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Optimization of helicopter hub locations and fleet composition in the Brazilian pre-salt fields

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MASTERKONTRAKT
- uttak av masteroppgave

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4. Underskrift

Student: Jeg erklærer herved at jeg har satt meg inn i gjeldende bestemmelser for mastergradsstudiet og at jeg oppfyller kravene for adgang til å påbegynne oppgaven, herunder eventuelle praksiskrav.

Partene er gjort kjent med avtalens vilkår, samt kapitlene i studiehåndboken om generelle regler og aktuell studieplan for masterstudiet.

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Abstract

When implementing oil and gas operations in the Santos Basin pre-salt fields, Brazilian energy corporation Petróleo Brasileiro S.A (Petrobras) faces some significant challenges. One of these is the large distance from shore: exceeding 300 km at the most, it is about three times the distance to oil fields explored in the past. This has great impact on the company's offshore helicopter operations. In its current form, helicopters transport employees directly between onshore airport bases and offshore installations. The question then arises as to whether this transportation model can and should be used in the future, or whether the installment and use of one or several offshore transhipment hubs is more appropriate from a cost or safety perspective.

Taking this query into consideration, this report presents three mathematical formulations for optimizing the number and locations of offshore transhipment hubs, as well as the size and mix of the helicopter fleet, required to support Petrobras' future helicopter operations in the Santos Basin area. Both mixed integer linear arc and path flow formulations are presented, as well as algorithms for generating various sets of predefined routes. The objective function in all models uses weighted linear combination to evaluate both the total investment and operational cost, and the total accident risk assessment of the system, as well as the relationship between the two.

Results show that if available helicopters are able to reach all offshore platforms to be installed in the Santos Basin, no offshore transhipment hub should be installed from both a cost and accident risk minimizing perspective. This means that Petrobras' current transportation model also should be used in future operations. If available helicopters are not able to reach all offshore platforms however, results suggest that one offshore transhipment hub should be installed. This hub should be located within a region laying close to the various offshore installations. The optimal size and composition of Petrobras' future helicopter fleet is greatly affected by the installment of an offshore transhipment hub. If this is performed, the fleet should mainly consist of Sikorsky S-92 aircraft. If the contrary were to occur on the other hand, mostly Sikorsky S-76 aircraft should be made use of.

In addition to presenting and implementing a mathematical model, the report also gives an overview of operations research literature related to the problem examined.

Preface

This report represents the author's dissertation for the degree of Master of Science in Industrial Economics and Technology Management at the Norwegian University of Science and Technology (NTNU). The thesis is an integrated part of an academic specialization in Managerial Economics and Operations Research. The work performed is a continuation of the author's specialization project completed in the autumn of 2012.

The report examines optimal helicopter hub locations and fleet composition in the Santos basin pre-salt fields for the Brazilian oil and gas company Petróleo Brasileiro S.A (Petrobras). The topic was provided by the Center for Integrated Operations in the Petroleum Industry (IO Center).

I would like to thank my academic supervisors Professor Kjetil Fagerholt and Associate Professor Henrik Andersson for all your guidance during the last year. Your help has been simply invaluable. I would also like to express my gratitude to my co-supervisors Eirik Fernández Cuesta and Vidar Gunnerud for always helping me whenever needed. Thank you to both of you.

Great and sincere gratefulness also goes to Felipe Augusto Coutinho Nascimento at Imperial College London for providing me with invaluable insight into Brazilian helicopter operations. You have helped me so much more than I could have ever expected.

Lastly, I would like to thank my friends and family for your unconditional support. In particular, a great thank you goes to my parents for being more amazing than I will ever understand.

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Ida Kristine Norddal

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AFF	Arc Flow Formulation
ANAC	Agência Nacional de Aviação Civil
B&B	Branch & Bound
BOE	Barrels of Oil Equivalents
BPD	Barrels Per Day
COMAR	Comando Aéreo Regional
DAESP	Departamento Aeroviário do Estado de São Paulo
DECEA	Departamento de Controle do Espaço Aéreo
FPSO	Floating Production, Storage and Offloading unit
FSMVRP	Fleet Size and Mix Vehicle Routing Problem
FSMVRPMD	Fleet Size and Mix Vehicle Routing Problem with Multiple Depots
ICAO	International Civil Aviation Organization
IFR	Instrumental Flight Rules
MILP	Mixed Integer Linear Programming
MIP	Mixed Integer Programming
MTOW	Maximum Take-Off Weight
OR	Operations Research

PFF	Path Flow Formulation
PKM	Passenger-kilometre
TAN	Tariff for communications use and aerial navigation assistance
TAT	Tariff for communication use and visual and radio assistance
TPL	Third Party Logistics provider
TSP	Travelling Salesman Problem
VFR	Visual Flight Rules
VRP	Vehicle Routing Problem

Chapter 1

Introduction

The oil and gas industry is the world's largest energy provider, with oil alone supplying over one third of the global energy consumption. The industry is a vital part of the world as we know it, and a sector in which many of the largest companies in the world operate. One of these companies is the Brazilian integrated energy corporation Petróleo Brasileiro S.A, also known as Petrobras. This is the company of interest in this report.

Petrobras is the world's third largest oil and gas-company in terms of market capitalization, but rank as number fourteen in terms of production volumes (MercoPress (2011), Forbes (2012)). Still, the discovery of reservoirs outside the Brazilian coast, containing billions of barrels of oil has provided a great opportunity for the future for the company. At present, the reserves are estimated containing up to 10 billion recoverable barrels of oil equivalents (BOE) (Petrobras (2011)). The findings can potentially transform Brazil into one of the world's leading oil producers and exporters.

However, significant challenges need to be overcome on the way. In the past, oil fields have been found at maximum 4000 meters below sea level, located underneath "normal" substances such as water, sand and rock. The new oil finds on the other hand are trapped under some additional 1000 meters of rock, as well as up to 2000 meters of compressed salt. Furthermore, the distance from the Brazilian coast exceeds 300 km, about three times the distance to oil fields explored in the past. These factors make the new oil finds incredibly difficult to reach. The pre-salt discoveries therefore require major investments in research and development, as well as extensive upgrades in platforms, refineries and pipeline network, to mention some.

This report addresses the issue of Petrobras' required upgrades in helicopter bases and fleet that follows from the future expansion of operations at the various pre-salt fields. In particular, future operations in the Santos Basin area will be investigated. The way investments in such equipment are made will have a significant impact on how efficiently Petrobras' operations can be performed in the future. As stated by Petrobras' executive manager Jose Miranda Formigli, "*It is absolutely fundamental...to come up with a flawless logistics solution for the pre-salt*" (Upstream (2011)). Following this, the purpose of this report is to develop decision support models that may aid these decision making processes by making use of mathematical optimization. The models should address the problem from both an economic and accident risk perspective, as well as being able to establish the connection between the two. The models are to be implemented in commercial optimization software and tested on various test instances. Following this, advice on how Petrobras' should organize its future helicopter operations are provided based on the results.

The outline of the report is presented in the following. Chapter 2 gives a further introduction to Petrobras' helicopter operations, while Chapter 3 provides an overview of relevant operations research (*OR*) literature. Chapter 4 thereafter defines the problem addressed in this report by words, while Chapter 5 presents three mathematical formulations of the same issue. Afterwards, algorithms for generating predefined routes used in two of the mathematical formulations are given in Chapter 6. The way in which the various mathematical formulations and algorithms have been implemented in commercial optimization software is then described in Chapter 7. Further, Chapter 8 presents the generation of parameters for various test instances. Technical, and economic and risk analyses of results are then given in Chapters 9 and 10 respectively. Lastly, suggestions for future research areas are presented in Chapter 11, while some concluding remarks are given in Chapter 12.

Chapter 2

Background

This chapter gives further insight into Petrobras' helicopter operations. The information obtained will be used in the subsequent chapters in order to make the decision support models developed well suited Petrobras' operations. Section 2.1 gives an overview of current helicopter operations at Petrobras in terms of its infrastructure and transportation model, while Section 2.2 takes a look at how these operations need to be altered when moving into the pre-salt fields. The cost and risk elements of the transportation system are established in Sections 2.3 and 2.4 respectively.

2.1 Current helicopter operations

In its current operations, Petrobras uses helicopters to transport employees between onshore airport bases and offshore platforms. About 1900 employees are transferred between these installations every day. This results in annual passenger traffic of over half a million passengers, which makes Petrobras' present helicopter activities one of the largest non-military helicopter operations in the world (Menezes et al. (2010)). The size of the transportation process is relatively stable throughout the year. As seen from Figure 2.1 on the following page, the monthly variations in the number of employees transported only vary in the interval from -6 % to 6 %.

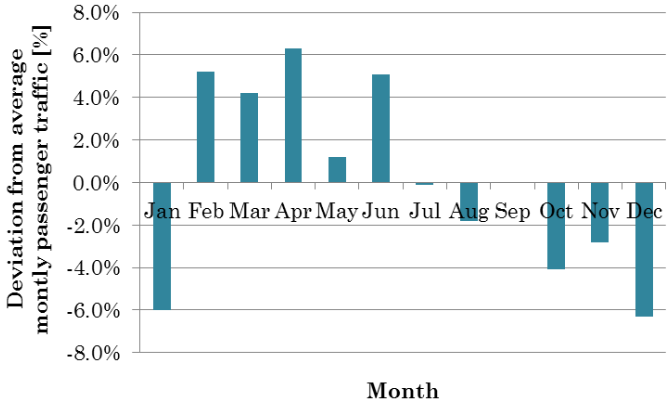

Figure 2.1: Monthly variation in passenger traffic, 2009 (Sena (2011))

There are three types of employees being transported. Firstly, shift workers are transferred between the onshore airport bases and offshore platforms on a regular basis. These are permanent employees working in various functions at the offshore installations, and that live on a platform for a given period of time according to their contractual agreements. Secondly, specialist workers performing maintenance activities, inspections etc. are transferred between the installations on an irregular basis. These are employees being sporadically transported to and between various offshore platforms according to the needs of their skills. Thirdly, managerial workers performing special visits and inspections are transferred between the installations on an irregular basis (Sena (2011)). To which platform employees are transported to or from can therefore be said to be determined by the pickup and delivery demand of the various types of employees at the various platforms.

Seven onshore airport bases are used in order to support these offshore helicopter operations. These bases are located in Navegantes, Itanhaém, Jacarepaguá, Cabo Frio, Macaé, Säo Tomé and Vitória. The largest bases are the ones located in Macaé and Säo Tomé, at which over 70 % of the total passenger traffic is handled. The smallest bases on the other hand are the ones located in Navegantes and Itanhaém, at which only 3 % of the total passenger traffic is handled. In addition to varying in size, the different bases are also managed by various operators. The onshore airport base in Säo Tomé is the only one

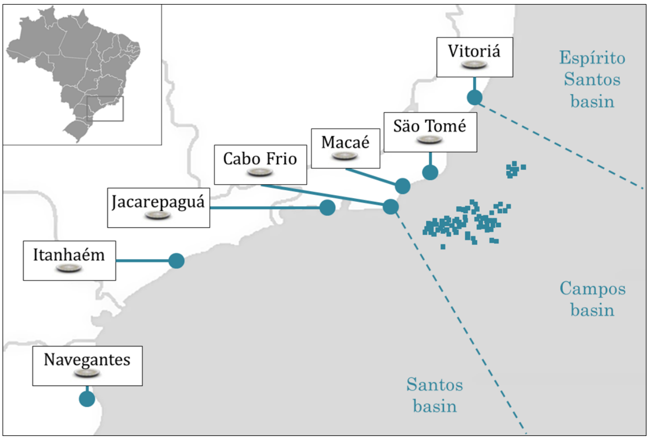

Figure 2.2: Current infrastructure (Menezes et al. (2010), Sena (2011))

operated by Petrobras itself. The bases in Itanhaém and Cabo Frio on the other hand are operated by Departamento Aeroviário do Estado de São Paulo (*DAESP*) and Costa do Sol respectively, while all other bases are operated by INFRAERO (Sena (2011)). Employees are not allowed to choose themselves from which base they would like to be transported to their destination.

The majority of platforms to which employees are transported to are currently located in the Campos Basin. This is an area located off the Brazilian coast south-east of Rio de Janeiro. Over 80 platforms of various types are currently situated in this basin. Both fixed and floating production units are in use, in addition to floating production, storage and offloading units (*FPSO*). Also, various drilling units for the drilling and completion of wells are utilized. Both ships and semi-submersible structures are used for this purpose (Sena (2013)). From which base(s) employees are to be transported to every platform is decided once a year. The geographical location of the various onshore airport bases and offshore platforms is depicted in Figure 2.2.

A fleet of helicopters is stationed at every onshore airport base. The sizes of these various fleets vary in accordance with a base's passenger traffic. In total however, about 50 helicopters are in use. The fleets allocated to the different bases consist of aircraft of various helicopter types. Currently, both medium and heavy twin engine helicopters are in use. The helicopter types also vary in terms of their passenger capacity, as well as other performance characteristics

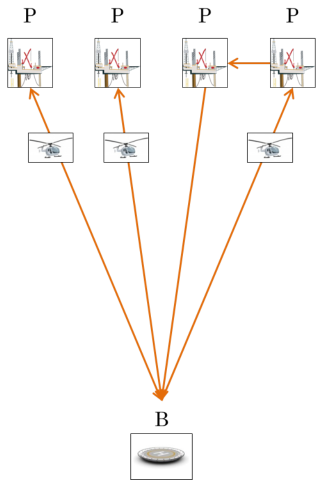

Figure 2.3: Current transportation model

such as their maximum take-off weight (*MTOW*) and maximum flying range. All helicopters in use are supplied by various third party logistics providers (*TPL*) through contractual agreements.

The transportation of employees between bases and platforms is performed directly. This means that no transhipment of passengers occur between the two locations. An example of this transportation process is illustrated in Figure 2.3. Notice that several platforms can be visited in one routing. Performing such a procedure is termed shuttling. Still, the maximum number of platform landings in a route is bounded. At present, normal practice is to have maximum three platform landings per flight (Sena (2013)). The upper limit however is five platform landings per flight (Menezes et al. (2010)).

All flights are subject to the rules of the National Civil Aviation Agency of Brazil (*ANAC*), the Brazilian Department of Airspace Control (*DECEA*) and the Regional Air Commands (*COMAR*). Also, the regulations given for flight in visual flight rules (*VFR*) or instrumental flight rules (*IFR*) must be followed. Offshore helicopter flights transporting employees of Petrobras use IFR for take-off and landing procedures on onshore installations, and VFR for all offshore operations (Sena (2013)). All flights performed in the Campos Basin are also subject to rules particular for the Macaé terminal control area (AIS (2008)).

2.2 Future helicopter operations

As presented in Chapter 1, initiating oil and gas operations in the pre-salt field is followed by significant challenges. Firstly, a considerable increase in production volumes will impact Petrobras' need for offshore transportation of employees. The company plans to reach a production level of 4.2 million barrels per day (BPD) by the year of 2020, which represents a doubling in the company's production volumes over a ten year period of time. The increase in volume is primarily to stem from establishment of production at the various pre-salt fields in the Santos Basin (Fraga (2012)). This significant change in daily output volumes will put great pressure on the company's existing infrastructure for helicopter operations. In particular, the capacity at the various onshore airport bases will be stressed.

Petrobras has therefore decided that its onshore facilities are to be expanded due to the development at the various pre-salt fields. The company is currently studying projects in Itaguaí and Guarujá, but has not yet decided at which locations bases are to be established. Additionally, the onshore airport bases in Jacarepaguá and Cabo Frio are to be used to support operations in the pre-salt fields. The geographical location of the pre-salt fields and the various onshore airport bases that are to or may be used to transport employees to and between installation at these locations is depicted in Figure 2.4 on the following page.

Secondly, the future helicopter operations are greatly affected by the long distances between the Brazilian coast and the various pre-salt fields. Estimates have shown that the characteristics of the new fields will double the cost of transporting employees directly between the onshore bases and the offshore installations by helicopters. In particular, the large distances will significantly increase the transport cost per passenger, as the increase in the required fuel will decrease the maximum passenger capacity (Vilameá et al. (2011)). In addition, if the present transportation model is to be used, the distances helicopters would need to travel would be close to their maximum range. This has great implications not only for the safety level of the operations, but also for the realization of such a solution.

A new model for the transportation of employees to offshore installations in the pre-salt fields is therefore desirable. Petrobras has initiated this work, and has already presented a solution. The key idea in their design is to locate one or several offshore transportation hubs between the bases and platforms.

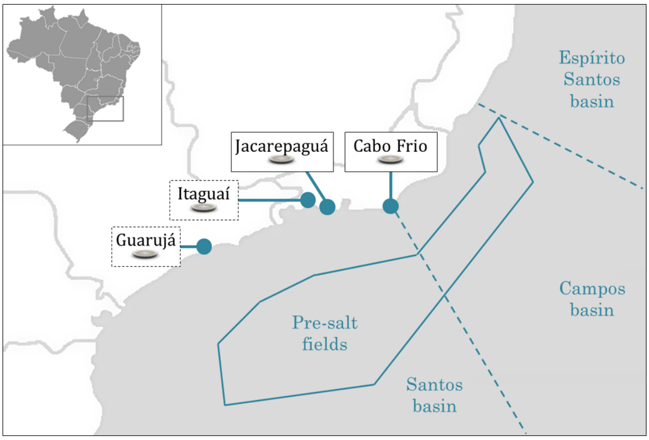

Figure 2.4: Future infrastructure in the pre-salt fields (Fraga (2012), Sena (2013))

Employees will then be transported to the hubs by high speed supply vessels, and transported from the hubs and to the offshore installations by helicopters. Such a design is believed to reduce transportation costs, with one of the reasons being that the helicopters are able to transport passengers at full capacity (Vilameá et al. (2011)). Also, the hubs will be able to serve as maintenance depots for helicopters and accommodation bases for offshore employees.

It is confirmed that this type of offshore transhipment hubs will be installed if doing so represent a cost and/or risk reduction of Petrobras' offshore helicopter operations (Sena (2013)). If this occurs, the installations are likely to be ordered under lease contracts (charter party). However, concerns exist regarding the transportation of employees by high speed supply vessels. This solution may not be realizable because of the difficulty of reaching an appropriate safety level in the offshore disembarking procedure due to the wave height at the open sea. An alternative solution is therefore to use helicopters as the only mode of transportation. An example of this optional transportation model is illustrated in Figure 2.5.

In this suggested future transportation model, both the onshore airport bases and the offshore hubs can be thought of as helicopter bases. Thus, when the number of helicopter bases and their position is to be addressed in this report, both of these installations need to be taken into consideration. A distinction is therefore made between the two from this point forwards.

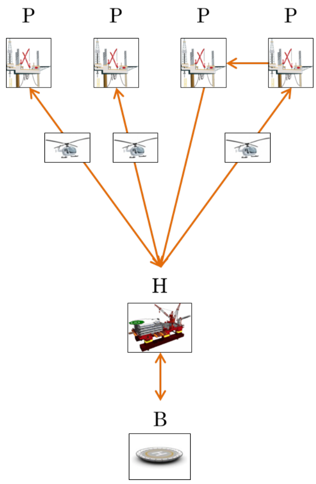

Figure 2.5: Suggested future transportation model in the pre-salt fields

2.3 Establishment of cost elements

In this section, the cost elements of Petrobras' helicopter operations are established. These costs are related to both the facilities and aircraft in use, in addition to the transportation process itself.

Firstly, let us take a look at the cost elements of the facilities in use. For onshore airport bases, various costs apply depending on whether a base is operated by Petrobras or by a third party. If a base is operated by Petrobras, investment costs associated with acquisition or lease of land, construction of passenger terminal, helipads, parking area, control tower etc. must be covered by the company itself at the time of the investment. If a base is operated by a third party on the other hand, such costs are normally amortized as operating costs over the contract's period. Thus, operating costs tend to increase when an onshore base is operated by a third party (Sena (2013)).

Additional variable costs also apply to the use of onshore airport bases. Boarding and landing costs apply to every flight performed, in addition to a tariff for communication use and visual and radio assistance (TAT) during take-off and landing procedures. Also, parking costs apply to every helicopter in use when this is stationed at the base to which it is allocated. These are costs that apply to Petrobras regardless of whether the base is managed by the company itself or by a third party. If the base is managed by a third party however, it should be mentioned that boarding, landing, parking and TAT costs vary between different bases and with the helicopter type in use (INFRAERO (2013)).

If offshore transhipment hubs are to be implemented on the other hand, only variable operating costs will apply to these facilities. This is due to the fact that such hubs are likely to be ordered under charter party (see Section 2.2). Investment costs are therefore probable to be amortized as operating costs over a contract's period in similar manner to the lease of base facilities operated by a third party. The variable cost will however vary with the type and size of the installation(s) in use. Also, the length of the lease contract is likely to have an impact on the price obtained.

Secondly, let us take a look at the cost elements of the aircraft in use. As all helicopters are supplied by various TPL's, only variable operating costs apply to these aircraft. Normally, the hiring party need to pay a daily charter cost for every helicopter rented, an hourly charter cost for every hour these helicopters are in use, and all fuel costs associated with the use of these helicopters. The two charter costs differ depending on both the helicopter type in use, and from which firm the service is hired (Romero et al. (2007)). Also, the length of the contractual agreement has an impact on the price obtained (Molvik (2013)).

Lastly, let us take a look at the cost elements associated with the transportation process itself. In addition to the costs already mentioned, a tariff for communications use and aerial navigation assistance (TAN) apply to every flight performed. This cost vary with different flight information regions and with various control areas (INFRAERO (2013)).

2.4 Establishment of accident risk elements

In this section, the risk elements of Petrobras' helicopter operations are established. When doing so, one must first specify what is meant by risk in this context, as this is a term that varies with different application areas. In finance for example, risk can denote market risk or credit risk to mention some, while the term has completely different interpretations in other industries. In this report, the risk of Petrobras' helicopter transportation system from an aviation safety perspective will be analysed. In particular, the risk of an accident to helicopter operations in the Santos Basin is investigated.

A standard definition of risk is that it is the combination of the probability of an event and its consequences (ISO (2002)). Still, when studying safety, risk can also be defined by the use of frequencies instead of probabilities. In the article published by Hokstad et al. (2001), the risk of offshore helicopter transportation is defined as the product of accident frequency (f) and the average consequence (C) of an accident. Following this, risk R can be quantified by using the equation $R = f \cdot C$. This approach of using accident frequencies instead of probabilities is a great simplification of accident causation. In reality, the likelihood of an accident is influenced by multiple causes and by human factors in particular. However, further analysis of this subject is beyond the scope of this report. The definition of risk given by Hokstad et al. (2001) is therefore used in the following.

Proper specifications of what is meant by an accident, its accident frequency f and its accident consequence C are therefore required. Regarding the first, it has been chosen in this report to use the definition of accident given by the International Civil Aviation Organization (*ICAO*). This is as follows: *"An accident is an occurrence associated with the operation of an aircraft . . . in which a person is fatally or seriously injured . . . or the aircraft sustains damage or structural failure . . . or the aircraft is missing or is completely inaccessible"* (ICAO (1994)). Thus, let f represent the frequency of all events that can be classified as accidents by the use of this definition. In quantitative terms, let this be equal to the product of the expected number of such accidents per 100k flight hours and the number of flight hours.

Let us then define what is meant by the accident consequence C. In Hokstad et al. (2001) and Qian et al. (2011) among others, accident consequence C in helicopter transportation is defined as the mean number of fatalities in one accident, which is equal to the product of the number of people on board and the probability of a fatal outcome for every passenger given that an accident has occurred. This is a common definition within the field of aviation safety (Herrera et al. (2010). Therefore, let this also be the definition of accident consequence C in this report.

Following the discussion in the previous paragraphs, the risk equation $R = f \cdot C$ can now be extended in the following manner:

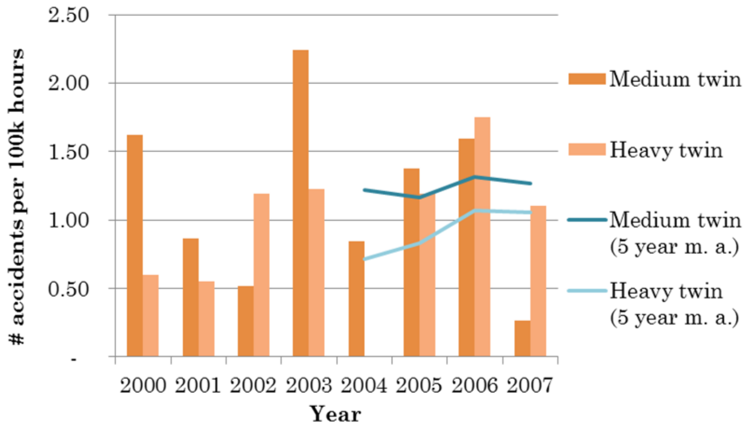

Figure 2.6: Offshore aviation accident rates for medium and heavy twin engine helicopters, 2000-2007 (OGP (2000, 2001, 2002, 2003, 2004, 2005, 2006, 2007))

$$\begin{gathered} R = f \cdot C \\ \mathrm{E}\,[\frac{\#\text{ of accidents}}{\text{100k flight hours}}] \cdot \#\text{ flight hours} \cdot \\ \mathrm{E}\,[\frac{\#\text{ fatalities per person on board}}{\text{accident}}] \cdot \#\text{ people on board} \end{gathered} \tag{2.1}$$

Having specified what is meant by risk in this report, the risk elements in Petrobras' helicopter operations can be established. When doing so, two important characteristics of helicopter accident frequencies and consequences are taken into consideration. Firstly, helicopter accident rates vary with different helicopter types. This is visualized in Figure 2.6, in which historical accident rates for medium and heavy twin engine helicopters are depicted in the columns and 5 year moving averages of the same values are shown in the lines. One can see from this figure that medium twin engine helicopters historically have had higher accident rates than heavy twin engine helicopters. This means that the type of helicopters in use influences the accident frequency and thus the safety of a transportation system using such aircraft as transportation mode. The selection of which helicopters to use therefore has an impact on the risk of Petrobras' helicopter operations.

Secondly, helicopter accident rates and consequences vary with different flight

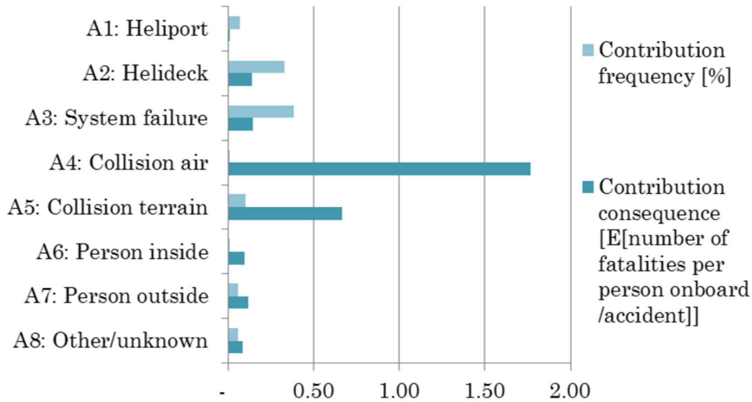

Figure 2.7: Accident classes' contribution to accident frequency and consequence (Herrera et al. (2010))

phases. This means that the likelihood and consequences of a helicopter accident are not constant during a flight. This fact was pointed out in the report developed by Herrera et al. (2010). In this study, statistics and expert judgements were used in order to quantify eight different accident classes' contribution to helicopter accident frequency and consequence. The result obtained is visualized in Figure 2.7. In this figure, one can see that the highest contribution to accident frequency is system failure (A3), and that offshore procedures (A2) have a higher contribution to accident frequency than onshore procedures (A1). It can also be noticed that if an accident has occurred, the expected number of fatalities is highest if this accident is a collision in air (A4). In this accident class, fatalities in two helicopters are taken into consideration. All eight accident classes are defined in Appendix A.

By categorizing these eight accident classes in terms of the flight stage to which they are related, one can obtain a relationship between flight phases, and accident frequency and consequence. When doing so, three different flight phases can be identified: take-off and landing at heliport (onshore airport bases), take-off and landing at helideck (offshore transhipment hubs and offshore platforms), and cruise procedures. Accident categories A1, A2 and A7 are related to take-off and landing at heliport and helideck, while all other accident categories are related to cruise procedures. The result of this process is that cruise procedures have an accident frequency contribution of 55 % , and an expected number of 0.55 fatalities per person on board if an accident were to happen. Take-off and landing procedures at heliport and helideck on the other hand have accident frequency contributions of 9 % and 36 %, and expected number of 0.04 and 0.13 fatalities per person on board if an accident were to happen, respectively. This indicates that the nature of risk for take-off and landing, and cruise procedures, are quite different. Thus, the way in which Petrobras' helicopter routing process is performed influences the risk of the transportation system.

Chapter 3

Literature

This chapter provides an overview of OR literature relevant to the problem addressed in this report.

To begin with, it can be noticed that the problem considered is a combination of two separate parts: finding the optimal number of bases and their locations, and finding the optimal size and mix of the helicopter fleet. The first part can from an optimization point of view be seen as a facility location problem. According to Lundgren et al. (2010), this problem class seeks to choose the optimal set of facilities to support a set of customers. The objective is to minimize the fixed cost of opening the facilities, and the cost related to transporting a given number of units between the facilities and the customers. Routing aspects however, are not considered.

Maranzana (1964) points out that *"the location of factories, warehouses and supply points in general . . . is often influenced by transport costs"*. Also, as stated by Lin and Lei (2009), *"incorporating the operational routing considerations into strategic location models can result in significant savings"*. Thus, in order to obtain good results when finding the optimal number of bases and their position, it would be wise to look at literature that combines facility location- and vehicle routing problems. The location-routing problem is such a problem class.

The second part of the problem can be seen as a fleet size and mix vehicle routing problem. This is a special class of optimization problems that simultaneously deals with determining the composition and routing of a heterogeneous fleet of vehicles. The problem is solved while fulfilling pre-specified delivery demands of a set of costumers (Liu and Shen (1999)).

In the following, articles addressing both of these two problem classes are therefore presented. First however, an overview of literature related to offshore helicopter transportation in the petroleum industry is given in Section 3.1. Literature related to location-routing and fleet size and mix problems is thereafter presented in Sections 3.2 and 3.3 respectively.

3.1 Offshore helicopter transportation in the petroleum industry

The majority of articles studying offshore helicopter operations examine safety aspects of the transportation process. Some articles of this kind, such as the ones by Gomes et al. (2009) and Nascimento et al. (2012a,b), have investigated factors that affect the likelihood of an accident. Other authors, such as Taber and McCabe (2006), have studied various elements that affect the consequences of an accident. Still, despite its importance, no further look will be taken at the safety aspects of offshore helicopter operations in this section as relevant accident risk elements of Petrobras transportation system were established in Chapter 2. Only articles concerning the helicopter transportation itself are therefore examined in the following. The problems studied in relevant articles are first presented, while a comparison is then made between these and the problem considered in this report.

Sierksma and Tijssen (1998) published one of the earlier works in the field of offshore helicopter transportation. In their article, the authors developed a model for creating transportation schedules for crew exchanges on platforms on the Dutch continental shelf of the North Sea. The same number of employees was to be picked up and delivered at each platform. The transportation process can therefore be defined as a *pickup and delivery problem* (Parragh et al. (2008)). The authors solved the problem by using a column-generation technique and a cluster-and-route heuristic.

Romero et al. (2007) and Qian et al. (2011) were also concerned with the pickup and delivery problem of employees. In their articles, the number and location of onshore bases and offshore platforms, as well as the number of homogeneous helicopters, were given. In the problem studied by Qian et al. (2011), all flights also had to start and end in one single onshore airport base. The objective of Romero et al. (2007) was to determine how the helicopter operations could be performed in the most cost efficient manner. Qian et al. (2011) on the other hand examined the safety of the transportation system in terms of its expected number of fatalities. Romero et al. (2007) solved the problem addressed by the use of genetic algorithms and heuristic optimization, while Qian et al. (2011) used commercial optimization software to obtain solutions for the arc flow-model developed.

In the model developed by Qian et al. (2011), every platform needed to be visited once. In order to relax this requirement, Qian et al. (2012) extended the model so that every platform could be visited either once or twice. After doing so, the problem was solved by the use of a tabu search algorithm. Genetic algorithms were later applied to a similar problem by Qian (2012).

In parallel with the works already presented, articles have been written about helicopter transportation at Petrobras in particular. Galvão and Guimarães (1990) published the first work of this kind. Their article was concerned with the development and implementation of a fully computerized system to control the firm's helicopter operations in the 1980s. The article did not present the model developed, but the problems that arose in the implementation process.

Moreno et al. (2006) and Menezes et al. (2010) on the other hand studied the construction of helicopter routes at Petrobras from a cost minimization perspective. The various routes were to be designed so that the pickup and delivery demands of employees at the various platforms were satisfied. Both articles formulated the problem considered as a mixed integer programming (*MIP*) model, which was solved by using a column generation based heuristic.

All articles presented in this section examined problems with several similarities to the one considered in this report. In general, all articles were concerned with offshore helicopter transportation processes in which employees are to be picked up and delivered at various offshore platforms. Still, differences are also to be found. Firstly, no article considered an infrastructure including a transhipment hub similar to the one proposed by Petrobras' for the pre-salt fields in the Santos Basin area. Secondly, authors such as Sierksma and Tijssen (1998), Qian et al. (2011) and Qian et al. (2012) considered a *single depot system*. This means that the helicopter operations considered only were supported by one airport base. On the contrary, multiple onshore airport bases are used to support Petrobras' helicopter operations.

Thirdly, authors such as Sierksma and Tijssen (1998), Romero et al. (2007), Qian et al. (2011) and Qian et al. (2012) considered a fixed and homogeneous helicopter fleet. This also greatly differs from the problem addressed in this report, in which the size and mix of the helicopter fleet is to be determined. Fourthly, Sierksma and Tijssen (1998) consider a scenario in which an equal number of employees are picked up and delivered at every platform. This is not a necessity for Petrobras' helicopter operations. Lastly, authors such as Romero et al. (2007) specifies between which base and platform every employee is to be brought. This is not the case for Petrobras' helicopter operations, in which employees are not able to decide themselves to and from which onshore airport base they would like to travel.

3.2 The location-routing problem

Location-routing is a relatively new branch within facility location analysis. It is not a well-defined problem such as the travelling salesman problem (*TSP*), or the vehicle routing problem (*VRP*). Instead, it can be thought of as an approach to modelling and solving location problems (Nagy and Salhi (2006)). Following Bruns (1998), it can be defined as *"location planning with tour planning aspects taken into account"*. From a mathematical point of view, the location-routing problem can be modelled as a combinatorial optimization problem, which combines the facility location problem and the VRP.

In the following, articles concerning the location-routing problem are reviewed. First, survey articles will be used in order to gain an overview of the topic. Thereafter, articles in which the problem studied has similar characteristics to the one studied in this report, are presented.

In recent literature, four relevant survey articles have been written about the location-routing problem. The first article of this kind was the one by Madsen (1983). His article was concerned with methods that can be used for solving the problem. Min et al. (1998) and Nagy and Salhi (2006) on the other hand later examined existing location-routing literature both in terms of the various problem types and solution methods. Liu et al. (2008) presented a study of the location-arc routing problem.

Both articles by Min et al. (1998) and Nagy and Salhi (2006) presented a classification scheme for location-routing problems. In this report, a further

Problem structure	Hierarchical	**Non-hierarchical/Other**
Type of input data	Stochastic	Deterministic
Planning period	Single-period	Multi-period
Objective	**Minimise cost**	**Other**
Number of depots	Single depot	**Multiple depots**
Vehicle fleet	Homogeneous	**Heterogeneous**
Route structure	Vehicle routing	**Other**

Table 3.1: Classification scheme for location-routing problems

look will be taken at the schema suggested by Nagy and Salhi (2006). By using this model and the information given in Chapter 2, a classification can be made of the problem addressed in this report. The result of this process is given in Table 3.1, in which the characteristics of the problem at hand are marked in bold. It should be noted that only categories concerning the problem structure are included. Also, no classification is made for "Type of input data" and "Planning period", as these are categories concerned with the mathematical formulation of the problem. This is first defined in Chapter 4.

In Table 3.1, a hierarchical structure means that the problem at hand consists of a set of facilities that each serves a set of customers via vehicle routes, and that no vehicle travels between two facilities. A non-hierarchical problem is thus any type of problem that does not have this type of structure. One can see from the table that the problem addressed in this report is defined as the latter. This is due to the fact that the transportation model proposed for helicopter operations in the pre-salt fields include one or several transhipment hubs. This makes Petrobras' potential future helicopter operations a transportation system with two routing levels. In other parts of literature, this type of structure is also termed a *two-echelon problem* (Boccia et al. (2010)).

Further, as the infrastructure supporting Petrobras' helicopter operations consists of several onshore airport bases, the problem addressed in this report is defined as having multiple depots. Also, in accordance with information given in previous chapters, the objective of the problem is classified to being both cost minimization and other, and the fleet is defined as heterogeneous. The problem is defined to not have a vehicle routing structure as both pickup and delivery of employees are performed at the various platforms.

Let us then a look at articles studying a two-echelon location-routing problem. Two examples of such are the ones by Nguyen et al. (2012a,b). In their articles, the optimal number and location of a set of hubs was to be determined. Also, the size of the vehicle fleet in use was undetermined. The problem addressed in these two articles therefore has several similar characteristics to the one examined in this report. Both articles formulated the problem at hand as a MIP model, and made use of heuristics in the solution procedure.

Further, Jin et al. (2009, 2010a,b) studied a two-echelon location-routing problem with multiple depots. Thus, in addition to deciding which hubs to open, these articles also examined which depots to open. Bookbinder and Reece (1988) and Ambrosino and Grazia Scutella (2005) studied a two-echelon location-routing problem heterogeneous vehicle fleet. The first of these two articles also incorporated the fleet size and mix problem into the location-routing problem. The problem addressed by Bookbinder and Reece (1988) therefore inhabits several of the characteristics of the problem examined in this report. Various solution methods were used to solve the problems examined in the articles presented in this paragraph. Jin et al. (2009) made use of a Branch and Bound (*B&B*) algorithm based on Lagrangean relaxation, while Jin et al. (2010a) used genetic algorithms to mention some. Bookbinder and Reece (1988) made use of Benders decomposition in their solution procedure.

Lastly, let us take a look at articles studying a location-routing problem with multiple depots. Some articles of this kind have already been mentioned in previous paragraphs. Other examples are the ones by Perl and Daskin (1985), Tavakkoli-Moghaddam et al. (2010) and Zarandi et al. (2011). Also, Wu et al. (2002) and Dawei et al. (2009) studied a location-routing problem with multiple depots and heterogeneous fleet. These authors therefore addressed a problem inhabiting several of the characteristics of one examined in this report in their work. The fleet size and mix problems addressed in the articles by Wu et al. (2002) and Dawei et al. (2009) were solved in a manner similar to the method used by Bookbinder and Reece (1988). A simulated annealing heuristic was also used in the solution procedure of both articles.

