NTNU – Trondheim
Norwegian University of
Science and Technology

VIX Exchange Traded Products

Performance, Price Discovery and Hedging

Christoffer Bordonado
Sven Richard Samdal

Industrial Economics and Technology Management
Submission date: June 2014
Supervisor: Peter Molnar, IØT

Norwegian University of Science and Technology
Department of Industrial Economics and Technology Management

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MASTERKONTRAKT

- uttak av masteroppgave

1. Studentens personalia

Etternavn, fornavn **Samdal, Sven Richard**	Fødselsdato **04. mar 1989**
E-post **svenris@stud.ntnu.no**	Telefon **97656494**

2. Studieopplysninger

Fakultet **Fakultet for samfunnsvitenskap og teknologiledelse**	
Institutt **Institutt for industriell økonomi og teknologiledelse**	
Studieprogram **Industriell økonomi og teknologiledelse**	Hovedprofil **Investering, finans og økonomistyring**

3. Masteroppgave

Oppstartsdato **15. jan 2014**	Innleveringsfrist **11. jun 2014**
Oppgavens (foreløpige) tittel **Investigation of volatility exchange traded products** **Performance, price discovery and hedging capabilities**	
Oppgavetekst/Problembeskrivelse Investigation of performance, price discovery and hedging capabilities of volatility exchange traded products.	
Hovedveileder ved institutt **Post doktor Peter Molnar**	Medveileder(e) ved institutt
Merknader **1 uke ekstra p.g.a påske.**	

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MASTERKONTRAKT

- uttak av masteroppgave

1. Studentens personalia

Etternavn, fornavn **Bordonado, Christoffer**	Fødselsdato **13. okt 1986**
E-post **bordonad@stud.ntnu.no**	Telefon **90101584**

2. Studieopplysninger

Fakultet **Fakultet for samfunnsvitenskap og teknologiledelse**	
Institutt **Institutt for industriell økonomi og teknologiledelse**	
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Trondheim 25/4/14
Sted og dato

C. Cordonado
Student

Hovedveileder

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Sammendrag

Denne artikkelen undersøker ytelsen, sikringsevnen og prisoppdagelsen til noen av de mest populære børshandlede produktene med volatilitetsindeksen VIX som den underliggende. Vi finner stor forskjell i prisoppdagelsesfunksjonen for de direkte ikke-girede VIX ETPene. De følger referanseindeksene sine tett, men lider av verditap over tid på grunn av terminstrukturformen på VIX futures. Dette verditapet gjør dem uegnede som langsiktige investeringer, men gir opphav til en lønnsom handlestrategi som bruker direkte og inverse VIX ETPer. Strategien er robust for transaksjonskostnader. Til tross for å ha høye negative korrelasjoner med S&P 500, fungerer ETPene dårlig som sikringsverktøy av en aksjeportefølje som følger S&P 500. Inkludering av VIX ETPer i porteføljen vil i stedet redusere risikojustert avkastning.

Preface

This master thesis marks the conclusion of the authors' Master of Science program in Industrial Economics and Technology Management at the Norwegian University of Science and Technology. The thesis was written during the spring semester of 2014.

The study is within the field of empirical finance and studies some of the most popular volatility exchange traded products by investigating their performance, price discovery and hedging ability. A simple trading strategy using some of these products is also implemented.

One person has been particularly valuable to the completion of this study. We would like to express gratitude to our academic supervisor post doctor Peter Molnár for his invaluable guidance, constructive feedback and supportive demeanor.

Trondheim, 11.06.2014

Sven Richard Samdal　　　Christoffer Bordonado

VIX exchange traded products: Performance, Price Discovery and Hedging

Christoffer Bordonado, Sven Richard Samdal

Norwegian University of Science and Technology
bordonad@stud.ntnu.no, svenris@stud.ntnu.no

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Abstract

This paper investigates the performance, hedging ability and price discovery relationship between some of the most popular exchange traded products with the volatility index VIX as the underlying. We find a large difference in the price discovery function for the direct unleveraged VIX ETPs. The VIX ETPs have good tracking performance, but suffer from time-decay due to the shape of the VIX futures term structure. This time-decay makes them unsuitable for buy-and-hold investments, but gives rise to a profitable trading strategy using direct and inverse VIX ETPs. The strategy is robust to transaction costs. Despite being negatively correlated with the S&P 500, the ETPs perform poorly as hedging tools. By using simple rebalancing rules, the inclusion of VIX ETPs in a portfolio tracking the S&P 500 will decrease the risk-adjusted performance of the portfolio.

1. Introduction

Financial research has made great strides in understanding risk and volatility. In recent years, the trading of volatility has also become widespread. A variety of methods and exchange traded products exist to facilitate volatility trading. Following the rise in methods and securities linked to volatility, several exchanges around the world now have listed volatility-dependent products. For instance, one can trade volatility derivatives on the S&P 500, the Euro Stoxx 50, Nikkei, emerging markets, oil, gold and many other. The introduction of exchange traded products (ETPs) has given market participants the ability to trade volatility without using futures and options. The term ETP refers to Exchange Traded Funds (ETFs) and a variation of ETFs known as Exchange Traded Notes (ETNs), as a whole. An ETF is a mini portfolio of equities, futures or other derivatives, which is designed to track an index or some other correlating investment, neatly packed into one product. These volatility ETPs provide retail investors with easy access to seemingly attractive hedging and diversification opportunities. However, many of these products are structured in such a way that their long term expected value is zero. Despite this fact, the popularity and trading volume in these products continue to rise.

Whaley (2013) explains how and why many VIX ETPs are virtually certain to lose money through time due to a "contango trap" in which VIX futures prices fall to the level of the VIX index. Deng et al. (2012) find that VIX ETPs are generally not effective hedges for stock portfolios because of a negative roll yield. However, they propose that medium term ETPs appear to both reduce the volatility and increase the return of stock portfolios. Furthermore, Alexander and Korovilas (2012b) point out that individual positions in VIX futures ETNs, including mid-term and longer-term trackers, offer no opportunities for diversification of equity exposure, except during the onset of a major crisis. However, they indicate that certain portfolios of VIX futures, or their ETNs, referred to as 'roll-yield arbitrage' portfolios, can offer unique risk and return characteristics and diversification opportunities. Simon and Campasano (2014), demonstrate that selling (buying) VIX futures contracts when the basis is in contango (backwardation) and hedging market exposure with short (long) S&P futures positions is highly profitable and robust to both conservative assumptions about transaction costs and the use of out of sample forecasts to set up hedge ratios.

Eraker and Wu (2013) propose an equilibrium model to explain the negative return premium for both VIX ETNs and futures. In this model, increases in volatility endogenously lead to decreasing stock prices. The negative return premium is an equilibrium outcome because long VIX futures positions hedge against low returns and high volatility states (i.e, financial crisis).

This paper investigates the performance, hedging ability and price discovery relationship between some of the most popular exchange traded products with the VIX as the underlying. For the price discovery analysis a classical measure called Common Factor Weights first introduced by Schwarz and Szakmary (1994) is used. The profitability of a simple trading strategy is also tested. The strategy exploits the difference between VIX futures and spot prices as entry point indicators for direct and inverse ETP trades.

This study contributes to the existing literature first and foremost by providing a comprehensive overview of different aspects of VIX ETPs. The tracking performance results of different VIX ETPs presented in Whaley (2013) are confirmed with a longer time period of data. The findings of Alexander and Korovilas (2012b) and Deng et al. (2012) regarding the hedging ability of VIX ETPs are confirmed as well. Further, this is, to the best of the authors' knowledge, the first study of price discovery between different VIX ETPs. Finally, similar results as the findings of Simon and Campasano (2014) are found when applying a similar trading strategy to VIX ETPs. However, when applied to VIX ETPs, an unhedged version of the strategy proves itself the most profitable.

The outline of this paper is as follows: Section 2 gives historical background of the VIX and a short explanation of how different VIX ETPs are constructed. Section 3 presents the data and summary statistics. Section 4 gives an assessment of the performance seen by the different ETPs relative to the indices they are designed to track. In Section 5 the price discovery relationship between different pairs of ETPs is studied. In Section 6 a trading strategy using direct and inverse VIX ETPs is proposed. Section 7 investigates the hedging ability of the different ETPs when included in an equity portfolio tracking the S&P 500. The final section summarizes and concludes.

2. S&P 500 and the VIX - My Fear Lady

The S&P 500 is an index of large cap U.S. equities, with index assets comprising approximately USD 1.6 trillion. The index includes 500 leading companies and captures approximately 80% coverage of available market capitalization in the US. Since it is considered one of the best representations of the U.S. stock market, and a bellwether for the U.S. economy, it is one of the most followed equity indices. Because of the size and importance of the S&P 500, investors have since 1983 used options on this benchmark for a variety of purposes, including investing, hedging, income, asset allocation, and the management of risk.

The idea of a volatility index was first presented by Brenner and Galai (1989). The same authors proposed a formula to compute the volatility index in 1993. The original formula for VIX was developed for the Chicago Board Options Exchange (CBOE) by Whaley (1993) and was based on CBOE S&P 100 Index (OEX) option prices. In 2003, CBOE collaborated with Goldman Sachs to update the VIX. They developed a new way to measure expected volatility and the underlying index was changed to the S&P 500 Index (SPX). The result of this work is the VIX as we know it today, which measures the implied volatility of S&P 500 index options. This is done by averaging the weighted prices of these options (SPX puts and calls) over a wide range of strike prices.

The derivation of the VIX formula is readily available.[1] It is calculated using the following formulae:

$$\sigma^2_{VIX} = \frac{2}{T}\sum_{i=1}^{N}\frac{\Delta X_i}{X_i^2} exp(rT)V(X_i) - \frac{1}{T}\left(\frac{F}{X_0} - 1\right)^2 \tag{1}$$

$$F = X_0 + exp(rT)(C_0 - P_0) \tag{2}$$

$$\Delta X_i = \frac{X_{i+1} + X_{i-1}}{2} \tag{3}$$

where r is the risk free rate, T is the expiration time (which CBOE calculates to the minute), F is the forward price of the index, X_0 is the strike price immediately below the forward price, X_i is the strike of the ith out-of-the-money option and V is the midprice of the corresponding option. Equations 1 to 3 are applied to the first two option expirations, T_1 and T_2. Interpolation is then used to find a constant 30-day volatility:

$$VIX = 100 * \sqrt{\left(T_1\sigma^2_{VIX1}\frac{N_{T2} - N_{T30}}{N_{T2} - N_{T1}} + T_2\sigma^2_{VIX2}\frac{N_{30} - N_{T1}}{N_{T2} - N_{T1}}\right)\frac{N_{365}}{N_{30}}}. \tag{4}$$

Commonly referred to as the fear gauge, the VIX is considered the leading barometer of investor sentiment and expectation of market volatility. It is annualized and quoted in percentage points. The VIX and the S&P 500 is shown in Figure 1.

The spikes in the VIX quite clearly correspond to price drawdowns in the S&P 500. From 1990 to 2014, there has been a correlation between the daily returns of the VIX and the S&P 500 of approximately -0.71. The VIX is therefore considered a useful tool to hedge against the potential downside of the broad equity market. Volatility does not measure the direction of price changes, merely their dispersion. Hence, large VIX readings mean investors see risk that the market will move sharply, downward *or* upward. However, markets typically fall faster than they rise. One possible explanation for the strong negative correlation between equity returns and the change in volatility, is the *leverage effect*.

A large and sudden fall in share price will increase the debt-to-equity ratio, i.e. the firm will instantly become more highly leveraged. This makes its future more uncertain and hence volatility tends to increase dramatically following a

[1] See for instance https://www.cboe.com/micro/vix/vixwhite.pdf

large fall in share price. The effect is not symmetric because large and sudden increase in share prices are rare, and even if they do occur this would be good news for the shareholders, so volatility should decrease. An exception is hyper-inflationary periods when rising volatility can be tied to rising prices. During the three-year period of hyperinflation in Germany (the Weimar Republic), realized volatility rose from around 15% in 1919 (similar to what is seen in US markets today), peaking at 2000% in 1923. However, the overall tendency of volatility is to increase when prices fall, which is evident in Figure 1.

