NTNU – Trondheim
Norwegian University of
Science and Technology

# A Comparison of Selected Real Options Valuation Approaches to the Net Present Value Method for an Investment Opportunity in Onshore Wind

An analysis of the specific case of Stokkfjellet
Wind Farm, Sør-Trøndelag, Norway

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| Oppgavetekst/Problembeskrivelse<br>This thesis evaluates how selected valuation methods can be used at an early stage of development to value a wind park investment opportunity, and compares the pros and cons of each method. The different methods are applied to the specific case of Stokkfjellet Wind Farm which is owned by TrønderEnergi AS. | |
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# Preface

This master's thesis has been prepared at the Norwegian University of Science and Technology, Department of Industrial Economics and Technology Management in the spring of 2014. The report is part of the specialisation project in TIØ4550 Financial Engineering and accounts for 30 ECTS credits.

We wish to thank our teaching supervisors, Associate Professor Verena Hagspiel and Professor Sjur Westgaard, for helpful discussions and guidance throughout the semester. We would also like to express our gratitude to TrønderEnergi AS for providing us with relevant data and industry insight.

Trondheim, June 10, 2014

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## Executive Summary

The industry standard among renewable energy companies is to value projects using the traditional discounted cash flow method. Unfortunately, discounted cash flows do not incorporate the value of flexibility. This makes the technique unsuitable for valuing investment opportunities related to wind farm development. Such investments are often highly uncertain, making flexibility valuable.

In this thesis we demonstrate the difference between some selected valuation methods, and discuss their suitability for valuing an investment opportunity in a wind power project. The focus is on real options analysis, and our hypothesis is that the use of real options valuation methods will improve the quality of the information available for decision makers, ultimately improving the quality of investment decisions in wind power.

The specific wind power project that we value is owned by TrønderEnergi AS, a renewable energy company based in Sør-Trønderlag county in Norway. The project is called Stokkfjellet Wind Farm, and the site is located in Selbu, a municipality in Sør-Trøndelag. TrønderEnergi has applied for a concession to build, and is expecting a decision by the Norwegian Water Resources and Energy Directorate (NVE) by the end of the summer of 2014.

To investigate this investment opportunity, we first consider the practical conditions affecting the investment decision. We move on to briefly present some theory of discounted cash flow analysis and the contingent net present value (NPV) approach. Then, a thorough introduction to real options theory and our assumptions is given, before we explain in detail the application of the binomial lattice and Monte Carlo valuation methods, which take a real options perspective. We next perform the actual valuations, and analyse the results from these as well as the methods used. The conclusions to be drawn are at last discussed.

For the development of the Stokkfjellet project, we assume there is uncertainty about events and markets. With events, we mean for example the outcome of the licence application processing by NVE. By market uncertainty we refer to the future development of the prices of power and tradable green certificates. Both prices influence the income from a wind farm directly.

For the real options valuations, we have modelled the power price as a mean-reverting process, and the price of tradable green certificates as a geometric Brownian motion. Modelling the electricity price as a mean-reverting process is an advantage as compared to modelling it as a geometric Brownian motion, since electricity prices show a clear tendency to revert back to a long-term level, rather than wander far away from its mean like a random

walk. The parameters of the stochastic processes have been estimated from historical electricity price data from January 1 2001 to December 31 2013 for the NO3 Elspot Market available at Nord Pool Spot, and historical prices of tradable green certificates from February 19 2003 to March 5 2014 as listed by NVE. The code developed for the real options valuations has been written in Visual Basic for Applications (VBA) for Excel, and is included in Appendices A-D in this thesis.

Throughout the report, we carefully present all the theory needed in order to perform the calculations for the valuations. For the most intricate mathematical discussions, is assumed that the reader is familiar with calculus, some basic statistics and stochastic processes. We have included a brief introduction to the concept of stochastic calculus in Appendix F.

Other than discuss each method theoretically, we also compare the methods for practical applications. We find that the most severe problem with discounted cash flows is that the method cannot handle the value of flexibility incorporated in wind projects, which could cause us to grossly undervalue a project when we use this method. The contingent NPV method succeeds in handling the uncertainty related to events, but is not by itself a sufficient tool for solving real options because of problems related to calculating the appropriate discount rate in every stage of the project. The binomial lattice approach avoids this problem by using so-called risk-neutral probabilities, which is a more accurate fashion than choosing a discount rate. The binomial lattice permits us to value the project as a compound option, which is beneficial considering that investments in the Stokkfjellet project are done in stages. This method handles both event and market uncertainty. The Monte Carlo valuation is a simple and flexible method that also exploits risk-neutral pricing. However, it is not straightforward to use Monte Carlo simulations to value compound options. This is a drawback with this method and we have to model the investment opportunity as a single European call option when using this method. In addition, this method does not cope well with event uncertainty. There will always be benefits and drawbacks with every method, thus it is in general better to refer to more than one valuation method when making important investment decisions.

The real option we decide to value is the option to invest. We model the real option as a European compound call option and a single European call option for the binomial lattice and Monte Carlo methods, respectively. The investment opportunity is valued at 29 MNOK with the binomial lattice approach, and 47 MNOK with the Monte Carlo method. The difference in the values assigned can be explained by the different option types used to model the project, among other factors. The value of the project has increased from a negative value of $-21$ MNOK for the contingent NPV. Thus, we demonstrate that valuing the investment opportunity while accounting for flexibility assigns the project a positive value whereas a contingent net present value approach assigns the investment opportunity a negative value. This is critical information that should be considered before making a decision about investing. We therefore recommend that a real options approach to valuing wind projects is adopted by TrønderEnergi.

In addition to our conclusions and the recommendations that we suggest in this thesis, we have created a program that is meant to be used by TrønderEnergi for valuing Stokkfjellet Wind Farm as well as other projects that are subject to similar risks and flexibility. The program is easy to use, flexible and effective, and constitutes an important part of the work done in relation to this thesis. This program is to be handed over to TrønderEnergi at the time when this thesis is submitted.

# Sammendrag

Det er i dag vanlig blant fornybare energiselskaper å verdsette prosjekter ved hjelp av den tradisjonelle diskonterte kontantstrømsmetoden. Dessverre inkluderer ikke diskonterte kontantstrømmer verdien av fleksibilitet. Denne teknikken er derfor å anse som uegnet ved verdsettelse av investeringsmuligheter knyttet til utvikling av vindkraftverk. Dette er prosjekter som typisk har mye usikkerhet knyttet til seg, noe som gjør fleksibilitet verdifullt.

I denne masteroppgaven vil vi demonstrere forskjeller mellom noen utvalgte verdsettelsesmetoder, og diskutere hvorvidt hver enkelt metode er egnet for å verdsette en investeringsmulighet i et vindkraftprosjekt. Hovedfokuset vil være på realopsjonsanalyse. Vår hypotese er at man ved å ta i bruk realopsjonsanalyse som et verktøy i verdsettelsen kan bedre beslutningsgrunnlaget, noe som igjen vil føre til at kvaliteten på beslutninger forbedres.

Prosjektet som blir studert i denne oppgaven eies av TrønderEnergi AS. TrønderEnergi er et fornybart energiselskap med hovedkontor i Trondheim. Det aktuelle vindkraftverket er lokalisert i Selbu kommune i Sør-Trøndelag og heter Stokkfjellet vindpark. TrønderEnergi har søkt Norges Vassdrag- og Energidirektorat (NVE) om konsesjon til å bygge, og det er ventet at svar vil foreligge i løpet av sommeren 2014.

Vi starter vår vurdering av investeringsmuligheten med å se på økonomiske, politiske og tekniske forhold knyttet til prosjektet. Deretter gjennomgår vi teori knyttet til de to tradisjonelle verdsettelsesmetodene diskontert kontantstrømsanalyse og beslutningstreanalyse. Vi går så nøye til verks med å introdusere realopsjonsteori og forklare våre antakelser. Vi forklarer deretter metodene for binomisk opsjonstre og opsjonsprising ved hjelp av Monte Carlo-simuleringer i et realopsjonsperspektiv. Selve verdivurderingen utføres systematisk for hver metode. Dette blir etterfulgt av en undersøkelse av resultatene, en sammenligning av metodene samt en generell analyse av arbeidet. Rapporten avsluttes med at vi drar de konklusjoner vi finner passende ut ifra våre funn.

Vi har antatt at Stokkfjellet vindpark under utvikling er eksponert for både hendelsesusikkerhet og markedsusikkerhet. Med hendelsesusikkerhet menes for eksempel usikkerheten knyttet til utfallet av konsesjonssøknaden som for tiden behandles av NVE. Markedsusikkerhet er knyttet til usikkerheten rundt fremtidig utvikling i kraftprisen og prisen på grønne sertifikater. Begge disse prisene påvirker inntektene til et vindkraftverk direkte.

I realopsjonsverdsettelsene har vi lagt til grunn en stokastisk kraftpris som reverterer mot et langsiktig likevektsnivå, og en stokastisk pris på de

grønne sertifikatene som er antatt å følge en geometrisk Brownsk bevegelse. Det å modellere strømprisen som snittreverterende heller enn en geometrisk Brownsk bevegelse er en klar fordel. Grunnen er at prisen på elektrisitet har en tydelig tendens til å falle tilbake til et langsiktig snittnivå, i motsetning til geometrisk Brownske bevegelser som kan vandre langt bort fra sin forventede verdi som en såkalt tilfeldig gang (random walk). Inngangsparameterne til prosessene er estimert fra historiske prisdata. Prisene for elektrisitet gjelder fra 1. januar 2001 til 31. desember 2013 for NO3 Elspot Marked tilgjengelig fra Nord Pool Spot, mens vi for grønne sertifikater anvender prisdata fra 19. februar 2003 til 5. mars 2014, som oppført av NVE. Programkoden som er utviklet for verdsettelsene er skrevet i Visual Basic for Applications (VBA) for Excel, og er inkludert i vedlegg A-D i denne avhandlingen.

Gjennom hele rapporten vil vi nøye presentere all teori som trengs for å utføre beregningene for de ulike verdsettelsene. For de mest intrikate matematiske diskusjonene forutsettes det at leseren er kjent med kalkulus, noe grunnleggende statistikk samt stokastiske prosesser. Vi har lagt ved en kort innføring i stokastisk kalkulus i vedlegg F.

I tillegg til å diskutere de ulike metodene på et teoretisk nivå, sammenligner vi også metodenes egnethet for praktiske anvendelser. Den største nedsiden til den statiske nåverdimetoden er at fleksibiliteten innlemmet i vindkraftprosjekter ikke blir håndtert. Dette kan føre til grove feil som gjør at man undervurderer verdien til prosjektet. I beslutningstreanalysen tar man hensyn til hendelsesusikkerheten, men dette er ikke i seg selv et tilstrekkelig verktøy for å regne på realopsjoner. Grunnen er at man vil støte på alvorlige problemer knyttet til beregning av diskonteringsraten i alle faser av prosjektet.

Binomisk opsjonstre er en praktisk metode for å løse sekvensielle sammensatte opsjoner, og ved bruk av denne metoden får vi tatt hensyn til både hendelsesusikkerhet og markedsusikkerhet. Opsjonsprising ved hjelp av Monte Carlo-simuleringer er derimot ikke velegnet for sammensatte opsjoner. Metodens styrke for denne anvendelsen er at den er enkel og fleksibel, og den tillater inkludering av flere stokastiske prosesser. Derfor behandler vi investeringsmuligheten som en enkel europeisk kjøpsopsjon i denne verdsettelsen. Monte Carlo-metoden tar kun hensyn til markedsusikkerheten.

Man vil kunne finne positive og negative sider ved alle verdsettelsesmetoder, og vi mener derfor at det på generell basis er fordelaktig å benytte seg av flere metoder for å sikre et solid beslutningsgrunnlag for viktige investeringsbeslutninger.

Realopsjonen vi velger å verdsette er opsjonen til å investere stegvis, altså muligheten til å revidere investeringsbeslutningen ved flere anledninger og eventuelt forlate prosjektet underveis. Dette modelleres ved hjelp av en europeisk sammensatt kjøpsopsjon i et binomisk tre, men forenkles til en enkel europeisk kjøpsopsjon i Monte Carlo-simuleringen. Opsjonsverdien vi finner er 29 MNOK med binomisk opsjontre-metoden, mens verdien settes til 47 MNOK når vi verdsetter opsjonen med Monte Carlo-simuleringer. Forskjellen i beregnet verdi skyldes blant annet at den sammensatte opsjonen alltid vil få en lavere verdi siden man trekker fra flere mindre investeringskostnader i utviklingsfasen av prosjektet. Beslutningstreanalysen som bruker diskonterte kontantstrømmer verdsetter prosjektet til den betydelig lavere summen −21 MNOK. Vi illustrerer dermed at å verdsette dette prosjektet med og uten fleksibilitet resulterer i to ulike anbefalinger for investeringsbeslutningen: Den tradisjonelle metoden konkluderer med å forlate prosjektet, mens metodene som inkluderer fleksibilitet antyder at man bør investere. Dette er viktig informasjon som burde foreligge når investeringsbeslutninger skal tas, og vi anbefaler derfor at TrønderEnergi tar i bruk realopsjonsmetoder ved verdsettelse av vindprosjekter.

I tillegg til selve oppgaven og det den har å tilby av konklusjoner og anbefalinger, har vi også utviklet et program som er ment å brukes til å verdsette prosjekter med tilsvarende egenskaper som Stokkfjellet vindpark. Programmet er lett å bruke, fleksibelt og effektivt, og utgjør en viktig del av arbeidet som er lagt ned som del av denne oppgaven. Programmet overleveres til TrønderEnergi på det tidspunktet denne oppgaven leveres inn.

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# 1 Introduction

With the growing international awareness to the development of more sustainable societies and the protection of the environment, the transition to renewable sources of energy is happening in many corners of the world. Furthermore, the high variability in fossil fuel prices and the pressure from governments to reduce dependence on conventional energy sources has contributed to increase the share of renewable energy sources in the global energy mix. In many countries, onshore wind power has established itself as the most mature and commercially viable renewable technology.

## 1.1 Background

Through the European Economic Area (EEA) cooperation, Norway is committed to increase its share of renewable energy production from 59.9 per cent in 2005 to 67.5 per cent in 2020. Norway is to achieve this goal by investing in new renewable electricity production, ultimately increasing the annual production of renewable power to the goal of 26.4 TWh. Due to low power prices, the government stimulates increased investment in renewable energy through a common tradable green certificates market with Sweden. The technology neutral market for tradable green certificates (TGC's) is a means of reaching the goal of 26.4 TWh of renewable power production by 2020.

## 1.2 Motivation

Wind power producers must account for long-term uncertainty in electricity prices, TGC prices and other volatile factors that affect the decision to invest. Due to the long lifetimes of such projects and the high degree of uncertainty, there is flexibility in managerial decisions as new information evolves. Flexibility allows managers to delay, rush or cancel investments and is thus of value to a project owner. Traditional net present value (NPV) methods that rely on discounted cash flows do not capture this value and are consequently not suitable for these investment decisions. In order to capture the value of such flexibility, it can be argued that a real options perspective is appropriate.

This thesis evaluates the applicability of different valuation methods for valuing an investment opportunity such as one TrønderEnergi AS currently holds with Stokkfjellet Wind Farm. TrønderEnergi have expressed that they wish to broaden their understanding for project valuation methods that could serve as an alternative to traditional discounted cash flow methods, with emphasis on real options analysis. This thesis demonstrates how these methods can be performed in practice, and assesses the suitability of two real options valuation methods as compared to two more traditional methods. We also create a program that is meant to be used by TrønderEnergi for valuing projects with similar risk characteristics as the project evaluated here.

To investigate the nature of the different valuation methods, we will apply them to the case of Stokkfjellet Wind Farm, an investment opportunity that TrønderEnergi currently holds in Sør-Trøndelag, Norway. This project is fully owned by TrønderEnergi, and the application to build is currently being processed by NVE.

The purpose of our work is to illustrate how real options analysis can be a useful tool in estimating the value of a certain wind project, and to highlight the differences between the traditional discounted cash flow approach and real options methods that incorporate the value of managerial flexibility. There is an extensive amount of literature available that treats the theoretical aspect of real options analysis; however, there seems to be a lack of studies demonstrating how this can be done in practice. Our contribution to the literature is thus that we illustrate through an actual case study how real options methods can be used to value a wind project, and what advantages and disadvantages are associated with the various approaches. One important contribution is a discussion on what characteristics make different valuation methods convenient for which purposes. We address practitioners who are in the position of analysing these investment opportunities in TrønderEnergi.

It is imporant to note that our goal is not to estimate the parameters needed for the valuations as accurately as possible. Rather, we discuss how these can be determined and assess the project value's sensitivity to some of the most important input parameters. Instead of going through the work of estimating parameters without the use of confidencial information, we develop a flexible valuation program where users can input their own parameter estimates.

Our hypothesis is that the use of real options valuation methods will increase the quality of the basis for decision making, when considering investment opportunities in onshore wind projects under development. This work was requested by TrønderEnergi, and the research has been shaped according to their preferences. Accordingly, relevant discussions will take their perspective.

## 1.3 Overview of chapters

The specific case study that we consider in this work is presented in Chapter 2. This is meant to provide a context and financial environment for the valuation. We briefly cover the theory of the traditional valuation methods that use discounted cash flows in Chapter 3. In Chapter 4, general real options theory is covered, and we discuss some important assumptions needed for the valuations. We then get more specific in Chapter 5, and describe carefully relevant theory and literature of the binomial lattice and Monte Carlo methods. The actual valuations of the project are carried out in Chapter 6. We start the chapter with determining the static NPV of the operational phase of the park, which is used as input for the other valuation methods. We then move on to the contingent NPV method, the binomial lattice approach and the real options valuation by Monte Carlo simulations, before the results are discussed. In Chapter 7 we analyse the results and compare and evaluate the methods used. This discussion will substantiate the conclusions drawn in Chapter 8. We focus on discussing the applicability, usefulness and precision of the methods as a practical tool for TrønderEnergi. We do provide some advice, but it is left up to TrønderEnergi to decide which methods they wish to adopt, if any.

# 2 Case Study Conditions

In order to make an informed investment decision, all important aspects of the relevant project must be considered. This chapter contains a discussion of different factors that together form the characteristics of the project and the investment decision. The Nordic power market, the subsidy scheme, the conditions for developing onshore wind power, company characteristics of TrønderEnergi AS and the details of Stokkfjellet Wind Farm are presented in detail below. This information forms the basis for the choice of methods selected for the valuation of the project.

## 2.1 The Nordic power market

Norway is part of a common Nordic power market. The electricity spot price is determined through trading on this Nordic power market, called Nord Pool Spot AS. Standardised financial contracts on future power prices are also traded on the exchange. Such financial contracts are offset by the spot price on Nord Pool Spot. Financial and physical contracts can also be traded on the OTC (over-the-counter) market, meaning that the participants trade directly between each other irrespective of any standardised marketplace.

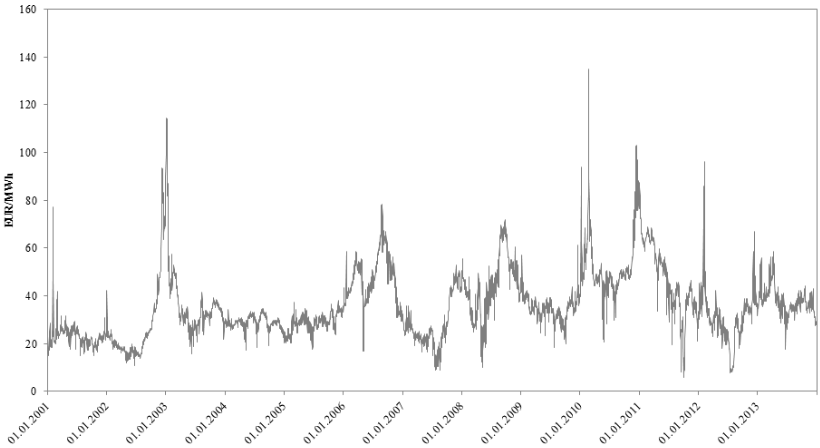



**Figure 1:** Daily historical real prices of power on the NO3 Elspot Market from January 1 2001 to December 31 2013. *Data source: Nord Pool Spot (2014).*

Forces of supply and demand establish the price on the exchange. The more power available, the lower the price will be, while a power shortage will put an upward pressure on the price. In the short-term, supply and demand are a result of the anticipated spot price, precipitation, temperature, reservoir levels, the marginal price of different production technologies, bottlenecks in the power grid, etc. In the long-term, expectations and ultimately power prices are determined by how much power generating capacity and transmission lines are built relative to the growth in consumer demand. The development of the regulatory conditions may also play an important role in the long-term. The historical daily electricity prices for this market is displayed in Fig. 1.

## 2.2 The tradable green certificates system

In 2012, Norway joined the already existing Swedish market for tradable green certificates. The Swedish market has existed since 2003, and has contributed to the building of renewable electricity generation at a relatively low cost compared to other subsidy regimes. Producers of renewable energy receive one certificate per MWh of power produced. The consumers then buy the green certificates at a rate determined by the government, while the price of a certificate is determined in the competitive market. The

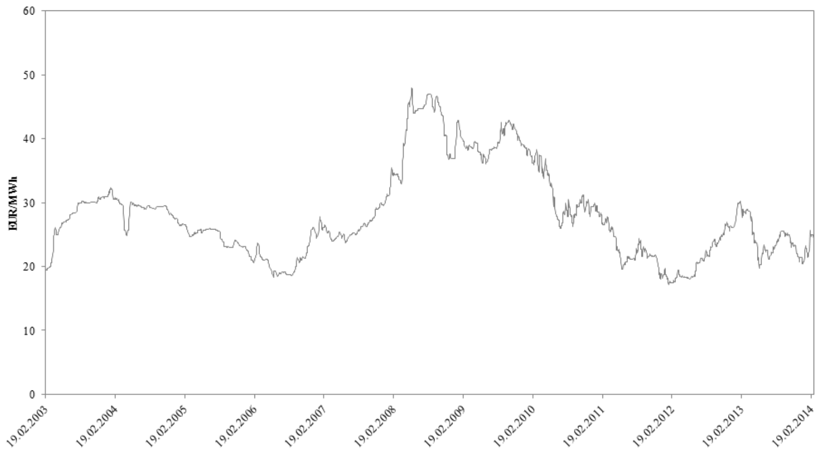



**Figure 2:** Daily historical real prices from the Swedish market for tradable green certificates from February 19 2003 to March 5 2014. In 2012, Norway joined the market to create a common certificate market for Sweden and Norway. *Data source: NVE (2014).*

sum of the price of electricity and green certificates is a means to make marginal renewable energy solutions profitable and increase investments. Daily historical prices are shown on the chart in Fig. 2.

The government makes regulations and issues certificates. There exists both a spot and forward market for tradable green certificates. The producers can speculate and save their certificates, but not lend them. A producer of renewable energy receives certificates for the power they generate over a period of 15 years. In Norway, the plant must be in operation by 2020 in order to receive green certificates, while in Sweden the producer can start operation any time before 2035. However, the certificates market will only be active until 2035. Currently, there is a surplus of certificates in the market due to a rapid development of renewable power generation in Sweden combined with a lower growth in power consumption than expected.

The certificates market is substantially different from the power market due to the absence of a marginal cost of production. Rather, the price formation is determined by factors such as the cost of investing in renewables, the power price, the size of the stock of certificates, expectations about the development of the stock of certificates, as well as political uncertainty.

In the coming years, the level of investment in renewable energy will continue to directly influence the prices in the certificate market. Investments in Sweden after 2020 will also affect the price of the certificates, since certificates will be issued to these producers. Uncontrollable factors such as weather will influence the production from several renewable energy sources, e.g. wind and hydro power, and thus the surplus or deficit of certificates. Looking beyond 2020, there is substantial uncertainty associated with the subsidy scheme. It might be that the system is expanded, modified, or it could be permanently shut down as planned. It is expected that the price will become either very low or very high as we approach the year 2035 in time (TrønderEnergi Kraft AS avd. Handel, 2014).

The recent trend of volatile and falling certificate prices in addition to decreasing power prices, has resulted in a lower sales price per MWh for renewable energy producers (TrønderEnergi Kraft AS avd. Handel, 2014).

## 2.3 Onshore wind power

Globally, wind power is the fastest growing energy technology (Wind Energy Foundation, 2014). The industry has evolved from being dominated by smaller local businesses to large corporations with a combined turnover of hundreds of millions of Euros (Sintef, 2012). By the end of 2012, the total installed capacity in Norway was 704 MW spread over 315 turbines. During that same year, 195.3 MW of new capacity was installed, which is a new domestic record. The average size of a turbine is currently 2.2 MW in Norway (fornybar.no, 2014).

The cost of onshore wind power technology has decreased over time due to a growing market and technological advances. From 1985 to 2004, the cost of production was more than halved. However, the last couple of years the total cost of a wind power plant has increased due to a higher demand for turbines and a general increase in the cost of raw materials, such as steel (OED, 2014). As for other renewable energy technologies, the investment cost represents the vast majority of the cost of developing a plant, particularly the cost of the turbines (fornybar.no, 2014).

The Norwegian Water Resources and Energy Directorate (NVE) has done a feasibility study and concluded that the potential for new wind power development in Norway is between 5800 MW and 7150 MW by 2025. Political conditions will be an important factor in terms of which projects will actually be realised (Waagaard et al., 2008).

## 2.4 TrønderEnergi AS

TrønderEnergi is organised as a group where TrønderEnergi AS is the parent company of a number of subsidiaries. The Group has three business areas: Energy, Networks, and Markets. The Group generates annual sales of approximately NOK 1.7 billion and has around 450 employees. 24 municipalities in Sør-Trøndelag and Nordmøre Energiverk AS own TrønderEnergi. TrønderEnergi Kraft is responsible for power generation and power sales to the wholesale market and produces around 2.1 TWh each year, of which 200 GWh originates from wind power (TrønderEnergi AS, 2014).

TrønderEnergi Kraft has the ambition to increase their production of renewable energy with 1 TWh by 2020. With the approval and construction of Stokkfjellet Wind Farm, a contribution of 243 MWh annually will help to achieve that goal. With an installed capacity of 80 MW, the final investment cost of the project is estimated at a cost of 890 MNOK (TrønderEnergi AS, 2013).

TrønderEnergi Kraft produces 2 TWh of renewable power annually, and 13 per cent of this arises from wind power. This makes TrønderEnergi the second largest wind power producer in Norway (TrønderEnergi AS, 2014).

## 2.5 Stokkfjellet wind park

TrønderEnergi considers the Stokkfjellet wind site to be feasible due to its wind conditions, topography and grid connection. The area is located in the municipality of Selbu in Sør-Trøndelag and covers approximately 5.8 $km^2$ as illustrated in Fig. 3. TrønderEnergi has applied for a concession to build up to 100 MW. The number of megawatts that will be granted will depend on the status of the ongoing applications for the nearby wind parks Eggjafjellet and Brungfjellet (TrønderEnergi AS, 2013). If these parks are also given concessions, an upgrade in the local power grid is necessary, and Stokkfjellet will be granted a capacity of 100 MW. Should one of these other applications not go through, then a new grid connection is not considered necessary by NVE. The current grid connection has limited capacity, thus Stokkfjellet will only get a concession for 80 MW if the grid is not upgraded. We will assume throughout this report that a concession for 80 MW is granted, if any. Key statistics for the project are shown in Table 1.

### 2.5.1 Project progress

In September 2013, TrønderEnergi submitted a licence application for Stokkfjellet Wind Farm including an environmental impact assessment. The application processing, which is done by NVE, is estimated to take about one year. However, recent events have indicated that it is very likely that NVE will grant a licence during the summer of 2014. The reason is that the county, Sør-Trøndelag, has unexpectedly expressed that they support the project.