Additionally, Dawei et al. (2009), Wasner and Zäpfel (2004), Karaoglan and Altiparmak (2010), Karaoglan et al. (2011) and Karaoglan et al. (2012) incorporated the pickup and delivery problem into the location-routing problem with multiple depots. However, contrary to one by Dawei et al. (2009), the last four articles considered a homogeneous vehicle fleet. When solving the problem, Karaoglan and Altiparmak (2010) made use of genetic algorithms and simulated annealing. Karaoglan et al. (2011) on the other hand made use of a branch and cut algorithm.

3.3 The fleet size and mix vehicle routing problem

The fleet size and mix vehicle routing problem (*FSMVRP*) is concerned with answering the following question: How many vehicles of which size are needed in order to accommodate demand at minimal cost? (Golden et al. (1984)) Such a decision is greatly interconnected with routing considerations. The FSMVRP is therefore closely related to the VRP. In the latter, the routing of a predetermined homogeneous vehicle fleet is to be determined. However, as pointed out by Salhi and Rand (1993), *"in real life, the appropriate fleet need not be homogeneous and a good vehicle fleet mix is likely to yield better results."* When relaxing the assumption of a homogeneous vehicle fleet of a predetermined size in the VRP, one deals with a FSMVRP. In literature, the problem is also named the vehicle fleet mix problem, as well as the fleet size and the fleet size and composition VRP (Taillard (1999)). It is also thought of as one class of problems relating to fleet composition and routing problems (Hoff et al. (2010)).

In the following, articles concerning the FSMVRP are reviewed. First, survey articles will be used in order to gain an overview of the topic. Thereafter, articles in which the problem examined is somewhat similar to the one studied in this report, are presented.

All survey articles written about the FSMVRP consider fleet composition and routing problems as a whole. The first review of this kind was one by Etezadi and Beasley (1983). Their article gives an overview of the existing literature in this field in the early 1980s. Of more recent works, the articles by Salhi and Rand (1993) and Hoff et al. (2010) can be mentioned. As the latter is quite new, it gives a thorough overview of existing literature on fleet composition problems. Problem solving techniques for the fleet size and mix problem were presented and compared by Gheysens et al. (1984).

Hoff et al. (2010) one the other hand also proposed a classification scheme for fleet composition problems. In this report, a further look will be taken at this schema. By using this model and the information given in Chapter 2, a classification can be made of the problem addressed in this report. The result of this process is given in Table 3.2 on the following page, in which the characteristics of the problem at hand are marked in bold. It should be noted that the fleet size is fixed or bounded by a maximum number in the heterogeneous fixed fleet vehicle routing problem. Also, the fleet size and mix

Name	*Abbreviation*
Standard fleet size and mix vehicle routing problem	FSMVRP
Heterogeneous fixed fleet vehicle routing problem	HFFVRP
Fleet size and mix vehicle routing problem with time windows	FSMVRPWT
Fleet size and mix vehicle routing problem with multiple depots	**FSMVRPMD**

Table 3.2: Classification scheme for fleet composition and routing problems

vehicle routing problem with multiple depots is similar to a pickup and delivery problem without a central depot.

From Table 3.2, one can see that the problem addressed in this report is defined as a fleet size and mix vehicle routing problem with multiple depots (*FSMVRPMD*). This is due to the fact that the aircraft serving Petrobras' helicopter operations are stationed at multiple onshore airport bases. The problem addressed in this report therefore greatly resembles a pickup and delivery problem without a central depot. Hoff et al. (2010) identified four articles addressing this problem. These articles are presented in the following.

In the works by Salhi and Fraser (1996), Salhi and Sari (1997) and Dondo and Cerdá (2007), the problems addressed were quite similar. The infrastructure consisted of sets of depots and customers, and the composition of a heterogeneous fleet, as well as the routes made by these vehicles in order to support customer demands, were to be determined. All objectives were to minimise total cost. Additionally, Salhi and Fraser (1996) also addressed the problem of deciding the number and location of depots. Dondo and Cerdá (2007) on the other hand, also introduced time windows. Still, the general problem addressed in all three papers had several similarities to the one studied in this report. Salhi and Fraser (1996), Salhi and Sari (1997) and Dondo and Cerdá (2007) all solved the problem at hand by using constructive heuristics.

Irnich (2000) on the other hand studied a somewhat different problem. In his article, requests had to be picked up at or delivered to one central location with the function of a consolidation point. Such a transportation sequence is similar to the second-echelon routing in Petrobras' helicopter operations, given that one or several of transhipment hubs are installed and helicopters are allowed to be stationed at the platforms. The problem was solved as a set partitioning problem, in which vehicle routes were generating in a heuristic manner.

Chapter 4

Description of the problem

This chapter presents the problem addressed in this report by words. The description is based on the information given in Chapter 2.

Some general premises are presented to begin with. Firstly, in accordance with Petrobras' current helicopter operations, the report only considers the transportation of employees between onshore airport bases and offshore platforms. The onshore transportation of employees to and from the bases is therefore not examined. Secondly, the transportation model for the pre-salt field suggested by Petrobras is used as a basis for how the transportation of employees between bases and platforms is performed. This means that the use of one or several transhipment hubs is taken into consideration. However, in order to appropriately limit the scope of the problem, this report will examine a scenario where helicopters are used as the only mode of transportation. Thirdly, no distinction is made between shift workers and other types of employees. The report therefore addresses the transportation of one type of "product".

Following these assumptions, the structure of the problem addressed in this report is established by three sets of facilities: a set of onshore airport bases, a set of offshore transhipment hubs, and a set of offshore platforms. The numbers of bases and platforms, as well as their locations, are given. The number of hubs and their location on the other hand, is undetermined. The daily operating cost of every installed hub however, is given. This cost encompasses both the hub's fixed daily chartered-in cost, as well as an estimate of its daily variable production cost. The helicopter parking capacity of every base, as well as the take-off and landing capacities for every base, hub and platform, are also given.

A fleet of helicopters is stationed at every onshore airport base. These fleets are heterogeneous, and consist of both medium and heavy sized twin-engine helicopters. The various helicopters differ in both passenger capacity and maximum range. The number and types of helicopters at each airport base is undetermined. However, the fixed investment- and variable operating costs of helicopters of various types are known. A helicopter's fixed investment cost corresponds to its monthly chartered-in cost. Its variable operating cost on the other hand encompasses its hourly chartered-in cost, fuel cost, TAN, TAT, as well as its barding, landing and parking costs at the onshore airport bases. In this report, these rates are converted to average costs per day and per flight kilometre respectively.

The helicopter fleets stationed at the various onshore airport bases are used to transport employees to and from the offshore platforms. There are no restrictions on from/to which onshore airport base an employee is transported to/from. This decision is therefore made based on what is optimal for the transportation process as a whole. From/to which platform employees are transported to/from on the other hand, is determined by a given pickup and delivery demand of employees at each platform. In this report, these demands are given as daily rates. All daily pickup and delivery demand must be satisfied.

A helicopter can transport employees between an onshore airport base and offshore platforms in two different manners. Firstly, a helicopter can make use of a direct routing policy. This means that the helicopter fly directly between an onshore airport base and an offshore platform. If optimal, the helicopter can thereafter visit additional platforms. No visit can however be made to any offshore transhipment hub during the flight. Secondly, a helicopter can make use of a hub connected routing policy. This means that the helicopter fly directly between an onshore airport base and an offshore transhipment hub (first echelon routing), for thereafter to visit one or several offshore platforms (second echelon routing). Only one hub can be visited during the flight, but several second echelon trips can be performed in the second echelon routing. The two types of helicopter routing policies are illustrated in Figure 4.1. All flights start and finish in the onshore airport base at which the helicopter performing the flight is stationed.

Several other constraints also apply to every flight. Firstly, assuming that every flight is performed by one pilot only, every flight is also subject to pilot flight time

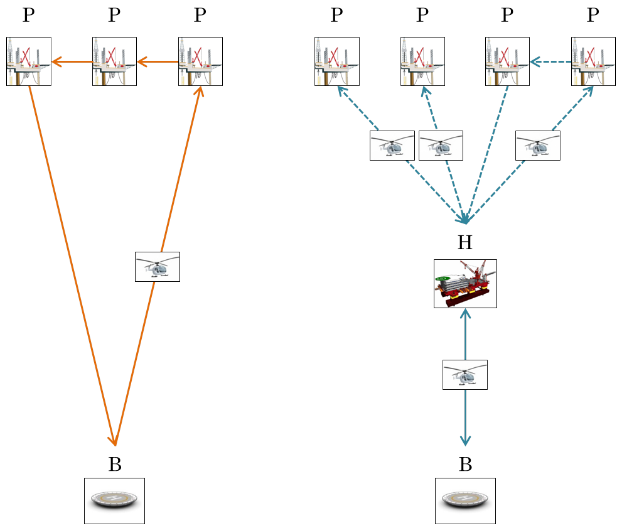

Figure 4.1: Helicopter routing policies

restrictions. Secondly, the maximum number of platform landings during every direct routing policy flight and second echelon trip, is limited to five. Thirdly, the range of every helicopter in use must be obeyed. Refuelling can be performed at every onshore base and offshore transhipment hub, but cannot be done at any offshore platforms. This means that if the direct routing policy is used, a helicopter's total flying distance during the flight cannot exceed its helicopter range. If the hub connected routing policy is used however, a helicopter's total flying distance during every first echelon trip between a base and a hub and every second echelon trip, cannot exceed its helicopter range. Fourthly, the maximum number of passengers during the course of a flight can never exceed the capacity of the helicopter in use.

It also exist a limitation to what type of flights are allowed to visit each platform. In general, every platform can be visited by multiple helicopters during several flights. However, if the direct routing policy is used, all flights visiting a platform must start from the same airport base. If the hub connecting routing policy is used, all flights visiting a platform must go via the same transhipment hub.

In addition, there are restrictions that apply to every helicopter in use. Firstly, a helicopter can perform maximum five flights per day in which various routing policies can be used, and maximum five second echelon trips per flight if a hub connected routing policy is used. Secondly, a helicopter's total usage time per day cannot exceed the maximum daily operating time. As night time offshore helicopter flying is banned in Brazil, the number of operating hours is limited to the number of hours between dusk and dawn (DECEA (2007)). A helicopter's total daily usage time encompass inspection and boarding time before every flight on the onshore airport base to which it is allocated, landing time on every offshore installation visited, as well as the helicopter's flying time between various installations. The latter is dependent of the daily flying distance and the cruise speed of the helicopter in use. Also, if more extensive maintenance inspections are required, the time spent in such procedures must also be included in a helicopter's usage time (Nascimento (2013)). However, due to their inconsistent nature and for simplicity reasons, such maintenance inspections are not further addressed in this report.

The objective is to determine at whether it is optimal for the system to implement one or several offshore transhipment hubs, where these hubs should be located, as well as finding the optimal size and mix of the various helicopter fleets from both a cost and risk perspective. The total cost of the system includes the investment cost for the hubs and helicopters use, the operational cost of every helicopter flight performed and the parking cost of having a helicopter stationed at an onshore base. The total risk of the system corresponds to the risk assessment of the helicopter routing performed. This risk encompasses both risks during cruise, and during take-off and landing procedures, which both vary for different helicopter types.

Chapter 5

Mathematical formulations

In this chapter, three mathematical formulations of the problem described in Chapter 4 are presented. An arc flow formulation is presented in Section 5.1, while two different path flow formulations are presented in Sections 5.2 and 5.3 respectively. The path flow formulation presented in Section 5.3 is an aggregated version of the first, which brings about a formulation with an considerably reduced number of variables and constraints.

Some important choices were made in the development process of the three mathematical formulations. First, all three models are formulated as mixed integer linear programming models (*MILP*). This decision was made for the purpose of being able to make use of MIP solution methods and available commercial optimization software. An implication of this choice is that the emplacements of offshore transhipment hubs have to be formulated in a non-continuous manner. This means that when determining the optimal location(s) of the offshore transhipment hub(s) installed, all three models have to choose the optimal solution from a predefined set of discrete locations. Due to this, the results that one obtains when solving the models are to a great extent dependent on the selection of predefined potential locations. The ramification of this characteristic can however be reduced by solving the models with an increasing number of predefined locations.

Second, all three models are formulated as deterministic programming models. This decision was made as it is realistic to assume that all data needed are available at the time of the decision making processes. This eliminates the need for stochastic programming formulations, as such models are only relevant when one would like to solve optimization problems incorporating uncertainty (Higle (2005)). By formulating the problem as deterministic models, one also avoids further complexifying an already intricate problem.

Third, all three models are formulated without the use of time periods. This decision was made due to the nature of the problem addressed in this report. Even though the ideal investment strategy for offshore transhipment hubs and helicopter fleet over time is to be determined, the problem can be decomposed into separate and subsequent decision making processes as all acquisitions are or are likely to be made under lease contracts. Because of this, every investment decision will be reversible after the expiration of its contract. Entering into a contract in one year may therefore not affect the decision making processes in future years depending on the length of the agreement. Also, the location of an offshore transhipment hub under lease may be altered before the expiration of the contract as semi-submersible vessels are movable. These characteristics eliminate the need for linkage between different time periods, and the models can be solved separately for various points in time. Because of this, and due to the fact that the size of Petrobras' helicopter operations is relatively stable throughout the year, all three models are formulated for helicopter operations at one particular day. This means that all parameter values must be given in daily units.

With regard to notation, decision variables and indices are denoted by lower-case letters in all three formulations. Sets, parameters and superscripts on the other hand, are denoted by capital letters.

5.1 Arc Flow Formulation (AFF)

In this section, an arc flow formulation of the problem addressed in this report is presented. This means that decision variables in the model correspond to the flow of helicopters or employees between two consecutive installations. The formulation is a continuation of the work performed by Norddal (2012). The greatest alterations made are that the formulation presented in this section allows a direct flow of helicopters between onshore airport bases and offshore platforms, as well as split pickup and deliveries of employees at the various installations. It also enable the practice of performing more than one flight per day for every helicopter in use, and allow the user to address the problem at hand from both an economic and risk perspective. Having performed these modifications makes the formulation presented in this section in much greater conformity with reality than the formulation developed by Norddal (2012), which significantly increases its applicability.

The various sets, indices, parameters and decision variables used in the arc flow formulation will be introduced throughout the section. Still, the way in which the formulation's decision variables relate to the addressed problem can already be seen in Figure 5.1 on page 31. A complete model is to be found in Appendix B.

First, constraints for the transportation network are presented in Section 5.1.1. Then, constraints for the direct and hub conneted routing policies are introduced in Sections 5.1.2 and 5.1.3. Pickup and delivery constraints are presented in Section 5.1.4, while the objective function is presented in Section 5.1.5. Lastly, some optional symmetry breaking constraints are introduced in Section 5.1.6.

5.1.1 Establishment of the transportation network

Let $\mathbf{B}$ be the set of onshore airport bases, indexed by b, i or j, $\mathbf{H}$ be the set of potential locations for offshore transhipment hub(s), indexed by h, i or j, and $\mathbf{P}$ be the set of offshore platforms, indexed by p, i or j. The elements in these three sets represent the physical points in the transportation system, and can be termed the nodes in the model. Notice how indices i and j can represent elements in all three sets $\mathbf{B}$, $\mathbf{H}$ and $\mathbf{P}$.

Further, let $\mathbf{K}$ be the set of available helicopter types, indexed by k, and $\mathbf{N_k}$ be the set of available helicopters of type k, indexed by n. The combination of elements in these two sets represents all helicopters that may be used in the transportation of employees between onshore airport bases and offshore platforms. Let $\mathbf{F}$ be the set of possible daily flights for every available helicopter, indexed by f, and $\mathbf{S}$ be the set of possible second echelon trips during every flight for every available helicopter, indexed by s.

A helicopter can perform two types of flights (see Chapter 4). Let $\mathbf{A^1}$ be the set of all feasible paths a helicopter may take between two nodes if using a direct routing policy. Similarly, let $\mathbf{A^2}$ and $\mathbf{A^3}$ be the set of all feasible paths a helicopter can take between two nodes if using a hub connected routing policy. The elements in $\mathbf{A^2}$ represent the feasible arcs in the first echelon, while the elements in $\mathbf{A^3}$ represent the feasible arcs in the second echelon of this routing policy. Let $\mathbf{A} = \mathbf{A^1} \cup \mathbf{A^2} \cup \mathbf{A^3}$ be the set of all feasible paths a helicopter may take between two nodes using both routing policies.

Following the establishment of these sets, the arc flow variables can be defined. The mathematical formulation in this section makes use of three binary variables for this purpose. Binary arc flow variable $x_{i,j,k,n,f}$, $(i,j) \in \mathbf{A^1}$, $k \in \mathbf{K}$, $n \in \mathbf{N_k}$, $f \in \mathbf{F}$, is equal to 1 if helicopter number n of type k uses a direct routing policy and travels between nodes i and j on flight number f, and otherwise 0. Binary arc flow variable $y_{i,j,k,n,f}$ on the other hand, $(i,j) \in \mathbf{A^2}$, $k \in \mathbf{K}$, $n \in \mathbf{N_k}$, $f \in \mathbf{F}$, is equal to 1 if helicopter number n of type k uses a hub connected routing policy and travels directly between nodes i and j during the first echelon trip on flight number f, and otherwise 0. Binary arc flow variable $z_{i,j,k,n,f,s}$, $(i,j) \in \mathbf{A^1}$, $k \in \mathbf{K}$, $n \in \mathbf{N_k}$, $f \in \mathbf{F}$, $s \in \mathbf{S}$, is equal to 1 if helicopter number n of type k uses a hub connected routing policy and travels directly between nodes i and j during second echelon trip number s on flight number f, and otherwise 0. All arc flow variables are asymmetric.

Some other variables are also needed in this section. Let binary variable $a^H_{b,k,n}$, $b \in \mathbf{B}$, $k \in \mathbf{K}$, $n \in \mathbf{N_k}$, be equal to 1 if helicopter number n of type k is assigned to onshore airport base b, and otherwise 0. Further, let binary variable $a^P_{i,p}$, $i \in \mathbf{B} \cup \mathbf{H}$, $p \in \mathbf{P}$, be equal to 1 if offshore platform p is assigned to onshore airport base or offshore transhipment hub i, and otherwise 0. Let binary variable o^H_h, $h \in \mathbf{H}$, be equal to 1 if an offshore transhipment hub is installed at location h, and otherwise 0. Lastly, let variable $t^P_{b,k,n}$, $b \in \mathbf{B}$, $k \in \mathbf{K}$, $n \in \mathbf{N_k}$, denote the time per day from which the parking cost on onshore airport base b should be derived for helicopter number n of type k.

Let us then introduce the required parameters. Q^H_b, $b \in \mathbf{B}$, gives the maximum number of helicopters that may be assigned to a base b. Q^{TB}_b, $b \in \mathbf{B}$, and Q^{TP}_p, $p \in \mathbf{P}$, on the other hand represent take-off and landing capacities of onshore airport bases b and offshore platforms p respectively. The take-off and landing capacity at every installed offshore transhipment hub is given by Q^{TH}. For all take-off and landing parameters, one unit capacity comprise one take-off and one landing of one particular helicopter. The passenger capacity of helicopters of type k is given by Q^P_k, $k \in \mathbf{K}$,.

Further, $T^{FA}_{i,j,k}$, $(i,j) \in \mathbf{A}$, $k \in \mathbf{K}$, denotes the total flying time on arc i,j for helicopters of type k. This parameter includes boarding time on node i if this is an onshore airport base and landing time on node j. The maximum allowed operating time per day for all available helicopters is given by T^{OD}, while the

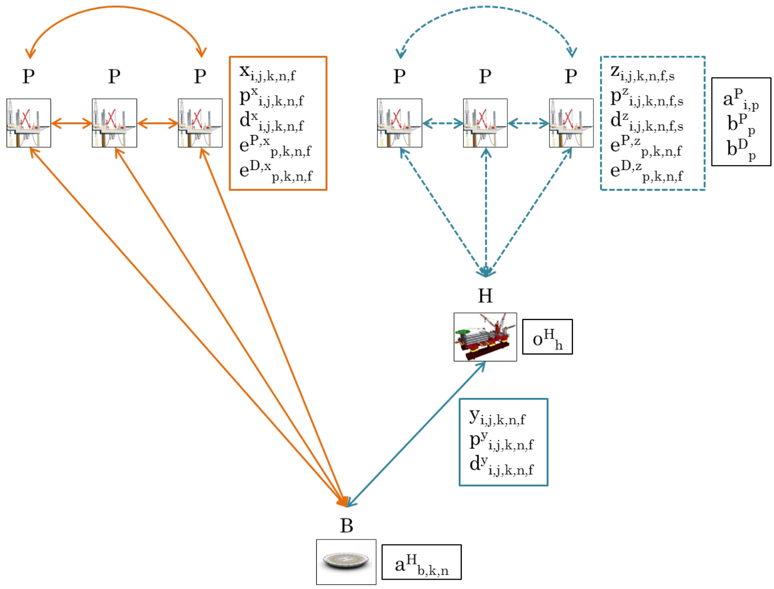

Figure 5.1: Decision variables, Arc Flow Formulation

maximum allowed operating time per flight for all available helicopters is given by T^{OF}. T^P gives the time per day from which a helicopter's parking cost on an onshore airport base is to be derived.

The general constraints of the transportation network can now be written as:

$$\sum_{b \in \mathbf{B}} a^H_{b,k,n} \leq 1, k \in \mathbf{K}, n \in \mathbf{N_k} \tag{5.1}$$

$$\sum_{k \in \mathbf{K}} \sum_{n \in \mathbf{N_k}} a^H_{b,k,n} \leq Q^H_b, b \in \mathbf{B} \tag{5.2}$$

$$\sum_{i \in \mathbf{B} \cup \mathbf{H}} a^P_{i,p} = 1, p \in \mathbf{P} \tag{5.3}$$

$$a^P_{h,p} - o^H_h \leq 0, h \in \mathbf{H}, p \in \mathbf{P} \tag{5.4}$$

$$\sum_{p \in \mathbf{P}} x_{b,p,k,n,f} + \sum_{h \in \mathbf{H}} y_{b,h,k,n,f} - a^H_{b,k,n} \leq 0, b \in \mathbf{B}, k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F} \tag{5.5}$$

$$\sum_{p \in \mathbf{P}} \sum_{k \in \mathbf{K}} \sum_{n \in \mathbf{N_k}} \sum_{f \in \mathbf{F}} x_{b,p,k,n,f} + \sum_{h \in \mathbf{H}} \sum_{k \in \mathbf{K}} \sum_{n \in \mathbf{N_k}} \sum_{f \in \mathbf{F}} y_{b,h,k,n,f} \leq Q^{TB}_b, b \in \mathbf{B} \tag{5.6}$$

$$\sum_{b\in\mathbf{B}}\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{f\in\mathbf{F}} y_{h,b,k,n,f}+ \\ \sum_{p\in\mathbf{P}}\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{f\in\mathbf{F}}\sum_{s\in\mathbf{S}} z_{h,p,k,n,f,s} - Q^{TH} o_h^H \leq 0, h \in \mathbf{H} \tag{5.7}$$

$$\sum_{j\in\mathbf{B}\cup\mathbf{P}}\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{f\in\mathbf{F}} x_{p,j,k,n,f}+ \\ \sum_{j\in\mathbf{H}\cup\mathbf{P}}\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{f\in\mathbf{F}}\sum_{s\in\mathbf{S}} z_{p,j,k,n,f,s} \leq Q_p^{TP}, p \in \mathbf{P} \tag{5.8}$$

$$\sum_{(i,j)\in\mathbf{A^1}} T_{i,j,k}^{FA} x_{i,j,k,n,f} + \sum_{(i,j)\in\mathbf{A^2}} T_{i,j,k}^{FA} y_{i,j,k,n,f}+ \\ \sum_{(i,j)\in\mathbf{A^3}}\sum_{s\in\mathbf{S}} T_{i,j,k}^{FA} z_{i,j,k,n,f,s} \leq T^{OF}, k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F} \tag{5.9}$$

$$\sum_{(i,j)\in\mathbf{A^1}}\sum_{f\in\mathbf{F}} T_{i,j,k}^{FA} x_{i,j,k,n,f} + \sum_{(i,j)\in\mathbf{A^2}}\sum_{f\in\mathbf{F}} T_{i,j,k}^{FA} y_{i,j,k,n,f}+ \\ \sum_{(i,j)\in\mathbf{A^3}}\sum_{f\in\mathbf{F}}\sum_{s\in\mathbf{S}} T_{i,j,k}^{FA} z_{i,j,k,n,f,s} \leq T^{OD}, k \in \mathbf{K}, n \in \mathbf{N_k} \tag{5.10}$$

$$\sum_{(i,j)\in\mathbf{A^1}}\sum_{f\in\mathbf{F}} T_{i,j,k}^{FA} x_{i,j,k,n,f} + \sum_{(i,j)\in\mathbf{A^2}}\sum_{f\in\mathbf{F}} T_{i,j,k}^{FA} y_{i,j,k,n,f}+ \\ \sum_{(i,j)\in\mathbf{A^3}}\sum_{f\in\mathbf{F}}\sum_{s\in\mathbf{S}} T_{i,j,k}^{FA} z_{i,j,k,n,f,s} + \sum_{b\in\mathbf{B}} t_{b,k,n}^P - \\ T^P \sum_{b\in\mathbf{B}} a_{b,k,n}^H = 0, k \in \mathbf{K}, n \in \mathbf{N_k} \tag{5.11}$$

$$t_{b,k,n}^P - T^P a_{b,k,n}^H \leq 0, b \in \mathbf{B}, k \in \mathbf{K}, n \in \mathbf{N_k} \tag{5.12}$$

$$x_{i,j,k,n,f} \in \{0,1\}, (i,j) \in \mathbf{A^1}, k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F} \tag{5.13}$$

$$y_{i,j,k,n,f} \in \{0,1\}, (i,j) \in \mathbf{A^2}, k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F} \tag{5.14}$$

$$z_{i,j,k,n,f,s} \in \{0,1\}, (i,j) \in \mathbf{A^3}, k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F}, s \in \mathbf{S} \tag{5.15}$$

$$a_{b,k,n}^H \in \{0,1\}, b \in \mathbf{B}, k \in \mathbf{K}, n \in \mathbf{N_k} \tag{5.16}$$

$$a_{i,p}^P \in \{0,1\}, i \in \mathbf{B} \cup \mathbf{H}, p \in \mathbf{P} \tag{5.17}$$

$$o_h^H \in \{0,1\}, h \in \mathbf{H} \tag{5.18}$$

$$t_{b,k,n}^P \geq 0, b \in \mathbf{B}, k \in \mathbf{K}, n \in \mathbf{N_k} \tag{5.19}$$

Constraints (5.1) ensure that every available helicopter is assigned to maximum one base, while constraints (5.2) make safe that the total number of helicopters assigned to every base is less than or equal to its helicopter parking capacity.

Constraints (5.3) ensure that every platform is assigned to one base or one hub, while constraints (5.4) make sure that an assignment to a hub can only occur if this hub is open. Constraints (5.5) guarantee that every helicopter can start a flight only at the base to which it is assigned. In addition, these constraints also make safe that every helicopter selects maximum one of the two routing policies on every flight. Constraints (5.6), (5.7) and (5.8) secures that the take-off and landing capacities at every base, hub and platform are not violated respectively. Constraints (5.7) also make sure that if one or several helicopters enter or leave a hub, this hub must be defined as open. Constraints (5.9) and (5.10) make safe that the maximum operating time per flight and per day for every helicopter in use is respected. Constraints (5.11) and (5.12) ensure that a helicopter's parking time at a base is assigned to variable $t^P_{b,k,n}$ if this helicopter is in use. All variables introduced in this section are defined in lines (5.13) to (5.19).

5.1.2 Direct routing policy

Let us now define the constraints for all direct routing policy flights. When doing so, some additional parameters must be introduced. $V^{P1,D}$ denotes the maximum number of platform visits during a direct routing policy flight. V^{P2} on the other hand gives the maximum number of visits to one particular platform during a direct routing policy flight, or during a second echelon trip of a flight using a hub connected routing policy. $L^{FA}_{i,j}$, $(i,j) \in \mathbf{A}$, denotes the flying length arc (i,j) for all available helicopters, while L^R_k, $k \in \mathbf{K}$, gives the maximum flying length for all helicopters of type k.

The direct routing policy constraints can now be written as:

$$\sum_{j \in \mathbf{B} \cup \mathbf{P}} x_{j,i,k,n,f} - \sum_{j \in \mathbf{B} \cup \mathbf{P}} x_{i,j,k,n,f} = 0, i \in \mathbf{B} \cup \mathbf{P}, k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F} \tag{5.20}$$

$$\sum_{i \in \mathbf{P}} \sum_{j \in \mathbf{P}} x_{i,j,k,n,f} - (V^{P1,D} - 1) \sum_{i \in \mathbf{B}} \sum_{j \in \mathbf{P}} x_{i,j,k,n,f} \leq 0, k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F} \tag{5.21}$$

$$\begin{gathered} \sum_{j \in \mathbf{P}} x_{b,j,k,n,f} + \sum_{i \in \mathbf{B} \cup \mathbf{P}} x_{i,p,k,n,f} - V^{P2} a^P_{b,p} \leq 1, \\ b \in \mathbf{B}, p \in \mathbf{P}, k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F} \end{gathered} \tag{5.22}$$

$$\sum_{(i,j) \in \mathbf{A^1}} L^{FA}_{i,j} x_{i,j,k,n,f} \leq L^R_k, k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F} \tag{5.23}$$

Constraints (5.20) ensure that every helicopter entering a node also leave this node. Constraints (5.21) on the other hand restrict the number of platform landings during every flight, while also securing that every helicopter visiting a platform must have departed from a base on that particular flight. Constraints (5.22) make sure that every helicopter can only visit a platform that is assigned to the same base. Constraints (5.23) make safe that a helicopter's range is not violated during any flight.

5.1.3 Hub connected routing policy

Let us now define the constraints for all hub connected routing policy flights. When doing so, $V^{P1,H}$ is introduced. This parameter denotes the maximum number of platform visits during a second echelon trip of a flight using a hub connected routing policy.

The hub connected routing policy constraints can now be written as:

$$\sum_{j \in \mathbf{B} \cup \mathbf{H}} y_{j,i,k,n,f} - \sum_{j \in \mathbf{B} \cup \mathbf{H}} y_{i,j,k,n,f} = 0, i \in \mathbf{B} \cup \mathbf{H}, k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F} \tag{5.24}$$

$$\begin{gathered} \sum_{j \in \mathbf{H} \cup \mathbf{P}} z_{j,i,k,n,f,s} - \sum_{j \in \mathbf{H} \cup \mathbf{P}} z_{i,j,k,n,f,s} = 0, \\ i \in \mathbf{H} \cup \mathbf{P}, k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F}, s \in \mathbf{S} \end{gathered} \tag{5.25}$$

$$\sum_{p \in \mathbf{P}} z_{h,p,k,n,f,s} - \sum_{b \in \mathbf{B}} y_{b,h,k,n,f} \le 0, h \in \mathbf{H}, k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F}, s \in \mathbf{S} \tag{5.26}$$

$$\begin{gathered} \sum_{i \in \mathbf{P}} \sum_{j \in \mathbf{P}} z_{i,j,k,n,f,s} - (V^{P1,H} - 1) \sum_{i \in \mathbf{H}} \sum_{j \in \mathbf{P}} z_{i,j,k,n,f,s} \le 0, \\ k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F}, s \in \mathbf{S} \end{gathered} \tag{5.27}$$

$$\begin{gathered} \sum_{b \in \mathbf{B}} y_{b,h,k,n,f} + \sum_{i \in \mathbf{H} \cup \mathbf{P}} z_{i,p,k,n,f,s} - V^{P2} a^{P}_{h,p} \le 1, \\ h \in \mathbf{H}, p \in \mathbf{P}, k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F}, s \in \mathbf{S} \end{gathered} \tag{5.28}$$

$$\sum_{b \in \mathbf{B}} \sum_{h \in \mathbf{H}} L^{FA}_{b,h} y_{b,h,k,n,f} \le L^{R}_{k}, k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F} \tag{5.29}$$

$$\sum_{(i,j) \in \mathbf{A^3}} L^{FA}_{i,j} z_{i,j,k,n,f,s} \le L^{R}_{k}, k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F}, s \in \mathbf{S} \tag{5.30}$$

Constraints (5.24) and (5.25) ensure that every helicopter entering a node also leave this node. Constraints (5.26) on the other hand make safe that every helicopter departing from a hub on a second echelon trip must have entered the same hub on the first echelon trip of the same flight. Constraints (5.27) restrict the number of platform landings during every flight. These constraints also secure that every helicopter visiting a platform must have departed from a hub on that particular flight. Constraints (5.28) make sure that every helicopter can only visit a platform that is assigned to the same hub. Constraints (5.29) and (5.30) ensure that a helicopter's range is not violated during any flight.

5.1.4 Pickup and deliveries with split demand

Let us now define the constraints for the pickup and delivery of employees. When doing so, some additional variables and parameters must be introduced. Variables $p^x_{i,j,k,n,f}$ and $d^x_{i,j,k,n,f}$, $(i,j) \in \mathbf{A^1}$, $k \in \mathbf{K}$, $n \in \mathbf{N_k}$, $f \in \mathbf{F}$, denotes the pickup and delivery load of employees on helicopter number n of type k if it uses a direct routing policy, and travels directly between nodes i and j, on flight number f respectively. Variables $p^y_{i,j,k,n,f}$ and $d^y_{i,j,k,n,f}$, $(i,j) \in \mathbf{A^2}$, $k \in \mathbf{K}$, $n \in \mathbf{N_k}$, $f \in \mathbf{F}$, on the other hand give the pickup and delivery load of employees on helicopter number n of type k if it uses a hub connected routing policy, and travels directly between nodes i and j, during the first echelon trip on flight number f respectively. Variables $p^z_{i,j,k,n,f,s}$ and $d^z_{i,j,k,n,f,s}$, $(i,j) \in \mathbf{A^3}$, $k \in \mathbf{K}$, $n \in \mathbf{N_k}$, $f \in \mathbf{F}$, denotes the pickup and delivery load of employees on helicopter number n of type k if it uses a hub connected routing policy, and travels directly between nodes i and j, during second echelon trip number s on flight number f. All pickup and delivery variables are asymmetric.

Further, variables $e^{P,x}_{p,k,n,f}$ and $e^{D,x}_{p,k,n,f}$, $p \in \mathbf{P}$, $k \in \mathbf{K}$, $n \in \mathbf{N_k}$, $f \in \mathbf{F}$, denote the total number of employees picked up and delivered at platform p by helicopter number n of type k during flight number f if a direct routing policy is used respectively. Similarly, variables $e^{P,z}_{p,k,n,f}$ and $e^{D,z}_{p,k,n,f}$, $p \in \mathbf{P}$, $k \in \mathbf{K}$, $n \in \mathbf{N_k}$, $f \in \mathbf{F}$, give the total number of employees picked up and delivered at platform p by helicopter number n of type k during flight number f if a hub connected routing policy is used respectively.

D^P_p, $p \in \mathbf{P}$, express the demand of employees to be picked up at platform p. D^D_p, $p \in \mathbf{P}$, on the other hand gives the demand of employees to be delivered up at platform p.

