methodology, however, it should be possible and would provide further insights to the NN debate.

7 References

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8 Appendix

8.1 Proofs

Splitting of networks, Proposition 1

Proposition 1 is only valid for systems where the assumptions of the M/M/1 queue are valid. If μ^T is the total capacity in a network, and λ^T is the average rate of arrivals, the average waiting time per package, w^T equals

$$w^T = \frac{1}{\mu^T - \lambda^T} \qquad (171)$$

We now split the capacity and arrival of packages into two separate networks, 1 and 2, with share of capacity and arrivals equal to β and $(1-\beta)$. The capacity, μ^1, of network 1 is thus $\beta\mu^T$ and the rate of arrivals, λ^1, equals $\beta\lambda^T$. The same can be done with network 2. The average waiting time per network is therefore

$$w^1 = \frac{1}{\mu^1 - \lambda^1} = \frac{1}{\beta\mu^T - \beta\lambda^T} = \frac{1}{\beta(\mu^T - \lambda^T)} \qquad (172)$$

$$w^2 = \frac{1}{\mu^2 - \lambda^2} = \frac{1}{(1-\beta)\mu^T - (1-\beta)\lambda^T} = \frac{1}{(1-\beta)(\mu^T - \lambda^T)} \qquad (173)$$

The average level of congestion after splitting the two networks, w^S, is thus

$$w^S = \beta w^1 + (1-\beta)w^2 = \frac{\beta}{\beta(\mu^T - \lambda^T)} + \frac{(1-\beta)}{(1-\beta)(\mu^T - \lambda^T)} = \frac{2}{\mu^T - \lambda^T} = 2w^T \qquad (174)$$

After splitting the networks, the average waiting time per package is twice as high, even though the total capacity and average arrival rate is the same. The reason is because when the whole network is not separated, the variation from λ^1 and λ^1 equalize each other, and more effectively make use of the capacity in the network. □

Optimal price with Bertrand competition and differentiation in an oligopoly

The utility of IUs connecting to a CSP j_S is modelled as in (67):

$$u_{xj_S}^{N/D} = \iota_{j_S}\lambda_{j_S}\left(1 - h_{j_S}w^{N/D}\right) - v_{j_S}^{N/D} - \frac{1}{N}\sum\nolimits_{i \neq j_S}\left(\lambda_i\left(1 - h_i w^{N/D}\right) - v_i^{N/D}\right) \quad (175)$$

We assume that all CPs have the same utility offer, $\lambda_{j_S}\left(1 - h_{j_S}w^{N/D}\right)$. By setting the utility function equal to zero we obtain an expression for the fraction ι_{j_S}that is indifferent to the offer by CP j_S:

$$\iota_{j_S} = \frac{v_{j_S}^{N/D}}{\lambda_{j_S}\left(1 - h_{j_S}w^{N/D}\right)} + 1 - \frac{1}{N} - \frac{\sum_{i \neq j_S} v_i^{N/D}}{\lambda_{j_S}\left(1 - h_{j_S}w^{N/D}\right)N} \quad (176)$$

Next we subtract v from 1 to obtain the fraction that is interested in the offer

$$1 - \iota_{j_S} = \frac{1}{N} - \frac{v_{j_S}^{N/D}}{\lambda_{j_S}\left(1 - h_{j_S}w^{N/D}\right)} + \frac{\sum_{i \neq j_S} v_i^{N/D}}{\lambda_{j_S}\left(1 - h_{j_S}w^{N/D}\right)N} \quad (177)$$

This is now the fraction of demand for CP j_S. If c is the marginal cost of serving demand, the profit function if CP j_S can be written

$$\Pi_{j_S} = (v_{j_S}^{N/D} - \mathrm{c})\frac{1}{N} - \frac{v_{j_S}^{N/D}}{\lambda_{j_S}\left(1 - h_{j_S}w^{N/D}\right)} + \frac{\sum_{i \neq j_S} v_i^{N/D}}{\lambda_{j_S}\left(1 - h_{j_S}w^{N/D}\right)N} \quad (178)$$

If we take the derivative and set it equal to zero we can obtain an expression for the optimal price

$$\frac{\partial \Pi_{j_S}}{\partial v_{j_S}^{N/D}} = \frac{1}{N} - \frac{2v_{j_S}^{N/D}}{\lambda_{j_S}\left(1 - h_{j_S} w^{N/D}\right)} + \frac{\sum_{i \neq j_S} v_i^{N/D}}{\lambda_{j_S}\left(1 - h_{j_S} w^{N/D}\right)N} + \frac{c}{\lambda_{j_S}\left(1 - h_{j_S} w^{N/D}\right)} = 0 \quad (179)$$

Now if we assume that all the CPs are also homogeneous in their cost preferences, in addition to their utility offer and demand functions, all the prices will be equal, so that $v_i = v_{js}$

$$\begin{aligned} 0 &= \frac{1}{N} - \frac{2Nv_{j_S}^{N/D} - (N-1)2Nv_{j_S}^{N/D}}{\lambda_{j_S}\left(1 - h_{j_S} w^{N/D}\right)N} + \frac{c}{\lambda_{j_S}\left(1 - h_{j_S} w^{N/D}\right)} \\ &= \frac{1}{N} - \frac{(N+1)v_{j_S}^{N/D}}{\lambda_{j_S}\left(1 - h_{j_S} w^{N/D}\right)N} + \frac{c}{\lambda_{j_S}\left(1 - h_{j_S} w^{N/D}\right)} \end{aligned} \quad (180)$$

Rearranging for v_{js} yields

$$v_{j_S}^{N/D} = \frac{\lambda_{j_S}\left(1 - h_{j_S} w^{N/D}\right) + Nc}{N+1} \square \quad (181)$$

Demand served by subscribing CPs as a function of competition

We have equations (68) and (69) that show the fraction of demand served by CP j_S and the price of the CP.

$$\eta_{j_S} = 1 - \iota_{j_S} = \frac{1}{N} - \frac{v_{j_S}^{N/D}}{\lambda_{j_S}\left(1 - h_{j_S} w^{N/D}\right)} + \frac{\sum_{i \neq j_S} v_i^{N/D}}{\lambda_{j_S}\left(1 - h_{j_S} w^{N/D}\right)N} \quad (182)$$

$$v_{j_S}^{N/D} = \frac{\lambda_{j_S}\left(1 - h_{j_S} w^{N/D}\right) + Nc}{N+1} \quad (183)$$

By inserting (183) into (182) we can obtain an expression for the demand served as a function of the utility, $\lambda_{j_S}\left(1 - h_{j_S} w^{N/D}\right)$, and the level of competition, N. The utility offer will be noted as b for better readability

$$\eta_{j_S} = \frac{1}{N} - \frac{1}{\mathrm{b}}\frac{\mathrm{b} + Nc}{N+1} + \frac{(N-1)}{\mathrm{b}N}\frac{\mathrm{b} + Nc}{N+1} \tag{184}$$

We have now already made the assumption that the CPs are homogeneous, and thus offer equal prices in a Nash equilibrium. The expression (184) can immediately be simplified by separating the fractions

$$\eta_{j_S} = \frac{1}{N} - \frac{1}{(\mathrm{N}+1)} - \frac{Nc}{b(N+1)} + \frac{(N-1)}{N(N+1)} + \frac{(N-1)Nc}{\mathrm{b}N(N+1)} \tag{185}$$

And by equalizing the denominators of the second and fourth term, and third and fifth term

$$\eta_{j_S} = \frac{1}{N} + \frac{(N-1)-N}{N(\mathrm{N}+1)} + \frac{(N-1-N)c}{\mathrm{b}(N+1)} = \frac{1}{N} - \frac{1}{N(\mathrm{N}+1)} - \frac{c}{\mathrm{b}(N+1)} \tag{186}$$

Then by equalizing all the denominators, we obtain

$$\begin{aligned}\eta_{j_S} &= \frac{1}{N} + \frac{(N-1)-N}{N(\mathrm{N}+1)} + \frac{(N-1-N)c}{\mathrm{b}(N+1)} = \frac{\mathrm{b}(N+1) - b - Nc}{bN(N+1)} \\ &= \frac{\mathrm{b}N - Nc}{bN(N+1)} = \frac{1}{N+1}\left(1 - \frac{c}{b}\right)\end{aligned} \tag{187}$$

By substituting the utility offer back into the equation we obtain

$$\eta_{j_S} = 1 - \iota_{j_S} = \frac{1}{N+1}\left(1 - \frac{c}{\lambda_{j_S}\left(1 - h_{j_S} w^{N/D}\right)}\right) \tag{188}$$

We see that the demand for content for the single CP is decreasing, approaching zero with perfect competition. This is from the assumption of consumers' time being a scarce resource, which is the reason why the CPs compete with price. □

8.2 Analytical Solutions

The following section contains the process of deriving the analytical solutions from the analytic model, as well as the modifications and second order optimum conditions of the reduced analytic model.

8.2.1 Analytic Model NN Regime

Description of information sets

0. Exogenous parameters set
    a. All players know this the whole game, including the optimal conditions of other players
1. Backbone and access ISPs decisions
    a. IUs connectivity price (access only)
        i. Know the demand for service, but are unfamiliar with the connectivity price set by the backbone ISP, and are unable to respond to this price once it has been set
    b. Investment levels (access only)
        i. Know all demand functions based on the connectivity prices. However, the prices are unknown and the ISP cannot respond to the prices once a decision to invest has been made
    c. Connectivity fee CPs (backbone only)
        i. Know the demand for CPs based on investment levels. However, do not know how large the investments from the access ISPs will be.
2. CPs and ISPs decisions
    a. CPs choose which access ISPs to connect to
        i. Know the price for connectivity from the backbone ISP and the investment levels by the access CPs. However, they are unfamiliar with the distribution of IUs on the ISPs and the final congestion levels
    b. IUs choose which platform, a or b, to connect to
        i. Know the price and investment levels of the ISPs, they also know their position within the city, biasing them towards either of the platforms. However, they do not yet know the distribution of CPs and the final congestion levels, but will favor ISPs with more content and low congestion

Stage 0: Exogenous parameters set

- $C^V(x_i^N) = C^V x_i^N$
- $C_c^V(\bar{y}_i^N) = C_c^V y_i^N$
- $C^I(\mu_i^N) = \frac{C^I}{2}\mu_i^{N\,2}$
- Other parameters to be set for numerical analysis

Congestion

$$\Lambda_i^N = \int_0^{\bar{y}_i^N} \lambda x_i^N dy_i = \lambda x_i^N \bar{y}_i^N \tag{189}$$

This can be inserted into the congestion formula

$$w_i^N = \frac{1}{\mu_i^N - \lambda x_i^N \bar{y}_i^N} \tag{190}$$

From stage 2: By substituting for the value of $\bar{y}_i^N$ and imposing the symmetric solution we obtain

$$w^N = \frac{r\lambda - 2f_c^N + 2rh}{2\mu_i^N rh} \tag{191}$$

Stage 2: The CPs and IUs decisions

CPs

The CP y^N will join as long as

$$\sum_{i\epsilon\{a,b\}} \left[r\lambda x_i^N\left(1 - hy_i^N w_i^N\right) - f_c^D\right] \geq 0 \tag{192}$$

The last CP $\bar{y}_i^N$ that connects to ISP i ϵ {a,b} can then be formulated as

$$r\lambda x_i^N\left(1 - h\bar{y}_i^N w_i^N\right) - f_c^N = 0 \tag{193}$$

The last CP interested in connecting is thus

$$\bar{y}_i^N = \frac{r\lambda x_i^N - f_c^N}{r\lambda h x_i^N}\frac{1}{w_i^N} \tag{194}$$

By substituting for w_i^N, this can be rearranged into

$$\bar{y}_i^N = \frac{\mu_i^N\left(r\lambda x_i^N - f_c^N\right)}{\lambda x_i^N\left(r\lambda x_i^N - f_c^N + rh\right)} \tag{195}$$

By imposing the symmetrical solution we get

$$\bar{y}^N = \frac{2\mu^N(r\lambda - 2f_c^N)}{\lambda(r\lambda - 2f_c^N + 2rh)} \tag{196}$$

Equation (194) and (195) provides us with a relation we need later

$$\frac{\mu_i^N rh}{r\lambda x_i^N - f_c^N + rh} = \frac{1}{w_i^N} \tag{197}$$

IUs

We assume there is enough numéraire utility, r, so that all IUs connect. If $x_a^N = \tilde{x}^N$ and $x_b^N = (1 - \tilde{x}^N)$ we get

$$\begin{aligned} U_{ax}^N = U_{ax}^N \rightarrow R + u_C\bar{y}_a^N + \frac{u_w}{w_a^N} - p_a^N - t\tilde{x}^N \\ = R + u_c\bar{y}_b^N + \frac{u_w}{w_b^N} - p_b^N - t(1 - \tilde{x}^N) \end{aligned} \tag{198}$$

This can be restructured into a function $\mathrm{F}(\tilde{x}^N, \bar{y}_a^N, \bar{y}_b^N, w_a^N, w_b^N, p_a^N, p_b^N)$

$$\mathrm{F}(\cdot) = u_C\left(\bar{y}_b^N - \bar{y}_a^N\right) + u_w\left(\frac{1}{w_b^N} - \frac{1}{w_a^N}\right) - \left(p_b^N - p_a^N\right) - t(1 - 2\tilde{x}^N) = 0 \tag{199}$$

By substituting (195) and (197) into we (199) we obtain

$$\begin{aligned} F(\cdot) \ = & \left(\frac{u_C(r\lambda(1-\tilde{x}^N)-f_c^N)}{\lambda(1-\tilde{x}^N)} + u_w rh\right)\frac{\mu_b^N}{r\lambda(1-\tilde{x}^N)-f_c^N+rh} \\ & -\left(\frac{u_C(r\lambda\tilde{x}^N-f_c^N)}{\lambda\tilde{x}^N} + u_w rh\right)\frac{\mu_a^N}{r\lambda\tilde{x}^N-f_c^N+rh} \\ & -\left(p_b^N - p_a^N\right) - t(1-2\tilde{x}^N) = 0 \end{aligned} \quad (200)$$

This function implicitly gives $\tilde{x}^N$ as a function $\tilde{x}^N(f_c^N, p_a^D, p_b^D, \mu_a^{NT}, \mu_b^{NT})$

