Hedge Fund Manager-Investor Conflicts of Interest

A Numerical Analysis with Loss-Aversion

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NTNU
Det skapende universitet

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- uttak av masteroppgave

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A Numerical Analysis with Loss-Aversion

Frederic Asheim

June 2, 2014

Preface

This thesis is the conclusion of the author's Master of Science degree in Industrial Economics and Technology Management at the Norwegian University of Science and Technology. It falls within the Group of Financial Engineering.

The text has been typeset in LaTeX. Simple quantitative analysis has been done with Microsoft Excel 2013, while more advanced numerical computations and simulations have been performed in MATLAB R2013a. Graphs and illustrations have been prepared in MATLAB R2013a. The thesis is written as an academic research paper in the article format. It therefore relies heavily on referencing and does not elaborate on well-known theory.

I would like to thank my supervisor, Associate Professor Einar Belsom at the Department of Industrial Economics and Technology Management, for providing excellent academic guidance and enthusiasm.

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Abstract

This paper investigates the dynamically optimal risk-taking by a loss-averse hedge fund manager who takes the possibility of fund liquidation into account. To achieve this, I develop a novel two-reference point utility-model based on Prospect Theory, and employ it within a numerical discrete time framework. The manager portrays complex risk-taking that varies considerably with fund value, time, fee rates, managerial ownership and optimization horizon. In many cases, the manager pursues excessively high risk-levels relative to those preferred by a loss-averse investor. The extent and economic significance of the resulting manager-investor conflict of interest is assessed for varieties of fee rates and managerial ownership. I find that the incentive fee option is the chief root of misalignments, and suggest using managerial fund shares to resolve the conflict, as investors otherwise may suffer substantial losses. The incentive fee is estimated to considerable value given low managerial ownership rates, thereby adding further to investors' costs.

Sammendrag

Denne artikkelen estimerer den dynamisk optimale risikotakingen til en tapsavers hedgefondforvalter som tar hensyn til muligheten for fondslikvidasjon. Jeg oppnår dette ved å utvikle en ny nyttemodell med to referansepunkter basert på prospektteori, og å anvende den i et numerisk tidsdiskret rammeverk. Forvalteren viser kompleks risikotaking som varierer betydelig med fondsverdien, tiden, honorarrater, forvalterens andel og optimeringshorisonten. I mange tilfeller forfølger forvalteren overdrevent høye risikonivåer i forhold til en tapsavers investor. Omfanget og den økonomiske signifikansen til den resulterende interessekonflikten evalueres for variasjoner av honorarrater og fondsandeler. Insentivhonoraret utpekes som konfliktens hovedkilde, og jeg foreslår å bruke fondsandeler som dempende faktor for å unngå reelle tap for investorer. Videre estimeres insentivhonoraret til å være av betydelig verdi gitt lave fondsandeler, noe som påfører investorer ytterligere kostnader.

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1 Introduction

> *"If you want to predict how people will behave, you only have to look at their incentives."*
>
> - Charlie Munger

In some categories of capital management, it is customary to include a so called incentive fee in the contract between manager and investor. Hedge funds represent one branch that utilize such contracts, which are often considered state of the art.

Incentive fees are essentially designed as periodical European call options on assets under management (AUM). In addition, hedge funds often require a periodical management fee amounting to a certain fraction of AUM.[1] The typical fee rates for the management- and incentive fee is 2 and 20 %, respectively. This fee structure is thus often dubbed "2/20", or "two and twenty". The underlying benchmark of the incentive fee frequently follows a so-called high-water mark mechanism, but may also be a simple constant rate or follow a reference index.

The intuitive perspective of the incentive fee as a deliverer of superior performance is frequently used in marketing toward potential investors. However, there is also some concern tied to the incentive fee, as it may give the manager an incentive to set hazardously high portfolio risks. This view is partly founded on Markowitz-portfolio theory (e.g. Markowitz, 1952), which implies that the manager is forced to set a portfolio risk that at least exceeds that of the benchmark in order to get the incentive fee.[2] This risk-level may in turn be higher than investors' preferred level.[3] Another argument supporting these concerns is the fact that the incentive fee has a strongly convex option gamma for relatively short time horizons, which intuitively should result in a distinctively non-linear managerial risk-taking.[4]

Much research on the topic of risk-taking in hedge funds (e.g. Carpenter, 2000; Goetzmann et al., 2003; Kouwenberg and Ziemba, 2007; Hodder and Jackwerth, 2007; Basak et al., 2007; Agarwal et al., 2009; Motaze, 2013) has agreed that there exists considerable misalignment of interest between manager and investors. In-

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[1] The actual payments are done monthly, quarterly or weekly at scaled rates.

[2] Strong-form market efficiency is inherently assumed in this deduction.

[3] Given that the investor is only looking to replicate the benchmark, it *must* be so. In the case of hedge funds however, that is not always the goal.

[4] Seeing as the option is a European call, the convexity of its gamma is generally very high for short time horizons, but declines with longer time horizons.

deed, concerns of this kind appear to be the foundation of the newly imposed bonus cap on risk-taking banking employees in the European Union (Kolinska, 2013).

This paper presents a theoretical analysis of the incentives that act upon a hedge fund manager and the resulting dynamic pattern of risk-taking. This problem bears similarities to the optimal asset allocation problem, as pioneered by Merton (1969), but is complicated by a range of necessary model extensions; most prominently the incentive option. The paper further evaluates the extent to which there exists manager-investor conflicts of interest, and investigates possible solutions to such misalignments.

Carpenter (2000)'s work is seminal to the research area. She uses a continuous-time martingale framework to analyse the risk-taking of a fund manager subject to the hyperbolic absolute risk-aversion (HARA) utility-function. The paper's main three findings are:

- The optimal risk taking for the manager is excessive compared to that of the HARA-investor when the incentive option is deeply out-of-the-money (OTM).

- The manager decreases the portfolio risk when the option is in-the-money (ITM) to a level *lower* than investors.

- The manager's optimal portfolio risk is negatively correlated with the incentive fee rate.

Furthermore, Carpenter accentuates the dynamic nature of managerial behaviour, which at any time depends on option moneyness. Following the third of said findings, a hedge fund manager could argue strongly in favour of high incentive fees in contract negotiations with investors, as this would increase alignment of interest. While Carpenter's model sheds light on several interesting aspects of incentive fees and yields some surprising findings, it also has a number of shortcomings. The paper's general model does not include the management fees popularly used in hedge fund's fee structures, nor does it take the fact that many hedge fund managers have considerable amounts of their own wealth invested in the fund into account.

Many subsequent papers extend on Carpenter (2000), and also find indications of conflicts of interest between hedge fund managers and investors through a broad array of methodologies and assumptions (e.g. Goetzmann et al., 2003; Kouwenberg and Ziemba, 2007; Hodder and Jackwerth, 2007; Basak et al., 2007; Agarwal et al.,

2009; Motaze, 2013). Yet there is major disagreement concerning in which situations managerial riskiness is *excessive* or *deficient* and the *severity* of the discrepancies. The recent papers of Panageas and Westerfield (2009) and Guasoni and Obloj (2013), represent the opposite side of the debate, as their methods suggest that hedge fund managers set constant portfolio risks in compliance with investor-preferences. Research outcomes consequently appear to be highly sensitive to the choice of assumptions and methods.

Hodder and Jackwerth (2007) make use of a discrete time-framework and flexible numerical methods to identify the optimal portfolio risk of a fund manager with constant relative risk aversion (CRRA).[5] The results describe a much more nuanced and varied behavioural pattern than most other research, which may be contributed to their unusual choice of methods. A manager who optimizes wealth at the end of a single fee evaluation period is found to set risk as follows:

- In the region immediately below the benchmark, risk is drastically increased in an attempt to get the option ITM. Similar propensities have been observed by Carpenter (2000); Kouwenberg and Ziemba (2007) and may be dubbed the break-even effect.

- Following an *increase* in moneyness beyond ATM, the optimal risk first plummets, and then stabilizes and flattens out as the option turns ITM. Intuitively, once the incentive option is quite safely ITM, the manager will try to minimize risk in order to retain the option fee. This is often called the lock-in effect.

- Following a *decrease* in moneyness quite far beneath ATM, the optimal risk first sees a drastic drop before it stabilizes and flattens out as the option turns OTM. This risk-averse behaviour is attributed to the manager's liquidation aversion, as implemented by Hodder and Jackwerth. In contrast, the models of Carpenter (2000); Kouwenberg and Ziemba (2007) do not include a mechanism of this kind, and their results therefore imply that the optimal risk level rises into theoretical infinity as moneyness decreases.

Kouwenberg and Ziemba (2007) extend on the work of Carpenter (2000) and Goetzmann et al. (2003), but due to their manager being described by loss-averseness, as defined by Prospect Theory (Kahneman and Tversky, 1979), rather than a risk-

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[5]Also known as isoelastic utility.

averse variant of von Neumann/Morgenstern utility theory[6], they find strongly
contradicting results: The two latter out of the three major findings of Carpen-
ter do not hold.[7] Their model indicates that the manager's optimal portfolio risk
is positively correlated with the incentive fee level, and that the manager's risk-
taking is excessive relative to the level preferred by investors for *all* states of op-
tion moneyness.

Another interesting finding by Kouwenberg and Ziemba is that hazardous risk-
taking can be greatly reduced by increasing managerial ownership in the fund.
They estimate that with a share of 30 % the behaviour of the manager will ap-
proximate that of a person operating a personal account.[8] On the other side of
the scale, they find incentives that pull the manager to extreme risk-taking when
he/she has a fund share below 10 %.

In addition to their theoretical analysis, Kouwenberg and Ziemba perform an em-
pirical study using a large European hedge fund database, which poses some in-
teresting observations. The relation between incentive fee rate and fund perfor-
mance (Sharpe ratio) is found to be negative. This stands in stark contrast to
the theoretical finding of Carpenter (2000). Additional results suggest that funds'
management fee rates are also negatively correlated with performance. With that,
the use of high management and incentive fees as selling points toward potential
investors appears unjustified.

The complexity of the topical problem makes a numerical approach particularly at-
tractive, as analytical solutions often require model simplifications that are detri-
mental to realism. One common such simplification, is to assume a continuous
time framework and fee accruements. My analysis of managerial risk-taking con-
versely employs a fundamental model based on Hodder and Jackwerth (2007),
using realistic discrete time portfolio revisions and fee accruements. The model is
used in combination with three different behavioural frameworks; CRRA-utility,
Prospect Theory (PT) (Kahneman and Tversky, 1979; Tversky and Kahneman,
1992) and a custom two-reference point (TRP) model based on Tversky and Kah-
neman (1992); Wang and Johnson (2012). Out of these, the latter is intended to
portray the most complete picture of managerial risk-taking, as it incorporates

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[6]Henceforth simply referred to as utility theory.

[7]In this text, "utility" will frequently be used irrespective of behavioural model, despite the popular
use of "value" in connection to Prospect Theory.

[8]Thus approximating the behaviour of his/her investors, given that they have the same *fundamental*
risk-preferences.

both loss-aversion and managerial aversion against fund liquidation. With that, the TRP model and its results represent the paper's foremost contribution to the literature. On one hand, it extends on Hodder and Jackwerth (2007), by taking the widely accepted shortcomings of utility theory into consideration by replacing it with the descriptively more accurate PT. On another hand, it extends on Kouwenberg and Ziemba (2007) by adding a time-dimension and managerial liquidation aversion to their analysis. In a separate part of the paper, I give a novel perspective to the assessment of a loss- and liquidation averse manager's behaviour by increasing his/her optimization horizon to span over *multiple* fee evaluation periods rather than the usual *single* period.[9]

Moreover, the paper presents an unprecedented focus on quantification of manager-investor interest discrepancies as a function of fee structure and managerial fund share varieties. This facilitates a deeper understanding of the underlying causes of misalignments, how they impact the investor and how they can be minimized within the prevalent contract framework. I give two different methods to measure the conflict of interest, each with its strengths and weaknesses. As it is useful for investors to know the costs of the incentive fee, the closing study estimates its value for a set of common assumptions.

The remaining part of the paper thus begins with the establishment of the fundamental numerical model used for most subsequent analysis. Second, each of the three behavioural models and their respective single-period horizon results are presented in sequence. Third, my approach for a multi-period assessment of risk-taking is introduced, followed by illustrative results using the TRP model. Fourth, the two methods for measuring manager-investor alignment are given, along with explanatory results. Fifth, the value of the incentive fee is estimated. Finally, I provide some concluding comments, including a summary of the most important results and implications. I also point out some promising perspectives for future investigations of the topical problem, and some alternative application areas for the model.

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[9]To my knowledge, the only papers giving an extensive review of multiple-period managerial risk-taking are Hodder and Jackwerth (2007); Panageas and Westerfield (2009); Guasoni and Obloj (2013); yet none of these take on the advancements of PT, which my paper does.