The pickup and delivery constraints can now be written as:

$$p^x_{i,j,k,n,f} + d^x_{i,j,k,n,f} - Q^P_k x_{i,j,k,n,f} \leq 0, (i,j) \in \mathbf{A^1}, k \in \mathbf{K} n \in \mathbf{N_k}, f \in \mathbf{F} \tag{5.31}$$

$$p^y_{i,j,k,n,f} + d^y_{i,j,k,n,f} - Q^P_k y_{i,j,k,n,f} \leq 0, (i,j) \in \mathbf{A^2}, k \in \mathbf{K} n \in \mathbf{N_k}, f \in \mathbf{F} \tag{5.32}$$

$$\begin{gathered} p^z_{i,j,k,n,f,s} + d^z_{i,j,k,n,f,s} - Q^P_k z_{i,j,k,n,f,s} \leq 0, \\ (i,j) \in \mathbf{A^3}, k \in \mathbf{K} n \in \mathbf{N_k}, f \in \mathbf{F}, s \in \mathbf{S} \end{gathered} \tag{5.33}$$

$$\sum_{b \in \mathbf{B}} \sum_{p \in \mathbf{P}} \sum_{k \in \mathbf{K}} \sum_{n \in \mathbf{N_k}} \sum_{f \in \mathbf{F}} p^x_{b,p,k,n,f} + \sum_{b \in \mathbf{B}} \sum_{h \in \mathbf{H}} \sum_{k \in \mathbf{K}} \sum_{n \in \mathbf{N_k}} \sum_{f \in \mathbf{F}} p^y_{b,h,k,n,f} = 0 \tag{5.34}$$

$$\sum_{b \in \mathbf{B}} \sum_{p \in \mathbf{P}} \sum_{k \in \mathbf{K}} \sum_{n \in \mathbf{N_k}} \sum_{f \in \mathbf{F}} d^x_{p,b,k,n,f} + \sum_{b \in \mathbf{B}} \sum_{h \in \mathbf{H}} \sum_{k \in \mathbf{K}} \sum_{n \in \mathbf{N_k}} \sum_{f \in \mathbf{F}} d^y_{h,b,k,n,f} = 0 \tag{5.35}$$

$$\sum_{p \in \mathbf{P}} \sum_{k \in \mathbf{K}} \sum_{n \in \mathbf{N_k}} \sum_{f \in \mathbf{F}} \sum_{s \in \mathbf{S}} p^z_{p,h,k,n,f,s} - \sum_{b \in \mathbf{B}} \sum_{k \in \mathbf{K}} \sum_{n \in \mathbf{N_k}} \sum_{f \in \mathbf{F}} p^y_{h,b,k,n,f} = 0, h \in \mathbf{H} \tag{5.36}$$

$$\sum_{b \in \mathbf{B}} \sum_{k \in \mathbf{K}} \sum_{n \in \mathbf{N_k}} \sum_{f \in \mathbf{F}} d^y_{b,h,k,n,f} - \sum_{p \in \mathbf{P}} \sum_{k \in \mathbf{K}} \sum_{n \in \mathbf{N_k}} \sum_{f \in \mathbf{F}} \sum_{s \in \mathbf{S}} d^z_{h,p,k,n,f,s} = 0, h \in \mathbf{H} \tag{5.37}$$

$$\sum_{j \in \mathbf{B} \cup \mathbf{P}} p^x_{p,j,k,n,f} - \sum_{i \in \mathbf{B} \cup \mathbf{P}} p^x_{i,p,k,n,f} - e^{P,x}_{p,k,n,f} = 0, p \in \mathbf{P}, k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F} \tag{5.38}$$

$$\sum_{i \in \mathbf{B} \cup \mathbf{P}} d^x_{i,p,k,n,f} - \sum_{j \in \mathbf{B} \cup \mathbf{P}} d^x_{p,j,k,n,f} - e^{D,x}_{p,k,n,f} = 0, p \in \mathbf{P}, k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F} \tag{5.39}$$

$$\begin{gathered} \sum_{j \in \mathbf{H} \cup \mathbf{P}} p^z_{p,j,k,n,f,s} - \sum_{i \in \mathbf{H} \cup \mathbf{P}} p^z_{i,p,k,n,f,s} - e^{P,z}_{p,k,n,f,s} = 0, \\ p \in \mathbf{P}, k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F}, s \in \mathbf{S} \end{gathered} \tag{5.40}$$

$$\begin{gathered} \sum_{i \in \mathbf{H} \cup \mathbf{P}} d^z_{i,p,k,n,f,s} - \sum_{j \in \mathbf{H} \cup \mathbf{P}} d^z_{p,j,k,n,f,s} - e^{D,z}_{p,k,n,f,s} = 0, \\ p \in \mathbf{P}, k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F}, s \in \mathbf{S} \end{gathered} \tag{5.41}$$

$$\sum_{k \in \mathbf{K}} \sum_{n \in \mathbf{N_k}} \sum_{f \in \mathbf{F}} e^{P,x}_{p,k,n,f} + \sum_{k \in \mathbf{K}} \sum_{n \in \mathbf{N_k}} \sum_{f \in \mathbf{F}} \sum_{s \in \mathbf{S}} e^{P,z}_{p,k,n,f,s} = D^P_p, p \in \mathbf{P} \tag{5.42}$$

$$\sum_{k \in \mathbf{K}} \sum_{n \in \mathbf{N_k}} \sum_{f \in \mathbf{F}} e^{D,x}_{p,k,n,f} + \sum_{k \in \mathbf{K}} \sum_{n \in \mathbf{N_k}} \sum_{f \in \mathbf{F}} \sum_{s \in \mathbf{S}} e^{D,z}_{p,k,n,f,s} = D^D_p, p \in \mathbf{P} \tag{5.43}$$

$$p^x_{i,j,k,n,f}, d^x_{i,j,k,n,f} \geq 0, (i,j) \in \mathbf{A^1}, k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F} \tag{5.44}$$

$$p^y_{i,j,k,n,f}, d^y_{i,j,k,n,f} \geq 0, (i,j) \in \mathbf{A^2}, k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F} \tag{5.45}$$

$$p^z_{i,j,k,n,f,s}, d^z_{i,j,k,n,f,s} \geq 0, (i,j) \in \mathbf{A^3}, k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F}, s \in \mathbf{S} \tag{5.46}$$

$$e^{P,x}_{p,k,n,f}, e^{D,x}_{p,k,n,f} \geq 0, p \in \mathbf{P}, k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F} \tag{5.47}$$

$$e^{P,z}_{p,k,n,f,s}, e^{D,z}_{p,k,n,f,s} \geq 0, p \in \mathbf{P}, k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F}, s \in \mathbf{S} \tag{5.48}$$

Constraints (5.31) to (5.33) ensure that the sum of a helicopter's pickup and delivery load does not exceed its passenger capacity. Constraint (5.34) make sure that the initial total pickup load on the helicopters departing from the bases is zero, while constraint (5.35) make sure that the total delivery load on the helicopters returning to the bases is zero. Constraints (5.36) to (5.37) secure that an equal number of pickup and delivery employees enter and depart from every hub respectively. Constraints (5.38) to (5.43) on the other hand balance the total load of employees picked up and delivered at every platform. According to Qian (2012) and Desrochers and Laporte (1991), these constraints will also eliminate subtours. All variables introduced in this section are defined in lines (5.44) to (5.48).

5.1.5 Objective function

Let us now define the objective function of the arc flow formulation. When doing so, some additional parameters must be introduced. C^{FH} denotes the fixed investment and operating cost per day for every installed offshore transhipment hub. C_k^{FK}, $k \in \mathbf{K}$, on the other hand denotes the fixed investment cost per day for helicopters of type k. $C_{i,j,k}^{VA}$, $(i,j) \in \mathbf{A}$, $k \in \mathbf{K}$, gives the variable operating cost on arc i,j for helicopters of type k, while $C_{b,k}^{VB}$, $b \in \mathbf{B}$, $k \in \mathbf{K}$, gives the variable parking cost on onshore airport base b for helicopters of type k.

$R_{i,j,k}^{A}$, $(i,j) \in \mathbf{A}$, $k \in \mathbf{K}$, denotes the risk assessment of arc (i,j) for helicopters of type k. This parameter includes the risk valuation of both take-off and landing, and cruise procedures. F^W gives the weight factor assigned to the total cost of the transportation system, while F^S gives the scale factor assigned to the total risk of the transportation system.

The objective function of the arc flow formulation can now be written as:

$$F^W\Big(\sum_{(i,j)\in\mathbf{A^1}} \sum_{k\in\mathbf{K}} \sum_{n\in\mathbf{N_k}} \sum_{f\in\mathbf{F}} C_{i,j,k}^{VA} x_{i,j,k,n,f} + \sum_{(i,j)\in\mathbf{A^2}} \sum_{k\in\mathbf{K}} \sum_{n\in\mathbf{N_k}} \sum_{f\in\mathbf{F}} C_{i,j,k}^{VA} y_{i,j,k,n,f} +$$

$$\sum_{(i,j)\in\mathbf{A^3}} \sum_{k\in\mathbf{K}} \sum_{n\in\mathbf{N_k}} \sum_{f\in\mathbf{F}} \sum_{s\in\mathbf{S}} C_{i,j,k}^{VA} z_{i,j,k,n,f,s} + \sum_{b\in\mathbf{B}} \sum_{k\in\mathbf{K}} \sum_{n\in\mathbf{N_k}} C_{b,k}^{VB} t_{b,k,n}^{P} +$$

$$\sum_{b\in\mathbf{B}} \sum_{k\in\mathbf{K}} \sum_{n\in\mathbf{N_k}} C_k^{FK} a_{b,k,n}^{H} + C^{FH} \sum_{h\in\mathbf{H}} o_h^H \Big) +$$

$$F^S(1-F^W)(\sum_{(i,j)\in\mathbf{A^1}}\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{f\in\mathbf{F}}R^A_{i,j,k}(p^x_{i,j,k,n,f}+d^x_{i,j,k,n,f})+ \sum_{(i,j)\in\mathbf{A^2}}\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{f\in\mathbf{F}}R^A_{i,j,k}(p^y_{i,j,k,n,f}+d^y_{i,j,k,n,f})+ \sum_{(i,j)\in\mathbf{A^3}}\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{f\in\mathbf{F}}\sum_{s\in\mathbf{S}}R^A_{i,j,k}(p^z_{i,j,k,n,f,s}+d^z_{i,j,k,n,f,s})) \tag{5.49}$$

The objective function (5.49) minimizes the weighted sum of the transportation system's total cost and total risk. The system's total cost consists of its routing cost and investment cost for the helicopters and offshore transhipment hubs in use. The system's total risk consists of the accident risk assessment of its routing process, and is formulated in line with equation (2.1) given in Chapter 2. Notice that only employees on board are included in the accident risk assessment. Only passenger accident risk is therefore addressed in the formulation.

5.1.6 Optional symmetry breaking constraints

In addition to the basic model, some optional symmetry breaking constraints are defined. These constraints are developed in order to eliminate symmetric solutions, which may have an impact on the model's solution time. No additional variables or parameters are needed when defining the constraints.

The optional symmetry breaking constraints are written as:

$$\sum_{b\in\mathbf{B}}a^H_{b,k,n}-\sum_{b\in\mathbf{B}}a^H_{b,k,n-1}\le 0, k\in\mathbf{K}, n=2\ldots\mid\mathbf{N_k}\mid \tag{5.50}$$

$$\sum_{(i,j)\in\mathbf{A^1}}\sum_{f\in\mathbf{F}}T^{FA}_{i,j,k}x_{i,j,k,n,f}+\sum_{(i,j)\in\mathbf{A^2}}\sum_{f\in\mathbf{F}}T^{FA}_{i,j,k}y_{i,j,k,n,f}+ \sum_{(i,j)\in\mathbf{A^3}}\sum_{f\in\mathbf{F}}\sum_{s\in\mathbf{S}}T^{FA}_{i,j,k}z_{i,j,k,n,f,s}-\sum_{(i,j)\in\mathbf{A^1}}\sum_{f\in\mathbf{F}}T^{FA}_{i,j,k}x_{i,j,k,n-1,f}- \sum_{(i,j)\in\mathbf{A^2}}\sum_{f\in\mathbf{F}}T^{FA}_{i,j,k}y_{i,j,k,n-1,f}-\sum_{(i,j)\in\mathbf{A^3}}\sum_{f\in\mathbf{F}}\sum_{s\in\mathbf{S}}T^{FA}_{i,j,k}z_{i,j,k,n-1,f,s}\le 0, \quad k\in\mathbf{K}, n=2\ldots\mid\mathbf{N_k}\mid \tag{5.51}$$

$$\sum_{(i,j)\in\mathbf{A^1}}T^{FA}_{i,j,k}x_{i,j,k,n,f}+\sum_{(i,j)\in\mathbf{A^2}}T^{FA}_{i,j,k}y_{i,j,k,n,f}+ \sum_{(i,j)\in\mathbf{A^3}}\sum_{s\in\mathbf{S}}T^{FA}_{i,j,k}z_{i,j,k,n,f,s}-\sum_{(i,j)\in\mathbf{A^1}}T^{FA}_{i,j,k}x_{i,j,k,n,f-1}-$$

$$\sum_{(i,j)\in\mathbf{A^2}} T^{FA}_{i,j,k} y_{i,j,k,n,f-1} - \sum_{(i,j)\in\mathbf{A^3}} \sum_{s\in\mathbf{S}} T^{FA}_{i,j,k} z_{i,j,k,n,f-1,s} \leq 0, \quad k \in \mathbf{K}, n \in \mathbf{N_k}, f = 2 \ldots \mid \mathbf{F} \mid \tag{5.52}$$

$$\sum_{(i,j)\in\mathbf{A^3}} (T^{FA}_{i,j,k} z_{i,j,k,n,f,s} - T^{FA}_{i,j,k} z_{i,j,k,n,f,s-1}) \leq 0, \quad k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F}, s = 2 \ldots \mid \mathbf{S} \mid \tag{5.53}$$

Constraints (5.50) ensure that helicopter number n of type k is assigned to a base before helicopter number $n+1$ of the same type k. Constraints (5.51) on the other hand make safe that the total operating time of helicopter number n of type k is lesser or equal to the total operating time of helicopter number $n-1$ of the same type k. Constraints (5.52) make sure that the total operating time of flight number f is lesser or equal to the total operating time of flight number $f-1$ for every helicopter in use. Constraints (5.53) make sure that the total operating time of second echelon trip number s is lesser or equal to the total operating time of second echelon trip number $s-1$ on every flight f of every helicopter in use.

5.2 Path Flow Formulation I (PFF1)

In this section, the first path flow formulation of the problem addressed in this report is presented. In this model, a path represents a predefined route that can be used by a particular helicopter type. The size of the model in terms of constraints is independent of the number of predefined routes. A route can be perfect or imperfect. Perfect routes represent routes that can be used if practising a direct routing policy, as well as first echelon routes that can be used if practising a hub connected routing policy. Imperfect routes represent second echelon routes that can be used if practising a hub connected routing policy. The latter routes are termed imperfect as they can only be used in unison with a first echelon route connected to the same offshore transhipment hub. All predefined routes contain information about to which onshore airport base and/or offshore transhipment hub the route is connected to, the order in which platforms are visited in the route, the cost assessment of the route, and time needed to perform the route for the given helicopter type. No routes contain information about the pickup and delivery of employees at the platforms visited. The generation of predefined routes is described in Chapter 6.

Several of the sets, indices, parameters and decision variables used in the arc flow formulation presented in Section 5.1 are also used in the path flow model introduced in this section. Additional and redefined notation is introduced throughout the section when needed. Still, the way in which the formulation's decision variables relate to the addressed problem can already be seen in Figure 5.2. A complete model is to be found in Appendix C.

First, constraints for the transportation network are presented in Section 5.2.1. Then, constraints for the direct and hub conneted routing policies are introduced in Sections 5.2.2 and 5.2.3. Pickup and delivery constraints are presented in Section 5.2.4, while the objective function is presented in Section 5.2.5. Lastly, some optional symmetry breaking constraints and cuts are introduced in Sections 5.2.6 and 5.2.7 respectively.

5.2.1 Establishment of the transportation network

In addition to the sets introduced in Section 5.1, let $\mathbf{R_k^1}$ be the set of all predefined, feasible, perfect routes using a direct routing policy for helicopters of type k, indexed by r. Further, let $\mathbf{R_k^2}$ and $\mathbf{R_k^3}$ be the set of all predefined, feasible, imperfect routes using a hub connected routing policy, also indexed by r. The elements in $\mathbf{R_k^2}$ represent predefined, feasible first echelon routes for helicopters of type k, while the elements in $\mathbf{R_k^3}$ represent predefined, feasible second echelon routes for helicopters of type k. Let $\mathbf{R_k} = \mathbf{R_k^1} \cup \mathbf{R_k^2} \cup \mathbf{R_k^3}$ be the set of all predefined, feasible routes for helicopters of type k, indexed by r.

Following the establishment of these sets, the path flow variables can be defined. The path flow formulation presented in this section makes use of two binary variables for this purpose. Variable $x_{k,n,r,f}$, $k \in \mathbf{K}$, $n \in \mathbf{N_k}$, $r \in \mathbf{R_k^1} \cup \mathbf{R_k^2}$, $f \in \mathbf{F}$, is equal to 1 if helicopter number n of type k travels directly between nodes i and j during a direct routing policy trip or during the first echelon trip of a hub connected routing policy, on flight number f, and otherwise 0. Variable $z_{k,n,r,f,s}$ on the other hand, $k \in \mathbf{K}$, $n \in \mathbf{N_k}$, $r \in \mathbf{R_k^3}$, $f \in \mathbf{F}$, $s \in \mathbf{S}$, is equal to 1 if helicopter number n of type k uses a hub connected routing policy, and travels directly between nodes i and j during second echelon trip number s, on flight number f, and otherwise 0.

The additional, required parameters are now introduced. $V_{i,k,r}$, $i \in \mathbf{B} \cup \mathbf{H} \cup \mathbf{P}$, $k \in \mathbf{K}$, $r \in \mathbf{R_k}$, is equal to 1 if onshore airport base, offshore transhipment hub or offshore platform i is visited on route r for helicopters of type k, and

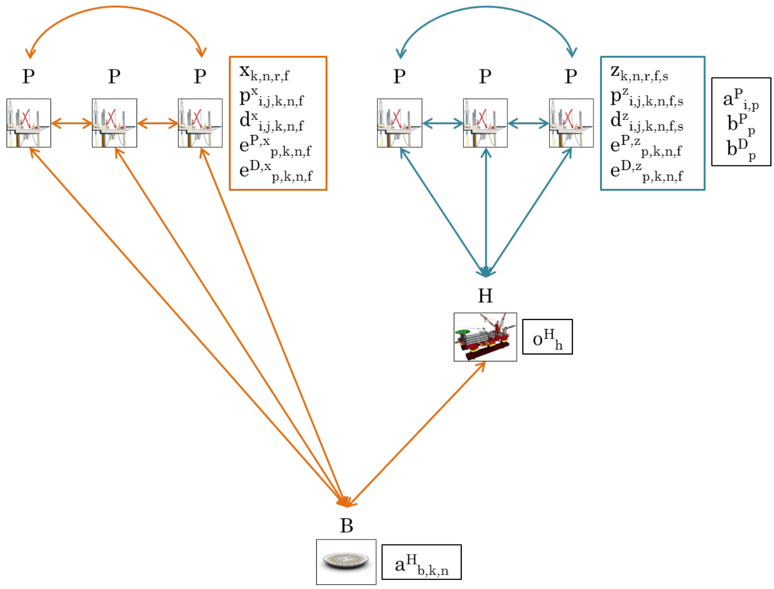

Figure 5.2: Decision variables, Path Flow Formulation I

otherwise 0. $L_{p,k,r}$ on the other hand, $p \in \mathbf{P}$, $k \in \mathbf{K}$, $r \in \mathbf{R_k}$, gives the number of times offshore platform i is visited on route r for helicopters of type k. $T^{OR}_{k,r}$, $k \in \mathbf{K}$, $r \in \mathbf{R_k}$, denote the operating time for route r for helicopters of type k.

Constraints (5.1) to (5.4), (5.12), and (5.16) to (5.19) presented in Section 5.1.1 also hold for the path flow formulation presented in this section. The additional, general constraints of the transportation network are written as:

$$\sum_{r \in \mathbf{R^1_k} \cup \mathbf{R^2_k}} V_{b,k,r} x_{k,n,r,f} - a^H_{b,k,n} \leq 0, b \in \mathbf{B}, k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F} \tag{5.54}$$

$$\sum_{k \in \mathbf{K}} \sum_{n \in \mathbf{N_k}} \sum_{r \in \mathbf{R^1_k} \cup \mathbf{R^2_k}} \sum_{f \in \mathbf{F}} V_{b,k,r} x_{k,n,r,f} \leq Q^{TB}_b, b \in \mathbf{B} \tag{5.55}$$

$$\begin{aligned} &\sum_{k \in \mathbf{K}} \sum_{n \in \mathbf{N_k}} \sum_{r \in \mathbf{R^2_k}} \sum_{f \in \mathbf{F}} V_{h,k,r} x_{k,n,r,f} + \\ &\sum_{k \in \mathbf{K}} \sum_{n \in \mathbf{N_k}} \sum_{r \in \mathbf{R^3_k}} \sum_{f \in \mathbf{F}} \sum_{s \in \mathbf{S}} V_{h,k,r} z_{k,n,r,f,s} - Q^{TH} o^H_h \leq 0, h \in \mathbf{H} \end{aligned} \tag{5.56}$$

$$\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{r\in\mathbf{R_k^1}}\sum_{f\in\mathbf{F}}L_{p,k,r}x_{k,n,r,f}+$$
$$\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{r\in\mathbf{R_k^3}}\sum_{f\in\mathbf{F}}\sum_{s\in\mathbf{S}}L_{p,k,r}z_{k,n,r,f,s}\leq Q_p^{TP}, p\in\mathbf{P} \tag{5.57}$$

$$\sum_{r\in\mathbf{R_k^1}\cup\mathbf{R_k^2}}T_{k,r}^{OR}x_{k,n,r,f}+\sum_{r\in\mathbf{R_k^3}}\sum_{s\in\mathbf{S}}T_{k,r}^{OR}z_{k,n,r,f,s}\leq T^{OF}, k\in\mathbf{K}, n\in\mathbf{N_k}, f\in\mathbf{F} \tag{5.58}$$

$$\sum_{r\in\mathbf{R_k^1}\cup\mathbf{R_k^2}}\sum_{f\in\mathbf{F}}T_{k,r}^{OR}x_{k,n,r,f}+\sum_{r\in\mathbf{R_k^3}}\sum_{f\in\mathbf{F}}\sum_{s\in\mathbf{S}}T_{k,r}^{OR}z_{k,n,r,f,s}\leq T^{OD}, k\in\mathbf{K}, n\in\mathbf{N_k} \tag{5.59}$$

$$\sum_{r\in\mathbf{R_k^1}\cup\mathbf{R_k^2}}\sum_{f\in\mathbf{F}}T_{k,r}^{OR}x_{k,n,r,f}+\sum_{r\in\mathbf{R_k^3}}\sum_{f\in\mathbf{F}}\sum_{s\in\mathbf{S}}T_{k,r}^{OR}z_{k,n,r,f,s}+$$
$$\sum_{b\in\mathbf{B}}t_{b,k,n}^{P}-T^{P}\sum_{b\in\mathbf{B}}a_{b,k,n}^{H}=0, k\in\mathbf{K}, n\in\mathbf{N_k} \tag{5.60}$$

$$x_{k,n,r,f}\in\{0,1\}, k\in\mathbf{K}, n\in\mathbf{N_k}, r\in\mathbf{R_k^1}\cup\mathbf{R_k^2}, f\in\mathbf{F} \tag{5.61}$$

$$z_{k,n,r,f,s}\in\{0,1\}, k\in\mathbf{K}, n\in\mathbf{N_k}, r\in\mathbf{R_k^3}, f\in\mathbf{F}, s\in\mathbf{S} \tag{5.62}$$

Constraints (5.54) to (5.60) correspond to constraints (5.5) to (5.11) in the arc flow formulation. The variables introduced in this section are defined in lines (5.61) and (5.62).

5.2.2 Direct routing policy

Let us now define the constraints for all direct routing policy flights. When doing so, no additional variables or parameters are needed.

Let all elements in set $\mathbf{R_k^1}$ satisfy constraints (5.20), (5.21) and (5.23) presented in Section 5.1.2. The direct routing policy constraints can now be written as:

$$\sum_{r\in\mathbf{R_k^1}}V_{b,k,r}V_{p,k,r}x_{k,n,r,f}-a_{b,p}^{P}\leq 0, b\in\mathbf{B}, p\in\mathbf{P}, k\in\mathbf{K}, n\in\mathbf{N_k}, f\in\mathbf{F} \tag{5.63}$$

Constraints (5.63) correspond to constraints (5.22) in the arc flow formulation.

5.2.3 Hub connected routing policy

Let us now define the constraints for all hub connected routing policy flights. When doing so, no additional variables or parameters are needed.

Let all elements in set $\mathbf{R_k^2}$ satisfy constraints (5.24) and (5.29) presented in Section 5.1.3. Additionally, let all elements in set $\mathbf{R_k^3}$ satisfy constraints (5.25), (5.27) and (5.30) introduced in the same section.

The hub connected routing policy constraints can now be written as:

$$\sum_{r \in \mathbf{R_k^3}} V_{h,k,r} z_{k,n,r,f,s} - \sum_{r \in \mathbf{R_k^2}} V_{h,k,r} x_{k,n,r,f} \leq 0, \quad h \in \mathbf{H}, k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F}, s \in \mathbf{S} \tag{5.64}$$

$$\sum_{r \in \mathbf{R_k^3}} V_{h,k,r} V_{p,k,r} z_{k,n,r,f,s} - a^P_{h,p} \leq 0, \quad h \in \mathbf{H}, p \in \mathbf{P}, k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F}, s \in \mathbf{S} \tag{5.65}$$

Constraints (5.64) and (5.65) correspond to constraints (5.26) and (5.28) in the arc flow formulation.

5.2.4 Pickup and deliveries with split demand

Let us now define the constraints for the pickup and delivery of employees. When doing so, some of the variables presented in previous sections are redefined. Some additional parameters must also be introduced. Let variables $p^x_{i,j,k,n,f}$ and $d^x_{i,j,k,n,f}$, $(i,j) \in \mathbf{A^1} \cup \mathbf{A^2}$, $k \in \mathbf{K}$, $n \in \mathbf{N_k}$, $f \in \mathbf{F}$, denote the pickup and delivery load of employees on helicopter number n of type k if it travels directly between nodes i and j during a direct routing policy trip or during the first echelon trip of a hub connected routing policy, on flight number f respectively. All of these pickup and delivery variables are asymmetric. Let parameter $A_{i,j,k,r}$, $(i,j) \in \mathbf{A}$, $k \in \mathbf{K}$, $r \in \mathbf{R_k}$, be equal to 1 if arc (i,j) is used in route r for helicopters of type k, and otherwise 0.

Constraints (5.38) to (5.43), and (5.46) to (5.48), presented in Section 5.1.4 also hold for the path flow formulation presented in this section. The additional pickup and delivery constraints are written as:

$$p^x_{i,j,k,n,f} + d^x_{i,j,k,n,f} - Q^P_k \sum_{r \in \mathbf{R_k^1} \cup \mathbf{R_k^2}} A_{i,j,k,r} x_{k,n,r,f} \leq 0, \quad (i,j) \in \mathbf{A^1} \cup \mathbf{A^2}, k \in \mathbf{K} n \in \mathbf{N_k}, f \in \mathbf{F} \tag{5.66}$$

$$p^z_{i,j,k,n,f,s} + d^z_{i,j,k,n,f,s} - Q^P_k \sum_{r \in \mathbf{R^3_k}} A_{i,j,k,r} z_{k,n,r,f,s} \leq 0, \tag{5.67}$$
$$(i,j) \in \mathbf{A^3}, k \in \mathbf{K} n \in \mathbf{N_k}, f \in \mathbf{F}, s \in \mathbf{S}$$

$$\sum_{b \in \mathbf{B}} \sum_{j \in \mathbf{P} \cup \mathbf{H}} \sum_{k \in \mathbf{K}} \sum_{n \in \mathbf{N_k}} \sum_{f \in \mathbf{F}} p^x_{b,j,k,n,f} = 0 \tag{5.68}$$

$$\sum_{i \in \mathbf{P} \cup \mathbf{H}} \sum_{b \in \mathbf{B}} \sum_{k \in \mathbf{K}} \sum_{n \in \mathbf{N_k}} \sum_{f \in \mathbf{F}} d^x_{i,b,k,n,f} = 0 \tag{5.69}$$

$$\sum_{p \in \mathbf{P}} \sum_{k \in \mathbf{K}} \sum_{n \in \mathbf{N_k}} \sum_{f \in \mathbf{F}} \sum_{s \in \mathbf{S}} p^z_{p,h,k,n,f,s} - \sum_{b \in \mathbf{B}} \sum_{k \in \mathbf{K}} \sum_{n \in \mathbf{N_k}} \sum_{f \in \mathbf{F}} p^x_{h,b,k,n,f} = 0, h \in \mathbf{H} \tag{5.70}$$

$$\sum_{b \in \mathbf{B}} \sum_{k \in \mathbf{K}} \sum_{n \in \mathbf{N_k}} \sum_{f \in \mathbf{F}} d^x_{b,h,k,n,f} - \sum_{p \in \mathbf{P}} \sum_{k \in \mathbf{K}} \sum_{n \in \mathbf{N_k}} \sum_{f \in \mathbf{F}} \sum_{s \in \mathbf{S}} d^z_{h,p,k,n,f,s} = 0, h \in \mathbf{H} \tag{5.71}$$

Constraints (5.66) correspond to constraints (5.31) and (5.32) in the arc flow formulation, while constraints (5.67) to (5.71) correspond to constraints (5.33) to (5.37) respectively.

5.2.5 Objective function

Let us now define the objective function of the first path flow formulation. When doing so, $C^{VR}_{k,r}$, $k \in \mathbf{K}$, $r \in \mathbf{R_k}$, is introduced. This parameter denotes the variable operating cost for route r for helicopters of type k.

The objective function of the first path flow formulation can now be written as:

$$\begin{aligned} F^W(& \sum_{k \in \mathbf{K}} \sum_{n \in \mathbf{N_k}} \sum_{r \in \mathbf{R^1_k} \cup \mathbf{R^2_k}} \sum_{f \in \mathbf{F}} C^{VR}_{k,r} x_{k,n,r,f} + \sum_{k \in \mathbf{K}} \sum_{n \in \mathbf{N_k}} \sum_{r \in \mathbf{R^3_k}} \sum_{f \in \mathbf{F}} \sum_{s \in \mathbf{S}} C^{VR}_{k,r} z_{k,n,r,f,s} + \\ & \sum_{b \in \mathbf{B}} \sum_{k \in \mathbf{K}} \sum_{n \in \mathbf{N_k}} C^{VB}_{b,k} t^P_{b,k,n} + \sum_{b \in \mathbf{B}} \sum_{k \in \mathbf{K}} \sum_{n \in \mathbf{N_k}} C^{FK}_k a^H_{b,k,n} + C^{FH} \sum_{h \in \mathbf{H}} o^H_h + \\ & F^S(1 - F^W)(\sum_{(i,j) \in \mathbf{A^1} \cup \mathbf{A^2}} \sum_{k \in \mathbf{K}} \sum_{n \in \mathbf{N_k}} \sum_{f \in \mathbf{F}} R^A_{i,j,k}(p^x_{i,j,k,n,f} + d^x_{i,j,k,n,f}) + \\ & \sum_{(i,j) \in \mathbf{A^3}} \sum_{k \in \mathbf{K}} \sum_{n \in \mathbf{N_k}} \sum_{f \in \mathbf{F}} \sum_{s \in \mathbf{S}} R^A_{i,j,k}(p^z_{i,j,k,n,f,s} + d^z_{i,j,k,n,f,s})) \end{aligned} \tag{5.72}$$

The objective function (5.72) corresponds to the objective function (5.49) in the arc flow formulation.

5.2.6 Optional symmetry breaking constraints

Some optional symmetry breaking constraints are also introduced for the first path flow formulation. As in Section 5.1.6, no additional variables or parameters are needed when defining these constraints.

Constraints (5.50) presented in Section 5.1.6 also hold for the path flow formulation presented in this section. The additional, optional symmetry braking constraints are written as:

$$\sum_{r\in\mathbf{R_k^1}\cup\mathbf{R_k^2}}\sum_{f\in\mathbf{F}} T_{k,r}^{OR} x_{k,n,r,f} + \sum_{r\in\mathbf{R_k^3}}\sum_{f\in\mathbf{F}}\sum_{s\in\mathbf{S}} T_{k,r}^{OR} z_{k,n,r,f,s} - \sum_{r\in\mathbf{R_k^1}\cup\mathbf{R_k^2}}\sum_{f\in\mathbf{F}} T_{k,r}^{OR} x_{k,n-1,r,f} - \sum_{r\in\mathbf{R_k^3}}\sum_{f\in\mathbf{F}}\sum_{s\in\mathbf{S}} T_{k,r}^{OR} z_{k,n-1,r,f,s} \leq 0, \quad k\in\mathbf{K}, n=2\ldots\mid\mathbf{N_k}\mid \tag{5.73}$$

$$\sum_{r\in\mathbf{R_k^1}\cup\mathbf{R_k^2}} T_{k,r}^{OR} x_{k,n,r,f} + \sum_{r\in\mathbf{R_k^3}}\sum_{s\in\mathbf{S}} T_{k,r}^{OR} z_{k,n,r,f,s} - \sum_{r\in\mathbf{R_k^1}\cup\mathbf{R_k^2}} T_{k,r}^{OR} x_{k,n,r,f-1} - \sum_{r\in\mathbf{R_k^3}}\sum_{s\in\mathbf{S}} T_{k,r}^{OR} z_{k,n,r,f-1,s} \leq 0, \quad k\in\mathbf{K}, n\in\mathbf{N_k}, f=2\ldots\mid\mathbf{F}\mid \tag{5.74}$$

$$\sum_{r\in\mathbf{R_k^3}} T_{k,r}^{O} z_{k,n,r,f,s} - \sum_{r\in\mathbf{R_k^3}} T_{k,r}^{O} z_{k,n,r,f-1,s} \leq 0, \quad k\in\mathbf{K}, n\in\mathbf{N_k}, f\in\mathbf{F}, s=2\ldots\mid\mathbf{S}\mid \tag{5.75}$$

Constraints (5.73) to (5.75) correspond to constraints (5.51) to (5.53) in the arc flow formulation.

5.2.7 Optional cuts

In addition to the optional symmetry breaking constraints, an optional cut can be defined for the path flow formulation. This constraint is developed in order to potentially reduce the solution space for the path flow model, which may have an impact on the model's solution time. No additional variables or parameters are needed when defining the constraint.

The optional cut is written as:

$$\sum_{k \in \mathbf{K}} \sum_{r \in \mathbf{R_k^1}} V_{p,k,r} + \sum_{h \in \mathbf{H}} o_h^H \geq 1, p \in \mathbf{p} \tag{5.76}$$

Constraint (5.76) ensure that an offshore transhipment hub must be opened if one or several platforms are not visited in any of the predefined, feasible routes using a direct routing policy.

5.3 Path Flow Formulation II (PFF2)

In this section, the second path flow formulation of the problem addressed in this report is presented. As in the first, a path represents a predefined route that can be used by a particular helicopter type. Also, the size of the model in terms of constraints is independent of the number of predefined routes. The predefined routes used in the formulation presented in Section 5.2, whose generation is described in Chapter 6, are also used in the formulation introduced in this section.

The path flow formulation presented in this section also makes use of the same sets, indices and parameters as the formulations presented in Sections 5.1 and 5.2. The number of variables on the other hand, is significantly reduced. This is mostly due to the fact that the set of possible second echelon trips during every flight for every available helicopter, **S**, is no longer used to describe the flights performed by the various helicopters in use. The path flow formulation presented in this section can therefore be said to be an aggregated version of the first. The change in notation is introduced throughout the section when needed. Still, the way in which the formulation's decision variables relate to the addressed problem can already be seen in Figure 5.3. A complete model is to be found in Appendix D.

First, constraints for the transportation network are presented in Section 5.3.1. Then, constraints for the direct and hub connected routing policies are introduced in Sections 5.3.2 and 5.3.3. Pickup and delivery constraints are presented in Section 5.3.4, while the objective function is presented in Section 5.3.5. Lastly, some optional symmetry breaking constraints and cuts are introduced in Sections 5.3.6 and 5.3.7 respectively.

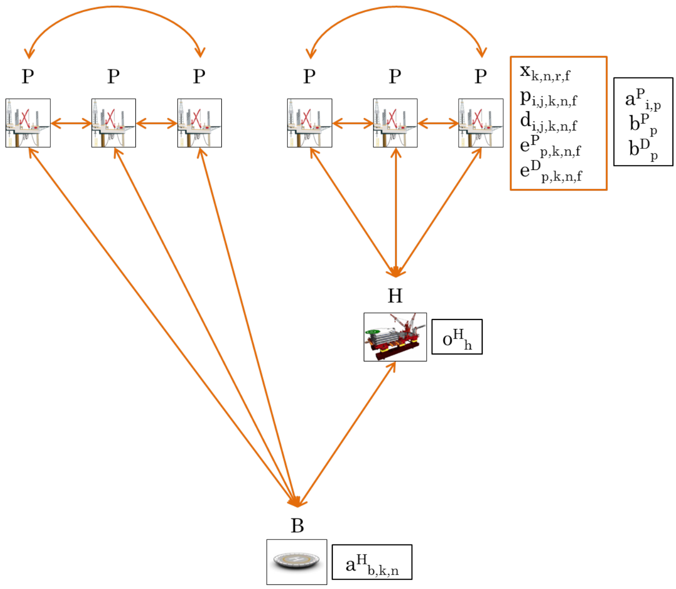

Figure 5.3: Decision variables, Path Flow Formulation II

5.3.1 Establishment of the transportation network

Contrary to the path flow formulation presented in Section 5.2, the formulation presented in this section makes use of only one path flow variable. This is integer variable $x_{k,n,r,f}$, $k \in \mathbf{K}$, $n \in \mathbf{N_k}$, $r \in \mathbf{R_k}$, $f \in \mathbf{F}$, who give the number of times helicopter number n of type k uses route r on flight number f.

Constraints (5.1) to (5.4), (5.12), and (5.16) to (5.19) presented in Section 5.1.1 also hold for the path flow formulation presented in this section. The additional, general constraints of the transportation network are written as:

$$\sum_{r \in \mathbf{R_k^1} \cup \mathbf{R_k^2}} V_{b,k,r} x_{k,n,r,f} - a^H_{b,k,n} \leq 0, b \in \mathbf{B}, k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F} \tag{5.77}$$

$$\sum_{k \in \mathbf{K}} \sum_{n \in \mathbf{N_k}} \sum_{r \in \mathbf{R_k^1} \cup \mathbf{R_k^2}} \sum_{f \in \mathbf{F}} V_{b,k,r} x_{k,n,r,f} \leq Q^{TB}_b, b \in \mathbf{B} \tag{5.78}$$

$$\sum_{k \in \mathbf{K}} \sum_{n \in \mathbf{N_k}} \sum_{r \in \mathbf{R_k^2} \cup \mathbf{R_k^3}} \sum_{f \in \mathbf{F}} V_{h,k,r} x_{k,n,r,f} - Q^{TH} o^H_h \leq 0, h \in \mathbf{H} \tag{5.79}$$

$$\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{r\in\mathbf{R_k^1}\cup\mathbf{R_k^3}}\sum_{f\in\mathbf{F}} L_{p,k,r}x_{k,n,r,f} \leq Q_p^{TP}, p \in \mathbf{P} \tag{5.80}$$

$$\sum_{r\in\mathbf{R_k}} T_{k,r}^{OR}x_{k,n,r,f} \leq T^{OF}, k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F} \tag{5.81}$$

$$\sum_{r\in\mathbf{R_k}}\sum_{f\in\mathbf{F}} T_{k,r}^{OR}x_{k,n,r,f} \leq T^{OD}, k \in \mathbf{K}, n \in \mathbf{N_k} \tag{5.82}$$

$$\sum_{r\in\mathbf{R_k}}\sum_{f\in\mathbf{F}} T_{k,r}^{OR}x_{k,n,r,f} + \sum_{b\in\mathbf{B}} t_{b,k,n}^{P} - T^{P}\sum_{b\in\mathbf{B}} a_{b,k,n}^{H} = 0, k \in \mathbf{K}, n \in \mathbf{N_k} \tag{5.83}$$

$$x_{k,n,r,f} \geq 0 \text{ and integer}, k \in \mathbf{K}, n \in \mathbf{N_k}, r \in \mathbf{R_k}, f \in \mathbf{F} \tag{5.84}$$

As in Section 5.2.1, constraints (5.77) to (5.83) correspond to constraints (5.5) to (5.11) in the arc flow formulation. The variables introduced in this section are defined in line (5.84).

5.3.2 Direct routing policy

Let us now define the constraints for all direct routing policy flights. When doing so, no additional variables or parameters are needed.

As in Section 5.2.2, let all elements in set $\mathbf{R_k^1}$ satisfy constraints (5.20), (5.21) and (5.23) presented in Section 5.1.2. The direct routing policy constraints can now be written as:

$$\sum_{r\in\mathbf{R_k^1}} V_{b,k,r}V_{p,k,r}x_{k,n,r,f} - a_{b,p}^{P} \leq 0, b \in \mathbf{B}, p \in \mathbf{P}, k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F} \tag{5.85}$$

As in Section 5.2.2, constraints (5.85) correspond to constraints (5.22) in the arc flow formulation.

5.3.3 Hub connected routing policy

Let us now define the constraints for all hub connected routing policy flights. When doing so, no additional variables or parameters are needed.

