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# Optimization Models and Algorithms for the Natural Gas Supply Chain

Thesis for the degree of Philosophiae Doctor

Trondheim, November 2013

Norwegian University of Science and Technology  
Faculty of Social Science and Technology Management  
Department of Industrial Economics and Technology  
Management



**NTNU – Trondheim**  
Norwegian University of  
Science and Technology

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Marte Fodstad

Trondheim, May 2013



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# Chapter 1

## Introduction

This thesis presents several applications of optimization within the natural gas supply chain, particularly focusing on issues relevant for Norway. That is, the main focus is on the upstream supply chain from production to market, and most models take a producer's perspective.

The main sponsor for this work has been the project RAMONA involving the University of Bergen, NTNU, SINTEF, the University of Stavanger, Cog-nIT, Gassco, Statoil, and the Research Council of Norway. The objective of this project was 'to develop new theory, methods and tools to optimize regularity and capacity utilization in gas production, processing and transportation systems'. Several other projects have also contributed to some of the papers in this thesis. These are the VENOGA project involving Statoil, SINTEF and the Research Council of Norway, 'Maritime transportation and logistics'<sup>1</sup> involving among others SINTEF, Statoil, and Suez Energy International, and LNGShipping involving GdFSuez, Statoil, NTNU, SINTEF and the Research Council of Norway.

The first chapter of the thesis describes the background for the work, with an introduction to the natural gas supply chain and an overview of operations research applied in the natural gas industry. The chapter is ended with a short presentation to each of the seven papers that form the research contributions in the thesis. The following seven chapters are reprints of the papers.

### 1.1 The Natural Gas Supply Chain

Natural gas is an important part of the global energy system, being the third largest primary energy source in 2010, after oil and coal (IEA 2012). Global gas production is expected to grow further, by 17 - 55 % from 2010 to 2035, driven by the general increase in global energy demand and through taking an even stronger position in the energy mix (IEA 2012). In the current debate on transforming the energy system to a more environmental friendly one, natural gas is frequently given a role as a transition fuel because of the reduced green

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<sup>1</sup>Original project title in Norwegian: 'Maritim transport og logistikk'

house gas emission levels compared to coal, and of its flexibility to balance more intermittent renewable sources in the system (EIA 2011). Another decisive factor is how production of unconventional gas, that has dramatically shifted the supply balance in North America, will propagate globally and thereby increase the natural gas reserves (IEA 2012). Norway was in 2011 the third largest gas exporting country in the world (Norwegian Ministry of Petroleum and Energy & Norwegian Petroleum Directorate 2013), exporting 100 bcm which corresponds to 20 % of EU consumption (British Petroleum 2011). Norwegian gas production is expected to slightly rise for at least another decade (Norwegian Ministry of Petroleum and Energy & Norwegian Petroleum Directorate 2013). Only 3 % of the Norwegian gas is consumed nationally.

## Supply Chain Components

The main components of the upstream part of a natural gas supply chain are exploration and production, processing, transportation, storage and sale. These will shortly be described here, taking a Norwegian perspective.

### Exploration and Production

Natural gas is found in reservoirs formed in permeable rocks below the surface of the earth. To locate petroleum reservoirs with certainty, drilling of exploration wells is a necessity. Since these are costly operations the ground is first examined with geophysical methods like seismology. (Gassco 2012, NaturalGas.org 2012) All Norwegian exploration activity is offshore on the Norwegian Continental Shelf (NCS).

The Norwegian gas is produced at approximately 50 fields, mainly located in the North Sea and the Norwegian Sea (Norwegian Ministry of Petroleum and Energy & Norwegian Petroleum Directorate 2012). In most fields gas production is associated with oil, natural gas liquid (NGL) or condensate production. Because of the superior market price of these products relative to gas, the operation of such fields treats gas as a by-product. Production in fields with mainly gas, often denoted swing fields, is on the other hand varied to balance gas demand and supply over the season. Such fields typically have a production license set by the authorities that limits the amount of gas that can be produced each year.

### Processing, Transportation and Storage

Natural gas extracted from reservoirs is rich gas, which is a mixture of hydrocarbons (methane, ethane, propane, n-butane, isobutane and naphtha) and some contaminants like carbon dioxide (CO<sub>2</sub>) and hydrogen sulfide (H<sub>2</sub>S). Through processing the gas is separated into dry gas and wet gas. Dry gas is traded in



1.1 The Natural Gas Supply Chain

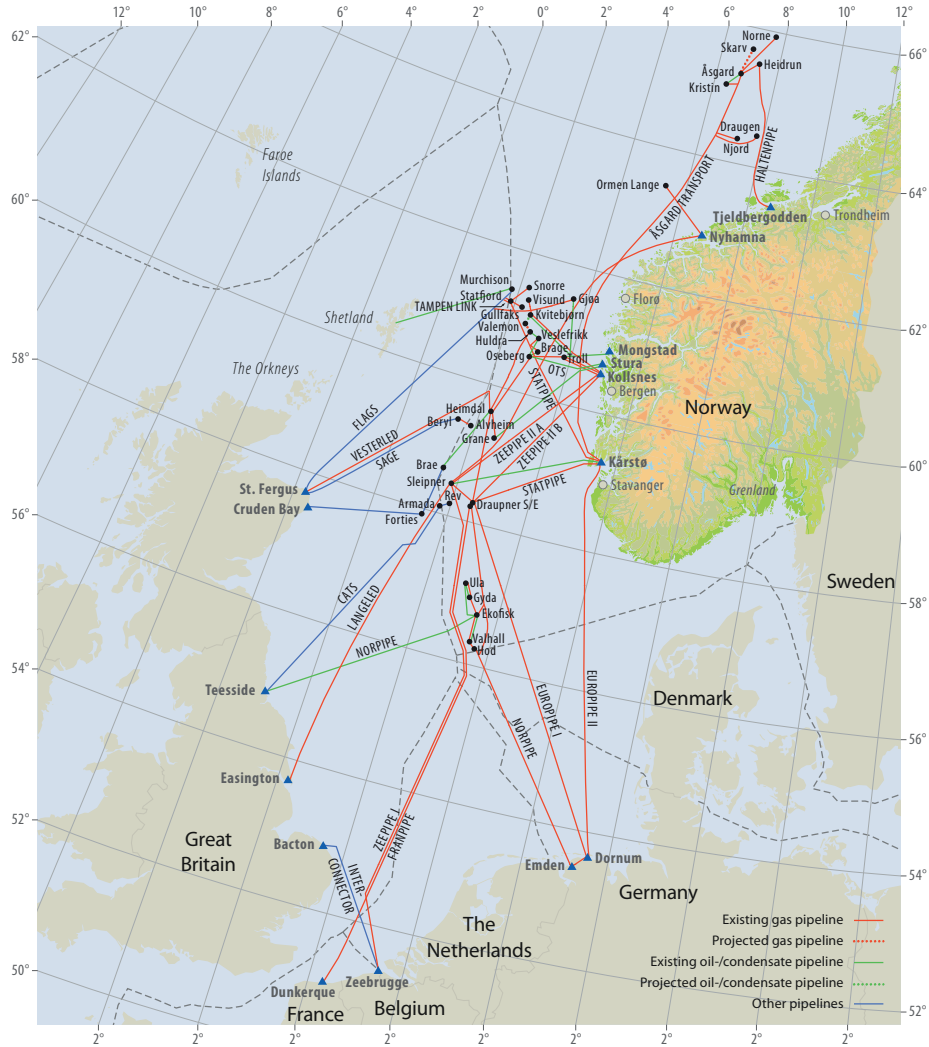
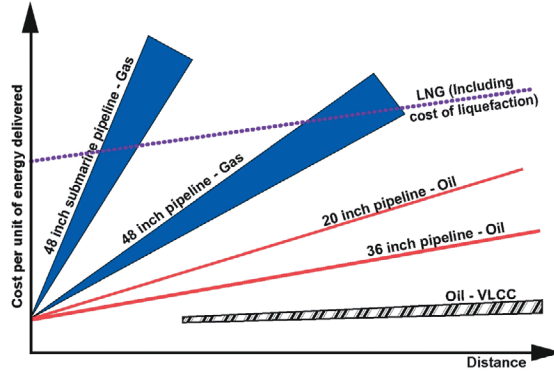


Figure 1.1: Existing and projected pipelines in the Norwegian petroleum export system. Source: The Norwegian Petroleum Directorate



**Figure 1.2:** Transportation costs depending on distance and scale. Source: Gas Strategies Consulting Ltd.

gas markets and contains mainly methane and some ethane. Wet gas contains the heavier components that can appear in liquid form under pressure. (Gassco 2012) These components can be fractionated into separate components, shipped by vessels and sold bilaterally (Myklebust, Tomasgard & Westgaard 2010).

Most Norwegian gas is transported in pipelines from fields to processing plants and further to gas markets in Continental Europe and United Kingdom. The transportation network, which consists of nearly 8000 km of offshore gas pipelines (Gassco 2012) is shown in Figure 1.1. Gas flow through the pipelines is driven by the pressure difference between the upstream inlets and the downstream outlets, which is controlled by compressors and valves.

The Snøhvit field with pipeline to Melkøya liquefaction plant is an independent system which exports liquefied natural gas (LNG) and covers 5 % of the Norwegian gas export (British Petroleum 2011). By cooling dry gas down to  $-160^{\circ}\text{C}$  it condenses into liquid form and improves its energy content per volume approximately 600 times relative to gaseous form. This process is costly, but on the other hand it enables transportation of the gas by specialized vessels over large distances (NaturalGas.org 2012) and provides market choice also after installation. The competitiveness of LNG compared to pipeline exports of gas depends on several parameters, two of them being distance and scale as illustrated in Figure 1.2.

There are both seasonal and short term variations in gas demand. Also on the supply side there are variations in the gas flows caused for instance by maintenance. Both to smooth out variations and to balance supply and demand, storages are needed. Storing gas is also a way of improving security of supply in case of unforeseen events that disrupt supply. Most commonly gas is stored

underground in depleted gas reservoirs, aquifers and salt caverns. LNG can be stored in LNG storage tanks onboard vessels and onshore. (NaturalGas.org 2012) Further, gas pipelines can provide storage capabilities by increasing the inlet pressure and inserting more gas to the pipeline than what is withdrawn at the outlet (Midthun, Nowak & Tomasgard 2007). Different storage types have different characteristics when it comes to storage capacity, injection rate, withdrawal rate and proximity to market, which make them fit different purposes.

### **Sale, Market and End-use**

Traditionally, natural gas has been traded in long-term contracts. Usually these contracts have prices linked to smoothed oil or coal indexes and a take-or-pay structure that lets the buyer decide on the daily delivered volume within upper and lower bounds. Contracts have been a means to share the risk coming from large investments costs. With the deregulation of natural gas markets, that first started in Northern America and United Kingdom, market hubs with standardized gas contracts on spot and derivate trade have emerged. This trend tends to increase the price volatility due to increased sensitivity to short term factors as weather-related demand variations and upstream supply problems (Jensen 2004, Heather 2010).

There are three main regional gas markets, the North American, the European and the Asian. The North American market is the most developed with mainly gas-to-gas competition and large traded volumes in the leading market, Henry Hub. On the other extreme is the Asian market that is dominated by oil-linked long-term contracts, even though increasing amounts of LNG spot trades are seen here as well. (Holmes 2007, IEA 2009) The European market is a bisected one, with large shares of oil-linked contracts on the Continent, but with UK's National Balancing Point (NBP) as a liquid trading hub. Several other hubs are seeing increasing trade as illustrated by the volumes reported in Figure 1.3 (IEA, IEF, IMF & OPEC 2011).

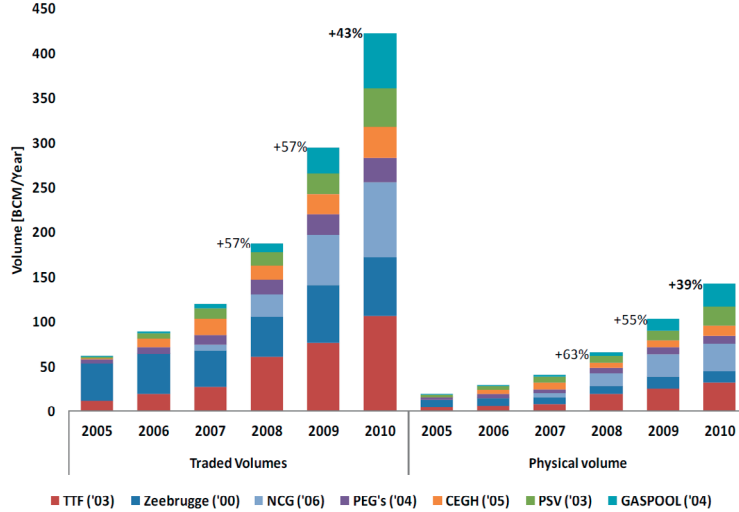
Natural gas has a range of end-uses, mainly as a source of energy through burning, but also as raw material for several products such as paint and plastics (NaturalGas.org 2012). The largest end-use sectors of OECD Europe are electric power generation, residential and commercial buildings and industry (EIA 2011).

### **Supply Chain Organization**

During the last 15 years the European natural gas industry has gone through a deregulation process driven by European Union legislations stated in three natural gas directives(1998/30/EC, 2003/55/EC, 2009/73/EC).<sup>2</sup> An overall goal

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<sup>2</sup>Note, deregulation of the gas industry in UK started earlier than in the rest of Europe, initiated by the Gas Act already in 1986 (Heather 2010).



**Figure 1.3:** Trade and physically delivered volumes on European hubs. Source: (IEA et al. 2011).

has been to form a single European gas market. A vital tool to achieve this is the opening of the transportation infrastructure with third party access and a cost-reflective and non-discriminatory tariffication system. (Neumann & Cullmann 2012)

As a consequence of deregulation the highly integrated Norwegian gas industry was divided to make different roles independent. The transmission system operator (TSO), Gassco, was founded in 2001 to facilitate third party access to the gas infrastructure. Gassco operates most of the Norwegian infrastructure on behalf of the joint venture Gassled who owns it. Gassco also administrates the allocation of transportation capacity among shippers through a primary market with allocation rules and preset tariffs, and they facilitate a secondary market where shippers can resell capacity bilaterally. (Gassco 2012) Due to the relatively limited upstream trade in the Norwegian system, the same company usually has the producer, the shipper and the marketer role in the supply chain. Both the terms producer or shipper will be used to represent these integrated companies in this thesis, depending on the context it is used. There are approximately 40 companies with shares in operating fields, with huge differences in the total production capacity between the different producers (Norwegian Ministry of Petroleum and Energy & Norwegian Petroleum Directorate 2012).

## 1.2 Operations Research in the Natural Gas Industry

‘In a nutshell, operations research<sup>3</sup> (OR) is the discipline of applying advanced analytical methods to help make better decisions.’ (INFORMS 2013) The field draws upon a broad range of disciplines, such as applied mathematics, economics, statistics, computer science and industrial engineering. The boundaries of the field is not clear cut, but ‘varies depending on the background and interests’ of the one asked. (Gass & Assad 2005) Mathematical programming is a cornerstone in the theoretical fundament of OR. *Programming* should in this setting be understood as *planning* (Williams 1999), and the terms *program*, *problem* and *model* is often used interchangeably.

In this section I will first present different model classes in OR defined by their mathematical properties, briefly mention algorithmic issues and exemplify with features in the natural gas supply chain that are typically modeled with the presented model classes. For a general and thorough introduction to OR, see for instance Hillier & Lieberman (2001). Further I give an overview of different OR applications in the natural gas industry. Finally, I discuss stochastic programming and portfolio and supply chain optimization which are perspectives particularly relevant for the papers presented in the coming chapters of the thesis. As this presentation takes multiple perspectives to the same topic, operations research in the natural gas industry, there are several parts of the literature that could fit in several subsections. In these situations I have chosen to only include references where I find them most relevant for my presentation to economize on the text and avoid repetitions.

### Model properties and tractability

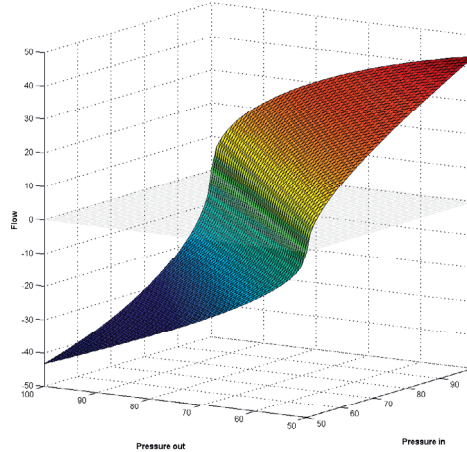
Model classes within OR define mathematical structures and properties of models that are decisive for tractability and algorithmic choice. Here some model classes and motivating features from the natural gas industry are presented.

*Linear programs (LP)* are a widely used problem class. LPs are convex problems since linear functions per definition are convex. They are easy to solve since an optimal solution will be at the intersection of some constraints. This is utilized by the Simplex algorithm that examines the intersections and disregards the remaining parts of the solution space. (Williams 1999) For further literature on linear programming, see Vanderbei (2008).

Several elements in the natural gas industry have a non-linear behavior, which give rise to *non-linear programs (NLP)*. A special case of NLP is convex non-linear programs, where the constraint set forms a convex solution space and the

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<sup>3</sup>[operational research in British English]



**Figure 1.4:** Plot of the Weymouth equation describing the relation between flow and pressure in and out for a pipeline. When pressure out exceeds pressure in, the flow will go in the opposite direction, represented as negative flows in the plot.

objective function is concave and maximized or convex and minimized. Convex problems are more tractable than non-convex problems, because a local optimum is always a global optimum in such problems. For an introduction to non-linear optimization, see Floudas (1995).

An example of a convex non-linear element in natural gas industry is Golombek et al.'s production cost function,  $a + bq + c \ln(1 - \frac{q}{Q})$  where  $q$  is the production decision variable,  $Q$  is the production capacity and  $a, b$  and  $c$  are parameters (Golombek, Gjelsvik & Rosendahl 1995). The pressure drop over a pipeline in steady state<sup>4</sup> is another example of a non-linear property that can be incorporated in convex models. The Weymouth equation that describes this property relates the flow ( $f_{ij}$ ) and inlet and outlet pressures ( $p_i$  and  $p_j$ ) in a pipeline,  $f_{ij} = K_{ij} \sqrt{p_i^2 - p_j^2}$ , where  $K_{ij}$  is a constant given by pipeline properties and the surrounding conditions (Campbell 1992). It can be seen from the plot in Figure 1.4 that the equation is non-linear and neither concave nor convex, but in the area where inlet pressure exceeds outlet pressure the function is concave. This means that single-directional pipelines can be modeled in a convex NLP problem.

<sup>4</sup>Steady (or stationary) state means a state where the flow is constant through out the pipeline and constant over time (Martin, Möller & Moritz 2006). For modeling of transient flows describing how changes in pressure, flow and density propagate in time and space, see for instance Moritz (2007).

Because of the preferable computational property of LPs, a NLP is sometimes formulated as a LP relaxation through *piecewise linear approximation*. This is how the Weymouth equation is modeled in, for example, Rømo, Tomasgard, Hellemo, Fodstad, Eidesen & Pedersen (2009), where the equation is replaced with a set of inequalities that represent a relaxation. Van der Hoeven (2004) presents several non-linear elements in a natural gas transportation network and ways of linearizing them.

*Quadratic programs (QP)* are a special case of convex NLP, where the objective function is a convex quadratic function and the constraint set is linear. Modeling trade in a market with a linear demand function will for instance give a quadratic objective function. Several special algorithms exist for this problem class, where a modified version of Simplex is one of them. (Hillier & Lieberman 2001)

Several elements in the natural gas supply chain have neither convex nor concave properties and give rise to *non-convex programs*. The compressor cost function, describing the compressor fuel cost as a function of input and output pressure (Martin et al. 2006) is such an example. Other examples are the Weymouth equation for bidirectional pipeline and the pooling problem. The pooling problem represents the splitting of a gas flow that consists of a mixture of different gas qualities. If for instance the flows from two fields meet in a node ( $i$ , the pool) that has two outgoing pipelines to nodes  $j$  and  $k$ , the challenge is to make sure these outgoing flows have the same gas quality. (1.1) represents this mathematically, where  $c_1 \in \mathcal{C}$  represents a gas component, for instance methane, from the set of all gas components  $\mathcal{C}$ .

$$\frac{f_{ijc_1}}{f_{ikc_1}} = \frac{f_{ijc}}{f_{ikc}} \quad c \in \mathcal{C} \tag{1.1}$$

Several approaches have been taken to solve this non-convex problem, see for instance Hellemo & Tomasgard (n.d.) for a recent overview.

*Mixed integer problems (MIP)* is a class of non-convex problems formed by introducing integrality or binary requirements on some of the variables of a LP. This ability to describe discreteness has a broad range of applications, for instance opening or closing a valve, deciding on the number of vessels to invest in, setting the direction of the flow in a bidirectional pipeline and thereby separating the non-convex Weymouth equation in two convex parts, and deciding a vessel's next destination on its route.

Vehicle routing and inventory routing problems has an extensive use of binary variables to decide on the routing and scheduling of a fleet of vehicles, for example LNG vessels. These are classical problems that are known to be hard to solve due to the exponential growth in possible ways of combining departures and arrivals in feasible schedules as the number of vehicles, locations and time periods grows. This is often referred to as the combinatorial explosion. Discrete variables can also be introduced in an approximation of non-linear functions. This is done by

Ulstein, Nygreen & Sagli (2007) and Rømo et al. (2009) who uses a discretization of possible split fractions between the outgoing flows in the pooling problem, and thereby transforms the problem into a MIP. The approach is developed further by Hellemo & Werner (2013).

A large variety of solution approaches has been proposed for MIP over the years. One category is heuristics, which is algorithms where the goal is to establish a good solution fast, but usually at the cost of not having any guarantee to ever converge to the globally optimal solution. The opposite of heuristics are exact approaches, where it can be mathematically proven that the solution process will find the optimal solution, but at the risk of a long solution time. Several approaches utilize decomposition techniques, where the problem is divided into smaller and easier solvable subproblems that can help to narrow the solution space for the original problem. Wolsey (1998) gives an introduction to integer programming and Conejo, Castillo, Minguez & García-Bertrand (2006) describes decomposition techniques for both MIP and general NLP problems.

The *Karush-Kuhn-Tucker optimality conditions (KKT)* are a constraint set derived from a problem. This set can be used to evaluate the optimality of a solution or guide the search for a solution. For most constrained differentiable NLPs, including LP and QP, a solution necessarily satisfies the KKT conditions to be a candidate for the optimal solution. For a convex problem satisfaction of the KKT conditions is sufficient to conclude that a solution is optimal.

KKT conditions take the form of a complementarity problem. A subclass, *mixed linear complementarity problems (MLCP)*, covers the KKT conditions of LP and QP and can be formulated as

$$\begin{aligned} Eu + Fv &\geq 0 \perp u \geq 0 \\ Gu + Hv &= 0, v \text{ free} \end{aligned} \tag{1.2}$$

where  $u$  and  $v$  are variable vectors and  $E, F, G$ , and  $H$  are parameter matrices. A generalization where some of the constraints include a nonlinear element is a *mixed complementarity problem (MCP)*, while a specialization without the equality constraint would turn the MLCP into a *linear complementarity problem (LCP)*. A defining feature of a complementarity problem is the complementarity constraint, represented with  $\perp$ , stating that either of the two expressions surrounding  $\perp$  should hold with equality. This constraint makes the problem non-convex and non-linear. Complementarity constraints can be reformulated with binary variables  $\gamma$ , a large constant  $M$ , and new constraints  $Eu + Fv \leq M\gamma$  and  $u \leq M(1-\gamma)$ . For an introduction to complementarity problems and the generalization called *variational inequalities (VI)*, see for instance Billups & Murty (2000) and Facchinei & Pang (2003).

Optimization models are best suited to model situations where all involved agents have a common overall objective or do not affect each others decisions (as



in a perfect competition situation). On the contrary, a complementarity problem can model equilibria between multiple agents with differing interests. This is achieved by letting the KKT conditions of agents' optimization problems, in addition to a set of market clearing conditions, form a complementarity problem. Gabriel & Smeers (2006) give a thorough explanation of how both a perfect competition equilibrium and a Nash-Cournot equilibrium for the natural gas supply chain can be formulated as complementarity problems.

*Bilevel programs (BP)*, or more generally *multilevel programs* are particularly suitable to model situations where different agents have different level of influence or different timing for their decisions. A classical example, that was the motivation for the first description of a BP, is the Stackelberg game where a leader makes a decision knowing the follower will observe his decision and optimize his own decision thereafter. The simplest form of a BP, which is a *linear bilevel program* can be formulated as

$$\begin{aligned} \max_x \quad & cx + dy \\ \text{s.t.} \quad & Ax + By \leq b \\ & y \in S(x) \end{aligned} \tag{1.3}$$

where

$$\begin{aligned} S(x) = \max_y \quad & fy \\ \text{s.t.} \quad & Gy \leq h - Kx \end{aligned} \tag{1.4}$$

(1.3) represents the leader's problem denoted upper level, and has the decision variable vector  $x$ . The upper level solution space and objective function are affected by the follower's decision problem that are represented by the lower level problem (1.4) with decision variable vector  $y$ .  $c, d, A, B, b, f, G, h$ , and  $K$  are parameters. Due to  $S(x)$  (1.3) can be non-convex even when all other elements of both upper and lower level are linear. A particular issue in bilevel programming is the ambiguity that arises if the lower level problem solution is not unique for a given  $x$ . This gives rise to the notion of optimistic and pessimistic solution concepts, where the lower level solution are selected from  $S(x)$  in accordance with or opposite to the upper level objective function, respectively. If the optimistic solution concept is chosen, (1.3) can be rephrased to a NLP by replacing  $y \in S(x)$  by the KKT conditions of (1.4). For further reading on BP see Dempe (2002) and Mersha (2008).

*Mathematical programs with equilibrium constraints (MPEC)* are a problem class closely related to BP. The difference is that the lower level problem of a MPEC is a variational inequality (the generalization of complementarity problems). Through a reformulation with KKT conditions, as mentioned above, a BP

is actually transformed into a MPEC. The ability to model lower level equilibria makes MPEC a generalization of BP, but on the other hand, the BP is more general in its ability to capture the pessimistic solution of a non-unique lower level (Dempe 2002). For a broader introduction to MPEC, see Luo, Pang & Ralph (1996).

Both BP and MPEC have been used in models of the natural gas business. This has become particularly relevant as the deregulation has given more agents with a clearer separation of roles in the supply chain. The cash-out problem is a typical example of a BP, where a natural gas shipper (leader) minimizes his imbalance penalty cost while taking into account how the system operator (follower) will try to balance the system and charge imbalance penalty costs (Kalashnikov, Pérez-Valdés, Tomasgard & Kalashnykova 2010, Dempe, Kalashnikov, Pérez-Valdés & Kalashnykova 2011). Siddiqui & Gabriel (2012) present an example of a MPEC used to model a Stackelberg game in the U.S. natural gas market, where the shale gas producers are Stackelberg leaders.

## Applications of OR

There is a broad variety of OR applications within the natural gas industry. I will here give an overview of different applications where optimization is a vital part. Hierarchical production planning, originating from management science, provides a taxonomy and framework that can be used to classify and characterize different applications. It defines three levels of decision making and planning, *strategic planning*, *tactical planning* and *operational control* initially named by Anthony (1965). I will include yet another level between tactical planning and operational control called *operational planning*. Strategic planning is characterized by long planning horizon, high level of aggregation and low frequency of replanning, as opposed to operational planning that is short term, detailed and frequent. Strategic planning typically relates to design and investment decisions, while tactical planning relates to resource allocation and planning for seasonal variations. Operational planning is the short-term planning of components in the supply chain and trade in short-term markets, seeking efficient adaptation to short-term variations. Operational control is about controlling the actual operation and is often part of automated systems for process control, for instance in closed-loop controllers. Planning horizons differs between industries. Within the natural gas industry strategic planning usually covers several decades with a yearly resolution, while tactical planning usually covers 1-3 years with weekly or monthly resolution. Operational planning usually has a horizon of weeks with hourly or daily resolution, and operational control usually cover minutes up to hours with resolutions down to less than a second. Naturally, long-term decision will affect the possibilities and constraints of decisions on shorter horizons, and

the operations constitute an evaluation of the tactical and strategic decisions. For a general introduction to hierarchical production planning, see Bitran & Tirupati (1993). Stremersch, Michalek & Hecq (2008) motivates for the different planning perspectives in the LNG industry seen from a business participant's perspective. Foss (2012) describes the planning hierarchy with focus on the production in fields. In the following overview I will not go further into operational control.

There is a substantial literature on gas market models, where the purpose is to analyze the impact of policies, competitive situations or large infrastructure developments. These are strategic issues and the models typically have a wide geographical coverage and models horizons of several decades. A heavily used perfect competition model is the The National Energy Modeling System (NEMS) for the U.S. energy sector that uses an iterative approach to integrate several modules, including a LP on natural gas (Gabriel, Kydes & Whitman 2001). Mathiesen, Roland & Thonstad (1987) and De Wolf & Smeers (1997) are early examples of a complementarity model and a stochastic MPEC, respectively, modeling imperfect supply side competition in the European natural gas industry. Boots, Rijkers & Hobbs (2004) present the GASTALE model for the European natural gas market with Cournot competition both among producers and traders, formulated as a MCP. This model is further extended by Lise & Hobbs (2008) who include transportation and storage capacity expansions in multiple years. The World Gas Model (WGM) is a MCP that covers the global natural gas market and assumes traders with market power. A thorough presentation of WGM is given in Egging (2010), and Gabriel, Rosendahl, Egging, Avetisyan & Siddiqui (2012) presents one of several analysis done by the model. Abada, Gabriel, Briat & Massol (2013) includes fuel substitution in a MCP for the European natural gas market.

The optimization of offshore oil and gas field infrastructure development is a strategic problem that has received considerable attention over several decades. Haugland, Hallefjord & Asheim (1988) and Sullivan (1988) focus on the modeling of reservoir and wells within a single field, while for instance Aboudi, Hallefjord, Helgesen, Helming, Jørnsten, Pettersen, Raum & Spence (1989) and Nygreen, Christiansen, Bjørkvoll, Haugen & Kristiansen (1998) and Carvalho & Pinto (2006) take a network perspective coordinating several fields with a common transportation and processing infrastructure. The models typically decide on which fields to develop at what time and with what capacity and planned production profile, and what pipelines and processing capacities to invest in. How uncertainty in market price and demand (Jørnsten 1992, Haugen 1996, Jonsbråten 1998) and in reservoir properties (Jonsbråten 1998, Goel & Grossmann 2004, Tarhan, Grossmann & Goel 2009) should affect these strategic decisions are also studied. Most commonly these models describe a steady state between each long-term decision, typically with a yearly granularity, while (Hellemo, Midthun,

Tomasgard & Werner 2013) points out the importance of also taking operational considerations and short-term variability and uncertainty into account when designing the infrastructure, and includes both strategic and operational decisions in their model. MIP is the dominating model class for this application, where discrete variables are used both for investment decisions, to approximate non-linear reservoir properties and to control the resolution of uncertainty when this is decision dependent.

Zhang & Zhu (1996) takes a narrower scope, discussing the pipeline dimensioning problem, and uses a BP reformulation to solve their problem. André (2010) extends this with several NLP and mixed integer non-linear problem formulations on pipeline network extensions and dimensioning for given supply and demand. A related MIP is described by Zheng & Pardalos (2010) who seek a cost minimizing plan for coordinated transmission network expansions and LNG regasification plant localization. Whereas Zheng & Pardalos (2010) jointly consider pipeline and LNG regasification investments, Werner, Uggen, Fodstad, Lium & Egging (n.d.) [Paper IV in this thesis] present a MIP for investment decisions solely within the LNG supply chain. This model includes both liquefaction, transportation and regasification investments, and accompanying the investments are decisions on contract mix for LNG purchase and sale. Sönmez, Kekre, Scheller-Wolf & Secomandi (2011) also model the LNG supply chain from liquefaction to regasification, however limited to one supply and one consumption location. They perform a strategic analysis of technology choice in capacity investment decisions where their technology options are conventional onshore regasification plants and onboard regasification. Simulation is used to evaluate the performance of the alternative investment options and MIP decides on technology choice and capacity.

Contract portfolio management and contract valuation are problems that can have both a strategic and tactical nature, depending on the duration of the contracts being analyzed. Also, it is common to use tactical models to evaluate the consequence of strategic choices, for instance evaluating the performance of a certain contract mix. There is an extensive literature on contract portfolio management problems seen from local distribution companies' point of view. These problems are linear problems minimizing supply costs while controlling the risk of being short on customer deliveries, usually modeling price or demand uncertainty (Avery, Brown, Rosenkranz & Wood 1992, Bopp, Kannan, Palocsay & Stevens 1996, Aouam, Rardin & Abrache 2010). Allevi, Bertocchi, Vespucci & Innorta (2007) and Maggioni, Vespucci, Allevi, Bertocchi & Innorta (2008) present both a deterministic and a stochastic contract portfolio management model that are nonlinear MIPs due to complex price structures and discrete customer choice. Guigues, Sagastizabal & Zubelli (2010) describes a similar stochastic MIP where the objective is to manage and price LNG contracts with cancellation options

as part of a supply mix together with other supply contracts, costly alternative fuels, and operation of storages and pipelines. Haurie, Smeers & Zaccour (1992) on the other hand, describes a stochastic LP and searches an optimal mix of gas contracts seen from a producer's point of view. They define a range of possible contracts with different duration, expected netback prices and price variations, and decide on contract mix to balance the portfolio's expected return and risk expressed as netback variance. Supply is limited by a production capacity that can be expanded cumulatively over time at a development cost, whereas no other transportation or production capacity limitations are explicitly addressed. The models in Tomasgard, Rømo, Fodstad & Midthun (2007), Fodstad, Midthun, Rømo & Tomasgard (2011), Fodstad, Uggen, Rømo, Lium, Stremersch & Hecq (2010/11) and Werner et al. (n.d.) [Papers I-IV] also use a portfolio approach seen from a producer's perspective.

A typical issue in tactical planning is how to handle elements that create dependencies between operations in different time periods. For instance Fodstad et al. (2011) [Paper II] discusses the allocation of natural gas over time in the Norwegian pipeline supply chain, where the cumulative daily production and transportation capacities exceeds yearly production concessions and future price and demand patterns are uncertain. A similar issue arises in the operation of seasonal storages, where one has to decide when to inject gas into and when to withdraw gas from storage taking into account price variations and uncertainty. Ejarque (2011) describes a dynamic programming model for a storage problem, and uses it to evaluate the cost of strategic restrictions from regulations requiring minimum safety stock levels.

In the LNG business the transportation by vessels creates a time dependency since a single voyage can take weeks. This motivates the use of annual delivery programs, which are yearly plans for how to fulfill contracted volumes with scheduled dates for loading and unloading in terminals, agreed upon by supplier and customer. Several MIPs have been developed to support the creation of such annual delivery programs based on variants of the inventory routing problem, that include upstream and downstream storage management in addition to vessel routing, see for instance Grønhaug & Christiansen (2009), Rakke, Stålhane, Moe, Christiansen, Andersson, Fagerholt & Norstad (2011) and Halvorsen-Weare, Fagerholt & Rönnqvist (2013). The LNGScheduler, presented in Fodstad et al. (2010/11) [Paper III] treats the same problem, but extends the scope along the supply chain to also capture interaction with downstream natural gas markets and gives a richer description of contracts.

Operational models often increase the level of detail when describing technical components and physical laws compared to tactical models. For instance Flores-Salazar, Vázquez-Román, Grossmann & Iglesias-Silva (2011) provide a detailed description of pressure dynamics in reservoirs, wells and pipelines. Their mixed

integer non-linear model describes a gas production system with multiple reservoirs, wells and platforms, and decides how to fulfill a daily demand through the decision of production rates on individual wells and pressure levels throughout the system. Selot, Kuok, Robinson, Mason & Barton (2008) and Foss & Halvorsen (2009) take a broader scope, modeling the upstream supply chain including a liquefaction plant. Selot et al. (2008) use a single period mixed integer non-linear non-convex model and describe both physical properties like pressure dynamics and gas blending and splitting, and contractual requirements in a network of fields and processing trains. Foss & Halvorsen (2009) on the other hand uses a NLP with a simpler network, but provide greater detail on the modeling of individual components in the liquefaction plant. Their planning horizon is 1-2 weeks with 3-6 hours time resolution.

Operational modeling of transportation and transmission network flows has received substantial attention. The objective of these models are usually to minimize operation costs by deciding flows and pressures in networks with sources, pipelines, valves, compressors and sinks. Two subgroups of network flow models are steady-state models and transient models, where the first ones require coarser time resolution to justify not modeling how pressure changes propagate through the pipelines (transient). For thorough presentations of network flow models, see Van der Hoeven (2004) and Moritz (2007). Rømo et al. (2009) present a MIP called GassOpt that extends the classical network flow models with fields and processing plants. GassOpt is designed to evaluate security of supply in the Norwegian supply chain in case of unplanned events in the system. Midthun et al. (2007) present a stochastic LP applied on the Norwegian supply chain. They evaluate the value of utilizing the capabilities of the transportation network as a flexible short-term storage, and thereby allow imbalance between injection and withdrawal. The possibility for a shipper of being imbalanced in the daily operations also gives rise to the previously presented cash-out problem presented in Kalashnikov & Ríos-Mercado (2006) and Kalashnikov et al. (2010).

## Optimization under uncertainty

An optimization model often requires a substantial amount of input data representing for instance capacities, prices, costs and efficiencies. Since the models usually are used to describe or plan for future situations, such data may be uncertain. In this situation a common approach is to do several model runs with different assumptions for the uncertain data, and through this search for insight in how to solve the underlying uncertain problem, by for instance looking for common properties in the solutions. Sensitivity analysis, what-if analysis, worst-case analysis and scenario analysis are all methods that build on this approach. A weakness of this approach is that the uncertainty is never explicitly repre-

sented in the model that is optimized, and as a consequence making decisions that provide flexibility or robustness for different future outcomes are not chosen unless this flexibility and robustness comes for free. A simple example illustrates this observation: if a person is offered to buy fire insurance for his house, where the period of insurance is the previous year, he would only be willing to pay for that insurance if his house actually did burn the previous year. Stochastic programming is an approach that explicitly represents uncertainty in the model formulation, and deterministic programming is the term representing the counterpart. The motivation for stochastic programming and a thorough discussion of the differences between stochastic and deterministic programming is given in Wallace (2002), while Kall & Wallace (1994) and Birge & Louveaux (1997) give more mathematical introductions to the topic.

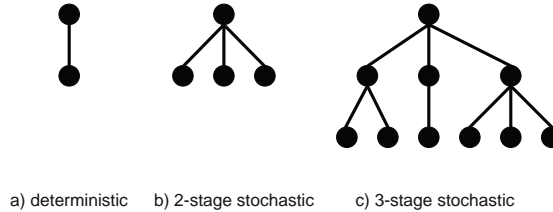
A deterministic linear programming program can for example have the form:

$$\begin{aligned}
 \max_{x,y} \quad & cx_1 + dx_2 \\
 \text{s.t.} \quad & Ax_1 \leq b \\
 & Tx_1 + Wx_2 \leq h \\
 & x_1 \geq 0 \\
 & x_2 \geq 0
 \end{aligned} \tag{1.5}$$

where  $x_1$  and  $x_2$  are vectors of decision variables for two different points in time,  $c$  and  $d$  are objective function coefficients,  $A, T$ , and  $W$  are constraint matrix coefficients, and  $b$  and  $h$  are right-hand-side coefficients. If  $d, T, W$ , and  $h$  are uncertain parameters, a stochastic programming program can be formulated as:

$$\begin{aligned}
 \max_{x,y_s} \quad & cx_1 + \sum_{s \in \mathcal{S}} p_s d_s x_{2,s} \\
 \text{s.t.} \quad & Ax_1 \leq b \\
 & Tx_1 + Wx_{2,s} \leq h_s \quad s \in \mathcal{S} \\
 & x_1 \geq 0 \\
 & x_{2,s} \geq 0 \quad s \in \mathcal{S}
 \end{aligned} \tag{1.6}$$

This problem is denoted a 2-stage stochastic problem or recourse problem. The set of scenarios,  $\mathcal{S}$ , represents a discretization of the possible outcomes of the uncertain parameters with matching probabilities,  $p_s$ . The first stage decisions that are taken before the outcome of the uncertain parameters becomes known are represented with  $x_1$ . The second stage decisions, or recourse decisions, can be adapted to each scenario since they are taken after the uncertainty is resolved, and are represented with  $x_{2,s}$ . (1.6) can be generalized to a multistage stochastic program, with repeated resolution of uncertainty and recourse decisions.



**Figure 1.5:** Examples of small scenario trees

The decision and information structure of a model can be illustrated by a scenario tree, where nodes represent decision points and branching illustrates uncertainty. Each branching separates one stage from the next. A path from the root node to a leaf node is called a scenario. Figure 1.5 illustrates examples of small scenario trees for a deterministic problem, a 2-stage and a 3-stage problem. See Kaut, Midthun, Werner, Tomasgard, Hellemo & Fodstad (2013) for a discussion of scenario trees structures.

Each node within a stage in a scenario tree contains an outcome of the uncertainty resolved in that stage and the probability of this outcome. Scenario generation is the process of filling the scenario tree with these outcomes, which usually implies making an approximation of the true probability distributions for the uncertain parameters and the interdependencies between them (like correlations). For multistage problems the scenario generation also needs to handle conditional distributions to capture the time dependency between outcomes within a scenario. Several methods have been suggested for scenario generation, see for instance Dupačová, Consigli & Wallace (2000), Kaut & Wallace (2007) and Pflug & Pichler (2011). The number of scenarios in a scenario tree directly affects the size of the stochastic model since a duplicate of the decision variables and constraints of a stage is needed for each node within that stage. Therefore a vital trade-off when generating scenario trees is to minimize the number of scenarios while still describing the important characteristics of the underlying uncertain parameters. Römisch (2009) and Morales, Pineda, Conejo & Carri (2009) present scenario reduction techniques that seek a guided reduction in scenario tree size. Lium & Kaut (2006) discuss evaluation of scenario generation approaches and scenario tree sizes, using measures of stability in the stochastic model results for different scenario trees.

In stochastic programming problem sizes grow rapidly as the number of uncertain variables or stages are increased, as can be seen even by the small examples in Figure 1.5. This has given rise to substantial work on development of solution methods particularly designed to solve stochastic programs, for instance Van Slyke & Wets (1969), Carøe & Schultz (1999), Kleywegt, Shapiro & Homem-



De-Mello (2001), Escudero (2009) and Kuhn, Wieseemann & Georghiou (2011). These often utilize the particular structure of stochastic programs, with similar subproblems for each node in a stage being linked together by common predecessor nodes. Scenario decomposition is an example of such an algorithm, which is described further and used in Shim, Fodstad, Gabriel & Tomasgard (2012) [Paper VII]. For introductions to stochastic programming algorithms, see for example Birge & Louveaux (1997) and Schultz (2003).

## Portfolio and Supply Chain Optimization

All applications presented in this thesis take a *portfolio* and *supply chain perspective* to the optimization. These two terms are closely related, but they have different origin and slightly different meaning. Portfolio optimization has its origin from finance, and its aim is to put together a portfolio of financial assets in a best possible way taking both value and uncertainty into account. Supply chain optimization has its origin from the introduction of OR methods into the supply chain management tradition. This tradition argues for integration along the supply chain to improve systemwide performance, with a main focus on efficient logistics and cost minimization.<sup>5</sup> Common for the two terms are the focus on seeing several units together in a system, rather than optimizing the performance of the units independently, and therefore *system perspective* will be used to describe the union of both terms here.

While assets are financial instruments in a financial portfolio, it can cover a whole range of physical and non-physical elements in the natural gas value chain. Assets treated in this thesis are production capacities, processing capacities, physical and contracted transportation capacities and natural gas sales and purchase contracts.

The deregulation processes in the British and European natural gas industries have brought about market hubs with short-term trade and a separation of responsibilities between the TSO and producers. At least in theory, this process could make the system perspective superfluous. That would be the situation if each asset could be planned and operated independently using the price in a perfect market as guidance, and without any limitations in the use of this market. Bjørkvoll, Fleten, Nowak, Tomasgard & Wallace (2001) and Wallace & Fleten (2003) illustrate this decoupling for the scheduling of power producing units. Despite this development towards a deregulated market, I argue that there are several issues that make a system perspective valuable for the participants in the Norwegian natural gas export system. Generally, these are bottlenecks that limit the access to a market or system effects that interlink assets in dynamic ways.

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<sup>5</sup>The term value chain optimization is largely overlapping with supply chain optimization, and these will be used as synonyms in this thesis.

A producer meets transportation capacity bottlenecks that separate the production capacities from markets. Certainly there is a transportation market that allocates transportation capacity between the producers, but most of the capacity is sold in the primary market where tariffs are set a priori, which means it does not give sufficient coordinating signals to operate the fields independently. Another market imperfection is contracts where the producer is obliged to deliver gas from own production, called equity gas, rather than gas bought in the market. There can also be delivery contracts located upstream of any market, which naturally means delivery in such contracts need to be coordinated with upstream production. And even when a market can be reached without any bottleneck, the liquidity in the market can be limited, which is the situation for several of the immature European markets. In these markets the amounts that can be sold without affecting the price is limited, and as pointed out by Wallace & Fleten (2003), a gap between achievable purchase and sales prices disqualifies the decoupling argument.

The two most important causes for system effects in the transportation network are pressure – flow dynamics and gas quality. Gas flows through pipelines from the high pressure end to the low pressure end. The capacity depends on pipeline properties and the pressure difference from inlet to outlet, and this relation is described in the Weymouth equations presented on page 8. As shown by Midthun, Bjørndal & Tomasgard (2009) changing the pressure and flow in one part of the network can change the capacity in other parts of the network, which gives dynamic dependencies between different parts of the system. In a similar way gas quality requirements create dependencies in the system. Different fields produce gas of different quality that is often off the specification for gas deliveries. In addition to processing, blending sour gas from one source with sweet gas from another source is a measure to reach the delivery requirements. Such dependencies, or system effects, requires a system perspective on the transportation network by the TSO to optimize the network performance.

Also the operation of a LNG supply chain is affected by bottlenecks and system effects. Scarce processing, transportation, or storage capacity can cause bottlenecks in the supply chain. Further, the discrete nature of transportation by vessels gives a combinatorial puzzle that provides profound system effects. For instance reducing the production rate in a liquefaction plant can delay a vessel which again delays delivery or can cause a rerouting to avoid emptying the storage in the regasification plant. Some contract clauses also cause dependencies in the supply chain. These are destination clauses that limit the share of LNG from some liquefaction plants that can be delivered to certain regasification plants. Further, there can be profit sharing or netback pricing mechanisms that make the upstream purchase price dependent on the destination or sales price (see Paper III page 123 for a detailed presentation of the contract clauses).

## 1.3 Papers and Contributions

A unifying property among the applications presented in this thesis is the system perspective, where we seek to integrate the business understanding found in the portfolio management tradition with the understanding of logistical challenges that is found both in the supply chain management tradition and in OR literature on gas transportation. In the part of the industry using pipeline transportation these perspectives are partly decoupled due to the deregulation with separation of shippers and TSO and an increasing number of market hubs. As previously argued, this deregulation is not sufficient to remove the value of a system perspective, but the separation of roles motivates new modeling that reflects the different roles and the transportation market that links the agents in the system.

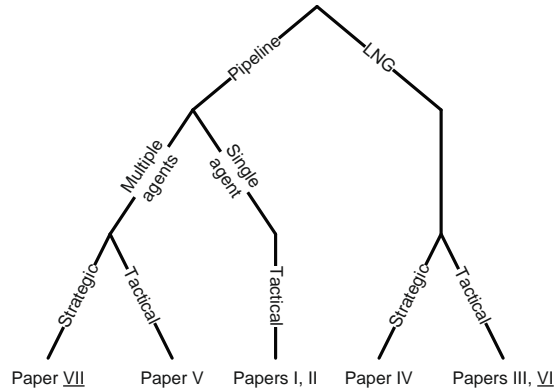
An overview of the papers presented in this thesis is given in Figure 1.6. Papers I-V focus on modeling, while the two last are algorithmic papers. There are four papers on natural gas transportation by pipeline [Papers I, II, V and VII] and three papers on LNG transportation [Papers III, IV and VI]. Both transportation modes are represented with both strategic and tactical models. The models for the pipeline supply chain can further be separated into those explicitly representing multiple agents with separate roles and those taking a single agent's perspective. The first algorithmic paper, Paper VI, solves a model on tactical planning in the LNG business that is presented in Paper III. The model motivating the algorithm presented in Paper VII is not presented in a separate paper, but a short description is given within the algorithmic paper.

All papers in the thesis are published, or in the review process, in international journals or books with peer review. A short presentation of each paper is given in this section, while each complete paper is presented in the next seven chapters.

### Paper I: Optimization Models for the Natural Gas Value Chain

In this paper we give an introduction to the natural gas supply chain. We present ways to model both physical and commercial parts of the supply chain and discuss the importance of using a portfolio perspective when planning in this supply chain. The modeling covers the pressure-flow dynamics in pipelines, bidirectional pipelines, compressors, gas quality and the pooling problem, processing plants, storages, long-term contracts, market trade and valuation of gas reserves in reservoirs. The paper is mainly based on two models for operational and tactical planning, GassOpt and Venoga, that are developed by SINTEF and NTNU. It does not contain any numerical analysis.

I did the implementation and contributed with an equal part of the modeling of Venoga, a stochastic portfolio model. The model unites technical and commercial modeling, both presented in the paper. Co-authors are my supervisor Asgeir



**Figure 1.6:** Overview of the application area for each of the papers presented in the following chapters of the thesis. The two underlined paper numbers indicate algorithmic papers, while the other papers have a modeling focus.

Tomasgard, and Frode Rømo and Kjetil Trovik Midthun in SINTEF. The paper is published in Hasle, Lie and Quak (eds.), *Geometric Modelling, Numerical Simulation and Optimization*, Springer Verlag, 2007.

## Paper II: Tactical Portfolio Planning in the Natural Gas Supply Chain

In this paper we present a version of the Venoga model that is designed for tactical planning seen from a producer’s perspective. The model covers production, booking of transportation capacity, long-term contract obligations and spot market trade. The main contribution in the paper is a thorough discussion of the value of incorporating short-term market trade in the tactical planning, and to use stochastic programming to properly describe market uncertainty. We show through numerical examples how spot trade can provide flexibility through geographical and time swaps, and that uncertain contract obligations, contract prices and spot prices require robust decisions and make flexibility particularly valuable.

I have had an equal part in modeling and implementing the model and in writing the paper. Additionally I have performed the analysis. Co-authors are Kjetil Trovik Midthun and Frode Rømo in SINTEF and my supervisor Asgeir Tomasgard. The paper is published in Bertocchi, Consigli, Dempster (eds.), *Stochastic Optimization Methods in Finance and Energy*, International Series in Operations Research & Management Science 163, Springer Science+Business

Media, 2011.

### **Paper III: LNGScheduler: A Rich Model for Coordinating Vessel Routing, Inventories and Trade in the LNG-Supply Chain**

In this paper we present a model for tactical planning in the LNG supply chain. Previous publications on this topic use vehicle routing problems (VRP), where the main focus is on the efficient utilization of the vessel fleet, or the extension of VRP into inventory routing problems, where also inventory management onboard and in the terminals are modeled. We extend the scope even further, by including decision on liquefaction rates, regasification rates, downstream spot markets and several contract types. We show through numerical analysis that taking a broader supply chain perspective can change the decisions relative to the traditional praxis and increase the profit margins.

I have done a major part of modeling and implementing the model, and an equal part in the analysis and writing the paper. Co-authors are Kristin Tolstad Uggen, Frode Rømo, and Arnt-Gunnar Lium in SINTEF and Geert Stremersch and Stéphane Hecq in GDFSuez. The paper is published in *The Journal of Energy Markets*, **3** (4), 31-64, Winter 2010/11.

### **Paper IV: Stochastic Mixed Integer Programming for Integrated Portfolio Planning in the LNG Business**

In this paper we present a model for strategic planning in the LNG supply chain. The model has a portfolio perspective and suggests investment decisions on liquefaction and regasification terminals, vessels and long-term contracts. It also approximates the operation of the resulting supply chain, with liquefaction, transportation, regasification, purchase and sales decisions. The future prices are treated as uncertain parameters in the stochastic model. We present two small numerical examples that, respectively, illustrate the effect of a portfolio view benchmarked with the traditional net present value method, and compare the difference between our stochastic model and a deterministic model.

I have contributed an equal part in modeling and writing the paper. Co-authors are Adrian Werner, Kristin Tolstad Uggen, and Arnt-Gunnar Lium in SINTEF and Ruud Egging in SINTEF and NTNU. The paper is accepted for publication in *Energy Journal*.

### **Paper V: Adding Flexibility in a Natural Gas Transportation Network Using Interruptible Transportation Services**

In this paper we analyze the effect of supplementing firm transportation services with interruptible transportation services in a natural gas network with capacity reducing events. We develop a model framework of stochastic optimization models that mimics a decision sequence for a shipper and a transportation system operator. We show through numerical analysis on data from the Norwegian Continental Shelf that offering interruptible services increases the throughput in the system.

I have done the implementation and analysis, and have contributed with an equal part in modeling and writing the paper. Co-authors are Kjetil Trovik Midthun in SINTEF and my supervisor Asgeir Tomasgard. The paper is submitted to an international journal.

### **Paper VI: Using and Extending Fix-and-Relax to Solve Maritime Inventory Routing Problems**

In this paper we present a new algorithm to solve inventory routing problems, such as the LNGScheduler model presented in Chapter 4. The algorithm is a heuristic based on fix-and-relax time decomposition, with extensions to reduce solution time and improve solution quality. Numerical tests on four cases for the LNGScheduler are presented and show that the algorithm reduces solution time considerably relative to solving the whole problem in a general MIP solver, at the cost of slightly worse objective function values.

I have contributed with a minor part of algorithmic development and implementation, and an equal part in analysis and writing the paper. Co-authors are Kristin Tolstad Uggen and Vibeke Stærkeby Nørstebø in SINTEF. The paper is published in *TOP*, and was made available online on March 2011.

### **Paper VII: A Branch-and-Bound Method for Discretely-Constrained Mathematical Programs with Equilibrium Constraints**

In this paper we develop an exact algorithm to solve mathematical programs with equilibrium constraints with discrete variables in the upper level. The algorithm decomposes a MIP representation of the problem into a Benders master and sub problem. A branch-and-bound scheme is used to partition the non-convex domain of the Benders sub problem into convex subdomains, and Lagrangean relaxation is used for bounding in this scheme. We show how scenario decomposition can be used to decompose the problem further in case of a stochastic lower level

problem. Numerical results are presented for random generated test instances, where the algorithm outperforms the previously published heuristic counterpart for instances of various sizes. We also briefly describe an application from the natural gas supply chain formulated as a discretely-constrained MPEC. This is a strategic model with investments in a natural gas pipeline network in the upper level. The lower level describes the operation of the network with a TSO and multiple shippers that face price and demand uncertainty. Numerical results are presented for small instances of this model.

I have contributed with a major part of the algorithm design and implementation for stochastic problems and the natural gas application, a minor part of the remaining parts of algorithm design, and an equal part in writing the paper. Co-authors are Yohan Shim and Steven A. Gabriel at University of Maryland, US, and my supervisor Asgeir Tomasgard. The paper is published in *Annals of Operations Research*, and was made available online from July 2012.





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## Paper I

Asgeir Tomasgard, Frode Rømo, Marte Fodstad and Kjetil Midthun:

### *Optimization Models for the Natural Gas Value Chain*

*Chapter in G. Hasle, K.-A. Lie, E. Quak (eds.): Geometric  
Modelling, Numerical Simulation and Optimization, Springer  
Verlag, 2007*



## Chapter 2

# Optimization Models for the Natural Gas Value Chain

### **Abstract:**

In this chapter we give an introduction to modelling the natural gas value chain including production, transportation, processing, contracts and markets. The presentation gives insight in the complexity of planning in the natural gas supply chain and how optimization can help decision makers in a natural gas company coordinate the different activities. We present an integrated view from the perspective of an upstream company. The chapter starts with describing how to model natural gas transportation and storage, and at the end we present a stochastic portfolio optimization model for the natural gas value chain in a liberalized market.

## 2.1 Introduction

The models in this chapter are based on the authors experience from making decision support tools for the Norwegian gas industry. Our focus is on describing modeling techniques and important technological issues, rather than a very detailed representation needed for commercial models. We study the natural gas value chain seen from the point of view of an upstream company with a portfolio of production fields. Such a company should plan its operations considering long term contract obligations, the short term markets and transportation capacity booking. In particular we describe how the operations and planning are influenced by the existence of spot markets and forward markets. For the models to make sense it is also critical to include the technological characteristics of natural gas transportation and processing. We therefore give a set of models where the interplay between the technological characteristics of natural gas and the markets are highlighted. In these models the economical content and the understanding of gas markets is essential.

We structure the paper by gradually introducing the different levels of the supply chain. We start by describing the most important components of the natural gas value chain in Section 2.2. Then in Section 2.3 we focus on how to model natural gas transportation in a steady-state situation. This is the type of transportation models suitable for planning problems with time resolution weeks,

months or years. In Section 2.4 we introduce gas storages and in Section 2.5 we describe a portfolio perspective and start investigating the integrated supply chain view. Here we introduce short term markets. In Section 2.6 we see how the spot-markets can be used to price natural gas storage capacity and indicate how to estimate the terminal value of natural gas still in storages or in reservoirs at the end of the planning horizon using the concept of an Expected Gas Value Function. An appendix describing all notation used in the paper is included at the end. All together these sections will give a supply chain optimization model with an integrated view of the value chain, from production, via transportation and processing to contract management and gas sales.

## 2.2 The Natural Gas Value Chain

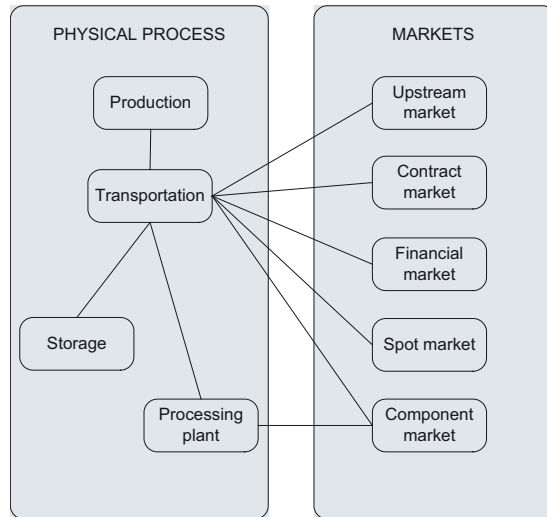
Here we give a brief description of the different elements of the natural gas value chain on the Norwegian continental shelf: production, transportation, processing, contract management and sales. The first action is to transport the natural gas from production fields to processing plants or transportation hubs where gas from different fields is mixed. Rich gas components are extracted and sold in separate markets. The remaining dry gas is transported to the import terminals in UK or on the European continent. In these hubs bilateral contracts and spot-trades are settled. Also upstream markets exist, where the gas is sold before it is transported to the import terminals. We focus on the value chain of a producing company, hence the issues of transmission and distribution to end customers are not considered.

In Figure 2.1 we show the main components of the natural gas export value chain. Before we go in detail on these we give a short summary of the main effects of liberalization and regulation in the European gas market.

### Production

Production of natural gas takes place in production fields. Often these fields have several owners, and each owner has production rights that are regulated by lifting agreements. Typically a producer's rights allow him to produce between a minimum level of production and a maximum level of production within a set of time periods of different length. This production band may be flexible so that gas can be transferred between periods within predefined limits. Normally such production intervals are defined for single days, for years, and for intermediate periods in between like weeks and months.

Much of the natural gas produced is traditionally committed to take-or pay contracts where the buyer has agreed to take a volume in a given import terminal for a sequence of years. Again there is flexibility on when to take the gas within a



**Figure 2.1:** Important objects in the natural gas value chain

year (or other time periods) and typically the daily offtake is within a minimum and maximum level. The customer nominates volumes within the take-or-pay agreements, and the producer has to deliver. These nominations are often done weekly, with final nomination the day before production. In take-or-pay contracts the price is usually indexed to other commodities like oil, to temperature and several other parameters.

## Transportation and Processing

Natural gas is transported in pipelines by using compressors to create a higher pressure in the originating end of a pipeline, so that molecules will flow towards the end. Several pipelines may meet in a node in the transportation network. They may have different pressure at the end of the pipeline, but the input pressure of all pipelines going out of a transportation node must be smaller than the smallest end pressure of pipelines coming into the node, unless there is a compressor in the node.

An example of an export network for natural gas is the one you find at the Norwegian continental shelf which consists of 6600 km of pipelines. Here natural gas from different fields have different quality, in terms of energy content and its chemical composition (methane, ethane, propane and several more). Hence when natural gas from different fields is blended in the transportation network, it is critical to either keep track of the energy content of the blend or the total

content of each natural gas component.

Some of the components can be extracted from the rich gas in processing plants. Processing facilities separate the rich gas into its various components. The components are liquefied petroleum gases like ethane, propane and butanes, which are exported by ship to separate commodity markets. The remaining dry gas (methane and some ethane) is transported in pipelines to import terminals in the UK, France, Belgium and Germany.

The organization of transportation markets varies a lot from region to region. One will often find that existing transportation rights already accounts for much of the available transportation capacity in the network. The Gas Directive (European Union 1998) enforces undiscriminating third party access to the remaining capacity (see Section *Liberalization and Regulation* on page 43 for a discussion of The Gas Directive). One way of resolving this is to introduce primary markets for transportation capacity where capacity can be booked. In some cases a fixed tariff is used for zones or for pipelines, in other cases bids are given for capacity and the market settled by some auction mechanism. In all cases the market is cleared and capacity is allocated by given transparent rules. In a secondary market with shorter time horizons transportation capacity is balanced with transportation needs for the different shippers.