After the licence has been granted, the procurement phase and further development of the project can commence. However, concessions for 9 out of 10 wind parks are appealed to the Oil and Energy Department (OED), usually by nature conservation associations. There is no reason to believe otherwise will be the case for Stokkfjellet. This will prolong the application processing time with at least one year. Nonetheless, only approximately 1 out of 5 appealed licences are withdrawn. Considering this, the planning of a project normally continues during appeal processing. These statistics are

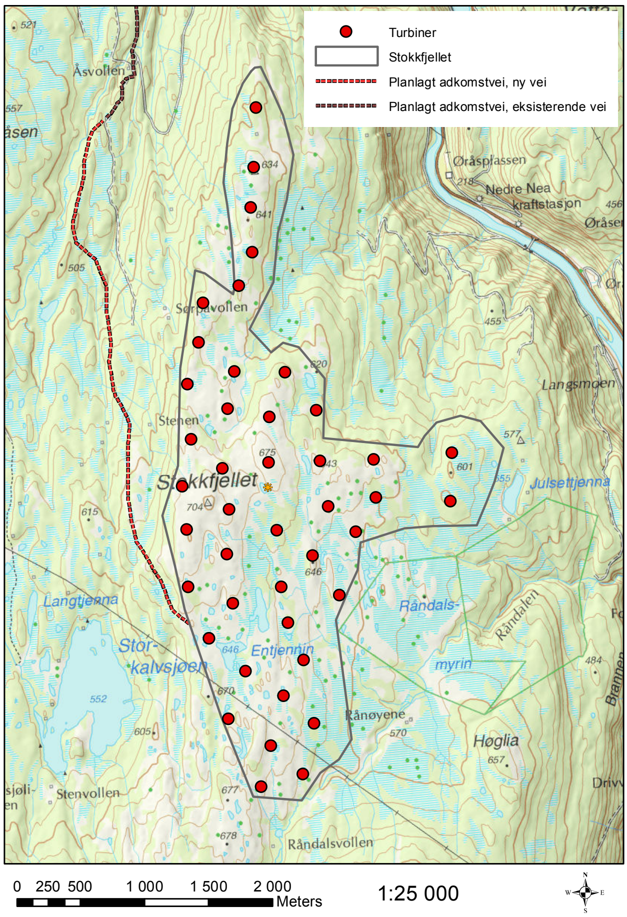



**Figure 3:** Map of the Stokkfjellet site. *Source: TrønderEnergi (2014).*

**Table 1:** Key statistics of the Stokkfjellet Wind Farm project.
*Source: TrønderEnergi AS (2013)*

| **Statistic** | **Value** |
|---|---|
| Hub height | 70-120 m |
| Wing diameter | 70-120 m |
| Nominal effect | 2-5 MW |
| Number of turbines | 20-43 |
| Estimated production (80 MW) | 250 GWh |
| Investment cost (80 MW) | 890 MNOK |
| Average wind speed | 7.8 m/s |

a result of discussions with TrønderEnergi and NVE.

The final investment decision will not be made until the appeal has been undisputedly declined and entrepreneurs have been selected. Only after this can the construction of the park begin. With the county's support, an optimistic estimate is that the final investment decision can be made by the end of 2015. This would give TrønderEnergi two years of flexibility with regards to timing the construction of the park before the issuing of tradable green certificates ends in 2020. Two summers will be required to complete the building of the park. The reason for the season specific timing is that the turbines must be installed during summertime.

Fig. 4 illustrates the projected timeline for the Stokkfjellet project, and also some other important characteristics of the investment decision. TrønderEnergi has performed all cost estimates. The costs listed in the three first stages are related to wind measurements and development costs, whereas the cost of the last stage is related to the building of the wind farm. The probabilities have been set in discussions with TrønderEnergi. The costs will be studied in more detail later.

### 2.5.2 Flexibility in decisions

In the case of Stokkfjellet Wind Farm, the owners have flexibility up until building starts. The most important flexibility is summarised below, and it is assumed that any decisions are revised only at the end of every stage. TrønderEnergi has confirmed that the decision to move on to the next stage is usually only considered after the last stage is completed.

- At the end of every stage, they can abandon the project.

| | Processing of license application | Complaint processing | Choose entrepreneur, make FID | Building |
|---|---|---|---|---|
| Time | 3 months | 12 months | 10 months | 14 months |
| Cost | ~0.5 MNOK | ~5 MNOK | ~4 MNOK | 890 MNOK |
| Probability of success | 1 | 0.8 | 1 | 1 |
| Accumulated probability | 1 | 0.8 | 0.8 | 0.8 |

May 2014 Aug 2014 Jul 2015 May 2016 Oct 2017

**Figure 4:** Illustration of decision gates that are left for the Stokkfjellet project, along with the time, nominal cost, and probability of success associated with each activity. All numbers are estimations done by TrønderEnergi. The time schedule is tentative. The valuations in this thesis will use May 1 2014 as the starting point. *Source: TrønderEnergi (2014).*

- At the end of every stage, the project can be shelved in anticipation of new information. However, two elements limit the time span for how long it may be shelved. First, launching the park is required within five years of getting a final concession from NVE. Second, green certificates will only be given to wind producers who start production before the year 2020, which in practice means that producers will not want to delay the launch beyond this point.

- After the planning process, the rights to the wind park can be sold. This could create liquidity for the owners, but will in general not increase the value of the isolated project because the market price should equal the net present value (NPV) of the expected future cash flows.

- The process of planning the wind park can be started even though the appeal has not yet been processed in order to rush the launching of the plant (this is indeed something that TrønderEnergi currently does).

- The size of the wind park can be downscaled.

After discussions with TrønderEnergi, it has become clear that the most valuable flexibility is assumably that related to the option to abandon the project at the end of every stage. This flexibility will be valued in the real options valuations in Sections 6.3 and 6.4.

The case study conditions form the basis for our valuation. The flexibility presented, the timeline and the two markets' properties are important determinants when valuing the investment opportunity. In the next section, we give an introduction on how to think about uncertainty, flexibility and the appropriateness of valuation methods.

## 2.6 Selecting valuation methods to apply

For wind parks, as much as 75 per cent of the capital expenditure can be scheduled up-front of launching the plant (Krohn et al., 2009). With such a cost structure, a careful analysis of the project becomes essential in order to avoid failures, which could potentially cause large losses. The flexibility incorporated in investment opportunities in wind farms typically includes flexibility in the timing of investments in different stages of building the park, supplier flexibility, option to extend the lifetime of the park and sometimes site flexibility. Typically, there is commercial, technological, and political risk associated with these projects.

In order to decide what risks impose the largest uncertainty on a project, we need to consider the risk profile of the investor. The question is which particular risk or group of risks could affect a project's cash flow to such an extent that it would alter management's future decisions.

As mentioned, an investment opportunity in onshore wind is exposed to commercial, political, and technological risk. Let us look more closely into this. There is uncertainty regarding the competitiveness of the marginal price of production for a power plant, and there is uncertainty about whether or not the project will receive the licences to operate. Furthermore, the technological performance of the park may deviate from the expected. That means there will be two types of underlying risk: Non-diversifiable and diversifiable risk. If commodity prices, such as the electricity price, are keys to future investment decisions, then the key underlying risk is not diversifiable because it is merely a result of market conditions. Alternatively, we are dealing with diversifiable risk if the key to future investment decisions is political or technological (Koller et al., 2010).

Managerial flexibility is not the same as uncertainty. Flexibility refers to choices between alternative plans that managers may make in response of new events. For cases where managers can freely respond to events that may occur as time passes, we need a special, contingent valuation approach that appreciates this flexibility (Koller et al., 2010). Valuing flexibility is typically most relevant when valuing individual businesses and projects, because there is flexibility in specific decisions such as investment timing, choice of technology, or commitment to the project that is to be valued. Koller et al. (2010) present two contingent valuation approaches: Real options valuation and decision tree analysis. These methods differ on some technical points, but both boil down to forecasting, implicitly or explicitly, the future free cash flows contingent on the future states of the world and management decision. What is left is then simply to discount these cash flows back to today.

Valuing flexibility does not *always* require sophisticated, formal option-pricing models. Koller et al. (2010) note that a decision tree analysis approach is an effective alternative for valuing flexibility related to e.g. technological risk, but not commodity risk. As will be discussed, electricity prices follow the typical commodity price pattern and are exposed to commodity risk. As a result, decision tree analysis can be useful for valuations where decisions are made in response to technological and political events. If we possess reliable estimates on the value and variance of the cash flows underlying the investment decision, then using sophisticated real option valuation approaches can be justified (Koller et al., 2010).

Theoretically, real options valuation is superior to decision tree analysis, but it cannot fully replace traditional discounted cash flow valuation, because valuing an option using real options still depends on knowing the value of the underlying assets. Unless the assets in question have an observable market price, the value will have to be estimated using traditional discounted cash flows (Koller et al., 2010). This will be discussed in Section 4.5.

The value of flexibility is related to the degree of uncertainty and the room for managerial reaction, as shown in Fig. 5. The value is largest when uncertainty is high and managers can react when new information evolves. The value is low when managers cannot react to new information, or when it is unlikely that new information that may become available will alter future decisions. It is worth noting that the decision to include flexibility in a project valuation is most important when the project's standard NPV is close to zero, i.e. the decision to invest is a close call. For renewable energy technology investments, we know that such margins are often small.

This thesis will evaluate different valuation methods' suitability for valuing a wind park. Starting with the discounted cash flow methods that TrønderEnergi currently uses, we move to the slightly more advanced contingent NPV approach, sometimes called decision tree analysis. We then

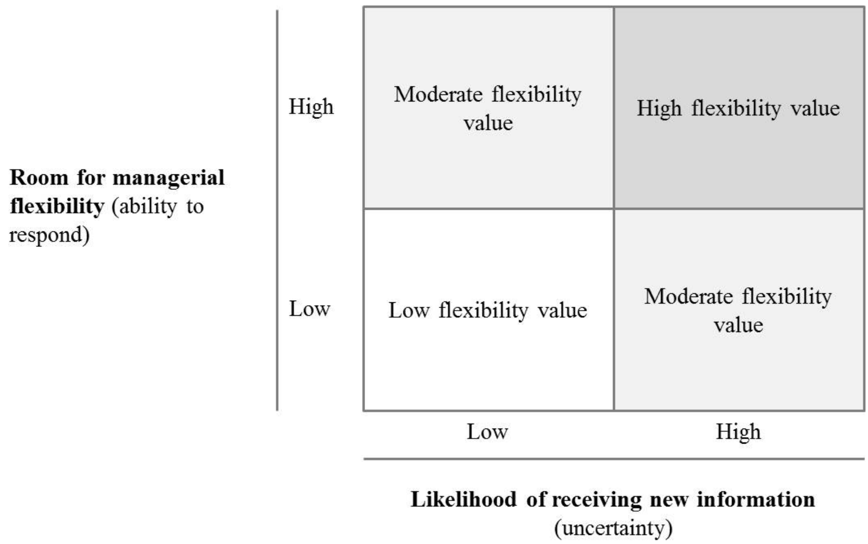



**Figure 5:** When is flexibility valuable? *Source: Koller et al. (2010).*

get more advanced and demonstrate the application of two different real options methods. First we do a binomial lattice valuation, then a valuation using Monte Carlo simulations. The two real options approaches are selected because of their ability to take into account important characteristics of the project. We have a real option that is similar to a European compound option, which can be valued conveniently using the binomial lattice approach with a single stochastic variable. Since we wish also to consider the uncertainty in the price of tradable green certificates, we have used the Monte Carlo method for simulating two different stochastic processes, this time valued as a single European call option. Relevant theory for the traditional valuation methods is presented next.

# 3 Traditional Valuation Methods

We will now present the theory behind the valuation methods that will be applied in this thesis to value Stokkfjellet Wind Farm. We will start with the traditional valuation methods in this chapter. We start by describing the discounted cash flow method, before we present the contingent net present value (NPV) approach, sometimes called decision tree analysis (DTA).

## 3.1 Static net present value

While pioneering the theory of interest and the value of time, Fisher (1907, 1930) developed the discounted cash flow (DCF) method, which is extensively adopted to evaluate financial investments and real asset investment decisions. The DCF method is based on the net present value method, which is characterised by its simple calculation and easily grasped logic. However, its analytical framework and assumptions are based on irreversible and non-deferrable investment. The method is thus applicable only for evaluating short-term investment projects with low uncertainty. Additionally, it cannot properly reflect managerial flexibility in decisions, and results often lead to underestimations of the opportunity value of an investment (Trigeorgis and Mason, 2003; Dixit and Pindyck, 1995). The DCF method is not the best choice for investments in renewable energy technology because it does not model high volatility and fails to include managerial flexibility (Deng and Oren, 1995).

Indeed, if we assume that a simple project follows a certain stochastic process and that decisions are at least partially irreversible, then it can be proven that the simple NPV rule is incorrect (unless, of course, if $\sigma = 0$). The proof can be found in the book by Dixit and Pindyck (1994) (pp. 140-142). The proof is derived for a project that follows a so-called geometric Brownian motion, but its validity is general. We will get back to geometric Brownian motions later in this thesis.

The simplest statement of the NPV rule is that one should undertake all projects with a positive NPV and discard all projects with a negative NPV. Also, a positive recommendation to take a project should only be made if the project does not prevent one from undertaking another more valuable project. The simple formula that works as the basis for the DCF methods is

$$NPV = I_0 + \sum_{t=1}^{\infty} \frac{FCF_t}{(1+r)^t}, \tag{3.1}$$

where $I_0$ is the initial investment, $FCF_t$ is the free cash flow in period $t$, $r$ is the appropriate discount rate and $t$ is the period. Eq. 3.1 assumes an infinite stream of cash flows, i.e. infinite operation. Wind farms are typically assumed to have an economic lifetime of 20 years. Of the factors that go into the equation, the investment cost and time horizon are typically given, while the cash flows must be forecasted and the discount rate estimated.

The cash flows should be discounted at the rate that reflects what investors expect to earn from investing in the project (Koller et al., 2010). The rate a certain investor demands on her capital is called the *cost of* *capital*. This rate may also be called the opportunity cost of capital, the required rate of return, or even expected return. The cost of capital can be defined as the expected rate of return forgone by not undertaking another potential investment activity for a given capital. It is a rate of return that investors expect to earn in financial markets while undertaking the same amount of risk for the same expected return. In other words, the investor will not accept a lower expected return than the cost of capital given the investment circumstances.

Since a project is typically financed with many different capital sources (e.g. common stock, preferred stock, bonds and other long-term debt) it is common to employ the *weighted average cost of capital* (WACC) for discounting the cash flows from a project, i.e. the rate that results from proportionally weighting each category of capital. It can thus be argued that the WACC should be used for the above calculation. This discount rate will be different for every company or even project, because the risk profile differs. This parameter can best be estimated internally in a company, because the company will best know its capital structure and investors' risk preferences.

The value of operations (which we will call often call *the NPV of the operational phase* in this thesis), is the discounted value of the future free cash flow. The free cash flow equals the cash generated by the company's operations, less any reinvestment back into the business. In other words, it is the cash flow available to all investors (equity holders, debt holders, and any other non-equity holding investor), and is thus independent of capital structure. Consistent with this definition, the free cash flow should be discounted at the weighted average cost of capital. The reason is that the WACC represents the rates of return required by the company's debt and equity holders blended together, essentially making it the company's opportunity cost of funds (Koller et al., 2010).

For each time period, we can calculate the free cash flow as [earnings before interest and tax]$*$[1-tax rate] + [depreciation and amortisation] - [change in net working capital] - [capital expenditure]. We will use this to calculate the value of operations in Section 6.1.

## 3.2 Decision tree analysis

The contingent NPV approach, commonly known as decision tree analysis (DTA), is on many points similar to the DCF method, but includes also the option to abandon an investment as information evolves. The key idea in decision tree analysis is that the decision maker, in each decision stage, can choose either to undergo the investment, or to abandon the investment opportunity, resulting in an NPV of 0. All previous costs are treated as sunk costs. The only case where the investor will choose abandonment is if the expected NPV of moving forward with the investment is negative. It is important to apply the correct cost of capital for this valuation.

Koller et al. (2010) recommend a DTA approach that uses different discount rates for the two components of the contingent cash flows, namely the development phase (the phase before operation commences) and the operational phase. In practice, this means that the operational phase of the project is discounted at the cost of capital, while the development phase is discounted at the risk-free interest rate. The reason is that the two phases are exposed to different kinds of risk. In the operational phase the project

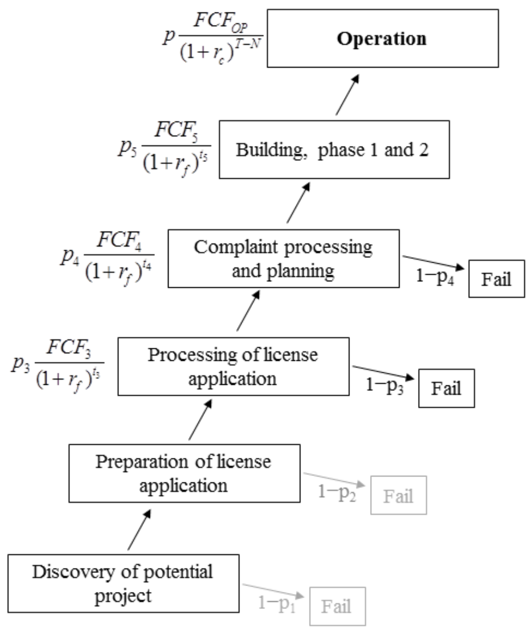



**Figure 6:** Decision tree for the investment opportunity of Stokkfjellet wind park. Failure in any phase will give a NPV equal to zero. The scenarios in grey have passed and are no longer possible outcomes, and will thus not be included in the calculations.

is exposed to commodity risk, and the cash flow should thus be discounted at the cost of capital. In the development phase, however, where the possible outcomes are reflected by the probabilities of success in each stage and cash flows are independent of market risk, we discount the cash flow at the risk-free rate.

The way we incorporate flexibility into a contingent NPV approach is by including the probability of success in each stage of development in the calculations. That is, for each stage, we must estimate the probability that we will move on to the subsequent phase of development. Moving on to the next phase is done by undertaking the cost of this stage. All future free cash flows must then be multiplied with this probability. In this thesis, we assume that TrønderEnergi's own estimates are the best probability estimates available.

Suppose we want to calculate the contingent NPV of a project with $n$ stages that could launch at time $N$ and terminate at time $T$. Then, the expression for the contingent NPV is

$$NPV_{contingent} = \left[ p_N \frac{FCF_{OP}}{(1+r_c)^{T-N}} - \sum_{n=0}^{N} p_n \frac{FCF_n}{(1+r_f)^{t_n}} \right], \tag{3.2}$$

where $FCF_{OP}$ is the future value of operation at the time that production commences, $FCF_n$ is the free cash flow in stage $n$, $p_n$ is the accumulated probability of succeeding in all stages up to and including stage $n$, $r_c$ is the cost of capital, and $r_f$ is the risk-free interest rate. See Fig. 6 for an illustration of the decision tree from Eq. (3.2) for the investment decision at hand.

# 4 Real Options Theory and Assumptions

The following chapter provides an introduction to real options and some of the mathematics and assumptions that are needed to perform the real options valuation of the wind farm.

## 4.1 Introduction to real options theory

As discussed in Section 2.6, the degree of flexibility and the risk profile of a project determine which valuation approach fits best. If a project's net present value (NPV) is large, so that the option to invest is deep in the money, and there is not much flexibility in decisions, it might not be worthwhile to perform a real options analysis. When an option is at the money or even slightly out of the money, if there is uncertainty and managerial flexibility, then a real options analysis can add enough value to the valuation that it could change a decision. It is still important to consider what type of real options analysis is appropriate. In this section we will introduce real options and explain some important assumptions. We will then introduce the two real options valuation methods that we have chosen for the valuations in this thesis.

When including the value of flexibility through real options analysis, the calculated market value of the project may increase. According to Copeland and Antikarov (2003), the value of the project is equal to the traditional NPV plus the value of any real options. Since the value of an option is always nonnegative, a real options analysis should never lead to a lower value than discounted cash flow methods when performed correctly.

The value of flexibility is always positive, but the value can vary a lot depending on how the uncertainty is related to the project value as well as decision makers' ability to respond as new information evolves.

### The history of real options

Real options theory was introduced as a new research area in the late 1970's due to growing discontent with the traditional methods for assessing projects under uncertainty, like the discounted cash flows method. Around the same time there were significant innovations in options pricing techniques such as the Black-Scholes formula (Black and Scholes, 1973), and later on the binomial lattice approach (Cox et al., 1979). In order to improve available methods, it was necessary to model flexibility as options to adjust projects in response to uncertainty. The introduction of the Black-Scholes formula provided a technique for valuing options, and soon followed the development of several techniques for the assessment of real options (Dixit and Pindyck, 1994; Trigeorgis, 1996). The identification and proper use of real options such as rushing, delaying, abandoning, or adjusting investment decisions, provides flexibility to projects and potentially increases their expected value and decreases their risk (Dixit and Pindyck, 1994).

### The analogy to financial options

Real options analysis has been accepted as a suitable tool to analyse investment decisions for renewable energy technology. Lee (2011) showed that the value of developing renewable energy technology increases with an increase in the underlying price, volatility, time to maturity and risk-free rate. Conversely, the value decreases with an increasing exercise price or investment cost. This is analogous to financial options.

A firm's investment opportunity is equivalent to a European call option: The right but not the obligation to buy a share of stock at a prescribed price (Dixit and Pindyck, 1994). Thus, the decision to invest corresponds to deciding whether or not to exercise such an option. More specifically, the investment opportunity is analogous to a perpetual call option on a dividend-paying stock, considering the pay out stream from the completed project equivalent to the dividend on the stock.

### Real options in relation to investments in wind

Important factors to take into account when valuing the option to invest in a wind farm is the policy uncertainty, and the significance of any subsidies that accrue to the project. Lee and Shih (2010) indicate through analytical results that the traditional NPV model underestimates the policy value of wind power, and that real options analysis more accurately reflects the actual policy value.

The uncertainty factors that will be taken into consideration in this thesis are the uncertainty in electricity price, uncertainty in success in the different stages of development and the unceratinty in the price of tradable green certificates.

Given the nature of our decision problem, we have selected two appropriate real option methods to evaluate the investment opportunity. First, the binomial lattice approach, which is suitable for pricing compound options. Second, a Monte Carlo simulation approach that is good for considering two stochastic, correlated processes.

### Sequential compound options valuation

Firms that are investing in new projects are in practice often considering a sequence of decisions and incurring a series of cash outflows. To improve the accuracy of an analysis, one should in such cases try to divide the lifetime of an option into smaller time steps.

Suppose we want to determine the value of holding the option to invest. The crucial aspect will be whether or not the decision maker can abandon the investment program once it begins (Guthrie, 2009). Suppose also that decisions of whether or not to pursue the investment series are only made at the end of each stage. This can be modelled as a series of European call options where each option is created when the previous option is exercised. The order in which the stages occur is fixed, while the timing is influenced by the decision maker and the project itself. The decision to move from one stage to another is completely irreversible. We are interested in determining the value added by the real options embedded in the project rights.

The development of the Stokkfjellet project can be treated as a sequential investment opportunity with a series of smaller investments in the developing phase, and a larger final investment cost for the option to build. The problem can be described as a series of European call options, with the timing of the exercise dates according to the information presented in Table 3. However, in the Monte Carlo valuation we must simplify the project to a single European call option in order to be able to take into account two stochastic prosesses.

## 4.2 No-arbitrage prices

The real options valuation models described in Chapter 5 rest on one important assumption: Assets are priced in such a way that arbitrage opportunities do not exist. The most important implication of this assumption, often called the law of one price, is that the price of any two portfolios that generate identical future cash flows must always be equal.

Say we are trying to estimate the market value of a cash flow that will be received after one period. This cash flow equals $Y_u$ in the up state and $Y_d$ in the down state. To do this, we suppose the existence of two assets: A one-period risk-free bond with a current price of 1 and a certain payoff of $R_f$ after one period, and a so-called spanning asset with a current price of $Z$. The spanning asset is a risky asset generating a payoff after one period equal to $X_u$ in the up state and $X_d$ in the down state. We use the information incorporated in $Z$ and $R_f$ to value the cash flow described by $(Y_u, Y_d)$. What we want to do is build a portfolio of just enough units of the risk-free bond and the spanning asset that the portfolio generates a cash flow of $Y_u$ in the up state and $Y_d$ in the down state. This is known as the cash flow's replicating portfolio. Guthrie (2009) shows that the replicating portfolio costs

$$V = \frac{\pi_u Y_u + \pi_d Y_d}{R_f}, \tag{4.1}$$

where

$$\pi_u = \frac{ZR_f - X_d}{X_u - X_d} \quad \text{and} \quad \pi_d = \frac{X_u - ZR_f}{X_u - X_d}. \tag{4.2}$$

When there are no arbitrage opportunities, the market value of the asset we are valuing must equal the cost of its replicating portfolio—i.e. the law of one price must hold. Therefore our estimate of the market value of the cash flow $(Y_u, Y_d)$ must be given by Eq. (4.1).

The law of one price assumption allows us to work as if we lived in a world where all investors are risk-neutral. Because risk-neutral investors are indifferent between receiving a risk-free cash flow and a risky one with the same expected value, all investors in this new world attach the same value to a risky cash flow $(Y_u, Y_d)$ as they would to a risk-free cash-flow of $\pi_u Y_u + \pi_d Y_d$. Since the latter flow is risk-free, its market value can be found by discounting the cash flow using the one-period risk-free interest rate, resulting in the expression in Eq. (4.1), which is known as the risk-neutral pricing formula. $\pi_u$ and $\pi_d$ are the risk-neutral probabilities of up and down moves.

It is worth noting that we have not hereby assumed that all investors are risk-neutral, merely that there are no arbitrage opportunities. The asset pricing formula in Eq. (4.1) still adjusts for risk, even though the risk-free interest rate (rather than the risk-adjusted discount rate) is used to discount the cash flows. The difference is that the risk adjustment happens in the numerator of Eq. (4.1), via the risk-neutral probabilities, rather than in the denominator, via the discount rate.

This result will be needed when we introduce the binomial lattice approach in Section 5.1.

## 4.3 Mean-reversion in prices

When valuing a power plant, the price of electricity is a critical variable, directly influencing the level of income for the producer. A realistic model for the price process will make the valuation more accurate, as it aims to capture some of the value of the inherent flexibility of the project. The trigger level of the electricity price to initiate investment has been found to be highly dependent on which stochastic price model one chooses to employ (Dixit and Pindyck, 1994).

There have been several proposals in the literature on how to model the electricity price. The statistical approaches model the electricity spot price process directly and parameters of the price processes are estimated from the available historical market data. Smith and Schwartz (2000) develop a two factor model consisting of short-term uncertainty about the deviations from the mean, and long-term uncertainty about the equilibrium price level to which commodity prices tend to revert. Lucia and Schwartz (2002) extend this model to capture seasonal variations in the Nordic power market, accounting for short-term mean-reversion, and an equilibrium price level in the long-run as well as seasonal price variations. This model is specifically built to capture the characteristics of the Nordic power market with its strong seasonality pattern in electricity prices. Davison et al. (2002) develop a hybrid, mean-reverting switching model, while Barlow (2002) introduce a non-linear Ornstein-Uhlenbeck model for spot power prices. Deng

(2000) models the electricity price as a mean-reverting process with jumps and spikes.

An extended lot of the literature has elaborated on the standard finance tool of geometric Brownian motion. A geometric Brownian motion, often referred to as a random walk, is an example of a diffusion process, with the diffusion coefficient being the volatility. Dixit and Pindyck (1994) apply a model first proposed by McDonald and Siegel (1984) to value the option to invest when the value of the firm follows a geometric Brownian motion. However, Brownian motion does not capture the commodity price pattern of mean-reversion, and for this reason it is not suitable for modelling electricity prices.

Rather than fluctuate far from the mean like a Brownian motion, a sudden increase in a commodity spot price will typically result in a supply increase, forcing the price to decrease as the price moves back towards the commodity's long-run marginal cost of production. Equivalently, a drop in the spot price of a commodity will typically cause some firms to shut down unprofitable production and other to exit the industry altogether, eventually resulting in an upwards pressure on the price as supply decreases (Guthrie, 2009). Another salient feature of energy commodity prices is the presence of price jumps and spikes. Since the supply and demand shocks cannot be smoothed by inventories, electricity spot prices are volatile (Deng, 2000). The profitability of a wind power plant is directly related to the relationship between electricity price and wind inflow, because neither wind nor electricity can be stored.

We will assume that the log of the power price follows a first-order autoregressive process (commonly referred to as an AR(1) process). According to Guthrie (2009) that is, if $p_j$ denotes the $j$th observation of the log price, then

$$p_{j+1} - p_j = \alpha_0 + \alpha_1 p_j + u_{j+1}, \quad u_{j+1} \sim N(0, \phi^2), \tag{4.3}$$

for some constants $\alpha_0$, $\alpha_1$, and $\phi$, where $\alpha_1$ is negative. This process exhibits mean-reversion. Suppose for example that $p_j$ is sufficiently large that $\alpha_0 + \alpha_1 p_j$ is negative. Then the expected value of $p_{j+1} - p_j$ is also negative, such that the log price is expected to decrease in the short-term. Similarly, suppose that $p_j$ is so small that $\alpha_0 + \alpha_1 p_j$ is positive. Then, we expect the log price to rise in the short-term. In either case, the log price tends to move towards a long-run level where $\alpha_0 + \alpha_1 p_j$ equals zero, and $p_j$ equals $\alpha_0/(-\alpha_1)$. As a result, if the log price follows an AR(1) process, sudden shocks to the price do not last: The price is gradually pulled back towards its long-run level.