2   Fundamental Model Setup

To estimate the dynamic risk-taking of a hedge fund manager, I consider his/her optimal portfolio problem within a single evaluation period consisting of T market days. In finance literature, it is standard to assume 252 market days in a year. As performance is often evaluated quarterly, I set $T = 252/4 = 63$.[10] This and all subsequently introduced parameters are listed in Appendix B with their standard case assumptions. The chosen evaluation period length is merely of practical relevance, as testing shows it has insignificant effects on results.

The optimization problem will be solved using a discrete time model, with a single market day as the smallest time unit. This limits the manager to a maximum portfolio revision frequency of once per day.[11] Note also that the model does not take transaction costs into account, which in practice would lay considerable restrictions on the manager's scope of action.[12]

The manager's initial wealth is $W(0)$, and the initial size of assets under management (AUM, or fund value) is $F(0)$. Oftentimes, the manager will own a fraction of AUM, denoted $0 \le S \le 1$. Thus, external investors own the remaining fraction of assets $(1 - S)$. At terminal time T, the manager's own share in the fund will then amount to $SF(T)$. Furthermore, the annual and quarterly management fee rate is denoted $\alpha \ge 0$ and $\alpha_q \ge 0$, respectively. It is assumed for the rest of the paper that management fees are paid at the end of each quarter.[13] This yields a quarterly management fee amounting to $\alpha_q(1 - S)F(T)$. The annual and quarterly incentive fee rate is given as β and β_q, respectively. Since this fee essentially is a European call option on AUM with the benchmark $B(T)$ as strike, it adds $\beta_q(1 - S)\max\{0, F(T) - B(T)\}$ to the manager's wealth at terminal time T. Throughout this paper it will be assumed that the manager does not hedge his/her ownership share with any external personal portfolio. In fact, hedge fund contracts commonly forbid such behaviour. Assuming that the manager's external wealth grows at the risk-free rate R_q each quarter, it amounts to $Z \equiv [1 + R_q][W(0) - SF(0)]$ at terminal time T. For simplicity, this parameter is set to zero in the standard case. As will become clear later on, the manager's external wealth is irrelevant under

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[10]Annual evaluations are also common.

[11]However, the model allows this frequency to be changed readily. Experiments were done with higher frequencies, but were not found to yield significant changes in results.

[12]The upside of ignoring transaction costs is that it enables the examination of managerial intensions to remain untouched by practical obstacles.

[13]The frequency of payments is also commonly monthly or weekly.

the loss-aversion assumption.

Note that the use of a single benchmark $B(t)$ represents a deviation from reality, as most hedge funds use personal benchmarks for each investor. The benchmark used here should thus be conceptualized as an aggregate of all investor benchmarks.

The total terminal time wealth of the manager can now be expressed

$$W(T) = SF(T) + \alpha_q(1 - S)F(T) + \beta_q(1 - S)\max\{0, F(T) - B(T)\} + Z, \qquad (1)$$

Appendix A provides a summary of explanations to the these and all subsequently introduced variables.

In order to facilitate a feasible solution to the optimization problem, I assume that the fund is invested in a risky asset (e.g. a stock) and/or a riskless asset (e.g. government bonds). Thus, the dynamic investment strategy of the fund manager consists purely of adjusting the balance of investment weights between these two assets, and the fund value depends only on their movements. The weight in the risky asset is denoted $\kappa(F, t) \equiv \kappa$, which makes the weight in the riskless asset $(1 - \kappa)$. For ease of exposition, the risky asset weight is referred to simply as kappa for the remainder of the paper. Note that the risky asset does not necessarily have to be a stock. It should instead be conceptualized as a compound of multiple more or less advanced investment assets known to be utilized by hedge funds.

The value of the riskless asset X_1, is assumed to follow the discrete time equivalent to the continuous time linear process

$$dX_1(t) = R_d X_1(t)dt, \qquad (2)$$

so that it has the constant daily risk-free rate R_d as drift and zero standard deviation.

It is standard for the risky asset value X_2, to follow a Brownian motion process so that its continuous time equivalent is given by

$$dX_2(t) = \mu X_2(t)dt + \sigma X_2(t)dz(t), \qquad (3)$$

where the constants μ and σ are the daily expected return and the daily standard deviation of the asset, respectively; and $z(t)$ is a Brownian motion. The standard settings of μ and σ are presented and reasoned for in Appendix B. With these

assumptions, the risky asset return follows the normal distribution. This may be criticized as an unrealistic model choice since hedge fund returns tend to be distinctively non-normal, with significant negative skewness and high kurtosis (see Asheim, 2013; Brooks and Kat, 2002; Getmansky et al., 2004; Fung and Hsieh, 1997; Malkiel and Saha, 2005).[14] The assumption is nevertheless made due to its simplifying properties. Note however that other distributions could be used with some relatively mild alterations to the model; the main disadvantage being longer runtimes. It is likely that the use of skewed and leptokurtic distributions would yield results suggesting either unchanged or increased risk-taking by the manager, depending primarily on the choice of behavioural model and fund share S. In particular, with utility theory the results would likely be near unchanged since the impact of a loss and a gain are quite equal with this model. On the other hand, with a behavioural model based on loss-aversion, the distribution's negative skewness would give an increased probability of ending up with a bad fund value at terminal time. This may in turn have significant effects on exhibited risk-taking. Berkelaar et al. (2004) show that in such cases, a loss-averse *investor* increases portfolio risk in the state space area just below the benchmark. By intuition, a hedge fund manager with a sufficient fund share S is likely to react in a similar manner.

I now return from the previous digression and conclude that with the chosen assumptions, $F(t)$ follows the discrete time equivalent of the process

$$
\begin{aligned}
dF(t) &= F(t)[(1-\kappa)dX_1(t) + \kappa dX_2(t)] \\
&= F(t)[(1-\kappa)R_d dt + \kappa(\mu dt + \sigma dz(t))].
\end{aligned} \tag{4}
$$

Changes in $\ln F(t)$ are consequently normally distributed.[15]

The next step is to establish a framework for asset movements and portfolio choices to work within. To enable dynamic choices, a dynamic programming approach is required. By discretizing and truncating the set of possible fund values and time points, the stochastic choice process of the manager becomes a finite-state Markov chain. In accordance with Hodder and Jackwerth (2007), a two-dimensional grid of possible fund values and time points is established, and thus represents the discrete state space. In contrast to Hodder and Jackwerth however, I truncate both

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[14]The dynamic investment strategies of hedge funds are likely to be the cause of this. Asheim (2013) suggests two-component normal mixture or Lévy alpha-stable as fitting distribution choices.

[15]Alternatively: Changes in $F(t)$ are lognormally distributed.

sides of the fund value-dimension.[16] Since changes in $\ln F(t)$ are normal, it is convenient to set the grid up so that the step between each fund value is logarithmically constant. Formally, $\Delta(\ln(F)) \equiv C = \text{constant}$. The grid's time steps are equal so that $\Delta t = \text{constant}$. Additionally, in order to ensure that the process reaches a grid point when the manager chooses to invest purely in the riskless asset (i.e. $\kappa = 0$), all fund values grow at the rate $\exp(R_d \Delta t)$ with each time step. The possible logarithmic changes in $F(t)$ then become $R_d(T-t) + iC$, where i is an integer index. My standard case uses $i = -150, -149, \ldots, 0, \ldots, 150$ and $C = \ln(10)/199 \approx 0.01157$. With $\min[F(0)] = 0.1$ this setup gives $\max[F(0)] \approx 3.21$. By standard, the initial fund value $F(0)$ and the benchmark $B(T)$ are set to 1 and $R_d(T-1) \approx 1.0125$ (the value of 1 at T), respectively.

By assuming that the fund manager seeks to maximize his/her future utility at terminal time, the expectation values of this as a function of the current state becomes the optimization problem's objective function. The sole decision variable is of course $\kappa(F,t)$. Formally, the optimization problem in each grid state, can be expressed $\max_{\kappa(F,t)} E[U(W(T))]$. Here $U(\cdots)$ is given by the manager's utility-function.

To find the expectation values, the probabilities of ending up at all possible terminal time fund values $\{F(T)\}$ are needed. These can be calculated via the probabilities of future fund value *movements*, which conveniently only depend on time t and kappa-choice $\kappa(F,t)$. A a three dimensional lookup-table of probabilities can thus be established and used in each grid state.

For a given κ and current time t, it is seen that

$$\Delta \ln(F) \sim N(\mu_{\kappa,t}, \sigma_{\kappa,t}) = N([\kappa\mu + (1-\kappa)R_d - \frac{1}{2}\kappa^2\sigma^2](T-t),\ \kappa\sigma\sqrt{T-t}), \quad (5)$$

where $\mu_{\kappa,t}$ and $\sigma_{\kappa,t}$ is the daily mean and standard deviation of fund returns given κ and remaining time $T - t > 0$.

The probability density $f_{i,\kappa,t}$ of the fund value moving $R_d(T-t) + iC$ logarithmically during the time period between t and T becomes

$$f_{i,\kappa,t} = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{\left(R_d(T-t) + iC - \mu\right)^2}{2\sigma^2}\right]. \quad (6)$$

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[16]This proves to be no issue as long as the grid boundaries are set broad enough.

The corresponding discrete probabilities $p_{i,\kappa,t}$ can then be estimated by dividing each probability density by its sum over all i:

$$p_{i,\kappa,t} = \frac{f_{i,\kappa,t}}{\sum_{i=-150}^{150} f_{i,\kappa,t}}. \tag{7}$$

The three-dimensional lookup table of probabilities is finally established using (7) with a discrete set of kappas. My standard parameter set uses

$$\{\kappa\} = \{0, 0.1, 0.2, 0.5, 1.0, 1.5, 2.0, 2.5, 3, 3.5, 4, 5, 6, 7, 8,$$
$$10, 20, 30, 40, 50, 60, 80, 100, 120, 140, 200, 250\},$$

prior to accuracy testing; a procedure that will be presented shortly. This is an extremely wide set of kappas, giving the manager a near unlimited degree of freedom in regard to risk adjustments. In reality, the upper end of the kappa set is unlikely to be conceivable. The assumption is nonetheless made to ensure a complete and unhindered picture of managerial intensions.

All necessary foundations for running the model are now in place. The first step of the optimization routine is to calculate the terminal time utilities for all possible terminal fund values in the grid. I then take one step backwards in time, so that $t = t - 1$, and use the lookup table of probabilities together with the vector of terminal utilities to compute expected utilities for each combination of current fund value $F(t)$ and kappa-choice $\kappa(F, t)$. The optimal kappa is then the one in the set that maximizes expected terminal utility. This routine is completed for all t, thus ultimately creating a matrix of optimal kappas as a function of current fund value and time. Formally, this matrix is denoted $K^* \equiv (\kappa_{F,t}^*)$.

The advantage of this optimization procedure over a standard gradient method is that it avoids the prospective problem of local extrema, and that the lookup-table of probabilities speeds up the process considerably.[17]

Since the probabilities of the model are discrete, it is important to check for reasonable accuracy. Low and high kappas in particular are expected to give the largest inaccuracies. Distributions for small kappas are prone to suffer from limited grid fineness due to low drift rates; and large kappas are likely to yield inaccuracies if the grid edges are too narrow, as they result in a loss of distribution tail.

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[17]The lookup-table is incompatible with gradient methods.

Using a goodness-of-fit test, the grid structure and the set of kappa can be calibrated so as to enable as precise estimations as possible. I use a test-statistic similar to the one suggested by Hodder and Jackwerth (2007, p. 825). It is based on the average squared difference between the 4 first raw moments of the approximated discrete distribution and the corresponding theoretical moments. Since estimates of the approximated distribution's moments are prone to inaccuracies themselves, the statistic is scaled by the variance of estimation (Stuart and Ord, 1994, p. 349):

$$\frac{1}{4}\sum_{r=1}^{4}\left[\frac{\hat{\mu}_r - \mu_r}{\frac{1}{n}\left(\mu_{2r} - \mu_r^2 + r^2\mu_2\mu_{r-1}^2 - 2r\mu_{r-1}\mu_{r+1}\right)}\right]^2, \tag{8}$$

where I set $n = 1$ and $\mu_0 = 0$.

The test is done for each possible combination of kappa-choice and number of remaining days to evaluation, thus giving good indications in regard to dismissable kappas.

After some testing, the interval $\kappa \in \langle 0, 1.0 \rangle$ is dismissed from the standard parameter set due to inaccuracies in their distribution approximations. There are also some relatively minor inaccuracies for the higher kappas, but these are restricted to very early times in the evaluation period. As will be seen in the coming results, the higher kappa values are most prominently chosen by the manager at later time points, which is why they are kept in the set.[18] The final set thus becomes

$$\{\kappa\} = \{0,\ 1.0,\ 1.5,\ 2.0,\ 2.5,\ 3,\ 3.5,\ 4,\ 5,\ 6,\ 7,\ 8,$$
$$10,\ 20,\ 30,\ 40,\ 50,\ 60,\ 80,\ 100,\ 120,\ 140,\ 200,\ 250\}.$$

To ensure satisfactory fineness in surface graphs of K^*, they use a more refined set of kappa.