In this paper we will only focus on the utilization of transportation capacity, while the capacity allocation regime and tariff regime is not discussed. For a further discussion on these topics see Dahl, Rømo & Tomasgard (2003).

## Storage

There exist several types of natural gas storages. Abandoned oil and gas fields have high capacity and thereby a cost advantage. They also have low risk as geological data are known. In aquifers water is replaced with gas. They have higher risk as seismic investigation is necessary. Salt caverns are underground storage tanks washed out from salt layers. They typically have high costs. Injection rates, capacities, withdrawal rates and characteristics depending on filling rate vary between the types. Storages are important in planning models because they allow us to store natural gas close to the market and thereby use them to exploit spot-market variations. They also allow producers to produce in time periods where demand is low and to thereby utilize the available transportation capacity. Also they can be used as seasonal storages to smooth out seasonal effects. Whether storage is used as to avoid bottlenecks in the system in high demand periods or to utilize market possibilities, today's storage capacity is very limited when compared to the total production volumes.



## **Import Terminals and Markets**

The import terminals are landing facilities for natural gas where the export pipelines end. Natural gas is delivered here according to specification on minimum and maximum pressure and energy content. These characteristics are often specified by the contracts as terms of delivery. Further transportation from the import terminals are taken on by the buyer using a transmission network to distribute the gas to the end customers.

Originally these terminals were the points of deliverance for the many take-or-pay contracts. Recently these terminals have also been the location of the growing spot markets for natural gas and for financial derivatives on the spot market. The leading European hubs in terms of liquidity are the National Balancing Point in Great Britain, TTF in the Netherlands and Zeebrugge in Belgium.

A different type of markets is also emerging upstream in the pipeline network. The main idea here is to have standardized trading mechanisms for natural gas at some important locations in the network to be able to provide an additional flexibility for the producers. Upstream markets are used to perform trades of natural gas before transportation takes place. They are useful because there is a need for having standardized mechanisms to exchange gas between producers. They include additional flexibility for producers in terms of being able to stay within the limits of their own lifting agreements, transportation capacities and contract commitments in case of unexpected events or in case overselling or underselling of natural gas has occurred. The buyer of gas upstream also has the responsibility to transport the gas to downstream markets. Upstream markets are not as well developed as the other markets. Still the idea is old and the former variant was the less standardized bilateral long term swing agreements between different producers, allowing fields with little flexibility an option to draw gas from fields with more flexibility in volumes.

## **Liberalization and Regulation**

The European natural gas industry has developed rapidly over the past thirty years. The European Commission has worked toward strengthening the competition within the complete gas- and energy value chain. A breakthrough in this process came on the 22nd of June 1998 when the gas directive was passed in the European Commission (European Commission & Transport 2002). In the directive a stepwise liberalization of the European gas market is described. The key components of the gas directive are third party access to all transportation installations, division of activities within the firms in the value chain (physically or by accounting) and the possibility for certain consumers to obtain their gas from the supplier of their choice. The directive was followed by a second gas directive in 2003 (European Union 2003) which moved another step towards liberalization.

Another implication of the gas directive and of EU competition laws was the 2002 closing down of the Gas Negotiation Committee (GFU), the forum for coordinated gas sales from the Norwegian continental shelf. The GFU formerly coordinated the supply of Norwegian natural gas producers Statoil and Hydro. Now the sales are company based and rarely linked to a specific production field.

An expected result from these changes is that short-term markets will evolve for natural gas. Though liquidity is still low, there are already clear signs indicating that short-term contracts and spot-trades will play an important role in the future. The prior market structure is dominated by long-term agreements and thus minimizes the uncertainty for the participants. In practice the producers take the price risk, as prices are fixed towards various indexes, while the buyers take the volume risks by going into long term agreements. The new markets will include short-term bilateral contracts, spot markets and financial markets. The introduction of short-term markets will most likely also lead to higher volatility and thus higher uncertainty.

Abolishment of the GFU-system and the introduction of a system where the individual companies are responsible for disposal of their own gas reserves called for a new access and tariff regime in the transportation network. The first step in the Norwegian transportation system was taken with the creation of Gassco AS in May 2001 under the provisions of a Norwegian White Paper. Gassco is assigned all the operator's responsibilities warranted in the Norwegian Petroleum Law and related Regulations. As a State owned company, Gassco AS should operate independently and impartially and offer equal services to all shippers. Systems operated by Gassco are the rich and dry gas systems previously operated by Statoil, Norsk Hydro and TotalFinaElf.

The models presented in this paper are simplified variants of models developed in co-operation with Gassco and Statoil to deal with the changes mentioned above.

## **2.3 A Natural Gas Transportation Model**

When modeling natural gas pipeline flow it is important to have a conscious view on how time and the dynamics of gas flow should be handled. The modeling of natural gas flow in continuous time has clear links to the process control paradigm (Hofsten 2000). Within this paradigm one normally uses active control, to operate the system according to a predetermined load and supply, finding a sequence of control actions which leads the system to a target state. The control regime often focuses on single processes or single components in the network. For our purpose we need to model a system of pipelines with a set of production fields, processing plants and markets. The natural choice is to look at mixed integer programming models from the modeling paradigm of mathematical pro-

gramming. Here time is discretized. If the resolution of time periods is minutes or hours there is a need to model the transient behavior of natural gas. Some attempts on optimizing the transient behavior of a system of natural gas pipelines are Westphalen (2004) and Nowak & Westphalen (2003), but only systems of limited size and complexity can be handled. To be able to handle the complexity needed for our models, we leave the concept of modeling the transient behavior of natural gas and approximate the time dimension by discrete time periods of such length that steady-state descriptions of the flow will be adequate. When the time resolution of the model are months, weeks, and maybe days, rather than minutes and hours, we can assume that the system is in a steady-state in each time period. The mathematical optimization models used to describe natural gas flow in the case of steady-state models are typical non-linear and non-convex. An approach using a non-linear formulation of the mathematical models is illustrated in De Wolf & Smeers (2000). We present here a linearized model based on mixed integer programming to optimize routing of natural gas in pipeline networks. We base our presentation on work done on linearization from Rømo, Tomasgard & Nowak (2004). Several examples on linearization of natural gas flow exist in the literature. For a recent PhD thesis on linearization of natural gas flow see Van der Hoeven (2004).

In this paper we describe the essential constraints needed to model the technological characteristics of natural gas flow in a steady-state setting. Issues like pressure, gas quality and gas components are dealt with from a pipeline transportation perspective. More detailed models very similar to the one we present here are today in use by Statoil and Gassco in the software package GassOpt developed by SINTEF. GassOpt is mainly used by the operator of the gas transportation system in the Norwegian sector of the North Sea. They are obliged to verify the delivery capabilities and robustness of the pipeline system transporting natural gas to European markets.

In the model presented in this section, we will focus on the transportation alone with the main purpose to meet demand for transportation generated by planned production profiles for the different fields. This typically represents the situation facing the neutral operator. The pipeline system is a natural monopoly, and is controlled by the authorities. This verification is also of strategic importance for the independent producers and customers in Germany, Belgium and France. The security of supply will influence the price possible to achieve for long term contracts, and contribute to infrastructure investment decisions, and GassOpt is one of the tools used to ensure maximum utilization of the infrastructure.

GassOpt itself focuses on analyzes of transportation possibilities. It can be used for optimal routing decisions from a flow maximization perspective. Also it is used to reroute natural gas when unexpected incidents lead to reduced capacity (in production units or pipeline). Thirdly, it can be applied at more tactical/op-

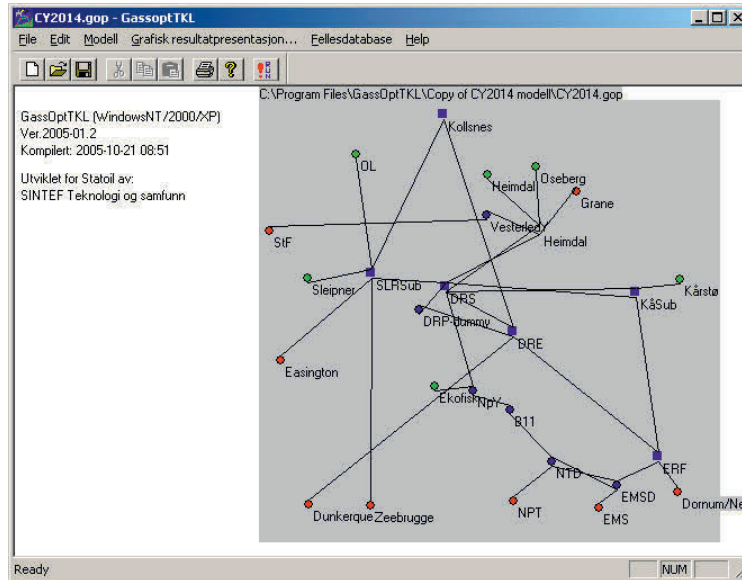


Figure 2.2: Network presentation in GassOpt

erational level by a commercial player in capacity planning and capacity booking.

In this section we present a static model of one period. Demand for natural gas in the import terminal is assumed to be aggregated over the contracts in the terminals and planned production volumes given as constants to represent the license holders' production plans. So the main task of this model is to operate the transportation network to make sure demand is met by the planned production. In Section 2.4 we extend the model with several time periods and storage capabilities. In Section 2.5 we include contracts, markets and a portfolio perspective on managing the natural gas supply chain with stochastic prices and demand.

### The GassOpt Modeling Interface

In GassOpt, the underlying physical network is represented in a graphical modeling environment with nodes and arcs. The modeling tool is hierarchical and applies to general network-configurations. Figure 2.2 indicates the network complexity for the North Sea network. The squared nodes contain subsystems with further nodes and pipelines. When modeling the North Sea system we need approximately 75 nodes and 100 arcs to represent the network.

## The GassOpt Mathematical Model

This network model includes flow balances, blending of different gas qualities from different fields, processing nodes for extracting components of the natural gas, compressor nodes, node pressures and the nonlinear nature of pressure drop in pipelines. The model describes a steady-state situation where the network is in equilibrium in terms of pressures and natural gas mix. It is typically the kind of model used to model situations where flows are aggregated over a given time period. When the time period gets short enough, for example hours or minutes, this steady-state description will not be good enough because of the need to describe the transient behavior of natural gas flow. The objective for the optimization model is to ensure optimal routing and mixing of natural gas.

The model should make sure the nominated volumes are delivered to the import terminals within a time period. This objective can be achieved in several ways. Penalties are introduced in the objective function to influence the impact of the following goals:

1. Maintain planned production from the producers, where this is physically possible.
2. Deliver natural gas which meets quality requirements in terms of energy content.
3. Deliver within the pressure requirements in the contracts.
4. Minimize the use of energy needed in order to deliver the natural gas to the customers by minimizing the pressure variables.

A typical optimization case describes a specified state of the network, including expected production and demand (characterized by volume and quality), shutdown situations and turn-up capacity (additional available but unplanned production capacity) from production fields. In a normal situation, there will be several possible strategies to deliver the maximum amount of gas to the customers. To make the model generate and report these realistic flows, we have introduced penalty costs in the objective function on deviation from planned production, quality requirements, pressure agreements and the energy use. These penalty costs can of course theoretically interfere with and prevent us to achieve the main goal, to deliver in accordance with the demand of the customers. The tests we have performed on the full North Sea network, show that this ‘multi-criteria’ aspect does not sacrifice much of the maximal flow potential, but is rather used to choose between alternative solutions with about the same flow. In a fault situation, for example if a field or pipeline is down, the model will prioritize to deliver the nominated volumes in the import terminals. For more information about multi-criteria decision making, see for instance Rardin (1998).

Seen from an operator's point of view the model tries to meet the customer's requirements for a given state of the network: either by optimal routing of gas or by turning up production in fields with flexibility on the production side. In the last case we say that we use turn-up capacity, which is available in some fields with flexible production characteristics.

## Sets

Below the sets used in the mathematical description of the model is presented.

- $\mathcal{N}$  The set of all nodes in the network.
- $\mathcal{B}$  The set of nodes where gas flows are splitted into two or more pipelines.
- $\mathcal{M}$  Nodes with buyers of natural gas: typically import terminals.
- $\mathcal{I}(n)$  The set of nodes with pipelines going into node  $n$ .
- $\mathcal{O}(n)$  The set of nodes with pipelines going out of node  $n$ .
- $\mathcal{R}$  The set of nodes with processing capabilities.
- $\mathcal{S}$  The set of nodes with storage facilities.
- $\mathcal{K}(b)$  The set of contracts in node  $b \in \mathcal{B}$ .
- $\mathcal{C}$  The set of components defining the chemical content of the natural gas.
- $\mathcal{T}$  The set of time periods included in the model.
- $\mathcal{L}$  The set of breakpoints used to linearize the Weymouth equation.
- $\mathcal{Z}$  The set of split percentages used to discretize possible split fractions in split-nodes of the network.
- $\mathcal{Y}$  The number of discretized storage and injection rate levels used to linearize storage characteristics.

## Objective Function

Our goal is to route the gas flow through the network, in order to meet demand in accordance with contractual obligations (volume, quality and pressure). In the formulation given below, variable  $f_{im}$  is the flow of gas from node  $i$  into market node  $m$ ,  $p_{ij}^{in}$  is the pressure into the pipeline going from node  $i$  to  $j$ ,  $\epsilon_m^+$  and  $\epsilon_m^-$  is the positive and negative deviation from the contracted pressure level respectively,  $\Delta_g^+$  and  $\Delta_g^-$  represents underproduction and the use of turn-up in relation to the planned production in field  $g$ ,  $\delta_m^{l-}$  is the negative deviation from the lower quality level limit, and  $\delta_m^{u+}$  is the positive deviation from the upper quality level limit in market node. The value of the flow to the customer nodes is given by the constant  $\omega$ . Furthermore,  $\kappa$  is the penalty cost for pressure level,  $\varpi$  is the penalty cost for deviation from contracted pressure level,  $\chi$  is the penalty

cost for deviation from contracted quality to customers and  $\iota$  for use of turn-up.

$$\begin{aligned} \max Z = & \sum_{i \in \mathcal{I}(m)} \sum_{m \in \mathcal{M}} \omega_m f_{im} - \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} \kappa p_{ij}^{in} - \sum_{m \in \mathcal{M}} \varpi (\epsilon_m^+ + \epsilon_m^-) \\ & - \sum_{g \in \mathcal{G}} \iota (\Delta_g^+ + \Delta_g^-) - \sum_{m \in \mathcal{M}} \chi (\delta_m^{l-} + \delta_m^{u+}) \end{aligned} \quad (2.1)$$

Energy consumption for transporting the natural gas is minimized through making the penalty cost ( $\kappa$ ) insignificant in size as compared to the value of the natural gas transported. This contributes to reduce the necessary build up of pressure to a minimum, without interfering with the correct volume, quality and pressure to the customer terminals. The penalty on using turn-up capacity will make sure that planned production in the fields is prioritized first, as long as it does not influence the throughput of the pipeline system. For most practical cases the contracted pressure level is not a soft constraint, and will then rather be put into a hard constraint instead of being penalized in the objective function.

### Constraints

**Production capacity** The following constraint says that the total flow out of a production node  $g$  cannot exceed the planned production of the field in that node. Here  $f_{gj}$  is the flow from production field  $g$  to node  $j$ :

$$\sum_{j \in \mathcal{O}(g)} f_{gj} \leq G_g, \quad g \in \mathcal{G}. \quad (2.2)$$

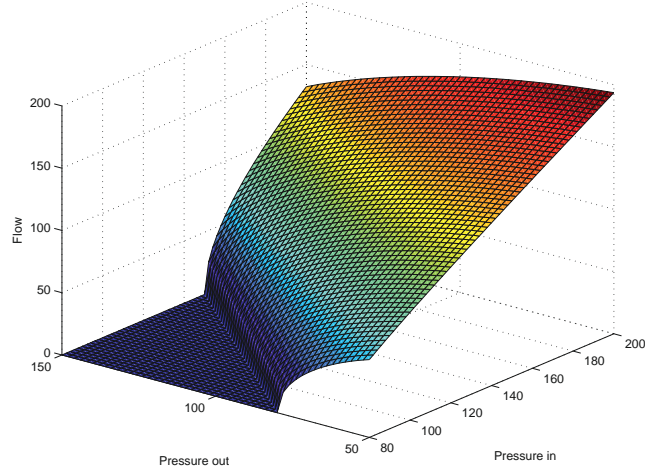
**Demand** This constraint says that the total flow into a node with customers for natural gas must not exceed the demand of that node:

$$\sum_{j \in \mathcal{I}(m)} f_{jm} \leq D_m, \quad m \in \mathcal{M}. \quad (2.3)$$

**Mass balance for node  $j$**  The following constraint ensures the mass balance in the transportation network. What flows into node  $j$  must also flow out of node  $j$ :

$$\sum_{i \in \mathcal{I}(j)} f_{ij} = \sum_{n \in \mathcal{O}(j)} f_{jn}, \quad j \in \mathcal{N}. \quad (2.4)$$

**Pressure constraints for pipelines** Offshore transportation networks often consist of very long pipelines without compression, where it is crucial to describe the pressure drops in the pipeline system. We use the Weymouth equation to



**Figure 2.3:** A three-dimensional illustration of how the Weymouth relates pressure at the inlet and outlet points to the capacity in the pipeline.

describe the flow in a pipeline as a function of input and output pressure. The Weymouth equation is described in e.g. Campbell (1992). In the Weymouth equation  $W_{ij}(p_{ij}^{in}, p_{ij}^{out})$  is the flow through a pipeline going from node  $i$  to node  $j$  as a consequence of the pressure difference between  $p_{ij}^{in}$  and  $p_{ij}^{out}$ :

$$W_{ij}(p_{ij}^{in}, p_{ij}^{out}) = K_{ij}^W \sqrt{p_{ij}^{in^2} - p_{ij}^{out^2}}, \quad j \in \mathcal{N}, i \in \mathcal{I}(j). \quad (2.5)$$

Here  $K_{ij}^W$  is the Weymouth constant for the pipeline going from  $i$  to  $j$ . This constant depends among others on the pipelines length and its diameter and is used to relate the correct theoretical flow to the characteristics of the specific pipeline. Figure 2.3 illustrates the Weymouth equation. The figure shows that the function in the interesting area (positive pressure levels) is one fourth of a cone. The cone starts in origo, and is limited by the inlet pressure axis, and the  $45^\circ$  line between the inlet pressure and outlet pressure axes.

Through Taylor series expansion it is possible to linearize Equation (2.5) around a point  $(PI, PO)$  representing fixed pressure into the pipeline and fixed pressure



out of the pipeline respectively:

$$\begin{aligned}
 W_{ij}(p_{ij}^{in}, p_{ij}^{out}) \leq & W_{ij}(PI, PO) + \frac{\partial W_{ij}}{\partial p_{ij}^{in}}(p_{ij}^{in} - PI) \\
 & + \frac{\partial W_{ij}}{\partial p_{ij}^{out}}(p_{ij}^{out} - PO), \quad j \in \mathcal{N}, i \in \mathcal{I}(j).
 \end{aligned} \tag{2.6}$$

We introduce a set of points to linearize this expression,  $(PI_l, PO_l)$ , where  $l = 1, \dots, L$ . Then we replace for each pipeline the nonlinear function (2.5) with  $L$  linear constraints of the type:

$$\begin{aligned}
 f_{ij} \leq & K_{ij}^W \frac{PI_l}{\sqrt{PI_l^2 - PO_l^2}} p_{ij}^{in} \\
 & - K_{ij}^W \frac{PO_l}{\sqrt{PI_l^2 - PO_l^2}} p_{ij}^{out}, \quad j \in \mathcal{N}, i \in \mathcal{I}(j), l = 1, \dots, L.
 \end{aligned} \tag{2.7}$$

For any given pipeline flow, only one of these  $L$  constraints will be binding, namely the one that approximates the flow best. The planes described in (2.7) will be tangent to the cone at the line where the ratio between pressure in and out of the pipeline is equal to the ratio between  $PI_l$  and  $PO_l$ . Together the planes give an outer approximation of the cone. This approximation will consist of triangular shapes defined by these planes.

**Pipelines without pressure drop** For physical pipelines between nodes where the distances are very limited it is not necessary to model pressure drops by the Weymouth equation. In this case a simple maxflow restriction is:

$$f_{ij} \leq F_{ij}, \quad j \in \mathcal{N}, i \in \mathcal{I}(j), \tag{2.8}$$

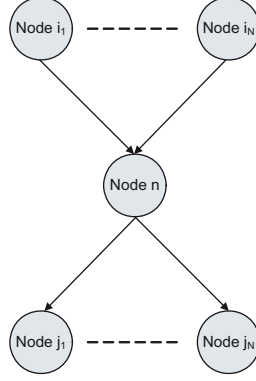
where  $F_{ij}$  is the capacity. In this case there is no pressure drop, so:

$$p_{ij}^{out} = p_{ij}^{in}, \quad j \in \mathcal{N}, i \in \mathcal{I}(j). \tag{2.9}$$

**Relationship between pressures into a node and out of a node** To achieve a relevant flow pattern, it is sometimes preferable to model the pressure out of all the pipelines going into the same node homogenously:

$$p_{in}^{out} = p_{jn}^{out}, \quad n \in \mathcal{N}, i \in \mathcal{I}(n), j \in \mathcal{I}(n). \tag{2.10}$$

Another important issue is the relationship between pressure in ingoing pipelines and the outgoing. In general for a node  $n$  the input pressure of all pipelines going out of  $n$  must be lower than the lowest pressure out of any pipeline going into



**Figure 2.4:** Example of a split node with the possibility to shut down operation of one of the pipelines. The upper nodes,  $i$ , have pipelines going to node  $n$ . The lower nodes,  $j$ , have pipelines coming from node  $n$ . The index on  $i$  and  $j$  goes from 1 to  $N$ , where  $N$  is the total amount of nodes.

node  $n$ , see Figure 2.4. There is one exception, and that is the case where a pipeline into the node has 0 flow. The end pressure of this arc is neglected. In the Equation (2.11) the variable  $\rho_{ij}$  is 0 for pipelines without flow and 1 for the others.  $M$  is a number which is large enough to not restrict the pressures when the flows are 0. Then the following constraints make sure that the input pressure of a pipeline leaving  $n$  is less than the output pressure of a pipeline ending in  $n$  as long as both pipelines have a flow larger than 0.

$$p_{nj}^{in} - p_{in}^{out} + M(\rho_{nj} + \rho_{in} - 1) \leq M, \quad n \in \mathcal{N}, i \in \mathcal{I}(n), j \in \mathcal{O}(n) \quad (2.11)$$

$$f_{nj} \leq M\rho_{nj}, \quad n \in \mathcal{N}, j \in \mathcal{O}(n) \quad (2.12)$$

$$\rho_{nj} = \begin{cases} i & \text{if flow from node } n \text{ to node } j \\ 0 & \text{otherwise.} \end{cases} \quad (2.13)$$

The Weymouth equation used gives an upper bound on the flow in a pipeline. This means that even if there is a pressure difference in a pipeline the flow can be zero. Because of this property it is not necessary to explicitly model the possibility of shutting down a pipeline. The model can simply put the flow to zero, and still keep the desired pressure. If omitting the constraints presented above one has to be aware of this when interpreting the results from the model.

**Modeling bidirectional pipelines** For pipelines designed to handle flows in both directions, the  $\rho_{ij}$  variable defined in the previous paragraph is used to determine

the direction of flow. Equations (2.14) and (2.15) make sure that there only flows gas in one direction in the pipeline.

$$f_{ij} \leq M\rho_{ij}, \quad i \in \mathcal{N}, j \in \mathcal{O}(i), \quad (2.14)$$

$$\rho_{jn} = 1 - \rho_{nj}, \quad n \in \mathcal{I}(j), j \in \mathcal{I}(n). \quad (2.15)$$

**Nodes with compression or pressure drop** In some cases we allow the pressure to increase in a node by using a compressor, or we force a pressure drop in the node. We here present a simplified formulation for modeling compression nodes where pressure can be built up or forced down. The compressor characteristics includes a compressor factor  $\Gamma$  used to limit how much the gas can be compressed in a node. If there is no compressor, this factor is 1. If there is a compressor, this  $\Gamma$  is a function of the flow  $f_n = \sum_{j \in \mathcal{I}(n)} f_{jn}$  into the node:

$$\Gamma_n(f_n) = \left( \frac{W^{max}\eta(K_a - 1)}{100K_a f_n} + 1 \right)^{\frac{K_a}{K_a - 1}}, \quad n \in \mathcal{N} \quad (2.16)$$

In this expression, the parameter  $K_a$  is the adiabatic constant for a certain gas type,  $W^{max}$  is the power output capacity of the compressor, and  $\eta$  is the compressor efficiency (Campbell 1992). Here we simplify this by using a constant compression factor independent of the flow. Then the pressure out of the compressor node  $n$  is limited by the compressor factor times the pressure into the node  $n$ :

$$\Gamma_n p_{jn}^{out} \geq p_{ni}^{in}, \quad n \in \mathcal{N}, j \in \mathcal{I}(n), i \in \mathcal{O}(n). \quad (2.17)$$

Pressure drop is modeled in the same way, but with a reduction factor  $\Theta_n$  instead of a compressor factor:

$$\Theta_n p_{jn}^{out} \geq p_{ni}^{in}, \quad n \in \mathcal{N}, j \in \mathcal{I}(n), i \in \mathcal{O}(n). \quad (2.18)$$

Here  $\Theta_n$  and  $\Gamma_n$  are constants, where  $0 < \Theta_n \leq 1$  and  $1 \leq \Gamma_n$ . The formulation is only meaningful if at most one of the factors is different from 1 in a node.

**Contracted pressure** It may be necessary to model the contracted pressure in nodes with customers. Most import terminals have a limited range around a target pressure  $P_m$  which they accept for incoming gas:

$$p_{im}^{out} + \epsilon_m^- - \epsilon_m^+ = P_m, \quad m \in \mathcal{M}, i \in \mathcal{I}(m). \quad (2.19)$$

Here  $\epsilon_m^-$  and  $\epsilon_m^+$  are negative and positive deviations from the target pressure. These deviations are penalized in the objective at a level reflecting how hard the pressure constraint is in practice.

It is also possible to specify restrictions for each pipeline for example for the pressure into and out of a given pipeline. Pressure restrictions often apply to nodes with compression or nodes where processing of the gas is being performed. These constraints are called technical pressure constraints. Examples are minimum and maximum pressure out of pipeline (represented by (2.20) and (2.21) respectively).

$$p_{ij}^{out} \geq P_{ij}^{min}, \quad j \in \mathcal{N}, i \in \mathcal{I}(j). \quad (2.20)$$

$$p_{ij}^{in} \leq P_{ij}^{max}, \quad j \in \mathcal{N}, i \in \mathcal{I}(j). \quad (2.21)$$

**Gas quality and energy content** In this model, gas quality can be specified in two different ways, focusing on combustion value (GCV) of the natural gas, or the content of  $CO_2$ . These properties are both technically and economically important for the customer. When dealing with  $CO_2$ , the customer accept a maximum content in terms of [mol %]. This is typically due to environmental taxes or to requirements related to avoiding corrosion in pipelines. If we focus on GCV, the customer accepts deliveries between a minimum and maximum combustion value. High GCV is in itself tractable as the energy content is higher, but in practice the plants using the natural gas are technically calibrated for a certain GCV-range. The quality is then measured in  $[MJ/Sm^3]$ . Here we only give the formulation for GCV:

$$Q_m^{min} \leq q_{im} \leq Q_m^{max}, \quad m \in \mathcal{M}, i \in \mathcal{I}(m), \quad (2.22)$$

where  $q_{im}$  is gas quality (GCV) in a pipeline going from node  $i$  to market node  $m$ . In practice we need more flexibility in the model by allowing reduced quality in order to increase the flow. Modeling this as hard constraints could lead to situations where unexpected shutdowns of production fields or pipelines may lead to a complete stop in deliveries to a customer due to the contractual quality. If it is an alternative to get some deliverances, outside the contracted limits, but within what is technically acceptable the latter will be chosen. This tradeoff will be valued in economical terms as reduction in the customer price. We need the variables  $\delta_m^{l+}$  and  $\delta_m^{l-}$  to indicate the positive and negative deviation from the lower quality limit  $Q_m^{min}$  of customer node  $m$ . Likewise we need  $\delta_m^{u+}$  and  $\delta_m^{u-}$  to indicate the positive and negative deviation from the upper quality limit  $Q_m^{max}$ :

$$q_{im} + \delta_m^{l-} - \delta_m^{l+} = Q_m^{min}, \quad m \in \mathcal{M}, i \in \mathcal{I}(m), \quad (2.23)$$

$$q_{im} + \delta_m^{u-} - \delta_m^{u+} = Q_m^{max}, \quad m \in \mathcal{M}, i \in \mathcal{I}(m). \quad (2.24)$$

**Gas quality and blending** Gas quality is a complicating element because we have to keep track of the quality in every node and pipeline, and this depends on the

flow. Where two flows meet, the gas quality out of the node to the downstream pipelines depends on flow and quality from all the pipelines going into the node. The flow in each pipeline is a decision variable in the model, and so is the quality out of each node. We assume that the resulting blending quality is common for all the downstream pipelines being connected to a node, and that it is decided by the convex combination of inflow qualities to the node:

$$q_{ij} = \frac{\sum_{n \in \mathcal{N}} q_{ni} f_{ni}}{\sum_{n \in \mathcal{N}} f_{ni}}, \quad i \in \mathcal{N}, j \in \mathcal{O}(i), \quad (2.25)$$

or:

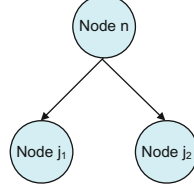
$$q_{ij} \sum_{n \in \mathcal{N}} f_{ni} - \sum_{n \in \mathcal{N}} q_{ni} f_{ni} = 0, \quad i \in \mathcal{N}, j \in \mathcal{O}(i). \quad (2.26)$$

This equation has two quadratic terms on the form  $q_{ni} f_{ni}$ . These terms can easily be reformulated in the following way: Define  $\alpha = q_{ni} - f_{ni}$  and  $\beta = q_{ni} + f_{ni}$ . Then  $q_{ni} f_{ni} = 1/4(\alpha^2 - \beta^2)$ . Linearizing  $\alpha^2$  and  $\beta^2$  is straightforward using Special Ordered Sets of type 2 (SOS2) (Williams 1999). In the SOS2 set at most two variables can be non-zero, and the two variables must be adjacent. Still this means that we need to move into solution techniques from integer programming, in particular branch and bound, so solution time will increase exponentially with the numbers of SOS2 sets needed.

**Modeling multi-component flows** If we model the flow of  $C$  components of the natural gas we require that the split fractions of the components going into the different pipelines out of the node  $n$  is equal for all components. For simplicity let us assume we always have only two pipelines out of a split node  $n \in \mathcal{N}$  going to node  $j_1$  and  $j_2$  (see Figure 2.5). Let us also denote the first component in the set  $\mathcal{C}$  of components for  $c_1$ . All components are indexed from  $c_1, \dots, c_C$ . Then the relation of the volume split between  $j_1$  and  $j_2$  is equal for all components:

$$\frac{f_{nj_1}^{c_1}}{f_{nj_2}^{c_1}} = \frac{f_{nj_1}^c}{f_{nj_2}^c}, \quad n \in \mathcal{N}, c \in \mathcal{C}. \quad (2.27)$$

This is a quadratic expression, and we reformulate it using the Equations (2.28) to (2.32). We need a set of binary variables  $\vartheta_{nz}$  where  $z = 1, \dots, Z$ , each representing the choice of a split percentage for the share of natural gas going to node  $j_1$ . The  $\vartheta_{nz}$  variable is modeled as a special ordered set of type 1 (SOS1), where only one variable can be non-zero (Williams 1999). For each  $\vartheta_{nz}$  we define a constant  $E_z$  giving the percentage related to the  $z$ . We also define a new variable  $e_{nz}$  representing the flow through node  $n$  of component  $c$  if  $\vartheta_{nz} = 1$ .



**Figure 2.5:** The flow is split in node  $n$  to node  $j_1$  and  $j_2$ .

The first constraint says that the flow from  $n$  to  $j_1$  of component  $c$  equals the percentage  $E_z$  multiplied with the total flow through node  $n$  of the component  $c$ .

$$f_{nj_1}^c = \sum_{z=1}^Z E_z e_{nz}^c, \quad n \in \mathcal{B}. \quad (2.28)$$

The set  $\mathcal{B}$  consists of all split nodes in the network. Then we need to restrict the formulation so that only one  $\vartheta_{nz}$  is positive for each node:

$$\sum_{z=1}^Z \vartheta_{nz} = 1, \quad z \in \{1, \dots, Z\}, n \in \mathcal{B}. \quad (2.29)$$

The  $e_{nz}^c$  variables giving the flow through the node of each component is constrained by the capacity of the node, corresponding to the active  $\vartheta_{nz}$ .

$$\sum_{c \in \mathcal{C}} e_{nz}^c \leq F_n \vartheta_{nz}, \quad z \in \{1, \dots, Z\}, n \in \mathcal{B}. \quad (2.30)$$

We also require that what flows through the node of each component either goes to node  $j_1$  or to node  $j_2$ :

$$\sum_{z=1}^Z e_{nz}^c = f_{nj_1}^c + f_{nj_2}^c, \quad n \in \mathcal{B}, c \in \mathcal{C}. \quad (2.31)$$

And to make sure that there does not flow more out of the node of each component than what comes in:

$$f_{nj_1}^c + f_{nj_2}^c = \sum_{i \in \mathcal{N}} f_{in}^c, \quad c \in \mathcal{C}, n \in \mathcal{B}. \quad (2.32)$$

**Processing plants** Some of the gas components are extracted and sold in separate component markets. The extraction is handled in processing plants in the network. In the modeling of this process it is assumed that the volume of each

component extracted is a constant fraction of the total volume of that component in a processing plant ( $A_r^c$ ). Hence, no decision on the configuration of the processing plant is made, but pressures and gas flows through a processing plant can be modeled by several processing nodes in sequence or parallel. This is expressed in Equation (2.33). The mass balance for the processing plant nodes can then be formulated as in Equation (2.34). The variable  $a_r^c$  is used to keep track of how much of component  $c$  is extracted from the flow in processing plant  $r$ .

$$a_r^c = A_r^c \sum_{i \in \mathcal{N}} f_{ir}^c, \quad c \in \mathcal{C}, r \in \mathcal{R} \quad (2.33)$$

$$\sum_{i \in \mathcal{N}} f_{ir}^c = \sum_{j \in \mathcal{N}} f_{rj}^c + a_r^c \quad (2.34)$$

**Modeling turn-up: flexibility in the production fields** Turn-up is an expression used for the flexibility present in some production fields. For example reduced transport capacity due to a shutdown in one part of the network may be compensated by turning up the planned production from other gas fields not directly affected by the reduced capacity. When modeling this turn-up capacity it is important to keep in mind that even if one are free to utilize this flexibility, it is not acceptable from a practical point of view that the model presents a flow allocation where fields with significant turn-up capacity will take over production from minor fields, which basically is not affected by the shutdown. The turn-up is only used to take over production from fields that for some reason are prevented to deliver. Hence, our first priority is to meet demand in the network and our second priority is to produce in accordance with the planned production at the fields.

We model this by adding a penalty cost for using turn-up in the objective to avoid turn-up to be used at the expense of normal production capacity in other fields. This works because not delivering gas to customers would generate a loss which is considerably higher than the small penalty put on using turn-up capacity.

The variables  $\Delta_g^-$  and  $\Delta_g^+$  represent underproduction and the use of turn-up in relation to the planned production of  $G_g$  for field  $g$ . As before  $f_{gj}$  is the flow from  $g$  to  $j$ :

$$\sum_{j \in \mathcal{O}(g)} f_{gj} + \Delta_g^- - \Delta_g^+ = G_g, \quad g \in \mathcal{G} \quad (2.35)$$

## 2.4 Management of Natural Gas Storages

As a consequence of the liberalization process in the natural gas industry, the natural gas markets have become more dynamic. The spot markets and the

possibility to trade gas in forward markets have increased the importance of gas storages. In this section we discuss models for gas storage operations in a market with uncertain demand.

In order to discuss the management of natural gas storages, a couple of terms need to be established (see Figure 2.6 for an illustration of the terms):

**Storage capacity** gives the maximal volume of natural gas in the storage facility. The storage capacity is limited by the physical properties of the storage.

**Volume of natural gas in the storage** is the total volume of natural gas in a given storage at a given time.

**Cushion gas** is the amount of gas needed to create necessary pressure in order to lift gas from the storage. The amount of cushion gas needed varies with the type of storage and the geological conditions at the storage location. For some types of storages the cushion gas requirement is as high as 80% of the total gas volume in the storage.

**Working gas** is the gas volume available during normal operation of the storage. This corresponds to the total amount of gas in the storage subtracted the cushion gas.

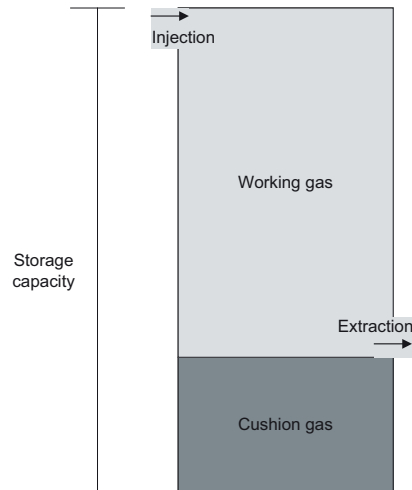
## Storage Facilities

The most common storage facilities are abandoned oil- and gas reservoirs, aquifers, salt caverns and LNG-storages. In the following, a short overview of advantages and disadvantages of these possibilities will be given. For further discussion of storage facilities, see Administration (2002).

**Abandoned oil- and gas reservoirs** are the most common storage facility. One reason for this is the relatively low startup costs. The storage facility is already in place, and so is most of the surface installations needed. Another advantage of this type of storage is the fact that infrastructure is normally already in place. One major drawback is the amount of cushion gas needed for operation.

**Aquifer** is a porous, underground water-bearing layer which can be transformed into a storage facility by replacing the water with natural gas. When using abandoned oil- and gas reservoirs the geological properties are known, this is not the case when using aquifers. This adds risk to the development of this type of storages. Cushion gas in the amount of 80 to 90 % is needed for operation, and the development takes time and is costly. These storages are normally only used in locations where no oil- and gas reservoirs





**Figure 2.6:** The complete square is the total storage capacity. The lower part of the figure is the cushion gas needed for operation of the storage, and the upper part of the figure is the gas currently available for extraction from the storage.

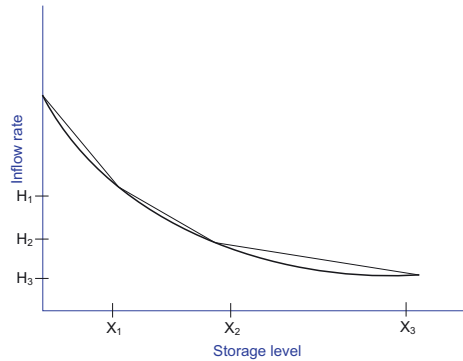
are available. One advantage of this type of storage is the relatively high delivery rate.

**Caverns** are created from underground salt or rock formations. In the salt caverns, water is used to dissolve halite and to shape cavities in natural salt formations. These cavities have the properties of a high-pressure gas container, with impenetrable walls. The storages have a high delivery capacity, and a cushion gas requirement of only approximately 25 %. The process of dissolving halite and shaping the cavities makes this alternative more expensive than the previous two alternatives.

**LNG-storages** are, in contrast to the previously presented alternatives, above-ground facilities. These storages consist of tanks containing liquefied natural gas (LNG) or liquefied petroleum gas (LPG). The capacity of these tanks is normally very limited compared to the other alternatives presented.

### Motivation for Utilization of Storage

The possibility of storing natural gas gives the participants increased flexibility with regards to production and transportation decisions. One important use of natural gas storages is to take advantage of the strong seasonal pattern in prices.



**Figure 2.7:** Illustration of the linearization of the injection rate of a storage.

Since the primary use of natural gas is for heating and production of electricity, the fundamental price determinant in the markets is the weather conditions. The demand is normally higher in winter than in summer, and the production capacity is also lower than the peak demand. This means that the monthly demand for natural gas may be much higher than the possible changes in production level can satisfy. The difference between production capacity and peak demand can to a certain degree be satisfied through utilization of storages. The use of storages can substitute for investments in new production fields and transportation capacity. Traditionally the storages have been used in order to ensure a high security of supply. When problems occurred either in the production or transportation facilities, storages could be used to supply the downstream participants. The storages operate as a security buffer in this case. With the development of short-term markets and volatile spot prices, the storages will be important for participants wanting to utilize the price fluctuations. Especially for gas producers not having a reservoir close to the market this will be important. It can take several days before a decision to change production level at the field will result in increased delivery in the market.

### Modeling Storage

The maximum in- and outflow rates of the storage varies with the current storage level. The maximal injection rate is a strictly decreasing convex function of the storage level. Likewise the outflow rate can be given as a strictly increasing convex function of the storage level. To be able to realistically represent the in- and outflow rates, the use of special ordered sets of type 2 is chosen (Williams 1999). An illustration of the implementation of the SOS2 is shown in Figure 2.7 for the injection rate. The storage levels are discretized by a set of constants

$X_1, \dots, X_Y$ , the corresponding injection rates are  $H_1, \dots, H_Y$  and the variables  $\nu_1, \dots, \nu_Y$  are used to give a convex combination of two of the points. This means that if  $\nu_y$  has a value different from 0, then only one additional variable can be non-zero. The only two candidates in this case are  $\nu_{y-1}$  or  $\nu_{y+1}$ . The storage level at a given time  $t$  is represented by  $x_s^t$ .

$$\sum_i f_{is}^t \leq \sum_{y=1}^Y \nu_{ys}^t H_{ys}, \quad s \in \mathcal{S}, \quad (2.36)$$

$$\sum_{y=1}^Y \nu_{ys}^t = 1, \text{ SOS2}, \quad s \in \mathcal{S}, \quad (2.37)$$

$$x_s^t = \sum_{y=1}^Y \nu_{ys}^t X_y, \quad s \in \mathcal{S}, \quad (2.38)$$

$$x_s^t = x_s^{t-1} + \sum_{i \in \mathcal{I}(s)} f_{is}^t - \sum_{i \in \mathcal{O}(s)} f_{si}^t, \quad s \in \mathcal{S}. \quad (2.39)$$

The maximum and minimum levels of storage are modeled implicitly with the representation given. The maximal level (equal to the total capacity of the storage) is restricted by the inflow function. When the storage reaches the upper capacity level, the associated inflow rate is equal to zero. The minimum level (coming from the requirement of a certain level of cushion gas in the storage) is handled in a similar way: when the minimum storage level is reached, the associated outflow rate will be equal to zero.

## 2.5 Value Chain Optimization and Portfolio Management

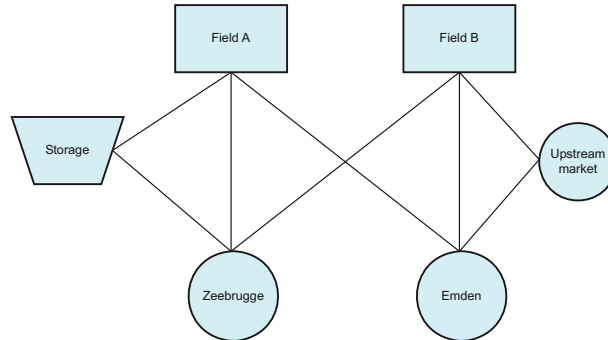
We will here give a short description on how to include markets and portfolio optimization in the natural gas value chain. For more advanced models on portfolio optimization in the natural gas value chain see Rømo, Tomasgard, Fodstad & Midthun (2004) from which most of the ideas presented here originate. Other relevant references are Nygreen, Christiansen, Bjørkvoll, Haugen & Kristiansen (1998) which consider portfolio optimization for oil and gas fields in a strategic horizon and Ulstein, Nygreen & Sagli (2004) which consider tactical value chain coordination, but without stochasticity and without multi-commodity flows.

## Different Levels of Portfolio and Value Chain Integration

The models presented here have both a portfolio and a value chain perspective. These are important properties of a natural gas optimization model. The importance of these perspectives can be realized when considering the complexity of the transportation system. Due to the technical nature of the gas network, several physical and technical threshold-values exist. If such values are trespassed, only minor incremental deliveries in one part can cause significant unintended reductions elsewhere. The bottlenecks in the transportation system make the flexibility incorporated in a system perspective valuable. We will not give all the previous formulations of the transportation network again, but each time period  $t$  in a value chain model will include transportation network constraints and variables like the ones from Section 2.3 with an additional index  $t$  on all variables.

The motivation behind the portfolio and value chain perspectives can be summarized by considering four levels of planning:

1. *Traditional production planning:* In this first level the model ensures balancing of the production portfolio with the contract portfolio. Stochastic demands and prices that are not perfectly correlated motivate a portfolio perspective on the planning, as the portfolio variation will be lower than the variation of the stochastic parameters of separate fields or contracts.
2. *Production and market optimization:* At this level markets are used to supplement the physical production in order to gain more from the physical production capabilities. The market can be used to resolve bottlenecks in the transportation network or on the production side. The purpose is to maximize the profit from production and contract obligations using also spot markets. At this level geographical swaps and time swaps of gas can be performed using the market, and they are used to fully utilize the flexibility in the system.
3. *Trading:* At this level contracts and financial instruments are traded independently of the physical production and contract obligations based on market opportunities. The trading is similar to the previous level in terms of using the spot market and financial instruments like futures and options, but the motivation is now speculation, not solving bottleneck problems. These trades are in no way connected to the physical production and contract obligations, unless the producer has market power.
4. *Risk management:* So far we have assumed the producer is risk neutral and tries to maximize expected profit. In that case it is enough to supplement physical production with trades in the spot market at level 2. If the producer is risk averse hedging the portfolio outcome using futures, forwards or options may be optimal.



**Figure 2.8:** Example of a natural gas network

The distinction between level 2 and 3 is clear in theory, but in practice the transition will be gradual.

### Utilization of Short-Term Markets in Value Chain Optimization

The use of short-term markets allows for considerable flexibility in the system. Consider the network in Figure 2.8. In a situation where field B needs to produce and the company has an obligation to deliver in a bilateral contract in Emden several possibilities exist:

- Field A supplies Emden, while field B sells spot in Zeebrugge
- The company may buy spot in Emden and the production from field B can be sold in the spot market in Zeebrugge.
- The company buys spot in Emden, while it sells the production from B spot in the upstream market.
- Storage might be used to supply Emden, while the production from field B is sold elsewhere.

These simple geographical swaps makes the system more flexible and gives the company the possibility to maximize the flow of natural gas (and the value of their production) beyond what traditional transportation planning would have done. For example bottlenecks in the production or in the transportation may be resolved or moved using the markets actively.

A different reason to use the markets is time swaps. Consider Figure 2.8 again. This time field B needs to produce in time 1, and the company has an obligation to deliver in time 2. Several options are then available to the company:

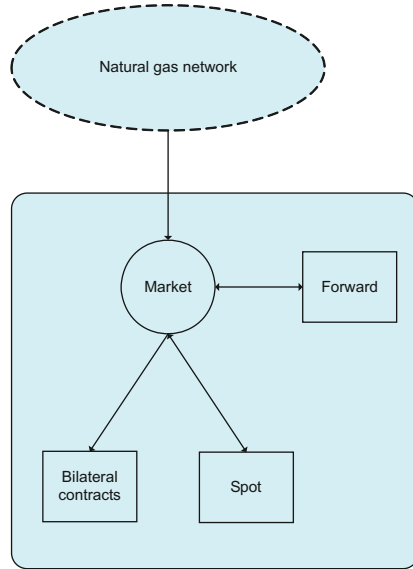
- In period 1 field B may supply storage, and in period 2 the storage supplies Emden.
- In period 1 field B can sell spot in Zeebrugge, and in period 2 either use a forward contract or buy spot in Emden.
- In period 1 field B can sell spot upstream, and then use either a forward contract or the spot market to supply Emden

This is just some of many possibilities that exist for geographical swaps and time swaps. The network considered is also very small. When expanding the network to, for instance, 20 fields, 80 pipelines and 10 markets, the number of possible routing decisions gets very large and the flexibility increases. It is this flexibility we try to capture when modeling the portfolios of production fields and contracts. The flexibility further increases when perfect spot markets are added. The need for flexibility comes from the fact that demands and prices are stochastic. The gain from portfolio thinking increases because they are not perfectly correlated. We assume the company is a price taker. For simplicity of notation, we assume there is only one company in the markets. If not, we would also need to model the other companies' transportation needs.

### Including Markets and Contracts

In Section 2.3 only aggregated deliveries to take-or-pay contracts in the different customer nodes  $m \in \mathcal{M}$  were considered. When including market transactions in the model a representation of the uncertainty in the price process is important. Based on this representation scenarios describing the uncertainty can be generated and optimal decisions in the interaction between the physical system and the market can be made. In this description some simplifications have been made. Only one company is considered, so no upstream market exists, the possibility of delaying production through lifting agreements will be disregarded, and only trades in the spot market will be considered. The possibility of trading forward contracts is only interesting for a risk averse company. This will be discussed shortly at the end of this section.

Figure 2.9 illustrates how the market nodes are included in the model. The arrows show that gas might flow from the transportation network to the market. There is no flow from the market to the network (as would be the case for an upstream market). In addition, transactions within the market node can be performed. In the spot market the company can purchase or sell volumes of natural gas. Obligations in the take-or-pay contracts can be fulfilled either by flow from the network to the market node, or by transactions within the market node.

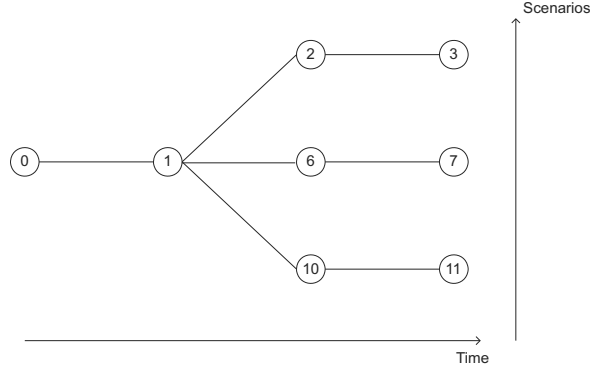


**Figure 2.9:** The market node

## Modeling Stochasticity

We use the modeling paradigm of stochastic programming to represent uncertainty in the models, see for example Kall & Wallace (1994). Uncertainty is then represented in a scenario tree, see Figure 2.10. The nodes in the scenario tree represent decision points, and uncertainty is resolved along the arcs going out of a node with several branches. In practice decisions are only made when new information becomes known. A stage is the set of time periods elapsing between each time information is learned by the decision maker. Each stage in the tree typically consists of several time periods, but only nodes after a branching are decision points, as they are the only time periods when new information about the future is resolved. Still, decision variables are present in time periods where information is not resolved, hence the time periodization using time periods  $t$  reflect in which time period the decision has effect. In Figure 2.10 there are 3 time periods. Time periods 1 and 2 are in stage 1, starting with the decision in node 0 and ending just before the new decisions at stage 2 (in nodes 2, 6 and 10).

In a two-stage stochastic programming model we define a set of time periods  $t \in \mathcal{T}_1 = \{t_1, \dots, T_1\}$  belonging to the first stage where information is deterministic, and a set of time periods  $t \in \mathcal{T}_2 = \{T_1 + 1, \dots, T_2\}$  where some parameters are stochastic (as seen from  $t \in \mathcal{T}_1$ ). When time passes on and one enters the



**Figure 2.10:** An example of a scenario tree

first  $t \in \mathcal{T}_2$ , uncertainty is resolved and also the remaining time periods can be considered deterministic.

In the model presented here we use a two-stage formulation for ease of notation. Several parameters are stochastic in reality. We will consider stochasticity in: contractual demands, contract prices and spot prices. We denote the stochastic contract price for contract  $k$  in customer node  $m$  at time period  $t \in \mathcal{T}_2$  as  $\tilde{\phi}_{mk}^t$ . Stochastic demand for contract  $k$  in customer node  $m$  at time period  $t$  is  $\tilde{\mu}_{mk}^t$ . The stochastic spot price is represented with  $\tilde{\psi}_m^t$ . The vector of all stochastic variables in time period  $t$  is  $\tilde{\xi} = (\tilde{\psi}^t, \tilde{\phi}^t, \tilde{\mu}^t)$ .

We use a tilde over the variable to reflect that it is stochastic (as seen from  $t \in \mathcal{T}_1$ ) and remove the tilde when the variable is deterministic. We then get the following:

$\tilde{\xi}_m^t$  Stochastic variables for customer node  $m$  in time period  $t \in \mathcal{T}_2$  seen from a point in time  $t' \in \mathcal{T}_1$ .

$\xi_m^t$  Deterministic parameters for customer node  $m$  in  $t \in \mathcal{T}_1$ , or  $t \in \mathcal{T}_2$  after uncertainty is resolved (Seen from a point in time  $t'$  where  $t' \in \mathcal{T}_2$ ).

A scenario tree can be constructed for example using price processes for natural gas or descriptions of the dynamic aspects of stochastic demand. We will not go in detail on how to do this here, but assume the scenario tree exists in the remaining part of this paper.

## The Objective

We describe the supply chain portfolio optimization model as a two-stage stochastic program with relatively complete recourse Kall & Wallace (1994). The only



stochasticity that is present is in the right hand side and in the objective. The typical length of a time period for a tactical planning model is one month, and the planning horizon would typically be 12 months, where for example the first 6 months would belong to  $\mathcal{T}_1$  and the last 6 months to  $\mathcal{T}_2$  in a two-stage formulation. The objective is to maximize expected profit taken into consideration cash flows and shortfall costs. Hence the objective can be described by summing the expected cash flow of the time periods. The cash flow of each time period  $t$  can be described as a function  $\Pi^t(x^{t-1}; \xi^t)$  (or  $\tilde{\xi}^t$  if stochastic) where  $x^{t-1}$  is the storage level in the start of the time period. The decision variables and constraints are equal in all time periods, except for initialization in time period 0 where only initial storage levels are defined  $x_{ms}^0$  and for the terminal conditions at the end of the model horizon. We use the vector  $x^0$  to denote the initial level of all storages and  $x^t$  to denote the level of all storages in time period  $t$ . The profit function for time period  $t \in \mathcal{T}_1 \cup \mathcal{T}_2$  can be formulated as:

$$\Pi^t(x^{t-1}; \xi^t) = \sum_{m \in \mathcal{M}} \sum_{k \in \mathcal{K}} \phi_{mk}^t \mu_{mk}^t + \sum_{m \in \mathcal{M}} \psi_m^t (\zeta_m^{t-} - \zeta_m^{t+}), \quad (2.40)$$

where the first term is the income from delivery in contract  $k$  in market  $m$  at time  $t$  and the second term gives the profit from trading in the spot market in node  $m$  in time period  $t$ .

The two-stage stochastic program with fixed relatively complete recourse is:

$$\max \sum_{t \in \mathcal{T}_1} \Pi^t(x^{t-1}) + \mathcal{Q}(x^{T_1}), \quad (2.41)$$

where

$$\mathcal{Q}(x^{T_1}) = \max_{E_{\tilde{\xi}}} E_{\tilde{\xi}} \left[ \sum_{t \in \mathcal{T}_2} \Pi^t(x^{t-1}; \tilde{\xi}^t) + EGV(x^{T_2}) \right], \quad (2.42)$$

subject to a set of constraints representing transportation, production and markets. These constraints are mainly the constraints described earlier in this paper, but we will look closer at the ones changing because of the introduction of markets and contracts. The constraint sets are identical for all time periods  $t \in \mathcal{T}_1 \cup \mathcal{T}_2$ . For the last time period the objective includes the terminal value of the natural gas in storages expressed by the Expected Gas Value function,  $EGV(x^{T_2})$ . This function is described in more detail in Section 2.6.

The solution resulting from maximizing expected profits will normally be different from the solution reached with the objective function presented in Section *Objective Function* on page 48. This means that the solution does not necessarily maximize the throughput in the network, or minimize the cost of achieving a given throughput. The solution will however show how the network should be managed in order to achieve the highest possible expected profit.

## Constraints Including Markets and Contracts

The mass balance in the market node for each time period and each scenario is expressed as:

$$\sum_{i \in \mathcal{I}(m)} f_{im}^t + \zeta_m^{t+} = \zeta_m^{t-} + \sum_{k \in \mathcal{K}(m)} \mu_{mk}^t, \quad \forall m \in \mathcal{M}, \forall t \in \mathcal{T}. \quad (2.43)$$

In (2.43),  $\zeta_m^t$  represent transactions in the spot market in node  $m$  in time period  $t$ . The + sign indicates purchases of natural gas whilst the – sign indicates sales. Delivery in contract type  $k$  in the node  $m$  in time period  $t$  are included in  $\mu_{mk}^t$ . The mass balance equation illustrates the flexibility gained by including markets in the model. It is no longer necessary to ship the gas to the market node in order to fulfill the contractual agreements, since the spot market can be utilized for this. This means that geographical and time swaps are now available to the company.

### Risk Aversion

In the discussion so far only the possibility for trading natural gas through the spot market has been discussed. For a risk neutral company that is maximizing expected profits this is an adequate approach. Since the forward price is a good predictor of the expected future spot price, trading in the forward market would on average be approximately equal to trading on the spot market (this is based on a simple arbitrage argument, see for instance Hull (2003)). The fact that natural gas is a commodity makes the argument less obvious, but under some assumptions still valid. In the case where the company is risk averse however the situation changes and some tools to handle risk management are needed. The inclusion of a forward market then gives the company the possibility to hedge, that is: to reduce the risk of their position. By trading forward contracts a given price can be locked in on advance.

In this case the company will no longer maximize expected profits from their operations, but rather maximize a utility function that incorporates the risk aversion of the company. Another way of doing this is to introduce a penalty function that will put extra cost in the objective function on deviations from some target profit value. In addition to the change in the objective function, the mass balance in the market node (see (2.43)) will be changed to incorporate the possibility to trade in the forward market.

### Solution Times

The complexity of the models introduced in this paper to a large extent depends on the modeling of the gas components. The inclusion of gas components adds

a large number of integer variables to the problem. When excluding the gas components, a stochastic model with a network consisting of approximately 80 nodes and 1000 scenarios, can be solved within an hour. This problem will have approximately one million rows, one and a half million columns, four million non-zero elements and fourteen thousand binary variables. When including gas components the solution time increases significantly, and it is difficult to find an optimal solution. For a physical system similar to the one above, with 100 scenarios and 10 breakpoints (see Section *Modeling multi-component flows* on page 55), a solution with an integrality gap of 4% to 5 % typically can be reached within 12 hours. If the objective is only to maximize flow in a static model, solution times are within minutes when components are omitted and increases correspondingly when components are added.

## 2.6 The Expected Gas Value Function (EGV)

So far no considerations have been made with respect to how the final period in the planning horizon will be handled. The model presented so far will most likely end up with a very low storage level, and the production might also be higher than optimal when considering a longer horizon (since the value of the gas still contained in the reservoirs is neglected).

In order to handle the end-of-horizon problem, several possibilities exist. One way of handling the storage problem is to set a target value for the storage level at the end-of-horizon, for instance the starting level.

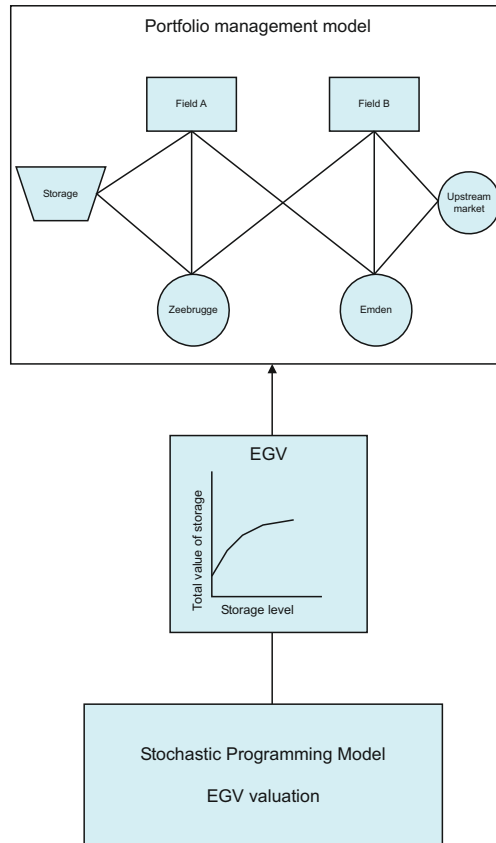
$$x_s^T \geq x_s^0 \tag{2.44}$$

This level might also be made dependent on various factors, such as the season in which the end-of-period belongs. This way of modeling the end-of-period however allows for limited flexibility and also neglects the true value of the gas contained in the storage. A way of determining the optimal level for the storages in the last period is by using the expected gas value function.

The Expected Gas Value function (*EGV*) gives an estimate of the value of a unit of gas in storage at some point in time  $t$ , based on expectations for the future development of the spot price of natural gas. When the *EGV* is used as a boundary value, the alternative value of the natural gas in storage is thereby included. This alternative value comes from the opportunities present after the end of the model horizon. Hence for each end-of-horizon storage level, the *EGV* must reflect the value of an optimal out-of-horizon strategy for injecting gas in the storage and selling gas from the storage.

If high prices are expected in the future, the *EGV* will encourage a high storage level in final time period  $T_2$ , whilst if lower prices are expected the optimal level

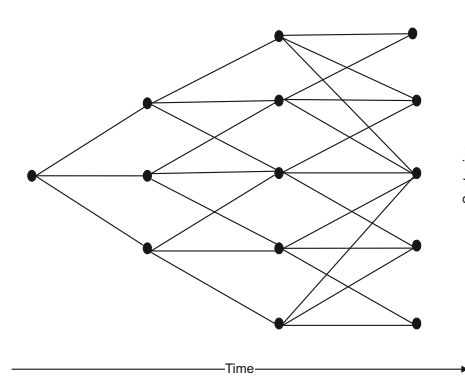
in period  $T_2$  may be lower. Figure 2.11 illustrates how the *EGV* is included in the existing optimization model. As the figure shows, the estimation of *EGV* is performed independently from the value chain model and the purpose is to give a boundary condition for the value of gas.