The so-called Ornstein-Uhlenbeck process with rate of mean-reversion

$a$, long-run level $b$, and volatility $\sigma$ generalises the AR(1) process to the situation where the log price is observed with arbitrary frequency. When viewed from date $t$, the change in the log price over the next $\Delta t$ units of time is normally distributed with a mean of $(1 - e^{-a\Delta t})(b - p_t)$ and a variance of $\sigma^2(1 - e^{-2a\Delta t})/2a$, where $a$, $b$, and $\sigma$ are constants. That is,

$$p_{t+\Delta t} - p_t \sim N((1 - e^{-a\Delta t})(b - p_t), \sigma^2(1 - e^{-2a\Delta t})/2a). \tag{4.4}$$

That is, if $p_t > b$, then the expected change in the log price is negative, while if $p_t < b$ the expected change is positive. In other words, the log price tends back towards $b$, indicating that $b$ is indeed the long-run level. If $a$ is large (so that $1 - e^{-a\Delta t}$ approaches unity), then the expected value of $p_{t+\Delta t} - p_t$ is close to $b - p_t$, so that the expected value of $p_{t+\Delta t}$ is close to $b$. That means that large values of $a$ indicate that the log price is strongly mean-reverting.

As Guthrie (2009) points out, two other things are worth noting. First, the variance term can be shown to be a decreasing function of the parameter $a$. This means that when the mean-reverting force is strong, the variance of changes in the log price is relatively low. When mean reversion is strong, the price has little opportunity to wander away. Second, when $\Delta t$ is very large, both $e^{-a\Delta t}$ and $e^{-2a\Delta t}$ approach zero, such that $p_{t+\Delta t}$ is normally distributed with mean $b$ and variance $\sigma^2/2a$. In other words, as we look further and further into the future, the distribution of the log price is well defined such that the distribution of the log price 50 years from now is practically identical to that 100 years ahead in time. This is perhaps the key difference between a mean-reverting process and a random walk: The variance of the geometric Brownian motion increases linearly with the time horizon, whereas that of a mean-reverting process levels off.

Under the process described by Eq. (4.3), changes in $p$ are normally distributed with mean $\alpha_0 + \alpha_1 p_j$ and variance $\phi^2$. Thus, the parameters $\alpha_0$ $\alpha_1$, and $\phi$ are related to the Ornstein-Uhlenbeck parameters by the equations

$$\alpha_0 = (1 - e^{-a\Delta t})b, \quad \alpha_1 = -(1 - e^{-a\Delta t}), \quad \phi^2 = \frac{\sigma^2}{2a}(1 - e^{-2a\Delta t}).$$

This result will be useful for building a binomial tree in Section 5.1 and for the theory on Monte Carlo valuation in Section 5.2.

## 4.4 Estimating volatility

The precise definition of the volatility of an asset is an annualised measure of dispersion in the stochastic process that is used to model the log returns (Alexander, 2008). That is, volatility is a measure for capturing the variation in the price of an underlying financial instrument over time. The traditional measure for dispersion from the mean is the standard deviation statistic, which is simply defined as the square root of the average squared deviation from the mean. Thus, the standard deviation of $N$ observed prices $P_1, P_2, ..., P_N$ with mean $\bar{P}$ is calculated as

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(P_i - \bar{P})^2}. \tag{4.5}$$

For the standard deviation to be an accurate measure of volatility, one has to assume that investment performance data is normally distributed. It is a common assumption that stock prices are log normally distributed, such that their log returns will follow the normal distribution.

Assume that one-period log returns are generated by a stationary identically and independently distributed (i.i.d.) process with mean $\mu$ and standard deviation $\sigma$. We denote by $p_{jt}$ the log return over the next $j$ periods observed at time $t$, i.e.

$$P_{jt} = \Delta_j \ln P_t = \ln P_{t+j} - \ln P_t. \tag{4.6}$$

In the case where one-period returns are believed to have some positive (or negative) correlation, the assumption that they are generated by an i.i.d. process is no longer valid. In particular, we assume returns are generated by a stationary AR(1) autoregressive process, i.e.

$$P_j = \alpha_0 + \alpha_1 P_{j-1} + u_j, \quad u_{j+1} \sim N(0, \sigma^2)\,, \; |\alpha_1| < 1, \tag{4.7}$$

where $\alpha_1$ is the autocorrelation, i.e. the correlation between the adjacent returns. Alexander (2008) demonstrates that a positive autocorrelation between the returns will lead to a larger volatility estimate, and a negative autocorrelation will lead to a lower volatility estimate, compared to the i.i.d. case. Indeed, we will model the mean-reverting behaviour of power prices as an AR(1) process in Chapter 6.

Alexander (2008) makes a remark about volatility and the fact that it is *unobservable*. We can only ever estimate and forecast volatility, and this only within the context of an assumed statistical model. Even if we knew for certain that our model was a correct representation of the data generating process, we could never measure the true volatility directly because it is not traded in the market.

The volatility estimate will be more precise the higher the number of data points, $n$, but true volatility may change over time, so that the distant past might not be good for predicting the future. Thus, the number of data points to be used is a matter of judgment.

For the Stokkfjellet project's volatility we will assume that the volatility of the commodity that it produces reflects well the volatility of the project. Obviously, this would be the volatility of the electricity price, but one can also consider the project to "produce" tradable green certificates, and thus this price process will be included when we perform a Monte Carlo valuation in Section 6.4. The project's volatility might slightly differ from the commodity price volatility, but this method is accepted and used by academia for real options analysis of commodity products. Certainly, Miller and Park (2002) argue that projects based on commodity products can utilise the commodity price pattern to estimate the project volatility, pointing out that this is the most important variable to evaluate when valuing projects of this nature. Fleten et al. (2007) and Bøckman et al. (2008) also apply this method for evaluating renewable energy projects using real options analysis.

## 4.5 Marketed asset disclaimer

Traditional options theory suggests the use of a twin-security for pricing an option, and by this essentially assumes that markets are complete. However, in practice, it is impossible to create a replicating portfolio with the exact risk profile as the underlying, and many believe that markets are incomplete (Dyer and Brãndao, 2005). In light of this, Copeland and Antikarov (2003) suggest another method for estimating the market value of the project. They make the assumption that the present value of the cash flows of the project without flexibility (i.e. the traditional NPV) is the best unbiased estimate of the market value of the project, had it been a traded asset. Assuming that the traditional NPV of the project can be used as the twin security is called the *marketed asset disclaimer*. As Copeland and Antikarov (2003) indicate, no portfolio is better correlated with the project than the project itself. This assumption makes it easy to value real options for which we have enough data to estimate the NPV without flexibility.

It is worth noting that the marketed asset disclaimer assumption is no stronger than assumptions used to estimate a project's NPV in the first place. By this reasoning, there is no reason why a decision maker currently using NPV to value a project without flexibility should use a different set of assumptions for real options analysis. The marketed asset disclaimer assumption simply comprises that the present value of the underlying risky asset without flexibility can be used as if it were a marketed security (Copeland and Antikarov, 2003). This assumption will be made in the real options valuation models described in Chapter 5.

# 5 Real Options Valuation Methods

We will now thoroughly describe the two real options approaches we have chosen to use for our valuation. The binomial lattice approach will be covered first, and the option pricing technique of Monte Carlo valuation will be explained afterwards.

## 5.1 Binomial lattice approach

The binomial lattice method takes the risk-neutral valuation approach as done by Cox et al. (1979). Their key insight was that because the value of an option is independent of an investor's risk preferences, the result from a valuation would be the same even when everyone is assumed to be risk-neutral. This critical assumption simplifies the calculations by eliminating the need to estimate the risk premium in the discount rate, as was discussed in Section 4.2.

Knowledge of some stochastic mathematics and Martingale processes is required in order to fully understand the complexities involved even in a simple binomial lattice. However, the more important aspect here is to understand intuitively how a lattice works. In this case, the underlying is assumed to follow the same process as its commodity price, which is expected to follow a mean reverting process. This is a reasonable assumption for electricity prices, as discussed in Section 4.3.

We construct a binomial tree using the logarithm of the price. The motivation for this is that the volatility of a price will tend to be higher when the price is high than when it is low (Mun, 2006). We can model such behaviour through building a model in which the logarithm of the price has constant volatility. Since changes in the log price correspond to continuously compounded rates of return, this assumption implies that rates of return have a constant standard deviation over time. The price will be more volatile when it is high than when it is low because the same rate of return is being applied to a higher base price.

The realism of our models and the accuracy of our results will be improved if the time steps in our binomial tree are small (Guthrie, 2009). Of course, it is not realistic for the price of a commodity to take one of exactly two possible values one year from now, as would be the case if our binomial tree had annual steps. It is preferable to have a large number of smaller steps. If we break the year into 12 monthly steps, then the price will take one of 13 possible values one year from now. Similarly, had we used 52 weekly time steps then the price has 53 possible values.

We want to avoid that the tree becomes too large, and at the same time have appropriately short time steps. Thus it seems reasonable to use monthly time steps. The time step of our raw electricity price data is daily, so we need to normalise the estimates from our data in order to use it for calibrating our binomial tree with monthly time steps.

As discussed in Section 4.3, we will assume that the log of the power price follows a first-order autoregressive process. Recall that Eq. (4.3) said that if $p_j$ denotes the $j$th observation of the log price, then

$$p_{j+1} - p_j = \alpha_0 + \alpha_1 p_j + u_{j+1}, \quad u_{j+1} \sim \mathcal{N}(0, \phi^2),$$

for some constants $\alpha_0$, $\alpha_1$, and $\phi$, where $\alpha_1$ is negative. This is a first-order autoregressive process, often called an AR(1) process.

Guthrie (2009) presents a procedure for constructing a binomial tree for the price and finding the risk neutral probabilities that we will now follow.

### 5.1.1 Obtaining normalised estimates for the parameters

If the data-generating process for the log price is described by Eq. (4.4), then the true values of $\alpha_0$, $\alpha_1$, and $\phi$ are related to the Ornstein-Uhlenbeck parameters by the equations

$$\alpha_0 = (1 - e^{-a\Delta t})b, \quad \alpha_1 = -(1 - e^{-a\Delta t}), \quad \phi^2 = \frac{\sigma^2}{2a}(1 - e^{-2a\Delta t}).$$

If a regression gives us estimates $\hat{\alpha}_0$, $\hat{\alpha}_1$, and $\hat{\phi}$, then sensible estimates of the parameters $a$, $b$, and $\sigma$ are the numbers $\hat{a}$, $\hat{b}$, and $\hat{\sigma}$ that satisfy

$$\hat{\alpha}_0 = (1 - e^{-\hat{a}\Delta t_d})\hat{b}, \quad \hat{\alpha}_1 = -(1 - e^{-\hat{a}\Delta t_d}), \quad \hat{\phi}^2 = \frac{\hat{\sigma}^2}{2\hat{a}}(1 - e^{-2\hat{a}\Delta t_d}).$$

Solving these equations for $\hat{a}$, $\hat{b}$, and $\hat{\sigma}$ gives us our normalised parameter estimates:

$$\hat{a} = \frac{-\ln(1 + \hat{\alpha}_1)}{\Delta t_d}, \quad \hat{b} = \frac{-\hat{\alpha}_0}{\hat{\alpha}_1}, \quad \hat{\sigma} = \hat{\phi}\left(\frac{2\ln(1 + \hat{\alpha}_1)}{\hat{\alpha}_1(2 + \hat{\alpha}_1)\Delta t_d}\right)^{1/2}. \tag{5.1}$$

Running a regression in Eq. (4.3) and substituting the parameter estimates into Eq. (5.1) will thus yield the normalised parameter estimates needed to describe the particular price process.

### 5.1.2 Building the tree for the price

The normalised estimates are used to build the binomial tree for the price. We let each period in our binomial tree represent $\Delta t_m$ years. The tree for the log price starts at $x(0, 0) = \log P_0$, which is the log price equal to the level it takes at the date that the real options analysis is carried out. We use the log price since it is commonly assumed that commodity prices are log normally distributed. An example of a tree of four periods is shown in Fig. 7. An increment in $n$ represents a step in time, while an increment in $i$ signals a down move in the price. In each subsequent period the log price increases or decreases by $\hat{\sigma}\sqrt{\Delta t_m}$ depending on whether an up or a down move occurs. Then, at node $(i, n)$ there have been $n - i$ up moves and $i$ down moves, so that the log price at this node is calculated as

$$\underbrace{\log P_0}_{\text{starting value}} + \underbrace{(n-i)(\hat{\sigma}\sqrt{\Delta t_m})}_{\text{effect of up moves}} + \underbrace{i(-\hat{\sigma}\sqrt{\Delta t_m})}_{\text{effect of down moves}} .$$

This simplifies to

$$x(i,n) = \log P_0 + (n-2i)\hat{\sigma}\sqrt{\Delta t_m}, \tag{5.2}$$

where $x(i, n) = \log X(i, n)$. Taking exponentials on both sides of this equation shows that the level of the price at node $(i, n)$ is

$$X(i,n) = e^{x(i,n)} = P_0 e^{(n-2i)\hat{\sigma}\sqrt{\Delta t_m}}. \tag{5.3}$$

This formula gives us a closed-form expression for the price at any node of our binomial tree. That means we can use it to calculate the price at any

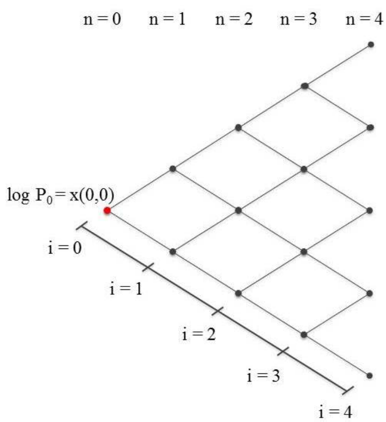



**Figure 7:** Binomial tree with $n = 4$ periods. An up move in the tree is characterised by keeping $i$ constant, whereas a down move is identified by an increment in $i$. The starting point of the tree is node $(0, 0)$, marked in red.

given node, without having to iterate through the tree from the first node and to the respective node.

To calculate the size of the up and down moves, consider that the price at node $(i, n)$ is given by Eq. (5.3), and that an up move will give a price of

$$X(i, n+1) = P_0 e^{((n+1)-2i)\hat{\sigma}\sqrt{\Delta t_m}} = X(i, n) e^{\hat{\sigma}\sqrt{\Delta t_m}}. \tag{5.4}$$

Thus, the size of an up move at this node must equal

$$U = \frac{X(i, n+1)}{X(i, n)} = e^{\hat{\sigma}\sqrt{\Delta t_m}}. \tag{5.5}$$

By the same line of reasoning, the size of a down move equals

$$D = \frac{X(i+1, n+1)}{X(i, n)} = e^{-\hat{\sigma}\sqrt{\Delta t_m}}. \tag{5.6}$$

This means that the sizes of the up and down moves are constant throughout the tree.

### 5.1.3 Calculating the probabilities of up and down moves

We need also to calculate the probability of an up move in the tree at each node. We choose the probabilities of up and down moves in such a way that the expected value of the change in the log price over the next period is equal to the value that is implied by our normalised parameter estimates as given by Eq. (5.1). If the probability of an up move at node $(i, n)$ equals

$$\theta_u(i, n) = \frac{1}{2} + \frac{(1 - e^{-\hat{a}\Delta t_m})(\hat{b} - x(i, n))}{2\hat{\sigma}\sqrt{\Delta t_m}}, \tag{5.7}$$

then the expected change in the log price is

$$(1 - e^{-\hat{a}\Delta t_m})(\hat{b} - x(i, n)),$$

which is the same as the expected value for the Ornstein-Uhlenbeck process presented in Eq. (4.4). A proof of this can be found in the book by Guthrie (2009) (pp. 275). The mean-reverting nature of the spot price is reflected in the formula for the probability of an up move: If the log price is currently higher than its long-run level (i.e. $x(i, n) > b$) then an up move is less likely than a down move (i.e. $\theta_u(i, n) < \frac{1}{2}$). Should the log price grow larger, then a down move becomes even more likely. Conversely, a log price that is currently lower than its long-run mean will have a greater probability of an up move. In this way the log price is drawn back towards its long-run average by our choice of a probability structure.

There are, however, some complications: $\theta_u(i,n)$ will become negative for sufficiently large values of $x(i,n)$, which is not compatible with $\theta_u(i,n)$ being a probability. This can be solved by resetting $\theta_u(i,n)$ to zero if the expression in Eq. (5.7) is negative and reset it to one should it be greater than unity. That is, we set the probability of an up move at node $(i,n)$ equal to

$$\theta_u(i,n) = \begin{cases} 0 & \text{if } \frac{1}{2} + \frac{(1-e^{-\hat{a}\Delta t_m})(\hat{b}-x(i,n))}{2\hat{\sigma}\sqrt{\Delta t_m}} \leq 0, \\ \frac{1}{2} + \frac{(1-e^{-\hat{a}\Delta t_m})(\hat{b}-x(i,n))}{2\hat{\sigma}\sqrt{\Delta t_m}} & \text{if } 0 < \frac{1}{2} + \frac{(1-e^{-\hat{a}\Delta t_m})(\hat{b}-x(i,n))}{2\hat{\sigma}\sqrt{\Delta t_m}} < 1, \\ 1 & \text{if } \frac{1}{2} + \frac{(1-e^{-\hat{a}\Delta t_m})(\hat{b}-x(i,n))}{2\hat{\sigma}\sqrt{\Delta t_m}} \geq 1. \end{cases} \tag{5.8}$$

Thus, at some nodes in the tree, we will know for certain that the price will go up or down in the next move. Some nodes in the tree will in this way be unreachable, because the price will never arrive at those nodes. However, it is important that we specify the probabilities throughout the tree in order to make the backwards induction procedure straightforward.

### 5.1.4 Calculating the risk-neutral probabilities

Up until this point we have been working with actual probabilities. We now wish to calculate risk-neutral probabilities of up and down moves at each node in the tree. An introduction to risk-neutral probabilities and the assumptions associated with this technique was given in Section 4.2. Recall from Eq. (4.2) that the risk-neutral probability of an up move is

$$\pi_u = \frac{ZR_f - X_d}{X_u - X_d}, \tag{5.9}$$

where $Z$ is the current price of the spanning asset. The asset will be worth $X_d$ if the price goes up and $X_d$ if a down move occurs. The risk-neutral probabilities may not be constant across the binomial tree due to the mean reversion effect of the prices. We calculate the risk-neutral probability of an up move at node $(i,n)$ of the tree according to

$$\pi_u = \frac{Z(i,n)R_f - X(i+1,n+1)}{X(i,n+1) - X(i+1,n+1)},$$

which is just the same expression as in Eq. (5.9), but with $X_d$ replaced by $X(i+1,n+1)$, $X_u$ replaced by $X(i,n+1)$, and $Z$ replaced by $Z(i,n)$. The risk-neutral probability of a down move is then

$$\pi_d(i,n) = 1 - \pi_u(i,n).$$

A few complications may arise when we allow the price to be mean-reverting. When the actual probability of an up move is zero or one, there is in fact no risk associated with the price development at the corresponding node. The risk-neutral probability should reflect this. Thus, the formula for the risk-neutral probability of an up move that will be used is

$$\pi_u(i,n) = \begin{cases} 0 & \text{if } \theta_u(i,n) = 0, \\ \frac{Z(i,n)R_f - X(i+1,n+1)}{X(i,n+1) - X(i+1,n+1)} & \text{if } 0 < \theta_u(i,n) < 1, \\ 1 & \text{if } \theta_u(i,n) = 1. \end{cases} \tag{5.10}$$

Using this equation, we will follow a standard valuation approach. First, we need to estimate the risk-neutral probabilities. To ensure simplicity, we will use the famous CAPM for this purpose. Alternative approaches proposed by Guthrie (2009) include matching the relationship between spot and futures prices and matching the current term structure of futures prices.

In estimating the risk-neutral probabilities we use the same set of historical data as before. As demonstrated by Guthrie (2009), the risk-neutral probability of an up move at node $(i,n)$ is equal to

$$\pi_u(i,n) = \theta_u(i,n) - \frac{(E[\tilde{R}_m] - R_f)\beta_x}{U - D}. \tag{5.11}$$

Then, the only additional information we need in order to estimate $\pi_u(i,n)$ is the market risk premium $(E[\tilde{R}_m] - R_f)$ and the price beta $(\beta_x)$. The task of estimating the market risk premium is not unique to real options analysis. It is a controversial issue, and describing this in detail is not the scope of our thesis. However, the market risk premium is not project specific, and there are many competing estimates available, so that it should not be too difficult to find a realistic estimate of the market risk premium.

The price beta is determined by the variance of the market return and the covariance between the project's return and the market return. Thus, an estimation of beta for an asset that is not traded, such as a wind power project, can be difficult.

An attempt at estimating these two parameters is not done in this thesis, and we merely discuss appropriate values qualitatively in Section 6.3. The reason is that we assume that TrønderEnergi possesses better estimates of these parameters than we will be able to easily derive here, keeping in mind that the estimation of these parameters is not unique to real options analysis.

Having completed the steps presented by Guthrie (2009), we now move on to building the tree for the underlying and describing how to derive the real option value.

### 5.1.5 Building the tree for the underlying

Before we can price the options, the tree for the value of the underlying, $V(i,n)$, must be computed. We assume that the underlying follows the same process as the commodity price, as argued in Section 4.4. Also, we will make the assumption that the ratio of the current power price to the average power price is the same as the ratio of the current net present value (NPV) of the operational phase to the average value of operation. This assumption is necessary since there is no market from which we can observe an average value of operation.

The development of the underlying is computed by starting in the first node, taking the present value of the underlying, $V(0,0)$, and multiplying it with the size of an up move, $U$. This will give the value of the underlying in node $(0,1)$. Consider also that a down move would indicate that the value at node $(1,1)$ is equal to $V(0,0)\,D$. We can generalise these observations to an equation for the value of the underlying at node $(n,i)$,

$$V(i,n) = V(0,0)U^{n-i}D^{i}. \tag{5.12}$$

When the tree has been constructed, the pricing of the compounded real option can commence.

### 5.1.6 Finding the option value

We have now described all the steps needed to build a tree for the underlying and how to find the risk-neutral probabilities associated with each node in the tree. In order to value the compound option, we will need to work backwards through the tree, using option pricing theory to derive the value of flexibility embedded in the project.

European compound options are valued by subtracting the respective exercise prices from the present value of the underlying at the relevant times of exercise. If this the intrinsic value is bigger than zero it will be valuable to exercise the option; otherwise, the project will be abandoned and the option to invest dies.

Suppose we have a set of possible exercise dates $T = \{t_1 < t_2 < ... < t_{N-1}\}$, as well as a set of investment costs $I = \{I_1, I_2, ..., I_{N-1}\}$, where $N$ is the number of possible exercise dates, and hence also the number of stages of investments. The very last possible exercise date is $t_N$ and represents a cost $I_N$. Note that $t_N$ must essentially be the final period included in the valuation. For convenience, we will keep the values corresponding to the final stage out of their respective sets, as the calculations differ depending on whether or not we are in the final stage. When we are in the final column of the tree, which is the last point where we have the option to exercise, the option value $C(i,n)$ is equal to the maximum of zero and the options intrinsic value, i.e.

$$C(i,n) = \max\{V(i,t_N) - I_N, 0\}.$$

We now move to the column to the left, and iterate backwards through the tree. At all nodes $(i, n \notin T)$, i.e. nodes corresponding to time steps that are not associated with an exercise date, we find the option value as

$$C(i,n) = \frac{\pi_u(i,n)C(i,n+1) + \pi_d(i,n)C(i+1,n+1)}{R_f}, \quad n \notin T.$$

At nodes corresponding to an exercise date, $n \in T$, the value of the option is given by

$$C(i,n) = \max\left\{\frac{\pi_u(i,n)C(i,n+1) + \pi_d(i,n)C(i+1,n+1)}{R_f} - I_n, 0\right\}, n \in T. \tag{5.13}$$

We can summarised the above conclusions as

$$C(i,n) = \begin{cases} \max\{V(i,t_N) - I_N, 0\} & \text{if } n = t_N \\ \frac{\pi_u(i,n)C(i,n+1)+\pi_d(i,n)C(i+1,n+1)}{R_f} & \text{if } n \notin T. \\ \max\left\{\frac{\pi_u(i,n)C(i,n+1)+\pi_d(i,n)C(i+1,n+1)}{R_f} - I_n, 0\right\} & \text{if } n \in T, \end{cases} \tag{5.14}$$

This will be our risk-neutral option pricing formula. Starting in the final column of the binomial tree for the project's market value, we work our way through the tree using this formula, going from right to left. We have to be careful so that we implement this procedure correctly: Different calculations apply depending on whether or not we have reached a point of exercise as we iterate through the binomial lattice. Finally, when we reach the first node in the tree, $(0,0)$, we have determined the present option value of our investment opportunity and thus arrived at the end point of the valuation.

### 5.1.7 Including the probability of success in each stage

Commodity risk is accounted for in the binomial lattice model that we have just described. Now, we want to include also diversifiable risk, such as the

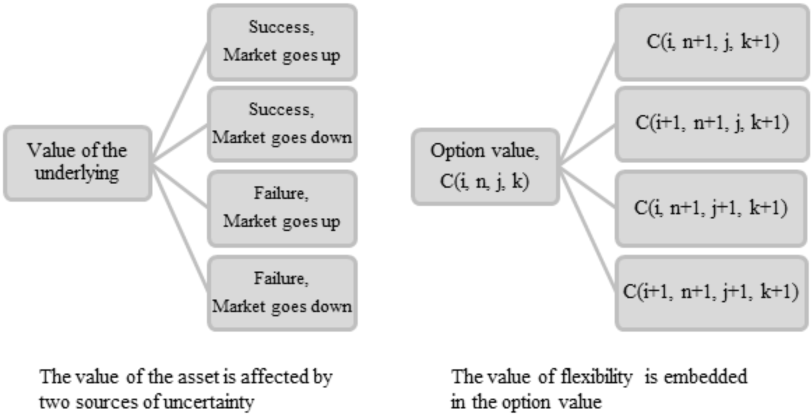



**Figure 8:** Quadranomial values of the underlying and the call option. *Source: Copeland and Antikarov (2003).*

event uncertainty that was considered in the decision tree analysis in Section 3.2. We will present a solution for including both the non-diversifiable commodity risk and the diversifiable political risk. To to this, we assume that the two uncertainties must be uncorrelated, which seems reasonable.

If the value of the underlying is affected by two sources of uncertainty, then there will be four possible outcomes from each relevant node in a tree instead of two. Copeland and Antikarov (2003) call this method the quadranomial approach, and suggest a solution for how to do such a valuation. Recall that we value the project as a European compound option, so that the exercise nodes are the ones that will be affected by the second source of uncertainty. Indeed, it is only at the end of every stage that you will know how the uncertain event turned out. At the exercise nodes, there will be four possible outcomes, as illustrated by Fig. 8. We modify Eq. (5.13) in order to account for the new possible outcomes. Let $j$ and $k$ indicate two new directions. $j$ will refer to the column number, while $k$ will indicate the row. The modified equation for the option value at a node associated with a date of exercise is then

$$\begin{aligned} C(i,n,j,k) = \quad & \max\{p_n \tfrac{\pi_u(i,n)C(i,n+1,j,k+1)+\pi_d(i,n)C(i+1,n+1,j+1,k+1)}{R_f} \\ & +(1-p_n)\tfrac{\pi_u(i,n)C(i,n+1,j+1,k+1)+\pi_d(i,n)C(i+1,n+1,j+1,k+1)}{R_f} \\ & -I_n, 0\}. \end{aligned}$$

However, since the project will have zero value if one stage fails, the second term is equal to zero. This leaves us with the expression

$$C(i,n,j,k) = \max\{p_n \tfrac{\pi_u(i,n)C(i,n+1,j,k+1)+\pi_d(i,n)C(i+1,n+1,j+1,k+1)}{R_f} - I_n, 0\}.$$

The modified version of Eq. (5.14) is then

$$C(i,n,j,k) = \begin{cases} \max\{V(i,t_N) - I_N, 0\} & \text{if } n = t_N, \\ \frac{\pi_u(i,n)C(i,n+1)+\pi_d(i,n)C(i+1,n+1)}{R_f} & \text{if } n \notin T, \\ \max\{p_n \frac{\pi_u(i,n)C(i,n+1,j,k+1)+\pi_d(i,n)C(i+1,n+1,j+1,k+1)}{R_f} - I_n, 0\} & \text{if } n \in T. \end{cases} \tag{5.15}$$

This will be our expression for the risk-neutral option pricing formula for our quadranomial valuation. We will apply the procedure described in this last section to the case of Stokkfjellet Wind Farm in Section 6.3.

## 5.2 The Monte Carlo method

The past two decades, much has been written on the benefits of using the contingent claims method to value real assets. However, limitations on solving procedures and computing power have forced academics to simplify such real option methods to an extent where they lose relevance for real-world decision making (Cortazar et al., 2008).