Stage 1: ISPs set the IUs price and SS price

Access ISPs

The first order optimal condition for the access ISPs are

$$\frac{\partial \Pi_i^N}{\partial p_i^N} = x_i^N + \left(p_i^N - C^V\right)\frac{\partial x_i^N}{\partial p_i^N} = 0 \quad (201)$$

$$\frac{\partial \Pi_i^N}{\partial \mu_i^N} = \left(p_i^N - C^V\right)\frac{\partial x_i^N}{\partial \mu_i^N} - C^I \mu_i^N = 0 \quad (202)$$

IUs price for connectivity on Access ISPs

To find the derivatives we use the implicit theorem

$$\frac{\partial x_a^N}{\partial p_a^N} = -\frac{\partial F / \partial p_a^N}{\partial F / \partial x_a^N} \quad (203)$$

Where

$$\frac{\partial F}{\partial p_a^N} = 1 \quad (204)$$

$$\frac{\partial F}{\partial x_a^N} = \frac{\partial F}{\partial \tilde{x}^D}$$
$$= \mu_b^T \left(\frac{r\lambda x_b^N \left(u_C \left(r\lambda x_b^D - 2f_c^N\right) + r\lambda x_b^N u_w h\right) + u_C f_c^N (f_c^N - rh)}{\lambda {x_b^N}^2 (r\lambda x_b^N - f_c^D + rh)^2} \right)$$
$$+ \mu_a^N \left(\frac{r\lambda x_a^N (u_C (r\lambda x_a^N - 2f_c^N) + r\lambda x_a^N u_w h) + u_C f_c^N (f_c^N - rh)}{\lambda {x_a^N}^2 (r\lambda x_a^N - f_c^N + rh)^2} \right) + 2t$$
$$= K^N > 0 \quad (205)$$

So we get

$$\frac{\partial x_a^N}{\partial p_a^N} = -\frac{1}{K^N} < 0 \quad (206)$$

By imposing the symmetrical solution we get

$$K^N = 8\mu^N \left(\frac{r\lambda (u_C (r\lambda - 4f_c^N) + r\lambda u_w h) + 4u_C f_c^N (f_c^N - rh)}{\lambda (r\lambda - 2f_c^N + 2rh)^2} \right) + 2t \quad (207)$$

Further, we have

$$p^N = \frac{K^N}{2} + C^V \quad (208)$$

By substituting for K^D and the y derivatives we get

$$p^N = 4\mu^N \left(\frac{r\lambda (u_C (r\lambda - 4f_c^N) + r\lambda u_w h) + 4u_C f_c^N (f_c^N - rh)}{\lambda (r\lambda - 2f_c^N + 2rh)^2} \right) + C^V + t \quad (209)$$

Investment levels

$$\frac{\partial \Pi_i^N}{\partial \mu_i^N} = \left(p_i^N - C^V\right) \frac{\partial x_i^N}{\partial \mu_i^N} - C^I \mu_i^N = 0 \quad (210)$$

To find $\frac{\partial x_i^N}{\partial \mu_i^N}$ we use the implicit derivation theorem $\frac{\partial x_i^N}{\partial \mu_i^N} = -\frac{\partial F/\partial \mu_i^N}{\partial F/\partial x_i^N} = -\frac{\partial F}{\partial \mu_i^N}\frac{1}{K^N}$, where we already have K^N. Further we have

$$\frac{\partial F}{\partial \mu_a^N} = -\left(\frac{u_A(r\lambda\tilde{x}^N - f_c^N) + u_w\lambda\tilde{x}^N rh}{\lambda\tilde{x}^N(r\lambda\tilde{x}^N - f_c^N + rh)}\right) \tag{211}$$

So we get

$$\frac{\partial x_a^D}{\partial \mu_a^N} = \frac{u_C(r\lambda\tilde{x}^N - f_c^N) + u_w\lambda\tilde{x}^N rh}{\lambda\tilde{x}^N(r\lambda\tilde{x}^N - f_c^N + rh)K^N} \tag{212}$$

By substituting (208) and (212) into the first order optimum condition for investment levels, and imposing the symmetrical solution, we obtain

$$\frac{u_C(r\lambda - 2f_c^N) + u_w\lambda rh}{\lambda(r\lambda - 2f_c^N + 2rh)} - C^I\mu_i^N = 0 \tag{213}$$

This can be rearranged into an expression for the investment level, μ^N

$$\mu^N = \frac{1}{C^I}\frac{u_C(r\lambda - 2f_c^N) + u_w\lambda rh}{\lambda(r\lambda - 2f_c^N + 2rh)} \tag{214}$$

Backbone ISP

The backbone cannot offer different prices based on which ISPs the CPs connect to. We know that the ISPs are symmetric, so by imposing symmetry we get

$$\Pi_c^D = 2f_c^D\bar{y}_i^D - 2C_c^V\bar{y}_i^D \tag{215}$$

Since $\bar{y}_i^N$ is a function of both f_c^N, and x_i^N, which is dependent on f_c^N, we have from the product and chain rule

$$\frac{\partial\bar{y}_i^N}{\partial f_c^N} = \frac{\partial\bar{y}_i^N}{\partial f_c^N} + \frac{\partial\bar{y}_i^N}{\partial x_i^N}\frac{\partial x_i^N}{\partial f_c^N} \tag{216}$$

This results in the following first order optimum condition for the backbone ISP

$$\frac{\partial \Pi_c^N}{\partial f_c^N} = \bar{y}_i^N + (f_c^N - C_c^V)\left(\frac{\partial \bar{y}_i^N}{\partial f_c^N} + \frac{\partial \bar{y}_i^N}{\partial x_i^N}\frac{\partial x_i^N}{\partial f_c^N}\right) = 0 \qquad (217)$$

Connectivity price

$$\frac{\partial \bar{y}_i^N}{\partial f_c^N} = -\frac{rh\mu_i^N}{\lambda x_i^N \left(\left(r\lambda x_i^N - f_c^N + rh\right)\right)^2} < 0 \qquad (218)$$

Further, because of symmetry, we have

$$\frac{\partial F}{\partial f_c^N} = 0 \rightarrow \frac{\partial x_i^N}{\partial f_c^N} = -\frac{\partial F}{\partial f_c^N}\frac{1}{K^N} = 0 \qquad (219)$$

If we impose the symmetric solution, we obtain

$$\frac{\partial \bar{y}^N}{\partial f_c^N} = -\frac{8rh\mu^N}{\lambda(r\lambda - 2f_c^N + 2rh)^2} < 0 \qquad (220)$$

So we can now write the first order optimum condition as

$$\frac{\partial \Pi_c^N}{\partial f_c^N} = \bar{y}_i^N + (f_c^N - C_c^V)\frac{\partial \bar{y}_i^N}{\partial f_c^N} = 0 \qquad (221)$$

By substituting for the derivative and $\bar{y}^N$, and equalizing the denominators, we obtain

$$\frac{\partial \Pi_c^N}{\partial f_c^N} = \frac{2\mu^N(r\lambda - 2f_c^N)(r\lambda - 2f_c^N + 2rh)}{\lambda(r\lambda - 2f_c^N + 2rh)^2} - \frac{8rh\mu^N(f_c^N - C_c^V)}{\lambda(r\lambda - 2f_c^N + 2rh)^2} = 0 \qquad (222)$$

This can be rearranged into an implicit expression for f_c^N

$$4{f_c^N}^2 - (8rh + 4r\lambda)f_c^N + (r\lambda)^2 + 2r^2\lambda h + 4rhC_c^V = 0 \qquad (223)$$

This expression can be solved with the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, which yields

$$f_c^N = \frac{(8rh + 4r\lambda) \pm \sqrt{(8rh + 4r\lambda)^2 - 16((r\lambda)^2 + 2r^2\lambda h + 4rhC_c^V)}}{8} = \frac{(2rh + r\lambda) \pm \sqrt{rh(4rh + 2r\lambda - 4C_c^V)}}{2} \quad (224)$$

This can be simplified into

$$f_c^N = \frac{2rh + r\lambda \pm \sqrt{rh(4rh + 2r\lambda - 4C_c^V)}}{2} \quad (225)$$

Welfare

The CPs

The profit of the CPs in equilibrium equals

$$\Pi_y^N = \begin{cases} \sum_{i\epsilon\{a,b\}} \left[\frac{r\lambda}{2}(1 - hy^N w^N) - f_c^N\right] & if \quad 0 \le y^N < \bar{y}^N \\ 0 & \quad\quad y^N \ge \bar{y}^N \end{cases} \quad (226)$$

Thus, the total profit of all CPs in equilibrium is

$$\Sigma\Pi_{CSP}^N = \int_0^{\bar{y}^N} [r\lambda(1 - hy^N w^N) - 2f_c^D]dy^N \quad (227)$$

Since all values except y^N is constant the total surplus equals

$$\Sigma\Pi_{CSP}^N = r\lambda\bar{y}^N - \frac{r\lambda h {\bar{y}^N}^2 w^N}{2} - 2f_c^D\bar{y}^N \quad (228)$$

By substituting for the values of $\bar{y}^N$ *and* w^N we obtain

$$\Sigma\Pi_{CSP}^{N} = \frac{2\mu^N(r\lambda - 2f_c^N)}{\lambda(r\lambda - 2f_c^N + 2rh)}\left(r\lambda - \frac{r\lambda h 2\mu^N(r\lambda - 2f_c^N)}{2\lambda(r\lambda - 2f_c^N + 2rh)}\frac{r\lambda - 2f_c^N + 2rh}{2\mu^N rh} - 2f_c^D\right) \quad (229)$$

$$\Sigma\Pi_{CSP}^{N} = \frac{\mu^N(r\lambda - 2f_c^N)^2}{\lambda(r\lambda - 2f_c^N + 2rh)} \quad (230)$$

The IUs

The IUs surplus, or consumer surplus, can be modelled as

$$CS_{IU}^{N} = 2\int_0^{1/2}\left(R + u_C\bar{y}_a^N + \frac{u_w}{w_a^N} - p_a^N - t\tilde{x}^N\right)d\tilde{x}^N \quad (231)$$

Since $\bar{y}^N, w^N and\ p^N$ are constant with respect to $\tilde{x}^N$, we get

$$CS_{IU}^{N} = R + u_C\bar{y}^N + \frac{u_w}{w^N} - p^N - t\frac{1}{4} \quad (232)$$

Substituting for $\bar{y}^N, w^N and\ p^N$ we obtain

$$CS_{IU}^{N} = R + 2\mu^N\left(\frac{u_C\left(r\lambda - 2f_c^N\right) + u_w r\lambda h}{\lambda\left(r\lambda - 2f_c^N + 2rh\right)}\right) - 4\mu^N\left(\frac{r\lambda\left(u_C\left(r\lambda - 4f_c^N\right) + r\lambda u_w h\right) + 4u_C f_c^N\left(f_c^N - rh\right)}{\lambda\left(r\lambda - 2f_c^N + 2rh\right)^2}\right) - C^V - t\frac{5}{4} \quad (233)$$

The access ISPs

There are two access ISPs, so the total surplus equals in equilibrium

$$\Sigma\Pi^N_{ISP_{access}} = \sum_{i\epsilon\{a,b\}} \left(p^N x_i^N - C^V x_i^N - C^I(\mu^N)\right) = p^N - C^V - 2C^I(\mu^N) \qquad (234)$$

This equals

$$\Sigma\Pi^N_{ISP_{access}} = 4\mu^N \left(\frac{r\lambda\left(u_c\left(r\lambda - 4f_c^N\right) + r\lambda u_w h\right) + 4u_c f_c^N\left(f_c^N - rh\right)}{\lambda\left(r\lambda - 2f_c^N + 2rh\right)^2}\right) + t - 2C^I(\mu^N) \qquad (235)$$

The backbone ISPs

There is only one backbone ISP, and by substituting the equilibrium solutions to the profit function we obtain

$$\Sigma\Pi^N_{ISP_{back}} = \sum_{i\epsilon\{a,b\}} \left[f_c^N \bar{y}^N - C_c^V \bar{y}^N\right] = \frac{4\mu^N\left(f_c^N - C_c^V\right)\left(r\lambda - 2f_c^N\right)}{\lambda\left(r\lambda - 2f_c^N + 2rh\right)} \qquad (236)$$

Total welfare

The total welfare is defined as the sum of the access and backbone ISPs profits, the CPs profits and the IUs consumer surplus

$$\begin{aligned} W^N &= \Sigma\Pi^N_{CSP} + CS^N_{IU} + \Sigma\Pi^N_{ISP_{access}} + \Sigma\Pi^N_{ISP_{back}} \\ &= R + (r\lambda - 2C_c^V)\bar{y}^N - \frac{r\lambda h \bar{y}^{N^2} w^N}{2} + u_C \bar{y}^N + \frac{u_w}{w^N} - t\frac{1}{4} \\ &- C^V - 2C^I(\mu^N) \end{aligned} \qquad (237)$$

By inserting for the profits and consumer surplus we obtain

$$W^N = R + 2\mu^N \frac{(r\lambda - 2C_c^V)\left(r\lambda - 2f_c^N\right)}{\lambda\left(r\lambda - 2f_c^N + 2rh\right)} - \mu^N \frac{\left(r\lambda - 2f_c^N\right)^2}{\lambda\left(r\lambda - 2f_c^N + 2rh\right)} + 2\mu^N \frac{u_C\left(r\lambda - 2f_c^N\right) + u_w r\lambda h}{\lambda\left(r\lambda - 2f_c^N + 2rh\right)} - t\frac{1}{4} - C^V - 2C^I(\mu^N) \qquad (238)$$