**Figure 2.11:** The estimation of the *EGV* is performed in a stochastic optimization model that is independent of the existing optimization model. The *EGV* is then used in the value-chain model as a boundary value on the gas in storage and reservoirs.

An important element in the model used to estimate *EGV* is the development of the natural gas spot price represented through spot price curves. These can be modeled using stochastic processes. Several versions of such models exist, for an overview of some of them, see Schwarz (1997). Based on the chosen price

model, scenarios describing possible future outcomes can be constructed (see Figure 2.12). Hence, for any given initial storage level a strategy is found for injection and withdrawal of natural gas based on a stochastic process for the gas price. In practice this is a real-options approach used to value the value of gas in the storage. The option value in gas storages comes from the operational flexibility. The company can switch between injection, withdrawal or idle modes, depending on the price development. For references on real-options, see for instance Hull (2003). It is possible to do this estimation both for individual storages, and also for the combination of all or some of the storages in the network. In the latter case a more complicated model is needed for estimation of the *EGV*.



**Figure 2.12:** Representation of the development of the spot price of natural gas. In this case a recombining trinomial tree. The arcs in the figure represent price movements, while the nodes represent different price scenarios.

In the following, an example of how the *EGV* can be calculated is given. The procedure is based on Scott, Brown & Perry (2000) and Manoliu (2004), and use a stochastic dynamic programming framework. For references to similar work in hydro power, see for instance Pereira, Campodónico & Kelman (1999) and Pereira & Pinto (1991). After choosing a stochastic model to represent the price of natural gas, a discrete approximation of the storage facility state space is made. A tree similar to the one constructed for the spot price (Figure 2.12) can be constructed also for the storage level. In this case the nodes represent different storage levels, while the arcs represent injection and withdrawal of natural gas in the storage. A multilevel tree representing the spot price and the amount in storage is then created. The valuation is performed by backward induction through the tree. The option value is calculated in each node by taking the maximum of the decision values of hold, injection and withdrawal. The hold

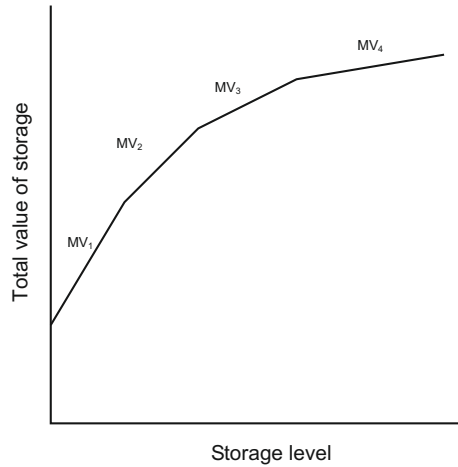
decision value is equal to the expectation of the option value of the next steps, when storage level is unaltered. The injection value is the negative value of gas injected in this period, plus the expected value of increased storage level in future nodes. The withdrawal value is then the value of releasing gas in this period, plus the expectation of option values of decreased storage levels in coming nodes. This can be illustrated by (2.45), which shows the value in a given node in the tree:

$$I^t(\tau^t) = \pi^t(\varphi^t) + I^{t+1}(\tau^{t+1}). \quad (2.45)$$

$I^t(\tau^t)$  is the value of storage level  $\tau$  in time period  $t$  in the node considered. This is determined by the value of flow  $\pi^t(\varphi^t)$ , (where  $\varphi^t$  is the volume injected or withdrawn in period  $t$ ) in the node in period  $t$  plus value of the storage level  $\tau^{t+1}$  in the next time period (in nodes following the considered one). The storage level is updated according to (2.46):

$$\tau^{t+1} = \tau^t + \varphi^t. \quad (2.46)$$

An illustration of a gas value function is given in Figure 2.13. The challenge is in finding the appropriate total value for each level of storage, as well as finding the breakpoints.



**Figure 2.13:** An example of an expected gas value function. The MVs shows the expected marginal value of gas for various levels of storage. This is the additional value of one more unit of natural gas in the storage.

Even though short-term markets for natural gas are developing in Europe, the liquidity in these markets is still very limited. This lack of liquidity makes

estimation of the spot-price process difficult, and therefore also estimation of the *EGV* difficult. Given that a spot-price model can be modeled for any given time horizon, a time-horizon of a couple of years may be appropriate for estimating the *EGV*. As the time-horizon for estimation is increased, the discount rate will make the gas value in the last periods decrease strongly.

## 2.7 Conclusions

In this paper we have gradually introduced the complexity of a stochastic optimization model for the natural gas value chain. We focus on coordination of the different levels of the chain and on a portfolio perspective. We started out by defining necessary constraints for a steady-state formulation of the underlying transportation network, supporting multi-commodity flows and pressures. Next we introduced the time aspect and the use of storages. Thereafter we introduced stochasticity in demands and prices and gave a stochastic programming formulation for a portfolio optimization model. Natural extensions of this model would be contract selection and more advanced modeling of the production flexibility reflected by lifting agreements. Finally we defined the Expected Gas Value function and explained its use for giving the terminal value of stored natural gas and indicated how to calculate it.

Most of the model formulations presented here are simplified variants of models that are implemented for commercial use on the Norwegian continental shelf. In this paper we have taken the position of a large producer, but many of the formulations would be relevant for more general models focusing on other parts of the natural gas value chain.

# Appendix

## 2.A Notation and Definitions

### Sets

- $\mathcal{N}$  The set of all nodes in the network.
- $\mathcal{G}$  The set of nodes in the network with production fields .
- $\mathcal{B}$  The set of nodes where gas flows are splitted into two or more pipelines.
- $\mathcal{M}$  Nodes with buyers of natural gas: typically import terminals.
- $\mathcal{I}(n)$  The set of nodes with pipelines going into node  $n$ .
- $\mathcal{O}(n)$  The set of nodes with pipelines going out of node  $n$ .
- $\mathcal{R}$  The set of nodes with processing capabilities.
- $\mathcal{S}$  The set of nodes with storage facilities.
- $\mathcal{K}(b)$  The set of contracts in node  $b \in \mathcal{B}$ .
- $\mathcal{C}$  The set of components defining the chemical content of the natural gas.
- $\mathcal{T}$  The set of time periods included in the model.
- $\mathcal{L}$  The set of breakpoints used to linearize the Weymouth equation.
- $\mathcal{Z}$  The set of split percentages used to discretize possible split fraction in split-node of the network.
- $\mathcal{Y}$  The number of discretized storage and injection rate levels used to linearize storage characteristics.

### Indexes

- $n$  Used for nodes in general.  $n \in \mathcal{N}$ . When more indexes are needed,  $i$  and  $j$  will be used.
- $g$  Used for nodes with production fields,  $g \in \mathcal{G}$ .
- $b$  Split nodes,  $m \in \mathcal{M}$ .
- $m$  Customer nodes  $b \in \mathcal{B}$ .
- $r$  Used for nodes with processing plants.
- $s$  Storage facility  $s \in \mathcal{S}$ .
- $k$  Contract  $k \in \mathcal{K}$ .
- $c$  Component  $c \in \mathcal{C}$ .
- $t$  Time period  $t \in \mathcal{T}$ .



- $l$  Breakpoints in linearized Weymouth restrictions.
- $z$  Breakpoints in linearization of split percentages in split nodes.
- $y$  Breakpoints for linearization of injection rate levels in storages.

### Constants

- $G_g$  Planned production [ $Sm^3/|t|$ ] in field  $g \in \mathcal{G}$ .
- $F_{ij}$  Upper limit for flow through the pipeline from node  $i \in \mathcal{N}$  to node  $j \in \mathcal{N}$ .
- $F_n$  Upper limit for flow through node  $n \in \mathcal{N}$ .
- $P_n^{max}$  Max pressure [bar] into a node  $n \in \mathcal{N}$ .
- $P_{ij}^{max}$  Max pressure [bar] into the pipeline from node  $i \in \mathcal{N}$  to node  $j \in \mathcal{N}$ .
- $P_{ij}^{min}$  Min pressure [bar] out of the pipeline from node  $i \in \mathcal{N}$  to node  $j \in \mathcal{N}$ .
- $P_b$  Target pressure [bar] for deliverances to a customer node  $b \in \mathcal{B}$ .
- $Q_n^{max}$  Max energy content requirement for gas deliverances to node  $n \in \mathcal{N}$ .  
Energy quality is given by (GCV or  $CO_2$ ) or  $\frac{[MJ/Sm^3]}{[mol\%]}$ , where GCV is the (Gross Caloric Value).
- $Q_n^{min}$  Min energy content requirement for gas deliverances to node  $n \in \mathcal{N}$ .
- $D_b$  Demand in standard cubic meter pr time unit [ $Sm^3/|t|$ ] for natural gas in node  $b \in \mathcal{N}$ .
- $S_l$  Storage capacity [ $Sm^3/|t|$ ] in node  $s \in \mathcal{S}$ .
- $K_{ij}^W$  The Weymouth constant is used as a constant in an empirical expression for linking flow and pressure in pipelines.
- $A_r^c$  Fraction of component  $c$  in processing plant  $r$  that is extracted from the flow.
- $PI$  Fixed point for pressure into a pipeline.
- $PO$  Fixed point for pressure out of a pipeline.
- $\Gamma_n$  Compressor factor in node  $n \in \mathcal{N}$ .
- $\Theta_n$  Pressure reduction factor in node  $n \in \mathcal{N}$ .
- $\eta$  Compressor efficiency.
- $K_a$  Adiabatic constant for a certain gas type.
- $W^{max}$  Power output capacity of the compressor.
- $\omega_b$  Value of gas to customer  $b$ .
- $\kappa$  Penalty cost for pressure level.
- $\varpi$  Penalty cost for deviation from contracted pressure level.
- $\iota$  Penalty cost for use of turn-up.
- $\chi$  Penalty cost for deviation from contracted quality to customers.
- $E_z$  Gives the split percentage related to a given  $z$  in linearization of split nodes.

$X_y$  Discrete representations of storage level in linearization of storages.  
 $H_y$  Discrete representations of injection rates in storages.

### Decision Variables

$f_{ij}^c$  Flow from node  $i \in \mathcal{N}$  to node  $j \in \mathcal{N}$  of component  $c$ .  
 In some cases index  $c$  is omitted when we do not consider multi-commodity flow.

$f_n$  Total flow into node  $n$ .

$e_{nz}$  Flow through node  $n$  of component  $c$  used for linearization in splitting nodes  $m \in \mathcal{M}$ .

$p_{ij}^{in}$  Pressure [bar] into the pipeline going from node  $i$  to node  $j$ .

$p_{ij}^{out}$  Pressure [bar] out of the pipeline going from node  $i$  to node  $j$ .

$q_{ij}$  Gas quality ( $GCV$  or  $CO_2$ ) in pipeline going from node  $i$  to node  $j$ .

$\nu_y$  Give convex combinations of  $X_y$  and  $H_y$ .

$\sigma_{in}$  Equal to 1 if flow from  $i$  to  $n$ , otherwise 0.

$\vartheta_{nz}$  Binary variable representing split percentage in node  $n$ .

$a_r^c$  Amount extracted of component  $c$  in plant  $r$ .

$\rho_{ij}$  Equal to 1 if flow goes from  $i$  to  $j$ , otherwise 0.

$\zeta_m^{t-}$  Volume sold in spot market  $m$  in time period  $t$ .

$\zeta_m^{t+}$  Volume bought in spot market  $m$  in time period  $t$ .

$\delta_b^{l+}$  Positive deviation from the lower quality limit  $Q_b^{min}$  of customer node  $b$ .

$\delta_b^{l-}$  Negative deviation from the lower quality limit  $Q_b^{min}$  of customer node  $b$ .

$\delta_b^{u+}$  Positive deviation from the upper quality limit  $Q_b^{max}$  of customer node  $b$ .

$\delta_b^{u-}$  Negative deviation from the upper quality limit  $Q_b^{max}$  of customer node  $b$ .

$x_s^t$  The storage level at a given time  $t$  in a storage  $s \in \mathcal{S}$ .

$\epsilon_b^+$  Positive deviation from the contracted pressure to customer  $b$ .

$\epsilon_b^-$  Negative deviation from the contracted pressure to customer  $b$ .

$\Delta_g^+$  Positive deviation from the planned production in field  $g$ .

$\Delta_g^-$  Negative deviation from the planned production in field  $g$ .

### Functions

$EGV^t(x_s)$  Expected gas value in time period  $t$  as a function of the storage level in storage  $s$ .

$W_{ij}(PI, PO)$  Flow resulting from pressure difference between Pressure in,  $PI$  and

	pressure out, $PO$ , of a pipeline according to the Weymouth equation.
$\Gamma(f_n)$	Compressor factor as a function of flow $f_n$ into the node $n \in \mathcal{N}$ .

### Stochastic Variables

$\tilde{\phi}_{bk}^t$	Contract price for contract $k$ in customer node $b$ in time period $t$ .
$\tilde{\mu}_{bk}^t$	Demand for contract $k$ in customer node $b$ in time period $t$ .
$\tilde{\psi}_m^t$	The spot price in market $m$ in time period $t$ .
$\tilde{\xi}^t$	The vector of all stochastic variables $\tilde{\phi}^t$ , $\tilde{\mu}^t$ and $\tilde{\psi}^t$ .

In time periods where these parameters are not stochastic or where uncertainty is resolved, the tilde is dropped in the variable name.

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## Paper II

Marte Fodstad, Kjetil Trovik Midthun, Frode Rømo and Asgeir  
Tomasgard:

# *Tactical Portfolio Planning in the Natural Gas Supply Chain*

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Management Science 163, Springer Science+Business Media,  
2011*





## Chapter 3

# Tactical Portfolio Planning in the Natural Gas Supply Chain

### **Abstract:**

We present a decision support tool for tactical planning in the natural gas supply chain. Our perspective is that of a large producer with a portfolio of production fields. The tool takes a global view of the supply chain, including elements such as production fields, booking of transportation capacity, bilateral contracts and spot markets. The bilateral contracts are typically take-or-pay contracts where the buyer's nomination and the prices are uncertain parameters. Also the spot prices in the market nodes are uncertain. To handle the uncertain parameters, the tool is based on stochastic programming.

The goal for the producer is to prioritize production over the planning period in a way that makes sure that both delivery obligations are satisfied and that profits are maximized. The flexibility provided by the short-term markets gives the producer a possibility to further increase his profits. Production and transportation booking decisions in the early periods are taken under the uncertainty of the coming obligations and prices which makes flexible and robust solutions important. There will be a trade-off between maximum profits and robustness with respect to delivery in long-term contracts.

## 3.1 Introduction

Portfolio optimization is commonly used to manage portfolios of financial assets (Ziemba & Vickson 2006, Mulvey 2001, Zenios 1993), but also physical asset portfolios can benefit from this methodology. We look at portfolio optimization applied for the natural gas supply chain, with a special focus on the sub sea system on the Norwegian Continental Shelf (NCS) which is illustrated in Figure 3.1. The basic components of this supply chain are production fields, intermediate nodes, storages, the contract delivery points and downstream spot markets, all connected with a grid of pipelines for transportation. Traditionally long term contracts have been most common and some large producers have dominated in this supply chain.

The portfolio perspective is particularly interesting given the liberalization process which the European natural gas business is going through at the moment. The process is mainly driven by two EU directives (European Union 1998, 2003,

European Commission & Transport 2002). This liberalization process has led to the emergence of new short-term market hubs, i.e. in Zeebrugge, and we also see developing derivative markets with natural gas as the underlying commodity, for instance the International Petroleum Exchange (IPE). It could be noted that the evolution of the UK market, NBP, which is the most developed European market, was mainly market driven and started prior to the EU directives (Heather 2010).

On the NCS, the main changes include the separation of transportation and production into separate companies. This is accompanied with third-party-access to the infrastructure. The tariffs for transportation are regulated by the Norwegian Ministry of Petroleum and Energy with the objective that profits should be generated in production and sale, not in transportation. Further, each producer now sells their gas independently, not through the mutual Gas Negotiating Committee as before (Dahl 2001).

In Ulstein, Nygreen & Sagli (2007) planning of offshore petroleum production is studied on a tactical level. The model has a supply chain approach where production plans, network routing, processing of natural gas and sales in the markets are considered. In addition, quality restrictions in the markets and multi-commodity flows are considered. The pressure constraints in the network are however not included in the model. The non-linear splitting for chemical processing is linearized with binary variables. The resulting model is a mixed integer programming model.

Selot, Kuok, Robinson, Mason & Barton (2008) presents an operational model for production planning and routing in the natural gas supply chain. The model combines a detailed infrastructure model with a complex contractual model. There is no market for natural gas included in the model. The infrastructure model includes non-linear equations for relating pressure and flow in wells and pipelines, multi-commodity flows and contractual agreements in the market nodes (delivery pressure and quality of the gas). The contractual model is based on a set of logical conditions for production sharing and customer requirements. The combined model is a mixed integer nonlinear programming model (MINLP). In addition, the model is non-convex due to the pressure-flow relationship and the modelling of multi-commodity flows.

A tactical portfolio optimization model with a focus on the physical properties of the natural gas transportation network is presented in Tomasgard, Rømo, Fodstad & Midthun (2007). The paper provides a stochastic formulation, but do not include any numerical examples. Midthun, Bjørndal & Tomasgard (2009) show how the properties of pressure and flow of gas in pipelines give system effects in a network that affect efficient utilization. Midthun, Nowak & Tomasgard (2007) present an operational portfolio optimization model where decisions are taken under uncertainty in demand and spot prices. Especially the paper focuses on the commercial value of utilizing line-pack, which is excess storage capacity



**Figure 3.1:** The transport network on the Norwegian Continental Shelf (Ministry of Petroleum and Energy / Norwegian Petroleum Directorate 2009)

in the pipelines system.

In this paper we focus on the business environment faced by a large natural gas producer: how can the portfolio of production rights, booking rights and market opportunities be handled in an optimal way? The physical network and routing are not included since these decisions are made by an independent system operator, and are out of the producers' control. This means that the most important decisions made by the producer are the booking of transportation capacity, distribution of production over the planning period, sales in spot markets and delivery in contracts. We present a multi-stage stochastic optimization model and provide numerical examples to illustrate the value of portfolio optimization in the natural gas supply chain.

In Section 3.2 we present the portfolio perspective in our model. The mathematical formulation is given in Section 3.3 before we discuss scenario generation for multi-stage stochastic models in Section 3.4. In Section 3.5 we provide some results and numerical examples before we give some conclusions in Section 3.6.

## **3.2 Portfolio Optimization**

Even though the natural gas producers do not control the routing in the network, they still face bottlenecks that make the portfolio perspective valuable:

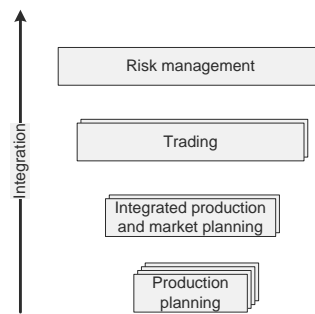
- Limited liquidity in the market nodes
- Equity gas requirements in the contracts
- Booking capacity
- Production capacity

The limited liquidity in the market nodes makes it challenging to match production plans from uncoordinated fields with the delivery obligations downstream. For some of the delivery contracts there may be requirements regarding equity gas. This means that a ratio of the total gas delivered should come from the producer's own production (and not from the spot markets). Lastly, limited booking and production capacity makes the coordination between markets favourable with respect to prioritizing between the fields.

### **Planning Perspectives**

As the operational framework and market structure evolve, also the producer's activities and organization may change. This is reflected in a evolution line of different planning perspectives illustrated in Figure 3.2 . Traditional production

planning has the focus on balancing the production portfolio with the contract portfolio. With the access to short-term and derivative markets, the possibility of combined production and market optimization is opened. At this level emphasis is on using the market flexibility to avoid physical bottlenecks and thereby maximize the total profit. This can evolve further into a trading level where transactions in the financial markets to maximize profits are done independently of the physical operations. Finally, for a risk averse company portfolio management can be integrated with risk management. Whether or not such integration is advisable depends amongst other on the completeness of the markets, existence of market power and organizational costs. This is discussed further in Bjørkvoll, Fleten, Nowak, Tomasgard & Wallace (2001) and Fleten, Wallace & Ziemba (2002) related to the electricity market. In this paper we will focus on a model for the production- and market optimization level, but with the greedy nature of a optimization model the border to trading is not as clear in the model as in the organizational structure and strategies of a company.



**Figure 3.2:** Evolution in planning perspectives

Anthony (1965) suggests to classify planning and control activities in three classes that are often named *strategic*, *tactical* or *operational*. This classification is frequently used in *hierarchical planning* (see e.g. Hax & Meal (1975), Bitran & Tirupati (1993)) that can also be applied on the natural gas supply chain. A producer's planning can be seen as a hierarchy of strategic, tactical and operational planning where the more long-term plans give limitations and guidelines for the more short-term plans. The strategic planning has several years horizon with a focus on investments, long-term contracts and energy allocation between the years. Tactical planning typically covers up to three years with a focus on energy allocation in a seasonal perspective, transportation capacity booking and positioning in the short-term markets. Operational planning relates to daily or weekly planning with short-term production planning based on market possibili-

ties, secondary market transportation booking and physical constraints. All the hierarchical levels can utilize portfolio planning to facilitate a global view of the available resources. In this paper the focus is on the tactical level.

A tactical portfolio optimization model gives decision support in several areas. It optimizes how to employ the production capacities of different fields and thereby helps on establishing production plans. Further it finds preferable transactions to make in the natural gas markets that can be used as input to tactical energy allocation. Similarly the model illustrates the need for transportation booking that can be input when booking decisions is to be made. Suggestions on booking and market transactions are useful both to initiate actions and as guidance for the operational planning. Besides the operational planning a tactical portfolio optimization model can be used to evaluate possible strategic decisions and for valuation of assets in the supply chain.

### 3.3 Model Description

The model presented here is a multi-period multi-stage linear programming problem. The period length can be chosen freely, but to support readability we assume all periods to have equal length in this presentation. The uncertainty is represented discretely by scenarios with outcomes for all the uncertain variables in each period.

The network that forms the basic structure of the model consists of fields, contract delivery nodes, spot markets and intermediate nodes. Any of these can be entry or exit nodes of the transportation market, here denoted as booking nodes. The possible flows are given by directed transportation links. Fields are sources that cannot have any inflow, whereas all other nodes can have both inflow and outflow. Market and delivery nodes can only have outflows going to other market or delivery nodes. This comes from the fact that there are no upstream markets and the direction of flow is determined in all the export pipelines from NCS. An example of a network is given in Figure 3.3.

We use a steady-state representation where the natural gas flows through this network without any time lag. In reality the time for a production rate change to be observable in the downstream markets can be several days, but this is still assumed to be neglectable in a tactical horizon.

#### Constraints and Objective

**Mass balance** In all nodes we have to make sure that the volumes entering the node correspond to the volumes leaving the node. Since fields are assumed to have no inflow this implies production should equal total outflow in each field:

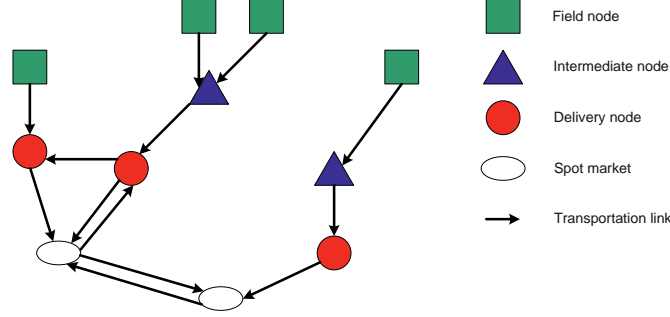


Figure 3.3: Example of a valid network

$$k_{gts} = \sum_{i \in \mathcal{O}(g)} f_{gits}, \quad g \in \mathcal{G}, t \in \mathcal{T}, s \in \mathcal{S}. \quad (3.1)$$

In other nodes than fields we require total inflow to equal total outflow minus what is sold or delivered to contracts within the node. Note that  $v_{cnts}$  and  $q_{nts}$  only exist for  $n \in \mathcal{D}$  and  $n \in \mathcal{M}$  respectively.

$$\sum_{i \in \mathcal{I}(n)} f_{ints} = \sum_{c \in \mathcal{C}(n)} v_{cnts} + q_{nts} + \sum_{i \in \mathcal{O}(n)} f_{nits} \quad (3.2)$$

$$n \in \mathcal{N} \setminus \mathcal{G}, t \in \mathcal{T}, s \in \mathcal{S}.$$

**Transportation capacity** We model a transportation market similar to the one existing on the NCS. The operations of the transportation network are unbundled from the production and marketing of natural gas, so physical requirements like pressure and gas blending are taken care of by an independent system operator (ISO). The network modelled here is a commercial network where booking nodes and transportation links are included according to how transportation capacities are made available by the ISO. Because of the ISO's flexibility of swapping different producers' gas the commercial network is typically more flexible than the underlying physical one.

The booking system on the NCS is closely related to a zonal system with entry and exit booking in each zone. It consists of a primary and a secondary market. In the primary market producers can buy capacity from the ISO within booking time windows at a fixed price. Each producer has an upper booking limit in each booking node that are calculated by the ISO based on the network capacity and the producer's production capacity and long term obligations. The secondary market is a bilateral market where a producer can resell booked capacity to

another producer. The secondary market is not included in the model presented here because this market has a very limited liquidity which makes it unreasonable to base tactical planning on the ability to trade in the market.

To model the transportation market we use two sets of variables, transaction variables  $h_{b\tau ts}$  representing the booking decisions and balance variables  $a_{b\tau ts}$  representing the amount that is booked so far. In each booking period the balance is updated according to the booking decisions and the balance from the previous period. The balance is initiated with the amount booked prior to the model horizon.

$$a_{b\tau ts} = a_{b,\tau-1,ts} + h_{b\tau ts}, \quad b \in \mathcal{B}, t \in \mathcal{T}^{booking}, \tau \in \mathcal{T}, t \geq \tau, s \in \mathcal{S}, \quad (3.3)$$

$$a_{b0ts} = X_{bt}, \quad b \in \mathcal{B}, t \in \mathcal{T}, s \in \mathcal{S}. \quad (3.4)$$

The booking should not be allowed to exceed the upper booking limit described above. Since selling transportation capacity is not included in the model, it is sufficient to make sure the balance in the period of transportation does not exceed the upper limit.

$$a_{btt s} \leq A_{bt}, \quad b \in \mathcal{B}, t \in \mathcal{T}, s \in \mathcal{S}. \quad (3.5)$$

At last we restrict the total flow into a booking node from exceeding the booked capacity. Since fields do not have any inflow this constraint relate to total outflow for fields.

$$\sum_{i \in \mathcal{O}(g)} f_{gits} \leq a_{gts}, \quad g \in \mathcal{G}, t \in \mathcal{T}, s \in \mathcal{S}, \quad (3.6)$$

$$\sum_{i \in \mathcal{I}(b)} f_{ibts} \leq a_{bts}, \quad b \in \mathcal{B} \setminus \mathcal{G}, t \in \mathcal{T}, s \in \mathcal{S}. \quad (3.7)$$

**Fields** The production in the field nodes is restricted by the minimum and maximum daily level. Some fields also have maximum yearly production limits which are concessions from the authorities. The flexibility of a field is reflected in how these levels relate. Many fields produce both gas, condensate and oil simultaneously, and the producer has very limited possibility to affect the ratio between the products. Since gas is the least valuable of these products typically the difference between daily minimum and maximum production is small for a field having a low gas-to-oil ratio. Fields mainly producing gas typically have a wider daily flexibility. The concessions are tighter than what could be achieved within the daily maximum production limits, which gives flexibility in how to allocate the gas within the year.



The daily and yearly limits are modelled below. The constants of the daily limits are aggregated to match the length of the periods in the model.

$$\underline{F}_{gt} \leq k_{gts} \leq \overline{F}_{gt}, \quad g \in \mathcal{G}, \quad t \in \mathcal{T}, \quad s \in \mathcal{S}, \quad (3.8)$$

$$\sum_{t \in \mathcal{T}(y)} k_{gts} \leq F_{gy}^{year}, \quad g \in \mathcal{G}, \quad y \in \mathcal{Y}, \quad s \in \mathcal{S}. \quad (3.9)$$

**Markets** *Trade limits* The market does not have perfect competition, but a constant price within a interval as modelled below. To approximate how large volumes would influence the price several price intervals can be used to give a piecewise linear convex function. Alternatively the price elasticity could be expressed in a quadratic objective.

$$q_{mts} \leq Q_m, \quad m \in \mathcal{M}, \quad t \in \mathcal{T}, \quad s \in \mathcal{S}, \quad (3.10)$$

$$q_{mts} \geq -Q_m, \quad m \in \mathcal{M}, \quad t \in \mathcal{T}, \quad s \in \mathcal{S}. \quad (3.11)$$

#### *Split contracts*

Some of the contracts give the producers the flexibility to choose which delivery point to send the gas to. This is constrained by upper and lower limits on the fraction of the delivery that can be sent to each delivery node:

$$v_{cdts} \leq C_{cd}^{max} V_{cts}, \quad d \in \mathcal{D}, \quad c \in \mathcal{C}(d), \quad t \in \mathcal{T}, \quad s \in \mathcal{S}, \quad (3.12)$$

$$v_{cdts} \geq C_{cd}^{min} V_{cts}, \quad d \in \mathcal{D}, \quad c \in \mathcal{C}(d), \quad t \in \mathcal{T}, \quad s \in \mathcal{S}. \quad (3.13)$$

#### *Meet demand*

The demand in each contract should always be met (either by equity gas or by utilizing the spot markets):

$$\sum_{d \in \mathcal{D}(c)} v_{cdts} = V_{cts}, \quad c \in \mathcal{C}, \quad t \in \mathcal{T}, \quad s \in \mathcal{S}. \quad (3.14)$$

**Equity gas** For some of the contracts there is a requirement that parts of the deliveries should be equity gas. This means that a fraction ( $\gamma_c$ ) can be sourced freely, while  $1 - \gamma_c$  must come from the producer's own production. The alternative to own production is gas bought in a spot market. According to the network definition spot gas can appear in market nodes and delivery nodes only. This means gas arriving the delivery node from fields or intermediate nodes, but not from other delivery nodes or market nodes is defined as equity gas.

The source requirement is modelled with two constraints in order to also take care of the both the contracts with single and multiple delivery nodes. We start with a formulation for the contracts single delivery nodes:

$$\sum_{n \in \mathcal{I}(d) \setminus \mathcal{M} \setminus \mathcal{D}} f_{ndts} - \sum_{c \in \mathcal{C}^{split}(d)} v_{cdts}^{eq} - \sum_{n \in \mathcal{O}(d)} f_{dnts} \geq \sum_{c \in \mathcal{C}(d) \setminus \mathcal{C}^{split}} (1 - \gamma_c) v_{cdts}, \quad (3.15)$$

$$d \in \mathcal{D}, t \in \mathcal{T}, s \in \mathcal{S}.$$

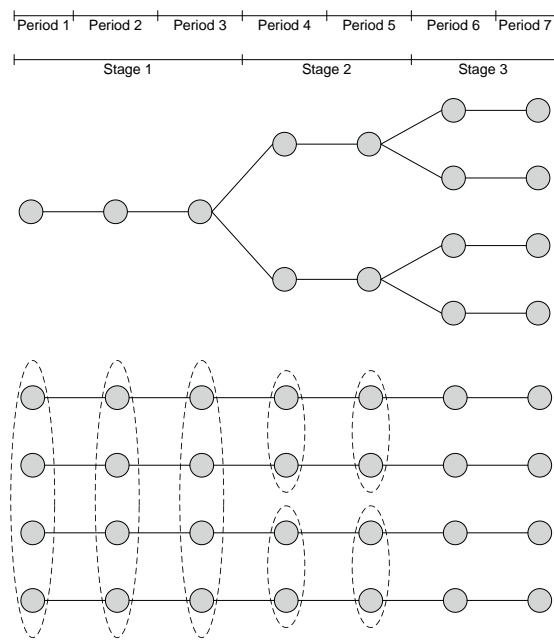
The equity gas available for delivery in the delivery node  $d$  is given by the inflows  $f_{ndts}$ . This gas can be used in two different ways; it can be delivered in a long-term contract or it can be transported to a connected downstream node. The contract deliveries can be divided further in single and multiple delivery node contracts. The right-hand side in Constraint (3.15) gives the total deliveries in contracts with one delivery node multiplied with the equity gas requirement. The second and third term on the left-hand side then gives the gas used for other purposes, the gas delivered in contracts with multiple delivery points and the gas transported out of the node, respectively. In sum, the constraint specifies that the delivery of equity gas in contracts with one delivery point has to satisfy the equity gas requirement in the contracts.

It then remains to take care of the contracts with multiple delivery points. Since  $v_{cdts}^{eq}$  is known to represent equity gas by the previous equations, we add up these equity gas deliveries from all the possible delivery nodes and require this sum to at least correspond to the required amount of equity gas.

$$\sum_{d \in \mathcal{D}(c)} v_{cdts}^{eq} \geq (1 - \gamma_c) \sum_{d \in \mathcal{D}(c)} v_{cdts}, \quad c \in \mathcal{C}^{split}, t \in \mathcal{T}, s \in \mathcal{S}. \quad (3.16)$$

**Non-anticipativity** We use a scenario tree, as illustrated in the upper part of Figure 3.4, to represent the information structure and possible outcomes of the stochastic variables. The information structure is the sequence of decision points and information flow telling what will be known and what will be uncertain at the time a decision should be taken. Branches in the scenario tree represent points where new information becomes available and some stochastic variables become certain. Decisions are taken in each node in the tree. A stage starts with a decision and ends with a branching.

Our model formulation corresponds to the scenario representation given in the lower part of Figure 3.4. To make sure the decisions taken in the early stages do not depend on foresight we need to add non-anticipativity constraints for all nodes for which the history of information are equal. These constraints force all decisions taken in one node to be equal for all scenarios containing that node (see Rockafellar & Wets (1991)).



**Figure 3.4:** Representation of uncertainty. The upper part as a scenario tree, the lower part as single scenarios with non-anticipativity constraints.

$$\frac{1}{|\mathcal{S}(z)|} \sum_{s' \in \mathcal{S}(z)} (k_{gts'}, q_{mts'}, f_{ijts'}, v_{cmts'}, v_{c'mts'}^{eq}, a_{b\tau ts'}, h_{b\tau ts'}) = \quad (3.17)$$

$$(k_{gts}, q_{mts}, f_{ijts}, v_{cmts}, v_{cmts}^{eq}, a_{b\tau ts}, h_{b\tau ts}), \quad z \in \mathcal{Z}, \quad s \in \mathcal{S}(z), \quad t \in T(z).$$

**Objective function** The objective is to maximize profits from contract sales, spot market trades, production and booking decisions. This leads to the following mathematical formulation:

$$\max \sum_{t \in \mathcal{T}} \sum_{s \in \mathcal{S}} \pi_{ts} \left( \sum_{c \in \mathcal{C}^{split}} P_{cts}^{contr} \sum_{d \in \mathcal{D}(c)} v_{cdts} + \sum_{m \in \mathcal{M}} P_{mts}^{spot} q_{mts} \right. \\ \left. - \sum_{g \in \mathcal{G}} K_g k_{gts} - \sum_{b \in \mathcal{B}} \sum_{\tau \in \mathcal{T}: \tau \leq t} H_b h_{b\tau ts} \right) \quad (3.18)$$

The first term gives the income from deliveries in the long-term contracts; the second term gives the income from trades in the spot market whilst the third term is the production costs and the fourth the costs from additional capacity booking. Only income from contracts with multiple delivery nodes are included in the objective, since there are no decision flexibility in the other contracts and the contract prices are given. Similarly the cost of booking decisions taken prior to the model horizon is left out. When the model is used for asset valuation these constant terms might be added after the optimization.

### 3.4 Scenario Generation

The uncertain parameters in our portfolio optimization model are the natural gas spot price in the markets, the demand in the bilateral contracts and the price in the bilateral contracts. The price in the contracts will typically depend on an underlying commodity such as the spot price of natural gas or of a competing fuel. In order to represent the uncertainty in our model, we construct scenario trees.

The structure of the scenario tree depends on the flow of information in our decision problem. In periods where we receive new information, we should include more than one branch. The size of the scenario tree will however directly influence the size of the optimization model. This means that we will have to keep the scenario trees at a reasonable size, and thus a trade-off between accurately describing the information flow and the total model size is important.

In addition, the properties of the stochastic parameters will influence how many branches we need to add for each stage. In the following, we give an introduction to how scenario generation can be performed for multi-stage models. A nice discussion and overview on scenario generation for multi-stage models are given in Dupačová, Consigli & Wallace (2000), and an evaluation of different methods can be found in Kaut & Wallace (2007).

There are two important elements to consider when building scenario-trees for multi-stage problems: a good representation of the properties of the stochastic parameters in each branching, and the linking of time periods. In a scenario-generation tool developed at SINTEF, the linking of time periods is done with a prediction function while the branching is done by moment-matching. The moment-matching technique is based on finding a discrete representation of continuous distributions, where the first four moments (expectation, variance, skewness and kurtosis) as well as the correlation between the different stochastic parameters are kept. The moment-matching procedure is based on Høyland, Kaut & Wallace (2003). The scenario generation is done in four steps:

1. Estimate prediction function
2. Find prediction errors
3. Build scenario tree for the prediction errors
4. Use the prediction function on the scenario tree

The procedure is independent of the chosen prediction function. This gives the user flexibility when it comes to representing the uncertainty in the given decision problem. For presentational purposes, we will here focus on an Ornstein-Uhlenbeck price process (Uhlenbeck & Ornstein 1930). The Ornstein-Uhlenbeck process is a mean reverting process that is given by the following stochastic differential equation:

$$dp_t = \eta(\bar{p} - p_t) dt + \sigma dW_t, \quad (3.19)$$

where  $p_t$  is the price in time  $t$ , the long-run equilibrium level is given by  $\bar{p}$ , the volatility by  $\sigma$  and the rate of mean reversion by  $\eta$ .  $W_t$  denotes the Wiener process.

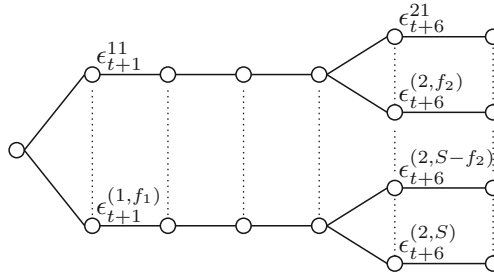
#### Example of a Scenario Generation Procedure

In the following example, we focus on the uncertainty in spot prices. In each node in the scenario-tree, there will then be a spot price for each of the market hubs in the network. We use the Ornstein-Uhlenbeck models to represent the spot price in all market nodes. A similar procedure can also be used for other

price processes / forecasting methods. The Ornstein-Uhlenbeck model can be discretized in the following manner:

$$p_t = e^{\ln [p_{t-1}]e^{-\eta\Delta t} + (1-e^{-\eta\Delta t})\left(\ln [\bar{p}] - \frac{\sigma^2}{4\eta}\right) + \sigma\sqrt{\frac{1-e^{-2\eta\Delta t}}{2\eta}}N(0,\sqrt{\Delta t})}, \quad (3.20)$$

where the last term,  $N(0, \sqrt{t})$ , represents sample from a normal distribution. It is this last term that forms the basis for our scenario generation. We use the general scenario generation procedure presented in Section 3.4 to generate scenarios for the normal distribution. The moment matching is performed with the given values for mean and variance, as well as the standard values for skewness and kurtosis. Figure 3.5 shows an example of a scenario tree for the standard normal distribution. The indexes  $f_1$  and  $f_2$  give the number of branches in the first and second stage, respectively. The value of  $\epsilon_t^{stage, branch}$  is zero in all nodes in the scenario tree, except for the nodes in the first period in a new stage (corresponding to period  $t+1$  and  $t+6$  in Figure 3.5). We generate  $S$  multivariate scenarios for the prediction error with the correct correlation between the markets and with correct moments for the individual error terms.



**Figure 3.5:** The scenario tree for the prediction errors.

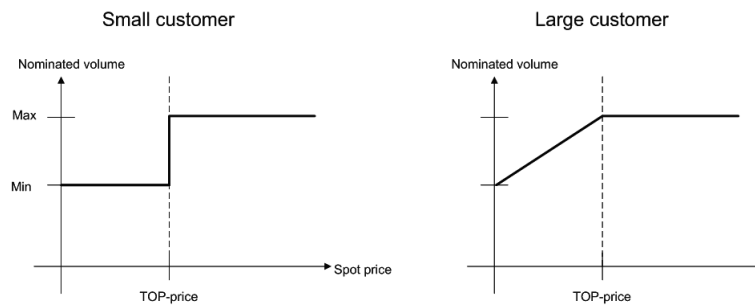
Finally, we combine the discretization of the Ornstein-Uhlenbeck process (Equation (3.20)) with the scenario tree for the prediction errors to one scenario tree. Each scenario presents a path from the root node to the leaf node (there are  $S$  unique paths through the tree). The value in each node in a path through the scenario tree can then easily be found by applying Equation (3.20). Hence we use the forecasting method to predict the expected price, and scenario generation to describe the variation (error) around this price.

### Uncertain Demand in the Bilateral Contracts

Traditionally, the bilateral contracts in the North-Sea have a take-or-pay structure where the yearly off-take is given. However, some of these contracts give

the customers substantial flexibility. This is true both with respect to the yearly volume, and the daily volume. The yearly and daily off-take must be within given limits. For instance, the daily off-take can be within 50 and 110% of a daily average contracted level. This means that for the producers, the volume uncertainty represents a challenge with respect to production and portfolio planning.

We model the uncertainty in the bilateral contracts is modelled by assuming that the customers in the contracts treat the contracts as real options. When the spot price is higher than the contract price, the customers will nominate a large volume of gas in the contract. On the other hand, when the spot price is lower than the contract price, the customer will nominate a small volume in the contract. Since some of the customers will have limited flexibility with respect to drastically changing their nominations based on the spread between spot price and contract price (due to limited liquidity in the markets and supply from their own customers), we include two different customer groups (this is a similar approach to the one used in Midthun et al. (2007)). The two customer groups are illustrated in Figure 3.6.



**Figure 3.6:** An illustration of two different customer types.

## Risk Aversion

Risk can be handled in several ways in the portfolio optimization model. In the version we present in this chapter we have focused on the risk of not being able to deliver according to the obligations in the bilateral contracts. Since the market liquidity is limited, this means that we have to be able to supply the customers mainly from our own production. This also means that we must distribute the production concessions over the planning period to be able to deliver in the contracts. In this perspective it may be interesting to evaluate the situation where the scenario tree do not represent the real underlying uncertainty well in the tails of the distributions. If the real demand outcome follows the spot price in

the manner expected during scenario generation, the decisions in the model will always make it possible to deliver according to the obligations. If however the demand turns out to be higher than we have anticipated in our scenario tree, we may risk having insufficient production concessions compared to the demand in the contracts. In order to handle this risk, we can add extreme scenarios to our scenario tree. This means that we introduce highly unlikely scenario (scenario with a zero probability of occurring) with maximum demand over the planning period. By including these scenarios we constrain our feasible region and thus we will also get a lower (or equally good) solution as before these scenarios were added. Since the probability of them occurring is zero, the profit in the scenarios will not be included in the objective function of the model. By running the model with and without these extreme scenarios, we can also find the cost of maintaining a high security of supply.

### **3.5 Numerical Examples**

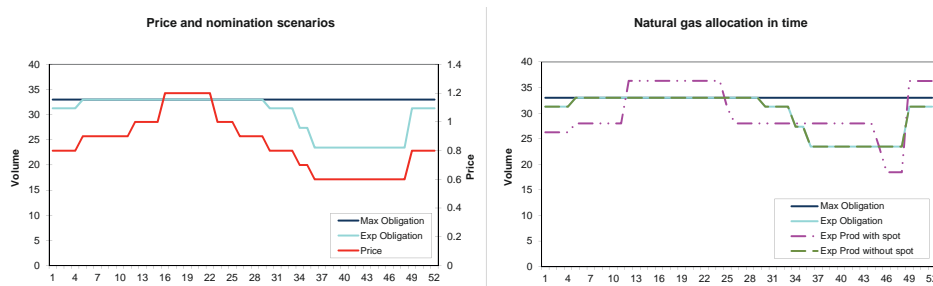
In this section we provide numerical examples to illustrate the importance of portfolio optimization and the use of a stochastic model to handle the uncertainty faced by the decision maker. We start with a simplified setting to show how the availability of short-term markets provides the producer with the flexibility to do profitable time-swaps and geographical swaps. Then we use a realistic data set to recognize these effect in a large-scale setting.

#### **Time-Swap**

We will illustrate how a producer can gain valuable flexibility through coordinated optimization of physical balancing and transactions in the spot market. To make the effects as visible as possible, we use a simplified example with one field, one market and no transportation limitations or costs. The producer has daily production capacity limitations and a concession limiting the yearly production. Further the producer has one take-or-pay contract where the buyer each morning decides on the daily volumes to be delivered. Let us assume for simplicity that the contract price is known with a seasonal variation, but the delivery obligation is a stochastic parameter since the producer does not know the buyer's nomination in advance. On the other hand, the producer and buyer have had a long-lasting business relation, so the producer is very confident with its delivery obligation forecast. Based on this we use only two scenarios in each stage, the forecast and an extreme scenario with a infinitely small probability and a volume given by the maximum contracted obligation. Both price and the two scenarios are plotted in Figure 3.7.



Let us analyze the producer's planning problem in the situation with and without a spot market. Without a spot market the producer has no real decisions to make. Each day he will have to produce and deliver the volume defined by the buyer which gives an expected production equal to the expected obligation. We include a spot market with a spot price equal to the contract price and a limitation on the volumes assumed available for that price. Figure 3.7 shows the expected production together with the two scenarios for obligations. When the production is below the expected obligation it means spot purchase and the other way around. As can be seen the producer will use the spot market to move the production capacity from periods with low price to periods with better price. The volumes to swap is limited by the assumed spot liquidity and the extreme scenario forcing the producer to hold back enough gas to fulfil the obligation if the unlikely extreme scenario is realized.

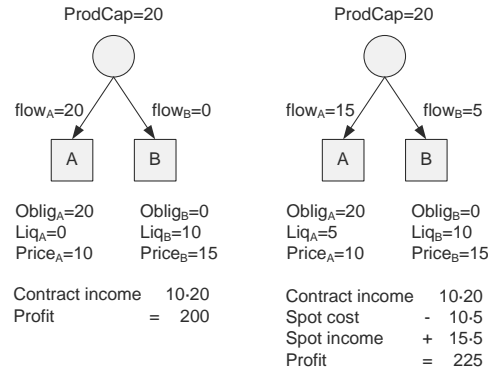


**Figure 3.7:** Illustration of the case study with and without a spot market. The figure on the left shows the price and demand scenarios, while the figure on the right shows the production results in the model version with and without the availability of a spot market.

### Geographical-Swap

To illustrate how the existence of short-term markets makes geographical swaps profitable, we use a similar setting to the one used for time-swaps. This time we consider a slightly larger network with one field node and two market nodes (see Figure 3.8). In market node A, the producer has a contract obligation. In the first case, there is a spot market available only in market node B. In the second case, there is a spot market available also in market node A. When there is a positive price difference between markets B and A, it will be profitable for the producer to use the spot market in node A to fulfil his commitments and sell a corresponding volume in market node B. The example is summarized in Figure 3.8. In networks where the producer is responsible for the routing in

addition to the production / booking decisions, the effect of geographical swaps will be even larger. The existence of booking limits will nevertheless give the possibility to perform geographical swaps a potential high value also in the setting that we present in this paper.



**Figure 3.8:** Illustration of the case study with varying liquidity in the spot markets.

### Comparison of Stochastic and Deterministic Models

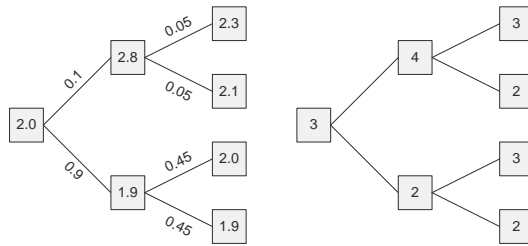
We will illustrate the difference in plans suggested by our multistage stochastic model and a corresponding deterministic model. For our tests we assume the scenario tree is an exact representation of the future. The deterministic model uses a single scenario with the expected values from the scenario tree. This corresponds to letting the deterministic model use the same forecasting method as the stochastic model. The deterministic model is ran in a dynamic way where the forecast is recalculated and the plan is reoptimized every time new information becomes available according to the scenario tree. In order to highlight the effects from including the stochasticity in the model, we use small cases. The structures will however appear (and often be enhanced) in real datasets.

When comparing stochastic and deterministic models the notion of *expected value of expected solution (EEV)* is frequently used. The expected solution is the solution of a deterministic model where all uncertain data are replaced with their expected values. EEV is defined for two-stage models as the expected objective value of the stochastic model if the first stage decisions are fixed according to the expected solution (Birge & Louveaux 1997). On the other hand the stochastic model (recourse problem) gives the stochastic solution and the objective value denoted *RP*. The *value of the stochastic solution (VSS)*, defined as  $VSS = RP -$

$EEV$  for maximization problems, is a measure of the expected objective value gain from using the stochastic model instead of the deterministic for the given description of the stochastic future.  $VSS$  is non-negative, since  $RP$  is optimizing its solution on the scenario tree while  $EEV$  is just evaluating the given expected solution on the same scenario tree.

Escudero, Garín, Merino & Pérez (2007) extends these concepts into a multi-stage setting through a dynamic way of defining the expected solution. For every node  $i$  in the scenario tree a solution is calculated for a problem where the future is described by the average value of the descendents of  $i$  and the decisions of the ancestors of  $i$  are fixed according to the previously calculated dynamic expected solutions of those nodes. Based on this procedure the expected value of the dynamic expected solution for a period  $t$  ( $EDEV_t$ ) can be calculated as the weighted average objective values of the scenario tree nodes of that period. The *dynamic value of the stochastic solution* is defined as  $(VSS^D) = RP - EDEV_T$  where  $T$  is the last time period. Analogous to  $VSS$ ,  $VSS^D$  is non-negative. We use this dynamic procedure to represent the deterministic model in our comparison.

We use a test case with three time periods, each with duration of 120 days. The network consists of one field, one contract and one spot hub. The contract can be supplied both from the field and the spot market in the hub. The production has a constant daily limitation of  $10 \text{ MSm}^3/\text{day}$  and a yearly limitation of  $1200 \text{ MSm}^3$ . No production cost is included. Transportation capacity booking is required for the exit from the field at a fixed price of  $0.01 \text{ MNOK}/\text{MSm}^3$ . Firm capacity equals the daily production capacity in the first period and is zero the two last periods. Until  $10 \text{ MSm}^3/\text{day}$  of capacity for each of the two last periods can be booked in the first period. The trade limit in the spot market is  $5 \text{ MSm}^3/\text{day}$ . The contract obligation and spot prices are uncertain and given in Figure 3.9.

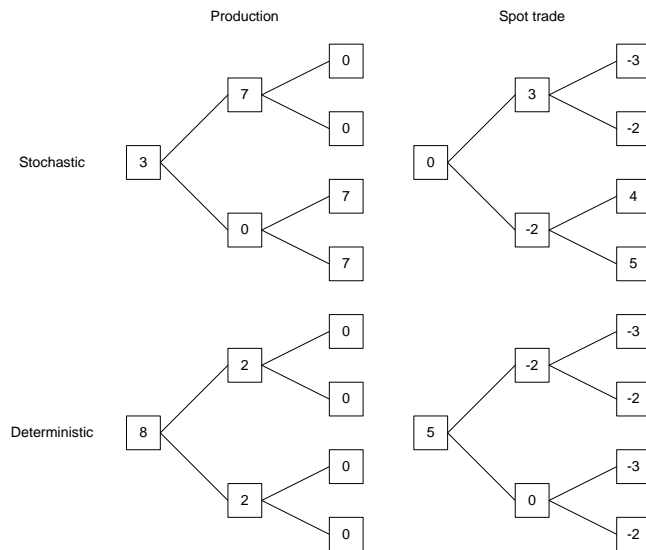


**Figure 3.9:** Input data for small test case. Left part: spot prices [ $\text{MNOK}/\text{MSm}^3$ ] and probabilities. Right part: Delivery obligations [ $\text{MSm}^3/\text{day}$ ]

Note that the scenario tree is not balanced in this case, but rather has an 'upside' scenario with high price and obligation at a low probability. The expected spot price is falling through out the model horizon. Further, note that the field

is very flexible in the sense that the daily production capacity is high compared to the yearly capacity.

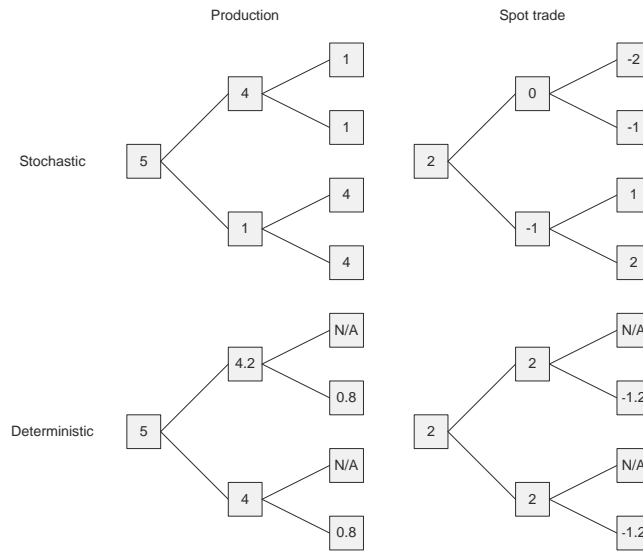
Let us look at how the yearly production capacity is allocated by the two models. The production and spot trade decisions are given in Figure 3.10. The deterministic model uses the production capacity as early as possible and utilizes the whole sales trade capacity the first period. This is reasonable, since the model takes its decisions based on the constantly falling expected spot price curve. On the contrary, the stochastic model saves capacity in the first period; to be able to utilize the high price in the second period if the 'upside' scenario is realized. If the 'upside' scenario is not realized the gas is sold in the last period since this gives a better expected price than the middle period. The value of using the stochastic model instead of the deterministic ( $VSS^D$ ) is for this test case a 3% addition to the expected profit achieved through exploiting the volatility of the spot prices. This might seem like a small payoff, but in a business where the profits are very large, the values can be substantial.



**Figure 3.10:** Result from the stochastic and deterministic models on the small test case with 5 MSm<sup>3</sup>/day as trade limit (all results are given as MSm<sup>3</sup>/day). Left part: production decisions. Right part: Spot trade decisions (positive means sale). Upper part: Stochastic model. Lower part: Deterministic model.

Now, let us change the trade limit in the spot market to 2 MSm<sup>3</sup>/day and

otherwise keep the test case unchanged. The new production and spot trade decisions are given in Figure 3.11. In this new situation the contract can no longer be fully supplied by the spot market which implies the model has no longer relative complete recourse. This causes the deterministic model to become infeasible in two of the four scenarios in the last stage. There are two decisions in the early stages that cause these infeasibilities, too little production capacity saved for the last period and too little transportation capacity booked for the last period.



**Figure 3.11:** Result from the stochastic and deterministic models on the small test case with 2 MSm<sup>3</sup>/day as trade limit. All results in MSm<sup>3</sup>/day. Left part: production decisions. Right part: Spot trade decisions (positive means sale). Upper part: Stochastic model. Lower part: Deterministic model.

To fulfil the obligation in the last period at least 1 MSm<sup>3</sup>/day of the production capacity must be available, but as the deterministic model bases the decisions on expected values it only sees the need for saving 0.5 MSm<sup>3</sup>/day. In the 'upside' scenario it can clearly be seen how the deterministic model saves less than 0.5 MSm<sup>3</sup>/day of the yearly production capacity for the last period and thereby becomes infeasible.

The first period is the only possible booking period in this test case. Table 3.1 contains the transportation booking decisions made by the two models. The de-

**Table 3.1:** Booking decisions [MSm<sup>3</sup>/day]

Model	Midle period	Last period
Deterministic	4.2	0.8
Stochastic	4	4

terministic model prefers early deliveries to late deliveries because of the falling expected spot price, which gives a similar pattern for the transportation booking. This implies booking enough transportation capacity to both fullfill the expected delivery obligation and fully exhausts the spot trade limit in the middle period. Booking for the last period corresponds to the remaining yearly production capacity that cannot be delivered the two first periods. Since this remainder is less than the 1 MSm<sup>3</sup>/day needed to fulfil the last period obligation in two scenarios this transportation booking decision makes the deterministic model infeasible in these two scenarios.

Theoretically we could argue that these infeasibilities mean the  $VSS^D$  is infinite in this situation. In real business, there typically are more instruments available to treat an 'infeasible' state, but these can be very expensive. Examples are buying replacement gas beyond the trade limit at a very high price or failing to fulfil an obligation with penalty fees and weakened reputation as a consequence.

In general, what we have seen in these two situations is how the stochastic model sees a value of making robust decisions in the early stages by making capacity (production and transportation) available till more information is available.

### Experiences with Large-Scale Realistic Data

In this section we use a large-scale example with realistic data for the NCS to show the same effects as illustrated in the previous sections. The data set represents a gas year with 6 time periods of 2 month each. The scenario tree is symmetric with 4 stages, 14 branches from each stage and 2744 scenarios. The example has 112 nodes, of which 35 are fields, 6 are spot markets and 7 have delivery obligations. We use real data describing fields, the spot prices are based on historical data while the data on contracts and transportation booking rights are sensitive data in the business and therefore substituted by synthetic but realistic data. In this section the model build from this large-scale example with the model description given in Section 3.3 is defined as the base case. All results reported are expected values.

The model is implemented in Mosel and solved by Xpress version 7.0.0 ([www.fico.com](http://www.fico.com)). The base case has approximately 386000 variables and 205000

**Table 3.2:** Effect of removing spot purchase possibility

Model	Profit	Spot income	Transportation cost
Without spot purchase	-4%	-10%	14%

constraints after presolve and is solved in 71 seconds on a computer with 2.33 GHz CPU and 3 GB RAM.

We will first analyse the value of coordinating market and production planning and thereby being able to use time and geographical swaps. The model is run with and without the possibility of buying spot gas in the markets, since this is a condition to be able to make swaps. The results reported in Table 3.2 show a 10% decrease in the spot income. The differences in total volume traded are only marginal, so the profit decrease is mainly a result of achieving lower prices for the gas. Totally the decrease in profit is 4% which in absolute values is in the order of 200 million Euro. (Note that only the decision dependent profit is included in the calculations. That includes transportation cost, production cost, spot sales income and income from contracts with optional delivery nodes.)

Further we look into the robustness and risk profile of different model structures. We use three models, the first iteration of the dynamic deterministic model (Deterministic, described in Section *Comparison of Stochastic and Deterministic Models* on page 100), the stochastic model (Base) and the stochastic model with a extended scenario tree (Stochastic extreme). The Deterministic model uses a single scenario given by the expected values from the scenario tree of the Base model. The extended scenario tree has a new extreme scenario after each node except the leaf nodes. These extreme scenarios have zero probability and contract obligations equal to the maximum level the customer can nominate within each contract. Adding extreme scenarios in the stochastic model corresponds to a risk averse policy where the probability of not being able to fulfilling an obligation is zero.

Table 3.3 shows how the expected value of the objective function components deviates from the Base model. The Deterministic model achieves better expected values and the Stochastic extreme model achieves worse than the Base model. There are no differences in contract income and only marginal differences in volumes traded spot in the three models. The extreme scenarios reduce the flexibility to make spot trade decisions based on preferable prices which limits the spot income in the Stochastic extreme model. Further the capacity booking increases with 177% compared to the Base model to be able to cover the obligations in the extreme scenarios, but because of low tariffs the effect on the total expected profit is limited.

**Table 3.3:** Deviations from base case with different models tested on the large-scale example.

Model	Profit	Spot income	Transportation cost
Deterministic	1%	1%	-29%
Stochastic extreme	-3%	-7%	177%

Note that these results orders the models according to the spread of the scenario tree optimized over, which can be interpreted as a ordering according to the level of risk aversion. The Deterministic model utilizes the resources very efficiently but the solution is less robust for deviations from the expected scenario. The Stochastic extreme model on the other hand can handle any outcome in the support of the stochastic parameters, but for this robustness a risk premium is paid.

The true values of these solutions are not realized until the actual outcome of the stochastic parameters is known. This is recognized in the procedure calculating EDEV (Section *Comparison of Stochastic and Deterministic Models* on page 100) where the solution of the Deterministic model is evaluated on an other tree than the single scenario tree it is optimized over. For the large-scale example the dynamic deterministic solution turns out to be infeasible in the last iteration, partly because of too aggressive spot sales in the spring and partly because of lacking transportation booking, which corresponds to the effect pointed out for the small examples. On the other hand, if a less than maximum offtake in the contracts is realized the Stochastic extreme model will prove unnecessarily cautious. The choice on which model to use should be taken based on the risk policy in the company and a judgement of the possibility and cost of handling high offtakes with means not included in the model.

### 3.6 Conclusions

We have presented a tactical portfolio optimization model for a natural gas producer. The model includes concession on fields, short-term markets, booking of transportation capacity, handling of contract commitments and uncertain parameters (price and demand in long-term contracts and price in the short-term markets). The model has been used for different analysis concerning portfolio optimization by a large natural gas producer for several years.

The numerical examples illustrate the potentially high value of utilizing short-term markets for geographical-swaps and time-swaps. In order to utilize this flexibility in an optimal manner, the portfolio view on the set of concessions,



contracts and market opportunities is vital. We have also included a large-scale example based on realistic data (market data, network data and field data), where we show a substantial economic potential of using stochastic programming and a portfolio perspective with coordinated production and trade decisions in planning.