The binomial lattice approach considers two sources of uncertainty, but does not include the uncertainty in the price development of tradable green certificates (TGC's). Another real options valuation method using Monte Carlo simulations is proposed in the following. This framework considers a European call option, as before, but the project will no longer be valued as a compound option. We have to simplify the option in order to simulate and value two stochastic processes.

Real options models will often be more challenging so solve than their financial option counterparts. This is mainly so for two reasons. First, many real options have a longer maturity, which makes risk modelling critical and may demand the use of several risk factors. Recall that the well-known option pricing model by Black and Scholes (1973) considers only one risk factor. Second, real options often consist of a complex set of nested and interacting American or Bermudan options. The introduction of multifactor price models into real options models with many interacting flexibilities, such as shutting down production, delay investments, or expanding capacity, complicates the solving process.

Schwartz (1997) shows that multifactor models perform significantly better than one factor models for commodity prices. Including several risk factors will allow for capturing observed behaviour of commodity prices, like mean-reversion and a declining volatility term-structure (Cortazar and Schwartz, 2003; Black and Scholes, 1973).

In Monte Carlo valuation we simulate future prices and use these simulated prices to compute the discounted future payoff of the real option. Risk-neutral pricing is a cornerstone of Monte Carlo valuation. Using the actual distribution instead of the risk-neutral distribution would create a complicated discounting problem (McDonald, 2013).

In this case, we will use two underlying stochastic processes to simulate the project value and ultimately obtain an option value for the investment opportunity. As before, the price of power is expected to follow a mean-reverting process. Now, the price of TGC's will also be taken into account, and we will assume that the price of TGC's follows a geometric Brownian motion, as was also done by Fleten and Ringen (2009). Considering the discussion in Section 2.2, the price formation of TGC's is difficult to understand because it is dependent on many factors, and it is believed that the price may increase or decrease significantly towards the end of the subsidy scheme period. This makes geometric Brownian motion an appropriate process for modelling this price development.

### 5.2.1 Computing the option price as a discounted expected value

The concept of risk-neutral valuation should be familiar from the discussion in Section 4.2 and the application in Section 5.1. We have seen that option valuation can be performed as if the underlying earned the risk-free rate of return and investors performed all discounting at this rate. Specifically, we compute the time price of a claim running from time 0 to time $T$ as

$$V_T = e^{-rT}\mathcal{E}_0^*[V_T], \tag{5.16}$$

where $\mathcal{E}_0^*$ is the expectation computed at time 0 using the risk-neutral distribution. Monte Carlo valuation exploits this procedure. We *assume* that the project will earn the risk-free rate of return and simulate its returns. For the project value 25 periods (months) from now, we can compute the payoff of the call option on the project's cash flows. We can then simulate the process many times and average the outcomes, which will give us an estimate of $\mathcal{E}_0^*[V_T]$. Since we are using risk-neutral valuation, we then discount the average payoff at the risk-free rate in order to arrive at the price. When we simulate such a process, we add a random element to the computation of the price one period ahead. We then create a path that is in part a result of chance. To do this, we need to draw random numbers from a suitable distribution, which would in this case be the normal distribution.

### 5.2.2 Drawing random numbers

Let us discuss how we can compute the normally distributed random numbers required for Monte Carlo valuation. The uniform distribution is defined on a specified range, over which probability is 1, and assigns equal probabilities to every interval of equal length in that range. A random variable, $u$, that is uniformly distributed on the interval $(a, b)$, has the distribution $\mathcal{U}(a, b)$. The uniform probability density function, $f(x; a, b)$, is defined as

$$f(x; a, b) = \frac{1}{b - a}; a \leq x \leq b \tag{5.17}$$

and is 0 otherwise. We want to use a random number drawn from the uniform probability density function to draw random numbers from the normal distribution (recall our normality assumption discussed in Section 4.2). Suppose that $u \sim \mathcal{U}(0, 1)$, and $z \sim \mathcal{N}(0, 1)$. The cumulative distribution function, denoted $U(w)$ for the uniform and $N(y)$ for the normal distribution, is the probability that $u < w$ or $z < y$, i.e.

$$U(w) = \Pr(u \leq w),$$
$$N(y) = \Pr(z \leq y).$$

We can interpret the randomly drawn number from the uniform distribution as a quantile for the normal distribution. Using the inverse distribution function and taking $N^{-1}(u)$ will yield the value from the normal distribution corresponding to that quantile. Now, to simulate a log-normal random variable, we can simulate a normal random variable and exponentiate the draws.

### 5.2.3 Simulating price processes

Simulating a price process involves calibrating the parameters of the process from historical data, and letting the actual development of the price be in part determined by a random variable, i.e. by a randomly drawn number such as one described in the above section. Soon, we will describe how the actual simulation of prices can be performed. First, it is necessary to explain how normalised estimates for the parameters of the price process of the TGC's can be obtained. Recall that we demonstrated how this could be done for a price that is mean-reverting in Section 5.1.

### Obtaining normalised estimates for the parameters

In order to simulate the price development of TGC's, we need to estimate the normalised parameters of the process. If $p_j$ denotes the $j$th observation of the log price, then the data-generating process for the log price is

$$p_{j+1} - p_j = \nu p_j + u_{j+1} p_j, \quad u_{j+1} \sim \mathcal{N}(0, \phi^2), \tag{5.18}$$

where $\nu$ and $\phi$ are constants and $u_{j+1}$ is a noise term. In other words, over each period the log price changes by an amount that is the product of the last log price and the sum of a constant ($\nu$) and a random term that is normally distributed with mean equal to zero and a standard deviation equal to another constant ($\phi$). Some points are worth noticing: First, the log price is more volatile when the price is high than when the price is low. Second, shocks to the log price are permanent, i.e. the process has no memory.

A stochastic process known as geometric Brownian motion with drift $\mu$ and volatility $\sigma$ generalises the process described by Eq. (5.18) to the situation where the log price is observed with arbitrary frequency. That is, we have

$$dp = \mu \, p \, dt + \sigma \, p \, dz, \tag{5.19}$$

where $dz$ is the increment of a Wiener process. Eq. (5.19) implies that the current price of TGC's is known, whereas future prices are uncertain and log normally distributed with a volatility that grows linearly with the time horizon. For an introduction to geometric Brownian motion and the concept of stochastic processes, we refer to Appendix F.

When viewed from date $t$, the change in $p$ over the next $\Delta t$ units of time is normally distributed with mean $(\mu - 0.5\sigma^2)\Delta t$ and variance $\sigma^2 \Delta t$, where $\mu$ and $\sigma$ are constants. That is,

$$p_{t+\Delta t} - p_t \sim \mathcal{N}((\mu - \frac{1}{2}\sigma^2)\Delta t, \sigma^2 \Delta t).$$

The most distinctive feature of this process is that the variance of the change in the log price continues to grow as we look further and further into the future. While the expected change equals $\mu - \frac{1}{2}\sigma^2$ per unit of time, there is no force keeping the log price from wandering far away from its expected path.

Under the process described by Eq. (5.18), changes in $p$ are normally distributed with mean $\nu - 0.5\phi^2$ and variance $\phi^2$. Thus, the parameters $\nu$ and $\phi$ in Eq. (5.18) are related to the drift and volatility of the geometric

Brownian motion by the equations

$$\nu = (\mu - \frac{1}{2}\sigma^2)\Delta t \text{ and } \phi^2 = \sigma^2 \Delta t.$$

Now, to ensure consistency with the binomial model, we will continue to use historical price data which has one observation every month, $\Delta t_d$. Suppose also that changes in the log price have sample mean $\hat{\nu}$ and standard deviation $\hat{\phi}$. We can estimate these parameters directly from the data. If the data-generating process for the log price is geometric Brownian motion with drift $\mu$ and volatility $\sigma$, then the population variance of changes in the log price is $\sigma^2 \Delta t_d$. Therefore, a sensible estimate of the volatility parameter $\sigma$ is the number $\hat{\sigma}$ that satisfies

$$\hat{\phi}^2 = \hat{\sigma}^2 \Delta t_d,$$

so that the population variance equals the sample variance. This implies that

$$\hat{\sigma} = \frac{\hat{\phi}}{\sqrt{\Delta t_d}}. \tag{5.20}$$

Likewise, the population mean of changes in the log price is $(\mu - 0.5\sigma^2)\Delta t_d$. Therefore, a sensible estimate of the drift parameter $\mu$ is the number $\hat{\mu}$ that makes the population mean equal to the sample mean. That is, we choose $\hat{\mu}$ so that it satisfies

$$\hat{\nu} = (\hat{\mu} - \frac{1}{2}\hat{\sigma}^2)\Delta t_d,$$

which implies that

$$\hat{\mu} = \frac{\hat{\nu}}{\Delta t} + \frac{1}{2}\hat{\sigma}^2. \tag{5.21}$$

This procedure will be employed in order to estimate the parameters of the price process of TGC's in Section 6.4.

### Simulating lognormal prices

In the Monte Carlo valuation in Section 6.4, we will assume that the price of TGC's follows the process of geometric Brownian motion. If $Z$ is a random normal variable, the lognormal certificate price can be written in discrete-time as

$$P_t = P_0 e^{(\hat{\mu} - 0.5\hat{\sigma}^2)\Delta t + \hat{\sigma} Z \sqrt{\Delta t}}, \tag{5.22}$$

assuming that the green certificate pays no dividends, where the parameters $\hat{\mu}$ and $\hat{\sigma}$ are estimated from historical data. This means that, over a finite time interval $t$, the change in the logarithm of $P_t$ is an i.i.d. process with a mean return of $(\hat{\mu} - 0.5\hat{\sigma}^2)t$ and a variance of $\hat{\sigma}^2 t$. This result will be used to generate price paths for the valuation.

### Simulating mean-reverting prices

Just as we did in Section 5.1, we will assume that the power price follows an Ornstein-Uhlenbeck process. If $Z$ is still a random normal variable, the power price can be written in discrete-time as

$$P_t = P_0 e^{-\hat{a}\Delta t} + \bar{P}(1 - e^{-\hat{a}\Delta t}) + \hat{\sigma} Z \sqrt{\frac{1 - e^{-2\hat{a}\Delta t}}{2\hat{a}}}, \tag{5.23}$$

where $\bar{P}$ is the exponentiated version of the long-run log price level $\hat{b}$ as determined in a regression along with the parameters $\hat{a}$ and $\hat{\sigma}$. The steps for how to determine the parameters of a mean-reverting process were described in Section 5.1.

### Simulating a correlation in the price of two different underlying processes

When two or more random variables are demonstrated not to be independent, one has to consider this when simultaneously simulating their processes. Let us first introduce The Pearson correlation coefficient, $\rho$, which measures the degree of correlation between random variables. The population correlation coefficient $\rho_{X,Y}$ between two random variables $X$ and $Y$ with expected values $\mu_X$ and $\mu_Y$ and standard deviations $\sigma_X$ and $\sigma_Y$ is defined as

$$\rho_{X,Y} = \frac{\text{cov}(X,Y)}{\sigma_X \sigma_Y} = \frac{\mathcal{E}[(X - \mu_X)(Y - \mu_Y)]}{\sigma_X \sigma_Y}, \tag{5.24}$$

where, as before, $\mathcal{E}$ is the expected value operator and cov means covariance.

For simulating systems with correlated variables, Cholesky decomposition can be used. Specifically, one needs to decompose the correlation matrix, which will yield the lower-triangular matrix $\mathbf{L}$ which contains real and positive diagonal entries. Applying this to a vector of uncorrelated samples, $\mathbf{u}$, produces a sample vector $\mathbf{Lu}$ with the covariance properties of the system being modelled. The goal here is not to give a full introduction to the concept of Cholesky decomposition, but rather to give an idea of the underlying mathematics applied for the Monte Carlo valuation.

Suppose we want to generate two correlated normal variables $x_1$ and $x_2$. In order to do this, generate two uncorrelated Gaussian random variables $z_1$ and $z_2$. Then, set

$$\begin{aligned} x_1 &= z_1, \\ x_2 &= \rho z_1 + \sqrt{1-\rho^2}\, z_2. \end{aligned} \tag{5.25}$$

This result can be used to ensure a correlation between two random processes, where the strength of the correlation is set by the parameter $\rho$.

### 5.2.4 Simulating the process for the underlying

In accordance with the marketed asset disclaimer assumption, we still assume that the value of the underlying is the value of operation without flexibility (i.e. the NPV of the operational phase). In addition, the earlier assumption that the projects volatility is equal to the price of electricity will be expanded to include the TGC's as well. The key idea is that we in each time step generate two correlated random variables that are used for determining the price in the subsequent time period of both processes. We will weight an average of the two simulated paths of the value of operation, and use this average value as the value of the underlying at exercise.

Suppose we are currently in time step $t$. Then, the NPV of the operational phase, $V_t$, is in time period $t$ determined as

$$V_{GBM,t} = V_{GBM,t-1}\, e^{(\hat{\mu}-0.5\hat{\sigma}^2)\Delta t + \hat{\sigma} x_1 \sqrt{\Delta t}},$$

for the value of operation that follows the geometric Brownian motion, and

$$V_{OU,t} = V_{OU,t-1}\, e^{-\hat{a}\Delta t} + \bar{V}_{OU}(1-e^{-\hat{a}\Delta t}) + \hat{\sigma} x_2 \sqrt{\frac{1-e^{-2\hat{a}\Delta t}}{2\hat{a}}},$$

for the Ornstein-Uhlenbeck process. Note that $Z$ has been replaced with $x_1$ and $x_2$ in the respective above equations. These equations will be used to perform the Monte Carlo simulations.

### 5.2.5 Finding the option value

It is the points where the simulated paths end, i.e. the set of all the different values of operation after 25 periods that we need for determining the option value. Suppose we run $S$ simulations through $T$ time periods. Let the value of operation resulting from simulation $s$ be denoted $V_{T,s}$. Then, the average option value resulting from simulating both processes can be found as

$$C_T = \frac{1}{S}\sum_{s=1}^{S} \frac{V_{GBM,T,s} + V_{OU,T,s}}{2} - I_T, \tag{5.26}$$

where $I_T$ is the investment cost incurring in period $T$. This is our estimate of the future value of the option. All that is left is then to find the current option value, $C_0$, which is done by discounting this value back to today, i.e. by taking

$$C_0 = e^{-rT} C_T,$$

where $r$ is the risk-free rate. The reasoning behind taking the average of the two end point values of operation in each simulation as done in Eq. (5.26) is that the price of power and the price of TGC's are both listed for 1 MWh, so that an equal weighting of the two processes can be justified.

The valuation in Section 6.4 will make use of the theory that has been presented in this section.

# 6 Valuation and Results

Our starting point for the valuation of Stokkfjellet Wind Farm is the discounted cash flow (DCF) method, performed in the same manner that TrønderEnergi currently does. From there on we want to perform the valuation using more advanced methods that include the value of the flexibility incorporated in the project. First, we compute the value of operation without flexibility from the static net present value (NPV) method. This will be used as input for the valuations performed. Second, we do a decision tree analysis, where we employ the statistical probabilities of success in the remaining stages of the development process. The first real options model takes the binomial lattice approach. The binomial model considers the power price as a stochastic variable, and uses a real options framework to find the path of possible scenarios and the value of the option. The two factor binomial lattice model will, in addition to the price uncertainty, take into account the probability of success in each stage, just as we did in the decision tree analysis. In order to consider the price development of tradable green certificates (TGC's), we end the chapter with a Monte Carlo valuation that takes into account the price process of electricity and TGC's, as well as their correlation.

May 2014 will be the starting point for the valuations that take a monthly perspective.

## 6.1 Discounted cash flow valuation

Currently, TrønderEnergi is applying the traditional discounted cash flow (DCF) method to find the NPV when evaluating their investment opportunities. We will use this method to determine the value of operation of

Stokkfjellet Wind Park, and this value will be used as input for the other valuation models. The future free cash flows are calculated by estimating the future income, costs and taxes. These free cash flows are then discounted at the projects rate of return to today's value.

### Income from operation

In this case study, the income from production will depend on three important factors: The amount of power produced, the spot price of electricity, and the market price of TGC's.

Kjeller Vindteknikk AS has measured the wind resources at the site of Stokkfjellet at 80 meters and 100 meters over a period of eight months. The data is corrected for long-term effects, and Kjeller Vindteknikk AS estimated a gross annual energy production of 338 GWh in the case of 43 Siemens SWT-2.3-93 2.3 MW turbines. Assuming that the total losses will average 28 per cent annually, the net annual energy production is calculated to 243 GWh and assumed constant throughout the lifetime of the plant.

The future development of the power price in the Nordic market must be estimated in order to find an expression for the income. The trading department of TrønderEnergi has provided a deterministic price path that has been applied in the DCF model, with one estimated value for each time step, which will here be yearly. Since our target is to illustrate the differences between valuation approaches, the price path provided has been accepted without further investigations.

As for the price of electricity, the price development of TGC's has been provided as a deterministic price path by the trading department of TrønderEnergi. As already mentioned in Section 2.2, the market for TGC's is a complicated market since there is no marginal cost of production.

### Costs

The actual size of costs are uncertain parameters until TrønderEnergi enters the procurement stage and receives actual tenders. Historical data from similar wind projects has been used to estimate the costs of operation as well as the investment cost. With the assumption of constant production over the lifetime of the plant, operational costs are also constant.

### Depreciation, amortisation, taxes, rate of return and inflation rate

Assets expected to have a lifetime of more than three years and a price of more than NOK 15,000 excluding taxes must by Norwegian law be depreciated. This implies that the investment cost is spread over several years

(Skatteetaten, 2014). Depreciation is done by the declining balance method, and the investment costs are divided into two depreciation groups, D and G. Depreciation group D is expenditure related to the turbines and related project management, and is discounted at 20 per cent over the lifetime of the plant, except the first year, where it is depreciated with 30 per cent. This is in accordance with guidelines set by Finansdepartementet (2014). Depreciation group G is depreciated at a rate of 5 per cent, which is the expenditure of the equipment for transmitting and distribution of electricity (Finansdepartementet, 2014). In the last year of production, the book values of the different depreciation groups are amortised to zero. This results in no taxes in the last year of operation and in principle negative tax for transfer. We do not take this into account because we are evaluating the isolated case of Stokkfjellet.

According to Finansdepartementet (2014), the tax rate on general income in Norway is 27 per cent. Property tax is treated as a fixed cost in the model. It is defined to be 0.7 per cent of the nominal investment cost over the lifetime of the project (Thema Consulting Group, 2012).

The real rate of return before taxes, $p_r$, is normally 8 per cent. This is a standard given by Enova for renewable energy investments (Gjølberg and Johnsen, 2007). Eq. (6.1) gives the expression for converting the real rate to nominal, after-tax values as

$$r_n = [p_r(1 + j) + j]\,(1 - s), \tag{6.1}$$

where $j$ is the inflation rate and $s$ is the taxation rate. This gives a nominal, after-tax required rate of return, $r_n$, of 7.8 per cent, which is rounded to 8 per cent in our model. The inflation rate used is 2.5 per cent, according to the Norwegian Governments monetary policy (Norges Bank, 2013).

It lies outside the scope of this thesis to accurately estimate the required rate of return for the wind park. Since we wish to illustrate the application of different valuation methods, we have not included derivation of the required rate of return, but rather applied the rate suggested by Gjølberg and Johnsen (2007). Also, we think TrønderEnergi is more qualified than us when it comes to estimating this parameter, given the confidential information they hold. Thus, we assume that the investment is funded by equity, and we have not evaluated the influence of the projects funding on its value.

### 6.1.1 Results

The value of the operational phase, hence excluding the initial investment costs, is found to be 873.8 MNOK. This value is the calculated DCF value

**Table 2:** Sensitivity analysis of the NPV of the operational phase of Stokkfjellet Wind Park. Values are listed in KNOK.

| | | | | | |
|---|---|---|---|---|---|
| **Rate of return** | | | | | |
| Deviation | −20 % | −10 % | - | 10 % | 20 % |
| Rate of return | 6,40 % | 7,20 % | 8,00 % | 8,80 % | 9,60 % |
| Contingent NPV | 1 018 554 | 942 568 | 873 816 | 811 488 | 754 878 |
| Relative change | 17 % | 8 % | - | −7 % | −14 % |
| **Production** | | | | | |
| Deviation | −20 % | −10 % | - | 10 % | 20 % |
| Production | 194 400 | 218 700 | 243 000 | 267 300 | 291 600 |
| Contingent NPV | 688 917 | 783 690 | 873 816 | 953 794 | 1 032 272 |
| Relative change | −21 % | −10 % | - % | 9 % | 18 % |
| **Power prices** | | | | | |
| Deviation | −20 % | −10 % | - | 10 % | 20 % |
| Contingent NPV | 736 700 | 807 582 | 873 816 | 933 368 | 922 038 |
| Relative change | −16 % | −8 % | - | 7 % | 6 % |

of operation, and will be used in the other valuation models as the value of the underlying. Take into account that the value is merely an estimate, due to possible inaccuracies in the input parameters. The most sensitive parameters are analysed in the following sensitivity analysis.

### 6.1.2 Sensitivity analysis of the static DCF results

One problem of the DCF model lies with the many economic factors that are input as assumptions. The motivation for performing the sensitivity analysis is to identify the most sensitive inputs to the model and this way evaluate the validity of the results given the estimated inputs. First, we vary all the relevant parameters to identify the most sensitive inputs, then we analyse them in detail. The relevant inputs were first varied by one per cent. This revealed that production, rate of return and the power price are the most sensitive parameters of the valuation. In Table 2 these inputs are analysed with 10 per cent and 20 per cent deviations. As can be seen, this greatly affects the value of operation.

Deviations in production will have the biggest impact on the NPV. A 20 per cent decrease in production will cause a negative change in the NPV of 21 per cent. This should be expected, given the natural correlation between these two factors.

Nevertheless, the rate of return is probably the most important input parameter, considering that the choice of the magnitude of this parameter is quite subjective. According to TrønderEnergi, the choice of the required rate of return is dependent on the different investors and their perception of which rate is appropriate. The rate of return is determined according to what type of project is considered in terms of risk and project lifetime. Hence, it is important to be aware that this is a parameter to which the NPV is very sensitive, so that an investment decision is not made on the wrong basis.

Power prices must be treated slightly differently than the two above parameters in the sensitivity analysis. The reason is that we have a whole price path as a starting point and not one single value. We have chosen to change each annual price estimate with a constant deviation. The effect will thus not be directly comparable to that of the above factors. Table 2 illustrates that small changes in the electricity price will have a significant impact on the NPV.

## 6.2 Contingent NPV – decision tree analysis

Decision makers do not know for certain if Stokkfjellet Wind Farm will reach its operational phase. The project can fail due to event outcomes, e.g. if the licence is withdrawn as a result of the appeal processing. Also, the market can go down such that the power price becomes too low for the investors to undergo the investment. This is important to take into account when valuing the project, and the static NPV fails to reflect the actual value embedded in the project. Since decision makers can revise their decisions at the arrival of new information, the static NPV is an unqualified valuation method for this purpose.

The contingent NPV approach, sometimes called decision tree analysis (DTA), is a method that includes diversifiable risk and allows for the use of two different discount rates. This model is thus more accurate than the static DCF method. However, DTA employs discounted cash flows to find the value of the project. The method weights every possible outcome with the related probability and discounts the value back to year 0 to find the present value of the wind farm under development. The information underlying the DTA is shown in Table 3.

### Risks

The diversifiable risk present in the Stokkfjellet project is the policy uncertainty, i.e. the probability of success in each stage. The uncertainty is related to the probability of success in the remaining stages of development

**Table 3:** Overview of the information associated with what is left of the development of Stokkfjellet Wind Farm. Here the stages as displayed in Fig. 4 have been divided into smaller steps, allowing for a more accurate valuation. All costs in KNOK.

| Action | Cost associated w/ action | Months from start | Prob. of success | Accumul. prob. |
|---|---|---|---|---|
| Identify potential project | | | 1 | 1 |
| Plan and prepare licence application | | | 1 | 1 |
| Processing of licence application (NVE) | 544 | 3 | 1 | 1 |
| Complaint processing (NVE) | 5 241 | 15 | 0.8 | 0.8 |
| Prepare tender enquiry, part 1 | 2 507 | 20 | 1 | 0.8 |
| Prepare tender enquiry, part 2 | 1 563 | 25 | 1 | 0.8 |
| Builing, phase 1 | 468 491 | 36 | 1 | 0.8 |
| Builing, phase 2 | 479 216 | 38 | 1 | 0.8 |

only. Estimating the probabilities of success for each stage is not an accurate science and a difficult task. There is not a sufficient amount of data available to allow us to accurately calculate the probabilities associated with success in each step of the decision tree for wind farm development. Fig. 6 shows the decision stages used by TrønderEnergi after a site has been identified as a potential project. A project currently in phase $i$ moves to the next phase of development $i + 1$ with probability $1 - \rho_i$. Probabilities supplied by TrønderEnergi and NVE will be used for the valuation. The probabilities are assumed independent of the development in the market, i.e. they are not exposed to commodity risk. This means that the outcome of events is independent of whether the price moves up or down (Méndez et al., 2009).

#### Applying different discount rates

Due to the different risk profiles for the project's development phase and operational phase, we apply different discount rates for these two phases. The development phase is not exposed to market risk and the costs associated with this phase should therefore be discounted at the risk-free rate of return. The operational phase is exposed to commodity risk, and is therefore dependent on the market. This phase must be discounted at the required rate of return. Recall that we in Section 6.1 defined the required rate of return to be 8 per cent. The ability to use two different discount rates is one of the benefits of DTA.

#### Risk-free rate

The risk-free interest rate over the lifetime of the project is assumed to be equal to the rate of 10-year treasury notes. We have taken the annual average of daily notations, which is 2.58 per cent (Norges Bank, 2014).

### 6.2.1 Determining the value of the park

We assume that the cost of each stage has to be paid on the day that the stage starts, while the decision to invest in the next stage and the outcome of any event fall on the last day of each stage. TrønderEnergi must pay $\sim 0.5$ MNOK to further develop the project in the phase of license processing, while the cost related to the complaint processing is $\sim 5$ MNOK. The cost of issuing tenders and making the final investment decision is $\sim 4$ MNOK, while the capital expenditure related to the building of the plant is estimated to be 890 MNOK. Cost estimates for each stage have been provided by TrønderEnergi, and include wind measurements as well as development costs until the building stage starts.

The probabilities of moving to the next phase of development have also been provided by TrønderEnergi, and rely on empirical observations. The tast of estimating such probabilities is a difficult one as each investment situation differs from another, but investigating the process behind these estimates lies outside the scope of this thesis.

Originally, the probability of success in the stage of the licence application was given to be 0.6. However, recently it became clear that Sør-Trøndelag County supports the project, and as a result it is now considered extremely likely that the concession will be given. However, the licence has not been officially granted, and the stage is not completed. We have chosen to take this information into account by adjusting the probability to

1. Support from the county can normally not be expected. Rather, power companies usually meet a lot of opposition when developing similar projects.

The probability that the appeal is declined by OED is set to 0.8. This too is based on empirical observations. Past events have indicated that there is a very good chance that the licence is granted after the appeal.

At last, the probability of reaching the operational phase for Stokkfjellet Wind Farm is set to 1. This means that if TrønderEnergi decides to build, we assume that they will reach the phase of operation. The accumulated probability of getting to the building stage is therefore 0.8 ($1 \times 0.8 \times 1$). This model assumes that TrønderEnergi proceeds to the next stage if the previous stage is successful.

The cost associated with each stage of the project is discounted back to today at the risk-free rate and multiplied with the accumulated probability of success up to that stage. These costs are then summarised, and the sum of the costs for the development phase is subtracted from the present value of the operational phase, which is discounted at the required rate of return.

### 6.2.2 Results

The model returns a negative value for the project of −20.8 MNOK. This valuation approach would thus advise the decision makers not to go through with the investment in Stokkfjellet Wind Farm.

### 6.2.3 Scenarios for the performance of the park after launch

TrønderEnergi has emphasised that there is uncertainty related to whether the park will operate as expected after it has been set in operation. The technological uncertainty related to performance could also be considered in a decision tree. We have done an analysis of parks currently in operation in Norway, and compared the estimated production to the actual produced amount of power. The results are shown in Appendix E. Our analysis shows that the biggest deviations in production come from the parks that were set in operation before the year 2000. Parks that launched after the year 2000 show a significantly better performance, so we consider these parks obsolete for comparison. The grand average deviation is only −1.46 per cent, so it does not seem meaningful to perform a scenario analysis on this matter.

### 6.2.4 Sensitivity analysis of the decision tree analysis

The probability of success in each stage might change during the project development, and the sensitivity of the model to potential changes to this factor is shown in Table 4.