As the goodness-of-fit test does nothing to remove probability distortions stemming from tail losses near the grid edges in the fund value-dimension, I further limit the functioning region of optimal kappa matrices K^* in that dimension. With the standard parameter set the imposed restriction is $F \in [0.60, 2.52]$. The lower limit is set equal to the later introduced liquidation boundary L for convenience, as optimal kappas below this limit are often incomprehensibly high.[19]

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[18]This is the case for all behavioural models.

[19]The manager will be willing to risk anything to beat the boundary.

3   CRRA-Utility Model

In the following three sections, I put the established fundamental model into use with three different behavioural models. To begin with, a similar study to the single-period analysis of Hodder and Jackwerth (2007) will be done, with constant relative risk aversion (CRRA) to model the manager's risk-preferences. This inquiry is made mostly to provide comparability of results.

The CRRA utility-function is given as

$$U[W(t)] = \frac{W(t)^{(1-\gamma)} - 1}{1 - \gamma} \qquad \text{if } W(t) > 0, W(t) \neq 1 \qquad (9)$$

$$= \ln[W(t)] \qquad \text{if } W(t) = 1$$

where γ is the risk-aversion coefficient set to 4 in the standard case. This is identical to what is used in Hodder and Jackwerth (2007).

My version of the CRRA-model uses a simpler kind of liquidation boundary than that of Hodder and Jackwerth (2007). It assumes that the fund cannot be liquidated *before* the terminal time, thus giving the manager time to surpass the boundary after having fallen behind. Following a liquidation the manager thus retrieves the period's full management fee and the remaining fund share. In the standard parameter set the liquidation boundary is set to 60 % of the benchmark, so that $L = 0.6B(T)$.

As Hodder and Jackwerth (2007) point out, it is useful to consider the seminal findings of Merton (1969) for the optimal asset allocation problem of a CRRA-investor before attempting novel inquiries. That paper concludes that for an independent investor, the lifetime optimal kappa is

$$\kappa^* = \frac{\mu_a - R_a}{\gamma \sigma_a{}^2}, \qquad (10)$$

where μ_a and σ_a is the risky asset annual mean and standard deviation; and R_a is the annual risk-free rate. With the standard set of parameters this yields a constant $\kappa^* = 2$. So what would the equivalent result be using my model? By setting $S = 1, \alpha = 0, \beta = 0$, and using (9) to compute utilities my model captures the behaviour of an independent CRRA-investor. With these values, the optimal kappa surface is found to be identical to that of Merton (1969) for all states ($K^* = 2$). For

the fund portfolio risk to fully comply with *rational* investor preferences, the fund kappa should thus be set to this constant value.[20]

With the investor's risk-preferences in mind, I explore the matter from the perspective of a CRRA-manager. Figure 1 shows the manager's optimal kappa surface assuming a 10 % fund ownership.
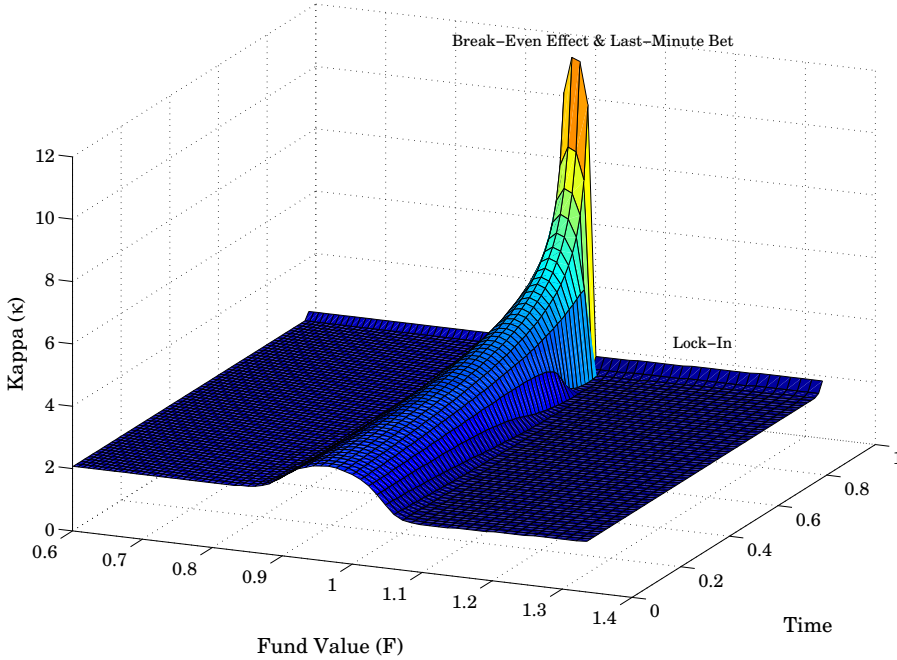

Figure 1: The optimal kappa surface K^* for a manager with CRRA utility-function, 10 % fund share.

Similarly to Hodder and Jackwerth (2007), the manager is found to set a relatively high risk-level in the vicinity of the benchmark (break-even effect). This tendency is further strengthened as the terminal date approaches (last-minute bet). The moment the incentive option goes ITM, the manager drastically reduces risk to a level lower than investors would prefer (the Merton constant). This is known as the lock-in effect, as it stems from the manager's attempt to secure option fees. As the option goes more deeply ITM however, the manager increases the risk to a level eventually equalling that of Merton.[21] In opposition, when the fund value declines starting from the benchmark, the manager reverts directly to a strategy

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[20]Assuming $\gamma = 4$.

[21]As this change of policy comes about very slowly, it is less visible in the graph.

consistent with Merton. Due to the aforementioned differences from Hodder and Jackwerth (2007) in implementation of the liquidation boundary, the observed behaviour surrounding it differs somewhat from what was found by them. Whereas their manager quickly reduces risk to zero as the liquidation boundary becomes perilously close, my manager maintains a constant Merton-risk even as the fund value falls slightly beneath the boundary.

In summary, the CRRA model suggests that the acceptability of fund risk from the point of view of investors is highly dependent on the current situation. The peak of risk-taking near the benchmark could be hazardous and should therefore be considered the most important finding.

4 Loss-Aversion Model

The concept of loss-aversion was introduced as a part of Prospect Theory (PT) (Kahneman and Tversky, 1979) and has since been recognized through a large collection of experimental and empirical research. As a descriptive theory, it models actual behaviour rather than rationally optimal behaviour.[22] A recent study by Haigh and List (2005) finds exceptionally loss-averse behaviour in a sample of option traders.[23] Given the similarities of their professions, hedge fund managers should consequently be susceptible to similar traits.

I proceed to run the fundamental model in combination with the PT framework as presented in Tversky and Kahneman (1992), but exclusive of their subjective probability weighting feature.[24] The utility-function of the manager can thus be modelled as

$$U[W(t)] = \begin{cases} -A[\theta - W(t)]^{\gamma_1} & \text{if } W(t) \leq \theta \\ [W(t) - \theta]^{\gamma_2} & \text{if } W(t) > \theta \end{cases}, \tag{11}$$

which is also used in Kouwenberg and Ziemba (2007). $\theta \equiv \theta(t)$ is the reference point that separates gains from losses in the manager's mind. It will henceforth often be referred to as the status quo. It is natural to assume that this personal threshold is given by the incentive fee benchmark. In my context of use, θ then becomes the terminal wealth level that the manager achieves when the terminal

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[22]As a consequence, the optimal kappa found with loss-aversion is not rationally optimal. Instead, it describes what the decision taker *considers* to be the optimal kappa in the given situation.

[23]Most other experimental tests of loss-aversion use students as subjects.

[24]I disregard this part of PT as I want to focus on the more consequential loss-aversion feature.

fund value is exactly at the benchmark:

$$\theta = SB(T) + \alpha(1-S)B(T) + Z, \tag{12}$$

With zero fund ownership, the manager does not take on a major loss of monetary wealth by underperforming the benchmark,[25] yet it is reasonable to assume that he/she suffers a relatively strong feeling of loss of opportunity and reputation. Furthermore, Getmansky (2012); Agarwal et al. (2004) show that future net fund flows are likely to be reduced following unsatisfactory performance, thereby leading to an undesirable decrease in future management fees.[26]

In my standard case, the manager is given the median preference parameters found experimentally by Tversky and Kahneman (1992). Additionally, it is assumed that all of the manager's initial wealth, if any, is invested in the fund. The value of this wealth then becomes $SF(0)$, and as a result $Z = 0$.[27]

Before analysing the behaviour of loss-averse managers, it is useful to build a basis of comparison by examining the risk-taking preferences of loss-averse investors. While *rational* investors may prefer the fund to follow the Merton constant (10), their realized risk-preferences are better described by PT. The risk-taking of loss-averse investors can easily be estimated by setting $S = 1, \alpha = 0, \beta = 0$ and running the fundamental model together with the PT utility-function. The resulting optimal kappa surface is given in Figure 2.

It immediately becomes clear that the behaviour of a loss-averse individual is very different from that of a risk-averse individual. Due to a stronger emotional impact felt from losses compared to gains, the investor exhibits irrationally prudent risk-taking whenever the fund value is near the benchmark (valley of loss-aversion).[28] Generally speaking, the investor's optimal risk is an increasing function of fund value-benchmark distance. As can be seen in the area below the benchmark, the investor becomes risk-seeking in the face of losses.[29] Conversely, in the face of gains, the portfolio risk is slowly increased due to a lowered probability of ending up below the benchmark. All of the aforementioned trends are consistent

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[25]Some direct losses will however incur through decreases in management fees.

[26]This particular eventuality is dependent on a broad range of external factors, the most important of which are identified by (Getmansky, 2012).

[27]As noted earlier, the value of Z is irrelevant for a loss-aversion analysis.

[28]The optimal kappa is actually set to zero in the valley's base.

[29]Put another way, once the portfolio is at a current loss, the actual magnitude of losses appear to become less important.
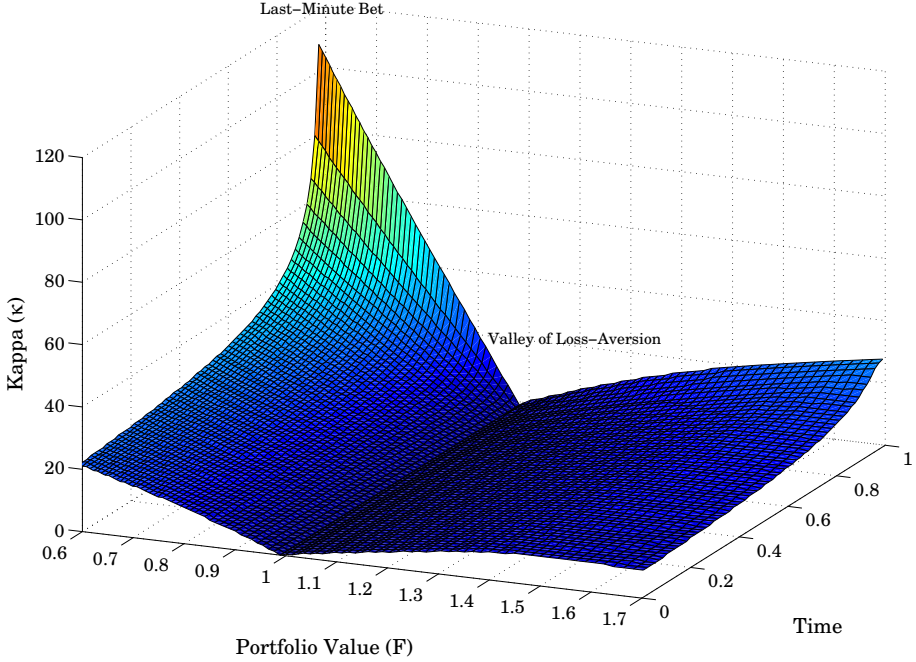
Figure 2: The optimal kappa surface K^* for a loss-averse investor, who behaves identical to a loss-averse manager with $S = 1, \alpha = 0, \beta = 0$. Standard parameters are used otherwise.

with Kouwenberg and Ziemba (2007). An interesting characteristic not captured by their model however, is the distinct increase in optimal risk with time. This phenomenon is explained by noting that as the evaluation date approaches, the probability for the portfolio value to get worse is substantially decreased; thereby encouraging gambling behaviour. The effect is most prominent for low portfolio values, as the aspiration to equal or beat the benchmark overwhelms the increased probability of diminishing losses. This gives origin to an area with especially risky behaviour (last-minute bet).