## **Acknowledgments**

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# Appendix

## 3.A Notation

### Sets

$\mathcal{N}$	The nodes in the transportation network
$\mathcal{B}$	Booking nodes, $\mathcal{B} \subseteq \mathcal{N}$
$\mathcal{G}$	Production fields, $\mathcal{G} \subseteq \mathcal{B}$
$\mathcal{D}$	Delivery nodes for the contracts, $\mathcal{D} \subseteq \mathcal{B}$
$\mathcal{M}$	Spot markets in the network, $\mathcal{M} \subseteq \mathcal{B}$ and $\mathcal{M} \cap \mathcal{D} = \emptyset$
$\mathcal{I}(n)$	Nodes with outflow going to node $n$
$\mathcal{O}(n)$	Nodes with inflow coming from node $n$
$\mathcal{C}$	The contracts in the portfolio
$\mathcal{C}^{split}$	Contracts in the portfolio with multiple delivery nodes, $\mathcal{C}^{split} \subseteq \mathcal{C}$
$\mathcal{C}(d)$	The contracts in delivery node $d$ , $\mathcal{C}(d) \subseteq \mathcal{C}$
$\mathcal{D}(c)$	The delivery nodes of contract $c$ , $\mathcal{D}(c) \subseteq \mathcal{D}$
$\mathcal{Y}$	The years included in the optimization horizon
$\mathcal{T}$	The time periods in the optimization horizon
$\mathcal{T}^{booking}$	The time periods where booking decisions can be made
$\mathcal{T}(y)$	The time periods included in year $y$
$\mathcal{S}$	The scenarios
$\mathcal{Z}$	Event nodes in the scenario tree
$\mathcal{S}(z)$	Scenarios passing through event node $z$

### Constants

$K_g$	The unit cost for production in field $g$
$H_b$	The per unit tariff in booking node $b$
$X_{bt}$	Booked firm capacity in booking node $b$ for transportation in time $t$
$A_{bt}$	Volume available for booking in node $b$ for transportation in time $t$
$Q_m$	The maximum trade in spot market $m$ , time $t$ and scenario $s$
$C_{cd}^{max}$	The maximum fraction of nominated gas in contract $c$ that can be delivered in delivery node $d$

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$C_{cd}^{min}$	The minimum fraction of nominated gas in contract $c$ that can be delivered in delivery node $d$
$\gamma_c$	The fraction of gas that can be sourced freely for delivery in contract $c$
$F_{ij}$	The flow capacity between the downstream nodes $i$ and $j$
$\bar{F}_{gt}$	The maximum daily production in field $g$ and time $t$ (Aggregated to match period length.)
$\underline{F}_{gt}$	The maximum daily production in field $g$ and time $t$ (Aggregated to match period length.)
$F_{gt}^{year}$	The maximum yearly production in field $g$ and year $y$
$T_z$	The time period of event node $z$

**Stochastic parameters**

$P_{mts}^{spot}$	The spot price in market $m$ in time $t$ in scenario $s$
$P_{cts}^{contr}$	The price in contract $c$ in time $t$ in scenario $s$
$V_{cts}$	The demand in take-or-pay contract $c$ in time $t$ in scenario $s$
$\pi_{ts}$	The probability of scenario $s$

**Variables**

$k_{gts}$	Production in field $g$ in time $t$ in scenario $s$
$q_{mts}$	Spot sale in time $t$ in scenario $s$ . Negative values represent purchase.
$v_{cdts}$	Volume delivered in take-or-pay contract $c$ in delivery node $d$ in time $t$ in scenario $s$
$v_{cdts}^{eq}$	Equity gas delivered in split contract $c$ in delivery node $d$ in time $t$ in scenario $s$
$a_{b\tau ts}$	The balance of transportation capacity booked from booking node $i$ to booking node $j$ at time $\tau$ for transportation in time $t$ in scenario $s$
$h_{b\tau ts}$	The booking of transportation capacity from booking node $i$ to booking node $j$ in time $\tau$ for transportation in time $t$ in scenario $s$
$f_{ijts}$	Flow from nodes $i$ to node $j$ in time $t$ and scenario $s$

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## Paper III

Marte Fodstad, Kristin Tolstad Uggen, Frode Rømo, Arnt-Gunnar Lium, Geert Stremersch and Stéphane Hecq:

### *LNGScheduler: A Rich Model for Coordinating Vessel Routing, Inventories and Trade in the LNG-Supply Chain*

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## Chapter 4

# LNGScheduler: A Rich Model for Coordinating Vessel Routing, Inventories and Trade in the LNG-Supply Chain

### **Abstract:**

Natural gas (NG) is one of the fastest growing sources of energy in the world and, due to liquefaction to liquefied natural gas (LNG), NG markets are becoming global and involve moving LNG over large distances by vessel. We present an optimization model that provides decision support for the LNG supply chain by coordinating vessel routing, inventory management upstream, onboard and downstream, trading, and contract obligations. The model maximizes profit by utilizing different trading contracts, seasonal variations in pricing, price differences between different markets and inventory routing. We look into how the model may change some of the business practices in the industry.

## 4.1 Introduction

Natural gas (NG) is one of the fastest growing sources of energy in the world. Due to the liquefaction of the natural gas into liquefied natural gas (LNG) which makes it easier to transport, the natural gas market is becoming global and involves moving LNG over large distances by vessel. In this market, players have to make decisions on how to manage their inventories of LNG upstream, onboard the vessels and downstream the supply chain. Further, they have to determine the rate of production of LNG upstream as well as the regasification rate of LNG into NG downstream. In addition the players need to coordinate the maintenance and routing of their own vessels as well as when to charter additional vessels and how to operate them. These decisions must be made in accordance with a plethora of constraints related to vessel operations, port operations and contractual obligations. To complicate planning further LNG and NG can be purchased and sold several times on its voyage from the producer to the consumer. As the players in the industry are growing with larger fleets serving more ports, the planning is becoming more complex and as a result, they are more and more interested in using optimization-based decision support systems when planning

their operations in order to obtain economies of scale.

Extensive research has been done on vehicle routing problems (VRP) since they were first described in a paper by Dantzig & Ramser (1959), and the interested reader is referred to Toth & Vigo (2002) and Crainic & Laporte (1997) for an introduction to the topic. However, the overwhelming majority of these papers address land-based transportation which has very different characteristics from deep sea transportation. This is especially true when it comes to dealing with the planning horizon. Land-based applications typically assume that operations happen instantaneously or that there is a somewhat natural way of partitioning the planning horizon, for example by planning one day at a time. These assumptions rarely hold in sea-based transportation, as highlighted by Christiansen, Fagerholt, Nygreen & Ronen (2006). Early examples of combining inventory management and vehicle routing can be found in Golden, Assad & Dahl (1984) and in Dror, Ball & Golden (1985). For an early example of inventory routing at sea, Miller (1987) addresses the problem of planning the transportation of chemicals. For an overview of papers on this topic the reader is referred to Andersson, Hoff, Christiansen, Hasle & Løkketangen (2007).

Review of the ship routing literature (see for example Christiansen et al. (2006) for an overview) reveals that there is usually a strong focus on how to improve the maritime operations and less on how to improve the overall performance of the supply chains these operations are a part of. When the scope is extended to inventory routing or on improving larger parts of the supply chain this is typically done with a focus on fulfillment of obligations and cost minimization (Andersson et al. 2007, Christiansen, Fagerholt & Ronen 2004). However, this is not always in line with the supply chain management philosophy presented by, e.g., Simchi-Levi, Kaminsky & Simchi-Levi (2003) that the objective is 'to be efficient and cost-effective across the entire system' to avoid sub-optimization based on differing objectives in different parts of the chain. Expecting the LNG markets to become more liquid and flexible, it is reasonable to look into the more market oriented OR literature, e.g. portfolio optimization. Here the focus is not on cost minimization, but on profit maximization in order to best utilize the possibilities given by the markets. A similar change in objective was seen earlier in models for the electricity business where the development of markets has come further (Wallace & Fleten 2003). An introduction to some of the issues that should be addressed in order to coordinate decisions across the LNG supply chain can be found in Stremersch, Michalek & Hecq (2008). The first attempt to bridge this gap between supply chain management, portfolio optimization and routing in the LNG supply chain was done by Grønhaug & Christiansen (2008) and Grønhaug, Christiansen, Desaulniers & Desrosiers (2008), who created schedules for a heterogeneous fleet of LNG-vessels, while at the same time addressing the inventory levels, so that profit was maximized.

The problem we address is a more extensive version of the LNG-Inventory Routing Problem (LNG-IRP) Grønhaug & Christiansen (2008) worked on. Our model addresses larger parts of the LNG-supply chain and incorporate more details compared to their model. Further, it also addresses the many facets associated with serving a wide range of contracts and trading in a spot market. Being able to fully use the difference in contract structures as well as trading in a spot market have a stronger impact on profit margins than just looking at transportation related costs.

We present a model for planning on the tactical level and maximizing profit throughout the LNG-supply chain from the liquefaction of NG upstream to NG sales downstream. We present a richer model compared to what is found in literature and give examples of how some of the new features change the solutions. We compare our results with those found in the literature and find our solution times to be competitive. In order to test the new features of our model, additional instances have been created. The model being presented is in its implementation phase at GDF SUEZ and Statoil, and is expected to be used in their planning on a regular basis.

The outline of the paper is as follows. Section 4.2 provides a more in-depth presentation of some of the most important features of the LNG- supply chain, the economical drivers in the LNG-industry as well as the most important features of the model. The experimental setting and the instances used are presented in Section 4.3, followed by the computational results. An analysis of our findings and their practical applications is presented in Section 4.4. We conclude in Section 4.5.

## 4.2 The LNG-Supply Chain

With an estimated growth in consumption from 153 million tons of LNG per year in 2006 to between 300 and 450 million tons per year by 2020, LNG/NG is becoming a more and more important source of energy. In 2007 there were 13 exporting and 17 importing countries worldwide. While there are several thousands of operational gas-wells worldwide, only 23 liquefaction plants were operational in 2006, supplying 58 regasification plants (Foss 2007). In March 2007 there were 224 LNG vessels operating around the world, with 145 on order. Indeed, this industry's strong growth is reflected in the composition of its fleet, where 40% of the vessels operated today are less than five years old.

The physical part of the LNG-supply chain starts with the exploration and development of natural gas fields. The gas is then brought to a liquefaction facility where most of the heavy hydrocarbons and contaminants are removed. This paper address the stages of the LNG-supply chain coming after this extraction process. The cleaned NG is cooled down to a temperature of approximately  $-162$  degrees Celsius, which makes it condense into a liquid (LNG). Liquefaction reduces the

volume of the gas by a factor of 610, making it economically feasible to transport by vessel over large distances where pipelines are not an option.

After liquefaction, LNG can be held in an inventory onshore before it is loaded onto a vessel or sold ‘free-on-board’ at the port, as indicated to the left in Figure 4.1.

Transport at sea is usually done at atmospheric pressure in specially built LNG vessels typically with a capacity of 135 000 to 155 000 cubic meters. During voyage some of the LNG is lost as boil-off. The LNG can be unloaded at a buoy port at sea or at a conventional port onshore. LNG can also be purchased from other companies’ vessels to fulfill contracts at the consumption ports. Buoy ports can only be served by vessels with onboard regasification equipment. After being converted from LNG to NG, the gas is sent through pipelines to a NG-hub for sale on a spot market or to fulfill some contract. The conversion from LNG to NG and the flow downstream the supply chain is shown in Figure 4.1 where the black arc represents the flow of LNG and the grey arcs represents the flow of NG.

Conventional consumption ports onshore is somewhat different from buoy ports as they are equipped with storage facilities for LNG, which means that there is no need for the regasification process to happen during the offloading of the vessel. This reduce the vessels time in port. At these ports LNG can be sold in some special ex-ship contracts to a shared storage, where the inventory level must be kept within certain levels. The LNG can also be sold in ex-ship contracts without any restrictions on storage capacities. A third alternative is for the LNG to be transferred to a storage tank onshore from which it can either be transferred directly to vehicles or converted into NG for further transport to a NG-hub, using pipelines, from which it can be sold on a spot market or used to fulfill some contract. The flow of LNG/NG through the LNG-supply chain is shown in Figure 4.1, while a more in depth description of modeling is given in Appendix 4.A.

## **Shipping**

Transportation is a vital part and a significant cost component in the LNG-industry. Overall growth for the LNG-industry combined with acquisitions and mergers (such as the merger between Suez and Gaz de France) result in larger fleets which in turn can result in economies of scale. Unfortunately a larger fleet also means increased complexity in the planning process; hence, making an optimization based decision support system invaluable to obtaining economies of scale. When planning their operations, companies have to create plans for how to trade and transport LNG. This involves not only negotiating contracts but also operating the company’s heterogeneous fleet of vessels as well as chartering extra capacity when required.

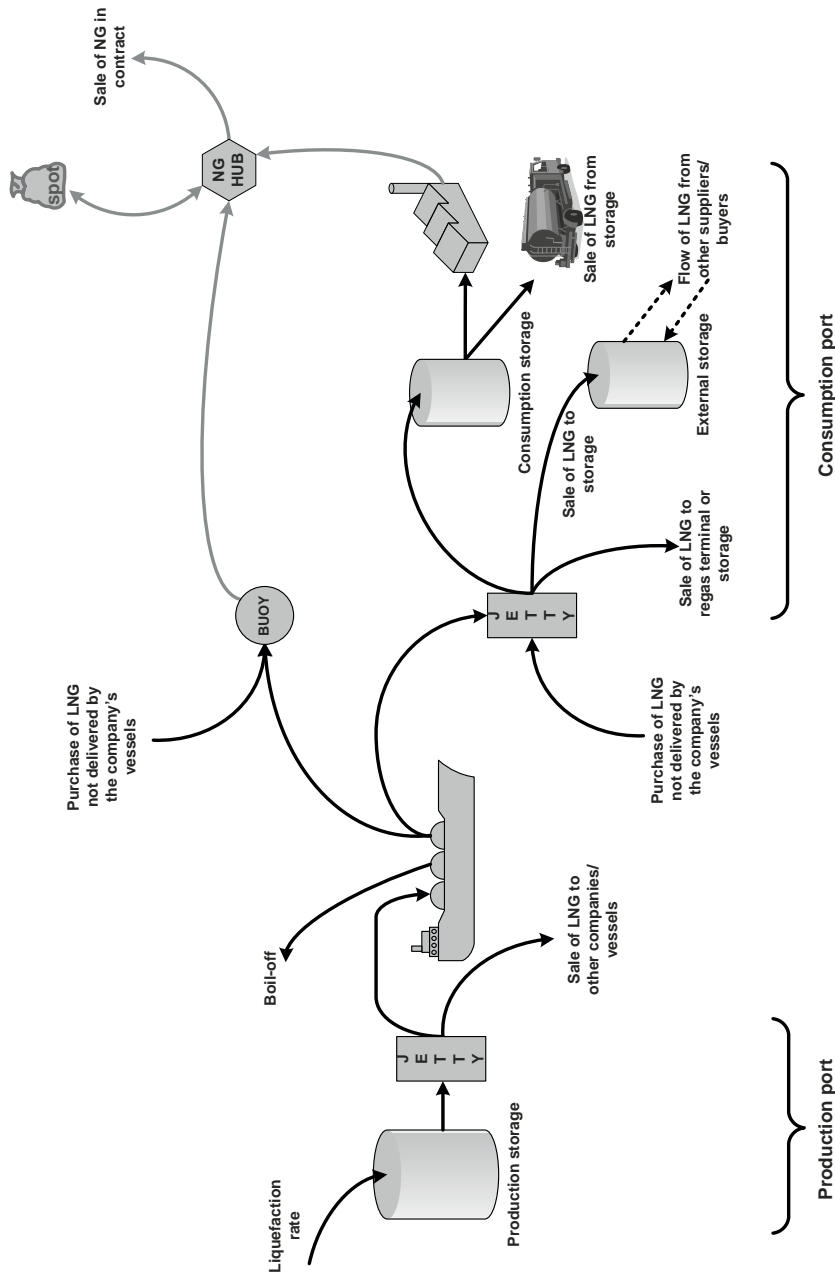
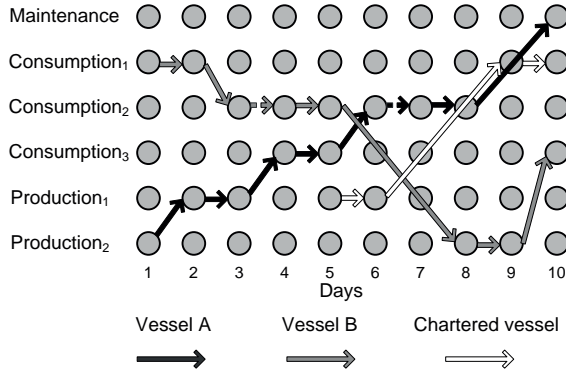


Figure 4.1: Overview of the LNG supply chain as it is modeled from liquefaction to sale at NG-hub



**Figure 4.2:** Possible routing decisions with diagonal links as traveling, horizontal solid links as port calls and horizontal broken links as waiting

In the model we have chosen to use an arc-flow formulation with a daily time granularity for vessel routing. Generally the vessels are allowed to travel to and from any port, but a link can be specified as inadmissible for one or more vessels. Further, any combination of vessels and ports can be defined as incompatible for example to represent a special draft or equipment requirements. A vessel is allowed to wait an optional number of days prior to a port call. Off-shore buoys are referred to as ports and treated as conventional ports when nothing else is mentioned. Figure 4.2 gives an illustration of the routing, where horizontal links represent waiting and port calls and diagonal links represent traveling.

Allowing the model to choose how many days the vessel should wait outside a production or a consumption port makes the problem significantly larger combinatorially due to the increased number of binary variables. However, it allows for some alternative routes for the vessel which the model otherwise would have been forced to ignore. This choice of modeling allows a company to let vessels wait outside a production or consumption port for as long as it is economically sensible. Although this may incur additional costs it also provides more flexibility for the vessels to deliver their cargo when prices are high, rather than being forced to deliver at lower rates due to some constraint limiting the waiting time.

Occasionally a vessel has to undergo planned maintenance at a maintenance port for a certain number of days, though this maintenance does not have to take place at strictly enforced intervals. This means that the company operating the vessels has to decide when the maintenance should take place. Removing a vessel from the operation reduces income as well as incurs costs related to maintenance and possibly creates the need to charter a replacement vessel. It is therefore vital that the planned maintenance is performed at the least inconvenient time in terms

of cost and lost revenue. In contrast to other ports, the number of waiting days in a maintenance port prior to maintenance is limited by a user-defined number of days in order to limit the number of discrete decisions in the model.

Some of the LNG onboard the vessel will be lost during transportation. This loss is caused by vaporizing LNG and is referred to as boil-off. As a general rule about 0.15 % of the storage content is lost every day. However, the boil-off rate depends on the vessel and the type of voyage. Laden voyages typically have a higher boil-off rate than ballast voyages. Consequently the difference between the volume loaded and unloaded increases when the travel time between production port and consumption port increases. If a cargo tank runs empty, the temperature will increase and the tank will need to be cooled down before LNG can be loaded again. This process is costly and time consuming and should therefore be avoided, which is why some of the cargo is always left in the tanks.

There are several costs related to operating the vessels that can affect their routing. One of them is the cost of port calls which are port dependent. Another cost component are the canal fees due, for example, at the Suez or Panama canal. We differentiate between four states of operation, each incurring different fuel costs: laden voyage, ballast voyage, port time and waiting. Some vessels are able to utilize the boil-off for propulsion, and thereby lower their fuel cost.

## Port and Terminal Operations

Terminals in an LNG-supply chain typically have multiple owners who all have certain rights to make decisions related to their operations. However, in order to reflect how the different stakeholders address the management of the terminals in a model, some generalizations have to be made. We define three basic levels of terminal influence in our model: 1) terminals where the model controls all aspects related to the operations of a terminal; 2) terminals where the model knows the inventory level at a terminal and have to make sure that their own cargoes do not cause a violation of storage limitations; or 3) terminals where the model only buys LNG from a liquefaction storage or sells LNG to a regasification storage without having to deal with inventories or other issues related to operating a terminal.

We define each port to have one function: a production port; a conventional consumption port; a buoy port or a maintenance port. Consumption port is used to refer to both buoy ports and conventional consumption ports. All ports have a travel time in relation to other ports, a compatibility state related to each vessel and a jetty capacity limiting the number of vessels that can visit the port simultaneously. These three characteristics are sufficient to describe a maintenance port; however, other ports have additional characteristics. A buoy port has a maximum unloading rate and a link to one or more NG hubs. A production

port and a conventional consumption port have a liquefaction terminal and a regasification terminal, respectively. These terminals are modeled with the same general properties but contrary to the buoy ports they have an inventory whose level must be kept between some upper and lower limits. The operating level for the process rate (liquefaction or regasification) is given, with the possibility to reduce the process rate below this level. It is not possible to increase the rate since the operating level typically is close to the maximum level and a tactical model should not plan to utilize this last margin. An interruption cost can be given for process rates below the operating level. Also, a minimum level for the process rate is given, since completely stopping the process should be avoided because of high start-up costs. During the regasification process there may be some loss caused by boil-off.

### **The LNG-Market and its Contracts**

Contrary to most internationally traded commodities, such as grain and crude oil, there exists no global price for NG or LNG. Most of the LNG is traded in long-term contracts without publicly known prices. Because of the limited availability of LNG to be traded on a spot market and the fact that it is a relatively infrequently traded product, one could argue that the LNG market is not an efficient one. One could say that the globe is divided into three NG-markets: North America, Europe and Asia. The North American market is the most developed with gas-to-gas competition while the Asian market is strongly affected by long-term contracts commonly linked to oil prices. Put in a tabloid way one could say that the European NG market is half way between the North American and the Asian NG market (Holmes 2007). However, the NG markets are undergoing a gradual deregulation, opening up for new entrants and an increased use of short-term contracts. Together with the growth in the LNG business this gives an evolution toward a global NG market.

To better deal with the risks associated with the high costs of constructing liquefaction plants, long-term contracts of 25-30 years have been and are still frequently used. Using long-term contracts has the advantage of creating stability for both sellers and buyers, at the cost of losing market opportunities due to the inflexibility such a system represents (Evan 2003). The short-term contracts that are becoming more common are typically of 2-5 years where prices, volumes and timing of deliveries are left open to be negotiated for each cargo. This, combined with increased trade in the spot markets, is making the industry more volatile. As a consequence it is becoming even more important to have the flexibility to adapt to changing market conditions. From an operational point of view, moving to more short term and more flexible contracts implies more frequent re-planning of vessel routes and inventory levels across the supply chain. Trading



is done to maximize profit, either by fulfilling the contracts yourself or by paying a competitor to fulfill the contracts for you. Such trading in the supply chain is typically done in relation to the liquefaction and regasification terminals and is usually done in bilateral contracts. The trade of NG is usually somewhat more centralized around NG hubs, where both bilateral trading and trading in standardized contracts like spot and forwards takes place. The relatively long planning horizon of this industry and the price difference in different markets create the possibility for arbitrage situations. Such arbitrage situations can be exploited using different contracts available to the players in the LNG business to obtain higher margins without having to be exposed to more risk.

In the model we assume that all contracts can be described in a general form with an exogenously given daily varying price. A contract can be seen from the buyer's or the seller's perspective and the location of their obligation are given. A contract can have upper and lower quantity limits within any user-defined time window.

When investing in facilities (such as liquefaction and regasification terminals) required to operate in the LNG-supply chain the two most significant risks are related to how much LNG/NG will be sold from the facility and at which price it is going to be sold. The industry uses different types of contracts to reduce the risk of opportunistic behavior. One such contract type is profit sharing. Somewhat simplified this implies that if the shipper wants to deliver LNG to another destination than the one in the contract to obtain a larger profit, he/she has to share the extra profit with the seller in the production port or the original buyer at a consumption port. This means the realized purchase price in the contract is:

$$\begin{aligned} \text{profit sharing purchase price} = & \text{purchase price} - \alpha - \beta (\text{sales price}) \\ & - \gamma (\text{sales price} - \text{reference sales price}) \end{aligned}$$

where  $\alpha$ ,  $\beta$ , and  $\gamma$  are parameters in the contract. Reference sales price is the sales price in the contracted destination and sales price is the sales price in the actual destination. Another type of contract used to reduce risk associated with price fluctuation is called netback pricing. The netback purchase price is calculated as follows:

$$\begin{aligned} \text{netback purchase price} = & \text{sales price} \\ & - \text{transportation costs} - \text{buyer's margin} \end{aligned}$$

This is often transformed into:

$$\text{netback purchase price} = (1 - \delta)\text{sales price}$$

where  $\delta$  covers the buyer's margin and a predefined transportation tariff.

The common property of these schemes is that the purchase price depends

upon the destination where the LNG is sold and is, hence, an important part of the decision process. Such dependencies between purchase and choice of destination also create a close connection between routing and trading. To enable modeling of this dependency the origin purchase contract of any LNG flowing in the supply chain is traced in the model. Purchase contracts with destination dependent pricing are split into several contracts serving one consumption port each. Together with quantity limitations this makes it possible to model the described pricing schemes.

Tracing the purchase contracts through the supply chain also enables the modeling of destination clauses. This is modeled as a set of constraints which stipulate that the quantity purchased in a set of purchase contracts and unloaded in a set of consumption ports within a time-window should be limited by upper and lower bounds.

Some contracts have clauses that are closely linked to the routing of vessels. There can be regulations on how much can be unloaded at any port within a time window for a set of contracts. Contracts can also limit which vessel can or cannot visit a port, the number of times vessels can visit a specific port and the frequency at which vessels should visit a port. Our model addresses all the before mentioned features.

In the hubs, downstream, trade is done in contracts with the general characteristics described previously. Additionally a hub has a spot market with a given daily varying price, and an upper limit on the amount allowed to be bought or sold within a day.

### 4.3 Experimental Setting and Computational Results

The model was implemented in Mosel reading data from an MS-Access database and solved using Xpress-MP 2008A. A graphical user interface to our model has been developed using C++ and C#. We later refer to this implementation as LNGScheduler. The computational tests have been done on an Intel Xeon 3 GHz computer with 3 GB RAM running Windows XP.

In order to benchmark the performance of the LNGScheduler we have used the same 27 test instances that were used in Grønhaug & Christiansen (2008) and Grønhaug et al. (2008), done some model adaptations and compared solution times. We have also added one large test instance based on the instances used by Grønhaug et al. (2008). Results from these tests as well as test instance descriptions are given in Appendix 4.B. However, our model addresses larger parts of the LNG-supply chain and provides additional features compared to what has previously been addressed in the literature. These changes are reflected

in the 16 new test instances that we have created. The additional features for the new instances and the computational results are reported in Section 3.1.

In general we observe that the results of the LNGScheduler are competitive when it comes to solution times for the test instances known from the literature. For further details, see Appendix app:results.

### **Addressing Larger and Richer Problems**

Compared to previous work, the LNGScheduler addresses larger sections of the LNG-supply chain, upstream as well as downstream. It also deals with a wide range of contractual issues which affect operations. Further, it provides a different way of addressing boil-off and gives the model flexibility to provide decision support on the number of days to wait outside a port. The LNGScheduler also allows for any combination of loading and unloading, such as loading in multiple ports before unloading in multiple ports (which makes the problem combinatorially significantly larger). Compared to previous work, such as Grønhaug et al. (2008), we have modeled the problem with more details which we believe makes it closer to the real problem. Although these new features make the problems computationally harder, they also make it possible to find solutions that would not be found using the models found in the literature. These new features can not only yield better solutions (with higher objective function values) compared to previous work but also have the potential to change common knowledge in the literature as well as in the industry.

In order to test how these extra features change the solutions as well as the computational performance, we have created 16 additional test instances. The new instances are significantly larger in terms of the number of vessels, the number of ports, the contracts addressed, the hubs served and in particular the length of the planning horizon addressed. As a consequence they have up to 4.7 times as many decision variables as the largest instance known from the literature. The new test instances have been created by GDF SUEZ and Statoil, and represents relevant cases for these companies. The only difference from real cases is that some of the prices and the actual contracts may not be the same as those presented here as they are considered business secrets. However, this is not expected to affect the general findings or the performance of the LNGScheduler.

The newly created instances can be divided into two groups which have very different characteristics. One group of instances which we refer to as ‘tight’ is very constrained with a plethora of contractual obligations and fixed production rates. The other group of instances is referred to as ‘loose’ and has few contractual obligations and is very flexible in terms of allowing reduction in production and regasification rates. Having instances that are as constrained as the ‘tight’ ones makes it very challenging to find a solution. However, the opposite is true for

**Table 4.1:** Overview of instances from GDF SUEZ and Statoil

Group	No. Vessels	No. Ports (Production, Consumption)	No. Contracts	No. NG Hubs	Time periods
tight	8	7(4,3)	7	4	181
loose	2	7(1,6)	2	6	181

the ‘loose’ instances where finding a feasible solution is fairly easy but provides challenges in terms of dealing with the many possible combinations related to operating on a spot market, serving a larger number of consumption ports and obtaining geographical arbitrage. An overview of the two groups of instances is shown in Table 4.1 below.

To be able to test the new features of our model we perform the tests on the new instances having some important features either turned on or off. We tested the effect of using the following three additional new features:

**Waiting days** on the consumption and the production side (off means waiting is only allowed on the production side)

**Partial loading and unloading** of the vessels (off means that the model is only allowed to load and unload full vessel loads)

**Spot markets** sale and purchase (off means that no NG spot hub is considered)

Both groups have been tested for all possible combinations of these three features. It can be noted that within a group (‘tight’ or ‘loose’) the only differences between the instances are that the three new features are being combined in different ways. The maximum computation time for each instance is set to 10 hours. The setup of the instances can be seen in Table 4.2. The column ID identifies the computational test, the Group column says which group the instance belongs to and the columns Waiting, Partial and Spot indicate whether the feature has been turned on or off during the test. It can be noted that if we did not allow any waiting days at all we would end up with infeasible solutions. To accommodate for this we have chosen to test with waiting days allowed outside all types of ports (on) and waiting allowed only outside production ports (off).

An overview of the results from the computational test of the 16 instances is shown in Table 4.3, below. The first column shows the ID of the instance, the second column shows the objective value after 10 hours, the third shows the optimality gap after 10 hours, the fourth shows the number of vessel voyages performed and the fifth shows the unloaded volume of LNG in the consumption ports.

**Table 4.2:** Test setup for instances from GDF SUEZ and Statoil

ID	Group	Waiting	Partial	Spot
29	tight	off	off	off
30	tight	off	off	on
31	tight	off	on	off
32	tight	off	on	on
33	tight	on	off	off
34	tight	on	off	on
35	tight	on	on	off
36	tight	on	on	on
37	loose	off	off	off
38	loose	off	off	on
39	loose	off	on	off
40	loose	off	on	on
41	loose	on	off	off
42	loose	on	off	on
43	loose	on	on	off
44	loose	on	on	on

It can be noted from Table 4.3 that the optimality gap is larger for all the tests with spot markets compared to the corresponding tests without spot markets, which indicates a slower convergence when using this feature.

Allowing for waiting days on both the production side and on the consumption side will result in fewer voyages since the vessels are allowed to wait if that is more profitable. Allowing for waiting days not only reduces the number of voyages it also reduces the port fees and costs related to bunkers and, to a large extent, allow the purchase of LNG when prices are low and its sale when prices are high.

Permitting partial loading, unloading and operating on a spot market has the opposite effect as it results in an increased number of voyages, *ceteris paribus*. Not surprisingly, allowing for partial loading, unloading and operating in the spot markets increases the volume unloaded (with the exception of one instance). It can be noted that for the group of ‘tight’ instances the volume unloaded decreased when allowing waiting days to be used. For the group of ‘loose’ instances we are not able to draw conclusions related to the use of waiting days. A general observation related to the objective function values shown in Table 4.3 is that there hardly seems to be any connection between the number of voyages, the unloaded volume and the objective function value. An example of this is instance 36 which obtained the highest objective function value in the group of ‘tight’ instances despite transporting the lowest volume of LNG.

**Table 4.3:** Results for instances from GDF SUEZ and Statoil

ID	Objective function value	Gap in %	No. voyages	Unloaded volume
29	5 147.3	7.44	97	6 465
30	5 243.6	9.65	99	6 564
31	5 354.4	5.55	110	6 586
32	5 481.3	7.58	111	6 799
33	5 129.6	9.06	95	6 431
34	5 271.6	10.06	97	6 554
35	5 317.2	5.97	101	6 470
36	5 507.2	7.54	104	6 325
37	597.1	0.00	18	1 253
38	893.7	9.00	19	1 253
39	634.9	1.10	25	1 286
40	1 067.3	11.13	46	1 395
41	597.1	0.00	18	1 268
42	893.7	12.80	19	1 277
43	634.9	1.80	25	1 286
44	1 058.7	12.25	42	1 340

Since the tests are not solved to optimality there is some uncertainty when it comes to indicating effects of the modeling features. For this reason a method from statistics, full factorial design, has been chosen for analyzing the effects. See for instance Box, Hunter & Hunter (2005) for an introduction to the topic. We use a  $2^4$ -factorial design with objective value as response and the factors waiting, partial, spot and group (off = ‘tight’). Three-factor and four-factor interaction effects are assumed to be insignificant. Figure 4.3 shows the normal plot of the main effects and two-factor interaction effects, with red squares indicating effects that are significant at any level  $> 0.05$ .

Not surprisingly, the group has a significant impact on the objective value. This is shown in the plot by the point labeled D. This supports the fact that the objective values in ‘tight’ are much larger than those in ‘loose’. Other significant features are partial loading and unloading and the existence of spot markets. The two-factor interaction effects show that the effects of spot and partial loading/unloading are larger in ‘loose’ than in ‘tight’. The features partial loading/unloading and spot markets strengthen each other (labeled BC). The ability to have waiting days on both sides in the supply chain does not have a significant impact on the objective function value. However, in practice this feature is required for some instances in order to maintain feasibility.

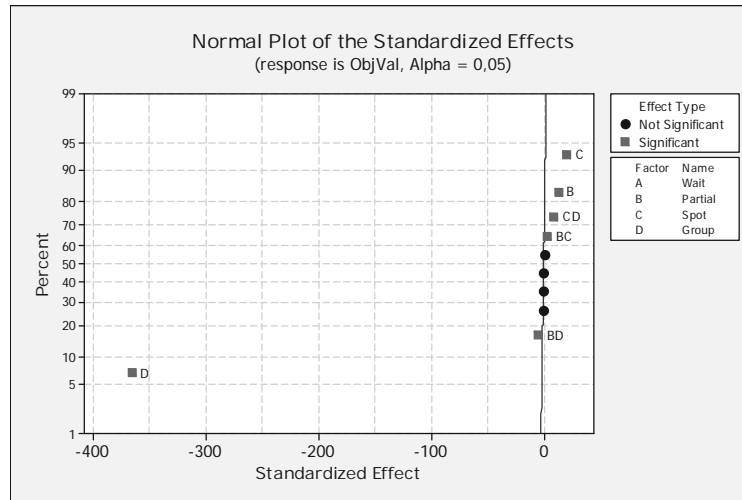


Figure 4.3: Normal plot of effects.

## 4.4 Practical Applications

### Using the LNGScheduler

Traditionally, planning in the LNG shipping industry has been done manually by experienced planners. The shipping companies have typically partitioned the planning problem in order to make it more manageable in term of size and complexity. This often implies having several groups of planners, where each group plans the operations of a limited number of vessels in the fleet operating in a specific geographical region. By partitioning the problem the planners have been able to create schedules within reasonable time. Using the LNGScheduler, planners are capable of obtaining economies of scale associated with coordinated planning and operations of a larger fleet. In addition to building more cost efficient routes for the vessels, global planning allows for profiting from geographical arbitrage, which would not have been found without using such a tool.

The cost of acquiring and operating vessels and terminals are significant. As a consequence, some of the players in this industry are running their operations with very little slack, making it a challenge to find feasible solutions and deal with unforeseen events. With the help of an optimization based tool, the user may be able to find feasible solutions that otherwise might not be found.

The LNGScheduler has proven to be useful in situations where plans had to be made or changed quickly. An example of this is when a consumption port operator plans the port operations for the following few months. During this

planning phase the various users of the port are asked to come up with a plan for when they wish to visit that specific port over the next months. The final joint plan is set in a meeting where adjustments have to be made on very short notice. This is somewhat challenging as the users not only have to take into account this specific port but their entire operation when accepting or rejecting a slot at the port at hand. Initial tests with LNGScheduler by GDF SUEZ in such situations give reason to expect useful decision support related to this planning in the future. The ability to make plans quickly is also useful when spot opportunities need to be evaluated on short notice, for unforeseen events such as the breakdown of a vessel, or in other situations that require new plans to be made quickly.

The LNGScheduler can be used to evaluate whether a fleet is sufficient to serve existing contracts. It can also give an indication of whether the capacity offered by the company's own vessels is appropriate or whether it should be changed, either to a lower level or to be increased to be able to handle more contracts and more deliveries on a spot market. The LNGScheduler provides information to the user on when maintenance should take place so that the negative economical impact of this activity is reduced to a minimum.

### **Changing Business Practices with LNGScheduler**

Planning in the LNG business is often separated in subtasks according to the different functions in the supply chain. In this setting the routing and scheduling has traditionally had a strong focus on utilizing their vessels as efficiently as possible. This has been obtained by operating with high load factors in combination with routing decisions that minimize distance traveled as well as waiting time at port. However, when changing focus from cost minimization to profit maximization for an entire supply chain, decisions which increase transportation costs can make sense as long as they are outweighed by an increase in revenue. The use of the LNGScheduler has provided several examples where it has been beneficial to abandon the traditional way of operating almost exclusively fully loaded vessels. When maximizing the profit of the entire supply chain, use the flexibility that comes with some of the contracts and the ability to trade NG, a very different business practice than the one of today appears. While the traditional way of operating vessels is with full loads, the LNGScheduler fairly often suggests that the vessels should leave the production port only partly loaded which incurs higher transportation costs (due to the reduced load factor). Nonetheless, this is outweighed by increased revenue due to the ability to perform more trips over the year and to better respond to high prices occurring at different points in time in different markets.

Another tradition in shipping, using a similar reasoning as operating full vessels only, is that vessels should never be idle. In LNGScheduler we have included



waiting days, which means that we move away from the idea that all vessels should travel unless they are in port. The inclusion of waiting days is, in the traditional cost minimization setting, not very efficient as you increase costs without doing anything. In a profit maximization setting, waiting can enable a company to use price differences and possibly provide fulfillment of contracts in a better way. These gains will outweigh the costs incurred during the waiting period as the LNGScheduler chooses to use waiting days only when it is profitable to do so. Introducing such flexibility can also make it easier to find schedules that are feasible because the number of possible solutions increases. On the other hand the increased solution space makes it harder to have a manual overview to find the best solutions and can also make it harder for the optimization tool to prove that the solution is optimal.

Different markets have different demand patterns over the year. These fluctuations mean that there is a higher demand for vessels at certain times (for example when demand is high in areas far from the liquefaction site). It also provides market opportunities in form of geographical arbitrage due to high prices in certain areas during their ‘peak’ season. As a consequence the LNGScheduler typically would suggest routes that involve few days not spent at sea during the ‘peak’ season.

Due to the capacity of the pipeline system there can be capacity constraints on the quantity of NG that can be delivered to the different markets at a given point in time. One way of overcoming this potential lack of capacity in parts of the pipeline system is to deliver additional energy to one or more of the hubs in the system using LNG-vessels. When delivering LNG by vessel to an NG hub, not only does it add transport capacity but more importantly it provides flexibility. This flexibility typically comes in the form of an inventory which provides a real option of being able to deliver LNG to more ports or having energy come to a NG-hub from more sources. It also provides additional energy, enhancing the possibility to ‘play the market’. The practical consequences of this is that a company may be able to serve one or more high yield markets without having to divert NG from other markets in the region. Not only does this allow for additional revenue from high yield markets, it also reduces the risk of not being able to deliver. One could say that it generates a larger potential for exploiting geographical arbitrage within the region served by a pipeline.

The users are, with the help of the LNGScheduler, able to create optimal or near optimal plans for how to operate vessels, ports and pipeline systems for all existing contracts within a few hours, and to give a good estimate on how much money the company can expect to make over the planning horizon. This feature of the LNGScheduler has already shown one of the industrial partners that their portfolio of contracts could potentially lead to future operational problems, and by acting on these findings, they have been able to reduce their costs significantly.

## **4.5 Conclusion and Future Work**

In this paper we have shown that it is possible to create richer models that cover larger parts of the LNG-supply chain, compared to what has been presented in the literature. Addressing larger parts of the supply chain, up- and downstream, is important as the user is able not just to look into more than just the routing of the vessels but also how to manage parts of the operations onshore. This enables the user to investigate the operations onshore for the LNG delivered by their own vessels, and it also makes it possible to buy or sell additional LNG/NG along the supply chain if this is necessary to fulfill contracts or if it turns out to be more profitable. The improved control over all parts of the supply chain makes it easier to evaluate the effect and value of current and future contracts. It also makes it easier to deal with more complex and shorter term contracts, often involving much larger flexibility with respect to price and volume, compared to what traditionally has been the case in the industry.

Our model has to some extent changed some of the ‘common knowledge’ in the industry where the best thing to do is to operate only fully loaded vessels to reduce the cost of transportation. Using the LNGScheduler we have shown that this approach may not always be the most profitable, and that there can indeed be situations where it is more profitable to operate vessels with less than full load. This is often the case when prices for natural gas are very high in certain markets for a limited number of weeks. Being able to deliver LNG when prices are high can more than outweigh the increased cost of transportation, not only due to increase revenue but also due to the fact that each vessel can do more trips during a year instead of being idle at ports waiting to be loaded.

As the players in the LNG-industry operate larger fleets visiting more ports, there is a need for algorithmic improvements to be able to tackle even larger problems than what is currently addressed. The LNGScheduler quickly calculates the costs and revenues associated with different decisions for a large part of the LNG-supply chain. However, the routing decisions are somewhat harder to address. One possible future development would be to combine exact methods and metaheuristics by having some metaheuristic attend to the routing decisions, while leaving the evaluation of the performance of the different routes to the LNGScheduler.

## **Acknowledgements**

During this project Bente Spissøy, Ingvar Skogseide and Alex Lewendon from Statoil have participated in fruitful discussions and provided us with insights on the challenges they face when planning the LNG-supply chain. This project has been supported by SINTEF, Statoil, GDF SUEZ and the Research Council of

Norway (project number 187340), for which we are grateful.

# Appendices

## 4.A Model Formulation

This appendix presents the mathematical formulation of the model. As described earlier this is a very rich model, so some functionality is only described in words to limit the size of the description. Also, special modeling needed to take care of start- and end-horizon effects are omitted for brevity's sake. In this appendix Greek letters are used to denote variables, calligraphic Latin letters represent sets, lower case Latin letters are indexes and upper case Latin letters are constants.

$\mathcal{N}$  is a set of physical ports, which is indexed by  $i, j$  and  $k$ . The set is separated into disjoint subsets representing different port types. Production ports are given by  $\mathcal{N}^P$ , conventional consumption ports by  $\mathcal{N}^C$ , buoy ports by  $\mathcal{N}^B$  and maintenance ports by  $\mathcal{N}^M$ . Consumption ports can be connected to one or more market hubs that are defined by the set  $\mathcal{H}$  and are indexed by  $h$ . Further,  $\mathcal{V}$ , indexed by  $v$ , represents a heterogeneous fleet of vessels. The days within the planning horizon are represented by the ordered set  $\mathcal{T}$ , indexed by  $t$  and  $t'$ .

Contracts are elements of the set  $\mathcal{W}$  indexed by  $w$  and  $u$ . Each contract is available in one and only one port.  $\mathcal{W}$  is split into disjoint subsets  $\mathcal{W}^{NG}$ ,  $\mathcal{W}^P$ , and  $\mathcal{W}^S$  representing NG sales contracts, LNG purchase contracts, and LNG sales contracts respectively. Each contract is described by its production or consumption port as well as delivery clauses in terms of price, quantity and other restrictions that can be applicable to a contract. There are four types of LNG sales contracts represented in different subsets of  $\mathcal{W}^S$ : 1) free-on-board sales  $\mathcal{W}^{SF}$ , 2) ex-ship sales  $\mathcal{W}^{SE}$ , 3) ex-ship sales with inventory management  $\mathcal{W}^{SI}$ , and 4) local sales  $\mathcal{W}^{SL}$ . Free-on-board contracts are related to production ports and the other three contract types are related to consumption ports.

The flow and storage of LNG and NG is given by continuous decision variables. An overview with the name and place in the supply chain is given in Figure 4.4. All the variables in the figure represent LNG except  $\chi_{iht}$ ,  $\psi_{ut}$ , and  $\omega_{ht}$  which represent NG. The variables are explained wherever they are used in the text. Almost all variables representing LNG and related mass conservation constraints throughout the supply chain have an index  $w$  which represents the purchase contract. This is done to keep track of the origin of the LNG, making it possible both to model dependencies between purchase and sales locations given in some contracts and to distinguish LNG with different energy content.  $Q_w$  gives the energy content of the LNG purchased in contract  $w$ , and is needed to convert

volumes into energy. All trading is in energy, not volume.

### Mass Conservation in the Supply Chain

The basis for the model is mass conservation through out the supply chain. Each junction in Figure 4.4 requires a mass conservation constraint. In this subsection mass conservations outside storages are described.  $\gamma_{wt}$  represents a purchase where the source is either a withdrawal from the inventory in a production port or an ex-ship purchase in a consumption port. LNG sales in contract  $u$  are represented with  $\sigma_{wut}$ , where  $w$  represents the purchase contract from where the LNG originates. LNG sales are possible in production ports in free-on-board contracts or in several different contract types in conventional consumption ports. Loading and unloading a vessel is represented with  $\eta_{ivwt}$  and  $\lambda_{ivwt}$ .  $\pi_{iwt}$  represents LNG from contract  $w$  sent to a conventional consumption port inventory and  $\phi_{iwt}$  is the corresponding withdrawal for regasification. NG from the regasification process sent to a hub is denoted  $\chi_{iht}$ . Sales of NG in contracts or spot are given by  $\psi_{ut}$  and  $\omega_{ht}$ , respectively.

$$\gamma_{wt} = \sum_{u \in \mathcal{W}^{SF}} \sigma_{wut} + \sum_{v \in \mathcal{V}} \eta_{ivwt} \quad (4.1)$$

$$\forall i \in \mathcal{N}^P, w \in \mathcal{W}^P, t \in \mathcal{T}$$

$$\sum_{v \in \mathcal{V}} \lambda_{ivwt} + \sum_{u \in \mathcal{W}^P} \gamma_{ut} = \pi_{iwt} + \sum_{u \in \mathcal{W}^{SE} \cup \mathcal{W}^{SI}} \sigma_{wut} \quad (4.2)$$

$$\forall i \in \mathcal{N}^C, w \in \mathcal{W}^P, t \in \mathcal{T}$$

$$\sum_{w \in \mathcal{W}^P} Q_w (1 - F_{it}^C) \phi_{iwt} = \sum_{h \in \mathcal{H}} \chi_{iht} \quad (4.3)$$

$$\forall i \in \mathcal{N}^C, t \in \mathcal{T}$$

$$\sum_{v \in \mathcal{V}} \sum_{w \in \mathcal{W}^P} Q_w (1 - F_{ivt}^{BV}) \lambda_{ivwt} + \sum_{w \in \mathcal{W}^P} Q_w (1 - F_{iwt}^{BW}) \gamma_{wt} = \sum_{h \in \mathcal{H}} \chi_{iht} \quad (4.4)$$

$$\forall i \in \mathcal{N}^B, t \in \mathcal{T}$$

$$\sum_{i \in \mathcal{N}^C \cup \mathcal{N}^B} \chi_{iht} = \sum_{w \in \mathcal{W}^{NG}} \psi_{wt} + \omega_{ht} \quad (4.5)$$

$$\forall h \in \mathcal{H}, t \in \mathcal{T}$$

Constraints (4.1) - (4.5) all represents mass conservation. Balance by the production port jetty and conventional consumption port jetty are given in constraints (4.1) and (4.2). Constraint (4.3) treats the regasification process in conventional consumption ports, where  $F_{it}^C$  is the loss factor during the process.

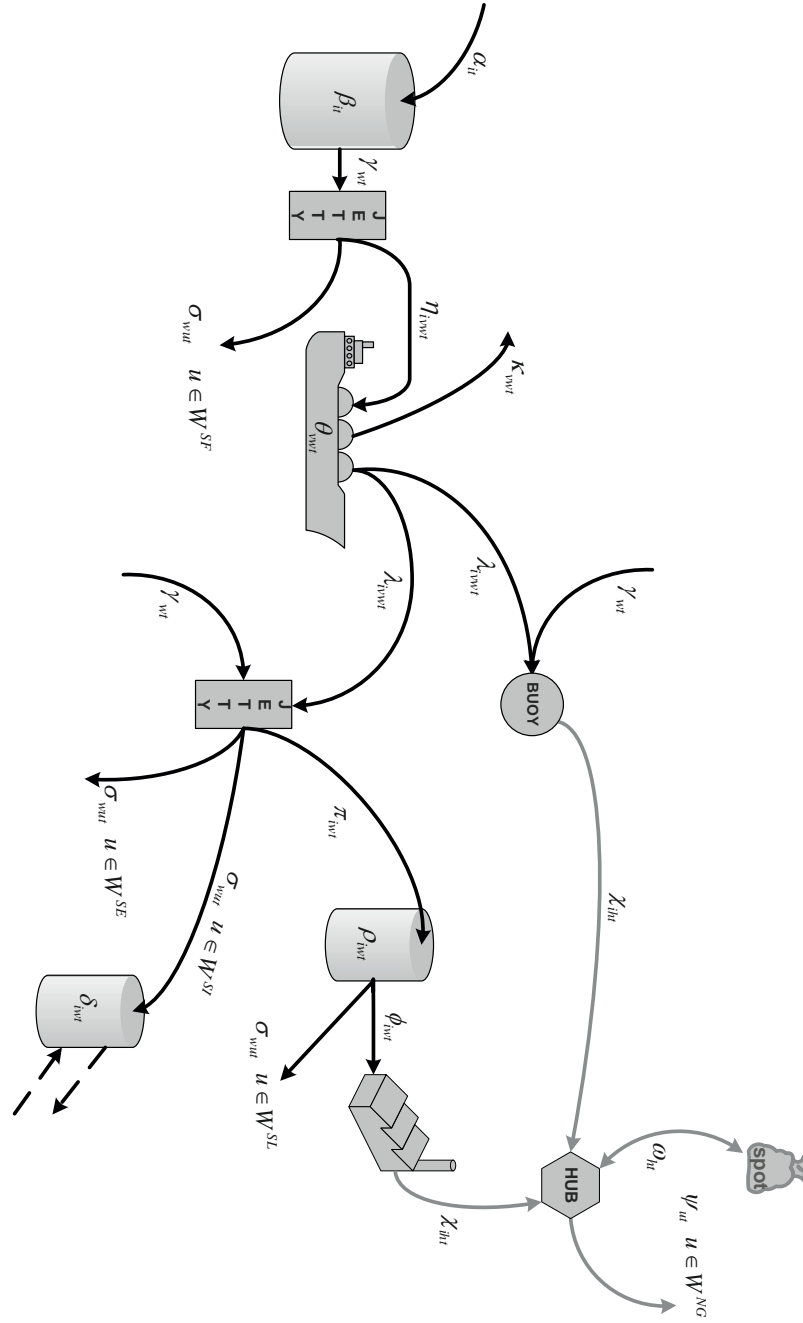


Figure 4.4: Overview of continuous LNG and NG flow variables

Constraint (4.4) enforces mass conservation in a buoy, where  $F_{iwt}^{BV}$  and  $F_{iwt}^{BW}$  are the loss factors from regasification from a vessel and from an ex-ship purchase, respectively. The mass conservation in a hub is modeled in constraints (4.5).

## Routing

A vessel can travel from one port to another, load or unload in a port or wait one or more days before entering a port. Traveling is represented with the binary variables  $\Delta_{ijvt}$  for vessel  $v$  leaving port  $i$  on day  $t$  heading for port  $j$ . The voyage will take  $T_{ijvt}^T$  days. After each voyage a vessel is assumed to visit a port for loading or unloading and the port stay will last for  $T_{iv}^P$  days. A port stay is also defined by the variable  $\Delta_{ijvt}$ . An exception is port stays in buoy ports where the duration is set by the model through the binary variables  $\Theta_{iwt}$  that is 1 if the vessel  $v$  stays in the port  $i$  at day  $t$ . This exception is included since the unloading rate in a buoy port is significantly lower than in conventional ports, which makes it reasonable to model a port stay that depends on the unloaded volume.

Between the voyage and the port stay any number of waiting days can be planned. Waiting days are given by  $\Phi_{iwt}$ . These variables take binary values, but do not need to be defined as binary variables since the constraints (4.6) and (4.7) defined below will make sure they take on binary values.

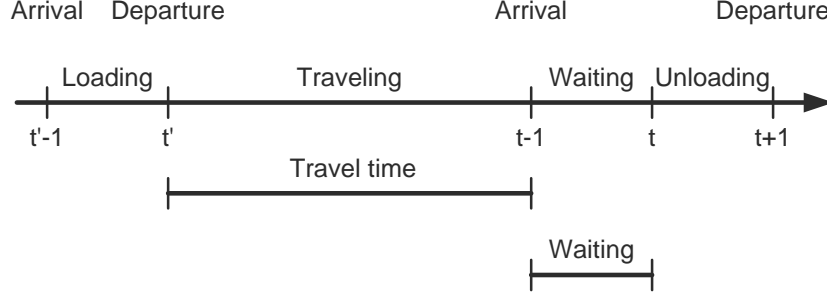
$$\sum_{j \in \mathcal{N}} \Delta_{ijvt} + \Phi_{iwt - T_{iv}^P} = \sum_{k \in \mathcal{N}} \sum_{\substack{t' \in \mathcal{T} \\ t = t' + T_{iv}^P + T_{kivt'}^T}} \Delta_{kivt'} + \Phi_{iwt - T_{iv}^P - 1} \quad (4.6)$$

$$\forall i \in \mathcal{N}^P \cup \mathcal{N}^C, v \in \mathcal{V}, t \in \mathcal{T}$$

$$\sum_{j \in \mathcal{N}} \Delta_{ijvt} + \Phi_{iwt - T_{iv}^P} + \Theta_{iwt} = \sum_{k \in \mathcal{N}} \sum_{\substack{t' \in \mathcal{T} \\ t = t' + T_{iv}^P + T_{kivt'}^T}} \Delta_{kivt'} \quad (4.7)$$

$$+ \Phi_{iwt - T_{iv}^P - 1} + \Theta_{iwt - 1} \quad \forall i \in \mathcal{N}^B, v \in \mathcal{V}, t \in \mathcal{T}$$

The constraints controlling the routing decisions in non-buoy ports and buoy ports are given in (4.6) and (4.7) respectively. Both represent conservation of flow for vessels, and make sure a vessel does not leave a port it has not arrived in. Figure 4.5 illustrates Constraint (4.6), where  $t' - 1$  is the time the vessel started to load/unload in the previous port (denoted  $i$ ),  $t'$  is the time when the vessel left the previous port and  $t - 1 = t' + T_{ijvt'}^T$  is the earliest possible time the ship can start loading/unloading in port  $j$ . Loading and unloading takes one day in this example. If the model decides that  $t - 1$  is a waiting day,  $t + 1$  will be the earliest possible departure day for the vessel, as  $t$  will be the unloading day.



**Figure 4.5:** Vessel activities along the time line as used in the routing constraints

### Maintenance

All required maintenances are defined in the set  $\mathcal{M}$ , indexed by  $m$ . The binary variable  $\Gamma_{mt}$  represents the start of a maintenance stay  $m$  at day  $t$  for the vessel  $V_m^M \in \mathcal{V}$  in the port  $N_m^M \in \mathcal{N}^M$ . This start time must be within the interval  $[T_m^{MF}, T_m^{ML}]$ , where  $T_m^{MF}$  and  $T_m^{ML}$  defines the time-window for maintenance start.  $T_m^M$  represents the required length of the maintenance. To limit the number of binary variables in the model, travel variables to a maintenance port and waiting variables in a maintenance port are only created in connection with maintenance stays.  $T_m^{MW}$  gives the number of such waiting variables prior to  $T_m^{MF}$ .

$$\begin{aligned} \sum_{i \in \mathcal{N}^C \cup \mathcal{N}^B} \sum_{\substack{t' \in \mathcal{T} \\ t = t' + T_{iN_m^M V_m^M}^T}} \Delta_{iN_m^M V_m^M t'} + \Phi_{N_m^M V_m^M t-1} \\ = \Gamma_{mt} + \Phi_{N_m^M V_m^M t-1} \end{aligned} \quad (4.8)$$

$$\forall m \in \mathcal{M}, t \in \{T_m^{MF} - T_m^{MW}, \dots, T_m^{ML}\}$$

$$\sum_{i \in \mathcal{N}^P} \Delta_{N_m^M i V_m^M t + T_m^M} = \Gamma_{mt} \quad \forall m \in \mathcal{M}, t \in \{T_m^{MF}, \dots, T_m^{ML}\} \quad (4.9)$$

Constraint (4.8) makes sure the vessel  $V_m^M$  is available in the maintenance port  $N_m^M$  within a time window, either through a direct travel from a consumption port to maintenance or through some waiting days by the maintenance port. The vessel is forced to leave the maintenance port  $T_m^M$  days after the maintenance starts and go to a production port, as described in Constraint (4.9). Vessels should



be empty during maintenance, which makes it reasonable to require a vessel to arrive at maintenance from a consumption port and depart to a production port.

### Vessel Inventory, Boil-off and Fuel

$\theta_{vwt}$  is the volume of LNG on board a vessel at the end of a day and  $\kappa_{vwt}$  is the boil-off. The total inventory level is limited by  $V_v$ , which is the storage capacity of the vessel.

$$\theta_{vwt-1} + \sum_{i \in \mathcal{N}^P} \eta_{ivwt} = \theta_{vwt} + \kappa_{vwt} + \sum_{i \in \mathcal{N}^C \cup \mathcal{N}^B} \lambda_{ivwt} \quad \forall v \in \mathcal{V}, w \in \mathcal{W}^P, t \in \mathcal{T} \quad (4.10)$$

$$\sum_{w \in \mathcal{W}^P} \theta_{vwt} \leq V_v \quad \forall v \in \mathcal{V}, t \in \mathcal{T} \quad (4.11)$$

Inventory balance for the vessels is described in constraint (4.10). The total volume on board a vessel should never exceed the storage capacity, as modeled in Constraint (4.11).

A vessel's minimum boil-off volume per day is given by  $G_v^B$  and  $G_v^L$  for vessels on ballast and laden voyages, respectively. Voyages heading for a production port are assumed to be ballast voyages and voyages heading for a consumption port are assumed to be laden voyages. Boil-off rates during waiting days are defined by the preceding voyage.

$$G_v^B \left( \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N} \setminus \mathcal{N}^M} \sum_{t'=t-T_{ijvt}^T-T_{jv+1}^P}^{t'=t} \Delta_{ijvt'} + \sum_{j \in \mathcal{N}^P} \Phi_{jvt-T_{jv}^P} \right) \leq \sum_{w \in \mathcal{W}^P} \kappa_{vwt} \quad \forall v \in \mathcal{V}, t \in \mathcal{T} \quad (4.12)$$

$$G_v^L \left( \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}^C} \sum_{t'=t-T_{ijvt}^T-T_{jv+1}^P}^{t'=t} \Delta_{ijvt'} + \sum_{j \in \mathcal{N}^C} \Phi_{ivt-T_{jv}^P} \right) + G_v^L \left( \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}^B} \sum_{t'=t-T_{ijvt}^T-T_{jv}^P}^{t'=t} \Delta_{ijvt'} + \sum_{j \in \mathcal{N}^B} \Phi_{jvt} + \sum_{j \in \mathcal{N}^B} \Theta_{jvt} \right) \leq \sum_{w \in \mathcal{W}^P} \kappa_{vwt} \quad \forall v \in \mathcal{V}, t \in \mathcal{T} \quad (4.13)$$

Constraints (4.12) and (4.13) force the total boil-off  $\sum_{w \in \mathcal{W}^P} \kappa_{vwt}$  never to fall below the minimum boil-off level in ballast and laden states respectively. However, they also provide flexibility to boil-off more than what is strictly required for physical reasons, if this should be more profitable. If the vessel has LNG from several purchase contracts on board, it is left to the model to decide from which contracts the boil-off volume should be taken.

Fuel consumption depends on whether a vessel is waiting in port, or in a laden or ballast voyage. It is modeled in a similar fashion as boil-off, but with extra constraints identifying port time and waiting. It is also possible to include boil-off as a source of fuel and thereby reduce fuel cost.

### Port Visits, Loading and Unloading

$U_{iv}$  is the maximum unloading rate for vessel  $v$  in buoy port  $i \in \mathcal{N}^B$ . For production ports and conventional consumption ports  $T_{iv}^P$  is the fixed port stay duration.

$$\sum_{w \in \mathcal{W}^P} \eta_{ivwt} \leq \sum_{j \in \mathcal{N}} \sum_{t'=t+1}^{t+T_{iv}^P} \Delta_{ijvt'} \frac{V}{T_{iv}^P} \quad \forall i \in \mathcal{N}^P, v \in \mathcal{V}, t \in \mathcal{T} \quad (4.14)$$

$$\sum_{w \in \mathcal{W}^P} \lambda_{ivwt} \leq \sum_{j \in \mathcal{N}} \sum_{t'=t+1}^{t+T_{iv}^P} \Delta_{ijvt'} \frac{V}{T_{iv}^P} \quad \forall i \in \mathcal{N}^C, v \in \mathcal{V}, t \in \mathcal{T} \quad (4.15)$$

$$\sum_{w \in \mathcal{W}^P} \lambda_{ivwt} \leq \Theta_{ivt} U_{iv} \quad \forall i \in \mathcal{N}^B, v \in \mathcal{V}, t \in \mathcal{T} \quad (4.16)$$

Constraints (4.14)-(4.16) make sure no loading or unloading takes place in a production port, conventional consumption port or buoy port, respectively, unless the vessel is in that port. They also make sure the quantities loaded do not exceed the maximum loading/unloading rates.