**Table 4:** Sensitivity analysis of important factors going into the contingent NPV valuation. Values are listed in KNOK.

| **Probability of success in licence application to NVE** | | | | | |
|---|---|---|---|---|---|
| Probability | 60 % | 70 % | 80 % | 90 % | 100 % |
| Contingent NPV | −12 490 | −14 571 | −16 653 | −18 735 | **−20 816** |
| Relative change | 40 % | 30 % | 20 % | 10 % | - |
| **Probability of success in processing of appeal to OED** | | | | | |
| Probability | 56 % | 64 % | 72 % | 80 % | 88 % |
| Contingent NPV | −14 734 | −16 762 | −18 789 | **−20 816** | −22 843 |
| Relative change | 29 % | 19 % | 10 % | − | −10 % |

The change from 0.6 to 1 in the probability of success for the first stage causes a relative change in the contingent NPV of 40 per cent. The contingent NPV increases in value when the probability of success decreases. Nonetheless, since the result will be negative for all probabilities, the project would be rejected in all cases.

## 6.3 Binomial lattice valuation

This method finds the project value of Stokkfjellet Wind Farm by modelling the investment opportunity as a sequential compound option. We explained why this way of modelling the investment opportunity was appropriate in Section 2.6. The model described in the following takes into account both commodity risk and policy risk.

At the end of every stage of the project, a decision about whether or not to move forward with the project has to be made. Since there are several stages, this investment pattern is analogous to a compound European call option. The exercise price of each single option is then the cost associated with the phase that we could proceed to. The cost of building the wind park is the final exercise price we would need to pay in order to be able to enter into the operational phase and receive the cash flows from the project. This cost constitutes the majority of the investment cost. When the largest cost comes last, the option value will be larger because we have the chance to avoid this cost in the case that the investment environment becomes more hostile so that we wish to abandon the investment opportunity. By exercising the last option, TrønderEnergi agrees to pay the strike price (cost of building the park) for the rights to receive the future cash flows generated by the project (net income of operational phase). We will value this real option using the binomial lattice approach that was presented in Section 5.1.

We first build a tree for the development of the electricity price. Based on this, we can calculate the risk-neutral probability of an up move in the price at each node in the tree. The next step is to compute the tree for the underlying, which we assume, is the NPV of the operational phase without flexibility. After generating the NPV tree, we take the intrinsic value of the NPV in the last column of the tree. From here we work our way backwards in the tree to find the real option value, treating the investment opportunity as a European compound call option. The option value reflects the market value of the project under development including the value of the flexibility associated with being able to revise the investment decision in stages.

We emphasise that our goal is to evaluate the project in the current phase of development, i.e. we consider past investments as sunk costs that do not affect our decision to invest. At this point, TrønderEnergi still has the possibility to make informed decisions as the market evolves.

As discussed in Section 4.3, we assume that the electricity price follows a mean-reverting process. The procedure for finding the real option value for a compound option with an underlying asset following a mean-reverting process can be reduced to seven steps:

1. Estimate the AR(1) model using historical log price data.
2. Find the Ornstein-Uhlenbeck parameters by substituting the AR(1) estimated parameters into Eq. (5.1).
3. Build the binomial tree for the price using Eq. (5.3).
4. Estimate the probability of an up move at each node in the tree using Eq. (5.8).
5. Find realistic estimations for the $\beta$ and the market price of risk and compute the risk neutral probabilities.
6. Compute the binomial tree for the underlying with the similar approach as for the price.
7. Find the intrinsic value of the option and from there on use backward induction and the option pricing formulas to find the present value of the real option.

After describing the data resources applied, we will present the steps in the procedure for finding the real option value.

### 6.3.1 Collecting the data

Daily electricity price data for the NO3 Elspot area from January 1 2001 to December 31 2013, available at Nord Pool Spot, is used for calibrating the price process. The real price data has been normalised to monthly prices. From Fig. 1, it can be seen that mean-reversion is evident in the electricity price data.

Deciding on the size of the steps in the binomial lattice is a trade-off between accuracy and simplicity. Increasing the number of time steps ($\Delta t \to 0$) will make the model close to continuous; however, if the tree becomes too large the computations will take longer, and at a certain point the benefit of using more steps becomes small. For this valuation, $\Delta t$ chosen to be one month (~0.833 years).

### 6.3.2 Computing the power price tree and the risk-neutral probabilitites

We start by running a regression in Eq. (4.3). The results are

$$\ln p_{j+1} - \ln p_j = \underset{(0.00057)}{0.14703} - \underset{(0.00058)}{0.04144} \ln p_j, \quad s = 0.05181, \tag{6.2}$$

where the numbers in parenthesis are the $P$ values. A $P$ value lower than 0.05 is generally accepted as the point where we reject the null hypothesis, i.e. the hypothesis that the coefficient is equal to zero (has no effect). As can be seen, we cannot reject the null hypothesis here, and we hereby assume that the coefficients are meaningful additions to our model because changes in the predictor's value are explained by changes in the response variable. Notice that $\alpha_1$ is indeed negative, which is a necessary condition by the discussion in Section 5.1. Generally, the coefficients in an AR(1) regression do not have causal interpretations.

**Table 5:** Relevant regression statistics. *Data source: Log price data from Nord Pool Spot (2014).*

| **Regression statistics** | |
|---|---|
| R Square | 0.0747 |
| Significance F | 0.0006 |
| Standard error | 0.0518 |
| Observations | 155 |

**Table 6:** Normalised estimates of the parameters. *Data source: Log price data from Nord Pool Spot (2014).*

| Parameter | Estimated value |
|---|---|
| $\hat{a}$, rate of mean reversion | 0.50784 |
| $\hat{b}$, long-run mean | 3.54839 |
| $\hat{\sigma}$, standard deviation | 0.18138 |

The key regression statistics are given in Table 5. $R^2$ takes on a value in the interval $[0, 1]$ and is a global measure of the fit of the model because it indicates how well the observed sample values are replicated by the model. The regression statistics indicate that the trajectory line is a poor fit for the sample, with $R^2 = 0.07$. In general, a low $R^2$ is not a problem unless the model is to be used for predicting values that are out of sample. The standard error $s$ of the estimate is the standard deviation of the residuals. The standard error is typically low for a large sample as it is inversely proportional to the sample size. The larger the sample size, the smaller the error because the statistic will approach the true value.

Substituting the regression coefficients and the standard error into Eq. (5.1) then yields the normalised parameter estimates needed to describe the particular price process. The normalised parameter estimates are given in Table 6.

The code for computing the tree for the price of power has been written in Visual Basic for Applications (VBA), and is included in Appendix A. Besides the normalised parameter estimates that describe the particular price process ($\hat{a}$, $\hat{b}$, $\hat{\sigma}$), we need also to input some other values to do the necessary calculations. The key inputs to the binomial tree for the power price and the risk neutral probabilities are the

- Current asset value, $p_0$
- Risk-free rate, $R_f$
- Company beta, $\beta$
- Market risk premium, $E[\tilde{R}_m] - R_f$
- Time to maturity, $T$

Each input parameter will be discussed briefly.

### Current asset value, $p_0$

For the price tree, the current asset value should in theory be the power price on the day that the valuation is performed. Nord Pool Spot lists the average power price in the Elspot area NO3 on May 1 2014 as 26.78 EUR/MWh. However, this is very low relative to the average price that has been observed in the not too distant past. In their own estimates, TrønderEnergi has assumed an average power price of 33 EUR/MWh for the year 2014. This is closer to the average price for the first 4 months of 2014. It is thus considered more appropriate to use this power price as the starting point for our valuation. This way we avoid that the NPV of the operational phase is treated to be as low relative to its long-run level as the current power price is to its long-run level. Recall that the value of the underlying, the NPV of operational phase, is assumed to follow the same process as the power price.

### Risk-free rate, $R_f$

As discussed in Section 6.2, the risk-free rate of return is assumed to be 2.58 per cent.

### Company beta, $\beta$

The parameters necessary for applying the CAPM were found in dialog with TrønderEnergi. Determining the company beta is not an exact science when the company is not listed, and there are actually few comparable observations in the market. The project beta will change internally in the company, depending on what technology the relevant project under consideration utilises. Hydropower projects will typically have a lower beta than wind power projects, and the company beta will be a weighted average of the different betas from the various business areas. Considering the relative risks between the business areas and the required rate of return, the beta for TrønderEnergi's onshore wind power projects will be set to 0.6. There is uncertainty related to this estimate due to little market data for wind power projects and few market transactions that can be used for comparison.

### Market risk premium, $E[\tilde{R}_m] - R_f$

The market risk premium is assumed to be 5 per cent, based on the report by PwC (2013) on the Norwegian market in 2013-14.

**Time to maturity,** $T$

The time to maturity of each of the options that together make up the compound real option is equal to the length of each stage in the development phase, i.e. 3, 12 and 10 months for the concession application, processing of the appeal and issuing of tenders and the final planning of the park, respectively. We therefore want to compute a tree for the price that consists of 25 time steps (months). Each individual option is purchased in the beginning of its stage and expires at the end of that stage, and on the date of expiration TrønderEnergi can choose to exercise that option in order to receive the next option.

### 6.3.3 Computing the risk-neutral probabilities of an up move

One important characteristic of the mean-reverting binomial tree is that the risk-neutral probabilities in each node are not necessarily constant. We therefore have to work through each node when computing the risk-neutral probabilities using Eq. (4.2). We use these probabilities to find the real option value that appreciates the flexibility embedded in the project.

### 6.3.4 Building the tree for the underlying

There are two important assumptions we must recall from Chapter 4 when we are to build the tree for the underlying. First, we have the marketed asset disclaimer assumption, i.e. the assumption that the value of the underlying is equal to the NPV of the operational phase without flexibility. The second assumption is that this underlying asset is assumed to follow the same process as the price process driving the value of the asset, namely the price of electricity. In accordance with this assumption, we can apply the same method as when we computing the price tree for computing the tree for the underlying. We are only changing the start value, taking as input the net present value of the operational phase, $V(0, 0)$, which was estimated in Section 6.1.1 to be approximately 873.8 MNOK.

### 6.3.5 Finding the real option value

We have arrived at the last stage in the process of finding the value of the project using the binomial lattice approach. After computing the respective NPV values, we work backwards in the tree in order to find the optimal decision for each outcome given the development of the underlying. In the last column of the tree we find the intrinsic value of the real option, and from here on we work our way backwards in the tree using the option pricing formula given in Eq. (5.15). The VBA code written for this procedure is found in Appendix B.

In order to do these calculations we need two more inputs, namely the

- Exercise price and dates, $I_n$ and $T_n$
- Probability of proceeding to the next stage, $p_n$.

#### Exercise price and dates, $I_n$ and $T_n$

The size and timing of the exercise prices, or the investment costs, are based on estimates given by TrønderEnergi, and all costs are assumed to be paid on the day that the stage it corresponds to starts. There is uncertainty related to both the timing and the size of the costs, but the estimates given by TrønderEnergi are considered deterministic in this thesis. The sensitivity analysis for the NPV valuation in Section 6.1 pointed out that the investment cost is a factor that the NPV is very sensitive to, so it is important to have realistic cost estimates to ensure accuracy in results.

The exercise dates are determined by the length of the stages, which are provided by TrønderEnergi based on their experience with similar projects. As mentioned, the first stage will is assumed to last for 3 months, stage number two for 12 months, while the third stage is assumed to last for 10 months. The final investment decision can therefore be made in 25 months, assuming that the other decisions stages have been completed on time and not abandoned.

#### Probability of proceeding to the next stage, $p_n$.

Earlier, we argued that the static NPV does not reflect the true value of the project because it does not take into account the uncertainty about events. By the same reasoning, it can be argued that the value of the real option without including event probabilities is not the most accurate. We should thus include the probabilities of moving on to the next stage in our binomial lattice model.

From Section 6.2, recall that the probabilities for the three next stages were set to 1, 0.8 and 1, respectively. The way to include this in the binomial lattice option valuation is by multiplying the option value at the exercise nodes with the probability of success in the relevant stage, i.e. by using Eq. (5.15). Note that we will be using the probability of success in each individual stage for the calculations, and not the accumulated probability, as we did in the DTA.

### 6.3.6 Results

The value of the investment opportunity is estimated at 28.7 MNOK. This can be seen as the green value furthest to the left in Fig. 9. This value reflects the market value of Stokkfjellet Wind Farm under development including the value of the flexibility of having the ability to revise the investment decision in stages.

This value is positive, in contrast to the result from the DTA. Thus the result from this valuation method would advise the decision makers at TrønderEnergi to undergo the investment. It is interesting to note that if the probability of success in the licence application stage would be set to 0.6 as it was in the original estimate by TrønderEnergi, the real option value would be reduced to 17.2 MNOK. That is, adjusting the probability from 0.6 to 1 resulted in a 67 per cent increase in the option value.

### 6.3.7 Sensitivity analysis of the binomial lattice model

According to Hartmann and Hassan (2006), volatility represents the key value driver in options pricing. It is therefore important to perform a sensitivity analysis of this factor in order to investigate the effect on the option value relative to the volatility. Table 7 displayes the changes in option value relative to changes in the volatility, and should be clear that a good estimation of the volatility is of great importance.

These values are positively correlated: If there is a higher chance that the stage is successful, the investor is more likely to receive the future cash flows from the project. This suggests that both commodity and political

**Table 7:** Sensitivity analysis for the volatility in the binomial lattice valuation. Values are listed in KNOK.

| **Sensitivity analysis of volatility** | | | | | |
|---|---|---|---|---|---|
| Deviation | -20 % | -10 % | - | 10 % | 20 % |
| Volatility | 14 % | 16 % | 18 % | 20 % | 22 % |
| Value of real option | 17 516 | 23 025 | 28 698 | 34 702 | 40 874 |
| Relative change | -39 % | -20 % | - | 21 % | 42 % |
| **Sensitivity analysis of final investment cost** | | | | | |
| Deviation | -20 % | -10 % | - | 10 % | 20 % |
| Investment cost | 908 875 | 1 022 485 | 1 136 094 | 1 249 703 | 1 363 313 |
| Value of real option | 126 317 | 66 523 | 28 698 | 9 681 | 2 276 |
| Relative change | 77 % | 57 % | - | -196 % | -1161 % |

| 0 | 1 | 2 | 3 | 4 | | 14 | 15 | 16 | | 23 | 24 | 25 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 873816 | 920475 | 969625 | 1021401 | 1075940 | ... | 1810128 | 1906784 | 2008601 | ... | 2890935 | 3045303 | 3207913 |
| 28698 | 34549 | 41817 | **50840** | 62028 | ... | 425284 | **500376** | 590814 | ... | 1668822 | 1940068 | **2261168** |
| | 829522 | 873816 | 920475 | 969625 | ... | 1631267 | 1718372 | 1810128 | ... | 2605279 | 2744393 | 2890935 |
| | 23037 | 28079 | **34429** | 42422 | ... | 340505 | **406826** | 487438 | ... | 1431645 | 1666766 | **1944190** |
| | | 787473 | 829522 | 873816 | ... | 1470080 | 1548578 | 1631267 | ... | 2347848 | 2473216 | 2605279 |
| | | 17725 | **21923** | 27298 | ... | 261976 | **319549** | 390981 | ... | 1215930 | 1419365 | **1658534** |
| NPV | | | 747556 | 787473 | ... | 1324819 | 1395561 | 1470080 | ... | 2115854 | 2228835 | 2347848 |
| Option value | | | **12909** | 16249 | ... | 191085 | **239262** | 301362 | ... | 1019740 | 1195357 | **1401103** |
| Option value at decision point | | | | 709662 | ... | 1193912 | 1257664 | 1324819 | ... | 1906784 | 2008601 | 2115854 |
| | | | | 8733 | ... | 129911 | **167826** | 219644 | ... | 841316 | 992537 | **1169109** |
| | | | | | ... | 1075940 | 1133393 | 1193912 | ... | 1718372 | 1810128 | 1906784 |
| | | | | | ... | 80474 | **107822** | 148335 | ... | 679056 | 808903 | **960039** |
| | | | | | | 969625 | 1021401 | 1075940 | ... | 1548578 | 1631267 | 1718372 |
| | | | | | | 43830 | **61433** | 90582 | ... | 531500 | 642645 | **771627** |
| | | | | | | 873816 | 920475 | 969625 | ... | 1395561 | 1470080 | 1548578 |
| | | | | | | 19456 | **29270** | 48528 | ... | 397322 | 492120 | **601833** |
| | | | | | | 787473 | 829522 | 873816 | ... | 1257664 | 1324819 | 1395561 |
| | | | | | | 5287 | **9840** | 21902 | ... | 275313 | 355845 | **448816** |
| | | | | | | 709662 | 747556 | 787473 | ... | 1133393 | 1193912 | 1257664 |
| | | | | | | 0 | **0** | 7826 | ... | 164374 | 232471 | **310919** |
| | | | | | | 639539 | 673689 | 709662 | ... | 1021401 | 1075940 | 1133393 |
| | | | | | | 0 | **0** | 1977 | ... | 71552 | 120780 | **186648** |
| | | | | | | 576345 | 607121 | 639539 | ... | 920475 | 969625 | 1021401 |
| | | | | | | 0 | **0** | 266 | ... | 16132 | 33940 | **74656** |
| | | | | | | 519396 | 547130 | 576345 | ... | 829522 | 873816 | 920475 |
| | | | | | | 0 | **0** | 0 | ... | 0 | 0 | **0** |
| | | | | | | 468074 | 493068 | 519396 | ... | 747556 | 787473 | 829522 |
| | | | | | | 0 | **0** | 0 | ... | 0 | 0 | **0** |
| | | | | | | 421823 | 444347 | 468074 | ... | 673689 | 709662 | 747556 |
| | | | | | | 0 | **0** | 0 | ... | 0 | 0 | **0** |
| | | | | | | | 400441 | 421823 | ... | 607121 | 639539 | 673689 |
| | | | | | | | **0** | 0 | ... | 0 | 0 | **0** |
| | | | | | | | | 380142 | ... | 547130 | 576345 | 607121 |
| | | | | | | | | 0 | ... | 0 | 0 | **0** |
| | | | | | | | | | ... | 493068 | 519396 | 547130 |
| | | | | | | | | | ... | 0 | 0 | **0** |
| | | | | | | | | | | 444347 | 468074 | 493068 |
| | | | | | | | | | | 0 | 0 | **0** |
| | | | | | | | | | | 400441 | 421823 | 444347 |
| | | | | | | | | | | 0 | 0 | **0** |
| | | | | | | | | | | 360873 | 380142 | 400441 |
| | | | | | | | | | | 0 | 0 | **0** |
| | | | | | | | | | | 325214 | 342580 | 360873 |
| | | | | | | | | | | 0 | 0 | **0** |
| | | | | | | | | | | 293079 | 308729 | 325214 |
| | | | | | | | | | | 0 | 0 | **0** |
| | | | | | | | | | | 264120 | 278223 | 293079 |
| | | | | | | | | | | 0 | 0 | **0** |
| | | | | | | | | | | | 250732 | 264120 |
| | | | | | | | | | | | 0 | **0** |
| | | | | | | | | | | | | 238022 |
| | | | | | | | | | | | | **0** |

**Figure 9:** Screenshot of selected columns of the NPV tree generated by the code presented in Appendix B. The green values represent the present value of the operational phase, while the blue are the option values for the periods not available for exercise. The purple values represent the option values in the exercise nodes.

risk should be included in the valuation.

Varying the size of the volatility results in the biggest relative change in the real option value compared to other factors, and it can be seen that a 20 per cent increase in the volatility results in a 42 per cent increase in option value. The higher the volatility, the more upside potential exists in the project, since volatility is the value driver in options. Recall that the potential downside of the option is reduced to the price of the option, since we are hedged towards a fall in the value of the underlying until the option expires. These effects are shown in Table 7.

The second factor we consider is the sensitivity of the option value to the gross investment cost. The analysis shows that a change in the capital expenditure causes large changes in the option value. The investment cost has such a significant effect on the option value because it is absolute, and not relative to the size of the option. Thus, when we increase this value and it is many times the size of the option value, the effect will be strong.

## 6.4 Valuation by Monte Carlo simulation

This Monte Carlo valuation will determine the real option value of the investment opportunity modelled as a single European call option. The option is the investment opportunity TrønderEnergi currently holds in Stokkfjellet Wind Farm. The option to invest, or the final investment decision, can be exercised 25 months from May 2014, which will be our starting point for this valuation as well. Compound options are hard to value with the use of simulations, but we are willing to make this simplification in order to be able to include two stochastic processes. In the valuation, we make use of the parameters that were estimated for the price of power in Section 6.3.2, where we assumed that the price follows an Ornstein-Uhlenbeck process. The estimated parameters were listed in Table 6. For the valuation in Section 6.3, recall that we assumed that the volatility of the power price was representative for the volatility of the project. In addition, we applied the marketed asset disclaimer assumption, taking the value of the underlying as representative for the value of operations. This assumption was discussed in Section 4.5. We will continue to make use of these assumptions for the following valuation.

To value the real option, we want to simulate the development of the value of operation (NPV of operational phase) as a weighted sum of two underlying processes, namely the processes for the price of power and TGC's. We implicitly assume that the value of operations, i.e. the underlying asset in this real options valuation, should follow the same development as the average of the two separate price processes. We justify this assumption by pointing out that 1 MWh of produced electricity will yield the exact prices listed for both power and TGC's, i.e. the weighting is given by the size of the prices themselves. We will here expand the assumption that the project follows the price process of electricity to include also the price process of TGC's.

As before, we begin our analysis on May 1 2014, and we will let the simulations run for 25 periods. This is the number of periods left before the final investment decision needs to be made, or the time to maturity of the real option. Once again, the goal is to value the investment decision in its current condition, i.e. past investments are treated as sunk costs and do not affect our decision to invest in the future.

The procedure for finding the option value for the investment decision at hand can be reduced to eight steps:

1. Estimate the AR(1) model using historical log price data.
2. Find the Ornstein-Uhlenbeck parameters by substituting the AR(1) estimated parameters into Eq. (5.1).
3. Estimate the parameters of the geometric Brownian motion (GBM) directly from the data using Eq. (5.21) and (5.20).
4. Estimate the correlation between the two processes using Eq. (5.24).
5. Establish a procedure to draw random numbers from the normal distribution and transform them using Eq. (5.25).
6. Perform many simulations. In each simulation, do the following:
   - Simulate the development of the underlying in two dimensions, one dimension where it follows the Ornstein-Uhlenbeck process, and one in which it follows the geometric Brownian motion. In each time step, collect the random normal variables for each process pairwise using the drawing procedure created in point 5.
   - In the end point of the simulated path, take the average price of the two dimensions.
   - Find the intrinsic value of the option by subtracting the investment cost from the average price resulting from the simulation in two dimensions.
7. Find the average option value resulting from the simulations.
8. Discount the option value back to today to get the present value of the option.

The number of simulations one should perform is a matter of judgment. In general, the more simulations, the better. However, since an infinite amount of time or computing power is not available, we must set a certain limit. These simulations are quite fast, so we will perform 10 000 simulations in order to determine the option value of the investment opportunity at hand.

Recall that we performed steps 1 and 2 in the valuation in Section 6.3, so we will not show these steps here.

### 6.4.1 Collecting the data

In Section 6.3, daily electricity price data from Nord Pool Spot in the NO3 Elspot area from January 1 2001 to December 31 2013 was used for calibrating the parameters of the process for the power price. The price data was normalised to monthly prices. In this section, daily prices of TGC's as listed by NVE from February 19 2003 up to March 5 2014 will be used for calibrating the parameters of the process for the TGC price. Afterwards, this data will be normalised to monthly time steps as well. For the correlation estimation, we use data corresponding to the same period for which we have collected electricity price data.

### 6.4.2 Computing the parameters

We estimate the parameters of the geometric Brownian motion directly from the data sample, and obtain the normalised parameter estimates by substituting the sample mean and sample variance of the data into Eq. (5.20) and Eq. (5.21). The results are shown in Table 8.

**Table 8:** Normalised estimates of the parameters. *Data source: Log of historical TGC prices from NVE (2014).*

| **Parameter** | **Estimated value** |
|---|---|
| $\hat{\sigma}$, sample standard deviation | 0.20883 |
| $\hat{\mu}$, sample mean | 0.02116 |

### 6.4.3 Determining the correlation between the two price processes

Analysing monthly prices of the two underlying processes reveal a correlation coefficient of -0.1, i.e. a weak, negative correlation. This is a very superficial investigation of the correlation coefficient, and is clearly not a satisfactory method of determination of the correlation between the two price processes. However, the scope of this thesis is not to determine the exact relationship between the electricity price and the price of TGC's. Determining this relationship in an exact manner would demand great attention to this question.

It is still interesting to discuss this relationship briefly. Many argue that the correlation between the power price and the price of TGC's is indeed negative, but that it is stronger than what we found here. In this case, the option value would be lower.

Suppose that the price of both power and TGC's were at some time very high. Soon, new power generation would become available. Then, after some time, the price of power would go down because of high availability. That is, a high TGC price should be associated with a low power price. Accordingly, if the price of both power and TGC's were at some point in time low, this would cause unprofitable production to shut down. After a while, the power price would then increase because of a lower availability. Then, a low TGC price should be associated with a high power price.

By this reasoning, the price of TGC's and power should not be very high or very low at the same time, i.e. there is a negative correlation between them. As established in Sections 2.1 and 2.2, there are many factors besides supply and demand that contribute to the formation of prices of power and TGC's, respectively. This could explain why only a weak correlation is evident on a monthly basis.

As mentioned, we will not put much effort into determining the strength of the correlation between the two price processes in this work. We understand from TrønderEnergi that they have an opinion on this price relationship, thus we leave it to the user of our simulation program to determine the most suitable correlation coefficient to apply.

### 6.4.4 Simulating the development of prices

For the simulations, we will need the normalised estimates for the parameters of the price processes, which we have already obtained. Almost all remaining parameters that are needed for the Monte Carlo valuation have already been discussed in Section 6.3, and we will not repeat the discussion of these parameters here. We will merely mention that the key inputs to the simulations for the price are:

- Current asset value (price of power/TGC's)
- Risk-free rate, $R_f$
- Company beta, $\beta$
- Market risk premium, $E[\tilde{R}_m] - R_f$
- Time to maturity, $T$
- Correlation coefficient, $\rho$ (new)
- Number of simulations (new)

The new parameters that are needed for these calculations are then $\rho$ as well as the number of simulations that we will perform. The correlation coefficient was discussed in Section 6.4.3, and the question about the number of simulations was briefly discussed at the start of this chapter. The way that we generate correlated random variables in each time step of the simulation is illustrated in Fig. 10.

With our approach, it is not necessary to simulate the development of the price processes as a separate step in order to arrive at an estimated price for the underlying. Therefore, we will move directly to the simulation of the underlying for the purpose of this report, but we have included some VBA code for simulating the development of the actual prices of power and TGC's in Appendix C for anyone interested in understanding how the actual prices can develop. We will now move on to build the simulation for the value of operation.

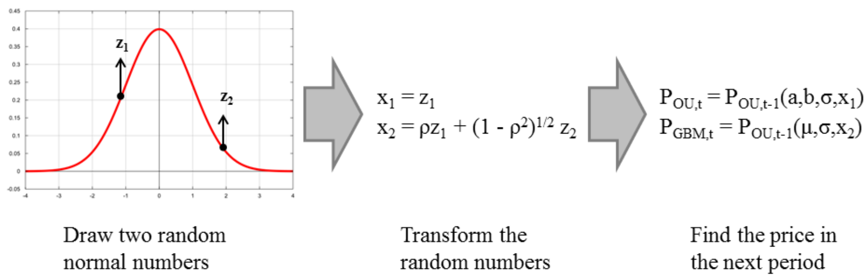



**Figure 10:** Illustration of how normal random numbers are drawn and used to generate two correlated normal variables, which are then used in the Monte Carlo simulation.

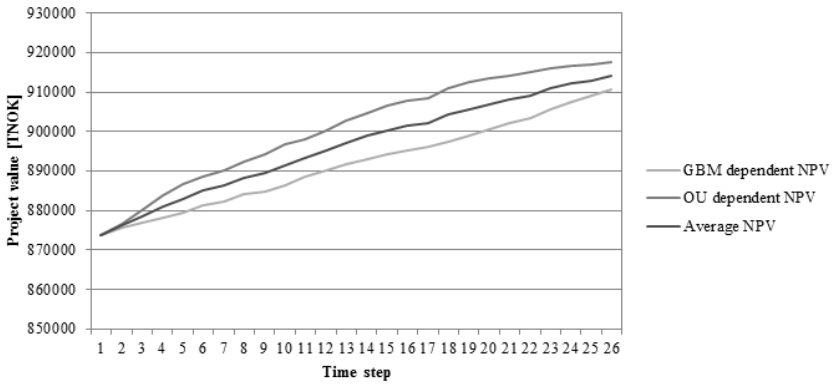



**Figure 11:** One possible simulation outcome when charting the values written out by the code in Appendix D.

### 6.4.5 Simulating the development of the value of operation

We now have all the parameters needed for the simulation, and are ready to simulate the development of the underlying, i.e. the NPV of the operational phase. In addition to the actual option value, we want to write out information about the average of all simulated values in each time step with each process, as well as the average of the two processes. This information can be used for charting the development of the processes.

