NTNU – Trondheim
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Day-ahead electricity market bidding for a cascaded reservoir system

A stochastic programming approach

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Industrial Economics and Technology Management
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Oppgavetekst/Problembeskrivelse Power producers sell their output through organized markets, where the day-ahead is the most important. This thesis will develop and evaluate decision support models for bidding into this market for a price-taking hydropower producer with cascaded reservoirs along a river. We will take into account the most important uncertainty factors and use a stochastic programming approach.	
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Sammendrag

Denne oppgaven implementerer og tester en stokastisk optimeringsmodell for anmelding og korttidsproduksjonsplanlegging gjennom en simuleringsalgoritme kjørt over tid og for et komplekst vassdrag. Den samme simuleringsalgoritmen er også implementert for en multiscenario deterministisk heuristikk tilsvarende den som brukes av mange produsenter i dag. Resultatene viser at den stokastiske modellen gir en forbedring i oppnådd gjennomsnittspris og totalverdi sammenlignet med den deterministiske modellen.

Abstract

In this thesis a stochastic model for bid optimization and short-term production scheduling has been implemented and tested trough a simulation procedure run over a longer period of time for a complex real-life river system. The same simulation procedure is also implemented for a multi scenario deterministic heuristic similar to what is used in the industry, and the results are compared. The comparison show that the stochastic model gives significant improvements in terms of higher obtained average price and higher total value than the equivalent deterministic model.

Preface

This thesis has been written for the degree of Master of Technology at the Norwegian University of Science and Technology (NTNU), Department of Industrial Economics and Technology Management within the field of Applied Economics and Operations Research. We would specially like to thank our teaching supervisor, Professor Stein-Erik Fleten for helpful assistance and valuable discussions. In addition, we owe Daniel Haugstvedt thanks for constructive feedback. Last but not least, we thank Jarand Røynstrand at Agder Energi who have provided us with invaluable data. Without their contribution this thesis would not have been possible.

Trondheim, June 5 2013

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1 Introduction

Agder Energi initiated this thesis in the fall of 2012 when they wanted to investigate the potential for using stochastic optimization models to solve the short-term production scheduling problem and specifically the determination of bids to Nord Pool. This thesis develops a stochastic model for optimal bidding and short-term production scheduling with the objective to maximize total profit from a given generation system. Our model is based on Fleten and Kristoffersen (2007), which develop a stochastic mixed-integer model for short-term production scheduling given uncertain prices. We also include uncertain inflow. The system presented in Fleten and Kristoffersen (2007) is a two-reservoir system without the option of bypass or spill. We, however, implement the stochastic model for a complicated real-life system, namely Mandalsvassdraget. This is a large hydropower cascade with several reservoirs, river courses and power stations linked together. A more complex system makes the formulation and implementation more challenging, due to added constraints related to the reservoir topology and additional dependence between hours.

Stochastic programming explicitly takes uncertainty into account and it is therefore expected that a stochastic model will give increased profits and a better reservoir management strategy than the currently used deterministic model. Our hypothesis, which is investigated throughout this thesis, is that a stochastic model for bid optimization will provide improved decision-making support for producers.

First, we set the stage by giving a brief review of the scheduling hierarchy for hydropower and the markets for power trade.

1.1 Hydropower scheduling

The objective of hydropower scheduling is to maximize the profits from available resources in the form of stored water and installed generation capacity. Hydropower scheduling involves a number of different problems spanning both short and long time horizons. In this thesis, short-term production scheduling is the main emphasis, but this task is difficult to accomplish without connections to longer-term models. Longer-term models calculate both the value of water and forecasts of price, and these are necessary inputs for short-term scheduling.

The hydropower scheduling process can be put into a hierarchy based on the time horizon of the scheduling tasks. Figure 1 shows the division of the scheduling process into long-term, seasonal-term and short-term scheduling, with a final simulation step to verify the resulting plans.

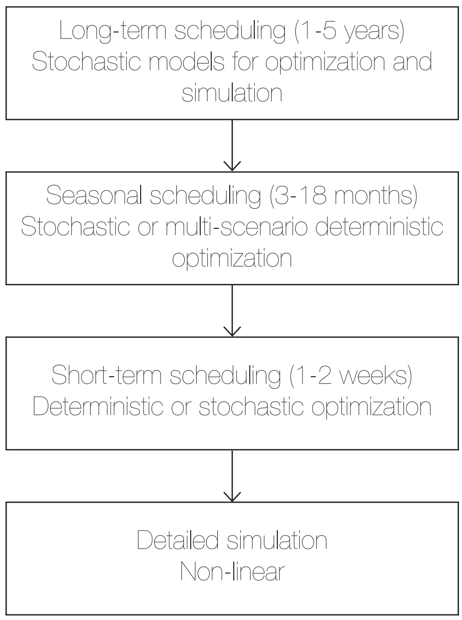

Figure 1: The hierarchy of hydropower scheduling.

The objective of long-term scheduling is to obtain optimal use of resources within a time horizon of 1 to 5 years. At this level statistical data is used to describe meteorological and hydrological phenomena that are vital inputs to the model. Forecast of demand, prices, inflow, planned outages, new plants and so on play a vital role in a stochastic optimization model that aims to optimize production resources on the basis of future prices. A major challenge in long term scheduling is that a large and very complex physical system is modelled over a long time horizon, leading to very large models. Aggregation of reservoirs, areas and time is therefore necessary to keep the models solvable. More information of long-term scheduling in the Norwegian case can be found in Fosso, Gjelsvik, Haugstad, Mo and Wangensteen (1999). The output from the long-term model is aggregated water values and target reservoir levels, as well as price forecasts.

The seasonal model acts as an intermediate step between the long- and short-term models. Aggregation is necessary in the long-term model while the short-term model requires detailed information for each reservoir for a shorter time step in order to successfully optimize individual resources. The seasonal model interprets the output from the long-term model into a form suitable for use in the short-term model. Particularly, the seasonal model is coupled to the long term model at certain times of the year where the reservoir levels are considered known, such as just before the spring flood when the reservoir levels should be low. From this coupling, water values for individual reservoirs can be calculated. Today seasonal scheduling is accomplished by running a deterministic optimization program several times with different scenarios as input. More information of seasonal scheduling can be found in Fosso et al (1999).

Finally, short-term scheduling involves the actual operation of available resources for the nearest hours and days. The short-term optimization model should result in an implementable production plan, and the model therefore needs to have a degree of detail that is adapted to the actual decisions to be taken. For instance, since market bidding is done at an hourly time step, the model also operates with hours. Today, deterministic optimization is normally used in scenario analysis, that is, several deterministic scenarios are used as input in the model and then the solutions are combined. Scenario analysis may be adequate in most cases, but we believe that a stochastic optimization model will give potential for increased profits and an increased understanding of the problem at hand. Literature covering the short-term scheduling process can be found in the literature section of this thesis and the references therein.

After the different optimization steps are accomplished, a final simulation is done to verify the resulting production plans. Optimization models for problems of this scale quickly become very large and hence have an unacceptable solution time, especially since some of the models, such as bidding determination, must be run on a daily basis. To overcome this difficulty, some simplifications of the real world must be done, and the simulation part verifies that the solution obtained is implementable in the real world. Simulation models can handle more details because the nature of the calculations is different than in an optimization model. The simulation model calculates each step separately, while an optimization tries to find the best solution for all time steps at once.

1.2 Power Market

Selling power into the day-ahead market constitutes a substantial part of the total produced volume for Nordic power producers (The Nordic Blueprint, 2011). Finding a good solution for the bidding problem is hence one of the most important tasks power producers are faced with. In the Nordic countries the day-ahead market for physical contracts is called the Elspot market, which is run by Nord Pool Spot AS. Buyers and sellers submit their bids for the day-ahead operations to Nord Pool. The participants deliver their bids in the form of a bidding matrix before 12:00 the day before operation, and

Nord Pool then calculates the spot price by aggregating sales and purchase curves for every hour of the following day. The spot price is found at the intersection between demand and supply, trough a mixed-integer program algorithm. A producer's bid for a given hour is accepted if the bid price is equal to or lower than the system spot price.

The transfer of power is subject to capacity constraints and if power flow in or out of one area exceeds the available transmission capacity, the prices are lowered in surplus areas and raised in deficit areas to facilitate the flow. This results in different area prices. The area prices are calculated by Nord Pool and are published together with the spot price every day before 13:30.

Once a producer has participated in the Elspot marked he is obligated to deliver the volume bid for the realized area price. This volume is calculated by interpolation between the two nearest price points in the bidding matrix for a given hour. The market is cleared up to 36 hours before actual delivery of power and due to uncertainties in the production situation the produced volume may not be in exact balance with the committed volume. This imbalance can be handled in the Elbas market, where continuous trading of physical power is available up to one hour prior to delivery. The Elbas market enables the producers to make trades much closer to the operating hour.

In the hour of operation, total production has to be instantly balanced by consumption, and this is the responsibility of the transmission system operator (in Norway Statnett). Producers may participate in the market for regulating power by committing to maintain available capacity to be called upon in case of unbalances. These obligations are committed after the spot market clears, but prior to the operating hour.

Both the Elbas market and the regulating power market are important properties of the Nordic power system. Still, the objective of this thesis is to optimize the bidding strategy for the Elspot market, and therefore neither Elbas nor the regulating market will be modelled in detail. The power system should be planned such that production is balanced by demand, and hence it is wanted by the TSO that producers trade their expected production in the Elspot market (Statnett, 2012).

1.3 Current practice at Agder Energi

Agder Energi and most other Nordic power producers use a deterministic model to solve the bidding problem, although future prices and inflow are stochastic variables in the short-term perspective. The producers use multi scenario analysis to create a bid matrix that is adapted to uncertainty in price. The multi scenario analysis is based on a single forecast of the prices of the coming operating day. This forecast is scaled with different weights to create a set of price scenarios. A deterministic model is run for each of the different price scenarios resulting in an optimal production volume for each hour. The volumes are then sorted by increasing price scenarios, resulting in a bid matrix.

The problem in Agder Energi's bidding strategy arises when the realized price profile does not match the profiles of the price scenarios. The multi scenario method cannot handle crossing price scenarios and it may happen that the realized price profile do not have the same profile as the scaled forecasted price. The realized price profile then crosses the price scenarios, as in Figure 2. This can cause unfortunate outcomes for the production plan, making costly adjustments for production scheduling necessary.

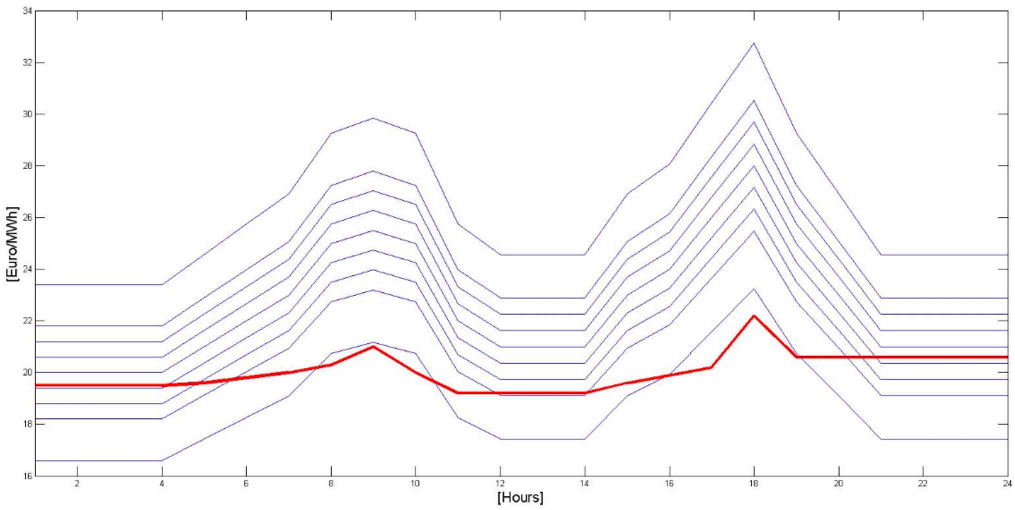

Figure 2: Scenario Price Profiles and realized price.

In the situation displayed in Figure 2, the realized price is intersecting a high price scenario for the first hours of the day and will therefore result in planned production in large parts of the system. But when the morning peak emerges the realized price increases less than the predicted profile of the price scenario, resulting in crossing of a lower price scenario. The price scenario that the realized price now intersects has over all such a low price that production is never started in this scenario. Hence one can risk that production is stopped even though the price actually increases in the morning hours. As shown in the figure, the same problem can occur for the afternoon peak. Situations like this is today sorted out by production planners at Agder Energi, but a model which can avoid such problems is desirable, and hence the motivation for this thesis.

1.4 Testing the short-term production scheduling model

In this thesis, a stochastic model for the short-term scheduling problem and specifically bid optimization is developed and tested trough a simulation procedure where the bid model is run over several weeks and the obtained profits, reservoir management strategy and other results are recorded. This is compared to a similar simulation using a deterministic model, and hence the performance of the stochastic model over a longer period of time can be assessed.

The simulation procedure is presented in Section 4 and cover the three last stages of the overall hydropower scheduling hierarchy presented in Figure 1, namely seasonal scheduling, short-term scheduling and a detailed simulation. Short-term scheduling is the main emphasis, whereas the seasonal model and the detailed simulation offer border conditions and verification. The hierarchy for scheduling used in this thesis is presented in Figure 3.

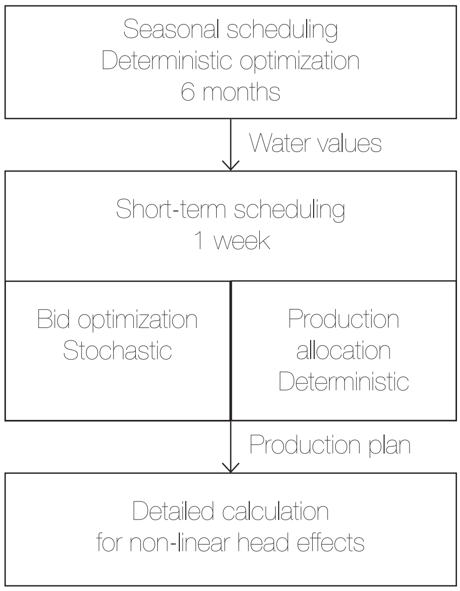

Figure 3: The scheduling hierarchy used in this thesis.

The seasonal model developed is a deterministic optimization model with a time horizon of 6 months, and is used to find the water values on a weekly time step. Short-term scheduling is divided into two optimization models; first, the bids are found in a stochastic bid optimization model and then, once the price and inflow has become known, production is allocated to specific stations and turbines by a deterministic optimization model. The bid optimization model has a time horizon of up to one week, while production allocation is done on a daily basis. The output from the short-term scheduling process is tested trough a final simulation that accounts for non-linear head effects. The most important final results are the obtained profits, production plans and reservoir management strategy.

The simulation procedure is developed for a stochastic bid optimization model with uncertain prices and inflow, since this is the main contribution of this thesis. In the case study, the simulation with the stochastic model is compared to a deterministic model, a stochastic model that allows for block bids and a stochastic model with uncertain prices and deterministic inflow. In addition, when use of the deterministic model is simulated, this is done for two different sets of input price scenarios. Finally, we also compare the stochastic model and the deterministic model when there are no time delays in the watercourses between reservoirs.

The layout of this report is as follows: In Section 2 we present related work on the short-term production scheduling problem, before theory of stochastic programming is presented in Section 3. Then the simulation procedure is explained in Section 4 and the following Sections 5 – 8 present the optimization programs developed and used in the simulation. The simulation procedure is implemented for a real-life reservoir system in the case study presented in Section 9 and finally the results from this is presented in Section 10.

2 Literature

In this section related work on the short-term production scheduling problem is presented. Our modeling approach is to a far extent based on Fleten and Kristoffersen (2007), where a stochastic mixed-integer model for optimizing bids and scheduling the short-term production is developed and compared to a deterministic approach were the uncertain prices are replaced by their expected values.

The model developed by Fleten and Kristoffersen (2007) is applicable to Nordic price-taking hydropower producers that participate in in a pool-based day-ahead market, and where the aim is to optimize the bid curve for both hourly and block bids under uncertainty in prices. The modeling of the hydropower production and reservoir topology is kept simple, but we have used the same principles in our model, adapting them to the case study. As a first instance, we disregard block bids in our model, although Fleten and Kristoffersen (2007) hold that block bids can be used to protect against major price fluctuations over time. In the case study, we implement a stochastic model that allows for block bids.

Belsnes, Fleten, Fleischmann, Haugstvedt and Steinsbø (2011) develop an extension of Fleten and Kristoffersen (2007) that looks at a time horizon from the day-ahead to the end of the drawdown season. This makes the model less dependent on input in the form of water values from other long- or intermediate-term models, but also creates a multistage problem where in every stage one must balance the value of producing water now against the value of storing it for later use. Our programming approach follow more on the lines of currently used models in the industry, where several hydro planning models with different time horizons are used, as explained in Section 1 and in Fosso, Gjelsvik, Haugstad, Mo and Wangensteen (1999) or Fosso and Belsnes (2004).

Löhndorf, Wozabal and Minner (2011) take yet another programming approach, where the bidding problem is solved using stochastic dual dynamic programming. This formulation integrates short-term intra-day decisions such as bidding and production scheduling with longer-term inter-day decisions of managing the reservoirs over time. Pritchard, Philpott and Neame (2005) also use a dynamic programming approach with stages. Here, a stage can have variable length and the first few stages represent a single trading period, whereas the later stages represent gradually longer time periods up to several days or even a week. The intra-stage sub problem computes the bids for every trading period in the current stage to give maximum expected revenue for a specified mean and variance of the water released over the current stage. The inter-stage problem then uses the values from the first sub-problem to choose the mean and variance of the water release over the stage to maximize revenue from the current stage plus the expected revenue from future stages. The two aforementioned papers also lists other examples where the short-term production scheduling is formulated and solved as a dynamic program. We, however, choose to follow the scenario tree approach presented by Fleten and Kristoffersen (2007) since it is easier to implement. For large systems it may have its disadvantages in terms of computational time, as explained by Cerisola, Fernández-Lópes, Ramos and Gollmer (2009) which also suggests other solution approaches that have potential for shorter solution times.

Ladurantaye, Gendreau and Potvin (2007) compares a stochastic model that optimizes bids in a deregulated electricity market with a model where the bidding process is disregarded as presented in Ladurantaye, Gendreau and Potvin (2005). Their results show that a model where the bid matrix is found with stochastic optimization is superior to a model that does not integrate bids in the optimization. These findings support our hypothesis that using a stochastic model when deciding what to bid in the spot market will result in higher profits for the producers than when only production allocation is optimized, not the bids.

In terms of approximations and limitations, Fleten and Kristoffersen (2007) assume that hydropower generation is proportional to discharge, which means that the relationship between produced power and discharged volume, often called the production function, is constant and linear. This is a simplification, as the generation efficiency is dependent on discharge level and water head, thus making the problem non-linear. The water head is the difference between the headwater elevation and the tailwater elevation, and whereas the former is a function of the reservoir level, the latter is a function of discharge. If head effects are neglected, the generation efficiency and hence the production function could be modeled by concave functions or piecewise-linear approximations of concave functions, such as in Fleten and Kristoffersen (2008), Faria and Fleten (2009) or Conejo, Arrayo, Contreras and Villamor (2002). We also choose piecewise linear curves to model the production functions. Catalao, Mariano, Mendes and Ferreira (2005) consider the short-term scheduling problem with head-dependency in a deterministic setting. The head-dependency makes the problem non-linear, but it is stated that for cascaded hydro systems formed by several small reservoirs, modeling of head-dependency gives more realistic results. Péréz-Diaz, Wilhelmi, Sánchez-Fernández (2010) also proposes a non-linear method to model head-dependency, and holds that more accurate modeling of these non-linear effects are most important when the reservoirs are small and their volumes can be significantly changed on a daily or hourly basis depending on the generation schedule.

Faria and Fleten (2009) consider the possibility of adjusting the dispatched power in the balancing market. After the spot market is cleared, some adjustments may be necessary due to uncertain prices, inflow and load, and the possibility of doing this can influence the bidding strategy for the day-ahead market. Bidding strategies with intentional imbalances is not wanted by the TSOs, as the spot market should be regarded as the "real" market. We, and Fleten and Kristoffersen (2007) model the balancing market by having penalties for producing in imbalance with the committed volume, even if the producer in fact may make a profit by selling balancing power. By having penalties, the producer still has the possibility to participate in the balancing market if necessary, but does not let this influence the bidding strategy as it is required that expected production is bid into the spot market. Boomsma, Juul and Fleten (2012) formulate the short-term production problem for coordinated bidding in sequential power markets.