Further, optional upper or lower limits on loaded/unloaded volume during a port call can be added. These are represented with  $\bar{L}_{ivt}$  and  $\underline{L}_{ivt}$  for loading and  $\bar{U}_{ivt}$  and  $\underline{U}_{ivt}$  for unloading and the constraints are given as (4.17) and (4.18).

$$\underline{L}_{ivt} \sum_{j \in \mathcal{N}} \Delta_{ijv t+T_{iv}^P} \leq \sum_{w \in \mathcal{W}^P} \sum_{t'=t}^{t+T_{iv}^P-1} \eta_{vw t'} \leq \bar{L}_{ivt} \sum_{j \in \mathcal{N}} \Delta_{ijv t+T_{iv}^P} \quad (4.17)$$

$$\forall i \in \mathcal{N}^P, v \in \mathcal{V}, t \in \mathcal{T}$$

$$\underline{U}_{ivt} \sum_{j \in \mathcal{N}} \Delta_{ijv} t + T_{iv}^P \leq \sum_{w \in \mathcal{W}^P} \sum_{t'=t}^{t+T_{iv}^P-1} \lambda_{ivwt'} \leq \bar{U}_{ivt} \sum_{j \in \mathcal{N}} \Delta_{ijv} t + T_{iv}^P \quad (4.18)$$

$$\forall i \in \mathcal{N}^C \cup \mathcal{N}^B, v \in \mathcal{V}, t \in \mathcal{T}$$

Ports can have limitations related to the minimum and maximum number of days between each port call within a given time window. This is used to represent contractual guidelines as ‘approximately evenly spread’ deliveries. A port has a finite capacity in terms of the number of vessels that can stay there at one time. Further, the model includes constraints that regulate the minimum and maximum number of visits during one or more time windows. This can be used to make sure that a vessel visits a port within a given time window or on a specific day. For brevity’s sake, the latter constraints have not been written algebraically.

### Inventories and Onshore Facilities

The processing capacity at production and consumption ports is given by  $A_{it}$  and the production is allowed to go  $B_{it}$  below  $A_{it}$ . There are also lower and upper bounds on the inventory levels at the different ports given by  $\underline{I}_{it}$  and  $\bar{I}_{it}$ , respectively.

The liquefaction rate and regasification rate are given as  $\alpha_{it}$  and  $\phi_{iwt}$ . Similarly, the inventory levels in production and conventional consumption ports are defined as  $\beta_{it}$  and  $\rho_{iwt}$ , respectively.

$$\alpha_{it} \leq A_{it} \quad \forall i \in \mathcal{N}^P, t \in \mathcal{T} \quad (4.19)$$

$$\sum_{w \in \mathcal{W}^P} \theta_{iwt} \leq A_{it} \quad \forall i \in \mathcal{N}^C, t \in \mathcal{T} \quad (4.20)$$

$$\alpha_{it} \geq (A_{it} - B_{it}) \quad \forall i \in \mathcal{N}^P, t \in \mathcal{T} \quad (4.21)$$

$$\sum_{w \in \mathcal{W}^P} \theta_{iwt} \geq (A_{it} - B_{it}) \quad \forall i \in \mathcal{N}^C, t \in \mathcal{T} \quad (4.22)$$

The constraints (4.19) and (4.20) enforce that the maximum processing limit for liquefaction and regasification is not exceeded. Limitations on how far these processing rates can be reduced below the maximum level are given by Constraints (4.21) and (4.22).

$$\underline{I}_{it} \leq \beta_{it} \leq \bar{I}_{it} \quad \forall i \in \mathcal{N}^P, t \in \mathcal{T} \quad (4.23)$$

$$\underline{I}_{it} \leq \sum_{w \in \mathcal{W}^P} \rho_{iwt} \leq \bar{I}_{it} \quad \forall i \in \mathcal{N}^C, t \in \mathcal{T} \quad (4.24)$$

Upper and lower limits on the inventory level at the end of each day are given for liquefaction and regasification inventories in the constraints (4.23) and (4.24).

$$\beta_{it-1} + \alpha_{it} = \beta_{it} + \sum_{w \in \mathcal{W}^P} \gamma_{wt} \quad \forall i \in \mathcal{N}^P, t \in \mathcal{T} \quad (4.25)$$

$$\begin{aligned} \rho_{iwt-1} + \pi_{iwt} &= \rho_{iwt} + \phi_{iwt} + \sum_{u \in \mathcal{W}^{SL}} \sigma_{wut} \\ &\forall i \in \mathcal{N}^C, w \in \mathcal{W}^P, t \in \mathcal{T} \end{aligned} \quad (4.26)$$

The inventory balances for liquefaction and regasification are given in Constraints (4.25) and (4.26). They say that the inventory level on day  $t$  equals the level of the previous day plus inflows minus outflows.

For ex-ship contracts with inventory management ( $\mathcal{W}^{SI}$ ), a separate storage is modeled with the storage level  $\delta_{iwt}$ . These storages have constraints similar to (4.24) and (4.26), but with additional fixed injections and withdrawals representing other companies' transactions at the storage facility.

### Contract Limitations

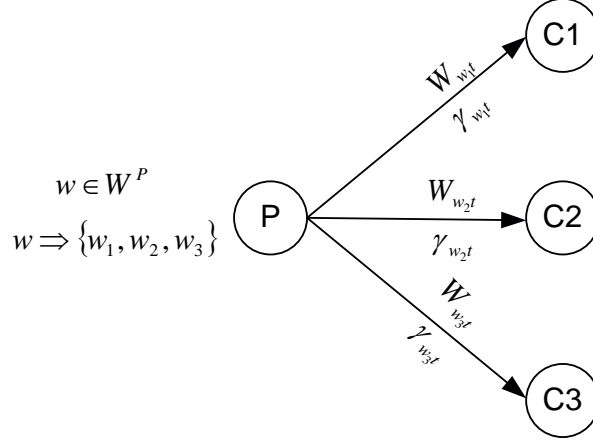
There are several volume limitations in the contracts, represented by  $d \in \mathcal{D}$ . These limits come in addition to physical limits of the system and are needed to ensure that the model respects contractual agreements. Three types of limitations are modeled, plain volume limits, destination clauses and destination dependent pricing, represented with the subsets  $\mathcal{D}^V$ ,  $\mathcal{D}^D$  and  $\mathcal{D}^P$ , respectively. The different types are described in detail below. The right-hand-sides of a limit  $d$  are given by  $\underline{D}_d$  and  $\overline{D}_d$  which represent the lower and an upper limit, respectively.

A plain volume limitation applies to one contract  $W_d$ , and sets lower and upper limits on the amount of energy that can be traded in this contract within a time-window,  $[T_d^{DS}, T_d^{DE}]$ .

$$\underline{D}_d \leq \sum_{t=T_d^{DS}}^{T_d^{DE}} \sum_{w \in \mathcal{W}^P} Q_w \sigma_w W_{dt} \leq \overline{D}_d \quad \forall d \in \mathcal{D}^V | W_d \in \mathcal{W}^S \quad (4.27)$$

Constraint (4.27) gives the equations necessary to limit the amount of LNG sold in a sale contract. Similar restrictions for purchase of LNG and sale of NG are included in the model, but omitted for brevity's sake.

$$\underline{D}_d \leq \sum_{i \in \mathcal{N}_d} \sum_{v \in \mathcal{V}} \sum_{w \in \mathcal{W}_d} \sum_{t=T_d^{DS}}^{T_d^{DE}} Q_w \lambda_{ivwt} \leq \overline{D}_d \quad \forall d \in \mathcal{D}^D \quad (4.28)$$



**Figure 4.6:** Example of destination dependent purchase prices modeled through a split of a purchase contract into several purchase contracts dedicated to different consumption ports

Constraint (4.28) models destination clauses where the choices of purchase contracts and consumption ports are made dependent. It states that the amount of energy purchased in a set of purchase contracts  $\mathcal{W}_d \subseteq \mathcal{W}^P$ , and unloaded in a set of consumption ports  $\mathcal{N}_d \subseteq \mathcal{N}^C \cup \mathcal{N}^B$  within a time-window should be limited by upper and lower bounds.

Destination dependent purchase price involves a purchase contract serving several different consumption ports with different purchase prices depending on which port the LNG is delivered to. There will also typically be restrictions on the amount of LNG transported to the different consumption ports. Figure 4.6 shows with an example how to model destination dependent purchase prices in a contract. In order to model this, we first split the original purchase contract  $w$  into three individual purchase contracts  $w_1, w_2, w_3$  that constitute the set  $\mathcal{W}_d$ . Each of these new contracts permits deliveries to one specific port only. For instance LNG bought in  $w_1$ ,  $\gamma_{w_1t}$ , can be unloaded in C1 and not in C2 or C3. Instead of having one purchase price  $W_{wt}$ , we can now have three different purchase prices, one for each of  $w_1, w_2, w_3$ .

To make sure transactions in the three contracts do not violate the volume-constraints of the original contract  $w$ , constraint (4.29) is applied.

$$\underline{D}_d \leq \sum_{t=T_d^{PS}}^{T_d^{DE}} \sum_{w \in \mathcal{W}_d} Q_w \gamma_{wt} \leq \overline{D}_d \quad \forall d \in \mathcal{D}^P \quad (4.29)$$

Constraint (4.29) states upper and lower limits on how much can be purchased from the subset of purchase contracts representing  $w$  in a time-window. Using constraint (4.27) we can put limits on volumes purchased in  $w_1, w_2$  and  $w_3$  separately in order to ensure any required volume distribution between the consumption ports.

The combination of constraints (4.27), (4.28) and (4.29) can thus be used to model different purchase prices for different consumption ports and also control volumes. These properties can then be utilized in order to model destination dependent pricing, netback pricing and even profit sharing clauses in contracts.

Some additional constraints mentioned in Section *The LNG-Market and its Contracts* on page 122 are omitted in this mathematical description.

## The Objective Function

The price of energy traded in a contract is given by  $W_{wt}$  and the spot price in a hub is  $H_{ht}$ . To denote various costs we use the parameter  $C$  with a superscript for each cost component, such as  $C_{ijv}^{CL}$  and  $C_{ijv}^{CB}$  to represent the canal cost from  $i$  to  $j$  with a loaded or empty vessel, respectively.  $C_{iv}^P$  denotes the cost for a port call. If the liquefaction or regasification rate drops below max capacity, an interruption cost  $C_{it}^I$  occurs.

The objective function is given in (4.30a)-(4.30g). This objective function maximizes the profit in the supply chain over the time horizon. (4.30a) represents the income and costs associated with LNG contract trades. Revenue from sales in contracts and net spot sales at NG hubs are given by (4.30b). The cost of visiting a port is given in (4.30c), with buoy ports treated separately in (4.30d). In conventional ports the cost of visits is vessel and port dependent. In buoy ports, the cost of visiting is vessel and port dependent as well as dependent on how many times a vessel enters the buoy during its visit (it can be profitable to leave and re-enter a port if there will be a price increase in the near future). There is also a cost if a vessel uses a canal on its voyage between two ports. This fee is dependent on the canal used and whether it is a laden or a ballast trip, and is formulated in (4.30e). (4.30f) and (4.30g) describe the cost associated with reducing the liquefaction and regasification rate below the maximum level, respectively. Fuel cost is also included in the objective as a given unit cost charged for the fuel consumption as described in Section *Vessel Inventory, Boil-off and*

*Fuel* on page 139, but this modeling is not included here.

$$\max \sum_{w \in \mathcal{W}^P} \sum_{u \in \mathcal{W}^S} \sum_{t \in \mathcal{T}} W_{ut} Q_u \sigma_{wut} - \sum_{w \in \mathcal{W}^P} \sum_{t \in \mathcal{T}} W_{wt} Q_w \gamma_{wt} \quad (4.30a)$$

$$+ \sum_{w \in \mathcal{W}^{NG}} \sum_{t \in \mathcal{T}} W_{wt} \Psi_{wt} + \sum_{h \in \mathcal{H}} \sum_{t \in \mathcal{T}} H_{ht} \omega_{ht} \quad (4.30b)$$

$$- \sum_{i \in \mathcal{N} \setminus \mathcal{N}^B} \sum_{j \in \mathcal{N}} \sum_{v \in \mathcal{V}} \sum_{t \in \mathcal{T}} C_{iv}^P \Delta_{ijv t + T_{iv}^P} \quad (4.30c)$$

$$- \sum_{i \in \mathcal{N}^B} \sum_{v \in \mathcal{V}} \sum_{t \in \mathcal{T}} C_{iv}^P \left( \Theta_{ivt} + \sum_{j \in \mathcal{N}} \sum_{t' = t - T_{ijv}^T}^{|T|} \Delta_{ijvt'} + \Phi_{iv t-1} - 1 \right) \quad (4.30d)$$

$$- \sum_{i \in \mathcal{N}} \sum_{v \in \mathcal{V}} \sum_{t \in \mathcal{T}} \left( \sum_{j \in \mathcal{N}^C \cup \mathcal{N}^B} C_{ijv}^{CL} \Delta_{ijvt} + \sum_{j \in \mathcal{N}^P \cup \mathcal{N}^M} C_{ijv}^{CB} \Delta_{ijvt} \right) \quad (4.30e)$$

$$- \sum_{i \in \mathcal{N}^P} \sum_{t \in \mathcal{T}} C_{it}^I (A_{it} - \alpha_{it}) \quad (4.30f)$$

$$- \sum_{i \in \mathcal{N}^C} \sum_{t \in \mathcal{T}} C_{it}^I \left( A_{it} - \sum_{w \in \mathcal{W}^P} \phi_{iwt} \right) \quad (4.30g)$$

## 4.B Computational Results

Our formulation deviates somewhat from the one described by Grønhaug & Christiansen (2008) and Grønhaug et al. (2008) as they divide the vessel into separate tanks and require each tank to be fully loaded. Our model assumes a single tank, but allows for partial loading. We have chosen, after advice from GDF SUEZ, not to split the load into several tanks when planning. However, to get a better comparison with Grønhaug & Christiansen (2008) and Grønhaug et al. (2008), the LNGScheduler assumes the use of separate tanks when tested on their instances. This is achieved through the use of bounds on the loaded and unloaded volumes as described in constraints (4.17) and (4.18) in Appendix 4.A. To make the comparison as fair as possible we have chosen to use the same approach for calculating boil-off as Grønhaug & Christiansen (2008) and Grønhaug et al. (2008) for instances 1-28. Further, we have turned off all constraints regarding the number and frequency of vessels visiting ports, all constraints related to contracts that are not ex-ship sales in consumption ports or free-on-board purchase in production port as well as the possibility to regasify the LNG, deliver NG to hubs and sell on the spot market, in order to better match the formulation by

Grønhaug & Christiansen (2008) and Grønhaug et al. (2008).

An overview of the test instances can be found in Table 4.4. The first column identifies the instances which were used by Grønhaug & Christiansen (2008) and Grønhaug et al. (2008), except from number 28 which is a newly created test instance. The instances have been divided into 7 groups (A to G), each assuming different number of vessels, tanks and ports. Each group consists of 4 test instances which only deviate in the length of the planning horizon. The instance IDs are equal to the IDs used by Grønhaug et al. (2008), while instance number 28 is the instance we have added.

When performing the test we have set the maximum computation time for each instance to ten hours, similar to Grønhaug et al. (2008). Table 4.5 shows the time to find the first integer solution (MIP1), the shortest time to find the first integer solution reported in the literature (MIP1-Best), the time to find the best solution (MIP\*), the time to find the best integer solution reported in the literature (MIP\*-Best), the total running time (Total), the total running time reported in the literature, the optimality gap for the best solution that we found (Gap) and the optimality gap for the best solution reported in the literature (Gap-Best). In the case where no optimal solution is found either in literature or using our model, we have assumed that the computational test with the smallest optimality gap is considered to be the best both in terms of total running time and best solution found.

As can be noted from Table 4.5 the LNGScheduler produces the first feasible integer solutions faster than what is currently reported in the literature for 9 of the 27 instances, while for 5 of the 27 test instances the LNGScheduler performs on par with what has previously been reported. When looking at computation times for optimal solutions we can note that the LNGScheduler reduced the solution time for 13 of the 27 instances, while being equally good for 5 of the 27 instances. When it comes to proving optimality the LNGScheduler has the lowest computational time for 15 of the 27 instances, and produces on par for 3 of the 27 test instances known from literature. When it comes to the different test instance groups, the LNGScheduler performs best for groups A, B, E and F except the 75 day instance for group F. The arc flow formulation from Grønhaug & Christiansen (2008) is by far the best for group C and also dominates group D and G, except for the 75 day instance of group D where the LNGScheduler is best. Even though the arc flow formulation gives good results, the Branch and Price approach from Grønhaug et al. (2008) gives more even results. If one looks at the different time horizons, the 30 day horizon has such low solution times that the differences are ignorable. When it comes to the 45 day and 60 day horizons the results are more spread among the different models. For the 75 day instances, the LNGScheduler is dominant, and also manages to find solutions for group G on a 75 day horizon.



**Table 4.4:** Instance overview

Group	ID	No. Vessels	No. Tanks	No. Ports (Production, consumption)	Time periods
A	1	1	2	4 (1,3)	30
A	2	1	2	4 (1,3)	45
A	3	1	2	4 (1,3)	60
A	22	1	2	4 (1,3)	75
B	4	2	2	3 (1,2)	30
B	5	2	2	3 (1,2)	45
B	6	2	2	3 (1,2)	60
B	23	2	2	3 (1,2)	75
C	7	2	2	4 (2,2)	30
C	8	2	2	4 (2,2)	45
C	9	2	2	4 (2,2)	60
C	24	2	2	4 (2,2)	75
D	10	2	1	5 (2,3)	30
D	11	2	1	5 (2,3)	45
D	12	2	1	5 (2,3)	60
D	25	2	1	5 (2,3)	75
E	13	2	2	5 (2,3)	30
E	14	2	2	5 (2,3)	45
E	15	2	2	5 (2,3)	60
E	26	2	2	5 (2,3)	75
F	16	3	1	4 (2,2)	30
F	17	3	1	4 (2,2)	45
F	18	3	1	4 (2,2)	60
F	27	3	1	4 (2,2)	75
G	19	5	1	6 (3,3)	30
G	20	5	1	6 (3,3)	45
G	21	5	1	6 (3,3)	60
G	28	5	1	6 (3,3)	75

**Table 4.5:** Computational results (in seconds). Values written in italic indicate that the LNGScheduler performed better than the benchmark.

ID	MIP1	MIP1 - Best	MIP*	MIP* - Best	Total	Total - Best	Gap [%]	Gap - Best [%]
1	0	0	0	0	0	0	0	0
2	0	0	0	0	2	3	0	0
3	2	11	8	15	12	32	0	0
22	5	104	5	112	38	233	0	0
4	0	0	0	0	1	1	0	0
5	0	0	3	13	4	26	0	0
6	2	4	12	1 393	32	1 580	0	0
23	3	4	152	18 657	389	26 527	0	0
7	2	0	2	1	2	1	0	0
8	17	1	76	3	118	4	0	0
9	4	155	2 138	454	3 819	456	0	0
24	7	1 877	3 900	19 285	30 213	19 707	0	0
10	0	0	0	0	1	0	0	0
11	1	0	4	3	11	9	0	0
12	3	2	47	27	107	56	0	0
25	7	2	334	1 756	334	2 889	0	0
13	1	0	1	1	1	5	0	0
14	3	0	6	63	9	95	0	0
15	7	63	76	1 599	102	3 107	0	0
26	17	11	558	4 148	565	18 315	0	0
16	1	0	1	10	2	10	0	0
17	60	0	134	206	214	1 219	0	0
18	8	0	883	3 354	11 242	36 000	0	16.60
27	1 323	10 826	29 650	33 342	36 000	36 000	317.10	183.71
19	1	0	5	3	11	14	0	0
20	7	11	15 248	3 074	36 000	36 000	23.19	0.06
21	178	41	33 173	12 214	36 000	36 000	34.66	25.10
28	798	n/a	25 248	n/a	36 000	n/a	48.48	n/a

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## Paper IV

Adrian Werner, Kristin Tolstad Uggen, Marte Fodstad, Arnt-Gunnar Lium and Ruud Egging:

# *Stochastic Mixed Integer Programming for Integrated Portfolio Planning in the LNG Business*

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## Chapter 5

# Stochastic Mixed-Integer Programming for Integrated Portfolio Planning in the LNG Supply Chain

### Abstract:

We present a new model to support the strategic planning by actors in the liquefied natural gas market. The model takes an integrated portfolio perspective and addresses uncertainty in future prices. Decision variables include investments and disinvestments in infrastructure and vessels, chartering of vessels, the timing of contracts, and spot market trades. The model accounts for various contract types and vessels, and addresses losses. The underlying mathematical model is a multistage stochastic mixed-integer linear problem. Industry-motivated numerical cases are discussed as benchmarks for the potential increases in profits that can be obtained by using the model for decision support. These examples illustrate how a portfolio perspective leads to decisions different than using the traditional net present value approach. We show how explicitly considering uncertainty affects investment and contracting decisions, leading to higher profits and better utilization of capacity. At the same time, model run times are very competitive with current business practices of manual planning.

## 5.1 Introduction

Stochastic mixed-integer programming has been used to examine a number of issues in energy economics, including investment planning in the natural gas industry (Guldmann and Wang 1999, Zheng and Pardalos 2010). The great majority of these applications have taken a cost-minimization approach. In this paper we take the perspective of firms that maximize expected profits. When doing so we look at the whole value chain in a portfolio perspective. We consider contracting decisions, while allowing for arbitrage trades in the spot market, whereby physical shipments can be re-directed to take advantage of geographical price differentials. This is illustrated using an example with price uncertainty inspired by a real-life application.

The increasing importance of liquefied natural gas (LNG) is illustrated by its rapid growth in recent years. In 2000, twelve LNG exporters traded 220 million

m<sup>3</sup> to ten LNG importing countries (BP 2001). By 2010, this had more than doubled, with 18 LNG exporting countries trading 483 million m<sup>3</sup> to 23 LNG importers (BP 2011, GIIGNL 2010).

The LNG supply chain typically includes the production of natural gas, transportation to a liquefaction terminal, the liquefaction process and loading of vessels, shipping of the LNG to regasification terminals, and finally regasification to natural gas for distribution through the pipeline grid. Figure 5.1 on page 159 provides an overview of the supply chain elements considered in our model, from liquefaction to transportation and regasification to markets for natural gas. For a thorough description of this chain we refer to Fodstad, Uggen, Rømo, Lium, Stremersch, and Hecq (2010).

Natural gas markets are dynamic and unpredictable both in the long and in the short term. Retrospectively, the US Energy Information Administration (EIA) observed that in their yearly energy outlooks the deviations between projections and market outcomes, regarding both volumes and prices, were larger for natural gas than for all other fuels (EIA 2010). Despite the significant uncertainty, lower operating and shipping costs and an increasing LNG market liquidity has induced a shift away from risk-reducing long-term contracts over the last decade. Due to a tenfold increase, spot and short-term trade accounted for 25% of total LNG trade in 2011 (GIIGNL 2011). Currently, an increasing share of short-term contracts and cargo re-routing is used to benefit from arbitrage opportunities in spot markets. These developments make it more difficult to devise profitable, yet flexible long-term strategies. It is, therefore, paramount to address uncertainty and recourse adequately when developing models to support investment and contract timing decisions.

The main contribution of the LNGPlanner model is to be a tool to perform integrated analysis of an industry actor's portfolio of both existing and potential investments along the LNG supply chain which accommodates price uncertainty. It focuses on both physical and economic aspects, allowing to exploit the flexibility in the supply chain to benefit from market opportunities while meeting operational criteria. The mathematical model forms a multistage stochastic mixed-integer linear programming (SMILP) problem, accommodating the uncertainty through a scenario tree approach. The model has been developed for and in close collaboration with several partners from the LNG industry to provide decision support for some of their strategic decisions.



## 5.2 Literature Review

There is an extensive base of literature on optimal<sup>1</sup> investment strategies; however, integrated approaches for LNG business that take uncertainty into account are underrepresented. Investment models such as the one in André (2010) tend to focus on deterministic cost minimization rather than stochastic profit maximization. In a recent paper, MirHassani and Noori (2011) explicitly addressed the drawbacks of the use of scenario analysis assuming perfect foresight for a realistic problem. Birge and Loveaux (1997) showed that stochastic optimization approaches are needed to make optimal decisions and to represent the hedging behavior of investors facing uncertainty.

Alternative means for evaluating investment opportunities are provided by real-options approaches. Murto and Keppo (2002), Klaassen, Kryazhinskii, and Tarasyev (2004), and Krey and Minullin (2005) emphasized game-theoretic aspects among the investors. Real option approaches for investment decisions in the oil and gas industry can be found in Smith and McCardle (1999), Bøckman, Fleten, Juliussen, Langhammer, and Revdal (2008), Kaminski, Feng, and Pang (2008), Thompson, Davison, and Rasmussen (2009), and Lai, Wang, Kekre, Scheller-Wolf, and Secomandi (2011). Such approaches provide useful insight into the timing of investments. However, they have limitations with regard to capturing the interrelations of multiple investment opportunities within the same modeling framework.

The stochastic dynamic programming (SDP) approaches for energy planning problems discussed in Botterud and Korpås (2007), Fleten (2000), and André (2010) potentially allow for the flexibility needed for optimal capacity and timing decisions that affect each other. However, due to the combinatorial characteristic of the problems, heuristics are needed to provide solutions within acceptable time limits. To circumvent these combinatorial challenges, Pereira and Pinto (1991), Granville, Oliveira, Thome, Campodonico, Latorre, Pereira, and Barroso (2003), Bezerra, Barroso, Kelman, Flach, Latorre, Campodonico, and Pereira (2010), and Aouam and Yu (2008) developed the concept of stochastic dual dynamic programs (SDDP). Although some of these concepts and insights can be transferred to other fields, the applicability of the SDDP approach to the problem studied in this paper is limited. SDDP requires a discretization of the potential values for

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<sup>1</sup>Throughout this chapter, we use the word optimal in the sense that it is used in Operations Research literature, namely as a solution that maximizes or minimizes a specified function. For an economist, “optimal” often means “Pareto optimal” or “efficient,” or at least utility, profit maximizing, cost, or expenditure minimizing. In particular, in the economics literature, maximization or minimization of an intertemporal objective function also entails specifying “transversality conditions” for the capital stocks or their shadow prices that follow from the underlying consumer or producer objectives and constraints. In our model, the continuation values are specified exogenously.

the decision variables while our approach includes continuous variables that may take a large range of values.

Other approaches have also been used for solving energy planning problems. Egging (2010) developed a stochastic mixed complementarity problem addressing optimal capacity expansion by various actors on the global natural gas market. The approach does not allow for integer variables and does not scale well in terms of the number of scenarios that can be accommodated. In a more operational setting, Tomasgard, Rømo, Fodstad, and Midthun (2007) presented an integrated operational and financial approach to manage and optimize the various elements of the natural gas supply chain from production to sales, taking into account uncertainty in both demand and prices in a two-stage recourse approach. Zheng and Pardalos (2010) propose a SMILP problem for location of LNG terminals and expansion of pipelines. Their model is highly relevant albeit complementary to the work presented here, as they minimize expected costs while we maximize expected net present value. The authors include pipeline expansions and regasification terminals in their model, while our model includes: regasification and liquefaction terminals, vessel investments and charter, contract decisions and spot markets, but not pipeline expansions.

Important stepping-stones to the model presented here are papers by Nygreen, Christiansen, Bjørkvoll, Haugen, and Kristiansen (1998), Fodstad et al. (2010), and Grønhaug, Christiansen, Desaulniers, and Desrosiers (2010). Nygreen et al. (1998) developed a model for optimally operating and expanding the pipeline network on the Norwegian continental shelf using a project-based approach for timing the start-up of production fields. The work of Fodstad et al. (2010) and Grønhaug et al. (2010) focused on tactical planning in the LNG business including routing of ships, typically within a yearly horizon, while LNGPlanner has a much longer planning horizon (typically 10–25 years).

The remainder of this paper is organized as follows. The next section presents the model. Sections 5.4 and 5.5 discuss two test cases, illustrating selected model features. The first case elaborates on uncertainty, hedging, and spot trading while the second one discusses the added value of the portfolio approach. Section 5.6 concludes and provides directions for future research. Appendix 5.A and 5.B provides more detailed results for the two test cases.

### 5.3 The Model

Our stochastic mixed-integer linear programming model covers the supply chain from liquefaction to shipping and regasification to natural gas markets. The objective of the model is to maximize the expected NPV of a company's portfolio of terminals, vessels, and contracts. This chapter gives a verbal description of the model, while the corresponding mathematical formulation can be found in

Werner, Tolstad Uggen, Fodstad, Lium, and Egging (2012).

### Strategic Decisions

The main decisions are investments and disinvestments that design the supply chain, and these constitute the integer and binary variables in the model. Investment opportunities are denoted as “projects” and cover liquefaction terminals, regasification terminals, vessels, and contracts. The timing of investments and disinvestments is chosen by the model, but is limited to a given time interval. The decisions typically make some capacity available in the time periods following the decision and generate a series of cash flows. In some situations, a project depends on other projects being started first; for instance, a terminal cannot be built unless the related feasibility study has been completed and the necessary permits are obtained. The model allows for mutually exclusive projects; for instance, it is not possible to choose two different sizes of terminal for a given location.

Vessel projects represent investments in different vessel types with varying capacities and costs. Furthermore, the fleet can be supplemented by chartering vessels. Unused vessels can be chartered out or sold. In contrast to buying and selling vessels, prematurely ending a charter is not possible.

The finite time horizon of a MILP model can often distort decisions close to the end of the horizon. For instance, it is unlikely for this model to invest in new assets in the last part of the time horizon because that would incur investment costs while the model only covers small parts of the operational period of the asset. A common way of handling this issue is to model and solve with a substantially longer time horizon than needed for the analysis. How much longer the time horizon should be depends on both the lifetime of the assets and the discount rate that reduces the importance of distant-future revenues and costs. It should also be kept in mind that for decision support only the first few periods are critical, since the model should be re-optimized as time goes by and new decision points are reached. To alleviate this distortion effect, we include an approximation of the discounted expected remaining value of the assets in the portfolio at the end of the horizon. For terminals, we assume that the remaining value is the investment cost less depreciation. Vessels have a predefined lifetime, and their remaining value is set to the charter rate for the remaining lifetime. Remaining values of contracts are approximated according to market prices. For sales contracts, a contract price increase relative to the market price gives a positive remaining value, and a decrease gives a negative value.

### Operational Decisions

To be able to evaluate the operational consequences of the strategic decisions, the supply chain operations are included in the model. Figure 5.1 illustrates

the operational supply chain as it is modeled; purchase and sales contracts in different parts of the supply chain are represented by stacks of rectangles.

Liquefaction terminals transform natural gas into LNG at a fixed unit cost, at a rate between the terminal's minimum production rate and maximum capacity. The produced volumes should match contractual purchases from the liquefaction terminal. By including both the production rate and the purchase contracts, it is possible to describe liquefaction terminals with different ownerships and levels of control. LNG storage is an important component in the daily operations of an LNG terminal. Typically, storage capacity covers a few days of operations with very limited flexibility to store for later months or (peak) seasons. Hence, storage has little relevance in a strategic context and is omitted here.

LNG bought at a liquefaction terminal can be either sold under contract at this terminal or shipped by the company's vessels. Vessel routing determines how much transportation capacity a fleet of vessels can give. It is a highly discrete operation that, from an optimization point of view, is known to be combinatorially challenging (Toth and Vigo 2002). To keep the model tractable, a continuous approximation has been chosen, matching transportation demand and total fleet capacity available. The former is determined from the amount of LNG to be transported and the travel time between each pair of terminals while the latter is described by the number of own and chartered vessels and their capacities, adjusted by a utilization factor reflecting ballast voyages. Additional limitations can be added to specific terminals and vessel types in order to address compatibility issues such as buoy ports requiring vessels with on-board regasification facilities. Some LNG is lost during transportation due to boil-off such that volumes available for regasification are lower than volumes sent out from liquefaction. In short time periods, part of the volumes sent out may arrive at a subsequent period due to long travel times.

LNG arriving at a regasification terminal can be either sold in a contract at the jetty or sent to the regasification facility. LNG can also be bought through a contract with delivery at the jetty. Regasification, converting LNG into natural gas, is subject to lower and upper limits on the production rate and happens at given unit costs. Losses during the process are described as a fraction of the regasification rate. Natural gas is sent to a market hub through pipelines that may also have capacity limitations. Similar to liquefaction terminals, storage facilities at regasification terminals are omitted from the model.

A market hub has a spot market for natural gas trading. The spot price is uncertain and, for a price-taker, independent of the traded volume. Because the real markets are not necessarily well-functioning, we include limits on the positions that can be taken on the purchases and the sales sides.

The model contains both purchase and sale contracts at different places in the supply chain as illustrated in Figure 5.1. They all have the same structure

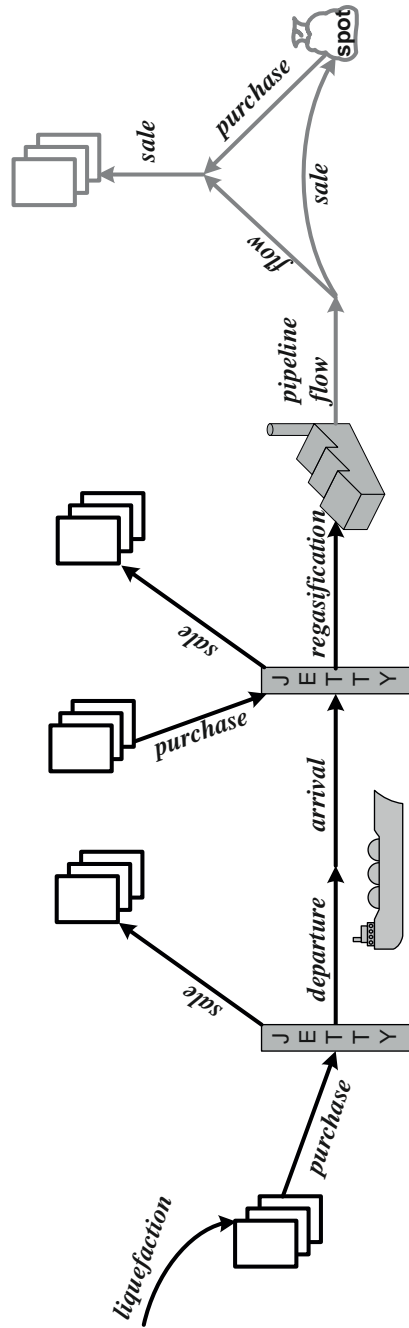


Figure 5.1: Overview of operational decisions.

and the commodity traded is LNG, except for contracts at the natural gas hubs which trade natural gas. All contracts can have uncertain prices. Furthermore, there can be limits on the amounts traded in the contracts. The simplest form of limitation is lower or upper bounds within a time interval. A single contract can have several such limits with overlapping time intervals. Destination and source clauses are limitations linking purchase and sales contracts. A destination clause states a lower or upper limit on the amounts that can be delivered from a given purchase contract to a set of sales contracts in a time interval. Similarly, source clauses restrict the sourcing from a set of purchase contracts to a given sales contract.

In the following two sections, we illustrate how our modeling choices with portfolio perspective, stochastic prices, and inclusion of spot markets can affect suggested decisions. Since our data sets are synthetic, our focus is not on the absolute values given by the tests, but rather on how different methods evaluate the projects and portfolios differently.

We implemented the model using off-the-shelf software, more precisely, XPress-MP. Obviously, solution times depend strongly on the size of the solution space (number and range of the variables), the length of the time horizon, and the description of the uncertainty, i.e., the size of the scenario tree. For test cases such as the ones presented in this paper, solution times were in the range of a few seconds to one minute. For larger realistic cases with up to 1600 scenarios, we experienced solution times of up to one hour. However, while we focused on efficient implementation, solution speed was not within the main scope of the development work. Moreover, such investment analyses are often performed only once in a while and, therefore, speed is not a major issue.

## **5.4 Test Case 1 – Valuing Robustness and Flexibility**

We use this test case to illustrate and discuss two model features, the use of stochastic programming to handle uncertain prices and the use of a supply chain perspective including spot markets where physical supply and trading are integrated.

### **Case Setup**

In order to make the discussion easier to follow, we simplify the physical value chain as much as possible. We consider a small system that consists of one liquefaction and one regasification terminal with a related natural gas hub. The existing terminals are assumed to have infinite capacity and there are no terminal

investment options. We assume sufficient transportation capacity, no transportation delay, and no natural gas or LNG losses in the supply chain. We also ignore all operational costs. The planning horizon has three periods of equal length and uncertain price information is revealed after each period, resulting in a three-stage problem.

Initially, no contract obligations exist, and all contracts must be sealed one period prior to any delivery. The liquefaction terminal has a single purchase contract option with a constant price of €6/MMBtu and contract limits of [250, 1000] MMBtu in each period. On the downstream side, there is a spot market (S) with an uncertain price that can be either low (€7.95/MMBtu) or high (€9.95/MMBtu) for each time period. Two types of natural gas contracts are also available in the market hub. A long-term contract (LC) can be entered in the first period, with deliveries within the range of [600, 700] MMBtu in the two following periods. Short-term contracts (SC) are available for entry in the two first periods, each with delivery within the range of [500, 525] MMBtu in the next period only. All contract prices are assumed to have a price formula with time lag, such that the price is known one period in advance. Spot and sales contract prices are presented in Figure 5.2 where all nodes in each time period have equal probability.

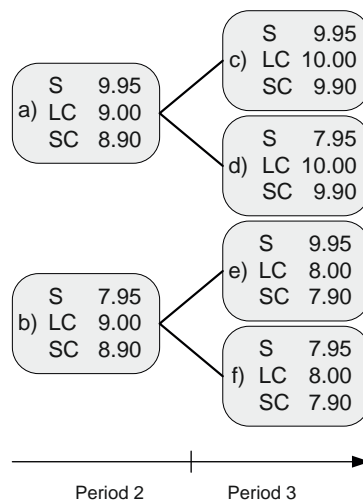


Figure 5.2: Prices in Test case 1 (€/MMBtu).

**Table 5.1:** Expected incomes from spot, long-term contract and short-term contract and profits for various planning approaches (€). All approaches have a expected purchase cost of 6,000 €.

Planning approach	E[Spot]	E[LC]	E[SC]	E[Profit]	% of STO
STO	5 370	12 600	0	6 150	100.0
DET	5 370	12 600	0	5 970	97.1
Avg DETd	6 116	11 950	0	6 066	98.6
SA_c	19 900	0	0	7 900	128.5
SA_d	13 726	0	5 198	6 924	112.6
SA_e	13 726	0	4 673	6 399	104.1
SA_f	4 770	11 900	0	4 670	75.9
Avg SA	13 031	2 975	2 468	6 473	105.3
EVPI	-2 582	2 975	-69	323	5.3
VSS	9 497	-11 950	2 537	83	1.4

## Dealing with Uncertainty

As discussed in Section 5.2, stochastic programming is often preferred over scenario analysis when uncertainty is an important characteristic of the problem. We investigate how the solutions for this test case change with different planning approaches such as stochastic programming (abbreviated to STO in Table 5.1), scenario analysis (SA\_c – SA\_f, where the last characters refer to leaf nodes representing scenarios in the scenario tree), and deterministic optimization assuming expected prices (DET). We also include dynamic deterministic optimization as described by Escudero, Garín, Merino, and Pérez (2007), where the strategy is re-optimized with updated expected prices as time goes by (Avg DETd). This mimics a decision maker who uses a deterministic decision support tool, but re-optimizes his decisions in each period.

Table 5.1 provides an overview of the objective function values for the different approaches. The bottom rows present the expected values from the scenario analysis (Avg SA), the expected value of perfect information (EVPI), and the value of the stochastic solution (VSS) (Birge and Loveaux 1997). The sales volumes corresponding to the results given in Table 5.1 are given in Appendix 5.A.

Table 5.1 shows that the stochastic solution yields a lower profit than the scenario-analysis solutions. This is hardly surprising as the scenario-analysis approach assumes perfect knowledge about the future and will plan accordingly. That is, for each of the four scenarios, a tailor-made solution is found. It is, however, worth noting that the deterministic solution performs worse than the stochastic solution. This seems somewhat surprising considering that the stochastic solution is not perfectly adapted to any single scenario. The explanation is that



**Table 5.2:** Contract commitment decisions. 1 means entering, 0 not entering.

Solution approach	1 <sup>st</sup> stage		2 <sup>nd</sup> stage	
	LC	SC_1	SC_2U	SC_2L
STO	0	0	1	0
DET	1	0		0
DETD	1	0	0	0
SA_c	0	0	0	n/a
SA_d	0	0	1	n/a
SA_e	0	1	n/a	0
SA_f	1	0	n/a	0

the stochastic solution is flexible enough to take advantage of upside variations of the prices while hedging for downside variations. The deterministic solution does not consider such price variations although the periodical contract limits still allow for some flexibility. The dynamic deterministic approach observes price variations as operational decisions are made, but it has limited ability to value future flexibility because future prices are seen as deterministic.

The differences in the strategic decisions are shown in Table 5.2, where SC\_1, SC\_2U, and SC\_2L refer to short-term contracts in period 1, the upper node in period 2, and the lower node in period 2, respectively. The deterministic model suggests to enter the long-term contract, because this is the alternative with highest price expectation, and thereby lock up more than half of the available gas. The stochastic model, on the other hand, sacrifices revenue from this expected high price, and thereby keeps the flexibility to choose between spot sales or the short-term contract when the contract prices are revealed in the second period. The dynamic deterministic approach is only partly able to offset the difference between the deterministic and stochastic models by allocating volumes to spot sales or to a long-term contract, depending on prices. This is because the contract obligations limit how large the volumes are that can be sold spot. The results in Table 5.2 illustrate a problem that can arise when using scenario analysis. The decisions in the four scenarios are all different, and it is hard to extract a pattern that provides a single optimal decision for the stochastic decision problem.

In our comparison of the stochastic and dynamic deterministic approaches, we evaluated both models with the data from the scenario tree in Table 5.2. Because the information in this tree was actually available to the stochastic model during optimization, this evaluation favors the stochastic approach. If the actual prices exactly matched the anticipated values, the deterministic model would perform better than the stochastic one. This shows that the performance of the different modeling approaches is highly dependent on the prices used for evaluation. Gen-

erally, a more balanced evaluation method for the model approaches would be to evaluate the model results on real-world outcomes over several time periods. Alternative evaluation methods use discrete-event simulation or truth trees (see Lium and Kaut (2006) for an example), both of which rely on the same assumptions on statistical properties for the distribution of the uncertain parameters as those used when generating the scenario tree. It should be noted, though, that these methods are more challenging in a multi-stage setting than for traditional two-stage problems. Because we study a synthetic case with a very small scenario tree, neither real outcomes nor evaluations based on the statistical properties in the tree are meaningful. The important aspect is not the quantitative performance of the different models, but rather the different structures of the found decisions. As long as there is uncertainty that can make flexibility valuable, a stochastic model is better suited to determine the most flexible position for realizing this value.

In this test we have used a risk-neutral stochastic model, which prefers to deliver predominantly to the spot market. In real life, many companies are risk averse and would see this strategy as too risky. However, for such companies stochastic optimization is still a good approach because it can express the value of flexibility in an uncertain environment and, therefore, allow for a quantified trade-off between risk and profit.

## Trading on the Spot Market

In this section we illustrate the value of flexibility from the possibility of trading on a spot market as a supplement to contractual deliveries. As a reference we use the stochastic-programming approach (here called Spot) from the previous section. We compare this with a test instance where no spot trade is possible (No spot) and an instance where spot trade is possible but the maximum purchase volume is limited to 700 MMBtu. The results are summarized in Table 5.3, while the complete set of results for these tests are presented in Appendix 5.A. Removing the opportunity to trade on the spot market forces the stochastic model to enter the long-term contract and deliver at its maximum level in each period, thereby reducing the expected NPV by €1 950. The major difference comes from a reduction from 1 000 MMBtu to 700 MMBtu in the volume delivered, which is caused by the lack of flexibility on the sales side to perfectly balance sales and purchase capacity. If the purchase capacity is restricted to a maximum of 700 MMBtu, the expected NPV is improved by €180 compared to the no-trade situation. Again the model avoids the long-term contract in order to exploit the price spread between spot and short-term contract sales. This results in an improved average sales price and, thereby, a higher expected profit.

It should be emphasized that not only spot sales, but also spot purchases

**Table 5.3:** Expected volumes, average prices, and NPV with different trading options.

Test	Expected volume (MMBtu)	Expected avg. price (€/MMBtu)	Expected NPV (€)
Spot	1 000	6.15	6 150
No spot	700	6.00	4 200
Max700	700	6.26	4 380

can add value. If contract obligations can be satisfied by spot purchases rather than own gas, volumes can be redirected to deliveries in other markets or in other time periods where prices are more favorable. Spot purchases enable geographical swaps and time swaps that make it possible to exploit price spreads for arbitrage. A consequence of a geographical swap is that transportation needs can change, which will increase or decrease fleet utilization, transportation costs, and boil-off depending on the relative distance from the source to each of the alternative markets.

## 5.5 Test Case 2 – Highlighting the Integrated Perspective

In this test case we focus specifically on the added value of the portfolio approach by comparing today’s planning practice with solutions achieved with the LNGPlanner framework.

### Case Setup

We consider a market with three liquefaction terminals (L1, L2, L3) and five regasification terminals (R1, ..., R5) over a twenty-period planning horizon where each period is one year. Each regasification terminal is connected to a market hub (H1, ..., H5) as illustrated in Figure 5.3. The economic life time of terminal L1 ends after period 5. There are two investment options, L1A and L1B, that can extend this terminal’s life. These two options differ in life time, up-front costs, and production costs as shown in Table 5.4. Although the accumulated production capacities are almost identical, the maximum yearly production capacity of option L1B is approximately 40% higher than that of option L1A.

Terminal L1 is owned by the considered company and has production costs of €0.5/MMBtu. The other two liquefaction terminals are not owned by the company, and production costs are internalized in the corresponding contract prices. All regasification terminals have production costs of €0.5/MMBtu.

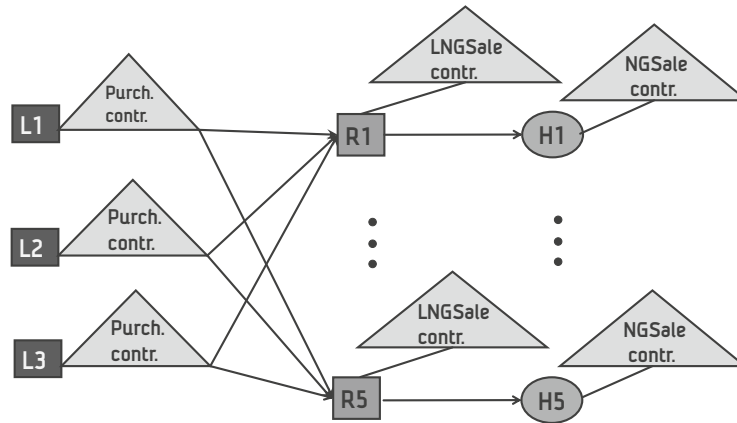
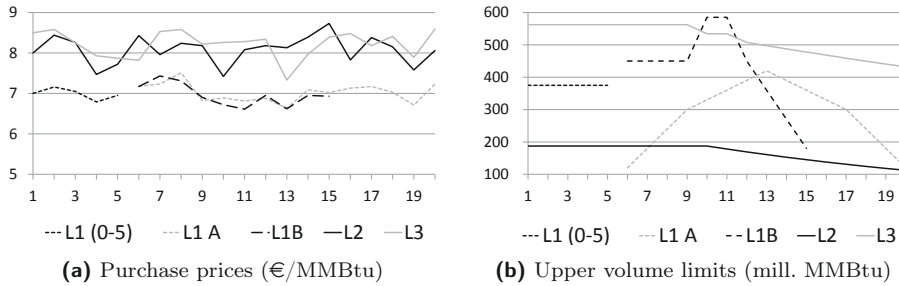


Figure 5.3: System layout in Test case 2.

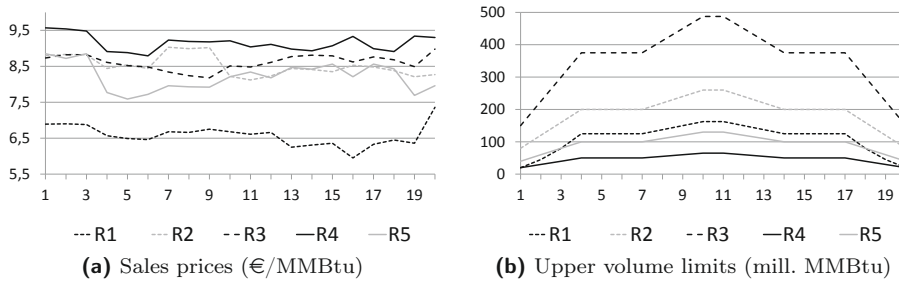
Table 5.4: Main characteristics of the two investment options.

Investment option	Life time years	Investment costs Mill. €	Production cost (change) €/MMBtu	Aggregated max. production volume Mill. MMBtu
L1A	15	1 200	0.3 (-40%)	4 260
L1B	10	800	0.6 (+20%)	4 230

5.5 Test Case 2 – Highlighting the Integrated Perspective



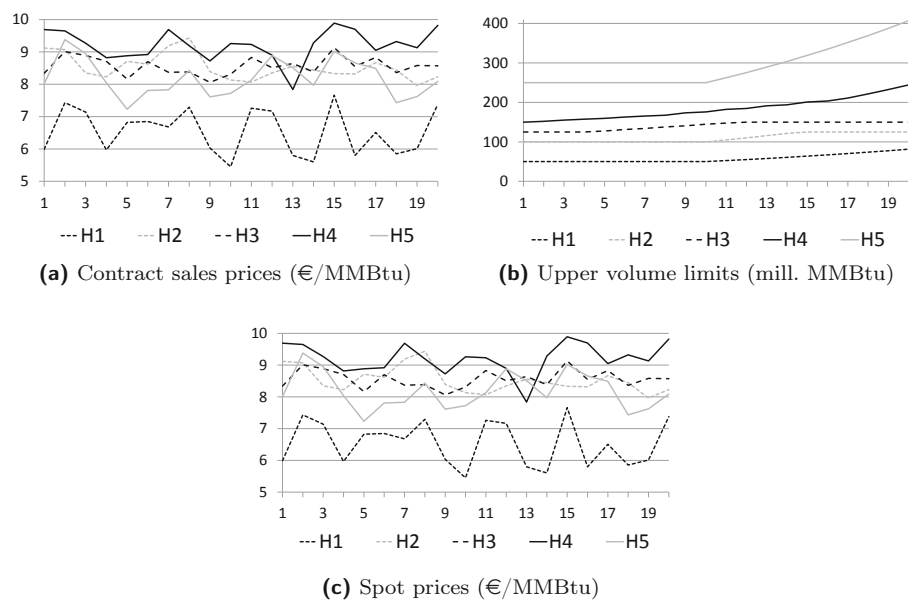
**Figure 5.4:** Prices and volume limits of LNG purchase contracts at liquefaction terminals, varying over 20 time periods.



**Figure 5.5:** Prices and volume limits of LNG sales contracts to regasification terminals, varying over 20 time periods.

There are LNG purchase contracts at all liquefaction terminals including the extension options, LNG sales supply contracts to all regasification terminals, and natural gas sales contracts at the hubs. The prices and volume limits of these contracts vary with both the location of the respective markets and the time as illustrated in Figures 5.4 – 5.6. Figure 5.4b shows that the purchase contracts at liquefaction terminals allow for ramp-up and -down periods of the corresponding terminal. Natural gas can also be sold spot at the hubs at prices illustrated in Figure 5.6c with yearly volume limits of 150 MMBtu, 250 MMBtu, 50 MMBtu, 125 MMBtu, and 100 MMBtu, respectively.

Transportation can be carried out using vessels of two sizes; small vessels that have a capacity of 120,000 m<sup>3</sup> and an investment cost of €220 million while large vessels can transport 220,000 m<sup>3</sup> with an investment cost of €300 million. Vessels can also be chartered in and out at yearly rates which start at €26.88



**Figure 5.6:** Prices and volume limits of natural gas sales contracts and spot prices at hubs, varying over 20 time periods.

**Table 5.5:** Revenues (Rev.), costs, and profits for the initial portfolio without extensions (No ext.), evaluation of each project in isolation (Isolated), keeping the existing portfolio fixed when including new projects (Fix.) and re-optimizing the whole portfolio including new projects (Re-op.) [Mill. €].

		No ext.	Isolated		Fix.		Re-op.	
			L1A	L1B	L1A	L1B	L1A	L1B
Rev.	Sale	18 404	9 069	11 622	30 629	33 804	29 386	32 354
	Charter	271			172	78	74	516
Purchase		15 933			25 002	28 639	23 549	25 611
Cost	Operations	951	864	554	1 465	1 592	1 379	1 487
	Charter	535			766	737	599	1 117
	Investm.	300	497	745	797	1 045	797	1 045
Profits		956	7 708	10 323	2 771	1 870	3 136	3 611

million and €18.85 million, respectively, and increase yearly by 2%. Initially, the company does not own any vessels and must decide whether to buy or charter vessels. Average transportation costs vary on the different links between liquefaction and regasification terminals, reflecting the terminals' locations. For example, it costs €6.18/m<sup>3</sup> to ship LNG from L1 to R1, €3.18/m<sup>3</sup> to R2, €4.07/m<sup>3</sup> to R3, €20.63/m<sup>3</sup> to R4, and €8.59/m<sup>3</sup> to R5. All costs and prices are in nominal terms and the discount rate is set at 10% annually.

It has to be decided which investment option to execute and which contracts to enter into. At the same time, an optimal fleet size and mix need to be determined.

In the remainder of this section, we discuss and compare several approaches to solving this planning problem. Table 5.5 shows revenues, costs, and profits achieved with these approaches while Appendix 5.B provides more detail about the amount of LNG and NG traded and the fleet sizes. For an easier comparison, the NPV of investment options L1A and L1B has been discounted to the first time period, too.

## The Current Planning Process

The current practice at a number of energy companies is to evaluate potential investments in several steps before they are implemented. At each step, the level of detail increases and the analysis becomes more precise. If a potential project appears feasible and economically sound, it will be considered for implementation. If there are mutually exclusive projects, the project with the highest NPV is chosen.

Manual planning typically analyzes each long-term investment opportunity in-

dividually and, to a lesser extent, how it affects the existing asset portfolio. This implies that the potential new portfolio is not fully re-optimized when investigating how the new investment will complement the existing portfolio, but the projects are rather evaluated isolated from the existing portfolio. We denote this approach “Isolated”. For the test case, excluding the other liquefaction terminals and focusing exclusively on the two options, the NPV of the longer running option, L1A, will be €7 708 million, and that of the shorter option, L1B, will be €10 323 million. Obviously, it is more profitable to invest in the shorter option that allows for higher production volumes in earlier years. Note, however, that this evaluation assumed that all LNG produced at the terminal can be sent to the most profitable market. This assumption is, in general, somewhat optimistic.

Analyzing the investment opportunities as complements to the existing portfolio of assets, keeping all other decisions fixed, and observing market limitations gives a solution where the purchase contract at the considered liquefaction terminal cannot be utilized to its full potential. We denote this approach “Fixed”. In this instance, the NPV of operating the whole system is €2 771 million if option L1A is chosen and €1 870 million for L1B. Choosing not to invest in either option yields a NPV of €956 million. This implies that option L1A would add a value of €1 815 million to the system and option L1B a value of €911 million. Consequently, the longer running option L1A appears more profitable now because it allows the full potential of the associated contracts to be realized.

In summary, analyzing investment options in a simplistic fashion, instead of re-optimizing the entire portfolio, can lead to an overly optimistic evaluation in some cases, while it may mean assets are not utilized optimally in other cases. In this particular example we end up in situations where an investment decision is based on flooding the premium market or where a new purchase contract cannot be fully utilized.

## From Individual Projects to Portfolio Management

An investment in one element of the supply chain is likely to affect other elements of the chain; however, often it is not immediately clear what those effects will be. Not only may it affect the utilization of physical assets such as terminals, vessels, pipelines, or hubs but it may also impact existing contracts. Furthermore, it will have an effect on the company’s ability to sign new contracts and to operate in the spot market. Re-optimizing the whole system considered in this test case instead of just trying to ‘fit in’ the new project, denoted “Re-optimized”, the system’s NPV increases to €3 136 million for option L1A and to €3 611 million for option L1B. Hence, the added value of option L1A is €2 180 million while that of the shorter option L1B is €2 655. Again, the shorter option appears more profitable.



In the previous section, we observed that the purchase contract at the terminal could not be fully utilized with the shorter option L1B. This is partially due to limitations from contracts in markets where the LNG could be sent without economic losses. A re-optimization of the decisions over the entire portfolio leads to a change of nearly all deliveries. Almost all sales contracts are affected, either through a changed source purchase contract or through changed volumes. This concerns not only assets directly linked to the two upgrade options, but also seemingly completely unrelated assets. This re-organization, however, allows for a complete and fully optimal utilization of the added LNG volumes.

Another observation is that the longer option, L1A, allows for profitable routing of LNG to more markets than option L1B because the volume is produced over a longer time span. This helps to avoid volume limitations in contracts in the more profitable markets or in transport capacity. But this positive effect is not sufficient to outweigh the lower investment cost associated with L1B.

Note that this solution is quite different compared to what manual planners would have found. In particular, since it requires changing the way delivery obligations are satisfied for nearly all contracts, the solution may never have been found with the current planning practice. The value of the best solution without re-optimizing the whole portfolio is €1 815 million. While re-optimizing the system not only leads to a different choice of upgrade option, it also increases the value of implementing this option to €2 655 million. Consequently, in the considered test instance, the evaluation of the investment opportunities using a complete portfolio management approach leads to a solution stipulating an €40 million increase of the system's NPV. Even if the sub-optimal option L1A were chosen, adapting all assets in the system to the new option would yield a €475 million higher NPV for the whole system compared to just fitting the new option to the existing system.

## 5.6 Conclusions and Perspectives

We presented a stochastic mixed-integer linear programming problem to support strategic planning processes in the LNG value chain. The model focuses on investments and disinvestments into infrastructure and vessels, on chartering decisions, and on decisions about purchases and sales of LNG and natural gas. Selected features of the model were illustrated by numerical case studies motivated by our industry partners: Explicitly taking into account uncertainty (for example, about the future price development) can lead to increased efficiency and higher profits. We also demonstrated that taking an integrated portfolio perspective may yield different solutions compared to traditional approaches.

Recent developments in liquefied natural gas technology may affect the market dramatically in the future. For instance, some terminals will become bi-

directional, so that both liquefaction and regasification can be performed. Also, larger plants and vessels are built in order to benefit from technological advances and economies of scale (see, e.g. Spilsbury, McLauchlin, and Kennington (2005)). On the other hand, small-scale plants are becoming economically viable. Floating liquefaction (FLNG) allows for the production from gas fields that were previously considered too small and too far away (GIIGNL 2010). These developments will doubtless challenge traditional planning and modelling approaches further, amplifying the need for further work on decision support tools based on mathematical programming approaches.

## **Acknowledgments**

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## Appendices

### 5.A Results of Test Case 1

This section supplements the results for Test case 1, with Table 5.6-5.8 presenting the amounts of LNG sold spot, delivered in the long-term contract and in the short-term contracts for each model run presented in Section *Dealing with Uncertainty* on page 162. Solution approach “STO” is identical with the test “Spot” and is therefore only listed once in the tables. The columns in the tables corresponds to the nodes in the scenario tree presented in Figure 5.2. Note that “DET” takes a single decision independent of the nodes in each period, which is represented with one centered value for each period in the tables. “—” indicates combinations of node and solution approach that is not defined.

All model runs give a purchased volume of 1 000 MMBtu except “No spot” and “Max700” that give a purchase of 700 MMBtu.

**Table 5.6:** Spot sale volume decisions for each solution approach and test in Section *Trading on the Spot Market* on page 164 [MMBtu].

Solution approach	Period 2		Period 3			
	Node a	Node b	Node c	Node d	Node e	Node f
STO	1 000	1 000	500	475	1 000	1 000
DET	300		300			
DETD	400	300	300	300	400	300
SA_c	1 000	—	1 000	—	—	—
SA_d	1 000	—	—	475	—	—
SA_e	—	475	—	—	1 000	—
SA_f	—	300	—	—	—	300
No spot	0	0	0	0	0	0
Max700	700	700	200	175	700	700

### 5.B Results of Test Case 2

This section gives an overview of results of the non-portfolio planning approaches for Test case 2 discussed in Section 5.5. Table 5.9 shows how much LNG is

**Table 5.7:** Long-term contract delivery volume decisions for each solution approach and test in Section *Trading on the Spot Market* on page 164 [MMBtu].

Solution approach	Period 2		Period 3			
	Node a	Node b	Node c	Node d	Node e	Node f
STO	0	0	0	0	0	0
DET	700		700			
DETD	600	700	700	700	600	700
SA_c	0	—	0	—	—	—
SA_d	0	—	—	0	—	—
SA_e	—	0	—	—	0	—
SA_f	—	700	—	—	—	700
No spot	700	700	700	700	700	700
Max700	0	0	0	0	0	0

**Table 5.8:** Short-term contract delivery volume decisions for each solution approach and test in Section *Trading on the Spot Market* on page 164 [MMBtu].