In each time step, we find the price for the subsequent period by using Eq. (5.22) and (5.23), generating correlated normal variables with the help of Eq. (5.24). VBA code for generating and writing out the simulated paths of the NPV of the operational phase can be found in Appendix D. Running the model once using 10,000 simulations gives the results shown in Fig. 11.

### 6.4.6 Results

The value of the wind park resulting from the simulation displayed in Fig. 11 is estimated at 914.2 MNOK, which yields an option value of 46.6 MNOK. That is, there is a significant positive value connected to the option to invest.

The value calculated here is higher than the option value resulting from the valuation using the binomial lattice approach in Section 6.3. This should be expected for three reasons. First, the binomial model takes in event probabilities that directly decrease the value of the option by the same amount that the event probability deviates from 1. Second, the option is not valued as a compound option, as was done in the binomial lattice approach. Using compound option valuation decreases the value of the option because several options have to be exercised in order to receive the cash flows from the project. Third, additional uncertainty is incorporated in the project valuation when there are two stochastic processes. Uncertainty increases the value of an option. For these reasons, it was expected there would be an increase in the value of the option value found using Monte Carlo simulations as compared to the binomial lattice valuation.

### 6.4.7 Sensitivity analysis of the Monte Carlo valuation

A sensitivity analysis of the volatilities of the two processes that are simulated in the Monte Carlo valuation is shown in Table 9. The analysis shows that the value of the option is more heavily dependent on the volatility of the TGC price (GBM process) than the power price (OU process). This is reasonable since a GBM will have a tendency to wander far from its mean, while an OU process reverts back to its mean after sudden shocks to the price.

Nonetheless, the observation is interesting, because the volatility connected to the TGC price is expected to go up in phases where political action is expected, which would indicate that the option value is positively correlated with political uncertainty. We will discuss this more closely in Chapter 7.

In addition to the analysis of the effect of volatility, we have analysed the effect that changes in the investment cost and the average NPV level have on

**Table 9:** Sensitivity analysis of the two volatilities going into the Monte Carlo simulations. Values are listed in KNOK.

| **Sensitivity analysis for GBM sigma** | | | | | |
|---|---|---|---|---|---|
| Deviation | −20 % | −10 % | - | 10 % | 20 % |
| Volatility | 16,71 % | 18,79 % | 20,88 % | 22,97 % | 25,06 % |
| Value of real option | 37506 | 43114 | 46607 | 51335 | 56520 |
| Relative change | −24 % | −8 % | - | 9 % | 18 % |
| **Sensitivity analysis for OU sigma** | | | | | |
| Deviation | −20 % | −10 % | - | 10 % | 20 % |
| Volatility | 14,42 % | 16,22 % | 18,02 % | 19,82 % | 21,62 % |
| Value of real option | 44093 | 45716 | 46607 | 47509 | 49999 |
| Relative change | −6 % | −2 % | - | 2 % | 7 % |

the option value. The sensitivity analysis of these parameters is displayed in Table 10. It can be seen that the investment cost has a significant effect on the option value, but it is not as significant as for the binomial approach. The reason is simple. The effect of the investment cost is absolute, and not relative to the size of the option. Thus, when we increase this value and it is many times the size of the option value, the effect will be stronger for a smaller option value, and the calculated binomial option value was indeed smaller.

The average NPV level is a parameter of the OU process, and since this parameter determines the level to which the NPV will tend to revert, it should be expected that the option value is very sensitive to this factor. This effect is evident in the sensitivity analysis, and we see that a 20 per cent reduction in the size of this factor results in a 73 per cent decrease in the option value. Recall our discussion about the assumption needed to find the average NPV level in Section 5.1.5. The average NPV level has a very significant effect on the option value that should not be underestimated; thus, attention should be given to the estimation of this parameter.

**Table 10:** Sensitivity analysis of the investment cost and the long-term NPV level of the Ornstein-Uhlenbeck process in the Monte Carlo simulations. Values are listed in KNOK.

| **Sensitivity analysis of investment cost** | | | | | |
|---|---|---|---|---|---|
| Deviation | −20 % | −10 % | - | 10 % | 20 % |
| Investment cost | 757 396 | 852 070 | 946 745 | 1 041 419 | 1 136 094 |
| Value of real option | 166 361 | 94 264 | 46 607 | 21 436 | 8 561 |
| Relative change | 72 % | 51 % | - | −117 % | −444 % |
| **Sensitivity analysis for avg. NPV level** | | | | | |
| Deviation | −20 % | −10 % | - | 10 % | 20 % |
| Avg. NPV level | 699 052 | 786 434 | 873 816 | 961 197 | 1 048 579 |
| Value of real option | 876 | 7 637 | 28 439 | 69 293 | 126 655 |
| Relative change | 97 % | 73 % | - | −144 % | −345 % |

## 6.5 Comparing the results of the different approaches

Table 11 summarises the results obtained from the valuations performed. The contingent NPV approach yielded a value of investment of approximately −21 MNOK, the two factor binomial lattice approach resulted in a value of approximately 29 MNOK, while the Monte Carlo simulation esti-

**Table 11:** Valuation results.

| Valuation methods | Uncertainty | Value [MNOK] |
|---|---|---|
| Contingent NPV | Events | −21 |
| Binomial tree (quadranomial) | Events and market | 29 |
| Monte Carlo valuation | Market | 47 |

mated the value of the wind park at approximately 47 MNOK. The most important take-away is that the value of the wind farm accounting for flexibility advices decision makers to proceed with the project, whereas the negative NPV from the DTA signals that the project is not profitable.

It is natural to question the validity of the numbers resulting from these valuations. The results are quite different due to the characteristics of the methods applied and the degree of flexibility and uncertainty included. The contingent NPV method assumes that all decisions about the future are already made when the project owner enters into the development phase of the wind park. Comparing the contingent NPV to the Monte Carlo valuation shows an added value of 68 MNOK. This is the calculated value of having the chance to revise decisions after information has been revealed in the future. When considering the magnitude of the final investment cost of the park, 890 MNOK, it seems reasonable that 68 MNOK could be the value of having the opportunity to avoid this huge cost in the case that the market conditions turn unfavourable.

The added value of flexibility as calculated by the binomial lattice approach is 50 MNOK. The framework for interpreting this value is the same as for the Monte Carlo valuation. The reason for the result being 18 MNOK lower is important to notice. First, the binomial lattice approach considers only an OU process, while the Monte Carlo valuation takes into account both an OU process and a GBM. As discussed in Section 4.3, a GBM will return a higher option value because its volatility is proportional to the time horizon, while the volatility of the OU process will level off. Second, the Monte Carlo valuation considers a single European call option, while the compounded binomial lattice evaluates three sequential call options. The two investment costs related to the first exercise dates in the compounded option are thus at no point subtracted from the option value in the Monte Carlo valuation because this method is not very compatible with compounded options. The third reason is that the event uncertainty, i.e. the probability of success in each stage of development, is not considered in the

Monte Carlo simulations, something that also contributes to a higher option value compared to the binomial option. As mentioned before, the Monte Carlo method is unsuited for these calculations. These arguments can together explain why the option value is calculated to be somewhat higher in the Monte Carlo valuation than the quadranomial lattice valuation.

# 7 Analysis

This thesis has applied the following three different valuation techniques to assess the investment opportunity that TrønderEnergi currently holds at Stokkfjellet: Decision tree analysis (DTA), the binomial lattice approach, and Monte Carlo valuation. As will always be the case, each method has its benefits and drawbacks. In general, it will almost always be better to use many different valuation methods rather than just one when making an investment decision. In this section, we will try to give an idea of the strengths and weaknesses of each of the methods applied, and thus how a decision maker can address these.

## 7.1 A general comparison of the methods

The results, as discussed in Section 6.5, suggest that using a real options approach for the valuation of Stokkfjellet Wind Farm is appropriate. This is in accordance with the literature that has been discussed in this thesis. First, because the value of the project is found to be a relatively close call in the DTA when comparing it to the magnitude of the investment cost. This could cause the value of flexibility to be decisive for this investment decision. Second, a real options approach is appropriate because of the commodity risk that the project is exposed to. The third reason is the fashion in which TrønderEnergi develop the wind farm, which can be modelled as a compound option.

Let us now compare the methods we have applied. We start with the discounted cash flow (DCF) method. This is the valuation method that TrønderEnergi currently uses, and we have thus used this method solely to determine the value of operation, often called the net present value (NPV) of the operational phase in this report. It should be clear from the discussions in this report that we do not recommend that decision makers use the sign of a traditional NPV alone when making a decision to invest in a wind farm. The reason is that flexibility incorporated in the investment opportunity is not captured by the DCF method. Let us still mention some advantages of the DCF method as pointed out by (Mun, 2006):

- Clear, consistent decision criteria for all projects.
- Results do not depend on investors' risk preferences.
- Quantitative, quite accurate, and economically rational.
- Not very vulnerable to accounting conventions (depreciation, inventory valuation, etc.)
- Considers the time value of money and risk structures.
- Simple, well-known, and widely accepted.
- Explaining it to management is straight-forward, if benefits outweigh the costs; invest.

Looking beyond these advantages, there are unfortunately several issues connected to using DCF methods. The most severe problem is the fact that there is risk and uncertainty associated with decision making, and that management has the strategic flexibility to make and revise decisions as these uncertainties become known over time (Mun, 2006). In the real world, using deterministic models like the DCF method could potentially grossly undervalue a particular project. A deterministic DCF model assumes that all future cash flows are known. Had this been the case, such that there were no fluctuations in business conditions, then the DCF model would correctly estimate the value of the project. Essentially, there would be no value to having flexibility. However, the real world is stochastic and the business environment is highly variable. If management has the flexibility to adapt their plans when conditions change, then flexibility is indeed valuable; a value that a DCF model does not attempt to capture.

We have now established that static DCF analysis should not be applied to situations involving dynamic decision making. In light of this, another relatively simple technique that appears somewhat more suitable is the contingent NPV approach, sometimes called decision tree analysis. The technique digs a little deeper than static DCF analysis when calculating future cash flows in that it views future cash flows as dependent on future actions and potentially information that may evolve. At each branch point in the decision tree the decision makers' future optimal actions, conditional on outcome, have already been established and have been incorporated in the future cash flow calculations (Mun, 2006). The market value is usually estimated with the use of a constant discount rate, but it is recommended that one uses different discount rates for different phases of the project when such is appropriate (Koller et al., 2010).

A decision tree analysis is not by itself a sufficient tool for solving real options (Mun, 2006). If such an analysis is used, then one must estimate different discount rates at each decision node at different times because different projects at different times have different risk properties. This will lead to estimation errors to be compounded on a large decision tree analysis. Binomial lattices, which use risk-neutral probabilities, avoid this error. In addition, risk-neutral probabilities are objective and easy to obtain, and the volatility can be estimated using commodity prices or Monte Carlo simulation. The imputed risk-neutral probabilities will thus be more accurate as compared to estimating a discount rate. The use of discount rates is further complicated as it requires a market benchmark that may or may not exist when we work with real options (Mun, 2006). Such a benchmark has been demonstrated in this work to be difficult to find in the Norwegian stock market. In fact, only one company that works with renewable electricity generation alone is publicly traded. This company was still not found to be a good match for the Stokkfjellet project as it deals largely with hydropower, which is still significantly less risky than wind power.

As Mun (2006) points out, the most important conclusion to be drawn from the binomial lattice approach is that risk-adjusting cash flows provides the exact same results as risk-adjusting the probabilities leading to those cash flows. Thus the results from a DCF analysis are identical to those generated using a binomial lattice in the case when we assume that the volatility of the cash flows is zero, i.e. the cash flows are assumed to be known with certainty. When zero uncertainty exists, there is no flexibility in decisions and zero strategic option value, meaning that we have calculated the static NPV. When we input a volatility of zero into our binomial lattice valuation, the value of the option becomes zero as well. This is in line with our results from the contingent NPV valuation, which gave a negative value, hence a project value of zero. In the option pricing model of the binomial tree, the model will return a value of zero for all negative project values. Intuitively, there will always be uncertainty about future cash flows. Indeed, the world and the market conditions are stochastic. By itself, this should justify the need for other methods besides the DCF method.

We use a Monte Carlo valuation approach for Stokkfjellet in order to take into account the uncertainty in the development of TGC prices in addition to the uncertainty about the power price, indicating that we are dealing with a two factor model. According to Hahn and James (2008), discrete time modelling of mean-reverting stochastic processes is problematic, and they suggest Monte Carlo simulation as a primary solution to this problem. Monte Carlo methods are able to accommodate many types of stochastic processes, and can used also for valuing early-exercise options, a shortcoming of traditional simulation-based methods. Nonetheless, a significant drawback of this valuation approach is that it is computationally intensive, and particularly so for problems with multiple concurrent options (Hahn and James, 2008).

Monte Carlo methods are straightforward to apply for European options. However, they can be difficult to apply to many complex real problems, such as a compound option. Different types of tree-building procedures, such as trinomial trees or time-dependent drift and volatility specifications, commonly result in trees that are computationally complex and/or path-dependent (Hahn and James, 2008).

## 7.2 Modelling the price processes

We have chosen to model the electricity price as an Ornstein-Uhlenbeck (OU) process, which is a so-called mean-reverting process. Compared to the common practice of using a geometric Brownian motion (GBM) for modelling the power price, a mean-reverting price process captures the clear tendency of electricity prices to revert towards a long-term level.

Even though modelling the power price as a mean-reverting process is considered more realistic compared to a price that follows a GBM, there are complications associated with this model. For instance, we need to estimate more parameters in order to describe the OU process. While we need only to estimate the volatility for a GBM (the mean is directly observable from the sample), we must estimate a long-term price level as well as a rate of mean reversion in order to capture the characteristics of an OU process.

In addition, a mean-reverting process does not capture all properties of electricity prices. For example, it is not successful in capturing the jumps that are often seen in electricity prices (this is evident in Fig. 1), and Blanco and Soronow (2001) point out that the rate of mean-reversion is not constant, but changes according to how large a particular jump is, in which direction it was and why it occurred. In modelling these processes the choice between accuracy and applicability will work in opposite directions. In order to create a binomial tree for the option in a relatively straightforward fashion, the price must follow a simple process. This is a goal in itself in this thesis, as we aim to make the models useful for practitioners who are not experts in the field. It would, however, be very interesting to try and model the rate of mean-reversion as parameter that can vary with time.

We model the price of tradable green certificates (TGC's) as a GBM. The reasoning was that there is no marginal cost of production for TGC's, so it does not tend to revert back to a mean like the price of power (this can be seen in Fig. 2). Additionally, a demand for renewable power is created artificially, and the demand is based on a quota defined by the government rather than by the individual consumer's desire for electricity from renewable sources. Given this, the price might become very high or very low at a certain point in time, e.g. when we approach the end of some period for which there exists a set goal for new renewable production. It is then very likely that the government will step in and prevent this price from becoming way too high or low. In that case, it can be hard to justify using a GBM, since in practice there would be a roof and a floor for the price.

Independently of high or low prices, political actions that will affect the subsidy scheme of TGC's are likely to occur from time to time. This possibility of political action aimed at building more renewable electricity production likely affects the volatility of the TGC price. The sensitivity analysis of volatility for the Monte Carlo valuation was shown in Table 9, and this analysis showed that the value of the real option is more dependent on changes in the volatility of the TGC price than that of the power price. Thus, if we assume that volatility of the TGC price is positively correlated with political intervention, then we can state that the option value will be higher in times where political action is expected or has just occurred and the market is still absorbing the implications of the amendments to the scheme.

It is important to be aware that the historical TGC price data used in this thesis does not fully represent the common market between Norway and Sweden that exists now. The common market was introduced in 2012, and the data from earlier years is from the Swedish TGC market that was in operation from 2003 throughout 2011. One can only speculate about how the market will evolve in the future compared to the past. Nonetheless, in this thesis we have assumed that the historical data will be representative of the future.

When we are already describing the underlying stochastic processes, it is natural also to discuss the correlation between them. As mentioned in Section 6.4.3, the correlation estimation done here is not one that can be trusted, as it was investigated only superficially. This is justified as this report aims to demonstrate the differences between valuation models, not to determine parameters as accurately as possible (except the parameters for the price processes). It is worth noting that TrønderEnergi is believed to have a broad understanding for the relationship between TGC and power prices, so we do not think that this is where we can make our biggest contribution.

## 7.3 The MAD assumption and choice of volatility

In contrast to financial options, historical data for the underlying value of a real option is often impossible to derive since real options are normally not traded in the marketplace. This complicates the estimation of the volatility for real options, and Shockley (2007) argues that this is one of the main reasons why the real options methodology is not applied more often in practice.

The marketed asset disclaimer (MAD) assumption is a means of solving the problem of incomplete markets. We assume that the value of the underlying is equal to the value of operation without flexibility. A problem with this assumption is that it may lead to errors in the valuation since the assumption cannot be tested in the market. For example, the appropriate choice of the discount rate for the project without flexibility is left to the discretion of the analyst, and the use of WACC may not be appropriate for all projects. Therefore, it is important to realise that the issue of determining the value of the underlying is not completely resolved by this methodology (Dyer and Brãndao, 2005).

For commodity based products, such as electricity, we have argued that the volatility of the project can be assumed equal to that of the price of the commodity on which it is based, namely electricity in the binomial tree and both electricity and TGC's in the Monte Carlo simulation. This might lead to errors in estimation of the project value. Copeland and Antikarov (2003) suggest that one can do better by using Monte Carlo simulations to estimate the projects volatility and build an event tree.

An alternative way to treat the value of the underlying and the stochastic process it follows would be to separate the operational income and expenditure. Instead of letting costs and production vary along with the income as a stochastic process, we could assume constant production and a fixed cost of operation per MWh produced. Then, only the operational income would follow the same process as the electricity price. However, neither costs nor production are in reality deterministic. Thus, a better extension to the models proposed in this thesis would be to perform a thorough analysis of the development of costs and production, and model these inputs as variables following their own appropriate processes.

## 7.4 The wind is stochastic

There are several uncertain factors going into the valuations that have been treated to be deterministic, but are in reality not so. One of these is the wind inflow, which directly influences the production and thereby the profit from a wind farm. Indeed, the weather is stochastic, and the variation in the wind speed at a certain site is typically assumed to follow the Weibull distribution (Sorokin et al., 2012). This variation is not treated in our models with respect to the goal of this thesis. It is, however, important to note that the wind variation is a crucial element that should be accounted for in a wind project valuation that tries to estimate the value of the wind farm in an accurate manner. As could be seen in Table 2 of the sensitivity analysis of the value of operation, this parameter was almost proportional to the production, as seems reasonable.

## 7.5 The investor's risk preferences

There exists no single right answer to the question of what is the correct required rate of return for a project, nor the project beta. The investor must have an opinion on what rate is preferred, and the total portfolio of investments and risks should also be taken into consideration. The division between the risk-free rate, market risk premium and beta is not crucial, but it is a way of arguing for the chosen required rate of return. We have applied values for which the CAPM does not hold. Rather, we have given solid arguments for the choice of all input parameters. Methods for determining the risk-neutral probabilities in the binomial lattice without using the CAPM were mentioned in Section 5.1.4. We emphasise once again that the scope of our thesis is not to evaluate the choice of the required rate of return, or any other parameter. However, TrønderEnergi treats the required rate of return as a confidential parameter, so we encourage the users of our program to utilise their own estimates to perform valuations.

## 7.6 Possible extensions

The volatility of the electricity price rises in some periods of our data sample. This may be in response to financial crises or other macroeconomic factors. When the volatility is higher in some periods, it could be that the demand is shifted to a steeper part of the convex supply curve, so that more volatility is realised from a given change in the power price. Whatever the cause, there is a clear tendency of volatility clustering, which indicates that the use of regime-switching volatility could lead to more accurate results. In addition to displaying mean-reverting behaviour, the Norwegian electricity price tends to have spikes, and it can be argued that the price process should also incorporate a jump process to be more realistic. Thus, we suggest that any work building onto this should consider the possibility of such extensions to the mean-reverting model for the electricity price that has been proposed in this thesis.

In order to obtain an even more accurate valuation, there are some other key points that should be paid attention to. In particular, the accuracy of a real options valuation would be significantly improved if the seasonal pattern of wind, the evolution in capital costs and other important but uncertain factors that are going into the valuation had been included as stochastic variables rather than being treated as deterministic. Investment costs have increased in the recent years, but are expected to fall over the next decade in response to the EU goals for renewable energy production set for 2020. The capital cost is by far the most significant cost going into the model, and a factor to which uncertainty is connected. Currently, the opportunity value of decreasing capital costs is not captured. Had we valued the option to delay investments in this work, then it would be a shortcoming that the model would not be able to weigh this opportunity value versus the decreasing value of the subsidies.

# 8 Conclusion

The goal for this thesis has been to demonstrate the difference between some selected valuation methods, and discuss their suitability for valuing the investment opportunity that TrønderEnergi currently holds in the Stokkfjellet Wind Farm project. TrønderEnergi has applied for a licence to build, and is expecting a decision by the end of the summer of 2014. TrønderEnergi currently uses the discounted cash flows method to value this and similar projects. Unfortunately, discounted cash flows do not incorporate the value of flexibility. This makes the technique unsuitable for valuing investment opportunities related to wind farm development, which are indeed highly uncertain. Particularly, there is uncertainty about events and markets. With events, we mean for example the outcome of the licence application processing. Also, the market prices of power and tradable green certificates directly influence the income from a wind farm. With this in mind, our hypothesis is that the use of real options valuation methods will improve the quality of the information available for decision makers, ultimately improving the quality of their investment decisions in wind power.

We have performed valuations using decision tree analysis, the binomial lattice approach and Monte Carlo simulations. The first method considers uncertainty about events, the second uncertainty about events and the market, and the third investigates solely market uncertainty. We have modelled the power price as a mean-reverting process, and the price of tradable green certificates as a geometric Brownian motion. Indeed, we have found that the literature supports our hypothesis that real options analysis is appropriate for investment decisions such as the one at hand, and also that flexibility generally adds a significant amount of value to the valuation of a wind park.

The real option we have valued is the option to invest, and the option has been valued at 29 MNOK and 47 MNOK, dependent on the real options method applied. We also saw that the contingent NPV returned a negative value of $-21$ MNOK. Even if real options methods add value, it is not always worthwhile to perform a time consuming real options analysis. In particular, if the NPV of a project has been found to be very high or very low, real options analysis will not affect the decision to invest. However, for investments in onshore wind in Norway, these decisions are often a close call, and a real options perspective will be useful. This was found to be the case for the Stokkfjellet project, and we have demonstrated that valuing the project with flexibility assigns the project a positive value whereas a contingent NPV approach assigns the investment opportunity a negative value. We therefore recommend that a real options approach to valuing wind projects be adopted by TrønderEnergi.

There is a number of other ways in which we could have evaluated the investment opportunity. Possible extensions to the model could consider a more complex electricity price process such as mean-reversion with jumps with regime-switching volatility. It would also be interesting to take into account the uncertainty about factors that are treated deterministically in these valuations, such as the wind inflow and the development of the capital costs.

In addition to the above findings, an important part of our work has been to create an effective, simple and flexible program that can be used for valuing Stokkfjellet as well as other projects that are subject to similar risks. The program also conveniently visualises the development of prices and project value. The program consists of four different Excel files, one for each type of valuation. The code has been written in Visual Basic for Applications (VBA) for Excel. This program was handed over to TrønderEnergi at the time when this thesis was submitted.

This thesis has offered a detailed introduction to three valuation methods that to a varying extent consider flexibility, as opposed to the discounted cash flow method. In particular, the report has given a thorough introduction to the concept of real options, and demonstrated why these methods are a better fit for wind projects. In doing this, the Stokkfjellet project has been analysed and valued. We provide academic insight and offer a mathematical framework for valuing the project, and we also create a program to complement this thesis. We expect that the discussions in this report and the program we have created will be of high value to decision makers at TrønderEnergi. We hope that decision makers have acquired an interest and basic understanding for the concept of real options analysis and the value associated with adopting these methods.

# References


Alexander, C. (2008). *Practical Financial Economics.* John Wiley & Sons, Ltd, West Sussex, England.

Barlow, M. T. (2002). A diffusion model for electricity prices. *Mathematical Finance*, 12(4):287–98.

Black, F. and Scholes, M. (1973). The pricing of options and corporate liabilities. *The Journal of Political Economy*, 81(3):637–54.

Blanco, C. and Soronow, D. (2001). Mean reverting processes: Energy price processes used for derivatives pricing and & risk management. *Commodities Now*, pages 68–72.

Bøckman, T., Fleten, S., Juliussen, E., Langhammer, H., and Revdal, I. (2008). Investment timing and optimal capacity choice for small hydropower projects. *European Journal of Operational Research*, 190(3):255–67.

Copeland, T. and Antikarov, V. (2003). *Real Options: A Practitioner's Guide.* Cengage Learning, New York, NY10010, U.S.

Cortazar, G., Gravet, M., and Urzua, J. (2008). The valuation of multidimensional American real options using the LSMC simulation method. *Applications of OR in Finance*, 35(1):113–29.

Cortazar, G. and Schwartz, E. (2003). Implementing a stochastic model for oil futures prices. *Energy Economics*, 25(3):215–38.

Cox, J., Ross, S., and Rubinstein, M. (1979). Option pricing: A simplified approach. *Journal of Financial Economics*, pages 229–63.

Davison, M., Anderson, C. L., Marcus, B., and Anderson, K. (2002). Development of a hybrid model for electrical power spot prices. *IEEE Transactions on Power Systems*, 17(2):257–64.

Deng, S.-J. (2000). Stochastic models of energy commodity prices and their applications: Mean-reversion with jumps and spikes. *POWER*, page 42.

Deng, S.-J. and Oren, S. (1995). Incorporating operational characteristics and start-up costs in option-based valuation of power generation capacity. *Probability in the Engineering and Informational Sciences*, 73(3):105–15.

Dixit, A. K. and Pindyck, R. (1995). The options approach to capital investments. *Harvard Business Review*, 73(3):105–15.

Dixit, A. K. and Pindyck, R. S. (1994). *Investment under Uncertainty.* Princeton University Press, Princeton, N.J.

Dyer, J. and Brãndao, L. (2005). Decision analysis and real options: A discrete time approach to real option valuation. *Annals of Operations Research*, 135:21–39.

Finansdepartementet (2014). Skattesatser 2014. `http://www.regjeringen.no/nb/dep/fin/tema/skatter_og_avgifter/skattesatser-2014.html?id=748052`.

Fisher, I. (1907). *The Rate of Interest: Its Nature, Determination, and Relation to Economic Phenomena.* Macmillan, New York.

Fisher, I. (1930). *The Theory of Interest.* Macmillan, New York.

Fleten, S., Maribu, K., and Wangensteen, I. (2007). Optimal investment strategies in decentralized renewable power generation under uncertainty. *Energy*, 32(5):803–15.

Fleten, S.-E. and Ringen, G. (2009). New renewable electricity capacity under uncertainty: The potential in Norway. *Journal of Energy Markets*, 2(1):71–88.

fornybar.no (2014). Produksjon og marked. `http://fornybar.no/vindkraft/produksjon-og-marked`.

Gjølberg, O. and Johnsen, T. (2007). Investeringer i produksjon av fornybar energi: Hvilket avkastningskrav bør Enova SF legge til grunn? `http://www2.enova.no/publikasjonsoversikt/publicationdetails.aspx?publicationID=282`.

Guthrie, G. (2009). *Real Options in Theory and Practice.* Oxford University Press, Inc., New York.

Hahn, J. and James, S. (2008). Discrete time modeling of mean-reverting stochastic processes for real option valuation. *European Journal of Operational Research*, 184(2):534 – 548.

Hartmann, M. and Hassan, A. (2006). Application of real options analysis for pharmaceutical R&D development project valuation – empirical results from a survey. *Research Policy*, 35:343–54.

Hofstad, K. (2012). Vindkraft – produksjonsstatistikk - 2011. `http://www.nve.no/PageFiles/14342/Vindkraftproduksjon_2011.pdf`.

Koller, T., Goedhart, M., and Wessels, D. (2010). *Valuation: Measuring and Managing the Value of Companies, Fifth Edition.* John Wiley & Sons, Inc., Hoboken, New Jersey.

Krohn, S., Morthorst, P.-E., and Awerbuch, S. (2009). The economics of wind energy. Technical report, European Wind Energy Association.

Lee, S. and Shih, L. (2010). Renewable energy policy evaluation using real option model — the case of Taiwan. *Energy Economics*, 32(1):67–78.