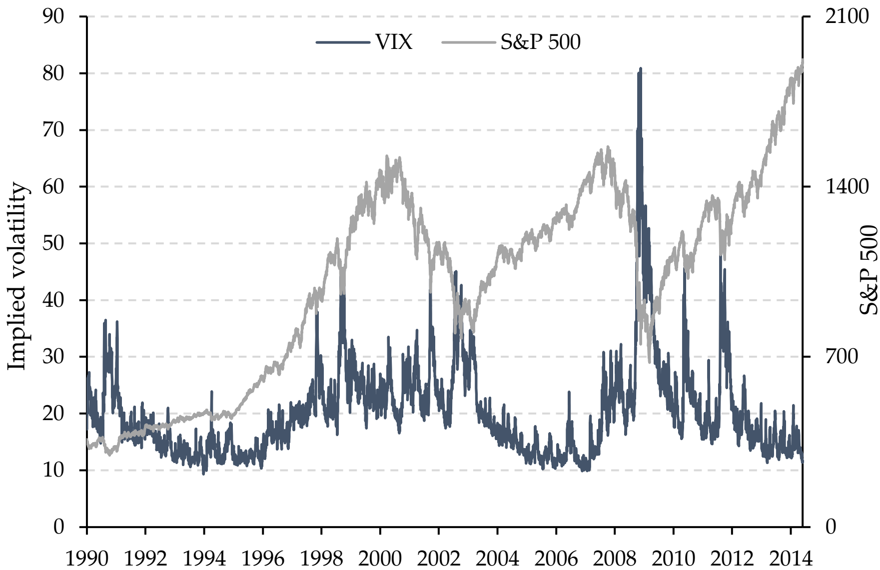

Figure 1: *VIX and S&P 500 from 1990 to 2014.*

While the VIX is not a traded entity itself, there is a market in VIX futures and options.[2] Trading of VIX futures began in March 2004 and VIX options in February 2006. VIX exchange traded products (ETPs), the second generation of volatility products, were introduced in 2009. ETPs are derivatively-priced and are traded on a securities exchange like a stock. Derivatively-priced means that the value is derived from another investment instrument such as a commodity, currency, stock price or interest rate. In the case of VIX ETPs this investment instrument is VIX futures. Exchange traded products include exchange traded

[2]See http://www.cboe.com/Strategies/VIXProducts.aspx

funds (ETFs), exchange traded vehicles (ETVs), exchange traded notes (ETNs) and certificates. The VIX ETPs used in this paper are either ETNs or ETFs. ETFs are transparent and specify exactly what instruments are used to generate the benchmark index return. ETNs, on the other hand, are a type of unsecured, unsubordinated debt security that promise the benchmark return over their stated maturity. Hence, the value of an ETN can be affected by the credit rating of the issuer and not just changes in the underlying index. For an in-depth explanation of how VIX ETFs and ETNs work, see for instance Whaley (2013) or Alexander and Korovilas (2012b).

ETPs have gained great popularity in recent years. Deutsche Bank estimates that the value of ETFs worldwide amount to over $2.3 trillion and have more than doubled since 2011, exceeding the $2.2 trillion pool of funds invested in private equity. Fueled by the growth of market participants who do not want or cannot trade in derivatives markets, more than twenty-eight VIX ETPs have been introduced in recent years. Figure 2 shows the average daily number of contracts traded in VIX ETPs, VIX futures and options per year. The number of ETP contracts traded is calculated by summing the average daily volume in seven of the most popular ETPs for the years 2009 to 2014 (see Table 1).

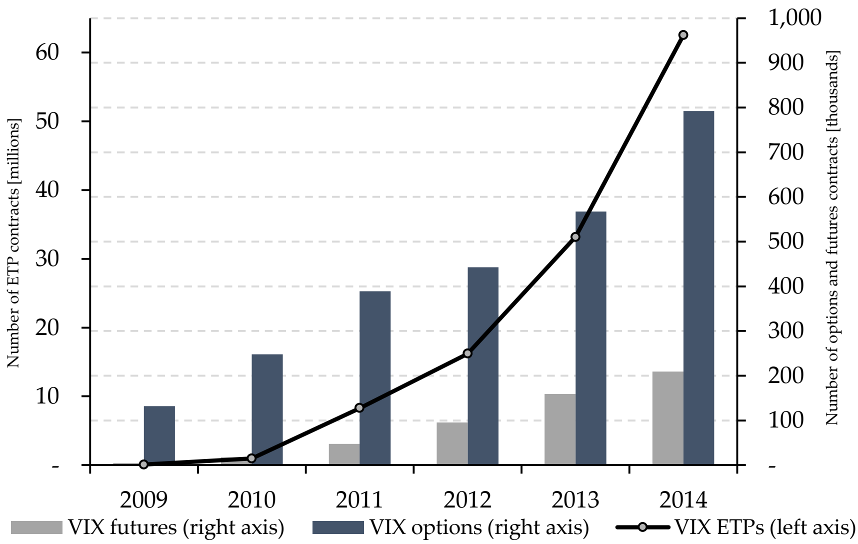

Figure 2: *The average number of contracts traded daily in VIX options, futures and ETPs from 2009 to 2014.*

It is important to emphasize that the ETPs are not designed to replicate the VIX itself, but rather futures indices on the VIX. This is because the VIX is difficult to replicate. One would need to continuously rebalance a basket of options due to concavity and to keep the maturity at 30 days, as evident from equations 1 to 4. Hence, no ETP sponsor is willing to do this because it would be too costly. VIX ETPs are instead created from VIX futures trading strategies designed to track the value of different S&P 500 VIX Futures Indices.[3] These indices track returns from VIX futures positions which are rolled daily throughout the period between expiration dates. The total return version of the indices incorporates interest accrual on the notional value of the indices and reinvestment into the indices. Interest is based on the 3-month US Treasury rate. For instance, the ETN VXX provides exposure to the value of the S&P 500 VIX Short-Term Futures Index Total Return, a one-month constant-maturity index that tracks a portfolio composed of first- and second-month VIX futures.

Cole (2014) gives an excellent analogy to volatility trading using the speech of Donald Rumsfeld, former United States Secretary of Defense. In a news briefing regarding the Iraq war, Rumsfeld stated:

"...there are known knowns; there are things that we know that we know. We also know there are known unknowns; that is to say we know there are some things we do not know. But there are also unknown unknowns, the ones we don't know we don't know."[4]
According to Cole, volatility trading is about putting a price on these *known* and *unknown* unknowns. A known unknown is a risk factor we are aware of, such as the Federal Reserve tapering bond purchases. The unknown unknowns on the other hand, are the true shocks, caused by risk factors that come by total surprise. Examples are the 9/11 terrorist attacks in 2001, the *Flash Crash*[5] in 2010 and the Tohoku earthquake in 2011. These market crashes are characterized by hyper-speed downturns and large volatility of volatility, meaning that the change in volatility itself is substantial.

With this as a backdrop, how VIX ETPs perform as investment and hedging tools is studied.

[3] http://us.spindices.com/index-family/strategy/vix

[4] http://www.defense.gov/transcripts/transcript.aspx?transcriptid=2636

[5] 2010 Flash Crash: A US stock market crash on Thursday May 6th, 2010 in which many indices fell about 9%, only to recover again minutes later.

3. Data and Descriptive Statistics

In order to evaluate the different aspects of the VIX ETPs, seven of the most traded ETPs are chosen. Table 1 presents an overview of these. The ETPs are sorted by the average number of units traded daily, given in the third column. Assets under management, date of inception and yearly fee are given in the three following columns. ST and MT indicate if the index tracked is based on short-term or mid-term futures. The column denoted TR/ER, indicates whether the ETPs are based on the Excess Return or the Total Return version of the index being tracked. The multiplier denotes the leverage of the ETP, with 1 indicating a direct ETP, -1 indicating an inverse ETP, and 2 indicating leverage with returns twice that of the underlying index. A direct ETP provides long exposure, while an inverse ETP provides short exposure to the underlying. All of these VIX ETPs are traded on NYSE Arca.

Table 1: *An overview of the VIX ETPs used in this paper*.*

Symbol	Name	Average volume [10^3]	Asset [mUSD]	Date of inception	Yearly fee [%]	Time horizon	Return type	Multiplier
VXX	iPath S&P 500 VIX Short-Term Futures ETN	20,900	919	29 Jan 2009	0.89	ST	TR	1
XIV	VelocityShares Daily Inverse VIX Short-Term ETN	13,100	591	29 Nov 2010	1.35	ST	ER	-1
TVIX	VelocityShares Daily 2x VIX Short-Term ETN	4,600	276	29 Nov 2010	1.65	ST	ER	2
UVXY	Proshares Ultra VIX Short-Term Futures ETF	2,400	238	04 Oct 2011	1.56	ST	ER	2
SVXY	Proshares Short VIX Short-Term Futures ETF	1,500	223	04 Oct 2011	1.32	ST	ER	-1
VIXY	ProShares VIX Short-Term Futures ETF	789	115	03 Jan 2011	0.83	ST	ER	1
VXZ	iPath S&P 500 VIX Mid-Term Futures ETN	708	77	29 Jan 2009	0.89	MT	TR	1

**Information taken from http://etfdb.com/etfdb-category/volatility/*

Since the inception of the first VIX ETPs in 2009, the S&P 500 has been trending upward, only interrupted by two periods with high volatility. The first was triggered by the European sovereign debt crisis in 2010 (see Figure 3) and caused the S&P 500 to fall about 15% over a two month period. The second high volatility period followed the downgrading of the US credit rating from AAA to AA+ in August 2011, causing the S&P 500 to fall nearly 11% in six days. What these two periods have in common is that the price fall, although dramatic, was relatively small and short lived. When backtesting different hedging and trading strategies it is desirable to investigate their performance during different market regimes, such as the market crash in 2008. Since the prices of the VIX ETPs are closely linked to the VIX futures indices they are designed to track, it

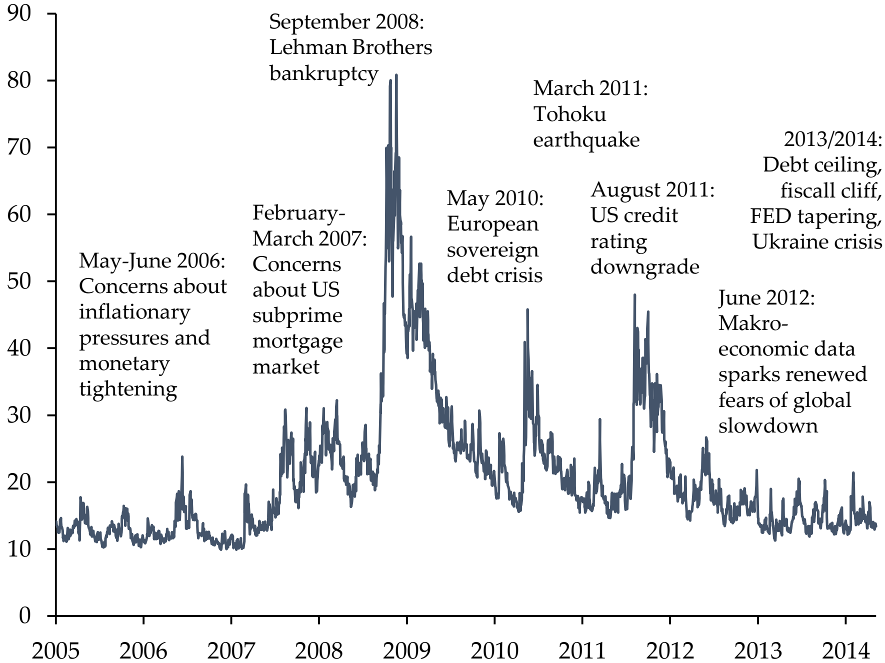

Figure 3: *Geopolitical events and VIX from 2005 to 2014.*

is possible to derive indicative closing values, i.e. *artificial prices*, on the ETPs in the time period before their inception. The prospectus for the VXX explains how the indicative value of the ETN is calculated.[6]

The daily ETP time series has been downloaded from Yahoo Finance. The artificial price data for the pre-inception period used in the analysis has been purchased from sixfigureinvesting.com, and is constructed in the manner described in the prospectus of each VIX ETP. Note however, that the closing market price of an ETP can deviate from its indicative closing price, and bias the analysis. To evaluate the bias, for the overlapping period in the time series, the artificial

[6]*The **closing indicative value** for a series of ETNs on any calendar day will be calculated in the following manner. The closing indicative value on the inception date will equal $100. On each subsequent calendar day until maturity or early redemption, the closing indicative value will equal (1) the closing indicative value for that series on the immediately preceding calendar day times (2) the daily index factor for that series on such calendar day (..) minus (3) the investor fee for that series on such calendar day. (...) The **daily index factor** for any series of ETNs on any index business day will equal (1) the closing level of the Index for that series on such index business day divided by (2) the closing level of the Index for that series on the immediately preceding index business day. (...) The **investor fee** for any series of ETNs is 0.89% per year times the applicable closing indicative value times the applicable daily index factor, calculated on a daily basis in the following manner. On each subsequent calendar day until maturity or early redemption, the investor fee will equal to (1) 0.89% times (2) the closing indicative value for that series on the immediately preceding calendar day times (3) the daily index factor for that series on that day divided by (4) 365.*

prices are regressed on the observed market prices. The regression is on the form

$$P_t^{obs} = \alpha + \beta P_t^{art} + e_t, \tag{5}$$

where P_t^{obs} is the observed ETP price at time t, P_t^{art} is the artificial ETP price at time t, and e_t is the residual at time t. The results are reported in Table 2, and it is clear that the artificial prices are close to the observed market prices. It is therefore concluded that the artificial prices are suitable to expand the time period used to evaluate the trading and hedging strategies.