As in Section 5.2.3, let all elements in set $\mathbf{R_k^2}$ satisfy constraints (5.24) and (5.29) presented in Section 5.1.2. Additionally, let all elements in set $\mathbf{R_k^3}$ satisfy constraints (5.25), (5.27) and (5.30) introduced in the same section. The hub connected routing policy constraints can now be written as:

$$\sum_{r \in \mathbf{R_k^3}} V_{h,k,r} x_{k,n,r,f} - \mid \mathbf{F} \mid \sum_{r \in \mathbf{R_k^2}} V_{h,k,r} x_{k,n,r,f} \leq 0, \quad h \in \mathbf{H}, p \in \mathbf{P}, k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F} \tag{5.86}$$

$$\sum_{r \in \mathbf{R_k^3}} V_{h,k,r} V_{p,k,r} x_{k,n,r,f} - \mid \mathbf{F} \mid a^P_{h,p} \leq 0, \quad h \in \mathbf{H}, p \in \mathbf{P}, k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F} \tag{5.87}$$

As in Section 5.2.3, constraints (5.86) and (5.87) correspond to constraints (5.26) and (5.28) in the arc flow formulation.

5.3.4 Pickup and deliveries with split demand

Let us now define the constraints for the pickup and delivery of employees. When doing so, some of the variables presented in previous sections are redefined. Variables $p_{i,j,k,n,f}$ and $d_{i,j,k,n,f}$, $(i,j) \in \mathbf{A}$, $k \in \mathbf{K}$, $n \in \mathbf{N_k}$, $f \in \mathbf{F}$, denote the pickup and delivery load of employees on helicopter number n of type k if it travels directly between nodes i and j on flight number f respectively. All of these pickup and delivery variables are asymmetric. Further, variables $e^P_{p,k,n,f}$ and $e^D_{p,k,n,f}$, $p \in \mathbf{P}$, $k \in \mathbf{K}$, $n \in \mathbf{N_k}$, $f \in \mathbf{F}$, indicate the total number of employees picked up and delivered at platform p by helicopter number n of type k during flight number f.

The pickup and delivery constraints can now be written as:

$$p_{i,j,k,n,f} + d_{i,j,k,n,f} - Q^P_k \sum_{r \in \mathbf{R_k}} A_{i,j,k,r} x_{k,n,r,f} \leq 0, \quad (i,j) \in \mathbf{A}, k \in \mathbf{K} n \in \mathbf{N_k}, f \in \mathbf{F} \tag{5.88}$$

$$\sum_{b \in \mathbf{B}} \sum_{j \in \mathbf{H} \cup \mathbf{P}} \sum_{k \in \mathbf{K}} \sum_{n \in \mathbf{N_k}} \sum_{f \in \mathbf{F}} p_{b,j,k,n,f} = 0 \tag{5.89}$$

$$\sum_{i \in \mathbf{H} \cup \mathbf{P}} \sum_{b \in \mathbf{B}} \sum_{k \in \mathbf{K}} \sum_{n \in \mathbf{N_k}} \sum_{f \in \mathbf{F}} d_{i,b,k,n,f} = 0 \tag{5.90}$$

$$\sum_{p \in \mathbf{P}} \sum_{k \in \mathbf{K}} \sum_{n \in \mathbf{N_k}} \sum_{f \in \mathbf{F}} p_{p,h,k,n,f} - \sum_{b \in \mathbf{B}} \sum_{k \in \mathbf{K}} \sum_{n \in \mathbf{N_k}} \sum_{f \in \mathbf{F}} p_{h,b,k,n,f} = 0, h \in \mathbf{H} \tag{5.91}$$

$$\sum_{b\in\mathbf{B}}\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{f\in\mathbf{F}} d_{b,h,k,n,f} - \sum_{p\in\mathbf{P}}\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{f\in\mathbf{F}} d_{h,p,k,n,f} = 0, h \in \mathbf{H} \tag{5.92}$$

$$\sum_{i\in\mathbf{B}\cup\mathbf{H}\cup\mathbf{P}} p_{p,i,k,n,f} - \sum_{i\in\mathbf{B}\cup\mathbf{H}\cup\mathbf{P}} p_{i,p,k,n,f} - e^P_{p,k,n,f} = 0, \quad p \in \mathbf{P}, k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F} \tag{5.93}$$

$$\sum_{i\in\mathbf{B}\cup\mathbf{H}\cup\mathbf{P}} d_{i,p,k,n,f} - \sum_{i\in\mathbf{B}\cup\mathbf{H}\cup\mathbf{P}} d_{p,i,k,n,f} - e^D_{p,k,n,f} = 0, \quad p \in \mathbf{P}, k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F} \tag{5.94}$$

$$\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{f\in\mathbf{F}} e^P_{p,k,n,f} = D^P_p, p \in \mathbf{P} \tag{5.95}$$

$$\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{f\in\mathbf{F}} e^D_{p,k,n,f} = D^D_p, p \in \mathbf{P} \tag{5.96}$$

$$p_{i,j,k,n,f}, d_{i,j,k,n,f} \geq 0, (i,j) \in \mathbf{A}, k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F} \tag{5.97}$$

$$e^P_{p,k,n,f}, e^D_{p,k,n,f} \geq 0, p \in \mathbf{P}, k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F} \tag{5.98}$$

Constraints (5.88) correspond to constraints (5.31) to (5.33) in the arc flow formulation, while constraints (5.89) to (5.92) correspond to constraints (5.34) to (5.37) respectively. Constraints (5.93) and (5.94) on the other hand correspond to constraints (5.38) to (5.39), while constraints (5.95) and (5.96) correspond to constraints (5.42) to (5.43). All variables introduced in this section are defined in lines (5.97) and (5.98).

5.3.5 Objective function

Let us now define the objective function of the second path flow formulation. When doing so, no additional parameters are needed.

The objective function of the second path flow formulation is written as:

$$\begin{aligned} F^W(&\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{r\in\mathbf{R_k}}\sum_{f\in\mathbf{F}} C^{VR}_{k,r} x_{k,n,r,f} + \sum_{b\in\mathbf{B}}\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}} C^{VB}_{b,k} t^P_{k,n} + \\ &\sum_{b\in\mathbf{B}}\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}} C^{FK}_k a^H_{b,k,n} + C^{FH}\sum_{h\in\mathbf{H}} o^H_h) + \\ F^S(1-F^W) &\sum_{(i,j)\in\mathbf{A}}\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{f\in\mathbf{F}} R^A_{i,j,k}(p_{i,j,k,n,f} + d_{i,j,k,n,f}) \end{aligned} \tag{5.99}$$

As in Section 5.2.5, the objective function (5.99) corresponds to the objective function (5.49) in the arc flow formulation.

5.3.6 Optional symmetry breaking constraints

Some optional symmetry breaking constraints are also introduced for the second path flow formulation. When doing so, no additional variables or parameters are needed.

Constraints (5.50) presented in Section 5.1.6 also hold for the path flow formulation presented in this section. The additional, optional symmetry braking constraints are written as:

$$\sum_{r \in \mathbf{R_k}} \sum_{f \in \mathbf{F}} T^{O}_{k,r} x_{k,n,r,f} - \sum_{r \in \mathbf{R_k}} \sum_{f \in \mathbf{F}} T^{O}_{k,r} x_{k,n-1,r,f} \leq 0, \quad k \in \mathbf{K}, n = 2 \dots \mid \mathbf{N_k} \mid \tag{5.100}$$

$$\sum_{r \in \mathbf{R_k}} T^{O}_{k,r} x_{k,n,r,f} - \sum_{r \in \mathbf{R_k}} T^{O}_{k,r} x_{k,n,r,f-1} \leq 0, \quad k \in \mathbf{K}, n \in \mathbf{N_k}, f = 2 \dots \mid \mathbf{F} \mid \tag{5.101}$$

Constraints (5.100) and (5.101) correspond to constraints (5.51) and (5.52) in the arc flow formulation respectively.

5.3.7 Optional cuts

The optional cut (5.76) presented in Section 5.2.7 also hold for the path flow formulation presented in this section.

It should also be mentioned that in the remaining chapters of this report, AFF, PFF1 and PFF2 formulations not including the optional symmetry breaking constraints and cut are termed basic models. AFF, PFF1 and PFF2 formulations including the optional symmetry breaking constraints and cut on the other hand are termed entire models.

Chapter 6

Generation of predefined routes

In this chapter, the generation of the predefined routes used in the path flow formulations PFF1 and PFF2 is described. An algorithm for generating all feasible routes performing maximum three offshore landings is presented in Section 6.1, while an algorithm for generating all feasible, non-dominated routes performing maximum three offshore landings is described in Section 6.2. Thereafter, a heuristic for generating feasible routes performing three or five offshore landings is presented in Section 6.3.

6.1 Generation of all feasible routes (RG1)

In this section, the generation of all feasible routes performing maximum three offshore landings is presented. The limit of three offshore landings is set as this adhere to Petrobras' present practise (see Section 2.1).

The generation of routes is carried out by iteratively creating and expanding routes starting from various initial start nodes. Three types of routes are created for every helicopter type: direct routing policy routes, and first and second echelon hub connected routing policy routes. The resulting three types of routes form the sets $\mathbf{R_k^1}$, $\mathbf{R_k^2}$ and $\mathbf{R_k^3}$ respectively. Depending on the type of route created, a route's start node is either an onshore airport base or an offshore transhipment hub. All direct and first echelon hub connected routing policy routes start in an onshore airport base. All second echelon hub connected routing policy routes on the other hand start in an offshore transhipment hub.

When performing the route generation, the following two terms are introduced for routes performing platform visits: partial routes and complete routes. A partial route is a route that does not end in the initial node from which it started, while a complete route is one that does end in its initial start node. These definitions are similar to the ones used by Andersson et al. (2011). Partial routes can be extended by either including another platform visit in the route, or by returning to the initial node from which the route started. When the latter is performed, the route becomes a complete route. Partial routes are therefore only used to develop complete routes, and only complete routes are used as predefined routes in the path flow formulations.

All routes created, both partial and complete, must fulfil a number of constraints in order to be classified as feasible. All direct routing policy routes created must satisfy Constraints (5.20), (5.21) and (5.23) presented in Section 5.1.2. Similarly, all hub connected routing policy routes created must satisfy Constraints (5.24), (5.29), (5.25), (5.27) and (5.30) presented in Section 5.1.3. As maximum three offshore landings is allowed for the routes generated in this section, $V^{P1,D}$ is set equal to three in Constraints (5.21) and $V^{P1,H}$ is set equal to two in Constraints (5.27). In addition, every route's total operating time cannot exceed the maximum operating time per flight. All complete routes must contain information about to which onshore airport base and/or offshore transhipment hub the route is connected to, the order in which platforms are visited in the route, the cost assessment of the route, and time needed to perform the route for the given helicopter type. No route is to contain information about the pickup and delivery of employees at the platforms visited.

The pseudo codes for the generation of all feasible direct and first echelon hub connected routing policy routes are given in Algorithms 1 and 2 on the following page respectively. All feasible second echelon hub connected routing policy routes can be generated by replacing the sets $\mathbf{R^1_k}$, $\mathbf{B}$ and $\mathbf{R^P_{b,k}}$ with the sets $\mathbf{R^3_k}$, $\mathbf{H}$ and $\mathbf{R^P_{h,k}}$ in the procedure depicted in Algorithm 1.

For all algorithms, the required input parameters are $C^{VA}_{i,j,k}$, $T^{FA}_{i,j,k}$, T^{OF}, $L^{FA}_{i,j}$ and L^R_k. The output of all algorithms are parameter matrices $A_{i,j,k,r}$, $V_{i,k,r}$, $L_{p,k,r}$, $C^{VR}_{k,r}$ and $T^{OR}_{k,r}$.

Algorithm 1 Generation of all feasible direct routing policy routes performing maximum three offshore landings (RG1)

```
for all $k \in \mathbf{K}$ do
    $\mathbf{R^1_k}$: Set of complete direct routing policy routes for helicopter type $k \in \mathbf{K}$
    $\mathbf{R^1_k} = \emptyset$
    for all $b \in \mathbf{B}$ do
        $\mathbf{R^P_{b,k}}$: Set of partial routes, start node $o(k) = b$, for helicopter type $k \in \mathbf{K}$
        while $\mathbf{R^P_{b,k}} \neq \emptyset$ do
            Select a partial route $r^P \in \mathbf{R^P_{b,k}}$
            for all $n \in \mathbf{P} \cup \{b\}$ do
                if $r^P$ can be extended by $n$ while remaining feasible then
                    $r_{new} = r^P$ extended by $n$
                    Create output matrices for $r_{new}$
                    if $n \in \mathbf{P}$ then
                        $\mathbf{R^P_{b,k}} = \mathbf{R^P_{b,k}} \cup \{r_{new}\}$
                    else if $n = b$ then
                        $\mathbf{R^1_k} = \mathbf{R^1_k} \cup \{r_{new}\}$
                    end if
                end if
            end for
            $\mathbf{R^P_{b,k}} = \mathbf{R^P_{b,k}} \setminus \{r^P\}$
        end while
    end for
end for
```

Algorithm 2 Generation of all feasible first echelon hub connected policy routes

```
for all $k \in \mathbf{K}$ do
    $\mathbf{R^2_k}$: Set of complete second echelon hub connected policy routes for helicopter type $k \in \mathbf{K}$
    $\mathbf{R^2_k} = \emptyset$
    for all $h \in \mathbf{H}$ do
        for all $p \in \mathbf{P}$ do
            if a route visiting nodes $h$ and $p$ is feasible then
                $r_{new}$ = route visiting nodes $h$ and $p$
                Create output matrices for $r_{new}$
                $\mathbf{R^2_k} = \mathbf{R^2_k} \cup \{r_{new}\}$
            end if
        end for
    end for
end for
```

6.2 Generation of all feasible, non-dominated routes (RG2)

In this section, the generation of all feasible, non-dominated routes performing maximum three offshore landings is presented. As in Section 6.1, the limit of three offshore landings is set as this adhere to Petrobras' present practise.

The background for this section is the following: When generating all feasible routes in Algorithms 1 and 2, the number of generated routes will enlarge as the number of elements in sets **B**, **H** and **P** increases. In particular, the number of predefined direct and second echelon hub connected routing policy routes will escalate as more and more offshore platforms are taken into consideration. This will greatly increase the number of variables in the two path flow formulations PFF1 and PFF2, as well as their computational complexity. This is likely to have a significant impact on the models solution time.

It is therefore of interest to reduce the number of predefined routes as much as possible, and especially direct and second echelon hub connected routing policy routes. This can be done by eliminating all routes that one knows will never be included in an optimal solution, in the generation process. When doing so, the following two terms are introduced: dominated routes and non-dominated routes. Dominated routes are routes that will never be included in an optimal solution, and that therefore can be excluded from the predefined set of feasible routes. Non-dominated routes on the other hand are routes that might be selected in an optimal solution, and that therefore cannot be excluded from the predefined set of feasible routes.

A route can be classified as dominated if it exists another route that can perform all combinations of pickup and delivery of employees performed in the first with a lower variable cost and a lower usage of time (Andersson et al. (2011)). Thus, dominated routes can be identified and eliminated from the set of predefined routes by iteratively comparing all complete routes visiting the same nodes in terms of these criteria. It should be noted that as all predefined routes are generated for a specific helicopter type, only routes created for the same helicopter type can be compared.

Still, by use of reason, some routes can be excluded even before the start of the generation process. Doing so is of great advantage as it limits the scale of the elimination procedure, and therefore also reduces its computational complexity.

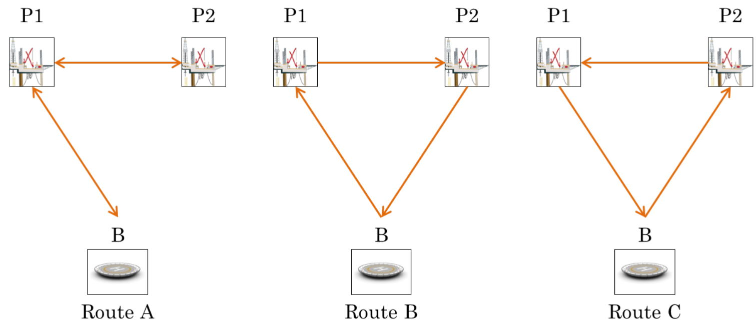

Figure 6.1: Possible flight routes visiting platforms P1 and P2

Algorithm 1 presented in Section 6.2 generates routes performing three platform landings on two different platforms. However, it can be proven that such routes always can be termed dominated, and can therefore be excluded from the route generation process. The proof is given in Proposition 6.1. For simplicity reasons, indices k, n and f are omitted from all variables and parameters in this proposition.

Suppose we have two platforms P1 and P2 that are to be visited during a particular flight. No other platforms are to be or can be visited during this flight. There are three manners in which the flight can be performed. Firstly, the helicopter in use can make use of a route A during which it first flies to platform P1, then flies to platform P2, and then flies back to platform P1 before returning to its original departure point. Secondly, the helicopter can make use of a route B during which it first flies to platform P1, and then flies to platform P2 before returning to its original departure point. Thirdly, the helicopter can make use of a route C during which it first flies to platform P2, and then flies to platform P1 before returning to its original departure point. These three routes are illustrated in Figure 6.1.

Route A is dominated by route B if and only if one can show that $C_A^V \geq C_B^V$, $T_A^{OR} \geq T_B^{OR}$, and that all combinations of pickup and deliveries that can be performed during route A also can be performed by route B when taking the passenger capacity of helicopter type k into consideration. Similarly, route A is dominated by route C if and only if the same can be proven when comparing routes A and C.

Proposition 1. *A helicopter will never make use of a route A, illustrated in Figure 6.1, during which three landings are made on two platforms P1 and P2. Such a route is always dominated by a route B or C during which the same platforms are visited, but only two landings are performed.*

Proof. Let us start by looking at the costs and operating times for routes A, B and C. We have that

$$\begin{aligned} C_A^V &= C_{B,P1}^{VA} + C_{P1,P2}^{VA} + C_{P2,P1}^{VA} + C_{P1,B}^{VA} \\ C_B^V &= C_{B,P1}^{VA} + C_{P1,P2}^{VA} + C_{P2,B}^{VA} \\ C_C^V &= C_{B,P2}^{VA} + C_{P2,P1}^{VA} + C_{P1,B}^{VA} \\ T_A^{OR} &= T_{B,P1}^{FA} + T_{P1,P2}^{FA} + T_{P2,P1}^{FA} + T_{P1,B}^{FA} \\ T_B^{OR} &= T_{B,P1}^{FA} + T_{P1,P2}^{FA} + T_{P2,B}^{FA} \\ T_C^{OR} &= T_{B,P2}^{FA} + T_{P2,P1}^{FA} + T_{P1,B}^{FA} \end{aligned} \tag{6.1}$$

It follows from the triangle inequality that $C_A^V \geq C_B^V$, $C_A^V \geq C_C^V$, $T_A^{OR} \geq T_B^{OR}$ and $T_A^{OR} \geq T_C^{OR}$.

Let us then take a look at the pickup and deliveries that can be performed during the various routes. During routes A, the pickup and delivery loads on the various arcs are

$$\begin{aligned} p_{B,P1} + d_{B,P1} &= e_{P1}^D + e_{P2}^D \leq Q^P \\ p_{P1,P2} + d_{P1,P2} &= e_{P2}^D \leq Q^P \\ p_{P2,P1} + d_{P2,P1} &= e_{P2}^P \leq Q^P \\ p_{P1,B} + d_{P1,B} &= e_{P1}^P + e_{P2}^P \leq Q^P \end{aligned} \tag{6.2}$$

It is assumed that no pickup of employees is perfomed during the first visit to platform P1. Doing so would only reduce the helicopter's passenger capacity on arcs (P1,P2) and (P2,P1), which could place an excessive constraint on e_{P2}^D.

Similarly, the pickup and delivery loads on the various arcs during route B are

$$\begin{aligned} p_{B,P1} + d_{B,P1} &= e_{P1}^D + e_{P2}^D \leq Q^P \\ p_{P1,P2} + d_{P1,P2} &= e_{P1}^P + e_{P2}^D \leq Q^P \\ p_{P2,B} + d_{P2,B} &= e_{P1}^P + e_{P2}^P \leq Q^P \end{aligned} \tag{6.3}$$

By comparing the two set of equations 6.2 and 6.3, one get that route B can perform all combinations of pickup and deliveries performed during route A if $e_{P1}^P + e_{P2}^D \leq Q^P$. Thus, if this equation holds, route A is dominated by route B. On the contrary, if $e_{P1}^P + e_{P2}^D \geq Q^P$, route A can perform a combination of pickup and deliveries that cannot be performed by route B. Thus, if this latter equation holds, route A is not dominated by route B.

Let us then look at route C. During this route, the pickup and delivery loads on the various arcs are

$$\begin{aligned} p_{B,P2} + d_{B,P2} &= e^D_{P1} + e^D_{P2} \leq Q^P \\ p_{P2,P1} + d_{P2,P1} &= e^D_{P1} + e^P_{P2} \leq Q^P \\ p_{P1,B} + d_{P1,B} &= e^P_{P1} + e^P_{P2} \leq Q^P \end{aligned} \tag{6.4}$$

By comparing the two set of equations 6.2 and 6.4, one get that route C can perform all combinations of pickup and deliveries performed during route A if $e^D_{P1} + e^P_{P2} \leq Q^P$. Thus, if this equation holds, route A is dominated by route C. Let us therefore examine whether this equation holds when route A is not dominated by route B, e.g. when $e^P_{P1} + e^D_{P2} \geq Q^P$. In such a scenario, the extreme values for all combinations of pickup and deliveries that can be performed by route A are

$$\begin{aligned} e^P_{P1} &= [(Q^P - e^D_{P2} + 1), Q^P] \\ e^P_{P2} &= [0, (Q^P - e^P_{P1})] \\ e^D_{P1} &= [0, (Q^P - e^D_{P2})] \\ e^D_{P2} &= [(Q^P - e^P_{P1} + 1), Q^P] \end{aligned} \tag{6.5}$$

This gives

$$\begin{aligned} & e^D_{P1} + e^P_{P2} \leq \\ & (Q^P - e^D_{P2}) + (Q^P - e^P_{P1}) = \\ & 2Q^P - (e^P_{P1} + e^D_{P2}) \leq Q^P_k \end{aligned} \tag{6.6}$$

Thus, if $e^P_{P1} + e^D_{P2} \leq Q^P$, route A is dominated by route B. If $e^P_{P1} + e^D_{P2} \geq Q^P$, route A is dominated by route C. Ergo, route A is always dominated by route B or C. □

Following the presentation of dominated and non-dominated routes and the proof given in Proposition 6.1, the pseudo code for the generation of all feasible, non-dominated direct routing policy routes is given in Algorithm 3 on the following page. All feasible, non-dominated second echelon hub connected routing policy routes can be generated by replacing the sets $\mathbf{R^1_k}$, $\mathbf{B}$ and $\mathbf{R^P_{b,k}}$ with the sets $\mathbf{R^3_k}$, $\mathbf{H}$ and $\mathbf{R^P_{h,k}}$ in the procedure depicted in the algorithm. First echelon hub connected routing policy routes on the other hand cannot be tested for dominance as they do not perform any offshore landings. Algorithm 2 presented in Section 6.1 is therefore still applicable for this type of routes.

The feasibility requirements for Algorithm 3 are equal to those for Algorithm 1. Similarly, the required input parameters are $C^{VA}_{i,j,k}$, $T^{FA}_{i,j,k}$, T^{OF}, $L^{FA}_{i,j}$ and L^R_k, and the output of the algorithm is parameter matrices $A_{i,j,k,r}$, $V_{i,k,r}$, $L_{p,k,r}$, $C^{VR}_{k,r}$ and $T^{OR}_{k,r}$.

Algorithm 3 Generation of all feasible, non-dominated direct routing policy routes performing maximum three offshore landings (RG2)

1: **for all** $k \in \mathbf{K}$ **do**
2: $\mathbf{R^1_k}$: Set of complete, non-dominated direct routing policy routes for helicopter type $k \in \mathbf{K}$
3: $\mathbf{R^1_k} = \emptyset$
4: **for all** $b \in \mathbf{B}$ **do**
5: $\mathbf{R^P_{b,k}}$: Set of partial routes, start node $o(k) = b$, for helicopter type $k \in \mathbf{K}$
6: **while** $\mathbf{R^P_{b,k}} \neq \emptyset$ **do**
7: Select a partial route $r^P \in \mathbf{R^P_{b,k}}$
8: **for all** $n \in \mathbf{P} \cup \{b\}$ **do**
9: **if** r^P can be extended by n while remaining feasible **then**
10: $r_{new} = r^P$ extended by n
11: Create output matrices for r_{new}
12: **if** $n \in \mathbf{P}$ **then**
13: $\mathbf{R^P_{b,k}} = \mathbf{R^P_{b,k}} \cup \{r_{new}\}$
14: **else if** $n = b$ **then**
15: **if** r_{new} is dominated by a route $r^1 \in \mathbf{R^1_k}$ **then**
16: $\mathbf{R^P_{b,k}} = \mathbf{R^P_{b,k}} \setminus \{r^P\}$
17: Return to line 6
18: **else if** r_{new} dominates a route $r^1 \in \mathbf{R^1_k}$ **then**
19: $\mathbf{R^1_k} = \mathbf{R^1_k} \setminus \{r^1\}$
20: **end if**
21: $\mathbf{R^1_k} = \mathbf{R^1_k} \cup \{r_{new}\}$
22: **end if**
23: **end if**
24: **end for**
25: $\mathbf{R^P_{b,k}} = \mathbf{R^P_{b,k}} \setminus \{r^P\}$
26: **end while**
27: **end for**
28: **end for**

6.3 Heuristic generation of feasible routes (RG3 and RG4)

In this section, a heuristic for generating feasible routes performing maximum three or five offshore landings is presented. The limit of five offshore landings is set as this adhere to Petrobras' maximum number of offshore landings in a flight (see Section 2.1).

The choice of also generating routes in a heuristic manner has been made due to reasons similar to the justification for eliminating dominated routes in Section 6.2. As the number of elements in set **P** increases, the number of predefined direct and second echelon hub connected routing policy routes will escalate. This will greatly increase the number of variables in the two path flow formulations, and is likely to have a significant impact on the models solution time. Additionally, it is possible that the process of removing routes by performing dominance testing will come short for certain types of routes. This is due to the formulation of the algorithm, which takes the passenger capacity of the helicopter type in use into consideration. This means that if the total pickup and/or delivery demand of passengers at the platforms to be visited is greater than the helicopter's passenger capacity, no routes can be termed dominated and thereby excluded from the generation process. This may occur for routes performing up to three offshore landings, and is even more likely to occur for routes performing up to five offshore landings.

In order to limit the number of predefined routes, a heuristic approach for generating routes performing maximum three or five offshore landings has therefore also been developed. When doing so, the geographic locations of the various offshore platforms are made use of. By assuming that shuttling is most likely to occur between platforms situated in the same area, the platforms visited during all generated routes are restricted to being placed in the same offshore oil and gas field. It is reasonable to believe that adding this restriction to the generation process will eliminate the number of predefined routes without to a great extent excluding routes that would have been included in an optimal solution.

Following this argumentation, the pseudo code for the heuristic generation of feasible direct routing policy routes performing maximum three offshore landings is given in Algorithm 4 on page 63. The heuristic is to a great extent similar to the one presented in Algorithm 3 in Section 6.2. However, as described in the previous paragraph, the predefined routes generated by Algorithm 4 are limited to only perform offshore landings on platforms situated in the same oil and gas field. This characteristic is described by using the set $\mathbf{F}$, whose elements represent the various offshore oil and gas fields. This set must not be misinterpret as the set $\mathbf{F}$ used in Chapter 5, which consisted of possible daily flights for every available helicopter. Similar to Algorithms 1 and 3, feasible second echelon hub connected routing policy routes performing maximum three offshore landings can be generated by replacing the sets $\mathbf{R^1_k}$, $\mathbf{B}$ and $\mathbf{R^P_{b,k,f}}$ with the sets $\mathbf{R^3_k}$, $\mathbf{H}$ and $\mathbf{R^P_{h,k,f}}$ in the procedure depicted in Algorithm 4. For reasons similar to those given in Section 6.2, no algorithm for heuristic generation of first echelon hub connected routing policy routes is given in this section. Algorithm 2 presented in Section 6.1 is therefore also here still applicable for this type of routes.

The pseudo code for the heuristic generation of feasible direct routing policy routes performing four and five offshore landings on the other hand, is given in Algorithm 5 on page 64. Also here, the set $\mathbf{F}$ is used to limit the offshore landings in a route to platforms located in the same oil and gas field. Notice that the creation of routes performing up to three offshore landings is not included in the algorithm. If a heuristic generation of routes performing up to five offshore landings is desirable, Algorithm 5 must therefore be used in unison with Algorithm 4. Similar to Algorithm 4, feasible second echelon hub connected routing policy routes performing four and five offshore landings can be generated by replacing the sets $\mathbf{R^1_k}$, $\mathbf{B}$ and $\mathbf{R^P_{b,k,f}}$ with the sets $\mathbf{R^3_k}$, $\mathbf{H}$ and $\mathbf{R^P_{h,k,f}}$ in the procedure depicted in Algorithm 5.

For Algorithm 4, the feasibility requirements are equal to those for Algorithm 1. For Algorithm 5 on the other hand, all feasibility requirements are identical exept for the ones of Constraints (5.21) and (5.27). As maximum five offshore landings are allowed for routes generated by this algorithm, $V^{P1,D}$ is set equal to five in Constraints (5.21) and $V^{P1,H}$ is set equal to four in Constraints (5.27). The required input parameters for all algorithms are $C^{VA}_{i,j,k}$, $T^{FA}_{i,j,k}$, T^{OF}, $L^{FA}_{i,j}$ and L^R_k. The output of all algorithms are parameter matrices $A_{i,j,k,r}$, $V_{i,k,r}$, $L_{p,k,r}$, $C^{VR}_{k,r}$ and $T^{OR}_{k,r}$.

Algorithm 4 Heuristic generation of feasible direct routing policy routes performing maximum three offshore landings (RG3)

1: $\mathbf{F}$: Set of offshore oil and gas fields
2: **for all** $k \in \mathbf{K}$ **do**
3:     $\mathbf{R^1_k}$: Set of complete direct routing policy routes performing maximum three offshore landings for helicopter type $k \in \mathbf{K}$
4:     $\mathbf{R^1_k} = \emptyset$
5:     **for all** $b \in \mathbf{B}$ **do**
6:         **for all** $f \in \mathbf{F}$ **do**
7:             $\mathbf{P^S_f}$: Subset of platforms located in the same field $f \in \mathbf{F}$, $\mathbf{P^S_f} \subseteq \mathbf{P}$
8:             $\mathbf{R^P_{b,k,f}}$: Set of partial routes, start node $o(k) = b$, performing maximum two offshore landings in field $f \in \mathbf{F}$ for helicopter type $k \in \mathbf{K}$
9:             **while** $\mathbf{R^P_{b,k,f}} \neq \emptyset$ **do**
10:                 Select a partial route $r^P \in \mathbf{R^P_{b,k,f}}$
11:                 **for all** $n \in \mathbf{P^S_f} \cup \{b\}$ **do**
12:                     **if** r^P can be extended by n while remaining feasible **then**
13:                         $r_{new} = r^P$ extended by n
14:                         Create output matrices for r_{new}
15:                         **if** $n \in \mathbf{P^S_f}$ **then**
16:                             $\mathbf{R^P_{b,k,f}} = \mathbf{R^P_{b,k,f}} \cup \{r_{new}\}$
17:                         **else if** $n = b$ **then**
18:                             **if** r_{new} is dominated by a route $r^1 \in \mathbf{R^1_k}$ **then**
19:                                 $\mathbf{R^P_{b,k,f}} = \mathbf{R^P_{b,k,f}} \setminus \{r^P\}$
20:                                 Return to line 9
21:                             **else if** r_{new} dominates a route $r^1 \in \mathbf{R^1_k}$ **then**
22:                                 $\mathbf{R^1_k} = \mathbf{R^1_k} \setminus \{r^1\}$
23:                             **end if**
24:                             $\mathbf{R^1_k} = \mathbf{R^1_k} \cup \{r_{new}\}$
25:                         **end if**
26:                     **end if**
27:                 **end for**
28:                 $\mathbf{R^P_{b,k}} = \mathbf{R^P_{b,k}} \setminus \{r^P\}$
29:             **end while**
30:         **end for**
31:     **end for**
32: **end for**

Algorithm 5 Heuristic generation of feasible direct routing policy routes performing four and five offshore landings (RG4)

$\mathbf{F}$: Set of offshore oil and gas fields
for all $k \in \mathbf{K}$ **do**
  $\mathbf{R^1_k}$: Set of complete direct routing policy routes performing maximum three platform visits for helicopter type $k \in \mathbf{K}$
  for all $b \in \mathbf{B}$ **do**
    for all $f \in \mathbf{F}$ **do**
      $\mathbf{P^S_f}$: Subset of platforms located in the same field $f \in \mathbf{F}$, $\mathbf{P^S_f} \subseteq \mathbf{P}$
      $\mathbf{R^P_{b,k,f}}$: Set of partial routes, start node $o(k) = b$, performing more than two offshore landings in field $f \in \mathbf{F}$ for helicopter type $k \in \mathbf{K}$
      while $\mathbf{R^P_{b,k,f}} \neq \emptyset$ **do**
        Select a partial route $r^P \in \mathbf{R^P_{b,k,f}}$
        for all $n \in \mathbf{P^S_f} \cup \{b\}$ **do**
          if r^P can be extended by n while remaining feasible **then**
            $r_{new} = r^P$ extended by n
            Create output matrices for r_{new}
            if $n \in \mathbf{P^S_f}$ **then**
              $\mathbf{R^P_{b,k,f}} = \mathbf{R^P_{b,k,f}} \cup \{r_{new}\}$
            else if $n = b$ **then**
              $\mathbf{R^1_k} = \mathbf{R^1_k} \cup \{r_{new}\}$
            end if
          end if
        end for
        $\mathbf{R^P_{b,k,f}} = \mathbf{R^P_{b,k,f}} \setminus \{r^P\}$
      end while
    end for
  end for
end for

Chapter 7

Implementation in commercial optimization software

In accordance with the purpose of this report, the mathematical formulations presented in Chapter 5 and the route generation algorithms presented in Chapter 6 have been implemented in commercial optimization software. In this chapter, this implementation process is described.

In this work, the author made use of Xpress-IVE Version 1.20.10, in which Xpress Mosel was used as modelling language. This selection of software was made as it is well suited for solving MILP problems. When running a model, Xpress-IVE start by reducing the problem at hand by applying various numerical methods in a process called pre-solve. Further, the dual simplex method is applied in order to find the LP relaxation of the problem. Lastly, B&B is applied in order to find improved lower bounds and feasible integer solutions. The relative difference between the best lower bound and best feasible integer solution is then expressed through the MIP gap. Thus, the best integer solution may have been found although a gap value greater than zero is obtained.

Each of the three mathematical formulations presented in Chapter 5 were implemented as six separate models named AFF, PFF1 and PFF2 Entire and

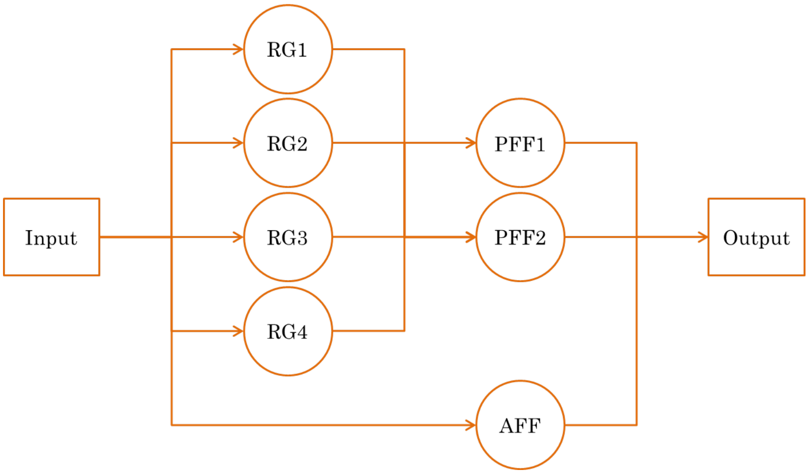

Figure 7.1: Data flow diagram for implemented models

Basic respectively in Xpress-IVE. The formulations were to a great extent modelled in the same way as they are formulated in Chapter 5. However, for the arc flow formulation, constraints (5.34) and (5.35) were defined when declaring the model's variables instead of being modelled as constraints. The same choice was made for constraints (5.68) and (5.69), and (5.89) and (5.90) in the first and second path flow formulations respectively.

Similarly, the four types of route generation algorithms presented in Chapter 6 were implemented as four separate models named RG1, RG2, RG3 and RG4 respectively. It should be noted however that Algorithm 2 was included in all four models, and Algorithm 4 was also included in RG4. This was done in order to avoid the need of linkage between the various models. A data flow diagram displaying the way in which all implemented models relate to one another and the process' input and output data, is presented in Figure 7.1.

After implementation, AFF, PFF1 and PFF2 Basic and Entire were run a selected number of times with various input data. For comparison reasons, all runs were made on the same computer. This was a HP dl165 G6 computer having two AMD Opteron 2431 processors with a clock speed of 2.4 GHz, and an installed memory of 24 GM (RAM). From this point forward, running the models with a particular set of input data is termed a test instance.

Chapter 8

Generation of test instances

No real input data has been available for use during the completion of this report. Therefore, reasonable parameters were developed by the author. In this chapter, this generation of parameters for various test instances is presented.

The creation of parameters relating to various bases, hubs and platforms is presented in Section 8.1. The establishment of parameters relating to various helicopter types on the other hand, is presented in Section 8.2, while the creation of risk parameters is presented in Section 8.3. Lastly, an overview of generated test instances is given in Section 8.4.

8.1 Establishment of base, hub and platform parameters

In this section, all parameters related to onshore airport bases, offshore transshipment hubs and offshore platforms are established. Firstly, the locations for these installations are determined. Two different sets of locations were established for this purpose. In the first set of locations, onshore airport bases that are to serve future helicopter operations in the Santos Basin for certain were used as bases. These are the two bases located in Jacarepaguá and Cabo Frio. Platform locations on the other hand were developed by using Petrobras' business plan for production units and rigs in the pre-salt field till

2020 (Petrobras (2012)). In this plan, the positions of these units in terms of oil and gas fields were given. The locations of future platforms in the Santos Basin were then approximated by using of the coordinates of the pre-salt fields in which production units and rigs are to be situated. This process resulted in total 41 platform locations.

Possible hub locations were selected by following three steps: First, the area in which any hub could be located was enclosed by a line connecting the most eastern base and platform, and a line connecting the most western base and platform. Installing a hub outside these borders will only cause an increase transportation costs. The author therefore found it unreasonable to include this option in the data set. Second, a set of hub locations were located on a line laying between the Brazilian coast and various platform locations. Third, a set of hub locations were located on a line laying in parallel to the former, but within the area in which the different platforms were located. A visualization of this first set of locations, as well as the lines used when selecting the various potential hub locations, is given in Figure 8.1. Due to the large number of platform locations, only the various oil and gas fields are depicted in the figure.