8.2.2 Analytic Model NNN Regime

Description of information sets

0. Exogenous parameters set
    a. All players know this the whole game, including the optimal conditions of other players
1. Backbone and access ISPs decisions
    a. IUs connectivity price (access only)
        i. Know the demand for service, but are unfamiliar with the connectivity price set by the backbone ISP, and are unable to respond to this price once it has been set
    b. Investment levels (access only)
        i. Know all demand functions based on the connectivity prices. However, the prices are unknown and the ISP cannot respond to the prices once a decision to invest has been made
    c. Connectivity fee CPs (backbone only)
        i. Know the demand for CPs based on investment levels. However, do not know how large the investments from the access ISPs will be.
    d. Connectivity fee for SS (both)
        i. Both access and backbone ISPs know the demand for SS. However, they do not know what price the other players will set, and thus participate in a double marginalization game, where the total demand is a result of their combined prices. They set prices simultaneously without the opportunity to change the price at a later stage
2. CPs and ISPs decisions
    a. CPs choose which access ISPs to connect to, and either SS or IAS
        i. Know the price for connectivity and SS, and the investment levels by the CPs. However, they are unfamiliar with the distribution of IUs on the ISPs and the final congestion levels
    b. IUs choose which platform, a or b, to connect to
        i. Know the price and investment levels of the ISPs, they also know their position within the city, biasing them towards either of the platforms. However, they do not yet know the distribution of CPs and the final congestion levels, but will favor ISPs with more content and low congestion

Stage 0: Exogenous parameters set

- $C^V(x_i^D) = C^V x_i^D$
- $C_c^V(\bar{y}_i^D) = C_c^V y_i^N$
- $C^I(\mu_i^D) = \frac{C^I}{2}\mu_i^{D\,2}$
- Other parameters to be set for numerical analysis

Congestion

$$\Lambda_i^D = \int_0^{\bar{y}_i^D} \lambda x_i^D \, dy_i = \lambda x_i^D \bar{y}_i^D \tag{239}$$

$$\Lambda_i^{DSS} = \int_{\tilde{y}_i^D}^{\bar{y}_i^D} \lambda x_i^D \, dy_i = \lambda x_i^D \left(\bar{y}_i^D - \tilde{y}_i^D\right) \tag{240}$$

$$\Lambda_i^{DIA} = \int_0^{\tilde{y}_i^D} \lambda x_i^D \, dy_i = \lambda x_i^D \bar{y}_i^D \tag{241}$$

$$\beta_i = \frac{\Lambda_i^{DSS}}{\Lambda_i^D} = \frac{\lambda x_i^D \left(\bar{y}_i^D - \tilde{y}_i^D\right)}{\lambda x_i^D \bar{y}_i^D} = 1 - \frac{\tilde{y}_i^D}{\bar{y}_i^D} \tag{242}$$

This can be inserted into the congestion formulas

$$w_i^D = \frac{1}{\mu_i^D - \lambda x_i^D \bar{y}_i^D} \tag{243}$$

$$w_i^D = \beta_i w_i^{DSS} + (1 - \beta_i) w_i^{DIA} \tag{244}$$

$$w_i^{DSS} = \frac{1}{\mu_i^D - \lambda x_i^D \left(\bar{y}_i^D - \tilde{y}_i^D\right)} \tag{245}$$

$$w_i^{DIA} = \frac{\mu_i^D}{\mu_i^D - \lambda x_i^D \bar{y}_i^D} w_i^{DSS} = \frac{\mu_i^D}{\mu_i^D - \lambda x_i^D \bar{y}_i^D} \frac{1}{\mu_i^D - \lambda x_i^D \left(\bar{y}_i^D - \tilde{y}_i^D\right)} \tag{246}$$

From stage 2: By substituting for the values of $\bar{y}_i^D$ and $\tilde{y}_i^D$ and imposing the symmetric solution we obtain

$$w^D = \frac{(r\lambda - 2f_c^D + 2rh)}{2rh\mu^D} \qquad (247)$$

$$w^{DSS} = \frac{\left(r\lambda - 2f_c^D - 2f_i^{DSS} - 2f_c^{DSS}\right)(r\lambda - 2f_c^D + 2rh)}{2rh\mu^D(r\lambda - 2f_c^D)} \qquad (248)$$

$$\begin{aligned} w^{DIA} &= \frac{(r\lambda - 2f_c^D + 2rh)}{(r\lambda - 2f_c^D + 2rh) - (r\lambda - 2f_c^D)} w^{DSS} \\ &= \frac{\left(r\lambda - 2f_c^D - 2f_i^{DSS} - 2f_c^{DSS}\right)(r\lambda - 2f_c^D + 2rh)^2}{4(rh)^2\mu^D(r\lambda - 2f_c^D)} \end{aligned} \qquad (249)$$

We can also formulate the relations

$$\begin{aligned} w^{DSS} &= \frac{(r\lambda - 2f_c^D - 2f_i^{DSS} - 2f_c^{DSS})w^D}{(r\lambda - 2f_c^D)}, w^D = \frac{(r\lambda - 2f_c^D + 2rh)}{2rh\mu^D}, w^{DIA} \\ &= \frac{(r\lambda - 2f_c^D - 2f_i^{DSS} - 2f_c^{DSS})(r\lambda - 2f_c^D + 2rh)w^D}{2rh(r\lambda - 2f_c^D)} \end{aligned} \qquad (250)$$

The relations can be ordered accordingly

$$\begin{aligned} &\frac{(r\lambda - 2f_c^D - 2f_i^{DSS} - 2f_c^{DSS})}{(r\lambda - 2f_c^D)} w^D \le w^D \\ &\le \frac{(r\lambda - 2f_c^D - 2f_i^{DSS} - 2f_c^{DSS})(r\lambda - 2f_c^D + 2rh)}{2rh(r\lambda - 2f_c^D)} w^D \end{aligned} \qquad (251)$$

Which is a necessary result for the SS scheme to be efficient. This always holds in equilibrium as long as

$$f_c^D, f_i^{DSS}, f_c^{DSS} \ge 0 \qquad (252)$$

Stage 2: The CPs and IUs decisions

CPs

The CP y_i^D will join as long as

$$\sum_{i\epsilon\{a,b\}}\left[r\lambda x_i^D\left(1-hy_i^D w_i^{DSS}\right)-f_c^D-f_i^{DSS}-f_c^{DSS}\right]\geq 0 \tag{253}$$

The last CP $\bar{y}_i^D$ that connects to ISP i϶{a,b} can then be formulated as

$$r\lambda x_i^D\left(1-h\bar{y}_i^D w_i^{DSS}\right)-f_c^D-f_i^{DSS}-f_c^{DSS}=0 \tag{254}$$

The CP in platform i϶{a,b} that is indifferent to SS can be formulated as

$$r\lambda x_i^D\left(1-h\tilde{y}_i^D w_i^{DSS}\right)-f_i^{DSS}-f_c^{DSS}=r\lambda x_i^D\left(1-h\tilde{y}_i^D w_i^{DIA}\right) \tag{255}$$

Equation (255) implies

$$f_c^{DSS}+f_i^{DSS}=r\lambda x_i^D h\tilde{y}^D\left(w_i^{DIA}-w_i^{DSS}\right) \tag{256}$$

This can be inserted into (193)

$$r\lambda x_i^D\left(1-h\bar{y}_i^D w_i^{DSS}\right)-f_c^D-r\lambda x_i^D h\tilde{y}^D\left(w_i^{DIA}-w_i^{DSS}\right)=0 \tag{257}$$

This can be simplified into

$$1-h\left(\left(\bar{y}_i^D-\tilde{y}^D\right)w_i^{DSS}+\tilde{y}^D w_i^{DIA}\right)=\frac{f_c^D}{r\lambda x_i^D} \tag{258}$$

If we divide (257) by $\bar{y}_i^D$ and apply relation (244) we obtain

$$\bar{y}_i^D=\frac{r\lambda x_i^D-f_c^D}{r\lambda x_i^D h}\frac{1}{w_i^D} \tag{259}$$

By substituting for w_i^D, this can be rearranged into

$$\bar{y}_i^D = \frac{\mu_i^D\left(r\lambda x_i^D - f_c^D\right)}{\lambda x_i^D\left(r\lambda x_i^D - f_c^D + rh\right)} \tag{260}$$

By imposing the symmetrical solution we get

$$\bar{y}^D = \frac{2\mu^D(r\lambda - 2f_c^D)}{\lambda(r\lambda - 2f_c^D + 2rh)} \tag{261}$$

Equation (259) and (195) provides us with a relation we need later

$$\frac{rh\mu_i^D}{r\lambda x_i^D - f_c^D + rh} = \frac{1}{w_i^D} \tag{262}$$

By rearranging (193), and substituting with (245), we get

$$\begin{aligned}\tilde{y}_i^D &= \frac{\mu_i^D rh\left(f_i^{DSS} + f_c^{DSS}\right)}{\lambda x_i^D\left(r\lambda x_i^D - f_c^D - f_i^{DSS} - f_c^{DSS}\right)\left(r\lambda x_i^D - f_c^D + rh\right)} \\ &= \frac{rh\left(f_i^{DSS} + f_c^{DSS}\right)}{\left(r\lambda x_i^D - f_c^D\right)\left(r\lambda x_i^D - f_c^D - f_i^{DSS} - f_c^{DSS}\right)}\bar{y}_i^D\end{aligned} \tag{263}$$

By imposing the symmetrical solution we get

$$\tilde{y}^D = \frac{4rh\left(f_i^{DSS} + f_c^{DSS}\right)}{(r\lambda - 2f_c^D)\left(r\lambda - 2f_c^D - 2f_i^{DSS} - 2f_c^{DSS}\right)}\bar{y}^D \tag{264}$$

IUs

We assume there is enough numéraire utility, R, so that all IUs connect. If $x_a^D = \tilde{x}^D$ and $x_b^D = (1 - \tilde{x}^D)$ we get

$$\begin{aligned}U_{ax}^D = U_{ax}^D \rightarrow R + u_c\bar{y}_a^D + \frac{u_w}{w_a^D} - p_a^D - t\tilde{x}^D \\ = R + u_c\bar{y}_b^D + \frac{u_w}{w_b^N} - p_b^D - t(1 - \tilde{x}^D)\end{aligned} \tag{265}$$

This can be restructured into a function $\mathrm{F}(\tilde{x}^D, \bar{y}_a^D, \bar{y}_b^D, w_a^D, w_b^D, p_a^D, p_b^D)$

$$\mathrm{F}(\cdot) = u_A\left(\bar{y}_b^D - \bar{y}_a^D\right) + u_w\left(\frac{1}{w_b^D} - \frac{1}{w_a^D}\right) - \left(p_b^D - p_a^D\right) - t(1 - 2\tilde{x}^D) = 0 \quad (266)$$

By substituting (195) and (197) into we (199) we obtain

$$\begin{aligned}\mathrm{F}(\cdot) = &\left(\frac{u_A(r\lambda(1-\tilde{x}^D) - f_c^D)}{\lambda(1-\tilde{x}^D)} + u_w rh\right)\frac{\mu_b^D}{r\lambda(1-\tilde{x}^D) - f_c^D + rh} \\ &- \left(\frac{u_A(r\lambda\tilde{x}^D - f_c^D)}{\lambda\tilde{x}^D} + u_w rh\right)\frac{\mu_a^D}{r\lambda\tilde{x}^D - f_c^D + rh} \\ &- \left(p_b^D - p_a^D\right) - t(1-2\tilde{x}^D) = 0\end{aligned} \quad (267)$$

This function implicitly gives $\tilde{x}^D$ as a function $\tilde{x}^D(f_c^D, p_a^D, p_b^D, \mu_a^{DT}, \mu_b^{DT})$

Stage 1: ISPs set the IUs price and SS price

Access ISPs

The first order optimal condition for the access ISPs are

$$\frac{\partial \Pi_i^D}{\partial p_i^D} = x_i^D + \left(p_i^D + \frac{\partial\left(f_i^{DSS}\left(\bar{y}_i^D - \tilde{y}_i^D\right)\right)}{\partial x_i^D} - C^V\right)\frac{\partial x_i^D}{\partial p_i^D} = 0 \quad (268)$$

$$\frac{\partial \Pi_i^D}{\partial f_i^{DSS}} = \left(\bar{y}_i^D - \tilde{y}_i^D\right) + f_i^{DSS}\frac{\partial\left(\bar{y}_i^D - \tilde{y}_i^D\right)}{\partial f_i^{DSS}} = 0 \quad (269)$$

$$\frac{\partial \Pi_i^D}{\partial \mu_i^D} = \left(p_i^D - C^V + \frac{f_i^{DSS}\,\partial\left(\bar{y}_i^D - \tilde{y}_i^D\right)}{\partial x_i^D}\right)\frac{\partial x_i^D}{\partial \mu_i^D} + \frac{f_i^{DSS}\,\partial\left(\bar{y}_i^D - \tilde{y}_i^D\right)}{\partial \mu_i^D} - C^I\mu_i^D = 0 \quad (270)$$

IUs price for connectivity on Access ISPs

To find the derivatives we use the implicit theorem

$$\frac{\partial x_a^D}{\partial p_a^D} = -\frac{\partial F/\,\partial p_a^D}{\partial F/\,\partial x_a^D} \quad (271)$$

Where

$$\frac{\partial F}{\partial p_a^D} = 1 \qquad (272)$$

$$\begin{aligned}
\frac{\partial F}{\partial x_a^N} &= \frac{\partial F}{\partial \tilde{x}^D} \\
&= \mu_b^D \left(\frac{r\lambda x_b^D \left(u_C \left(r\lambda x_b^D - 2f_c^D\right) + r\lambda x_b^D u_w h\right) + u_C f_c^D (f_c^D - rh)}{\lambda {\tilde{x}_b^D}^2 (r\lambda \tilde{x}_b^D - f_c^D + rh)^2} \right) \\
&+ \mu_a^D \left(\frac{r\lambda x_a^D (u_C (r\lambda x_a^D - 2f_c^D) + r\lambda x_a^D u_w h) + u_C f_c^D (f_c^D - rh)}{\lambda {x_a^D}^2 (r\lambda x_a^D - f_c^D + rh)^2} \right) + 2t \\
&= K^D > 0
\end{aligned} \qquad (273)$$