Figure 4 reports the evolution of the different VIX ETPs and the S&P 500 index. The prices are rebased to 100 at the beginning of the period. The ETPs which track the same underlying VIX futures index have been merged, since their price series are identical in the period leading up to their inception, and more or less identical after inception. For instance, VXX/VIXY in Figure 4 depicts the rebased prices of both VXX and VIXY in one, which track the short term VIX futures index. When the market crashed in 2008, the direct ETPs doubled many times over, while the inverse ETPs were almost wiped out. In addition, the constant rolling of the VIX futures contracts used to construct the ETPs represent a massive drag on the performance of the direct ETPs, causing them to decline rapidly in stable periods. On the other hand, this contributes to the outperformance of the inverse VIX ETPs on the S&P 500.

Table 2: *Results when artificial VIX ETP prices are regressed on observed VIX ETP prices.*

Ticker	Start date	No. of obs.	R^2	Adj. R^2	α	t-stat α	β	t-stat β
VXX	30 Jan 09	1256	0.9999	0.9999	-0.191	-0.3956	1.00	4502
VXZ	20 Feb 09	1242	0.9999	0.9999	0.010	0.5490	0.99	3404
TVIX	30 Nov 10	794	0.9991	0.9991	23.732	7.7996	0.96	932
XIV	30 Nov 10	794	0.9995	0.9995	-0.002	-0.1572	0.99	1322
UVXY	04 Oct 11	581	0.9996	0.9996	-25.376	-1.8432	1.02	1224
VIXY	04 Jan 11	770	0.9997	0.9997	0.107	0.7614	1.00	1695
SVXY	04 Oct 11	581	0.9995	0.9995	-0.014	-0.3864	0.99	1093

Tables 3, 4 and 5 report descriptive statistics for three versions of the data used in this paper. Table 3 shows the descriptive statistics for daily returns of the VIX ETPs for the period 21 June 2006 to 25 April 2014. In the time series for this period, returns on the ETPs are calculated based on their artificial prices until their date of inception. From their date of inception, and until the end of the period, the observed closing prices downloaded from Yahoo Finance are used. This data set is used for testing the trading and hedging strategies presented in Sections 6 and 7 respectively. Table 4 reports descriptive statistics for the returns of the observed ETP market prices. These prices are used to see how well the ETPs track their respective indices in Section 4, and finally Table 5 reports descriptive statistics for the one-minute prices that will be used to examine price discovery in Section 5. The one-minute data set has been downloaded from the WRDS TAQ database. To reduce noise that might occur during opening and closing trading minutes, the observations from the first and last fifteen minutes of each day have been removed from this data set.

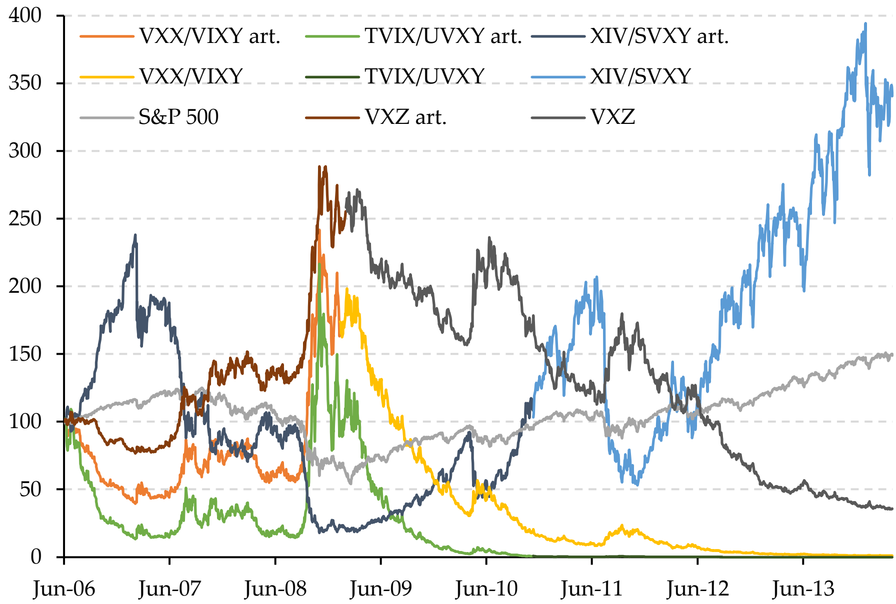

Figure 4: *Rebased prices of VIX ETPs and the S&P 500 index. ETPs tracking the same index are more or less identical in price, so their series have been merged. The time before and after inception is shown by dividing the series for each ETP into a series for the artificial (art.) prices, and the observed closing prices.*

All return distributions exhibit leptokurtosis, as one would expect for this group of derivatives. In other words, there is a high probability for extreme values to occur. The direct VIX ETPs have a right skewed distribution, where most values are concentrated to the left of the mean, and the extreme values are located to the right. The positive skewness in the direct VIX ETPs illustrates the consistent negative drag due to the constant rolling of the underlying futures contracts and fees, as well as extraordinary returns when volatility spikes. The direct ETPs yield positive returns when volatility rises. Volatility often spikes following a negative surprise to the market, and this causes the direct VIX ETPs to gain in value over a short time period. The opposite holds for the inverse VIX ETPs (XIV and SVXY).

Table 3: *Descriptive statistics for daily VIX and S&P 500 ETP returns from 21 Jun. 2006 to 25 Apr. 2014. The VIX series are based on artificial and observed VIX ETP returns.*

	Mean	Median	Std. Dev.	Variance	Kurtosis	Skewness	Min	Max	No. of obs.
VXX	-0.16%	-0.54%	3.83%	0.0015	3.02	0.85	-14.26%	24.55%	1974
VIXY	-0.16%	-0.54%	3.84%	0.0015	3.08	0.87	-14.10%	24.55%	1974
VXZ	-0.03%	-0.18%	1.99%	0.0004	3.19	0.68	-8.21%	11.14%	1974
TVIX	-0.35%	-1.11%	7.44%	0.0055	3.60	0.86	-29.80%	49.06%	1974
UVXY	-0.33%	-1.14%	7.75%	0.0060	3.29	0.87	-32.24%	49.06%	1974
XIV	0.14%	0.54%	3.83%	0.0015	3.05	-0.88	-24.54%	14.08%	1974
SVXY	0.14%	0.56%	3.88%	0.0015	3.31	-0.88	-24.54%	16.11%	1974
SPY	0.04%	0.07%	1.40%	0.0002	13.48	0.23	-9.84%	14.51%	1974
SH	-0.03%	-0.08%	1.39%	0.0002	9.42	0.10	-11.02%	9.42%	1974

Table 4: *Descriptive statistics for daily VIX ETP returns from the date of their inception until 25 Apr. 2014.*

	Start date*	Mean	Median	Std. Dev.	Variance	Kurtosis	Skewness	Min	Max	No. of obs.
VXX	02 Feb 2009	-0.32%	-0.71%	3.77%	0.0014	2.915	0.808	-14.26%	20.70%	1338
VIXY	05 jan 2011	-0.24%	-0.52%	4.01%	0.0016	2.880	0.802	-13.13%	20.68%	831
VXZ	23 Feb 2009	-0.13%	-0.25%	1.91%	0.0004	2.888	0.568	-8.21%	10.26%	1303
TVIX	01 Dec 2010	-0.60%	-1.11%	7.43%	0.0055	4.122	0.767	-29.80%	40.79%	855
UVXY	05 Oct 2011	-0.83%	-1.11%	7.74%	0.0060	2.022	0.465	-26.66%	37.03%	642
XIV	01 Dec 2010	0.22%	0.52%	3.96%	0.0016	2.863	-0.818	-19.98%	13.18%	855
SVXY	05 Oct 2011	0.35%	0.55%	3.88%	0.0015	2.086	-0.502	-19.20%	13.15%	642

Time series downloaded from finance.yahoo.com. Note that start date might differ from inception date in Table 1 because of unavailable data.

Table 5: *Descriptive statistics for the log returns of 1-minute price data used in the price discovery section. The series end 31 Dec. 2012.*

	Startdate	Mean	Median	Stdev	Variance	Kurtosis	Skewness	Min	Max	Nobs
VXX	30 Jan 2009	-0.0010%	0.0000%	0.200%	0.000004	519	-0.68	-19.26%	12.80%	409646
VIXY	04 Jan 2011	-0.0008%	0.0000%	0.222%	0.000005	556	1.49	-14.00%	13.57%	201768
TVIX	30 Nov 2010	-0.0023%	0.0000%	0.428%	0.000018	290	0.42	-22.66%	24.39%	216636
UVXY	04 Oct 2011	-0.0037%	0.0000%	0.415%	0.000017	342	-0.06	-22.15%	23.27%	128756
XIV	30 Nov 2010	0.0002%	0.0000%	0.218%	0.000005	341	-3.86	-14.47%	9.74%	215511
SVXY	03 Jan 2012	0.0008%	0.0000%	0.197%	0.000004	228	-3.33	-9.30%	5.54%	100816

4. Performance and Tracking Error - Good, Bad and Ugly

In this section the performance of the different VIX ETPs are assessed relative to the benchmark they are designed to track. That is, either the short-term (ST) or medium-term (MT) S&P 500 VIX Futures Indices.[7]. Alexander (2008b) gives a good introduction to models for evaluating portfolio performance. The observed prices since inception are used. The analyzed ETPs can be found in

[7]See http://us.spindices.com/index-family/strategy/vix

Table 1. First, the active return is calculated by taking the difference between the ETP and index daily returns. That is, the active log return is the ETP log return minus the benchmark log return. Using this time series R_t consisting of T active returns, the tracking error is computed as

$$TE = \sqrt{\frac{1}{T-1}\sum_{t=1}^{T}(R_t - \hat{R})^2} \tag{6}$$

where $\hat{R}$ is the average active return. Similarly, the mean absolute deviation (MAD) and root mean square error is calculated as follows

$$MAD = \frac{1}{T}\sum_{t=1}^{T}|R_t - \hat{R}| \tag{7}$$

$$RMSE = \sqrt{\frac{\sum_{t=1}^{T}(\hat{R} - R_t)^2}{T}}. \tag{8}$$

To further evaluate the performance of the VIX ETPs, their daily returns are regressed on the daily returns of their respective benchmark, that is

$$R_{i,t} = \alpha + \beta R_{BM_i,t} + \epsilon_{i,t} \tag{9}$$

where i denotes the i-th ETP whose day t return is $R_{i,t}$ and whose day t benchmark return is $R_{BM_i,t}$. For the levered and inverse products, the underlying index returns are scaled by their multipliers. The results of these calculations are reported in Table 6, which summarizes the performance measures of how the different ETPs track their respective indices.

If the ETP follows the benchmark exactly, the intercept α should be zero and the slope β should be one. However, because of fees, transaction costs and other expenses the intercept is negative and because of tracking errors the slope is lower than one.

Looking at Table 6, all ETPs have a relatively high adjusted R-squared, in the range 0.90 to 0.93, which means they track their indices quite well. VXX tracks its index the best, while TVIX and VXZ tracks poorest with values 0.86 and 0.89 respectively. All ETPs have negative intercepts (α) close to zero. TVIX and UVXY have the most negative intercepts, which is expected since these ETPs also have the highest fees (1.65% and 1.54% respectively). Also as expected, the slope (β) is lower than one for all ETPs, again with TVIX and UVXY having the lowest coefficients. The slightly poorer tracking performance and higher fee in these two products can be attributed to the fact that they both are leveraged (2X).