In the second set of locations, onshore airport bases that are to or might serve future helicopter operations in the Santos Basin, were used as bases. These are the four bases located in Guarujá, Itaguaí, Jacarepaguá and Cabo Frio. The same set of platform locations as used in the first set of locations was used. Also, all potential hub locations were found in the manner described in the previous paragraph. A visualization of the second set of locations, as well as the lines used when selecting the potential hub locations, is given in Figure 8.2. Also in this figure, only the various oil and gas fields are depicted due to the large number of platform locations.

Secondly, other parameters related to onshore airport bases, offshore transshipment hubs and offshore platforms are determined. For the bases that might be built in Guarujá and Itaguaí, their available parking capacity Q_b^H was set to 20 as this was in accordance with Petrobras' requirements for parking positions at a new airport in Farol de São Tomé (Correia (2006)). For the existing bases in Jacarepaguá and Cabo Frio on the other hand, this parameter was set to 10. This number was selected in order to properly differentiate between parking capacities at bases operated by Petrobras and by other parties.

The fixed investment cost C^{FH} of every hub installed was selected by looking at daily rates for offshore floatels. These are semi-submersible vessels used as accommodation bases for offshore workers, and is therefore an installation

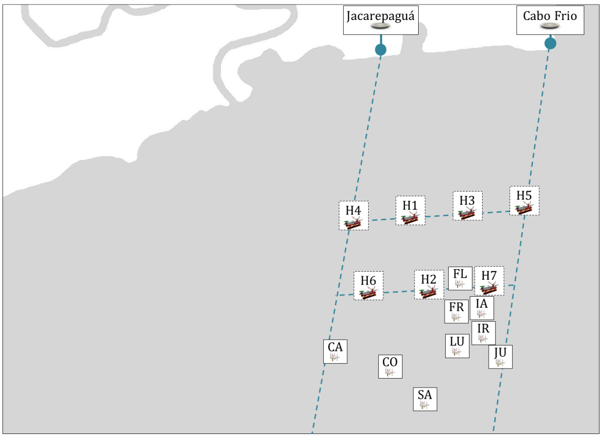

Figure 8.1: Visualisation of first set of base, hub and platform locations (CA:Carcará, CO:Carioca, FL:Florim, FR:Franco, IA:Iara, IR:Iracema, JU:Júpiter, LU:Lula, SA:Sapinhoá)

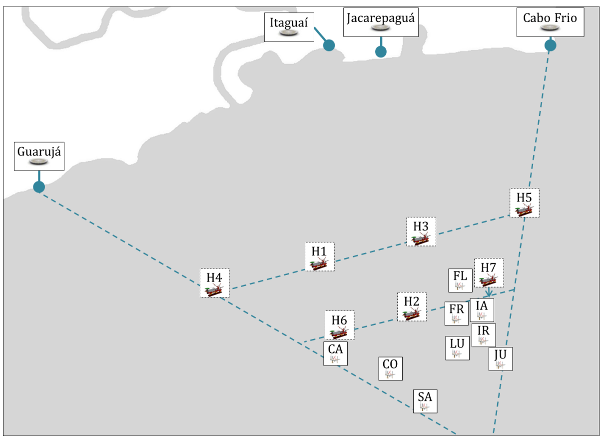

Figure 8.2: Visualisation of second set of base, hub and platform locations (CA:Carcará, CO:Carioca, FL:Florim, FR:Franco, IA:Iara, IR:Iracema, JU:Júpiter, LU:Lula, SA:Sapinhoá)

type inhabiting several characteristics similar to the offshore transhipment hubs proposed to be used in Petrobras' future helicopter operations. Also, as it is likely that hubs will be ordered under chartered parties, the author found it reasonable to estimate the fixed daily cost for every installed hub by using average international floatel day rates. This gave C^{FH} equal to \$ 150 000.

Pickup and delivery demands of employees D_p^P and D_p^D at every platform were generated by using the same mechanism for creating random demands as used by Dethloff (2001) and Qian et al. (2011) among others. The procedure was as follows: For every platform p, the delivery demand D_p^D was generated as an integer uniformly distributed in the interval [0,M]. The pickup demand D_p^P was then calculated by using the formula $D_p^P = D_p^D\ (0.5 + \alpha)$, in which α is an integer uniformly distributed in the interval [0,1]. Two different values were selected for the upper demand limit M, thus creating two different sets of pickup and delivery demands. For the first set, an upper limit of 36 was chosen in order to obtain average demands approximately equal to the average number of employees transported to and from platforms in the Campos Basin every day. For the second set, an upper limit of 36 multiplied by 1.06 was chosen in order to obtain average demands approximately equal to the maximum number of employees transported in the Campos Basin every day (see Section 2.1).

For all bases, hubs and platforms, the installations take-off and landing capacities Q_b^{TB}, Q^{TH} and Q_p^{TP} were set equal to 48. This number was selected by dividing the maximum operating time per day by the estimated time for every take-off and landing procedure. The flying lengths $L_{i,j}^{FA}$ between various installations were estimated by using their respective coordinates and calculating the Euclidean distances between them by the using the formula for great-circle distances. The generated values for pickup and delivery demands, as well as all distance matrices, are to be found in Appendix E.

8.2 Establishment of helicopter parameters

In this section, all parameters related to the various helicopter types are established. Firstly, the set of helicopter types are determined. When doing so, the author made use of the aircraft currently in use by Petrobras. These are the helicopter types Sikorsky S-76, Agusta Westland AW139, Sikorsky S-92 and Eurocopter EC-225 (Sena (2013)). Although other aircraft types also are available in the Brazilian helicopter market, this choice was made in order to

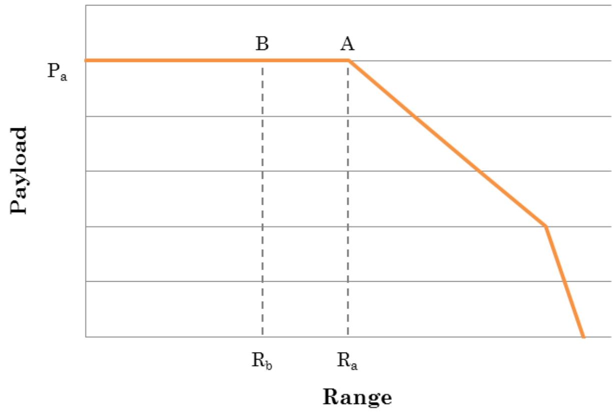

Figure 8.3: Typical relationship between helicopter payload and range (adapted from Horonjeff et al. (2010))

obtain relevant results. In accordance with the information given in Chapter 2, the four helicopter types selected are both medium and heavy twin engine helicopters. S-76, AW139 and S-92 are medium twin engine helicopters, while EC-225 is a heavy twin engine helicopter.

Secondly, the various parameters for these helicopter types are determined. The helicopter passenger capacities Q_k^P were set equal to the maximum passenger capacities of the various helicopter types, which gave values varying between 12 and 19 passengers. Further, the maximum ranges of the aircraft when carrying this amount of passengers were used as values for the helicopter ranges L_k^R. These values correspond to the point A in Figure 8.3. This procedure resulted in helicopter range values varying between 300 and 870 km. For sensitivity analysis purposes, helicopter ranges corresponding to increasing flying distances were also developed. These values correspond to the point B in Figure 8.3. This procedure resulted in helicopter range values varying between 205 and 595 km.

Further, the required landing and inspection times at the various installations were both set to 15 minutes as offshore turnaround time normally varies between 10 and 15 minutes (Qian (2012)). Together with the cruise speeds of the various helicopter types, these values were used in order to estimate the helicopter types' flying time $T_{i,j,k}^{FA}$ between various installations. Moreover, as Brazil is located quite close to the equator, the maximum operating time per day T^{OD} for every helicopter in use was set equal to 12 as this is an appropriate approximation of the time between dusk and dawn. The maximum operating time per flight T^{OF} on the other hand was set equal to 6 in accordance with the standard set by Oil & Gas UK (OGUK (2012)).

The cost parameters for the various helicopter types were determined by using various sources. The fixed investment costs C_k^{FK} for the various helicopter types were estimated to being in the range between $ 15 000 and $ 23 000 after consulting an industry representer (Molvik (2013)). The variable operating and parking costs $C_{i,j,k}^{VA}$ and $C_{b,k}^{VB}$ on the other hand were determined by using publicly available data from the U.S. Forest Service and INFRAERO (USFS (2013), INFRAERO (2013)). The time per day from which a helicopter's parking cost on an onshore airport base is to be derived, T^P, was set equal to 21 as this cost normally apply to an aircraft from the first three hours after landing.

8.3 Establishment of risk parameters

In this section, all risk parameters are established. In this work, the author made use of the information given in Chapter 2.

By assuming that the world-wide offshore aviation accident rates given by OGP (2000, 2001, 2002, 2003, 2004, 2005, 2006, 2007) are applicable to Brazilian helicopter operations, accident rates for the various helicopter types were developed by taking average values for medium and heavy twin engine helicopters. In order to obtain accident rates varying with both helicopter type and flight phases, these values were then combined with the accident frequency contributions of the various flight phases. Values for take-off and landing at heliport and helideck were converted to expected number of accidents per 100k flight stages by making use of the fact that average flight length in the OGP data set was 20 minutes, and assuming that each flight equals one flight stage.

In order to estimate the expected number of fatalities per person on board per 100k flight hours and 100k flight stages for cruise and take-off and landing procedures respectively, the obtained accident rates were then combined with the accident consequence contributions of the various flight phases. In order to obtain rates for the expected number of fatalities per person on board for cruise procedures between various installations, these values were combined with the number of flight hours between various installations for every helicopter type. The risk assessment $R^A_{i,j,k}$ of transporting an employee between various installations by a particular helicopter type, was then found by combining the various rates for the expected number of fatalities per person on board for various flight phases in an additive manner. It follows that $R^A_{i,j,k}$ is equal to the expected number of fatalities per person on board.

8.4 Overview of test instances

Following the establishment of the various input parameters, five different sets of test instances were developed. The first set of test instances was created for use in a technical analysis, while the latter four were developed for use in economic and accident risk analyses.

The various test instances differed in the number of onshore airport bases, potential locations for offshore transshipment hubs and offshore platforms taken into consideration, as well as in their use of pickup and delivery demands and helicopter ranges. All elements in the first set of test instances however made use of two airport bases and eight helicopters. Also, the first sets of pickup and delivery demands and helicopter ranges were always used. For all elements in the second to fifth sets of test instance, seven potential hub locations were always used. Also, the number of helicopters taken into consideration was always adjusted so that this would not place restrictions on the optimal solutions obtained. An overview of test instances for technical, and economic and accident risk analyses are given in Tables 8.1 and 8.2 respectively.

Set #	*Test instance*	*# of bases*	*# of hubs*	*# of platforms*	*# of helicopters*	*Set of D^P_p & D^D_p*	*Set of L^R_k*	F^W
1	S1_H3_P5	2	3	5	8	1	1	1.0
1	S1_H5_P5	2	5	5	8	1	1	1.0
1	S1_H7_P5	2	7	5	8	1	1	1.0
1	S1_H3_P10	2	3	10	8	1	1	1.0
1	S1_H5_P10	2	5	10	8	1	1	1.0
1	S1_H7_P10	2	7	10	8	1	1	1.0
1	S1_H3_P15	2	3	15	8	1	1	1.0
1	S1_H5_P15	2	5	15	8	1	1	1.0
1	S1_H7_P15	2	7	15	8	1	1	1.0

Table 8.1: Test instances for technical analysis

Set #	*Test instance*	*# of bases*	*# of hubs*	*# of platforms*	*# of helicopters*	*Set of D_p^P & D_p^D*	*Set of L_k^R*	F^W
2	S2_B2_2014	2	7	2	x	2	1	1.0
2	S2_B2_2015	2	7	3	x	2	1	1.0
2	S2_B2_2016	2	7	17	x	2	1	1.0
2	S2_B2_2017	2	7	28	x	2	1	1.0
2	S2_B2_2018	2	7	36	x	2	1	1.0
2	S2_B2_2019	2	7	39	x	2	1	1.0
2	S2_B2_2020	2	7	41	x	2	1	1.0
3	S3_B4_2014	4	7	2	x	2	1	1.0
3	S3_B4_2015	4	7	3	x	2	1	1.0
3	S3_B4_2016	4	7	17	x	2	1	1.0
3	S3_B4_2017	4	7	28	x	2	1	1.0
3	S3_B4_2018	4	7	36	x	2	1	1.0
3	S3_B4_2019	4	7	39	x	2	1	1.0
3	S3_B4_2020	4	7	41	x	2	1	1.0
4	S4_B2_2014	2	7	2	x	2	2	1.0
4	S4_B2_2015	2	7	3	x	2	2	1.0
4	S4_B2_2016	2	7	17	x	2	2	1.0
4	S4_B2_2017	2	7	28	x	2	2	1.0
4	S4_B2_2018	2	7	36	x	2	2	1.0
4	S4_B2_2019	2	7	39	x	2	2	1.0
4	S4_B2_2020	2	7	41	x	2	2	1.0
5	S5_H7_P10	2	7	10	x	1	1	0.0
5	S5_H7_P10	2	7	10	x	1	1	0.1
5	S5_H7_P10	2	7	10	x	1	1	0.2
5	S5_H7_P10	2	7	10	x	1	1	0.3
5	S5_H7_P10	2	7	10	x	1	1	0.4
5	S5_H7_P10	2	7	10	x	1	1	0.5
5	S5_H7_P10	2	7	10	x	1	1	0.6
5	S5_H7_P10	2	7	10	x	1	1	0.7
5	S5_H7_P10	2	7	10	x	1	1	0.8
5	S5_H7_P10	2	7	10	x	1	1	0.9
5	S5_H7_P10	2	7	10	x	1	1	1.0

Table 8.2: Test instances for economic and accident risk analysis

Chapter 9

Technical analysis

In this chapter, the results obtained when running AFF, PFF1 and PFF2 with the first set of test instances in Xpress-IVE are presented and discussed. The aim of the chapter is to examine the performance of the various models, as well as establishing how this is affected by the predefined routes and constraints in use.

Section 9.1 examines the results of the various route generation algorithms presented in Chapter 6. Thereafter, the performances of the three basic models are studied in Section 9.2. The effects of using various sets of predefined routes and optional constraints are then investigated in Sections 9.3 and 9.4 respectively. Based on these analyses, a conclusion is drawn regarding which model to pursue in the remaining chapters of this report. Section 9.5 then examines the effect of increasing the maximum number of allowed offshore landings in a route. Lastly, the effect of aggregating platform data is studied in Section 9.6.

9.1 Results generation of predefined routes

In this section, the results obtained when running the four route generation models RG1, RG2, RG3 and RG4 in Xpress-IVE are analysed. In accordance with the discussion in Chapter 6, maximum of three offshore landings were

Test instance	*RG1*	*RG2*	*RG3*	*RG4*
S1_H3_P5	867	765	183	183
S1_H5_P5	1,057	955	269	269
S1_H7_P5	1,273	1,171	357	357
S1_H3_P10	6,099	5,703	632	992
S1_H5_P10	6,828	6,423	853	1,437
S1_H7_P10	7,644	7,229	1,077	1,885
S1_H3_P15	20,419	19,300	1,933	5,533
S1_H5_P15	22,100	20,942	2,412	7,676
S1_H7_P15	23,916	22,718	2,894	9,822

Table 9.1: Number of generated routes RG1, RG2 and RG3

allowed for routes generated by the first three models RG1, RG2 and RG3. This corresponds to maximum three platform visits for direct routing policy routes, and maximum two platform visits for second echelon hub connected policy routes. For the fourth route generation model RG4 on the other hand, maximum of five offshore landings were allowed for the generated routes. This corresponds to maximum five platform visits for direct routing policy routes, and maximum four platform visits for second echelon hub connected policy routes.

The number of predefined routes generated by each of the route generation models is presented in Table 9.1. By examining the results, one can firstly notice that the number of predefined routes is much greater affected by the number of offshore platforms than the number of potential locations for offshore transhipment hubs. If comparing the number of routes generated by RG1 for test instances H3_P5 and H7_P5, one can observe that the number of generated routes is multiplied by 1.5 when the number of hub locations is increased by five. However, if comparing the number of routes generated by the same model for test instances H3_P5 and H3_P10, results show that the number of generated routes is multiplied by 7 when the number of platforms is increased by five. Thus the number of platforms has a significant impact on the number of routes generated.

Secondly, by comparing the number of routes generated by RG1 and RG2, one can notice that eliminating dominated routes causes only a small reduction in the number of predefined routes. For the various test instances, the reductions lay in the interval 5 to 12 %. It is realistic to assume that this occurrence is caused by the input data used. As seen from the pickup and delivery demands depicted in Appendix E, the majority of the platforms used in the various test instances have demands surpassing the maximum passenger capacity of all helicopter types. Because of this, there exists a great amount of routes in which one or several of these platforms are visited, that can never be termed dominated and thus never be eliminated from the generation process (see discussion in Section 6.3). Thus eliminating routes by dominance testing only has a small impact on the total number of generated routes for the problem examined in this report.

Thirdly, by comparing the number of routes generated by RG1, RG3 and RG4, one can notice that generating routes in a heuristic manner significantly reduces the number of predefined routes. For the various test instances in which maximum number of allowed offshore landings was set to three, the reductions lays in the interval 72 to 91 %. Increasing the maximum number of platform landings in a route causes the number of predefined routes to increase. Still, fewer routes are generated by RG4 in which maximum five offshore landings are allowed in a route than by both models RG1 and RG2 in which maximum three offshore landings are allowed.

Lastly, it should be mentioned that all predefined routes were generated within one second. Thus the need to pre-generate routes for the path flow models PFF1 and PFF2 has an insignificant impact on the models total solution time.

9.2 Results basic models

In this section, the results obtained when running the three models AFF, PFF1 and PFF2 Basic in Xpress-IVE are analysed. For the two path flow formulations, the routes generated by RG1 were used as predefined routes. For all runs, maximum allowed solution time was 2 hours.

Firstly, let us take a look at the sizes of the various models. These are illustrated in Table 9.2 on the following page, in which the number of rows and columns after pre-solve for all three models are given. From this table, one can notice that the number of rows in the two models AFF and PFF1 are approximately the same. However, the number of rows is significantly reduced in PFF2. For the various test instances, the number of rows in this second path flow model is about on fifth of the number of rows in AFF and PFF1. Also, by comparing the number of columns in the various models, one can see that the number of

	# rows after pre-solve			# columns after pre-solve		
Test instance	*AFF*	*PFF1*	*PFF2*	*AFF*	*PFF1*	*PFF2*
S1_H3_P5	20,334	17,754	4,614	29,888	38,372	12,958
S1_H5_P5	49,719	45,939	10,519	93,505	165,595	73,077
S1_H7_P5	90,704	85,774	18,454	191,830	431,320	228,100
S1_H3_P10	27,350	24,190	6,170	38,134	51,362	15,950
S1_H5_P10	62,645	58,235	13,255	109,567	209,447	82,249
S1_H7_P10	109,640	104,080	22,400	215,902	526,782	247,602
S1_H3_P15	34,566	30,926	7,786	46,784	65,854	19,242
S1_H5_P15	75,871	70,981	16,081	126,229	257,949	92,349
S1_H7_P15	128,876	122,836	26,436	240,574	629,294	268,514

Table 9.2: Rows and columns after pre-solve AFF, PFF1 and PFF2 Basic (RG1)

columns is considerably reduced in PFF2 compared to PFF1. For the various test instances, the second path flow model has about one third the number of columns in the first. These results confirm that PFF2 is an aggregated version of PFF1.

The number of rows and columns before pre-solve are not shown in Table 9.2. However, it should be mentioned that the number of rows and columns eliminated in the various pre-solve processes was maximum 11 and 3 % respectively. No significant variance in the percentage of constraints and variables eliminated was observed between the various models. Still, no columns were removed in the pre-solve process of PFF2.

Secondly, let us take a look at the objective values obtained in the LP relaxations of the various models when running the different test instances. These are given in Table 9.3. From the table, one can see that all three models have the same LP relaxation values for the various test instances. This is an important observation, as it signifies that all models solve the same problem and that their results therefore are comparable. Still, it is surprising to observe that the LP relaxations of the path flow models are not stronger than the one of the arc flow model. This indicates that the arc flow formulation obtained after pre-solve is a tight formulation.

Thirdly, let us take a look at the solution times and gap values obtained for the various models. In Table 9.3, one can see that only AFF is able to solve test instances to optimality within the allowed solution time. This occurs for

	Solution time [s]			*Gap [%]*			*LP*		
Test instance	*AFF*	*PFF1*	*PFF2*	*AFF*	*PFF1*	*PFF2*	*AFF*	*PFF1*	*PFF2*
S1_H3_P5	4,776.7	>7200	>7200	0.0	0.9	1.0	88,680	88,680	88,680
S1_H5_P5	6,356.3	>7200	>7200	0.0	2.9	2.6	88,680	88,680	88,680
S1_H7_P5	>7200	>7200	>7200	0.1	1.3	2.1	88,680	88,680	88,680
S1_H3_P10	>7200	>7200	>7200	10.2	10.8	4.3	172,352	172,351	172,351
S1_H5_P10	>7200	>7200	>7200	25.8	13.6	4.4	172,352	172,351	172,351
S1_H7_P10	>7200	>7200	>7200	16.9	14.3	9.1	172,352	172,351	172,351
S1_H3_P15	>7200	>7200	>7200	N/A*	N/A*	15.5	231,739	231,739	231,739
S1_H5_P15	>7200	>7200	>7200	18.5	53.2	17.7	231,739	231,739	231,739
S1_H7_P15	>7200	>7200	>7200	33.8	29.2	19.8	231,739	231,739	231,739

Table 9.3: Solution time, gap and LP AFF, PFF1 and PFF2 Basic (RG1)
*No MIP solution was found within maximum allowed solution time

the two smallest instances, H3_P5 and H5_P5. However, PFF2 is the only model that is able to obtain MIP solutions for all test instances within the maximum allowed solution time. Also, the gap values obtained for this model are generally lower than the ones obtained for AFF and PFF1 when the number of platforms increases. These results indicate that the reduction in model size in the second path flow model is beneficial for larger instances. This reduces the computational effort needed in every node in the B&B tree, which again increases the number of nodes that can be explored within the given time limit. Still, no model outperforms the others in terms of both solution time and gap values obtained.

Lastly, let us take a look at the objective values obtained by the various models. These are given in Table 9.4 on the following page. From the table, one can notice that all models obtain equal objection function values only for test instances H3_P5, H3_P10 and H3_P15. This result is not a necessity, but is also not surprising as small gap values were only obtained for all models for these three instances. For larger test instances, no consistency can be found in the MIP values obtained for similar reason.

It may therefore be of greater interest to look at the charter-in cost obtained by the various models for the all test instances. This cost encompasses all investment costs in helicopters and offshore transhipment hubs, and can therefore be said to address the objective of this report to a greater extent than the MIP values obtained. From Table 9.4, one can see that the same charter-in

	MIP			Charter-in cost		
Test instance	*AFF*	*PFF1*	*PFF2*	*AFF*	*PFF1*	*PFF2*
S1_H3_P5	104,209	104,209	104,209	36,857	36,857	36,857
S1_H5_P5	104,209	104,209	104,209	36,857	36,857	36,857
S1_H7_P5	104,209	104,209	104,209	36,857	36,857	36,857
S1_H3_P10	201,641	205,121	203,189	73,714	73,714	73,714
S1_H5_P10	238,746	206,489	204,492	85,072	73,714	73,714
S1_H7_P10	213,039	208,962	202,921	73,714	73,714	73,714
S1_H3_P15	N/A	N/A	291,524	N/A	N/A	99,745
S1_H5_P15	291,371	513,254	290,738	99,745	282,729	99,745
S1_H7_P15	358,818	339,114	300,962	125,776	125,776	99,745

Table 9.4: MIP and charter-in cost AFF, PFF1 and PFF2 Basic (RG1)

costs were obtained by all models for all test instances solved to a gap value less than 20 %. This means that even though optimality is not proven, the results obtained can speak greatly about the way in which Petrobras' should address its future upgrades in helicopter bases and fleet. The charter-in costs obtained will therefore also be used in the subsequent sections of this chapter when comparing the various results obtained.

9.3 Effect of reducing the number of predefined routes

In this section, the results obtained when running the two models PFF1 and PFF2 Basic in Xpress-IVE with routes generated by RG1, RG2 and RG3 are analysed. In particular, a look is taken at the effect of reducing the number of predefined routes by using the models RG2 and RG3. For all runs, the maximum allowed solution time was 2 hours.

Firstly, let us take a look at how the sizes of the various models are affected by a varying number of predefined routes. This can be seen in Table 9.5, in which the number of columns after pre-solve for models PFF1 and PFF2 is given. One can see from the table that removing dominated routes has little impact on the number of columns in both models. For all instances, the maximum reduction in number of columns is 8 %. However, by generating routes in a heuristic manner,

Test instance	PFF1			PFF2		
	RG1	*RG2*	*RG3*	*RG1*	*RG2*	*RG3*
S1_H3_P5	38,372	37,312	16,678	12,958	11,898	4,758
S1_H5_P5	165,595	159,395	43,167	73,077	67,957	12,333
S1_H7_P5	431,320	413,350	92,700	228,100	214,450	29,702
S1_H3_P10	51,362	50,302	24,470	15,950	14,890	6,710
S1_H5_P10	209,447	202,797	61,617	82,249	77,039	16,425
S1_H7_P10	526,782	506,862	128,062	247,602	233,562	37,184
S1_H3_P15	65,854	64,794	32,562	19,242	18,182	8,722
S1_H5_P15	257,949	250,799	80,519	92,349	87,039	20,607
S1_H7_P15	629,294	607,374	163,874	268,514	254,074	44,754

Table 9.5: Columns after pre-solve PFF1 and PFF2 Basic (RG1, RG2, and RG3)

the number of columns is reduced with up to 90 %. These results are in line with the variance in number of routes generated by RG1, RG2 and RG2 (see Section 9.1).

The number of rows in the various path flow models are not depicted in Table 9.5 as this is not affected by a varying number of predefined routes. Still, it should be mentioned that pre-solve was able to remove more rows when the predefined routes generated by RG3 were used than when the routes generated by RG1 and RG2 were applied. The greatest difference in reduction of rows was obtained for test instance H5_P5, for which pre-solve was able to remove over 50 % of the original rows when the RG3 routes were used and only about 9 % of the original rows when RG1 and RG2 routes were used.

Secondly, let us take a look at the objective values obtained in the LP relaxations of the PFF1 and PFF2 when run with different sets of predefined routes. These values can be found in Tables 9.6 and 9.7 on pages 82 and 83 respectively. From these tables, one can see that both PFF1 and PFF2 obtained the same LP values when solving the same test instances with the same set of predefined routes. This confirms that the two models solve the same problem. Further, one can notice that the LP values are generally higher when the predefined routes generated by RG3 are used. This indicates that when generating routes in a heuristic manner, one or several routes that would have been included in an optimal solution are not created.

Thirdly, let us take a look at the solution times and gap values obtained for

	Solution time [s]			*Gap [%]*			*LP*		
Test instance	*RG1*	*RG2*	*RG3*	*RG1*	*RG2*	*RG3*	*RG1*	*RG2*	*RG3*
S1_H3_P5	>7200	>7200	364.2	0.9	1.6	0.0	88,680	88,680	90,121
S1_H5_P5	>7200	>7200	325.2	2.9	1.1	0.0	88,680	88,680	90,121
S1_H7_P5	>7200	>7200	217.1	1.3	1.6	0.0	88,680	88,680	90,121
S1_H3_P10	>7200	>7200	>7200	10.8	12.5	2.5	172,351	172,351	175,227
S1_H5_P10	>7200	>7200	>7200	13.6	17.4	2.7	172,351	172,351	175,227
S1_H7_P10	>7200	>7200	>7200	14.3	22.3	7.7	172,351	172,351	175,227
S1_H3_P15	>7200	>7200	>7200	N/A	29.2	14.9	231,739	231,739	237,419
S1_H5_P15	>7200	>7200	>7200	53.2	34.0	52.0	231,739	231,739	237,419
S1_H7_P15	>7200	>7200	>7200	29.2	N/A	N/A	231,739	231,739	237,415

Table 9.6: Solution time, gap and LP PFF1 Basic (RG1, RG2 and RG3)

PFF1 and PFF2 when run with different sets of predefined routes. These values are also to be found in Tables 9.6 and 9.7. Let us first compare the results when running PFF1 and PFF2 with routes generated by RG1 and RG2. From the tables, one can notice that removing dominated routes has a positive impact on the solution times for PFF2. When this is done, the model is able to solve test instance H3_P5 to optimality within the allowed solution time. Still, for all other instances, using the routes generated by RG2 instead of the ones generated by RG1 causes little or no reduction in the gap values obtained for both models. For some test instances, the gap values even increase. These results are surprising, as reducing the sizes of the models reduces the computational complexity in every node in the B&B tree. This should again increase the number of iterations performed within the allowed solution time, which is expected to reduce the gap values obtained. However, due to the randomness of the B&B process, optimal nodes may be found both early and late in the tree search procedure. Thus although the increase in certain gap values is surprising, it is not illogical for test instances not solved to optimality.

Let us then compare the computational results obtained when running PFF1 and PFF2 with routes generated by RG1 and RG2, and RG3. From Tables 9.6 and 9.7, one can see that generating routes in a heuristic manner has a positive impact on the solution times for both PFF1 and PFF2. When this is done, the models are able to solve all test instances with five platforms to optimality within the allowed solution time. Also, the gap values obtained for all test instances are generally lower when this third set of predefined routes is used.

	Solution time [s]			*Gap [%]*			*LP*		
Test instance	*RG1*	*RG2*	*RG3*	*RG1*	*RG2*	*RG3*	*RG1*	*RG2*	*RG3*
S1_H3_P5	>7200	1,215.4	3,201.7	1.0	0.0	0.0	88,680	88,680	90,121
S1_H5_P5	>7200	>7200	3,529.1	2.6	1.1	0.0	88,680	88,680	90,121
S1_H7_P5	>7200	>7200	733.1	2.1	0.8	0.0	88,680	88,680	90,121
S1_H3_P10	>7200	>7200	>7200	4.3	9.5	2.5	172,351	172,351	175,227
S1_H5_P10	>7200	>7200	>7200	4.4	8.1	2.7	172,351	172,351	175,227
S1_H7_P10	>7200	>7200	>7200	9.1	8.1	3.2	172,351	172,351	175,227
S1_H3_P15	>7200	>7200	>7200	15.5	16.5	4.1	231,739	231,739	237,419
S1_H5_P15	>7200	>7200	>7200	17.7	11.7	4.6	231,739	231,739	237,419
S1_H7_P15	>7200	>7200	>7200	19.8	14.8	4.8	231,739	231,739	237,415

Table 9.7: Solution time, gap and LP PFF2 Basic (RG1, RG2 and RG3)

These results show that reducing the sizes of the models by limiting the number of predefined routes has a positive impact on the computational performance of the two path flow models.

Lastly, let us take a look at the objective values obtained by PFF1 and PFF2 when run with different sets of predefined routes. These values are given in Tables 9.8 and 9.9 on the following page. From the tables, one can see that the MIP values obtained when running the two models with routes generated by RG3 are lower than the MIP values obtained when using routes generated by RG1 and RG2 for all instances solved to gap values less than 10 %. Similar to the LP values, this result indicate that one or several routes that would have been included in an optimal solution are not created in the third route generation model. This limits the solution space for the two path flow models, which again causes an increase in the optimal MIP value. Still, one can also see from the tables that the charter-in costs obtained are not affected by the reduction in pre-defined routes for all test instances solved to gap values less than 20 %. This result on the other hand suggest that the heuristic generation of routes developed in Section 6.3 can be applied to the path flow models developed in this report without affecting the variables of interest. Due to this and its positive impact on the computational performance of the two path flow models, the heuristic route generation model RG3 is therefore used in all remaining parts of this report.

	MIP			*Charter-in cost*		
Test instance	*RG1*	*RG2*	*RG3*	*RG1*	*RG2*	*RG3*
S1_H3_P5	104,209	104,209	108,926	36,857	36,857	36,857
S1_H5_P5	104,209	104,209	108,926	36,857	36,857	36,857
S1_H7_P5	104,209	104,209	108,926	36,857	36,857	36,857
S1_H3_P10	205,121	207,843	205,338	73,714	73,714	73,714
S1_H5_P10	206,489	216,121	205,338	73,714	73,714	73,714
S1_H7_P10	208,962	230,999	206,778	73,714	77,561	73,714
S1_H3_P15	N/A	345,551	297,847	N/A	125,776	99,745
S1_H5_P15	513,254	365,256	529,472	282,729	125,776	308,760
S1_H7_P15	339,114	N/A	N/A	125,776	N/A	N/A

Table 9.8: MIP and charter-in cost PFF1 Basic (RG1, RG2 and RG3)

	MIP			*Charter-in cost*		
Test instance	*RG1*	*RG2*	*RG3*	*RG1*	*RG2*	*RG3*
S1_H3_P5	104,209	104,209	108,926	36,857	36,857	36,857
S1_H5_P5	104,209	104,209	108,926	36,857	36,857	36,857
S1_H7_P5	104,209	104,209	108,926	36,857	36,857	36,857
S1_H3_P10	203,189	203,928	205,338	73,714	73,714	73,714
S1_H5_P10	204,492	201,950	205,338	73,714	73,714	73,714
S1_H7_P10	202,921	204,330	205,338	73,714	73,714	73,714
S1_H3_P15	291,524	290,665	287,321	99,745	99,745	99,745
S1_H5_P15	290,738	287,686	287,268	99,745	99,745	99,745
S1_H7_P15	300,962	296,146	287,695	99,745	99,745	99,745

Table 9.9: MIP and charter-in cost PFF2 Basic (RG1, RG2 and RG3)

9.4 Effect of using optional constraints

In this section, the results obtained when running the three models AFF, PFF1 and PFF2 Entire in Xpress-IVE are analysed. In particular, a look is taken at the effect of adding the various optional constraints to the basic models discussed in Section 9.2. For the two path flow formulations, the routes generated by RG3 were used as predefined routes. For all runs, maximum allowed solution time was 2 hours.

Firstly, let us take a look at how the sizes of the various models are affected by adding the various optional constrains. This can be seen in Table 9.10 on the following page, in which the number of rows after pre-solve for all models is given. The number of columns is not depicted in this table as this is not affected by adding additional constraints to the model. One can see from the table that although adding constraints increases the number of rows in the original models, this is not always the case after the pre-solve process for the two models PFF1 and PFF2. For several test instances, the number of rows after pre-solve is greater for the basic path flow models than for the entire ones.

Secondly, let us take a look at the objective values obtained in the LP relaxations of the various entire models when running the different test instances. These values are not given in any tables, as the results obtained were equal to the LP values obtained for the basic models. Still, this is an important result and should be mentioned. The result confirms that the various optional constraints are feasible, and that adding them to the AFF, PFF1 and PFF2 models does not cut away any optimal solutions.

Thirdly, let us take a look at the solution times and gap values obtained for the three entire models. These values can to be found in Table 9.12 on page 87. Comparable values for the three basic models are to be found in Table 9.11 on the same page. By examining the values in these two tables, one can see that adding the various optional constraints to the models has a positive impact on the solution time for all instances solved to optimality for the three models AFF, PFF1 and PFF2. This indicates that adding the various optional constraints have a positive effect on all complete B&B tree search processes. This is a reasonable result, as adding these constraints allow additional feasible cuts to be made in the search tree. Such a procedure is likely to reduce the number of nodes that needs to be explored, which presumably also reduces the process' total solution time.

	Basic			*Entire*		
Test instance	*AFF*	*PFF1*	*PFF2*	*AFF*	*PFF1*	*PFF2*
S1_H3_P5	20,334	17,754	4,614	20,620	13,810	3,919
S1_H5_P5	49,719	45,939	10,519	49,919	28,222	7,435
S1_H7_P5	90,704	85,774	18,454	90,904	46,264	11,603
S1_H3_P10	27,350	24,190	6,170	27,375	20,342	5,480
S1_H5_P10	62,645	58,235	13,255	62,845	40,495	10,171
S1_H7_P10	109,640	104,080	22,400	109,840	64,570	15,549
S1_H3_P15	34,566	30,926	7,786	34,766	27,078	7,094
S1_H5_P15	75,871	70,981	16,081	76,071	53,241	12,997
S1_H7_P15	128,876	122,836	26,436	129,076	83,326	19,586

Table 9.10: Rows after pre-solve AFF, PFF1 and PFF2 Basic and Entire (RG3)

However, one can also see from the tables that adding the various optional constraints to the models shows negative effects on the results obtained when solving larger test instances. This can be seen by the gap values obtained, which are significantly larger for the entire AFF and PFF1 models than for the basic ones. Similar results can also be seen for the PFF2 model when solving test instances H3_P15 and H7_P15. These results are surprising, as one would expect the solution times and gap values to reduce when adding various symmetry breaking constraints since additional cuts can be made in the B&B tree. Still, doing so also increases the computational complexity in every node. In addition, multiple solutions that could have been included in an optimal solution in the basic models are now termed infeasible. In tree search processes during which good solutions are not found rapidly, adding the various optional constraints may therefore increase the processes' solution time. This result is also reflected in the general increase in time to find the first MIP solutions for the entire AFF, PFF1 and PFF2 models.

Nevertheless, the results given in Tables 9.11 and 9.12 indicate that the second path flow model PFF2 shows significantly better results than AFF and PFF1. For smaller instances, PFF2 Entire is able to obtain optimal solutions within a much smaller time window than all other models. For larger instances on the other hand, PFF2 Basic is able to find better solutions within the maximum allowed solution time. The latter is reflected in the lower gap values obtained. Therefore, the second path flow model PFF2 is used in all remaining parts of this report.