Berkelaar et al. (2004) perform a similar analysis for a loss-averse investor, yet with a very different methodology. They find that the initial risk-level is set high when the portfolio value starts off lower than the PT reference-point, which is coherent with what is found here.[30]

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[30]Their model is time static, and is thus unable to say anything about the tendency to increase risk as terminal time approaches. Moreover, they do not consider the case when the reference-point initially is set lower than the portfolio value, thus yielding no comparability in the region above the reference point. It is also worthwhile to mention that differences in kappa-level between that paper and those

I now turn to analyse managerial behaviour, starting off with the optimal kappa surface of a manager with 10 % fund share (Figure 3).
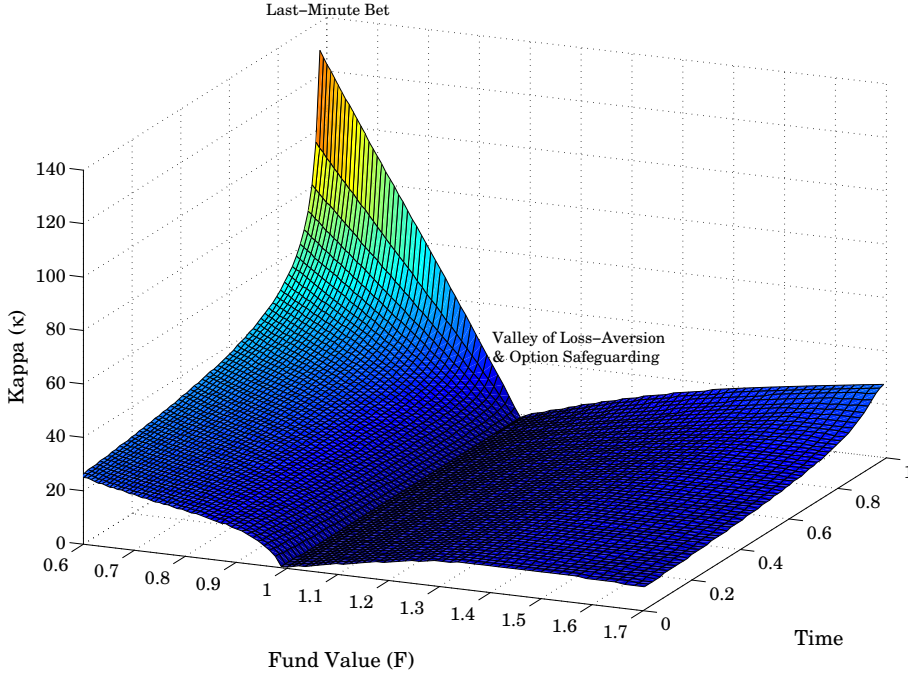

Figure 3: The optimal kappa surface K^* for a loss-averse manager with fund share $S = 0.1$.

There are evidently only subtle differences relative to a loss-averse investor. Both parties exhibit the same propensities, but the manager's overall optimal risk-level is slightly higher. This is in perfect alignment with Kouwenberg and Ziemba (2007), who find that risk-seeking increases with the incentive fee rate irrespective of fund value.

In Kouwenberg and Ziemba (2007), the fund share of the manager is emphasized as the most important tool available for interest alignment between manager and investor. With that in mind, I introduce the optimal kappa surface of a manager with zero fund share and otherwise unchanged parameters (Figure 4).

The decrease in fund share is seen to have a profound effect on risk-taking. Extremely high levels of risk are observed in almost all regions of the state space except for very high fund values, thereby confirming said proposal from Kouwenberg

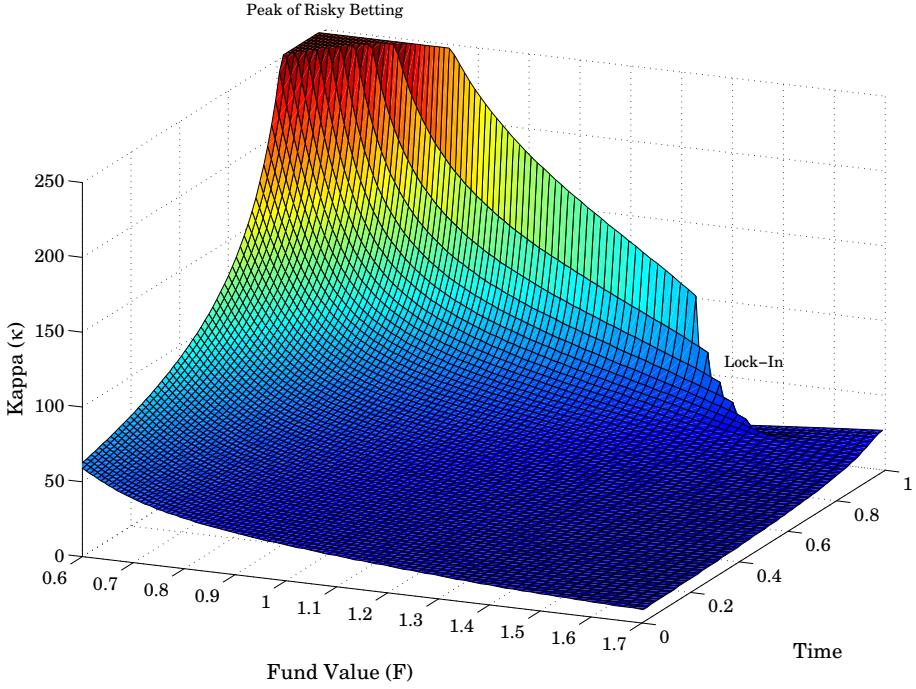found here are likely to stem from differences in Sharpe ratio.

Figure 4: The optimal kappa surface K^* for a loss-averse manager with fund share $S = 0$.

and Ziemba (2007). Yet, the pattern of risk-taking bear some of the same tendencies as with 10 % fund share. Most noticeably, there is still a peak of risk-taking for low fund values and short time left to evaluation (peak of risky betting). However, it is now a near ubiquitous phenomenon, as its influence stretches far beyond the benchmark and past the half-time spot. In conjunction with this, the valley of loss-aversion and option safeguarding has vanished. The overall risk-level can be described as a monotonously declining function of fund value. The risky behaviour accordingly diminishes once the benchmark has been exceeded greatly (lock-in). What is more, the inclination toward increasing riskiness as time passes is still very much in place.

5 Two-Reference Point Model: Loss- and Liquidation Aversion

A realistic complication to the loss-averse model is the possibility of fund liquidation following bad performance (e.g. Goetzmann et al., 2003; Hodder and Jackwerth, 2007; Liang, 2000). This eventuality is likely driven by both exterior and interior factors. Investors are expected to withdraw their money more frequently when the fund performs badly (Getmansky, 2012). This lowers management fees, making fixed fund costs harder to pay; thus strengthening the manager's incentive to perform an endogenous liquidation. A loss of reputation more severe than with only small losses in AUM should also be expected, thereby resulting in decreased future net fund flows (Getmansky, 2012).

Goetzmann et al. (2003) use a predetermined constant boundary in their implementation of the phenomenon; quite similar to one used in Section 3. In their most advanced implementation, Hodder and Jackwerth (2007) give the manager a choice between carrying on managing the fund and endogenously liquidating it in favour of outside compensation opportunities. This leads him/her to liquidation whenever expected fees become lower than the outside compensation. In the following, I alter the standard PT utility-function so that it takes the possibility of fund liquidation into account. I then use this novel function to analyse the behaviour of a more realistic manager than those portrayed in previous sections and much of the existing literature.

A considerable amount of research suggests that decision makers often are better modelled as considering more than just one reference-point - such as the status quo of PT - during the evaluation process (see Wang and Johnson, 2012; Koop and Johnson, 2012; Ordónez et al., 2000; Sullivan and Kida, 1995). Wang and Johnson (2012) suggest a model that extends on PT, but uses three reference-points rather than one, and refer to them as the "goal", the "status quo" and the "minimum requirement". These three reference-points divide risky outcomes into four functional regions: success, gain, loss and failure. In the context of this paper, such a model proves particularly useful as it allows for realization of liquidation risk in the form of a "minimum requirement" reference-point.

Guided by Wang and Johnson (2012) and Tversky and Kahneman (1992), I introduce a utility model founded on two reference-points (TRP) dubbed the status

quo and the liquidation point/boundary.[31] The status quo of this new model corresponds to the one used in PT. Here too, it represents the separation of gains and losses in the mind of the manager. Similarly to PT in Section 4, the new model will use θ to denote the location of the status quo in terms of managerial wealth $W(t)$.

The novel liquidation point is intended to represent the verge of failure in the mind of the manager, chiefly caused by the possibility of a complete fund liquidation. While the manager is assumed to retain all of his fund share and possibly parts of the management fees accrued through the current period upon liquidation, there can be no doubt that such an event has a severe psychological impact on him/her. Brown et al. (2001) find that in the hedge fund industry, a return to fund management following a liquidation is unlikely, although it has been known to happen.[32] It is thus reasonable to envision major changes in a manager's career following a liquidation. In wealth and fund value terms the liquidation point is denoted λ and L, respectively. With that, I have

$$\lambda = SLF(0) + \alpha(1-S)LF(0) + Z. \tag{13}$$

The basic utility-function suggested by Wang and Johnson (2012) splits evaluation at each reference-point with a piecewise linear formula, but does not include standard PT's convex/concave shape over losses/gains. To ensure compatibility with PT and comparability to Kouwenberg and Ziemba (2007), the TRP-function therefore combines the functions of Tversky and Kahneman (1992) and Wang and Johnson (2012) to create

$$U[W(t)] = \begin{cases} \tau_\lambda \lambda & \text{if } W(t) < \lambda \\ \tau_- \left[\theta - W(t)\right]^{\delta_-} & \text{if } \lambda \leq W(t) \leq \theta \\ \tau_+ \left[W(t) - \theta\right]^{\delta_+} & \text{if } W(t) > \theta \end{cases}, \tag{14}$$

with the restrictions

$$\tau_\lambda < \tau_- \leq \tau_+,$$
$$\tau_\lambda < 0, \quad \tau_- < 0, \quad \tau_+ \geq 0$$

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[31] The "goal" reference-point of Wang and Johnson (2012) is not included in the model as it arguably has no useful purpose in the context of a fund manager.

[32] In situations where the manager has fallen far behind on many high-water marks, making the prospects of earning incentive fees slim, a last resort could be to do an endogenous liquidation and start a new fund from scratch.

imposed.[33]

Here, δ_- and δ_+ are the respective curvature coefficients over losses and gains; while τ_λ, τ_- and τ_+ are coefficients representing the respective psychological impact of liquidation, losses and gains.[34] Since the liquidation point is to be considered the absolute low, the evaluated value beneath that point is constant, as shown in the formula's first case. This is intuitive given that the fund is assumed to be liquidated with certainty for those values.

To facilitate compatibility with the standard PT parameters of Tversky and Kahneman (1992), I have

$$\delta_- = \gamma_1 = 0.88, \quad \delta_+ = \gamma_2 = 0.88, \quad \tau_- = -A = -2.25, \quad \tau_+ = 1.$$

τ_λ can be set so that the value of (14) is equal to its PT equivalent (11) at the lowest possible $W(t)$-value, zero. It is then seen that $\tau_\lambda = \frac{-A\theta^{\gamma_1}}{\lambda}$. This sets the impact of a liquidation measured by TRP equal to the impact of zero wealth measured by PT. An alternative approach is to calibrate τ_λ so that the TRP-utility equals the PT-utility at a predetermined wealth-level, thereby assuming that the manager has external compensation opportunities. Such an approach is comparable to the endogenous liquidation mechanism of Hodder and Jackwerth (2007). However, testing with their external compensation of 0.0045 of initial AUM for a manager with 10 % fund share showed no significant changes in behaviour. Additionally, the method is unapplicable for low fund share values, since utility values then, in opposition with intentions, become *higher* once the fund value creeps below the liquidation boundary. For simplicity this alternative practice is therefore not taken into use.

With these assumptions, variable substitution yields

$$U[W(t)] = \begin{cases} -A\theta^{\gamma_1} & \text{if } W(t) < \lambda \\ -A[\theta - W(t)]^{\gamma_1} & \text{if } \lambda \le W(t) \le \theta \\ [W(t) - \theta]^{\gamma_2} & \text{if } W(t) > \theta \end{cases} . \tag{15}$$

It may be noted that the TRP-function bears similarities to the utility-function

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[33] As Barberis et al. (2001) point out, the curvature coefficients are strictly speaking unnecessary in this area of application, as they are most relevant for decision processes involving *only* gains or *only* losses. The main reason for keeping them is therefore comparability concerns.

[34] In Wang and Johnson (2012), they are denoted β_F, β_- and β_+.

employed by Barberis et al. (2001), who alter the original PT-function so that it
takes an investor's past-dependent sensitivity to losses into account.