In the last four columns the daily return deviations are given. The mean of the active returns are all negative and very close to zero. The tracking

Table 6: *Results from the tracking performance analysis.*

Ticker	Obs.	R^2	Adj. R^2	α	t-stat α	β	t-stat β	Daily return deviations			
								Mean	TE	MAD	RMSE
VXX	1258	0.93	0.93	-0.0004	-1.61	0.90	134	-0.00004	0.010	0.007	0.010
VXZ	1258	0.88	0.88	-0.0002	-0.86	0.92	99	-0.00003	0.006	0.005	0.006
TVIX	855	0.86	0.86	-0.0033	-3.53	0.79	73	-0.00189	0.033	0.020	0.027
XIV	855	0.91	0.91	-0.0016	-3.95	0.89	93	-0.00202	0.013	0.008	0.012
UVXY	642	0.91	0.92	-0.0032	-3.67	0.87	85	-0.00198	0.025	0.017	0.022
VIXY	831	0.92	0.93	-0.0004	-1.19	0.88	104	-0.00004	0.012	0.008	0.011
SVXY	642	0.90	0.90	-0.0013	-2.63	0.87	76	-0.00191	0.014	0.009	0.012

error (TE), mean absolute deviation (MAD) and root mean square error (RMSE) are consistent with the regression results, with TVIX and UVXY having the largest deviations. Perfect tracking would require the ETP sponsor to perfectly hedge their futures position using the same prices used by S&P in their index computations. This is not possible due to the fact that S&P uses settlement prices, which are set after the market is closed, while hedging/rebalancing must happen during trading hours. As explained in Section 3, the ETP manager uses closing prices when calculating the daily index factor, which can be both larger and smaller than the settlement price. However, the results show that all the ETPs show a good tracking performance.

The main reason for the grim performance of the direct VIX ETPs is the contango in the VIX futures market. It is the futures prices curve which determines whether the futures market is in contango or backwardation. Backwardation refers to a downward sloping term structure of futures prices and contango refers to an upward sloping term structure. Figure 5 shows the term structure in contango on 21 April 2011 and 12 April 2013. On 19 August 2011 and 27 October 2008 the term structure was in backwardation and downward sloping.

The ETPs incurr small losses from constantly rolling over the VIX futures. Because the slope of the term structure is steeper at the short end, the rolling over of futures contracts incurs smaller losses for the MT ETPs than the ST ETPs. In spite of this, ST ETPs remain the most popular.

Figure 6 shows the *average* VIX futures term structure per year since 2007. The data has been normalized by setting the front month VIX futures contract to 100 and expressing the averages of the second through seven months as multiples of the front month. It is clear that during most market regimes the term structure of VIX futures is in contango. Since 2004, the price of the back month futures contract has been higher than the the front month contract 83% of the time. When volatility is at a low level, fear held by investors that it will soon increase, keeps the term structure in contango. If one is holding a rolling 30-day VIX future, each day 1/30th of the front future is rolled and the next second future bought. When the market is in contango, the second future is more expensive, so the long position loses to the rolling process. When quoted in percentage-points, the cost of rolling the futures is commonly referred to as the "roll-yield". During low points in the volatility cycle the returns on VIX futures and their ETNs are heavily eroded by the roll-yield, which is particularly high at the short end of the term structure.

Figure 6 shows that the average term structure was steeply upward sloping during 2012. From January 3 to December 31 2012, the return of UVXY and VXX was -96.80% and -76.38%, respectively. In 2008 however, the average term structure was steeply *downward* sloping. The artificial ETP prices indicate that the return of holding UVXY and VXX from January 3 to December 31 2008 would have been 214% and 121%, respectively. This indicates that it might be profitable to buy direct VIX ETPs when the term structure is in contango, and buy inverse VIX ETPs when the term structure is in backwardation.

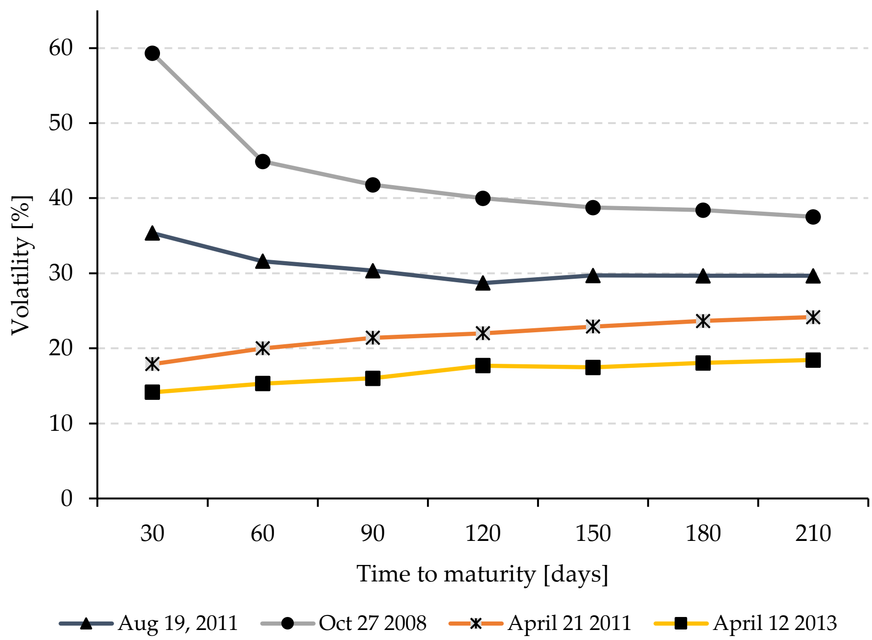

Figure 5: *Examples of different degrees of contango and backwardation in the VIX futures term structure.*

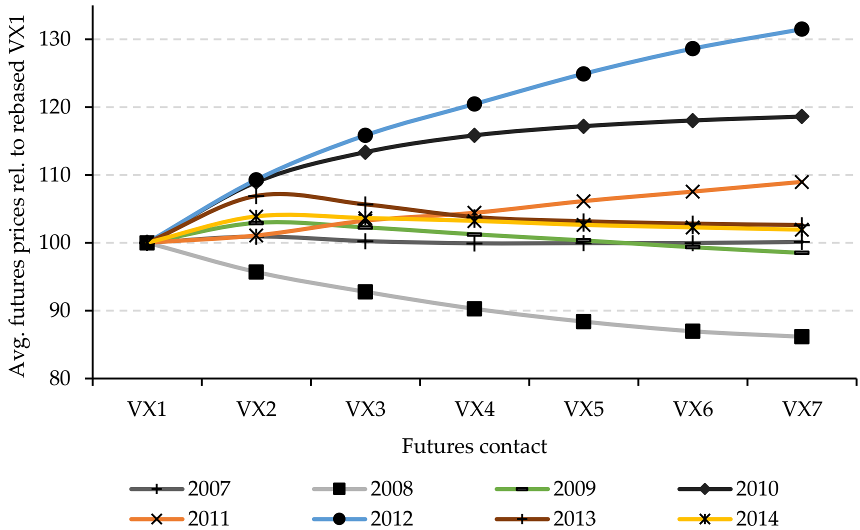

Figure 6: *Average VIX futures term structure per year, 2007 - 2014.*

5. Price Discovery

In this section, pairs of ETPs tracking the different VIX futures indices are analyzed in order to decide which of the ETPs within each category is the most efficient at absorbing new information. The answers obtained are then compared with the average daily number of units traded and the age of each ETP. Although some ETPs are more popular than others, all ETPs considered in this paper are sufficiently liquid to expect that they are equally good at absorbing new information arriving at the market place. If discrepancies are found, this might give some insight into the preferences of informed traders, which in turn can be used in future research to investigate possible high frequency trading strategies. This first part is devoted to presenting the framework applied. The findings are then presented in the second part.

When prices in different markets are cointegrated, the error correction mechanism has become the focus of research into the price discovery relationship, that is, the process in which information implicit in investor trading is incorporated into market prices, see for instance Alexander (2008a).

Several measures exist to assess the relative contributions of cointegrated assets to price discovery. The most widely used are Granger causality of Granger (1986), the common factor weights of Schwarz and Szakmary (1994) and Hasbrouck (1995) information share (IS). Gonzalo and Granger (1995) extended the causality framework to a common factor model (CF). This model focuses on the components of the common factor and the error correction process, while the Hasbrouck model considers each market's contribution to the variance of the innovations to a common factor.

Baillie et al. (2002) show that these two models are directly related and provide similar results if the residuals are uncorrelated between markets. Comparing the three measures, Theissen (2002) shows that the Schwarz and Szakmary measure can be derived from the Gonzalo-Granger framework and conclusions are qualitatively similar to those based on information shares.

In this paper, the Engle and Granger framework is applied to estimate the error correction process between the pairs of ETPs considered. The estimated coefficients of these models are then applied to determine the Schwarz Szakmary common factor weights.

Since the ETPs considered in this paper are either leveraged differently, or track different indices, they are not all cointegrated with one another. Because of this, the space of VIX ETPs has been divided into three categories, and the price discovery relationship between the two ETPs within each category is analyzed. The categories are direct, leveraged and inverse VIX ETPs. Three pairs are formed for further analysis:

- VXX and VIXY
- TVIX and UVXY
- XIV and SVXY

Notice that the VXZ mid term ETN presented in Section 3 is not among the ETPs mentioned above. Since VXZ is the only ETP in this study which tracks the S&P 500 VIX Mid-Term Futures Index, it is left out of this part of the paper. Since corrections between the prices of different ETPs occur rapidly, the one-minute time series described in Section 3 is used. The one-minute data span the period from the date of inception of the last ETP in each pair until 31 December 2012. For descriptive statistics, please refer to Table 5 in Section 3.

5.1. Cointegration

Prices of individual assets tend to wander about at random, yet some pairs of series are expected to move so they do not drift too far apart. Engle and Granger (1987) showed that a class of models, known as error correction models (ECMs), allows long-run components of variables to obey equilibrium constraints while short-run components have a flexible dynamic specification. A condition for this to be true, called cointegration, was introduced by Granger (1981).

The concept behind cointegration is the existence of a long-run equilibrium relationship between two or more variables. The series may deviate in the short-run, but market forces bring them back together. With the error correction representation, a portion of the disequilibirum in one period is expected to be corrected in the next. In other words, if two VIX ETPs are cointegrated, it means that they have a long-term relationship which prevents them from wandering apart without bound. Furthermore, when VIX ETP prices are cointegrated, an error correction mechanism can be used to uncover the price discovery relationship between them.

Since the different ETPs are constructed to track the same underlying futures index, they are tied together and the difference between them should be mean reverting.

The Engle-Granger two-step procedure is applied to the different VIX ETP pairs. First the log price of one variable is regressed on the log price of the other variable:

$$Y_t = z + \gamma X_t + \varepsilon_t \tag{10}$$

Here, $Y = Y_{t=1}, Y_{t=2}, ..., Y_{t=T}$ and $X = X_{t=1}, X_{t=2}, ..., X_{t=T}$ are the log of the prices of two ETPs both tracking the same VIX futures index. Once the cointegrating regression in Equation (10) is run and estimates of γ and z are obtained, the Engle-Granger test is performed. This is a unit root test on the residuals:

$$\Delta\varepsilon_t = \varphi\varepsilon_{t-1} + \epsilon_t. \tag{11}$$

If the unit root test indicates that the residuals are stationary then the prices are cointegrated with cointegrating vector [1 -γ], and the long run equilibrium is z.

5.2. Estimating the Error Correction Models

The Granger Representation Theorem states that the cointegrated series X and Y have an error correction representation of order k, given by

$$\Delta Y_t = \alpha_Y (Y_{t-1} - \gamma X_{t-1} - z) + \sum_{i=1}^{k} \beta_{11}^{i} \Delta Y_{t-i} + \sum_{i=1}^{k} \beta_{12}^{i} \Delta X_{t-i} + \varepsilon_{1t} \tag{12}$$

$$\Delta X_t = \alpha_X (Y_{t-1} - \gamma X_{t-1} - z) + \sum_{i=1}^{k} \beta_{21}^{i} \Delta Y_{t-i} + \sum_{i=1}^{k} \beta_{22}^{i} \Delta X_{t-i} + \varepsilon_{2t} \tag{13}$$

where the coefficients of Equations (12) and (13) can be estimated using OLS. The appropriate number of lags to use in each model is determined by using the Bayesian information criterion. Hence, the value of k might be different in Equations (12) and (13). The alphas (α) describe the speed of adjustment back to equilibrium. For this to be true, the alphas must have opposite signs.