Test instance	*Solution time [s]*			*Gap [%]*			*Time to first MIP [s]*		
	AFF	*PFF1*	*PFF2*	*AFF*	*PFF1*	*PFF2*	*AFF*	*PFF1*	*PFF2*
S1_H3_P5	4,776.7	364.2	3,201.7	0.0	0.0	0.0	4.6	1.9	0.4
S1_H5_P5	6,356.3	325.2	3,529.1	0.0	0.0	0.0	49.0	9.5	0.5
S1_H7_P5	>7200	217.1	733.1	0.1	0.0	0.0	8.3	6.1	0.8
S1_H3_P10	>7200	>7200	>7200	10.2	2.5	2.5	158.8	126.9	3.2
S1_H5_P10	>7200	>7200	>7200	25.8	2.7	2.7	197.5	260.5	4.6
S1_H7_P10	>7200	>7200	>7200	16.9	7.7	3.2	240.1	42.8	10.4
S1_H3_P15	>7200	>7200	>7200	N/A	14.9	4.1	N/A	34.2	130.8
S1_H5_P15	>7200	>7200	>7200	18.5	52.0	4.6	675.7	2,068.4	151.2
S1_H7_P15	>7200	>7200	>7200	33.8	N/A	4.8	1,005.6	N/A	407.0

Table 9.11: Solution time, gap and time to first MIP AFF, PFF1 and PFF2 Basic (RG3)

Test instance	*Solution time [s]*			*Gap [%]*			*Time to first MIP [s]*		
	AFF	*PFF1*	*PFF2*	*AFF*	*PFF1*	*PFF2*	*AFF*	*PFF1*	*PFF2*
S1_H3_P5	2,419.7	73.4	19.4	0.0	0.0	0.0	26.1	29.1	13.5
S1_H5_P5	774.5	122.6	23.0	0.0	0.0	0.0	4.7	12.4	1.5
S1_H7_P5	1,128.6	206.7	35.8	0.0	0.0	0.0	51.7	199.9	4.6
S1_H3_P10	>7200	>7200	>7200	N/A	7.0	1.7	N/A	355.2	42.6
S1_H5_P10	>7200	>7200	>7200	N/A	3.0	2.0	N/A	844.4	50.4
S1_H7_P10	>7200	>7200	>7200	N/A	53.9	2.1	N/A	4,104.5	73.4
S1_H3_P15	>7200	>7200	>7200	N/A	N/A	8.1	N/A	N/A	372.0
S1_H5_P15	>7200	>7200	>7200	N/A	N/A	4.3	N/A	N/A	1,174.2
S1_H7_P15	>7200	>7200	>7200	N/A	N/A	14.9	N/A	N/A	1,252.2

Table 9.12: Solution time, gap and time to first MIP AFF, PFF1 and PFF2 Entire (RG3)

9.5 Effect of increasing the maximum number of offshore landings

In this section, the results obtained when running the model PFF2 Basic in Xpress-IVE with the set of routes generated by RG3 and RG4 are analysed. In particular, a look is taken at the effect of increasing the maximum number of offshore landings in the predefined routes. As additional routes were only created for test instances with 10 and 15 platforms when increasing the maximum number of offshore landings (see Section 9.1), only such test instances are examined. This is also why PFF2 Basic was used, as this model has shown the best computational results for large test instances in the previous sections of this chapter. For all runs, the maximum allowed solution time was 2 hours.

The results obtained when running PFF2 Basic with the two sets of routes generated by RG3 and RG4 are given in Table 9.13. From this table, one can see that increasing the maximum number of offshore landings has no impact on the LP values and charter-in costs obtained. The same result also occurs for the MIP values for all test instances solved to a gap value less than 4 %. These results indicate that increasing the maximum number of offshore landings beyond three has no impact on the solutions obtained. This result is further confirmed by the fact that maximum three offshore landings were performed in all routes included in the solutions obtained after a 2 hour run time.

The values obtained suggest that setting an upper limit of three offshore landings is appropriate for the predefined routes generated for the problem addressed in this report. Therefore, including routes during which additional offshore landings are performed is not further investigated in this report.

	Gap [%]		*LP*		*MIP*		*Charter-in cost*	
Test instance	*RG3*	*RG4*	*RG3*	*RG4*	*RG3*	*RG4*	*RG3*	*RG4*
S1_H3_P10	2.5	2.5	175,227	175,227	205,338	205,338	73,714	73,714
S1_H5_P10	2.7	2.7	175,227	175,227	205,338	205,338	73,714	73,714
S1_H7_P10	3.2	2.5	175,227	175,227	205,338	205,338	73,714	73,714
S1_H3_P15	4.1	6.3	237,419	237,419	287,321	288,467	99,745	99,745
S1_H5_P15	4.6	4.0	237,419	237,419	287,268	287,385	99,745	99,745
S1_H7_P15	4.8	9.6	237,415	237,415	287,695	292,156	99,745	99,745

Table 9.13: Gap, LP, MIP and charter-in cost PFF2 Basic (RG3 and RG4)

9.6 Effect of aggregating platform data

In this section, the results obtained when running the model PFF2 Entire in Xpress-IVE with aggregated platform data are analysed. When doing so, all platforms located in the same oil and gas field were clustered together and replaced by one platform node. Consequently, the pickup and delivery demands of the various platforms were combined into one single point. This type of aggregation technique is commonly used within the field of forecasting to mention some. By making use of the procedure, the complexity of the model in use can be reduced while at the same time increasing the accuracy of the demand forecasts (Chopra and Meindl (2010)). For the problem addressed in this report, aggregating platform data is thus synonym to decreasing the number of platform nodes in the model in use while taking the same amount of real platforms into consideration. PFF2 Entire was therefore used as this model has shown the best computational results for small test instances in the previous sections of this chapter. For all runs, the routes generated by RG3 were used as predefined routes and the maximum allowed solution time was 2 hours.

Research has shown that aggregating demand data has a minor impact on model accuracy (Simchi-Levi et al. (2009)). Still, one should be aware of the effects of making use of the procedure. For the problem addressed in this report, aggregating platform data is likely to reduce the number of platform landings required to fulfil all pickup and delivery demand. This is due to the fact that all shuttling processes between platforms located in the same field are eliminated. If this were to occur, the time and cost of the helicopter routing process as a whole will decrease. The extent of this effect can however not be determined beforehand.

Let us then take a look at the results obtained when running PFF2 Entire with aggregated platform data. Firstly, the effect on the size of the model is examined. This can be seen from Table 9.14 on the following page, in which the number of rows and columns of the model with and without aggregated data is given. By examining the values, one can notice a significant decrease in model size when the latter data set is used. On average, the number of rows and columns is reduced by 50 % and 30 % respectively.

Secondly, let us examine the effect that this reduction in model size has on the model's computational results. When doing so, the solution times, gap and LP values obtained when running PFF2 Entire with and without aggregated

	# rows after pre-solve		*# columns after pre-solve*	
Test instance	*Original data*	*Aggregated data*	*Original data*	*Aggregated data*
S1_H3_P5	3,919	2,356	4,766	2,304
S1_H5_P5	7,435	3,420	12,361	3,364
S1_H7_P5	11,603	3,830	29,709	3,795
S1_H3_P10	5,480	3,428	6,732	3,452
S1_H5_P10	10,171	4,876	16,453	4,918
S1_H7_P10	15,549	5,713	37,191	5,761
S1_H3_P15	7,094	4,529	8,741	4,648
S1_H5_P15	12,997	6,524	20,635	6,651
S1_H7_P15	19,586	7,580	44,754	7,704

Table 9.14: Rows and columns after pre-solve PFF2 Entire (RG3) with and without aggregated data

platform data are compared. These values can be found in Table 9.15. By examining the results, one can notice that aggregating platform data results in a significant decrease in solution time. When using this data set, all test instances are solved to optimality within 720 seconds. This is the first time that this occurs for the models examined in this chapter.

Thirdly, let us take a look at the objective values obtained when running PFF2 Entire with aggregated platform data. From Table 9.15, one can see that the values of the LP relaxations are reduced for most test instances compared to the results obtained with the original data set. Similar effects can be found for the MIP values obtained, which can be seen in Table 9.16. These results are in consistency with the discussion in former paragraph. When aggregating platform data, elements of the helicopter routing process are disregarded, which causes an reduction of the total cost of the system. Still, the decrease in LP and MIP values is less than 1 % for all test instances.

It also appears that the charter-in costs obtained are unaffected by whether or not data aggregation is performed. This latter observation can be seen from Table 9.16, in which the same charter-in costs are given for all test instances solved to gap values less than 10 %. These results indicate that aggregation of platform data can be applied to the problem addressed in this report without affecting the variables of interest. This is an important finding, and will be made use of when performing economic and accident risk analyses in Chapter 10.

		Solution time [s]		Gap [%]		LP	
Test instance	*# of fields*	*Original data*	*Aggregated data*	*Original data*	*Aggregated data*	*Original data*	*Aggregated data*
S1_H3_P5	3	19.4	4.1	0.0	0.0	90,121	90,004
S1_H5_P5	3	23.0	9.1	0.0	0.0	90,121	90,004
S1_H7_P5	3	35.8	7.5	0.0	0.0	90,121	90,088
S1_H3_P10	5	>7200	17.7	1.7	0.0	175,227	171,444
S1_H5_P10	5	>7200	41.0	2.0	0.0	175,227	173,059
S1_H7_P10	5	>7200	49.4	2.1	0.0	175,227	171,444
S1_H3_P15	6	>7200	243.8	8.1	0.0	237,419	239,996
S1_H5_P15	6	>7200	225.0	4.3	0.0	237,419	236,730
S1_H7_P15	6	>7200	717.5	14.9	0.0	237,415	236,725

Table 9.15: Solution time, gap and LP PFF2 Entire (RG3) with and without aggregated data

		MIP		Charter-in cost	
Test instance	*# of fields*	*Original data*	*Aggregated data*	*Original data*	*Aggregated data*
S1_H3_P5	3	108,926	108,208	36,857	36,857
S1_H5_P5	3	108,926	108,208	36,857	36,857
S1_H7_P5	3	108,926	108,208	36,857	36,857
S1_H3_P10	5	205,338	200,258	73,714	73,714
S1_H5_P10	5	205,338	200,258	73,714	73,714
S1_H7_P10	5	205,338	200,258	73,714	73,714
S1_H3_P15	6	296,023	280,937	99,745	99,747
S1_H5_P15	6	288,464	280,937	99,745	99,747
S1_H7_P15	6	318,025	280,937	111,103	99,747

Table 9.16: MIP and charter-in cost PFF2 Entire (RG3) with and without aggregated data

Chapter 10

Economic and accident risk analysis

In this chapter, the results obtained when running PFF2 with the second to fifth set of test instances in Xpress-IVE are presented and discussed. The aim of the chapter is to provide insight into how Petrobras should organize its future helicopter operations in the Santos Basin area by looking at this issue from both a cost and accident risk perspective.

Both PFF2 Basic and Entire were used when running the various test instances in commercial optimization software. All instances were solved with both models, and the best objective values and lower bounds obtained were then selected. Aggregated platform data was used for all test instances as this method showed improved computational results without affecting variables of interest in Section 9.6. It appeared however that the models solution time significantly increased as the total pickup and delivery demand of the system expanded. In order to obtain results with low gap values, maximum allowed run time was thus set to 20 hours. Still, several test instances were solved to optimality within shorter time, and in particular runs in which weight was given to accident risk.

Section 10.1 examines Petrobras' required future upgrades in helicopter bases and fleet from a cost perspective. A sensitivity analysis of the results obtained is thereafter presented in Section 10.2. Lastly, the relationship between cost and accident risk of the transportation system is established in Section 10.3.

10.1 Optimal structure of future helicopter operations

In this section, Petrobras' required upgrades in helicopter bases and fleet that follows from the future expansion of operations at the pre-salt fields in the Santos Basin area are investigated. The issue is examined from a cost perspective.

The discussion stems from the results obtained when running PFF2 with the second set of test instances. These can be seen in Table 10.1, in which the onshore airport bases, offshore transhipment hubs and helicopter fleet in use in the optimal solution obtained when running the model with the various test instances are given. In the table, helicopter bases number 1 and 2 correspond to the existing facilities in Jacarepaguá and Cabo Frio respectively.

Firstly, the results show that no offshore transhipment hub should be implemented in order to support Petrobras' helicopter operations in future years if minimal operational cost of the transportation system is to be attained. This means that if available helicopters are able to reach all offshore platforms installed, Petrobras' current transportation model should also be used in future operations. It is likely that these results are caused by the significant cost of implementing an offshore transhipment hub. In this report, the daily price of this investment was set to \$ 150 000. This is a notable value, and significant cost reductions must be achieved in the helicopter routing process should it be cost efficient to implement an offshore transhipment hub. Results suggest that this type of cost reduction cannot be obtained by implementing an offshore hub, and that no investment should therefore be made in this type of installation.

Secondly, the results show that a consolidation of onshore activities is desirable. This can be seen from the fourth column in Table 10.1, in which the results suggest that helicopter base in Jacarepaguá should be used in all future years, and that capacity should be expanded at the base in Cabo Frio from 2017. It is apparent that a second onshore base is opened when the helicopter capacity of the first is maximised. The question is then why fully exploiting the capacity at the base in Jacarepaguá causes more cost efficient helicopter operations than also making use of the base in Cabo Frio. Some answers to this query can be found by examining the input data used. The sums of the boarding and landings fees for the various helicopters taken into consideration in this report are lower at Jacarepaguá airport than at Cabo Frio. Also, the distances from Jacarepaguá to the various offshore platforms are in general shorter than the distance from

Test instance	*# of platforms*	*Hub(s) opened*	*Bases used*	*# of AW139*	*# of EC-225*	*# of S-76*	*# of S-92*
S2_B2_2014	2	-	1	-	-	-	1
S2_B2_2015	3	-	1	-	-	2	-
S2_B2_2016	17	-	1	-	-	6	1
S2_B2_2017	28	-	1,2	-	-	9	2
S2_B2_2018	36	-	1,2	-	-	8	5
S2_B2_2019	39	-	1,2	-	-	8	6
S2_B2_2020	41	-	1,2	-	-	7	7

Table 10.1: Proposed hub and helicopter fleet for second set of test instances

Cabo Frio to the installations. These points suggest that onshore activities should be consolidated at the onshore base located closest to the offshore platforms in terms of flying distances. By following this strategy, the distances needed to be travelled by the various helicopters are reduced, which again causes a decrease in the total routing cost of the transportation system.

Thirdly, the results show that some helicopter types are better suited for Petrobras' future helicopter operations than others if minimal operational cost is to be attained. In particular, helicopter types Sikorsky S-76 and S-92 are preferred over Agusta Westland AW139 and Eurocopter EC-225. As for the onshore airport bases, the causes behind this observation can be detected by examining the input data used. For example, both Sikorsky helicopters are long range aircraft. This makes these helicopter types particularly appropriate when reduction in routing costs cannot be obtained by installing offshore transhipment hubs, and a direct routing policy is applied to all flights. Also, if comparing the operational costs of S-92 to the ones of EC-225 which is an helicopter type with the same passenger capacity, one find that that the operational costs of the first aircraft type is lower than the ones of the second. Thus the selection of helicopter types S-76 and S-92 can be explained by their cost and operational characteristics.

Let us then take a further look at the composition of helicopter types S-76 and S-92 in the optimal helicopter fleet. This is visualized in Figure 10.1 on the following page. From the figure, one can see that the use of S-92 is only appropriate as the number of platforms, and thus the pickup and delivery demands in the transportation system, expands. This result suggests that transporting employees by helicopters with high passenger capacity is

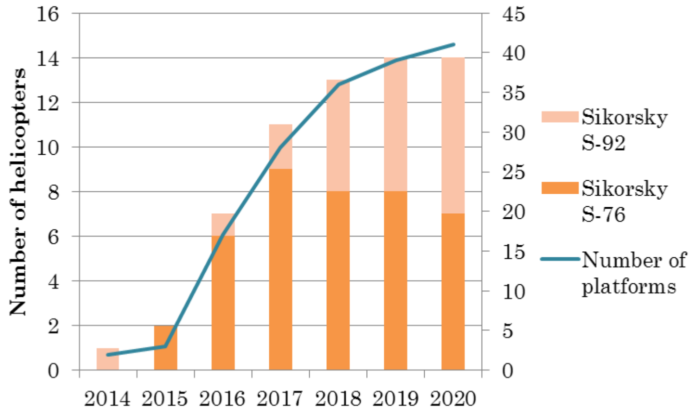

Figure 10.1: Optimal composition of helicopter fleet, 2014-2020

only relevant for a system with considerable pickup and delivery demands of employees. For a network in which the number of transported passengers is small on the other hand, lowest operational cost is achieved by using aircraft with a smaller passenger capacity. These results are likely to stem from the effects caused by economies of scale. With higher passenger demand in the transportation system, the capacity of helicopters with high passenger capabilities can be better utilized, which reduces the charter-in cost per passenger for high capacity helicopters to an acceptable level.

Still, results indicate that no cost advantages can be obtained in the transportation system as the size of helicopter operations increases. This can be seen from Figure 10.2, in which the system's total daily cost is portrayed as a function of total passenger-kilometres (pkm). From the figure, one can see that a linear relationship is obtained between these two variables. This result suggests that Petrobras' helicopter transportation system obtains no scale effects on cost as the volume of transported employees increases when no onshore transhipment hub is installed. The result is likely to stem from the composition of investments made in this scenario. As only helicopter investments are made and the passenger capacity every aircraft is relatively small, the number of required investments is highly adjustable to the system's pickup and delivery demands of employees. Still, it should be noted that the models developed in this report take into account a constant marginal charter-in cost. In reality,

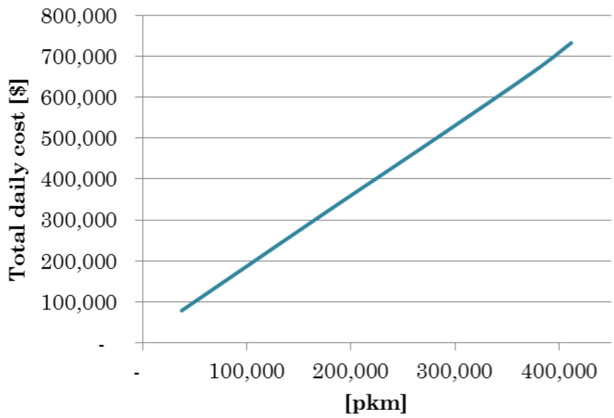

Figure 10.2: Daily cost of the system as a function of passenger-kilometre [pkm]

this is likely to be a decreasing function of helicopter quantity. Thus, although the result depicted in Figure 10.2 suggests the contrary, Petrobras' helicopter operations may still experience economies of scale in reality.

Lastly, let us take a look at the cost estimates obtained for Petrobras' future helicopter operations in the Santos Basin area. These values can be seen in Figure 10.3 on the following page, in which both the system's total cost and disaggregated routing and charter-in values are visualized. From the figure, one can see that the largest cost component is routing costs. In fractional values, this constituent represents about one third of the system's total cost. This result signify the importance of having an infrastructure that allows efficient routing processes in the transportation system.

In quantitative terms, the estimated daily cost of Petrobras' future helicopter operations in the Santos Basin area obtained in this section is about $ 70 000 and $ 750 000 in the years 2014 and 2020 respectively. These values represent an annual charter-in cost of about $ 8 million and $ 95 million in the years 2014 and 2020 respectively. If calculating annual routing costs on the other hand, the use of pickup and delivery demands being 6 % higher than the systems average demand must be taken into consideration. Thus, estimated annual routing costs are calculated by multiplying daily values by 365 and 94 %. Performing this process results in annual routing costs of about $ 15 million and $ 170 million in the years 2014 and 2020 respectively.

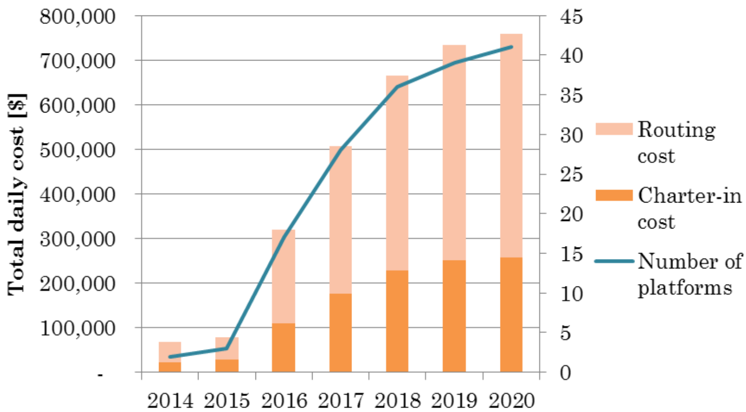

Figure 10.3: Daily routing and charter-in costs of the transportation system, 2014-2020

10.2 Sensitivity analysis

In this section, a sensitivity analysis of the results obtained in Section 10.1 is performed. In particular, the effect of changing two important properties in Petrobras' offshore helicopter transportation system is investigated. In Section 10.2.1, the cost reductions obtained by opening new onshore airport bases in Guarujá and Itaguaí is analysed. In Section 10.2.2 on the other hand, the implications of reducing the range of the various helicopter types are examined. All analyses in this section investigate the offshore helicopter operations from a cost perspective.

10.2.1 Effect of opening new onshore airport bases

In this section, the effects of opening new onshore airport bases in Guarujá and Itaguaí are investigated. The discussion stems from the results obtained when running PFF2 with the third set of test instances. These can be seen in Figure 10.4, in which the total cost of the transportation system obtained when running this model with the second and third group of test instances is compared. Notice that all costs are disaggregated into their lower bound and

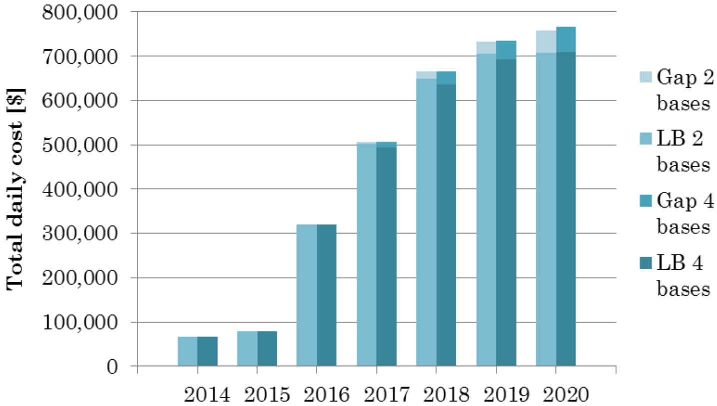

Figure 10.4: Daily cost of the transportation system with two and four available onshore airport bases, 2014-2020

gap values in order to display the model's reliability of the values obtained.

The results indicate that no cost reduction can be obtained by implementing new onshore airport bases as long as the capacity of existing facilities is large enough to handle the increase in transportation volume caused by the future expansion of oil and gas operations in the Santos Basin area. For all years, an identical selection of onshore airport bases and fleet composition was obtained when allowing activity at bases in Jacarepaguá, Cabo Frio, Guarujá and Itaguaí, and when only allowing activity at the first two. The cause behind this observation can be detected by examining the input data used. For the majority of offshore platforms, the distances to the existing bases in Jacarepaguá and Cabo Frio are shorter than the distances to Guarujá and Itaguaí. This means that allocating aircraft to these latter bases would result in an increase in helicopter transportation costs. Results show that this increase is larger than the cost savings obtained by reduced boarding, landing and parking costs, and that no investment should therefore be made in this type of facilities if not necessary.

Further, should capacity restriction at existing facilities arise, results indicate that expanding capacity at these locations may be more cost effective than implementing new onshore airport bases in Guarujá and Itaguaí. This strategy also eliminates the need for large investment costs. Still, it should be mentioned that qualitative aspects not investigated in this report may suggest otherwise.

10.2.2 Effect of reducing helicopter range

In this section, the implications of reducing the range of the various helicopter types are examined. The discussion stems from the results obtained when running PFF2 with the fourth set of test instances.

Multiple reasons exist for performing this kind of analysis. Firstly, it may be that the helicopter ranges provided by the various producers are longer than realistic values. Secondly, pilot regulations such as IFR and VRF place additional constraints on the possible flying length during every flight. In the former, fuel should be enough to fly the planned route to the intended destination, plus the route to an alternative destination and 30 minutes in reserves. In the latter, fuel should be enough to fly the planned route to the intended destination, plus 20 minutes in reserves and 10 % of the total flight time (ANAC (2010)). It is unclear whether this type of restrictions is taken into consideration in the maximum flying ranges specified by various helicopter producers.

Thirdly, one can imagine a scenario in which rules similar to the ones particular for the Macaé terminal control area are imposed on helicopter operations in the Santos Basin area. Due to the congested airspace surrounding the airport in Macaé, helicopters operating in this area must make use of special airways when transporting employees between onshore facilities and offshore installations. At present, this means that helicopters cannot fly directly between their origin and destination points, but must pass via different notification points before flying to the platform to which they are intended to go. This increases the distances travelled by the various helicopters. The imaginary hub and spoke system in the Campos Basin area is visualised in Figure 10.5. In the figure, the three offshore oil and gas fields Albacora, Marlin and Enchova can be seen to the right, while the two transition areas Central and Tomé that helicopters are supposed to overfly can be seen to the left.

The results obtained when running PFF2 with the fourth set of test instances are given in Table 10.2 on page 102. In the table, the onshore airport bases, offshore transhipment hubs and helicopter fleet in use in the model's optimal solutions are given. Helicopter bases number 1 and 2 correspond to the existing facilities in Jacarepaguá and Cabo Frio respectively.

Firstly, the results show that one offshore transhipment hub should be implemented in order to support Petrobras' helicopter operations in future years if helicopter ranges are reduced and minimal operational cost of the

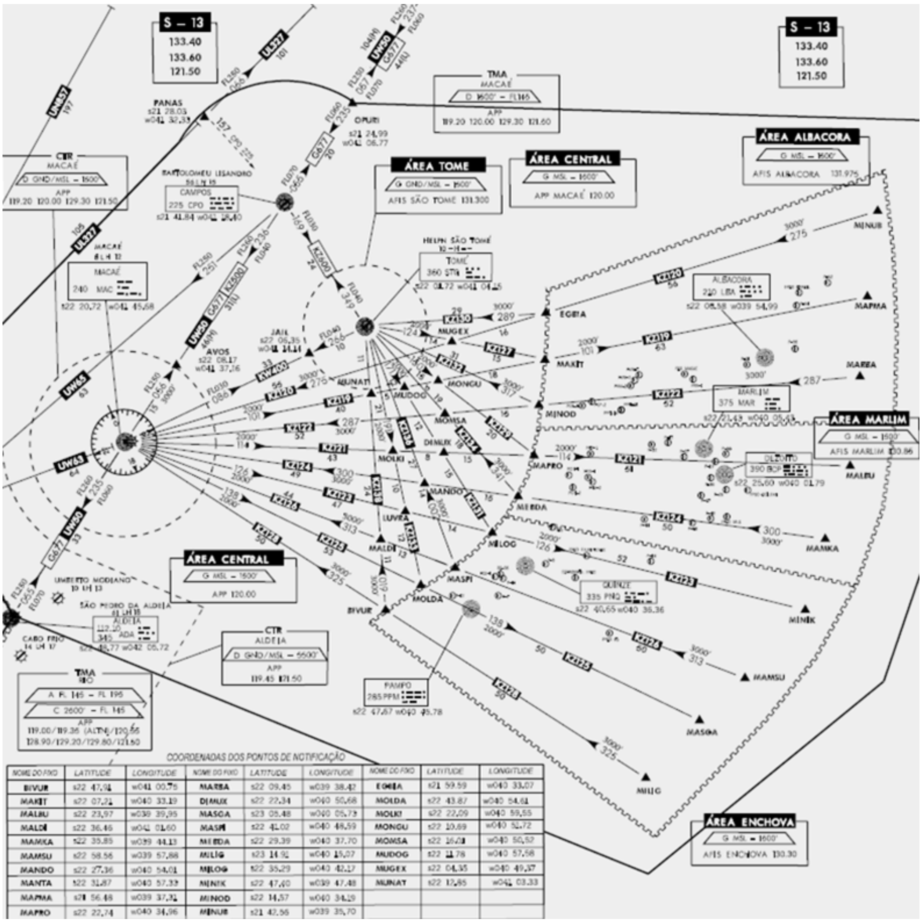

Figure 10.5: Air circulation chart Macaé (AIS (2008))

Test instance	*# of platforms*	*Hub(s) opened*	*Bases used*	*# of AW139*	*# of EC-225*	*# of S-76*	*# of S-92*
S4_B2_2014	2	1	1	-	-	-	1
S4_B2_2015	3	2	1	1	-	1	-
S4_B2_2016	17	2	1	1	-	-	4
S4_B2_2017	28	2	1	2	-	-	6
S4_B2_2018	36	2	1,2	2	-	1	8
S4_B2_2019	39	2	1,2	3	-	1	9
S4_B2_2020	41	2	1,2	1	-	-	11

Table 10.2: Proposed hub and helicopter fleet for fourth set of test instances

transportation system is to be attained. This means that if available helicopters are not able to reach all offshore platforms installed, the installation of an offshore transhipment hub is necessary. Results suggest that the optimal location of this hub varies over the timespan investigated in this report. Location number one is preferred in 2014, while the second location should be used in the years 2015 to 2020.

Let us then take a further look at these two locations for the offshore transhipment hub. The way in which these are situated relative to the offshore oil and gas field at which platforms are to be installed, is visualized in Figures 10.6 and 10.7. From the figures, one can see that the location selected for 2014 is positioned in a region laying between the onshore facilities and offshore installations. The location selected for all other years on the other hand, is positioned in a region laying closer to the offshore installations. The latter result is interesting. Intuitively, one would have expected that the optimal hub location would be as close as possible to the onshore facilities. By using this strategy, the offshore transhipment hub installed would correspond to an offshore airport base located so that a direct routing policy could be performed on every flight. However, results suggest that the optimal location of the offshore transhipment hub is relatively close to the oil and gas fields. This result indicates that the optimal configuration of the transportation system's infrastructure is influenced by complex routing processes. Mathematical optimization is therefore an appropriate tool for guiding such decision making processes.

Secondly, the results show that the optimal composition of Petrobras' helicopter fleet is influenced by whether or not an offshore transhipment hub is installed. When this is required, results suggest that all three helicopter types AW139, S-

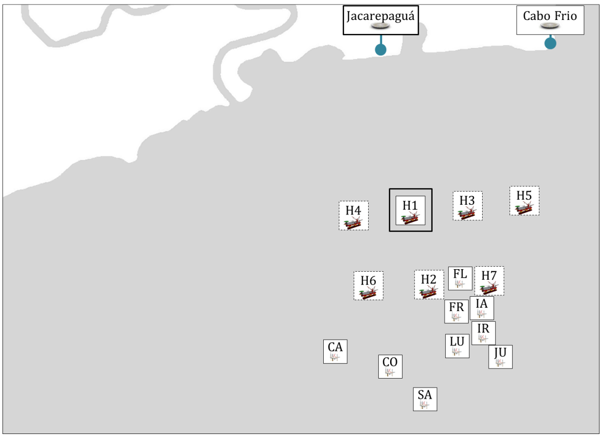

Figure 10.6: Selection of offshore transhipment hub when helicopter ranges are reduced 2014 (CA:Carcará, CO:Carioca, FL:Florim, FR:Franco, IA:Iara, IR:Iracema, JU:Júpiter, LU:Lula, SA:Sapinhoá)

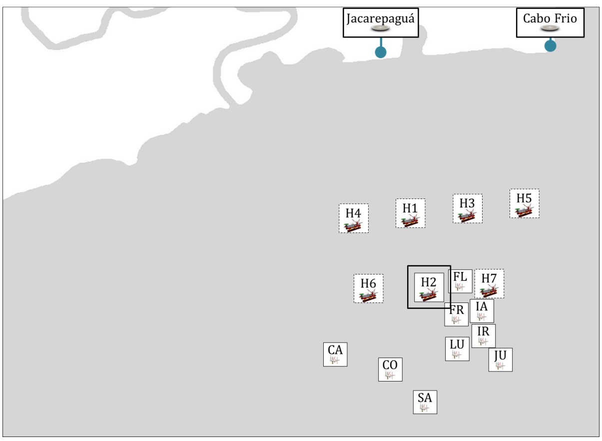

Figure 10.7: Selection of offshore transhipment hub when helicopter ranges are reduced 2015-2020 (CA:Carcará, CO:Carioca, FL:Florim, FR:Franco, IA:Iara, IR:Iracema, JU:Júpiter, LU:Lula, SA:Sapinhoá)

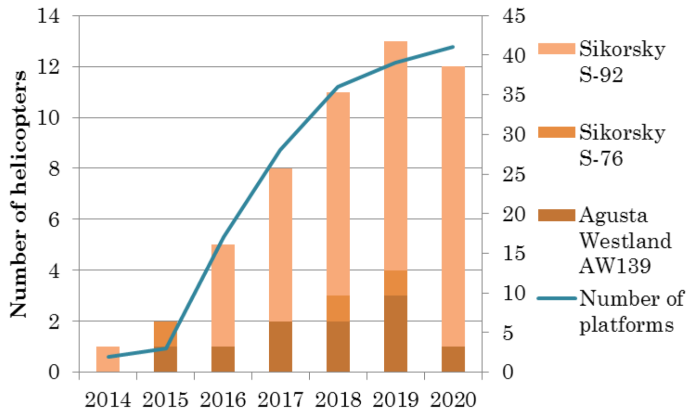

Figure 10.8: Composition of helicopter fleet when installing offshore transhipment hub, 2014-2020

76, and S-92 should be made use of in Petrobras' future helicopter operations if minimal operational cost is to be attained. This differs from the results obtained for the scenario in which an offshore transhipment hub was not implemented in Section 10.1. In this previous section, only the use of helicopter types S-76 and S-92 was suggested. The cause behind this modification of optimal helicopter fleet can be detected by examining the input data used. For example, AW139 is a shorter range aircraft with lower operational costs per passenger capacity than both S-76 and S-92. This makes this helicopter type more appropriate when an offshore transhipment hub is installed, as the implementation of this installation reduces the required flying distances of the helicopters in use.

Let us then take a further look at the composition of the various helicopter types in the optimal helicopter fleet when an offshore transhipment hub is installed. This is visualized in Figure 10.8. From the figure, one can see that the majority of the optimal helicopter fleet is composed of S-92 aircraft. As in the previous paragraph, the reason behind this observation can also be detected by examining the input data used. If doing so, one can see that S-76 and S-92 are the only aircraft able to reach the majority of platforms by using direct routing policy flights. It also becomes apparent that a high percentage of the system's pickup and delivery demand is located at offshore oil and gas fields reachable by these helicopter types. This means that a direct routing policy can be applied to

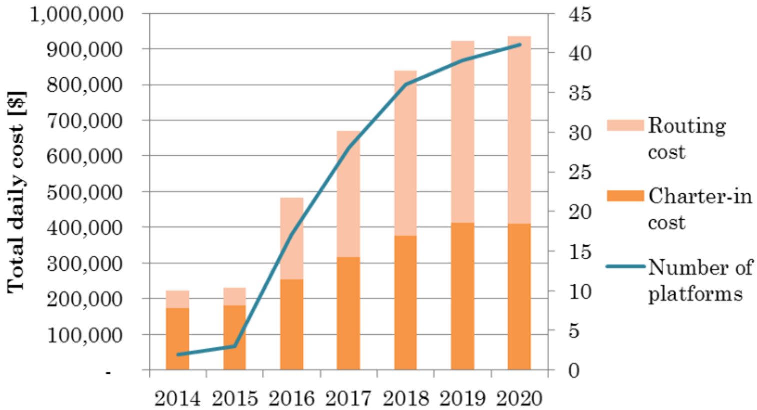

Figure 10.9: Daily routing and charter-in costs of the transportation system when opening hub, 2014-2020

numerous flights if making use of these two helicopter types. However, as a high utilization of the passenger capacity can be obtained, the charter-in cost per passenger for the S-92 becomes lower than the one for S-76. This makes the S-92 the best suited aircraft for the routing process.

Lastly, let us take a look at the cost estimates obtained for Petrobras' future helicopter operations in the Santos Basin area in this section. These values can be seen in Figure 10.9, in which both the system's total cost and disaggregated routing and charter-in values are visualized. From the figure, one can see that when implementing an offshore transhipment hub, charter-in costs is the largest cost component in the first three years. However, as the scale of operations increases, routing costs becomes the largest cost component from 2017.

In quantitative terms, the estimated daily cost of Petrobras' future helicopter operations in the Santos Basin area obtained in this section is about \$ 220 000 and \$ 930 000 in the years 2014 and 2020 respectively. These values represent an annual charter-in cost of about \$ 63 million and \$ 150 million in the years 2014 and 2020 respectively. Also, by multiplying daily charter-in values by 365 and 94 %, estimated annual routing costs of about \$ 17 million and \$ 180 million in the years 2014 and 2020 respectively are obtained. One can see that the costs of Petrobras' offshore helicopter operations significantly increase if the installation of an offshore transhipment hub is required.

10.3 Relationship between cost and accident risk

In this section, Petrobras' future helicopter operations in the Santos Basin are investigated from both a cost and accident risk perspective. In particular, the relationship between these two objectives is established. The discussion stems from the results obtained when running PFF2 with the fifth set of test instances.

Firstly, the results show that no offshore transhipment hub should be implemented in order to support Petrobras' helicopter operations in future years if minimizing both the total cost and accident risk of the transportation system. This means that if available helicopters are able to reach all offshore platforms installed, Petrobras' current transportation model should also be used in future operations. The reasons why the installation of an offshore hub is unsuitable from a cost perspective are already investigated in previous sections of this chapter, and are therefore not addressed in this section. The causes why such an implementation also is unsuitable from an accident risk perspective on the other hand, are likely to derive from the way risk is modelled in this report. In particular, as every offshore take-off and landing procedure performed causes an increase in the system's total accident risk assessment, it is desirable to reduce the number of such operations. Installing a transhipment hub and performing hub connected routing policy flights however increases the number of offshore take-off and landing procedures. Thus, also from an accident risk perspective it is of interest to perform direct routing policy flights whenever possible.

Secondly, the results show that when optimizing Petrobras' future helicopter operations in the Santos Basin, minimizing accident risk can to some extent be included as an objective without significantly increasing the total cost of the transportation system. This is visualized in Figure 10.10, in which the Pareto frontier of the two objectives is given. From the figure, one can see that reducing the total accident risk of the system by 15 % causes about a 10 % increase in total cost. This means that minimizing the total cost and total accident risk of Petrobras' helicopter operations are to some degree aligned objectives. This observation is likely to be caused by the fact that all cruise procedures performed increase both the total cost and accident risk of the system. Thus, from both a cost and accident risk perspective, it is of interest to reduce the total distance travelled by the helicopters in use.

The question is then why minimizing the accident risk of Petrobras' helicopter operations causes an increase in the total cost of the transportation system.

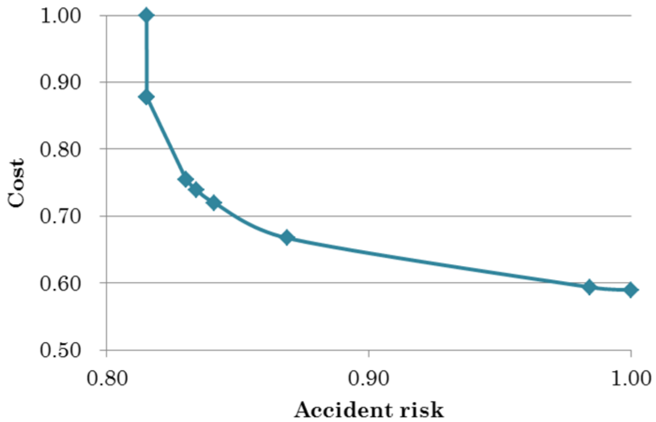

Figure 10.10: Pareto frontier cost-accident risk

The answer to this query can be found by examining the results obtained when running PFF2 with the fifth set of test instances. If doing so, it is becomes apparent that only EC-225 aircraft are used if the system's total accident risk is to be minimized. However, this helicopter type is never selected if minimizing the system's total cost. This latter result is in line with observations made in previous sections of this chapter. It is likely that EC-225 is selected to be the most suitable aircraft from a safety perspective because of the way risk is modelled in this report. As this is the only heavy twin-engine helicopter in use by Petrobras, and because this helicopter type historically has had lower offshore aviation accident rates than medium twin-engine helicopters, making use of this aircraft type reduces the total accident risk assessment of the transportation system. This result is in line with the information given in Section 2.4.