Lee, S.-C. (2011). Using real option analysis for highly uncertain technology investments: The case of wind energy technology. *Renewable and Sustainable Energy Reviews*, 15:4443–50.

Lucia, J. and Schwartz, E. (2002). Electricity prices and power derivatives. evidence from the nordic power exchange. *Review of Derivatives Research*, 5(1):5–115.

McDonald, R. and Siegel, D. (1984). Option pricing when the underlying asset earns a below-equilibrium rate of return: A note. *Journal of Finance*, 39(1):261–5.

McDonald, R. L. (2013). *Derivatives Markets.* Pearson Education, Inc., Upper Saddle River, New Jersey.

Miller, L. and Park, C. (2002). Real options to the rescue. *The Engineering Economist*, 47(2):145–49.

Mun, J. (2006). *Real Options Analysis: Tools and Techniques for Valuing Strategic Investments and Decisions.* John Wiley & Sons, Inc., Hoboken, New Jersey.

Muñoz, J., Contreras, J., Caamaño, J., and Correia, P. (2011). A decision making tool for project investments based on real options: The case of wind power generation. *Annals of Operations Research*, 186:465–90.

Méndez, M., Lamothe, P., and Goyanes, A. (2009). Real options valuation of a wind farm. `http://www.realoptions.org/papers2009/46.pdf`.

Norges Bank (2013). Inflasjon.
`http://www.norges-bank.no/no/prisstabilitet/inflasjon/`.

Norges Bank (2014). Statsobligasjoner. Årsgjennomsnitt.
`http://www.norges-bank.no/no/prisstabilitet/rentestatistikk/statsobligasjoner-rente-arsgjennomsnitt-av-daglige-noteringer`.

OED (2014). Om lov om fornybar energiproduksjon til havs (Havenergilova). http://www.regjeringen.no/nn/dep/oed/dokument/proposisjonar-og-meldingar/odelstingsproposisjonar/2008-2009/otprp-nr-107-2008-2009-/4/4.html?id=569864.

PwC (2013). Risikopremien i det norske markedet, 2013-14. http://www.pwc.no/no/publikasjoner/deals/risikopremien-2013-2014.pdf.

Schwartz, E. (1997). The stochastic behavior of commodity prices implications for valuation and hedging. *Journal of Finance*, (52):923–73.

Shockley, R. (2007). *An Applied Course in Real Options Valuation*. Thomson, Mason, U.S.

Sintef (2012). Vind – landbasert og offshore. http://www.sintef.no/SINTEF-Energi-AS/Prosjektarbeid/Vind/.

Skatteetaten (2014). Avskrivninger. http://www.skatteetaten.no/no/Bedrift-og-organisasjon/Drive-bedrift/Bokforing-og-regnskap/Kostnader/Avskrivninger/.

Smith, J. and Schwartz, E. (2000). Short-term variations and long-term dynamics in commodity prices. *Management Science*, 46(7):893–911.

Sorokin, A., Rebennack, S., Pardalos, P., Iliadis, N., and Pereira, M. (2012). *Handbook of Networks in Power Systems I*. Springer, Heidelberg, Berlin.

Thema Consulting Group (2012). Skattemessige avskrivinger for kraftproduksjon. http://www.t-cg.no/userfiles/THEMA_R-2012-10%20Skattemessige%20avskrivninger%20for%20kraftproduksjon.pdf.

Trigeorgis, L. (1996). *Real Options: Managerial Flexibility and Strategy in Resource Allocation*. MIT Press, Cambridge, MA.

Trigeorgis, L. and Mason, S. (2003). Valuing managerial flexibility and strategy in resource. *Midland Corporate Finance Journal*, 17(2):155–81.

TrønderEnergi AS (2013). Konsesjonssøknad, Stokkfjellet Vindkraftverk. webfileservice.nve.no/API/PublishedFiles/Download/201106956/793053.

TrønderEnergi AS (2014). About TrønderEnergi. http://tronderenergi.no/english.

TrønderEnergi Kraft AS avd. Handel (2014). Elsertifikatmarkedet. Technical report, TrønderEnergi AS.

Waagaard, I., Christophersen, E., and Slungård, I. (2008). Mulighetsstudie for landbasert vindkraft 2015 og 2025. http://www.nve.no/Global/Publikasjoner/Publikasjoner%202008/Rapport%202008/rapport18-08.pdf.

Weir, D. E. (2013). Vindkraftproduksjon i 2012. http://webby.nve.no/publikasjoner/rapport/2013/rapport2013_13.pdf.

Weir, D. E. (2014). Vindkraftproduksjon i 2013. http://webby.nve.no/publikasjoner/rapport/2014/rapport2014_20.pdf.

Wind Energy Foundation (2014). About wind energy. http://www.windenergyfoundation.org/about-wind-energy.

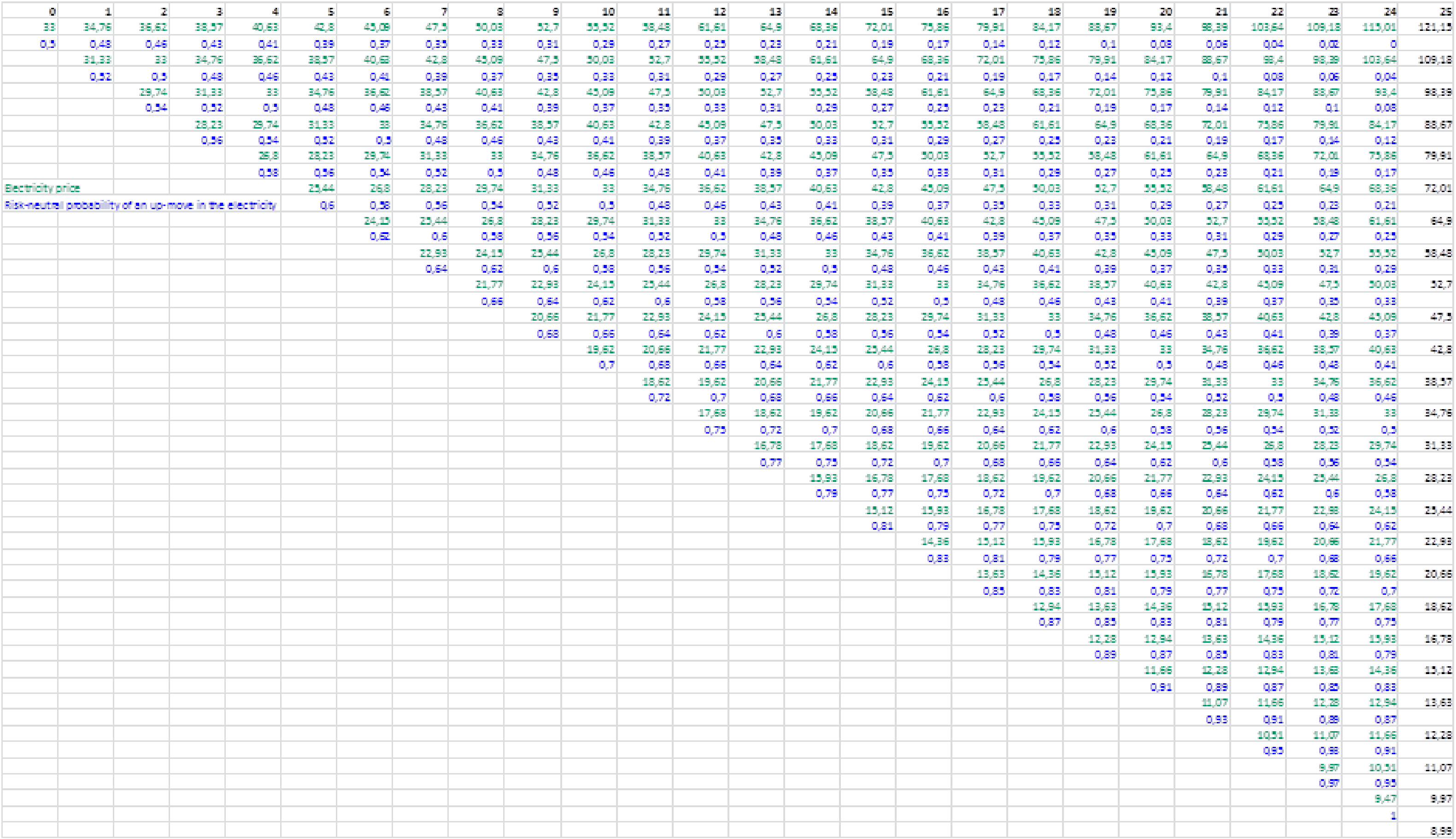
# A A binomial tree for the price (VBA code)

This code generates a tree for the development of the power price and lists the risk-neutral probability of an up move at each node. The resulting output is displayed after the code.

Cell referrals and output formatting is not included in the code.

```
Sub OneFactorBinomialTree()
    Worksheets("StartSheet").Activate

    ... loading cell values ...

    Worksheets("PriceTree").Cells.Clear
    Worksheets("PriceTree").Activate

    Dim price(), priceNew() As Double

    ReDim price(0 To n)
    ReDim priceNew(0 To n)

    'Calculate real probabilities
    up = Exp(sig * Sqr(dt))
    down = 1 / up

    Dim i, j As Integer

    'Adds an informative row at the top of the output sheet
    For i = 0 To n
        Cells(1, i + 1).Value = i
    Next i

    'Calculate the price at the end nodes (column n + 1)
    For i = 0 To n
        price(i) = startprice * Exp((n - 2 * i) * sig * Sqr(dt))
        Cells(2 * i + 2, n + 1).Value = price(i)
    Next i

    'Work backwards in the tree to find prices
    For j = (n - 1) To 0 Step -1
        For i = 0 To j
            priceNew(i) = price(i) / up
            Prob = theta_up(i, j, a, b, dt, startprice, sig, up, _
                    down, beta, mr)

            'Inputs the calculated values into the sheet
            Cells(2 * i + 2, j + 1).Value = priceNew(i)
            Cells(2 * i + 3, j + 1).Value = Prob
        Next i
```

```
        For i = 0 To j
            price(i) = priceNew(i)
        Next i
    Next j
End Sub

'Get risk-neutral probability of an up-move at specified node
Function theta_up(i, j, a, b, dt, startprice, sig, u, d, beta, mr)

    'Get the actual probability of an up move
    theta_up = 0.5 + (1 - Exp(-a * dt)) * (b - (Log((startprice) *_
    Exp((j - 2 * i) * sig * Sqr(dt))))) / (2 * sig * Sqr(dt))

    'Get the risk-neutral probability of an up-move
    If theta_up < 0 Then
        theta_up = 0
    ElseIf theta_up > 1 Then
        theta_up = 1
    Else
        theta_up = theta_up - ((mr * beta) / (u - d))
        If theta_up < 0 Then
            theta_up = 0
        End If
    End If
End Function
```



| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 33 | 34,76 | 36,62 | 38,57 | 40,63 | 42,8 | 45,09 | 47,5 | 50,03 | 52,7 | 55,52 | 58,48 | 61,61 | 64,9 | 68,36 | 72,01 | 75,86 | 79,91 | 84,17 | 88,67 | 93,4 | 98,39 | 103,64 | 109,18 | 115,01 | 121,15 |
| 0,5 | 0,48 | 0,46 | 0,43 | 0,41 | 0,39 | 0,37 | 0,35 | 0,33 | 0,31 | 0,29 | 0,27 | 0,25 | 0,23 | 0,21 | 0,19 | 0,17 | 0,14 | 0,12 | 0,1 | 0,08 | 0,06 | 0,04 | 0,02 | 0 | |
| | 31,33 | 33 | 34,76 | 36,62 | 38,57 | 40,63 | 42,8 | 45,09 | 47,5 | 50,03 | 52,7 | 55,52 | 58,48 | 61,61 | 64,9 | 68,36 | 72,01 | 75,86 | 79,91 | 84,17 | 88,67 | 93,4 | 98,39 | 103,64 | 109,18 |
| | 0,52 | 0,5 | 0,48 | 0,46 | 0,43 | 0,41 | 0,39 | 0,37 | 0,35 | 0,33 | 0,31 | 0,29 | 0,27 | 0,25 | 0,23 | 0,21 | 0,19 | 0,17 | 0,14 | 0,12 | 0,1 | 0,08 | 0,06 | 0,04 | |
| | | 29,74 | 31,33 | 33 | 34,76 | 36,62 | 38,57 | 40,63 | 42,8 | 45,09 | 47,5 | 50,03 | 52,7 | 55,52 | 58,48 | 61,61 | 64,9 | 68,36 | 72,01 | 75,86 | 79,91 | 84,17 | 88,67 | 93,4 | 98,39 |
| | | 0,54 | 0,52 | 0,5 | 0,48 | 0,46 | 0,43 | 0,41 | 0,39 | 0,37 | 0,35 | 0,33 | 0,31 | 0,29 | 0,27 | 0,25 | 0,23 | 0,21 | 0,19 | 0,17 | 0,14 | 0,12 | 0,1 | 0,08 | |
| | | | 28,23 | 29,74 | 31,33 | 33 | 34,76 | 36,62 | 38,57 | 40,63 | 42,8 | 45,09 | 47,5 | 50,03 | 52,7 | 55,52 | 58,48 | 61,61 | 64,9 | 68,36 | 72,01 | 75,86 | 79,91 | 84,17 | 88,67 |
| | | | 0,56 | 0,54 | 0,52 | 0,5 | 0,48 | 0,46 | 0,43 | 0,41 | 0,39 | 0,37 | 0,35 | 0,33 | 0,31 | 0,29 | 0,27 | 0,25 | 0,23 | 0,21 | 0,19 | 0,17 | 0,14 | 0,12 | |
| | | | | 26,8 | 28,23 | 29,74 | 31,33 | 33 | 34,76 | 36,62 | 38,57 | 40,63 | 42,8 | 45,09 | 47,5 | 50,03 | 52,7 | 55,52 | 58,48 | 61,61 | 64,9 | 68,36 | 72,01 | 75,86 | 79,91 |
| | | | | 0,58 | 0,56 | 0,54 | 0,52 | 0,5 | 0,48 | 0,46 | 0,43 | 0,41 | 0,39 | 0,37 | 0,35 | 0,33 | 0,31 | 0,29 | 0,27 | 0,25 | 0,23 | 0,21 | 0,19 | 0,17 | |
| Electricity price | | | | | 25,44 | 26,8 | 28,23 | 29,74 | 31,33 | 33 | 34,76 | 36,62 | 38,57 | 40,63 | 42,8 | 45,09 | 47,5 | 50,03 | 52,7 | 55,52 | 58,48 | 61,61 | 64,9 | 68,36 | 72,01 |
| Risk-neutral probability of an up-move in the electricity | | | | | 0,6 | 0,58 | 0,56 | 0,54 | 0,52 | 0,5 | 0,48 | 0,46 | 0,43 | 0,41 | 0,39 | 0,37 | 0,35 | 0,33 | 0,31 | 0,29 | 0,27 | 0,25 | 0,23 | 0,21 | |
| | | | | | | 24,15 | 25,44 | 26,8 | 28,23 | 29,74 | 31,33 | 33 | 34,76 | 36,62 | 38,57 | 40,63 | 42,8 | 45,09 | 47,5 | 50,03 | 52,7 | 55,52 | 58,48 | 61,61 | 64,9 |
| | | | | | | 0,62 | 0,6 | 0,58 | 0,56 | 0,54 | 0,52 | 0,5 | 0,48 | 0,46 | 0,43 | 0,41 | 0,39 | 0,37 | 0,35 | 0,33 | 0,31 | 0,29 | 0,27 | 0,25 | |
| | | | | | | | 22,93 | 24,15 | 25,44 | 26,8 | 28,23 | 29,74 | 31,33 | 33 | 34,76 | 36,62 | 38,57 | 40,63 | 42,8 | 45,09 | 47,5 | 50,03 | 52,7 | 55,52 | 58,48 |
| | | | | | | | 0,64 | 0,62 | 0,6 | 0,58 | 0,56 | 0,54 | 0,52 | 0,5 | 0,48 | 0,46 | 0,43 | 0,41 | 0,39 | 0,37 | 0,35 | 0,33 | 0,31 | 0,29 | |
| | | | | | | | | 21,77 | 22,93 | 24,15 | 25,44 | 26,8 | 28,23 | 29,74 | 31,33 | 33 | 34,76 | 36,62 | 38,57 | 40,63 | 42,8 | 45,09 | 47,5 | 50,03 | 52,7 |
| | | | | | | | | 0,66 | 0,64 | 0,62 | 0,6 | 0,58 | 0,56 | 0,54 | 0,52 | 0,5 | 0,48 | 0,46 | 0,43 | 0,41 | 0,39 | 0,37 | 0,35 | 0,33 | |
| | | | | | | | | | 20,66 | 21,77 | 22,93 | 24,15 | 25,44 | 26,8 | 28,23 | 29,74 | 31,33 | 33 | 34,76 | 36,62 | 38,57 | 40,63 | 42,8 | 45,09 | 47,5 |
| | | | | | | | | | 0,68 | 0,66 | 0,64 | 0,62 | 0,6 | 0,58 | 0,56 | 0,54 | 0,52 | 0,5 | 0,48 | 0,46 | 0,43 | 0,41 | 0,39 | 0,37 | |
| | | | | | | | | | | 19,62 | 20,66 | 21,77 | 22,93 | 24,15 | 25,44 | 26,8 | 28,23 | 29,74 | 31,33 | 33 | 34,76 | 36,62 | 38,57 | 40,63 | 42,8 |
| | | | | | | | | | | 0,7 | 0,68 | 0,66 | 0,64 | 0,62 | 0,6 | 0,58 | 0,56 | 0,54 | 0,52 | 0,5 | 0,48 | 0,46 | 0,43 | 0,41 | |
| | | | | | | | | | | | 18,62 | 19,62 | 20,66 | 21,77 | 22,93 | 24,15 | 25,44 | 26,8 | 28,23 | 29,74 | 31,33 | 33 | 34,76 | 36,62 | 38,57 |
| | | | | | | | | | | | 0,72 | 0,7 | 0,68 | 0,66 | 0,64 | 0,62 | 0,6 | 0,58 | 0,56 | 0,54 | 0,52 | 0,5 | 0,48 | 0,46 | |
| | | | | | | | | | | | | 17,68 | 18,62 | 19,62 | 20,66 | 21,77 | 22,93 | 24,15 | 25,44 | 26,8 | 28,23 | 29,74 | 31,33 | 33 | 34,76 |
| | | | | | | | | | | | | 0,75 | 0,72 | 0,7 | 0,68 | 0,66 | 0,64 | 0,62 | 0,6 | 0,58 | 0,56 | 0,54 | 0,52 | 0,5 | |
| | | | | | | | | | | | | | 16,78 | 17,68 | 18,62 | 19,62 | 20,66 | 21,77 | 22,93 | 24,15 | 25,44 | 26,8 | 28,23 | 29,74 | 31,33 |
| | | | | | | | | | | | | | 0,77 | 0,75 | 0,72 | 0,7 | 0,68 | 0,66 | 0,64 | 0,62 | 0,6 | 0,58 | 0,56 | 0,54 | |
| | | | | | | | | | | | | | | 15,93 | 16,78 | 17,68 | 18,62 | 19,62 | 20,66 | 21,77 | 22,93 | 24,15 | 25,44 | 26,8 | 28,23 |
| | | | | | | | | | | | | | | 0,79 | 0,77 | 0,75 | 0,72 | 0,7 | 0,68 | 0,66 | 0,64 | 0,62 | 0,6 | 0,58 | |
| | | | | | | | | | | | | | | | 15,12 | 15,93 | 16,78 | 17,68 | 18,62 | 19,62 | 20,66 | 21,77 | 22,93 | 24,15 | 25,44 |
| | | | | | | | | | | | | | | | 0,81 | 0,79 | 0,77 | 0,75 | 0,72 | 0,7 | 0,68 | 0,66 | 0,64 | 0,62 | |
| | | | | | | | | | | | | | | | | 14,36 | 15,12 | 15,93 | 16,78 | 17,68 | 18,62 | 19,62 | 20,66 | 21,77 | 22,93 |
| | | | | | | | | | | | | | | | | 0,83 | 0,81 | 0,79 | 0,77 | 0,75 | 0,72 | 0,7 | 0,68 | 0,66 | |
| | | | | | | | | | | | | | | | | | 13,63 | 14,36 | 15,12 | 15,93 | 16,78 | 17,68 | 18,62 | 19,62 | 20,66 |
| | | | | | | | | | | | | | | | | | 0,85 | 0,83 | 0,81 | 0,79 | 0,77 | 0,75 | 0,72 | 0,7 | |
| | | | | | | | | | | | | | | | | | | 12,94 | 13,63 | 14,36 | 15,12 | 15,93 | 16,78 | 17,68 | 18,62 |
| | | | | | | | | | | | | | | | | | | 0,87 | 0,85 | 0,83 | 0,81 | 0,79 | 0,77 | 0,75 | |
| | | | | | | | | | | | | | | | | | | | 12,28 | 12,94 | 13,63 | 14,36 | 15,12 | 15,93 | 16,78 |
| | | | | | | | | | | | | | | | | | | | 0,89 | 0,87 | 0,85 | 0,83 | 0,81 | 0,79 | |
| | | | | | | | | | | | | | | | | | | | | 11,66 | 12,28 | 12,94 | 13,63 | 14,36 | 15,12 |
| | | | | | | | | | | | | | | | | | | | | 0,91 | 0,89 | 0,87 | 0,85 | 0,83 | |
| | | | | | | | | | | | | | | | | | | | | | 11,07 | 11,66 | 12,28 | 12,94 | 13,63 |
| | | | | | | | | | | | | | | | | | | | | | 0,93 | 0,91 | 0,89 | 0,87 | |
| | | | | | | | | | | | | | | | | | | | | | | 10,51 | 11,07 | 11,66 | 12,28 |
| | | | | | | | | | | | | | | | | | | | | | | 0,95 | 0,93 | 0,91 | |
| | | | | | | | | | | | | | | | | | | | | | | | 9,97 | 10,51 | 11,07 |
| | | | | | | | | | | | | | | | | | | | | | | | 0,97 | 0,95 | |
| | | | | | | | | | | | | | | | | | | | | | | | | 9,47 | 9,97 |
| | | | | | | | | | | | | | | | | | | | | | | | | 1 | |
| | | | | | | | | | | | | | | | | | | | | | | | | | 8,99 |

**Figure A.1:** Screenshot of the price tree generated by the code in Appendix A with input parameters as calculated for the Stokkfjellet project. Green cells give the electricity price, and blue cells give the probability that the price will move up in the next period corresponding to a certain node. Values are listed in NOK.

# B A binomial tree for the NPV (VBA code)

This code generates a tree for the development of the NPV of the operational phase and lists the corresponding option price at each node.

Cell referrals and output formatting is not included in the code.

```
Sub NPVtree()
    Worksheets("StartSheet").Activate

    ... loading cell values ...

    Dim aNpv(), npvNew(), optionValue(), optionVNew(), times(),
        Inv(), successprob() As Double

    ReDim aNpv(0 To n)
    ReDim npvNew(0 To n)
    ReDim optionValue(0 To n)
    ReDim optionVNew(0 To n)
    ReDim times(0 To stages + 1)
    ReDim Inv(0 To stages + 1)
    ReDim successprob(0 To stages + 1)

    'Calculate real probabilities
    up = Exp(sig * Sqr(dt))
    down = 1 / up

    'Loads the investment costs into the Inv(.) array
    Inv(0) = 0
    For i = 1 To stages
        Inv(i) = Cells(13 + i, 5).Value
    Next i

    'Loads the times of investments into the times(.) array
    times(0) = -1
    For i = 1 To stages
        times(i) = Cells(13 + i, 6).Value
    Next i

    'Loads the prob. of success into the successprob(.) array
    successprob(0) = 1
    For i = 1 To stages
        successprob(i) = Cells(13 + i, 7).Value
    Next i

    Worksheets("NPVtree").Cells.Clear
    Worksheets("NPVtree").Activate

    'Adds an informative row at the top of the output sheet
    For i = 0 To n
        Cells(1, i + 1).Value = i
    Next i
```

```
Dim j, k As Integer

For i = 0 To n
    'Calculate price level at node (i, n)
    aNpv(i) = start_npv * Exp((n - 2 * i) * sig * Sqr(dt))

    'Take the max of intrinsic value and zero
    optionValue(i) = WorksheetFunction.Max(aNpv(i) - Inv(k) *_
            successprob(k), 0)

    Cells(2 * i + 2, n + 1).Value = aNpv(i)
    Cells(2 * i + 3, n + 1).Value = optionValue(i)
Next i

k = k - 1

For j = (n - 1) To 0 Step -1 'Start in second to last column

    'While we have not yet reached the next point of exercise
    If j > times(k) Then

        'Work backwards in the tree to find NPV values
        For i = 0 To j
            npvNew(i) = aNpv(i) / up
            Prob = theta_up(i, j, a, b, dt, startprice, sig, up,_
            down, beta, mr)
            optionVNew(i) = ((Prob * optionValue(i) +_
            (1 - Prob) * optionValue(i + 1)) / rf)

            'Inputs the values into the sheet
            Cells(2 * i + 2, j + 1).Value = npvNew(i)
            Cells(2 * i + 3, j + 1).Value = optionVNew(i)
        Next i

        'Saves calculated values to array
        For i = 0 To j
            aNpv(i) = npvNew(i)
            optionValue(i) = optionVNew(i)
        Next i

    'When we have reached a point of exercise
    ElseIf k > 0 Then

        'Go to the next point of exercise, times(k - 1)
        k = k - 1

        'Find NPV value and take max of intrinsic value and zero
        For i = 0 To j
            npvNew(i) = aNpv(i) / up
            Prob = theta_up(i, j, a, b, dt, startprice, sig, up,_
            down, beta, mr)
```

```
            optionVNew(i) = WorksheetFunction.Max(((((Prob *_
            optionValue(i) + (1 - Prob) * optionValue(i + 1))_
            / rf) - Inv(k))) * successprob(k), 0)

            Cells(2 * i + 2, j + 1).Value = npvNew(i)
            Cells(2 * i + 3, j + 1).Value = optionVNew(i)
        Next i

        'Save calculated values to arrays
        For i = 0 To j
            aNpv(i) = npvNew(i)
            optionValue(i) = optionVNew(i)
        Next i
    End If
Next j

'Write out the option value.
MsgBox ("The value of holding the option to invest is "_
& Round(optionValue(0)) & " KNOK.")
End Sub


'Get risk-neutral probability of an up-move at specified node
Function theta_up(i, j, a, b, dt, startprice, sig, up, down,_
beta, mr)

    'Obtaining the actual probability of an up move
    theta_up = 0.5 + (1 - Exp(-a * dt)) * (b - (Log((startprice)_
    * Exp((j - 2 * i) * sig * Sqr(dt))))) / (2 * sig * Sqr(dt))

    'Get the risk-neutral probability
    If theta_up < 0 Then
        theta_up = 0
    ElseIf theta_up > 1 Then
        theta_up = 1
    Else
        theta_up = theta_up - ((mr * beta) / (up - down))
        If theta_up < 0 Then
            theta_up = 0
        End If
    End If

End Function
```