So we get

$$\frac{\partial x_a^D}{\partial p_a^D} = -\frac{1}{K^D} < 0 \qquad (274)$$

Further we have

$$\frac{\partial \left(f_i^{DSS} \left(\bar{y}_i^D - \tilde{y}_i^D \right) \right)}{\partial x_i^D} = f_i^{DSS} \left(\frac{\partial \bar{y}_i^D}{\partial x_i^D} - \frac{\partial \tilde{y}_i^D}{\partial x_i^D} \right) \qquad (275)$$

$$\frac{\partial \bar{y}_i^D}{\partial x_i^D} = \mu_i^D \, \frac{rhf_c^D - \left(r\lambda x_i^D - f_c^D\right)^2}{\lambda \left(x_i^D \left(r\lambda x_i^D - f_c^D + rh \right) \right)^2} \qquad (276)$$

$$\frac{\partial \tilde{y}_i^D}{\partial x_i^D} = \mu_i^D rh\left(f_i^{DSS} + f_c^{DSS}\right)\left(\frac{\left(f_c^D + f_i^{DSS} + f_c^{DSS}\right)(rh - f_c^D)}{\lambda\left(x_i^D\left(r\lambda x_i^D - f_c^D - f_i^{DSS} - f_c^{DSS}\right)\left(r\lambda x_i^D - f_c^D + rh\right)\right)^2} - \frac{r\lambda x_i^D\left(3r\lambda x_i^D - 4f_c^D - 2f_i^{DSS} - 2f_c^{DSS} + 2rh\right)}{\lambda\left(x_i^D\left(r\lambda x_i^D - f_c^D - f_i^{DSS} - f_c^{DSS}\right)\left(r\lambda x_i^D - f_c^D + rh\right)\right)^2}\right) \quad (277)$$

For easier notation, we write $\left(f_i^{DSS} + f_c^{DSS}\right) = f$. By imposing the symmetrical solution we get

$$\frac{\partial \bar{y}_i^D}{\partial x_i^D} = \mu^D \frac{16rhf_c^D - (2r\lambda - 4f_c^D)^2}{\lambda(r\lambda - 2f_c^D + 2rh)^2} \quad (278)$$

$$\frac{\partial \tilde{y}_i^D}{\partial x_i^D} = 16\mu_i^D rhf \frac{4(f_c^D + f)(rh - f_c^D) - r\lambda(3r\lambda - 8f_c^D - 4f + 4rh)}{\lambda\left((r\lambda - 2f_c^D - 2f)(r\lambda - 2f_c^D + 2rh)\right)^2} \quad (279)$$

$$\frac{\partial \bar{y}_i^D}{\partial x_i^D} - \frac{\partial \tilde{y}_i^D}{\partial x_i^D} = \mu^D\left(\frac{(16rhf_c^D - (2r\lambda - 4f_c^D)^2)(r\lambda - 2f_c^D - 2f)^2}{\lambda\left((r\lambda - 2f_c^D - 2f)(r\lambda - 2f_c^D + 2rh)\right)^2} - \frac{16rhf\left(4(f_c^D + f)(rh - f_c^D) - r\lambda(3r\lambda - 8f_c^D - 4f + 4rh)\right)}{\lambda\left((r\lambda - 2f_c^D - 2f)(r\lambda - 2f_c^D + 2rh)\right)^2}\right) = Y^D \quad (280)$$

$$K^D = 8\mu^D\left(\frac{r\lambda(u_C(r\lambda - 4f_c^D) + r\lambda u_w h) + 4u_C f_c^D(f_c^D - rh)}{\lambda(r\lambda - 2f_c^D + 2rh)^2}\right) + 2t \quad (281)$$

This gives

$$p^D = \frac{K^D}{2} - f_i^{DSS}\frac{\partial\left(\bar{y}_i^D - \tilde{y}_i^D\right)}{\partial x_i^D} + C^V \quad (282)$$

By substituting for K^D and the y derivatives we get

$$p^D = 4\mu^D\left(\frac{r\lambda(u_C(r\lambda - 4f_c^D) + r\lambda u_w h) + 4u_C f_c^D(f_c^D - rh)}{\lambda(r\lambda - 2f_c^D + 2rh)^2}\right) - f_i^{DSS}Y^D + C^V + t \quad (283)$$

Price for SS access ISPs

$$\frac{\partial\left(\bar{y}_i^D - \tilde{y}_i^D\right)}{\partial f_i^{DSS}} = -\frac{\partial\tilde{y}_i^D}{\partial f_i^{DSS}} = -\frac{rh\bar{y}_i^D}{\left(r\lambda x_i^D - f_c^D - f_i^{DSS} - f_c^{DSS}\right)^2} \quad (284)$$

$$\frac{\partial\Pi_i^D}{\partial f_i^{DSS}} = \left(\bar{y}_i^D - \tilde{y}_i^D\right) - \frac{f_i^{DSS}rh\bar{y}_i^D}{\left(r\lambda x_i^D - f_c^D - f_i^{DSS} - f_c^{DSS}\right)^2} = 0 \quad (285)$$

This can be simplified by substituting for $\tilde{y}_i^D$

$$\left(r\lambda x_i^D - f_c^D\right)\left(r\lambda x_i^D - f_c^D - f_i^{DSS} - f_c^{DSS}\right)^2 - rh\left(f_i^{DSS} + f_c^{DSS}\right)\left(r\lambda x_i^D - f_c^D - f_i^{DSS} - f_c^{DSS}\right) - rhf_i^{DSS}\left(r\lambda x_i^D - f_c^D\right) = 0 \quad (286)$$

By imposing the symmetrical solution we get

$$f_i^{DSS}: (r\lambda - 2f_c^D)\left(r\lambda - 2f_c^D - 2f_i^{DSS} - 2f_c^{DSS}\right)^2 - rh\left(f_i^{DSS} + f_c^{DSS}\right)\left(4r\lambda - 8f_c^D - 8f_i^{DSS} - 8f_c^{DSS}\right) - rhf_i^{DSS}(4r\lambda - 8f_c^D) = 0 \quad (287)$$

This expression can implicitly be solved for f_i^{DSS}

Investment levels

We already have $\frac{f_i^{DSS}\,\partial(\bar{y}_i^D-\tilde{y}_i^D)}{\partial x_i^D}$ from (280).

To find $\frac{\partial x_i^D}{\partial \mu_i^D}$ we use the implicit derivation theorem $\frac{\partial x_i^D}{\partial \mu_i^D}=-\frac{\partial F/\partial \mu_i^D}{\partial F/\partial x_i^D}=-\frac{\partial F}{\partial \mu_i^D}\frac{1}{K^D}$, where we already have K^D. Further we have

$$\frac{\partial F}{\partial \mu_a^D}=-\left(\frac{u_C(r\lambda\tilde{x}^D-f_c^D)+u_w\lambda\tilde{x}^D rh}{\lambda\tilde{x}^D(r\lambda\tilde{x}^D-f_c^D+rh)}\right) \qquad (288)$$

So we get

$$\frac{\partial x_a^D}{\partial \mu_a^D}=\frac{u_C(r\lambda\tilde{x}^D-f_c^D)+u_w\lambda\tilde{x}^D rh}{\lambda\tilde{x}^D(r\lambda\tilde{x}^D-f_c^D+rh)K^D} \qquad (289)$$

$$\frac{\partial \bar{y}_i^D}{\partial \mu_i^D}=\frac{r\lambda x_i^D-f_c^D}{\lambda x_i^D\left(r\lambda x_i^D-f_c^D+rh\right)} \qquad (290)$$

$$\frac{\partial \tilde{y}_i^D}{\partial \mu_i^D}=\frac{rh\left(f_i^{DSS}+f_c^{DSS}\right)}{\lambda x_i^D\left(r\lambda x_i^D-f_c^D-f_i^{DSS}-f_c^{DSS}\right)\left(r\lambda x_i^D-f_c^D+rh\right)} \qquad (291)$$

By imposing the symmetrical solution we get

$$\frac{\partial \bar{y}_i^D}{\partial \mu_i^D}=\frac{2r\lambda-4f_c^D}{\lambda(r\lambda-2f_c^D+2rh)} \qquad (292)$$

$$\frac{\partial \tilde{y}_i^D}{\partial \mu_i^D}=\frac{8rh\left(f_i^{DSS}+f_c^{DSS}\right)}{\lambda\left(r\lambda-2f_c^D-2f_i^{DSS}-2f_c^{DSS}\right)(r\lambda-2f_c^D+2rh)} \qquad (293)$$

Now $\frac{\partial(\bar{y}_i^D-\tilde{y}_i^D)}{\partial \mu_i^D}$ can be formulated by combining the two terms

$$\frac{\partial\left(\bar{y}_i^D-\tilde{y}_i^D\right)}{\partial \mu_i^D}=\frac{(2r\lambda-4f_c^D)\left(r\lambda-2f_c^D-2f_i^{DSS}-2f_c^{DSS}\right)-8rh\left(f_i^{DSS}+f_c^{DSS}\right)}{\lambda\left(r\lambda-2f_c^D-2f_i^{DSS}-2f_c^{DSS}\right)(r\lambda-2f_c^D+2rh)} \qquad (294)$$

By substituting (208) and (212) into the first order optimum condition for investment levels, and imposing the symmetrical solution, we obtain

$$\frac{u_A(r\lambda - 2f_c^D) + u_w \lambda rh}{\lambda(r\lambda - 2f_c^D + 2rh)} + f_i^{DSS}\frac{(2r\lambda - 4f_c^D)\left(r\lambda - 2f_c^D - 2f_i^{DSS} - 2f_c^{DSS}\right) - 8rh\left(f_i^{DSS} + f_c^{DSS}\right)}{\lambda\left(r\lambda - 2f_c^D - 2f_i^{DSS} - 2f_c^{DSS}\right)(r\lambda - 2f_c^D + 2rh)} - C^I \mu_i^D = 0 \quad (295)$$

This can be rearranged into an expression for the investment level, μ^D

$$\mu^D = \frac{1}{C^I}\left(\frac{u_A(r\lambda - 2f_c^D) + u_w \lambda rh}{\lambda(r\lambda - 2f_c^D + 2rh)} + f_i^{DSS}\frac{(2r\lambda - 4f_c^D)\left(r\lambda - 2f_c^D - 2f_i^{DSS} - 2f_c^{DSS}\right) - 8rh\left(f_i^{DSS} + f_c^{DSS}\right)}{\lambda\left(r\lambda - 2f_c^D - 2f_i^{DSS} - 2f_c^{DSS}\right)(r\lambda - 2f_c^D + 2rh)}\right) \quad (296)$$

In addition to the first order optimum condition, the optimal investment levels are bound by the regulators restriction

$$w_i^{DIA} \leq \Psi \quad (297)$$

By imposing the symmetrical solution, this can be rewritten as

$$\mu^D \geq \frac{1}{\Psi}\frac{\left(r\lambda - 2f_c^D - 2f_i^{DSS} - 2f_c^{DSS}\right)(r\lambda - 2f_c^D + 2rh)^2}{4(rh)^2(r\lambda - 2f_c^D)} \quad (298)$$

Backbone ISP

The backbone cannot offer different prices based on which ISPs the CPs connect to. We know that the ISPs are symmetric, so by imposing symmetry we get

$$\Pi_c^D = 2f_c^D \bar{y}_i^D + 2f_c^{DSS}\left(\bar{y}_i^D - \tilde{y}_i^D\right) - 2C_c^V \bar{y}_i^D \quad (299)$$

Since $\bar{y}_i^D$ and $\tilde{y}_i^D$ are functions of both f_c^D, and x_i^D, which is dependent on f_c^D, we have from the product and chain rule

$$\frac{\partial \bar{y}_i^D}{\partial f_c^D} = \frac{\partial \bar{y}_i^D}{\partial f_c^D} + \frac{\partial \bar{y}_i^D}{\partial x_i^D}\frac{\partial x_i^D}{\partial f_c^D}, \frac{\partial \tilde{y}_i^D}{\partial f_c^D} = \frac{\partial \tilde{y}_i^D}{\partial f_c^D} + \frac{\partial \tilde{y}_i^D}{\partial x_i^D}\frac{\partial x_i^D}{\partial f_c^D} \tag{300}$$

This results in the following first order optimum conditions for the backbone ISP

$$\begin{aligned}\frac{\partial \Pi_c^D}{\partial f_c^D} &= \bar{y}_i^D + (f_c^D - C_c^V)\left(\frac{\partial \bar{y}_i^D}{\partial f_c^D} + \frac{\partial \bar{y}_i^D}{\partial x_i^D}\frac{\partial x_i^D}{\partial f_c^D}\right) \\ &+ f_c^{DSS}\left(\frac{\partial \bar{y}_i^D}{\partial f_c^D} + \frac{\partial \bar{y}_i^D}{\partial x_i^D}\frac{\partial x_i^D}{\partial f_c^D} - \frac{\partial \tilde{y}_i^D}{\partial f_c^D} - \frac{\partial \tilde{y}_i^D}{\partial x_i^D}\frac{\partial x_i^D}{\partial f_c^D}\right) = 0\end{aligned} \tag{301}$$

$$\frac{\partial \Pi_c^D}{\partial f_c^{DSS}} = \left(\bar{y}_i^D - \tilde{y}_i^D\right) - f_c^{DSS}\frac{\partial \tilde{y}_i^D}{\partial f_c^{DSS}} = 0 \tag{302}$$

We can rewrite the first condition as

$$\begin{aligned}\frac{\partial \Pi_c^D}{\partial f_c^D} &= \bar{y}_i^D + (f_c^D - C_c^V)\left(\frac{\partial \bar{y}_i^D}{\partial f_c^D} + \frac{\partial \bar{y}_i^D}{\partial x_i^D}\frac{\partial x_i^D}{\partial f_c^D}\right) \\ &+ f_c^{DSS}\left(\frac{\partial \bar{y}_i^D}{\partial f_c^D} - \frac{\partial \tilde{y}_i^D}{\partial f_c^D} + \left(\frac{\partial \bar{y}_i^D}{\partial x_i^D} - \frac{\partial \tilde{y}_i^D}{\partial x_i^D}\right)\frac{\partial x_i^D}{\partial f_c^D}\right) = 0\end{aligned} \tag{303}$$

Connectivity price

$$\frac{\partial \bar{y}_i^D}{\partial f_c^D} = -\frac{rh\mu_i^D}{\lambda x_i^D\left(r\lambda x_i^D - f_c^D + rh\right)^2} < 0 \tag{304}$$

$$\begin{aligned}&\frac{\partial \tilde{y}_i^D}{\partial f_c^D} \\ &= \frac{rh\left(f_i^{DSS} + f_c^{DSS}\right)\left(2r\lambda x_i^D - 2f_c^D - f_i^{DSS} - f_c^{DSS}\right)}{\left(\left(r\lambda x_i^D - f_c^D\right)\left(r\lambda x_i^D - f_c^D - f_i^{DSS} - f_c^{DSS}\right)\right)^2}\frac{\mu_i^D\left(r\lambda x_i^D - f_c^D\right)}{\lambda x_i^D\left(r\lambda x_i^D - f_c^D + rh\right)} \\ &- \frac{rh\left(f_i^{DSS} + f_c^{DSS}\right)}{\left(r\lambda x_i^D - f_c^D\right)\left(r\lambda x_i^D - f_c^D - f_i^{DSS} - f_c^{DSS}\right)}\frac{rh\mu_i^D}{\lambda x_i^D\left(r\lambda x_i^D - f_c^D + rh\right)^2}\end{aligned} \tag{305}$$