3 Stochastic programming

In this section we present the stochastic programming approach as a decision support tool for hydropower producers. Hydropower producers face uncertainty in electricity market prices and reservoir inflows. Therefore, they must constantly evaluate the opportunity to release the available water and produce electricity today against the opportunity to save water for future use. In fact, all users of natural resources face this trade-off between the value of using the resource now and the value of saving it for use in the future. This is a decision made under uncertainty of future development, and for hydropower this relates to uncertain prices and availability of water, and hence a stochastic solution method is deemed appropriate.

3.1 Converting a real-life system into a model

When converting a complex system in to a model it is challenging to know which parts of the system needs to be described in detail and which parts can be approximated by easier expressions. How detailed and how close to the real world the model is, does not necessarily measure the quality of the model. Instead one has to look at the purpose of the model and find out which parts are essential to getting a satisfying result. In production scheduling it is important to model the bidding process as accurate as possible, since the bids in fact are sent to the market operator and used to allocate production. It is also important to get a good description of the physical system and what restrictions has to be followed in regards of reservoir storage level and water flow in the river courses. Finally, for stochastic programming, it is also of crucial importance to accurately model the uncertainty inherit in the real-world problem. The quality of the solution to a stochastic program heavily depends on the quality of the representation of uncertainty used as input to the model. The model formulation itself and the input scenarios are independent of each other, and the mathematical formulation presented in Section 5 can be used with any preferred scenario generation method, but the output will be of little use if the scenarios used do not adequately represent the stochastic elements faced in the real-world.

3.2 Why stochastic programming?

There are two main ways to solve an optimization problem; deterministic and stochastic modelling. A deterministic model builds on the assumption that every parameter in the model is predictable and known with certainty. On the other hand, a stochastic model takes uncertain parameters into account when solving the problem, under the assumption that the probability distribution of the uncertain parameters is known. Most real life decision problems are made under uncertainty. For the short-term production scheduling problem were we regard the spot market price and inflow to the reservoirs to be the most important stochastic parameters.

Today's practice is to solve the production scheduling problem by running a deterministic model several times for different set of input prices, and then combine the solutions of these models, in what is called scenario analysis. This is done at Agder Energi as explained in Section 1 and 9.2, and although this approach solves the problem sufficiently in most cases, there are cases where a deterministic model does not give good enough answers, as in the case where the realized price profile does not match the one scaled price scenario used by the deterministic model. Our hypothesis is that a stochastic model will get reasonable results in the situations where today's model fails to give an adequate answer.

A stochastic approach explicitly accounts for uncertainty while solving the problem, and thus offers a flexible solution, were the potential for profit in good scenarios is weighted against the potential for loss in bad scenarios. Looking at a deterministic future is far too optimistic, because if *every parameter is known with certainty*, the optimal decision will always be perfectly adapted to these values and there is no need for contingencies. There are no rewards for flexibility if flexibility is never needed. But what happens then, when the realized price profile does in fact not match the input scenario? Flexibility *is* needed in the real-life situation, and hence the stochastic solution is better suited to handle a larger range of possible events. When decision problems are solved as deterministic problems, odd and special situations are excluded from consideration, which can be dangerous since what appears to be a detail in the time of analysis, may appear to have major effect on future development (Wallace and Fleten, 2003).

A drawback is that stochastic programs usually are hard to solve for large problems, and that the simpler deterministic program may be used with acceptable losses in terms of solution quality. The differences between stochastic and deterministic model formulation of the bidding problem is further discussed in Section 10.

3.3 Deterministic equivalent

Another distinguishing feature between deterministic and stochastic models is the fact that *time* is very important in stochastic models, whereas for deterministic models time is irrelevant, since all information is known with certainty from the start. Time is here related to the information flow, that is, the gradual revealing of uncertain parameters, and it becomes important if a decision is to be made before or after uncertainty is revealed. For stochastic programs, the word stage is used as the period between two points in time were new information is uncovered. The bidding problem can be seen as a problem with multiple stages, as the knowledge of prices over the week gradually becomes known when the spot market is cleared every day.

To describe that uncertainty gradually becomes known, a set of finite scenarios with known probabilities are used, were a scenario is a path of realizations of the uncertain parameters over the time horizon of the model. To represent the information flow, the scenarios are clustered together in a scenario tree, and the structure of the tree and the stages when decisions have to be made corresponds to each other. More on the tree structure for our problem can be found in Section 6 and for general problems in Römish and Schultz (2001) and Follestad, Wolfgang and Belsnes (2011).

In terms of mathematical formulation, we solve the stochastic short-term production scheduling problem by solving its deterministic equivalent (Kall and Wallace, 1994). When solving the problem stochastically the realization of uncertain parameters, the scenarios, are described by ξ, where the probability distribution of ξ is assumed known as stated above. The decision variables, x, are independent of the distribution of ξ, but the decision on x has to be made before knowing the realization of ξ; we have to bid in the market before knowing the price. The general stochastic formulation can be presented as (Kall and Wallace, 1994, Wets, 1974)

$$\begin{aligned} &\min g_0(x, \tilde{\xi}) \\ s.t \quad &g_i(x, \tilde{\xi}) \leq 0, \; i = 1...m \\ &x \in X \in \mathbb{R}^n \end{aligned} \tag{1}$$

The model needs to have a way to describe how to take good decisions on x, before knowing the realization of ξ. It is therefore necessary to do a revision of the equations above. The model needs some kind of recourse action to compensate for the deviation between the decision made under uncertainty and the optimal solution when ξ is revealed. The difference between this recourse or second stage activity and the choice made under uncertainty has an extra cost or penalty, q, which will be minimized in the objective function (Kall and Wallace, 1994, Wets 1974).

$$\begin{aligned} Q(x, \xi) = &\min_y \sum_{i=1}^{m} q_i y_i \\ s.t \quad &y_i(\xi) \geq g_i^+(x, \tilde{\xi}), \; i = 1...m \end{aligned} \tag{2}$$

This gives the total cost to be minimized in the objective function

$$f_0(x, \xi) = g_0(x, \xi) + Q(x, \xi) \tag{3}$$

We can easily relate this to the short-term production planning problem were the producer face a cost for producing in imbalance with committed volume. The possibility of participating in the balancing market can hence be seen as the recourse activity for the bidding problem, since this is the action that is used to compensate for not knowing the price when the bid decision is taken. If the power company have a committed volume lower than the produced volume there is a surplus of power that can be sold in the balancing market. The cost associated with this is due to the fact that the price the production company get in the balancing market is lower then the spot price. If the power company have more committed power than the volume produced, power has to be bought in the balancing market to a higher price than the spot price. So either way, if there is an imbalance between committed volume and produced volume the power company will loose profit in our model formulation. In the real situation, however, the producer may make a profit from selling regulating power.

The above pattern of bidding, market clearing, production and trading in the balance market is repeated every day of the week, and hence we have a multi-stage stochastic decision problem.

4 Program Flow

The complexity of hydropower scheduling may make it difficult to follow the simulation procedure. To ease the understanding of the program flow, a flow chart is presented and explained in this section. The different programs will be explained more thoroughly in separate sections, but it is important to know how they communicate with each other and what values are changed day by day in the simulation. The procedure will be explained for one day, but in the case study the simulation is done over all consecutive days in a period of seven weeks. Through the simulation we get results for reservoir management and profits when using the stochastic model over time.

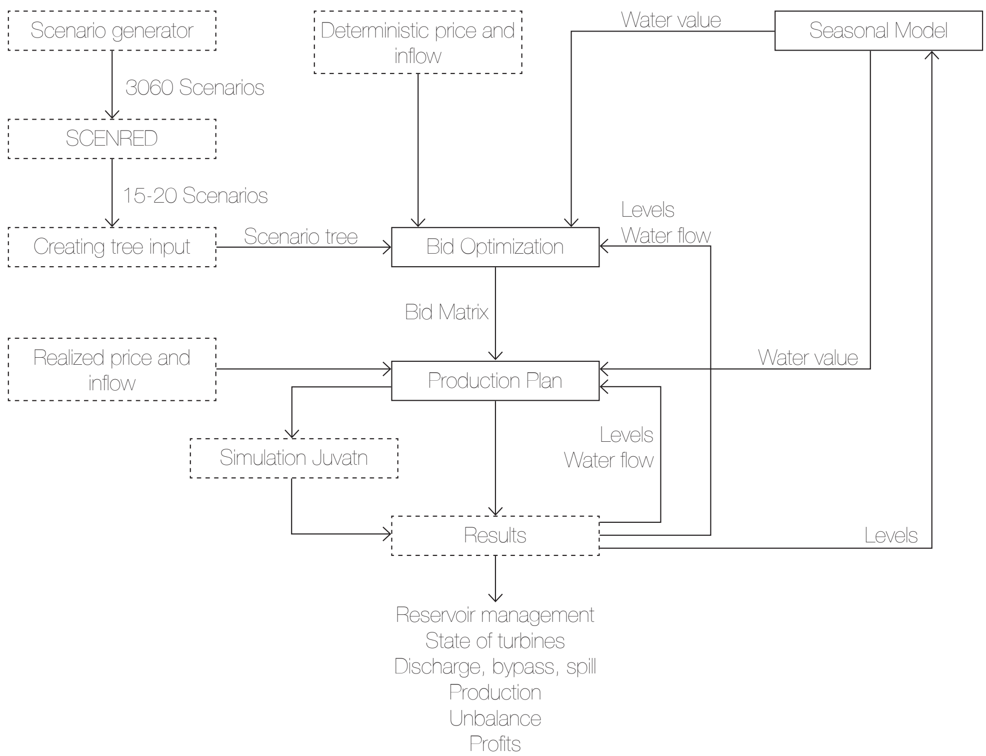

Figure 4: Flow chart for the simulation procedure.

The simulation procedure starts by setting initial values for the day counter and the day-of-the-week counter. We need to keep track of both of these counters due to the variable horizon of the bid optimization model, and the interplay with the seasonal model to get water values at the end of each week. Before the simulation is started, the seasonal model also needs to be run one time to get initial parameters for the water values.

On each day, the algorithm starts with sampling of individual scenarios for price and inflow. This is based on price scenarios from SKM Market Predictor AS and Agder Energi's ensemble scenarios for inflow. A given number of individual price scenarios are obtained as explained in Section 6.2 and combined with inflow scenarios. These are then sent trough the SCENRED algorithm (Heitsch and Rømisch, 2006) to obtain a scenario tree needed as input to the stochastic optimization model. The SCENRED algorithm also reduces the tree according to reduction parameters settings as explained in Section 6, so that the resulting tree is as small as possible according to the settings and the statistical properties of the individual scenario input. To get the tree into a form that is suitable as input to the bid optimization model, a program that detects the structure of the tree and processes the results is run. This program also adds a maximum and minimum scenario to the tree from SCENRED, in order to be certain that no information is lost through the reduction algorithm.

Meanwhile, the bid optimization model also needs as input deterministic prices and inflow for the days that are modelled as deterministic, see Section 6. These data do not need to be put trough SCENRED, since they do not represent any tree structure but only a single realization of the price and inflow in each hour. We choose random realizations of the prices and inflow scenarios as the deterministic values. The deterministic prices and inflow is sent as input to the bid optimization model.

Finally, the bid model needs the resource cost of water as input in the form of water values. Water values for each reservoir is output from the seasonal model as explained in the introduction. Water values are dependent on reservoir levels and time of year, but we have no guarantee that our stochastic model will result in the same reservoir levels at the same time of the year as the model currently used by Agder Energi, and hence we cannot use historical water values directly. A simplified version of the seasonal model is developed as explained in Section 8 and used to generate cuts to constrain the value of the water left in the reservoirs after each week. It is common practice in the industry to update the water values once a week, and this is hence what we choose to do in our model. The seasonal model is therefore only run each seventh day, and then it uses reservoir levels from the previous day as input and calculates cuts based on the production capacity for each reservoir. In addition, the seasonal model requires forecasts of future inflow and future prices as input. For this we use historical data obtained from Agder Energi and Nord Pool.

When all required input is generated, the bid optimization model is ready to be run. This results in an optimal bid matrix with bids for each hour of the following day. The bid matrix is sent to the optimization model for production allocation. In this optimization, the spot price is known and inflow is considered certain and hence this is a deterministic optimization program. This optimization results in a realistic production schedule. After the production allocation model is run, a final simulation procedure is run that accounts for head effects at the reservoirs. This is done to verify the results of the optimizations and make sure that the production plan is implementable for the real life system.

The output from the production allocation in terms of reservoir levels, daily production, daily profits and unit commitment will be considered the main output from the simulation and are saved after each run. Reservoir levels after each day and the state of the reservoirs in the last hour of the day are sent as input to the bid optimization model for the next day. The day counter then increases by one, and the whole simulation process is run again for the next day. When the simulation has been run for all days in one week, reservoir levels at the end of the week are sent as input to the seasonal model for the next week and the week counter is increased by one.

The different programs and algorithms involved in the simulation process are now presented separately. Some of the programs, such as the bid optimization model and the scenario generation process are quite involved, but also they constitute the largest theoretical effort of this thesis.

5 Bid optimization model

In this section the mathematical formulation of the bid optimization model is presented. For the sake of clarity, the model is first introduced without uncertainty. The objective function is explained first, then the equations related to the power market and finally the equations for the production system. Uncertainty is introduced after all relevant explanations are given, and then the equations for the stochastic model are given in full.

5.1 Objective function

The objective of the bid model is to maximize the profit from selling power in the day-ahead market. The profit is the income from spot market sales less the start-up costs and the penalty for using the balancing market. The value of saving water for future use must also be taken into consideration, and this is done by the water value. The short-term scheduling problem can hence be described as finding the optimal balance between water used today and water stored for future use, and also the bids that represent this balance.

The income from the day-ahead market is denoted by $\rho_h y_h$ where the index h is defined for every hour of the operating day, ρ_h is the spot market price and y_h is the volume sold.

In hydropower production the variable costs of production are negligible (Alnæs, Grøndahl, Boomsma and Fleten, 2013), and the real resource cost of power generation is the opportunity cost of water, represented by the marginal water value. The water value is denoted by ν, and is a non-linear function of reservoir levels, future inflow and prices and also the time of year as will be explained in Section 8. To keep the formulation linear, the value of water is constrained by cuts given by the seasonal model presented in Section 8. The objective function strikes a balance between profits from using water this week against saving water for next week, which is represented by the water value at the end of the week.

Other costs associated with hydropower generation are start-up costs for the turbines, S_t. These are subtracted from the revenue in hours where a turbine is turned on, denoted by a binary variable δ. The start-up costs are important for the bidding problem since it is undesirable to have frequent starts and stops of turbines, due to tear on the turbines. The start-up cost should reflect the fact that whenever there is a start or stop of production, water is lost. In addition, frequent starts and stops cause unnecessary exhaustion of the turbines, increases the risks of component failure and requires more work from the operator.

The last term in the objective function is the penalties for using the balancing market. The spot market facilitates day-ahead trade of expected production, but real-time imbalances may still occur, and the trading of these imbalances is done in the balancing market. Both buying and selling power in the balancing market is penalized in the objective function, even though producers may make a profit by selling balancing power, see for instance Boomsma, Juul and Fleten (2012) for an analysis of bidding strategies in sequential power markets. We, however, want to optimize the bids for the day-ahead market where the expected generation is traded, and hence use the balancing market only when needed. In terms of stochastic modelling, the volume bought or sold in the balancing market is the recourse action that compensates for the difference between volume bid under uncertainty and the optimal volume when actual inflow and price becomes known. The balancing volume is denoted by z^+ for upregulation and z^- for downregulation, and are both penalized with the same penalty μ. The objective function of the bidding model is hence

$$\max \sum_{h \in H} \rho_h y_h + \nu^{End} - \mu \sum_{h \in H} (z_h^+ + z_h^-) - \sum_{h \in H} \sum_{t \in T} S_t \delta_{th} \tag{4}$$

Where the symbols have the following meaning:
ρ_h : The spot market price in hour h
y_h : The committed volume in hour h
ν^{End} : The water value at the end of the week
μ : The penalty for using the balancing market
z_h^+ : The positive unbalance; volume sold in the balancing market
z_h^- : The negative unbalance; volume bought in the balancing market
S_t : Start-up costs for turbine t
δ_{th} : Binary variable with value 1 if turbine t is turned on in hour h and 0 otherwise

5.2 Markets and bidding

5.2.1 Modelling bids to Nord Pool

There are several ways to bid in the Nord Pool spot market; the common one-hour bid, flexible bids and block bids. We first concentrate on one-hour bids, where the market participants submit a set of price-volume bids for every hour of the following operating day. The market operator then interprets the bids as piecewise linear functions between the price-volume points.

The problem of finding optimal price and volume points simultaneously results in a non-linear problem. We avoid this non-linearity by fixing price points in advance and optimizing only the volume corresponding to each of the fixed prices (Fleten and Pettersen, 2005). When the market is cleared, the supply and demand curves for all market participants are aggregated and the equilibrium price is determined at the intersection of the aggregated curves. The producers are notified of their committed volume and are obliged to deliver this volume. For a single producer, the committed volume is found by interpolation between neighbouring price points. The bidding curve and the committed volume can be found as

$$y_h = \frac{\rho_h - P_{i-1}}{P_i - P_{i-1}} x_{ih} + \frac{P_i - \rho_h}{P_i - P_{i-1}} x_{i-1h}, \quad P_{i-1} \leq \rho_h \leq P_i, \quad h \in H, i \in I \quad (5)$$

Where y_h is the committed volume from the bids in hour h that is decided by the market clearing spot price ρ_h. As mentioned above, y_h is found by linear interpolation between the two nearest price-volume points (P_i, x_{ih}) and (P_{i-1}, x_{i-1h}) where the bid volumes x are decided by the optimization.

Due to the rules of how to submit bids in the spot market, the bids have to represent monotone increasing curves. Because of this we get an additional constraint for the volume points

$$x_{hi} \leq x_{hi+1}, \quad h \in H, i \in I \setminus \{I\} \quad (6)$$

A question arises as to how many bid points are suitable in the model formulation. In practical terms, the maximum number of bid points is limited to 64 by NordPool, where two of these are reserved for pre-set maximum and minimum prices. The number of bid points increase the computation time and is also limited by the number of scenarios used as input. In all cases, at least one scenario price has to fall between every set of price points, or else the model will be indifferent of what to bid between these points. In a stochastic model, there has to be more than one scenario price between price points, or the model will in fact be deterministic, since the price realization between two points then will be known with certainty. As Löhndorf, Wozabal and Minner (2011) we choose the number of price points after the following rule

$$I = \max\big(64, \frac{S-2}{2}\big) \quad (7)$$

Where I is the number of bid points and S is the number of scenarios. Having 64 price points then gives too many scenarios to handle in terms of computational time and hence we let the number of scenarios be the determinant of the number of price points, not the other way around. The number of scenarios may change every day, but for ease of comparison we choose to have the

Table 1: Values of price points.

Number	1	2	3	4	5	6	7
Price	0	15	20	23	27	30	100

same number of bid points every day. The minimum number of scenarios in our input data is 16, so we get 7 points to bid for. Agder Energi currently use between 10 and 20 price points, but these are used for all reservoirs systems owned by Agder Energi. When bidding only for Mandalsvassdraget, 7 price points are sufficient.

Choosing these price points can be done in many ways, but they must at least cover all possible outcomes for the spot price. In this thesis the price points are selected based on an analysis of what effect different sets of price points have on the result of the optimization model. The analysis shows that as long as the area where a change in spot price means a significant change in the bid volume has an adequate resolution of price points, the difference in results are low. This is shown in Appendix B. The chosen bid points have smaller intervals between 20 and 30 €/MWh where most of the spot prices during the simulation period occur. For prices over 30 €/MWh the optimal bid volume equals maximum production as long as there is enough water available. Hence, there are no price points chosen between 30 €/MWh and the upper boundary set at 100 €/MWh. The price points used can be seen in Table 1. The price points used in this thesis is set somewhat arbitrarily, but ideally the price points should reflect the marginal cost of water in the reservoirs.

5.2.2 Modelling Balancing Power

As discussed briefly in association with the objective function, the model needs a way to handle differences between committed volume and volume actually produced. If there is imbalance between the committed and the produced volume the producer has the opportunity to sell or buy this capacity in the balancing market closer to real time (Fleten and Kristoffersen, 2007). The variables z_h^+ and z_h^- represent the imbalance in the system and are calculated as slack variables in the balance between the committed volume, y_h, and the sum of production from the different power stations. As shown in the objective function in Equation (4), the balance variables are multiplied with a cost to penalize imbalances, since the aim is to obtain optimal bidding for the day-ahead market. The penaties is chosen to have high values compared to the spot price to reflect the fact that intentional imbalances is not wanted. The volume balance equation is shown as Equation (8).

$$\sum_{r \in R} w_{rh} - y_h + z_h^+ - z_h^- = 0, \quad h \in H \tag{8}$$

5.3 Production System

The system consists of several reservoirs and power stations, each station containing one or more turbines. The reservoirs are also connected to each other according to the topology of the river system. Several constraints are needed to model this; for instance how the stations and reservoirs are linked together, how the discharge from one reservoir leads to production in the underlying station and a reservoir balance for each reservoir. In our system, each reservoir is connected to at most one station, so the index r is used for both reservoirs and stations. A general system is pictured in Figure 5.