Figure 5 graphs a typical TRP-function and its corresponding PT-function for a
linear $W(t)$ interval, under the standard parameter assumptions presented above.
It is seen that the two curves perfectly coincide for $W(t) \leq \lambda$. For $W(t) < \lambda$ how-
ever, the TRP-utility immediately drops to the level held by the the PT-function at
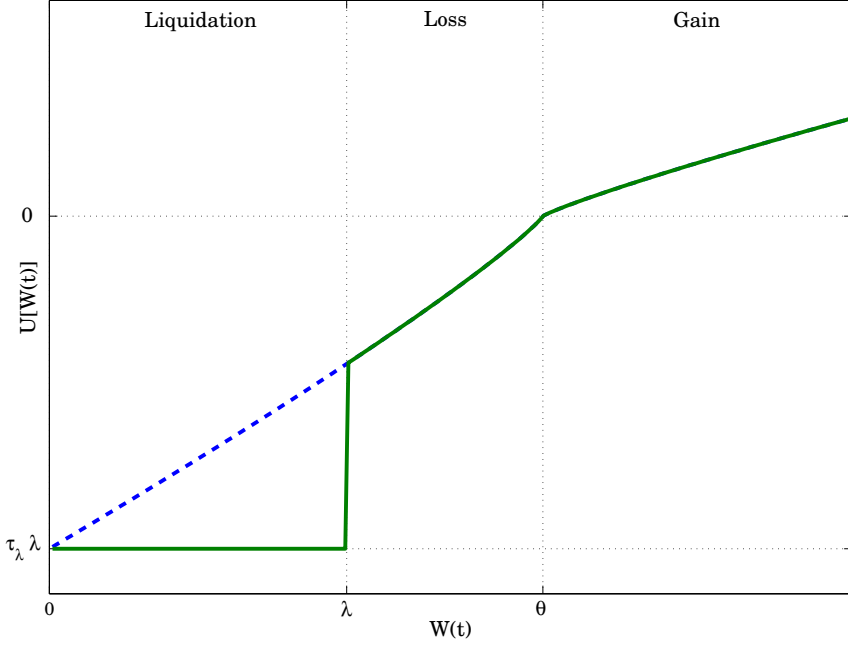$W(t) = 0$.

Figure 5: The solid green curve shows a typical TRP utility-function, while the
dotted blue curve shows its corresponding PT utility-function. Note that the two
curves coincide perfectly in the interval $\lambda \leq W(t) < \infty$. The concave/convex prop-
erty of the curves is in place, but difficult to see due to relatively high settings for
γ_1 and γ_2.

Figure 6 shows the estimated optimal kappa surface for a TRP-manager with zero
fund share. Comparison to the corresponding PT results in Figure 4 indicates no
major differences caused by the added possibility of liquidation. As the manager
has no large fund wealth to lose, the added emotional impact is not enough to
stagger the risky betting taking place near the boundary and terminal time.
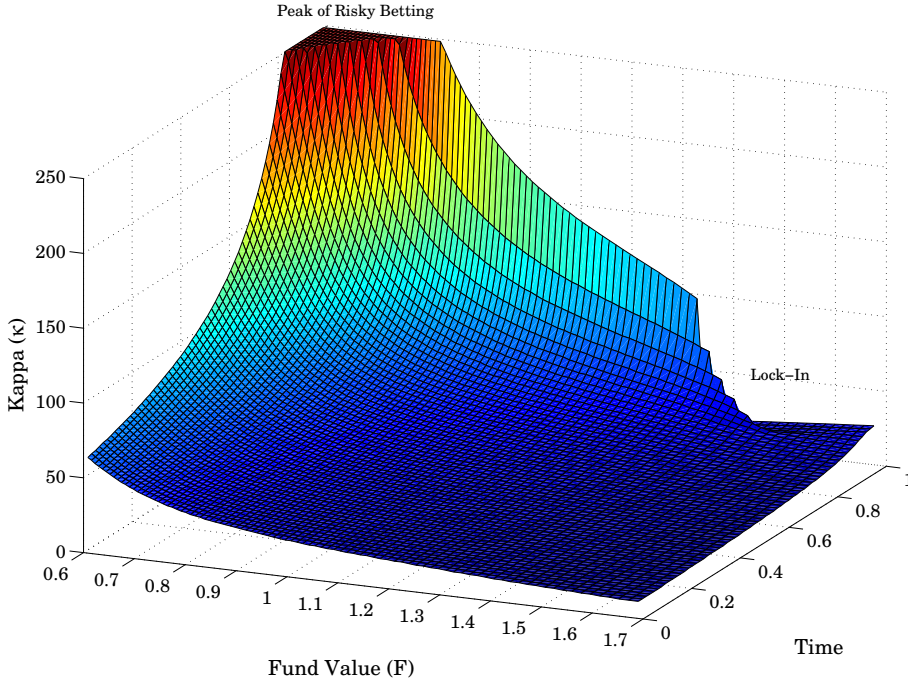
Figure 6: The optimal kappa surface a manager subject to TRP with a fund share of $S = 0$.

In line with the previous section, I now graph the estimated kappa surface for a manager with 10 % fund share (Figure 7).

It can immediately be seen that a significant fund share introduces far more interesting dynamism and richness to the manager's behaviour. There are few traces left of the monotonic tendencies found for a manager with zero fund share. The overall risk-taking is considerably reduced across the state space compared to both the corresponding PT results in Figure 3 and the TRP results with zero fund share.

A closer analysis using intuition may help identify the rationale behind the manager's behaviour. What first catches the eye may be the two independent peaks of riskiness, separated only by the deep valley of prudence in the vicinity of the benchmark. In the valley, the loss-aversion of the manager causes a minimized portfolio risk in an attempt to *avoid* fund share losses and *gain* option fees. This same tendency was also found with PT (see Figure 3). The two peaks represent gambles taken by the manager in situations where the fund value is sufficiently far from either the liquidation point or the benchmark, while also being low enough
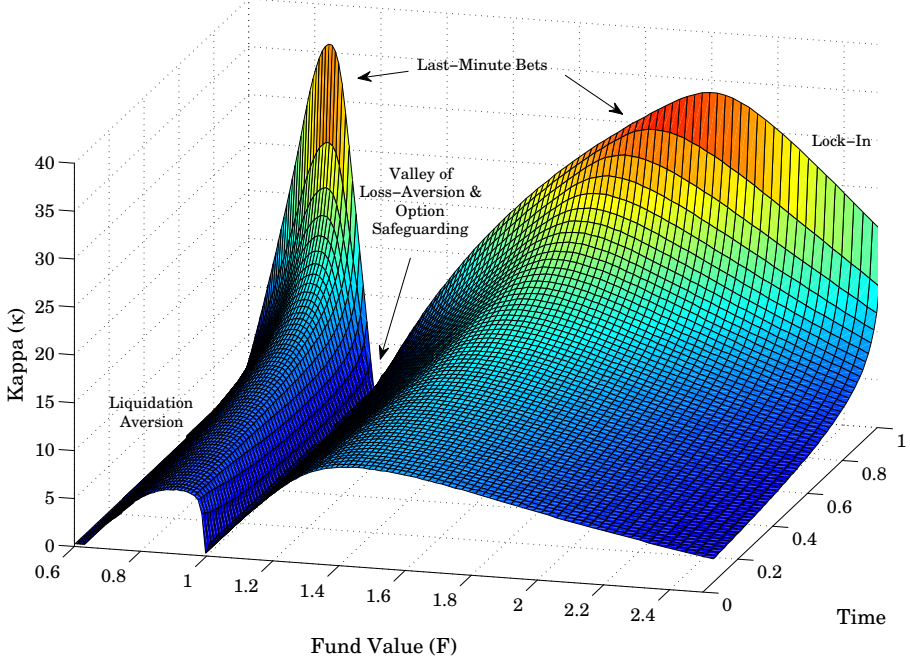
Figure 7: The optimal kappa surface for a manager subject to TRP with a fund share of $S = 0.1$.

for a lock-in to remain unattractive. In coherence with previous results, both peaks gain steepness as time passes due to a "last-minute betting"-effect stemming from the decreased probability of ending up in a worse position.

The leftmost peak appears for the same reasons as the "last-minute bet" peak found for a PT-manager with 10 % share and a PT-investor (see Figures 3, 2). However, the added liquidation boundary and increases in fund share has pushed its summit down and effectively curbed risk-taking on its left side (liquidation aversion). In the region between the liquidation boundary and the benchmark, this manager can generally be said to prefer a risk-level *lower* than the PT-investor.

The rightmost peak represents a largely new phenomenon. It rises in the area where the probability of ending up below the boundary is considered small and the preliminary size of fund share- and incentive fee gains are small enough to induce some greediness into the manager. Starting from the rightmost peak's summit, further increases in fund value slowly satisfies the greed of the manager, who ultimately reverts to a mild lock-in like behaviour.

As a final remark, it should be stressed that TRP is unapplicable to an investor, since he/she naturally does not suffer a negative psychological impact from the possibility of fund liquidation *beyond* what is already captured by PT.[35] As such, the appropriate utility-function and kappa surface for a realistic investor remains that of (11) and Figure 2, respectively.

6 The Multi-Period Case

The results presented hereto assumes a manager that maximizes expected utility at the termination of a single evaluation period. Such a manager may be characterized as a myopically optimizing individual, in the sense that he/she uses a relatively short optimization horizon.[36] On the contrary, it may be argued that hedge fund managers hold a multi-period perspective, which is expected to dampen extreme risk-taking (e.g. Panageas and Westerfield, 2009; Guasoni and Obloj, 2013). This is particularly true when the benchmark has a high-water mark mechanism: With it, the manager knows that he/she will have to regain any losses that are sustained in the current period in subsequent ones in order to accrue incentive fees. The negative consequences of getting far out-of-the-money thus become more severe.

Further indications of the risk-dampening effects of long horizons can be found by considering the incentive fee option in isolation. As stressed earlier, it can be modelled as a European call option on AUM. It is known that the gamma of such an option is a decreasing function of time-to-maturity, which in this case translates to the length of the manager's optimization horizon. Hence the option delta is increasingly linearized for longer horizons. As a consequence, an incentive option with a very long time horizon is expected to affect current managerial behaviour in a manner that is analogous to management fees and fund shares. Considering only the incentive fee option, which may be the chief root of risk-taking, risky behaviour should therefore be dampened in a multi-period model.

Panageas and Westerfield (2009) and Guasoni and Obloj (2013) find that a manager using a perpetual utility optimization process is likely to set a constant weight to the risky asset. However, Hodder and Jackwerth (2007) questions the realism

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[35]Except for the minor opportunity losses that may incur from having to reinvest funds.

[36]In the following, I will often use "horizon" as short form for "optimization horizon", which is analogous to the "evaluation period" used by Benartzi and Thaler (1995).

of their use of continuous time fee accruements and state space structures. They present, with their discrete time framework, a comparative analysis suggesting that CRRA-managers *do* use a more prudent risk-taking in a multi-period setting, but that it takes a horizon of several decades for the optimal risk-level to approach that of investors.[37]

With PT or TRP, building a multi-period model is particularly challenging due to the path dependency of the status quo and the high-water mark. More importantly however, there is a considerable amount of research suggesting that investors are myopic (see Benartzi and Thaler, 1995; Thaler et al., 1997; Gneezy and Potters, 1997; Gneezy et al., 2003; Haigh and List, 2005). Research has also found a positive correlation between portfolio monitoring frequencies and the exhibited degree of myopia. It is reasonable to assume that myopia applies to hedge fund managers as well, thereby giving some validation to the single-period analysis of previous sections.[38] Conversely, a multi-period framework may yield unrealistic results, as the myopic property of hedge fund managers is likely to be predominantly determined by each single fee evaluation period.

A relevant perspective on short and long term risk attitudes is that of Levy and Wiener (2013). They recognize investors as myopically loss-averse in the short run, but argue that investors turn to the traditional utility theory framework in the long run. Assuming a completely non-myopic manager, their supposition thus agrees with the multi-period CRRA analysis of Hodder and Jackwerth (2007).

In spite of said disagreements, I will in the following present the procedure for a multi-period analysis of managerial behaviour subject to PT or TRP. The benchmark is assumed to follow a high-water mark, and the initial fund value is set equal to it. A feasible implementation requires the following assumptions:

- The manager is able to estimate expected utility in future periods as a function of the current period's developments.

- The status quo θ is always equal to the high-water mark of the current period.

- The liquidation boundary L is always a constant fraction of the current pe-

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[37]As they assume annual evaluation periods, it should be noted that with quarterly periods the required horizon would be correspondingly shorter.

[38]Haigh and List (2005) confirms myopic loss-aversion for a sample of professionals with many similarities to hedge fund managers (option traders).

riod's initial fund value.

The first and second assumption implicitly assumes that the manager rationally forecasts the loss-aversion he/she will feel in the future as well as changes in his/her personal status quo θ. In regard to the first issue, Loewenstein et al. (2003) suggest a more complex reality in which people have trouble forecasting their feelings about future events. The reasonability of the second assumption is also often disputed when downward adjustments of the status quo are possible, as it may be inconsistent with loss-aversion itself (see Levy and Wiener, 2013). However, it poses less of a problem here since the status quo is assumed to follow the fund's high-water mark, and can thus never be reduced, only increased.