Because of the symmetry, we have

$$\frac{\partial x_i^D}{\partial f_c^D} = -\frac{\partial F}{\partial f_c^D}\frac{1}{K^D} = 0 \quad (306)$$

This makes it possible to simplify the first order optimal condition significantly. We can now write the first order optimum condition as

$$\frac{\partial \Pi_c^D}{\partial f_c^D} = \bar{y}_i^D + (f_c^D + f_c^{DSS} - C_c^V)\frac{\partial \bar{y}_i^D}{\partial f_c^D} - f_c^{DSS}\frac{\partial \tilde{y}_i^D}{\partial f_c^D} = 0 \quad (307)$$

Further we can impose the symmetrical solution on (218) and (305) to obtain

$$\frac{\partial \bar{y}^D}{\partial f_c^D} = -\frac{8rh\mu^D}{\lambda(r\lambda - 2f_c^D + 2rh)^2} < 0 \quad (308)$$

$$\frac{\partial \tilde{y}^D}{\partial f_c^D} = \frac{16rh\left(f_i^{DSS} + f_c^{DSS}\right)\left(2r\lambda - 4f_c^D - 2f_i^{DSS} - 2f_c^{DSS}\right)(r\lambda - 2f_c^D)\mu^D}{\lambda\left((r\lambda - 2f_c^D)\left(r\lambda - 2f_c^D - 2f_i^{DSS} - 2f_c^{DSS}\right)\right)^2(r\lambda - 2f_c^D + 2rh)} - \frac{32(rh)^2\left(f_i^{DSS} + f_c^{DSS}\right)\mu^D}{\lambda(r\lambda - 2f_c^D)\left(r\lambda - 2f_c^D - 2f_i^{DSS} - 2f_c^{DSS}\right)(r\lambda - 2f_c^D + 2rh)^2} \quad (309)$$

By substituting for the derivatives and $\bar{y}^D$ we obtain

$$\frac{\partial \Pi_c^D}{\partial f_c^D} = \frac{2\mu^D(r\lambda - 2f_c^D)}{\lambda(r\lambda - 2f_c^D + 2rh)} - (f_c^D + f_c^{DSS} - C_c^V)\frac{8rh\mu^D}{\lambda(r\lambda - 2f_c^D + 2rh)^2} - f_c^{DSS}\left[\frac{16rh\left(f_i^{DSS} + f_c^{DSS}\right)\left(2r\lambda - 4f_c^D - 2f_i^{DSS} - 2f_c^{DSS}\right)(r\lambda - 2f_c^D)\mu^D}{\lambda\left((r\lambda - 2f_c^D)\left(r\lambda - 2f_c^D - 2f_i^{DSS} - 2f_c^{DSS}\right)\right)^2(r\lambda - 2f_c^D + 2rh)} - \frac{32(rh)^2\left(f_i^{DSS} + f_c^{DSS}\right)\mu^D}{\lambda(r\lambda - 2f_c^D)\left(r\lambda - 2f_c^D - 2f_i^{DSS} - 2f_c^{DSS}\right)(r\lambda - 2f_c^D + 2rh)^2}\right] = 0 \quad (310)$$

This can be rearranged into an implicit expression for f_c^D

$$f_c^D: 2[(r\lambda - 2f_c^D)(r\lambda - 2f_c^D + 2rh) - 4rh(f_c^D + f_c^{DSS} - C_c^V)]\left[(r\lambda - 2f_c^D)\left(r\lambda - 2f_c^D - 2f_i^{DSS} - 2f_c^{DSS}\right)\right]^2 - 16f_c^{DSS}\left[rh\left(f_i^{DSS} + f_c^{DSS}\right)\left(2r\lambda - 4f_c^D - 2f_i^{DSS} - 2f_c^{DSS}\right)(r\lambda - 2f_c^D)(r\lambda - 2f_c^D + 2rh) - 2(rh)^2\left(f_i^{DSS} + f_c^{DSS}\right)(r\lambda - 2f_c^D)\left(r\lambda - 2f_c^D - 2f_i^{DSS} - 2f_c^{DSS}\right)\right] = 0 \quad (311)$$

Backbone SS price

We have

$$\frac{\partial \tilde{y}_i^D}{\partial f_c^{DSS}} = \frac{rh\bar{y}_i^D}{\left(r\lambda x_i^D - f_c^D - f_i^{DSS} - f_c^{DSS}\right)^2} \quad (312)$$

By substituting for $\tilde{y}_i^D$ and its derivative, and dividing by $\bar{y}_i^D$, we get

$$\frac{\partial \Pi_c^D}{\partial f_c^{DSS}} = \left(1 - \frac{rh\left(f_i^{DSS} + f_c^{DSS}\right)}{\left(r\lambda x_i^D - f_c^D\right)\left(r\lambda x_i^D - f_c^D - f_i^{DSS} - f_c^{DSS}\right)}\right) - f_c^{DSS} \frac{rh}{\left(r\lambda x_i^D - f_c^D - f_i^{DSS} - f_c^{DSS}\right)^2} = 0 \quad (313)$$

This can be rearranged into

$$f_c^{DSS}: \left(r\lambda x_i^D - f_c^D\right)\left(r\lambda x_i^D - f_c^D - f_i^{DSS} - f_c^{DSS}\right)^2 - rh\left(f_i^{DSS} + f_c^{DSS}\right)\left(r\lambda x_i^D - f_c^D - f_i^{DSS} - f_c^{DSS}\right) - f_c^{DSS} rh\left(r\lambda x_i^D - f_c^D\right) = 0 \quad (314)$$

This function can implicitly be solved for f_c^{DSS}. By imposing the symmetrical solution we get

$$f_c^{DSS}: (r\lambda - 2f_c^D)\left(r\lambda - 2f_c^D - 2f_i^{DSS} - 2f_c^{DSS}\right)^2 - rh\left(f_i^{DSS} + f_c^{DSS}\right)\left(4r\lambda - 8f_c^D - 8f_i^{DSS} - 8f_c^{DSS}\right) - rh(4r\lambda - 8f_c^D)f_c^{DSS} = 0 \quad (315)$$

8.2.3 Analytical Results Modifications in the Reduced Simplified Model

To conduct a numerical analysis of the mathematical model, the backbone SS price, f_c^{DSS} had to be removed from the model. This only affected the NNN regime. Without the backbone ISP's opportunity to claim extra profits from SS, the whole last term disappears, and the implicit function f_c^D consists of two products, where either has to be zero.

$$f_c^D: [(r\lambda - 2f_c^D)(r\lambda - 2f_c^D + 2rh) - 4rh(f_c^D - C_c^V)]\left[(r\lambda - 2f_c^D)\left(r\lambda - 2f_c^D - 2f_i^{DSS}\right)\right]^2 = 0 \quad (316)$$

The last product consists of two products inside the bracket, which gives two opportunities.

$$(r\lambda - 2f_c^D) = 0 \rightarrow f_c^D = \frac{r\lambda}{2} \quad (317)$$

And

$$\left(r\lambda - 2f_c^D - 2f_i^{DSS}\right) = 0 \rightarrow f_c^D = \frac{r\lambda}{2} - f_i^{DSS} \qquad (318)$$

The first term is obviously too high, as it implies that no profit would be left for the CPs, even if none of them were congestion sensitive. The last term implies that the access ISP could force the backbone to pay for providing connectivity. So these solutions are either not feasible, or not maxima, as the Hessian analysis will show. The optimal solution is then then derived from the first bracket.

$$(r\lambda - 2f_c^D)(r\lambda - 2f_c^D + 2rh) - 4rh(f_c^D - C_c^V) = 0 \qquad (319)$$

This yields

$$f_c^D = \frac{2rh + r\lambda}{2} - \frac{\sqrt{rh(4rh + 2r\lambda - 4C_c^V)}}{2} \qquad (320)$$

This shows that the solution is independent of regime, and will be the same in either NN or NNN. As in the NN regime, the other solution, with a positive square root, is a local minimum rather than maximum. The solution is neither feasible for the system of equations to have a solution. The price for SS is more complex.

$$f_i^{DSS}: (r\lambda - 2f_c^D)\left(r\lambda - 2f_c^D - 2f_i^{DSS}\right)^2 - rh\left(f_i^{DSS}\right)\left(4r\lambda - 8f_c^D - 8f_i^{DSS}\right) - rh(4r\lambda - 8f_c^D)f_i^{DSS} = 0 \qquad (321)$$

However, from maple a similar solution to that of the backbone connectivity price, based on the quadratic formula, can be obtained

$$f_i^{DSS} = \frac{\left(r\lambda - 2f_c^D + 2rh - \sqrt{2r^2\lambda h + 4r^2h^2 - 4rhf_c^D}\right)(r\lambda - 2f_c^D)}{2(2rh - 2f_c^D + r\lambda)} \qquad (322)$$

Welfare from the solutions by the reduced analytical model

The CPs

The profit of the CPs in equilibrium equals

$$\Pi_y^D = \begin{cases} \sum_{i\epsilon\{a,b\}} \left[\frac{r\lambda}{2}(1 - hy^D w^{DSS}) - f_c^D - f_i^{DSS} - f_c^{DSS}\right] & \tilde{y}^D < y^D < \bar{y}^D \\ \sum_{i\epsilon\{a,b\}} \left[\frac{r\lambda}{2}(1 - hy^D w^{DIA}) - f_c^D\right] & if \quad 0 \le y^D \le \tilde{y}^D \\ 0 & y^D \ge \bar{y}^D \end{cases} \quad (323)$$

Thus, the total profit of all CPs in equilibrium is

$$\Sigma\Pi_{CSP}^D = \int_0^{\tilde{y}^D} [r\lambda(1 - hy^D w^{DIA}) - 2f_c^D] dy^D + \int_{\tilde{y}^D}^{\bar{y}^D} \left[r\lambda(1 - hy^D w^{DSS}) - 2f_c^D - 2f_i^{DSS} - 2f_c^{DSS}\right] dy^D \quad (324)$$

Since all values except y^D is constant the total surplus equals

$$\Sigma\Pi_{CSP}^D = r\lambda\tilde{y}^D - \frac{r\lambda h{\tilde{y}^D}^2 w^{DIA}}{2} - 2f_c^D\tilde{y}^D + r\lambda(\bar{y}^D - \tilde{y}^D) - \frac{r\lambda h\left({\bar{y}^D}^2 - {\tilde{y}^D}^2\right) w^{DSS}}{2} - 2f_c^D(\bar{y}^D - \tilde{y}^D) - 2f_i^{DSS}(\bar{y}^D - \tilde{y}^D) - 2f_c^{DSS}(\bar{y}^D - \tilde{y}^D) \quad (325)$$

This can be simplified into

$$\Sigma\Pi_{CSP}^D = r\lambda\bar{y}^D - \frac{r\lambda h{\tilde{y}^D}^2 w^{DIA}}{2} - \frac{r\lambda h\left({\bar{y}^D}^2 - {\tilde{y}^D}^2\right) w^{DSS}}{2} - 2f_c^D\bar{y}^D - 2\left(f_i^{DSS} + f_c^{DSS}\right)(\bar{y}^D - \tilde{y}^D) \quad (326)$$

By substituting the solution from the reduced simplified model, we obtain

$$\Sigma\Pi_{CSP}^{D} = \frac{2\mu^D(r\lambda-2f_c^D)^2}{\lambda(r\lambda-2f_c^D+2rh)} - \frac{8rh\mu^D f_i^{DSS^2}}{\lambda(r\lambda-2f_c^D)\left(r\lambda-2f_c^D-2f_i^{DSS}\right)} - \frac{\mu^D\left(\left((r\lambda-2f_c^D)\left(r\lambda-2f_c^D-2f_i^{DSS}\right)\right)^2-16(rh)^2f_i^{DSS^2}\right)}{\lambda(r\lambda-2f_c^D+2rh)(r\lambda-2f_c^D)\left(r\lambda-2f_c^D-2f_i^{DSS}\right)} - \frac{4\mu^D f_i^{DSS}\left((r\lambda-2f_c^D)\left(r\lambda-2f_c^D-2f_i^{DSS}\right)-4rhf_i^{DSS}\right)}{\lambda(r\lambda-2f_c^D+2rh)\left(r\lambda-2f_c^D-2f_i^{DSS}\right)} \quad (327)$$

Results are provided by numerical experimentation.

The IUs

The IUs surplus, or consumer surplus, can be modelled as

$$CS_{IU}^{D} = 2\int_0^{1/2}\left(R+u_c\bar{y}_a^D+\frac{u_w}{w_a^N}-p_a^N-t\tilde{x}^N\right)d\tilde{x}^D \quad (328)$$

Since $\bar{y}^N, w^N and\ p^N$ are constant with respect to $\tilde{x}^N$, we get

$$CS_{IU}^{D} = R+u_c\bar{y}^D+\frac{u_w}{w^D}-p^D-t\frac{1}{4} \quad (329)$$

By substituting the reduced simplified solution we obtain

$$CS_{IU}^{D} = R+2\mu^D\left(\frac{u_c(r\lambda-2f_c^D)+u_w r\lambda h}{\lambda(r\lambda-2f_c^D+2rh)}\right) - 4\mu^D\left(\frac{r\lambda(u_C(r\lambda-4f_c^D)+r\lambda u_w h)+4u_C f_c^D(f_c^D-rh)}{\lambda(r\lambda-2f_c^D+2rh)^2}\right) + f_i^{DSS}Y^D - C^V - t\frac{5}{4} \quad (330)$$

Results calculated by numerical experiments.