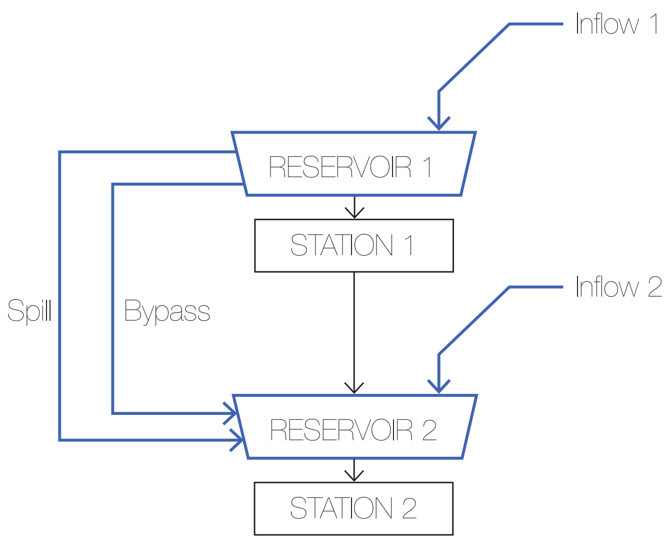

Figure 5: General system setup for a two-reservoir system.

5.3.1 Production

Each power station has one or more turbines where power is produced. The turbines have bounds on minimum and maximum capacity, and this is modelled as in Equation (9), stating that if there is production in a turbine, the binary variable u_{th} is 1 and the production has to be within the bounds.

$$u_{th} W_t^{min} \leq w_{th} \leq u_{th} W_t^{max}, \quad t \in T, h \in H \tag{9}$$

How much power a turbine generates, w_{th}, from one unit of water discharged, v_{th}, is dependent on the efficiency of the turbine. Generally, this is a non-linear relationship, due to dependency on both water head and discharge level. To keep the model linear, the production functions for the turbines are approximated by piecewise linear functions and head effects are disregarded in the optimization model. For Mandalsvassdraget, this approximation is appropriate, as most of the reservoirs are not affected by head effects. For the reservoirs where head effects are important, the results are validated trough detailed simulations as will be explained in Section 7.4. The production function relates power produced to discharged volume. An example of a production function can be seen in Figure 6, with the linearization also indicated.

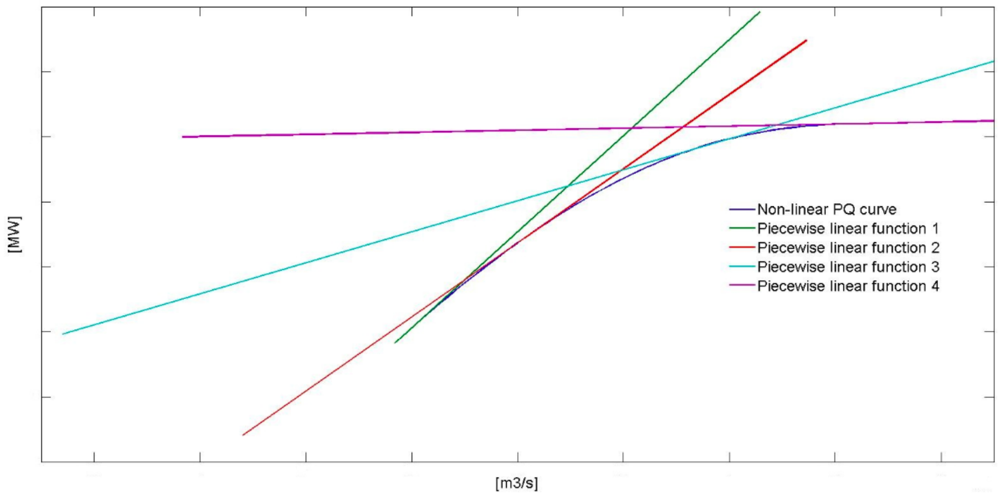

Figure 6: Piesewise linear approximation of the production function.

For the linearization we define a set of discharge-production points of the form (V_{ti}, W_{ti}) and constrain the production, w_{ht}, to always be below the curve between these points. Hence, we have

$$w_{th} \leq \frac{v_{th} - V_{i-1}^t u_{th}}{V_i^t - V_{i-1}^t}(W_i^t - W_{i-1}^t) + W_{i-1}^t u_{th}, \tag{10}$$

$$V_{i-1}^t \leq v_{rh} \leq V_t^t, \quad t \in T, h \in H, i \in I$$

Where V_{hi}^t and W_{hi}^t are known volumes on the curve and w_{th} is the produced volume corresponding to the discharged volume v_{th}. The binary variable u_{th} is included in the equation to make sure that the production functions also are valid when production is turned off. This is diffrent from the formulation in Fleten and Kristoffersen (2007). The breakpoints are selected so that they cover the entire range of possible values for discharge and production, that is, one breakpoint each for maximum and minimum. To best capture the effects of the efficiency curves on the bidding strategy, one of the breakpoints should be the best-point volume. More breakpoints give a better approximation of the non-linear production functions, but also increase the solution time.

For each station, the sum of the power produced for all turbines in the station is the total production for that station

$$w_{rh} = \sum_{t \in T(R)} w_{th}, \quad h \in H, r \in R \tag{11}$$

The total amount of water discharged at each station is also equal to the sum of water used in each turbine in that station

$$v_{rh} = \sum_{t \in T(R)} v_{th}, \quad h \in H, r \in R \tag{12}$$

5.3.2 Other discharge

There are other forms of discharge from a reservoir than simply production discharge, such as bypass or spill. Bypass is controlled flow of water leaving the reservoir not used for production. Some reservoirs have bypass restrictions so that the river does not run dry or flood. Spill is uncontrolled water flow from a reservoir and happens when the reservoir is over-full. The different discharges may have different watercourses according to the topology of the reservoir cascade, and have the following restrictions

$$V_r^{Bypass,min} \leq v_{hr}^{Bypass} \leq V_r^{Bypass,max}, \quad r \in R, h \in H \tag{13}$$

5.3.3 Reservoir Balance

The reservoirs are modelled as units with constraints on minimum and maximum storage, and a balance equation that connects discharge and inflow to the change in storage level. The storage level at the end of any given hour is the reservoir level at the start of the hour minus the discharge used for production, bypass and spill, plus inflow and water released from upstream reservoirs. The water from upstream reservoirs arrives at the current reservoir after a given time delay. Time delays means that hours are more dependent on each other and hence the degree of freedom when making bid or production allocation decisions is reduced. Inflow to reservoir r in hours h is denoted as r_{rh}.

The reservoir topology may be more complex than just one reservoir directly below the next as in figure 5. A river system can have parallel or crossing river courses and it is therefore not given that the discharge from reservoir r ends up in reservoir $r+1$, as in Figure 7 which is a schematic of Mandalsvassdraget. Hence, a matrix that gives which reservoirs are connected to each other is needed, denoted by C_{rk}. This matrix consists of binary parameters, equal to 1 if there is a direct waterway between reservoir r and k, and zero otherwise. Since there can be different waterways for discharge from production, bypass and spill, we need different connection matrixes. Hence, the reservoir balance equation is

$$\begin{aligned} l_{hr} - l_{h-1r} + \sum_{k \in R} (v_{hk}^{Prod} * C_{rk}^{Prod} + v_{hk}^{Bypass} * C_{rk}^{Bypass} + v_{hk}^{Spill} * C_{rk}^{Spill}) \\ + r_{rh} - v_{hr}^{Prod} - v_{hr}^{Bypass} - v_{hr}^{Spill} = 0, \quad s \in S, r \in R, h \in H \end{aligned} \tag{14}$$

The storage level in the reservoirs can only be regulated within an upper and lower boundary. These boundaries are either set by the physical constraints of the reservoir or by regulations set by the government through the Norwegian Water Resources and Energy Directorate, NVE. Hence, we have

$$L^{min} \leq l_{rh} \leq L_r^{max}, \quad r \in R, h \in H \tag{15}$$

5.3.4 Modelling start-up costs

Start-up costs are included in the model to represent the loss of water connected to start and stops of turbines. It is undesirable to have frequent starts and stops because this causes unnecessary exhaustion of the turbines and requires attention from the operator. A production schedule where the same

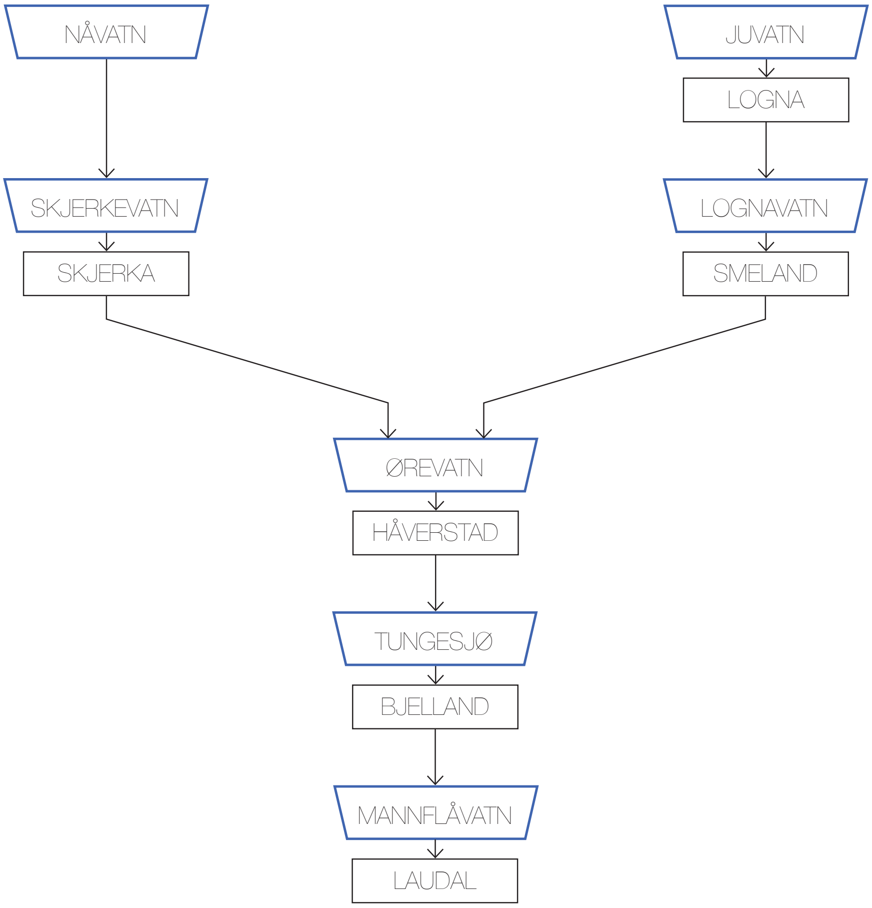

Figure 7: Schematic of the reservoir topology in Mandalsvassdraget.

turbine is turned on and off very frequently is not wanted. To model this a binary variable, u_{th}, for each turbine is created, having the value 1 if the turbine is on and 0 otherwise. To find out if a turbine has changed state from one hour to another, a new binary variable, δ_{th} is introduced. This variable has value 1 if a turbine has started up in hour t, and zero otherwise, according to Equation (16).

$$\delta_{th} \geq u_{th} - u_{th-1}, \quad t \in T, h \in H \tag{16}$$

5.3.5 Modelling the water value

The water value is the marginal opportunity cost of water in the reservoirs, and hence the resource cost of power generation. The water value is a function of future development depending on demand, market prices and inflow. The short-term production scheduling problem is to choose the cheapest reservoir to produce from, and the loss of water value represent this cost. The water value is known from longer term models with a time resolution of one week, as will b explained in Section 8. The water value function is in general non-linear and is therefore approximated by adding cuts to constain the value of water using information from the seasonal model. The cuts are on the form of

$$\nu \leq P_c - \sum_{r=1}^{n} \alpha_{rc}(L_{rc} - l_r) \tag{17}$$

where P_c is the future profits for cut c, α_{rc} is the dual variable for reservoir r in cut c, L_{rc} is the storage level in reservoir r used in cut c and finally l_r is the current storage level. Finally, ν is the value of the water left in the reservoir at the end of the current week. The generation of water value cuts is explained in Section 8.

In addition, we need to give value to the water that is discharged from an upstream reservoir but has not reached the downstream reservoir before the end of the short-term planning horizon, due to time delay between reservoirs. This water needs to be valued to avoid end of horizon effects, which for instance may be that stations are turned off in the last hours of the day to save water in the overlying reservoir, where is has value. If the water in the watercourses at the end has value zero, the optimization will avoid releasing water in the last hours. We let all water in the waterways at the end, that is, production discharge, bypass and spill, have the same value as the water in the reservoir where it is heading.

5.4 Stochastic Model

Thus far, we have presented a general model for the short-term production scheduling problem without uncertainty. In this section the stochastic model is presented in full. A scenario tree is given as input to the model, with each scenario having a known probability of π_s. Then each parameter and variable in the model that may have different realizations in the different scenarios has to be defined over all scenarios in addition to whatever indices they already have. For instance, the committed volume will vary with scenario and is now denoted by y_{sh} instead of just y_h as before. The input parameters that differ in the different scenarios is the scenario price ρ_h and inflow, r_{rh}^{Inflow}. Every variable calculated in the model, y_h, l_{rh}, z_h^+, z_h^-, δ_{th}, u_{th}, w_{rh}, v_{rh}, v_{rh}^{Bypass} and v_{rh}^{Spill} are all dependent on inflow and price, which vary in the different scenarios. Therefore all the variables calculated in the model have to be denoted with s. The stochastic model is thus given by Equations (18) - (36).

$$\max \sum_{s\in S} p_s \Big(\sum_{h\in H} \rho_{sh} y_{sh} + \nu_s^{End} - \mu \sum_{h\in H} (z_{sh}^+ + z_{sh}^-) - \sum_{h\in H} \sum_{t\in T} S_t \delta_{sht} \Big) \quad (18)$$

$$y_{sh} = \frac{\rho_{sh} - P_{i-1}}{P_i - P_{i-1}} x_{ih} + \frac{P_i - \rho_h}{P_i - P_{i-1}} x_{i-1h}, \quad (19)$$

$$P_{i-1} \le \rho_h \le P_i, \quad s \in S, h \in H, i \in I$$

$$x_{ih} \le x_{i+1h}, \quad h \in H, i \in I \setminus \{I\}, \quad (20)$$

$$\sum_{r\in R} w_{shr} - y_{sh} + z_{sh}^+ - z_h^- = 0, \quad s \in S, h \in H \quad (21)$$

$$u_{sht} W_t^{min} \le w_{sht} \le u_{sht} W_t^{max}, \quad s \in S, t \in T, h \in H \quad (22)$$

$$w_{sht} \le \frac{v_{sht} - V_{i-1}^t u_{sht}}{V_i^t - V_{i-1}^t} (W_i^t - W_{i-1}^t) + W_{i-1}^t u_{sht}, \quad (23)$$

$$V_{i-1}^t \le v_{shr} \le V_t^t, \quad s \in S, t \in T, h \in H, i \in I$$

$$w_{srh} = \sum_{t \in T(R)} w_{sht}, \quad s \in S, h \in H, r \in R \tag{24}$$

$$v_{srh} = \sum_{t \in T(R)} v_{sht}, \quad s \in S, h \in H, r \in R \tag{25}$$

$$V_r^{Bypass,min} \leq v_{shr}^{Bypass} \leq V_r^{Bypass,max}, \quad s \in S, r \in R, h \in H \tag{26}$$

$$\begin{aligned} l_{shr} - l_{sh-1r} + \sum_{k \in R} (v_{shk}^{Prod} * C_{rk}^{Prod} + v_{shk}^{Bypass} * C_{rk}^{Bypass} + v_{shk}^{Spill} * C_{rk}^{Spill}) \\ + R_{rh} - v_{shr}^{Prod} - v_{shr}^{Bypass} - v_{shr}^{Spill} = 0, \quad s \in S, r \in R, h \in H \end{aligned} \tag{27}$$

$$L^{min} \leq l_{srh} \leq L_r^{max}, \quad s \in S, t \in T, h \in H \tag{28}$$

$$\delta_{sht} \geq u_{sht} - u_{sh-1t}, \quad s \in S, t \in T, h \in H \tag{29}$$

$$\nu_s \leq P_c - \sum_{r=1}^{n} \alpha_{rc}(L_{rc} - l_{rs}) \tag{30}$$

$$y_{sh} \geq 0, \quad s \in S, h \in H \tag{31}$$

$$x_{hi} \geq 0, \quad h \in H, i \in I \tag{32}$$

$$u_{sth} \in (0,1), \quad s \in S, t \in T, h \in H \tag{33}$$

$$\delta_{sth}, \in (0,1) \quad s \in S, t \in T, h \in H \tag{34}$$

$$z_{sh}^{+} \geq 0, \quad s \in S, h \in H \tag{35}$$

$$z_{sh}^{-} \geq 0, \quad s \in S, h \in H \tag{36}$$

6 Scenario generation

The quality of the solution to a stochastic program depends on how well the uncertainty inherent in the real-world problem is modeled. Integrating too little uncertainty will give too optimistic results, while on the other hand, incorporating too much uncertainty will lead to prohibitively large models, that are either unsolvable, too complex to be informative, or both.

For our approach, uncertainty is represented trough scenario trees, which can be generated by many different methods. A review of methods most commonly used for scenario generation is given in Mitra (2006) or Kaut and Wallace (2003). In addition, Kaut and Wallace (2003) also give quality and suitability measures for different scenario-generation methods. The method we have chosen is only a suggestion for how the scenario generation could be done, and we empathize that the model itself and the input scenarios are independent from each other, as the model can be used with any scenario generation method available.

6.1 Generating the scenario tree

The scenario tree is used to represent how the future prices and inflow develops over time. What we bid in the market for tomorrow is dependent on the balance between expected profits for tomorrow versus expected profits for the rest of the short-term planning period, and the scenarios represent the uncertainty in prices and inflow for tomorrow and the days after. The expected profits after the short-term horizon are represented by the water value. The bid volumes and produced volumes are dependent on the uncertain prices, which again are dependent on uncertain inflow, and hence the profits are themselves uncertain. Our objective is to find the bidding volumes that give the optimal balance between quantity of power produced tomorrow and quantity of water stored for later use.

Short-term scheduling of hydropower production has a time horizon of up to seven days, which is also the longest horizon in our model. As explained in Section 8, the water value is known from the seasonal model for a time step of one week, so the short-term problem deals with production scheduling within the week. If the water values are calculated each Sunday, then the scenario tree needs to cover all possible realizations of prices and inflow for the remaining days until next Sunday. The resulting scenario tree consists of nodes representing different realizations of the future prices and inflow, and one path trough the tree is equivalent to one specific realization of prices and inflow for the rest of the week, which is called a scenario. If scheduling is done for Monday, then all days up to Sunday have to be represented in the tree, and hence the tree will have 7 stages.

The tree structure is needed to represent the information flow inherent in the problem; that is, it represents the timing of when new information becomes available to the decision maker. On any given day, the bidding decisions for the day ahead are dependent on the price realizations of all following days in the week, not just the day one is actually bidding for. This is exactly what the tree represents; it takes into account all future possible realizations of prices for the remaining days of the week, and also the timing of when these prices become known.