Hodder and Jackwerth (2007) provide the basis of my approach for the multi-period model. I first consider the initial period in a two-period model. To include the second period into the manager's decision process, the future consequences of asset movements and risk choices in the first period have to be included in first period's decision foundation. Thus, the manager's terminal time compensation has to be augmented by the certainty equivalent of the expected utility of the second period, thereby altering risk-taking in period one. This certainty equivalent is dependent on status at the first period's terminal time T_1:

- $F(T_1) < L$: The fund is liquidated, yielding a certainty equivalent of zero from period two, and an unaltered terminal wealth for period one.

- $L < F(T_1) \leq B$: The high-water mark B remains unchanged in the next period, but incurred losses in AUM have to be regained in the next period for incentive fees to be earned. The terminal compensation for period one therefore has to be augmented by the certainty equivalent of a period two with initial fund value equal to the terminal fund value of period one. Formally, $F(0_2) = F(T_1)$.

- $F(T_1) > B$: The high-water mark is reset for the next period. The terminal wealth for period one has to be augmented by the certainty equivalent of a period two with initial fund value equal to period one's initial value ($F(0_2) = F(0_1)$).

The certainty equivalents for the two latter cases are found through Monte Carlo simulation using the second period optimal kappa matrix with necessary varia-

tions of $F(0_2)$.[39] That particular matrix is identical to the single-period kappa matrix since it is the last period on record. With PT, certainty equivalents are given by

$$W(U) = \begin{cases} \theta - \left(-\frac{1}{A}U\right)^{1/\gamma_1} & \text{if } U < 0 \\ \theta + U^{1/\gamma_2} & \text{if } U \leq 0 \end{cases};$$ (16)

and correspondingly, with TRP

$$W(U) = \begin{cases} 0 & \text{if } U \ < \ \tau_-(\theta - \lambda)^{\delta_-} \\ \theta - \left(\frac{U}{\tau_-}\right)^{1/\delta_-} & \text{if } \tau_-(\theta - \lambda)^{\delta_-} \ \leq \ U \ \leq \ 0 \\ \theta + \left(\frac{U}{\tau_+}\right)^{1/\delta_+} & \text{if } U \ > \ 0 \end{cases}.$$ (17)

In the first case of (17), the mathematical solution is actually an infinite set. However, as it corresponds to $F(T_1) < L$, the certainty equivalent is simply set to zero there (which is one of the possible solutions). Note that since the status quo is updated for each new period, there is no need to scale the certainty equivalent of future periods with the increasing value of fund shares and fees.[40] Also, since the fund value grid for a single period already grows at the risk-free rate, there is no need to manually scale the certainty equivalents for decreasing time value.

With a recursive algorithm, the results for any number of evaluation periods can be found by starting at the last evaluation period and doing a backward sweep, performing the given procedure for each extra period.

Figure 8 shows the initial quarter optimal kappa surface of a zero fund share-manager subject to TRP who optimizes four quarters ahead.[41]

The surface shows a remarkable similarity to Figure 7, in which the single-period TRP manager has a 10 % fund share. As predicted by the time-declining convexity of the incentive option-gamma, the introduction of multiple long horizon options into the manager's decision basis has given an effect analogous to that of fund share increases.

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[39]By assuming a constant κ until the initial period terminal time and over the second period, the certainty equivalents can be computed directly. I do not make that assumption. By using Monte Carlo simulations, I instead assume that the manager has accrued, through some experience, some intuition concerning the probability of future period outcomes.

[40]On the contrary, this is a necessity in Hodder and Jackwerth (2007), as they use CRRA-utility.

[41]This choice of optimization period is partly founded on Benartzi and Thaler (1995), as their average evaluation period length is estimated to a year.
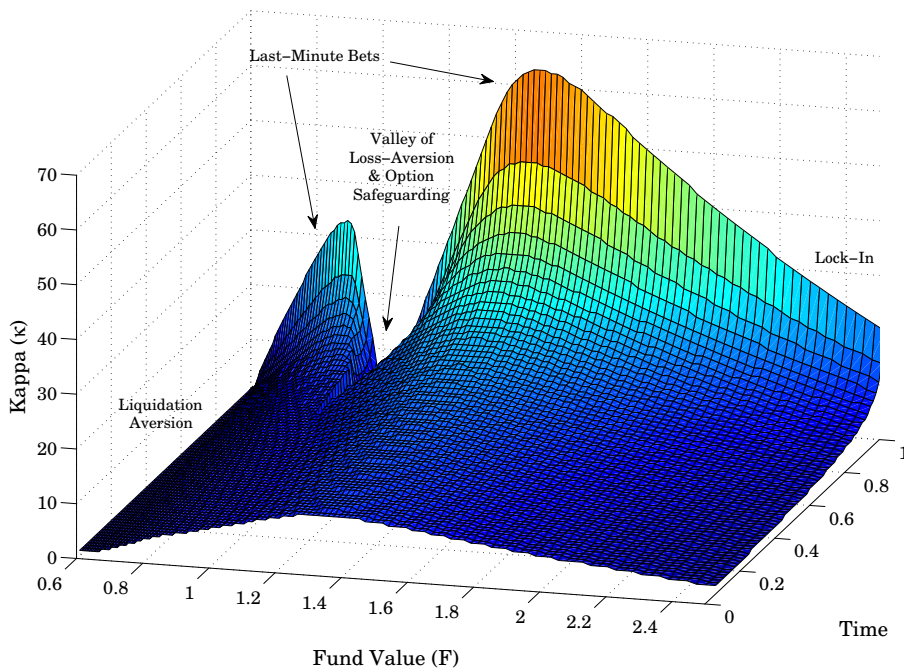
Figure 8: The initial quarter optimal kappa surface for the TRP-manager with an optimization horizon of four quarters and fund share $S = 0$.

More generally, the average optimal kappa-level of a TRP-manager is found to be a decreasing function of horizon-length.[42] The state space region between the liquidation boundary and the high-water mark is heavily affected, as the consequences of a liquidation become increasingly severe with longer horizons (liquidation aversion): In the event of a fund liquidation, the manager is guaranteed a compensation of zero in all of the remaining recorded periods.[43]

For the state space region above the high-water mark, changes in risk-level are not as significant for longer horizons. Even with a horizon of 16 quarters (four years), the pattern of risk-taking in that area remains largely unchanged from the fourth quarter (Figure 9). This is quite intuitive since the probability of underachieving the high-water mark is relatively low there. Beneath the benchmark however, the risk is reduced to a level close to the Merton constant (10).

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[42]The same is found for a PT-manager.

[43]Using a lower compensation limit instead, the changes in risk-level as a function of horizon are mildly dampened.

On an important note, the trends observed here are highly dependent on the length of fee-evaluation periods (T). Using annual fee-evaluations,[44] what takes four *quarters* with this paper's assumptions, would take roughly four *years* instead.
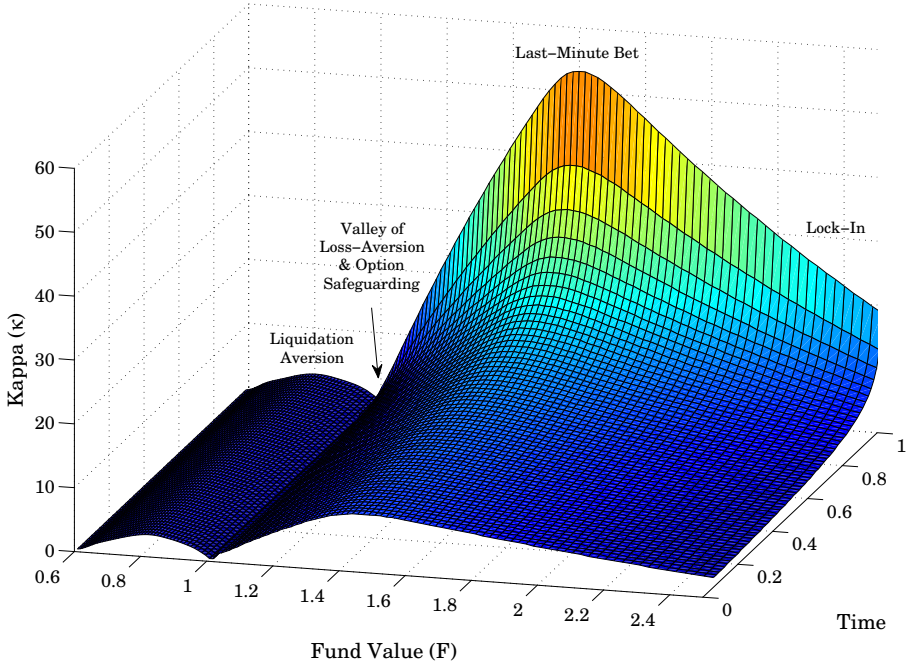
Figure 9: The initial quarter optimal kappa surface for the TRP-manager with an optimization horizon of 16 quarters and fund share $S = 0.1$. Note that due to the decreasing significance of S with longer horizons, the surface is practically unchanged with $S = 0$.

Although the multi-period results are highly interesting, the difficulties with realistic assumptions stressed earlier in the section should not be forgotten. The multi-period assumption is consequently abandoned in favour of the single-period one for the remainder of the paper.

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[44]Such as Hodder and Jackwerth (2007); Kouwenberg and Ziemba (2007).

7 Extent and Economic Significance of Misalignments

Given the indications of manager-investor interest conflicts in previous sections, one may ask whether it generally is sensible to invest in hedge funds. Due to the relatively high fees used in the industry, it is clear that investors must believe strongly in the fund manager's abilities to create excess value compared to the overall market.

Research shows no consensus regarding hedge funds' ability to attain alpha. See for instance Fung et al. (2008); Amin and Kat (2003*a*); Edwards and Caglayan (2001); Brown et al. (1999). The same goes for persistency and the existence of manager-skill (Agarwal and Naik, 2000; Kat and Menexe, 2003; Baquero et al., 2005). Yet hedge funds have grown in popularity among investors, and the industry is set to reach $3 trillion under management in 2014 (*Hedge fund assets seen reaching $3 trillion this year - survey*, 2014). The discussion concerning performance, persistency and manager-skill is beyond the purview of this paper. Nonetheless, it is useful to explore exactly how strong excess performance a hedge fund has to deliver to be able to justify possible interest-misalignments.

Seemingly, the trivial solution to any misalignment problem is to remove the incentive fee option completely from the fee structure. This is in line with Kouwenberg and Ziemba (2007), who find that the severity of the misalignment increases monotonically with higher incentive fee rates. As argued by Goetzmann et al. (2003) however, the incentive fee has become the industry standard; making it very difficult to remove. A more realistic measure could thus be to either increase managerial ownership, or increase management fees in exchange for lower incentive fees.

By intuition, both fund share and management fees should lead managerial risk-taking closer to that of a corresponding investor. As the wealth function (1) shows, the manager's earnings from either of them are a linear function of AUM. Their effect on behaviour are thus expected to be equal, but with different strength of impact. Through the option gamma argument of Section 6, it is seen that the earnings from incentive fees are highly non-linear, thereby yielding an opposing effect.

With that, the interplay between the three determining variables α, β and S

should be a prime research interest. In the following, I therefore present two different methods aiming to quantify the extent of manager-investor conflicts of interest, as a function of these variables. Pros and cons of each method and illustrative results are also given and discussed.

7.1   Manager-Investor Kappa-Difference

A straight-forward way to measure misalignments is to compute the optimal kappa-difference between investor and manager investment policies for varieties of α, β and S:

$$\text{abs}[K_M^*(\alpha,\beta,S) - K_I^*(\alpha,\beta,S)]. \tag{18}$$

Here, $K_M^*(\alpha,\beta,S) = (\kappa_{ij})$ and $K_I^* = K_M^*(0,0,1)$ is the manager and investor's optimal kappa matrix, respectively.

For convenience, a scalar measure can be given through the use of a matrix norm. A suitable norm for the current application is

$$M(\alpha,\beta,S) = \frac{1}{G(T-1)} \sum_{i=1}^{G} \sum_{j=1}^{(T-1)} \text{abs}[(\kappa_{ij})], \tag{19}$$

where G denotes the fund value grid height, and (κ_{ij}) is the i,j-th entry of (18).[45] By dividing with $G(T-1)$ the measure becomes interpretable as the mean difference in kappa irrespective of the probabilities of paths.

In the following I present and discuss estimations of $M(\alpha,\beta,S)$ for varieties of input parameters using the TRP model for the manager and PT for the investor. By doing this, the kappa-profile of the PT-investor is implicitly taken on as the manager's reference point for perfect interest alignment. Seeing as PT is a psychological model capturing human irrationality, it might not represent the investor's ideal preferences. An alternative could thus be to use the lifetime optimal risk of Merton (1969) given by (10) as reference for kappa. On the other hand, this approach may be considered unfair on the manager's behalf. Additionally, and in line with the discussion in the beginning of Section 6, the performance demands

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[45]The sum is from 1 to $T-1$ and not T because the model is set up so that the period ends on the T-th day (the terminal day), which makes kappa unchangeable on this day.

from fund investors are likely to be shaped by myopic loss-aversion.[46] In light of these considerations, I find it most reasonable to use the investment policy of a PT-investor as point of reference.

