NTNU – Trondheim
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Hydropower Bidding Using Linear Decision Rules

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Oppstartsdato	Innleveringsfrist
15. jan 2013	**11. jun 2013**
Oppgavens (foreløpige) tittel	
Multimarket hydropower bidding using linear decision rules	
Oppgavetekst/Problembeskrivelse	
Power producers sell their output through organized markets such as the day-ahead, intraday, balancing, and ancillary services markets. This thesis will develop decision support models for bidding into such markets for a price-taking hydropower producer with reservoir capacity. The approach will take into account the most important uncertainty factors and use linear decision rules as an approximation.	
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Abstract

This thesis investigates the Linear Decision Rule (LDR) approach applied to the bidding problem of a Nordic hydropower producer with reservoir capacity. A stochastic programming model with piecewise LDR in the spot prices is developed. A comprehensive case study with uncertain spot prices conducted for the fall of 2012 shows that the LDR model performs equally well as a scenario based model on expectation, yet with a smaller standard deviation in the profits. The runtime of the LDR model is substantially longer than the runtime of the scenario based model. Therefore, promising techniques to reduce the runtime are developed and presented.

Sammendrag

Denne avhandlingen undersøker om budgivningsproblemet til en nordisk vannkraftsprodusent bør løses ved hjelp av lineære beslutningsregler (LDR). En stokastisk programmeringsmodell med stykkevis lineære beslutningsregler i spotprisene er utviklet. En omfattende case-studie med usikre spotpriser viser at en LDR-modell og en scenariobasert modell har tilnærmet lik forventet fortjeneste. Standardavviket fra LDR-modellen er mindre, men kjøretiden er betydelig lengre sammenlignet med den scenariobaserte modellen. Lovende teknikker for å redusere kjøretiden er derfor utviklet og presentert.

Preface

This dissertation is written as a Master's thesis for the Master of Science degree at the Norwegian University of Science and Technology (NTNU), department of Industrial Economics and Technology Management within the field of Managerial Economics and Operations Research.

First and foremost, we would like to thank our supervisor, Professor Stein-Erik Fleten for helpful supervision, and PhD candidate Gro Klæboe at the Department of Electric Power Engineering at NTNU for hours with valuable discussions. We would also like to give sincere thanks to Dr. Daniel Kuhn at the Imperial College London for guidance on the Linear Decision Rule models and helpful inputs throughout the semester. We would further like to thank Lars Thore Wibe Aarrestad and the good folks at Powel AS Smart Generation, and Pål Otto Eide and Knut-Harald Bakke at Norsk Hydro ASA for helpful comments and for providing us with data.

Trondheim, June 7, 2013

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Nomenclature

a_{it}	Bid volume at bid point $i \in \mathcal{I}_t^S$ at time $t \in \mathcal{T}^B$ in the deterministic equivalent.
$b_{\mathfrak{s}}(\xi^{\mathfrak{s}})$	Right-hand side vector in stage $\mathfrak{s} \in \mathcal{S}$.
$c_{\mathfrak{s}}(\xi^{\mathfrak{s}})$	Coefficients in objective function in stage $\mathfrak{s} \in \mathcal{S}$.
d_{rt}	Spill from reservoir $r \in \mathcal{R}$ at time $t \in \mathcal{T}$.
e	Vector of ones.
e_1	Unit vector.
f_{rt}	Reservoir level in reservoir $r \in \mathcal{R}$ at the end of period $t \in \mathcal{T}$.
h	Used to describe the polyhedron $\hat{\Xi}$.
$\hat{h}$	Subvector of h.
h'	Used to describe the polyhedron $\hat{\Xi}'$.
$\tilde{h}$	Used to describes the polyhedron containing $\tilde{\Xi}$.
k	Dimension of ξ.
$k_{\mathfrak{s}}$	Dimension of $\xi_{\mathfrak{s}}$ in stage $\mathfrak{s} \in \mathcal{S}$.
$k^{\mathfrak{s}}$	Dimension of $\xi^{\mathfrak{s}}$ in stage $\mathfrak{s} \in \mathcal{S}$.
l	Dimension of h.
$\tilde{l}$	Dimension of $\tilde{h}$.
m	The total number of constraints in the original problem.
$m_{\mathfrak{s}}$	Dimension of $b_{\mathfrak{s}}(\xi^{\mathfrak{s}})$, i.e., the number of constraints in stage $\mathfrak{s} \in \mathcal{S}$.
n	The total number of decision variables in the original problem.
$n_{\mathfrak{s}}$	Dimension of $x_{\mathfrak{s}}(\xi^{\mathfrak{s}})$, i.e., the number of decision variables in stage $\mathfrak{s} \in \mathcal{S}$.
o_{gt}	Imposed start-up cost for generator $g \in \mathcal{G}$ at time $t \in \mathcal{T}$.
q_{gt}	Water discharge for generator $g \in \mathcal{G}$ at time $t \in \mathcal{T}$.
r_i	The number of line segments used for ξ_i, $i \in \{1, \dots, k\}$ in the piecewise LDR model.
$\mathfrak{r}_{\mathfrak{s}i}$	Folding direction, used in the piecewise LDR model with general segmentation, for $i \in \{1, \dots, k_{\mathfrak{s}}\}$ and stage $\mathfrak{s} \in \mathcal{S} \setminus \{1\}$.
$s_{\mathfrak{s}}(\xi^{\mathfrak{s}})$	Slack variables in stage $\mathfrak{s} \in \mathcal{S}$.
$\mathfrak{s}_t$	Stage corresponding to time period $t \in \mathcal{T}^B$.
u_{gt}	State variable, equal to 1 if generator $g \in \mathcal{G}$ is operation at time $t \in \mathcal{T}$ and 0 otherwise.
v	Value of stored water at the end of the planning horizon.

w_{gt}	Production from generator $g \in \mathcal{G}$ at time $t \in \mathcal{T}$.
w_{gt}^{+}	Increased production from generator $g \in \mathcal{G}$ at time $t \in \mathcal{T}$, compared to production at time $t-1$.
w_{gt}^{-}	Decreased production from generator $g \in \mathcal{G}$ at time $t \in \mathcal{T}$, compared to production at time $t-1$.
Δw_t^{+}	Production surplus at time $t \in \mathcal{T}^B$.
Δw_t^{-}	Production deficit at time $t \in \mathcal{T}^B$.
$x_{\mathfrak{s}}(\xi^{\mathfrak{s}})$	Decision variables in stage $\mathfrak{s} \in \mathcal{S}$.
y_t	Spot market volume commitment at time $t \in \mathcal{T}$.
$A_{\mathfrak{s}\tau}$	Parameters of $x_\tau(\xi^\tau)$ for $\tau \in \mathcal{S}$ in the constraints in stage $\mathfrak{s} \in \mathcal{S}$.
$B_{\mathfrak{s}}$	Right-hand side matrix for stage $\mathfrak{s} \in \mathcal{S}$.
$C_{\mathfrak{s}}$	Coefficient matrix in the objective function in stage $\mathfrak{s} \in \mathcal{S}$.
$E_{g\mathfrak{h}}$	Constant in the production-discharge curve for generator $g \in \mathcal{G}$ and cut $\mathfrak{h} \in \mathcal{H}$.
$\hat{E}_{g\mathfrak{h}}$	Constant in the production-discharge curve for generator $g \in \mathcal{G}$ and cut $\mathfrak{h} \in \mathcal{H}$.
$F_{r\mathfrak{p}}$	Constant (reservoir filling) in water value cut $\mathfrak{p} \in \mathcal{P}$ for reservoir $r \in \mathcal{R}$.
$\underline{F}_r$	Lower bound on the reservoir level in reservoir $r \in \mathcal{R}$.
$\overline{F}_r$	Upper bound on the reservoir level in reservoir $r \in \mathcal{R}$.
K_g	Start-up cost for generator $g \in \mathcal{G}$.
$L_{\mathfrak{p}}$	Constant (future value) in the water value curve for cut $\mathfrak{p} \in \mathcal{P}$.
M	Moment matrix of the uncertain parameters.
$\tilde{M}$	Moment matrix of the principal components.
$P_{\mathfrak{s}}$	Truncation operator in stage $\mathfrak{s} \in \mathcal{S}$, i.e., $\xi^{\mathfrak{s}} = P_{\mathfrak{s}}\xi$.
$\underline{Q}_g$	Lower bound on the discharge for generator $g \in \mathcal{G}$.
$\overline{Q}_g$	Upper bound on the discharge for generator $g \in \mathcal{G}$.
$R_{\mathfrak{s}}$	Matrix with columns equal to the eigenvectors of $\Sigma_{\mathfrak{s}}$ in stage $\mathfrak{s} \in \mathcal{S}$.
$R_{\mathfrak{s}}^{\star}$	Reduced coefficient matrix in stage $\mathfrak{s} \in \mathcal{S}$, used in the principal component reduction, submatrix of $R_{\mathfrak{s}}$.
$R^{\mathfrak{s}}$	Coefficient matrix that converts $\tilde{\xi}$ into $\xi^{\mathfrak{s}}$ in stage $\mathfrak{s} \in \mathcal{S}$.
$S_{\mathfrak{s}}$	Decision variables for slack variables in the LDR problem in stage $\mathfrak{s} \in \mathcal{S}$.
$V_{r\mathfrak{p}}$	Marginal water value for reservoir $r \in \mathcal{R}$ and cut $\mathfrak{p} \in \mathcal{P}$.
W	Used to describe the polyhedron $\hat{\Xi}$.
$\hat{W}$	Submatrix of W.
W_i'	Used to describe the polyhedron $\hat{\Xi}'$ for $i \in \{2,\ldots,k\}$.
$\tilde{W}$	Used to describe the polyhedron that contains $\tilde{\Xi}$.
$\underline{W}_g$	Lower bound on the generation from generator $g \in \mathcal{G}$.
$\overline{W}_g$	Upper bound on the generation from generator $g \in \mathcal{G}$.
ΔW_t^{+}	Decision rule for Δw_t^{+} at time $t \in \mathcal{T}^B$.
ΔW_t^{-}	Decision rule for Δw_t^{-} at time $t \in \mathcal{T}^B$.
$X_{\mathfrak{s}}$	Decision variables in the LDR problem in stage $\mathfrak{s} \in \mathcal{S}$.
$\mathcal{G}$	Set of generators.

$\mathcal{G}_r$	Set of generators belonging to reservoir $r \in \mathcal{R}$, subset of $\mathcal{G}$.
$\mathcal{H}_g$	Set of cuts in the production-discharge curve for generator $g \in \mathcal{G}$.
$\mathcal{I}_t$	Set of break points (bid points) in the bid curve at time $t \in \mathcal{T}^B$ in the piecewise LDR model.
$\mathcal{I}_t^S$	Set of bid points in the bid curve at time $t \in \mathcal{T}^B$ in the deterministic equivalent.
$\mathcal{M}$	Set of macroperiods, used in the stage aggregation.
$\mathcal{P}$	Set of cuts in the water value curve.
$\mathcal{R}$	Set of reservoirs.
$\mathcal{R}_r^A$	Set of reservoirs immediately above reservoir $r \in \mathcal{R}$, subset of $\mathcal{R}$.
$\mathcal{S}$	Set of stages in LDR model.
$\mathcal{T}$	Set of time periods in the planning horizon.
$\mathcal{T}^B$	Set of time periods with spot market bidding (24 hours).
$\mathcal{T}^T$	Set of time periods in the training period for the ARMA parameter estimation.
$\mathcal{T}_\mathfrak{m}$	Set of time periods in macroperiod $\mathfrak{m} \in \mathcal{M}$, subset of $\mathcal{M}$.

$\beta_\mathfrak{s}$	Share of the variance of $\xi_\mathfrak{s}$ captured by the reduced vector of principal components in stage $\mathfrak{s} \in \mathcal{S}$.
γ_t^-	Demanded volume in the balancing market for ramping down production at time $t \in \mathcal{T}^B$.
γ_t^+	Demanded volume in the balancing market for ramping up production at time $t \in \mathcal{T}^B$.
δ	Small real number.
ϵ_t	Forecast error for the spot price at time $t \in \mathcal{T} \cup \mathcal{T}^T$.
$\bar{\epsilon}_\mathfrak{m}$	Average spot price forecast error in macroperiod $\mathfrak{m} \in \mathcal{M}$.
ζ_j^i	Break point $j \in \{1,\dots,r_i - 1\}$ in ξ_i, $i \in \{1,\dots,k\}$, used in the piecewise LDR.
$\zeta_{\mathfrak{s}j}$	Vector of break point $j \in \{1,\dots,r_i\}$ in $\xi_\mathfrak{s}$ in stage $\mathfrak{s} \in \mathcal{S}$.
$\tilde{\zeta}_{\mathfrak{s}j}$	Vector of break point $j \in \{1,\dots,r_i\}$ in $\tilde{\xi}_\mathfrak{s}$ in stage $\mathfrak{s} \in \mathcal{S}$.
$\tilde{\zeta}_j^i$	Break point $j \in \{1,\dots,\tilde{r}_i - 1\}$ in ξ_i, $i \in \{1,\dots,\tilde{k}\}$, used in the piecewise LDR with PCR.
κ_{rt}	Inflow to reservoir $r \in \mathcal{R}$ at time $t \in \mathcal{T}$.
$\mu^\mathfrak{s}$	Expected values of $\xi^\mathfrak{s}$ in stage $\mathfrak{s} \in \mathcal{S}$, seen from the first stage.
$\nu_i^\mathfrak{s}$	Eigenvalue $i \in \{1,\dots,k_\mathfrak{s}\}$ of $\Sigma_\mathfrak{s}$ in stage $\mathfrak{s} \in \mathcal{S}$.
ξ	Vector of all uncertain parameters.
$\xi_\mathfrak{s}$	Vector of uncertain parameters observed in stage $\mathfrak{s} \in \mathcal{S}$.
$\xi^\mathfrak{s}$	Vector of uncertain parameters observed up to stage $\mathfrak{s} \in \mathcal{S}$, i.e., $\xi^\mathfrak{s} \triangleq (\xi_1^\top,\dots,\xi_\mathfrak{s}^\top)^\top$.
$\underline{\xi}_i$	Lower bound on ξ_i for $i \in \{1,\dots,k\}$.
$\bar{\xi}_i$	Upper bound on ξ_i for $i \in \{1,\dots,k\}$.
ξ'	Uncertain parameters in the lifted space, i.e., the uncertain parameters in the piecewise LDR problem.
ξ_i'	Lifted parameters of ξ_i for $i \in \{2,\dots,k\}$.
$\tilde{\xi}$	Vector of all principal components.

$\tilde{\xi}_{\mathfrak{s}}$	The principal components of $\xi_{\mathfrak{s}}$ in stage $\mathfrak{s} \in \mathcal{S}$.
$\tilde{\xi}^{\star}_{\mathfrak{s}}$	The reduced vector of principal components in stage $\mathfrak{s} \in \mathcal{S}$.
ρ_t	Spot price at time $t \in \mathcal{T} \cup \mathcal{T}^T$.
ρ_t^D	Deterministic spot price forecast at time $t \in \mathcal{T} \cup \mathcal{T}^T$.
σ_t^-	The imbalance price of production deficit at time $t \in \mathcal{T}^B$.
σ_t^+	The imbalance price of production surplus at time $t \in \mathcal{T}^B$.
ω_t	Residual in the ARMA model for the spot forecast error at time $t \in \mathcal{T} \cup \mathcal{T}^T$.
Θ_1	AR1 coefficient in the ARMA model for the spot price forecast error.
Θ_1	AR2 coefficient in the ARMA model for the spot price forecast error.
$\Lambda_{\mathfrak{s}}$	Decision variables associated with the support description in the LDR problem in stage $\mathfrak{s} \in \mathcal{S}$.
$\tilde{\Lambda}_{\mathfrak{s}}$	Decision variables associated with the support description in the LDR problem with PCR in stage $\mathfrak{s} \in \mathcal{S}$.
Ξ	The support of ξ.
Ξ'	The support of ξ'.
$\tilde{\Xi}$	The support of $\tilde{\xi}$.
$\hat{\Xi}$	A bounded polyhedron containing the convex hull of Ξ.
$\hat{\Xi}'$	A bounded polyhedron containing the convex hull of Ξ'.
$\Sigma_{\mathfrak{s}}$	The covariance matrix of $\xi_{\mathfrak{s}}$ in stage $\mathfrak{s} \in \mathcal{S}$.
Φ_1	MA1 coefficient in the ARMA model for the spot price forecast error.
$\mathbb{B}$	Retraction operator.
$\mathbb{B}_i$	Retraction operator, transforms ξ' into ξ_i for $i \in \{1, \ldots, k\}$.
$\mathbb{E}$	Expected value operator.
$\mathbb{L}_{ij}$	Lifting operator, transforms ξ into ξ_{ij} for $i \in \{1, \ldots, k\}$ and $j \in \{1, \ldots, r_i\}$.
$\mathbb{Z}^+$	Set of positive integers.

Contents

1. Introduction

Electricity producers with portfolios consisting of reservoir hydropower must decide in which time periods they should produce the energy stored in their reservoirs. Each producer determines a value of the stored water and submits bids into wholesale electricity markets in order to maximize profits. We refer to the problem of optimizing the bidding decisions in the day-ahead market as the *bidding problem* of a hydropower producer. This thesis evaluates a new framework for solving the bidding problem, using the Linear Decision Rules (LDR) approach.

The main marketplace for Nordic power producers is the day-ahead auction, Elspot, organized by Nord Pool Spot (NPS) [1]. This is the largest physical electricity market in the World, including Norway, Sweden, Finland, Denmark, Estonia and Lithuania. In 2012, the total turnover in the Elspot market was 334 TWh, and 77 % of all electricity in the Nordic region was traded in this market. Prior to 12:00 noon producers and consumers submit bids for selling and purchasing electricity for the next day, that is, for the next 12 to 36 hours. The Nordic exchange area is divided into price zones and NPS calculates the zone prices for each hour and zone.

Single hourly bids represent the largest share of the trading in Elspot, in which the participants specify the purchase and sales volumes for each hour. A single hourly bid may consist of up to 64 price points. Furthermore, the bidding curve must be nondecreasing[1]. A participant accepts that NPS will make a linear interpolation of volumes between each adjacent pair of submitted price points.

The Transmission System Operators (TSOs) are responsible for the grid stability and the power balance in the system. Imbalances caused by deviations from the production plans and load forecasts must be leveled out in order to maintain the instant balance in the system at any point of time. Hence, the TSOs need access to balancing power, which is procured in the balancing markets. The Nordic tertiary Balancing Market (BM)[2] gives the producers the opportunity to ramp up production at a price higher than the spot price, and ramp down production at a price lower

[1] A nondecreasing bid curve consists of bids that yield nondecreasing bid volumes with increasing bid prices.