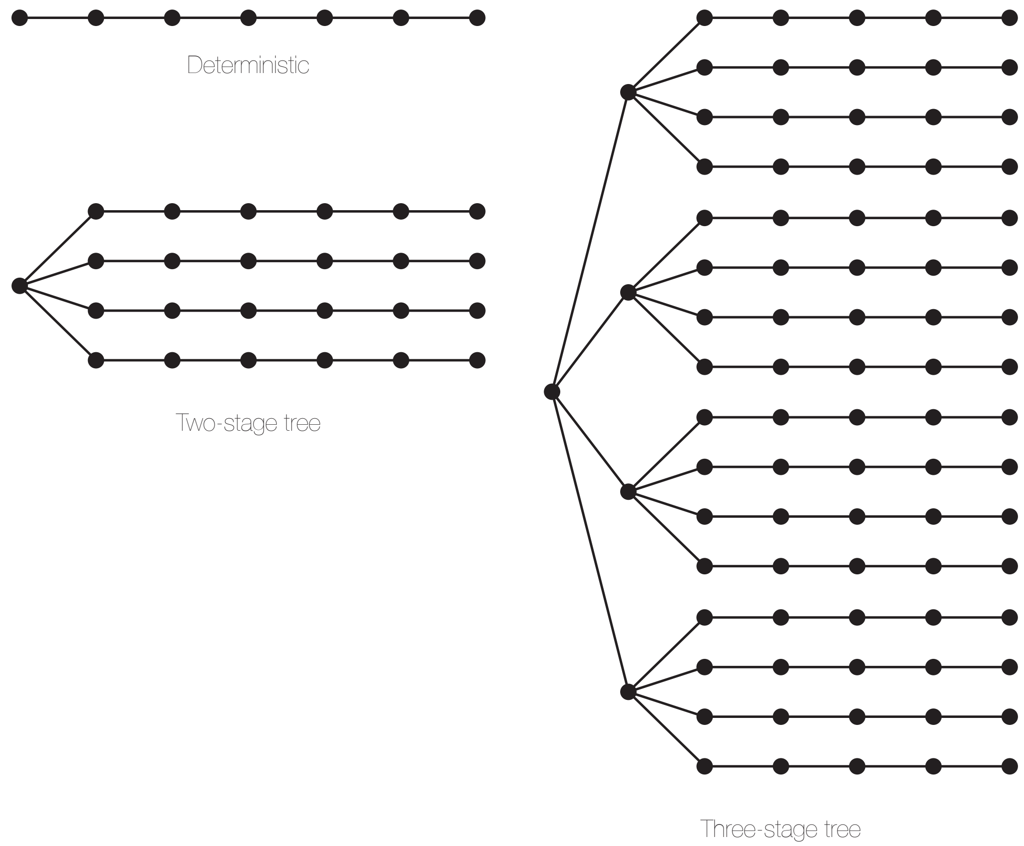

Figure 8: Development of scenario trees with none, one and two stages.

Looking at Figure 8, the scenario tree approach can be explained more thoroughly. First, one individual scenario for prices and inflow in the 7 days to come is shown to the upper left. If uncertainty of prices and inflow is represented in this way, we actually have a deterministic problem since this single scenario is the only one that can occur and hence it has a probability of 1. Down to the left in Figure 8 is shown a scenario tree with 4 different possible realizations for the price and inflow tomorrow. After the price tomorrow becomes known, the prices for the remaining days also become known, and there is no more uncertainty. Hence this is a two-stage tree; we only face uncertainty once while moving in the tree. Finally, the figure to the right shows a tree with three stages and 16 scenarios in total. There are 4 different possible realizations for the price tomorrow, and then there are 4 different realizations for the day after tomorrow. At this point the prices for the remaining days are assumed certain.

The tree to the right in Figure 8 can be expanded to include more stages and more realizations per stage. In our case we need up to 7 stages in the tree, since the water value is known on a weekly time step. If we only have 4 realizations in each stage as in the tree above, we get $4^7 = 16384$ scenarios. This illustrates that the tree structure grows rather quickly, and considering that we need much more than 4 realizations per stage to adequately represent the uncertainty in the real world problem, it is easy to imagine that the problem can become too large to solve.

The tree structure suitable for our model will look different from the trees in Figure 8, and an example of our tree structure is given in Figure 9. The number of realizations in each stage will vary according to the individual input scenarios and the reduction parameter settings in SCENRED. Another feature is that our tree will have several time steps in each stage, representing the 24 hours of a day. The next stage occurs when the market is cleared and the prices for all hours of the next day are revealed. Having up to seven stages means that the tree will become too big for the model to be solved every day. A choice is therefore made of having at most three stochastic days and the rest of the days until the water value is updated modeled with known prices and inflow. This means that once the spot price and inflow is revealed on the third day, then all future prices and inflow also is known, and hence there is no more uncertainty in the tree. This is illustrated in Figure 9 by a scenario tree covering the first three days and then the rest of the days of the week all have the same realization of price and inflow. This can also be seen as a way of adapting the tree structure to the time horizon of the seasonal model, since the value of available water in the last days of the week is conditional on the price realization for these days. In practice, only the first-stage solution, that is, the bid decisions for the first day, will be used and the conditional decisions in later stages are only made in order to find the right values for the first-stage solutions.

In our approach, we generate a scenario tree using a scenario tree generation algorithm called SCENRED, developed by Heitsch and Römisch (2006).

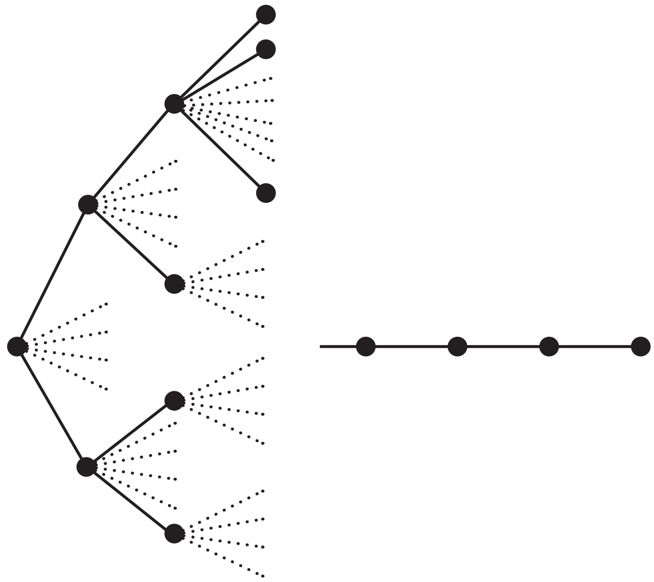

Figure 9: Scenario tree structure used as input to the bid optimization model.

See also GAMS Software (2002) for more information about the algorithm. First, the scenarios for price are combined with scenarios for inflow, which give us a discrete realization of the prices and inflow throughout the planning horizon, but not in the form of a tree where the realizations gradually become known. The SCENRED algorithm takes the individual scenarios as input and generates a scenario tree where new information is revealed at the appropriate time. After the tree is created the algorithm reduces the tree to the minimum number of nodes still needed for the tree to be representative of the statistical properties of the input scenarios.

6.2 Price scenarios

There are different ways to generate scenarios for the future spot price, all with its pros and cons. The current practice among hydropower producers is to create price scenarios by weighting the price forecast by different factors, and thereby produce price percentiles. This method is common because most producers use a deterministic model for optimizing bids, which cannot take crossing price scenarios as input. A stochastic model, however, can deal with scenarios crossing each other; this opens for creating more realistic scenarios.

We have looked at two methods for creating crossing price scenarios; one based on historical data and one based on the physical properties of the power system.

Price scenarios generated from historical data use historical price forecasts and realized prices for the same day. From this a distribution of the forecasting error is calculated; from which random error terms can be drawn and added to the latest forecast as an easy way to get different scenarios as in equation (37).

$$\rho_{sh} = PriceForecast + \epsilon_h \tag{37}$$

Using historical data and the statistical method above we can secure correlations both between hours within a day and between different days by drawing a series of errors spanning over several days. This method requires a thorough collection and analysis of historical errors securing an acceptable error distribution. The advantage of this method is that a producer relatively easy can obtain and analyze its own forecasting errors and produce crossing price scenarios. However, the method creates non-physical scenarios based purely on statistical properties of historical errors. Scenarios with similar statistical properties can be detected by SCENRED, and thus will be reduced to one or just a few scenarios. This leaves a scenario tree not covering the entire span of the input scenarios and so small that it almost can be considered a deterministic scenario.

Price scenarios based on physical properties of the power system are a more lifelike method for producing scenarios. In reality the spot price is dependent on a range of different fundamental events occurring simultaneously. Such events can be different consumption patterns for electricity, bottlenecks in the power grid, different temperature profiles or the state of other generating units in the system. Some of these fundamental events are chosen and a model for the entire power system is run for different combinations of events producing a spot price series for each combination. The advantage of the fundamental method is that each scenario can be tracked back to a specific combination of physical events. The counter-argument is that the method requires a large and complex model that requires resources from the producer or a separate company.

We have chosen to use fundamental scenarios because this is the most realistic representation of the spot price. Because we neither have access to a model that calculates how the spot price is affected by changes in the physical properties of the power system, nor the capacity to make such a model, we have to use price scenarios made by an external company. Our scenarios are obtained from SKM Market Predictor AS for the NO2 area in the period 16.8.2012 to 1.10.2012. SKM produce price scenarios to several producers and retailers in the Nordic power system, and we consider it a strength of our modeling approach that it directly can take input currently used in the industry. The scenarios are a combination of 5 different consumption patterns for electricity, 3 scenarios for flow on the NorNed connection and 3 scenarios of nuclear power production. In addition we have 3 different scenarios for temperature profiles that is matched with a base scenario for transmission on NorNed and nuclear power and 5 different cases of consumption patterns. This gives a total of 60 price scenarios that we match with the inflow scenarios, resulting in the individual scenarios used as input to SCENRED.

6.3 Inflow scenarios

The inflow scenarios are given by Agder Energi. They obtain a forecast of inflow from the ECMWF model, a complex metrological model forecasting the global weather up to 15 days ahead in time. From this forecast, 50 ensemble scenarios are generated using a less complex model. Hence we get 51 scenarios for inflow, 50 ensemble scenarios plus the one base scenario from the ECMWF model. These 51 scenarios are combined with the scenarios for price, making 3060 input scenarios for SCENRED.

6.4 Variable horizon

As mentioned in Section 6.1, short-term scheduling can have a time horizon of up to seven days; in stochastic programming, this corresponds to up to seven stages. In this thesis, the number of stages is chosen to be three as a trade-off between correct modeling of uncertainty and computation time, and hence the scenario tree covers the first three days of the week. For the remaining days, the possible revenues from the spot market is determined by the price realization for these days, which is the same regardless of what scenario occurs for the first days. In this thesis the seasonal model is run every Sunday, producing new water values after the price is reveled midday Sunday. Thus the number of days represented in the model depends on which day the bidding is done for. Bidding for Monday will have a total horizon of 7 days, where 3 days have uncertain prices and inflow and 4 days have known prices and inflow. Bidding for Tuesday will have a horizon of 6 days; 3 days with uncertain prices and 3 days with known prices, and so on throughout the week. Figure 10 shows how the scenario tree looks each day of the week. Even if the tree has three stages, each stage represents several time steps since the prices for all hours of the following day are revealed at the same

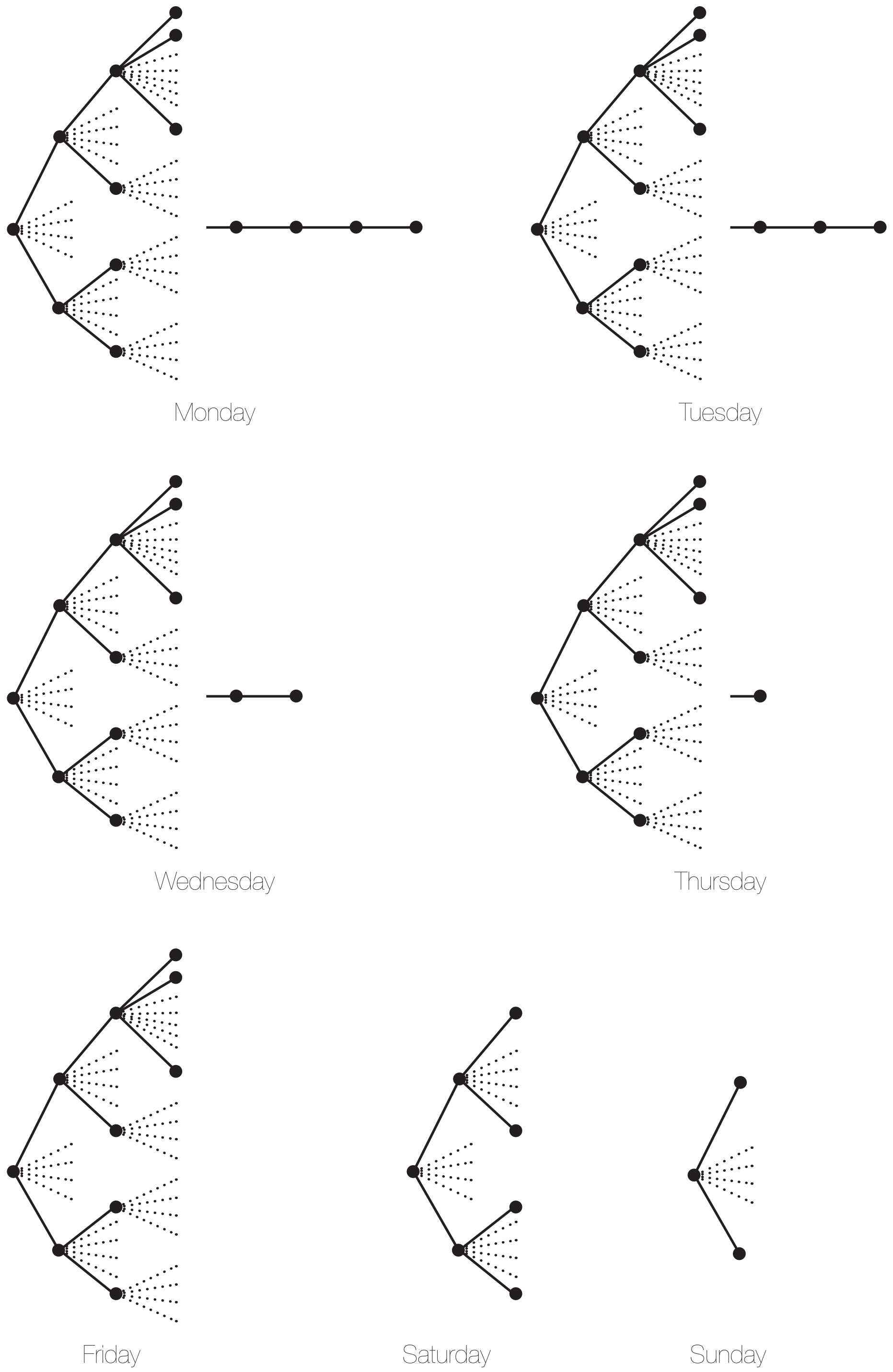

Figure 10: The tree structure and its development over the week as we move closer and closer to the new realization of water values at the end of the week.

time when the market clears. Each node in the tree in Figure 10 therefore represents the price and inflow realization of all the 24 hours in that stage.

6.5 Securing the span of the feasible region of scenarios

When the scenario tree is created and reduced in SCENRED the span of the feasible range of the scenarios may decrease. SCENRED reduces the scenario tree based on the assumption that the input scenarios cover a large area, where the tails of the scenario distribution are extreme cases. A problem occurs when SCENRED tries to reduce the price scenarios from SKM and inflow scenarios from Agder Energi, where the tails of the distribution do not take on extreme values because the scenarios are made by combinations of physical events. Therefore, scenarios that are not extreme are reduced together by SCENRED, and hence we lose the span of the region where the scenarios can occur. To avoid the problem with a too small range we add two extreme scenarios to the scenario tree given by SCENRED. It is most important to maintain the span of scenarios the first 24 hours, since this are the hours where the bids found by the bid optimization model are actually implemented. Hence it is here the results will suffer the most damage if the span of scenarios is too small. The extreme scenarios that are added to the scenario tree are called the minimum and the maximum scenario. The minimum scenario is created by taking the lowest values of price and inflow in each of the 24 first hours and the maximum scenario is created with the highest values.

The disadvantage of the method of adding extreme scenarios is that we get scenarios that are not real scenarios in the sense that they are not made from combinations of physical events. Adding the two scenarios is hence a trade-off between securing the range of scenarios and having scenarios made from real events. We choose to add the extreme scenarios to the scenario tree because the weight, or probability, of these scenarios will be so small that their presence will influence the solution less than what having a too small range would.

Once the extreme scenarios are generated the weight of these has to be decided. The goal when finding the weights is that the kurtosis and mean of the scenario tree should be as close as possible to the same properties before reduction in SCENRED. The probability of the highest scenario after the reduction is split by the factor α, so the new weight of this scenario is $(1-\alpha)$ multiplied with the old probability. α multiplied with the probability of the highest scenario is then used as the weight of the maximum scenario, which is added to the scenario tree as seen in Figure 11. The same is done when finding the weight of the minimum scenario; we split the weight of the lowest scenario after reduction by a factor β. The value of α and β are found by minimizing the difference in kurtosis and mean of the scenario tree before reduction and the new scenario tree with the added extreme scenarios.

In hours 25 to 72 the maximum and minimum scenarios is equal to the highest and lowest scenarios in the scenario tree before adding the extreme scenarios, hence the structure of the scenario tree is preserved in hours 25 to 72 when adding the extreme scenarios. This is indicated in Figure 11, where a copy of the subtree of the highest scenario is added to the new maximum scenario.

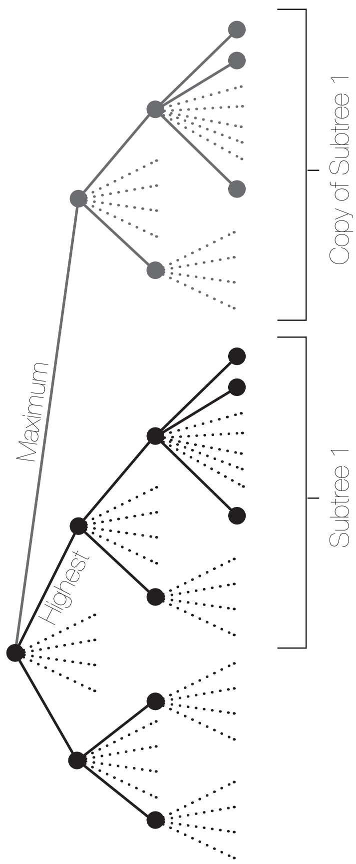

Figure 11: A new extreme scenario is added to the redced tree to secure the span of possible prices.

7 Production Allocation Model

After the bid optimization is done for one day, the bids are sent to Nord Pool and the market is cleared. The spot price and the committed volume for each market participant then become known. It is now up to the producer to assign the committed volume to the specific stations and turbines where it is to be produced. The production allocation model optimizes the unit commitment decisions based on the spot price realization, the water value for the reservoirs and the costs for each turbine. The production allocation model presented here is based on the formulation in Fleten and Kristoffersen (2008) and is equivalent to the currently much used Short-term Hydro Operation Planning (SHOP) model (Flatab, Haugstad, Mo and Fosso, 2002, Fosso, Belsnes, 2004 and Fosso et al, 1999). The production allocation model is based on the same equations and assumptions as the bid model, but with slight modifications. The modelling of the production plan is model presented below.

7.1 Time horizon

The production allocation model has a time horizon of 24 hours. This is due to the fact that even if the bid model takes decisions on the basis of a one-week horizon, only the bids for the first 24 hours are actually implemented. The day-ahead market clears for 24 hours at a time, and it is the bids for these hours that determine the volume to be produced the next day. The spot price is known when the production plan model is run, and following currently used models, we also take inflow to be certain. Hence the production allocation model is a deterministic mixed-integer program.

7.2 Objective function

The objective of the production allocation model is to choose a unit commitment schedule that covers the committed volume while minimizes costs due to start-ups and loss of water value. The production allocation model also has the option of trading in the balancing market, but this is penalized as in the bid model. The reason for this is that we want to fill the spot market obligations from our own generation resources and that the use of the balancing market is regarded as the recourse action that compensates between the volume bid under uncertainty and the truly optimal volume if the spot price had been known in advance. The objective function of the production allocation model is hence

$$\max \sum_{h \in H} \rho_h y_h - \nu^{End} + \mu \sum_{h \in H} (z_h^+ + z_h^-) - \sum_{h \in H} \sum_{t \in T} S_r \delta_{ht} \tag{38}$$

where the symbols have the same meaning as for the bid optimization model.

7.3 Modelling of the production allocation model

The only equation that is different from the bid optimization model presented in Section 5 is the interpolation between price points to find the committed volume. Now, in addition to the price points being known, the bid volumes and spot price are also known. Hence the linear interpolation is now

$$\begin{gathered} y_h = \frac{\rho_h^{Realized} - P_{i-1}}{P_i - P_{i-1}} X_{ih} + \frac{P_i - \rho_h^{Realized}}{P_i - P_{i-1}} X_{i-1h}, \\ P_{i-1} \leq \rho_h \leq P_i, \quad i \in I, h \in H \end{gathered} \tag{39}$$

with y_h as the only variable, and the volume points are taken as the optimal volumes from the bid model.The modelling of the production system follows the same equations as in the bid optimization model, and hence we do not repeat them here but refer to Equations (9) – (17).