The access ISPs

There are two access ISPs, so the total surplus equals in equilibrium

$$\Sigma\Pi^{D}_{ISP_{access}} = \sum_{i\epsilon\{a,b\}} \left(p^D x_i^D + f_i^{DSS}\left(\bar{y}_i^D - \tilde{y}_i^D\right) - C^V x_i^D - C^I(\mu^D)\right) = p^D + 2f_i^{DSS}(\bar{y}^D - \tilde{y}^D) - C^V - 2C^I(\mu^D) \quad (331)$$

This by substituting the values for $p^D, \bar{y}^D$ *and* $\tilde{y}^D$ we obtain

$$\Sigma\Pi^{D}_{ISP_{access}} = 4\mu^D\left(\frac{r\lambda(u_C(r\lambda - 4f_c^D) + r\lambda u_w h) + 4u_C f_c^D(f_c^D - rh)}{\lambda(r\lambda - 2f_c^D + 2rh)^2}\right) + t + f_i^{DSS}\left[2\left(1 - \frac{4rh\left(f_i^{DSS} + f_c^{DSS}\right)}{(r\lambda - 2f_c^D)\left(r\lambda - 2f_c^D - 2f_i^{DSS} - 2f_c^{DSS}\right)}\right)\frac{2\mu^D(r\lambda - 2f_c^D)}{\lambda(r\lambda - 2f_c^D + 2rh)} - Y^D\right] - 2C^I(\mu^D) \quad (332)$$

Results calculated by numerical experiments.

The backbone ISPs

There is only one backbone ISP, and by substituting the equilibrium solutions to the profit function we obtain

$$\Sigma\Pi^{D}_{ISP_{back}} = \sum_{i\epsilon\{a,b\}} \left[f_c^D \bar{y}_i^D + f_c^{DSS}\left(\bar{y}_i^D - \tilde{y}_i^D\right) - C_c^V \bar{y}_i^D\right] = 2f_c^D\bar{y}^D + 2f_c^{DSS}(\bar{y}^D - \tilde{y}^D) - 2C_c^V\bar{y}^D \quad (333)$$

In the reduced simplified model, we obtain

$$\Sigma\Pi^{D}_{ISP_{back}} = \sum_{i\epsilon\{a,b\}} \left[f_c^D \bar{y}_i^D - C_c^V \bar{y}_i^D\right] = \frac{4\mu^D(f_c^D - C_c^V)(r\lambda - 2f_c^D)}{\lambda(r\lambda - 2f_c^D + 2rh)} \quad (334)$$

Total welfare

The total welfare, or social welfare, is defined as the sum of the access and backbone ISPs profits, the CPs profits and the IUs consumer surplus

$$\begin{aligned}W^D &= \Sigma\Pi_{CSP}^D + CS_{IU}^D + \Sigma\Pi_{ISP_{access}}^D + \Sigma\Pi_{ISP_{back}}^D \\ &= R + (r\lambda - 2C_c^V)\bar{y}^D \\ &- \frac{r\lambda h}{2}\left(\bar{y}^{D^2} w^{DSS} + \tilde{y}^{D^2}(w^{DIA} - w^{DSS})\right) + u_c\bar{y}^D + \frac{u_w}{w^D} \\ &- t\frac{1}{4} - C^V - 2C^I(\mu^D)\end{aligned} \tag{335}$$

By substituting the analytic results we obtain

$$\begin{aligned}W^D &= R + \frac{2\mu^D(r\lambda - 2f_c^D)(r\lambda - 2C_c^V)}{\lambda(r\lambda - 2f_c^D + 2rh)} \\ &- \frac{\mu^D\left((r\lambda - 2f_c^D - 2f_i^{DSS})^2(r\lambda - 2f_c^D) + 8rhf_i^{DSS^2}\right)}{\lambda(r\lambda - 2f_c^D + 2rh)(r\lambda - 2f_c^D - 2f_i^{DSS})} \\ &+ 2\mu^D\frac{u_c(r\lambda - 2f_c^D) + u_w r\lambda h}{\lambda(r\lambda - 2f_c^D + 2rh)} - t\frac{1}{4} - C^V - C^I\mu^{D^2}\end{aligned} \tag{336}$$

8.2.4 Second Order Conditions Reduced Simplified Model

For the equilibrium results in the analytical solution and numerical analysis to be valid, the second order conditions for local optimum must also be valid. The previous discussion, based on feasible solutions, also shows that this is the global maxima to the optimization problem of the players in the game. The results are listed with the imposed symmetrical solution. This simplifies many of the expression after the derivatives have been derived, so the expressions are easier to analyze.

Net neutrality

The solutions in equilibrium are a maximum of the profit functions if the corresponding Hessian Matrix is negative definite. For the access ISP that is if

$$\boldsymbol{H}(p_i^N, \mu_i^N) = \begin{bmatrix} \frac{\partial^2\Pi_i^N}{\partial p_i^{N^2}} & \frac{\partial^2\Pi_i^N}{\partial p_i^N \partial\mu_i^N} \\ \frac{\partial^2\Pi_i^N}{\partial\mu_i^N \partial p_i^N} & \frac{\partial^2\Pi_i^N}{\partial\mu_i^{N^2}} \end{bmatrix} \tag{337}$$

is negative definite. And for the backbone ISP that is if

$$\boldsymbol{H}(f_c^N) = \frac{\partial \Pi_c^N}{\partial {f_c^N}^2} < 0 \tag{338}$$

Access ISP

$$\frac{\partial \Pi_i^N}{\partial {p_i^N}^2} = 2\frac{\partial x_i^N}{\partial p_i^N} + \left(p_i^N - C^V\right)\frac{\partial^2 x_i^N}{\partial {p_i^D}^2} = -\frac{2}{K^N} = H_{1,1}^N < 0 \tag{339}$$

$$\frac{\partial^2 \Pi_i^N}{\partial p_i^N \partial \mu_i^N} = \frac{\partial^2 \Pi_i^N}{\partial \mu_i^N \partial p_i^N} = \frac{\partial x_i^N}{\partial \mu_i^N} + \left(p_i^N - C^V\right)\frac{\partial^2 x_i^N}{\partial \mu_i^N\, \partial p_i^N} = H_{1,2}^N \tag{340}$$

$$H_{1,2}^N = \frac{2r\lambda(u_C + u_w h)(2r\lambda - 2f_c^N + 2rh) - 4u_C f_c^N(3r\lambda - 4f_c^N + 4rh)}{\lambda(r\lambda - 2f_c^N + 2rh)^2 K^N} \tag{341}$$

$$\frac{\partial \Pi_i^N}{\partial {\mu_i^N}^2} = \left(p_i^N - C^V\right)\frac{\partial^2 x_i^N}{\partial {\mu_i^N}^2} - C^I = H_{2,2}^N \tag{342}$$

$$H_{2,2}^N = \frac{4\left[u_C\left(r\lambda - 2f_c^N\right) + u_w\lambda rh\right]\left[8f_c^N\left(u_C r(\lambda + h) - f_c^N\right) - 2(r\lambda)^2(u_C + u_w h)\right]}{\lambda\left(r\lambda - 2f_c^N + 2rh\right)\left(8\mu^{NT} r\lambda\left(u_C\left(r\lambda - 4f_c^N\right) + r\lambda u_w h\right) + 4u_C f_c^N\left(f_c^N - rh\right) + 2t\lambda\left(r\lambda - 2f_c^N + 2rh\right)^2\right)} - C^I \tag{343}$$

The Hessian is negative definite, and thus the problem concave, if

$$H_{1,1}^N < 0 \tag{344}$$

$$H_{2,2}^N < 0 \tag{345}$$

$$|\boldsymbol{H}^N| = H_{1,1}^N H_{2,2}^N - H_{1,2}^N H_{1,2}^N > 0 \tag{346}$$

This is always true as long as the cost for making investments, C^I, and transporting cost, t, are sufficiently high.

Backbone ISP

$$\frac{\partial \Pi_c^N}{\partial {f_c^N}^2} = 8f_c^N - (8rh + 4r\lambda) \tag{347}$$

This is < 0 when

$$f_c^N < \frac{r\lambda + 2rh}{2} \tag{348}$$

This is also a necessary condition, as seen by observation, of the solution formulas. Results also verified from numerical analysis.

Non-net neutrality

The solutions in equilibrium are a maximum of the profit functions if the corresponding Hessian Matrix is negative definite. For the access ISP that is if

$$\boldsymbol{H}(p_i^D, f_i^{DSS}, \mu_i^D) = \begin{bmatrix} \frac{\partial^2 \Pi_i^D}{\partial {p_i^D}^2} & \frac{\partial^2 \Pi_i^D}{\partial p_i^D \partial f_i^{DSS}} & \frac{\partial^2 \Pi_i^D}{\partial p_i^D \partial \mu_i^D} \\ \frac{\partial^2 \Pi_i^D}{\partial f_i^{DSS} \partial p_i^D} & \frac{\partial^2 \Pi_i^D}{\partial {f_i^{DSS}}^2} & \frac{\partial^2 \Pi_i^D}{\partial f_i^{DSS} \partial \mu_i^D} \\ \frac{\partial^2 \Pi_i^D}{\partial \mu_i^D \partial p_i^D} & \frac{\partial^2 \Pi_i^D}{\partial \mu_i^D \partial f_i^{DSS}} & \frac{\partial^2 \Pi_i^D}{\partial {\mu_i^D}^2} \end{bmatrix} \tag{349}$$

is negative definite. And for the backbone ISP that is if

$$\boldsymbol{H}(f_c^D) = \frac{\partial \Pi_c^D}{\partial {f_c^D}^2} < 0 \tag{350}$$

This is the same as in NN.

Access ISP

$$\frac{\partial^2 \Pi_i^D}{\partial {p_i^D}^2} = \left(2 + f_i^{DSS} \frac{\partial^2 (\bar{y}_i^D - \tilde{y}_i^D)}{\partial {x_i^D}^2} \frac{\partial x_i^D}{\partial p_i^D}\right) \frac{\partial x_i^D}{\partial p_i^D} + \left(p_i^D + \frac{\partial \left(f_i^{DSS} (\bar{y}_i^D - \tilde{y}_i^D)\right)}{\partial x_i^D} - C^V\right) \frac{\partial^2 x_i^D}{\partial {p_i^D}^2} = H_{1,1}^D \quad (351)$$

$$
\begin{aligned}
H_{1,1}^{D} \\
= -\Bigg(2 \\
+ f_i^{DSS} 8\mu_i^{D} \Bigg[\frac{r\lambda^2 (r\lambda - 2f_c^D)^2 (r\lambda - 2f_c^D + 2rh)^2 + 4\lambda(4rhf_c^D - (r\lambda - 2f_c^D)^2)(r\lambda - 2f_c^D + 2rh)(r\lambda - f_c^D + rh)}{\lambda^2 (r\lambda - 2f_c^D + 2rh)^4} \\
- 4rhf_i^{DSS} \Bigg(\frac{r\lambda\big(3r\lambda - 4f_c^D - 2f_i^{DSS} + 2rh\big)\big(r\lambda - 2f_c^D - 2f_i^{DSS}\big)(r\lambda - 2f_c^D + 2rh)}{\lambda\Big((r\lambda - 2f_c^D - 2f_i^{DSS})(r\lambda - 2f_c^D + 2rh)\Big)^3} \\
+ 2 \frac{\big[4\big(f_c^D + f_i^{DSS}\big)(rh - f_c^D) - r\lambda\big(3r\lambda - 8f_c^D - 4f_i^{DSS} + 4rh\big)\big]\Big(\big(2r\lambda - 2f_c^D - 2f_i^{DSS}\big)(r\lambda - 2f_c^D + 2rh) + \big(r\lambda - 2f_c^D - 2f_i^{DSS}\big)r\lambda\Big)}{\lambda\Big((r\lambda - 2f_c^D - 2f_i^{DSS})(r\lambda - 2f_c^D + 2rh)\Big)^3} \Bigg) \Bigg] \frac{1}{K^D} \Bigg) \frac{1}{K^D} \\
< 0
\end{aligned}
\tag{352}
$$

$$\frac{\partial^2 \Pi_i^D}{\partial p_i^D \partial f_i^{DSS}} = \frac{\partial^2 \Pi_i^D}{\partial f_i^{DSS} \partial p_i^D} = \frac{\partial}{\partial p_i^D}\left[\left(\bar{y}_i^D - \tilde{y}_i^D\right) - \frac{f_i^{DSS} r h \bar{y}_i^D}{\left(r\lambda x_i^D - f_c^D - f_i^{DSS}\right)^2}\right] = H_{1,2}^D \tag{353}$$

$$\begin{aligned} H_{1,2}^D = \mu^D \Bigg[f_i^{DSS} \Bigg(& \frac{32 r^2 \lambda h (r\lambda - 2f_c^D)}{\lambda (r\lambda - 2f_c^D + 2rh)\left(r\lambda - 2f_c^D - 2f_i^{DSS}\right)^3} \\ & + \frac{4rh(16rhf_c^D - (2r\lambda - 4f_c^D)^2)}{\lambda(r\lambda - 2f_c^D + 2rh)^2\left(r\lambda - 2f_c^D - 2f_i^{DSS}\right)^2}\frac{1}{K^D}\Bigg) \\ & - \Bigg(\frac{(16rhf_c^D - (2r\lambda - 4f_c^D)^2)(r\lambda - 2f_c^D - 2f)^2}{\lambda\left((r\lambda - 2f_c^D - 2f)(r\lambda - 2f_c^D + 2rh)\right)^2} \\ & - \frac{16rhf\left(4(f_c^D + f)(rh - f_c^D) - r\lambda(3r\lambda - 8f_c^D - 4f + 4rh)\right)}{\lambda\left((r\lambda - 2f_c^D - 2f)(r\lambda - 2f_c^D + 2rh)\right)^2}\Bigg)\frac{1}{K^D}\Bigg] > 0 \end{aligned} \tag{354}$$

$$\frac{\partial^2 \Pi_i^D}{\partial {f_i^{DSS}}^2} = -\frac{2rh\bar{y}_i^D(r\lambda x_i^D - f_c^D)}{(r\lambda x_i^D - f_c^D - f_i^{DSS})^3} = H_{2,2}^D < 0 \quad (355)$$

$$\frac{\partial^2 \Pi_i^D}{\partial p_i^D \partial \mu_i^D} = \frac{\partial^2 \Pi_i^D}{\partial \mu_i^D \partial p_i^D}$$