[2] *NO: Regulerkraftmarkedet.*

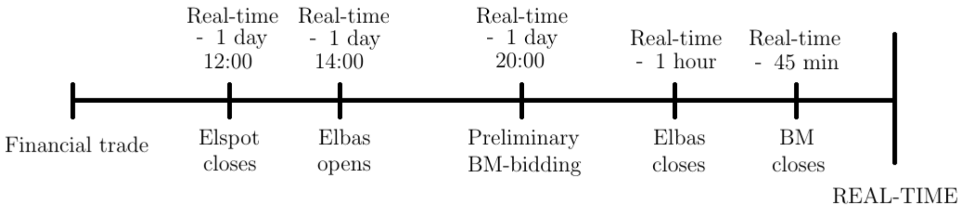


Fig. 1.1: Trading routines in the Nordic electricity markets. Markets for primary and secondary reserves are not included.

than the spot price[3]. Further, NPS organizes the continuous bilateral Elbas market, which is active after the spot market has been cleared. Trading routines are shown in Fig. 1.1.

Hydropower producers often base their bidding decisions on deterministic models, such as the Short-term Hydro Optimization (SHOP) [2], which is commonly used in the Nordic region. A deterministic model does not provide decision support for constructing a bid curve, the bidding decisions hence rely on the skills and experience of the production planners. Furthermore, the level of uncertainty in the power system is currently increasing as a result of a rapidly growing share of nonflexible renewable power [3]. The increased uncertainty might result in more volatile prices in the electricity markets. Accounting for uncertainty in the bidding decisions using stochastic optimization tools might therefore increase the profits of electricity producers substantially. Hydropower producers also face uncertainty associated with the inflows to the reservoirs. In the literature, electricity producers are commonly modeled as risk neutral in the bidding decisions, whereas risk management is conducted in the financial markets [4].

The literature on solving the stochastic bidding problem using scenario based solution methods is extensive. A stochastic dynamic programming model was presented in [5], solving a convex quadratic programming in each stage. In [6], a stochastic integer programming model was suggested. The deterministic equivalent of the bidding problem solved in [7], using mixed-integer linear programming. Bender's decomposition was used to solve the bidding problem in [8] and Approximate Dual Dynamic Programming was used in [9]. Bidding in sequential markets has gained some recent attention [10, 11, 12, 13, 14], in which the balancing market or intraday trading has been taken into account in the spot bidding decisions. A review of the literature on bidding strategy optimization in a general context can be found in [15].

Solving stochastic programming problems is computationally demanding. The general two-stage problem is proven to be #P-hard [16], and the multi-stage problem

[3] Note that there is an essential difference between offering volumes in the BM at a better price than or equal to the spot price and "buying" balancing services from the TSO because of own imbalances. In the latter case the price in the BM is always worse or equal to the spot price.

is even harder to solve [17]. This indicates that stochastic programming problems need to be simplified in order to obtain tractability. The classical approach is to represent the underlying stochastic processes of the random parameters by a finite scenario tree [18, 19]. The runtime of scenario based models is to a large extent dependent on the size of the scenario tree, which grows rapidly with an increasing number of time stages. This means that the number of scenarios should be limited to a small number, while sufficiently representing the underlying uncertainty. The performance of a scenario based model might be very sensitive with respect to the construction of the scenario tree. That is, the modeler risks to "overfit" the solution to the scenario tree, resulting in poor performance in an out-of-sample evaluation [20].

The LDR approach provides an approximation of a stochastic programming problem. This solution method allows for continuous probability distributions of the uncertain parameters and yields a tractable linear programming problem. The concept of LDR is old [21], but it has gained increased attention through recent developments [22, 23, 24, 25, 26, 27, 28], mainly in the framework of Robust Optimization. There is no need for state discretization or scenario generation in the LDR approach, as opposed to scenario based solution methods. To our knowledge the LDR approach has never been tested for the bidding problem. This thesis investigates the LDR framework and the adaptability to the bidding problem of a Nordic hydropower producer. The results are compared to results from a scenario based model and a deterministic model.

The contribution of this thesis is threefold:

1. An LDR model of the bidding problem of a Nordic hydropower producer is developed, taking into account uncertainty in inflows, spot prices and prices in an intraday market.
2. The performance of the LDR model is compared to the performance of a scenario based model and a deterministic model in a case study using stochastic spot prices, deterministic inflows and deterministic intraday market prices.
3. Methods to reduce the runtime are investigated and tested. In addition, a new methodology using Principal Component Analysis is developed and evaluated.

The outline of this thesis is the following. Chapter 2 provides a general presentation of the LDR theory and develops a problem reduction method using Principal Component Analysis. The bidding problem is presented in Chapter 3. A comprehensive case study is performed in Chapter 4, in which methods that reduce the runtime are investigated, and comparisons with a scenario based model and a deterministic model are conducted. Finally, Chapter 5 concludes and suggests future work.

Notation We denote the trace of a square matrix $A \in \mathbb{R}^{m \times m}$ by $\mathrm{Tr}(A)$. We use

$\geq$ and $\leq$ as componentwise inequalities for matrices and vectors. Moreover, $\operatorname{conv}\Xi$ denotes the *convex hull* of the set Ξ, i.e., the smallest convex set that contains Ξ.

We will apply the *cyclicity property* of the trace operator, that is,

$$\operatorname{Tr}(BC) = \sum_{\mathtt{i}=1}^{m} \sum_{\mathtt{j}=1}^{n} B_{\mathtt{ij}} C_{\mathtt{ji}} = \operatorname{Tr}(CB), \tag{1.1}$$

for some matrices $B \in \mathbb{R}^{m \times n}$ and $C \in \mathbb{R}^{n \times m}$.

2. Theory

The Linear Decision Rules (LDR) approximation of a stochastic programming problem provides a tractable Linear Programming problem (LP) at the cost of a potential loss of optimality. The LDR approach does however not require discrete distributions of the uncertain parameters or state discretization, in contrast to scenario based optimization models. This chapter derives the LDR approximation for linear stochastic programming problems with fixed recourse. An improvement of the approximation using piecewise linear decision rules is presented later in this chapter. We further suggest a method to reduce the problem size, called *finite memory.* In addition, a new technique used to reduce the problem size applying principal component analysis is developed.

We seek to solve the following multistage stochastic programming problem

$$\min \; \mathbb{E}\left[\sum_{\mathfrak{s}\in\mathcal{S}} c_{\mathfrak{s}}(\xi^{\mathfrak{s}})^\top x_{\mathfrak{s}}(\xi^{\mathfrak{s}})\right] \tag{2.1}$$

$$\text{s.t.} \quad \sum_{\tau=1}^{\mathfrak{s}} A_{\mathfrak{s}\tau} x_\tau(\xi^\tau) \le b_{\mathfrak{s}}(\xi^{\mathfrak{s}}), \qquad \forall \mathfrak{s} \in \mathcal{S}, \xi \in \Xi, \tag{2.2}$$

where $\mathcal{S}$ denotes the set of time stages and Ξ denotes the support of the uncertain parameters $\xi \in \mathbb{R}^k$. Further, $\xi^{\mathfrak{s}} \in \mathbb{R}^{k^{\mathfrak{s}}}$ denotes the uncertain parameters that are observed up to stage $\mathfrak{s} \in \mathcal{S}$, $x_{\mathfrak{s}}(\xi^{\mathfrak{s}}) \in \mathbb{R}^{n_{\mathfrak{s}}}$ denote the decision variables and $c_{\mathfrak{s}}(\xi^{\mathfrak{s}}) \in \mathbb{R}^{m_{\mathfrak{s}}}$, $A_{\mathfrak{s}\tau} \in \mathbb{R}^{m_{\mathfrak{s}} \times n_\tau}$ and $b_{\mathfrak{s}}(\xi^{\mathfrak{s}}) \in \mathbb{R}^{m_{\mathfrak{s}}}$ denote the problem parameters. Note that $A_{\mathfrak{s}\tau}$ is independent of ξ, that is, the problem has *fixed recourse.* Applying the LDR approximation to a problem with random recourse is more intricate and results in a semidefinite programming problem [22]. Moreover, $x_{\mathfrak{s}}$ is a function of $\xi^{\mathfrak{s}}$ only, which assures nonanticipativity.

We introduce slack variables $s_{\mathfrak{s}}(\xi^{\mathfrak{s}}) \in \mathbb{R}^{m_{\mathfrak{s}}}$ in stage $\mathfrak{s} \in \mathcal{S}$, and write problem (2.1)-

(2.2) on the form

$$\min \mathbb{E}\left[\sum_{\mathfrak{s}\in\mathcal{S}} c_{\mathfrak{s}}(\xi^{\mathfrak{s}})^\top x_{\mathfrak{s}}(\xi^{\mathfrak{s}})\right] \tag{2.3}$$

$$\text{s.t.} \sum_{\tau=1}^{t} A_{\mathfrak{s}\tau} x_\tau(\xi^\tau) + s_{\mathfrak{s}}(\xi^{\mathfrak{s}}) = b_{\mathfrak{s}}(\xi^{\mathfrak{s}}), \qquad \forall \mathfrak{s}\in\mathcal{S}, \xi\in\Xi, \tag{2.4}$$

$$s_{\mathfrak{s}}(\xi^{\mathfrak{s}}) \geq 0, \qquad \forall \mathfrak{s}\in\mathcal{S}, \xi\in\Xi. \tag{2.5}$$

2.1 The Linear Decision Rules Approximation

This section derives the LDR approximation, based on the derivation in [26], yet adjusted to the problem presented above.

First, we need some assumptions for the problem parameters. Without loss of generality, we let $\xi^1 = 1$ almost surely for technical reasons. Furthermore, we assume that the convex hull of Ξ is contained in a compact polyhedron $\hat{\Xi}$ on the form

$$\operatorname{conv}\Xi \subseteq \hat{\Xi} \triangleq \left\{\xi\in\mathbb{R}^k : W\xi \geq h, W\in\mathbb{R}^{l\times k}, h\in\mathbb{R}^l, l\in\mathbb{Z}^+\right\}. \tag{2.6}$$

The system $W\xi \geq h$ is assumed[1] to be strictly feasible for ξ_i for all $i\in\{2,\ldots,k\}$, i.e., let

$$W = \begin{pmatrix} e_1^\top \\ -e_1^\top \\ \hat{W} \end{pmatrix} \quad \text{and} \quad h = \begin{pmatrix} 1 \\ -1 \\ \hat{h} \end{pmatrix}, \tag{2.7}$$

for some $\hat{W}\in\mathbb{R}^{(l-2)\times k}$ and $\hat{h}\in\mathbb{R}^{l-2}$, where $e_1 \triangleq (1,0,\ldots,0)^\top \in\mathbb{R}^k$ is a unit vector. Then,

$$\exists \delta\in\mathbb{R}, \hat{\xi}\in\hat{\Xi} : \delta > 0, \hat{W}\hat{\xi} \geq \hat{h} + \delta e, \tag{2.8}$$

where δ is any small positive number and $e\in\mathbb{R}^{l-2}$ is a vector of ones. In essence, the strict feasibility requirement ensures that none of the stochastic parameters are deterministic. This requirement implies that the linear span of Ξ coincides with $\mathbb{R}^k$.

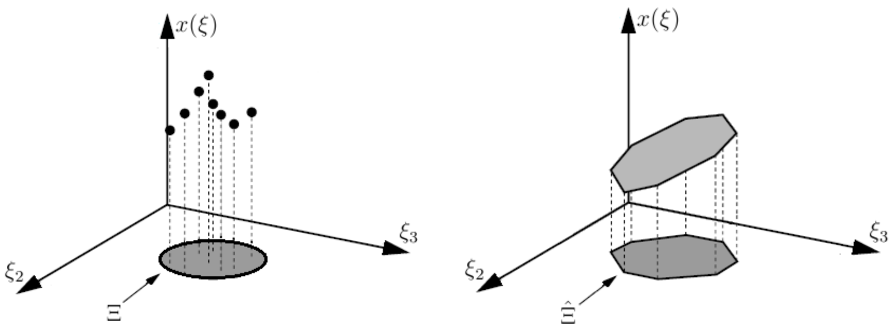
For notational purposes we introduce the truncation operator $P_{\mathfrak{s}} : \mathbb{R}^k \to \mathbb{R}^{k^{\mathfrak{s}}}$ such that

$$\xi^{\mathfrak{s}} = P_{\mathfrak{s}}\xi, \quad \forall \mathfrak{s}\in\mathcal{S}. \tag{2.9}$$

We further assume[2] that $c_{\mathfrak{s}}(\xi^{\mathfrak{s}})$ and $b_{\mathfrak{s}}(\xi^{\mathfrak{s}})$ can be written as

$$c_{\mathfrak{s}}(\xi^{\mathfrak{s}}) = C_{\mathfrak{s}}\xi^{\mathfrak{s}} = C_{\mathfrak{s}}P_{\mathfrak{s}}\xi \quad \text{and} \quad b_{\mathfrak{s}}(\xi^{\mathfrak{s}}) = B_{\mathfrak{s}}\xi^{\mathfrak{s}} = B_{\mathfrak{s}}P_{\mathfrak{s}}\xi, \quad \forall \mathfrak{s}\in\mathcal{S}, \tag{2.10}$$

[1] This assumption is nonrestrictive, because we are free to reduce the dimension of ξ.
[2] This assumption is also nonrestrictive because $\xi^{\mathfrak{s}}$ can contain $c_{\mathfrak{s}}(\xi^{\mathfrak{s}})$ and $b_{\mathfrak{s}}(\xi^{\mathfrak{s}})$ as subvectors.



(a) Scenario based model. *(b)* LDR model.

Fig. 2.1: Illustration of feasible decisions in scenario based models and in the LDR approximation.

for some matrices $C_{\mathfrak{s}} \in \mathbb{R}^{n_{\mathfrak{s}} \times k^{\mathfrak{s}}}$ and $B_{\mathfrak{s}} \in \mathbb{R}^{m_{\mathfrak{s}} \times k^{\mathfrak{s}}}$.

The approximation lies in the *decision rules* that we impose on $x_{\mathfrak{s}}(\xi^{\mathfrak{s}})$. We restrict the decisions such that the following linear dependency on the observed uncertain parameters is satisfied

$$x_{\mathfrak{s}}(\xi^{\mathfrak{s}}) = X_{\mathfrak{s}}\xi^{\mathfrak{s}} = X_{\mathfrak{s}}P_{\mathfrak{s}}\xi, \quad \forall \mathfrak{s} \in \mathcal{S}, \tag{2.11}$$

for some matrices $X_{\mathfrak{s}} \in \mathbb{R}^{n_{\mathfrak{s}} \times k^{\mathfrak{s}}}$. The $X_{\mathfrak{s}}$ matrices are the new decision variables, independent of $\xi^{\mathfrak{s}}$. These variables are hence *deterministic*. Imposing the above decision rules on the problem is a restriction and might therefore result in a loss of optimality. Fig. 2.1 illustrates how the LDR restrict the decisions to lie on a hyperplane imposed by the uncertain parameters, as opposed to scenario based solution methods. However, the LDR approximation enables us to transform the stochastic program into a finite, tractable LP without discretizing the support or the solution space of the decision variables.

We substitute the decision rules into the original problem (2.3)-(2.5).

$$\min \mathbb{E}\left[\sum_{\mathfrak{s} \in \mathcal{S}} (C_{\mathfrak{s}}P_{\mathfrak{s}}\xi)^\top X_{\mathfrak{s}}P_{\mathfrak{s}}\xi\right] \tag{2.12}$$

$$\text{s.t.} \quad \sum_{\tau=1}^{\mathfrak{s}} A_{\mathfrak{s}\tau}X_\tau P_\tau \xi + S_{\mathfrak{s}}P_{\mathfrak{s}}\xi = B_{\mathfrak{s}}P_{\mathfrak{s}}\xi, \qquad \forall \mathfrak{s} \in \mathcal{S}, \xi \in \Xi, \tag{2.13}$$

$$S_{\mathfrak{s}}P_{\mathfrak{s}}\xi \geq 0, \qquad \forall \mathfrak{s} \in \mathcal{S}, \xi \in \Xi, \tag{2.14}$$

where we have imposed the following decision rules on the slack variables

$$s_{\mathfrak{s}}(\xi^{\mathfrak{s}}) = S_{\mathfrak{s}}\xi^{\mathfrak{s}} = S_{\mathfrak{s}}P_{\mathfrak{s}}\xi, \quad \forall \mathfrak{s} \in \mathcal{S}, \tag{2.15}$$

for some matrices $S_{\mathfrak{s}} \in \mathbb{R}^{m_{\mathfrak{s}} \times k^{\mathfrak{s}}}$.

We define the moment matrix $M \in \mathbb{R}^{k\times k}$ associated with the uncertain parameters through

$$M \triangleq \mathbb{E}\left[\xi\xi^\top\right]. \tag{2.16}$$

Problem (2.12)-(2.14) can now be converted into

$$\begin{aligned}
\min\ & \sum_{\mathfrak{s}\in\mathcal{S}} \mathrm{Tr}\left(P_{\mathfrak{s}} M P_{\mathfrak{s}}^\top C_{\mathfrak{s}}^\top X_{\mathfrak{s}}\right) && && (2.17)\\
\text{s.t.}\ & \left(\sum_{\tau=1}^{\mathfrak{s}} A_{\mathfrak{s}\tau} X_\tau P_\tau + S_{\mathfrak{s}} P_{\mathfrak{s}} - B_{\mathfrak{s}} P_{\mathfrak{s}}\right)\xi = 0, && \forall \mathfrak{s}\in\mathcal{S}, \xi\in\Xi, && (2.18)\\
& S_{\mathfrak{s}} P_{\mathfrak{s}} \xi \geq 0, && \forall \mathfrak{s}\in\mathcal{S}, \xi\in\Xi, && (2.19)
\end{aligned}$$

where we have used the cyclicity property of the trace operator in the objective function[3].