7.4 Accounting for head variations

For one of the reservoirs in Mandalsvassdraget, head effects are important for the efficiency curves of the turbines in the belonging station. This non-linearity is not taken into consideration in the optimization models, and hence it is crucial to check if the actual results are feasible for the real situation. This could be done for all reservoirs, but in our case we only have data for head-dependent curves for this one reservoir, so running a simulation for all stations would only increase computational time without any additional insights. However, the importance of a final simulation to verify results still remains, and if necessary the general algorithm explained here for Juvatn could be done for every reservoir where head effects are present.

When the production allocation model is run, a volume for each hour is assigned to Juvatn on the basis of the piecewise-linear production curves used in the production allocation model. These curves do not account for head effects, and may therefore not calculate the correct volume of water released for a given amount of power produced. If the current head is lower than the middle value that is used in the optimization, then more water is actually needed and the real storage level at the end of the day will be lower than calculated. If only the calculated level is used as input for the next day's bidding model, the bid may be too large to be covered by the real reservoir level. A simulation procedure that do account for head effects is therefore run after the production allocation model is finished and the correct reservoir storage level is then sent as input to the bid model for the next day.

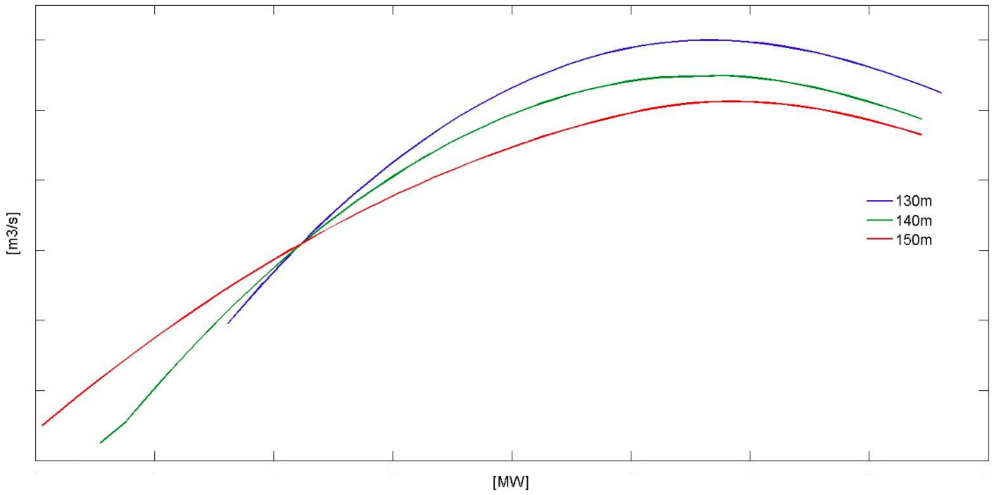

Figure 12: Plot of the production function at Juvatn for different heads.

The simulation procedure takes as input the current level in the reservoir and the committed volume to be produced at Juvatn for all hours of the following day. An interpolation on a curve of known points for storage level and reservoir head is performed, in order to find the present water head. The equation for finding the present head is

$$h_h = \frac{l_h - L_{i-1}}{L_i - L_{i-1}}(H_i - H_{i-1}) + H_{i-1}, \quad L_{i-1} \leq l_h \leq L_i, \quad i \in I, h \in H \quad (40)$$

where (L_i, H_i) are known points on a curve for storage level and head, l_h is the current storage level and h_h is the calculated head.

When the current head is known, the current production curve can be found by interpolation between the known curves for different water heads. Volume-discharge points for the production curves are known for three different heads, so the interpolation is done between the neighbouring points on each side of the current head as in

$$V_i^{new} = \frac{h_h - H_{i-1}}{H_i - H_{i-1}}(V_i^{H_i} - V_{i-1}^{H_{i-1}}) + V_{i-1}, \quad H_{i-1} \leq h_h \leq H_i, \quad i \in I \quad (41)$$

where V_i^{new} is the new volume point corresponding to the current head, h_h. $(V_i^{H_i}, H_i)$ are known points on the production curves for different heads. A corresponding interpolation is also done to find the volume for produced power, W_i^{new}, for the different heads.

Finally, one last interpolation is done to find the actual released volume for the committed generation volume at Juvatn. The committed volume for each hour is known from the production allocation optimization, and the now known head-dependent production curve is used to find the volume of water released.

$$
\begin{aligned}
v_{ht} &= \frac{w_{ht} - W_{i-1}^{new}}{W_i^{new} - W_{i-1}^{new}}(V_i^{new} - V_{i-1}^{new}) + V_{i-1}^{new}, \\
W_{i-1}^{new} &\leq w_{ht} \leq W_i^{new}, \quad i \in I, h \in H
\end{aligned}
\tag{42}
$$

Using the above interpolations, a more realistic value of water released is found for each hour of the operating day, and hence the correct reservoir level can be sent as input to the bid optimization for the next day.

8 Seasonal Model

The main role of the seasonal model is to establish boundary conditions for decisions within the short-term horizon. The time horizon of the seasonal model is up to 18 months, depending on the time of year and topology of the system. With such a long time horizon, uncertainty must be modeled in some way, either through an explicit stochastic model or using dynamic programming. In this thesis, a simplified deterministic seasonal model is used to calculate individual water values for each reservoir, but a more complex model than ours could also be used for the calculation of forecasts of reservoir levels, generation and spillage, or for maintenance planning or risk analysis. More information on seasonal scheduling can be found in Fosso and Belsnes(2004), Mo, Haugstad and Fosso (1997), Gjelsvik(1982) or Røtting and Gjelsvik (1992).

The seasonal model has a time step of one week, since some aggregation is necessary due to the longer time horizon. The resulting output is information about the water values for each reservoir on a time step of one week. This is in line with the time steps and variable horizon of the bidding model, where we at most look at one-week horizon, as explained in Section 6. The water value is the marginal opportunity cost of water in the reservoirs, and hence it is the resource cost of water. This cost must be compared with the spot price in order to determine the optimal bids to NordPool. The water value is hence a key input to the bidding model.

The water value is a function of future development depending on load, market prices and inflow, which all are uncertain. The water value is hence not a deterministic quantity; it changes over the year, with reservoir levels and market conditions. What we call the water value is actually the function of the expected marginal value of the water stored in the reservoirs.

The water value function is both non-linear and uncertain, which leads to a very complicated model. Some approximations are therefore necessary. Specifically, we approximate the water value function by developing cuts to constrain the value of the water left in the reservoir at the end of the week based on the reservoir level at the beginning of the week and installed production capacity. The cuts are made with output from the seasonal model and added as constraints in the bidding and production plan models.

It is challenging to find the correct cuts to value the water stored in the reservoirs. A commonly used method is to use the dual variables of the reservoir balance constraints in the seasonal model (Mo, Haugstad and Fosso, 1997). We assume that the reservoir levels at the end of the seasonal scheduling period are far enough ahead in time to be independent of the current reservoir levels. For instance, we choose the end of the seasonal planning horizon to be in April, so that the reservoir should be nearly empty after the winter to have enough capacity to store all the inflow coming when the snow melts. We do not set the storage level specifically to minimum and let the model choose this level in an optimal way, but end-of-horizon effects will leave the reservoir nearly empty.

The objective of the seasonal model is to maximize profit from selling power in the spot market. It is assumed that all power produced in a given week is sold at the average price that week. It is also assumed that inflow is equal to average inflow for that week. We use historical data for prices obtained from NordPool and historical data for inflow from Agder Energi. With known prices and inflow the model finds the optimal way of using the water available. The profit is bounded by the reservoir balance constraints and the dual values for these constraints will give the shadow price of water; that is, the profit if one more unit of water was available at that time. The dual variables hence gives the cost of using one more unit of water this week compared to saving it for later weeks, and it is therefore the opportunity cost of water. Water – or rain – is basically free, but it is limited by its availability, and hence the opportunity cost of using water is the true resource cost of hydropower generation.

The seasonal model used in this thesis is based on the same equations as the bidding or production plan model presented earlier, so we do not repeat the mathematical presentation. The most important differences are that the time step is changed from hours to weeks, and that all power is sold to the average price. Hence the model involves no bidding and no balancing market, and can be roughly explained as finding the optimal production schedule on a weekly basis when generations is constrained by efficiency curves, reservoir balances and generation capacities. The dual variables of the balance constraints and the expected future profits are the outputs that are used to generate the cuts, as in Equation (43) for one reservoir.

$$\nu \leq P_c - \alpha_c(L_c - l) \tag{43}$$

P_c is the future profits for cut c, α_c is the dual variable for the reservoir in cut c, L_c is the storage level used in cut c and finally l_r is the current storage level. ν is the value of the water left in the reservoir at the end of the current week.

The water value function gets even more complex with a cascaded river system as Mandalvassdraget. The water value in one reservoir is dependent on the amount of water in all the other reservoirs in the system. A cut is therefore defined by a combination of reservoir levels, and the equation above needs to be changed to

$$\nu \leq P_c - \sum_{r \in R} \alpha_{rc}(L_{rc} - l_r) \quad (44)$$

Where:
ν : Future profit [€]
P_c: Future income for cut c [€]
α_{rc}: Dual values for reservoir r in cut c [€/kWh]
L_{rc}: Initial reservoir level for reservoir r in cut c [kWh]
l_r: Reservoir level in reservoir r [m^3]

Figure 13 illustrates the water value function for a system with two reservoirs. Mandalsvassdraget has seven reservoirs, which makes it difficult to represent the water value function graphically.

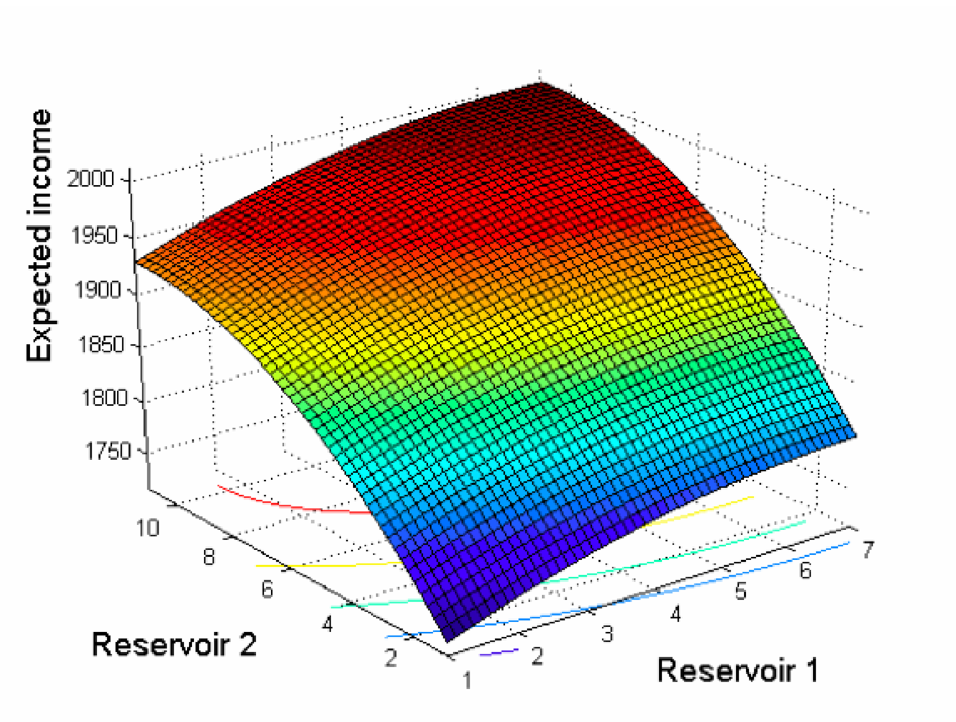

Figure 13: Plot of water value function for two reservoirs.

Table 2: Cuts used for water value approximation

Cut	Name	Production schedule	Inflow
1	Maximum	All turbines at maximum	Average
2	Minimum	All turbines at minimum	Zero
3	Left Side	West side at maximum	Average
4	Right Side	East side at maximum	Average
5	Lower	3 lower stations at maximum	Average
6	Upper Left	2 west stations at maximum	Average
7	Upper Right	2 east stations at maximum	Average
8	Best point	All turbines at best-point	Average
9	Large Inflow	All turbines at minimum	High

From Figure 13, we see that the relation between reservoir levels and future profits is increasing, since higher initial reservoir levels give possibilities for more profits in the long run. The relation is also concave, since the more water available, the lower the average price will be. The water value is approximated by this non-linear function. We need to generate cuts so that the water value function in the neighborhood of the current reservoir level is well represented, while the approximation could be coarser for reservoir levels far from the current level to keep the number of cuts and hence the size of the model as small as possible. The neighborhood where the water value function needs to be well approximated is defined by the weekly production capacity since this is the maximum change that can happen from the initial reservoir level. For instance, if the maximum production capacity for one of the reservoirs corresponds to releasing 200.000 m^3 of water this week, than this is the maximum the reservoir level can change. The reservoir level could also increase with maximum weekly inflow and no generation.

To generate cuts, we could let the reservoir level in each reservoir vary with for instance minimum generation, maximum generation, generation at best point, average inflow, no inflow, large inflow or any combination of these. Unfortunately, for a seven-reservoir system as Mandalsvassdraget, the cut generation is severely affected by the curse of dimensionality since all reservoirs change simultaneously. We therefore chose an approach were the cuts are generated on the basis of some commonly occurring production schedule due to the topology of the reservoir cascade. For instance we generate a cut consisting of all reservoirs producing at maximum capacity, or by letting only the west or east side produce while the other side is turned off. This gives us a manageable number of cuts as shown in Table 2.

A question arises as to how many cuts are necessary to approximate the water value function. Are the cuts listed in Table 2 adequate and by what measure is this to be decided? As shown in the analysis of how many cuts are necessary in Appendix C, the all nine different cuts result in a nearly linear segment for all reservoirs. The difference in slope of the cuts are marginal, and hence it would suffice to use just the most extreme cuts in the optimization models since this creates a lower bound for the water value. How many cuts are necessary depends on the cascade topology. Our implementation of the seasonal model is deterministic and linear, so in terms of computational time, one cut does not take long to execute by itself, but each cut means an extra constraint in the stochastic mixed-integer bidding model.

A drawback of our seasonal model is that it is deterministic, and that the prices and inflow used are historical data for the actual development of prices and inflow in 2012 and 2013. The actual development of price and inflow over the seasonal horizon is shown in Figures 14 and 15. This information was not available for the production schedulers at Agder Energi at the time of operations, and hence our water values are not comparable to the water values used by Agder Energi. The reservoirs may therefore be managed diffrently than they would have been under actual conditions. Because we do not face any unceratinty in prices or inflow in the long run we have no reason to manage our reservoirs moderately in anticipation of future adverse developments. Our water value is perfectly aligned with the actual future development, and hence water is released in an optimal way which only could have been found in hindsight. Remodelling of the seasonal model in terms of incorporating uncertainty would be beyond the scope of this text, but a small improvement could be made by using prices for forward contracts instead of the historical price.

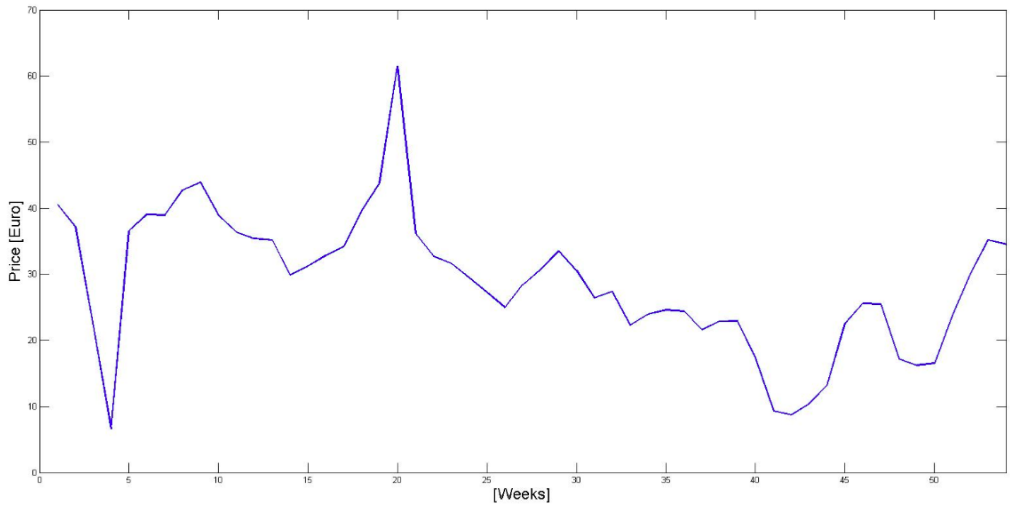

Figure 14: Plot of the prices for the NO2 area from 12.Aug 2012 - 1.April 2013.

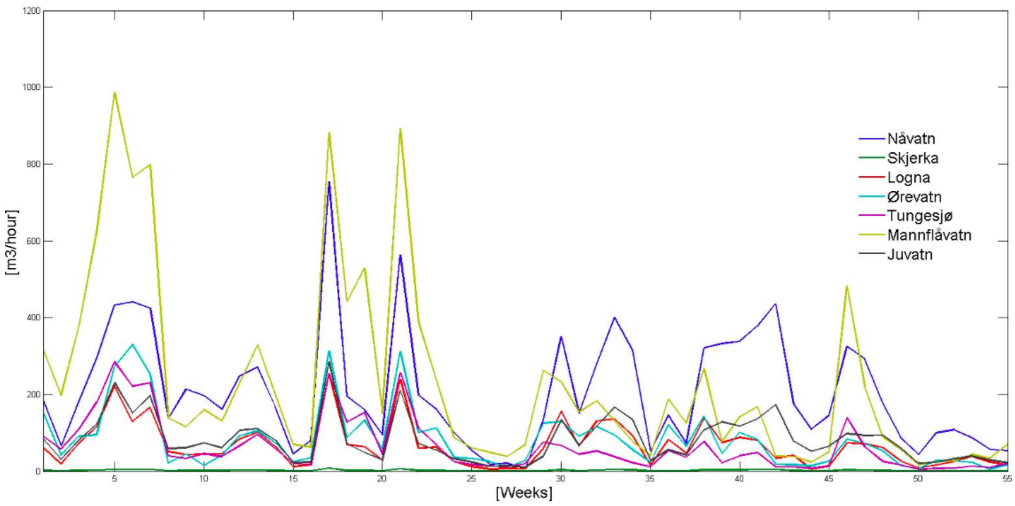

Figure 15: Plot of the local inflow from 12.Aug 2012 -1 april.2013.

9 Case study

The simulation procedure presented so far is implemented for Mandalsvassdraget, a large Norwegian hydro system owned by Agder Energi. The system contains twelve reservoirs and six power plants. When the system is modeled some of the reservoirs, namely Storevatn, Kværnevatn, Langevatn, Stegil and Nåvatn are accumulated into one big reservoir called Nåvatn. The reason for doing this is that the reservoirs above Nåvatn are small and have a low degree of regulation, which means that the water contained in these reservoirs will end up in Nåvatn within a short period of time. Also, Agder Energi has plans to build a bigger dam at Nåvatn, and demolish the dams of the overlaying reservoirs (NVE, 2011). A simple representation of the topology of Mandalsvassdraget is found in Figure 7 in Section 5. The system also includes other smaller river courses that have their outlets in Mandalsvassdraget and these are included as inflow in the model.

All simulations are carried out over the same period of time and covers 46 days in late summer and early fall from 16. August 2012 til 30. September 2012. This period is chosen due to availability of data from Agder Energi. The seasonal model used for all simulations has a time horizon of 34 weeks, that is, until april 2013.

The stochastic bid optimization model is compared to a deterministic model that uses the same bidding method as Agder Energi applies today. The deterministic model is tested with two sets of input developed from different methods of price forecasting; one set of Agder Energi's own forecast and one set of forecasts obtained from SKM Market Predictor AS. In addition, a simulation were the stochastic bid model allows for block bids is run, as well as a simulation of the stochastic model with only uncertainty in prices. Finally, the stochastic model is implemented for the same river cascade without time delays in the watercourses.

The test runs will first be presented on their own before a general comparison of the main results is done in Section 10. The stochastic model simulation is presented in Section 9.1, the two deterministic runs in Section 9.2, the block bid simulation in Section 9.3, the stochastic model with only price uncertainty in Section 9.4 and finally both the stochastic model and the deterministic model simulation without time delays in Section 9.5.

9.1 Simulation with the stochastic bid optimization model

The stochastic bid optimization model is presented in full in Section 5 and the simulation procedure follows the flow chart of Section 4. The main output from the simulation is the obtained average price per MWh produced and how the reservoirs are managed.