$$= \frac{\partial}{\partial p_i^D}\left[\left(p_i^D - C^V + \frac{f_i^{DSS}\,\partial(\bar{y}_i^D - \tilde{y}_i^D)}{\partial x_i^D}\right)\frac{\partial x_i^D}{\partial \mu_i^D} + \frac{f_i^{DSS}\,\partial(\bar{y}_i^D - \tilde{y}_i^D)}{\partial \mu_i^D} - C^I \mu_i^D\right] = H_{1,3}^D \quad (356)$$

$$
\begin{aligned}
H_{1,3}^{D} \\
= \Bigg(& 1 \\
& + f_i^{DSS} 8\mu_i^{D} \Bigg[\frac{r\lambda^2 (r\lambda - 2f_c^D)^2 (r\lambda - 2f_c^D + 2rh)^2 + 4\lambda(4rhf_c^D - (r\lambda - 2f_c^D)^2)(r\lambda - 2f_c^D + 2rh)(r\lambda - f_c^D + rh)}{\lambda^2 (r\lambda - 2f_c^D + 2rh)^4} \\
& - 4rhf_i^{DSS} \Bigg(\frac{r\lambda(3r\lambda - 4f_c^D - 2f_i^{DSS} + 2rh)(r\lambda - 2f_c^D - 2f_i^{DSS})(r\lambda - 2f_c^D + 2rh)}{\lambda\big((r\lambda - 2f_c^D - 2f_i^{DSS})(r\lambda - 2f_c^D + 2rh)\big)^3} \\
& + 2 \frac{[4(f_c^D + f_i^{DSS})(rh - f_c^D) - r\lambda(3r\lambda - 8f_c^D - 4f_i^{DSS} + 4rh)]\big((2r\lambda - 2f_c^D - 2f_i^{DSS})(r\lambda - 2f_c^D + 2rh) + (r\lambda - 2f_c^D - 2f_i^{DSS})r\lambda\big)}{\lambda\big((r\lambda - 2f_c^D - 2f_i^{DSS})(r\lambda - 2f_c^D + 2rh)\big)^3} \Bigg) \Bigg] \frac{1}{K^D} \Bigg) \frac{2u_C(r\lambda - 2f_c^D) + 2u_w \lambda rh}{\lambda(r\lambda - 2f_c^D + 2rh)K^D} \\
& + \frac{2(r\lambda)^2 (u_C + u_w rh) - 4u_C f_c^D (2r\lambda - 2f_c^D + 2rh)}{\lambda(r\lambda - 2f_c^D + 2rh)^2 K^D} \\
& - f_i^{DSS} \left(\frac{(16rhf_c^D - (2r\lambda - 4f_c^D)^2)(r\lambda - 2f_c^D - 2f)^2 - 16rhf\big(4(f_c^D + f)(rh - f_c^D) - r\lambda(3r\lambda - 8f_c^D - 4f + 4rh)\big)}{\lambda\big((r\lambda - 2f_c^D - 2f)(r\lambda - 2f_c^D + 2rh)\big)^2} \right) \frac{1}{K^D}
\end{aligned}
\tag{357}
$$

$$\frac{\partial^2 \Pi_i^D}{\partial f_i^{DSS} \partial \mu_i^D} = \frac{\partial^2 \Pi_i^D}{\partial \mu_i^D \partial f_i^{DSS}} = \frac{\partial\left(\bar{y}_i^D - \tilde{y}_i^D\right)}{\partial \mu_i^D} + \frac{\partial\left(\bar{y}_i^D - \tilde{y}_i^D\right)}{\partial x_i^D}\frac{\partial x_i^D}{\partial \mu_i^D} - \frac{\bar{y}_i^D \partial}{\partial \mu_i^D}\frac{f_i^{DSS} rh}{\left(r\lambda x_i^D - f_c^D - f_i^{DSS}\right)^2} + \frac{f_i^{DSS} rh}{\left(r\lambda x_i^D - f_c^D - f_i^{DSS}\right)^2}\left(\frac{\partial \bar{y}_i^D}{\partial \mu_i^D} + \frac{\partial \bar{y}_i^D}{\partial x_i^D}\frac{\partial x_i^D}{\partial \mu_i^D}\right) = H_{2,3}^D \tag{358}$$

$$H_{2,3}^D = \frac{(2r\lambda - 4f_c^D)\left(r\lambda - 2f_c^D - 2f_i^{DSS}\right) - 8rhf_i^{DSS}}{\lambda\left(r\lambda - 2f_c^D - 2f_i^{DSS}\right)(r\lambda - 2f_c^D + 2rh)} + \mu^D\left(\frac{(16rhf_c^D - (2r\lambda - 4f_c^D)^2)(r\lambda - 2f_c^D - 2f)^2}{\lambda\left((r\lambda - 2f_c^D - 2f)(r\lambda - 2f_c^D + 2rh)\right)^2} - \frac{16rhf\left(4(f_c^D + f)(rh - f_c^D) - r\lambda(3r\lambda - 8f_c^D - 4f + 4rh)\right)}{\lambda\left((r\lambda - 2f_c^D - 2f)(r\lambda - 2f_c^D + 2rh)\right)^2} + \frac{32f_i^{DSS} r^2 h(r\lambda - 2f_c^D)}{(r\lambda - 2f_c^D + 2rh)\left(r\lambda - 2f_c^D - 2f_i^{DSS}\right)^3}\right)\frac{2u_C(r\lambda - 2f_c^D) + 2u_w\lambda rh}{\lambda(r\lambda - 2f_c^D + 2rh)K^D} + \frac{4f_i^{DSS} rh}{\left(r\lambda - 2f_c^D - 2f_i^{DSS}\right)^2}\left(\frac{2r\lambda - 4f_c^D}{\lambda(r\lambda - 2f_c^D + 2rh)} + \mu^D\frac{(16rhf_c^D - (2r\lambda - 4f_c^D)^2)(2u_C(r\lambda - 2f_c^D) + 2u_w\lambda rh)}{\lambda^2(r\lambda - 2f_c^D + 2rh)^3 K^D}\right) > 0 \tag{359}$$

$$\frac{\partial^2 \Pi_i^D}{\partial {\mu_i^D}^2} = \frac{\partial K^D}{2\partial \mu_i^D}\frac{\partial x_i^D}{\partial \mu_i^D} + \frac{K^D}{2}\frac{\partial^2 x_i^D}{\partial {\mu_i^D}^2} + \frac{f_i^{DSS}\,\partial^2\left(\bar{y}_i^D - \tilde{y}_i^D\right)}{\partial {\mu_i^D}^2} + \frac{f_i^{DSS}\,\partial^2\left(\bar{y}_i^D - \tilde{y}_i^D\right)}{\partial \mu_i^D\,\partial x_i^D}\frac{\partial x_i^D}{\partial \mu_i^D} - C^I = H_{3,3}^D \tag{360}$$

$$
\begin{aligned}
H_{3,3}^{D} = {} & \frac{2r\lambda\left(u_C\left(r\lambda - 4f_c^D\right) + r\lambda u_w h\right) + 8u_C f_c^D\left(f_c^D - rh\right)}{\lambda\left(r\lambda - 2f_c^D + 2rh\right)^2} \frac{2u_C\left(r\lambda - 2f_c^D\right) + 2u_w\lambda rh}{\lambda\left(r\lambda - 2f_c^D + 2rh\right)K^D} \\
& + 4\frac{\left(u_C\left(r\lambda - 2f_c^D\right) + u_w\lambda rh\right)\left(2u_C f_c^D\left(2r\lambda - 2f_c^D + 2rh\right) - (r\lambda)^2\left(u_C + u_w h\right)\right)}{\lambda^2\left(r\lambda - 2f_c^D + 2rh\right)^3 K^D} \\
& - 4\frac{2\left((r\lambda)^2\left(u_C + u_w h\right) - 4r\lambda u_C f_c^D\right) + 8u_C f_c^D\left(f_c^D - rh\right)}{\lambda^2\left(r\lambda - 2f_c^D + 2rh\right)^3 K^D} \\
& + f_i^{DSS}\left(\frac{\left(16rhf_c^D - \left(2r\lambda - 4f_c^D\right)^2\right)\left(r\lambda - 2f_c^D - 2f\right)^2}{\lambda\left(\left(r\lambda - 2f_c^D - 2f\right)\left(r\lambda - 2f_c^D + 2rh\right)\right)^2}\right. \\
& \left. - \frac{16rhf\left(4\left(f_c^D + f\right)\left(rh - f_c^D\right) - r\lambda\left(3r\lambda - 8f_c^D - 4f + 4rh\right)\right)}{\lambda\left(\left(r\lambda - 2f_c^D - 2f\right)\left(r\lambda - 2f_c^D + 2rh\right)\right)^2}\right)\frac{2u_C\left(r\lambda - 2f_c^D\right) + 2u_w\lambda rh}{\lambda\left(r\lambda - 2f_c^D + 2rh\right)K^D} - C^I
\end{aligned}
\tag{361}
$$

The Hessian is negative definite, and thus the problem concave, if

$$H^D_{1,1} < 0 \tag{362}$$

$$H^D_{2,2} < 0 \tag{363}$$

$$H^D_{3,3} < 0 \tag{364}$$

$$\left|h^D_{2,1}\right| = H^D_{1,1}H^D_{3,3} - H^D_{1,3}H^D_{1,3} > 0 \tag{365}$$

$$\begin{aligned}|\boldsymbol{H}^D| = H^D_{1,1}\left(H^D_{2,2}H^D_{3,3} - H^D_{2,3}H^D_{2,3}\right) - H^D_{1,2}\left(H^D_{1,2}H^D_{3,3} - H^D_{2,3}H^D_{1,3}\right)\\ + H^D_{1,3}\left(H^D_{1,2}H^D_{2,3} - H^D_{2,2}H^D_{1,3}\right) < 0\end{aligned} \tag{366}$$

This is true in the symmetric equilibrium solutions as long as C^I and t are sufficiently high.

Backbone ISP

On order to find the second derivative, we include all possible solutions. That is, both brackets as discussed from deriving the solution. We then obtain

$$\begin{aligned}\frac{\partial^2 \Pi^N_c}{\partial {f^N_c}^2} &= -4(r\lambda - 2f^D_c + 2rh)(r\lambda - 2f^D_c)^2\left(r\lambda - 2f^D_c - 2f^{DSS}_i\right)^2\\ &\quad - 4\left((r\lambda - 2f^D_c)(r\lambda - 2f^D_c + 2rh) - 4rh(f^D_c - C^V_c)\right)(r\lambda\\ &\quad - 2f^D_c)\left(r\lambda - 2f^D_c - 2f^{DSS}_i\right)\left(2r\lambda - 4f^D_c - 2f^{DSS}_i\right)\end{aligned} \tag{367}$$

Since we know that $(r\lambda - 2f^D_c + 2rh), (r\lambda - 2f^D_c - 2f^{DSS}_i)\ and\ (r\lambda - 2f^D_c)$ is positive in the feasible solution, we can obtain

$$\begin{aligned}&(r\lambda - 2f^D_c)(r\lambda - 2f^D_c + 2rh)\left(3r\lambda - 6f^D_c - 4f^{DSS}_i\right)\\ &\quad > 8rh(f^D_c - C^V_c)\left(r\lambda - 2f^D_c - f^{DSS}_i\right)\end{aligned} \tag{368}$$

This is true in the equilibrium solutions.

8.3 Additional Numerical Results

Additional results from Scenario 1: Attractiveness of online advertising

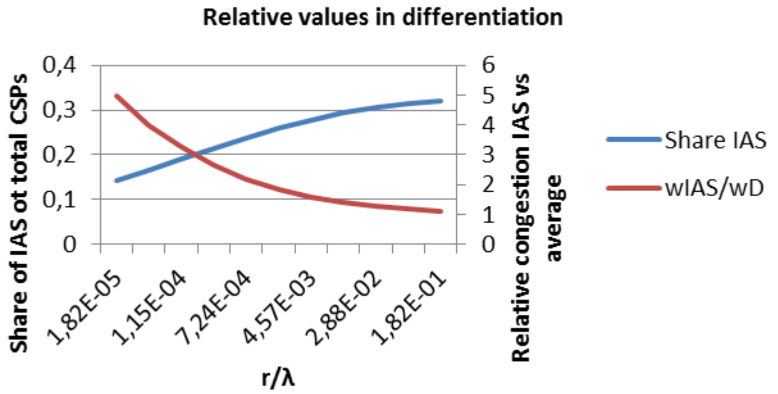

Figure 47: Share of IAS and w^{DIA}/w^{D} in the attractiveness of online advertising scenario's NNN regime

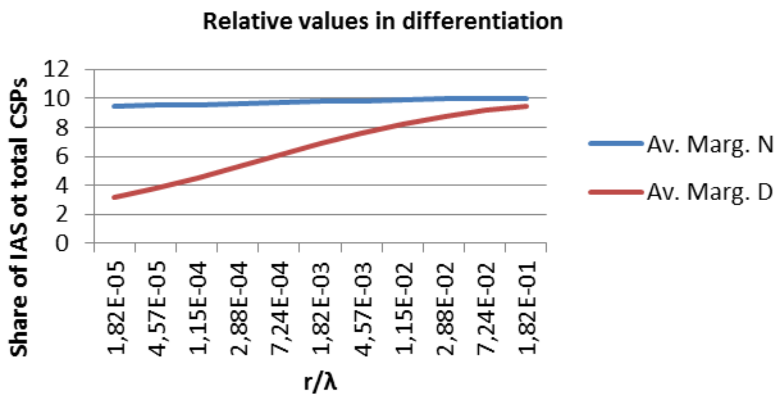

Figure 48: Average margin of CPs in the attractiveness of online advertising scenario. One should consider this as margin per volume of content and not per CSP.