Proposition 1. *For any support Ξ that satisfies assumption (2.8) and for any $A \in \mathbb{R}^{m\times k}$ and $m \in \mathbb{Z}^+$ we have that*

$$\begin{aligned}
& A\xi = 0, \quad \forall \xi \in \Xi\\
\Leftrightarrow\ & A = 0.
\end{aligned}$$

Proof. The linear span of Ξ belongs to the null space of A. The support Ξ spans $\mathbb{R}^k$, thus, $\mathrm{null}(A) = \mathbb{R}^k$ and $A = 0$. □

Proposition 2. *For any support Ξ that satisfies assumption (2.6) and for any $a \in \mathbb{R}^k$ we have that*

$$\begin{aligned}
& \exists \lambda \in \mathbb{R}^l : h^\top\lambda \geq 0, W^\top\lambda = a, \lambda \geq 0\\
\Longrightarrow\ & a^\top\xi \geq 0,\ \forall \xi \in \Xi.
\end{aligned}$$

Proof. We first prove that

$$\begin{aligned}
& \exists \lambda \in \mathbb{R}^l : h^\top\lambda \geq 0, W^\top\lambda = a, \lambda \geq 0\\
\Leftrightarrow\ & a^\top\xi \geq 0,\ \forall \xi \in \hat{\Xi},
\end{aligned}$$

[3] We have $\mathbb{E}\left[(C_{\mathfrak{s}}P_{\mathfrak{s}}\xi)^\top X_{\mathfrak{s}}P_{\mathfrak{s}}\xi\right] = \mathrm{Tr}\left(\mathbb{E}\left[(C_{\mathfrak{s}}P_{\mathfrak{s}}\xi)^\top X_{\mathfrak{s}}P_{\mathfrak{s}}\xi\right]\right)$ since $(C_{\mathfrak{s}}P_{\mathfrak{s}}\xi)^\top X_{\mathfrak{s}}P_{\mathfrak{s}}\xi \in \mathbb{R}$. Further, $\mathrm{Tr}\left(\mathbb{E}\left[(C_{\mathfrak{s}}P_{\mathfrak{s}}\xi)^\top X_{\mathfrak{s}}P_{\mathfrak{s}}\xi\right]\right) = \mathrm{Tr}\left(\mathbb{E}\left[(\xi^\top P_{\mathfrak{s}}^\top C_{\mathfrak{s}}^\top)(X_{\mathfrak{s}}P_{\mathfrak{s}}\xi)\right]\right) = \mathrm{Tr}\left(\mathbb{E}\left[(X_{\mathfrak{s}}P_{\mathfrak{s}}\xi)(\xi^\top P_{\mathfrak{s}}^\top C_{\mathfrak{s}}^\top)\right]\right) = \mathrm{Tr}\left(X_{\mathfrak{s}}P_{\mathfrak{s}}MP_{\mathfrak{s}}^\top C_{\mathfrak{s}}^\top\right) = \mathrm{Tr}\left(P_{\mathfrak{s}}MP_{\mathfrak{s}}^\top C_{\mathfrak{s}}^\top X_{\mathfrak{s}}\right)$.

using strong duality in Linear Programming. That is,

$$\begin{aligned}
& a^\top \xi \geq 0,\ \forall \xi : W\xi \geq h \\
\Leftrightarrow\ & \min_{\xi \in \mathbb{R}^k} \left\{ a^\top \xi : W\xi \geq h \right\} \geq 0 \\
\Leftrightarrow\ & \max_{\lambda \in \mathbb{R}^l} \left\{ h^\top \lambda : W^\top \lambda = a, \lambda \geq 0 \right\} \geq 0 \\
\Leftrightarrow\ & \exists \lambda \in \mathbb{R}^l : h^\top \lambda \geq 0, W^\top \lambda = a, \lambda \geq 0.
\end{aligned}$$

We have that

$$\begin{aligned}
& a^\top \xi \geq 0, \quad \forall \xi \in \hat{\Xi} \\
\Longrightarrow\ & a^\top \xi \geq 0, \quad \forall \xi \in \Xi,
\end{aligned}$$

because the support Ξ is a subset of the polyhedron $\hat{\Xi}$. □

Applying Proposition 1 to the equality constraints (2.18) and Proposition 2 to the inequality constraints (2.19) we are able to transform the problem into the finite, tractable LP

$$\begin{aligned}
\min\ & \sum_{\mathfrak{s} \in \mathcal{S}} \operatorname{Tr}\left(P_{\mathfrak{s}} M P_{\mathfrak{s}}^\top C_{\mathfrak{s}}^\top X_{\mathfrak{s}} \right) && && (2.20) \\
\text{s.t.}\ & \sum_{\tau=1}^{\mathfrak{s}} A_{\mathfrak{s}\tau} X_\tau P_\tau + S_{\mathfrak{s}} P_{\mathfrak{s}} = B_{\mathfrak{s}} P_{\mathfrak{s}}, && \forall \mathfrak{s} \in \mathcal{S}, && (2.21) \\
& \Lambda_{\mathfrak{s}} W = S_{\mathfrak{s}} P_{\mathfrak{s}}, && \forall \mathfrak{s} \in \mathcal{S}, && (2.22) \\
& \Lambda_{\mathfrak{s}} h \geq 0, && \forall \mathfrak{s} \in \mathcal{S}, && (2.23) \\
& \Lambda_{\mathfrak{s}} \geq 0, && \forall \mathfrak{s} \in \mathcal{S}, && (2.24)
\end{aligned}$$

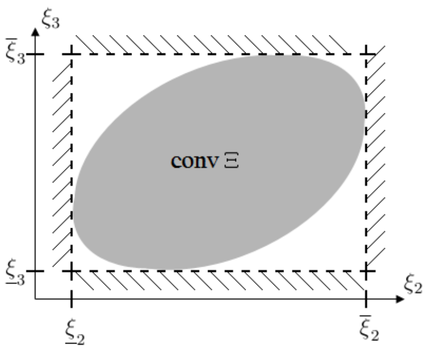
where $X_{\mathfrak{s}}$, $S_{\mathfrak{s}}$ and $\Lambda_{\mathfrak{s}} \in \mathbb{R}^{m_{\mathfrak{s}} \times l}$ are the decision variables.

We substitute $S_{\mathfrak{s}} P_{\mathfrak{s}} = \Lambda_{\mathfrak{s}} W$ into constraints (2.21) because the slack variables are nonnegative on $\hat{\Xi}$

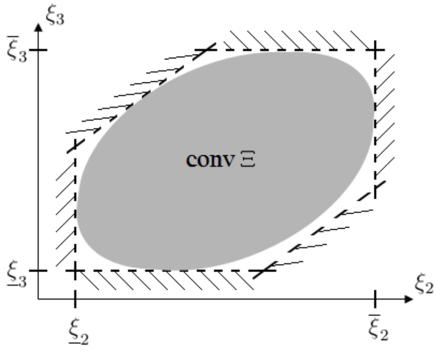
$$S_{\mathfrak{s}} P_{\mathfrak{s}} \xi = \Lambda_{\mathfrak{s}} W \xi = \Lambda_{\mathfrak{s}} (W\xi - h) + \Lambda_{\mathfrak{s}} h \geq 0, \quad \forall \xi : W\xi \geq h. \tag{2.25}$$

Thus, we end up with the LDR approximation

$$\begin{aligned}
\min\ & \sum_{\mathfrak{s} \in \mathcal{S}} \operatorname{Tr}\left(P_{\mathfrak{s}} M P_{\mathfrak{s}}^\top C_{\mathfrak{s}}^\top X_{\mathfrak{s}} \right) && && (2.26) \\
\text{s.t.}\ & \sum_{\tau=1}^{\mathfrak{s}} A_{\mathfrak{s}\tau} X_\tau P_\tau + \Lambda_{\mathfrak{s}} W = B_{\mathfrak{s}} P_{\mathfrak{s}}, && \forall \mathfrak{s} \in \mathcal{S}, && (2.27) \\
& \Lambda_{\mathfrak{s}} h \geq 0, && \forall \mathfrak{s} \in \mathcal{S}, && (2.28) \\
& \Lambda_{\mathfrak{s}} \geq 0, && \forall \mathfrak{s} \in \mathcal{S}. && (2.29)
\end{aligned}$$



(a) Uncertainty box.



(b) Tight approximation.

Fig. 2.2: Illustration of a "uncertainty box", i.e., a polyhedron that consists of upper and lower bounds on the uncertain parameters, and a tight approximation of the convex hull of Ξ for $k = 3$.

This problem is polynomial in k, l, $m \triangleq \sum_{\mathfrak{s}\in\mathcal{S}} m_{\mathfrak{s}}$ and $n \triangleq \sum_{\mathfrak{s}\in\mathcal{S}} n_{\mathfrak{s}}$.

Note that if we use a compact polyhedron $\hat{\Xi}$ on the form (2.6) which is not equal to the convex hull of Ξ, i.e., $\hat{\Xi} \supsetneq \operatorname{conv}\Xi$, we impose a restriction on the problem that might result in a loss of optimality. However, the LDR approximation still provides an upper bound for the original problem. Thus, we want a polyhedron that is a tight approximation of the convex hull of Ξ, as illustrated in Fig. 2.2.

An optimistic bound on the optimal solution can be obtained by solving the LDR approximation of the dual of the original problem, as pointed out in [28]. An alternative upper bound is developed in [26], using Lagrangian duality.

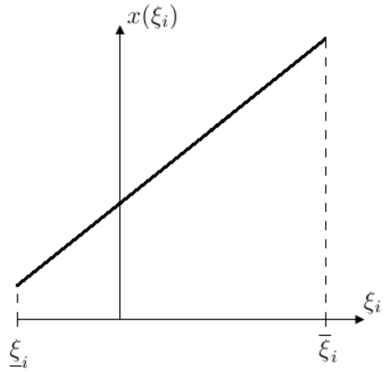
2.2 Piecewise Linear Decision Rules

Approximating a stochastic programming problem using the LDR approach might result in a large approximation error. One way to improve the performance of the LDR approximation is to augment the vector of uncertain parameters ξ, in order to allow for higher flexibility in the decisions. The increased flexibility does however come at a cost of an increased problem size. In this section we present piecewise linear continuous decision rules with axial segmentation, as developed in [27].

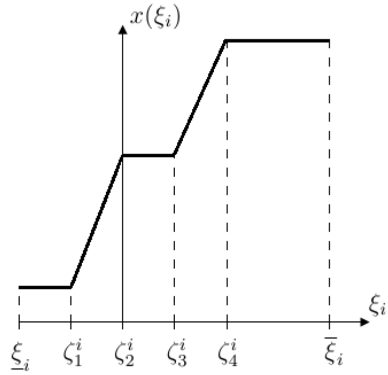
The idea of piecewise linear decision rules is to expand the sample space of the uncertain parameter ξ_i into r_i line segments, with $r_i - 1$ break points ζ_j^i for $j \in \{1, \ldots, r_i - 1\}$ and $i \in \{2, \ldots, k\}$, such that

$$\underline{\xi}_i < \zeta_1^i < \cdots < \zeta_{(r_i-1)}^i < \overline{\xi}_i, \quad \forall i \in \{2, \ldots, k\}, \tag{2.30}$$

where $\overline{\xi}_i$ and $\underline{\xi}_i$ are the upper and lower bounds on ξ_i, respectively.



(a) Linear decision rules.



(b) Piecewise Linear decision rules.

Fig. 2.3: Illustration of linear decision rules and piecewise linear decision rules when one uncertain parameter is observed.

We introduce the *lifted space* $\mathbb{R}^{k'}$ of the piecewise linear parameters $\xi_i' \in \mathbb{R}^{r_i}$, in the *lifted support* Ξ', i.e., $\xi' \in \Xi'$, where $\xi' \triangleq (1, \xi_2'^\top, \ldots, \xi_k'^\top)^\top$. Further, we define the *lifting operator* $\mathbb{L}_{ij} : \mathbb{R}^k \to \mathbb{R}$, such that

$$\xi_{ij}' = \mathbb{L}_{ij}[\xi] \triangleq \begin{cases} \xi_i & \text{if } r_i = 1, \\ \min\{\xi_i, \zeta_1^i\} & \text{if } r_i > 1 \text{ and } j = 1, \\ \max\{\min\{\xi_i, \zeta_j^i\} - \zeta_{(j-1)}^i, 0\} & \text{if } r_i > 1 \text{ and } j \in \{2, \ldots, r_i - 1\}, \\ \max\{\xi_i - \zeta_{(j-1)}^i, 0\} & \text{if } r_i > 1 \text{ and } j = r_i, \end{cases} \tag{2.31}$$

for all $i \in \{2, \ldots, k\}$ and $j \in \{1, \ldots, r_i\}$. Note that if r_i equals 1, there are no break points, which is equivalent to the pure LDR approach, as derived in Section 2.1.

Fig. 2.3 illustrates the decisions taken with LDR and piecewise LDR. The figure shows that one or more break points may increase the flexibility of the decisions substantially.

The retraction operator $\mathbb{B}_i : \mathbb{R}^{k'} \to \mathbb{R}$ converts the lifted parameters into the original parameters, i.e.,

$$\mathbb{B}_i[\xi_i'] \triangleq \sum_{j=1}^{r_i} \xi_{ij}' = \xi_i, \quad \forall i \in \{1, \ldots, k\}. \tag{2.32}$$

Proposition 3. *Let $\hat{\Xi}'$ be defined through*

$$\hat{\Xi}' \triangleq \left\{ \xi' \in \mathbb{R}^{k'} : W\mathbb{B}[\xi'] \geq h, W_i' \begin{pmatrix} 1 \\ \xi_i' \end{pmatrix} \geq 0, \forall i \in \{2, \ldots, k\} \right\},$$

where $\mathbb{B} \triangleq (\mathbb{B}_1,\ldots,\mathbb{B}_k)$ *and* $W_i' \in \mathbb{R}^{(r_i+1)\times(r_i+1)}$ *is defined through*

$$W_i' \triangleq \begin{pmatrix} \frac{\zeta_1^i}{\zeta_1^i-\underline{\xi}_i} & -\frac{1}{\zeta_1^i-\underline{\xi}_i} & & & & \\ -\frac{\underline{\xi}_i}{\zeta_1^i-\underline{\xi}_i} & \frac{1}{\zeta_1^i-\underline{\xi}_i} & -\frac{1}{\zeta_2^i-\zeta_1^i} & & & \\ & & \frac{1}{\zeta_2^i-\zeta_1^i} & \ddots & & \\ & & & \ddots & -\frac{1}{\zeta_{(r_i-1)}^i-\zeta_{(r_i-2)}^i} & \\ & & & & \frac{1}{\zeta_{(r_i-1)}^i-\zeta_{(r_i-2)}^i} & -\frac{1}{\overline{\xi}_i-\zeta_{(r_i-1)}^i} \\ & & & & & \frac{1}{\overline{\xi}_i-\zeta_{(r_i-1)}^i} \end{pmatrix}.$$

Then

$$\hat{\Xi}' \supseteq \operatorname{conv}\Xi'.$$

Proposition 3 is proven in [27].

The compact polyhedron $\hat{\Xi}'$ contains the convex hull of the lifted support Ξ'. We can thus apply the LDR approximation derived in Section 2.1 to the problem with a lifted support.

2.3 Finite Memory

When expanding a problem using piecewise LDR, one might experience large problem sizes and long runtimes. Using a *finite memory* might yield substantially shorter runtimes.

The decision variable $x_{\mathfrak{s}\mathfrak{j}}(\xi^{\mathfrak{s}})$ is given as

$$x_{\mathfrak{s}\mathfrak{j}}(\xi^{\mathfrak{s}}) = \sum_{i=1}^{k^{\mathfrak{s}}} X_{\mathfrak{s}\mathfrak{j}i}\xi_i, \quad \forall \mathfrak{s} \in \mathcal{S}, \mathfrak{j} \in \{1,\ldots,n_{\mathfrak{s}}\}, \tag{2.33}$$

when using the LDR approximation. The notation $x_{\mathfrak{s}\mathfrak{j}}(\xi^{\mathfrak{s}})$ refers to element $\mathfrak{j}$ of $x_{\mathfrak{s}}(\xi^{\mathfrak{s}})$ and $X_{\mathfrak{s}\mathfrak{j}i}$ refers to element $(\mathfrak{j},i)$ of $X_{\mathfrak{s}}$.

We introduce the *information basis* $\mathcal{I}_{\mathfrak{s}\mathfrak{j}}$ of decision variable $x_{\mathfrak{s}\mathfrak{j}}(\xi^{\mathfrak{s}})$. The information basis contains the indices of the uncertain parameters on which the decision variable should depend. Thus, $\mathcal{I}_{\mathfrak{s}\mathfrak{j}}$ is a subset of $\{1,\ldots,k^{\mathfrak{s}}\}$. We express the decision variables as

$$x_{\mathfrak{s}\mathfrak{j}}(\xi^{\mathfrak{s}}) = \sum_{i\in\mathcal{I}_{\mathfrak{s}\mathfrak{j}}} X_{\mathfrak{s}\mathfrak{j}i}\xi_i, \quad \forall \mathfrak{s} \in \mathcal{S}, \mathfrak{j} \in \{1,\ldots,n_{\mathfrak{s}}\}. \tag{2.34}$$

We remove some of the decision rules $X_{\mathfrak{s}\mathfrak{j}i}$ from the basis, and might therefore experience a loss of optimality. However, the problem size is decreased, which might reduce the runtime.

2.4 Principal Component Reduction

Large stochastic programming problems with many uncertain parameters in each stage give long runtimes. We will now develop a method to reduce the problem size using Principal Component Analysis (PCA), and denote this method Principal Component Reduction (PCR).

PCA theory gives that the vector of uncertain parameters $\xi_{\mathfrak{s}} \in \mathbb{R}^{k_{\mathfrak{s}}}$ observed in stage $\mathfrak{s} \in \mathcal{S}$ can be expressed through

$$\xi_{\mathfrak{s}} = R_{\mathfrak{s}} \tilde{\xi}_{\mathfrak{s}}, \quad \forall \mathfrak{s} \in \mathcal{S}, \tag{2.35}$$

where $\tilde{\xi}_{\mathfrak{s}} \in \mathbb{R}^{k_{\mathfrak{s}}}$ is a vector of the principal components of $\xi_{\mathfrak{s}}$ and $R_{\mathfrak{s}} \in \mathbb{R}^{k_{\mathfrak{s}} \times k_{\mathfrak{s}}}$ is an orthogonal matrix with columns equal to the eigenvectors of the covariance matrix $\Sigma_{\mathfrak{s}}$ of $\xi_{\mathfrak{s}}$ in stage $\mathfrak{s} \in \mathcal{S}$ [29]. Moreover, the variance of the principal components are given by the eigenvalues $\nu_i^{\mathfrak{s}}$ of $\Sigma_{\mathfrak{s}}$ for $i \in \{1, \ldots, k_{\mathfrak{s}}\}$ in stage $\mathfrak{s} \in \mathcal{S}$.

By removing the least important components of $\tilde{\xi}_{\mathfrak{s}}$, i.e., the elements corresponding to the smallest eigenvalues, we obtain an approximation of $\xi_{\mathfrak{s}}$ that keeps the majority of the variability in the original parameters. We denote the reduced vector of principal components by $\tilde{\xi}_{\mathfrak{s}}^{\star} \in \mathbb{R}^{\tilde{k}_{\mathfrak{s}}}$, with a corresponding coefficient matrix $R_{\mathfrak{s}}^{\star} \in \mathbb{R}^{k_{\mathfrak{s}} \times \tilde{k}_{\mathfrak{s}}}$, where $\tilde{k}_{\mathfrak{s}}$ is the number of principal components used in stage $\mathfrak{s} \in \mathcal{S}$. We approximate $\xi_{\mathfrak{s}}$ using

$$\xi_{\mathfrak{s}} \approx R_{\mathfrak{s}}^{\star} \tilde{\xi}_{\mathfrak{s}}^{\star}, \quad \forall \mathfrak{s} \in \mathcal{S}. \tag{2.36}$$

The share $\beta_{\mathfrak{s}}$ of the variance that is captured by the principal components can now be expressed through

$$\beta_{\mathfrak{s}} = \frac{\sum_{i=1}^{\tilde{k}_{\mathfrak{s}}} \nu_i^{\mathfrak{s}}}{\sum_{i=1}^{k_{\mathfrak{s}}} \nu_i^{\mathfrak{s}}}, \quad \forall \mathfrak{s} \in \mathcal{S}, \tag{2.37}$$

where the eigenvalues are ordered such that $\nu_1^{\mathfrak{s}} > \cdots > \nu_{k_{\mathfrak{s}}}^{\mathfrak{s}} > 0$, for all $\mathfrak{s} \in \mathcal{S}$.