9.1.1 Results from the stochastic model

The average price per MWh is a measure of performance for the short-term scheduling model and shows how improvements in modelling may increase the price at which a producer can sell its power. The obtained average price is the sum of all spot market revenues during the simulated period divided by the total produced volume over the same period. Even though use of the balancing market is penalized in the bid optimization model as explained in Section 5, the producer may make a profit by participating in the balancing market. When income from the market is calculated, real historical costs for up regulation and profits from down regulation is included in the measure.

The obtained average price may not be an adequate measure of performance, as it does not account for operational costs. The costs of hydropower are related to start-ups of turbines and loss of water value. Hence another measure of performance is the sum of total obtained profits over the simulated period and the total value of the water left in the reservoirs. This is a measure of the total value of water used in the simulation period and water saved for later use. In this thesis, this is referred to as the total value.

Another measure of performance is defined here as odd starts, and is related to start-up costs. The hydropower producer incurs costs when turbines are turned on or off. All start-ups have associated costs, but it is particularly undesirable to have very frequent starts and stops of the turbines, as this causes tear on the turbine, higher risk of damage and more attention from the operator as explained in Section 5. We define a measure for these unwanted starts as odd starts where we record the number of times a given turbine is started and then stopped again after only one or two hours, and then add these numbers for all turbines in the system. A low number of odd starts is an indicator of a realistic and implementable production plan.

Finally, the managing of reservoir is important. The measure of total value is a trade-off between using water now and saving it for later, since releasing water and producing at high prices now means that less water will be available for production later. Mandalsvassdraget as we model it here has seven reservoirs, where Nåvatn and Juvatn are the largest and have the highest degree of regulation. These reservoirs are used for storing water over seasons, and we call this long-term reservoir management. The other reservoirs have a small degree of regulation and can be emptied or filled within the week and hence we refer to their management as short-term reservoir management.

The aforementioned measures of performance are the main emphasis of the comparison between models in Section 10.

9.2 Simulation with the deterministic multi scenario bid optimization model

To evaluate the stochastic model it is compared with results from a model similar to what Agder Energi currently use for bid optimization. As mentioned in Section 1, Agder Energi, like most producers, use a deterministic model with scenario analysis, where they run a deterministic model with different scenarios for price and linearize between the results from the different scenarios to create a bid matrix. As Agder Energi we use 9 different price scenarios; the forecasted price itself plus the forecasted price scaled by eight different weights. The result from each deterministic run is a value for optimal production given the deterministic price for each of the 24 hours of the following operating day. With nine deterministic runs this results in a $24 * 9$ matrix that is used as a bid matrix to Nord Pool. The problem with this matrix is that we cannot be certain that the bid volumes are increasing for increasing price points, as is required by NordPool.

Producers commonly solve this problem using heuristics; solving the deterministic problem several times with restrictions of increasing bid volumes. In our model we include a new restriction stating that the bid for a given hour and price point has to be higher or equal to the bid given for all lower price points. We start with the lowest price scenario, with no restriction on the bids, and iterate over the increasing price scenarios taking the bids from the previous price as input. This gives a bid matrix were legal bids are secured.

9.2.1 Two sets of input

Another feature of the deterministic test run is that we test the deterministic model for two types of input. First, we use the actual forecasted prices developed and used by Agder Energi and weights of this as explained above. In addition, we also use price forecasts from SKM and the same weighting of this. The initial forecast is then the SKM scenario with average values for consumption, nuclear power and transmission on the NorNed connection, see Section 6.

The specific weights are found in Table 3, and are the same as currently used by Agder Energi. This method of simply scaling the price profile may not give very realistic results, since the rate of change may be dependent on the overall price level. For instance, if the overall price level is moderate the

Table 3: Weight factors used in making the deterministic scenarios

Scenario	1	2	3	4	5	6	7	8	9
Percent	-17	-9	-6	-3	0	+3	+6	+9	+17

morning peak also rises only moderately, while if the overall level is high it may happen that the morning peak spikes to very high levels. These and similar effects will never be captured by the scaling method, because the profile and hence the rate of change is the same for all scenarios. One could solve this by letting the weights be dependent on price level or time of day. An alternative would be to use deterministic scenarios with different profiles, but as the current model cannot take crossing scenarios, the only valid alternative is to use a stochastic model. How to better decide the weights is not tested in this thesis, as we use the same weights for both runs of the deterministic model.

The choice of having two sets of input to the deterministic model is made for two reasons. First, we wanted deeper insight into current practice at Agder Energi, and hence we use their forecasted prices. Second, for the stochastic model, scenarios based on fundamental events were preferred over purely statistically generated scenarios. Then, for fair comparison, the deterministic model also has to be tested with the fundamental scenarios as input. The different sets of input scenarios are used to test if there is any gain from switching price forecasting technique and if the stochastic model is truly better or if it just has more accurate input.

9.2.2 Formulating the deterministic model

The deterministic simulation procedure follows the same simulation flow chart presented in Section 4 for the stochastic case, but some algorithms are unnecessary. The deterministic model does not require any scenario tree as input, so all the programs related to the tree are replaced by algorithms that construct suitable deterministic input. The other programs follow the same simulation procedure.

The deterministic bid optimization model roughly follows the formulation of the stochastic model, with the modifications presented below. The objective function is given as

$$\max \sum_{h \in H} \rho_h y_h + \nu^{End} - \sum_{h \in H} \sum_{t \in T} S_t \delta_{tr} \tag{45}$$

The objective function is the same as for the stochastic model, but without the scenario indexes and without the penalty for using the balancing market. The deterministic bid optimization model will never use the balancing market since it sees the future prices with certainty, and thus the model will always give a solution were committed volume is the same as the produced volume. The production plan model, however, still has the possibility to use the balancing market since this may be needed when the realized price differs from the prices used in the bid model.

Since the deterministic model finds the optimal volume to produce given a known market price there is no use for actual modeling of the bids. Hence, Equations (5) and (6) are not necessary in the deterministic model. The reservoir balance, and production and startup of turbines are the same as in the stochastic model, and are not repeated here.

As mentioned the balancing market is not used in the deterministic model, hence the sum of produced volume in all stations has to be equal to the committed volume

$$\sum_{r \in R} w_{hr} - y_h = 0, \quad h \in H \tag{46}$$

The new restriction used to secure increasing bids is given as

$$y_h^j \geq Y_h^{j-1}, \quad h \in H \tag{47}$$

Where the restriction bid matrix, Y_h^j, in the first iteration is zero for all 24 hours, and is updated with the committed volume for each hour when the deterministic model is run for j increasing price points.

9.2.3 Results from the deterministic model

For the deterministic test runs we use the same measures of performance as explained for the stochastic model. That is, the long- and short-term management of reservoirs, the obtained average price and the total value are most important. In addition, we also look at odd starts.

9.3 Simulation with the stochastic model using block bids

In addition to only using hourly bids we test how block bids affects the results of the stochastic model. Block bids are bids that span over at least two consecutive hours. A block bid is bid with an all-or-nothing condition, that is; a bid is to be accepted as a whole or not accepted at all. A block is accepted if the average spot price in the hours of the block is lower than the bidding price of the block. We assume that the producer is a price taker and thus the spot price will not be affected by the amount bid in either hourly nor block bids. Hence we avoid the problem regarding paradoxically rejected blocks (Meeus, 2006). A paradoxically rejected block is when a given block with block bid price lower then the spot price is rejected because accepting the block would cause the spot price to decrease below the bid block price. Such issues can occur in markets where the spot price depends on the bids given, for instance if one of the market participants has market power.

Block bids are used in day-ahead market bidding as a way of solving the problem of making decisions between hours that are dependent on each other due to for instance time delay in the watercourses between reservoirs. It is undesirable to have rapid changes in production even though the spot price can change rapidly from hour to hour. One way of handling this is to have a cost related to the start-up of new turbines, which has already been implemented in the bid optimization model. Another embellishment is to include the possibility of block bids as is done here. If the price peaks in one of the hours in the block, the bid will not result in a peak in the produced volume. Hence the use of block bids can give a more stable production plan when the spot price has high peaks during the day. Another reason for using blocks is that the criterion for acceptance of a block is related to the average price over the hours of the block, which is less volatile then the price in one single hour.

9.3.1 Formulating the stochastic model with block bids

When modeling block bids we include a new set of parameters $B = \{b_1 \dots b_B\}$ where b_j is a given block covering some predetermined hours. Each block bid is bid at bid price P_i, which are the same price points used for hourly bids. For each price point, a volume bid is given for the block and if the average price over the hours of the block is lower than the spot price in these hours, the block volume is accepted.

$$y_b = \sum_{i \in I | P_i \leq \rho_{h \in b}} x_{bi}, \quad i \in I, b \in B \tag{48}$$

where y_b is the block committed volume, x_{bi} is the volume bidden for block b at price point i and $\rho_{h \in b}$ is the average spot price over the hours of block b. As the equation above states, the committed volume from block bids is the sum over the volumes bid at all price points where the bid price is lower than the average spot price. The volume bid at each price points is therefore the incremental volume between two price points.

For all hours within a block, the bid volume from block bids for each hour is equal to the sum of the volume bid for all blocks on lower price points.

$$x_h^{Block} = \sum_{i \in I, b \in B | h \in b} x_{bi}, \quad h \in H \tag{49}$$

The total committed volume in each hour is the volume committed from hourly bids and block bids in that hour

$$y_h^{Total} = y_h + y_h^{Block} \tag{50}$$

This total volume is sold to the spot market price and hence the income is

$$\sum_{h \in H} \rho_h y_h^{Total} \tag{51}$$

which is part of the objective function, Equation (18). The total committed volume is also used in the equation for finding the balancing volumes, (21), since total production has to cover all our commitments. In addition, for every hour, the volume bid as hourly bids and the volume bid as block bids cannot be more than the maximum production capacity

$$x_h^{Block} + x_{hi} \leq \sum_{t \in T} W_t^{Max}, \quad h \in H \tag{52}$$

In the stochastic model the block bid volume x_{bi}, the committed volume from block bids y_h^{Block} and the average spot price $\rho_{h \in b}$ are scenario-dependent, and thus also has to be indexed with $s \in S$, where S is the set of all scenarios.

9.3.2 Including block bids in the study case

Before running the simulation with the possibility of block bids the set of block needs to be determined. A block has to span over at least 3 hours, and for a 24-hour horizon this means that there are 253 possible blocks. Because each new block corresponds to a new variable in the bid optimization problem the number of blocks has to be limited. We chose one block for each of the following set of hours; morning, midday, afternoon, evening and night. Hence:

$$B = 5,$$
$$b_1 = \{1 \ldots 6\}, b_2 = \{7 \ldots 10\}, b_3 = \{11 \ldots 14\}, b_4 = \{15 \ldots 18\}, b_5 = \{19 \ldots 24\}$$

The model formulation opens for crossing blocks, that is; one single hour could be included in several blocks. This is not implemented at present. The performance of the model would be improved if the optimization model also chose the combination of blocks (Alnæs et al, 2013).

The block bid simulation procedure follows the exact same flow chart as the stochastic model, see Section 4 with the only addition that the stochastic bid optimization model opens for block bids as explained in Equations (48) – (52).

9.3.3 Results from stochastic model with block bids

From the simulation with block bids we look at obtained average price, but the main effort is concerned with the number of odd starts. The purpose of block bids is to lock in a more stable production schedule, which means less frequent starts and stops. In addition, we look at how much block bids are used, and if this has any additional benefits in terms of obtained average price or total value.

When block bids are allowed, a substantial part, about 65%, of the total volume is committed as blocks. This is more than the about 20 % which is usual in the industry and sometimes leaves us with committments that are difficult to cover.

9.4 Simulation with the stochastic model having only price uncertainty

The stochastic model presented so far face uncertainty in both prices and inflow. We want to investigate what effects the uncertainty in inflow actually has, in particular when it comes to reservoir management. With uncertain inflow, the reservoirs should be handled more moderately and not tend towards the reservoir boundaries. If the reservoir level is high or maximum over a long period of time there is increased risk of spillage. Spillage is equivalent with loss and should therefore be limited. A stochastic representation of inflow may lead to a better reservoir management, and we want to investigate this effect separate from the result when having uncertainty in both prices and inflow. Thus, we implement a test run where the stochastic model only has uncertain prices.

The simulation procedure follows the same flow chart as in Section 4 and the scenario tree structure is not altered other than the fact that every scenario in the tree now has the same scenario for inflow. The probability of the scenarios adds to one, and hence the one inflow realization is in principle deterministic.

9.4.1 Results from using only uncertain prices

From this simulation we particularly look at the short-term reservoir management and spillage. The obtained average price and the total value are also used in the comparison.

9.5 Simulation without time delays

To investigate how the complexity of the river cascade affects the results, the stochastic model is implemented for the same reservoir topology as Mandalsvassdraget, but without time delays in the watercourses between reservoirs. Time delay is the time it takes the water that is released from an upstream reservoir to reach the downstream reservoir. Having time delays means that hours are more strongly dependent on each other, because large discharged volumes from upstream reservoirs in high priced hours may force production in downstream reservoirs a few hours later when the price is lower. An extreme case would be that the downstream reservoirs are flooded and water is lost. These dependencies must be taken into consideration when the bids and the following production allocation are determined.

Our hypothesis is that a stochastic model is even more applicable when the reservoir topology is complex. With a complex river system, and hours being more dependent on each other as explained above, it is crucial that the bid decisions are flexible enough to account for possible adverse developments.

We run both the stochastic model with uncertain price and inflow and the deterministic model with prices from SKM without time delays. This is because these are the two runs that are most fairly compared since they use the same input. Both simulation runs follows exactly the same program flow as explained in Section 9.1 for the stochastic model and in Section 9.2 for the deterministic model, with the only difference that the values for time delays in the different water courses is set to zero.

9.5.1 Results without time delays

From this simulation run we compare the obtained average profit and the total value for the stochastic and deterministic run without time delays. Without time delays, it is expected that the difference between the stochastic and the deterministic model is smaller since the system is less constrained.

10 Results

In this section, the main results from the different simulations presented in Setion 9 is compared to each other and explained.

10.1 Average price and total value

A measure of performance of the bid model is the obtained average price the model achieves when bidding in the spot market. As explained in Section 3 the average price is found as the sum of total revenue from the spot and balancing market divided by the total produced volume. Table 4 shows the results for obtained average price for some of the simulation runs done in this case study.

The main result is to compare the average price obtained by the stochastic model with the average price obtained with the two runs of the deterministic model. The improvement in average price over the entire simulation period with the stochastic model is 0,69% and 4,39% compared to the deterministic model with prices from SKM and Agder Energi, respectively. A comparison of the obtained average profits each week can be found in appendix A.

A high average price indicates a good fit between the spot price and how production is allocated; that is, production is allocated to the hours where the price is at its highest. To get a good fit between volume produced and the spot price profile the model needs to have a good representation the prices for tomorrow and the days to come. Because the stochastic model has a more accurate description of the future spot price than the deterministic model using scenario analysis, it is as expected that the stochastic model achieves a higher average price. Figures 16, 17 and 18 shows the fit between produced volume and the spot price for the stochastic model and the deterministic model with prices from both SKM and Agder Energi.

As we see in the figures, all three models give a reasonably good fit between the produced volume and the price, with the stochastic model slightly better. Our simulations may give an unrealistically good fit between price and production, due to the fact that our models use water values from a deterministic seasonal model that is based on historical prices and inflow. Hence, in principle we make decisions based on information that is not available at the time of operations. However, all of the models use the same water value, so the result of a better fit between prices and production by the stochastic model remains valid. This means that the stochastic model better exploits the high-price hours and release more water during these hours.

Table 4: Results for the obtained average price per MWh for the different models.

	Stochastic	Deterministic SKM	Deterministic AE	Block bids	Deterministic inflow
Euro	23.084	22.925	22.070	21.281	22.939
Percent		-0.686	-4.391	-7.814	-0.627

Table 5: Results for the total value of production and water left in the reservoirs.

	Stochastic	Deterministic SKM	Deterministic AE	Block bids	Deterministic inflow
Euro	14442784	14354771	14322764	13690314	14363333
Percent		-0.61	-0.83	-5.21	-0.55

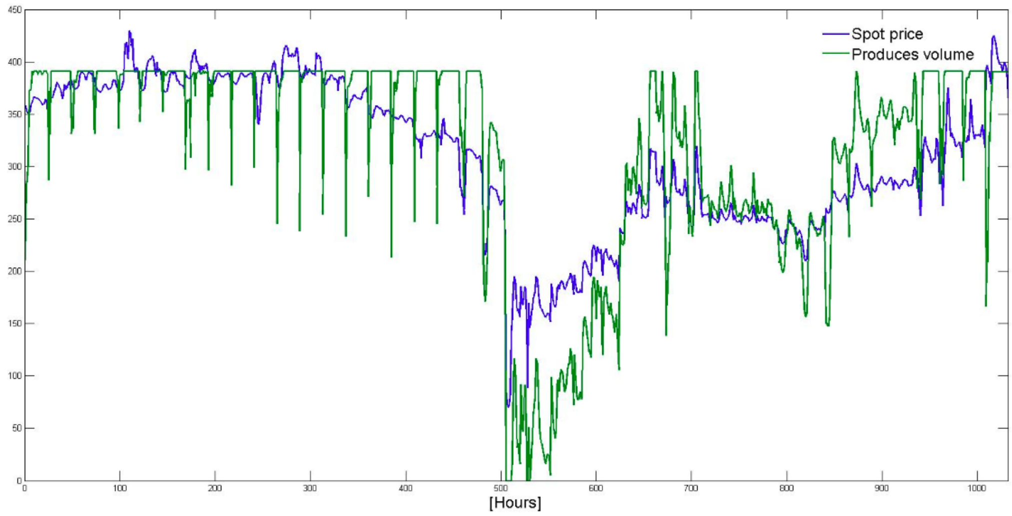

Figure 16: Plot of prices and production for the stochastic model.

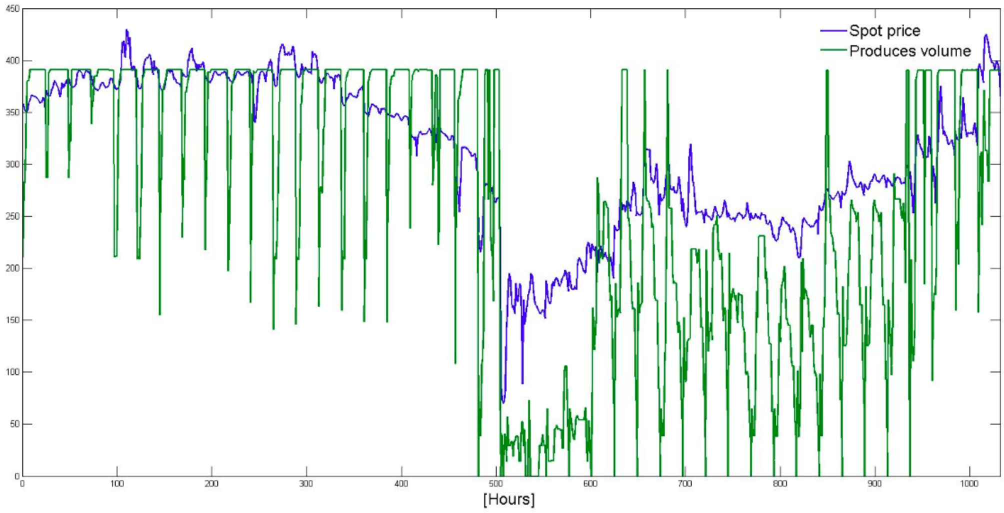

Figure 17: Plot of prices and production for the deterministic model with prices from SKM.

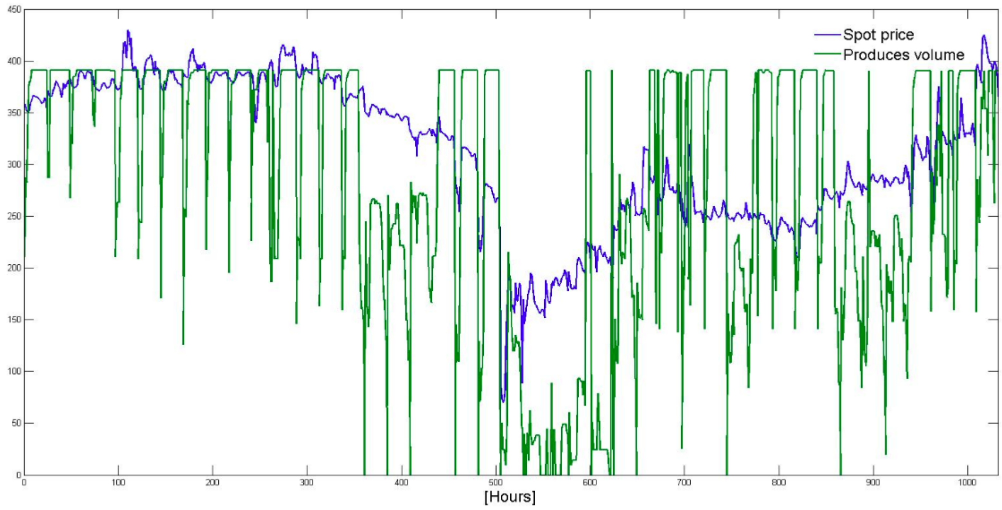

Figure 18: Plot of prices and production for the deterministic model with prices from Agder Energi.