5.3. Common Factor Weights

Schwarz and Szakmary (1994) proposed a *common factor measure* to assess the contribution of each market to the price discovery process. The measure is easy to compute and has an intuitive interpretation. A formal justification for the measure is derived from the work of Gonzalo and Granger (1995) and given by Theissen (2002).

The common factor weights are calculated using the estimated coefficients of each of the error correction models. The magnitude of each weight (θ_i) then indicates which market mainly initiates the price discovery process, and which market mainly follows as a result of price correction. The equations for the weights are calculated as follows:

$$\theta_X = \frac{|\alpha_Y|}{|\alpha_Y| + |\alpha_X|} \tag{14}$$

$$\theta_Y = 1 - \theta_X = \frac{|\alpha_X|}{|\alpha_X| + |\alpha_Y|} \tag{15}$$

where α_Y and α_X are the error correction coefficients from equation (12) and (13) respectively. As discussed in Section 5.2, the alphas give the magnitude of speed at which the ETPs adjust to changes in equilibrium. Conversely, a low magnitude indicates the ETP is leading the process of price formation and therefore initiates the deviation from the equilibrium.

5.4. Results

Table 7 reports the results of the unit root tests of the ETPs. The results suggest that the prices of TVIX, VXX, and UVXY are stationary. However, we argue that all VIX ETPs are non-stationary if measured over a longer time period. By reviewing the figures of the price plots since year 2006 in Section 3, prices do not seem stationary. The time frame of the prices used in this section are from the inception of the different ETPs through 2012, periods which are characterized by an upward trending market, interrupted by the market corrections described in Section 3. Hence, the decay on direct VIX ETPs discussed in Section 4 represents a considerable role in the evolution of the prices. Moreover, Equations 12 and Equations 13 can be estimated for stationary time series also.

Table 8 reports the results of the Engle Granger cointegrating regression. *Regression Statistics* reports the results of the regression, and *Test Statistics* reports the results of the Augmented Dickey Fuller (ADF) test with two control lags on the regression residuals. The intercept term represents the long run equilibrium between the two ETPs which is subtracted from the spread between the ETPs in the ECM. The γ's represent the second element in the cointegrating vector multiplied by -1. These are all approximately equal to 1. All the intercepts and the γ coefficients are highly significant. $h = 1$ indicates rejection of the null of no cointegration, and as reported by the column *Test Statistic*, all null hypotheses are strongly rejected.

Table 7: *ADF(2) stationarity test with constant and trend. $h = 0$ indicates failure to reject unit root. Critical value for this test statistic is -1.94.*

	h	pValue	Test Statistic
TVIX	1	0.001	-3.26
VXX	1	0.001	-3.80
VIXY	0	0.165	-1.35
UVXY	1	0.001	-5.84
XIV	0	0.705	0.14
SVXY	0	0.870	0.71
TVIX rtn	1	0.001	-272.66
VXX rtn	1	0.001	-371.45
VIXY rtn	1	0.001	-260.58
UVXY rtn	1	0.001	-207.98
XIV rtn	1	0.001	-269.17
SVXY rtn	1	0.001	-184.66

Table 8: *Engle-Granger cointegration test. $h = 0$ indicates failure to reject the null hypothesis of no cointegration. The critical value for this test is -1.94.*

	Regression Statistics				Test Statistics		
Pair	z (intercept)	t-stat z	γ	t-stat γ	h	p-Value	Test Statistic
VXX, VIXY	-2.13	-17894	0.99	32399	1	0.001	-29.55
TVIX, UVXY	1.04	3943	0.98	6098	1	0.001	-4.96
XIV, SVXY	0.03	49	1.05	8205	1	0.001	-12.40

Table 9 reports the results of the estimated error correction model for each VIX ETP, as well as its common factor weight (CFW). It is concluded that for the VXX and VIXY, VXX is the price leader with a common factor weight of 75%. With VXX being one of the most traded VIX ETPs, this result is in line with what one might expect. XIV and SVXY incorporate new information about just as quickly, with SVXY adjusting slightly more to changes in XIV than the other way around. Of the two, XIV is the oldest. It has a larger average daily volume and measured in assets under management it is about 2.5 times larger than the SVXY. For TVIX and UVXY, the price discovery is approximately the same for both, as one would expect since these ETPs are similar in size.

Notice that there are a few other signs indicating the price leadership. In each pair, the leading ETP has fewer significant lagged returns (k) than the following, and its adjusted R-squared is worse than the following ETP. For all of the ECMs, the magnitude of the error correction coefficient is very small. This means that it takes some time to adjust for disequilibrium in prices.

Table 9: *Results of the estimated error correction models and Common Factor Weights.*

ETP	lags	R^2	$adj.R^2$	α	SE	t-stat	p-value	CFW
VXX	1	0.002	0.002	-0.0015	0.0006	-2.54	0.01	0.75
VIXY	15	0.053	0.053	0.0045	0.0006	7.38	0.01	0.25
XIV	2	0.003	0.003	-0.0006	0.0004	-1.70	0.08	0.54
SVXY	5	0.062	0.062	0.0007	0.0004	2.07	0.03	0.46
TVIX	9	0.039	0.039	-0.0003	0.0001	-1.72	0.05	0.43
UVXY	5	0.010	0.010	0.0002	0.0001	1.31	0.19	0.57

6. Trading Strategy - Reaping the Market Fear

In this section a trading strategy intended to extract the roll-yield inherent in the VIX ETPs is presented . A similar strategy using VIX futures was studied by Simon and Campasano (2014), Chan (2013) and Sinclair (2013). Since VIX ETPs track VIX futures indices quite well, it is expected that this trading strategy should also work using VIX ETPs.

According to the rational expectations hypothesis, VIX futures prices should be unbiased predictors of the future value of the spot VIX index. This means that the slope of the VIX term structure today, constructed from VIX futures prices, should be concurrent with the direction and magnitude of subsequent changes of the VIX index itself. However, Asensio (2013) shows that VIX futures are consistently overpriced relative to the subsequent moves in the underlying VIX index. Furthermore, Simon and Campasano (2014) present evidence that the futures prices can be predicted by looking at the difference between the VIX front month futures price and the VIX Index. If the futures are trading over the VIX, the futures tend to fall and if the futures are trading below the VIX they tend to rise.

The term structure can possibly be explained by the fact that most investors are risk-averse. They are therefore willing to pay a premium for VIX exposure because it represents an insurance against losses in the equity market. This means that the expected return when going long a VIX futures is typically negative, which is consistent with the typical contango pattern. In a sense, it is this premium that is being harvested in the trading strategy.

As described in Section 4, the roll-yield is caused by the rolling of futures contracts necessary when constructing the ETPs. If the total return of the futures is equal to the spot return plus the roll-yield, the roll-yield can be extracted by shorting (buying) the futures and buying (shorting) the underlying spot when the market is in contango (backwardation). However, because it is unfeasible to trade the VIX index directly, the usual futures pricing relationship based on cash-and-carry arbitrage does not hold. However, it is not necessary to use the spot itself, as long as the instrument used has a high correlation with the spot. As mentioned in Section 2, the S&P 500 has a high negative correlation with the

VIX. Hence, an instrument with the S&P 500 as the underlying can be used.

To implement the trading strategy the following ETPs are used. SPDR S&P 500 Trust ETF (SPY) and ProShares Short S&P 500 ETF (SH) provide long and short exposure to the S&P 500, respectively.[8] iPath S&P 500 VIX ST Futures ETN (VXX) and VelocityShares Daily Inverse VIX ST ETN (XIV) provide long and short exposure to short-term VIX futures, respectively. In other words, when the market is in contango, XIV and SH are bought, and when the market is in backwardation, VXX and SPY are bought.

First, the relative difference between the front month VIX futures and spot VIX is calculated. This is defined as the *relative basis*, R^B. If it is positive and exceeds a certain upper buy-threshold B^U set by the trader, a position is opened by buying the XIV (short volatility exposure). The XIV position is hedged by buying SH (short S&P 500 exposure). The size of the hedge is also determined by the trader, depending on her risk aversion. This position will make money as long as the market stays in contango, and the VIX futures term structure does not shift downward. The position is closed when the relative basis falls below an upper sell-threshold, S^U, which may be set equal to, or lower than the buy-threshold. A reason why one might want the upper sell-threshold lower than the upper buy-threshold is that the relative basis might oscillate closely around the buy-threshold. This will cause consecutive "false" buy and sell signals. The same strategy applies when the relative difference between the front month VIX futures and spot VIX is negative and falls below a certain lower buy-threshold, B^L. Then a position is opened where the trader buys VXX (long volatility exposure) and hedges part of the position by also buying SPY (long S&P 500 index ETF). The position is closed when the relative difference between the front month VIX futures contract and the spot VIX rises above the lower sell-threshold S^L. The upper and lower buy and sell threshold are illustrated in Figure 7.

The time series used to test the trading strategy contain the closing prices of the ETPs and spot VIX, and the settle prices after closing of the front month VIX futures. Since the strategy involves changing positions right before the markets close, it is assumed that the prices of the ETPs do not change much right before closing, and that the price of the front month futures contract right before closing is close to the settle price. In periods with no trading, it is assumed that no interests are earned.

[8]Prospectus for these products are readily available on www.spdr.com and www.proshares.com

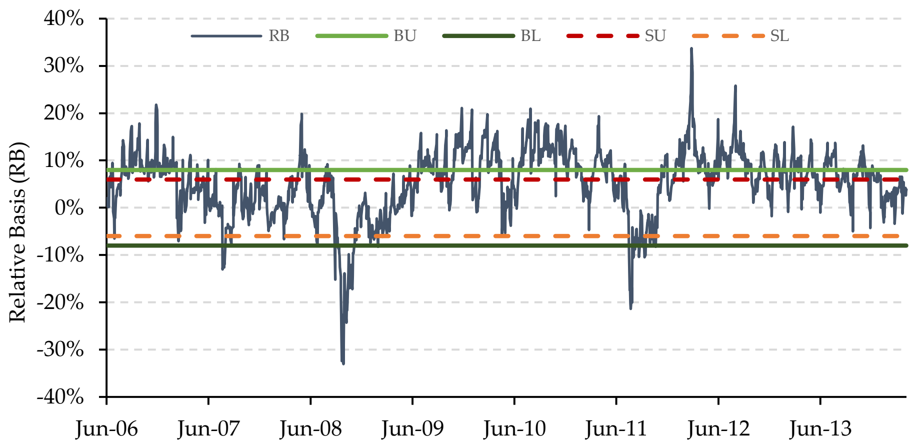

Figure 7: *Relative basis throughout the period, and the upper and lower buy and sell thresholds.*

In order to take into account trading costs and spread, buying and selling prices are calculated as follows:

$$\begin{aligned} P^{BUY} &= P^{close}(1 + spread)(1 + fee) \\ P^{SELL} &= P^{close}(1 - spread)(1 - fee) \end{aligned} \tag{16}$$

where *fee* represents a percentage paid per trade, and *spread* is expressed as a percentage of the closing price. These two terms are important as they might influence the performance of the trading strategy substantially. Ideally, these two terms should have been expressed in dollar terms, but since this analysis has not considered the splits and reverse splits performed on the VIX ETPs on a regular basis, this has not been possible. In practice, the fee paid when opening positions will vary depending on the price of the ETPs bought. The same challenge applies to estimating the average spread in percentage terms.

The brokerage fee is set to 0.20%. Several brokerages provide access to US equity markets for fees lower than this. [9] The bid-ask spread is based on historical average and is set to 0.15%.[10]

MATLAB is used to model the performance of the trading strategy. The flowchart of the algorithm is shown in Figure 8. Right before the end of each trading day, the relative basis R^B, i.e. the percentage difference between the price of the front month VIX futures contract and VIX spot is calculated. If the relative basis is above (below) an upper (lower) buy threshold, B^U (B^L) determined by the trader, it indicates that the market is in contango (backwardation) and that

[9] See for instance http://www.netfonds.no/priser.php and https://www.nordnet.no/mux/web/nordnet/ALLcourtage.html

[10] See for instance http://www.cboe.com/rmc/2013/Day3-Session-1ACross.pdf

one should hold XIV (VXX) and hedge with SH (SPY).

First it is checked whether any positions are open. If not, the cash on hand is used to buy XIV (VXX) and SH (SPY) with the appropriate weights. If a position in XIV (VXX) and SH (SPY) is already open, nothing is done and the position is held through the next trading day. If a position in VXX (XIV) and SPY (SH) is held, this is closed and a position in XIV and SH (VXX and SPY) is opened. If the relative basis lies within the interval of the lower sell and upper sell, S^L and S^U thresholds (also determined by the trader), it is checked whether there are any open positions. If not, the cash on hand is kept until the end of the next trading day, and if there are any open positions these are closed. One then waits until right before the end of the next trading day, and repeats the process.