Further, we write the approximation of all observable uncertain parameters $\xi^{\mathfrak{s}} = (\xi_1, \ldots, \xi_{\mathfrak{s}})^{\top}$ up to stage $\mathfrak{s} \in \mathcal{S}$ as

$$\xi^{\mathfrak{s}} \approx R^{\mathfrak{s}} \tilde{\xi}, \quad \forall \mathfrak{s} \in \mathcal{S}, \tag{2.38}$$

where $\tilde{\xi} \in \mathbb{R}^{\tilde{k}}$ is defined as $\tilde{\xi} \triangleq (1, \tilde{\xi}_2^{\star\top}, \ldots, \tilde{\xi}_{|\mathcal{S}|}^{\star\top})^{\top}$, and $R^{\mathfrak{s}} \in \mathbb{R}^{k^{\mathfrak{s}} \times \tilde{k}}$ is defined through

$$R^{\mathfrak{s}} \triangleq \begin{pmatrix} 1 & 0 & \cdots & \cdots & \cdots & 0 \\ 0 & R_2^{\star} & & & & \vdots \\ \vdots & & \ddots & & & \vdots \\ \vdots & & & R_{\mathfrak{s}-1}^{\star} & 0 & 0 \\ 0 & \cdots & \cdots & 0 & R_{\mathfrak{s}}^{\star} & 0 \end{pmatrix}, \quad \forall \mathfrak{s} \in \mathcal{S}. \tag{2.39}$$

We want to express the LDR approximation through the principal components. Still, the decision variables should depend on the original uncertain parameters[4], i.e., $x_{\mathfrak{s}} = x_{\mathfrak{s}}(\xi^{\mathfrak{s}})$. We therefore express the decision rules as

$$x_{\mathfrak{s}}(\xi^{\mathfrak{s}}) = X_{\mathfrak{s}} R^{\mathfrak{s}} \tilde{\xi}, \quad \forall \mathfrak{s} \in \mathcal{S}. \tag{2.40}$$

Notice that we do not reduce the size of the $X_{\mathfrak{s}}$ matrices. Yet, the dimension of the vector of uncertain parameters in each stage can now be reduced. Thus, the dimensions of the matrices used in the description of the support, and hence the size of the resulting LP, will be reduced.

We assume that the convex hull of the support $\tilde{\Xi}$ of $\tilde{\xi}$ can be expressed as a bounded polyhedron on the form

$$\operatorname{conv} \tilde{\Xi} \subseteq \left\{ \tilde{\xi} \in \mathbb{R}^{\tilde{k}} : \tilde{W}\tilde{\xi} \geq \tilde{h}, \tilde{W} \in \mathbb{R}^{\tilde{l} \times \tilde{k}}, \tilde{h} \in \mathbb{R}^{\tilde{l}}, \tilde{l} \in \mathbb{Z}^{+} \right\}. \tag{2.41}$$

By applying the theory presented in Section 2.1, we express the LDR approximation using the principal components through

$$\begin{aligned}
\min \ & \sum_{\mathfrak{s} \in \mathcal{S}} \operatorname{Tr}\left(R^{\mathfrak{s}} \tilde{M} R^{\mathfrak{s}\top} C_{\mathfrak{s}}^{\top} X_{\mathfrak{s}} \right) && && (2.42) \\
\text{s.t.} \ & \sum_{\tau=1}^{\mathfrak{s}} A_{\mathfrak{s}\tau} X_{\tau} R^{\tau} + \tilde{\Lambda}_{\mathfrak{s}} \tilde{W} = B_{\mathfrak{s}} R^{\mathfrak{s}}, && \forall \mathfrak{s} \in \mathcal{S}, && (2.43) \\
& \tilde{\Lambda}_{\mathfrak{s}} \tilde{h} \geq 0, && \forall \mathfrak{s} \in \mathcal{S}, && (2.44) \\
& \tilde{\Lambda}_{\mathfrak{s}} \geq 0, && \forall \mathfrak{s} \in \mathcal{S}, && (2.45)
\end{aligned}$$

in which $X_{\mathfrak{s}} \in \mathbb{R}^{n_{\mathfrak{s}} \times k^{\mathfrak{s}}}$ and $\tilde{\Lambda}_{\mathfrak{s}} \in \mathbb{R}^{m_{\mathfrak{s}} \times \tilde{l}}$ are the decision variables.

Piecewise LDR can be applied to the PCR problem. The location of the break points in the original formulation might however be of great importance. Thus, we will choose break points in each of the original parameters ξ_i and convert a vector of break points $\zeta_{\mathfrak{s}j} \in \mathbb{R}^{k_{\mathfrak{s}}}$ into a vector of break points in the principal component $\tilde{\zeta}_{\mathfrak{s}j} \in \mathbb{R}^{\tilde{k}_{\mathfrak{s}}}$. That is,

$$\tilde{\zeta}_{\mathfrak{s}j} = R_{\mathfrak{s}}^{\star\top} \zeta_{\mathfrak{s}j}, \quad \forall \mathfrak{s} \in \mathcal{S}, j \in \{1, \ldots, r_{\mathfrak{s}}\}, \tag{2.46}$$

where $r_{\mathfrak{s}}$ denotes the number of line segments in the decision variables in stage $\mathfrak{s} \in \mathcal{S}$.

The theory presented in Section 2.2 can now be applied to the PCR problem with piecewise LDR. However, the break points in the original uncertain parameters will be located at locations approximately corresponding to the original break points,

[4] We want the decision variables to be dependent on the original uncertain parameters because then we will be able to express the bid curves as piecewise LDR, see Chapter 3.

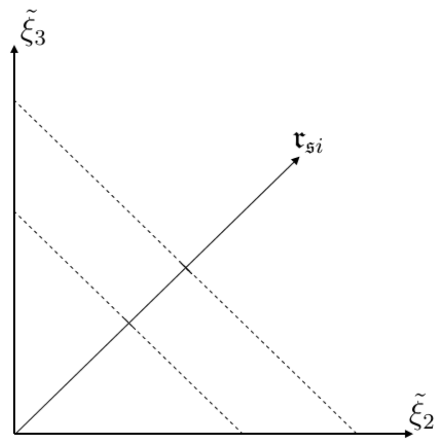


Fig. 2.4: Illustration of piecewise LDR with general segmentation in two dimensions and with two break points in the folding direction $\mathfrak{r}_{\mathfrak{s}i}$.

because the reduced coefficient matrix gives an approximation of the original parameters. Moreover, the location of the break points in the original parameters ζ_j^i will become dependent on the realizations of the principal components, so the locations of these break points are no longer fixed. The lifting operators presented in Section 2.2 might therefore perform poorly.

To remedy the latter shortcoming, break points that are fixed in the original uncertain parameters ξ rather than the principal components $\tilde{\xi}$ can be used. This can be obtained through piecewise LDR with general segmentation, developed in [27]. We define *folding directions* $\mathfrak{r}_{\mathfrak{s}i} \in \mathbb{R}^{\tilde{k}_{\mathfrak{s}}}$ as the rows in $R^\star_{\mathfrak{s}}$ for all $i \in \{1, \dots, k_{\mathfrak{s}}\}$, i.e., $R^\star_{\mathfrak{s}} = (\mathfrak{r}_{\mathfrak{s}1}, \dots, \mathfrak{r}_{\mathfrak{s}k_{\mathfrak{s}}})^\top$, in each stage $\mathfrak{s} \in \mathcal{S} \setminus \{1\}$. Note that the folding directions are not necessarily parallel to the original axis, thus, we have *general segmentation*.

We define break points in the folding directions, corresponding to ξ, which yields kinks in the $\mathbb{R}^{\tilde{k}}$ space that are perpendicular to the folding directions. Fig. 2.4 illustrates the kinks in the principal components with general segmentation in two dimensions.

An outer approximation of the convex hull of the lifted support with general segmentation, expressed as a bounded polyhedron, can be found in [27]. We are hence able to express the PCR problem with general segmentation as a finite, tractable LP, following the theory presented in Section 2.1.

3. The Hydropower Bidding Problem

This chapter presents the stochastic programming formulation of a hydropower producer and its piecewise LDR approximation. A scenario based approach to the bidding problem is also discussed.

3.1 Hydropower Scheduling

The short-term hydropower scheduling problem is studied in detail in the literature, e.g., the deterministic case is discussed in [30], and the stochastic case is discussed in [31]. Below is a brief description of the hydropower scheduling problem.

The production-discharge relationship for each generator g in the set of generators $\mathcal{G}$ is described by a set of linear cuts $\mathcal{H}_g$, i.e.,

$$w_{gt} \le E_{g\mathfrak{h}} q_{gt} + \hat{E}_{g\mathfrak{h}}, \quad \forall g \in \mathcal{G}, \mathfrak{h} \in \mathcal{H}_g, t \in \mathcal{T}, \tag{3.1}$$

where w_{gt} is the production at time t in the set of time periods $\mathcal{T}$, q_{gt} is the discharge, and $E_{g\mathfrak{h}}$ and $\hat{E}_{g\mathfrak{h}}$ denote the constants that describe cut $\mathfrak{h} \in \mathcal{H}_g$ for generator $g \in \mathcal{G}$.

The reservoir balance is given by

$$f_{rt} - f_{r(t-1)} + \sum_{g \in \mathcal{G}_r} q_{gt} + d_{rt} - \sum_{\mathfrak{y} \in \mathcal{R}_r^A} \left(d_{\mathfrak{y}t} + \sum_{g \in \mathcal{G}_{\mathfrak{y}}} q_{gt} \right) = \kappa_{rt}, \quad \forall r \in \mathcal{R}, t \in \mathcal{T}, \tag{3.2}$$

where f_{rt}, d_{rt} and κ_{rt} denote the discharge, spill and inflow, respectively, to reservoir r in the set of reservoirs $\mathcal{R}$ at time $t \in \mathcal{T}$. The set $\mathcal{G}_r$ denotes the generators belonging to reservoir $r \in \mathcal{R}$ and $\mathcal{R}_r^A$ denotes the set of reservoirs immediately above reservoir $r \in \mathcal{R}$.

The value of the water stored in the reservoirs in the end of the planning horizon is determined by a long-term scheduling model, and is dependent on the reservoir level [32]. The relationship between the value of the reservoirs and the reservoir levels is described by a set $\mathcal{P}$ of linear cuts,

$$v \leq F_{\mathfrak{p}} - \sum_{r \in \mathcal{R}} V_{r\mathfrak{p}}(F_{r\mathfrak{p}} - f_{r|\mathcal{T}|}), \quad \forall \mathfrak{p} \in \mathcal{P}, \tag{3.3}$$

where v denotes the value of the reservoirs, and $L_{\mathfrak{p}}$, $V_{r\mathfrak{p}}$ and $f_{r\mathfrak{p}}$ denote the constants describing cut $\mathfrak{p} \in \mathcal{P}$. These cuts are assumed exogenously given.

Starting a generator results in increased maintenance costs [33]. Start-up costs are often modeled using binary variables, see e.g. [34]. The state variable u_{gt} equals 1 if generator $g \in \mathcal{G}$ operates at time $t \in \mathcal{T}$, i.e., if $w_{gt} > 0$, and u_{gt} equals 0 otherwise. Let the cost of starting generator $g \in \mathcal{G}$ equal K_g, then the imposed start-up cost o_{gt} can be modeled in the following way

$$\underline{W}_g u_{gt} \leq w_{gt} \leq \overline{W}_g u_{gt}, \qquad \forall g \in \mathcal{G}, t \in \mathcal{T}, \tag{3.4}$$

$$o_{gt} \geq K_g \left(u_{gt} - u_{g(t-1)}\right), \qquad \forall g \in \mathcal{G}, t \in \mathcal{T}, \tag{3.5}$$

$$o_{gt} \geq 0, \qquad \forall g \in \mathcal{G}, t \in \mathcal{T}, \tag{3.6}$$

$$u_{gt} \in \{0,1\}, \qquad \forall g \in \mathcal{G}, t \in \mathcal{T}, \tag{3.7}$$

and the sum $\sum_{g \in \mathcal{G}} \sum_{t \in \mathcal{T}} o_{gt}$ is subtracted from the objective function. Constraints (3.4) forces u_{gt} to be equal to 1 if generator $g \in \mathcal{G}$ is operating at time $t \in \mathcal{T}$, and equal to 0 otherwise. Constraints (3.5) assign start-up costs to variable o_{gt} if generator $g \in \mathcal{G}$ is operating at time $t \in \mathcal{T}$ and is not operating at time $t-1$. The nonnegativity requirements (3.6) are needed in order to avoid negative start-up costs in time periods without start-ups.

The modeling presented above requires binary variables in all time stages. This is not compatible with the LDR approximation. Note that the minimum production bound cannot be imposed on a generator without binary variables.

The simplest way to avoid binary requirements is to relax the integer requirement (3.7) into

$$0 \leq u_{gt} \leq 1, \quad \forall g \in \mathcal{G}, t \in \mathcal{T}. \tag{3.8}$$

This relaxation is a simplification that underestimates the imposed start-up costs o_{gt}.

An alternative relaxation of the binary requirement is suggested in [35], in which *ramping up* production, i.e., increasing production, is penalized, rather than actual start-ups. This can be modeled through

$$w_{gt} = w_{g(t-1)} + w_{gt}^{+} - w_{gt}^{-}, \qquad \forall g \in \mathcal{G}, t \in \mathcal{T}, \tag{3.9}$$

$$w_{gt}^{+}, w_{gt}^{-} \geq 0, \qquad \forall g \in \mathcal{G}, t \in \mathcal{T}, \tag{3.10}$$

$$0 \leq w_{gt} \leq \overline{W}_g, \qquad \forall g \in \mathcal{G}, t \in \mathcal{T}, \tag{3.11}$$

where w^+_{gt} and w^-_{gt} denote the increased and decreased production for generator $g \in \mathcal{G}$ at time $t \in \mathcal{T}$, respectively, compared to the production level at time $t-1$. The sum

$$\sum_{g\in\mathcal{G}}\sum_{t\in\mathcal{T}} K_g \frac{w^+_{gt}}{\overline{W}_g} \tag{3.12}$$

should be subtracted from the objective function. This modeling is suited for thermal scheduling, because there is a cost of ramping up production. There is however no large costs associated with ramping up production of a hydro turbine that is already operating. Penalizing increased production rates will restrict the advantageous flexibility of hydropower, because a penalty is assigned to ramping up production rather than start-ups.

Another way of dealing with binary variables in the LDR approach is moving these variables to the first stage. First stage binary variables can be handled by the LDR approximation [28]. This simplification implies that the model determines in which time periods each generator should produce in the first stage, and then allows for production rates on the interval $\left[\underline{W}_g, \overline{W}_g\right]$ at time $t \in \mathcal{T}$ if u_{gt} is set to 1. If u_{gt} is set to 0 in the first stage, generator $g \in \mathcal{G}$ will not be allowed to produce at time $t \in \mathcal{T}$. Hydropower is very flexible when it comes to starting and stopping production, compared to thermal generation. Moreover, the start-up costs are small for hydropower. Thus, deciding in which time periods to produce in the first stage will restrict much of the advantageous flexibility of hydropower. We therefore do not find this modeling suitable.

In the rest of this chapter we model start-up costs using the pure LP-relaxation, i.e., constraints (3.4)-(3.6) and (3.8), because it is the best approximation of start-up costs while providing high flexibility.

Upper and lower bounds might constrain the discharges and the reservoir levels, i.e.,

$$\underline{Q}_g \le q_{gt} \le \overline{Q}_g, \quad \forall g \in \mathcal{G}, t \in \mathcal{T}, \tag{3.13}$$

$$\underline{F}_r \le f_{rt} \le \overline{F}_r, \quad \forall r \in \mathcal{R}, t \in \mathcal{T}, \tag{3.14}$$

where $\overline{Q}_g$ and $\underline{Q}_g$ are the upper and lower bounds on the discharge of generator $g \in \mathcal{G}$, respectively, and $\overline{F}_r$ and $\underline{F}_r$ are the upper and lower bounds on the reservoir level in reservoir $r \in \mathcal{R}$, respectively.

3.2 Market Modeling

There is no evidence of significant systematic use of market power in the Nordic spot market [36]. Moreover, market regulations forbid abuse of market power. The producer is therefore modeled as a price taker.

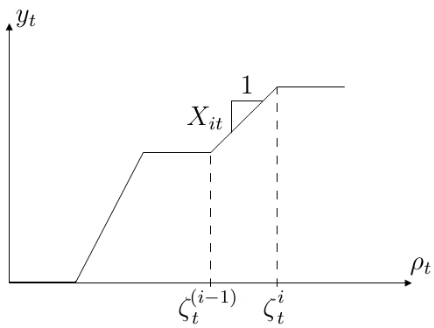


Fig. 3.1: Spot bid curve with linear interpolation.

The volume commitment in the Nordic spot market (Elspot) is determined by a linear interpolation between the price points and bid volumes. The price points and the bid volumes are set by the producer. The TSO requires unbiased bidding, i.e., a producer should not anticipate any deviations from the committed spot volumes. We therefore add constraints that require that the expected deviations from the committed spot volumes equal zero in each time period.

The linear interpolation in the spot market can be expressed through piecewise linear decision rules, because the bidding decisions in the first day are taken in the first stage. Fig. 4.14 illustrates the spot bidding curve with piecewise linear decision rules X_{it}. The decision variable X_{it} is the slope of the bid curve between price points $\zeta_t^{(i-1)}$ and ζ_t^i for break point i in the set of break points $\mathcal{I}_t$ at time t in the set of time periods with bidding $\mathcal{T}^B$. The volume commitment y_t can thus be expressed through

$$y_t = X_{1t} + \sum_{i \in \mathcal{I}_t} \rho'_{it} X_{it}, \quad \forall t \in \mathcal{T}^B, \tag{3.15}$$

where ρ_t is the spot market clearing price at time $t \in \mathcal{T}^B$. The variable X_{1t} is the constant in the bid curve, i.e., the bid volume at $\rho_t = 0$. We only model the bidding decisions in the first day (24 hours), whereas spot commitments in subsequent time periods follow standard linear decision rules. Further, ρ'_{it} refers to the piecewise linear spot prices, defined through

$$\rho'_{it} = \mathbb{L}_{it}(\rho_t) \triangleq \begin{cases} \min\{\rho_t, \zeta_t^1\} & \text{if } i = 1, \\ \max\{\min\{\rho_t, \zeta_t^i\} - \zeta_t^{(i-1)}, 0\} & \text{if } i \in \{2, \dots, |\mathcal{I}_t| - 1\}, \\ \max\{\rho_t - \zeta_t^{|\mathcal{I}_t|}, 0\} & \text{if } i = |\mathcal{I}_t|, \end{cases} \tag{3.16}$$

for all $t \in \mathcal{T}^B$ and $i \in \mathcal{I}_t$.

In order to obtain a nondecreasing bid curve, we restrict the slopes X_{it} to be nonnegative for all $i \in \mathcal{I}_t$ and $t \in \mathcal{T}^B$. This will also prevent negative spot market commitments.