To start with, I am interested in the interplay between the two fee rates. As fund share is expected to dominate the influence of management fee rates, I ensure isolation of the two fee rates by setting $S = 0$ while varying α and β. The resulting estimates for $M(\alpha, \beta, 0)$ are graphed in Figure 10.
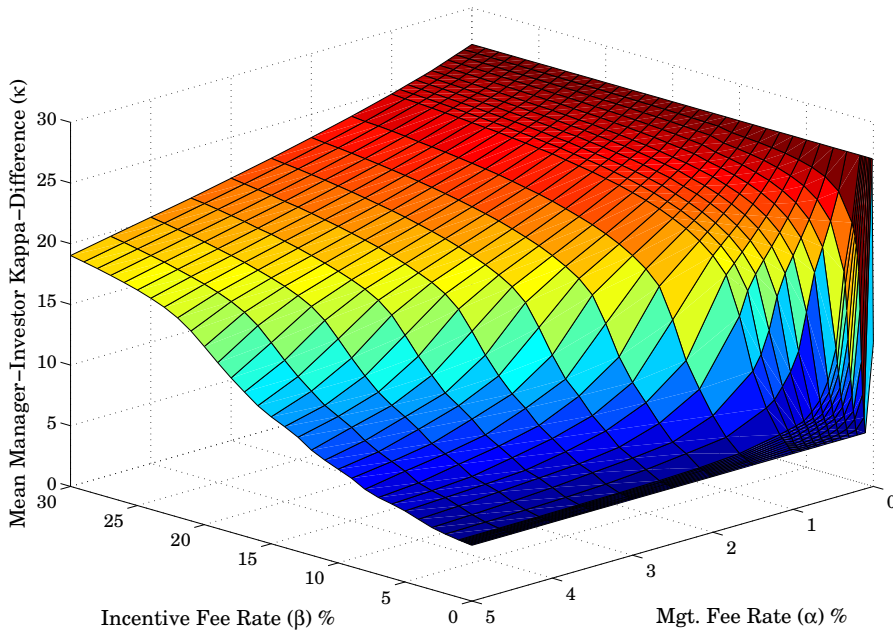

Figure 10: The mean manager-investor kappa-difference $M(\alpha, \beta, 0)$ for a set of $\alpha \in [0, 0.05]$ and $\beta \in [0, 0.30]$.

The graph suggests that M quickly grows with increases in β. Furthermore, management fees are shown to be an inhibitor to this relation, but a weak one at that. Management fees accordingly do not appear to be an effective tool for interest alignment. The necessity of major jumps in charged management fees would certainty also contribute to this view.

By setting $S = 0.1$ and rerunning the estimation, it is further verified that the behavioural effect of fund shares strongly dominates that of management fees.

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[46]This is particularly true if investors monitor fund performance with relatively high frequencies.

With this setting, the influence of management fees is insignificant.

I now turn to look into the incentive effects of managerial fund share by graphing $M(0.02, 0.20, S)$ in Figure 11.
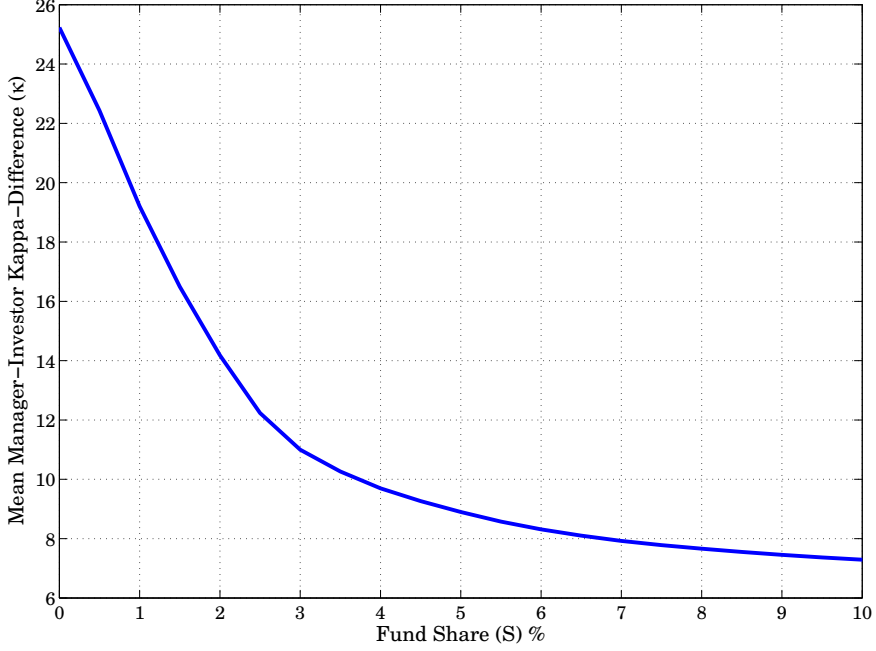

Figure 11: The mean manager-investor kappa-difference $M(0.02, 0.20, S)$ graphed over $S \in [0, 0.1]$.

The graph goes on to confirm the managerial ownership rate as a strong alignment tool, as its introduction into the fund structure has an immediate and profound effect on managerial behaviour. A relatively low rate therefore goes a long way in reducing adverse risk-taking. With 3 % fund ownership, almost all of the positive effects of managerial share have been used up, and beyond 10 % the benefits from fund share increases are negligible.

Note that with the current assumptions, $M(\alpha, \beta, S)$ can never be zero because the manager has a liquidation boundary, which the investor does not. As hinted in the graph, its minimum value, at $S = 1$, is approximately 6.

Due its equal weighting property, $M(\alpha, \beta, S)$ measures misalignments irrespective of the probability attributed to entering each state. This property is a two

edged-sword: On one hand, it ensures a certain neutrality to the measure. On another hand, one may ask why investors should be concerned with unfavorable managerial risk-taking in situations that are unlikely to ever take place. Another disadvantage is the measure's non-monetary dimension.

7.2 Costs of Hedge Fund Investing

The investor may be best served with a probabilistic measure that captures possible utility-losses. It would then be possible to assess the economic significance of adverse managerial behaviour, thereby giving an answer to how much excess return a hedge fund should deliver to justify its risk-taking. The following presents the procedure for such a measure, as well as illustrative results.

The mean terminal time investor utility can be computed with Monte Carlo methods: A large sample of stochastic paths through the fund value grid is generated, regulating risk at each time step in accordance with the predetermined optimal kappa matrix. Each sample path ends up with a terminal time investor-utility U_I; the occurrence of which is counted.[47] Thus, with a large enough sample, the expected investor-utility $E(U_I)$ can be estimated.

This simulation procedure is performed twice, using either investor- or manager risk-taking policies, represented by their respective optimal kappa matrix, in each run. The difference between the two expected investor-utilities is then the investor's loss (or gain) in utility caused by managerial control. The economic significance of the utility-loss is given by the investor's loss in certainty equivalent, as found with (16). I now take on the usual TRP-manager/PT-investor assumption and present some interesting results.

The investor's expected utility using his/her own risk policies - is estimated to $E(U_I) = 0.0211$. Conversely, with managerial control and *zero* managerial ownership, the same number is found to be -0.1740. These are quite surprising results. Not only is there a discrepancy in preferred risk-taking; the investor's expected utility is even negative due to managerial incentives. In other words, the investor should be discouraged from placing an investment at all. Remember that this happens in spite of the higher expected return yielded by managerial risk-preferences. As such, the investor appears to prefer a relatively low risk-profile to a relatively

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[47]As fund values are discretized, terminal utilities are too.

higher expected return.

To put the consequences of the manager's adverse behaviour into perspective, consider an investor whose initial hedge fund investment amounts to $10m. The loss in certainty equivalents is then an estimated $ 670,000. The manager would thus have to achieve a pure alpha return of 6.7 pp *in excess of current expected returns* through managerial expertise, technology and effort to justify the fund's risk-level to investors. This corresponds to a 40 % raise in Sharpe ratio. On top of this, the expected cost of fees comes to an additional 2.6 pp, resulting in a total cost at 9.3 % of the initial investment.[48]

By running the simulation for a variety of fee structures a study corresponding to that which was done for the mean kappa-difference of Section 7.1 can be done.[49] The results show similar tendencies: The utility divergency increases with the incentive fee rate. As was also found with previous analysis, the management fee rate moderates this propensity by raising the point of impact. Illustratively, by raising the management fee rate from 0 to 5 %, the major leap in utility divergency occurs at an incentive fee rate of about 15 % rather than immediately. For practical purposes however, management fees again appear to confirmed to be too minor to make it interesting for use in alignment issues.

With that, I once more turn to managerial fund share as a restraining instrument. Figure 12 shows the investor's costs as a function of S, before (solid) and after fees (dotted). The results suggest that with an 8 % fund share or higher, the manager-investor conflict of interest is completely eliminated.[50]

The reader is encouraged to remember that since this analysis is based on expectation values, the regions of the state space that exhibit the stronger deviances are averaged out by the areas with weaker deviances. The fact the that deviations in kappa-preference across the state space sometimes pull in opposite directions also contributes to this effect.

As a final remark, it should be remembered that there have been put no practical limitations on the kappa of the manager. Complications such as transaction

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[48]The methods for this calculation are given later.

[49]Fund share is set to zero for the same reasons as there.

[50]At that point, the manager's optimal initial day risk, is zero. As can be seen in Figure 7, this locks him/her into the so-called valley for the remainder of the period. Since the PT-investor's optimal risk is also zero in that area, the conflict principally speaking no longer exists. This "lock-in" tendency can be avoided by forcing the kappa to a minimum of 1, but the resulting changes in certainty equivalent-loss are very minor, due to the probability of large movements still being quite low.
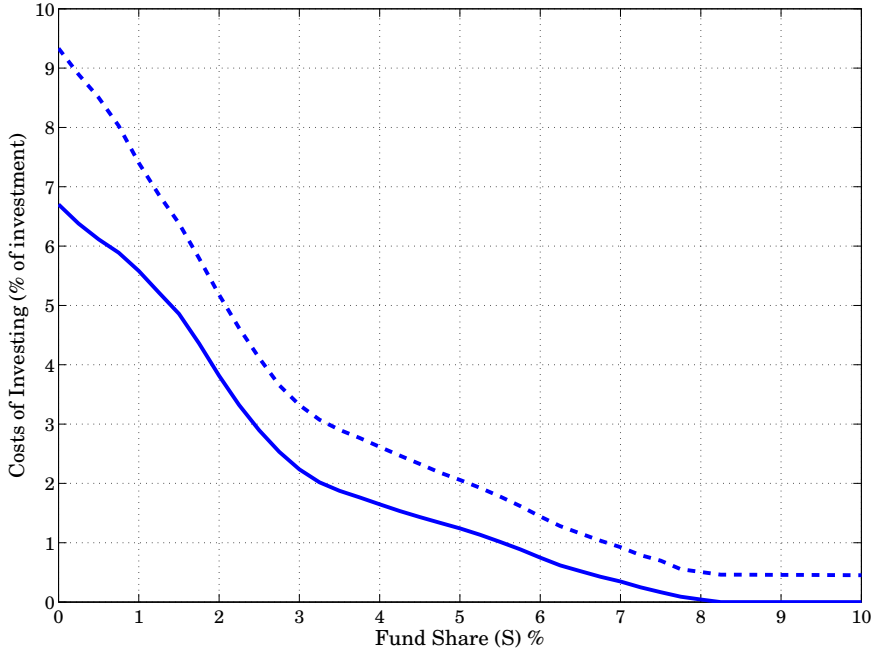
Figure 12: The solid curve graphs the expected quarterly loss in certainty equiv-
alent, measured in percentage of initial investment. The dotted curve augments
the certainty equivalent-loss with expected fee costs, thereby giving the total costs
of investing. S is capped at 10 % as both curves go flat from 8 % and onwards.
Incentive fees are computed as in Section 8.

costs and a roof on leverage would thus be expected to limit the manager-investor
conflict of interest somewhat.

8   Incentive Fee Option Value

For hedge fund investors it is useful to know the isolated cost of the incentive fee
option. Using the TRP-manager's optimal kappa matrix and Monte Carlo meth-
ods, I estimate the quarterly ATM option value as a function of fund share S for
β = 0.1, 0.2, 0.3 and otherwise standard parameters (Figure 13). As a point of
reference, the option value for a manager with a constant kappa of 1 and $\beta = 0.2$
is also graphed.