Solution approach	Period 2		Period 3			
	Node a	Node b	Node c	Node d	Node e	Node f
STO	0	0	500	525	0	0
DET	0		0			
DETD	0	0	0	0	0	0
SA_c	0	—	0	—	—	—
SA_d	0	—	—	525	—	—
SA_e	—	525	—	—	0	—
SA_f	—	0	—	—	—	0
No spot	0	0	0	0	0	0
Max700	0	0	500	525	0	0

purchased at the liquefaction terminals and sold at the regasification terminals in all analyzed time periods. The amounts of natural gas sold in a contract or spot at a hub are indicated in Tables 5.10 and 5.11, respectively. Finally, Table 5.12 lists the fleet size and composition needed to transport the LNG from liquefaction to regasification.

Empty cells indicate zero purchases or sales – despite having the possibility – while “—” entries in Table 5.9 mark the availability of the option in the given time period.

We compare the planning approaches of choosing not to extend terminal L1's lifetime ("No extension"), evaluating the NPV of both extension options L1A and L1B in isolation ("Isolated"), fitting the extension into the existing fixed system ("Fixed"), and re-optimizing all decisions when phasing in an extension option ("Re-optimized"). Note that for the latter two approaches, only the results of selecting the shorter extension option L1B are shown as this option yields a higher profit than option L1A.

Evidently, the solution in the "Re-optimized" approach differs slightly from the "No extension" and "Fixed" approach solutions also in periods 1 – 5 where a potential investment in an extension should not make any difference. This may be due to different profit-maximizing solutions for one of the approaches having the same objective function value despite different variable values during the periods.

**Table 5.9:** LNG purchased at liquefaction terminals L1 – L3 and sold at regasification terminals R1 – R3, choosing option LIB for the “Fixed” (Fix.) and “Re-optimized” (Re-op.) approaches. Values for “Re-optimized” are given in *italics* if values are the same as for “No extension” but from different sources (purchase contract). [MMBtu]

Time	L1		L1A		LIB		L2		L3		R1		R2		R3	
	All	Isolated	Fix.	Re-op.	No ext. & Fix.	Re-op.	No ext. & Fix.	Re-op.	No ext. & Fix.	Re-op.	All	No ext.	Fix.	Re-op.	No ext.	Re-op.
1	375	—	—	—	—	—	5.1	—	9	6.2	5	80	80	80	—	—
2	375	—	—	—	—	—	—	—	2.4	2.4	11.3	30	30	30	30	—
3	375	—	—	—	—	—	—	—	—	12	20	160	160	160	160	—
4	375	—	—	—	—	—	85	—	—	—	31.3	200	200	50	50	—
5	375	—	—	—	—	—	34.6	—	—	—	31.3	135.3	135.3	50	50	—
6	—	120	450	450	450	450	82.1	—	14.1	11.5	<i>31.3</i>	50	145	50	—	—
7	—	180	450	450	450	450	132.9	129.2	—	—	31.3	50	200	198.4	—	—
8	—	240	450	450	450	450	132.4	39.7	—	—	34.4	55	220	97.3	—	—
9	—	300	450	450	450	450	100.5	—	—	—	<i>37.5</i>	62	240	240	240	6.4
10	—	330	585	585	530.6	585	133.6	28.7	—	—	<i>40.6</i>	65	260	79.4	—	—
11	—	360	585	585	564.5	585	133.6	—	—	14	<i>40.6</i>	65	260	65	—	—
12	—	390	450	450	450	450	98.5	—	—	—	<i>37.5</i>	60	139.8	60	—	—
13	—	420	360	360	360	360	68.8	—	21.4	3.4	<i>34.4</i>	55	55	55	55	13.9
14	—	390	270	270	270	270	82.1	—	14.1	16.6	<i>31.3</i>	50	200	50	200	36.4
15	—	360	180	180	180	180	82.1	—	37	24.6	<i>31.3</i>	50	93.2	55	55	—
16	—	330	—	—	—	—	82.1	137.8	14.1	50.1	31.3	50	50	50	50	—
17	—	300	—	—	—	—	82.1	130.9	—	46	31.3	50	50	50	50	3.7
18	—	240	—	—	—	—	60.6	124.4	3.7	16	20	40	40	40	40	—
19	—	180	—	—	—	—	69.5	110.6	—	—	11.3	30	30	30	30	8.3
20	—	120	—	—	—	—	25.3	72.2	8.4	—	5	20	20	20	20	—

**Table 5.10:** Natural gas sold in contracts at hubs H2 – H5, choosing option L1B for the “fixed” (Fix.) and “re-optimized” (Re-op.) approaches. There are no NG sales in contracts at hub H1 or after period 16. [MMBtu]

Time	Hub 2			Hub 3			Hub 4	Hub 5	
	No ext.	Fix.	Re-op.	No ext.	Fix.	Re-op.	No ext. & Fix.	No ext.	Fix.
1	248.8	248.8					8.8		
2				50	50		2.4		
3	57.9	57.9		50	50	50		32.4	32.4
4	72.3	72.3		50	50	50			
5									
6		248.8	169.1		50	50	13.9		
7	50.1	248.8	248.8		50				
8					21.8				10
9									
10					50				
11					52.5	52.5			
12		248.8			55.1				
13		246.7			57.9				
14									
15					63.8				
16							13.9		

**Table 5.11:** Natural gas sold spot at hubs H2 – H5, choosing option L1B for the “fixed” (Fix.) and “re-optimized” (Re-op.) approaches. There are no spot sales at hub H1. [MMBtu]

Time	Hub 2			Hub 3			Hub 4			Hub 5	
	No ext.	Fix.	Re-op.	No ext.	Fix.	Re-op.	No ext.	Fix.	Re-op.	Fix.	Fix.
1			248.8	36.1	36.1						
2	248.8	248.8	248.8	29.6	29.6	23.6					
3				50	50	50					
4				50	50	50					
5	248.8	248.8	229.3								
6						50					
7					45.4						
8	41.5	248.8	248.8		50						
9		248.8	55.2		17.4						
10		248.8		26.5	50		6.4				
11		248.8		26.5	50	50			14		37.3
12					50						
13					50						
14		116.7						13.9	13.9	16.3	
15						3.2		36.4	36.4	24.2	20.5
16											
17											
18				27.4	27.4	12		3.7	3.7	4	
19								8.3	8.3		
20											



**Table 5.12:** Fleet sizes – number of vessels of each type.

Time	Large vessel			Small vessel		
	No ext.	Fix.	Re-op.	No ext.	Fix.	Re-op.
1	2	2	2	1	1	1
2	2	2	2	1	1	1
3	3	3	3	0	0	0
4	3	3	3	0	0	2
5	3	3	3	0	0	0
6	1	4	3	1	1	1
7	2	5	4	0	0	1
8	2	5	3	0	0	1
9	1	4	3	1	1	1
10	2	5	5	0	1	0
11	2	6	5	0	0	0
12	1	4	4	1	1	0
13	1	4	3	1	0	0
14	1	3	2	1	1	1
15	2	3	2	0	1	0
16	1	1	3	1	1	0
17	1	1	3	1	1	0
18	1	1	1	0	0	2
19	1	1	1	0	0	1
20	0	0	1	1	1	0

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Paper V

Marte Fodstad, Kjetil Trovik Midthun, Asgeir Tomasgard:

*Adding Flexibility in a  
Natural Gas  
Transportation Network  
Using Interruptible  
Transportation Services*

*Submitted to international journal with peer review.*





## Chapter 6

# Adding Flexibility in a Natural Gas Transportation Network Using Interruptible Transportation Services

### **Abstract:**

We present a modeling framework for analyzing if the use of interruptible transportation services can improve capacity utilization in a natural gas transportation network. The network consists of two decision makers: the transmission system operator (TSO) and a shipper of natural gas. The TSO is responsible for the routing of gas in the network and allocates capacity to the shipper to ensure that the security of supply in the network is within given bounds. The TSO can offer two different types of transportation services: firm and interruptible. Only firm services have a security of supply measure, while the interruptible services can freely be interrupted whenever the available capacity in the transportation network is not sufficiently large. We apply our modeling framework on a case study with realistic data from the Norwegian Continental Shelf. The results indicate substantially increased throughput and profits with the introduction of interruptible services.

## 6.1 Introduction

In this paper we discuss whether the introduction of interruptible transportation services in a natural gas network can increase throughput without deteriorating the security of supply. In our modeling framework we include both firm and interruptible transportation services, where firm services are characterized by a guaranteed level of security of supply while interruptible services are delivered provided there is available capacity on the given day. We present a general model framework and a case study based on realistic data from the Norwegian natural gas transportation system that covers nearly 20% of European gas consumption (Norwegian Petroleum Directorate 2012).

Interruptible transportation services are well known within the natural gas supply chain, as they are available in the US and in several European systems (including the Norwegian). These services allow the TSO to resell capacity that is booked firm but not nominated, as described in Doane, McAfee, Nayyar &

Williams (2008). It is usually required that all firm capacity, defined by a predefined static limit, is sold before any capacity can be resold as interruptible. The intention of the interruptible services is to improve the short term redistribution of transportation capacity to support an efficient use of the network (Ruff 2012). Our motivation for introducing interruptible transportation services is different. We focus on increasing the capacity initially made available by the TSO rather than redistribution. We are not aware of any natural gas network or examples in the literature where interruptible services are used for this purpose.

A high level of security of supply is important on the market side, for the shippers to be able to deliver in long-term contracts. It is also important on the production side, to ensure that the oil production on the fields with associated gas will not be decreased. In order to maintain a high level of security of supply, it is necessary for the system operator to withhold some flexibility in the system at the time of booking to handle uncertainties in the final operation. This withheld flexibility can decrease the capacity utilization in the network. Security of supply can be expressed through different measures. Within the power sector the N-1 method, requiring feasible operation even if one element in the network goes down, is a traditional way of providing robustness in case of contingencies (see for instance Vournas 2001). Bopp, Kannan, Palocsay & Stevens (1996) use a set of business rules to achieve satisfactory security of supply when optimizing the planning problem of a local natural gas distribution company. Guldmann & Wang (1999) include a curtailment cost on not satisfied demand for a similar problem. In the stochastic programming literature a variety of risk measures are presented (see for example Rockafellar 2007), but so far these are rarely applied in natural gas applications. We define the security of supply level as the expected transportation capacity offered to the shippers in the whole system relative to the total firm booking. This is the same definition as used by Hellemo, Midthun, Tomasgard & Werner (2013), but in contrast to them we also report numerical analysis.

Unplanned events, such as outages and technical failure, cause uncertainty in the available capacity in the transportation network. Furthermore, the system operator must take into account system effects that make it impossible to a priori determine fixed capacities (see Midthun, Bjørndal & Tomasgard 2009). This corresponds to the arguments by Vazquez, Hallack & Glachant (2012) who point out that the shipper's simplified view on the transportation network, only acting in accordance with entry and exit booking points, requires the TSO to withhold some capacity to match the booking obligations with the physical network capabilities.

The short-term system flexibility comes from the possibility to increase production levels in some fields, to reroute the gas, and from the storage capabilities in the pipelines (linepack). Midthun, Nowak & Tomasgard (2007) and Keyaerts,

Hallack, Glachant & D’haeseleer (2011) show that linepack also has a commercial value that introduces a trade-off in relation to security of supply. We focus our analysis on the effect of interruptible services and have not included linepack and this trade-off in our analysis.

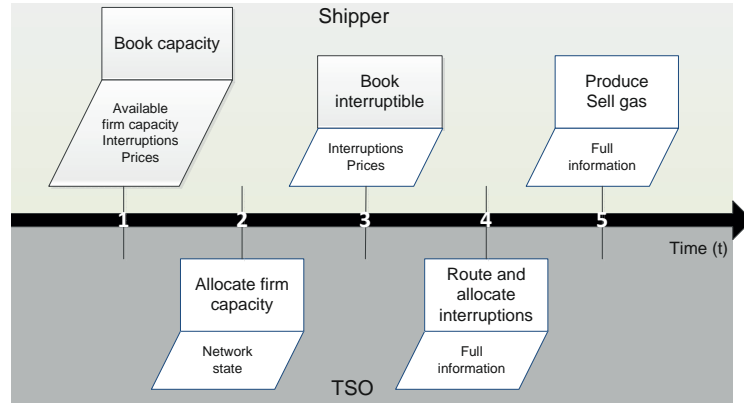
Modeling the physics of gas transportation in pipeline networks is challenging, mainly due to nonlinear properties in pressure dynamics in pipelines, compressor efficiency and gas quality management. Martin, Möller & Moritz (2006) and Tomasgard, Rømo, Fodstad & Midthun (2007) optimizes a steady-state representation of gas network pressures and flows. Moritz (2007) models transient flows, while Ulstein, Nygreen & Sagli (2007), Selot, Kuok, Robinson, Mason & Barton (2008) and Li, Armagan, Tomasgard & Barton (2011) models gas quality issues. We use a linear steady-state approximation of the pressure dynamics in pipelines. We assume a homogeneous gas quality, and thereby avoid the nonlinearities from gas quality management. This assumption is a reasonable approximation in networks with small quality variations, for instance downstream of processing, but it is otherwise a simplification.

Our contribution is both a modeling framework that allows for detailed analysis of interruptible services to address uncertainty in network capacity availability in the natural gas transportation network, as well as a case study based on realistic data and topology from the Norwegian Continental Shelf (NCS). In addition, we introduce a new production cost function for natural gas fields that takes into account associated oil production. Our models are based on stochastic programming and do not include strategic behavior of the participants. The validity of this will be discussed in further detail when we introduce our models.

In Section 6.2 we describe in more detail the decision sequence as well as some of our assumptions. We then present the modeling framework in Section 6.3, before we move on to the case study in Section 6.4. Finally, we conclude in Section 6.5.

## 6.2 Problem Statement

We establish a decision sequence involving the agents in the supply chain (see Figure 6.1). At  $t = 1$  the shippers submit their booking requests under the uncertainty of available firm capacity and at  $t = 2$  the TSO allocates the capacity between the shippers. The TSO tries to minimize the deviation between requested booking and allocated capacity while meeting the security of supply requirements. At  $t = 3$ , when the allocated firm booking is known by the shippers, they book interruptible capacity. The interruptible capacity is unlimited, but the shippers will recognize the probability of not receiving this capacity. At  $t = 4$  uncertainty is resolved and the network state and market prices are known. The TSO then allocates interruptions based on a feasible routing pattern. Finally, at  $t = 5$



**Figure 6.1:** Illustration of the decision sequence for a shipper and the TSO. The squares show the decisions that are made at each point, while the trapezoids show the uncertain parameters at that stage.

the shippers produce and do short term trades. The decision sequence we have described is similar to the one used in the current system on the NCS, apart from the difference in use of interruptible services.

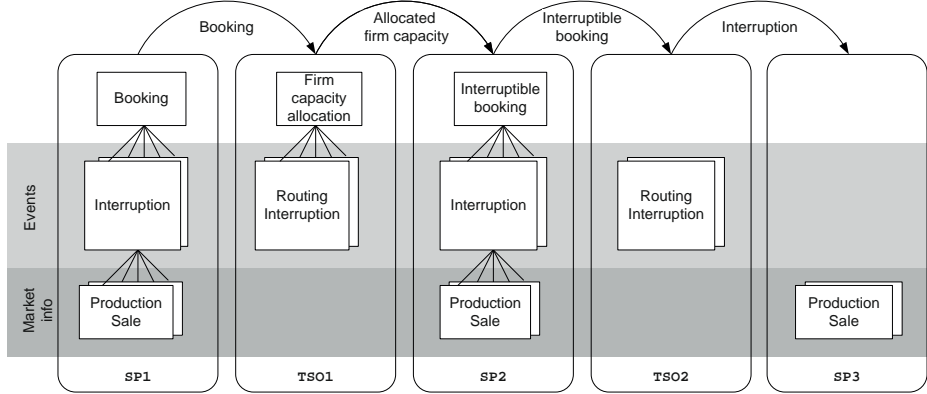
The objective of this work is to evaluate the potential from introducing interruptible transportation services for the network as a whole, while recognizing that different agents in the system have different incentives. We have made some important modeling choices: 1) We model all shippers as a single agent, which gives us an optimistic view of the system performance. It implies that any strategic behavior that could improve a single shipper's performance, but reduce the overall supply chain performance, is not captured. The aggregation of all shippers into a single agent also reduces the uncertainty seen by the shippers, since the modeled shipper knows the total booking in the system. 2) We model booking of firm and interruptible capacity by a sequential procedure, in order to give realistic incentives for firm services relative to interruptible services. In reality interruptible capacity is available only if firm capacity is already fully utilized. 3) We model the TSO and the shipper as independent agents rather than an integrated agent even though we seek a supply chain perspective. This is done to be able to observe how the two agents' different objectives and access to information about different parts of the supply chain affect the overall performance. 4) We assume that both shipper and TSO base their expectations of network events on statistical properties from historical observations which are common knowledge for both.

There are other modeling techniques that are commonly used to explicitly represent different agents and their interaction in a single model. Generalized Nash equilibria, modeled through quasi-variational inequalities, typically used to describe the relation between agents taking simultaneous decisions, as done in Midthun, Bjørndal, Tomasgard & Smeers (2007). Bilevel or multilevel modeling are relevant techniques to capture interaction between agents in a decision sequence (see Kalashnikov, Pérez-Valdés, Tomasgard & Kalashnykova 2010, for an example). A common feature for these techniques is the possibility to model gaming, such as an agent’s ability to make strategic decisions to deliberately affect other agents’ decisions. Using such techniques to model the interactions between TSO and shippers and between different shippers both within and across decision levels would give us a mathematical program with equilibrium constraints (MPEC) or equilibrium problem with equilibrium constraints (EPEC) that would be complex to design, solve and analyze (see Hobbs, Metzler & Pang 2000, Ehrenmann & Neuhoff 2009, for application examples of the two problem classes from the power industry). It is also not clear if such gaming is present in the transportation market on the NCS. Thus we have decided not to model gaming for our initial study, but rather leave that for possible future work.

**Booking tariffs** To keep our model in accordance with the current regime on the NCS, we use fixed tariffs and the tariffs for interruptible services are lower than for firm (Gassco 2012). From a shipper’s perspective it is reasonable to expect interruptible tariffs to be less than firm tariffs, but that might not be the result when tariffs are based on costs. E.g. if interruptible capacity is seen as additional capacity ‘on top of’ the firm capacity, the interruptible capacity will typically be priced by a more expensive part of the compressor cost curve (which has a growing marginal cost).

## 6.3 Model Descriptions

This section provides a mathematical formulation of the models. A model system overview is given in Figure 6.2, and the full notation is provided in Appendix 6.B. The model system consists of five optimization models, three shipper problems, SP1-SP3, and two models for the TSO, TS01-TS02. Among these, three models are stochastic problems, SP1, SP2 and TS01, while two models are deterministic problems, SP3 and TS02. Figure 6.2 shows the sequence the models are run in (from left to right), the decisions that are taken in each model and the information flow (division into stages). The text within each box belonging to a model describes the decisions taken by the model. Results from each problem except SP3 are used by all the following models, as illustrated by the curved arcs on top. The three horizontal areas in the illustration, white, light gray, and dark gray,

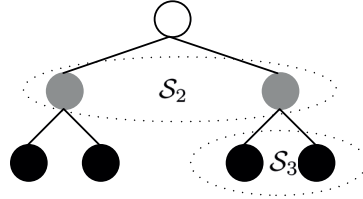


**Figure 6.2:** Illustration of the stage structure of each model and the sequence of model runs, starting with SP1 and ending with SP3. Each rounded square represent one model run, while the squares within represent stages with associated decisions in the model. Each row represents a level of information, where the events are known in the light gray area and market information is known in the dark gray area.

indicates the information set that each decision is based on, and corresponds to the scenario tree illustrated in Figure 6.3. The outcomes in the second stage (light gray) represent network events,  $s_2 \in \mathcal{S}_2$  and the outcomes in the third stage (dark gray) represent market prices,  $s_3 \in \mathcal{S}_3$ . These outcomes are assumed to be independent of each other. A scenario then consists of a combination of an event and a price outcome. The outcomes have the corresponding probabilities  $\pi_{s_2}$  and  $\pi_{s_3}$ .

In the first shipper problem, SP1, the producer requests booking based on event and market outcomes and estimates of future interruptions, production and sales decisions. At this stage the shipper does not see the possibility to book interruptible capacity later. Based on the booking request the TSO allocates firm capacity in TS01. In the second shipper problem, SP2, interruptible booking decisions are taken. This problem is similar to SP1, except that firm capacity is given and interruptible booking capacity is made available. When all booking is decided and the events in the transportation network have become known the TSO decides how much interruptible and firm capacity he needs to interrupt in TS02. Based on the final available capacity and the realized market prices the shipper decides on the amounts to produce and sell in SP3.

SP1 and SP2 are three stage quadratic stochastic programs, while SP3 is a deterministic model that is run for all nodes in the last stage of the scenario tree.



**Figure 6.3:** The three-stage scenario tree. In the second stage (the gray nodes) the uncertainty with regards to network availability is revealed, whilst the uncertainty with respect to market prices is revealed in the third stage (the black nodes). The network availability and the market uncertainty are assumed to be independent of each other, all nodes in the set  $\mathcal{S}_2$  is therefore linked to the same set of nodes  $\mathcal{S}_3$ .

The first stage decision in SP1 and SP2 is capacity booking, whilst in the second stage the shipper's estimates of the TSO's interruption decisions are made based on the event outcomes. The third stage is the actual operation with production and sales. SP2 and SP3 are variants of SP1, where firm booking is fixed in SP2 while both booking types are fixed in SP3. SP3 is the same problem as the last stage of SP1 and SP2.

The TSO problems use a network composed of field nodes,  $g \in \mathcal{G}$ , market nodes,  $m \in \mathcal{M}$ , intermediate nodes,  $i \in \mathcal{I}$  and connecting pipelines. The field nodes and market nodes constitute the booking nodes,  $n \in \mathcal{N} = \mathcal{G} \cup \mathcal{M}$ , that are the network nodes in the shipper's problems. In accordance with the separate responsibilities of TSO and shipper, the shipper does not see the pipelines, but rather a fully connected network.

### First Shipper Problem, SP1

In SP1 interruptible booking is set to zero. This means that the shipper does not foresee the possibility to book interruptible capacity as he makes his initial booking request. This mimics the market design rule that interruptible capacity should only be made available if all firm capacity is allocated.

**Objective function** The objective of a shipper is to maximize his expected profit, that is the expected income from selling gas in the spot markets less

the expected production costs and the transportation cost (booking tariffs):

$$\begin{aligned} \max \sum_{s_2 \in \mathcal{S}_2} \pi_{s_2} \sum_{s_3 \in \mathcal{S}_3} \pi_{s_3} & \left( \sum_{m \in \mathcal{M}} (P_{m,s_3} x_{m,s_2,s_3}) - \sum_{g \in \mathcal{G}} F_g(y_{g,s_2,s_3}) \right) \\ & - \sum_{n \in \mathcal{N}} (T_n^F z_n^F + T_n^I z_n^I), \end{aligned} \quad (6.1)$$

where  $P_{m,s_3}$  is the market price in market  $m$  in market outcome  $s_3$ ,  $x_{m,s_2,s_3}$  is the volume sold in market  $m$  in the scenario given by event outcome  $s_2$  and market outcome  $s_3$ . The production cost is given by the cost function  $F_g(\cdot)$  which is a function of the production  $y_{g,s_2,s_3}$  in field  $g$ . Lastly, the booking costs are given by the sum of costs for firm and interruptible booking. The cost of firm booking is given by the tariff  $T_n^F$  in node  $n$  and the booking level  $z_n^F$ , while the cost of interruptible booking is similarly given by the tariff  $T_n^I$  in node  $n$  and the booking level  $z_n^I$ . Booking costs are independent of whether the capacity is interrupted or not.

**Production cost** The production cost function consists of two parts, one for production with associated oil, called ‘must-take’, and one for the swing field production. The reasoning behind this is that each of the field nodes in our network represents a set of fields, each with different properties. In the must-take area, the natural gas is closely linked to the oil production. If the gas production is decreased, the oil production must also be decreased. Such a decrease will then lead to a substantial loss for the shipper. For the swing fields, the gas production will not influence the oil production. We represent the production costs for these fields with a quadratic cost function based on a modification of the function provided by Golombek, Gjelsvik & Rosendahl (1995). We omit the logarithmic part that, in effect, provides maximum production levels. Instead we have implemented a fixed upper limit for the production in each field node. The production cost function is represented by (6.2)-(6.5) and is illustrated in Figure 6.4. The production cost for the must-take production,  $c_{g,s_2,s_3}^{MT}$ , is modeled by a set of  $Q$  linear constraints:

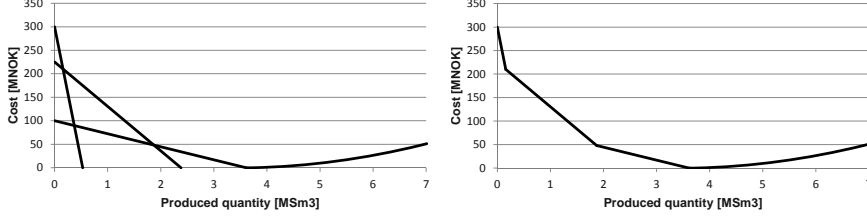
$$c_{g,s_2,s_3}^{MT} \geq C_{g,q}^{AMT} + C_{g,q}^{BMT} y_{g,s_2,s_3} \quad g \in \mathcal{G}, q \in \mathcal{Q}, s_2 \in \mathcal{S}_2, s_3 \in \mathcal{S}_3, \quad (6.2)$$

where  $C_{g,q}^{AMT}$  and  $C_{g,q}^{BMT}$  are cost parameters for field  $g$  and linear constraint  $q$ . In addition, there is the quadratic production cost function in the swing fields:

$$c_{g,s_2,s_3}^{SW} = C_g^{BSW} y_{g,s_2,s_3}^{SW} + C_g^{CSW} y_{g,s_2,s_3}^{SW^2} \quad g \in \mathcal{G}, q \in \mathcal{Q}, s_2 \in \mathcal{S}_2, s_3 \in \mathcal{S}_3, \quad (6.3)$$

where  $C_g^{BSW}$  is the cost parameter for the linear part of the cost function in field  $g$  while  $C_g^{CSW}$  is the cost parameter for the quadratic part of the cost function.





**Figure 6.4:** The production cost function used for the aggregated field nodes. Looking at the left hand side of the figure, the cost of reduced oil production is decreasing as the production increases, and this causes the negatively sloped part of the production cost function. The different lines represent the different fields that are aggregated into the field node. The steeper the line, the higher the oil to natural gas ratio is. The positively sloped part of the production cost function represents the gas production that is independent of oil production, where increasing production level gives increasing cost. The right hand side of the figure illustrates the resulting production cost function that we use in our model.

The production related to swing capacity in a field is given by  $y_{g,s_2,s_3}^{SW}$  which is the difference between the total production in a node,  $y_{g,s_2,s_3}$  and the defined must-take level  $Y_g^{MT}$ :

$$y_{g,s_2,s_3}^{SW} \geq y_{g,s_2,s_3} - Y_g^{MT} \quad g \in \mathcal{G}, s_2 \in \mathcal{S}_2, s_3 \in \mathcal{S}_3. \quad (6.4)$$

The cost function  $F(y_{g,s_2,s_3})$  is then given as:

$$F(y_{g,s_2,s_3}) = c_{g,s_2,s_3}^{MT} + c_{g,s_2,s_3}^{SW} \quad g \in \mathcal{G}, s_2 \in \mathcal{S}_2, s_3 \in \mathcal{S}_3. \quad (6.5)$$

Production should stay within the production capacity limits:

$$y_{g,s_2,s_3} \leq Y_{g,s_2} \quad g \in \mathcal{G}, s_2 \in \mathcal{S}_2, s_3 \in \mathcal{S}_3. \quad (6.6)$$

**Capacity booking** The shipper has to book capacity to inject gas into the network from the field nodes and to extract gas from the network to the market nodes. Firm capacity is booked in the first stage. When only firm capacity is available we require that the requested booking for entry capacity in the fields should equal the requested booking for exit capacity in the markets:

$$\sum_{g \in \mathcal{G}} z_g^F = \sum_{m \in \mathcal{M}} z_m^F. \quad (6.7)$$

The TSO will always allocate capacity such that the firm booking is balanced. When interruptible services are available however, we do not include this constraint, since the first-stage booking request from the shipper will then signal how much total capacity he would like to have. The TSO can then decide what portion of this is firm capacity. Since there is no requirement for the interruptible booking to be balanced, the total first stage booking request from the shipper does not have to be balanced either. The balance in firm booking is necessary to make it possible for the TSO to evaluate the capacity in each node, which depends on the flow pattern in the system.

In reality the shipper has historical data and observations on network capacities and events. In our model we need to approximate this in a transportation capacity,  $H_{n,s_2}$ , that depends on the event outcomes. This is done by a pre-processing procedure described in Appendix 6.A. The transportation capacity is static in the sense that it does not take into account how the shipper's own production and delivery decisions affect the flow patterns and thereby the capacity. The shipper's estimate for total interruption,  $a_{n,s_2}$  can then be described by the difference between the transportation capacity and the booking  $z_n^F$ :

$$a_{n,s_2} \geq z_n^F - H_{n,s_2} \quad n \in \mathcal{N}, s_2 \in \mathcal{S}_2. \quad (6.8)$$

**Mass balances** Gas is sold in spot markets with perfect competition. The total sales by the shipper are then limited by the total production:

$$\sum_{g \in \mathcal{G}} y_{g,s_2,s_3} = \sum_{m \in \mathcal{M}} x_{m,s_2,s_3} \quad s_2 \in \mathcal{S}_2, s_3 \in \mathcal{S}_3, \quad (6.9)$$

where  $x_{m,s_2,s_3}$  is the volume sold in market  $m$  in event outcome  $s_2$  and market outcome  $s_3$ . In addition, the sale in market  $m$  is limited by the uninterrupted booking into this node:

$$x_{m,s_2,s_3} \leq z_m^F - a_{m,s_2} \quad m \in \mathcal{M}, s_2 \in \mathcal{S}_2, s_3 \in \mathcal{S}_3. \quad (6.10)$$

Finally, we must make sure that the uninterrupted booking is sufficient for the production levels in the field nodes:

$$y_{g,s_2,s_3} \leq z_g^F - a_{g,s_2} \quad g \in \mathcal{G}, s_2 \in \mathcal{S}_2, s_3 \in \mathcal{S}_3. \quad (6.11)$$

### First TSO Problem, TS01

The TSO allocates firm capacity to the shipper based on the booking requests from SP1. The TSO seeks to satisfy the requests, but are limited by the transportation capacities in the network and a requirement on expected security of supply. TS01 is a two-stage quadratic stochastic program, where allocations are

given in the first stage; whilst the routing decisions are made in the second stage after the network capacity availability is known. The objective of this problem is to minimize the square deviation from booking requests, such that the allocation stays close to the requests from the shipper:

$$\min \sum_{n \in \mathcal{N}} (\bar{z}_n^F - z_n^F)^2, \quad (6.12)$$

where  $\bar{z}_n^F$  is the booking requests made by the shipper while  $z_n^F$  is the allocated capacity to the shipper from the TSO. We must make sure that the shipper capacity allocation is no more than he requested  $\bar{z}_n^F$ :

$$z_n^F \leq \bar{z}_n^F \quad n \in \mathcal{N}. \quad (6.13)$$

On expectation, the security of supply should be at least  $R$ , where the security of supply is defined as the total expected delivery rate by the TSO over total firm booking. This limits the expected interruptions, which are represented by the difference between the allocated firm bookings to a market node  $m$ ,  $z_m^F$  and the sum of flows  $f_{j,m,s_2}$  from nodes  $j$  to market  $m$ :

$$\sum_{s_2 \in \mathcal{S}_2} \pi_{s_2} \sum_{m \in \mathcal{M}} \left( z_m^F - \sum_{j \in \mathcal{I}(m)} f_{j,m,s_2} \right) \leq (1 - R) \sum_{m \in \mathcal{M}} z_m^F. \quad (6.14)$$

We then add constraints to make sure that the flows are limited by the allocated booking in the booking nodes (fields  $g$  and markets  $m$ ). The set  $\mathcal{O}(j)$  gives the nodes which are connected directly downstream to a pipeline going from node  $j$ , whilst the set  $\mathcal{I}(j)$  gives the nodes that are connected directly upstream to a pipeline in node  $j$ .

$$\sum_{j \in \mathcal{O}(g)} f_{g,j,s_2} \leq z_g^F \quad g \in \mathcal{G}, s_2 \in \mathcal{S}_2, \quad (6.15)$$

$$\sum_{j \in \mathcal{I}(m)} f_{j,m,s_2} \leq z_m^F \quad m \in \mathcal{M}, s_2 \in \mathcal{S}_2. \quad (6.16)$$

For the intermediate nodes (that are not booking nodes) we must take care of the mass balance:

$$\sum_{i \in \mathcal{I}(j)} f_{i,j,s_2} = \sum_{k \in \mathcal{O}(j)} f_{j,k,s_2} \quad j \in \mathcal{J} \setminus \mathcal{N}, s_2 \in \mathcal{S}_2. \quad (6.17)$$

The events are modeled by reduced capacities  $K_{i,j,s_2}$  in pipelines between nodes  $i$  and  $j$ . In fields and markets the reduced capacity is given by  $K_{j,s_2}$ :

$$f_{j,i,s_2} \leq K_{j,i,s_2} \quad j \in \mathcal{J}, i \in \mathcal{O}(j), s_2 \in \mathcal{S}_2, \quad (6.18)$$

$$\sum_{j \in \mathcal{O}(g)} f_{g,j,s_2} \leq K_{g,s_2} \quad g \in \mathcal{G}, s_2 \in \mathcal{S}_2, \quad (6.19)$$

$$\sum_{j \in \mathcal{I}(m)} f_{j,m,s_2} \leq K_{m,s_2} \quad m \in \mathcal{M}, s_2 \in \mathcal{S}_2. \quad (6.20)$$

The dynamic capacity of the network depends on the pressure in each node, and is described through a Weymouth equation for each pipeline. The Weymouth equation has the following form:

$$f_{j,i,s_2} = K_{j,i}^W \sqrt{p_{j,s_2}^2 - p_{i,s_2}^2} \quad j \in \mathcal{J}, i \in \mathcal{O}(j), s_2 \in \mathcal{S}_2, \quad (6.21)$$

where  $K_{j,i}^W$  is the Weymouth constant for the pipeline going from node  $j$  to node  $i$ . This constant depends on characteristics of the pipeline such as diameter, length and roughness (for more details, see for instance Campbell (1992)). The pressure in node  $j$  in event outcome  $s_2$  is then given by  $p_{j,s_2}$ . Since (6.21) is not linear, an outer approximation derived by Taylor series expansion around fixed pressure points  $PI_l$  and  $PO_l$  is used (Rømo, Tomasgard, Hellemo, Fodstad, Eidesen & Pedersen 2009):

$$f_{j,i,s_2} \leq K_{j,i}^W \left( \frac{PI_l}{PI_l^2 - PO_l^2} p_{i,s_2} - \frac{PO_l}{PI_l^2 - PO_l^2} p_{j,s_2} \right) \quad l \in \mathcal{L}, s_2 \in \mathcal{S}_2, \quad (6.22)$$

where the set  $\mathcal{L}$  gives the fixed points used for the linearization. In our experience around 20 of these constraints are needed to assure a good fit to the Weymouth equation. Finally, we must make sure that the pressure in each node  $j$  is within its upper and lower limit ( $\bar{P}_j$  and  $\underline{P}_j$ ):

$$\underline{P}_j \leq p_{j,s_2} \leq \bar{P}_j \quad j \in \mathcal{J}, s_2 \in \mathcal{S}_2. \quad (6.23)$$

## Second Shipper Problem, SP2

In SP2 the objective function is the same as in SP1 (see Equation (6.1)). The main change is that the allocation of firm capacity has been done by the TSO and is now input to the shipper's optimization problem,  $\bar{z}_n^F$ . In the first stage in this three-stage stochastic program the shipper decides how much interruptible capacity to book,  $z_n^I$ . In the second stage the network capacity availability becomes known and finally, in the third stage, the market prices become known.

When the network capacity availability becomes known in the second stage, the TSO decides upon routing and thus on how much of the shipper's booked capacity that must be interrupted. The shipper estimates the event dependent interruption,  $a_{n,s_2}$ , as the difference between the total booking and the shipper's estimate for available capacity in event outcome  $s_2$ ,  $H_{n,s_2}$ :

$$a_{n,s_2} \geq (\bar{z}_n^F + z_n^I) - H_{n,s_2}, \quad n \in \mathcal{N}, s_2 \in \mathcal{S}_2. \quad (6.24)$$

Finally, the shipper assumes that the TSO will allocate capacities such that the final booking is balanced in the sense that entry (field) capacity equals exit (market) capacity:

$$\sum_{g \in \mathcal{G}} (\bar{z}_g^F + z_g^I - a_{g,s_2}) = \sum_{m \in \mathcal{M}} (\bar{z}_m^F + z_m^I - a_{m,s_2}) \quad s_2 \in \mathcal{S}_2. \quad (6.25)$$

We also need to include the mass balances in this problem, now including both firm and interruptible booking. The injections and extractions are not allowed to exceed the allocated capacity in each booking node, that is the booked firm ( $\bar{z}_g^F$  and  $\bar{z}_m^F$ ) and interruptible capacity ( $z_g^I$  and  $z_m^I$ ) less the estimated interruptions ( $a_{g,s_2}$  and  $a_{m,s_2}$ ).

$$y_{g,s_2,s_3} \leq \bar{z}_g^F + z_g^I - a_{g,s_2}, \quad g \in \mathcal{G}, s_2 \in \mathcal{S}_2, s_3 \in \mathcal{S}_3, \quad (6.26)$$

$$x_{m,s_2,s_3}^S \leq \bar{z}_m^F + z_m^I - a_{m,s_2}, \quad m \in \mathcal{M}, s_2 \in \mathcal{S}_2, s_3 \in \mathcal{S}_3. \quad (6.27)$$

## Second TSO Problem, TS02

TS02 is a quadratic mixed integer linear program run for each of the event outcomes in  $\mathcal{S}_2$ . When the network capacity availability is known, the TSO decides how much firm and interruptible capacity that can be delivered. The allocation of firm capacity has been determined in TS01, and the interruptible booking by the shipper was determined in SP2.

The objective is to minimize the square of interruptions weighted by the booking tariffs to make sure firm capacity is given priority:

$$\min \sum_{n \in \mathcal{N}} \left( T_n^F a_{n,s_2}^{F^2} + T_n^I a_{n,s_2}^{I^2} \right). \quad (6.28)$$

The TSO must then make sure that there is a balance between firm booking  $\bar{z}_g^F$ , interruptible booking  $\bar{z}_g^I$ , interruption of firm and interruptible capacity ( $a_{g,s_2}^F +$

$a_{g,s_2}^I$ ) and the flow  $f_{g,j,s_2}$  in each booking node:

$$\sum_{j \in \mathcal{O}(g)} f_{g,j,s_2} = \bar{z}_g^F + \bar{z}_g^I - a_{g,s_2}^F - a_{g,s_2}^I, \quad g \in \mathcal{G}, \quad (6.29)$$

$$\sum_{j \in \mathcal{I}(m)} f_{j,m,s_2} = \bar{z}_m^F + \bar{z}_m^I - a_{m,s_2}^F - a_{m,s_2}^I, \quad m \in \mathcal{M}. \quad (6.30)$$

The TSO must also make sure that the interruption of capacity is not larger than the allocated booking of firm and interruptible services:

$$a_{n,s_2}^F \leq \bar{z}_n^F, \quad n \in \mathcal{N}, \quad (6.31)$$

$$a_{n,s_2}^I \leq \bar{z}_{n,s_2}^I, \quad n \in \mathcal{N}. \quad (6.32)$$

No firm booking should be interrupted unless all interruptible booking in the same node is interrupted. The two following constraints enforce this by the use of the binary variable  $\beta_{n,s_2}$  that is 1 if some firm booking is interrupted:

$$a_{n,s_2}^F \leq \bar{z}_n^F \beta_{n,s_2}, \quad n \in \mathcal{N}, \quad (6.33)$$

$$a_{n,s_2}^I \geq \bar{z}_{n,s_2}^I \beta_{n,s_2}, \quad n \in \mathcal{N}. \quad (6.34)$$

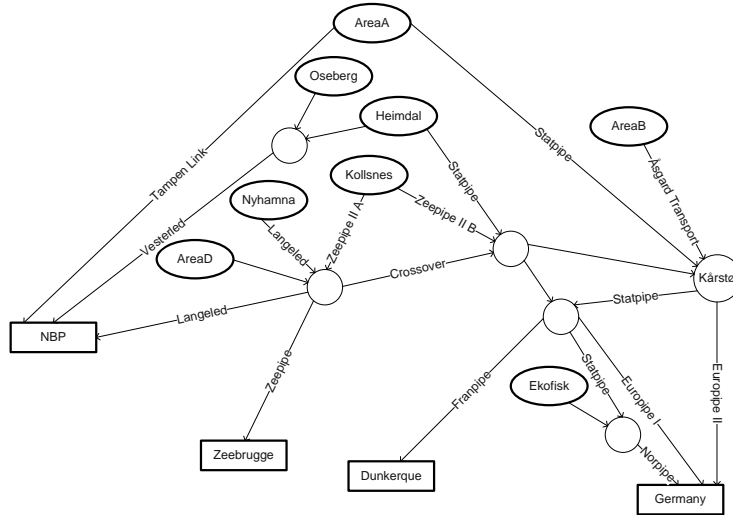
The flows are limited by the capacities in the network, with constraints equal to (6.17)-(6.23) of TS01.

## 6.4 Numerical Analysis and Discussion

In this section we first present the data and the assumptions for our case study. We then discuss the numerical results and our main findings.

### Input Data and Assumptions

The topology in our case study is based on the topology on the Norwegian Continental Shelf (NCS), and is illustrated in Figure 6.5. The basis for the topology is given in Norwegian Petroleum Directorate (2011), while details on production and transportation capacities and plans are confidential and provided by the Norwegian system operator, Gassco. The fields that we use in our case study are aggregates of real fields in the same region. These aggregated fields can cover both must-take fields and swing fields. All fields and markets are booking nodes in the network, such that they require booking of transportation capacity corresponding to their production and sale. The booking tariffs for firm transportation capacity correspond to the real tariffs on the NCS, and are available at Gassco (2012). We have assumed the booking tariff for interruptible services to be half

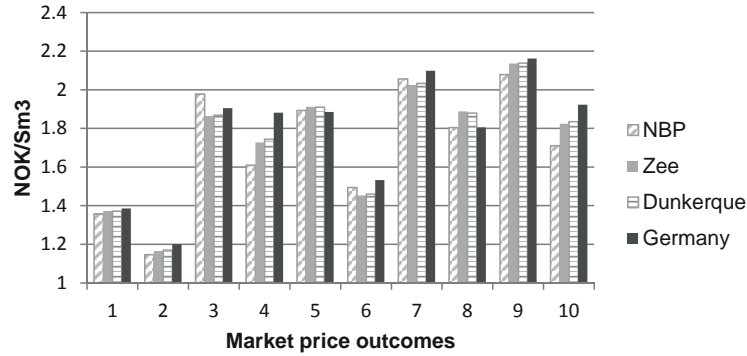


**Figure 6.5:** Topology for the NCS test case. The ovals represent fields, the circles are intermediate nodes, the rectangles are markets and the arrows are pipelines.

the price of firm services. The model has been tested with security of supply requirements for firm services in the range from 0.99 to 1 (where 1 indicates that all firm capacity must be delivered in all scenarios).

Real production cost data are not easily available, so we have based our estimates on different sources. The production costs of must-take production are estimated based on gas-to-oil ratios in Norwegian Petroleum Directorate (2011), where a larger oil share gives a larger marginal production cost. The oil price is taken from the Norwegian national budget for 2012. For the production cost in the swing fields, we have based our parameters on Golombek, Gjelsvik & Rosendahl (1998) and Kon-Kraft (2003).

**Scenarios** We have generated price outcomes for the markets based on real spot prices from 2010 and 2011 for the market hubs NBP (UK), Zeebrugge (Belgium), Gas Pool (Germany) and NetConnect (Germany). Since we only have one node representing the markets in Germany, we have defined the price for the market node ‘Germany’ as the average of the two German market hubs. The Dunkerque price is estimated as 10% of the GasPool price and 90% of the Zeebrugge price. The market prices are represented by 10 outcomes that are generated with the moment-matching procedure described in Høyland, Kaut & Wallace (2003). Fig-



**Figure 6.6:** The 10 spot price outcomes used in our analysis. The figure shows that the prices are volatile and highly correlated.

ure 6.6 shows the prices for the 10 outcomes.

We have defined 19 events with reduced capacity, each corresponding to a separate outcome. In addition, we have a default outcome where the system operates at full capacity. We do not consider multiple simultaneous events in any of the outcomes. This means that each event outcome only gives capacity reduction in a single node. The event outcomes are constructed such that all markets and fields have capacity reduction in one outcome, while the two processing plants Kollnes and Kårstø (in our test case a field and an intermediate node, respectively) have four events each. The probability and extent of the capacity reductions are calibrated so that the availability corresponds to the average availability figures reported by Gassco in 2010 and 2011 (Gassco 2010, 2011). Since our field nodes represent an aggregate of several smaller fields we have distinguished between field nodes that represent only a few underlying fields and field nodes that represent many. The capacity reduction is larger for field nodes with few underlying fields and smaller but more likely for field nodes with several underlying fields. Table 6.1 lists the events with the affected node, probability and the capacity reduction.

In total the 10 market outcomes and 20 event outcomes give a three-stage scenario tree with 200 scenarios.

## Results and Discussion

The model is implemented in Mosel version 3.2.2 and solved by Xpress Optimizer version 22.01.10 on a 2.80GHz dual core computer with 4 GB RAM. The whole sequence of shipper and TSO problems were solved within seconds. A benchmark



Outcome	Node	Probability	Capacity reduction
0	No event	0.631	0 %
1	NBP	0.001	35 %
2	Zeebrugge	0.001	35 %
3	Dunkerque	0.001	35 %
4	Germany	0.001	35 %
5	AreaD	0.007	50 %
6	Nyhamna	0.004	75 %
7	Heimdøl	0.007	50 %
8	Oseberg	0.005	70 %
9	AreaA	0.011	30 %
10	AreaB	0.013	25 %
11	Ekofisk	0.003	100 %
12	Kollsnes	0.069	25 %
13	Kollsnes	0.020	50 %
14	Kollsnes	0.010	75 %
15	Kollsnes	0.001	100 %
16	Kårstø	0.076	25 %
17	Kårstø	0.020	50 %
18	Kårstø	0.010	75 %
19	Kårstø	0.001	100 %

**Table 6.1:** The event outcomes that we use in our analysis. Only one node has a capacity reduction in each of the outcomes. The numbers are calibrated to match the reported availability figures from Gassco.

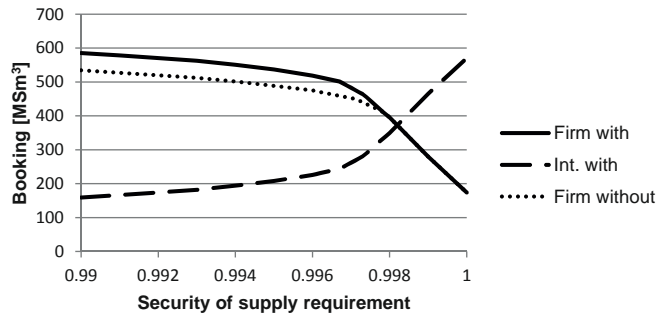
is calculated with the same model framework. The difference is that there are no interruptible services available in the benchmark, which also means that SP2 is superfluous. Furthermore, we require that the booking request for entry and exit booking in SP1 in the benchmark analysis is balanced. This is due to the requirement for firm services to be balanced, while the interruptible services can be unbalanced. In the following we use the label ‘Without’ for the benchmark solution, while ‘With’ indicates tests with interruptible services. We test the two model setups for increasing security of supply requirements, and compare the effects on booking levels, total throughputs, incomes and costs.

**Booking levels** Our *first observation* is that the total booking stays constant independent of the security of supply level when interruptible services are available. This can be seen in Figure 6.7 where the firm booking decreases, while the interruptible booking increases with the same amount. This result is intuitive, since the producer’s preference for transportation capacity is unchanged from SP1 to SP2. The producer will then seek to obtain the same total amount of capacity by increasing the interruptible booking.

Figure 6.7 also shows our *second observation*, that allocated firm booking is slightly reduced when interruptible services are not available. This is the effect of requiring booking requests to be balanced in SP1, which reduces the booking request and thereby limits the TSO’s possibility to adapt allocation to the network capacities. When allowed, the shipper consistently books nearly 90 MSm<sup>3</sup> more entry capacity than exit capacity, even though it implies paying for some transportation capacity that necessarily will be interrupted. This comes from the ability to adapt to events by substituting production with fields that are not affected by an event. It should be noted that the tariffs are very small, less than 13% of the average spot price, so the option cost of this flexibility is very low.

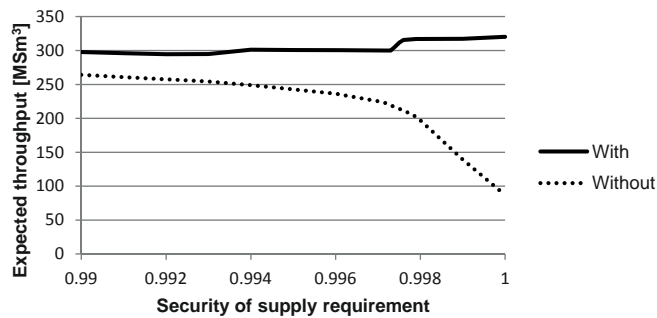
**Total expected throughput** The allocated firm capacity is falling with increasing security of supply level, which comes natural since increasing buffers are needed to withstand the events. Our *third observation* is that the benchmark has a falling expected throughput as the security of supply requirement increases, as can be observed in Figure 6.8. More surprisingly, according to our *fourth observation* the expected throughput is increasing with the security of supply requirement when interruptible services are available. Since interruptible services do not have any security of supply requirement this shift from firm to interruptible capacity seen in Figure 6.7 increases the flexibility for the TSO and makes it possible to better adapt the flow to the network capacities with its system effects. The expected throughput increases by 7% when security of supply requirements increases from 0.99 to 1 and interruptible services are available.

These two last observations together confirms our hypothesis, that including

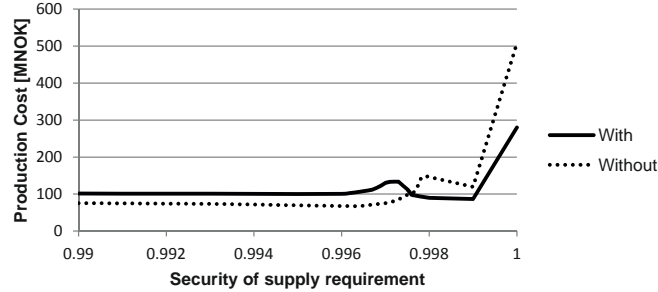


**Figure 6.7:** Interruptible and allocated firm booking decisions. Since there are both entry and exit booking, total flow cannot exceed half the total booking.

interruptible services to the transportation service regime will increase the efficiency by enabling a larger expected throughput in the transportation network without reducing the security of supply. The expected throughput increases with 13% when introducing interruptible services at the lowest security of supply level (0.99), and the difference increases to over 250% when the security of supply requirement is 1.



**Figure 6.8:** The total expected throughput in the system. The dotted line shows the results for the model without interruptible services, while the full line shows the results for the model with interruptible services.



**Figure 6.9:** The final production cost in the fields in our model runs. The full line shows the results when interruptible booking is available, while the dotted line is the result of the model where interruptible booking is not available.

**Income and costs** Resulting income shows a pattern similar to the total flow, with an increase of between 13% (with security of supply level of 0.99) to 274% (with security of supply level of 1) compared to the benchmark. Differences in average achieved spot prices are small, which is reasonable since both booking and interruption are allocated before spot prices become known. The ability to adapt to the varying spot prices is therefore limited. Our *fifth observation* on the other hand, is a dramatic increase in production cost due to decreased oil production for the benchmark as security of supply increases, as can be seen in Figure 6.9. When the transportation capacity is reduced, production of natural gas at the field must be reduced and therefore also the oil production. This leads to the largest changes in the benchmark since the transportation capacity falls below the must-take production limits in these fields when the security of supply requirements is high. Since we do not have real production cost functions available there is substantial uncertainty with respect to the true monetary cost of this decreased oil production. The profit margins of oil is however substantially larger than for gas, so the shape of the production cost functions are representative. We therefore also argue that the improved ability of stable oil production through introduction of interruptible services is valid. The non-monotone shape of the curves in Figure 6.9 is due to our model framework with a sequence of models with objectives that are not fully aligned, where an early booking or allocation decision can be non-optimal for the final supply chain performance.

**Discussion of TSO behavior** In our modeling framework the TSO is a non-profit agent who minimizes the deviation between the shipper's booking requests

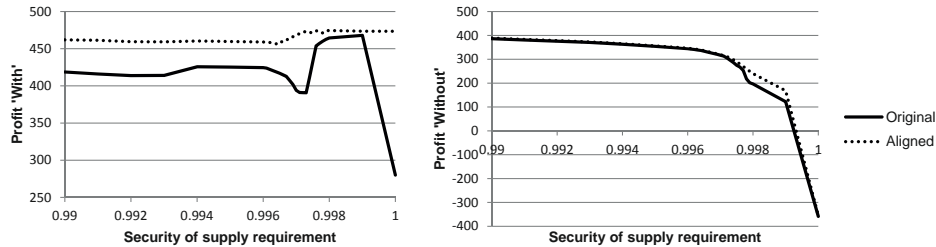
and the flow in the network. The shipper on the other hand is profit maximizing. This corresponds well to the incentives of the agents in the Norwegian system. These non-aligned objectives can however cause inefficiencies seen from a system perspective. The TSO do neither have information about spot prices nor production costs. The TSO might therefore give priority to a swing field and interrupt must-take production, or interrupt a high price market rather than a low price market.<sup>1</sup> In the modeled booking regime, which corresponds closely to the real Norwegian system, there is no way for the shippers to signal priorities between different booking nodes, which leave a risk for inefficient operation when prices or costs vary.

To test the significance of the agents' differing incentives on the supply chain performance, we ran our case study with an alternative set of TSO models where the incentives were more in line with the shipper's incentives. That is, the original objective functions were replaced with an objective where social surplus in the network were maximized, and the flows were allowed to fall below the uninterrupted capacities. This corresponds to an idealized situation where the TSO has all price and production cost information. Figure 6.10 compares profits with the social surplus maximizing TSO to the original formulation. The *sixth observation* is that the profits can increase if the TSO maximizes social surplus rather than taking market signals from the booking requests only. Due to increased flow and income there is approximately 10 % increase of profit for security of supply requirements less than 0.997 when interruptible services are available. For stricter security of supply requirements the profit increases are less regular. With interruptible services the model where the TSO maximizes social surplus avoids withholding must-take production with valuable associated oil, which causes a major profit increase of 69 % when security of supply is 1.

**Implications of using a three-stage model** We have considered two types of uncertainty in our model: events in the network and market prices. The flexibility offered by the interruptible services can be used to increase the throughput in the system due to uncertainty regarding events. The interruptible services also allow the shipper to have unbalanced booking which could be valuable with volatile market prices. While we have demonstrated that the effect on throughput is indeed significant, we have not been able to demonstrate any significant value of including interruptible services when unknown market prices are the only source of uncertainty. This is however as expected since interruptions are allocated before market uncertainty is resolved in our model framework. To analyze the effect of interruptible services on market uncertainty, we would have to increase

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<sup>1</sup>In real life operation the TSO naturally have more insight in the system than what is provided to him through the bookings, so it is unlikely that swing production would be given priority over must-take production in actual operation.



**Figure 6.10:** Comparison of profit (income less production cost) from model runs with the original TSO objective functions ('Original') and the modified TSO objective functions that are more aligned with the producer's objective function ('Aligned'). The left part shows results with interruptible services and the right part is without interruptible services available.

the number of stages in our models. This is due to the number of iterations used on the NCS to allocate firm capacity and the possibility to adapt the capacity allocation to market prices as the uncertainty is gradually reduced.

## 6.5 Conclusions

In this paper we have developed a framework for analysis of interruptible transportation services in the natural gas system. We have also developed a new production cost function where the effects of reduced gas production on oil production in the same field are incorporated. We have tested the modeling framework on a case study with realistic input data, and a topology that is similar to the topology on the Norwegian Continental Shelf. The results from our case study show that there is a substantial gain in efficiency in the network when interruptible services are introduced to address network events. Both total flow and income in the system is drastically increased compared to the benchmark solution where interruptible services are not available. For the highest level of security of supply, the increase in flow and income in the system is as large as 250%.

## Acknowledgements

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for providing us with valuable input and Dr. Ruud Egging for useful discussions on model formulation.

# Appendices

## 6.A Input Data

In real life, a shipper bases his expectation for network capacity availability on historical observations and knowledge about the system. To generate the shipper's estimates to network capacity we have used an optimization model where the objective is to find the best possible utilization of the network in each scenario  $(s_2, s_3) \in (\mathcal{S}_2, \mathcal{S}_3)$ :

$$\max \sum_{m \in \mathcal{M}} P_{m,s_2} x_{m,s_2,s_3} - \sum_{g \in \mathcal{G}} c_{g,s_2,s_3}. \quad (6.35)$$

The production cost  $c_{g,s_2,s_3}$  is represented by the set  $Q$  of linear constraints describing must-take production as in the shipper problems and a constraint representing the linear production cost of swing production.

$$c_{g,s_2,s_3} \geq C_{g,q}^{AMT} + C_{g,q}^{BMT} y_{g,s_2,s_3} \quad g \in \mathcal{G}, q \in \mathcal{Q}, \quad (6.36)$$

$$c_{g,s_2,s_3} \geq C_g^{BSW} (y_{g,s_2,s_3} - Y_g^{MT}) \quad g \in \mathcal{G}. \quad (6.37)$$

The following two constraints make sure the flows correspond to production and sales decisions:

$$y_{g,s_2,s_3} = \sum_{j \in \mathcal{O}(g)} f_{g,j,s_2,s_3} \quad g \in \mathcal{G}, \quad (6.38)$$

$$x_{m,s_2,s_3} = \sum_{j \in \mathcal{I}(m)} f_{j,m,s_2,s_3} \quad m \in \mathcal{M}. \quad (6.39)$$

The production is limited by the production capacities as in constraint (6.6) in SP1 and the flows are limited by the network capacities with constraints equal to (6.17)-(6.23) in TSO1.

The problem utilizes the TSO's knowledge of the physical network structure and capacities, and the shipper's knowledge of production costs and market prices. The problem has a hybrid objective, where we seek to maximize throughput in the system, but with a realistic distribution between the fields and the markets. To achieve this we maximize sales income less production cost as defined in SP1<sup>2</sup>.

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<sup>2</sup>The quadratic part of the production cost is removed to avoid that production is limited by high production costs.



For each event outcome  $s_2 \in \mathcal{S}_2$  we set the shipper's estimate for available capacity based on the market outcome  $\hat{s}_3 \in \mathcal{S}_3$  with the largest total throughput  $\sum_{m \in \mathcal{M}} x_{m,s_2,s_3}$ . This gives the following definition of the estimated capacity  $H_{n,s_2}$ :

$$H_{g,s_2} = y_{g,s_2,\hat{s}_3}, \quad g \in \mathcal{G}, s_2 \in \mathcal{S}_2, \hat{s}_3 = \operatorname{argmax}_{s_3 \in \mathcal{S}_3} \sum_{m \in \mathcal{M}} x_{m,s_2,s_3}, \quad (6.40)$$

$$H_{m,s_2} = x_{m,s_2,\hat{s}_3}, \quad m \in \mathcal{M}, s_2 \in \mathcal{S}_2, \hat{s}_3 = \operatorname{argmax}_{s_3 \in \mathcal{S}_3} \sum_{m \in \mathcal{M}} x_{m,s_2,s_3}. \quad (6.41)$$

## 6.B Notation

Name	Description	Type
$g$	Field	index
$m$	Market	index
$n$	Node with transportation booking	index
$j, i$	Nodes in general	index
$s, s_2, s_3$	Outcome	index
$q$	Linearization of production cost function	index
$\mathcal{J}$	Nodes	set
$\mathcal{I}(j)$	Nodes with pipeline going into node $j$	set
$\mathcal{O}(j)$	Nodes with pipeline going out from node $j$	set
$\mathcal{G}$	Fields, $\mathcal{G} \subset \mathcal{J}$	set
$\mathcal{M}$	Markets, $\mathcal{M} \subset \mathcal{J}$	set
$\mathcal{N}$	Booking nodes, $\mathcal{N} = \mathcal{G} \cup \mathcal{M}$	set
$\mathcal{S}_2$	Scenario nodes in second stage, event outcomes	set
$\mathcal{S}_3$	Scenario nodes in third stage, market price outcomes	set
$\mathcal{Q}$	Linear pieces in production cost function	set

Name	Description	Type
$\pi_{s_2}$	Probability of event outcome	constant
$\pi_{s_3}$	Probability of market outcome	constant
$R$	Requirement on security of supply for firm services	constant
$Y_{g,s_2}$	Production capacity (uncertain)	constant
$Y_{MT}^g$	Must-take production capacity	constant
$T_n^F$	Firm tariff	constant
$T_n^I$	Interruptible tariff	constant
$H_{n,s_2}$	Transportation capacity through node $n$ assumed by the shipper (uncertain)	constant
$P_{m,s_3}$	Spot price (uncertain)	constant
$C_{g,q}^{AMT}$	Coefficient of constant part of must-take production cost	constant
$C_{g,q}^{BMT}$	Coefficient of linear part of must-take production cost, $< 0$	constant
$C_{g,q}^{BSW}$	Coefficient of linear part of swing production cost	constant
$C_{g,q}^{CSW}$	Coefficient of quadratic part of swing production cost	constant
$K_{i,j}^W$	Weymouth constant	constant
$K_{i,j,s_2}$	Static pipeline capacity under event	constant
$K_{j,s_2}$	Static node capacity under event	constant
$\overline{P}_j$	Maximum pressure in node	constant
$\underline{P}_j$	Minimum pressure in node	constant
$z_n^F$	Firm booking	variable
$z_n^I$	Interruptible booking	variable
$a_{n,s_2}^F$	Interruption of firm capacity	variable
$a_{n,s_2}^I$	Interruption of interruptible capacity	variable
$a_{n,s_2}$	Total interruption of booked capacity	variable
$x_{m,s_2,s_3}$	Spot sale	variable
$y_{g,s_2,s_3}$	Production	variable
$c_{g,s_2,s_3}^{MT}$	Must take production cost	variable
$c_{g,s_2,s_3}^{SW}$	Swing production cost	variable
$f_{j,jj,s_2}$	Flow	variable
$p_{j,s_2}$	Pressure	variable
$\beta_{n,s_2}$	Binary variable that is 1 if firm booking is interrupted	variable
$c_{g,s_2,s_3}$	Production cost	variable

If a variable is overlined in a problem description, it is a constant in that problem.