The unbiased spot market bidding requirement can be expressed mathematically as

$$\mathbb{E}\left[\Delta w_t^+ - \Delta w_t^- \,\middle|\, \xi^1\right] = 0, \quad \forall t \in \mathcal{T}^B, \tag{3.17}$$

where Δw_t^+ and Δw_t^- denote the production surplus and production deficit, respectively, at time $t \in \mathcal{T}^B$. The imbalances are nonnegative.

By introducing the decision rules $\Delta w_t^+ = \Delta W_{\mathfrak{s}_t}^+ \xi^{\mathfrak{s}_t}$ and $\Delta w_t^- = \Delta W_{\mathfrak{s}_t}^- \xi^{\mathfrak{s}_t}$ for some matrices $\Delta W_{\mathfrak{s}_t}^+ \in \mathbb{R}^{1\times k^{\mathfrak{s}_t}}$ and $\Delta W_{\mathfrak{s}_t}^- \in \mathbb{R}^{1\times k^{\mathfrak{s}_t}}$, where $\mathfrak{s}_t$ denotes the stage corresponding to time period $t \in \mathcal{T}^B$, i.e., $\mathfrak{s}_t = t+2$, we express the expected value constraints as

$$\mathbb{E}\left[\Delta W_{\mathfrak{s}_t}^+ \xi^{\mathfrak{s}_t} - \Delta W_{\mathfrak{s}_t}^- \xi^{\mathfrak{s}_t} \,\middle|\, \xi^1\right] = \left(\Delta W_{\mathfrak{s}_t}^+ - \Delta W_{\mathfrak{s}_t}^-\right) \mu^{\mathfrak{s}_t} = 0, \quad \forall t \in \mathcal{T}^B, \tag{3.18}$$

where $\mu^{\mathfrak{s}_t} \in \mathbb{R}^{k^{\mathfrak{s}_t}}$ denotes the expected value of the vector of uncertain parameters $\xi^{\mathfrak{s}_t}$ seen from the first stage, i.e.,

$$\mu^{\mathfrak{s}_t} \triangleq \mathbb{E}\left[\xi^{\mathfrak{s}_t} \,\middle|\, \xi^1\right], \quad \forall t \in \mathcal{T}^B. \tag{3.19}$$

We only allow for imbalances in the bidding period. The relationship between the spot market commitment and the production levels and imbalances are

$$y_t = \sum_{g\in\mathcal{G}} w_{gt} - \Delta w_t^+ + \Delta w_t^-, \quad \forall t \in \mathcal{T}^B \tag{3.20}$$

in the bidding period, and

$$y_t = \sum_{g\in\mathcal{G}} w_{gt}, \quad \forall t \in \mathcal{T} \setminus \mathcal{T}^B \tag{3.21}$$

in the remaining period.

Accounting for intraday trading is possible in an LDR model. In the Nordics the tertiary balancing market and the bilateral continuous intraday market Elbas are both active after the spot market has been cleared. An LDR model supports a large number of time stages, thus, intraday trading can be included in the stage following the spot market clearing. The imbalances Δw_t^+ and Δw_t^- can be interpreted as commitment in an intraday market. The imbalance price might for instance be set to the price in the Elbas market.

The balancing market is more complicated. The system operator demands balancing power in hours with imbalances in the system only. In Southern Norway, imbalances occur in about fifty percent of all hours [1]. A producer might only be committed to offered volumes if the system operator demands balancing power in the applicable hour and direction (ramping up or down production or consumption). Hence, whether the balancing market exists or not in a given hour is uncertain. This can be modeled using a discrete Markov process as done in [37], or by determining the state of the system using a SARIMA model, as shown in [38]. The LDR approximation, however, requires a continuous support of the uncertain parameters. There is therefore no straight forward way of modeling the uncertainty in the balancing market using the LDR approach.

It is possible to design decision rules that make the volume commitment in the balancing market dependent on all the uncertain parameters in a beneficial way., and at the same time restrict the volume commitment to be no larger than the demanded volume in this market. Let γ_t^+ and γ_t^- be the demanded volume in the balancing market in the upward and downward directions, respectively, at time $t \in \mathcal{T}^B$. We add constraints that restrict the volume commitment in this market to be no larger than the demanded volume, i.e.,

$$0 \leq \Delta w_t^+ \leq \gamma_t^+, \quad \forall t \in \mathcal{T}^B, \tag{3.22}$$

$$0 \leq \Delta w_t^- \leq \gamma_t^-, \quad \forall t \in \mathcal{T}^B. \tag{3.23}$$

We might still use the decision rules $\Delta w_t^+ = \Delta W_t^+ \xi^{\mathfrak{s}_t}$ and $\Delta w_t^- = \Delta W_t^- \xi^{\mathfrak{s}_t}$ for the imbalances. However, these constraints will in most cases not allow for any dependency on the uncertain parameters other than γ_t^+ and γ_t^-, because constraints (3.22) and (3.23) will not be satisfied for $\gamma_t^+ = 0$ and $\gamma_t^- = 0$, respectively. Note that the most common state in the Norwegian balancing market is no demanded volume.

The decision rules can be improved by creating a dependency on the product of the demanded balancing volume times the other uncertain parameters, i.e., setting $\Delta w_t^+ = \Delta W_t^+ (\gamma_t^+ \xi^{\mathfrak{s}_t})$ and $\Delta w_t^- = \Delta W_t^- (\gamma_t^- \xi^{\mathfrak{s}_t})$ at time $t \in \mathcal{T}^B$. However, the decision rules are no longer linear in the uncertain parameters. Thus, we have *polynomial* decision rules, which requires us to solve a Semidefinite Programming problem [39]. Applying polynomial decision rules to the bidding problem is beyond the scope of this thesis.

In the remainder we will model the producer to act only as a purchaser of balancing services. That is, the producer does not offer volumes into the balancing market, but is able to buy balancing volumes at a price worse than the spot price if imbalances occur.

3.3 The Stochastic Programming Problem Formulation

Fig. 3.2 shows the uncertainty structure of the bidding problem. Note that the spot market bidding decisions are taken in the first stage, and not shown in the figure. These decisions are modeled through piecewise linear decision rules, as explained in the previous section. Furthermore, the bidding decisions for the days following the first day are not modeled accurately. That is, the prices in our model are observed in sequence for each hour, whereas all prices in the Elspot market for the next day are revealed at the same time in reality. The bidding restrictions, i.e., the requirement that the producer must submit a piecewise linear bid curve the day prior to real-

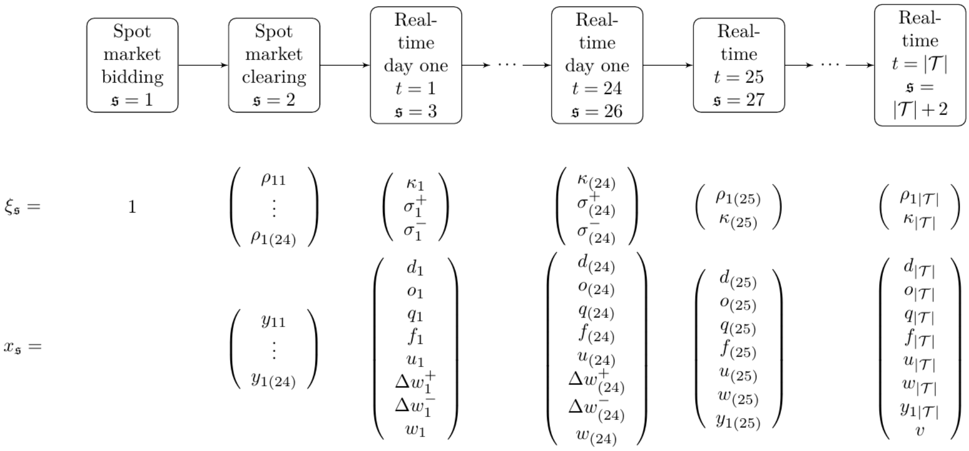


Fig. 3.2: Uncertainty structure, showing revealed uncertain parameters and decision taken in stage $\mathfrak{s} \in \mathcal{S}$. For notational convenience, indices for reservoirs and generators are omitted, and $\kappa_t, d_t, o_t, q_t, f_t, u_t, w_t$ represent vectors of all generators or reservoirs, e.g., $\kappa_t = \left(\kappa_{1t}, ..., \kappa_{|\mathcal{R}|t}\right)^\top$. The vector $\xi^{\mathfrak{s}}$ can be expressed as $\left(\xi_1^\top, \dots, \xi_{\mathfrak{s}}^\top\right)^\top$.

time, are not modeled for the days following the first day. Relaxing these restrictions yields increased flexibility in the decisions and the model is able to adapt perfectly to the observed prices.

We now present the suggested stochastic programming problem formulation that can be solved using the LDR approach. The objective is to maximize revenues minus start-up costs plus revenues from imbalances (in the intraday market) plus the value of the reservoirs in the end of the planning horizon. The resulting optimization model yields

$$\max z = \mathbb{E}\left[\sum_{t\in\mathcal{T}} \rho_t y_t - \sum_{g\in\mathcal{G}}\sum_{t\in\mathcal{T}} o_{gt} + \sum_{t\in\mathcal{T}^B} \left(\sigma_t^+ \Delta w_t^+ - \sigma_t^- \Delta w_t^-\right) + v\right] \tag{3.24}$$

s.t.

$$w_{gt} \leq E_{g\mathfrak{h}} q_{gt} + \hat{E}_{g\mathfrak{h}}, \qquad \forall g \in \mathcal{G}, \mathfrak{h} \in \mathcal{H}_g, t \in \mathcal{T}, \tag{3.25}$$

$$f_{rt} - f_{r(t-1)} + \sum_{g\in\mathcal{G}_r} q_{gt} + d_{rt} - \sum_{\mathfrak{y}\in\mathcal{R}_r^A}\left(d_{\mathfrak{y}t} + \sum_{g\in\mathcal{G}_\mathfrak{y}} q_{gt}\right) = \kappa_{rt}, \qquad \forall r \in \mathcal{R}, t \in \mathcal{T}, \tag{3.26}$$

$$v \leq L_\mathfrak{p} - \sum_{r\in\mathcal{R}} V_{r\mathfrak{p}}(F_{f\mathfrak{p}} - f_{r|\mathcal{T}|}), \qquad \forall \mathfrak{p} \in \mathcal{P}, \tag{3.27}$$

$$\underline{W}_g u_{gt} \leq w_{gt} \leq \overline{W}_g u_{gt}, \qquad \forall g \in \mathcal{G}, t \in \mathcal{T}, \tag{3.28}$$

$$o_{gt} \geq K_g\left(u_{gt} - u_{g(t-1)}\right), \qquad \forall g \in \mathcal{G}, t \in \mathcal{T}, \tag{3.29}$$

$$\mathbb{E}\left[\Delta w_t^+ - \Delta w_t^- \,\middle|\, \xi^1\right] = 0, \qquad \forall t \in \mathcal{T}^B, \tag{3.30}$$

$$y_t = \sum_{g\in\mathcal{G}} w_{gt} - \Delta w_t^+ + \Delta w_t^-, \qquad \forall t \in \mathcal{T}^B, \tag{3.31}$$

$$y_t = \sum_{g\in\mathcal{G}} w_{gt}, \qquad \forall t \in \mathcal{T} \setminus \mathcal{T}^B, \tag{3.32}$$

$$o_{gt} \geq 0, \qquad \forall g \in \mathcal{G}, t \in \mathcal{T}, \tag{3.33}$$

$$0 \leq u_{gt} \leq 1, \qquad \forall g \in \mathcal{G}, t \in \mathcal{T}, \tag{3.34}$$

$$d_{rt} \geq 0, \qquad \forall r \in \mathcal{R}, t \in \mathcal{T}, \tag{3.35}$$

$$\underline{Q}_g \leq q_{gt} \leq \overline{Q}_g, \qquad \forall g \in \mathcal{G}, t \in \mathcal{T}, \tag{3.36}$$

$$\underline{F}_r \leq f_{rt} \leq \overline{F}_r, \qquad \forall r \in \mathcal{R}, t \in \mathcal{T}, \tag{3.37}$$

$$\Delta w_t^+, \Delta w_t^- \geq 0, \qquad \forall t \in \mathcal{T}^B. \tag{3.38}$$

All constraints must be valid for all realizations of the uncertain parameters[1], i.e., for all $\xi \in \Xi$. Constraints (3.25)-(3.29) describe the short-term scheduling problem, as presented in Section 3.1, constraints (3.30)-(3.32) describe the market modeling, as presented in Section 3.2 and constraints (3.33)-(3.38) give the bounds on the variables.

Note that the equality constraints (3.32) can be substituted directly into the objective function. The equality constraints (3.26) can be substituted into Constraints

[1] Note that, for notational convenience, the decision variables and the uncertain parameters are not written as functions of the vector of uncertain parameters ξ.

(3.27) and (3.37) using

$$f_{rt} = f_{r0} - \sum_{\tau=1}^{t} \left(\sum_{g \in \mathcal{G}_r} q_{g\tau} - d_{r\tau} + \sum_{\eta \in \mathcal{R}_r^A} \left(d_{\eta\tau} + \sum_{g \in \mathcal{G}_\eta} q_{g\tau} \right) + \kappa_{r\tau} \right), \tag{3.39}$$

in order to reduce the numbers of constraints and variables.

3.4 A Scenario Based Model

Scenario based models comprise a commonly used alternative solution method for the bidding problem, particularly the *deterministic equivalent* [19]. In order to solve the deterministic equivalent we generate a finite set of scenarios that represents the uncertainty. A scenario tree must be constructed such that the information is revealed gradually according to the information structure of the problem. The underlying probability distributions of the uncertain parameters in the bidding problem are continuous. Hence, representing the uncertainty by a scenario tree is a simplification. A scenario based model thus gives an approximation of the bidding problem.

In order to model the bidding decisions, the following constraints must be added to problem (3.24)-(3.38). An in-depth presentation of these constraints can be found in [7]. The linear interpolation in the spot curve can be expressed through

$$\begin{aligned} y_t = \frac{\rho_t - \zeta_t^i}{\zeta_t^{(i+1)} - \zeta_t^i} a_t^{(i+1)} + \frac{\zeta_t^{(i+1)} - \rho_t}{\zeta_t^{(i+1)} - \zeta_t^i} a_{(i+1)t}, \\ \text{if } \zeta_t^i \le \rho_t \le \zeta_t^{(i+1)}, \quad \forall i \in \mathcal{I}_t^S \setminus \left\{ |\mathcal{I}_t^S| \right\}, t \in \mathcal{T}^B, \end{aligned} \tag{3.40}$$

where a_{it} is the bidding volume at price ζ_t^i for bid point i in the set of bid points $\mathcal{I}_t^S$ at time $t \in \mathcal{T}^B$.

The bid volumes must be nondecreasing[2], i.e.,

$$a_{(i-1)t} \le a_{it}, \quad \forall i \in \mathcal{I}_t^S \setminus \{1\}, t \in \mathcal{T}^B. \tag{3.41}$$

Solving a multistage stochastic programming problem using a deterministic equivalent also requires nonanticipaticity constraints, see [19].

[2] We order the bid points such that $\zeta_t^1 < \cdots < \zeta_t^{|\mathcal{I}_t^S|}$ for all $t \in \mathcal{T}^B$.

4. Case Study

This chapter presents a case study performed for a Norwegian watercourse during the period October 4 through October 7 2012. The results from the LDR approximation is compared to the results from a scenario based model and a deterministic model. Moreover, methods to reduce the runtime of the LDR model are tested. The optimization models in this chapter are implemented in Mosel/Xpress MP [40]. The scenario generation is done in Matlab, and Matlab generates the relevant input files for the models. The code can be found at `http://folk.ntnu.no/andereri/`. Appendix A provides details on the implementation. The different models presented in this chapter are compared using an out-of-sample simulation.

4.1 Case Description

The modeled watercourse is located in the Norwegian price area NO2 and is operated by Norsk Hydro ASA. The model consists of two reservoirs in cascade, each connected to a power station, as shown in Fig. 4.1. The upper reservoir is large ($178 \cdot 10^6\,\mathrm{m}^3$), whereas the lower reservoir is small ($1.6 \cdot 10^6\,\mathrm{m}^3$) and has little flexibility. The risk of spill is significant for the lower reservoir. The upper reservoir is an aggregation of three reservoirs, as shown in the figure. The aggregation is done because the reservoirs in reality are controlled such that the upper reservoirs store an increasing amount of water if there is a risk of spill in the lower reservoir. The aggregation does hence not represent any major simplification, whereas only one variable for the upper three reservoirs is needed in the model. The marginal water values for the upper reservoir are estimated as the volume weighted average of the individual water values in the original three reservoirs. The differences in the marginal water values are however small. Further, the two stations are parts of a larger watercourse, but the discharges from the two stations are not affected by the rest of the watercourse. There are no restrictions on the discharges and no time delay between the reservoirs.

The head is assumed to be constant, which is a minor simplification in the short-term scheduling process. The upper reservoir is kept at approximately constant head by controlling the upper two reservoirs. Head changes in the lower reservoir are small. Thus, the constant head assumption will not cause major errors. The

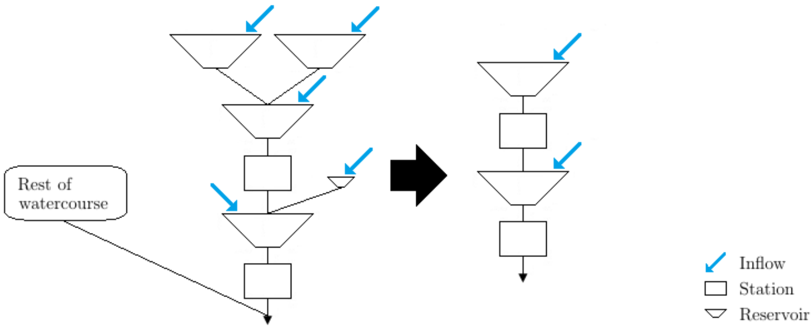


Fig. 4.1: Illustration of the watercourse in reality (left) and the simplified watercourse (right).

production from each generator is modeled with three linear cuts, as shown in Eq. (3.1). There is one generator (Fig. 4.2a) in the upper station, with a maximum production of 45 MW. The lower station contains two generators (Fig 4.2b-c) with a maximum production capacity of 80 MW and 70.5 MW, respectively. The cuts in the production-discharge curve for each generator are chosen such that the curve intersects the origin and the maximum production point, and such that there is a break point at the point with the highest turbine efficiency. In order to reduce the problem size, start-up costs are not accounted for in this case study. Start-up costs are small for hydropower, and two variables for each generator are needed for each hour to model start-up costs. Thus, almost half of the decision variables are associated with the modeling of start-up cost, which yields a large problem size and only a small improvement in the solution.