The thresholds described above must satisfy the following conditions:

$$\begin{aligned} |B^L| &\geq |S^L| \\ |B^U| &\geq |S^U| \\ S^U &\geq B^L \\ S^L &\leq B^U \end{aligned} \tag{17}$$

The first two conditions in Equation 17 imply that the magnitude of the sell-threshold must be less than or equal to the buy-threshold. The last two conditions must be met in order to avoid having open positions that are betting on opposite directions of the futures curve.

Table 10 reports the results of the trading strategies with different amounts of the ETP position hedged with the S&P 500 direct and inverse ETPs. The values in the column *Hedge* report the share of total capital allocated to the hedge (SPY or SH). Surprisingly, the strategy that performs the best is the unhedged. This indicates that the relative basis does a good job in predicting subsequent price moves. By only switching the position between XIV and VXX, the trader would have earned an annualized return of 69% with a yearly volatility of 39%. The strategy proved itself to be robust to transaction costs and spread. One reason for this might be that the average duration of each trade reported in the column *Avg. dur.* is 18 trading days. This equates to 106 changes in position over the whole period. Figure 9 shows the portfolio value for two of the variants of the strategy.

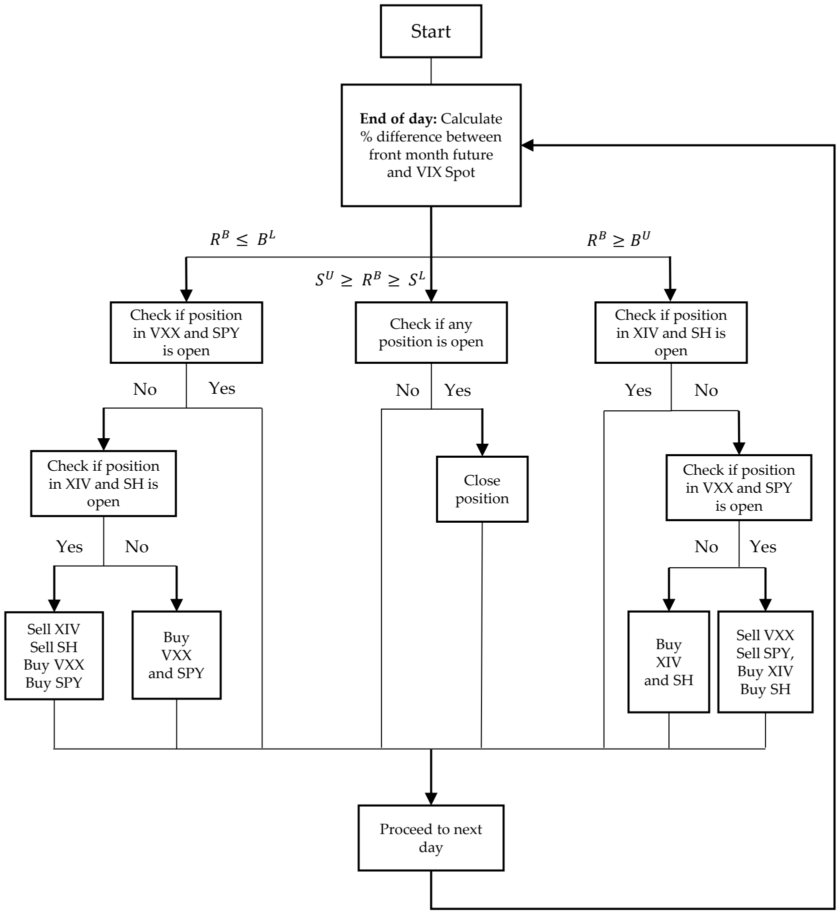

Figure 8: *Flowchart illustrating the trading strategy algorithm.*

Table 10: *Results of the trading strategy when the following buy and sell conditions are set:* $B^L = -8\%$, $B^U = 8\%$, $S^L = -6\%$, $S^U = 6\%$, $fee = 0.20\%$ *and spread* $= 0.15\%$. *The total number of changes in position is 106.*

Hedge ratio	Avg. return per trade	Winners/ losers	Avg. no. holding days	Total return	Annualized return	Volatility	Sharpe ratio	Sortino ratio	Largest drawdown
0%	5.24%	53/53	18	5919%	69%	39%	2.11	3.17	-24.5%
10%	4.64%	53/53	18	4027%	61%	35%	2.02	3.04	-22.3%
20%	4.04%	52/54	18	2668%	53%	31%	1.94	2.91	-19.9%
30%	3.43%	50/56	18	1711%	45%	27%	1.86	2.78	-17.5%
40%	2.81%	52/54	18	1052%	37%	23%	1.77	2.64	-14.9%
50%	2.19%	50/56	18	610%	28%	18%	1.66	2.48	-12.2%
60%	1.56%	46/60	18	322%	20%	14%	1.53	2.27	-9.3%
70%	0.93%	42/64	18	140%	12%	10%	1.26	1.89	-6.3%
80%	0.28%	34/72	18	30%	3.0%	7.0%	0.51	0.77	-3.8%
90%	-0.37%	28/78	18	-33%	-5.0%	9.0%	-0.51	-0.73	-6.0%
100%	-1.03%	35/71	18	-69%	-14%	16%	-0.82	-1.14	-9.8%

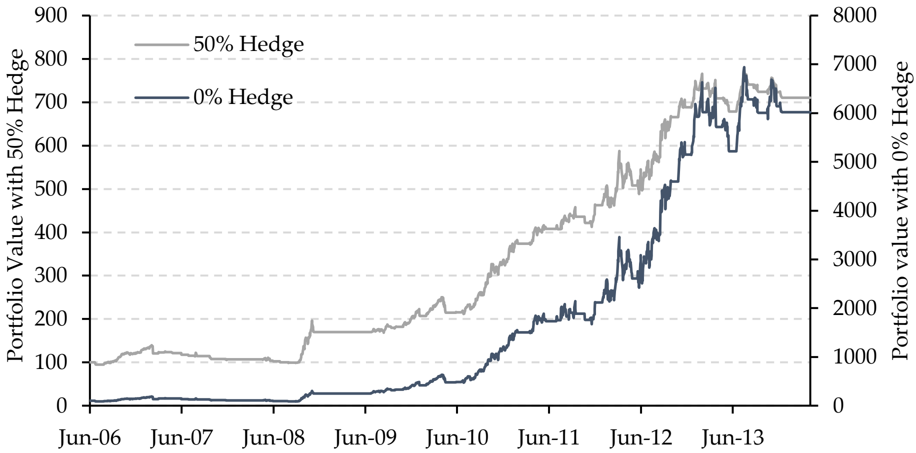

Figure 9: *Performance of the trading strategy with 0% and 50% hedge.*

7. Hedging Capabilities - Black Swan Hunting

This section investigates the effect of including a VIX ETP in a portfolio tracking the S&P 500 index. As discussed in Section 2, the VIX usually spikes during market drawdowns. Hence, the VIX has a high negative correlation with the S&P 500 and exposure to VIX ETPs might mitigate the effects of tail events.

Table 11 reports the correlation between the returns of the S&P 500 index and the VIX ETP returns. As expected, all the direct VIX ETPs are strongly negatively correlated with the S&P 500, which indicates that they might be suitable instruments for hedging a portfolio tracking the S&P 500.

Table 11: *Correlation between returns. 21 June 2006 to 25 April 2014*

	VXX	VIXY	VXZ	TVIX	UVXY
S&P 500	-0.78	-0.78	-0.76	-0.78	-0.78

Large corrections and market crashes occur relatively seldom, and are characterized by a sudden and dramatic decline in prices. When they do occur they can cause significant losses to even the most well diversified equity portfolio. The reason for this is that during a crisis, the systematic risk swamps the unsystematic risk. For instance, Szado (2009) shows that the increased correlations among diverse asset classes in the latter half of 2008 generated significant losses for many investors who had previously considered themselves well diversified.

VIX ETP positions might therefore provide diversification benefits to an equity portfolio. However, hedging a portfolio with VIX ETPs pose several challenges. Firstly, direct ETPs experience extremely high returns in periods with large market drawdowns, but these periods are short lived and occur seldom. The value of the position will therefore be eroded between each occurrence, as explained in Section 4.

Secondly, choosing the most suitable ETP for hedging is not straightforward. Lastly, determining the optimal hedge ratio by using for instance modern portfolio theory is challenging. Since the direct ETPs on average have very high negative expected returns, many approaches will suggest that no capital should be allocated to the VIX ETPs. Although this might indicate that VIX ETPs are not suitable for hedging any portfolio, part of this study is to investigate how the hedge performs in different market regimes.

Figure 10 reports the returns of the different VIX ETPs against the returns of S&P 500. All ETPs show signs of a linear relationship in the center of the distribution, with some non-linearity in the tails.

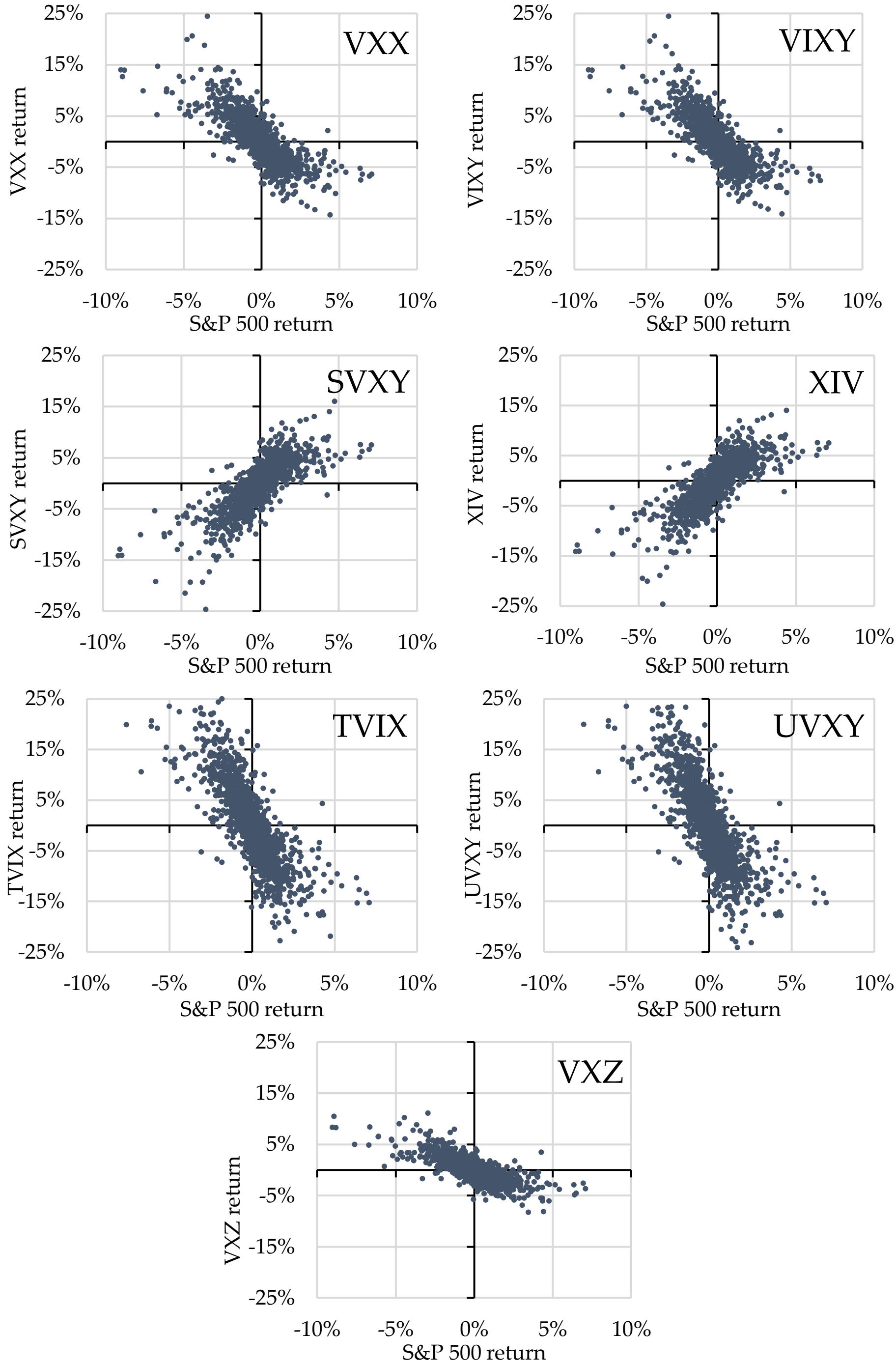

Figure 10: *Scatterplot of the various VIX ETP returns versus S&P 500 returns from 22 Jun. 2006 to 25 Apr. 2014.*

Table 12 reports the value at risk (VaR) and conditional value at risk (CVaR) in the upper and lower tails for the VIX ETPs and S&P 500 index. The leveraged ETPs (TVIX, UVXY) have the most extreme returns in both ends of the distribution. As expected, at the 1% level the magnitude of the returns are larger in the upper tails than in the lower tails for the direct VIX ETPs. However, the fact that the returns in the lower tails are still quite large is a bit surprising. One reason for this might be that volatility has a tendency to decrease rather quickly after a spike, causing the direct VIX ETPs to decrease in value just as quickly.