Lastly, a remainder is made of the use of world-wide offshore aviation accident rates in this report. Due to this selection of data, local factors affecting the accident risk of Petrobras' helicopter operations are not taken into consideration. Examples of such are the quality of pilot training, environmental conditions, and organisational issues to mention some. It is highly unlikely that Brazilian offshore helicopter operations represent the norm of all these elements. Still, going into the depths of factors affecting accident frequencies is an exercise beyond the scope of this report. Interested readers are however referred to work performed by authors as Gomes et al. (2009) and Nascimento et al. (2012a,b).

Chapter 11

Future research

The work presented in this report is a continuation of the work performed by Norddal (2012), and thus represents a second step in developing a decision support model for Petrobras' future upgrades in helicopter bases and fleet. Still, although several improvements of previous work have been made during the development of this report, additional extensions of interest also exist. In this chapter, potential future research areas for the topic addressed in this report are presented.

From an operations research point of view, there are essentially three directions one may take. Firstly, a validation can be made of the AFF, PFF1 and PFF2 models developed in this report by comparing their quantitative outputs with historical accounting information. This type of work could not be performed during the completion of this thesis as no real input data have been available for use. Still, the importance of performing such a procedure should not be underestimated. By validating the models developed, extensive insight can be obtained as to whether or not the important aspects of the transportation system have been taken into consideration. This may again reveal areas for improvements of the mathematical formulations.

This discussion leads us to the second option for future research area. This is to expand or change the mathematical formulations presented in Chapter 5 so that they better describe reality. One flaw of the models was already pointed out in Section 10.1. This was the use of a constant marginal charter-in cost.

One area for future improvement is therefore to formulate the cost function of investments in various helicopter types and hubs in a more detailed manner. By doing so, the models would to a greater extent portray the decision making process faced by Petrobras' administrators. Also, more accurate cost estimates are likely to be obtained.

Further, as seen in Section 10.2, whether or not an offshore transhipment hub should be opened is greatly affected by the ranges of the helicopters in use. Thus, incorporating the relationship between helicopter ranges and number of passengers into the mathematical formulations could be of great interest (see Figure 8.3 on page 71). By doing so, more accurate investment decisions are likely to be obtained regarding the opening of one or several offshore transhipment hubs. Similar result can also be obtained by incorporating the possibility of opening hubs with various capacities in the models.

Thirdly, the last option for future research area is to develop methods for reducing the solution times of the various models. As noted in Chapters 10 and 9, these times significantly increase as the pickup and delivery demands of the transportation system expands. This characteristic of is likely to reduce the usability of the models developed. It is therefore of interest to limit this effect as much as possible. One way of doing so is to reduce the complexity of the mathematical formulations by relaxing constraints and/or variables that have a limited influence on the results obtained. This includes improving the generation procedure for predefined routes. Also, better solution methods can be developed by the use of heuristics, or by applying a decomposition method to the problem.

Chapter 12

Concluding remarks

The purpose of this report was three folded: Firstly, develop decision support models addressing Petrobras' required upgrades in helicopter bases and fleet that follows from the future expansion of operations in the Santos Basin pre-salt fields by the use of mathematical optimization. Secondly, implement the models developed in commercial optimization software, and test them on various test instances. Thirdly, provide advices on how Petrobras' should organize its future helicopter operations based on the results.

In the previous eleven chapters of this report, these objectives have been addressed one by one. Results obtained indicate that if available helicopters are able to reach all offshore platforms that are to be installed in the Santos Basin, no offshore transhipment hub should be implemented in order to support Petrobras' helicopter operations. This conclusion is made from both a cost minimizing and accident risk minimizing perspective. The result means that if helicopter ranges are not a limitation, Petrobras' current transportation model in which direct flights are made between various installations also should be used in future operations.

If available helicopters are not able to reach all offshore platforms however, results suggest that one offshore transhipment hub should be installed. Doing so causes though a significant increase in the transportation system's total operational cost. If installing an offshore hub, the total cost of Petrobras' helicopter operations are estimated to increase by a factor varying between

1.2 and 3.4 over the years 2014 to 2020. Also, contrary to what one could have expected, the optimal location of this hub has shown to be within a region laying close to the various offshore installations. This result indicates that the optimal configuration of the infrastructure supporting Petrobras' helicopter operations is influenced by complex routing processes. This shows that mathematical optimization is an appropriate tool for aiding such decision making processes.

The optimal size and composition of Petrobras' future helicopter fleet has demonstrated to be greatly affected by the installment of an offshore transhipment hub. If this implementation is not performed, results indicate that the fleet should mainly consist of Sikorsky S-76 aircraft. If a hub is installed on the other hand, results show that mostly Sikorsky S-92 aircraft should be made use of. This shows that the investment decisions in helicopter bases and fleet are greatly interconnected, and that it is appropriate to address these issues collectively as done in this report.

Further, results point out that a consolidation of onshore activities is desirable. In particular, such operations should be strengthened at the onshore airport base located closest to the offshore installations in terms of flying distances. By doing so, the required total travel distance of the helicopters in can be reduced. This causes a decrease in the transportation system's routing cost. Also, results show that if capacity restrictions arise at existing facilities, it may be more cost effective to expand the capacity at these locations than to implement new onshore airport bases in Guarujá and Itaguaí. Following this strategy also eliminates the need for large investment costs.

From an optimization point of view, results have shown that a path flow formulation of the problem at hand obtains significantly better computational results than an arc flow formulation when generating predefined routes in a heuristic manner. By adding optional symmetry breaking constraints and cuts to the model, the solution time can also be further reduced for all complete B&B processes. Additionally, results indicate that setting an upper limit of three offshore landings is appropriate for the predefined routes generated for the path flow formulation. Also, aggregating platform data has shown to be applicable to the problem addressed in this report without affecting the variables of interest.

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Appendix A

Definition of accident classes A1-A8

A1: Accident during take-off or landing at heliport/airport [Heliport]
Accident which occurs after passengers have boarded the helicopter and before TPD (Takeoff Decision Point) or after LDP (Landing Decision Point) and before passengers have left the heliport/airport.

A2: Accident during take-off or landing on helideck [Helideck]
Accident which occurs after passengers have boarded the helicopter and before TDP (Takeoff Decision Point) or after LDP (Landing Decision Point) and before passengers have left the helideck.

A3: Accident caused by critical failure in helicopter during flight [System failure]
Accident caused by critical system failure in the helicopter after TDP (Take-off Decision Point) and before LDP (Landing Decision Point), for example in the main rotor, tail rotor, engine, gearbox, etc. When a critical system failure occurs, the craft (pilots/passengers) can only be saved through a successful emergency landing.

A4: Collision with another aircraft [Mid-air collision]
Collision with another aircraft during flight, without any critical failure occurring. (Mid-Air Collision; MAC)

A5: Controlled flight into terrain, sea or building [Terrain collision]
Accident caused by collision into terrain, sea, or building after TDP (Take-off Decision Point) and before LDP (Landing Decision Point), with no critical failure occurring. (Controlled Flight Into Terrain, sea or building; CFIT)

A6: Accident with risk for persons in the helicopter [Person inside]
Accident involving danger to persons (pilots/passengers) located in the helicopter, for example caused by toxic gases due to a baggage or cargo fire.

A7: Accident with danger for persons outside helicopter [Person outside]
Accident involving danger to persons (pilot/passengers) located outside the helicopter, for example, the tail rotor strikes a person. (Note that danger to other persons than helicopter pilots and passengers, for example helideck personnel, is not included.)

A8: Accident caused by weather conditions, surrounding environment, or other [Other/unknown]
Accident caused by weather conditions (for example lightning strike), surrounding environment (for example collision with a vehicle at the heliport/airport), or other (for example an act of terror), in addition to accidents with unknown causes.

Appendix B

Arc Flow Formulation (AFF)

Indices

b:	Onshore airport base.
h:	Potential location for offshore transhipment hub.
p:	Offshore platform.
i, j:	Onshore airport base, potential location for offshore transhipment hub, or offshore platform.
k:	Helicopter type.
n:	Identity number helicopter.
f:	Identity number helicopter flight.
s:	Identity number second echelon trip.

Sets

$\mathbf{B}$:	Set of onshore airport bases.
$\mathbf{H}$:	Set of potential locations for offshore transhipment hub(s).
$\mathbf{P}$:	Set of offshore platforms.
$\mathbf{K}$:	Set of available helicopter types.
$\mathbf{N_k}$:	Set of available helicopters of type k.
$\mathbf{F}$:	Set of possible daily flights for every available helicopter.
$\mathbf{S}$:	Set of possible second echelon trips during every flight for every available helicopter.

$\mathbf{A}$: Set of all feasible arcs.
$\mathbf{A^1}$: Set of feasible arcs in the direct routing policy.
$\mathbf{A^2}$: Set of feasible arcs in the first echelon of the hub connected routing policy.
$\mathbf{A^3}$: Set of feasible arcs in the second echelon of the hub connected routing policy.

Parameters

C^{FH}: Fixed investment and operating cost per day for every installed offshore transhipment hub.
C_k^{FK}: Fixed investment cost per day for helicopters of type k.
$C_{i,j,k}^{VA}$: Variable operating cost arc (i,j) for helicopters of type k.
$C_{b,k}^{VB}$: Variable parking cost on onshore airport base b for helicopters of type k.
$R_{i,j,k}^{A}$: Risk assessment arc (i,j) for helicopters of type k.
F^{W}: Weight factor assigned to the total cost of the transportation system.
F^{S}: Scale factor assigned to the total risk of the transportation system.
Q_b^{H}: Available helicopter parking capacity at onshore airport base b.
Q_b^{TB}: Available take-off and landing capacity at onshore airport base b.
Q^{TH}: Take-off and landing capacity at every installed offshore transhipment hub.
Q_p^{TP}: Take-off and landing capacity at offshore platform p.
Q_k^{P}: Passenger capacity of helicopters of type k.
$T_{i,j,k}^{FA}$: Flying time arc (i,j), including boarding time on node i if this is an onshore airport base and landing time on node j, for helicopters of type k.
T^{OD}: Maximum operating time per day for all available helicopters.
T^{OF}: Maximum operating time per flight for all available helicopters.
T^{P}: Time per day from which a helicopter's parking cost on an onshore airport base is to be derived.
$V^{P1,D}$: Maximum number of platform visits during a direct routing policy flight.
$V^{P1,H}$: Maximum number of platform visits during a second echelon trip of a flight using a hub connected routing policy.

V^{P2}: Maximum number of visits to one particular platform during a direct routing policy flight or during a second echelon trip of a flight using a hub connected routing policy.

$L^{FA}_{i,j}$: Flying length arc (i,j) for all available helicopters.

L^{R}_{k}: Maximum flying length (range) for helicopters of type k.

D^{P}_{p}: Demand of employees to be picked up at platform p.

D^{D}_{p}: Demand of employees to be delivered at platform p.

Decision variables

$x_{i,j,k,n,f}$: 1 if helicopter number n of type k uses a direct routing policy, and travels directly between nodes i and j, on flight number f. Otherwise 0.

$y_{i,j,k,n,f}$: 1 if helicopter number n of type k uses a hub connected routing policy, and travels directly between nodes i and j during the first echelon trip, on flight number f. Otherwise 0.

$z_{i,j,k,n,f,s}$: 1 if helicopter number n of type k uses a hub connected routing policy, and travels directly between nodes i and j during second echelon trip number s, on flight number f. Otherwise 0.

$a^{H}_{b,k,n}$: 1 if helicopter number n of type k is assigned to onshore airport base b, otherwise 0.

$a^{P}_{i,p}$: 1 if offshore platform p is assigned to onshore airport base or offshore transhipment hub i, otherwise 0.

o^{H}_{h}: 1 if an offshore transhipment hub is installed at location h, otherwise 0.

$t^{P}_{b,k,n}$: Time per day from which the parking cost on onshore airport base b should be derived for helicopter number n of type k.

$p^{x}_{i,j,k,n,f}$: Pickup load of employees on helicopter number n of type k if it uses a direct routing policy, and travels directly between nodes i and j, on flight number f. Otherwise 0.

$d^{x}_{i,j,k,n,f}$: Delivery load of employees on helicopter number n of type k if it uses a direct routing policy, and travels directly between nodes i and j, on flight number f. Otherwise 0.

$p^{y}_{i,j,k,n,f}$: Pickup load of employees on helicopter number n of type k if it uses a hub connected routing policy, and travels directly between nodes i and j during the first echelon trip, on flight number f. Otherwise 0.

$d^{y}_{i,j,k,n,f}$: Delivery load of employees on helicopter number n of type k if it uses a hub connected routing policy, and travels directly between nodes i and j during the first echelon trip, on flight number f. Otherwise 0.

$p^{z}_{i,j,k,n,f,s}$: Pickup load of employees on helicopter number n of type k if it uses a hub connected routing policy, and travels directly between nodes i and j during second echelon trip number s, on flight number f. Otherwise 0.

$d^{z}_{i,j,k,n,f,s}$: Delivery load of employees on helicopter number n of type k if it uses a hub connected routing policy, and travels directly between nodes i and j during second echelon trip number s, on flight number f. Otherwise 0.

$e^{P,x}_{p,k,n,f}$: The total number of employees picked up at platform p by helicopter number n of type k during flight number f if a direct routing policy is used.

$e^{D,x}_{p,k,n,f}$: The total number of employees delivered at platform p by helicopter number n of type k during flight number f if a direct routing policy is used.

$e^{P,z}_{p,k,n,f}$: The total number of employees picked up at platform p by helicopter number n of type k during flight number f if a hub connected routing policy is used.

$e^{D,z}_{p,k,n,f}$: The total number of employees delivered at platform p by helicopter number n of type k during flight number f if a hub connected routing policy is used.

Model

Minimize

$$\begin{aligned}
F^W(\sum_{(i,j)\in\mathbf{A^1}}\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{f\in\mathbf{F}} C^{VA}_{i,j,k}x_{i,j,k,n,f} + \sum_{(i,j)\in\mathbf{A^2}}\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{f\in\mathbf{F}} C^{VA}_{i,j,k}y_{i,j,k,n,f} + \\
\sum_{(i,j)\in\mathbf{A^3}}\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{f\in\mathbf{F}}\sum_{s\in\mathbf{S}} C^{VA}_{i,j,k}z_{i,j,k,n,f,s} + \\
\sum_{b\in\mathbf{B}}\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}} C^{VB}_{b,k}t^P_{b,k,n} + \sum_{b\in\mathbf{B}}\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}} C^{FK}_k a^H_{b,k,n} + C^{FH}\sum_{h\in\mathbf{H}} o^H_h) + \\
F^S(1-F^W)(\sum_{(i,j)\in\mathbf{A^1}}\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{f\in\mathbf{F}} R^A_{i,j,k}(p^x_{i,j,k,n,f} + d^x_{i,j,k,n,f}) + \\
\sum_{(i,j)\in\mathbf{A^2}}\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{f\in\mathbf{F}} R^A_{i,j,k}(p^y_{i,j,k,n,f} + d^y_{i,j,k,n,f}) + \sum_{(i,j)\in\mathbf{A^3}}\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{f\in\mathbf{F}}\sum_{s\in\mathbf{S}} R^A_{i,j,k}(p^z_{i,j,k,n,f,s} + d^z_{i,j,k,n,f,s}))
\end{aligned} \tag{B.1}$$

Subject to

Network constraints:

$$\sum_{b\in\mathbf{B}} a^H_{b,k,n} \leq 1, \qquad k\in\mathbf{K}, n\in\mathbf{N_k} \tag{B.2}$$

$$\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}} a^H_{b,k,n} \leq Q^H_b, \qquad b\in\mathbf{B} \tag{B.3}$$

$$\sum_{i\in\mathbf{B}\cup\mathbf{H}} a^P_{i,p} = 1, \qquad p\in\mathbf{P} \tag{B.4}$$

$$a^P_{h,p} - o^H_h \leq 0, \qquad h\in\mathbf{H}, p\in\mathbf{P} \tag{B.5}$$

$$\sum_{p\in\mathbf{P}} x_{b,p,k,n,f} + \sum_{h\in\mathbf{H}} y_{b,h,k,n,f} - a^H_{b,k,n} \leq 0, \qquad b\in\mathbf{B}, k\in\mathbf{K}, n\in\mathbf{N_k}, f\in\mathbf{F} \tag{B.6}$$

$$\sum_{p\in\mathbf{P}}\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{f\in\mathbf{F}} x_{b,p,k,n,f} + \sum_{h\in\mathbf{H}}\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{f\in\mathbf{F}} y_{b,h,k,n,f} \leq Q^{TB}_b, \qquad b\in\mathbf{B} \tag{B.7}$$

$$\sum_{b\in\mathbf{B}}\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{f\in\mathbf{F}} y_{h,b,k,n,f} + \sum_{p\in\mathbf{P}}\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{f\in\mathbf{F}}\sum_{s\in\mathbf{S}} z_{h,p,k,n,f,s} - Q^{TH} o^H_h \leq 0, \qquad h\in\mathbf{H} \tag{B.8}$$

$$\sum_{j\in\mathbf{B}\cup\mathbf{P}}\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{f\in\mathbf{F}} x_{p,j,k,n,f} + \sum_{j\in\mathbf{H}\cup\mathbf{P}}\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{f\in\mathbf{F}}\sum_{s\in\mathbf{S}} z_{p,j,k,n,f,s} \leq Q^{TP}_p, \qquad p\in\mathbf{P} \tag{B.9}$$

$$\sum_{(i,j)\in\mathbf{A^1}} T^{FA}_{i,j,k} x_{i,j,k,n,f} + \sum_{(i,j)\in\mathbf{A^2}} T^{FA}_{i,j,k} y_{i,j,k,n,f} + \sum_{(i,j)\in\mathbf{A^3}}\sum_{s\in\mathbf{S}} T^{FA}_{i,j,k} z_{i,j,k,n,f,s} \leq T^{OF}, \qquad k\in\mathbf{K}, n\in\mathbf{N_k}, f\in\mathbf{F} \tag{B.10}$$

$$\sum_{(i,j)\in\mathbf{A^1}}\sum_{f\in\mathbf{F}} T^{FA}_{i,j,k} x_{i,j,k,n,f} + \sum_{(i,j)\in\mathbf{A^2}}\sum_{f\in\mathbf{F}} T^{FA}_{i,j,k} y_{i,j,k,n,f} + \sum_{(i,j)\in\mathbf{A^3}}\sum_{f\in\mathbf{F}}\sum_{s\in\mathbf{S}} T^{FA}_{i,j,k} z_{i,j,k,n,f,s} \leq T^{OD}, \qquad k\in\mathbf{K}, n\in\mathbf{N_k} \tag{B.11}$$

$$\sum_{(i,j)\in\mathbf{A^1}}\sum_{f\in\mathbf{F}} T^{FA}_{i,j,k} x_{i,j,k,n,f} + \sum_{(i,j)\in\mathbf{A^2}}\sum_{f\in\mathbf{F}} T^{FA}_{i,j,k} y_{i,j,k,n,f} + \sum_{(i,j)\in\mathbf{A^3}}\sum_{f\in\mathbf{F}}\sum_{s\in\mathbf{S}} T^{FA}_{i,j,k} z_{i,j,k,n,f,s} + \sum_{b\in\mathbf{B}} t^P_{b,k,n} - T^P \sum_{b\in\mathbf{B}} a^H_{b,k,n} = 0, \qquad k\in\mathbf{K}, n\in\mathbf{N_k} \tag{B.12}$$

$$t^P_{b,k,n} - T^P a^H_{b,k,n} \leq 0, \qquad b\in\mathbf{B}, k\in\mathbf{K}, n\in\mathbf{N_k} \tag{B.13}$$

Direct routing policy constraints:

$$\sum_{j \in \mathbf{B} \cup \mathbf{P}} x_{j,i,k,n,f} - \sum_{j \in \mathbf{B} \cup \mathbf{P}} x_{i,j,k,n,f} = 0, \qquad i \in \mathbf{B} \cup \mathbf{P}, k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F} \tag{B.14}$$

$$\sum_{i \in \mathbf{P}} \sum_{j \in \mathbf{P}} x_{i,j,k,n,f} - (V^{P1,D} - 1) \sum_{i \in \mathbf{B}} \sum_{j \in \mathbf{P}} x_{i,j,k,n,f} \leq 0, \qquad k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F} \tag{B.15}$$

$$\sum_{j \in \mathbf{P}} x_{b,j,k,n,f} + \sum_{i \in \mathbf{B} \cup \mathbf{P}} x_{i,p,k,n,f} - V^{P2} a^{P}_{b,p} \leq 1, \qquad b \in \mathbf{B}, p \in \mathbf{P}, k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F} \tag{B.16}$$

$$\sum_{(i,j) \in \mathbf{A^1}} L^{FA}_{i,j} x_{i,j,k,n,f} \leq L^{R}_{k}, \qquad k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F} \tag{B.17}$$

Hub connected routing policy constraints:

$$\sum_{j \in \mathbf{B} \cup \mathbf{H}} y_{j,i,k,n,f} - \sum_{j \in \mathbf{B} \cup \mathbf{H}} y_{i,j,k,n,f} = 0, \qquad i \in \mathbf{B} \cup \mathbf{H}, k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F} \tag{B.18}$$

$$\sum_{j \in \mathbf{H} \cup \mathbf{P}} z_{j,i,k,n,f,s} - \sum_{j \in \mathbf{H} \cup \mathbf{P}} z_{i,j,k,n,f,s} = 0, \qquad i \in \mathbf{H} \cup \mathbf{P}, k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F}, s \in \mathbf{S} \tag{B.19}$$

$$\sum_{p \in \mathbf{P}} z_{h,p,k,n,f,s} - \sum_{b \in \mathbf{B}} y_{b,h,k,n,f} \leq 0, \qquad h \in \mathbf{H}, k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F}, s \in \mathbf{S} \tag{B.20}$$

$$\sum_{i \in \mathbf{P}} \sum_{j \in \mathbf{P}} z_{i,j,k,n,f,s} - (V^{P1,H} - 1) \sum_{i \in \mathbf{H}} \sum_{j \in \mathbf{P}} z_{i,j,k,n,f,s} \leq 0, \qquad k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F}, s \in \mathbf{S} \tag{B.21}$$

$$\sum_{b \in \mathbf{B}} y_{b,h,k,n,f} + \sum_{i \in \mathbf{H} \cup \mathbf{P}} z_{i,p,k,n,f,s} - V^{P2} a^{P}_{h,p} \leq 1, \qquad h \in \mathbf{H}, p \in \mathbf{P}, k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F}, s \in \mathbf{S} \tag{B.22}$$

$$\sum_{b\in\mathbf{B}}\sum_{h\in\mathbf{H}} L^{FA}_{b,h} y_{b,h,k,n,f} \le L^R_k, \qquad k\in\mathbf{K}, n\in\mathbf{N_k}, f\in\mathbf{F} \tag{B.23}$$

$$\sum_{(i,j)\in\mathbf{A^3}} L^{FA}_{i,j} z_{i,j,k,n,f,s} \le L^R_k, \qquad k\in\mathbf{K}, n\in\mathbf{N_k}, f\in\mathbf{F}, s\in\mathbf{S} \tag{B.24}$$

Pickup and delivery constraints:

$$p^x_{i,j,k,n,f} + d^x_{i,j,k,n,f} - Q^P_k x_{i,j,k,n,f} \le 0, \qquad (i,j)\in\mathbf{A^1}, k\in\mathbf{K} n\in\mathbf{N_k}, f\in\mathbf{F} \tag{B.25}$$

$$p^y_{i,j,k,n,f} + d^y_{i,j,k,n,f} - Q^P_k y_{i,j,k,n,f} \le 0, \qquad (i,j)\in\mathbf{A^2}, k\in\mathbf{K} n\in\mathbf{N_k}, f\in\mathbf{F} \tag{B.26}$$

$$p^z_{i,j,k,n,f,s} + d^z_{i,j,k,n,f,s} - Q^P_k z_{i,j,k,n,f,s} \le 0, \qquad (i,j)\in\mathbf{A^3}, k\in\mathbf{K} n\in\mathbf{N_k}, f\in\mathbf{F}, s\in\mathbf{S} \tag{B.27}$$

$$\sum_{b\in\mathbf{B}}\sum_{p\in\mathbf{P}}\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{f\in\mathbf{F}} p^x_{b,p,k,n,f} + \sum_{b\in\mathbf{B}}\sum_{h\in\mathbf{H}}\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{f\in\mathbf{F}} p^y_{b,h,k,n,f} = 0 \tag{B.28}$$

$$\sum_{b\in\mathbf{B}}\sum_{p\in\mathbf{P}}\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{f\in\mathbf{F}} d^x_{p,b,k,n,f} + \sum_{b\in\mathbf{B}}\sum_{h\in\mathbf{H}}\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{f\in\mathbf{F}} d^y_{h,b,k,n,f} = 0 \tag{B.29}$$

$$\sum_{p\in\mathbf{P}}\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{f\in\mathbf{F}}\sum_{s\in\mathbf{S}} p^z_{p,h,k,n,f,s} - \sum_{b\in\mathbf{B}}\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{f\in\mathbf{F}} p^y_{h,b,k,n,f} = 0, \qquad h\in\mathbf{H} \tag{B.30}$$

$$\sum_{b\in\mathbf{B}}\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{f\in\mathbf{F}} d^y_{b,h,k,n,f} - \sum_{p\in\mathbf{P}}\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{f\in\mathbf{F}}\sum_{s\in\mathbf{S}} d^z_{h,p,k,n,f,s} = 0, \qquad h\in\mathbf{H} \tag{B.31}$$

$$\sum_{j\in\mathbf{B}\cup\mathbf{P}} p^x_{p,j,k,n,f} - \sum_{i\in\mathbf{B}\cup\mathbf{P}} p^x_{i,p,k,n,f} - e^{P,x}_{p,k,n,f} = 0, \qquad p\in\mathbf{P}, k\in\mathbf{K}, n\in\mathbf{N_k}, f\in\mathbf{F} \tag{B.32}$$

$$\sum_{i\in\mathbf{B}\cup\mathbf{P}} d^x_{i,p,k,n,f} - \sum_{j\in\mathbf{B}\cup\mathbf{P}} d^x_{p,j,k,n,f} - e^{D,x}_{p,k,n,f} = 0, \qquad p\in\mathbf{P}, k\in\mathbf{K}, n\in\mathbf{N_k}, f\in\mathbf{F} \tag{B.33}$$

$$\sum_{j\in\mathbf{H}\cup\mathbf{P}} p^z_{p,j,k,n,f,s} - \sum_{i\in\mathbf{H}\cup\mathbf{P}} p^z_{i,p,k,n,f,s} - e^{P,z}_{p,k,n,f,s} = 0, \qquad p\in\mathbf{P}, k\in\mathbf{K}, n\in\mathbf{N_k}, f\in\mathbf{F}, s\in\mathbf{S} \tag{B.34}$$

$$\sum_{i\in \mathbf{H}\cup \mathbf{P}} d^z_{i,p,k,n,f,s} - \sum_{j\in \mathbf{H}\cup \mathbf{P}} d^z_{p,j,k,n,f,s} - e^{D,z}_{p,k,n,f,s} = 0, \quad p\in \mathbf{P}, k\in \mathbf{K}, n\in \mathbf{N_k}, f\in \mathbf{F}, s\in \mathbf{S} \tag{B.35}$$

$$\sum_{k\in \mathbf{K}}\sum_{n\in \mathbf{N_k}}\sum_{f\in \mathbf{F}} e^{P,x}_{p,k,n,f} + \sum_{k\in \mathbf{K}}\sum_{n\in \mathbf{N_k}}\sum_{f\in \mathbf{F}}\sum_{s\in \mathbf{S}} e^{P,z}_{p,k,n,f,s} = D^P_p, \quad p\in \mathbf{P} \tag{B.36}$$

$$\sum_{k\in \mathbf{K}}\sum_{n\in \mathbf{N_k}}\sum_{f\in \mathbf{F}} e^{D,x}_{p,k,n,f} + \sum_{k\in \mathbf{K}}\sum_{n\in \mathbf{N_k}}\sum_{f\in \mathbf{F}}\sum_{s\in \mathbf{S}} e^{D,z}_{p,k,n,f,s} = D^D_p, \quad p\in \mathbf{P} \tag{B.37}$$

Constraints on variables:

$$x_{i,j,k,n,f} \in \{0,1\}, \quad (i,j)\in \mathbf{A^1}, k\in \mathbf{K}, n\in \mathbf{N_k}, f\in \mathbf{F} \tag{B.38}$$

$$y_{i,j,k,n,f} \in \{0,1\}, \quad (i,j)\in \mathbf{A^2}, k\in \mathbf{K}, n\in \mathbf{N_k}, f\in \mathbf{F} \tag{B.39}$$

$$z_{i,j,k,n,f,s} \in \{0,1\}, \quad (i,j)\in \mathbf{A^3}, k\in \mathbf{K}, n\in \mathbf{N_k}, f\in \mathbf{F}, s\in \mathbf{S} \tag{B.40}$$

$$a^H_{b,k,n} \in \{0,1\}, \quad b\in \mathbf{B}, k\in \mathbf{K}, n\in \mathbf{N_k} \tag{B.41}$$

$$a^P_{i,p} \in \{0,1\}, \quad i\in \mathbf{B}\cup \mathbf{H}, p\in \mathbf{P} \tag{B.42}$$

$$o^H_h \in \{0,1\}, \quad h\in \mathbf{H} \tag{B.43}$$

$$t^P_{b,k,n} \geq 0, \quad b\in \mathbf{B}, k\in \mathbf{K}, n\in \mathbf{N_k} \tag{B.44}$$

$$p^x_{i,j,k,n,f}, d^x_{i,j,k,n,f} \geq 0, \quad (i,j)\in \mathbf{A^1}, k\in \mathbf{K}, n\in \mathbf{N_k}, f\in \mathbf{F} \tag{B.45}$$

$$p^y_{i,j,k,n,f}, d^y_{i,j,k,n,f} \geq 0, \quad (i,j)\in \mathbf{A^2}, k\in \mathbf{K}, n\in \mathbf{N_k}, f\in \mathbf{F} \tag{B.46}$$

$$p^z_{i,j,k,n,f,s}, d^z_{i,j,k,n,f,s} \geq 0, \quad (i,j)\in \mathbf{A^3}, k\in \mathbf{K}, n\in \mathbf{N_k}, f\in \mathbf{F}, s\in \mathbf{S} \tag{B.47}$$

$$e^{P,x}_{p,k,n,f}, e^{D,x}_{p,k,n,f} \geq 0, \quad p\in \mathbf{P}, k\in \mathbf{K}, n\in \mathbf{N_k}, f\in \mathbf{F} \tag{B.48}$$

$$e^{P,z}_{p,k,n,f,s}, e^{D,z}_{p,k,n,f,s} \geq 0, \qquad p \in \mathbf{P}, k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F}, s \in \mathbf{S} \tag{B.49}$$

Optional symmetry breaking constraints:

$$\sum_{b \in \mathbf{B}} a^H_{b,k,n} - \sum_{b \in \mathbf{B}} a^H_{b,k,n-1} \leq 0, \qquad k \in \mathbf{K}, n = 2 \ldots |\, \mathbf{N_k} \,| \tag{B.50}$$

$$\begin{aligned} &\sum_{(i,j) \in \mathbf{A^1}} \sum_{f \in \mathbf{F}} T^{FA}_{i,j,k} x_{i,j,k,n,f} + \sum_{(i,j) \in \mathbf{A^2}} \sum_{f \in \mathbf{F}} T^{FA}_{i,j,k} y_{i,j,k,n,f} + \\ &\sum_{(i,j) \in \mathbf{A^3}} \sum_{f \in \mathbf{F}} \sum_{s \in \mathbf{S}} T^{FA}_{i,j,k} z_{i,j,k,n,f,s} - \sum_{(i,j) \in \mathbf{A^1}} \sum_{f \in \mathbf{F}} T^{FA}_{i,j,k} x_{i,j,k,n-1,f} - \\ &\sum_{(i,j) \in \mathbf{A^2}} \sum_{f \in \mathbf{F}} T^{FA}_{i,j,k} y_{i,j,k,n-1,f} - \sum_{(i,j) \in \mathbf{A^3}} \sum_{f \in \mathbf{F}} \sum_{s \in \mathbf{S}} T^{FA}_{i,j,k} z_{i,j,k,n-1,f,s} \leq 0, \qquad k \in \mathbf{K}, n = 2 \ldots |\, \mathbf{N_k} \,| \end{aligned} \tag{B.51}$$

$$\begin{aligned} &\sum_{(i,j) \in \mathbf{A^1}} T^{FA}_{i,j,k} x_{i,j,k,n,f} + \sum_{(i,j) \in \mathbf{A^2}} T^{FA}_{i,j,k} y_{i,j,k,n,f} + \\ &\sum_{(i,j) \in \mathbf{A^3}} \sum_{s \in \mathbf{S}} T^{FA}_{i,j,k} z_{i,j,k,n,f,s} - \sum_{(i,j) \in \mathbf{A^1}} T^{FA}_{i,j,k} x_{i,j,k,n,f-1} - \\ &\sum_{(i,j) \in \mathbf{A^2}} T^{FA}_{i,j,k} y_{i,j,k,n,f-1} - \sum_{(i,j) \in \mathbf{A^3}} \sum_{s \in \mathbf{S}} T^{FA}_{i,j,k} z_{i,j,k,n,f-1,s} \leq 0, \qquad k \in \mathbf{K}, n \in \mathbf{N_k}, f = 2 \ldots |\, \mathbf{F} \,| \end{aligned} \tag{B.52}$$

$$\sum_{(i,j) \in \mathbf{A^3}} (T^{FA}_{i,j,k} z_{i,j,k,n,f,s} - T^{FA}_{i,j,k} z_{i,j,k,n,f,s-1}) \leq 0, \qquad k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F}, s = 2 \ldots |\, \mathbf{S} \,| \tag{B.53}$$

Appendix C

Path Flow Formulation I (PFF1)

Indices

b:	Onshore airport base.
h:	Potential location for offshore transhipment hub.
p:	Offshore platform.
i, j:	Onshore airport base, potential location for offshore transhipment hub, or offshore platform.
k:	Helicopter type.
n:	Identity number helicopter.
r:	Predefined, feasible route.
f:	Identity number helicopter flight.
s:	Identity number second echelon trip.

Sets

$\mathbf{B}$:	Set of onshore airport bases.
$\mathbf{H}$:	Set of potential locations for offshore transhipment hub(s).
$\mathbf{P}$:	Set of offshore platforms.
$\mathbf{K}$:	Set of available helicopter types.
$\mathbf{N_k}$:	Set of available helicopters of type k.

$\mathbf{R_k}$: Set of predefined, feasible routes for helicopters of type k.
$\mathbf{R_k^1}$: Set of predefined, feasible routes using a direct routing policy for helicopters of type k .
$\mathbf{R_k^2}$: Set of predefined, feasible first echelon routes for a hub connected routing policy for helicopters of type k .
$\mathbf{R_k^3}$: Set of predefined, feasible second echelon routes for a hub connected routing policy for helicopters of type k.
$\mathbf{F}$: Set of possible daily flights for every available helicopter.
$\mathbf{S}$: Set of possible second echelon trips during every flight for every available helicopter.
$\mathbf{A}$: Set of all feasible arcs.
$\mathbf{A^1}$: Set of feasible arcs in the direct routing policy.
$\mathbf{A^2}$: Set of feasible arcs in the first echelon of the hub connected routing policy.
$\mathbf{A^3}$: Set of feasible arcs in the second echelon of the hub connected routing policy.

Parameters

$A_{i,j,k,r}$: 1 if arc (i, j) is used in route r for helicopters of type k, otherwise 0.
$V_{i,k,r}$: 1 if onshore airport base, offshore transhipment hub or offshore platform i is visited on route r for helicopters of type k, otherwise 0.
$L_{p,k,r}$: Number of times offshore platform i is visited on route r for helicopters of type k.
C^{FH}: Fixed investment and operating cost per day for every installed offshore transhipment hub.
C_k^{FK}: Fixed investment cost per day for helicopters of type k.
$C_{k,r}^{VR}$: Variable operating cost for route r for helicopters of type k.
$C_{b,k}^{VB}$: Variable parking cost on onshore airport base b for helicopters of type k.
$R_{i,j,k}^{A}$: Risk assessment arc (i, j) for helicopters of type k.
F^W: Weight factor assigned to the total cost of the transportation system.
F^S: Scale factor assigned to the total risk of the transportation system.
Q_b^H: Available helicopter parking capacity at onshore airport base b.

Q_b^{TB}: Available take-off and landing capacity at onshore airport base b.

Q^{TH}: Take-off and landing capacity at every installed offshore transhipment hub.

Q_p^{TP}: Take-off and landing capacity at offshore platform p.

Q_k^P: Passenger capacity of helicopters of type k.

$T_{k,r}^{OR}$: Operating time route r for helicopters of type k.

T^{OD}: Maximum operating time per day for all available helicopters.

T^{OF}: Maximum operating time per flight for all available helicopters.

T^P: Time per day from which a helicopter's parking cost on an onshore airport base should be derived.

D_p^P: Demand of employees to be picked up at platform p.

D_p^D: Demand of employees to be delivered at platform p.

Decision variables

$x_{k,n,r,f}$: 1 if helicopter number n of type k travels directly between nodes i and j during a direct routing policy trip or during the first echelon trip of a hub connected routing policy, on flight number f. Otherwise 0.

$z_{k,n,r,f,s}$: 1 if helicopter number n of type k uses a hub connected routing policy, and travels directly between nodes i and j during second echelon trip number s, on flight number f. Otherwise 0.

$a_{b,k,n}^H$: 1 if helicopter number n of type k is assigned to onshore airport base b, otherwise 0.

$a_{i,p}^P$: 1 if offshore platform p is assigned to onshore airport base or offshore transhipment hub i, otherwise 0.

o_h^H: 1 if an offshore transhipment hub is installed at location h, otherwise 0.

$t_{b,k,n}^P$: Time per day from which the parking cost on onshore airport base b should be derived for helicopter number n of type k.

$p_{i,j,k,n,f}^x$: Pickup load of employees on helicopter number n of type k if it travels directly between nodes i and j during a direct routing policy trip or during the first echelon trip of a hub connected routing policy, on flight number f. Otherwise 0.