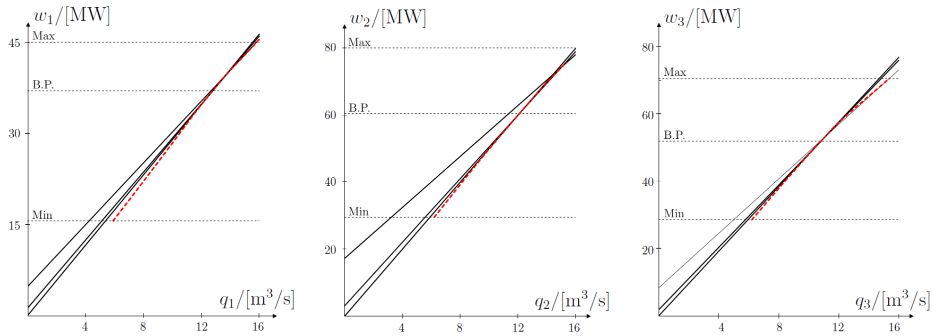
There are 18 water value cuts implemented in the model, valid for the end of October 7. The reservoir levels in the beginning of the period are high, and one should hence anticipate high production rates. The currency used in the model is Euro (€).

In this case study we will assume that the producer is not participating in any other physical electricity market than the spot market and the balancing market. We further only model single hourly bids[1].

4.2 Uncertainty Modeling

In order to simplify this analysis, deterministic inflow is used, shown in Fig. 4.3, equal to the expectation of 50 given inflow scenarios provided by the producer Norsk Hydro ASA.

[1] The market operator Nord Pool Spot also accepts *block bids*, that is, bids that set an all-or-nothing condition for a set of at least three consecutive hours. However, block bids do not provide any advantage to single hourly bids when start-up costs are not taken into account.



(a) Generator 1. *(b)* Generator 2. *(c)* Generator 3.

Fig. 4.2: Production-discharge curves with minimum production (Min), production at the highest turbine efficiency (B.P.) and maximum production (Max). The dotted red lines are the original production-discharge curves.

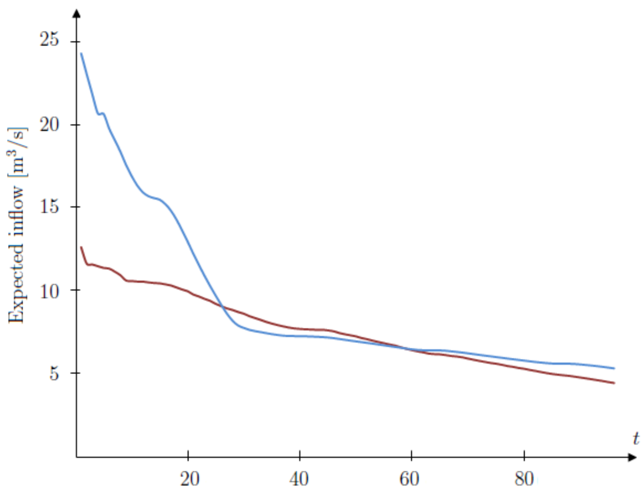


Fig. 4.3: Expected inflow in the planning horizon for the upper reservoir (red line) and the lower reservoir (blue line).

	Θ_1	Θ_2	Φ_1
Parameter value	0.8950	-0.0062	0.1237
Standard error	0.1827	0.1672	0.1805

Tab. 4.1: Parameters in the ARMA model for the spot forecast error.

The difference between the spot price and the imbalance price is modeled as deterministic in order to reduce the number of uncertain parameters and to focus attention on the spot market. The imbalance price is set equal to the spot price plus €6 for production deficits and equal to the spot price minus €6 for production surpluses. This is done because the expected premium in the balancing market in the price area NO2 is €6 for the eight weeks preceding the start of the modeled period [1]. That is,

$$\sigma_t^+ = \rho_t - €6, \quad \forall t \in \mathcal{T}^B, \tag{4.1}$$

$$\sigma_t^- = \rho_t + €6, \quad \forall t \in \mathcal{T}^B. \tag{4.2}$$

The spot price is modeled as a stochastic parameter. The spot prices for the next day (24 hours) are revealed at the same time, the hourly spot price is hence not a time series. The autocorrelation in the forecast error is however large, and an ARMA(2,1) model [41] is therefore found suitable to model the error term ϵ_t from a deterministic forecast ρ_t^D. The stochastic spot price can be expressed through

$$\rho_t = \rho_t^D + \Theta_1 \epsilon_{t-1} + \Theta_2 \epsilon_{t-2} + \Phi_1 \omega_{t-1} + \omega_t, \quad \forall t \in \mathcal{T}^T, \tag{4.3}$$

where

$$\epsilon_t \triangleq \rho_t - \rho_t^D, \quad \forall t \in \mathcal{T}^T, \tag{4.4}$$

and ω_t denotes the residual and Θ_1, Θ_2 and Φ_1 denote the parameters in the ARMA model. The set $\mathcal{T}^T$ denotes the hours in the training period, consisting of eight weeks, as found appropriate in [42]. The deterministic forecast ρ_t^D is provided by the producer Norsk Hydro ASA. The data source for historical spot prices is [1], and *R* [43] is used in the parameter estimation, using the maximum likelihood method. Fig. 4.4 shows the expected spot price with upper and lower bounds. The two last days constitute a weekend, with lower expected prices than the first two days. Tab. 4.1 shows the parameters in the ARMA model.

A set of 3000 scenarios is generated using Eq. (4.3), in which one of the residuals from the training period is drawn randomly in each hour in each scenario. The matrices in the support description, i.e., W, h and M, are estimated empirically from these scenarios. The resulting support description does not change much when the simulation is repeated. The 3000 scenarios are therefore assumed to represent the uncertainty sufficiently.

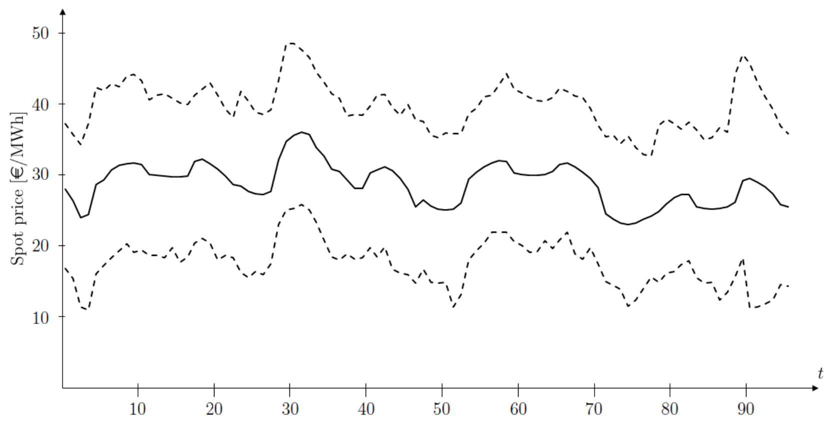


Fig. 4.4: Spot price mean (solid line) and upper and lower bounds (dotted lines).

4.3 Stage Aggregation

The multistage stochastic programming model consists of one stage for the spot market bidding, one stage for the spot market clearing, and one stage for each subsequent hour in the planning horizon, as shown in Fig. 3.2. A planning horizon of four days, i.e., 96 hours, thus comprises numerous time stages. One uncertain parameter in each stage yields a large problem, because the problem size is dependent on the dimension of the vector of uncertain parameters $\xi \in \mathbb{R}^k$. It is however possible to reduce the number of uncertain parameters, and thus the problem size, as suggested in [44].

In order to reduce the dimension of ξ we aggregate a number of hours into one *macroperiod*. The prices in the first day, i.e., stage two, are not aggregated. The aggregation of spot prices ρ_t is done such that the same error term $\bar{\epsilon}_m$ is added to all hours in macroperiod $\mathfrak{m}$ in the set of macroperiods $\mathcal{M}$, i.e.,

$$\rho_t = \mathbb{E}[\rho_t] + \bar{\epsilon}_{\mathfrak{m}}, \quad \forall t \in \mathcal{T}_{\mathfrak{m}}, \mathfrak{m} \in \mathcal{M}, \tag{4.5}$$

where $\mathcal{T}_{\mathfrak{m}}$ denotes the set of time periods in macroperiod $\mathfrak{m} \in \mathcal{M}$. The error term $\bar{\epsilon}_{\mathfrak{m}}$ denotes the average forecast error in macroperiod $\mathfrak{m} \in \mathcal{M}$, i.e.,

$$\bar{\epsilon}_{\mathfrak{m}} \triangleq \frac{\sum_{t \in \mathcal{T}_{\mathfrak{m}}} \epsilon_t}{|\mathcal{M}|}, \quad \forall \mathfrak{m} \in \mathcal{M}. \tag{4.6}$$

Fig. 4.5 illustrates the stage aggregation in a scenario tree.

The results from aggregating the information structure into three or more stages are shown in Fig. 4.6a-b, when applying LDR to the bidding problem. Three time stages represent the smallest obtainable problem size in our approach. The size of the LDR problem depends on the number of stochastic parameters and not the

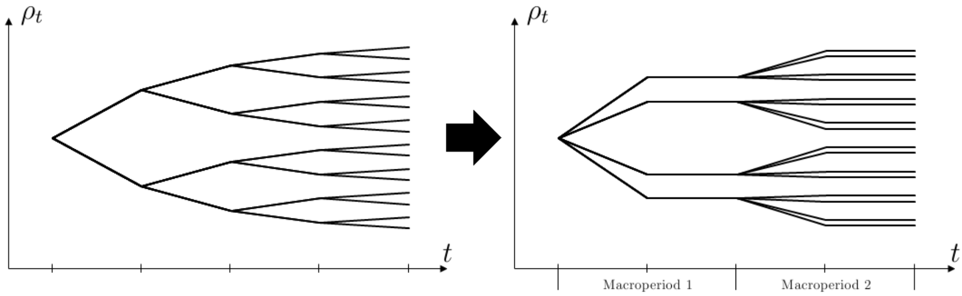


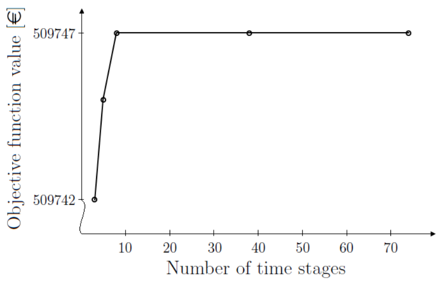
Fig. 4.5: Illustration of the stage aggregation. The left figure illustrates the scenario tree before the stage aggregation and the right figure illustrates the aggregated scenario tree. Note that this figure is for illustration of the scenario generation only, since scenarios are not used in the LDR model.

number of time stages. We keep all the spot prices in the first day as separate stochastic parameters. Moreover, the forecast errors in the subsequent spot prices are aggregated into one stochastic parameter in a three stage model, yielding the smallest possible problem size.

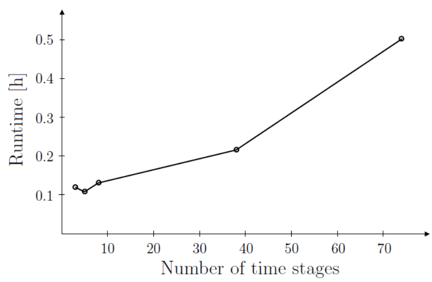
Piecewise LDR is not used in this analysis. Note that the maximum number of stages is 74 when the spot price is the only uncertain parameter, because there is no information observed in the hours in the first day. The prices in the first day are observed in stage two. The loss from aggregating the model into three time stages is only 0.001 %, whereas the runtime for three time stages is 23.8 % of the runtime with the maximum number of time stages.

A similar analysis is conducted with an extended planning horizon of eleven days, in order to analyze whether three stages is sufficient for longer time horizons. The results are shown in Fig. 4.6c-d. There is no substantial loss when aggregating the problem into three time stages. This indicates that the bidding problem has a three-stage structure, and that there is no need to use more stages.

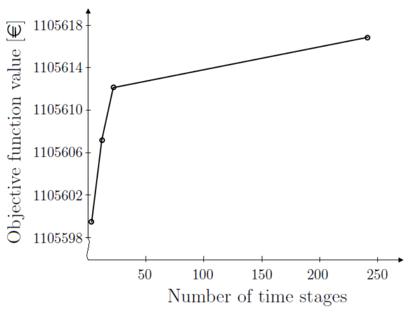
Note that the LDR model gives an approximation of the objective function, and that using more stages might give better results in a more accurate model, e.g., a piecewise LDR model. However, we have no indication that there is a gain from using more than three stages, and the runtime is significantly reduced compared to a model with more stages. We will therefore use three time stages in the rest of this case study. Note also that aggregating all hours within one day into one stage is not a simplification in the information structure. The prices in the spot market for each day (24 hours) are observed at the same time.



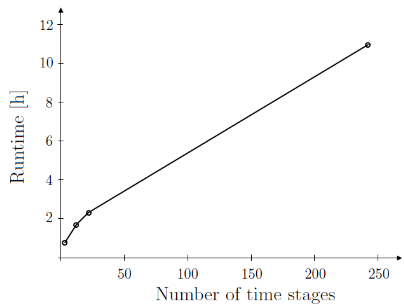
(a) Objective function value, four days in the planning horizon.



(b) Runtime, four days in the planning horizon.



(c) Objective function value, eleven days in the planning horizon



(d) Runtime, eleven days in the planning horizon

Fig. 4.6: Results from the LDR model with stage aggregation for different number of time stages.

4.4 Piecewise Linear Decision Rules

The LDR model is implemented with piecewise linear spot prices in the first day using three time stages. Fig. 4.7 shows the results for different number of line segments. Using r_i line segments corresponds to $r_i + 1$ price points in the bid curve, because each break point (a total of $r_i - 1$) gives one price point, and the upper and lower bounds on the spot price give one price point each. The locations of the break points are crucial, because a producer wants to start production when the spot price exceeds the marginal water values, corresponding to the marginal cost of production. Thus, the break points are centered around the marginal water values, with a spacing of €2.

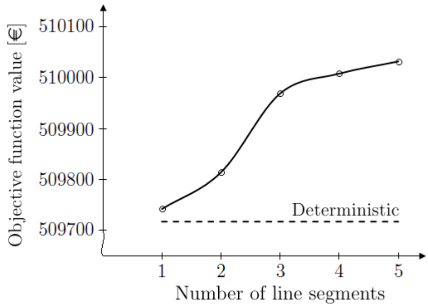
The results show that there is a significant gain from using three or more line segments. The model is able to start the majority of the production when the spot price exceeds the marginal water values with three line segments. The runtime is however very long for many line segments. Using three line segments results in a runtime of about 60 hours, which is overly long for the model to be used as operational decision support.

4.5 Finite Memory

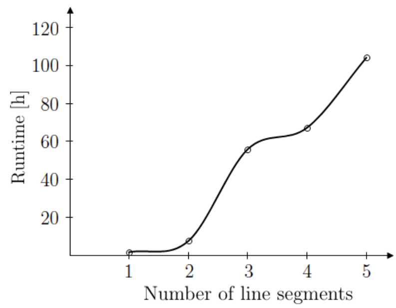
The results from Section 4.4 show that the decisions mainly depend on the observations of a few characteristic spot prices in the first day, namely the spot prices in hours with spikes and troughs. Using the method of finite memory might therefore be useful for the bidding problem. Fig. 4.8 shows the results with three line segments (four price points), and different sizes of the information basis. The figure shows that it is possible to obtain a significant reduction in the runtime with a small loss of optimality. Note that the information basis in these results was picked out using the results from the full model. When running the model for operational decision support, the modeler does not have access to the perfect solution, and the modeler is therefore not able to pick out the information basis that gives the best solution. However, since the solution is mainly dependent on the spikes and troughs, one can pick the hours with the highest and lowest expected spot prices. An alternative, yet simpler, information basis can be obtained by choosing a random selection of the spot prices in the first day, or choosing every $24/|\mathcal{I}_{\mathbf{j}\mathfrak{s}}|$ hour in the first day as the information basis of $x_{\mathbf{j}\mathfrak{s}}(\xi^{\mathfrak{s}})$.

4.6 Principal Component Reduction

Principal Component Reduction (PCR) is implemented with axial break points in the principal components. There is only one uncertain parameter in each stage, except stage two, hence the PCR is only used for the spot prices in the first day, that is ξ_2. The support description of the principal components are estimated by performing a principal component analysis on the 3000 scenarios generated as described in Section 4.2. Fig. 4.9 shows the value of β_2 with an increasing number of principal

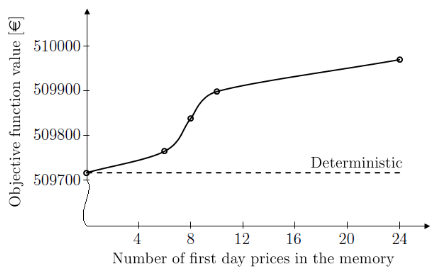


(a) Objective function value.

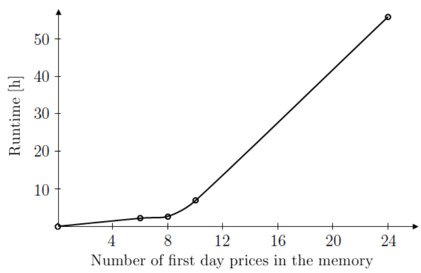


(b) Runtime.

Fig. 4.7: Results from the piecewise LDR model for different number of line segments.



(a) Objective function value.



(b) Runtime.

Fig. 4.8: Results from the piecewise LDR model with finite memory for four price points.

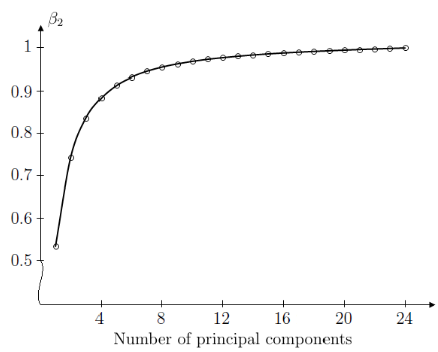
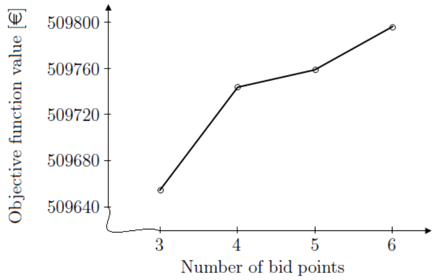
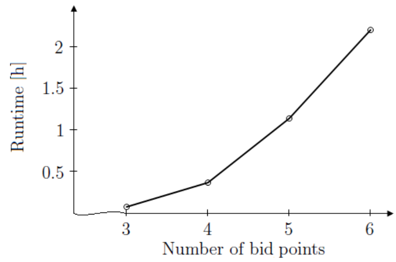


Fig. 4.9: The share β_2 of the variance of ξ_2 captured by the principal components.

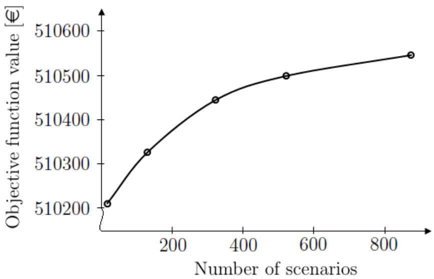


(a) Objective function value.

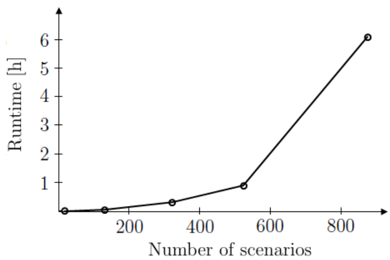


(b) Runtime.