Although the obtained average price over the simulation period is a common measure of model performance, the amount of water used and the value of stored water needs to be taken into account. A model can achieve a very high average price if it chooses to produce with full capacity in hours with high price and not care about the value of water left in the reservoirs after the simulation period. As mentioned in Section 9 the obtained average price may not be an adequate measure of performance, as it does not account for the operational costs and loss of water value. Hence we use another measure of performance, namely the total value, which is the sum of total profits obtained over the simulation period and the value of the water left in the reservoirs at the end of the period. The total value for the different runs is given in Table 5.

In Table 5 we see that the stochastic model has a total value that is 0,61% higher than the deterministic model using prices from SKM and 0,83% higher than the deterministic model using prices from Agder Energi. The stochastic model uses more water than the deterministic models and hence has a lower value of the water left in the reservoirs, but as a total achieves a higher total value.

The run with the stochastic model with deterministic inflow achieves a 0,67% lower average price and a 0,55% lower total value than the stochastic model. This is due to the deterministic description of inflow, which leads to somewhat more spill than the model with stochastic inflow, as described in Section 10.3. The model also achieves a lower total value since spill equals loss of water value.

When we include the possibility of block bids in the stochastic model, the result is a significant lower average price and total value. As mentioned in Section 9.3, about 65% of our total produced volume is bid as block bids when this is allowed. This locks in a stable production schedule, but leaves little room for exploiting high price hours resulting in a lower average price. The reason that such a large volume is bid as blocks is that the model with block bid makes decisions that are too adapted to specific high probability scenarios, which has adverse effects when this scenario does not occur.

10.2 Long-term reservoir management

How the reservoirs are managed is important for the validation of the obtained average price and other results. The reservoirs should not reach undesirable or unrealistic levels, and for the reservoirs with a higher degree of regulation the ability to store water over seasons should be maintained. For Mandalsvassdraget, Nåvatn and Juvatn have the highest degrees of regulation, and have enough capacity to cover several weeks of full production.

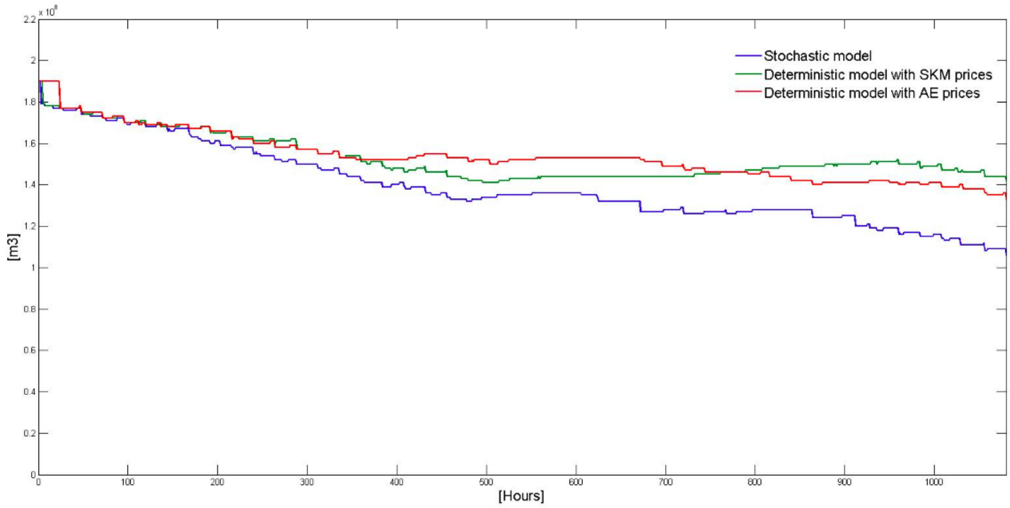

Figure 19: Plot of reservoir level at Nåvatn over the simulation period.

Figures 19 and 20 show how the level in Nåvatn and Juvatn develop over the simulated weeks. From these reservoirs, quite large amounts of water are drawn and this might not be realistic for the time of year as the reservoirs should be quite full before the winter sets in. When our simulation procedure ends in September, there may not come enough inflow to fill the reservoirs before the winter season when inflow comes as snow and electricity consumption and therefore also prices are at their highest. For reservoirs with a high degree of regulation, we expect the reservoir levels to follow a yearly profile with a filling season during summer and fall, and a drawdown season during winter until the reservoirs are nearly empty before the spring flood. The weekly water value given by the seasonal model should represent the value of storing water over seasons; for instance, the water should be more worth saving in the fall in expectation of higher prices and less inflow in the winter months, than in the spring when large inflows are expected.

Nåvatn and Juvatn do not show the behaviour just explained following a yearly profile. This is mainly due to the simplified seasonal model used in the simulation, which give unrealistically good water values. The water values we use are based on perfect information of future development over the seasonal horizon, and this is information that is not available to Agder Energi or any other hydropower producer at the time of production scheduling. A plot of the real historical prices used in the seasonal model is shown is Figure 14 in Section 8 along with the historical weekly average inflow in Figure 15.

Knowing inflow and prices, the seasonal model correctly allocates water to the highest priced weeks trough the water value.

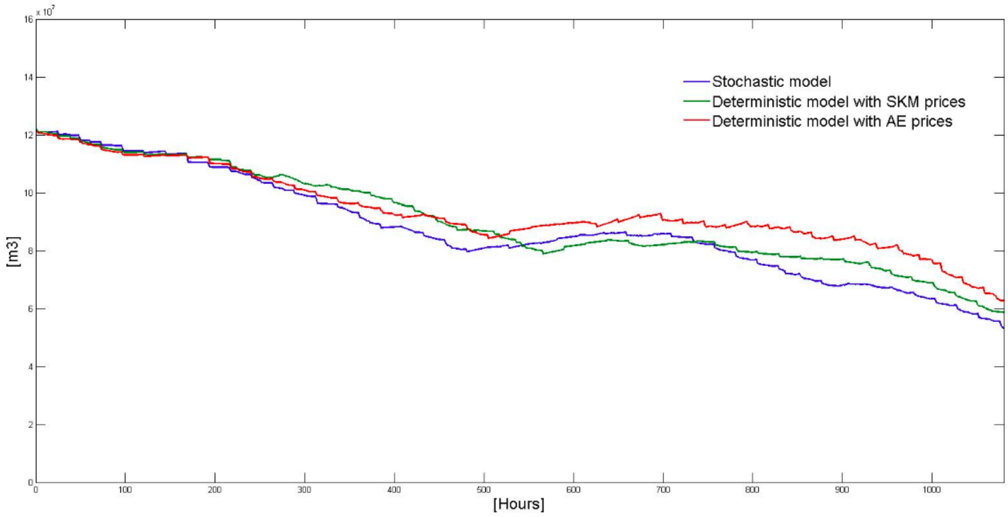

Figure 20: Plot of reservoir level at Juvatn over the simulation period.

The seasonal price has a high peak around week 20, and then decreases towards spring. Taking this into account, as long as there is enough water to cover full production in the peak weeks, it is actually better to release water in the first few weeks of the seasonal horizon, since besides the dip around week 5 prices are actually higher here, and this is the reason why Nåvatn and Juvatn release so much of their capacity.

Agder Energi, however, could not have anticipated this rather unusual development of the spot price, as the expectation is that the prices during the winter months are higher than in the fall. The water value used by Agder would have reflected this and the actual reservoir management would have been different, and our results are therefore not directly comparable to the real situation. For the purpose of comparing the stochastic and deterministic models, the simplified seasonal model is deemed sufficient since the same water values are used in both models.

Looking again at Figure 19, the resulting reservoir levels at Nåvatn for the simulation using the stochastic model and the simulations using the two sets of input for the deterministic model is shown. The level decreases at first but stabilizes towards the end of the period. The realized spot price is highest in the beginning, has a dip around week three and then increases again with more fluctuation than before the dip. The reservoir level at Nåvatn is controlled by the water value and even though this may not be a realistic development of the reservoir level at Nåvatn, it is the correct behaviour according the water values used. The stochastic model always has the lowest reservoir level, and hence releases more water. The reason for this is that the stochastic model has more production at maximum capacity to exploit the high price hours, as explained more thoroughly in Section 10.5.

Juvatn is drawn down to about half of its starting reservoir, which is perhaps the most unrealistic of our results. Juvatn is the largest reservoir in Mandalsvassdraget, and in reality it would be beneficial to store water here in anticipation of higher prices during winter. This is not the case in the simulation, and water is drawn from the reservoir as long as the price is higher than the water value even though this means very low reservoir levels. Juvatn exhibits the same general behaviour as Nåvatn, where the level decreases at first and then stabilizes. The stochastic model often has the lowest level, but not always as for Nåvatn.

10.3 Short-term reservoir management

The reservoirs with a smaller degree of regulation are managed differently than the larger reservoirs. The smaller reservoirs can be filled and emptied within the week, and does not have the capability to store large amounts of water for longer periods of time without overflow. The water in these reservoirs hence has to be produced within a shorter period of time to leave room for new expected inflow.

For Mandalsvassdraget, Skjerkevatn, Lognavatn, Tungesjø and Mannflåvatn all have small degrees of regulation, but we look specifically at Skjerkevatn and Lognavatn as these are the reservoirs where the differences between modelling the inflow as stochastic or deterministic are most evident. We look at the results from the stochastic model with uncertainty in both prices and inflow and the stochastic model where inflow is modelled as deterministic. The uncertainty in prices is kept for both models, since we here try to capture the effect of the inflow uncertainty by itself. The pure deterministic model and the differences regarding the bidding matrix are independent from the reservoir management, and so the results for reservoir management from the purely deterministic model will be comparable the stochastic model with certain inflow.

Figure 21 show the reservoir levels for Skjerkevatn for the stochastic model with uncertainty in prices and inflow and the stochastic model with deterministic inflow. There is a regular pattern where the reservoir level is decreased when production is on during the day, and the rises again during night hours where the prices are lower. This regular pattern is evident for both models. The interesting result is in the period where this pattern is broken, as is the case around week three where the prices are lower and more fluctuating than usual. In this period, production is stopped and the reservoir is at its maximum level for a longer period of time. Higher levels mean a higher risk of spillage, and as spillage is equivalent with loss, this behaviour should be limited.

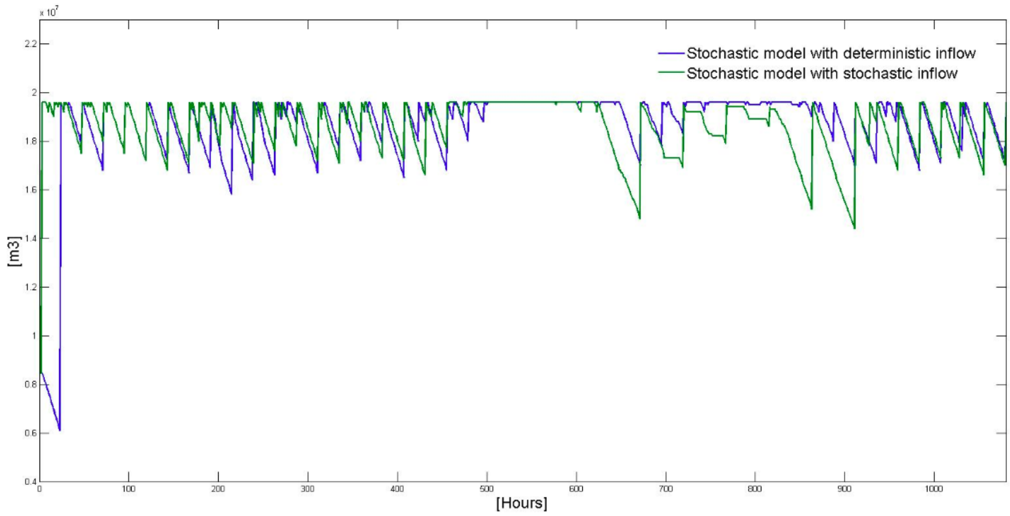

Figure 21: Plot of reservoir level at Skjerkevatn over the simulation period.

The model with uncertain inflow is more moderate in its reservoir management than the model with deterministic inflow, as can be seen in the figure where the stochastic inflow model has lower reservoir levels than the deterministic one towards the end of the period. Also, in Table 6, the number of hours where the reservoir is at its maximum is presented, and the stochastic inflow model has about 25% less hours at maximum, indicating a reservoir management strategy that does not tend as much towards the boundaries of the reservoir as the deterministic inflow model.

The same results are evident for Lognavatn, as can be seen in Figure 22 which shows the equivalent plot for the reservoir levels at Lognavatn over the simulated period. Here, the reservoir level results are further apart than at Skjerkevatn, and from Table 6, the stochastic inflow model has about 32 % less hours at its maximum level than the model with deterministic inflow.

Table 6: Results for the number of hours where the reservoir level is at maximum.

		Stochastic	Deterministic inflow
Skjerkevatn	Hours	135	181
	Percent		25%
Lognavatn	Hours	112	165
	Percent		32%

Table 7: Results for the number of hours and the amount of spill.

		Stochastic	Deterministic inflow
Skjerkevatn	Hours	4	6
	m^3	3156	5039
Lognavatn	Hours	8	12
	m^3	201338	354784

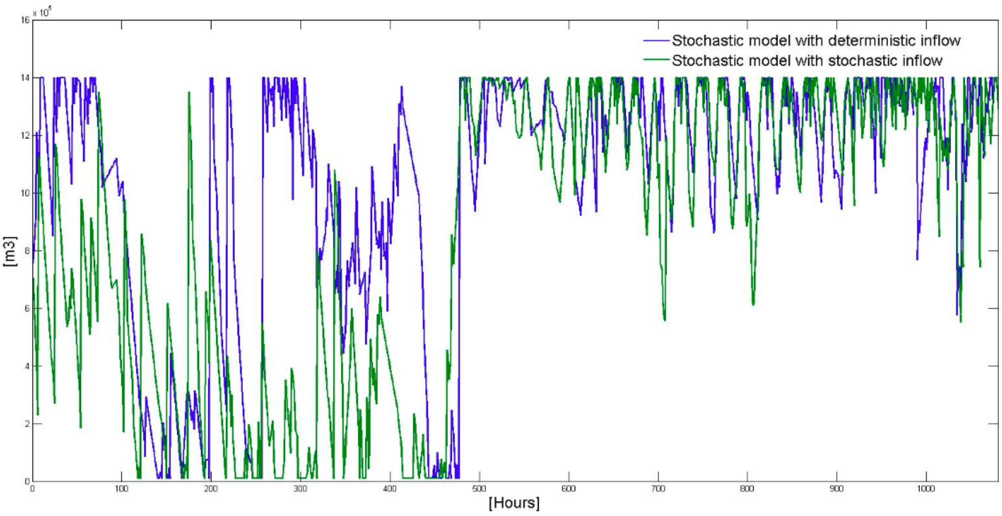

Figure 22: Plot of reservoir level at Lognavatn over the simulation period.

Table 7 shows the actual number of hours with spill and the volume of water lost at Skjerkevatn and Lognavatn for the simulated period. The results from above are strengthened as spill actually occurs more often when inflow is modelled as deterministic. Skjerkevatn has few incidents of spill and low values, whereas Lognavatn spills rather frequently in comparison. As spill is unwanted, we deem the stochastic representation of inflow to yield the best reservoir management strategy since it avoids extreme reservoir levels to a larger degree than the model with deterministic inflow.

10.4 Odd starts

A result that directly relates to the original problem received from Agder Energi is the number of odd starts and stops of the turbines. To turn the stations on or off for just a few hours at a time is an undesirable production schedule due to the fact that frequent start and stops cause tear on turbines and loss of water. Table 8 shows the number of odd starts of all nine turbines over the whole simulation period.

The stochastic model has fewer odd starts and stops than both the deterministic model with SKM prices and with prices from Agder Energi. Odd starts often come from sudden changes in the spot price over few hours. When the stochastic model finds the bid matrix it takes into account scenarios with different price profiles and hence different scenarios for where the price peaks can occur. The deterministic model, on the other hand, sees the same profile scaled equally in all hours, resulting in a bid matrix that is not as robust to sudden changes in price. The simulation over seven weeks shows a 17,03% improvement in odd starts when using the stochastic model versus the deterministic model with prices from SKM, and a 30,41% improvement versus the deterministic model with prices from Agder Energi. This is a significant improvement of how the turbines are run, but if we include the possibility of block bids in the stochastic model, we see an even more radical change in the number of odd starts and stops.

When including the option of block bids in the stochastic model, a more stable production plan may be achieved. If price peaks occur in hours with block bids, the bid will not result in a peak in the produced volume. As shown in Table 8 the amount of odd starts and stops improve drastically when including block bids.

Table 8: Results for the total number of odd starts over the simulated period.

	Stochastic	Deterministic SKM	Deterministic AE	Block bids
Hours	151	182	217	33
Percent		-17.03	-30.42	78.15

10.5 Best-point and maximum-point production

A result relating to both production and reservoir management is the percentage of time the turbines are run at best-point or maximum production, as this can be an indicator of how well the model exploits high price hours.

Best-point production is production as the best possible efficiency for the turbine, while maximum production is production at the maximum capacity. The production curves used in the optimization models are linear approximations of concave efficiency functions for each turbine, as explained in Section 5.3. One of the breakpoints of the linear curves approximating the efficiency curves is chosen to be the best-point production, and another one is chosen to be the maximum. From this we can compare how often production is run at best-point or maximum by noting which breakpoint is actually used.

For our data set, the production function for almost all turbines is nearly linear, due to the actual turbines installed in Mandalsvassdraget. This means that best-point production is less frequent, since in most cases there is really no gain from scheduling production at this breakpoint compared to other points. The loss of efficiency due to moving from best-point to maximum-point is offset by the fact that more power can be produced and sold at a high price.

Regardless, the different models show slightly different tendencies when it comes to choosing breakpoints for production, as shown in Table 9 and 10 which show the percentage of time the turbines are run at best-point production and maximum production, respectively.

The deterministic model, regardless of whether the prices from Agder Energi or SKM are used, always has a larger percentage of the time where the turbines are run at best-point than the stochastic model. This may stem from the fact that the scenarios used for the different runs of the deterministic model are a pure weighting of the same base price profile, and hence the price always change at the same time and with the same rate for all scenarios. This makes it possible to lock in a specific schedule where the turbines are steadily run at best-point. In the hours where the price peaks, the turbines are turned up to maximum or other turbines are turned on.

Overall, maximum production is far the most common, but the stochastic model always has a higher percentage of time where the turbines are run at

Table 9: Results for the number of hours where the turbines produce at best-point.

	Stochastic	Deterministic SKM	Deterministic AE
Skjerkevatn	0.72	2.81	1.54
Juvatn	8.24	14.13	14.76
Lognavatn	2.54	4.35	1.27
Ørevatn 1	2.08	10.05	5.43
Ørevatn 2	1.00	2.99	1.81
Tungesjø 1	2.17	7.70	6.70
Tungesjø 2	2.99	6.97	3.08
Mannflåvatn 1	8.70	8.70	7.25
Mannflåvatn 2	4.80	5.80	5.07

Table 10: Results for the number of hours where the turbines produce at maximum capacity.

	Stochastic	Deterministic SKM	Deterministic AE
Skjerkevatn	62.77	46.74	54.17
Juvatn	78.17	68.30	69.02
Lognavatn	74.46	6.21	73.82
Ørevatn 1	56.16	44.20	51.45
Ørevatn 2	57.88	51.09	56.79
Tungesjø 1	82.25	66.58	74.73
Tungesjø 2	81.43	67.39	78.62
Mannflåvatn 1	49.09	37.23	44.20
Mannflåvatn 2	51.36	39.04	44.47

maximum. The price scenarios used in the stochastic model has a different nature than the deterministic scenarios; that is, they can be crossing, and the prices do not peak in the same hours for all of them, and not with same rate. The stochastic model hence have a better understanding of how the price may realize, and exploits this by scheduling maximum production in the hours where the price is expected to be high. This may lead to a production schedule with more variation in production volumes, and it is more difficult to lock in a steady schedule. The larger variation in prices leads to a more extreme unit commitment schedule where the volume may change quite rapidly to exploit high prices.