Additional results from Scenario 3: Varying performance of the ISPs

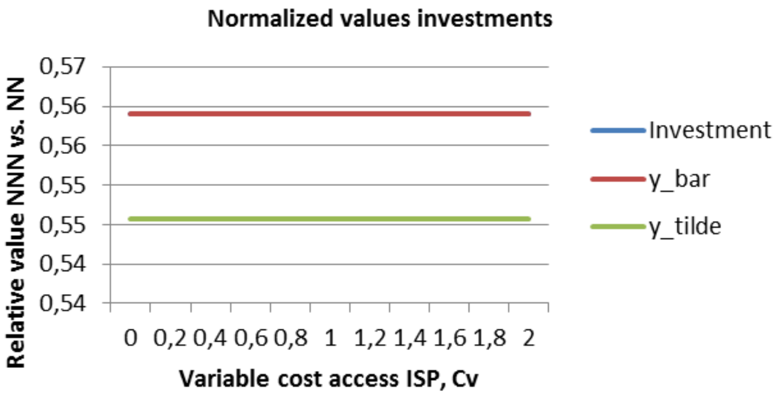

Figure 49: Normalized values of investments and innovation in the operational performance of the access ISPs scenario

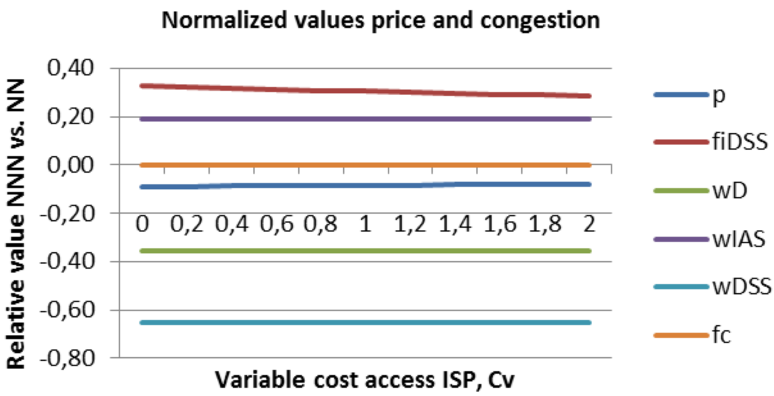

Figure 50: Normalized values of price and congestion in the operational performance of the access ISPs scenario

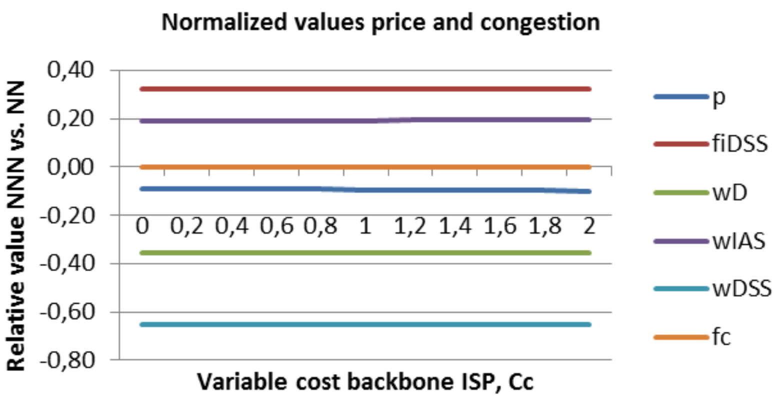

Figure 51: Normalized values of price and congestion in the operational performance of the backbone ISP scenario

A regulated network – basic scenario

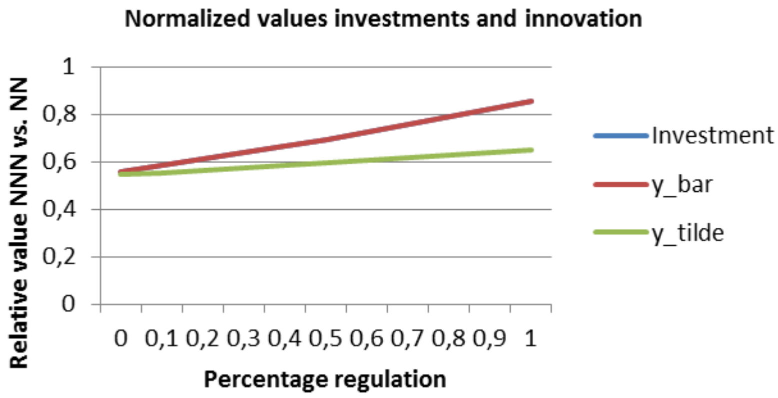

Figure 52: Normalized values of investments and innovation in the regulated network – basic scenario

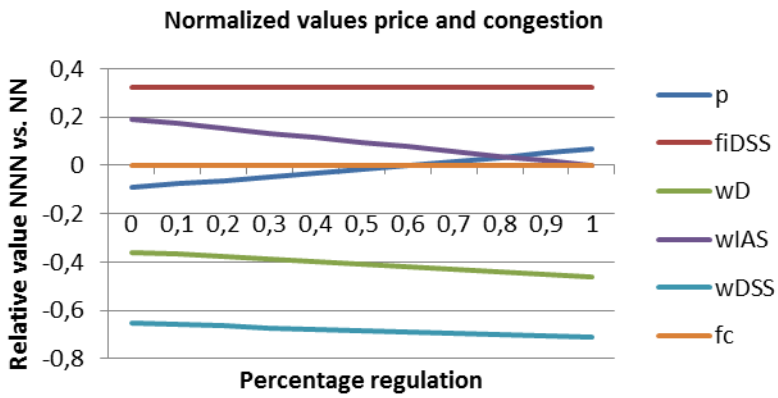

Figure 53: Normalized values of price and congestion in the regulated network – basic scenario

A regulated network – high IU utility on CPs and QoS

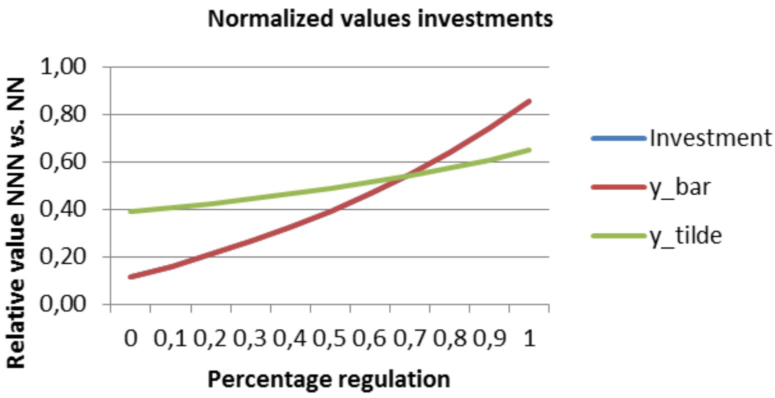

Figure 54: Normalized values of investments and innovation in the regulated network – high IU utility on CPs and QoS scenario

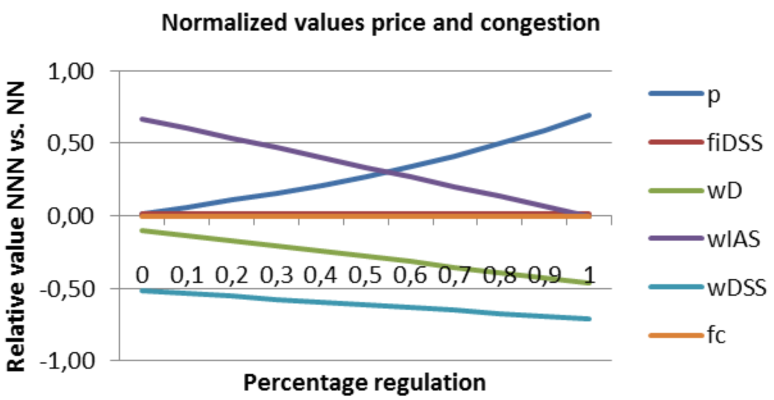

Figure 55: Normalized values of price and congestion in the regulated network – high IU utility on CPs and QoS scenario

A regulated network– low IU utility on CPs and QoS

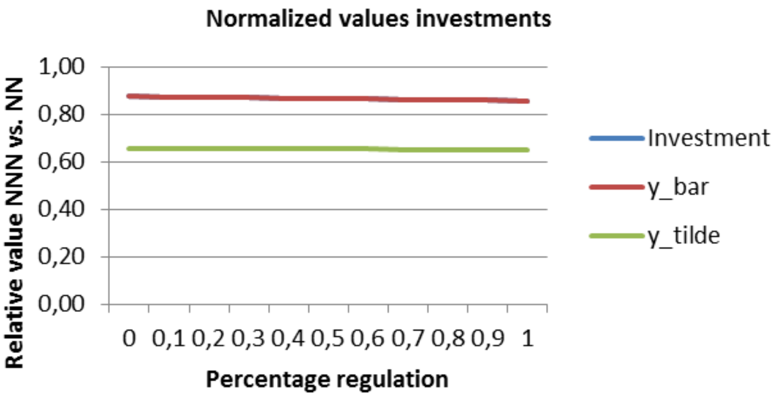

Figure 56: Normalized values of investments and innovation in the regulated network– low IU utility on CPs and QoS scenario

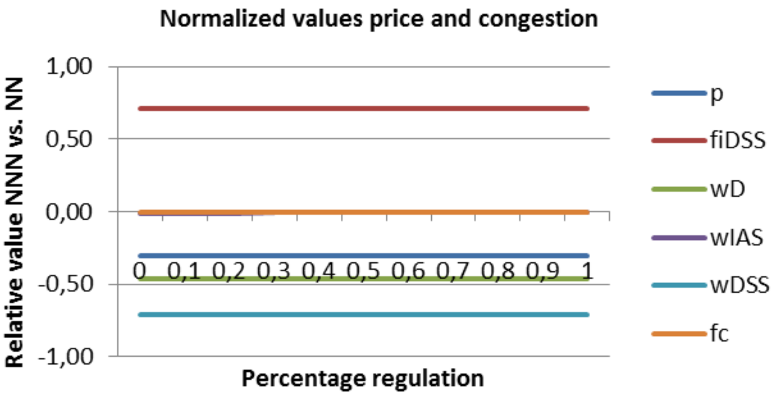

Figure 57: Normalized values of price and congestion in the regulated network– low IU utility on CPs and QoS scenario

A regulated network– low r/λ

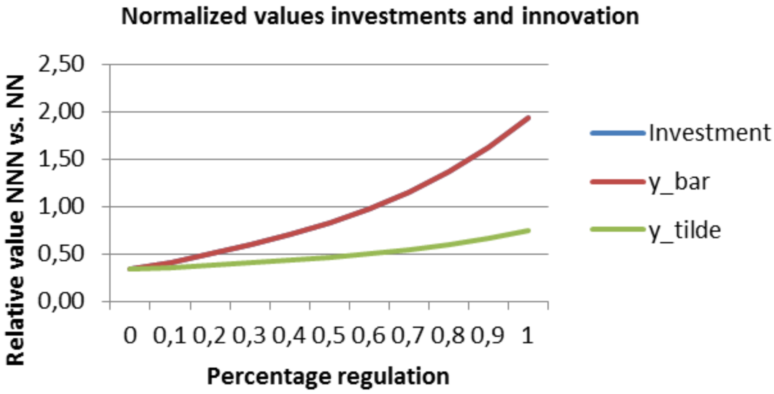

Figure 58: Normalized values of investments and innovation in the regulated network– low r/λ scenario

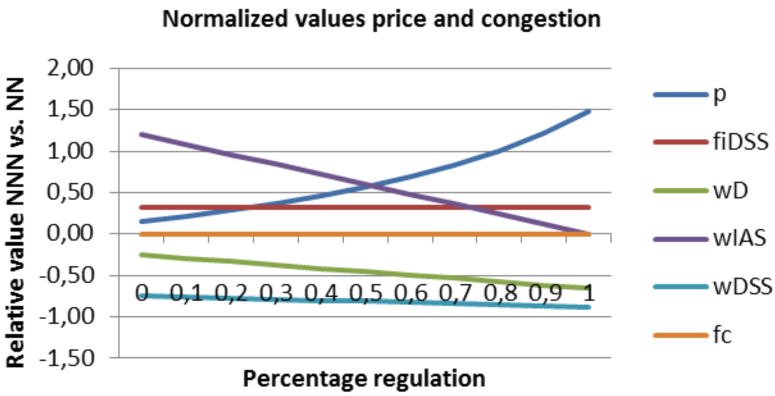

Figure 59: Normalized values of price and congestion in the regulated network– low r/λ scenario

A regulated network– low IU utility on CPs and QoS

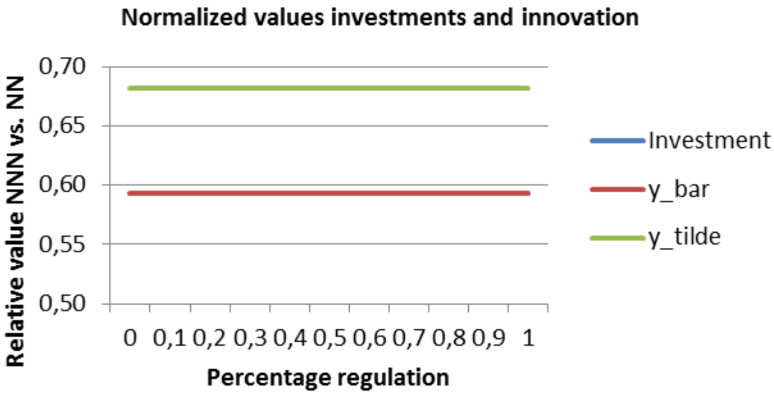

Figure 60: Normalized values of investments and innovation in the regulated network– high r/λ scenario

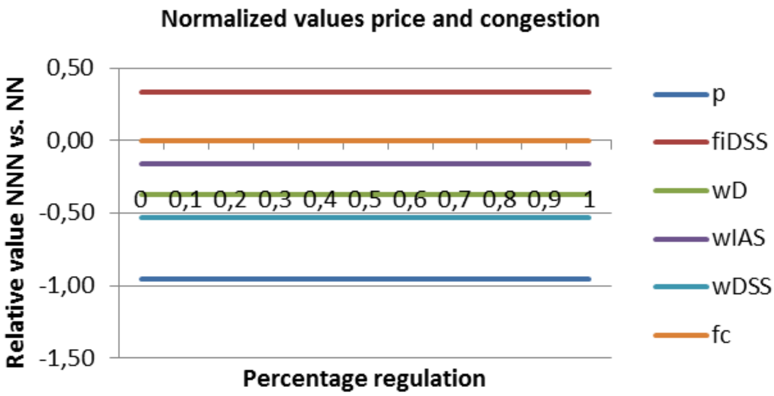

Figure 61: Normalized values of price and congestion in the regulated network– high r/λ scenario