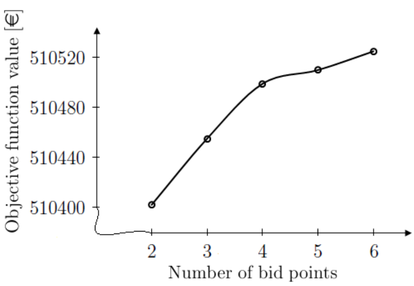
Fig. 4.10: Results from the piecewise LDR model with PCR.



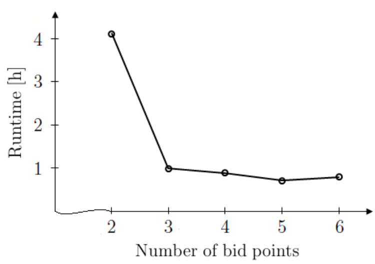
(a) Objective function value, four bid points, different number of scenarios.



(b) Runtime, four bid points, different number of scenarios.



(c) Objective function value, 523 scenarios, different number of bid points.



(d) Runtime, 523 scenarios, different number of bid points.

Fig. 4.11: Results from the deterministic equivalent.

components, that is, the share of the variance in ξ_2 that is represented by the principal components. The figure shows that four principal components capture 88 % of the original variance, and that the additional gain is very small when increasing the number of principal components. We therefore use four principal components in the rest of this case study.

The results for different number of break points in the spot prices are shown in Fig. 4.10. The figure shows that the runtime is greatly reduced, compared to the original LDR model. On average, the runtime of the PCR model is only 1.4 % of the runtime of the original LDR model. Note that the objective function values are not representing the real objective function, because the principal components slightly change the expected values of the spot prices. The performance of the PCR model with axial segmentation is evaluated in Section 4.8.

The results presented in this section are derived from a piecewise LDR model with axial segmentation in the principal components. Hence the price points in the spot prices are not fixed. Applying piecewise LDR with general segmentation yields break points in the spot prices, as explained in Section 2.4, and might therefore perform better than the axial segmentation. However, implementing the PCR model with general segmentation is left to future research.

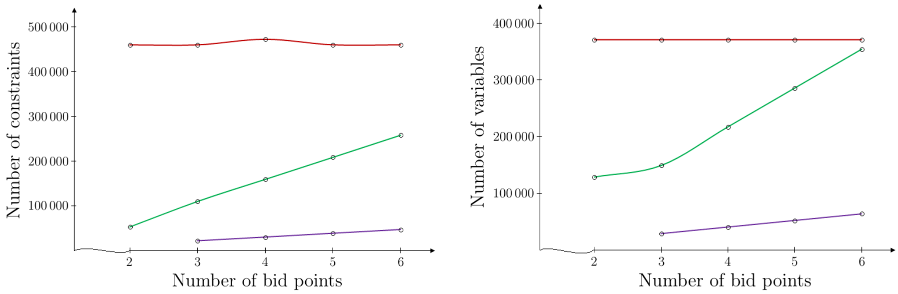
4.7 Scenario Based Model

In order to determine whether the LDR approach is an appropriate solution method for the bidding problem, the deterministic equivalent of the bidding problem is also implemented. An alternative method to evaluate the performance of the LDR approach is to solve the dual of the problem, see Appendix B.

The deterministic equivalent is a scenario based stochastic programming model. Scenario based models represent the most common approach to the bidding problem in the literature [8, 10, 11, 12, 13, 14]. The stochastic programming problem analyzed in this case study is a three stage model. Hence, we use nonanticipativity constraints for the first and second stage variables.

First, we determine an appropriate size of the set of scenarios used in the deterministic equivalent. The 3000 scenarios generated as described in Section 4.2 are reduced, using the methodology described in [45]. The results from the deterministic equivalent of the bidding problem with four bid points with different number of scenarios are shown in Fig. 4.11a-b. The figures show that using 523 scenarios results in a small loss of optimality, while keeping the solution time at an acceptable level below one hour. We therefore use 523 scenarios in the rest of this case study.

Fig. 4.11c-d show the results from the deterministic equivalent using different numbers of price points. In contrast to the piecewise LDR model, the runtime does not



(a) Number of constraints. *(b)* Number of variables.

Fig. 4.12: Number of constraints and variables in the presolved problems, showing the LDR model (green lines), the scenario based model (red lines) and the PCR model (purple lines).

increase with an increasing number of bid points. In particular, a model with two price points gives substantially longer runtime than a model with three or more bid points. The upper and lower bounds on ξ_2 and the break points used in Section 4.4 are the price points used in the scenario based model. Thus, the deterministic equivalent generates a bid curve with the same price points as the piecewise LDR model.

4.8 Comparison

Fig. 4.12 shows the number of constraints and variables in the piecewise LDR model, scenario based model and the PCR model, respectively. The figure shows the problem sizes after Xpress MP has conducted a presolve method that removes redundant variables and constraints. The problem size of the LDR model is smaller than the problem size of the scenario based model. Still, the solution time of the LDR model is substantially longer. This indicates that the LDR model is harder to solve. Moreover, the problem size of the scenario based model does not increase notably when increasing the number of bid points. This is however not the case for the piecewise LDR model, whose problem size is largely dependent on the number of bid points. The figure also shows that the PCR with axial segmentation reduces the size of the LDR problem substantially.

In order to compare performance of the different models presented in this chapter, we perform an out-of-sample simulation on each of the bid curves constructed by the different models. The simulation is conducted in the following way:

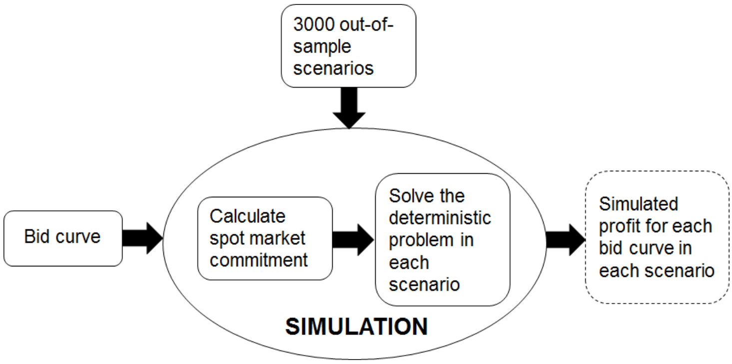


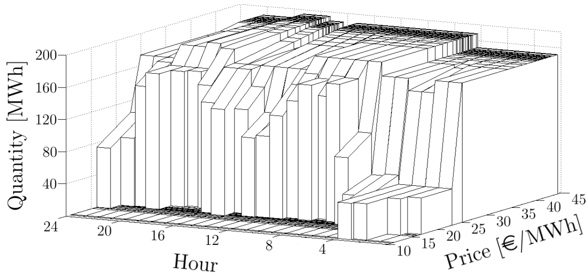
Fig. 4.13: Illustration of the simulation procedure.

Generate 3000 new scenarios based on the methodology presented in Section 4.2.
For each model do
  Solve the optimization problem and generate a bid curve.
  For each scenario do
    Calculate the spot market volume commitment based on the bid curve from the model.
    Solve the deterministic version of problem (3.24)-(3.38) using fixed $y_t \in \mathcal{T}^B$ as calculated above.
  End for
End for.

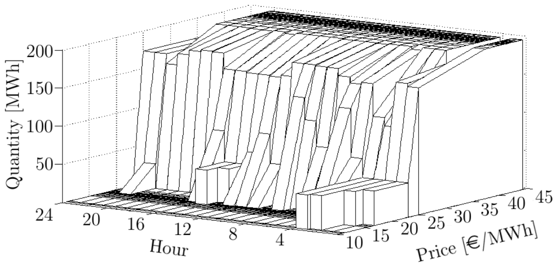
The simulation procedure is illustrated in Fig. 4.13. The simulation estimates the future profits in 3000 scenarios for each model. This number of scenarios is assumed to represent the underlying probability distribution of the spot prices sufficiently. Note that we do not evaluate the price model. Our model is assumed to be a perfect representation of the uncertainty in the system. The focus in this case study is on the solution methods.

The procedure presented above solves a two-stage problem, in which the spot market bidding is performed in the first stage, and the simulation performs the second stage. This is a simplification compared to the three-stage model that was originally solved, because the information in stage three is now observed in stage two. That is, the spot prices for all future time periods are observed prior to the first day. However, this minor simplification should not cause any fundamental errors when comparing the results from the different models using the same assumptions.

Fig. 4.14 shows the bid curves generated by the LDR model and the scenario based



(a) Piecewise LDR model.



(b) Scenario based model.

Fig. 4.14: Bid curves using four price points.

model using four price points. The figure shows that the scenario based model yields steeper kinks in the bid curve, compared to the LDR model. The bid curve from the LDR model therefore yields higher volume commitments at low prices than the bid curve from the scenario based model.

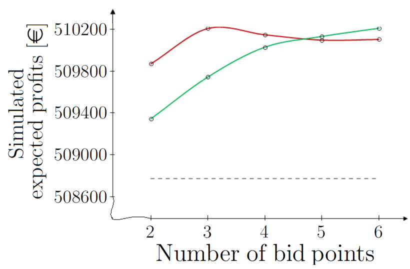
Fig. 4.15a shows the simulation results using the bid curves from the piecewise LDR model and the deterministic equivalent. The results from the deterministic version of the bidding problem are also shown, that is, problem (3.24)-(3.38) solved for the expected values of the uncertain parameters. The bid curve for each hour in the deterministic version consists of a price independent volume commitment, equal to the production rate given in the solution of the deterministic model. The figure shows that the expected profits in the scenario based model stabilize at about €510 150 for more than two price points. In fact, the scenario based model performs slightly worse with four or more price points compared to the performance at three price points. The piecewise LDR model performs monotonically better with an increasing number of bid points. Both the scenario based model and the piecewise LDR model performs better than the deterministic model for all price points.

Fig. 4.15b shows the standard deviations from the different models, using different number of price points. The figure shows that the standard deviation is smaller for the piecewise LDR model compared to the scenario based model for all number of price points. Moreover, Fig. 4.16 shows the cumulative distribution functions for five price points. Both the scenario based model and the piecewise LDR model dominate the deterministic model. The distributions of the scenario based model and the piecewise LDR model are very similar. Yet, the range is smaller in the results from the piecewise LDR model. The piecewise LDR model dominates the scenario based model in the lower region of the realized profits, whereas the scenario based model dominates the piecewise LDR model in the upper region of the realized profits. These results are valid for all number of price points greater than three. This indicates that the piecewise LDR model tends to provide the same expected profits as the scenario based model, yet with a smaller standard deviation. Thus, the piecewise LDR model reduces the short-term risk, compared to a scenario based model.

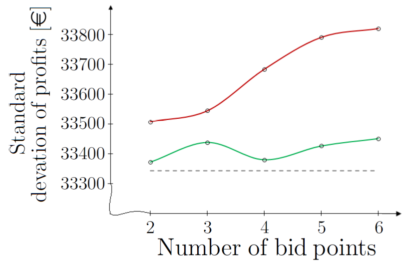
Fig. 4.15c shows the simulation results from the finite memory model, and compares these results to the results from the piecewise LDR model and the scenario based model. The results show that there is no substantial loss of profits when using a memory consisting of ten characteristic hours from the first day, rather than the entire memory of 24 hours. The runtime when using ten hours in the memory is only 6.6 % of the runtime of the original LDR model (see Fig. 4.8b). Thus, the combination of stage aggregation and finite memory provides an LDR approximation that is competitive with a scenario based model and with a smaller volatility in the profits.

Fig. 4.15d shows the simulation results from the PCR model with axial segmentation, and compares these results to the results from the piecewise LDR model and the deterministic model. The figure shows that the PCR model performs better than the deterministic model, yet substantially worse than the piecewise LDR model. For five bid points the PCR model yields an expected increase of profits of €813 (0.16 %) compared to the deterministic model and an expected decrease of €543 (0.11 %) compared to the piecewise LDR model. Note that the percentwise differences in the parentheses are expressed with all future profits as the reference. The differences are small compared to the total future profits since we only model the bidding decisions for one day. In order to increase the understanding of how much the total profits are increased one should perform a simulation with rolling horizon, see e.g. [46]. That is, one should model the bidding decisions each day for several weeks.

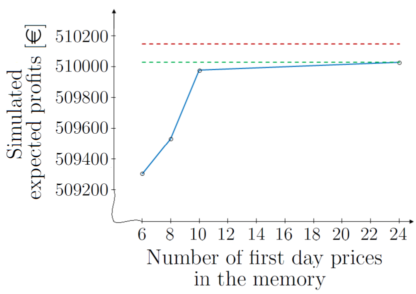
One characteristic that separates the solutions from one another is the use of imbalances, i.e., the balancing market. Fig. 4.17 shows the distributions of the use of the balancing market in the simulations, using five bid points. The figure shows that the LDR model yields higher production deficits and lower production surpluses than the scenario based model. Moreover, the LDR model gives more use of the balancing market, compared to the scenario based model. However, the balancing market volumes are small, and remain less than 1 MWh in each hour on expectation in every model. We anticipated small volumes in the balancing market, because there is no rational reason for an extensive use of the balancing market when the price in this



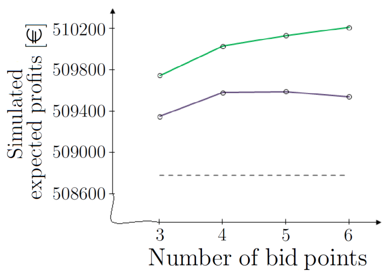
(a) Simulated expected future profits, different number of bid points.



(b) Simulated standard deviation of profits, different number of bid points.



(c) Simulated expected future profits, four bid points, different sizes of the memory (blue line).



(d) Simulated expected future profits, different number of bid points, with PCR (purple line).

Fig. 4.15: Results from the simulations for the piecewise LDR model (green lines), the scenario based model (red lines), the finite memory model (blue line), PCR (purple line) and the deterministic model (grey lines).

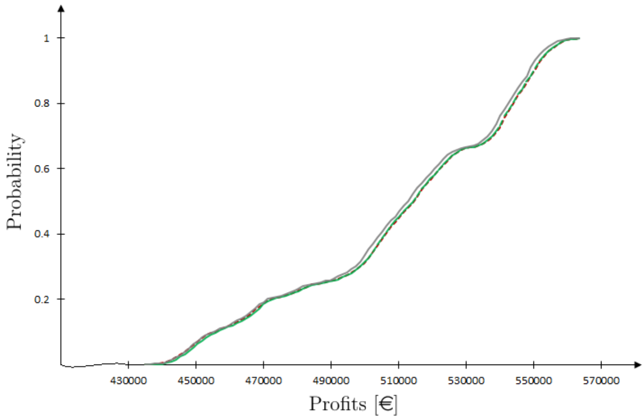
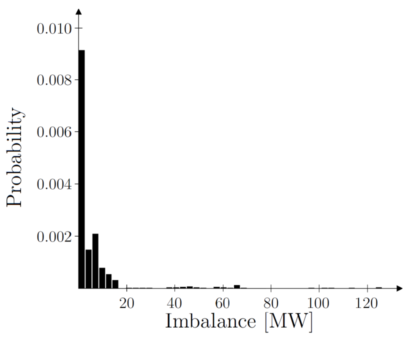
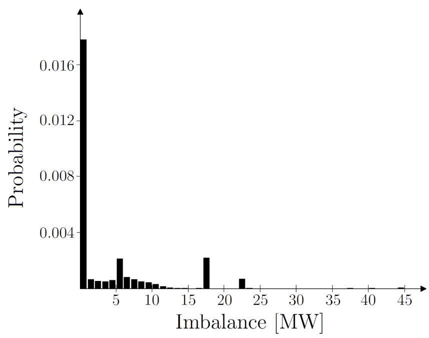


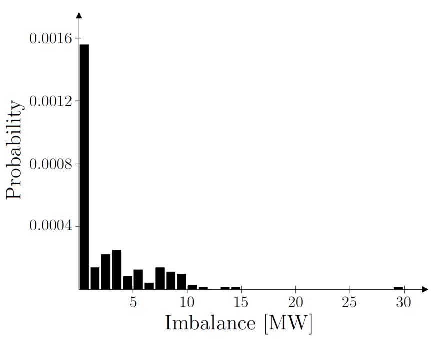
Fig. 4.16: Cumulative distributions of profits for the piecewise LDR model (green line), the scenario based model (red line) and the deterministic model (grey line).



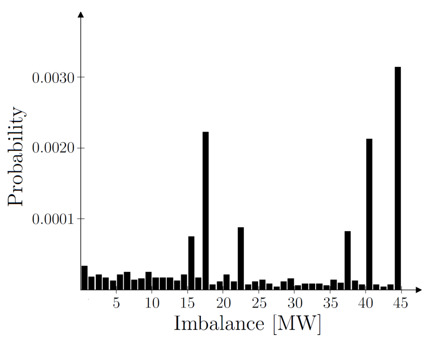
(a) Piecewise LDR, production deficit.



(b) Piecewise LDR, production surplus.



(c) Scenario based model, production deficit.



(d) Scenario based model, production surplus.

Fig. 4.17: Distributions of imbalances for five price points. The cases with no imbalances are removed from the distributions. Note that the axes are different.

market is worse than the spot price by construction.

4.9 Discussion

The simulations show that the scenario based model overestimates the expected profits, as pointed out as a problem in [20]. The piecewise LDR model tends to slightly underestimate the future profits from the bid curve. This result is expected because some of the flexibility is lost in the LDR model due to the restrictive decision rules that reduce the solution space of the decision variables. These restrictions are removed in the simulation. The bid curve from the LDR model therefore performs slightly better in the simulation than the objective function value predicts.

The main disadvantage of the piecewise LDR model is the runtime. The bidding problem is found to have no more than three important time stages. When using a three-stage LDR model, all of the uncertain parameters except one are revealed in the second stage. Because there are no first stage decisions in the model, all the decision variables are dependent on almost all of the uncertain parameters. Thus, the problem size is large taking into account that we are using the LDR approach. Using piecewise LDR on the spot prices in the first day enhances this effect, because the number of uncertain parameters in the second stage is vastly increased. The result is that the scenario based model is solved substantially faster than the piecewise LDR model. This occurs although the problem size of the LDR model is smaller than the size of the scenario based model. Hence, the LDR model is harder to solve.

The LDR approach provides a tractable approximation of the multistage stochastic programming problem. In particular, the LDR approximation remains tractability for a large number of time stages. This case study indicates that three time stages are sufficient to model the bidding problem. In contrast, the problem size of a scenario based model grows rapidly with an increasing number of time stages. The LDR model does not gain any benefit from adapting to multiple time stages because we only model three time stages. This effect contributes to the large difference in the runtimes of the two models.

When using many bid points, the piecewise LDR model performs equally well as the scenario based model on expectation. However, the range and the standard deviation of the profits are smaller in the piecewise LDR model. The LDR model hence provides more robust results, yielding lower short-term risk than a scenario based model.