$d_{i,j,k,n,f}^x$: Delivery load of employees on helicopter number n of type k if it travels directly between nodes i and j during a direct routing policy trip or during the first echelon trip of a hub connected routing policy, on flight number f. Otherwise 0.

$p^z_{i,j,k,n,f,s}$: Pickup load of employees on helicopter number n of type k if it used a hub connected routing policy, and travels directly between nodes i and j during second echelon trip number s, on flight number f. Otherwise 0.

$d^z_{i,j,k,n,f,s}$: Delivery load of employees on helicopter number n of type k if it uses a hub connected routing policy, and travels directly between nodes i and j during second echelon trip number s , on flight number f. Otherwise 0.

$e^{P,x}_{p,k,n,f}$: The total number of employees picked up at platform p by helicopter number n of type k during flight number f if a direct routing policy is used.

$e^{D,x}_{p,k,n,f}$: The total number of employees delivered at platform p by helicopter number n of type k during flight number f if a direct routing policy is used.

$e^{P,z}_{p,k,n,f}$: The total number of employees picked up at platform p by helicopter number n of type k during flight number f if a hub connected routing policy is used.

$e^{D,z}_{p,k,n,f}$: The total number of employees delivered at platform p by helicopter number n of type k during flight number f if a hub connected routing policy is used.

Model

Minimize

$$
\begin{aligned}
&F^W(\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{r\in\mathbf{R^1_k}\cup\mathbf{R^2_k}}\sum_{f\in\mathbf{F}} C^{VR}_{k,r}x_{k,n,r,f} + \sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{r\in\mathbf{R^3_k}}\sum_{f\in\mathbf{F}}\sum_{s\in\mathbf{S}} C^{VR}_{k,r}z_{k,n,r,f,s} + \\
&\sum_{b\in\mathbf{B}}\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}} C^{VB}_{b,k}t^P_{b,k,n} + \sum_{b\in\mathbf{B}}\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}} C^{FK}_k a^H_{b,k,n} + C^{FH}\sum_{h\in\mathbf{H}} o^H_h) + \\
&F^S(1-F^W)(\sum_{(i,j)\in\mathbf{A^1}\cup\mathbf{A^2}}\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{f\in\mathbf{F}} R^A_{i,j,k}(p^x_{i,j,k,n,f} + d^x_{i,j,k,n,f}) + \\
&\sum_{(i,j)\in\mathbf{A^3}}\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{f\in\mathbf{F}}\sum_{s\in\mathbf{S}} R^A_{i,j,k}(p^z_{i,j,k,n,f,s} + d^z_{i,j,k,n,f,s}))
\end{aligned}
\tag{C.1}
$$

Subject to

Network constraints:

$$\sum_{b\in\mathbf{B}} a^H_{b,k,n} \leq 1, \qquad k\in\mathbf{K}, n\in\mathbf{N_k} \tag{C.2}$$

$$\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}} a^H_{b,k,n} \leq Q^H_b, \qquad b\in\mathbf{B} \tag{C.3}$$

$$\sum_{i\in\mathbf{B}\cup\mathbf{H}} a^P_{i,p} = 1, \qquad p\in\mathbf{P} \tag{C.4}$$

$$a^P_{h,p} - o^H_h \leq 0, \qquad h\in\mathbf{H}, p\in\mathbf{P} \tag{C.5}$$

$$\sum_{r\in\mathbf{R^1_k}\cup\mathbf{R^2_k}} V_{b,k,r}x_{k,n,r,f} - a^H_{b,k,n} \leq 0, \qquad b\in\mathbf{B}, k\in\mathbf{K}, n\in\mathbf{N_k}, f\in\mathbf{F} \tag{C.6}$$

$$\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{r\in\mathbf{R^1_k}\cup\mathbf{R^2_k}}\sum_{f\in\mathbf{F}} V_{b,k,r}x_{k,n,r,f} \leq Q^{TB}_b, \qquad b\in\mathbf{B} \tag{C.7}$$

$$\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{r\in\mathbf{R_k^2}}\sum_{f\in\mathbf{F}}V_{h,k,r}x_{k,n,r,f}+$$

$$\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{r\in\mathbf{R_k^3}}\sum_{f\in\mathbf{F}}\sum_{s\in\mathbf{S}}V_{h,k,r}z_{k,n,r,f,s}-Q^{TH}o_h^H\leq 0,\qquad h\in\mathbf{H}\tag{C.8}$$

$$\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{r\in\mathbf{R_k^1}}\sum_{f\in\mathbf{F}}L_{p,k,r}x_{k,n,r,f}+$$

$$\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{r\in\mathbf{R_k^3}}\sum_{f\in\mathbf{F}}\sum_{s\in\mathbf{S}}L_{p,k,r}z_{k,n,r,f,s}\leq Q_p^{TP},\qquad p\in\mathbf{P}\tag{C.9}$$

$$\sum_{r\in\mathbf{R_k^1}\cup\mathbf{R_k^2}}T_{k,r}^{OR}x_{k,n,r,f}+\sum_{r\in\mathbf{R_k^3}}\sum_{s\in\mathbf{S}}T_{k,r}^{OR}z_{k,n,r,f,s}\leq T^{OF},\qquad k\in\mathbf{K},n\in\mathbf{N_k},f\in\mathbf{F}\tag{C.10}$$

$$\sum_{r\in\mathbf{R_k^1}\cup\mathbf{R_k^2}}\sum_{f\in\mathbf{F}}T_{k,r}^{OR}x_{k,n,r,f}+\sum_{r\in\mathbf{R_k^3}}\sum_{f\in\mathbf{F}}\sum_{s\in\mathbf{S}}T_{k,r}^{OR}z_{k,n,r,f,s}\leq T^{OD},\qquad k\in\mathbf{K},n\in\mathbf{N_k}\tag{C.11}$$

$$\sum_{r\in\mathbf{R_k^1}\cup\mathbf{R_k^2}}\sum_{f\in\mathbf{F}}T_{k,r}^{OR}x_{k,n,r,f}+\sum_{r\in\mathbf{R_k^3}}\sum_{f\in\mathbf{F}}\sum_{s\in\mathbf{S}}T_{k,r}^{OR}z_{k,n,r,f,s}+$$

$$\sum_{b\in\mathbf{B}}t_{b,k,n}^P-T^P\sum_{b\in\mathbf{B}}a_{b,k,n}^H=0,\qquad k\in\mathbf{K},n\in\mathbf{N_k}\tag{C.12}$$

$$t_{b,k,n}^P-T^Pa_{b,k,n}^H\leq 0,\qquad b\in\mathbf{B},k\in\mathbf{K},n\in\mathbf{N_k}\tag{C.13}$$

Direct routing policy constraints:

$$\sum_{r\in\mathbf{R_k^1}}V_{b,k,r}V_{p,k,r}x_{k,n,r,f}-a_{b,p}^P\leq 0,\qquad b\in\mathbf{B},p\in\mathbf{P},k\in\mathbf{K},n\in\mathbf{N_k},f\in\mathbf{F}\tag{C.14}$$

Hub connected routing policy constraints:

$$\sum_{r \in \mathbf{R_k^3}} V_{h,k,r} z_{k,n,r,f,s} - \sum_{r \in \mathbf{R_k^2}} V_{h,k,r} x_{k,n,r,f} \leq 0, \qquad h \in \mathbf{H}, k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F}, s \in \mathbf{S} \tag{C.15}$$

$$\sum_{r \in \mathbf{R_k^3}} V_{h,k,r} V_{p,k,r} z_{k,n,r,f,s} - a^P_{h,p} \leq 0, \qquad h \in \mathbf{H}, p \in \mathbf{P}, k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F}, s \in \mathbf{S} \tag{C.16}$$

Pickup and delivery constraints:

$$p^x_{i,j,k,n,f} + d^x_{i,j,k,n,f} - Q^P_k \sum_{r \in \mathbf{R_k^1} \cup \mathbf{R_k^2}} A_{i,j,k,r} x_{k,n,r,f} \leq 0, \qquad (i,j) \in \mathbf{A^1} \cup \mathbf{A^2}, k \in \mathbf{K} n \in \mathbf{N_k}, f \in \mathbf{F} \tag{C.17}$$

$$p^z_{i,j,k,n,f,s} + d^z_{i,j,k,n,f,s} - Q^P_k \sum_{r \in \mathbf{R_k^3}} A_{i,j,k,r} z_{k,n,r,f,s} \leq 0, \qquad (i,j) \in \mathbf{A^3}, k \in \mathbf{K} n \in \mathbf{N_k}, f \in \mathbf{F}, s \in \mathbf{S} \tag{C.18}$$

$$\sum_{b \in \mathbf{B}} \sum_{j \in \mathbf{P} \cup \mathbf{H}} \sum_{k \in \mathbf{K}} \sum_{n \in \mathbf{N_k}} \sum_{f \in \mathbf{F}} p^x_{b,j,k,n,f} = 0 \tag{C.19}$$

$$\sum_{i \in \mathbf{P} \cup \mathbf{H}} \sum_{b \in \mathbf{B}} \sum_{k \in \mathbf{K}} \sum_{n \in \mathbf{N_k}} \sum_{f \in \mathbf{F}} d^x_{i,b,k,n,f} = 0 \tag{C.20}$$

$$\sum_{p \in \mathbf{P}} \sum_{k \in \mathbf{K}} \sum_{n \in \mathbf{N_k}} \sum_{f \in \mathbf{F}} \sum_{s \in \mathbf{S}} p^z_{p,h,k,n,f,s} - \sum_{b \in \mathbf{B}} \sum_{k \in \mathbf{K}} \sum_{n \in \mathbf{N_k}} \sum_{f \in \mathbf{F}} p^x_{h,b,k,n,f} = 0, \qquad h \in \mathbf{H} \tag{C.21}$$

$$\sum_{b \in \mathbf{B}} \sum_{k \in \mathbf{K}} \sum_{n \in \mathbf{N_k}} \sum_{f \in \mathbf{F}} d^x_{b,h,k,n,f} - \sum_{p \in \mathbf{P}} \sum_{k \in \mathbf{K}} \sum_{n \in \mathbf{N_k}} \sum_{f \in \mathbf{F}} \sum_{s \in \mathbf{S}} d^z_{h,p,k,n,f,s} = 0, \qquad h \in \mathbf{H} \tag{C.22}$$

$$\sum_{j \in \mathbf{B} \cup \mathbf{P}} p^x_{p,j,k,n,f} - \sum_{i \in \mathbf{B} \cup \mathbf{P}} p^x_{i,p,k,n,f} - e^{P,x}_{p,k,n,f} = 0, \qquad p \in \mathbf{P}, k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F} \tag{C.23}$$

$$\sum_{i \in \mathbf{B} \cup \mathbf{P}} d^x_{i,p,k,n,f} - \sum_{j \in \mathbf{B} \cup \mathbf{P}} d^x_{p,j,k,n,f} - e^{D,x}_{p,k,n,f} = 0, \qquad p \in \mathbf{P}, k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F} \tag{C.24}$$

$$\sum_{j \in \mathbf{H} \cup \mathbf{P}} p^z_{p,j,k,n,f,s} - \sum_{i \in \mathbf{H} \cup \mathbf{P}} p^z_{i,p,k,n,f,s} - e^{P,z}_{p,k,n,f,s} = 0, \qquad p \in \mathbf{P}, k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F}, s \in \mathbf{S} \tag{C.25}$$

$$\sum_{i \in \mathbf{H} \cup \mathbf{P}} d^z_{i,p,k,n,f,s} - \sum_{j \in \mathbf{H} \cup \mathbf{P}} d^z_{p,j,k,n,f,s} - e^{D,z}_{p,k,n,f,s} = 0, \qquad p \in \mathbf{P}, k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F}, s \in \mathbf{S} \tag{C.26}$$

$$\sum_{k \in \mathbf{K}} \sum_{n \in \mathbf{N_k}} \sum_{f \in \mathbf{F}} e^{P,x}_{p,k,n,f} + \sum_{k \in \mathbf{K}} \sum_{n \in \mathbf{N_k}} \sum_{f \in \mathbf{F}} \sum_{s \in \mathbf{S}} e^{P,z}_{p,k,n,f,s} = D^P_p, \qquad p \in \mathbf{P} \tag{C.27}$$

$$\sum_{k \in \mathbf{K}} \sum_{n \in \mathbf{N_k}} \sum_{f \in \mathbf{F}} e^{D,x}_{p,k,n,f} + \sum_{k \in \mathbf{K}} \sum_{n \in \mathbf{N_k}} \sum_{f \in \mathbf{F}} \sum_{s \in \mathbf{S}} e^{D,z}_{p,k,n,f,s} = D^D_p, \qquad p \in \mathbf{P} \tag{C.28}$$

Constraints on variables:

$$x_{k,n,r,f} \in \{0,1\}, \quad k \in \mathbf{K}, n \in \mathbf{N_k}, r \in \mathbf{R^1_k} \cup \mathbf{R^2_k}, f \in \mathbf{F} \tag{C.29}$$

$$z_{k,n,r,f,s} \in \{0,1\}, \quad k \in \mathbf{K}, n \in \mathbf{N_k}, r \in \mathbf{R^3_k}, f \in \mathbf{F}, s \in \mathbf{S} \tag{C.30}$$

$$a^H_{b,k,n} \in \{0,1\}, \quad b \in \mathbf{B}, k \in \mathbf{K}, n \in \mathbf{N_k} \tag{C.31}$$

$$a^P_{i,p} \in \{0,1\}, \quad i \in \mathbf{B} \cup \mathbf{H}, p \in \mathbf{P} \tag{C.32}$$

$$o^H_h \in \{0,1\}, \quad h \in \mathbf{H} \tag{C.33}$$

$$t^P_{b,k,n} \geq 0, \quad b \in \mathbf{B}, k \in \mathbf{K}, n \in \mathbf{N_k} \tag{C.34}$$

$$p^x_{i,j,k,n,f}, d^x_{i,j,k,n,f} \geq 0, \quad (i,j) \in \mathbf{A^1} \cup \mathbf{A^2}, k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F} \tag{C.35}$$

$$p^z_{i,j,k,n,f,s}, d^z_{i,j,k,n,f,s} \geq 0, \quad (i,j) \in \mathbf{A^3}, k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F}, s \in \mathbf{S} \tag{C.36}$$

$$e^{P,x}_{p,k,n,f}, e^{D,x}_{p,k,n,f} \geq 0, \quad p \in \mathbf{P}, k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F} \tag{C.37}$$

$$e^{P,z}_{p,k,n,f,s}, e^{D,z}_{p,k,n,f,s} \geq 0, \qquad p \in \mathbf{P}, k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F}, s \in \mathbf{S} \quad \text{(C.38)}$$

Optional symmetry breaking constraints:

$$\sum_{b \in \mathbf{B}} a^H_{b,k,n} - \sum_{b \in \mathbf{B}} a^H_{b,k,n-1} \leq 0, \qquad k \in \mathbf{K}, n = 2 \ldots \mid \mathbf{N_k} \mid \quad \text{(C.39)}$$

$$\sum_{r \in \mathbf{R^1_k} \cup \mathbf{R^2_k}} \sum_{f \in \mathbf{F}} T^{OR}_{k,r} x_{k,n,r,f} + \sum_{r \in \mathbf{R^3_k}} \sum_{f \in \mathbf{F}} \sum_{s \in \mathbf{S}} T^{OR}_{k,r} z_{k,n,r,f,s} - \sum_{r \in \mathbf{R^1_k} \cup \mathbf{R^2_k}} \sum_{f \in \mathbf{F}} T^{OR}_{k,r} x_{k,n-1,r,f} - \sum_{r \in \mathbf{R^3_k}} \sum_{f \in \mathbf{F}} \sum_{s \in \mathbf{S}} T^{OR}_{k,r} z_{k,n-1,r,f,s} \leq 0, \qquad k \in \mathbf{K}, n = 2 \ldots \mid \mathbf{N_k} \mid \quad \text{(C.40)}$$

$$\sum_{r \in \mathbf{R^1_k} \cup \mathbf{R^2_k}} T^{OR}_{k,r} x_{k,n,r,f} + \sum_{r \in \mathbf{R^3_k}} \sum_{s \in \mathbf{S}} T^{OR}_{k,r} z_{k,n,r,f,s} - \sum_{r \in \mathbf{R^1_k} \cup \mathbf{R^2_k}} T^{OR}_{k,r} x_{k,n,r,f-1} - \sum_{r \in \mathbf{R^3_k}} \sum_{s \in \mathbf{S}} T^{OR}_{k,r} z_{k,n,r,f-1,s} \leq 0, \qquad k \in \mathbf{K}, n \in \mathbf{N_k}, f = 2 \ldots \mid \mathbf{F} \mid \quad \text{(C.41)}$$

$$\sum_{r \in \mathbf{R^3_k}} T^{O}_{k,r} z_{k,n,r,f,s} - \sum_{r \in \mathbf{R^3_k}} T^{O}_{k,r} z_{k,n,r,f-1,s} \leq 0, \qquad k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F}, s = 2 \ldots \mid \mathbf{S} \mid \quad \text{(C.42)}$$

Optional cuts:

$$\sum_{k \in \mathbf{K}} \sum_{r \in \mathbf{R^1_k}} V_{p,k,r} + \sum_{h \in \mathbf{H}} o^H_h \geq 1, \qquad p \in \mathbf{p} \quad \text{(C.43)}$$

Appendix D

Path Flow Formulation II (PFF2)

Indices

b:	Onshore airport base.
h:	Potential location for offshore transhipment hub.
p:	Offshore platform.
i, j:	Onshore airport base, potential location for offshore transhipment hub, or offshore platform.
k:	Helicopter type.
n:	Identity number helicopter.
r:	Predefined, feasible route.
f:	Identity number helicopter flight.

Sets

$\mathbf{B}$:	Set of onshore airport bases.
$\mathbf{H}$:	Set of potential locations for offshore transhipment hub(s).
$\mathbf{P}$:	Set of offshore platforms.
$\mathbf{K}$:	Set of available helicopter types.
$\mathbf{N_k}$:	Set of available helicopters of type k.
$\mathbf{R_k}$:	Set of predefined, feasible routes for helicopters of type k.

$\mathbf{R_k^1}$:	Set of predefined, feasible routes using a direct routing policy for helicopters of type k .
$\mathbf{R_k^2}$:	Set of predefined, feasible first echelon routes for a hub connected routing policy for helicopters of type k .
$\mathbf{R_k^3}$:	Set of predefined, feasible second echelon routes for a hub connected routing policy for helicopters of type k.
$\mathbf{F}$:	Set of possible daily flights for every available helicopter.
$\mathbf{A}$:	Set of all feasible arcs.

Parameters

$A_{i,j,k,r}$:	1 if arc (i,j) is used in route r for helicopters of type k, otherwise 0.
$V_{i,k,r}$:	1 if onshore airport base, offshore transhipment hub or offshore platform i is visited on route r for helicopters of type k, otherwise 0.
$L_{p,k,r}$:	Number of times offshore platform i is visited on route r for helicopters of type k.
C^{FH}:	Fixed investment and operating cost per day for every installed offshore transhipment hub.
C_k^{FK}:	Fixed investment cost per day for helicopters of type k.
$C_{k,r}^{VR}$:	Variable operating cost for route r for helicopters of type k.
$C_{b,k}^{VB}$:	Variable parking cost on onshore airport base b for helicopters of type k.
$R_{i,j,k}^{A}$:	Risk assessment arc (i,j) for helicopters of type k.
F^W:	Weight factor assigned to the total cost of the transportation system.
F^S:	Scale factor assigned to the total risk of the transportation system.
Q_b^H:	Available helicopter parking capacity at onshore airport base b.
Q_b^{TB}:	Available take-off and landing capacity at onshore airport base b.
Q^{TH}:	Take-off and landing capacity at every installed offshore transhipment hub.
Q_p^{TP}:	Take-off and landing capacity at offshore platform p.
Q_k^P:	Passenger capacity of helicopters of type k.
$T_{k,r}^{OR}$:	Operating time route r for helicopters of type k.
T^{OD}:	Maximum operating time per day for all available helicopters.

T^{OF}:	Maximum operating time per flight for all available helicopters.
T^{P}:	Time per day from which a helicopter's parking cost on an onshore airport base should be derived.
D_p^P:	Demand of employees to be picked up at platform p.
D_p^D:	Demand of employees to be delivered at platform p.

Decision variables

$x_{k,n,r,f}$:	Number of times helicopter number n of type k uses route r on flight number f.
$a^H_{b,k,n}$:	1 if helicopter number n of type k is assigned to onshore airport base b, otherwise 0.
$a^P_{i,p}$:	1 if offshore platform p is assigned to onshore airport base or offshore transhipment hub i, otherwise 0.
o^H_h:	1 if an offshore transhipment hub is installed at location h, otherwise 0.
$t^P_{b,k,n}$:	Time per day from which the parking cost on onshore airport base b should be derived for helicopter number n of type k.
$p_{i,j,k,n,f}$:	Pickup load of employees on helicopter number n of type k if it travels directly between nodes i and j on flight number f.
$d_{i,j,k,n,f}$:	Delivery load of employees on helicopter number n of type k if it travels directly between nodes i and j on flight number f.
$e^P_{p,k,n,f}$:	Number of employees picked up at platform p by helicopter number n of type k on flight number f.
$e^D_{p,k,n,f}$:	Number of employees delivered up at platform p by helicopter number n of type k on flight number f.

Model

Minimize

$$F^W\Big(\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{r\in\mathbf{R_k}}\sum_{f\in\mathbf{F}} C^{VR}_{k,r} x_{k,n,r,f} + \sum_{b\in\mathbf{B}}\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}} C^{VB}_{b,k} t^P_{k,n} + \sum_{b\in\mathbf{B}}\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}} C^{FK}_k a^H_{b,k,n} + C^{FH}\sum_{h\in\mathbf{H}} o^H_h\Big) + F^S(1-F^W)\sum_{(i,j)\in\mathbf{A}}\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{f\in\mathbf{F}} R^A_{i,j,k}(p_{i,j,k,n,f} + d_{i,j,k,n,f}) \tag{D.1}$$

Subject to

Network constraints:

$$\sum_{b\in\mathbf{B}} a^H_{b,k,n} \le 1, \qquad k\in\mathbf{K}, n\in\mathbf{N_k} \tag{D.2}$$

$$\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}} a^H_{b,k,n} \le Q^H_b, \qquad b\in\mathbf{B} \tag{D.3}$$

$$\sum_{i\in\mathbf{B}\cup\mathbf{H}} a^P_{i,p} = 1, \qquad p\in\mathbf{P} \tag{D.4}$$

$$a^P_{h,p} - o^H_h \le 0, \qquad h\in\mathbf{H}, p\in\mathbf{P} \tag{D.5}$$

$$\sum_{r\in\mathbf{R^1_k}\cup\mathbf{R^2_k}} V_{b,k,r} x_{k,n,r,f} - a^H_{b,k,n} \le 0, \qquad b\in\mathbf{B}, k\in\mathbf{K}, n\in\mathbf{N_k}, f\in\mathbf{F} \tag{D.6}$$

$$\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{r\in\mathbf{R^1_k}\cup\mathbf{R^2_k}}\sum_{f\in\mathbf{F}} V_{b,k,r} x_{k,n,r,f} \le Q^{TB}_b, \qquad b\in\mathbf{B} \tag{D.7}$$

$$\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{r\in\mathbf{R^2_k}\cup\mathbf{R^3_k}}\sum_{f\in\mathbf{F}} V_{h,k,r} x_{k,n,r,f} - Q^{TH} o^H_h \le 0, \qquad h\in\mathbf{H} \tag{D.8}$$

$$\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{r\in\mathbf{R_k^1}\cup\mathbf{R_k^3}}\sum_{f\in\mathbf{F}} L_{p,k,r}x_{k,n,r,f} \leq Q_p^{TP}, \qquad p\in\mathbf{P} \tag{D.9}$$

$$\sum_{r\in\mathbf{R_k}} T_{k,r}^{OR}x_{k,n,r,f} \leq T^{OF}, \qquad k\in\mathbf{K}, n\in\mathbf{N_k}, f\in\mathbf{F} \tag{D.10}$$

$$\sum_{r\in\mathbf{R_k}}\sum_{f\in\mathbf{F}} T_{k,r}^{OR}x_{k,n,r,f} \leq T^{OD}, \qquad k\in\mathbf{K}, n\in\mathbf{N_k} \tag{D.11}$$

$$\sum_{r\in\mathbf{R_k}}\sum_{f\in\mathbf{F}} T_{k,r}^{OR}x_{k,n,r,f} + \sum_{b\in\mathbf{B}} t_{b,k,n}^{P} - T^{P}\sum_{b\in\mathbf{B}} a_{b,k,n}^{H} = 0, \qquad k\in\mathbf{K}, n\in\mathbf{N_k} \tag{D.12}$$

$$t_{b,k,n}^{P} - T^{P}a_{b,k,n}^{H} \leq 0, \qquad b\in\mathbf{B}, k\in\mathbf{K}, n\in\mathbf{N_k} \tag{D.13}$$

Direct routing policy constraints:

$$\sum_{r\in\mathbf{R_k^1}} V_{b,k,r}V_{p,k,r}x_{k,n,r,f} - a_{b,p}^{P} \leq 0, \qquad b\in\mathbf{B}, p\in\mathbf{P}, k\in\mathbf{K}, n\in\mathbf{N_k}, f\in\mathbf{F} \tag{D.14}$$

Hub connected routing policy constraints:

$$\sum_{r\in\mathbf{R_k^3}} V_{h,k,r}x_{k,n,r,f} - \mid\mathbf{F}\mid \sum_{r\in\mathbf{R_k^2}} V_{h,k,r}x_{k,n,r,f} \leq 0, \qquad h\in\mathbf{H}, p\in\mathbf{P}, k\in\mathbf{K}, n\in\mathbf{N_k}, f\in\mathbf{F} \tag{D.15}$$

$$\sum_{r\in\mathbf{R_k^3}} V_{h,k,r}V_{p,k,r}x_{k,n,r,f} - \mid\mathbf{F}\mid a_{h,p}^{P} \leq 0, \qquad h\in\mathbf{H}, p\in\mathbf{P}, k\in\mathbf{K}, n\in\mathbf{N_k}, f\in\mathbf{F} \tag{D.16}$$

Pickup and delivery constraints:

$$p_{i,j,k,n,f} + d_{i,j,k,n,f} - Q_k^{P}\sum_{r\in\mathbf{R_k}} A_{i,j,k,r}x_{k,n,r,f} \leq 0, \qquad (i,j)\in\mathbf{A}, k\in\mathbf{K}n\in\mathbf{N_k}, f\in\mathbf{F} \tag{D.17}$$

$$\sum_{b\in\mathbf{B}}\sum_{j\in\mathbf{H}\cup\mathbf{P}}\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{f\in\mathbf{F}} p_{b,j,k,n,f} = 0 \tag{D.18}$$

$$\sum_{i\in\mathbf{H}\cup\mathbf{P}}\sum_{b\in\mathbf{B}}\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{f\in\mathbf{F}} d_{i,b,k,n,f} = 0 \tag{D.19}$$

$$\sum_{p\in\mathbf{P}}\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{f\in\mathbf{F}} p_{p,h,k,n,f} - \sum_{b\in\mathbf{B}}\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{f\in\mathbf{F}} p_{h,b,k,n,f} = 0, \qquad h \in \mathbf{H} \tag{D.20}$$

$$\sum_{b\in\mathbf{B}}\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{f\in\mathbf{F}} d_{b,h,k,n,f} - \sum_{p\in\mathbf{P}}\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{f\in\mathbf{F}} d_{h,p,k,n,f} = 0, \qquad h \in \mathbf{H} \tag{D.21}$$

$$\sum_{i\in\mathbf{B}\cup\mathbf{H}\cup\mathbf{P}} p_{p,i,k,n,f} - \sum_{i\in\mathbf{B}\cup\mathbf{H}\cup\mathbf{P}} p_{i,p,k,n,f} - e^P_{p,k,n,f} = 0, \qquad p \in \mathbf{P}, k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F} \tag{D.22}$$

$$\sum_{i\in\mathbf{B}\cup\mathbf{H}\cup\mathbf{P}} d_{i,p,k,n,f} - \sum_{i\in\mathbf{B}\cup\mathbf{H}\cup\mathbf{P}} d_{p,i,k,n,f} - e^D_{p,k,n,f} = 0, \qquad p \in \mathbf{P}, k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F} \tag{D.23}$$

$$\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{f\in\mathbf{F}} e^P_{p,k,n,f} = D^P_p, \qquad p \in \mathbf{P} \tag{D.24}$$

$$\sum_{k\in\mathbf{K}}\sum_{n\in\mathbf{N_k}}\sum_{f\in\mathbf{F}} e^D_{p,k,n,f} = D^D_p, \qquad p \in \mathbf{P} \tag{D.25}$$

Constraints on variables:

$$x_{k,n,r,f} \geq 0 \text{ and integer}, \qquad k \in \mathbf{K}, n \in \mathbf{N_k}, r \in \mathbf{R_k}, f \in \mathbf{F} \tag{D.26}$$

$$a^H_{b,k,n} \in \{0,1\}, \qquad b \in \mathbf{B}, k \in \mathbf{K}, n \in \mathbf{N_k} \tag{D.27}$$

$$a^P_{i,p} \in \{0,1\}, \qquad i \in \mathbf{B} \cup \mathbf{H}, p \in \mathbf{P} \tag{D.28}$$

$$o^H_h \in \{0,1\}, \qquad h \in \mathbf{H} \tag{D.29}$$

$$t^P_{b,k,n} \geq 0, \qquad b \in \mathbf{B}, k \in \mathbf{K}, n \in \mathbf{N_k} \tag{D.30}$$

$$p_{i,j,k,n,f}, d_{i,j,k,n,f} \geq 0, \qquad (i,j) \in \mathbf{A}, k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F} \tag{D.31}$$

$$e^P_{p,k,n,f}, e^D_{p,k,n,f} \geq 0, \qquad p \in \mathbf{P}, k \in \mathbf{K}, n \in \mathbf{N_k}, f \in \mathbf{F} \tag{D.32}$$

Optional symmetry breaking constraints:

$$\sum_{b \in \mathbf{B}} a^H_{b,k,n} - \sum_{b \in \mathbf{B}} a^H_{b,k,n-1} \leq 0, \qquad k \in \mathbf{K}, n = 2 \dots |\, \mathbf{N_k} \,| \tag{D.33}$$

$$\sum_{r \in \mathbf{R_k}} \sum_{f \in \mathbf{F}} T^O_{k,r} x_{k,n,r,f} - \sum_{r \in \mathbf{R_k}} \sum_{f \in \mathbf{F}} T^O_{k,r} x_{k,n-1,r,f} \leq 0, \qquad k \in \mathbf{K}, n = 2 \dots |\, \mathbf{N_k} \,| \tag{D.34}$$

$$\sum_{r \in \mathbf{R_k}} T^O_{k,r} x_{k,n,r,f} - \sum_{r \in \mathbf{R_k}} T^O_{k,r} x_{k,n,r,f-1} \leq 0, \qquad k \in \mathbf{K}, n \in \mathbf{N_k}, f = 2 \dots |\, \mathbf{F} \,| \tag{D.35}$$

Optional cuts:

$$\sum_{k \in \mathbf{K}} \sum_{r \in \mathbf{R^1_k}} V_{p,k,r} + \sum_{h \in \mathbf{H}} o^H_h \geq 1, \qquad p \in \mathbf{p} \tag{D.36}$$

Appendix E

Input parameters

Platform #	*Name of platform*	*Oil & gas fiel*	*Installation year*	*Pickup demand*	*Delivery demand*
1	Cidade de Iilhabela	Sapinhoá	2014	29	28
2	Cidade de Mangaratiba	Iracema	2014	33	31
3	Z1	Iracema	2015	4	4
4	P-66	Lula	2016	29	27
5	P-67	Lula	2016	14	18
6	P-68	Lula	2016	32	23
7	P-74	Franco	2016	12	12
8	Z2	Carioca	2016	30	34
9	P-69	Lula	2016	6	10
10	P-75	Franco	2016	25	20
11	Arpoador	Lula	2016	12	19
12	Copacobana	Franco	2016	26	23
13	Urca	Franco	2016	2	2
14	Guarapari	Lula	2016	21	21
15	Ondina	Iara	2016	14	14
16	Cassino	Lula	2016	4	7

Table E.1: First set of pickup and delivery demands, part 1

Platform #	*Name of platform*	*Oil & gas fiel*	*Installation year*	*Pickup demand*	*Delivery demand*
17	Grumari	Carcará	2016	37	26
18	P-70	Lula	2017	37	35
19	P-71	Iara	2017	27	34
20	P-76	Lula	2017	8	14
21	P-72	Iara	2017	14	11
22	P-77	Franco	2017	22	32
23	Frade	Júpiter	2017	16	31
24	Camburi	Sapinhoá	2017	6	5
25	Pituba	Iara	2017	6	4
26	Ipanema	Júpiter	2017	17	13
27	Bracuhy	Carcará	2017	20	22
28	Itaoca	Iara	2017	20	22
29	CO5	Franco	2018	43	31
30	CO6	Sapinhoá	2018	35	32
31	-	Júpiter	2018	1	2
32	P-73	Carcará	2018	20	15
33	CO7	Franco	2018	24	24
34	Leblon	Franco	2018	5	6
35	Portogalo	Iara	2018	33	25
36	Leme	Franco	2018	17	12
37	CO8	Iara	2019	42	36
38	Marambaia	Iara	2019	12	22
39	Botinas	Florim	2019	20	24
40	Itapema	Iara	2019	1	1
41	CO9	Florim	2020	21	29

Table E.2: First set of pickup and delivery demands, part 2

Platform #	*Name of platform*	*Oil & gas fiel*	*Installation year*	*Pickup demand*	*Delivery demand*
1	Cidade de Iilhabela	Sapinhoá	2014	31	30
2	Cidade de Mangaratiba	Iracema	2014	35	33
3	Z1	Iracema	2015	4	4
4	P-66	Lula	2016	31	29

Table E.3: Second set of pickup and delivery demands, part 1

Platform #	*Name of platform*	*Oil & gas fiel*	*Installation year*	*Pickup demand*	*Delivery demand*
5	P-67	Lula	2016	15	19
6	P-68	Lula	2016	34	24
7	P-74	Franco	2016	13	13
8	Z2	Carioca	2016	32	36
9	P-69	Lula	2016	6	11
10	P-75	Franco	2016	27	21
11	Arpoador	Lula	2016	13	20
12	Copacobana	Franco	2016	28	24
13	Urca	Franco	2016	2	2
14	Guarapari	Lula	2016	22	22
15	Ondina	Iara	2016	15	15
16	Cassino	Lula	2016	4	7
17	Grumari	Carcará	2016	39	28
18	P-70	Lula	2017	39	37
19	P-71	Iara	2017	29	36
20	P-76	Lula	2017	8	15
21	P-72	Iara	2017	15	12
22	P-77	Franco	2017	23	34
23	Frade	Júpiter	2017	17	33
24	Camburi	Sapinhoá	2017	6	5
25	Pituba	Iara	2017	6	4
26	Ipanema	Júpiter	2017	18	14
27	Bracuhy	Carcará	2017	21	23
28	Itaoca	Iara	2017	21	23
29	CO5	Franco	2018	46	33
30	CO6	Sapinhoá	2018	37	34
31	-	Júpiter	2018	1	2
32	P-73	Carcará	2018	21	16
33	CO7	Franco	2018	25	25
34	Leblon	Franco	2018	5	6
35	Portogalo	Iara	2018	35	27
36	Leme	Franco	2018	18	13
37	CO8	Iara	2019	45	38
38	Marambaia	Iara	2019	13	23
39	Botinas	Florim	2019	21	25
40	Itapema	Iara	2019	1	1
41	CO9	Florim	2020	22	31

Table E.4: Second set of pickup and delivery demands, part 2

	Jacarepaguá	*Cabo Frio*
H1	137	181
H2	201	227
H3	150	153
H4	140	213
H5	172	138
H6	202	255
H7	212	208

Table E.5: Euclidean distances between onshore airport bases and potential locations for offshore transhipment hubs, first set of locations

	Jacarepaguá	*Cabo Frio*	*Guarujá*	*Itaguaí*
H1	184	263	242	188
H2	217	253	324	237
H3	155	191	322	179
H4	246	347	166	235
H5	172	138	406	211
H6	235	295	276	244
H7	214	215	377	243

Table E.6: Euclidean distances between onshore airport bases and potential locations for offshore transhipment hubs, second set of locations

	Jacarepaguá	*Cabo Frio*	*Guarujá*	*Itaguaí*
Carcará	259	317	285	269
Carioca	271	307	333	287
Florim	208	214	370	237
Franco	233	243	370	259
Iara	234	237	380	262
Iracema	253	254	389	280
Júpiter	273	268	410	302
Lula	258	269	374	282
Sapinhoá	298	321	369	318

Table E.7: Euclidean distances between onshore airport bases and offshore oil and gas fields, both sets of locations

	H1	*H2*	*H3*	*H4*	*H5*	*H6*	*H7*
Carcará	136	98	168	119	201	62	141
Carioca	136	80	154	135	180	71	112
Florim	77	33	64	108	81	82	15
Franco	98	38	93	121	108	79	37
Iara	101	45	90	127	101	88	30
Iracema	119	60	109	142	117	98	47
Júpiter	141	83	127	165	129	120	63
Lula	121	58	120	138	134	86	63
Sapinhoá	161	99	169	167	187	105	115

Table E.8: Euclidean distances between potential locations for offshore transhipment hubs and offshore oil and gas fields, first set of locations

	H1	*H2*	*H3*	*H4*	*H5*	*H6*	*H7*
Carcará	82	71	126	119	201	25	132
Carioca	113	56	122	167	180	59	102
Florim	128	57	60	216	81	115	8
Franco	129	47	80	211	108	104	28
Iara	138	57	83	221	101	114	23
Iracema	149	66	101	228	117	118	41
Júpiter	171	87	123	247	129	137	60
Lula	138	54	104	211	134	101	55
Sapinhoá	150	81	145	203	187	97	107

Table E.9: Euclidean distances between potential locations for offshore transhipment hubs and offshore oil and gas fields, second set of locations

	Carcará	*Carioca*	*Florim*	*Franco*	*Iara*	*Iracema*	*Júpiter*	*Lula*	*Sapinhoá*
Carcará	-	49	127	110	121	121	136	101	84
Carioca	49	-	99	75	85	80	91	58	38
Florim	127	99	-	29	26	45	65	56	107
Franco	110	75	29	-	11	22	44	27	79
Iara	121	85	26	11	-	19	40	33	85
Iracema	121	80	45	22	19	-	23	22	73
Júpiter	136	91	65	44	40	23	-	36	74
Lula	101	58	56	27	33	22	36	-	53
Sapinhoá	84	38	107	79	85	73	74	53	-

Table E.10: Euclidean distances between oil and gas fields, both sets of locations