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## Paper VI

Kristin Tolstad Uggen, Marte Fodstad and Vibeke Stærkebye  
Nørstebø:

# *Using and Extending Fix-and-Relax to Solve Maritime Inventory Routing Problems*

*Accepted in TOP<sup>a</sup>, and published online on March 2011.*

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## Chapter 7

# Using and Extending Fix-and-Relax to Solve Maritime Inventory Routing Problems

### **Abstract:**

The paper presents a new way of optimising maritime inventory routing problems (IRP) by using a heuristic approach based on fix-and-relax time decomposition extended with two new features. The purpose of the extensions is to reduce computation time during the fix-and-relax process and to improve solution quality after a first solution is found. The feature which improves solution quality is independent of the method used for calculating the first solution. In this study, the algorithm and extensions have been tested on four liquefied natural gas (LNG) cases and the impacts on computational time and objective function value are reported. The results show that using fix-and-relax reduces computing time considerably while the objective function value is only slightly worse compared to a general MILP solver. Furthermore, the results confirm that the extensions work according to the intentions when compared to the original fix-and-relax heuristic. For relatively complex cases, it appears advantageous to use the developed extensions.

## 7.1 Introduction

In industrial supply chains, inventory management and routing have traditionally been managed separately. Combining the management of inventory and routing leads to an inventory routing problem (IRP). In maritime IRP the transportation mode is vessels. In such problems, products are produced and stored in inventories at loading ports and are transported by sea to unloading ports where they are stored prior to consumption. Christiansen & Fagerholt (2009) formulate a basic maritime IRP model and discuss different model extensions. The toughest computational challenges for optimising IRP come from the routing which is known to be a NP-hard binary linear problem. Because of the complexity of such problems, almost all solution approaches concerning IRP are heuristics.

The objective of this paper is to present a new heuristic method for solving a maritime IRP based on fix-and-relax time decomposition, an approach originally developed by Dillenberger, Escudero, Wollensak & Zhang (1994). To obtain

better and faster solutions we have extended this approach by reducing size of sub-problems by simplifying the modelling of the last intervals and adding an Improvement Phase after a feasible solution is found. We have not found any literature where the fix-and-relax method is used for a routing problem.

We test the method on a maritime IRP where the product is liquefied natural gas (LNG). In addition to routing and storage management, the problem addresses onshore processes, contracts, and markets. One major difference to more basic IRPs is that in our problem some of the cargo (LNG) is lost during transport. An in-depth description of the model is given in Fodstad, Tolstad Uggem, Rømo, Lium, Stremersch & Hecq (2010).

The paper is organised as follows: The next section reviews existing research on IRP, especially maritime IRP, and on the fix-and-relax method. These research studies provide a theoretical foundation for the extended fix-and-relax approach described in the subsequent section. Thereafter, a presentation of the LNG case study and the test setup is given, and results from running the LNG-IRP model with the various extensions are presented and discussed. The last section concludes the work.

## **7.2 Theoretical Foundation and Related Research**

This section presents a short literature overview regarding the problem class IRP focusing on maritime transportation and a description of work done on the fix-and-relax methodology.

### **Maritime Inventory Routing Problem**

Most of the IRP literature considers land-based problems. Since the problem class addressed in this paper is maritime IRP, we have focused on IRP literature dealing with the maritime sector. For papers on land-based IRP, see for example (Golden, Assad & Dahl 1984, Dror, Ball & Golden 1985, Campbell, Clarke, Kleywegt & Savelsbergh 1998). In addition Andersson, Hoff, Christiansen, Hasle & Løkketangen (2010) and Cordeau, Laporte, Savelsbergh & Vigo (2007) describe both land-based and maritime IRP, point out industrial aspects of the problems and give an overview of literature. Differences between land-based and maritime transportation are described in Christiansen, Fagerholt, Nygreen & Ronen (2007). There are several similarities between IRP and vendor managed inventory (VMI) problems. Al-Ameri, Shah & Papageorgiou (2008) make a review of literature concerning VMI, including a shipping-based VMI system.

Christiansen, Fagerholt & Ronen (2004) present a review of literature concerning maritime IRP. The problem is mainly discussed with regards to tactical and operational problems in industrial shipping. Some examples of applications

**Table 7.1:** Notation for basic IRP

Name	Description	Type
$N$	All ports	Set
$N_v$	Ports compatible with vessel $v$ , $N_v \subseteq N$	Set
$T$	All time periods	Set
$V$	All vessels	Set
$\tau_{i,j,t,v}$	$= \{t' \in T \mid t = t' + TT_{i,j,t',v} + 1\}$ , $\tau_{i,j,t,v} \subset T$	Set
$x_{i,j,t,v}$	Travel from port $i$ to port $j$ with vessel $v$ leaving on $t$	Variable
$w_{i,t,v}$	Vessel $v$ waiting by port $i$ in time period $t$	Variable
$q_{i,t,v}$	Amount loaded/unloaded by vessel $v$ in port $i$ in time period $t$	Variable
$l_{t,v}$	Storage level of vessel $v$ in the end of time period $t$	Variable
$s_{i,t}$	Storage level in port $i$ in the end of time period $t$	Variable
$C_{i,j,v}$	Cost for vessel $v$ travelling from port $i$ to port $j$	Constant
$TT_{i,j,t,v}$	Travel time from port $i$ to port $j$ with vessel $v$ leaving on $t$	Constant
$Q_{i,v}$	Loading/unloading capacity for vessel $v$ in port $i$	Constant
$L_v$	Storage capacity of vessel $v$	Constant
$S_i$	Storage capacity in port $i$	Constant
$I_i$	Indicator. 1 for loading ports and -1 for unloading ports	Constant
$R_i$	Production/consumption rate in port $i$	Constant

are (Christiansen 1999, Christiansen & Nygreen 1998*a,b*, Flatberg, Haavardtun, Kloster & Løkketangen 2000) who formulate and solve a combined inventory management and ship scheduling problem for a company in the fertilizer industry. Miller (1987) and Dauzère-Pérès, Nordli, Olstad, Haugen, Koester, Myrstad, Teistklub & Reistad (2007) discuss the problem with sloshing of chemical and frozen products in tanks, which is a critical aspect in maritime transportation. Dauzère-Pérès et al. (2007) also present implementation of a decision support system. Persson & Göthe-Lundgren (2005) and Al-Khayal & Hwang (2007) deal with multiple products, more exactly bitumen and petrochemical products, respectively. Grønhaug, Christiansen, Desaulniers & Desrosiers (2008) and Grønhaug & Christiansen (2009) present models and solution methods to solve an LNG supply chain problem. A rich model for solving the LNG supply chain problem can also be found in Fodstad et al. (2010).

A basic maritime IRP consists of operating a given fleet of vessels between different ports, taking into account travel time and capacities on vessels and ports. Equations (7.1)-(7.9) present a mathematical formulation of a basic IRP, with the notation and basic elements given in Table 7.1. The objective function

(1) is to minimise the variable costs of operating the vessel fleet. Constraints (2) take care of routing and scheduling of the vessels, including idle periods. The port calls are assumed to be one day at the end of each voyage. Constraints (3) make sure loading and unloading only takes place during a port call. The storage levels for vessels are updated in constraints (4) and storage capacities are taken care of in constraints (5). Similar constraints for onshore storages are given in constraints (6) and (7), where fixed production and consumption rates represents the interface to surrounding other parts of the value chain. Constraints (8) and (9) require  $x$ -variables to be binary and all other variables to be non-negative. Note that the  $w$ -variables will take binary values without a binary requirement because the other variables in constraints (2) are binary. This formulation resembles the basic IRP described by Christiansen & Fagerholt (2009), but leaving out start and end conditions and modelling discrete time steps instead of continuous time

$$\min \sum_{v \in V} \sum_{i \in N_v} \sum_{j \in N_v} \sum_{t \in T} C_{i,j,v} x_{i,j,v,t} \quad (7.1)$$

$$\sum_{j \in N_v} \sum_{t' \in \tau_{i,j,t,v}} x_{j,i,t',v} + w_{i,t-1,v} = \sum_{j \in N_v} x_{i,j,t,v} + w_{i,t,v} \quad \forall v \in V, i \in N_v, t \in T \quad (7.2)$$

$$q_{i,t,v} \leq Q_{i,v} \sum_{j \in N_v} x_{i,j,t+1,v} \quad \forall v \in V, i \in N_v, t \in T \quad (7.3)$$

$$l_{t,v} = l_{t-1,v} + \sum_{i \in N_v} I_i q_{i,t,v} \quad \forall v \in V, t \in T \quad (7.4)$$

$$l_{t,v} \leq L_v \quad \forall v \in V, t \in T \quad (7.5)$$

$$s_{i,t} = s_{i,t-1} - \sum_{v \in V} I_i q_{i,t,v} + I_i R_i \quad \forall i \in N, t \in T \quad (7.6)$$

$$s_{i,t} \leq S_i \quad \forall i \in N \quad (7.7)$$

$$x_{i,j,t,v} \in \{0, 1\} \quad \forall v \in V, i \in N_v, j \in N_v, t \in T \quad (7.8)$$

$$w_{i,t,v}, q_{i,t,v}, l_{t,v}, s_{i,t} \geq 0 \quad \forall i \in N, t \in T, v \in V \quad (7.9)$$

Several approaches have been used for solving IRPs. One approach that is widely used in the maritime sector is based on the path-flow formulation. Path-flow models divide the solution process into two phases. In the first phase, feasible ship schedules are generated. In the second phase, the model selects among these feasible schedules. Appelgren (1969, 1971) presents the first studies using this approach in the maritime sector. Christiansen (1999) uses Dantzig-Wolfe decomposition to obtain a path-flow formulation with column generation, and branch-and-price in the problem solving. In Christiansen & Nygreen (2005), this problem is extended to deal with uncertainties in sailing and port time. Persson & Göthe-Lundgren (2005) include varying production rates at loading ports and

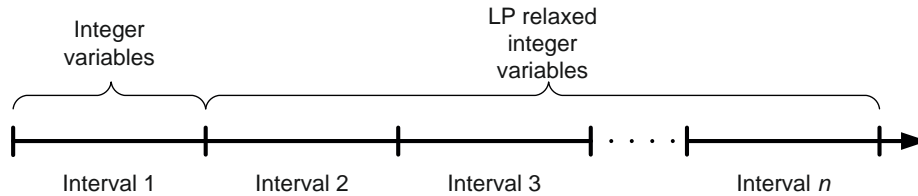
consider a multi-product case when they apply a path-based formulation on a maritime IRP. The approach is based on column generation and tree search. A branch-and-price-and-cut method is used by Grønhaug et al. (2008) on their path-flow formulation.

Another approach is the arc-flow formulation, where a feasible schedule is explicitly given through arc constraints in the model. Miller (1987) uses an arc flow formulation to evaluate changes that are made to an existing solution of a maritime IRP, generated by a constructive heuristic. In Al-Khayal & Hwang (2007), the formulation by Christiansen (1999) is extended and comprises multiple products and ships with multiple compartments. The extended problem is solved by branch-and-bound methodology. Grønhaug & Christiansen (2009) compare solution times for an arc-flow and a path-flow formulation of the same problem.

Many variants of heuristics have been proposed for the maritime IRP, some of them are briefly described here: Flatberg et al. (2000) decompose the problem discussed in Christiansen (1999) and propose an algorithm which combines an iterative heuristic with an exact LP (Linear Programming) solution approach. The algorithm uses a combinatorial local search heuristic to solve the ship routing part of the problem. LP is applied to optimise the scheduling problem with given vessel routes. Dauzère-Pérès et al. (2007) formulate a less complex model for describing the maritime IRP by including practical knowledge in the modelling. However, only an approximation of the problem can be solved because the objective function is non-linear. A greedy heuristic and a genetic local search are applied. Another approach for solving this problem is formulated by Moscato & Cotta (2003). They develop a genetic algorithm to determine the best order of visit for the problem.

### **Fix-and-Relax Heuristic**

The fix-and-relax time decomposition heuristic was originally described by Dillenberger et al. (1994). Since then, it is applied widely, most notably in production planning, in particular lot-sizing and scheduling. Examples are Dillenberger et al. (1994), Stadtler (2003), Kelly & Mann (2004), Beraldi, Ghiani, Grieco & Guerriero (2006, 2008), Absi & Kedad-Sidhoum (2007), Federgruen, Meissner & Tzur (2007), de Araujo, Arenales & Clark (2007, 2008), Akartunali & Miller (2009), Pochet & Warichet (2008), Mohammadi, Fatemi Ghomi, Karimi & Torabi (2008) and Ferreira, Morabito & Rangel (2009). Other applications are Ouhimmou, D'Amours, Beaugard, Ait-Kadi & Singh Chauhan (2008) on tactical supply chain planning, Alonso-Ayuso, Escudero & Pizarro (2009) and Alonso-Ayuso, Escudero, Pizarro, Romeijn & Romero Morales (2006) on a multi-period single-source assignment problem, Escudero & Salmeron (2005) on project scheduling,



**Figure 7.1:** Fix-and-relax - iteration one

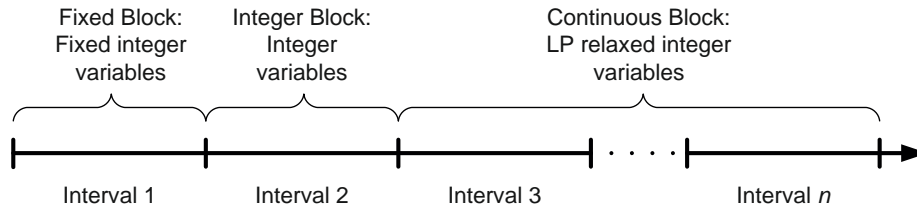
Alonso, Escudero & Ortuno (2000) and Marín (2006) on airport traffic management and Aytekin (2002) on capital investment planning in the U.S. Navy. In several examples of fix-and-relax applied to stochastic programming problems, the structure of a scenario tree is utilised to achieve separable sub-trees as in Alonso et al. (2000) and Beraldi et al. (2006).

The basic idea of the method as described by Dillenberger et al. (1994) is as follows: the planning horizon is divided into a finite number of time intervals  $n$ , as illustrated in Figure 7.1. The problem is then decomposed into  $n$  sub-problems, and solved in iterations corresponding to the time intervals.

In the first iteration, the iteration counter  $v$  is set to 1. The sub-problem is solved with the integer variables in the first interval, while the integer variables in the remaining time intervals are replaced by continuous variables, here referred to as an LP relaxation of the integer variables. (All variables that are continuous in the model description are kept continuous throughout the solution process.)

In each of the next iterations,  $v$  is increased by 1. The integer variables from interval  $v - 1$  are fixed to the solution values from the previous iteration. Integrality constraints are reintroduced for the integer variables of interval  $v$  while all other variables are kept non-fixed and continuous (see Figure 7.2, where  $v = 2$ ). After solving the new sub-problem, the iteration is completed. The process is then repeated until interval  $v = n$  is completed. After solving iteration  $n$ , a complete solution to the original problem is found. The procedure groups the intervals in three blocks: the Fix Block, the Integer Block and the Continuous Block as illustrated in Figure 7.2.

Wolsey (1998) gives a more general presentation of the heuristic under the name *Relax-and-Fix*. He splits the integer variables into “important” and “less important” variables. The important integer variables are solved first while the less important are LP relaxed. In the next iteration the important integer variables are fixed and the integrality constraints of the less important variables are reintroduced. Kelly & Mann (2004) extend this to several groups of variables, and let the groups represent the different levels in a production process. Beraldi et al. (2008) use both time and product as basis for the grouping of variables. Ferreira et al. (2009) show how any index of the integer variables can be used as



**Figure 7.2:** Fix-and-relax - iteration two

criteria for partitioning the variables into groups.

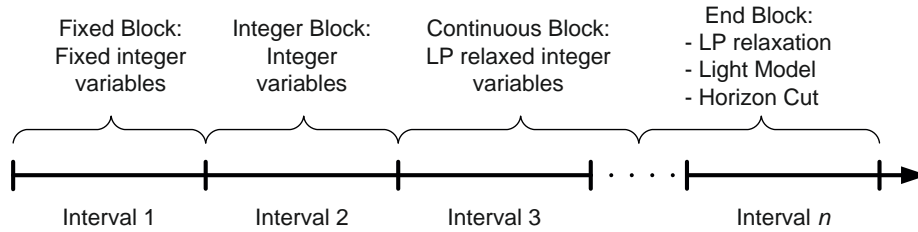
Another generalization of the heuristic is given by Mohammadi et al. (2008). They replace the Continuous Block with a simplified linear model formulation.

There are several possible ways to specify the fixing principle. With binary variables, Akartunali & Miller (2009) observed no significant difference between fixing all binary values from an intermediate solution, fixing only the 1-values or fixing only the 0-values. Federgruen et al. (2007), amongst others, discuss whether to fix the continuous variables or not.

Since the overall algorithm is a heuristic with limited ability to see how an early decision affects the later intervals, it might not be worth solving each sub-problem to optimality. Both Absi & Kedad-Sidhoum (2007) and Akartunali & Miller (2009) use time limits and MIP gap limits as stopping criteria in each iteration. Akartunali & Miller (2009) point out the need for greater time limits for early iterations due to the change in sub-problem complexity during the algorithm.

The basic fix-and-relax algorithm, as described above, may fail to produce a feasible solution even if one exists. Several proposals have been made to reduce or remove this weakness. Dillenberger et al. (1994) fix the second best solution of the previous iteration if a sub-problem (except the first) turns out infeasible. Stadtler (2003) and Absi & Kedad-Sidhoum (2007) use overlap between the intervals so that some variables with integrality requirement in the previous iteration are kept unfixed. Pochet & Warichet (2008) use another known method, local branching, where a limited number of binary variables from the previous intervals are allowed to change value. If a sub-problem is infeasible, Escudero & Salmeron (2005) unfix the variables of previous intervals and re-optimize until the problem becomes feasible. Beraldi et al. (2006) point out that this algorithm may cancel out the whole decomposition, ending up with the original problem. Instead, for their stochastic lot-sizing problem they show that adding “worst case” scenarios derived from their scenario tree in a structured way under certain conditions guarantees feasibility.

Fix-and-relax can be combined with several other methods. For instance, de Araujo et al. (2007, 2008) and Beraldi et al. (2008) combine fix-and-relax



**Figure 7.3:** Fix-and-relax with four blocks - iteration two,  $v = 2$

with a rolling horizon approach. As previously mentioned, Pochet & Warichet (2008) use local branching together with fix-and-relax, both to avoid infeasibility and to improve the solution within an interval. Akartunali & Miller (2009) try to find solutions of the original problem using LP-and-fix to get cut-off values that can be used in the branch-and-bound process of later iterations.

### 7.3 Extended Fix-and-Relax Approach

We have developed some extensions to the original fix-and-relax heuristic that are presented here.

#### Simplified Model in the End Block

The integer variables are the main difficulty for the tractability of the IRP model, while a significant number of continuous variables add to the computation time. Our aim is to reduce the effort from repeatedly solving many continuous variables at the end of the model horizon. We separate a new block from the last part of the Continuous Block as illustrated in Figure 7.3, and call this the End Block. There are two ways of treating this End Block: either using a simplified linear model called the Light Model or completely omitting this part of the model horizon which implies a Horizon Cut. Both Light Model and Horizon Cut reduce the number of variables compared to the original fix-and-relax.

A pseudo code for the fix-and-relax procedure including Horizon Cut is given in Algorithm 7.1. Here  $P$  represents the total problem defined over  $\tau = \{0, \dots, T\}$  time periods with integer variables  $x$  and continuous variables  $y$ .  $n$  is the number of intervals and  $u$  is the size of the overlap as a percentage of the interval length.

The Light Model is similar to what is used by Mohammadi et al. (2008), but we combine this with the Continuous Block instead of replacing the Continuous Block with the Light Model. The Light Model is built based on problem specific knowledge and contains continuous variables and constraints that are assumed to



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**Algorithm 7.1** FixAndRelax( $P(x(\tau), y(\tau)), n, u$ )

---

```

1: RelaxIntegrityCondition( $x(\tau)$ )
2: for  $v = 1, \dots, n$  do
3:    $t_1 := \{\frac{T}{n}(v-1), \dots, \frac{T}{n}v\}$ 
4:   AddIntegrityCondition( $x(t_1)$ )
5:    $t_2 := \{0, \dots, \min [T; \frac{T}{n}(v+f)]\}$ 
6:   SolveMIP( $P(x(t_2), y(t_2))$ )
7:    $t_3 := \{\frac{T}{n}(v-1)(1 - \frac{u}{100}), \dots, \frac{T}{n}v(1 - \frac{u}{100})\}$ 
8:   FixToLastSolution( $x(t_3)$ )
9: end for
10: return GetLastSolution( $P$ )

```

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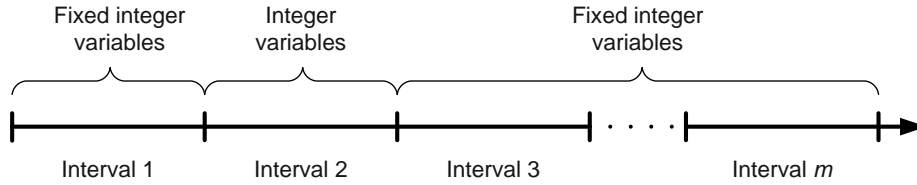
significantly affect decisions in previous periods. Typically these constraints link the time periods before and after the horizon cut. One example is a multi-time period bound limiting total deliveries in a year.

Let  $f$  be the number of intervals in the Continuous Block which in each iteration leaves  $n - f - v$  intervals in the End Block, where  $n$  is the total number of intervals and  $v$  is the iteration counter. The length of the End Block will decrease as the iterations go by, and setting  $f = n - 1$  would completely remove the End Block already in the first iteration. Through making  $f$  depend on the iteration number,  $fv$ , the partition between the Continuous Block and the End Block can be tailored to the problem at hand by, for instance, avoiding a partition leaving a small part of a multi-time period constraint out of the Continuous Block. Our only requirement is that  $v + fv$  should not decrease as  $v$  increases.

It is possible to limit the length of the Fix Block in a similar way as we do with the End Block using Horizon Cut. The computational effect of this is assumed to be considerably less than using an End Block since the fixed integer decisions leave a small solution space for this part of the problem. This is confirmed by tests showing that the last iterations of the fix-and-relax algorithm, where the length of the Fix Block is large, are solved very fast.

### Improvement Phase

Having produced a feasible solution for the original problem by using one of the fix-and-relax versions described above, another fix-and-relax inspired heuristic can be used in order to improve this solution. In this Improvement Phase, the time horizon of the model is again divided into intervals. The total number of these intervals,  $m$ , will usually be smaller than the  $n$  intervals in the previous fix-and-relax run. All integer variables are fixed to the existing solution, denoted the incumbent. The algorithm loops through the  $m$  intervals in consecutive order,



**Figure 7.4:** Re-optimising a feasible solution - iteration two

removes the fixing of the integer variables in this interval (see Figure 7.4) and re-optimises. The continuous variables are kept free through the whole process. If a better solution is found during re-optimisation this is kept as the incumbent. Before stepping to the next iteration treating the next interval, the integer solutions are fixed according to the incumbent. The algorithm continues to loop until one  $f$  run through the whole time horizon is completed without improvement in the objective function value. With time limitation on each iteration and an upper bound on the number of loops, one can control the maximum time spent on the Improvement Phase.

A pseudo code for the Improvement Phase of a maximisation problem is given in Algorithm 7.2. Here  $P$  represents the total problem defined over  $\tau = \{0, \dots, T\}$  time periods with integer variables  $x$  and continuous variables  $y$ .  $S$  is the initial solution to  $P$  and  $m$  is the number of intervals for the Improvement Phase.

It should be pointed out that this Improvement Phase also can be used on solutions generated in other ways than one of the described fix-and relax variants. For instance, a manually found solution can be fed into the model to search for improvements.

## 7.4 Test Case Description and Setup

In order to evaluate the impact on computational time and objective function value, the fix-and-relax heuristic with the extensions described previously has been applied to an IRP for the liquefied natural gas business (IRP-LNG). This IRP and the setup for the experiments are described in the following.

### The LNG Problem

Natural gas is one of the fastest growing sources of energy in the world. By liquefying the natural gas into LNG, it can be transported over large distances by vessels. The capacity for production and transportation of LNG is expected to increase substantially in the years to come, while recent shifts in world economy and natural gas sources have given expectations for “a glut of gas (...) with

**Algorithm 7.2** ImprovementPhase( $P(x(\tau), y(\tau)), S, m$ )

---

```

1:  $obj := \text{GetObjVal}(S)$ 
2:  $prevObj := -\infty$ 
3:  $cnt := 0$ 
4:  $\text{FixToSolution}(x(\tau), S)$ 
5: while  $obj > prevObj$  and  $cnt < \text{IterationMax}$  do
6:   for  $v = 1, \dots, m$  do
7:      $t_1 := \{\frac{T}{m}(v-1), \dots, \frac{T}{m}v\}$ 
8:      $\text{RelaxFixing}(x(t_1))$ 
9:      $\text{SolveMIP}(P(x(\tau), y(\tau)))$ 
10:     $t_2 := \{\frac{T}{m}(v-1), \dots, \frac{T}{m}v\}$ 
11:     $\text{FixToLastSolution}(x(t_2))$ 
12:   end for
13:    $prevObj := obj$ 
14:    $obj := \text{GetObjVal}(\text{GetLastSolution}(P))$ 
15:    $cnt := cnt + 1$ 
16: end while
17: return  $\text{GetLastSolution}(P)$ 

```

---

far-reaching implications for gas pricing” (International Energy Agency 2009). This means it will become increasingly important to be able to operate larger fleets and more terminals in an efficient manner, and thereby creating the need for decision support tools. We have developed an optimisation based decision support tool in close cooperation with our industrial partners GDF SUEZ and Statoil, and this paper is based on testing with this tool.

The problem used for the numerical tests in this paper is described in Fodstad et al. (2010) and is an extension of the IRP model on LNG introduced by Grønhaug & Christiansen (2009). The core of the model is an IRP with profit maximization, single commodity, heterogeneous fleet, and multiple loading and unloading ports. In addition, the model has a rich description of contract requirements and pricing, and includes downstream natural gas markets. Further, liquefaction and regasification rates are decisions in the model since flexibility in these rates can be highly valuable in smoothing out the discreteness in the transportation system. A distinct characteristic of LNG is evaporation that causes losses in the system, which is also reflected in the model.

The LNG system we deal with has mainly three types of sales; sale of LNG, sale of natural gas (regasified LNG), and natural gas spot sale. (See Fodstad et al. 2010.) In the case of natural gas sales, LNG is regasified and stored in a tank before it is sold to the customers or spot markets. The customers typically have a daily or monthly demand within a certain range. Contract demand must

**Table 7.2:** Characteristics of test cases

Case ID	Vessel	Ports	Contracts	Spot markets	Contract limit tightness
1	2	7	1	6	Very loose
2	8	7	7	4	Tight
3	5	10	7	8	Very tight
4	2	8	2	6	Loose

be fulfilled given the importance of security of supply in the energy business and the high costs associated with shutting down production in the LNG-business.

In Grønhaug & Christiansen (2009) an arc-flow and a path-flow formulation of the LNG-IRP problem are tested, and in Grønhaug et al. (2008) the same instances are solved with a branch-and-price-and-cut approach. The use of the general MILP solver Xpress is shown to be equally good (Fodstad et al. 2010). To the best of our knowledge, these are the only attempts to solve this problem, so we have chosen to use the Xpress solver as the benchmark when evaluating computational time and objective function value of our fix-and-relax heuristics.

## Test Setup

Four test cases have been analyzed, each over a period of half a year (180 days). All the cases are realistic, inspired by problems from our industrial partners. Table 7.2 lists the most important parameters of the cases. The number of vessels and number of ports are crucial for the number of binary variables in the model, while the number of contracts and spot markets indicates the size of the continuous part of the model beyond a general IRP.

In addition, we vary the tightness of the restrictions regarding contractual obligations, where tightness measures the model's freedom in choosing when and where to deliver the gas. Very tight restrictions give a smaller solution space in the continuous dimensions and thereby possibly faster solution of each node in the branch-and-bound tree but at the expense of frequent cut-offs prior to finding the first feasible solution. On the contrary loose contract restrictions usually make it easier to find a first feasible solution, but hard to prove optimality because of many feasible combinations of the binary variables and a relatively flat objective function.

The model and heuristics have been implemented in Mosel and solved by Xpress Optimizer version 2008a ([www.fico.com](http://www.fico.com)). Computations are performed on a 3.0 GHz computer with 8Gb RAM and Rock Cluster version 5.1 operating system. Fix-and-relax parameters are varied with respect to the aspects described below for all four test cases and evaluated with respect to computational time and objective function value. The total number of test runs per case is 52, which

**Table 7.3:** Test setup for interval overlap and MIP gap acceptance

Number of intervals, $n$	Overlap size [%]	Initial MIP gap [%]
5,10	10,30,50	10,15,20

means 208 test runs in total.

For benchmark we have used the Xpress Optimizer without any decomposition tool. The solver is run for 100 hours, unless an optimality gap less than 1% is reached earlier.

### Original Fix-and-Relax

To analyse how to tune the fix-and-relax heuristic described in the literature for our IRP problem, all combinations of values for number of intervals, overlap, and initial MIP gap given in Table 7.3 are tested. Five intervals give 36 days per interval which corresponds approximately to a long roundtrip, while 10 intervals of 18 days correspond approximately to a short roundtrip. Our implementation uses a more flexible version of the overlap described by Stadtler (2003) and Absi & Kedad-Sidhoum (2007), where the size of overlap is defined as a percentage of the previous interval. The overlap can therefore include any number of days, and not only entire fix-and-relax intervals. Initial tests indicated that overlap less than 10% and above 50% were not suitable because of infeasibility or long computation times. We use the MIP gap as stopping criteria, and since the problem tends to solve faster in later iterations we reduce the MIP gap limit as iterations goes by. The MIP gap limit of the first iteration is given in Table 7.3, while we have used a 1% gap as limit for the last iteration in all tests. For all intervals in-between, the MIP gap requirement is linearised between these two values. In addition, a time limit of 3600 seconds for each interval is used. The number of test runs for the original fix-and-relax then sums up to 18.

### Modelling in the End Block

The two alternative ways of simplifying the model in the End Block, Horizon Cut and Light Model, are analysed with different lengths of the End Block. An overlap of 10% and an initial MIP gap of 20% are used in these tests, based on the experiences from the tests described above. We have one interval in the Integer Block and at least one interval in the Continuous Block, which in the first iteration leaves at most  $n - 2$  intervals in the End Block. The number of test runs for End Block modelling then sums up to 22. Table 7.4 lists all the different End Block lengths tested.

**Table 7.4:** Test setup for the End Block model

Number of intervals, $n$	End Block model	End Block length
5	Horizon Cut, Light Model	1,2,3
10	Horizon Cut, Light Model	1,2,3,4,5,6,7,8

**Table 7.5:** Test setup for the Improvement Phase

Number of intervals, $n$	Number of Improvement Phase intervals, $m$
5	2,3,4
10	2,3,4,5,6,7,8,9

### Improvement Phase

The Improvement Phase is tested with solutions from both 5 and 10 intervals in the original fix-and-relax runs. Similar to the test of End Block modelling, an overlap of 10% and an initial MIP gap of 20% are used in these tests. We assume the model to be easier to solve in the Improvement Phase and therefore use a smaller number of intervals in the Improvement Phase than in the corresponding original fix-and-relax run. The number of test runs for modelling of the Improvement Phase sums up to 11. All different interval lengths tested are listed in Table 7.5.

The aim of the Improvement Phase is to improve the objective function value, without too much additional computation time. Hence, a time limit of 600 seconds for one whole loop that is divided equally at all intervals,  $m$ , is considered sufficient. The results support this since it turns out that the time limit in most cases is not the critical stop criterion. The MIP gap requirement for all iterations is set to 0.1%.

## 7.5 Computational Results and Discussion

In the following, the impacts of the original fix-and-relax heuristic on computational time and objective function value compared to the benchmark are described. These runs also form a starting point for further analysis of the extensions that we have developed to this heuristic, namely End Block modelling and the Improvement Phase, which are emphasized subsequently.

Based on the test results we give some guidelines on how to use the heuristics that have been presented. These seek a trade-off between solution time and quality which typically will be chosen in an interactive analysis and planning situation.

**Table 7.6:** Summary of the results for all test cases

		Time [sec]	Objective function value	Normalised objective func. value	
Case 1	Benchmark	First solution	3 807	1 261	0.82
		Last solution	36 431	1 450	0.94
	Fix-and-relax	Shortest comp. time	89	1 397	0.91
		Largest objective function value	89	1 397	0.91
		Average values	205	1 364	0.89
	End Block modelling	Shortest comp. time	15	1 233	0.80
		Largest objective function value	48	1 418	0.92
		Average values	56	1 360	0.88
	Improvement Phase	Shortest comp. time	93	1 397	0.91
		Largest objective function value	140	1 440	0.93
		Average values	329	1 405	0.91
	Upper bound	Initial	11	1 632	1.06
		End	360 000	1 541	1.00
	Case 2	Benchmark	First solution	929	4 758
Last solution			305 407	5 195	0.93
Fix-and-relax		Shortest comp. time	1 060	4 402	0.79
		Largest objective function value	3 140	4 842	0.86
		Average values	1 432	4 655	0.83
End Block modelling		Shortest comp. time	115	4 562	0.81
		Largest objective function value	203	4 764	0.85
		Average values	467	4 511	0.80
Improvement Phase		Shortest comp. time	1 108	4 805	0.86
		Largest objective function value	4 775	5 119	0.91
		Average values	2 033	4 923	0.88
Upper bound		Initial	154	5 743	1.03
		End	360 000	5 604	1.00

Table 7.6: continued

		Time [sec]	Objective function value	Normalised objective func. value	
Case 3	Benchmark	First solution	72	7 312	0.99
		Last solution	76	7 312	0.99
	Fix-and-relax	Shortest comp. time	176	7 182	0.97
		Largest objective function value	287	7 267	0.98
		Average values	244	7 211	0.98
	End Block modelling	Shortest comp. time	83	7 277	0.99
		Largest objective function value	187	7 279	0.99
		Average values	174	7 234	0.98
	Improvement Phase	Shortest comp. time	189	7 273	0.98
		Largest objective function value	212	7 312	0.99
		Average values	279	7 300	0.99
	Upper bound	Initial	40	7 759	1.05
End		6 002	7 385	1.00	
Case 4	Benchmark	First solution	8 995	3 019	0.84
		Last solution	301 125	3 257	0.90
	Fix-and-relax	Shortest comp. time	355	3 145	0.87
		Largest objective function value	4 142	3 248	0.89
		Average values	2 161	3 192	0.88
	End Block modelling	Shortest comp. time	58	3 154	0.87
		Largest objective function value	865	3 231	0.89
		Average values	482	3 129	0.87
	Improvement Phase	Shortest comp. time	363	3 154	0.87
		Largest objective function value	2 373	3 248	0.90
		Average values	1 255	3 192	0.88
	Upper bound	Initial	113	3 659	1.01
End		360 000	3 611	1.00	



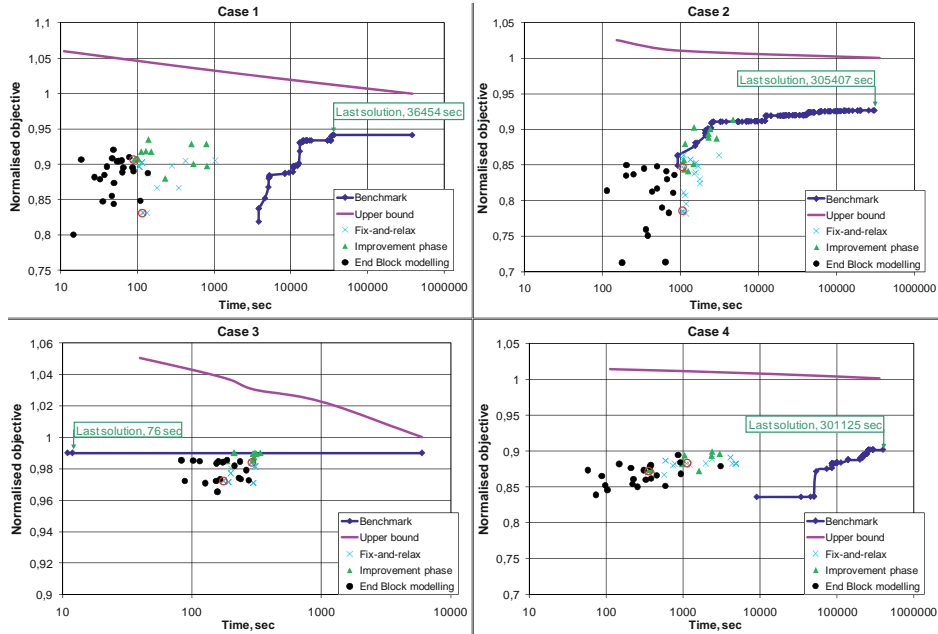


Figure 7.5: Results for all test cases

Figure 7.5 presents normalised objective function values as a function of time for all results from the benchmark, all original fix-and-relax, End Block modelling and use of Improvement Phase. Selected fix-and-relax solutions (10% overlap and 20% initial MIP gap) are framed with circles. Normalisation is done relative to the final upper bounds given by Xpress during the benchmark runs. Also the upper bound as a function of time from the branch-and-bound process of Xpress is plotted. The computational times reported for the Improvement Phase include the times used by the preceding fix-and-relax process. A summary of these results can be found in Table 7.6. In the table, the runs are grouped by case and whether original fix-and-relax, End Block modelling or Improvement Phase have been used. For each of these three modelling methods, the test run with the shortest computational time and corresponding objective function value, the test run with the highest objective function value and corresponding computational time, and the average computational time and average objective function value over all test runs are given. All combinations of overlap and initial MIP gap sizes are included in the original fix-and-relax results. Table 7.7 gives an overview of the model dimensions.

**Table 7.7:** Model dimensions

Case	Number of variables	Number of binary variables	Number of restrictions	Number of nonzero in the matrix
1	18 127	4 372	20 648	76 600
2	45 182	10 002	47 065	267 354
3	64 721	24 210	55 443	314 295
4	30 799	8 720	29 360	141 073

## Results from Original Fix-and-Relax

The crosses in Figure 7.5 represent the results from the original fix-and-relax tests. For cases 1 and 4 the solutions by original fix-and-relax are found 19 and 4 times faster on average, respectively, than the first solution from the benchmark. For case 2, these two solutions are found at approximately equal time. In case 3, which is a very tight case, the first, and best, solution is obtained almost immediately in the benchmark. The best bound decreases steadily, so this solution proves to be optimal relatively fast. Hence, for case 3, it does not seem necessary to use any fix-and-relax-heuristic. The average objective function value obtained by using the original fix-and-relax-heuristic is between 1 and 10 percentage points lower than the best objective function value from the benchmark for all cases. Finally, all heuristic solutions are found within 5 000 seconds.

Further, the results indicate no clear relation between the number of fix-and-relax intervals and computational time or objective function value. By increasing the number of intervals, the objective function value and computational time increase for some cases, while they decrease for others. When increasing the overlap, the computational time increases by 50% and 100% on average for an overlap size of 30% and 50%, respectively. The objective function value, however, increases in some cases and decreases in others although these variations are relatively small, mostly within 2%. Hence, no clear relation between size of the overlap and objective function value is evident. The results show that by increasing the initial MIP gap from 10% to 15%, the computational time decreases by 10-20%, and the time decreases by up to 35% for a MIP gap of 20% compared to 10%. However, there are only minor and unstructured changes in the objective function value, mostly within 1%, when the initial MIP gap is changed. These results show that a low overlap and a high initial MIP gap are the best choice in order to achieve reduced computing time and a good objective function value.

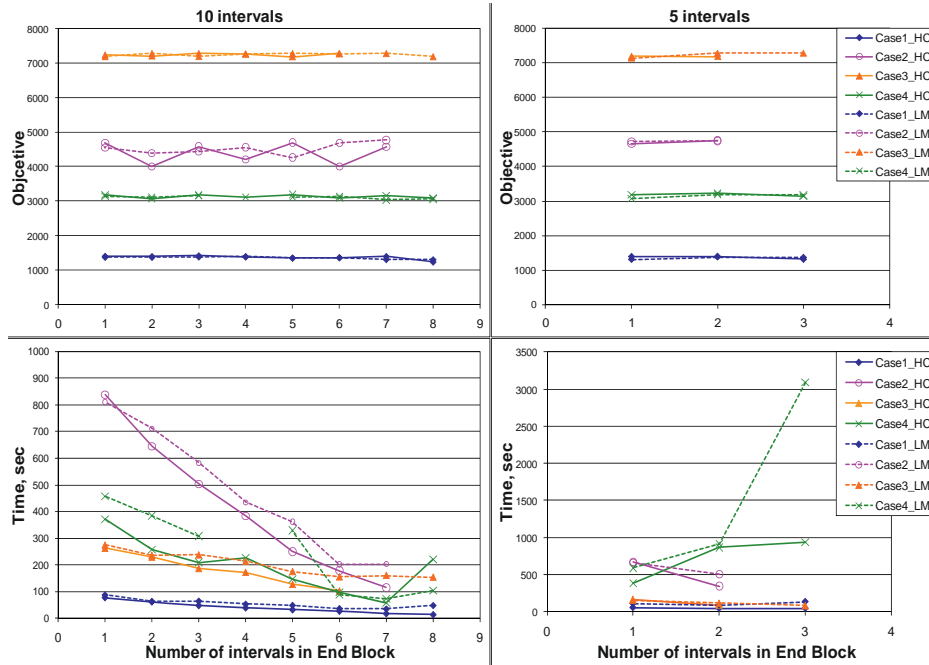
**Table 7.8:** Average normalised objective function values and computing times for Horizon Cut and Light Model runs. Infeasible runs excluded in calculations

Case	Method	Normalised objective	Time [sec]
1	Horizon Cut	0.89	43
1	Light Model	0.88	70
2	Horizon Cut	0.80	437
2	Light Model	0.81	497
3	Horizon Cut	0.98	167
3	Light Model	0.98	195
4	Horizon Cut	0.87	354
4	Light Model	0.86	634

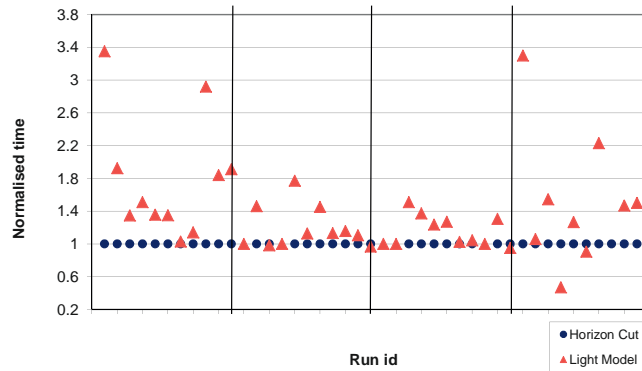
## Effects of End Block Modelling

The filled circles in Figure 7.5 represent the results from analyses with End Block modelling. Simplifying this part reduces computing time by 45, 56, 26, and 35% on average for cases 1 to 4 respectively, compared to the respective original fix-and-relax solutions. The first solution in the simplified models is obtained six to nine times faster than for the original fix-and-relax heuristic for cases 1, 2, and 4, and twice as fast for case 3. The average objective function values for each case deviate from the original fix-and-relax solutions by  $-2-2\%$  when applying End Block modelling. Figure 7.6 shows how the length of the End Block affects the objective function value and computing time. Horizon Cut (HC) is illustrated with a solid line, whereas Light Model (LM) is illustrated with a dotted line. The trend is that a longer End Block and short Continuous Block, in most cases  $f$  equal to two or three intervals, result in the lowest computing time. Using only one interval in the Continuous Block, however, frequently leads to infeasibility. The objective function values are relatively unaffected by varying the End Block length.

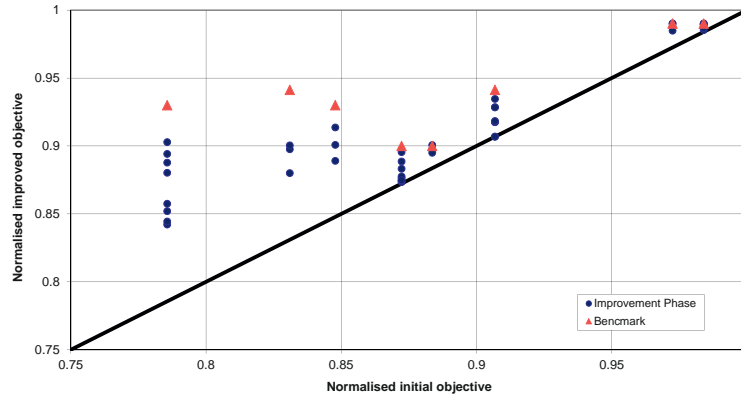
Furthermore, Figure 7.7 compares the solution times with Horizon Cut and Light Model, both normalised relative to the values of Horizon Cut. The figure shows a shorter computing time for Horizon Cut in most runs. On the contrary, none of the methods prove favourable over the other on objective function value. These observations are confirmed by the average normalised objective function values and solution times given in Table 7.8.



**Figure 7.6:** Effects of Horizon Cut and Light Model in End Block. Lacking values represent infeasible model runs



**Figure 7.7:** Comparison of Horizon Cut vs Light Model on solution time normalised relative to the values of Horizon Cut. Lacking values represent infeasible model runs

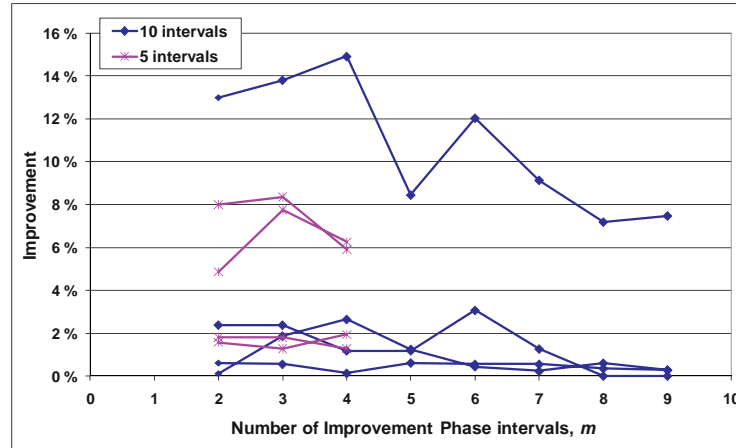


**Figure 7.8:** Improvement in Improvement Phase relative to upper bound and benchmark

### Effects of the Improvement Phase

The triangles in Figure 7.5 represent the results from the tests with the Improvement Phase. These lie above the results from the original fix-and-relax runs, indicating improved solutions. The computing time with the Improvement Phase is limited to maximum 600 seconds per loop through the whole model horizon, and is therefore relatively low. Figure 7.8 plots the normalised objective function values after the Improvement Phase (y-axis) against the normalised objective function values prior to the Improvement Phase. Triangles indicate the normalised objective function values achieved by the benchmark. The figure shows that the biggest improvements are achieved where the potential relative to both the upper bound and the benchmark is the biggest. The average objective function values for all cases by using the Improvement Phase are increased by approximately 5% for case 1, 8% for case 2, 1% for case 3 and 1% for case 4 compared to their respective initial solutions. However, average computing time is also increased when applying Improvement Phase, by 221, 92, 19, and 70% for cases 1 to 4 respectively.

Figure 7.9 shows the improvement in objective function value by applying the Improvement Phase, as a function of number of intervals of this phase,  $m$ . The results indicate that increasing the number of Improvement Phase intervals has a relatively small effect on the objective function value. However, in some cases an increase in the number of intervals results in a decrease in the objective function value. The computing time is unaffected by varying the number of intervals.



**Figure 7.9:** Effects on objective function value of including the Improvement Phase

## 7.6 Conclusion

We have applied fix-and-relax time decomposition heuristics to solve a maritime IRP problem and this is a new solution approach for this problem class. The heuristic was extended with an End Block that simplifies the end of the model horizon to speed up the solution process. Further, an Improvement Phase was added to improve the objective function value after an initial feasible solution is found. This Improvement Phase could also be combined with other construction heuristics or manual planning. The heuristic with its extensions were tested on cases from the LNG business, and computational times and objective function values were reported. The general MILP solver Xpress Optimizer was used as benchmark. The results from the tests are summarised in Table 7.9. The table shows the average values over all cases. The objective function values are normalised relative to the best bound found by the benchmark.

Applying original fix-and-relax heuristics to the LNG-IRP model reduces computational time, often considerably, compared to the use of Xpress without decomposition. On average, fix-and-relax reduced the computing time by 71% compared to the time of the first benchmark solution, and is 160 times faster compared to the time of the best benchmark solution. The objective function values are on average 2% higher than the first benchmark solution, but are decreased compared to the best benchmark solution by 5% on average. Our results show that a low overlap and a high initial MIP gap for the fix-and-relax heuris-

**Table 7.9:** Average normalised objective function values and computing times over all cases

	Normalised objective	Time [sec]
First benchmark solution	0.87	3 451
Last benchmark solution	0.94	160 760
All fix-and-relax	0.89	1 011
Selected fix-and-relax <sup>1</sup>	0.89	534
End Block modelling	0.88	288
Improvement Phase	0.92	974

tics are the best choices in order to achieve reduced computing time and a good objective function value.

By including either an End Block model or an Improvement Phase in the fix-and-relax method, the results can be improved further. An Improvement Phase is applied in order to improve the objective function value after a feasible solution is found, while an End Block model aims at reducing computing time compared to the original fix-and-relax runs.

Applying an End Block model reduces computing time by 46% on average over all cases compared to the respective selected fix-and-relax solutions with small impact on objective function value, as shown in Table 7.9. Therefore, using this extension is recommended. Horizon Cut gives smaller computing time than Light Model and is therefore recommended. Furthermore, using two or three intervals in the Continuous Block is recommended, since this results in the lowest computing time for most cases.

Applying the Improvement Phase improves objective function values on average by 3% compared to the respective original fix-and-relax results, with an increase in computing time of 82% on average. However, as shown in Table 7.9, the computing time is still small compared to the benchmark. Generally, a low number of Improvement Phase intervals seems to be most suitable.

These results confirm that both the developed extensions, End Block modelling and Improvement Phase, help to achieve the intended goals, reduced computing time and improved objective function values, respectively.

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## Paper VII

Yohan Shim, Marte Fodstad, Steven A. Gabriel and Asgeir  
Tomasgard:

# *A Branch-and-Bound Method for Discretely-Constrained Mathematical Programs with Equilibrium Constraints*

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## Chapter 8

# A Branch-and-Bound Method for Discretely-Constrained Mathematical Programs with Equilibrium Constraints

### Abstract:

We present a branch-and-bound algorithm for discretely-constrained mathematical programs with equilibrium constraints (DC-MPEC). This is a class of bilevel programs with an integer program in the upper-level and a complementarity problem in the lower-level. The algorithm builds on the work by Gabriel, Shim, Conejo, de la Torre & Garcia-Bertrand (2010) and uses Benders decomposition to form a master problem and a subproblem. The new dynamic partition scheme that we present ensures that the algorithm converges to the global optimum. Partitioning is done to overcome the non-convexity of the Benders subproblem. In addition Lagrangean relaxation provides bounds that enable fathoming in the branching tree and warm-starting the Benders algorithm. Numerical tests show significantly reduced solution times compared to the original algorithm. When the lower level problem is stochastic our algorithm can easily be further decomposed using scenario decomposition. This is demonstrated on a realistic case.

## 8.1 Introduction

In this paper, we focus on bilevel programming problems where the upper-level deals with discrete decisions and the lower-level is a mixed complementarity problem (MCP). It is a variant of the traditional mathematical program with equilibrium constraints (MPEC) where the leader is only allowed to make discrete decisions. We call the whole formulation DC-MPEC (Gabriel et al. 2010).

Gabriel et al. (2010) propose a heuristic to solve the DC-MPEC problem based on Benders decomposition. They rephrase the problem as a mixed integer linear problem (MILP) and decompose the problem by placing all constraints and objective elements containing lower-level variables in the Benders sub problem. The master problem domain is a priori heuristically partitioned into subdomains of  $x$  with the aim of finding subdomains where the lower level objective is convex. Afterwards each subdomain is solved by Benders decomposition method. It is shown that the heuristic can give a sub-optimal solution unless all subdomains

are convex.

In our new approach we develop an idea mentioned in Gabriel et al. (2010) with a dynamic branching procedure that partitions the subdomains as the algorithm proceeds. We branch until the subdomains which have candidates for the global solution are convex and thereby guarantee to find the optimal solution. As opposed to the branching on single variables as employed in many branch-and-bound approaches, we use intersection of Benders cuts to partition the upper-level decision domain. The branching procedure is supported by lower bounding based on Lagrangean relaxation, which makes it possible to cut off parts of the master problem domain and thereby increase the efficiency of the algorithm compared to the static version. Using LR to accelerate the branch-and-bound procedure was introduced in both Geoffrion (1974) for MILPs and Falk (1969) for non-convex programs.

Several papers contribute to improving the convergence properties of Benders decomposition, for a nice review see Saharidis & Ierapetritou (2010). One common approach is to add cuts to the relaxed master problem, as for instance Saharidis, Boile & Theofanis (2011) do. Similarly we utilize the solution value from the Lagrangean relaxation as a bound in the Benders decomposition somewhat inspired by cross decomposition (van Roy 1983, 1986). To the best of our knowledge this is a new way of utilizing Lagrangean relaxation results in bilevel programming: using it both in the lower bounding in the branch-and-bound procedure and to accelerate the Benders decomposition used to find the upper bound.

We also show how the addition of strong duality constraints, enabled by the Benders decomposition, increases the robustness of the transformation from DC-MPEC to MILP. When the lower-level is a two-stage stochastic MCP, we show how the lower bounding method can be adapted using scenario decomposition (Carøe & Schultz 1999) to achieve further decomposition, and test this on a natural gas application.

Our computational results show that using the dynamic partitioning algorithm supported by the strong duality constraints considerably reduces the partitioning work needed compared to the static version of the algorithm.

Even the continuous linear bilevel programming problem has been shown to be NP-hard (Hansen, Jaumard & Savard 1992) and the discrete nature of upper-level variables and their related constraints would make the DC-MPEC problem even more intractable. A substantial number of contributions exist for the different problem classes within bilevel programming, and we will point out the ones closest related to our DC-MPEC problem. For a broader overview see for instance Dempe (2002) or Colson, Marcotte & Savard (2007) on bilevel programming and Luo, Pang & Ralph (1996), Outrata, Kocvara & Zowe (1998) or Fukushima & Lin (2004) on MPEC.



Gabriel & Leuthold (2010) formulate a Stackelberg game within the electric power market as a DC-MPEC and provide exact solutions with standard branch-and-bound after reformulating to a MILP. On the contrary most solution procedures for DC-MPEC are heuristics. Meng, Huang & Cheu (2009) and Meng & Wang (2011) use genetic algorithms supplemented with suitable procedures for solving lower level parametric VIs for facility location and service network design applications, respectively. Another DC-MPEC application is presented by Wang & Lo (2008) who transforms their problem into a mixed integer nonlinear program solved with an application specific heuristic.

Mesbah, Sarvi, Ouveysi & Currie (2011) use generalized Benders decomposition to solve a bilevel problem for transportation network design. Their upper level has binary variables and a non-linear objective function. The lower level consists of three parts, two optimization problems and an equilibrium problem. Lagrangean relaxation is used to solve the optimization lower-level problems.

Saharidis & Ierapetritou (2009) propose Benders decomposition for problems closely related to our DC-MPEC, but with lower level limited to LPs. They decompose the problem into a master problem containing all integer variables and pure integer constraints and a bilevel subproblem. The subproblem is transformed into a single level problem by using the KKT conditions and provides a feasibility cut or optimality cut for the master problem in each iteration. The integrality conditions are handled by adding integer exclusion cuts to the master problem. This work differs from ours in different ways of decomposing the problem and different ways of treating the integrality requirements.

Wen & Yang (1990) also solve a DC-MPEC with the lower level limited to LPs. They do not use the common reformulation to MILP based on KKT conditions, but develop valid bounds adapted to the bilevel structure and apply these in branch-and-bound. A similar strategy is used by Moore & Bard (1990) for bilevel problem with integrality constraints in both upper and lower level, and they also point out why standard bounding and fathoming rules for branch-and-bound in integer programming do not apply for their problem.

The rest of this paper is organized as follows: First we present the basic ideas of the algorithm, with lower bounding, upper bounding and dynamic partitioning. Then follows a pseudocode overview of the total algorithm and proofs for the validity of bounds and overall convergence. The section ends with a description on how to adapt the lower bounding method to stochastic programs. Next follows numerical results on general problems with randomly generated data and on a natural gas supply chain problem before we conclude.

## 8.2 Dynamic Algorithm

The overall discretely-constrained mathematical program with equilibrium constraints (DC-MPEC) is given as follows:

$$\begin{aligned}
 & \min_{x,y} c^\top x + d^\top y \\
 & \text{s.t. } Qx \leq q \\
 & \quad Ax + By \geq a \\
 & \quad y \in S(x)
 \end{aligned} \tag{8.1}$$

where  $x \in Z^{n_x}$  and  $y \in R^{n_y}$  are integer upper-level variables and continuous lower-level variables, respectively. The constraints  $Qx \leq q$  contain the bounds on the  $x$ 's and other linear constraints with only  $x$  variables;  $Ax + By \geq a$  are the joint linear constraints upon  $x$  and  $y$ . The solution set of the lower-level MCP is given by

$$S(x) = \left\{ (y, z, w) \left| \begin{array}{l} 0 \leq y \perp Ey + e - M^\top z - D^\top w \geq 0 \\ 0 \leq z \perp My + Nx - k \geq 0 \\ Dy + Fx = g \end{array} \right. \right\} \tag{8.2}$$

where  $z \in R^{n_z}$ ,  $z \geq 0$  and  $w \in R^{n_w}$ .  $z$  and  $w$  are lower-level variables that typically correspond to dual variables of a underlying optimization problem. It is assumed that the dimension of each data element (i.e.  $c, d, Q, q, A, B, a, E, e, M, N, k, D, F, g$ ) agrees with its associated variables.  $E$  is a symmetric and positive semi-definite matrix of the convex quadratic function  $\frac{1}{2}y^\top Ey + e^\top y$  so that the KKT conditions are necessary and sufficient optimality conditions. Note that the lower-level problem also covers linear problems (LP) and quadratic convex problems (QP) since the KKT optimality conditions of these problem classes are MCP problems. We assume that a solution to Problem (8.1) exists.

As previously shown by amongst others Fortuny-Amat & McCarl (1981) a MPEC can be rephrased to a MILP through replacing the lower-level complementarity conditions by disjunctive constraints, binary variables and a large constant  $C$ . This reformulation implies an optimistic view on the lower-level in the sense that if multiple equilibria exist in lower-level the most favorable according to the upper-level objective is chosen. We denote the problem resulting from this reformulation (MILP).

We decompose (MILP) with Benders decomposition into a master problem:

$$\begin{aligned}
& \text{(B - MILP)} \\
& \min_x z_{\text{B-MILP}} = c^\top x + \alpha(x) \\
& \text{s.t. } Qx \leq q \\
& \quad \check{Q}x \leq \check{q}
\end{aligned} \tag{8.3}$$

and a subproblem:

$$\begin{aligned}
\alpha(x) &= \min_{y,z,w,\bar{b},\tilde{b}} d^\top y \\
& \text{s.t. } a \leq Ax + By \\
& \quad 0 \leq y \leq C(1 - \bar{b}) \\
& \quad 0 \leq Ey + e - M^\top z - D^\top w \leq C\bar{b} \\
& \quad 0 \leq z \leq C(1 - \tilde{b}) \\
& \quad 0 \leq My + (Nx - k) \leq C\tilde{b} \\
& \quad Dy + Fx = g \\
& \quad \bar{b}, \tilde{b} : \text{binary} \\
& \quad y, z \geq 0
\end{aligned} \tag{8.4}$$

Only  $x$  variables are placed in the master problem and the other variables are in the subproblem. Because of the disjunctive variables  $\bar{b}$  and  $\tilde{b}$  the function  $\alpha(x)$  is piecewise linear but not in general convex in  $x$  (Gabriel et al. 2010). The non-convexity means Benders decomposition method is not guaranteed to converge to an optimal solution for (MILP) (Benders 1962). The main idea of the dynamic partitioning algorithm is to partition the domain of  $X = \{x \in Z^{n_x} | Qx \leq q\}$  into subdomains where  $\alpha(x)$  is convex, as illustrated in Figure 8.1. The partitioning is controlled by upper and lower bounds that will converge as the non-convexities are removed in exchange for an increasing number of subdomains.  $\check{Q}x \leq \check{q}$  is a set of linear partitioning constraints defining a subdomain. An overview of the problems used in this paper and their relations is given in Figure 8.2.