In order to cope with the long runtimes, using a finite memory in the piecewise LDR model represents a promising methodology. By using a “smart” memory, we are able to design a model that is both efficient in terms of runtime and performance. The challenge lies in determining the information basis.

The method of Principal Component Reduction (PCR) does not perform as well as the piecewise LDR model or the scenario based model when using axial segmentation in the principal components. The locations of the break points are crucial in the bidding model, because the model should provide a bid curve that starts the majority of the production when the price exceeds the marginal cost of production, in this case the marginal water value. Thus, when the break points in the spot price are not fixed, the model does not completely achieve this. However, by reducing the dimension of the vector of uncertain parameters we reduce the runtime substantially. Therefore, using fixed break points in the spot prices through piecewise LDR with general segmentation, as proposed in Section 2.4, might be a promising technique that gives a tractable problem that performs well.

This case study does not take start-up costs into account. Start-up costs might impact the solution, because it might no longer be optimal to start and stop production rapidly. Start-up costs can be accounted for using an LP-relaxation in the LDR model, which will underestimate the real start-up costs. The deterministic equivalent of the bidding problem is able to model start-up costs more accurately using binary variables. However, solution methods that solve scenario based models efficiently might not be compatible with integer decisions in stages following the first stage, like the LDR approach. This is the case for Bender's Decomposition [19], used to solve the bidding problem in [8].

The simulations show that the piecewise LDR model and the scenario based model use the intraday market opportunity differently. Moreover, using multiple price points, the piecewise LDR model yields slightly higher expected profits and significantly smaller volatility, compared to the scenario based model. One possible explanation might be that the piecewise LDR model is able to adapt to the intraday market in a more beneficial way. Investigating the performance of an LDR model with an improved description of the intraday markets, such as uncertain intraday prices, is left to future research.

5. Conclusion

This thesis investigates the Linear Decision Rule (LDR) approach applied to the bidding problem of a Nordic hydropower producer. A stochastic programming model with piecewise LDR in the spot prices is developed. Furthermore, a comprehensive case study is conducted, using uncertainty in the spot prices. Methods to reduce the runtime are presented and tested. In particular, a new method named *Principal Component Reduction* (PCR) is developed. Moreover, the performance of the LDR model is compared to a scenario based model through an out-of-sample simulation.

We find that the LDR model performs equally well as a scenario based model on expectation. The standard deviation and the range of the simulated profits from the LDR model are substantially smaller. This implies that the LDR model yields a more robust solution, and that the short-term risk is reduced. The runtime of the piecewise LDR model using many price points is however overly long. Methods to reduce the runtime are therefore needed. Computational results indicate that stage aggregation and finite memory are promising techniques that reduce the runtime without a major loss of optimality. The PCR method with axial segmentation yields a short runtime and a small loss of optimality.

Taking into account that producers commonly perform risk management activities in the financial markets, the results from this case study does not show that the LDR approach are preferred over a scenario based model. Yet, this case study only account for uncertainty in the spot prices. Constructing a scenario tree with uncertain inflow and imbalance prices is more challenging and might require a larger number of scenarios in order to represent the distributions sufficiently. Therefore, future studies might show that the LDR approach outperforms a scenario based model in certain cases.

5.1 Future Work

In order to determine whether the LDR approach is the most favorable solution method for the bidding problem, more detailed studies should be conducted. These studies should take effects such as start-up costs, stochastic inflow and stochastic intraday prices into account. The results should be compared to the results from scenario based models solved using efficient solution methods. Moreover, simulations with rolling time horizons should be performed for different cases, in order to improve the understanding of the performance of the LDR model. Further investigations of methods to reduce the runtime should be carried out, using the principal component reduction in combination with piecewise LDR with general segmentation will particularly be of great interest.

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Appendix A

Details on the Implementation

We will now briefly describe how the implementations of the LDR models were done in Mosel/Xpress MP.

We divided the coefficient matrices $A_{\mathfrak{s}\tau}$ into five submatrices, in order to remove many empty elements (zeros) of the matrices, and hence reduce the preprocessing time. The problem can be written as

$$\min z = \mathbb{E}_{\xi}\left[\sum_{\mathfrak{s}\in\mathcal{S}} c_{\mathfrak{s}}^{\top}(\xi^{\mathfrak{s}})x_{\mathfrak{s}}(\xi^{\mathfrak{s}})\right] \tag{A.1}$$

$$\text{s.t. } \sum_{\tau=1}^{\mathfrak{s}} A_{\mathfrak{s}\tau}^{1} x_{\tau}(\xi^{\tau}) \leq b_{\mathfrak{s}}^{1}(\xi^{\mathfrak{s}}), \qquad \forall \mathfrak{s}\in\mathcal{S}, \tag{A.2}$$

$$\sum_{\tau=\mathfrak{s}-1}^{\mathfrak{s}} A_{\mathfrak{s}\tau}^{2} x_{\tau}(\xi^{\tau}) \leq b_{\mathfrak{s}}^{2}(\xi^{\mathfrak{s}}), \qquad \forall \mathfrak{s}\in\mathcal{S}, \tag{A.3}$$

$$A_{\mathfrak{s}}^{3} x_{\mathfrak{s}}(\xi^{\mathfrak{s}}) \leq b_{\mathfrak{s}}^{3}(\xi^{\mathfrak{s}}), \qquad \forall \mathfrak{s}\in\mathcal{S}, \tag{A.4}$$

$$\sum_{\tau=1}^{\mathfrak{s}} A_{\mathfrak{s}\tau}^{Eq} x_{\tau}(\xi^{\tau}) = 0, \qquad \forall \mathfrak{s}\in\mathcal{S}, \tag{A.5}$$

$$\mathbb{E}_{\xi}\left[A_{\mathfrak{s}}^{EV} x_{\mathfrak{s}}(\xi^{\mathfrak{s}}) \,|\, \xi^{1}\right] = 0, \qquad \forall \mathfrak{s}\in\mathcal{S}, \tag{A.6}$$

where $A_{\mathfrak{s}\tau}^{\mathfrak{t}} \in \mathbb{R}^{m_{\mathfrak{s}}^{\mathfrak{t}}\times n_{\tau}}$, where $\mathfrak{t}$ refers to the constraint, i.e., $\mathfrak{t}\in\{1,2,3,Eq,EV\}$, and where $b_{\mathfrak{s}}^{\mathfrak{t}}(\xi^{\mathfrak{s}}) \in \mathbb{R}^{m_{\mathfrak{s}}^{\mathfrak{t}}}$.

The new constraints (A.2) represent the reservoir bounds constraints (3.37) and the water value cuts constraints (3.27). The new constraints (A.3) represent the modeling of the start-up costs, i.e., constraints (3.29). Further, the new constraints (A.4) represent the original constraints in stage $\mathfrak{s}\in\mathcal{S}$ that are only dependent on the variables in stage $\mathfrak{s}$, i.e., constraints (3.25), (3.28), (3.33)-(3.36) and (3.38). The new constraints (A.5) represent the equality constraints (3.31) and finally constraints (A.6) represent the expectation constraints (3.30).

The LDR approximation of this problem is

$$
\begin{aligned}
\min \sum_{\mathfrak{s}\in\mathcal{S}} & \operatorname{Tr}\left(P_{\mathfrak{s}} M P_{\mathfrak{s}}^{\top} C_{\mathfrak{s}}^{\top} X_{\mathfrak{s}}\right) && && \text{(A.7)}\\
\text{s.t. } \sum_{\tau=1}^{\mathfrak{s}} & A_{\mathfrak{s}\tau}^{1} X_\tau P_\tau + \Lambda_{\mathfrak{s}}^{1} W = B_{\mathfrak{s}}^{1} P_{\mathfrak{s}}, && \forall \mathfrak{s}\in\mathcal{S}, && \text{(A.8)}\\
& \sum_{\tau=\mathfrak{s}-1}^{\mathfrak{s}} A_{\mathfrak{s}\tau}^{2} X_\tau P_\tau + \Lambda_{\mathfrak{s}}^{2} W = B_{\mathfrak{s}}^{2}, && \forall \mathfrak{s}\in\mathcal{S}, && \text{(A.9)}\\
& A_{\mathfrak{s}}^{3} X_{\mathfrak{s}} P_{\mathfrak{s}} + \Lambda_{\mathfrak{s}}^{3} W = B_{\mathfrak{s}}^{3} P_{\mathfrak{s}}, && \forall \mathfrak{s}\in\mathcal{S}, && \text{(A.10)}\\
& \sum_{\tau=1}^{\mathfrak{s}} A_{\mathfrak{s}\tau}^{Eq} X_\tau P_\tau = 0, && \forall \mathfrak{s}\in\mathcal{S}, && \text{(A.11)}\\
& A_{\mathfrak{s}}^{EV} X_{\mathfrak{s}} \mu^{\mathfrak{s}} = 0, && \forall \mathfrak{s}\in\mathcal{S}, && \text{(A.12)}\\
& \Lambda_{\mathfrak{s}}^{1} h \geq 0, && \forall \mathfrak{s}\in\mathcal{S}, && \text{(A.13)}\\
& \Lambda_{\mathfrak{s}}^{2} h \geq 0, && \forall \mathfrak{s}\in\mathcal{S}, && \text{(A.14)}\\
& \Lambda_{\mathfrak{s}}^{3} h \geq 0, && \forall \mathfrak{s}\in\mathcal{S}, && \text{(A.15)}\\
& \Lambda_{\mathfrak{s}}^{1}, \Lambda_{\mathfrak{s}}^{2}, \Lambda_{\mathfrak{s}}^{3} \geq 0, && \forall \mathfrak{s}\in\mathcal{S}, && \text{(A.16)}
\end{aligned}
$$

where $X_{\mathfrak{s}} \in \mathbb{R}^{n_{\mathfrak{s}} \times k^{\mathfrak{s}}}$ and $\Lambda_{\mathfrak{s}}^{\mathfrak{t}} \in \mathbb{R}^{m_{\mathfrak{s}}^{\mathfrak{t}}}$ are the decision variables for $\mathfrak{s} \in \mathcal{S}$ and $\mathfrak{t} \in \{1,2,3\}$, and where $b_{\mathfrak{s}}^{\mathfrak{t}}(\xi^{\mathfrak{s}})$ is written as $B_{\mathfrak{s}}^{\mathfrak{t}} P_{\mathfrak{s}} \xi$ for $\mathfrak{s} \in \mathcal{S}$ and $\mathfrak{t} \in \{1,3\}$.

In order to implement this problem in Mosel/Xpress MP, we write the problem on summation form.

For notational convenience, we define the matrix $D_{\mathfrak{s}} \in \mathbb{R}^{k^{\mathfrak{s}} \times n_{\mathfrak{s}}}$ through

$$D_{\mathfrak{s}} \triangleq P_{\mathfrak{s}} M P_{\mathfrak{s}}^{\top} C_{\mathfrak{s}}^{\top}, \quad \forall \mathfrak{s} \in \mathcal{S}, \tag{A.17}$$

corresponding to the coefficients in the objective function of the LDR approximation.

The LDR approximation can now be written as

$$\min \sum_{\mathfrak{s}\in\mathcal{S}} \sum_{\mathfrak{i}=1}^{k^{\mathfrak{s}}} \sum_{\mathfrak{j}=1}^{n_{\mathfrak{s}}} D_{\mathfrak{s}\mathfrak{i}\mathfrak{j}} X_{\mathfrak{s}\mathfrak{j}\mathfrak{i}} \tag{A.18}$$

s.t.

$$\sum_{\tau=1}^{\mathfrak{s}}\sum_{\mathfrak{j}=1}^{n_\tau} A^{\mathfrak{t}}_{\mathfrak{s}\tau\mathfrak{r}\mathfrak{j}} X_{\mathfrak{s}\mathfrak{j}\mathfrak{i}} + \sum_{\mathfrak{u}=1}^{l} \Lambda^{\mathfrak{t}}_{\mathfrak{s}\mathfrak{l}\mathfrak{u}} W_{\mathfrak{u}\mathfrak{i}} = B^{\mathfrak{t}}_{\mathfrak{s}\mathfrak{r}\mathfrak{i}},$$

$$\forall \mathfrak{s} \in \mathcal{S}, \mathfrak{l} \in \{1,\dots,m^{\mathfrak{t}}_{\mathfrak{s}}\}, \mathfrak{i} \in \{1,\dots,k^{\mathfrak{s}}\}, \mathfrak{t} \in \{1,\dots,3\}, \quad \text{(A.19)}$$

$$\sum_{\mathfrak{u}=1}^{l} \Lambda^{\mathfrak{t}}_{\mathfrak{s}\mathfrak{l}\mathfrak{u}} W_{\mathfrak{u}\mathfrak{i}} = 0, \quad \forall \mathfrak{s} \in \mathcal{S}, \mathfrak{l} \in \{1,\dots,m^{\mathfrak{t}}_{\mathfrak{s}}\}, \mathfrak{i} \in \{k^{\mathfrak{s}}+1,\dots,k\}, \mathfrak{t} \in \{1,\dots,3\}, \quad \text{(A.20)}$$

$$\sum_{\tau=1}^{\mathfrak{s}}\sum_{\mathfrak{j}=1}^{n_\tau} A^{Eq}_{\mathfrak{s}\tau\mathfrak{r}\mathfrak{j}} X_{\mathfrak{s}\mathfrak{j}\mathfrak{i}} = 0, \quad \forall \mathfrak{s} \in \mathcal{S}, \mathfrak{l} \in \{1,\dots,m^{Eq}_{\mathfrak{s}}\}, \mathfrak{i} \in \{1,\dots,k^{\mathfrak{s}}\}, \quad \text{(A.21)}$$

$$\sum_{\mathfrak{j}=1}^{n_{\mathfrak{s}}}\sum_{\mathfrak{i}=1}^{k^{\mathfrak{s}}} A^{EV}_{\mathfrak{s}\mathfrak{r}\mathfrak{j}} X_{\mathfrak{s}\mathfrak{j}\mathfrak{i}} \mu_{\mathfrak{i}} = 0, \quad \forall \mathfrak{s} \in \mathcal{S}, \mathfrak{l} \in \{1,\dots,m^{EV}_{\mathfrak{s}}\}, \quad \text{(A.22)}$$

$$\sum_{\mathfrak{u}=1}^{l} \Lambda^{\mathfrak{t}}_{\mathfrak{s}\mathfrak{l}\mathfrak{u}} h_{\mathfrak{u}}, \quad \forall \mathfrak{s} \in \mathcal{S}, \mathfrak{l} \in \{1,\dots,m^{\mathfrak{t}}_{\mathfrak{s}}\}, \mathfrak{t} \in \{1,\dots,3\}, \quad \text{(A.23)}$$

$$\Lambda^{\mathfrak{t}}_{\mathfrak{s}\mathfrak{l}\mathfrak{u}} \geq 0, \quad \forall \mathfrak{s} \in \mathcal{S}, \mathfrak{l} \in \{1,\dots,m^{\mathfrak{t}}_{\mathfrak{s}}\}, \mathfrak{u} \in \{1,\dots,l\}, \mathfrak{t} \in \{1,\dots,3\}. \quad \text{(A.24)}$$

Note that constraints (A.8)-(A.10) are split into constraints (A.19) and (A.20), because $P_{\mathfrak{s}} = (I_{k^{\mathfrak{s}}}\ 0)$.

We have implemented the above problem in the Mosel script `LDR_primal.mos`. The Matlab script `matwriter.m` generates the relevant input data to the Xpress model.

The PCR problem with axial segmentation is implemented in the script `LDR_primal_PCR.mos`. The deterministic equivalent is implemented in the script `scenarioFormulation.mos`. The simulation is performed by the script `simulator.mos`.

The deterministic model is implemented in two different ways, namely, in the script `matrix_det.mos` and in the script `exp_deterministic.mos`.

Appendix B

The Dual Problem

As mentioned in Section 2.1 one can obtain an optimistic bound on the objective function value by solving the dual approximation presented in [26]. The dual approximation of problem (A.1)-(A.6) is

$$\min z_D = \sum_{\mathfrak{s} \in \mathcal{S}} \mathrm{Tr}\left(P_{\mathfrak{s}} M P_{\mathfrak{s}}^{\top} C_{\mathfrak{s}}^{\top} X_{\mathfrak{s}} \right) \tag{B.1}$$

$$\text{s.t.} \quad \sum_{\tau=1}^{\mathfrak{s}} A_{\mathfrak{s}\tau}^{1} X_{\tau} P_{\tau} + S_{\mathfrak{s}}^{1} P_{\mathfrak{s}} = B_{\mathfrak{s}}^{1} P_{\mathfrak{s}}, \qquad \forall \mathfrak{s} \in \mathcal{S}, \tag{B.2}$$

$$\sum_{\tau=\mathfrak{s}-1}^{\mathfrak{s}} A_{\mathfrak{s}\tau}^{2} X_{\tau} P_{\tau} + S_{\mathfrak{s}}^{2} P_{\mathfrak{s}} = 0, \qquad \forall \mathfrak{s} \in \mathcal{S}, \tag{B.3}$$

$$A_{\mathfrak{s}}^{3} X_{\mathfrak{s}} P_{\mathfrak{s}} + S_{\mathfrak{s}}^{3} P_{\mathfrak{s}} = B_{\mathfrak{s}}^{3} P_{\mathfrak{s}}, \qquad \forall \mathfrak{s} \in \mathcal{S}, \tag{B.4}$$

$$\sum_{\tau=1}^{t} A_{\mathfrak{s}\tau}^{Eq} X_{\tau} P_{\tau} = 0, \qquad \forall \mathfrak{s} \in \mathcal{S}, \tag{B.5}$$

$$A_{\mathfrak{s}}^{EV} X_{\mathfrak{s}} P_{\mathfrak{s}} M P_{1}^{\top} = 0, \qquad \forall \mathfrak{s} \in \mathcal{S}, \tag{B.6}$$

$$(W - h e_1^{\top}) M P_{\mathfrak{s}}^{\top} S_{\mathfrak{s}}^{1\top} \geq 0, \qquad \forall \mathfrak{s} \in \mathcal{S}, \tag{B.7}$$

$$(W - h e_1^{\top}) M P_{\mathfrak{s}}^{\top} S_{\mathfrak{s}}^{2\top} \geq 0, \qquad \forall \mathfrak{s} \in \mathcal{S}, \tag{B.8}$$

$$(W - h e_1^{\top}) M P_{\mathfrak{s}}^{\top} S_{\mathfrak{s}}^{3\top} \geq 0, \qquad \forall \mathfrak{s} \in \mathcal{S}, \tag{B.9}$$

where $X_{\mathfrak{s}} \in \mathbb{R}^{n_{\mathfrak{s}} \times k^{\mathfrak{s}}}$ and $S_{\mathfrak{s}} \in \mathbb{R}^{m_{\mathfrak{s}} \times k^{\mathfrak{s}}}$ for all $\mathfrak{s} \in \mathcal{S}$ are the decision variables.

The dual approximation is implemented in the file `LDR_dual.mos`. The results are however not presented in this thesis, because we wanted to compare the primal LDR solution with a scenario based model. We also encountered some problems with ill-conditioned moment matrices when using piecewise LDR. Using a moment matrix that is close to singular yields an unbounded dual problem.