As expected due to the close connection between option value and managerial be-
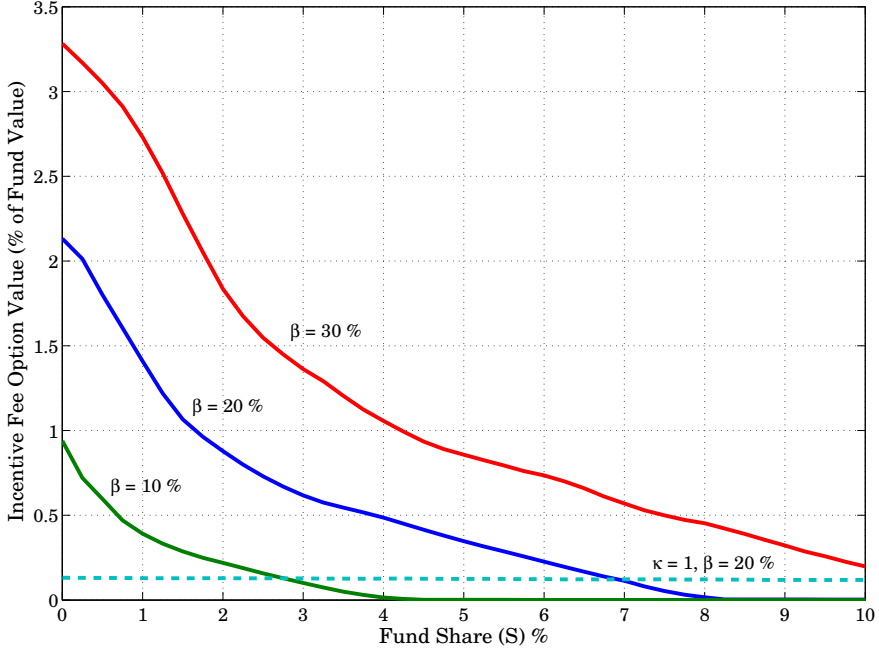
Figure 13: The at-the-money 10, 20 and 30 % incentive fee option values as a percentage of initial AUM for the interval $S \in [0, 0.1]$. Additionally, the dotted curve graphs the corresponding value for a manager with a 20 % incentive option fee and a constant kappa of 1.

haviour, the graph's curves are quite similar to that those of Figure 12. They show a sharp decline in option value with increasing fund share. For comparison, the management fees with zero managerial fund share amount to 0.5 % of AUM.

9   Concluding Comments

The novel TRP model and the fundamental numerical approach of this paper provide the means to analyse the dynamic risk-taking of a realistic hedge fund manager. It extends current research by combining contemporary loss-aversion, a new implementation of the concept of managerial fund liquidation aversion and an accurate discrete time framework.

With a single-period horizon, the TRP-manager generally portrays more complex and dynamic behavioural patterns than the corresponding loss- or risk-averse

manager. An absence of fund ownership inclines the manager to take risks according to a monotonically decreasing function of fund value. For all but exceptionally high fund values, this manager's risk-level is shown to be hazardously high compared to the preferences of a loss-averse investor.

Perhaps the most interesting behaviour however, is observed when the manager has a significant fund ownership. His/her risk-taking then becomes distinctively bimodal. Near the benchmark, when the incentive option is ATM, the manager shows characteristically loss-averse prudence. Similar tendencies are also induced by the manager's liquidation aversion for very low fund values. In the state space regions sufficiently detached from the benchmark and the liquidation boundary on the other hand, the manager dramatically increases the the fund's risk-level. A final interesting and systematic characteristic is the loss-averse *individual's* propensity to increase risk as the terminal time approaches nearer. The observed risky behaviour by managers may provide a reasonable explanation to the relatively low survival rates of hedge funds, as documented by Amin and Kat (2003*b*) and Brown et al. (1999).

In the paper's examination of incentive effects for longer optimization horizons than a single evaluation period, the manager is found to reduce the overall risk-level with longer horizons. This is consistent with the time-decreasing convexity of the incentive option gamma. Importantly however, the risk-taking of a manager with a very long horizon is still found to be excessive compared to that of investors when the option is ITM and the terminal time is near. Although it yields some interesting results, the multi-period model is omitted from extensive consideration because it requires some arguably unrealistic assumptions.

From an external perspective, the fund's risk should principally follow the preferences of its investors. A primary interest should thus be to identify ways to reduce adverse managerial behaviour. I explore this issue using a probabilistic approach estimating the investor's utility-loss from leaving control of his/her assets to the fund manager. By varying fee rates and managerial fund share fraction, I find that the easiest way to achieve manager-investor interest alignment is to increase the latter. In a fund with the typical 2/20 fee setup, a fund share at 8 % of AUM or more would yield near perfect alignment. Some hedge funds already comply with this requirement: The hedge fund sample of Agarwal et al. (2009) has a managerial fund share mean of 7.1 %. However, their bottom quartile is at 0.1 %, thereby implying that a great number of hedge funds use perverse managerial incentives.

Given the relatively high fees of hedge funds, it is clear that investors must believe quite strongly in their ability to deliver performance in excess of the market. My analysis suggests that, depending mostly on managerial fund share and in addition to fees, investors may suffer substantial economic losses from adverse managerial incentives. In the case of zero fund share, the mean total costs of investing are estimated to 9.3 % quarterly.

A common perception expressed by hedge funds is that the incentive fee helps *align* interest with investors.[51] I suggest the strict opposite by pointing it out as the chief root of misalignments. As I see it, there are two basic explanations for this incoherence. The first is given by considering the option contract demanded by the manager as part of the agency problem, rather than its solution. The second implies that my model fails to capture some influential aspect of managerial incentives. One possible such aspect is given by the recent long-term perspective of Levy and Wiener (2013). Another may be the addition of practical limitations such as transaction costs and leverage restrictions.

While this paper has focused on the situation of a hedge fund manager, it is clear that its general approach is applicable to any process where an individual holds control over a stochastic process determining that same individual's utility. With some additional model complications, mutual fund managers and some bank trader types easily fit the bill. By conceptualizing a firm's more or less risky value creation process as being manipulated by a stock option holding CEO, the methodology could also be used to analyse his/her incentives.

In relation to mutual fund managers, there are now especially good reasons to apply the model perspective, as a growing number of them add incentive options to their compensation structure.[52] This aptly raises concerns regarding their risk-taking. These concerns are somewhat reduced by the stringent leverage regulations put on mutual funds,[53] but a mutual fund can often circumvent these limitations to some degree through derivative investments.[54]

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[51]This notion is supported by Carpenter (2000).

[52]An example is the Norwegian Skagen Fondene, which recently introduced a 20 % high-water mark to some of their funds.

[53]In the US for example, there is a 300 % asset coverage requirement.

[54]Given that the fund is allowed to invest in them.

A Variable Notation

Table 1: Variable notation

$W(t)$	Fund manager wealth at time t
$F(t)$	Assets under management (AUM)/fund value at time t
$B(t)$	Benchmark at time t
Z	External wealth of the manager at terminal time T
$\kappa(F,t)$	Risky asset weight/kappa
$X_1(t)$	Riskless asset value
$X_2(t)$	Risky asset value
$z(t)$	Brownian motion process
θ	PT/TRP status quo
λ	TRP wealth liquidation point

B Standard Parameters

The table at the end of this appendix presents the standard case parameters used for illustrative results throughout the paper. The rationales behind chosen parameters for asset returns and the liquidation boundary are given in the following discussion.

To achieve realistic results it is essential to find reasonable values for μ, σ and R_d, which jointly constitute the movements of the risky and riskless asset. In this regard, the Sharpe ratio is very useful. As a measure of risk-adjusted performance, it can be used to calibrate the balance of attractiveness between the risky and riskless asset.

I start the calibration process by setting the annual risk-free rate to 5 %. The daily risk-free rate is then $R_d = 0.05/252 \approx 2 \times 10^{-4}$.

Hodder and Jackwerth (2007) use an annual expected return of 7 % and volatility of 5 % for the risky asset, which together with a risk-free rate at 5 % gives a Sharpe ratio of 0.42. A way to check the rationality behind this choice of parameters is to compare it to the Sharpe ratio of a hedge fund index. A simple analysis of the

Credit Suisse Broad Hedge Fund Index[55] implies an annual 9.05 % mean and 9.48
% volatility.[56] At first glance then, the risky mean and volatility used by Hodder
and Jackwerth both appear to be misestimated. However, the index does in fact
have a Sharpe ratio of 0.43,[57] which is almost identical to that of Hodder and
Jackwerth. For the purpose of portfolio selection, the only measure that matters is
the risk-premium to deviation ratio. Therefore, the actual means and volatilities
of the two assets are irrelevant, as long as they jointly yield a reasonable Sharpe
ratio.

On an additional note, it is interesting to consider the effect that different market
regimes have on the typical hedge fund Sharpe ratio. The Credit Suisse index'
statistics includes data from the Dot-com bubble and the 2007/2008 financial cri-
sis. Using a Sharpe ratio of 0.43 would thus assume that the model hedge fund
manager takes the possibility of future market extremities into account. During
relatively calm periods, higher Sharpe ratios are likely.[58]

There are a few other aspects worthy of consideration when choosing an appropri-
ate Sharpe. The ratio assumes normal returns, arguably making it unsuitable for
use with hedge fund returns, as is also discussed in the main paper. The proper-
ties of hedge fund returns generally bias the Sharpe ratio high, thus calling for a
lower setting of Sharpe than what is empirically measured (e.g. Brooks and Kat,
2002). As a final remark, all indices, including the Credit Suisse index used here,
are prone to diversification effects, thus biasing the volatility low.[59] This also calls
for a conservative Sharpe choice.

In light of the aforementioned perspectives, the asset settings of Hodder and Jack-
werth (2007) are replicated for the standard parameters. This gives a risky asset
daily mean and standard deviation of

$$\mu = 0.07/252 \approx 2.8 \times 10^{-4}$$
$$\sigma = 0.05/\sqrt{252} = 3.1 \times 10^{-3},$$

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[55]A global index comprised of over 9000 hedge funds, beginning in January 1994. Monthly data and
additional information can be found at www.hedgeindex.com.

[56]Assuming normal and independent returns.

[57]Assuming the same risk-free rate of 5 %.

[58]For instance, in the period 2002-2007; the Credit Suisse index had a 9.54 % mean, 3.92 % volatility
and a Sharpe of 1.15 (given a 5 % risk-free rate). Brooks and Kat (2002); Brown et al. (1999) reveal
similar findings.

[59]This is particularly true for the *broad* Credit Suisse index.

with the assumption of normal and independent returns.

The predetermined liquidation boundary used by Hodder and Jackwerth (2007) is at 50 % of the benchmark. Goetzmann et al. (2003) do their computations with a liquidation boundary at 0, 50 and 80 % of the individual high-water mark. They argue that most hedge fund investors would retract their funds following a fall of 15 to 25 %, but give no empirical data to support their estimation. Based on the aforementioned two papers and my quarterly evaluation period I set the predetermined and constant liquidation boundary L to 60 % of the benchmark in the standard case.

Table 2: Standard parameters

Standard Parameters

Fundamental Model Setup

Period length & terminal time	T	63 market days
Initial fund value	F(0)	1
Fund value grid step	C	$\ln(10)/199 \approx 0.0116$
Grid, lowest theoretical value	-	0.1
Grid, fund value height	G	301
Grid, practical lower value	-	0.60
Grid, practical upper value	-	2.52
Possible risky asset weights/kappa	$\{\kappa\}$	{0, 1, 1.5, 2, 2.5, 3, 3.5, 4, 5, 6, 7, 8, 10, 20, 30, 40, 50, 60, 70, 80, 100, 120, 140, 200, 250}

Asset Return and Volatility

Risky asset, annual expected returns	μ_a	0.07
Risky asset, daily expected return	μ	2.8×10^{-4}
Risky asset, annual standard deviation	σ_a	0.05
Risky asset, daily standard deviation	σ	0.0031
Risk free annual rate	R_a	0.05
Risk free quarterly rate	R_q	0.0125
Risk free daily rate	R_d	2×10^{-4}

Fund Structure

Management fee rate, annual	α	0.02
Management fee rate, quarterly	α_q	$0.02 \times (63/252) \approx 0.0050$
Incentive fee rate, annual	β	0.20
Incentive fee rate, quarterly	β_q	$0.20 \times (63/252) \approx 0.0500$
Benchmark	B	$R_d(T-1) \approx 1.0125$
Managerial Fund Share	S	(no std. value)

(. . .)

Standard Parameters

(. . .)

CRRA-Utility Model

CRRA gamma	γ	4

Loss-Aversion Model

Loss curvature constant	γ_1	0.88
Gain curvature constant	γ_2	0.88
Loss-aversion constant	A	2.25
Initial manager wealth	-	$SF(0)$

Two-Reference Point Model

Liquidation impact coefficient	τ_λ	$-A\theta^{\gamma_1}/\lambda$
Loss impact coefficient	τ_-	$-A = -2.25$
Gain impact coefficient	τ_+	1
Loss curvature coefficient	δ_-	$\gamma_1 = 0.88$
Gain curvature coefficient	δ_+	$\gamma_2 = 0.88$

Miscellaneous

Liquidation boundary	L	$0.6B(T)$

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