The use of the balancing market is connected to the choice of production at best-point or maximum-point by the fact that if production is scheduled below maximum, it is easier and cheaper to regulate the produced volume by changing the level of the producers own generation resources, instead of buying up regulation. Hence, with a tendency to production below maximum capacity, the deterministic model uses less regulating power than expected. The balancing volume is still larger for the deterministic model, and both up and down regulation are used. This indicates that the price scenarios used in the deterministic model not always captures the actual realization of the spot price.

The stochastic model use almost only up regulation, which follows from the fact that if the committed volume is the maximum volume, and it is not possible to produce this volume with own resources, then the missing volume has to be bought in the regulating market. Since the stochastic model more often produce at maximum, it is clear that this effect is more present than in the deterministic model.

10.6 Simulation without time delays

The simulation without time delays in the watercourses between reservoirs results in a higher obtained average price and total value for both the stochastic and the deterministic model. Without time delays both the bid optimization and the production allocation model are less constrained when finding the optimal solution. The models without time delay can move water from an upstream to a downstream reservoir within the hour, resulting in a more flexible system. The obtained average price using the stochastic model for bid optimization increased with 0,16 % to 23,12 €/MWh when no time delays are present. The deterministic model's obtained average price increased from 22,93 €/MWh to 22,98 €/MWh; an improvement of 0,22 %. As seen in Table 11 and 12, the difference in both average price and total value between the stochastic and deterministic model decrease when the time delay is removed

Table 11: Results for the obtained average price per MWh for the stochastic and deterministic models with and without time delays.

	Stochastic	Deterministic SKM	Stochastic without delays	Deterministic without delays
Euro	23.08	22.93	23.12	22.98
Percent		-0.69		-0.61

Table 12: Results for the total value for the stochastic and deterministic models with and without time delays.

	Stochastic	Deterministic SKM	Stochastic without delays	Deterministic without delays
Euro	14442784	14354771	14467069	14384291
Percent		-0.61		-0.57

from the formulation. For instance, the difference in obtained average profit between the stochastic and deterministic model is 0.67 % with time delays, and 0.61 % without. Our results show that the potential improvement by using the stochastic model is larger when the system is more complex.

11 Conclusion

11.1 General results

In this thesis, a stochastic model for bid optimization and short-term production scheduling has been implemented and tested trough a simulation procedure run over a longer period of time for a complex real-life river system. The results show that the stochastic model gives significant improvements in terms of higher obtained average price and higher total value than an equivalent deterministic model.

Our results also indicate that the reservoir management strategy is improved, as the stochastic model obtains a 0.6 % higher total value than the deterministic model. This is due to the fact that more water is scheduled for production now when prices are higher and not saved for later. This leaves lower reservoir levels at the end of the simulated period. Hence the stochastic model has a slight shift towards producing now instead of saving the water for later in comparison with the deterministic strategy. The balance between producing now and saving water for later is controlled by the water value and the results may have been different if other water values were used. As the two models tested in this thesis see the same water values, and the stochastic model has higher total value, our results indicate that the implementation of a stochastic bid optimization model may give hydropower producers a potential for increased profits.

In addition, the unit commitment results in a steadier schedule when using the stochastic model. In this thesis, this is measured by the number of odd starts, which is decreased by 17 % when using the stochastic model. This number may be further reduced by including the possibility of block bids in the stochastic model, but for our case this as adverse effects on the obtained average price and the total value. This is due to an unusual large volume committed as blocks, and hence a rather inflexible production schedule.

The stochastic model also results in a more moderate reservoir management strategy where the risk of spillage is reduced. The stochastic model has a less percentage of time where the reservoir level is at its maximum, and hence leaves room for flexibility. The number of hours with maximum reservoir level is reduced by about 20 – 30 % depending on the reservoir.

Having uncertainty in both market prices and inflow means a more realistic representation of the actual conditions when the bid decision is to be made. The output from the model is therefore more reliable than the output from a deterministic model, which requires more analysis from the operator. The stochastic model gives a weighted decision that accounts for a larger range of possible future events than the deterministic model, and hence the solution is better suited for both good and bad outcomes, whereas the deterministic model is optimally adapted to only one outcome. The deterministic model may perform adequately in most cases, but over time we see that it is outperformed by the stochastic model.

The fact that the use of the stochastic model is tested over a longer time period gives validity to the results. Still, some moderation is necessary since the procedure is not tested for various times of year or other reservoir systems. Hydropower production is very dependent on time of year and the amount of available water. The time period used in this thesis was characterized by large amounts of inflow and high prices. The results could have been different depending on various conditions related to the power market or the meteorological situation.

It is also a strength that the model is implemented for a complex real-life system, and that the degree of physical detail is not compromised in the modeling. Our results show that the potential for inceased profits by using a stochastic model is larger when the system is more complex. In addition, the stochastic model use inputs that are currently available to the actors in the industry, namely price scenarios developed by market analysts or the power company itself. The algorithm is also fast enough to be used on a daily basis and is tailored to be included in the scheduling hierarchy used by most producers today. This makes the shift to a stochastic model easier since only small changes in the routines for bidding and price forecasting is required.

A last result evident from our case study is that there may be potential for increased profits by analyzing and improving the price forecasting and scenario generation method used with the deterministic model. This would be a smaller step than implementing a new stochastic bid optimization tool, and may lead to both a better understanding of uncertain parameters and increased profits.

11.2 Suggestions for further studies

The single largest drawback of the simulation procedure developed in this thesis is the seasonal model that uses historical values for price and inflow. This means that the water value used by the bid optimization model is too good; that is, the available water is scheduled in an optimal way that only could have been found in hindsight and not at the time of operations. Agder Energi should test a stochastic model over time using actual water values which would lead to a reservoir management strategy that is directly comparable to the real situation, and hence the other results would also be more comparable.

The simulation procedure should also be tested for other data sets, other reservoir systems or other times of the year. High reservoir levels and large inflows characterize the time of year when the simulation in this thesis is performed. To get a good comparison between the stochastic and the deterministic model the simulation should cover a longer period of time with more variation in regards to the availability of water or market prices. Agder Energi should hence start collecting data on price forecasts, bid volumes and water values in order to make future studies with more realistic results possible.

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A Statistical test of the weekly obtained average profits

All bid strategies have a variance that is dependent on what the price actually turns out to be and other specific conditions at the time of operations. This makes the comparison between the deterministic and stochastic model more difficult, since it is hard to judge if the results show an actual significant difference or if they are just due to the specific situation depending on reservoir levels, price, water values and inflow. This should be judged by statistical methods, but with only seven weeks of observations, the data set is so small that only some statistical methods are appropriate.

We have used the paired sample Wilconxon signed rank test (Wilconxon, 1945) to test if the difference in profits between the stochastic and the deterministic model each week is significantly different from each other. This is done for the stochastic model and the deterministic model with input from SKM since these are the runs that are most fairly compared. The calculations are shown in Tables 13 and 14.

The test statistic W is calculated as the sum of the values of the sign times the rank of the observed difference and has to be larger than a critical W to reject the null hypothesis of the differences being equal. The test statistic for our observations is 26, and the critical value for a 0.05 significance level is 22, so the conclusion is that the stochastic and the deterministic model have significant different obtained profits each week.

$$W = \sum_{N} Sign * Rank = 26 \tag{53}$$

$$W \geq W_{Critical} = 22 \tag{54}$$

The number of observations is small, and we would be more confident if the result were obtained from a larger data set. To do this the simulation has to be run over a longer time period, and also for different times of the year and other reservoir systems as suggested in suggestions for further work in Section 11.2.

Table 13: Test for difference in obtained average profits.

Week	Stochastic Average Price	Deterministic Average Price	Difference	Sign	Absolute value
1	25.59	24.89	0.71	1	0.71
2	26.55	25.84	0.71	1	0.71
3	25.76	24.34	1.41	1	1.41
4	18.75	19.14	-0.39	-1	0.39
5	17.60	16.93	0.68	1	0.68
6	17.96	17.50	0.46	1	0.46
7	25.67	24.45	1.22	1	1.22

Table 14: Test for difference in obtained average profits - sorted table.

Week	Difference	Sign	Absolute value	Rank	Sign $*$ Rank
6	-0.39	-1	0.39	1	-1
4	0.46	1	0.46	2	2
5	0.68	1	0.68	3	3
2	0.71	1	0.71	4	4
1	0.71	1	0.71	5	5
7	1.22	1	1.22	6	6
3	1.41	1	1.41	7	7

B Analysis of the choice of price points

To analyze the effect different sets of price points have on the results of the bid optimization model the simulation is run for five different sets of price points, see Table 15. The analysis is done for an eleven-day period, optimizing the bid volume for different price points, and allocating production after the spot price is realized. The sets represent different strategies for bidding in the spot market. The difference in obtained revenue from the spot market is shown in Table 16 and the spot price realization for the first 11 days in Table 17.

In the first run quite large intervals between the price points is chosen, with a higher density in the area between 20 €/MWh and 30 €/MWh, which is the area where most of the realized prices occur. This is the base run with which the results from the runs with other sets of price points will be compared.

The second run has a high density of price points in the small area between 20 €/MWh and 23 €/MWh, which represents a strategy with more price points in the area where it is expected that the optimal volume differs the most with small changes in price. Throughout the entire simulation period of seven weeks the low water values results in high or maximum production if the price exceeds the mid-twenties and it is therefore expected that optimal produced volume will differ the most in the area just below.

The third run has a poor choice of price point since the area with high density of price points is chosen for very high prices, so that the optimal volume will be maximum capacity for all bid points. Interpolation between the high points is unnecessary; they all result in the same committed volume. It is therefore the line segment between 0 €/MWh and 30 €/MWh that decide the results of this run. We chose to include this run to prove that even for poorly chosen price points, the obtained revenue from the spot market differs by at most 0,25 %. As expected it is the day with the lowest average price that differs the most form run 1, because this day the weight of price point 0 €/MWh have a greater influence on the committed volume.

Run four is a linear line between price point 1 and 7 with equal intervals between the points. This run results in small differences between obtained revenue in the spot market from run 1. Since bid point 33,33 €/MWh almost always results in maximum production it is the linearization on the line segment between price points 2 and 3 that results in the change of committed volume from run 1.

The last run has a big leap from price point one to two, with a 25 €/MWh difference. In hours with spot price lower than 25, the slope of the line segment is higher than the slope between line segments under 25 €/MWh in

Table 15: Set of price points for the different runs of the stochastic model

Price Point	1	2	3	4	5	6	7
Run 1	0	15.00	20.00	23.00	27.00	30.00	100.00
Run 2	0	20.00	21.00	22.00	23.00	30.00	100.00
Run 3	0	30.00	40.00	41.00	42.00	43.00	100.00
Run 4	0	16.67	33.33	50.00	66.67	83.33	100.00
Run 5	0	25.00	28.75	32.50	36.25	40.00	100.00

Table 16: Difference in revenue for the different sets of price points

Run	1	2	3	4	5
Day 1	0,0000	-0,0022	0,2535	0,0045	0,0125
Day 2	0,0000	0,0000	0,1535	0,0465	0,0113
Day 3	0,0000	0,0000	0,1535	0,0456	0,0078
Day 4	0,0000	0,0000	0,1500	0,0156	0,0036
Day 5	0,0000	0,0000	0,1523	0,0427	0,0000
Day 6	0,0000	0,0000	0,1500	0,0025	0,0089
Day 7	0,0000	0,0000	0,1505	0,0264	0,0065
Day 8	0,0000	0,0000	0,1504	0,0017	0,0000
Day 9	0,0000	-0,0016	0,1502	0,0136	0,0000
Day 10	0,0000	0,0000	0,1505	0,0123	0,0000
Day 11	0,0000	0,0000	0,1545	0,0125	0,0019

run 1. The committed volume will therefore differ some for spot prices below 25 €/MWh. But the line segments between bid point 2 and 4 in run 5 has a slope almost equal to the slope of the line segment between bid point 4 to 6 in run 1, hence for spot prices over 25 €/MWh run 1 and 5 gives quite equal results.

This analysis proves that the committed volume from the spot market is quite stable if the price points are chosen to cover the possible area for the spot price and has a higher density of price points in the area where it is most likely that variation in spot prices results in large changes in the committed volume.

Table 17: The spot market price in each hour of the first 11 days of the simulation period.

Day	1	2	3	4	5	6	7	8	9	10	11
Hour 1	23.87	24.77	25.53	25.69	25.14	25.56	24.97	26.22	25.89	25.56	25.88
Hour 2	23.60	24.50	25.21	25.28	24.89	24.96	24.86	26.02	25.73	25.57	25.67
Hour 3	23.43	24.41	25.03	24.99	24.83	24.87	24.77	25.84	25.64	25.36	25.06
Hour 4	23.39	24.37	24.98	24.93	24.81	24.85	24.74	25.79	25.61	25.31	23.47
Hour 5	23.40	24.37	24.96	24.82	24.86	24.87	24.78	25.85	25.60	25.30	22.78
Hour 6	23.53	24.45	24.94	24.80	24.95	24.97	24.81	26.02	25.73	25.23	22.67
Hour 7	23.99	24.81	24.99	24.75	25.24	25.30	25.18	26.63	26.68	25.37	23.19
Hour 8	24.15	24.84	25.04	24.74	25.51	26.24	25.55	26.55	26.67	25.43	24.22
Hour 9	24.31	25.18	25.23	24.82	26.89	26.51	25.91	26.83	27.26	25.64	25.11
Hour 10	24.42	25.36	25.77	24.92	27.81	27.73	26.04	26.87	27.16	25.82	25.60
Hour 11	24.40	25.24	25.98	25.13	27.91	26.81	25.98	26.85	27.47	25.94	25.85
Hour 12	24.37	25.15	26.00	25.25	26.92	26.81	25.85	26.83	26.67	25.97	25.93
Hour 13	24.34	25.00	25.98	25.22	26.87	26.84	26.08	26.96	26.69	25.87	25.94
Hour 14	24.31	24.86	25.92	25.11	28.64	27.02	26.07	27.13	26.47	25.76	25.93
Hour 15	24.28	24.53	25.79	24.98	28.50	26.97	25.75	26.90	26.46	25.70	25.82
Hour 16	24.19	24.33	25.57	24.91	27.72	26.47	25.45	26.50	25.98	25.73	25.50
Hour 17	24.12	24.69	25.53	24.94	28.14	26.02	25.48	26.41	26.39	25.72	25.31
Hour 18	24.02	24.22	25.83	24.95	26.99	25.76	25.38	26.23	26.16	25.80	25.92
Hour 19	23.93	24.94	25.96	25.04	26.09	26.66	25.22	26.30	26.41	25.85	26.07
Hour 20	23.97	25.12	25.96	25.21	26.00	26.03	25.18	26.50	26.21	25.86	26.29
Hour 21	24.03	25.01	25.93	25.50	26.27	26.08	25.14	26.63	26.36	25.94	26.57
Hour 22	24.27	25.30	25.93	25.49	26.29	26.02	25.13	25.67	26.44	25.92	26.71
Hour 23	24.21	25.43	25.89	25.48	25.93	25.98	25.14	25.20	26.54	25.82	26.53
Hour 24	23.67	24.93	25.42	25.11	25.24	25.34	24.78	25.41	25.91	25.48	25.69
Average	24.00	24.87	25.55	25.09	26.35	25.99	24.34	25.47	26.34	25.6	25.32

C Analysis of the number of cuts used to approximate the water value

To analyze the cuts used to constrain the water value in the bid optimization and production allocation model, the cuts are plotted for the two largest reservoirs in the system, Nåvatn and Juvatn. The smaller reservoirs can be filled or emptied within the week and hence the water has the same marginal value within the same week, since this is the time step of the seasonal model. Plot of the cuts for Nåvatn and Juvatn with 25 % of the reservoir capacity are shown in Figure 23 and 24, respectively. We choose to present the cuts with low reservoir levels because it is in these situations that the marginal value of water changes the most depending on the cut.

As seen in the plots, the cuts represent linear segments. For Juvatn, Cut 1 is restricting the water value for low reservoir levels, but for higher reservoir levels Cut 3 is the active restriction. No other cuts are active, and hence in this situation it would have been sufficient with two cuts. For Nåvatn, Cut 1 is the active cut for all reservoir levels. Cut 1 is made based in the extreme event of maximum production and no inflow throughout the week.

Generally, for the reservoirs in Mandalsvassdraget and all combinations of start reservoirs, the value of water is restricted by a nearly linear segment. The marginal value of water in the reservoirs, or equivalently the dual variable of the reservoir balance in the seasonal model, differs depending on the cut, as seen in Table 18. The change of the dual variables is not enough to make more than two cuts necessary to restrict the water value. It would therefore be sufficient to include only the most extreme cuts from the seasonal model in the bid optimizing or production allocation for Mandalsvassdraget.

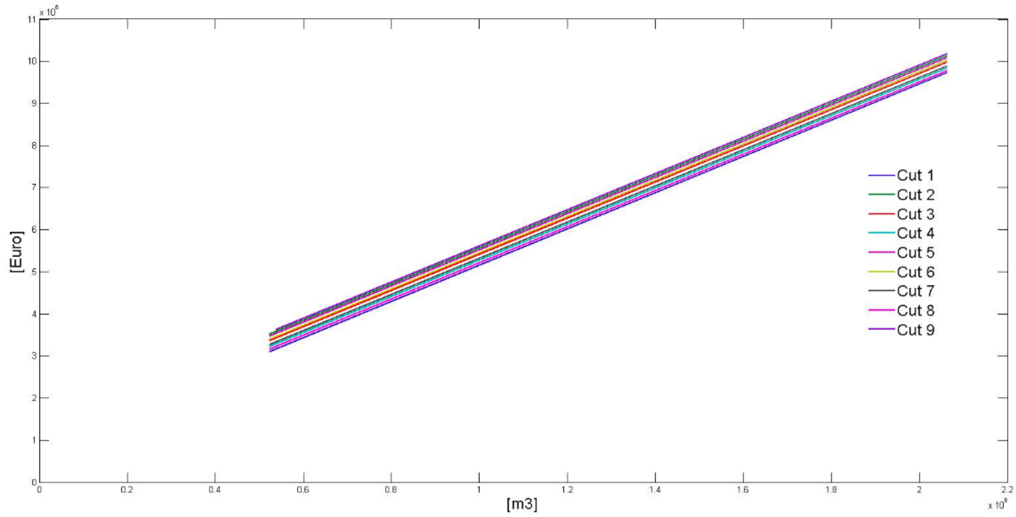

Figure 23: Plot of the cuts for the water value at Nåvatn.

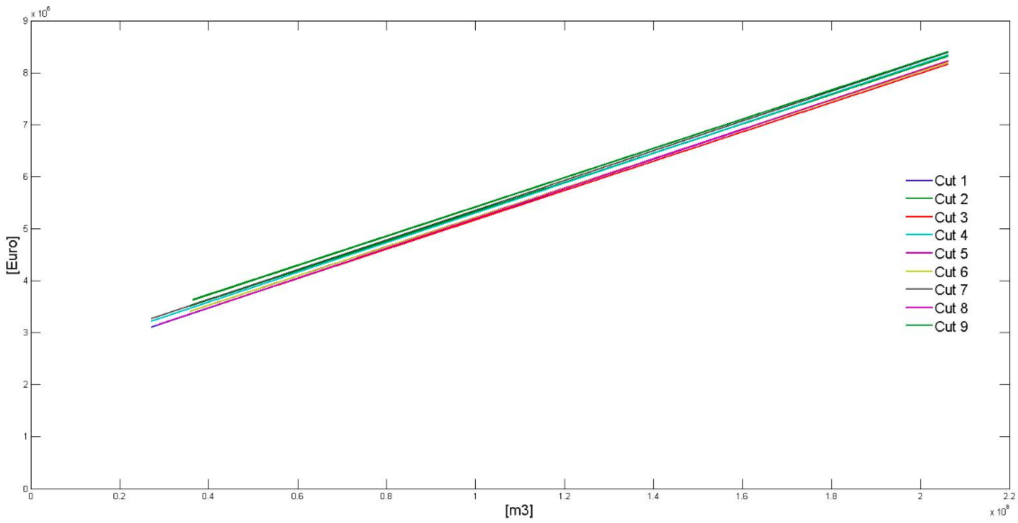

Figure 24: Plot of the cuts ofr the water value at Juvatn.

Table 18: Values of the dual variables for Nåvatn and Juvatn for the different cuts

	Unit	Nåvatn	Juvatn
Cut 1	€ $/m^3$	0.0365	0.0245
Cut 2	€ $/m^3$	0.0360	0.0238
Cut 3	€ $/m^3$	0.0365	0.0244
Cut 4	€ $/m^3$	0.0360	0.0244
Cut 5	€ $/m^3$	0.0360	0.0239
Cut 6	€ $/m^3$	0.0365	0.0239
Cut 7	€ $/m^3$	0.0360	0.0240
Cut 8	€ $/m^3$	0.0365	0.0245
Cut 9	€ $/m^3$	0.0360	0.0238