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European Power Market Model

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Problem Description

Present a model of the European power market with emphasis on prices, generation dispatch, consumption and transmission.

Focus areas:

- Study the power market today and in future scenarios
- Study the role of different technologies in the power market
- Study the robustness of the system by including stochastic variables
- Consider different market power representations
- Collect data to the model and implement it.

Since the model and data set are large, we will consider decomposition techniques and other ways to reduce the solution time.

The master's thesis contracts are attached after the appendices.

Preface

This report represents our Master's Thesis in Managerial Economics and Operations Research at the Department of Industrial Economics and Technology Management (IØT), at the Norwegian University of Science and Technology (NTNU).

During the last months we have learned a lot about electricity market optimization problems and equilibrium modeling. The Thesis is a continuation of the work presented in our Specialization Project (TIØ4500) last fall, which focused on a deterministic power market model. The report also presented a stochastic model as a proof of concept.

Many persons have supported us during the last months, and we want to thank our advisors Professor Asgeir Tomasgard and Post doc. Rudolf G. Egging for their enthusiastic directions and feedback. We also would like to thank Maria Eknes Stagrum for letting us occupy the apartment and use it as our office the last year.

Eivind Samseth and Geir Anders Haga

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Abstract

We develop and present a stochastic power market model to study the short-term spot market in Northern Europe. The model is formulated as a Mixed Linear Complementarity Problem using Conjectural Variations for the market power representation. The power producers and the transmission system operator simultaneously solve a profit maximization problem to reach a Nash-Equilibrium.

The model results show dispatch, transmission and prices in 2010 and 2020 for different stochastic wind forecasts. We see a clear distinction between technologies used for base load and for balancing the market. Prices in Norway and Denmark will increase towards 2020 and decrease in Germany and the Netherlands. We study how our market power representation can be adjusted to be as realistic as possible, the effect of wind uncertainty, how robust the power system is with respect to volatile prices and the effect of the carbon price and bio power feed-in tariffs. Because of the uncertain wind there is an increase in natural gas dispatch compared to a deterministic case.

The model is also decomposed with Dantzig-Wolfe decomposition technique. Unfortunately the decomposed model is not able to solve as big problems as the non-decomposed model. However we show significant algorithmic improvements and are able to reduce the number of iterations on a small problem from 183 to 17.

Sammendrag

Vi utvikler og presenter en stokastisk modell av elektrisitetsmarkedet for å kunne studere spot-markedet i Nord-Europa. Modellen er formulert som en likevektsmodell med bruk av Conjectural Variations for å representere markedsmakt. Kraftprodusentene og systemoperatøren løser sine profittmaksimeringsproblemer samtidig for å nå en Nash-likevekt.

Resultatet fra modellen viser kraftproduksjon, kraftoverføring og priser for 2010 og 2020 for forskjellige stokastiske vindvarsler. Vi ser en klar forskjell mellom produksjonsteknologier brukt som grunnlast og for å balansere markedet. Strømprisene vil trolig gå noe opp i Norge og Danmark mot 2020, mens de vil gå ned i Nederland og Tyskland. Vi studerer hvordan vi kan justere den modellerte markedsmakten slik at resultatene gjenspeiler virkeligheten i så stor grad som mulig, hvilken effekt usikkerheten i vindproduksjonen har og hvor robust kraftmarkedet er i forhold til prisvolatilitet og til slutt effekten av endrede karbonpriser og av bioenergi. Usikkerheten i vindkraften fører til at det blir brukt mer gasskraft enn det ville gjort med deterministisk vindkraft.

Vi dekomponerer modellen ved hjelp av Dantzig-Wolfe dekomponeringsmetode. Dessverre klarer vi ikke få algoritmen til å kjøre like store og større datasett enn det den udekomponerte modellen klarer. Men vi viser signifikante forbedringer i algoritmen når det gjelder antall iterasjoner og kjøretid. Vi reduserer antall iterasjoner fra 183 til 17 for et gitt eksempel.