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 873816 | 920475 | 969625 | 1021401 | 1075940 | 1133393 | 1193912 | 1257664 | 1324819 | 1395561 | 1470080 | 1548578 | 1631267 | 1718372 | 1810128 | 1906784 | 2008601 | 2115854 | 2228835 | 2347848 | 2473216 | 2605279 | 2744393 | 2890935 | 3045303 | 3207913 |
| 28698 | 34549 | 41817 | **50840** | 62028 | 75864 | 92914 | 113824 | 139299 | 170078 | 206870 | 250300 | 300881 | 359045 | 425284 | **500376** | 590814 | 687785 | 798562 | 925614 | 1071965 | 1241340 | 1438365 | 1668822 | 1940068 | **2261168** |
| | 829522 | 873816 | 920475 | 969625 | 1021401 | 1075940 | 1133393 | 1193912 | 1257664 | 1324819 | 1395561 | 1470080 | 1548578 | 1631267 | 1718372 | 1810128 | 1906784 | 2008601 | 2115854 | 2228835 | 2347848 | 2473216 | 2605279 | 2744393 | 2890935 |
| | 23037 | 28079 | **34429** | 42422 | 52471 | 65070 | 80806 | 100358 | 124494 | 154030 | 189763 | 232403 | 282509 | 340505 | **406826** | 487438 | 573083 | 670737 | 782499 | 910934 | 1059187 | 1231144 | 1431645 | 1666766 | **1944190** |
| | | 787473 | 829522 | 873816 | 920475 | 969625 | 1021401 | 1075940 | 1133393 | 1193912 | 1257664 | 1324819 | 1395561 | 1470080 | 1548578 | 1631267 | 1718372 | 1810128 | 1906784 | 2008601 | 2115854 | 2228835 | 2347848 | 2473216 | 2605279 |
| | | 17725 | **21923** | 27298 | 34179 | 42974 | 54183 | 68406 | 86358 | 108859 | 136817 | 171141 | 212649 | 261976 | **319549** | 390981 | 466392 | 552229 | 650275 | 762696 | 892141 | 1041872 | 1215930 | 1419365 | **1658534** |
| NPV | | | 747556 | 787473 | 829522 | 873816 | 920475 | 969625 | 1021401 | 1075940 | 1133393 | 1193912 | 1257664 | 1324819 | 1395561 | 1470080 | 1548578 | 1631267 | 1718372 | 1810128 | 1906784 | 2008601 | 2115854 | 2228835 | 2347848 |
| Option value | | | **12909** | 16249 | 20614 | 26318 | 33762 | 43441 | 55965 | 72065 | 92616 | 118621 | 151123 | 191085 | **239262** | 301362 | 367162 | 442370 | 528121 | 626243 | 738961 | 869004 | 1019740 | 1195357 | **1401103** |
| Option value at decision point | | | | 709662 | 747556 | 787473 | 829522 | 873816 | 920475 | 969625 | 1021401 | 1075940 | 1133393 | 1193912 | 1257664 | 1324819 | 1395561 | 1470080 | 1548578 | 1631267 | 1718372 | 1810128 | 1906784 | 2008601 | 2115854 |
| | | | | 8733 | 11232 | 14581 | 19074 | 25095 | 33130 | 43785 | 57799 | 76073 | 99712 | 129911 | **167826** | 219644 | 275409 | 340537 | 415281 | 500648 | 598504 | 711127 | 841316 | 992537 | **1169109** |
| | | | | | 673689 | 709662 | 747556 | 787473 | 829522 | 873816 | 920475 | 969625 | 1021401 | 1075940 | 1133393 | 1193912 | 1257664 | 1324819 | 1395561 | 1470080 | 1548578 | 1631267 | 1718372 | 1810128 | 1906784 |
| | | | | | 5328 | 7038 | 9406 | 12695 | 17261 | 23574 | 32230 | 43954 | 59639 | 80474 | **107822** | 148335 | 192717 | 246921 | 311051 | 385055 | 469722 | 566949 | 679056 | 808903 | **960039** |
| | | | | | | 639539 | 673689 | 709662 | 747556 | 787473 | 829522 | 873816 | 920475 | 969625 | 1021401 | 1075940 | 1133393 | 1193912 | 1257664 | 1324819 | 1395561 | 1470080 | 1548578 | 1631267 | 1718372 |
| | | | | | | 2784 | 3804 | 5280 | 7426 | 10559 | 15121 | 21697 | 31000 | 43830 | **61433** | 90582 | 122466 | 163906 | 215936 | 278675 | 351651 | 435287 | 531500 | 642645 | **771627** |
| | | | | | | | 607121 | 639539 | 673689 | 709662 | 747556 | 787473 | 829522 | 873816 | 920475 | 969625 | 1021401 | 1075940 | 1133393 | 1193912 | 1257664 | 1324819 | 1395561 | 1470080 | 1548578 |
| | | | | | | | 1125 | 1606 | 2343 | 3488 | 5286 | 8127 | 12596 | 19456 | **29270** | 48528 | 68370 | 96017 | 133548 | 182564 | 243409 | 315062 | 397322 | 492120 | **601833** |
| | | | | | | | | 576345 | 607121 | 639539 | 673689 | 709662 | 747556 | 787473 | 829522 | 873816 | 920475 | 969625 | 1021401 | 1075940 | 1133393 | 1193912 | 1257664 | 1324819 | 1395561 |
| | | | | | | | | 269 | 406 | 634 | 1023 | 1707 | 2950 | 5287 | **9840** | 21902 | 32114 | 47353 | 69834 | 102222 | 147031 | 205286 | 275313 | 355845 | **448816** |
| | | | | | | | | | 547130 | 576345 | 607121 | 639539 | 673689 | 709662 | 747556 | 787473 | 829522 | 873816 | 920475 | 969625 | 1021401 | 1075940 | 1133393 | 1193912 | 1257664 |
| | | | | | | | | | 0 | 0 | 0 | 0 | 0 | 0 | **0** | 7826 | 11903 | 18383 | 28725 | 45174 | 70917 | 109761 | 164374 | 232471 | **310919** |
| | | | | | | | | | | 519396 | 547130 | 576345 | 607121 | 639539 | 673689 | 709662 | 747556 | 787473 | 829522 | 873816 | 920475 | 969625 | 1021401 | 1075940 | 1133393 |
| | | | | | | | | | | 0 | 0 | 0 | 0 | 0 | **0** | 1977 | 3105 | 4999 | 8241 | 13882 | 23807 | 41293 | 71552 | 120780 | **186648** |
| | | | | | | | | | | | 493068 | 519396 | 547130 | 576345 | 607121 | 639539 | 673689 | 709662 | 747556 | 787473 | 829522 | 873816 | 920475 | 969625 | 1021401 |
| | | | | | | | | | | | 0 | 0 | 0 | 0 | **0** | 266 | 430 | 717 | 1239 | 2221 | 4134 | 8001 | 16132 | 33940 | **74656** |
| | | | | | | | | | | | | 468074 | 493068 | 519396 | 547130 | 576345 | 607121 | 639539 | 673689 | 709662 | 747556 | 787473 | 829522 | 873816 | 920475 |
| | | | | | | | | | | | | 0 | 0 | 0 | **0** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | **0** |
| | | | | | | | | | | | | | 444347 | 468074 | 493068 | 519396 | 547130 | 576345 | 607121 | 639539 | 673689 | 709662 | 747556 | 787473 | 829522 |
| | | | | | | | | | | | | | 0 | 0 | **0** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | **0** |
| | | | | | | | | | | | | | | 421823 | 444347 | 468074 | 493068 | 519396 | 547130 | 576345 | 607121 | 639539 | 673689 | 709662 | 747556 |
| | | | | | | | | | | | | | | 0 | **0** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | **0** |
| | | | | | | | | | | | | | | | 400441 | 421823 | 444347 | 468074 | 493068 | 519396 | 547130 | 576345 | 607121 | 639539 | 673689 |
| | | | | | | | | | | | | | | | **0** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | **0** |
| | | | | | | | | | | | | | | | | 380142 | 400441 | 421823 | 444347 | 468074 | 493068 | 519396 | 547130 | 576345 | 607121 |
| | | | | | | | | | | | | | | | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | **0** |
| | | | | | | | | | | | | | | | | | 360873 | 380142 | 400441 | 421823 | 444347 | 468074 | 493068 | 519396 | 547130 |
| | | | | | | | | | | | | | | | | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | **0** |
| | | | | | | | | | | | | | | | | | | 342580 | 360873 | 380142 | 400441 | 421823 | 444347 | 468074 | 493068 |
| | | | | | | | | | | | | | | | | | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | **0** |
| | | | | | | | | | | | | | | | | | | | 325214 | 342580 | 360873 | 380142 | 400441 | 421823 | 444347 |
| | | | | | | | | | | | | | | | | | | | 0 | 0 | 0 | 0 | 0 | 0 | **0** |
| | | | | | | | | | | | | | | | | | | | | 308729 | 325214 | 342580 | 360873 | 380142 | 400441 |
| | | | | | | | | | | | | | | | | | | | | 0 | 0 | 0 | 0 | 0 | **0** |
| | | | | | | | | | | | | | | | | | | | | | 293079 | 308729 | 325214 | 342580 | 360873 |
| | | | | | | | | | | | | | | | | | | | | | 0 | 0 | 0 | 0 | **0** |
| | | | | | | | | | | | | | | | | | | | | | | 278223 | 293079 | 308729 | 325214 |
| | | | | | | | | | | | | | | | | | | | | | | 0 | 0 | 0 | **0** |
| | | | | | | | | | | | | | | | | | | | | | | | 264120 | 278223 | 293079 |
| | | | | | | | | | | | | | | | | | | | | | | | 0 | 0 | **0** |
| | | | | | | | | | | | | | | | | | | | | | | | | 250732 | 264120 |
| | | | | | | | | | | | | | | | | | | | | | | | | 0 | **0** |
| | | | | | | | | | | | | | | | | | | | | | | | | | 238022 |
| | | | | | | | | | | | | | | | | | | | | | | | | | **0** |

**Figure B.1:** Screenshot of the NPV tree generated by the code in Appendix B with input parameters as calculated for the Stokkfjellet project. Green cells show the development of the NPV, blue cells list the option value, and pink cells give the option value at a time of exercise. Values are listed in KNOK.

# C Monte Carlo simulation of prices (VBA code)

This code simulates the price development of power and TGC's, and writes out the simulated price at each time step of the Monte Carlo simulation of both prices as well as the average of the two at each time step of the Monte Carlo simulation.

Cell referrals and output formatting is not included in the code.

```
Sub simulation()
    Worksheets("StartSheet").Activate

    ... loading cell values ...

    Dim GBMprices(), GBMpriceNew(), OUprices(), OUpriceNew(), _
    sum(), avgGBMprice(), avgOUprice()  As Double

    ReDim GBMprices(0 To N)
    ReDim GBMpriceNew(0 To N)
    ReDim OUprices(0 To N)
    ReDim OUpriceNew(0 To N)
    ReDim avgGBMprice(0 To sims, 0 To N)
    ReDim avgOUprice(0 To sims, 0 To N)

    Dim i, j As Integer

    'Activates cells
    Worksheets("AvgCalculations").Cells.Clear
    Worksheets("AvgCalculations").Activate

    'Adds an informative row at the top
    For i = 0 To N
        Cells(1, i + 1).Value = i
    Next i

    For j = 1 To sims
        'Input start values to arrays
        GBMprices(0) = TGCstartprice
        OUprices(0) = startprice

        For i = 1 To N
            'Draw two random numbers from the normal distribution
            e1 = WorksheetFunction.NormInv(Rnd(), 0, 1)
            Z = rho * e1 + WorksheetFunction.NormInv(Rnd(), 0, 1) _
            * Sqr(1 - rho ^ 2)

            'Get the next price of TGC's
            GBMprices(i) = GBMprice_next(GBMprices(i - 1), mu, _
            GBMsigma, dT, e1)
            avgGBMprice(j, i) = GBMprices(i)
```

```
            'Get the next price of electricity
            OUprices(i) = OUprice_next(OUprices(i - 1), a, b, _
            dT, OUsigma, Z)
            avgOUprice(j, i) = OUprices(i)
        Next i

        GBMsum = GBMsum + GBMprices(N)
        OUsum = OUsum + OUprices(N)
    Next j

    'Print average prices generated above
    For i = 1 To N
        GBMtemp = 0
        OUtemp = 0

        For j = 1 To sims
            GBMtemp = GBMtemp + avgGBMprice(j, i)
            OUtemp = OUtemp + avgOUprice(j, i)
        Next j

        GBMtemp = GBMtemp / sims
        OUtemp = OUtemp / sims

        Cells(2, i).Value = GBMtemp
        Cells(3, i).Value = OUtemp
    Next i

    GBMsum = GBMsum / sims              'Get the average value
    OUsum = OUsum / sims                'Get the average value
    Totsum = GBMsum + OUsum             'Get the average total sum
    Totsum = Totsum * Exp(-rf * N)      'Discount the option value
    Totsum = Round(Totsum, 0)           'Rounding to no decimals

    MsgBox ("The avg. profit after " & N & " periods and " & sims _
    & " simulations is " & Totsum)

End Sub

'Get the next NPV level when the NPV follows a GBM
Function GBMprice_next(last_price, mu, sigma, dT, e1)
    GBMprice_next = last_price * Exp((mu - 0.5 * sigma ^ 2) * dT _
    + sigma * Sqr(dT) * e1)
End Function


'Get the next NPV level when the NPV follows an OU process
Function OUprice_next(last_price, a, b, dT, sigma, Z)
    OUprice_next = WorksheetFunction.Ln(last_price) * Exp(-a *dT) _
    + b * (1 - Exp(-a * dT)) + sigma * Sqr((1 - Exp(-2 * a * dT)) _
    / (2 * a)) * Z
    OUprice_next = Exp(OUprice_next)
End Function
```

# D Monte Carlo simulation of NPV (VBA code)

This code simulates the development of the value of operation as a two-dimensional process that follows the processes of the power price and the price of TGC's, respectively, and writes out the value of the NPV for both price processes as well as the average of the two at each time step of the Monte Carlo simulation.

Cell referrals and output formatting is not included in the code.

```
Sub NPVsimulation()
    Worksheets("StartSheet").Activate

    ... loading cell values ...

    Dim GBMnpv(), OUnpv(), TOTnpv(), avgGBMnpv(), avgOUnpv(), _
    avgnpv() As Double

    ReDim GBMnpv(0 To N)
    ReDim OUnpv(0 To N)
    ReDim TOTnpv(0 To N)
    ReDim avgGBMnpv(0 To sims, 0 To N)
    ReDim avgOUnpv(0 To sims, 0 To N)
    ReDim avgnpv(0 To sims, 0 To N)

    Dim i, j As Integer

    Worksheets("NPVcalculations").Cells.Clear
    Worksheets("NPVcalculations").Activate

    'Adds an informative row at the top
    For i = 0 To N
        Cells(1, i + 1).Value = i
    Next i

    'Input initial NPV value into the first column
    Cells(2, 1).Value = startnpv
    Cells(3, 1).Value = startnpv
    Cells(4, 1).Value = startnpv

    'Estimate average NPV
    avg_npv = ratio * startnpv
    avg_npv = WorksheetFunction.Ln(avg_npv)

    For j = 0 To sims
        'Input start values to arrays
        GBMnpv(0) = startnpv
        OUnpv(0) = startnpv
        TOTnpv(0) = startnpv
```

```
    For i = 1 To N
        'Draw two random numbers from the normal distribution
        e1 = WorksheetFunction.NormInv(Rnd(), 0, 1)
        Z = rho * e1 + WorksheetFunction.NormInv(Rnd(), 0, 1) _
        * Sqr(1 - rho ^ 2)

        'Get the next price of TGC's
        GBMnpv(i) = GBMnpv_next(GBMnpv(i - 1), mu, GBMsigma, _
        dT, e1)
        avgGBMnpv(j, i) = GBMnpv(i)

        'Get the next price of electricity
        OUnpv(i) = OUnpv_next(OUnpv(i - 1), a, avg_npv, dT, _
        OUsigma, Z)
        avgOUnpv(j, i) = OUnpv(i)

        'Avg. NPV calculations for simulation j
        TOTnpv(i) = (GBMnpv(i) + OUnpv(i)) / 2
        avgnpv(j, i) = TOTnpv(i)
    Next i

    'Saving NPV levels at the end of the jth simulation
    GBMsum = GBMsum + GBMnpv(N)
    OUsum = OUsum + OUnpv(N)

    'Find NPV value and take max of intrinsic value and zero
    TOToption = TOToption + WorksheetFunction.Max(TOTnpv(N) -_
    inv, 0)
Next j

'Save values into relevant variables
GBMsum = GBMsum / sims
OUsum = OUsum / sims
TOToption = TOToption / sims
TOToption = Round(TOToption, 0) 'Rounding to no decimals

'Get the total average NPV value
Totsum = (GBMsum + OUsum) / 2

'Discount the option value back to today
Totsum = Totsum * Exp(-rf * N)
Totsum = Round(Totsum, 0) 'Rounding to no decimals

'Print average values to cells
For i = 1 To N
    GBMtemp = 0
    OUtemp = 0
    TOTtemp = 0
```

```
        'Save all levels of the NPV generated at a certain time
        For j = 1 To sims
            GBMtemp = GBMtemp + avgGBMnpv(j, i)
            OUtemp = OUtemp + avgOUnpv(j, i)
            TOTtemp = TOTtemp + avgnpv(j, i)
        Next j

        'Get the avg. of all the NPV levels at a certain time
        GBMtemp = GBMtemp / sims
        OUtemp = OUtemp / sims
        TOTtemp = TOTtemp / sims
        TOTtemp = Round(TOTtemp, 0) 'Rounding to no decimals

        'Print average prices in each time step n
        Cells(2, i + 1).Value = GBMtemp
        Cells(3, i + 1).Value = OUtemp
        Cells(4, i + 1).Value = TOTtemp
    Next i

    MsgBox ("The avg. NPV after " & N & " periods and " & sims & _
    " simulations is " & Totsum & _
    ". The corresponding option value is " & TOToption & ".")

End Sub

'Get the next NPV level when the NPV follows a GBM
Function GBMnpv_next(last_npv, mu, sigma, dT, e1)
    GBMnpv_next = last_npv * Exp((mu - 0.5 * sigma ^ 2) * dT + _
    sigma * Sqr(dT) * e1)
End Function


'Get the next NPV level when the NPV follows an OU process
Function OUnpv_next(last_npv, a, avg_npv, dT, sigma, Z)
    OUnpv_next = WorksheetFunction.Ln(last_npv) * Exp(-a * dT) + _
    avg_npv * (1 - Exp(-a * dT)) + sigma * Sqr((1 - Exp(-2 * a * _
    dT)) / (2 * a)) * Z
    OUnpv_next = Exp(OUnpv_next)
End Function
```

# E Analysis of production from Norwegian wind farms

An analysis of the performance of Norwegian wind farms built between 1991 and 2012 is shown in Table E.1. As can be seen, the deviation in real production from estimated production is significantly larger for wind farms built before the year 2000. This is assumably so because the technology has improved and because the oldest wind farms have only one or a few turbines, so that the standard deviation in production will typically be higher.

The grand average deviation calculation does not include the deviations from the oldest wind farms because these numbers are considered to be out of date. The result indicates that there is only a small negative deviation from estimated production for the wind farms. As a result, no scenario analysis is performed in this thesis because, in any case, there are other parameters going into the valuations to which a lot more uncertainty is connected.

**Table E.1:** This table shows estimated production vs. real production for all Norwegian wind farms built between 1991 and 2012. The years 2011, 2012 and 2013 have been considered. (For some wind farms, the number of operating hours could not be retrieved for 2011 and 2012.) *Source: Hofstad (2012), Weir (2013) and Weir (2014).*

| **Wind plant** | **Owner** | **Launch year** | **No. of turbines** | **Installed capacity [MW]** | **Normal production [GWh]** | **Est. no. of operating hours** | **Operating hours** | | | **Avg. dev.** |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | ***2013*** | ***2012*** | ***2011*** | |
| Andøya | Andøya Energi | 1991 | 1 | 0.4 | 1 | 2 500 | 1 478 | 2 064 | 1 650 | (-31 %) |
| Hovden | Vesterålskraft produksjon AS | 1991 | 1 | 0.4 | 1 | 2 500 | 1 495 | 1 015 | 1 025 | (-53 %) |
| Vikna | NTE | 1992 | 2 | 0.9 | 2.4 | 2 667 | 1 643 | - | 2 288 | (-26 %) |
| Fjeldskår | Norsk Miljø Energi | 1998 | 5 | 3.75 | 8.5 | 2 267 | 1 957 | 1 447 | 1 631 | (-26 %) |
| Harøy | Sandøy Energi | 1999 | 5 | 3.75 | 10 | 2 667 | 2 202 | 2 564 | 2 481 | (-9 %) |
| Mehuken | Kvalheim Kraft | 2001 | 13 | 22.65 | 65 | 2 870 | 2 998 | 2 989 | - | 4 % |
| Smøla I&II | Statkraft | 2002 | 68 | 150.4 | 356 | 2 367 | 2 033 | 2 455 | 2 494 | -2 % |
| Havøygavlen | Artic Wind | 2002 | 16 | 40.5 | 100 | 2 469 | 2 185 | 2 612 | 2 608 | 0 % |
| Utsira I&II | Solvind Prosjekt | 2004 | 2 | 1.2 | 3.5 | 2 917 | 2 925 | 3 580 | 3 208 | 11 % |
| Hitra | Statkraft | 2004 | 24 | 55.2 | 138 | 2 500 | 2 400 | 2 762 | 2 713 | 5 % |
| Nygårdsfjellet | Nordkraft Vind AS | 2005 | 14 | 32.2 | 104 | 3 230 | 2 687 | 2 783 | 2 928 | -13 % |
| Kjøllefjord | Statkraft | 2006 | 17 | 39.1 | 119 | 3 043 | 3 049 | 3 326 | 3 270 | 6 % |
| Valsneset | TE | 2006 | 5 | 11.5 | 35 | 3 043 | 2 579 | 2 716 | 3 200 | -7 % |
| Bessakerfjellet | TE | 2008 | 25 | 57.5 | 175 | 3 043 | 2 795 | 2 957 | 3 163 | -2 % |
| Hywind | Statoil | 2009 | 1 | 2.3 | 8 | 3 478 | 3 601 | 3 257 | 4 391 | 8 % |
| Høg-Jæren | Jæren Energi | 2011 | 32 | 73.6 | 186.4 | 2 533 | 3 052 | - | - | 21 % |
| Åsen | Solvind Åsen AS | 2012 | 2 | 1.6 | 4.9 | 3 063 | 2 431 | - | - | -21 % |
| Fakken | Troms Kraft | 2012 | 18 | 54 | 139 | 2 574 | 2 378 | - | - | -8 % |
| Ytre Vikna | Sarepta Energi AS | 2012 | 17 | 39.1 | 127 | 3 248 | 2 656 | - | - | -18 % |
| Lista | Lista Vindkraftverk | 2012 | 31 | 71.3 | 220 | 3 086 | 2 921 | - | - | -5 % |
| **Grand average deviation** | | | | | | | | | | **-1.46 %** |

# F A brief introduction to stochastic calculus

For the convenience of readers interested in a brief introduction to the mathematical concepts used in this thesis, we have included an introduction to relevant stochastic calculus below.

## Stochastic processes

Due to the inherent uncertainty of the parameters involved in the estimation of the NPV of a project, it is recommended to reproduce its behavior by means of a stochastic process (Muñoz et al., 2011). A stochastic process is a variable that evolves over time in a way that is at least partially random.

## The Wiener process

A Wiener process, sometimes called Brownian Motion (BM), is a continuous-time stochastic process. It is well suited for modelling fluctuations in any real-life process that has some special characteristics (Dixit and Pindyck, 1994). Three important properties of the process are:

1. The Markov property
2. Independent increments
3. Normally distributed changes in the process over any finite interval of time

The Markov property implies that the probability distribution for all future values are dependent only on the current value, and is thus unaffected by past values of the process. Independent increments means that the change in the process over any time interval is independent of any other (non-overlapping) time interval. We need the changes to be normally distributed over any finite interval of time, so lastly we must be dealing with a standard normal random variable ($\mu = 0$, $\sigma = 1$).

The increment of a Wiener process, $dz$, is represented in continuous time by

$$dz = \epsilon_t \sqrt{dt} \tag{F.1}$$

where $\epsilon_t$ is a standard normal random variable that is serially uncorrelated, such that $\mathcal{E}[\epsilon_t \epsilon_s] = 0$ for $t \neq s$. It is worth noting that since $\epsilon_t$ has zero mean and unit standard deviation, $\mathcal{E}(dz) = 0$ and $\mathcal{V}(dz) = \mathcal{E}[(dz)^2] = dt$. Evidently, the variance of the change in a Wiener process grows linearly with the time horizon. Accordingly, the standard deviation of the process grows as the *square root* of time passing. The Wiener process can easily be generalized into more complex processes, such as the Brownian motion with drift.

### Brownian Motion with drift

The simplest generalisation of Eq. (F.1) is the *Brownian motion with drift:*

$$dx = \alpha\, dt + \sigma\, dz \tag{F.2}$$

where $dz$ is the increment of a Wiener process as defined above. Here, $\alpha$ is called the drift parameter, and $\sigma$ is the variance parameter. The change in $x$, $\Delta x$, over any time interval $\Delta t$, is normally distributed and has expectation $\mathcal{E}(\Delta x) = \alpha \Delta t$ and $\mathcal{V}(\Delta x) = \sigma^2 \Delta t$. The changes being normally distributed can be demonstrated by allowing a BM process to be represented as a random walk and letting the number of steps become very large.

## Generalised Brownian Motion-Itō processes

Now, we wish to use the Wiener process as a building block to model more stochastic variables. Let now the simple BM be represented by

$$dx = a\,(x,t)\,dt + b\,(x,t)\,dz \tag{F.3}$$

where $dz$ is defined as before and $a\,(x,t)$ and $b\,(x,t)$ are known functions. Note that the drift and variance terms are now dependent on the current state and time. The continuous-time stochastic process $x(t)$ represented by Eq. (F.3) is called an *Itō process.*

Consider the mean and variance of the increments of this process. Since $\mathcal{E}[dz] = 0$, $\mathcal{E}[dx] = a\,(x,t)\,dt$. The variance of $dx$ is $\mathcal{E}[dx^2] - (\mathcal{E}[dx]^2)$. Terms including $(dt)^2$ and $(dt)(dz) = (dt)^{3/2}$ are very small and can be ignored, leaving us with $\mathcal{V}[dx] = b^2\,(x,t)\,dt$. Let us call $a\,(x,t)$ the expected instantaneous *drift rate* of the Itō process, and $b^2\,(x,t)$ the instantaneous *variance rate.*

### Geometric Brownian motion

An important special case of Eq. (F.3) is the *geometric Brownian motion (GBM) with drift* (Dixit and Pindyck, 1994). Here $a\,(x,t) = \alpha x$ and $b\,(x,t) = \sigma x$, $\alpha$ and $\sigma$ being constants. That gives us

$$dx = \alpha x\,dt + \sigma x\,dz. \tag{F.4}$$

We know from the section about the Brownian motion with drift that percentage changes in $x$, $\Delta x/x$, are normally distributed. Since these are changes in the natural logarithm of $x$, we know that absolute changes in $x$, $\Delta x$, must be *lognormally* distributed. It can be shown that if $x(t)$ is given by Eq. (F.4), then $F(x) = log\,x$ is the following simple BM with drift:

$$dF = (\alpha - \frac{1}{2}\sigma^2)\,dt + \sigma x\,dz \tag{F.5}$$

so that over a finite time interval $t$, the change in the logarithm of $x$ is normally distributed with mean $(\alpha - \frac{1}{2}\sigma^2)t$ and variance $\sigma^2 t$. Considering $x$ itself, it can be shown that if the current value of $x(0) = x_0$, the expectation of $x(t)$ is given by

$$\mathcal{E}[x(t)] = x_0 e^{\alpha t}, \tag{F.6}$$

and the variance of $x(t)$ is given by

$$\mathcal{V}[x(t)] = x_0^2\,e^{2\alpha t}(e^{\sigma^2 t} - 1). \tag{F.7}$$

This result for the expected value of a geometric Brownian motion can be used to calculate the expected present discounted value of $x(t)$ over some time interval. It is interesting to note that

$$\mathcal{E}\left[\int_0^\infty x(t)\, e^{-rt}\, dt\right] = \mathcal{E}\int_0^\infty x_0\, e^{-(r-\alpha)t}\, dt = x_0/(r-\alpha) \tag{F.8}$$

assuming that the discount rate $r$ is larger that the growth rate $\alpha$.

### Mean-reverting processes

The simplest mean-reverting process, called the *Ornstein-Uhlenbeck process*, is given by

$$dx = \eta\,(\bar{x} - x)\, dt + \sigma\, dz, \tag{F.9}$$

where $\eta$ is the speed of reversion towards the mean and $\bar{x}$ is the 'normal' level of $x$, i.e. the level to which $x$ tends to revert (e.g. the marginal production cost of a commodity for which $x$ is the price). We note that the expected change in $x$ depends on the current difference between $x$ and $\bar{x}$. This means that, depending on the current position of $x$ relative to $\bar{x}$, the price is more likely to rise (if $x < \bar{x}$) or fall (if $x > \bar{x}$) over the next short interval of time.

If the value of $x(0) = x_0$ and $x$ follows Eq. (F.9), then its expected value at any future time $t$ is equal to

$$\mathcal{E}[x(t)] = \bar{x} + (x_0 - \bar{x})\, e^{-\eta t}, \tag{F.10}$$

and the variance of $x(t) - \bar{x}$ is given by

$$\mathcal{V}[x(t) - \bar{x}] = \frac{\sigma^2}{2\eta}(1 - e^{-2\eta t}). \tag{F.11}$$

For the derivation of Eq. (F.10) and (F.11), we refer to Dixit and Pindyck (1994) (pp. 90-92). From these equations, we can observe that the expectation of $x(t)$ converges to $\bar{x}$ as $t$ goes to infinity, while the variance converges to $\sigma/2\eta$. On the other hand, in the special case where we let the speed of reversion, $\eta$, become sufficiently large, the variance goes to zero, so that $\bar{x}$ can never deviate from $x$. Finally, as $\eta$ goes to zero, $x$ becomes a simple Brownian motion, and the variance converges towards $\sigma^2 t$.