Table 12: *Value at Risk and Conditional Value at Risk for the ETPs used in this study, as well as for the S&P500.*

Pecentile	VXX	VIXY	VXZ	TVIX	UVXY	XIV	SVXY	S&P 500
Lower Tail VaR								
1%	-8.64%	-8.69%	-4.89%	-17.17%	-17.38%	-11.87%	-11.66%	-4.48%
5%	-5.78%	-5.80%	-3.11%	-11.05%	-11.63%	-6.76%	-6.86%	-2.22%
10%	-4.30%	-4.31%	-2.14%	-8.34%	-8.76%	-4.63%	-4.70%	-1.39%
Upper Tail VaR								
1%	11.78%	11.68%	6.05%	22.52%	23.34%	8.52%	8.59%	4.08%
5%	6.67%	6.78%	3.34%	13.04%	13.64%	5.76%	5.81%	1.95%
10%	4.69%	4.65%	2.33%	8.88%	9.32%	4.31%	4.31%	1.35%
Lower Tail CVaR								
1%	-10.41%	-10.37%	-5.91%	-21.33%	-21.58%	-15.18%	-15.59%	-6.04%
5%	-7.51%	-7.53%	-4.12%	-14.87%	-15.23%	-9.84%	-9.98%	-3.52%
10%	-6.23%	-6.24%	-3.33%	-12.20%	-12.59%	-7.75%	-7.85%	-2.63%
Upper Tail CVaR								
1%	14.91%	15.13%	8.09%	29.80%	31.02%	10.24%	10.68%	5.80%
5%	9.77%	9.85%	5.08%	19.09%	19.90%	7.43%	7.57%	3.32%
10%	7.69%	7.75%	3.91%	14.90%	15.63%	6.18%	6.25%	2.45%

In order to select which ETP to use as the hedge, regressions are performed with the ETP returns on the returns of the S&P 500. The preferred ETP is chosen by looking at the estimated coefficients. To minimize negative drag on the portfolio, the ETP with the largest constant term is chosen with a hedge ratio indicated by the estimated β. The results are reported in Table 13. The alphas are negative for all the ETPs except for that of the VXZ, which is the ETP based on the mid-term VIX futures index. However, by looking at the t-statistics of the reported alphas, it cannot be concluded that none of them are significantly different from zero. However, because the VXZ is based on longer term futures (where the term structure is flatter), it suffers less from time-decay. This is consistent with the findings reported in Section 3. Also, Figure 4 shows that the VXZ is the direct ETP with the lowest decay.

The hedging ability of the chosen VIX ETPs is evaluated from the viewpoint of a retail investor holding a portfolio tracking the S&P 500 index. Because the investor is risk averse, she wants to protect her holdings from tail events by allocating a proportion of her assets to VXZ. The regression results in Table 13 indicate an initial holding of approximately 35%. Based on this, three simple strategies are proposed and compared to holding an unhedged portfolio of S&P 500.

Table 13: *Regression results indicating that VXZ constitues the best hedging instrument among the VIX ETPs considered.* $rSP500_t = \alpha + \beta \times rETP_t + u_t$.

ETP	R^2	$Adj.R^2$	α	t-stat α	β	t-stat β
VXX	0.60	0.60	-0.015%	-0.74	-0.286	-54.82
VIXY	0.60	0.60	-0.015%	-0.73	-0.286	-54.93
VXZ	0.58	0.58	0.012%	0.61	-0.540	-52.18
UVXY	0.61	0.61	-0.016%	-0.82	-0.142	-55.14
TVIX	0.61	0.61	-0.022%	-1.11	-0.148	-55.40

The first strategy is called *The Passive Investor*. With this strategy, a portfolio of S&P 500 and VXZ is set up at the appropriate weights, and never rebalanced. This implies that the amount of portfolio protection will vary as time passes, depending on the performance of the VXZ and the S&P 500. With the poor performance of the VXZ in times with low volatility, it is expected that the amount of portfolio protection will decline as the price of VXZ declines, and spike when volatility rises. The second strategy is called *The Semi-Passive*. This strategy differs from the first one in that the portfolio weights are rebalanced every 90 trading days. This number is chosen arbitrarily and implies that the portfolio will be rebalanced roughly every four and a half months. It is expected that the weight of VXZ in the portfolio will diverge from the initial 35% between each rebalancing point. The final strategy is called *The Active Investor*. In this strategy, the portfolio is rebalanced at the end of every day, to keep the weight of VXZ close to 35%. Note however that costs of rebalancing the portfolios have been neglected. Incurring these costs would have affected the performance of all hedgeing strategies negatively, especially *The Active Investor*. However, later in this section it is shown that this hedging strategy performs quite poorly, even without transaction costs.

The results of the strategies are reported in Table 14 and compared to the performance of an unhedged investment in the S&P 500 index. Figure 11 reports the evolution of the portfolio with different strategies compared with S&P 500. Figure 12 show how the weight of VXZ in the portfolio changes through time.

In terms of hedging against tail events, all strategies perform as intended. For instance, when the S&P 500 fell 37.6% in 2008, the portfolios of the Passive Investor, Semi-Passive and Active Investor hedge strategies returned 8%, 3.8% and -3%, respectively. However, during the course of 2013 the market gained 26.4% while the strategies returned 0%, -0.2% and -1.6%. So the VXZ does a good job in removing the large negative returns during market drawdowns, but the cost of being hedged in upward moving markets removes the potential positive returns of the portfolio.

Table 14: *Results and returns statistics of three simple hedging strategies, as well as an unhedged postion.*

	The Passive	The Semi-Passive	The Active	Unhedged
Initial ETP holdings	35%	35%	35%	0%
Rebalancing	Never	Every 90 days	Every day	Never
Std. dev.	0.58%	0.53%	0.59%	1.41%
Avg. Rtn.	0.006%	0.012%	0.008%	0.030%
Max	3.845%	3.884%	5.915%	11.580%
Min	-3.187%	-2.839%	-3.466%	-9.035%
Median	0.000%	-0.005%	-0.010%	0.082%
Sharpe Ratio	0.0105	0.0225	0.0136	0.0213
Sortino Ratio	0.0153	0.0335	0.0207	0.0298
Correlation w/ S&P 500	-0.06	0.42	0.65	1.00
Total return	9%	29%	13%	49%
Annualized return	1.1%	3.3%	1.6%	5.1%

Looking at Figure 12, it is clear that the weight of VXZ for the *Passive Investor* strategy, has quite extreme values in both ends. The weight of VXZ rises rapidly during the onset of the financial crisis. It then decreases steadily from mid 2008 until the end of the period, only interrupted by the volatile periods in 2010 and 2011. It is clear that, given enough time before a major market crisis, the value of the VXZ holdings will eventually decline towards zero. By then, the protection it offers to the portfolio will be negligible. Thus the performance of this strategy is biased by when the original portfolio is set up, and how long time period one considers.

The weight of VXZ in the portfolio *The Semi-Passive* is corrected back to 35% of the total portfolio value every 90 trading days. During the fall of 2008, the value of the holdings in VXZ rose above 60% of total portfolio value. This drastic shift in portfolio weights is caused by two factors. First, as the S&P 500 index falls rapidly, so does its value in the portfolio. In turn, this decreases its portfolio weight. The decreasing weight of the S&P 500 in the portfolio implies that the weight of VXZ increases. However, since the value of the VXZ holdings rises due to the rising price of VXZ, its weight in the portfolio increases even further.

By comparing Figures 11 and 12, it is clear that the gain in the *Semi-Passive* portfolio is biased by the fact that the portfolio was rebalanced right before this large and sudden market decline. Again, it is seen that the results are biased by when the original portfolio is set up. Since the final strategy, *The Active Investor*, involves rebalancing the portfolio every day, it is not affected by the same bias as the other two. When investigating hedging strategies, the starting point of the time series and time between rebalancing the portfolio should be considered carefully. Especially if the time between portofolio rebalancing is constant and not close to the increments of the observed prices.

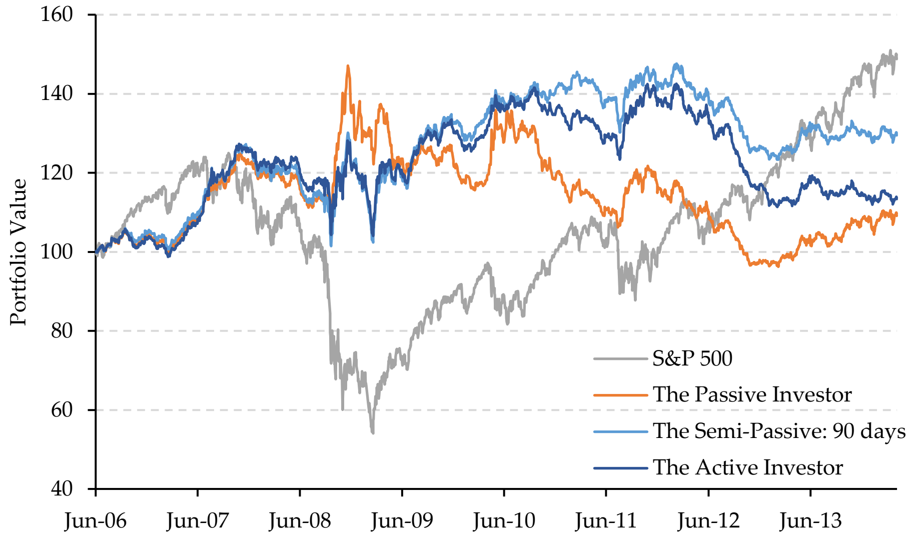

Figure 11: *The performance of the Passive, Semi-Passive and Active hedging strategies compared with the S&P 500.*

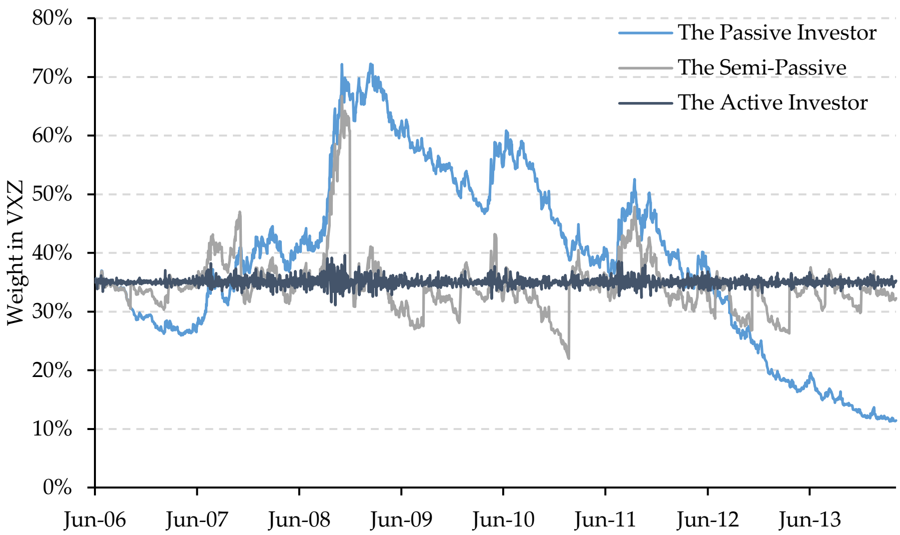

Figure 12: *The evolution of weights in VXZ for the three hedging strategies.*

Interestingly, the Semi-Passive hedging strategy indicates some degree of increased risk-adjusted performance. Table 14 reports that this strategy has better Sharpe- and Sortino-ratios than the unhedged position. The maximum draw-down is also quite high for the unhedged position relative to the Semi-Passive (-9.04% vs. -2.83%).