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Abbreviations

AVI	Affine Variational Inequalities
BD	Benders Decomposition
CG	Convergence Gap
CP	Complementarity Problem
CQ	Constraint Qualification
CVE	Conjectural Variations-based Equilibrium
DWD	Dantzig-Wolfe Decomposition
EPEC	Equality Programs with Equality Constraints
HHI	Herfindahl-Hirschman Index
KKT	Karush-Kuhn-Tucker
LR	Lagrangian Relaxation
MAE	Mean Absolute Error
MCP	Mixed Complementarity Problem
MLCP	Mixed Linear Complementarity Problem
MP	Market Power
MPEC	Mathematical Programs with Equality Constraints
NA	Non-Anticipativity
NCP	Nonlinear Complementarity Problem
NWP	Numerical Weather Prediction
PX	Power Exchange
SFE	Supply Function Equilibrium
TSO	Transmission System Operator
VI	Variational Inequalities

Chapter 1

Introduction

In the last decade there has been a widespread deployment of renewable energy sources in the European electricity market. This development has been promoted by EU Directives on the promotion of electricity from renewable energy sources, last through Directive 2009/28/EC, setting binding national targets to fulfill the EU's "20-20-20"-goals of a 20 % renewables share in electricity production in 2020. Wind power will play a major role in fulfilling the goal, with wind contributing 19 % and 21 % to the electricity production in Germany and Great Britain in 2020. Germany is well underway in reaching the goal, with wind power responsible for 7.8 % of the electricity generation in 2011[1]. This poses challenges to the electricity system due to the intermittent and uncertain nature of wind power, and there have been several instances where consumers have been paid to consume electricity in the northern parts of Germany (Bloomberg, 2010).

At the same time, the EU has worked on opening and integrating the internal electricity markets, which previously had been characterized by isolated vertical utilities owning both generation and distribution. The liberalization is still ongoing, but the Commission is disappointed with the progress (Commission, 2011b). The concentration among electricity generators is high, with an average HHI[2] of 4,177 in the EU (Commission, 2011a).

This master's thesis is motivated by the mentioned growth of renewables and the possibility of exertion of market power in the electricity market, and we will therefore propose a power market model that can analyze the power market subject to stochastic generation technologies and producers with market power.

1.1 Main Contributions

Our master's thesis has three main contributions.

[1]Statistics from `http://www.wind-energie.de/infocenter/statistiken`

[2]The HHI Index is an indicator of the concentration in a market, a figure above 1,800 is considered to be a concentrated market, see Section 2.4

1.1.1 Presentation of a New Stochastic Power Market Model

We present a stochastic equilibrium electricity market model. As we will see, the proposed model is unique in its combinations of modeling aspects. The model consists of one or more power producers and a transmission system operator that all maximize their individual profit. The producers can be active in one or more nodes; the nodes are connected by a transmission network. Several different power generation technologies can be included, and the output may be modeled as either deterministic or stochastic. Other important aspects in the model are ramping costs and restrictions, seasonal pump storage, and a market power representation that can represent situations from perfect competition to Cournot competition. The model can be formulated both as a Mixed Complementarity Problem (MCP) and a Variational Inequality (VI) problem.

1.1.2 Application of the Model on the Northern European Electricity Market

We apply the model on the Northern European power market today and in 2020. The 2020 case corresponds to industry estimates and the National Renewable Action Plans that outline the 20-20-20 goals for each EU country. The model is run with stochastic wind, and we generate wind scenarios based on historical data from Denmark. The model is applied on various cases to show the effect of market power, stochastic wind, and the robustness of the power system to different wind forecasts. Changes in the prices, dispatch, consumption and transmission are highlighted.

1.1.3 Solution Procedure with Dantzig-Wolfe Decomposition

We decompose the problem to try to reduce the solution time, since computation times grow quickly when trying to solve the model with a large data set. The model is decomposed with Dantzig-Wolfe decomposition because of the structure of the problem. The problem is divided into a master problem and several subproblems which are iteratively solved. The decomposition is done on the non-anticipativity constraints, that binds the scenarios into a scenario tree. Thus each subproblem solves one scenario. We show the implementation of this method and its applicability on smaller problems. A great amount of effort has been spent to increase the rate of convergence and speed of the algorithm.