## Lower Bounding

Traditionally the LP relaxation is used for bounding in branch-and-bound, but the MILP reformulation with binary variables and large constants gives weak LP bounds. Instead we apply the Lagrangean relaxation algorithm (LR) as lower bound of (MILP) relaxing  $Qx \leq q$  with  $\mu$  as the Lagrangean multiplier. The mathematical formulation of the Lagrangean subproblem (MILP( $\mu$ )) is given as

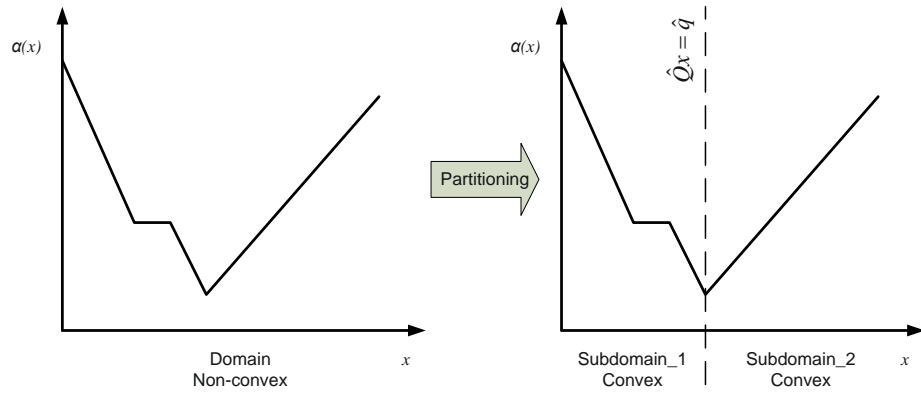


Figure 8.1: Illustration of how partitioning transforms a domain with a non-convex function into two subdomains with convex functions

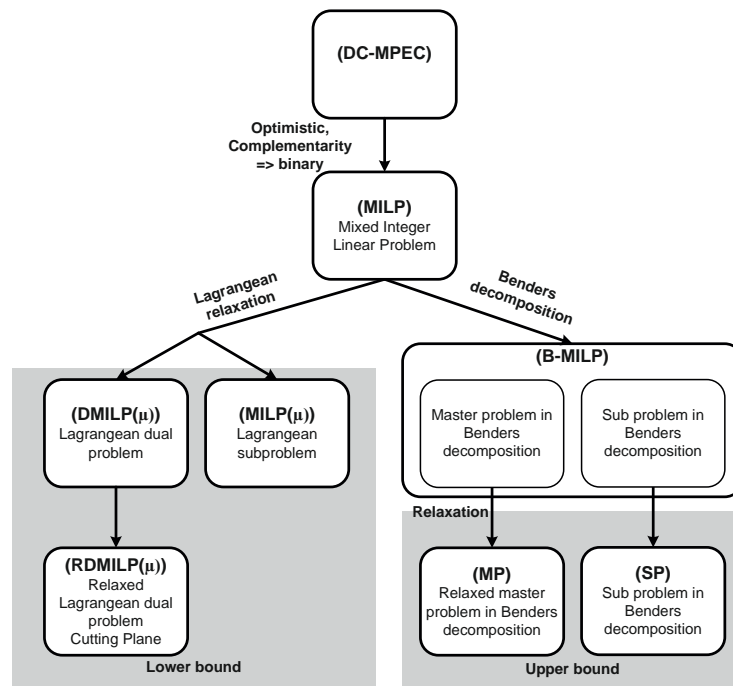


Figure 8.2: Problem and subproblem overview

Problem (8.5) and its dual problem is defined as  $z_{LD} = \max_{\mu} \{\phi(\mu) | \mu \geq 0\}$ . (MILP( $\mu$ )) is a relaxation of (MILP) as proved in Geoffrion (1974).

$$\begin{aligned}
 & \text{(MILP}(\mu)\text{)} \\
 \phi(\mu) = & \min_{x,y,z,w,\bar{b},\tilde{b}} z_{\text{MILP}(\mu)} = c^\top x + d^\top y + \mu^\top (Qx - q) \\
 & \text{s.t. } \check{Q}x \leq \check{q} \\
 & a \leq Ax + By \\
 & 0 \leq y \leq C(1 - \bar{b}) \\
 & 0 \leq Ey + e - M^\top z - D^\top w \leq C\bar{b} \\
 & 0 \leq z \leq C(1 - \tilde{b}) \\
 & 0 \leq My + (Nx - k) \leq C\tilde{b} \\
 & Dy + Fx = g \\
 & x : \text{integer} \\
 & \bar{b}, \tilde{b} : \text{binary} \\
 & y, z \geq 0
 \end{aligned} \tag{8.5}$$

MILP( $\mu$ ) is a mixed integer linear program that usually will be solved repeatedly to make the Lagrangean process converge. This means (MILP( $\mu$ )) needs to be significantly simpler than (MILP) to solve to make the lower bounding worthwhile. Generally that means there should be a substantial number of constraints  $Qx \leq q$  or these constraints should have a complicating structure (see Conejo, Castillo, Minguez & García-Bertrand 2006) as in the example of stochastic programming in Section *Application to a Natural Gas Supply Chain* on page 276.

In our implementation of LR, the Lagrangean multipliers ( $\mu$ ) are updated by a cutting plane method (Conejo et al. 2006). The Lagrangean iterations are stopped whenever the gap between the cutting plane problem (relaxed Lagrangean dual problem) and the Lagrangean subproblem (MILP( $\mu$ )) are sufficiently small. Also a limit on the number of iterations is implemented to avoid any cycling. A duality gap can occur because the Lagrangean subproblem has integral variables, as shown in Geoffrion (1974). This represents a non-convexity domain for  $\alpha(x)$  that will cause further partitioning.

## Upper Bounding

We apply Benders decomposition method (BD) as described in Conejo et al. (2006) to measure the upper bound (UB) of (MILP). The function  $\alpha(x)$  in the

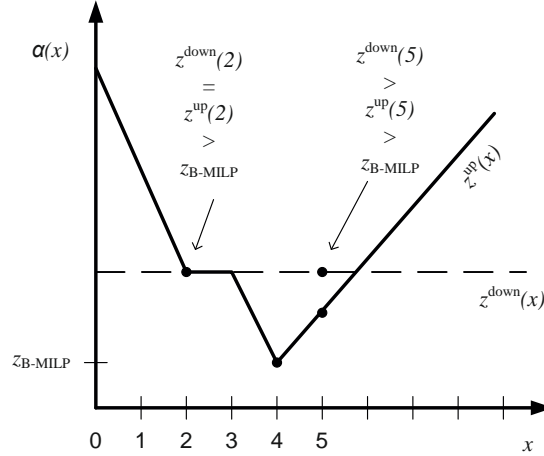
master problem (8.3) is relaxed, and through the solution procedure rebuilt by iteratively solving the relaxed master problem (MP) and sub problem (SP) below.

$$\begin{aligned}
 & \text{(MP)} \\
 & \min_x c^\top x + \alpha \\
 & \text{s.t. } Qx \leq q \\
 & \quad \check{Q}x \leq \check{q} \\
 & \quad \alpha \geq \alpha(x^{(k)}) + \lambda^\top (x - x^{(k)}) \\
 & \quad \alpha \geq \alpha^{\text{down}} \\
 & \quad k = 1, \dots, v - 1
 \end{aligned} \tag{8.6}$$

$$\begin{aligned}
 & \text{(SP)} \\
 & \alpha(x) = \min_{y, z, w, \bar{b}, \tilde{b}} d^\top y \\
 & \text{s.t. } a \leq Ax + By \\
 & \quad 0 \leq y \leq C(1 - \bar{b}) \\
 & \quad 0 \leq Ey + e - M^\top z - D^\top w \leq C\bar{b} \\
 & \quad 0 \leq z \leq C(1 - \tilde{b}) \\
 & \quad 0 \leq My + (Nx - k) \leq C\tilde{b} \\
 & \quad Dy + Fx = g \\
 & \quad \bar{b}, \tilde{b} : \text{binary} \\
 & \quad y, z \geq 0 \\
 & \quad x^{(v)} : (\lambda : \text{free})
 \end{aligned} \tag{8.7}$$

In each Benders iteration,  $k$ , the solution of (SP) for a given  $x^{(k)}$  gives a new Benders cut  $\alpha \geq \alpha(x^{(k)}) + \lambda^\top (x - x^{(k)})$  that is added to (MP) to approximate  $\alpha(x)$ . Let  $z^{\text{down}}(x^{(v)}) = c^\top x^{(v)} + \alpha(x^{(v)})$  and  $z^{\text{up}}(x^{(v)}) = c^\top x^{(v)} + d^\top y^{(v)}$ . Figure 8.3 illustrates a non-convex  $\alpha(x)$  function and a master problem approximation based on a single Benders cut (broken line).

- If  $\alpha(x)$  is convex for a given subdomain, Benders cuts create a lower envelope of  $\alpha(x)$ . In each iteration  $v$ , (MP) and (SP) provide lower and upper bound on  $z_{\text{B-MILP}}$ , respectively,  $z^{\text{down}}(x^{(v)}) \leq z_{\text{B-MILP}} \leq z^{\text{up}}(x^{(v)})$  and these bounds iteratively converge (Conejo et al. 2006).
- If  $\alpha(x)$  is non-convex for a given subdomain, the Benders cuts may overestimate  $\alpha(x)$  and eliminate a true optimum in the subdomain. This implies



**Figure 8.3:** Illustration of non-convex  $\alpha(x)$  and a single Benders cut (broken line)

that either  $z^{\text{down}}(x^{(v)})$  and  $z^{\text{up}}(x^{(v)})$  converge to a value greater than the true optimum or  $z^{\text{down}}(x^{(v)}) > z^{\text{up}}(x^{(v)})$  which stops the iterations. These situations are illustrated in Figure 8.3 for  $x = 2$  and  $x = 5$ , respectively.

In either case  $z^{\text{up}}(x^{(v)})$  gives a valid upper bound, which is justified by the proof in the Appendix. Because of assumption (A2) that follows on page 264, we do not consider feasibility cuts.

### Accelerating the Benders Decomposition by Including the Lower Bound

We could utilize the solution from  $(\text{MILP}(\mu))$  to warm-start the Benders iterations seeking to reduce the number of Benders iterations. The solution of  $x$  from  $(\text{MILP}(\mu))$  are set as the initial solution of  $(\text{MP})$  and the objective value of  $(\text{MILP}(\mu))$  forms a lower bound for  $(\text{MP})$ , expressed by the optimality cut  $c^\top x + \alpha \geq LB$ . The optimality cut removes solutions inferior to the incumbent in the current iteration of the Benders decomposition, thereby reducing the search space and potentially producing faster convergence of the algorithm. Table 8.1 compares the number of iterations and computation time of BD for two cases: without and with the warm-start (ws). Ten test problems were solved. All data were generated from the intervals  $[0, 100]$  with uniform distributions. The upper-level decision variables were all binary; the lower-level problems are built by deriving the KKT conditions of LP problems. Matlab (ver. 7.0) and Xpress-MP (ver. 2006) were used to implement both BD and LR algorithms. LB and

**Table 8.1:** Numerical results: Speeding up UB measure by warm-starting with LB solution. The left part gives the dimensions of (MP) and (SP), where  $\dim(b) = \dim(\bar{b}) + \dim(\tilde{b})$ .  $C$  is the value of the disjunctive constant. The right part gives the number of iterations and solution time in seconds for computing UB without and with warm-start (ws).

MP $x$	SP			$C$	UB without ws		UB with ws	
	$y$	$z$	$b$		Iter	Time	Iter	Time
20	100	100	200	1E5	14	18	6	7
20	100	100	200	1E5	14	17	6	7
20	100	100	200	1E5	14	18	5	6
20	100	100	200	1E5	10	12	4	5
20	100	100	200	1E5	10	12	4	5
20	1 000	1 000	2 000	1E6	130	938	74	470
20	1 000	1 000	2 000	1E6	77	508	46	307
20	1 000	1 000	2 000	1E6	14	87	3	18
20	1 000	1 000	2 000	1E6	23	144	3	19
20	1 000	1 000	2 000	1E6	118	854	37	251

UB converged to the same value for all instances. As can be seen from the last two columns, the warm-start greatly reduced the number of iterations as well as the computation time.

### Strong Duality Constraint

The complementarity constraints in the lower-level problem were linearized with disjunctive binary variables and big constant  $C$  in Gabriel et al. (2010), Gabriel & Leuthold (2010), Labbé, Marcotte & Savard (1998), Hu, Mitchell, Pang, Bennett & Kunapuli (2008), Mitsos (2010), Saharidis & Ierapetritou (2009) and DeNegre & Ralphs (2009), where their common question was the value of  $C$  for which the feasible region formed by complementarity constraints is not altered. Hu et al. (2008) proposed a solution method which does not require knowing the big constants, but the method is limited to linear programs with linear complementarity constraints (LPCC). Gabriel et al. (2010) shows that  $C$  can be chosen by a sensitivity analysis or when the matrix  $M$  has a specific property the constants can be chosen analytically.

For lower-level problems with  $E = 0$  (defined in Problem (8.2)), which for instance correspond to the KKT conditions of an LP, we impose the strong duality constraint  $(e^\top - w^\top D)y = z^\top(k - Nx)$  to (B-MILP). The constraint was induced from the two complementarity constraints  $y^\top(e - M^\top z - D^\top w) = 0$  and



$z^\top(My + Nx - k) = 0$ . A similar strong duality constraint cannot be imposed when  $E \neq 0$  since that would give a non-linear constraint. This constraint is also not applicable if the lower-level problem does not contain the pair of complementarity constraints, which may be the situation if the problem has equilibria that are not derived from an underlying optimization problem.

Tables 8.2 and 8.3 contain results from testing the use of the strong duality constraint. We compare the valid range of  $C$  in (B-MILP) with strong duality constraint and (MILP) without this constraint. The valid range is the range where the optimal objective value of the original bilevel problem is reproduced. GAMS (ver. 23.0) and Xpress-MP (ver. 2006) were used to compute the solutions by enumeration. The bar graphs in the tables represent the effective range of  $C$  for each test problem. The strong duality constraint played a major role in making (B-MILP) almost insensitive to the choice of  $C$ .

The single-level approach in Gabriel & Leuthold (2010) and Audet, Savard & Zghal (2007), which corresponds to (MILP) may not use the strong duality constraint since it would make the problem non-linear and non-convex because  $x$  variables in the constraint cannot be fixed. Hence the working range of  $C$  for the single level approach would be narrow compared to the decomposed problem.

## Branch-and-Bound

Each node in the branch-and-bound tree represents a subdomain for the upper-level variables  $x$ . The literature shows several ways to do branching of the subdomains, for instance in Bard & Moore (1990), Hansen et al. (1992) and Gabriel et al. (2010). We have chosen to follow Gabriel et al. (2010). Two sample points  $s_i$  (for  $i = 1, 2$ ) are picked and Benders cuts,  $\alpha(x) \geq \alpha(s_i) + \lambda_i^\top(x - s_i)$ , are calculated for each point.  $\lambda_i$  is the dual variable vector to the linear constraint  $x = s_i$  of (SP). Two planes are obtained by changing the inequality symbols of the Benders cuts to equality, that is,  $\alpha(x) = \alpha(s_i) + \lambda_i^\top(x - s_i)$ , and where these planes intersect,  $\alpha(s_1) + \lambda_1^\top(x - s_1) = \alpha(s_2) + \lambda_2^\top(x - s_2)$ , the domain is partitioned. Since  $\alpha(x)$  can be non-convex, the two Benders cuts might not partition the domain into two non-empty subdomains. In that case new sample points are chosen a limited number of times, and if proper branching is still not achieved an arbitrary partition of the subdomain into two non-empty subdomains is chosen.

We use the following branching and fathoming rules:

- Branch if  $|(UB - LB)/LB| > TOL$  and  $LB < Incumbent$
- Pruning by optimality if  $|(UB - LB)/LB| \leq TOL$
- Pruning by bound if  $LB \geq Incumbent$

Here *Incumbent* is the value of the best solution found so far and *TOL* is a user-specific tolerance.

**Table 8.2:** The working range for  $C$  increases when strong duality constraints are added. For each instance the objective value, shape of the  $\alpha(x)$ -function and working range within  $[1E4, 1E10]$  for (B-MILP) with strong duality constraints and (MILP) is given. The integer upper-level variable is limited to  $[-10, 10]$ , all model parameters are in  $[0, 10]$  and  $\dim(x) = 1, \dim(y) = 10, \dim(z) = 5, \dim(\bar{b}) + \dim(\tilde{b}) = 15$

ObjVal	Formulation	$\alpha(x)$	C ( $1E+n$ for n given below)							
			3	4	5	6	7	8	9	10
-37.00	B-MILP	non-convex	[3, 10]							
	MILP	-	[3, 6.5]							
-0.33	B-MILP	non-convex	[3, 10]							
	MILP	-	[3, 7.5]							
-33.75	B-MILP	non-convex	[3, 10]							
	MILP	-	[3, 6.5]							
3.00	B-MILP	non-convex	[3, 10]							
	MILP	-	[3, 7.5]							
-54.17	B-MILP	non-convex	[3, 10]							
	MILP	-	[3, 6.5]							
2.52	B-MILP	non-convex	[3, 10]							
	MILP	-	[3, 6.5]							
-1.33	B-MILP	non-convex	[3, 10]							
	MILP	-	[3, 3.5]							
-0.67	B-MILP	non-convex	[3, 10]							
	MILP	-	[3, 4.5]							
-6.53	B-MILP	non-convex	[3, 10]							
	MILP	-	[3, 3.5]							
-5.58	B-MILP	non-convex	[3, 10]							
	MILP	-	[3, 6.5]							
-0.88	B-MILP	non-convex	[3, 10]							
	MILP	-	[3, 4.5]							
-7.76	B-MILP	non-convex	[3, 10]							
	MILP	-	[3, 6.5]							
-19.52	B-MILP	non-convex	[3, 10]							
	MILP	-	[3, 6.5]							
0.00	B-MILP	non-convex	[3, 10]							
	MILP	-	[3, 4.5]							
-10.96	B-MILP	non-convex	[3, 10]							
	MILP	-	[3, 6.5]							

**Table 8.3:** The working range for  $C$  increases when strong duality constraints are added. For each instance the objective value, shape of the  $\alpha(x)$ -function and working range within  $[1E4, 1E10]$  for (B-MILP) with strong duality constraints and (MILP) is given. The integer upper-level variables are limited to  $[-10, 10]$ , all model parameters are in  $[0, 100]$  and  $\dim(x) = 2$ ,  $(y) = 400$ ,  $\dim(z) = 200$ ,  $\dim(\bar{b}) + \dim(\tilde{b}) = 600$

ObjVal	Formulation	$\alpha(x)$	C ( $1E+n$ for n given below)						
			4	5	6	7	8	9	10
302.72	B-MILP	non-convex	[4, 10]						
	MILP	-	[4, 7]						
517.00	B-MILP	convex	[4, 8]						
	MILP	-	[4, 5]						
2440.00	B-MILP	non-convex	[4, 8]						
	MILP	-	[4, 7]						
454.54	B-MILP	convex	[4, 9]						
	MILP	-	[4, 6]						
882.32	B-MILP	non-convex	[4, 10]						
	MILP	-	[4, 9]						
172.21	B-MILP	convex	[4, 9]						
	MILP	-	[4, 6]						
216.35	B-MILP	non-convex	[4, 10]						
	MILP	-	[4, 7]						
181.90	B-MILP	convex	[4, 9]						
	MILP	-	[4, 6]						
145.35	B-MILP	convex	[4, 9]						
	MILP	-	[4, 6]						
1423.00	B-MILP	non-convex	[4, 9]						
	MILP	-	[4, 6]						

## Pseudocode for the Algorithm

Algorithm 8.1 is a pseudocode that describes the overall algorithm. The main workload is the branch-and-bound process, where  $N$  refers to the index of current node,  $L$  is the list of active nodes in the partitioning tree and  $D(N)$  refers to the subdomain defined by node  $N$ . Three subroutines are used for solving the subproblems, making bounding decisions and branching and these are described in Subroutines 8.1-8.3. `SelectNextNode(L)` is a subroutine that selects next node from  $L$ . In our tests `SelectNextNode(L)` has applied the depth-first principle.

---

### Algorithm 8.1 Main

---

```

1:  $L := \{0\}$ 
2:  $D(0) := X$ 
3:  $Incumbent := \infty$ 
4: while  $L \neq \emptyset$  do
5:    $N := \text{SelectNextNode}(L)$ 
6:    $L := L \setminus \{N\}$ 
7:    $(Decision, Incumbent) := \text{SolveAndBound}(N, D(N), Incumbent)$ 
8:   if  $Decision = \text{BRANCHING}$  then
9:      $(N+1, N+2, D(N+1), D(N+2)) := \text{DecomposeDomain}(N, D(N))$ 
10:     $L := L \cup \{N+1, N+2\}$ 
11:   end if
12: end while
13: return  $(Incumbent, (x^*, y^*, z^*, \bar{b}^*, \tilde{b}^*))$ 

```

---



---

### Subroutine 8.1 ComputeBounds( $N, D(N)$ )

---

```

1: Compute  $LB$  with Lagrangean relaxation algorithm by solving  $\text{MILP}(\mu)$ 
2: Compute  $UB$  with the Benders decomposition algorithm by iteratively solving (MP) and (SP). Include the strong duality constraint  $(e^\top - w^\top D)y = z^\top(k - Nx)$  to (MP) if valid for the problem. Warm-start with the optimal solution from Step 1 as the initial starting point and lower bound for (MP) if it exists
3: return  $(UB, LB)$ 

```

---

## Convergence of the Dynamic Algorithm

To prove that the algorithm in the previous section converges, we make the following assumptions:

---

**Subroutine 8.2** SolveAndBound( $N, D(N), Incumbent$ )

---

```

1:  $(UB, LB) := \text{ComputeBounds}(N, D(N))$ 
2: if  $UB < Incumbent$  then
3:    $Incumbent := UB$ 
4:   Record  $Incumbent$  and the optimal solution  $(x^*, y^*, z^*, \bar{b}^*, \tilde{b}^*)$  of BD.
5: end if
6: if  $\left| \frac{UB-LB}{LB} \right| > TOL$  and  $LB < Incumbent$  then
7:    $Decision := \text{BRANCHING}$ 
8: else
9:    $Decision := \text{FATHOMING}$ 
10: end if
11: return  $(Decision, Incumbent)$ 

```

---



---

**Subroutine 8.3** DecomposeDomain( $N, D(N)$ )

---

```

1:  $Count := 0$ 
2:  $Branched := \text{false}$ 
3: while not  $Branched$  and  $Count < CountLimit$  do
4:   Get two sample points  $s_1$  and  $s_2$  from  $D(N)$  and compute their associated  $(\lambda_i^\top, \bar{b}_i^\top, \tilde{b}_i^\top)$  values ( $i = 1, 2$ ). The two sample points can be chosen randomly within  $D(N)$  for instance by solving the problems  $\{x | \min_x c_0 x \text{ s.t. } x \in D(N)\}$  and  $\{x | \max_x c_0 x \text{ s.t. } x \in D(N)\}$ , where  $c_0$  is a random cost vector
5:   Compute their intersection hyperplane as  $\alpha(s_1) + \lambda_1^\top(x - s_1) = \alpha(s_2) + \lambda_2^\top(x - s_2)$ 
6:    $D(N+1) := D(N) \cap \{x | \alpha(s_1) + \lambda_1^\top(x - s_1) \leq \alpha(s_2) + \lambda_2^\top(x - s_2)\}$  and  $D(N+2) := D(N) \cap \{x | \alpha(s_1) + \lambda_1^\top(x - s_1) \geq \alpha(s_2) + \lambda_2^\top(x - s_2) + TOL\}$ 
7:   if  $D(N+1) \neq \emptyset$  and  $D(N+2) \neq \emptyset$  then
8:      $Branched := \text{true}$ 
9:   else
10:     $Count+ = 1$ 
11:   end if
12: end while
13: if  $Branched = \text{false}$  then
14:   Select arbitrary intersecting hyperplane within  $D(N)$  and define  $D(N+1)$  and  $D(N+2)$  accordingly
15: end if
16: return  $(N+1, N+2, D(N+1), D(N+2))$ 

```

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**Assumption 1 (A1)** The feasible region of (MP),  $X = \{x \in Z^{n_x} | Qx \leq q\}$ , is a bounded, non-empty set.

**Assumption 2 (A2)** The feasible region of (SP) for a given  $x$ ,  $\Omega_{\text{SP}}(x) \neq \emptyset$  when  $x \in X$ .

**Theorem 1.** *Suppose that assumptions (A1) and (A2) hold and we are able to solve all sub problems within tolerance  $TOL$ . Then, the above dynamic DC-MPEC algorithm converges to a global optimum of problem (MILP), within the accuracy  $TOL$ , in a finite number of iterations.*

*Proof.* This theorem is proved in two steps. First, we prove that we cannot prune the subdomain containing the optimal solution. Next, we prove that the algorithm in a finite number of iterations will be able to partition in such a way that the optimal solution is in a subdomain with convex  $\alpha(x)$ .

1. We prune by bound when the solution of the Lagrangean subproblem  $z_{\text{MILP}(\mu)} \leq \text{Incumbent}$ . Geoffrion (1974) proves in Theorem 1a) that the Lagrangean subproblem is a valid lower bound for (MILP) which means no optimal solution can be lost by pruning.
2. Since we have a limit on the number of Lagrange iterations, the computation of lower bound from (MILP( $\mu$ )) ends in a finite number of iterations. The computation of the upper bound  $z_{\text{B-MILP}}$  using Benders decomposition also terminates in a finite number of steps according to Benders (1962). By assumption (A1) there are a finite number of points in the domain  $X$  and each branching is forced to leave at least one point in each subdomain, which means branch-and-bound can reach subdomains containing single points within a finite number of partitions. By definition  $\alpha(x)$  is convex in subdomains containing a single point.

From the following three observations, we know that the algorithm will find the global optimal solution: (i) the subdomain containing the global optimal solution cannot be pruned by 1.; (ii) we can find the subdomain containing the global optimal solution where  $\alpha(x)$  is convex by 2.; and (iii) according to Benders (1962), Benders decomposition algorithm provides the global solution for a convex  $\alpha(x)$ .  $\square$

## Scenario Decomposition

In the setting where the lower-level is a two-stage stochastic complementarity program the structure of the problem can be utilized when solving the Lagrangean subproblem (MILP( $\mu$ )) using scenario decomposition (Carøe & Schultz 1999).

The uncertainty is described by a set of scenarios  $j$  with the probability  $p_j$ ,  $j = 1, \dots, J$ , where  $\sum_{j=1}^J p_j = 1$ . For a problem with a two-stage stochastic program in the lower-level, Problem (8.1) can be written as:

$$\begin{aligned}
 \min_{x,y} \quad & \sum_{j=1}^J p_j \left( c^\top x_j + \check{d}^\top \check{y}_j + \hat{d}_j^\top \hat{y}_j \right) \\
 \text{s.t.} \quad & Qx_j \leq q \quad \text{for } j = 1, \dots, J \\
 & \check{Q}x_j \leq \check{q} \quad \text{for } j = 1, \dots, J \\
 & A_j x_j + \check{B}_j \check{y}_j + \hat{B}_j \hat{y}_j \geq a_j \quad \text{for } j = 1, \dots, J \\
 & x_j = x_{j-1} \quad \text{for } j = 2, \dots, J \\
 & (\check{y}_1, \dots, \check{y}_J, \hat{y}_1, \dots, \hat{y}_J) \in S(x_1, \dots, x_J)
 \end{aligned} \tag{8.8}$$

where  $x_j \in Z^{n_x}$ ,  $\check{y}_j \in R^{n_{\check{y}}}$  and  $\hat{y}_j \in R^{n_{\hat{y}}}$  for  $j = 1, \dots, J$  and

$$\begin{aligned}
 S(x_1, \dots, x_J) = & \left\{ \begin{array}{l} (\check{y}_1, \dots, \check{y}_J, \\ \hat{y}_1, \dots, \hat{y}_J, \\ \check{z}_1, \dots, \check{z}_J, \\ \hat{z}_1, \dots, \hat{z}_J, \\ \hat{w}_1, \dots, \hat{w}_J) \end{array} \left| \begin{array}{l} 0 \leq \check{E}\check{y}_j + \check{e} - M^\top \check{z}_j - \sum_{j=1}^J \check{M}^\top \hat{z}_j \perp \check{y}_j \geq 0 \\ 0 \leq p_j \hat{E}_j \hat{y}_j + p_j \hat{e}_j - \hat{M}_j^\top \hat{z}_j - D_j^\top \hat{w}_j \perp \hat{y}_j \geq 0 \\ 0 \leq k - Nx_j - M^\top \check{y}_j \perp \check{z} \geq 0 \\ 0 \leq M^\top \check{y}_j - \hat{M}_j^\top \hat{y}_j - \hat{k}_j + \hat{N}_j x_j \perp \hat{z}_j \geq 0 \\ D\hat{y}_j + Fx_j = g \\ \text{for } j = 1, \dots, J \\ \check{y}_j = \check{y}_{j-1} \\ \check{z}_j = \check{z}_{j-1} \\ \text{for } j = 2, \dots, J \end{array} \right. \right\} \tag{8.9}
 \end{aligned}$$

Here  $\check{y}_j$  and  $\check{z}_j$  are the first-stage decisions and  $\hat{y}_j$ ,  $\hat{z}_j$  and  $\hat{w}_j$  are the second-stage decisions of the lower-level for scenario  $j$ . The two last equation sets of  $S(x_1, \dots, x_J)$  are non-anticipativity constraints (Rockafellar & Wets 1976) included to make sure the first-stage decisions are equal in all scenarios.

We transform the complementarity conditions of Problem (8.9) into disjunctive constraints as described earlier, transforming the problem into a MILP. The underlying stochastic program gives the resulting MILP matrix a structure of nearly separable blocks for each scenario. To achieve separability in the scenarios the non-anticipativity constraints are dualized using Lagrangean relaxation. Since one set of optimality conditions contain a sum over all scenarios also these

constraints need to be dualized to achieve separability. Note that also the upper-level variables and some disjunctive binary variables act as first-stage variables in this setting.

Scenario decomposition corresponds to Lagrangean relaxation of an integer program. As stated in Theorem 1 in Geoffrion (1974) this gives a lower bound on the original problem which makes it valid for our lower bounding purpose, but does not guarantee a tight bound.

### 8.3 Results

The dynamic algorithm has been tested both on randomly generated test data for the general model formulation and on three cases for an application of the DC-MPEC problem from the Norwegian natural gas supply chain. For all tests the tolerance was set to  $TOL = 10^{-5}$ .

#### Results on Randomly Generated Data

In this section, we present our computational results with the dynamic DC-MPEC algorithm based on randomly generated data. Each lower-level problem is created by deriving the KKT conditions from a LP or convex QP problem. 59 LP-based problems are generated by random data from the interval  $[-500,500]$  with a uniform distribution. Similarly, data for 40 QP-based problems are sampled from a uniform distribution  $[-100,100]$ . The test problems are grouped according to the dimension of the variable vectors and underlying problem type, as summarized in Table 8.4. Sensitivity analysis has been conducted to find the working range for the disjunctive constant  $C$ , and the values reported in Table 8.4 is within this range and corresponds to the results reported in the following tables. Data for Groups 1 to 4 is provided in the online appendix for Gabriel et al. (2010), while the rest are new test problems. All tests were conducted on a computer with 2.34 GHz processor and 23.55 GB memory. The algorithm was implemented with MATLAB (ver. 7.0) and GAMS (ver. 23.6) interfacing where GAMS used Xpress-MP for its MILP solver. Test runs using more than 10 hours were stopped, giving rise to “n/a” in the result tables.

Table 8.5 lists the number of subdomains and the number of sampling and branching attempts for the alternative algorithms. Results are given for the original static algorithm (“Stat”) as in Gabriel et al. (2010) and the dynamic algorithm with and without warm-starting the Benders algorithm with the solution from Lagrangean relaxation (“Dyn ws” and “Dyn”, respectively). For the dynamic algorithm the number of subdomains corresponds to leaf nodes in the branch-and-bound tree, while in the static algorithm subdomains are identified by comparing  $(\lambda^\top, \bar{b}^\top, \tilde{b}^\top)$  as described in Gabriel et al. (2010). We observe that



**Table 8.4:** Random generated test problem groups. MP and SP give the dimension of the variable vector. The value of the disjunctive constant used for the tests are given by  $C$ . Whether the lower level problem is derived from a LP or QP is indicated by the problem type.

Gr	MP		SP			$C$	Probl type
	$x$	type	$y$	$z$	$b$		
1	2	int	5	2	7	1.E+03	LP
2	5	int	5	2	7	1.E+03	LP
3	10	bin	10	5	15	300	LP
4	10	bin	15	10	25	1.E+06	LP
5	2	int	400	200	600	1.E+06	LP
6	20	bin	100	100	200	1.E+05	LP
7	2	int	100	50	150	1.E+06	QP

the dynamic algorithm reduces the number of subdomains and samplings compared to the static algorithm. In Table 8.6 the solution times for the same test problems and algorithms are given. “Bounding time” covers the Lagrangean and Benders algorithms (Subroutines 8.1-8.2), “Sampling time” sampling and branching the  $x$ -domain (Subroutine 8.3), and “Total time” is the sum of the two. The dynamic algorithm shows a significant reduction in solution time compared to the static algorithm, mainly caused by a major reduction in “Sampling time”. This corresponds well to the reduction in number of subdomains and sampling, and shows that the gain from fewer subdomains because of dynamic branching is larger than the added work on computing bounds. We observe that including the warm-start of the Benders algorithm makes the dynamic algorithm able to solve the problem in the root node for all test problems, also those with non-convex  $\alpha(x)$ , which indicates that Lagrangean relaxation gives a very strong lower bound for the problem. We have not been able to prove that this result is guaranteed for the problem class in general. The tables also show that the dynamic algorithm with warm-start solves all test problems, while the dynamic algorithm without warm-start leaves two problems unsolved and the static leaves five unsolved due to long solution times.

**Table 8.5:** Number of subdomains and samplings for random generated test problem solved by static algorithm and dynamic algorithm without and with warm-start (ws). # subdomains refers to the finest partitioning of the  $x$ -domain, which corresponds to the leaf nodes of the branch-and-bound tree for the dynamic algorithm. For the static algorithm the subdomains are identified through sampling and comparing  $(\lambda^\top, \bar{b}^\top, \tilde{b}^\top)$  as described in Gabriel et al. (2010). The true number of subdomains are identified by enumeration, where a subdomain is defined as a set of integer  $x$  points where  $(\lambda^\top, \bar{b}^\top, \tilde{b}^\top)$  is the same. Mean and median values cover all test problems that were solved by all three algorithms.

Gr	ID	# subdomains				# samplings			
		True	Stat	Dyn	Dyn ws	Stat	Dyn	Dyn ws	
	mean	19.5	19.7	3.9	1	40.6	58.5	0	
	median	14	4	1	1	4.5	2.5	0	
1	1	4	1	1	1	2	0	0	
1	2	4	1	1	1	2	0	0	
1	3	6	5	2	1	10	4	0	
1	4	5	4	1	1	2	2	0	
2	1	4	1	1	1	2	8	0	
2	2	4	1	1	1	2	0	0	
2	3	6	1	1	1	2	0	0	
2	4	7	198	1	1	382	0	0	
3	1	6	1	3	1	2	24	0	
3	2	16	47	11	1	123	139	0	
3	3	10	61	4	1	195	37	0	
3	4	20	29	1	1	71	3	0	
3	5	20	50	1	1	115	14	0	
3	6	7	17	17	1	40	446	0	
3	7	14	1	3	1	2	100	0	
3	8	8	16	1	1	31	17	0	
3	9	22	26	1	1	74	7	0	
3	10	9	1	1	1	2	0	0	
3	11	9	4	3	1	4	100	0	
3	12	10	1	2	1	2	22	0	
3	13	16	11	3	1	27	92	0	
3	14	14	77	2	1	174	136	0	
3	15	14	1	1	1	2	5	0	
3	16	33	65	5	1	188	62	0	
3	17	32	63	1	1	168	2	0	
3	18	20	22	11	1	26	247	0	

Table 8.5: continued

Gr	ID	# subdomains				# samplings		
		True	Stat	Dyn	Dyn ws	Stat	Dyn	Dyn ws
4	1	38	4	13	1	15	149	0
4	2	41	1	35	1	1	708	0
4	3	27	1	29	1	1	831	0
4	4	12	99	35	1	223	395	0
4	5	52	2	1	1	1	6	0
4	6	6	12	13	1	48	502	0
4	7	13	20	52	1	39	552	0
4	8	60	105	3	1	259	70	0
4	9	7	1	2	1	2	58	0
4	10	33	41	5	1	97	33	0
4	11	24	14	5	1	12	138	0
4	12	15	24	3	1	47	0	0
4	13	28	79	5	1	206	36	0
5	1	16	19	2	1	37	2	0
5	2	6	12	2	1	23	2	0
5	3	6	1	1	1	2	6	0
5	4	11	2	1	1	3	0	0
5	5	42	33	2	1	78	2	0
5	6	92	22	1	1	43	0	0
5	7	21	9	1	1	23	0	0
5	8	42	26	1	1	51	0	0
5	9	21	3	1	1	5	0	0
5	10	23	24	1	1	49	0	0
6	1	n/a	1	1	1	2	0	0
6	2	n/a	3	1	1	5	5	0
6	3	n/a	2	1	1	3	0	0
6	4	n/a	1	1	1	2	0	0
6	5	n/a	1	1	1	2	0	0
6	6	n/a	1	1	1	2	0	0
6	7	n/a	1	1	1	2	0	0
6	8	n/a	1	1	1	2	0	0
6	9	n/a	2	1	1	3	0	0
6	10	n/a	n/a	n/a	1	n/a	n/a	0

Table 8.5: continued

Gr	ID	# subdomains				# samplings		
		True	Stat	Dyn	Dyn ws	Stat	Dyn	Dyn ws
7	1	n/a	3	1	1	5	0	0
7	2	n/a	5	1	1	9	2	0
7	3	n/a	1	3	1	4	59	0
7	4	n/a	2	5	1	3	62	0
7	5	n/a	10	5	1	19	41	0
7	6	n/a	7	1	1	19	0	0
7	7	n/a	1	1	1	2	0	0
7	8	n/a	1	3	1	2	34	0
7	9	n/a	1	1	1	2	0	0
7	10	n/a	1	1	1	2	2	0
7	11	n/a	2	2	1	3	6	0
7	12	n/a	1	1	1	2	0	0
7	13	n/a	2	2	1	4	20	0
7	14	n/a	1	1	1	2	0	0
7	15	n/a	1	1	1	2	0	0
7	16	n/a	3	1	1	3	0	0
7	17	n/a	2	1	1	3	0	0
7	18	n/a	1	1	1	2	0	0
7	19	n/a	26	1	1	34	0	0
7	20	n/a	37	3	1	55	17	0
7	21	n/a	5	1	1	7	2	0
7	22	n/a	40	2	1	121	44	0
7	23	n/a	12	3	1	87	58	0
7	24	n/a	1	1	1	2	0	0
7	25	n/a	1	1	1	2	0	0
7	26	n/a	1	1	1	2	0	0
7	27	n/a	10	4	1	27	45	0
7	28	n/a	68	1	1	313	0	0
7	29	n/a	53	1	1	111	4	0
7	30	n/a	53	1	1	116	0	0
7	31	n/a	72	2	1	147	36	0
7	32	n/a	12	4	1	30	47	0
7	33	n/a	65	2	1	152	50	0
7	34	n/a	5	4	1	10	9	0
7	35	n/a	75	1	1	156	0	0
7	36	n/a	1	1	1	2	0	0
7	37	n/a	n/a	n/a	1	n/a	n/a	0
7	38	n/a	n/a	1	1	n/a	0	0
7	39	n/a	n/a	1	1	n/a	0	0
7	40	n/a	n/a	1	1	n/a	0	0

**Table 8.6:** Solution times for random generated test problem solved by static algorithm and dynamic algorithm without and with warm-start (ws). The convexity of  $\alpha(x)$  is checked through plotting, and is therefore only available for 2-dimensional problems. Bounding time covers Benders and Lagrangean algorithm, Sampling time is time for sampling and branching while Total time is the sum Bounding and Sampling. Mean and median values covers all test problems that were solved by all three algorithms. All times in seconds.

Gr	ID	Convex	Total time			Sampling time			Bounding time		
			Stat	Dyn	Dyn ws	Stat	Dyn	Dyn ws	Stat	Dyn	Dyn ws
mean			614.03	296.23	42.34	505.43	110.83	0	108.60	185.40	42.34
median			113.78	61.37	2.64	85.57	10.47	0	36.85	37.93	2.64
1	1	Yes	3.65	0.86	0.78	2.62	0	0	1.03	0.86	0.78
1	2	Yes	3.08	0.84	0.78	2.38	0	0	0.7	0.84	0.78
1	3	No	53.83	7.15	0.71	45.98	4.45	0	7.85	2.71	0.71
1	4	No	21.97	3.74	0.78	13.72	2.22	0	8.25	1.52	0.78
2	1	n/a	3.69	18.31	0.81	2.36	9.00	0	1.33	9.31	0.81
2	2	n/a	3.23	6.41	0.80	2.24	0	0	0.99	6.41	0.80
2	3	n/a	3.25	3.65	0.73	2.26	0	0	0.99	3.65	0.73
2	4	n/a	4372.40	10.95	0.79	3461.7	0	0	910.70	10.95	0.79
3	1	n/a	3.72	66.07	0.78	2.59	29.01	0	1.12	37.06	0.78
3	2	n/a	602.74	308.42	0.83	526.43	167.58	0	76.31	140.84	0.83
3	3	n/a	949.43	80.54	0.86	856.59	43.99	0	92.84	36.55	0.86
3	4	n/a	357.67	6.72	0.78	308.12	3.66	0	49.55	3.06	0.78
3	5	n/a	589.89	30.83	0.84	502.13	16.70	0	87.76	14.13	0.84
3	6	n/a	204.17	916.14	0.79	178.06	543.31	0	26.11	372.83	0.79
3	7	n/a	3.67	268.67	0.84	2.54	127.19	0	1.13	141.48	0.84
3	8	n/a	191.79	39.64	0.77	146.61	21.01	0	45.18	18.63	0.77
3	9	n/a	373.93	14.21	0.85	325.22	8.64	0	48.71	5.57	0.85

Table 8.6: continued

Gr	ID	Convex $\alpha(x)$ ?	Total time		Sampling time		Bounding time				
			Stat	Dyn	Stat	Dyn	Stat	Dyn	Stat	Dyn	ws
3	10	n/a	2.99	1.31	0.85	2.58	0	0.40	1.31	0.85	
3	11	n/a	34.60	258.38	1.04	21.72	125.08	0	12.88	133.30	1.04
3	12	n/a	4.78	59.84	0.78	2.51	27.33	0	2.26	32.51	0.78
3	13	n/a	135.42	204.19	0.89	116.71	112.34	0	18.71	91.85	0.89
3	14	n/a	938.08	314.31	0.84	784.07	171.50	0	154.01	142.81	0.84
3	15	n/a	2.93	11.29	0.82	2.52	6.07	0	0.41	5.22	0.82
3	16	n/a	938.34	137.76	0.76	824.76	76.34	0	113.58	61.43	0.76
3	17	n/a	862.79	5.93	0.83	733.41	2.50	0	129.38	3.43	0.83
3	18	n/a	223.64	593.27	0.91	152.38	303.82	0	71.26	289.45	0.91
4	1	n/a	74.55	370.99	0.93	67.16	195.94	0	7.39	175.05	0.93
4	2	n/a	2.8	2286.52	1.69	1.93	891.61	0	0.87	1394.91	1.69
4	3	n/a	2.68	2239.25	1.69	1.74	1060.1	0	0.94	1179.15	1.69
4	4	n/a	1276.04	1010.31	1.18	1074.4	513.2	0	201.64	497.11	1.18
4	5	n/a	13.19	20.14	0.88	4.86	7.45	0	8.33	12.69	0.88
4	6	n/a	250.85	1100.45	0.81	213.46	609.88	0	37.39	490.57	0.81
4	7	n/a	231.46	1567.46	0.87	180.94	703.81	0	50.52	863.65	0.87
4	8	n/a	1366.17	269.18	1.8	1158.9	90.51	0	207.27	178.67	1.8
4	9	n/a	16.21	179.64	1.02	8.53	73.59	0	7.68	106.05	1.02
4	10	n/a	514.46	125.42	1.92	427.97	43.06	0	86.49	82.37	1.92
4	11	n/a	227.28	295.99	0.97	128.81	176.47	0	98.47	119.52	0.97
4	12	n/a	284.97	2.53	0.81	221.23	0	0	63.74	2.53	0.81
4	13	n/a	1105.88	123.33	2.12	954.53	44.86	0	151.35	78.48	2.12

Table 8.6: continued

Gr	ID	Convex $\alpha(x)$ ?	Total time		Sampling time		Bounding time				
			Stat	Dyn	Stat	Dyn	Stat	Dyn			
5	1	No	1985.52	251.12	30.47	1717.40	157.75	268.12	93.37	30.47	30.47
5	2	Yes	1372.76	294.16	30.22	1077.00	149.88	295.76	144.28	30.22	30.22
5	3	No	104	669.86	30.40	92.24	461.78	11.76	208.08	30.40	30.40
5	4	Yes	231.12	148.27	30.56	137.98	0	93.14	148.27	30.56	30.56
5	5	No	4307.94	596.21	31.96	3806.40	156.53	501.54	439.68	31.96	31.96
5	6	Yes	2418.82	134.59	30.7	2094.50	0	324.32	134.59	30.70	30.70
5	7	No	1339.73	180.39	34.50	1083.8	0	255.93	180.39	34.5	34.5
5	8	Yes	2878.06	128.22	30.47	2496.20	0	381.86	128.22	30.47	30.47
5	9	Yes	365	165.89	30.37	226.44	0	138.56	165.89	30.37	30.37
5	10	No	2704.98	33.52	32.36	2373.90	0	331.08	33.52	32.36	32.36
6	1	n/a	58.17	42.82	4.11	16.6	0	41.57	42.82	4.11	4.11
6	2	n/a	87.13	170.17	4.28	41.59	42.58	45.54	127.59	4.28	4.28
6	3	n/a	115.3	58.37	4.08	24.63	0	90.67	58.37	4.08	4.08
6	4	n/a	52.64	38.32	4.26	16.37	0	36.27	38.32	4.26	4.26
6	5	n/a	39.93	30.87	4.31	16.61	0	23.32	30.87	4.31	4.31
6	6	n/a	52.66	45.05	4.33	16.35	0	36.32	45.05	4.33	4.33
6	7	n/a	52.44	37.55	3.99	16.4	0	36.04	37.55	3.99	3.99
6	8	n/a	41.94	29.72	4.13	16.33	0	25.62	29.72	4.13	4.13
6	9	n/a	65.5	24.91	4.04	24.66	0	40.84	24.91	4.04	4.04
6	10	n/a	n/a	n/a	3.97	n/a	n/a	n/a	n/a	3.97	3.97

Table 8.6: continued

Gr	ID	Convex $\alpha(x)$ ?	Total time		Sampling time		Bounding time			
			Stat	Dyn	Stat	Dyn	Stat	Dyn	Stat	Dyn
7	1	Yes	112.27	12.05	2.99	78.89	0	33.38	12.05	2.99
7	2	Yes	169.52	30.07	2.95	135.99	12.09	33.53	17.98	2.95
7	3	Yes	75.91	761.02	2.94	62.47	374.09	13.44	386.93	2.94
7	4	Yes	66.31	769.70	2.95	45.27	381.47	21.05	388.23	2.95
7	5	Yes	358.20	542.91	3.06	288.53	245.89	69.67	297.02	3.06
7	6	Yes	402.86	16.17	3.43	318.02	0	84.84	16.17	3.43
7	7	Yes	26.45	13.22	3.22	14.33	0	12.13	13.22	3.22
7	8	Yes	39.64	777.80	3.15	23.77	325.77	15.87	452.03	3.15
7	9	Yes	36.30	15.65	3.26	21.28	0	15.02	15.65	3.26
7	10	Yes	30.37	50.27	3.21	19.10	21.21	11.27	29.06	3.21
7	11	Yes	33.56	66.52	2.60	15.92	31.87	17.64	34.66	2.60
7	12	Yes	22.31	13.30	2.73	10.80	0	11.51	13.30	2.73
7	13	Yes	35.92	215.91	2.60	21.41	106.08	14.51	109.83	2.60
7	14	Yes	22.36	10.97	2.58	10.78	0	11.58	10.97	2.58
7	15	Yes	26.65	16.85	2.60	11.14	0	15.50	16.85	2.60
7	16	Yes	35.95	15.63	2.63	21.33	0	14.62	15.63	2.63
7	17	Yes	34.25	14.31	2.69	16.12	0	18.13	14.31	2.69
7	18	Yes	22.33	12.61	2.61	10.80	0	11.53	12.61	2.61
7	19	Yes	335.77	15.05	2.70	182.85	0	152.92	15.05	2.70
7	20	Yes	404.77	170.31	2.54	294.86	90.35	109.91	79.96	2.54



Table 8.6: continued

Gr	ID	Convex $\alpha(x)$ ?	Total time		Sampling time		Bounding time		
			Stat	Dyn	Stat	Dyn	Stat	Dyn	
7	21	Yes	69.07	35.48	40.93	11.93	28.14	23.54	3.84
7	22	Yes	2222.41	454.72	1986.00	218.75	236.41	235.97	3.65
7	23	Yes	2282.70	691.04	2147.00	335.19	135.70	355.85	3.07
7	24	Yes	22.19	12.47	11.58	0	10.61	12.47	3.75
7	25	Yes	26	15.87	11.74	0	14.26	15.87	3.62
7	26	Yes	25.88	15.87	11.69	0	14.20	15.87	3.12
7	27	Yes	688.69	521.51	600.03	275.15	88.66	246.36	3.66
7	28	Yes	3073.97	16.16	2887.40	0	186.57	16.16	3.90
7	29	Yes	1015.68	62.91	874.39	27.78	141.29	35.13	3.59
7	30	Yes	1039.43	14.00	914.59	0	124.84	14.00	3.74
7	31	Yes	2708.87	655.22	2312.60	193.22	396.27	462.00	2.64
7	32	Yes	550.99	435.16	433.41	249.03	117.58	186.13	2.65
7	33	Yes	2748.08	752.89	2423.50	289.52	324.58	463.37	2.87
7	34	Yes	70.94	113.12	59.45	47.91	11.50	65.21	2.69
7	35	Yes	1082.78	11.66	942.78	0	140.00	11.66	2.54
7	36	No	3273.79	4456.41	1487.50	0	1786.29	4456.41	3485.45
7	37	No	n/a	n/a	n/a	n/a	n/a	n/a	6141.19
7	38	No	n/a	6134.12	n/a	0	n/a	6134.12	6134.52
7	39	No	n/a	3372.82	n/a	0	n/a	3372.82	2407.15
7	40	No	n/a	2414.44	n/a	0	n/a	2414.44	1801.42

## Application to a Natural Gas Supply Chain

### Background

The presented algorithm is tested on a stochastic two-stage problem from the Norwegian natural gas supply chain. For this problem we use the scenario decomposition described earlier. Because of high investment costs, thorough planning is required for field and infrastructure developments in the natural gas industry. This planning typically has a bilevel structure where the investment level should be coordinated within the network, taking into account the competitive behavior of multiple producers in the operational level. Further, the investments are binary decisions taken under both long-term and operational uncertainty.

### Model Description

The model presented here is a MPEC that can be written in this form:

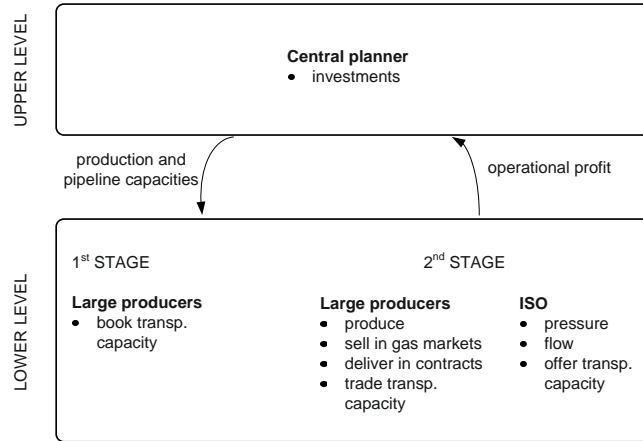
$$\begin{aligned} \min_{x,y} \quad & c^\top x + d^\top y \\ \text{s.t.} \quad & Qx \leq q \\ & y \in S(x) \end{aligned} \quad (8.10)$$

where  $x \in Z^{n_x}$ ,  $y \in R^{n_y}$ , and

$$S(x) = \left\{ y \mid \begin{array}{l} 0 \leq z \perp My + Nx - k \geq 0 \\ 0 \leq y \perp Ey + e - M^\top z \geq 0 \\ Ax + By \geq a \end{array} \right\} \quad (8.11)$$

The upper-level takes the perspective of a central planner making binary decisions on which fields and pipelines to invest in and when to invest. These decisions are represented by  $x$ . The investments have investment costs,  $c$ , and there can be dependencies between the possible investments given by  $Qx \leq q$ . The central planner maximizes the long term profits generated in the supply chain taking into account the short term operations,  $y$ .

For the lower-level problem we use a multi-period version of the model by Midthun, Bjørndal, Tomasgard & Smeers (2007) with some extension to facilitate a connection to the upper-level decisions. Midthun et al. (2007) describes a stochastic mixed complementarity problem where several producers make production decisions, trade in a transportation market, deliver natural gas in long term contracts and sell natural gas on the spot market. The producers maximize their profit consisting of natural gas sales income, transportation costs and production cost. The natural gas spot market has exogenously given stochastic prices and delivery obligations in contracts are stochastic. The independent system operator (ISO) routes the gas and makes transportation capacity available



**Figure 8.4:** Overview over natural gas value chain model

in the transportation market according to the physical capacities of the network. The market for transportation capacity is modeled endogenously in two stages. In the primary market large prequalified producers book capacity at a fixed price before the uncertainty is resolved. In the secondary market the ISO offers spare capacity, the large producers trade and the optimality conditions of a competitive fringe give a demand curve that clears the market. One producer will not know the other producers' primary booking in the second stage, an assumption that makes it possible to formulate the whole operational level as a Generalized Nash equilibrium. Figure 8.4 gives an overview of this model.

Midthun et al. (2007) shows that a competitive transportation market where decisions partly are taken under uncertainty results in some inefficiency compared to centralized coordination maximizing the social surplus of the supply chain. These losses are caused by excess transportation booking that carries forward to excess production and sale. The investment decisions by a centralized upper level planner in our model will typically search for network structures that counteract these losses by providing a flexible network design.

### Numerical Results

Three small instances of this problem are solved by the dynamic DC-MPEC algorithm. Two datasets (Datasets 1 and 2) are entirely synthetic, while the last (Dataset 3) has production capacities from an extract of the existing fields. Each dataset have several instances with different number of scenarios.

For the natural gas application, the main coordination of the dynamic DC-

MPEC algorithm is implemented in Matlab. Scenario decomposition was used for lower bounding, implemented with Mosel/Xpress to solve subproblems. To speed up the upper bounding the mixed complementarity problem from the lower-level was solved by GAMS/PATH and provided an initial solution for the binary variables of the Benders subproblem solved with GAMS/Xpress. As benchmark the whole bilevel problem is formulated as a single MILP and solved with Mosel/Xpress.

Table 8.7 presents the datasets and results from the tests. The disjunctive constant  $C$  was  $10^5$  for Dataset 1 and  $10^6$  for Dataset 2 and 3. Dataset 1 and 2 were tested on a HP dl160 G3 with 2x3.0GHz Intel E5472 Xeon processors and 16Gb RAM and Dataset 3 was tested on a Pentium 4 with 3.6GHz processor and 3.0Gb RAM.

**Table 8.7:** Numerical results for natural gas application. Three datasets are tested, each for several different numbers of scenarios. MP and SP give the dimension of the variable vectors. For the dynamic algorithm with warm-start the upper and lower bound and solution time in seconds is reported. For the benchmark, Xpress solving (MILP) is the objective value and solution time in seconds reported.

Dataset		Dynamic with warm-start						Benchmark		
#	scen	MP		SP		UB	LB	Time	Obj value	Time
		$x$	$y$	$z$	$b$					
1	10	12	324	604	688	1.09832E6	1.09832E6	11.2	1.09832E6	2.1
1	15	12	484	904	1 028	1.11765E6	1.11765E6	12.9	1.11765E6	2.2
1	20	12	644	1 204	1 368	1.19232E6	1.19232E6	14.4	1.19232E6	2.2
1	25	12	804	1 504	1 708	1.22435E6	1.22435E6	16.7	n/a <sup>a</sup>	>2h
1	30	12	964	1 804	2 048	1.26057E6	1.26057E6	17.5	n/a	n/a
1	100	12	3 204	6 004	6 808	1.20326E6	1.20326E6	50.4	n/a	n/a
1	500	12	16 004	30 004	34 008	1.14888E6	1.14888E6	472.2	n/a	n/a
1	1 000	12	32 004	60 004	68 008	1.14710E6	1.14710E6	7454.6	n/a	n/a

<sup>a</sup>The benchmark code did not complete it running in 2 hours, so we stopped it.

**Table 8.7: continued**

Dataset		Dynamic with warm-start						Benchmark		
#	scen	MP	SP			UB	LB	Time	Obj value	Time
		$x$	$y$	$z$	$b$					
2	10	6	762	1392	1644	-4.41152	-4.41152	16.9	-4.41152	1.1
2	25	6	1887	3462	4074	-4.50951	-4.50951	39.4	-4.50951	3.2
2	50	6	3762	6912	8124	-4.57339	-4.57339	115.7	-4.57339	6.0
2	75	6	5637	10362	12174	-4.48222	-4.48222	170.8	-4.48222	11.4
2	100	6	7512	13812	16224	-4.53883	-4.53883	336.2	-4.53883	16.9
2	300	6	22512	41412	48624	-4.50862	-4.50862	4589.0	-4.50862	128.0
2	500	6	37512	69012	81024	-4.50097	-4.50097	17114.4	-4.50097	414.5
2	700	6	52512	96612	113424	-4.49860	-4.49861	27201.4	-4.49861	647.0
3	10	6	762	1392	1644	-0.819261	-0.819261	44.6	-0.819261	2.7
3	25	6	1887	3462	4074	-0.821139	-0.821139	87.8	-0.821139	36.0
3	50	6	3762	6912	8124	-0.766766	-0.766766	234.1	n/a <sup>b</sup>	>23h
3	75	6	5637	10362	12174	-0.784225	-0.784223	508.6	n/a	n/a
3	100	6	7512	13812	16224	-0.783515	-0.783515	707.7	n/a	n/a

<sup>b</sup>The benchmark code did not complete it running in 23 hours, so we stopped it. Until then, best solution and best bound found were 0.765686 and 0.766766, respectively.

The benchmark was faster in the small instances, but our algorithm was able to solve substantially larger instances for Dataset 1 and 3. It is not surprising to find the benchmark faster on small instances, since our research code combining several software has a significant overhead in data transfer. We observe that in all tests where the real optimal value was known from the benchmark, the dynamic DC-MPEC algorithm provided an optimal solution, as expected based on the theory.

We have experienced challenges related to numerical stability in the testing of our algorithm. In several occasions the solver provided a solution to a MILP problem, but when asking the solver to fix the binary variables according to the solution and resolve, the solver claimed the resulting LP to be infeasible. To overcome this situation the tests on Dataset 2 and 3 were run without the presolve function activated in the Xpress solver ([www.gams.com/dd/docs/solvers/xpress.pdf](http://www.gams.com/dd/docs/solvers/xpress.pdf)) for upper bounding, and on Dataset 3 we also had to use different values for MIP tolerance on different instances. Naturally, deactivating presolve gives a disadvantage to the dynamic DC-MPEC algorithm when it comes to solution time.

## 8.4 Conclusions

We have presented a dynamic DC-MPEC algorithm to solve discretely constrained mathematical programs with equilibrium constraints (DC-MPEC) which is a class of bilevel program with integer program in the upper-level and mixed complementary problem in the lower-level. We develop a new branch-and-bound method for DC-MPEC problems applying Benders decomposition and Lagrangean relaxation methods. We provide convergence theory for the new method showing that it will find the global optimum and implement the new dynamic DC-MPEC algorithm on a set of test problems for both convex and non-convex domains. The numerical results show that the dynamic DC-MPEC algorithm outperforms the static counterpart presented by Gabriel et al. (2010) due to reduced sampling and branching efforts. The dynamic algorithm is further improved by warm-starting the Benders algorithm with the solution found by Lagrangean relaxation. We enhance the new method with the scenario decomposition method (Carøe & Schultz 1999) for two-stage stochastic DC-MPEC problems with discrete probability space. Then we compare the stochastic DC-MPEC algorithm with the single level approach by Fortuny-Amat & McCarl (1981) for an application for the Norwegian natural gas value chain. These numerical results present the effectiveness of our branch-and-bound algorithm and demonstrate the potential of the algorithm for a decision support tool for upper-level planners whose decisions are discrete.

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# Appendix

## 8.A Theorem

**Theorem 2.** *The solution of (SP) from the Benders decomposition algorithm provides an upper bound on  $z_{\text{MILP}} = d^\top x + d^\top y$  for any given partition, and the optimal solution  $z_{\text{MILP}} = d^\top x^* + d^\top y^*$  for any partition where  $\alpha(x)$  is convex.*

*Proof.* By the assumption (A2) any  $x^{(v)}$  which is a feasible in (MP) is also feasible in (MILP). For a given  $x^{(v)}$  the feasible region of (SP) is identical to the feasible region for the variables  $y, z, \bar{b}$  and  $\hat{b}$  of (MILP), which makes a solution of (SP) feasible in (MILP). And since the function  $z^{\text{up}}(x^{(v)})$  is identical to the objective function of (MILP), it is an upper bound of (MILP). Benders (1962) proves that Benders decomposition algorithm converges to the optimal solution in the case of a convex  $\alpha(x)$ .  $\square$

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