To further evaluate the performance of the Semi-Passive hedging strategy, a sensitivity analysis is performed to see how the risk-adjusted return varies when the portion of VXZ is changed. The risk-adjusted returns are measured by calculating the Sharpe Ratio and the average return per tail risk (average daily return divided by 1% daily VaR) for different holdings. Then the same is done for the VXX, also using the Semi-Passive strategy. Strictly for illustrative purposes, the same analysis is performed when pretending as if the spot VIX could actually have been bought and sold just like any other VIX ETP.

Since the financial crisis following the Lehman Brothers bankruptcy was particularly severe, this may skew the results in favor of ETP hedging. Because of this, the same calculations are done with the real prices starting 20 Feb 2009 for the VXZ and 20 Jan 2009 for the VXX and spot VIX. This way the major VIX spike in September 2008 is filtered out, but the period still covers the big market drawdowns during the European sovereign debt crisis in May 2010 and the US credit downgrade in August 2011. The results can be seen in Figure 13.

When evaluating the whole period, icluding a small portion of VXZ in the portfolio would have improved its risk adjusted returns when measured both with the Sharpe Ratio and the average return per 1% daily VaR. As for including the VXX in the portfolio, it would not have improved its Sharpe Ratio. However, its average return per 1% daily VaR would have improved slightly. When looking at the period after the inception of each ETP, after the 2008 crisis, it is clear that the inclusion of VIX ETPs in the portfolio would have offered no improvement in the risk adjusted returns.

The two top plots in Figure 13 reports the results when it is pretended that hedging with spot VIX is a viable option. In terms of the Sharpe Ratio, it is clear that the inclusion of the spot VIX would have offered a considerable improvement in risk adjusted returns, compared with VXZ and VXX. In terms of the average return per 1% daily VaR, some improvement would have been offered when evaluating the whole period. However, when evaluating the period after the 2008 crisis, this ratio indicates that no improvement is offered when including the spot VIX in the portfolio.

The results indicate that VIX ETPs may have some potential in hedging against large drawdowns on a portfolio. However, holding VIX ETPs at all times creates a substantial drag on a portfolio. The difference between the VIX ETPs and the VIX index are also indicated. Had one been able to buy and sell the actual VIX index, the results indicate that implementing a simple hedging strategy would have improved the risk adjusted returns of the portfolio considerably. Further, since VIX ETPs prove themselves useful during periods with large market declines, successfully implementing a *tactical hedging* strategy will smooth out the bumps in the returns of a portfolio over time. Yet tactical hedging is in some way an oxymoron. Tail events are extremely hard to predict, so to be secured from the substantial draw-down a tail event incurs on a portfolio, one should in fact at all times hold some protection. However, this protection is so expensive that it takes away much of the portfolio upside potential when markets do not crash.

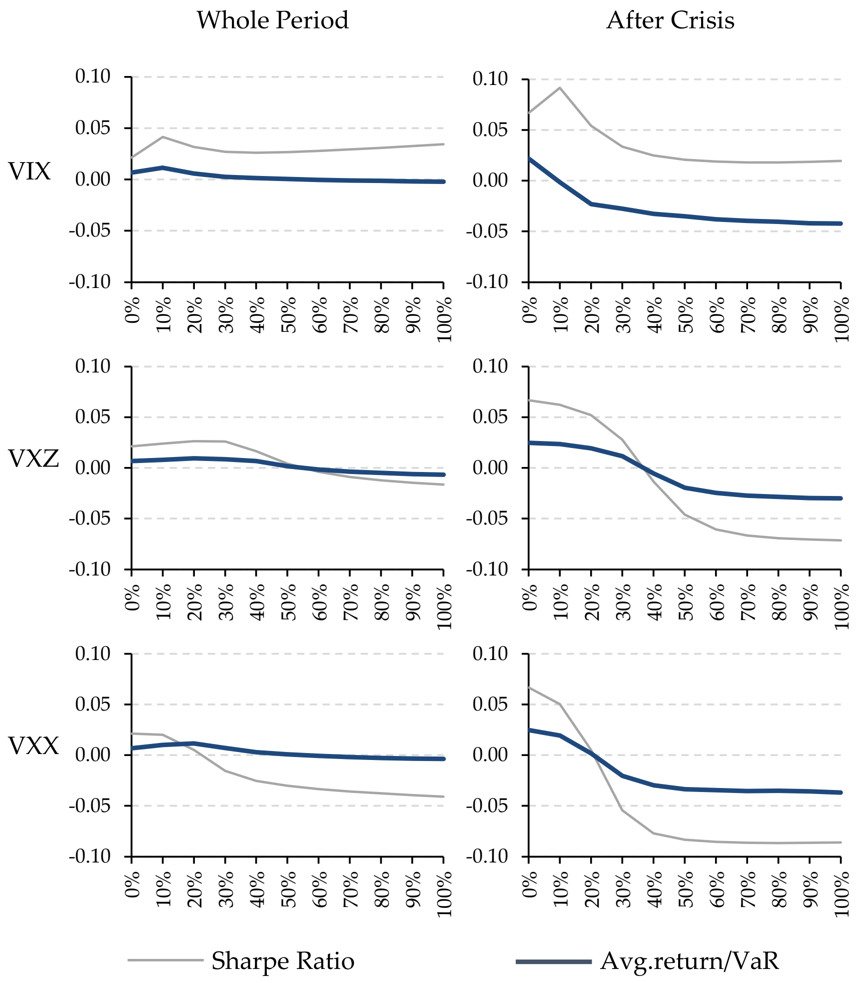

Figure 13: *Risk adjusted performance measures for different weights of Spot VIX and two ETPs, during the whole sample period, and the period after inception of each ETP. The X-axis represents the relative amount of VIX ETPs (or spot VIX) included in the portfolio, the Y-axis gives the value of the ratios.*

To justify the use of VIX ETPs as a hedging tool, one should possess some ability or tool for prediction. And if one is able to successfully predict extreme tail events, hedging for protection would not be necessary.

These results confirm the findings of Alexander and Korovilas (2012a) which show that single positions on direct VIX futures ETNs of any maturity could only provide a hedge of equity exposure during the first few months of a great crisis. Also, for diversification to be profitable outside of a period of excessive volatility, the investor must have precise forecasts of VIX futures returns.

8. Conclusion

This paper investigates VIX exchange traded products with the purpose of revealing their performance, price discovery and usefulness as hedging and trading instruments. It is found that all VIX ETPs do what they promise quite well, namely to track direct or inverse VIX short or medium term futures indices. Direct and inverse VIX ETPs mimic being long and short the underlying indices, respectively.

By measuring the price discovery between the different ETPs it is found that they impound information at different speeds. The VXX impounds information faster than the VIXY, which one might expect since it has higher volume and has existed longer. Similarly for the most popular inverse ETPs, the slightly older and more traded XIV impounds information somewhat faster than SVXY. However, for the leveraged ETPs, TVIX and UVXY, the younger and less traded UVXY is leading the price formation process.

The VIX futures term structure makes direct positions in VIX ETPs unsuitable for buy-and-hold positions due to the time-decay it usually causes. For instance, a position in the most popular direct VIX ETP, the VXX, would have lost over 99.5% of its value if opened in January 2009 and held to April 2014. The returns of direct ETPs are particularly bad when the market is in contango. During periods when the term structure is in backwardation, the direct VIX ETPs perform well. But, backwardation occurs seldom, and when it does, it is short-lived. Hence, ETPs which offer long volatility exposure are too expensive to be treated as buy-and-hold investments.

However, time-decay is not a feature exclusive to VIX ETPs. The presence of volatility risk premiums will likely impose time-decay on investors looking for volatility exposure using other products too, such as put options, variance swaps, volatility futures, or other related products. Insurance comes at a price.

Furthermore, the time-decaying feature of the VIX ETPs gives rise to a trading opportunity by buying inverse VIX ETPs (effectively obtaining short exposure to VIX futures). By buying inverse (direct) VIX ETPs when the term structure is sufficiently in contango (backwardation), the trading strategy proves itself highly profitable and robust to transaction costs. With conservative assumptions about trading costs, the strategy yielded annual returns of over 60% and Sharpe Ratio above 2. The strategy confirms previous findings of Simon and Campasano (2014) and contributes additional evidence that the strategy is profitable also when using VIX ETPs instead of futures. However, the strategy might be prone to in-sample biases.

The present study provides additional evidence with respect to how VIX ETPs perform as hedging tools. Despite using VIX ETPs based on mid term futures (which suffer less from time-decay), VIX ETPs seem to offer little or no improvement in risk-adjusted return, even when the sample period covers severe volatility spikes and market drawdowns.

Future research should assess the price discovery process among the VIX ETPs when more historical data is available. In addition, researching the price discovery process between VIX ETP and futures prices would be interesting. The rising popularity of VIX ETPs and their connection with VIX futures has probably affected the market microstructure. This might be revealed by a price discovery analysis. Furthermore, evidence of price leadership between one of the two might give rise to profitable trading strategies. In terms of the trading strategy investigated in this paper, the upper and lower thresholds were set more or less arbitrarily. Future research should investigate if an optimal threshold for buying and selling exists, and whether the thresholds are constant or vary with time. It is also suggested that VIX ETPs hedging ability is investigated further, by applying more sophisticated methods and tactical hedging strategies.

References

Carol Alexander. *Market Risk Analysis*. John Wiley & Sons Ltd., West Sussex PO10 8SQ, England, 2008a.

Carol Alexander. Market risk analysis: Practical financial econometrics, 2, 2008b.

Carol Alexander and Dimitris Korovilas. Diversification of equity with vix futures: Personal views and skewness preference. *Available at SSRN*, 2012a.

Carol Alexander and Dimitris Korovilas. Understanding etns on vix futures. *Available at SSRN 2043061*, 2012b.

Ivan O Asensio. The vix-vix futures puzzle. 2013.

Richard T. Baillie, G. Geoffrey Bootha, Yiuman Tseb, and Tatyana Zabotinac. Price discovery and common factor models. *Journal of Financial Markets*, 5: 309–321, 2002.

Menachem Brenner and Dan Galai. New financial instruments for hedging changes in volatility. *Financial Analysts Journal*, pages 61–65, 1989.

Ernie Chan. *Algorithmic Trading: Winning Strategies and Their Rationale*, volume 625. John Wiley & Sons, 2013.

Christopher R. Cole. Volatility: The market price of uncertainty. *CFA Institute Conference Proceedings Quarterly*, 31(1):36–47, 2014. doi: 10.2469/cp.v31.n1.1.

Geng Deng, Craig McCann, and Olivia Wang. Are vix futures etps effective hedges? *The Journal of Index Investing*, 3(3):35–48, 2012.

Robert F. Engle and C. W. J. Granger. Co-integration and error correction: Representation, estimation, and testing. *Econometrica*, 55:251–276, 1987.

Bjørn Eraker and Yue Wu. Explaining the negative returns to vix futures and etns: An equilibrium approach. *Available at SSRN 2340070*, 2013.

Jesus Gonzalo and Clive Granger. Estimation of common long-memory components in cointegrated systems. *Journal of Business & Economic Statistics*, 13: 27–35, 1995.

C. W. J. Granger. Some properties of time series data and their use in econometric model specification. *Journal of Econometrics*, 16:121–130, 1981.

Joel Hasbrouck. One security, many markets: Determining the contributions to price discovery. *The Journal of Finance*, 50:1175–1199, 1995.

T. Schwarz and A. Szakmary. Price discovery in petroleum markets: Arbitrage, cointegraion and the time interval of analysis. *Journal of Futures Markets*, 14: 147–167, 1994.

David P Simon and Jim Campasano. The vix futures basis: Evidence and trading strategies. *The Journal of Derivatives*, 21(3):54–69, 2014. doi: 10.3905/jod.2014.21.3.054#sthash.BtDsXIis.hh1PQSqh.dpuf.

Euan Sinclair. *Volatility trading*. John Wiley & Sons, 2013.

Edward Szado. Vix futures and options: A case study of portfolio diversification during the 2008 financial crisis. *The Journal of Alternative Investments*, 12(2): 68–85, 2009.

Erik Theissen. Price discovery in floor and screen trading systems. *Journal of Empirical Finance*, 9:455–474, 2002.

Robert E Whaley. Derivatives on market volatility: Hedging tools long overdue. *The journal of Derivatives*, 1(1):71–84, 1993.

Robert E Whaley. Trading volatility: At what cost? *The Journal of Portfolio Management*, 40(1):95–108, 2013.