1.2 Thesis Overview

The master's thesis is a further development of the work presented in our project report in our Specialization Project (TIØ4500, fall 2011). The report focused mainly on developing a deterministic power market model, but a stochastic model was also presented as a proof of concept. The stochastic model used a scenario tree formulation, while we in this thesis improve the formulation, present a non-anticipativity formulation and a decomposition approach. In addition we use a scenario generation method to get a realistic wind forecasts, instead of entirely fictional scenarios in the project.

The remainder of the master's thesis is organized as follows. Chapter 2 contains the necessary background information for this thesis. At first, in Section 2.1 there is a short literature review of current modeling trends and how our model fits in with the existing literature, before the characteristics of the electricity market is explained in Section 2.2. Section 2.3 presents relevant theory of equilibrium modeling. Since these topics are the same, some parts are reused from the project report. Section 2.4 discusses how market power can be modeled and Section 2.5 stochastic programming. In Section 2.6 there is a brief overview on wind forecasting. At last in the background chapter, in Section 2.7, we present theory on decomposition and how it is applied on equilibrium problems.

In Chapter 3 the stochastic model is presented. The formulation is based on the deterministic model from the project, but is now stochastic and uses non-anticipativity constraints. Next the decomposed model is presented in Chapter 4. Chapter 5 presents the data set that is used to run the model and Chapter 6 provides some implementation details. Chapter 7 provides the results of our case studies, and Chapter 8 presents the results of the decomposed model. A general discussion about the model is done in Chapter 9, before the conclusion in Chapter 10.

An overview of all the different formulations of our model can be seen in Figure 1.1.

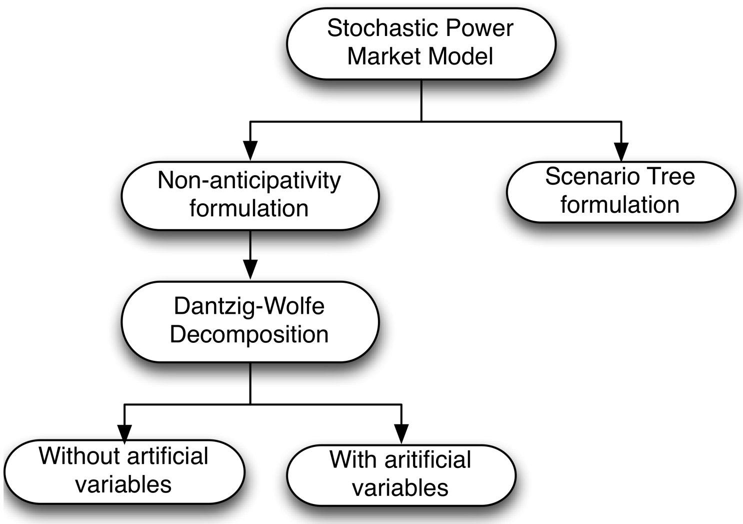

Figure 1.1: Overview of the presented models

Chapter 2

Background Information

This chapter presents an overview of relevant literature and theories on modeling electricity markets, with a special focus on equilibrium programming.

2.1 Current modeling trends

Ventosa et al. (2005) provides an overview of recent trends in electricity market modeling. They note that the models can be divided into three types:

- Optimization for one firm
- Market equilibrium considering all firms
- Simulation models

Single firm optimization models are used when a high representation of detail is necessary, such as the unit commitment problem, when a power plant should start and stop. The firm maximizes the profit and the price is either exogenous or dependent on the firm's production as in Varian (1992). In Gross and Finlay (1996) the exogenous price is deterministic and the simple comparison of each generator's marginal cost and the price determine the production. The model can also include an uncertain price as in Rajaraman et al. (2002).

For long-term planning or market power analysis, equilibrium models have been widely used, but also simulation models since they allow for a greater complexity. Equilibrium problems includes both models based on Cournot competition (includes conjectural variations approach) and supply function equilibrium approach (SFE) which are based on the concept of Nash-equilibrium. Hobbs (2001) is a good example of the former, where two Cournot models of imperfect competition among electricity producers are formulated as mixed linear complementarity problems. The SFE approach was introduced by Klemperer and Meyer (1989) and is much harder to compute. It turns out that the mathematical structure of Cournot models is a set of algebraic equations while the mathematical structure of SFE is a set of differential equations (Ventosa et al., 2005).

Equilibrium models are based on a formal definition of equilibrium, which is mathematically expressed by equations. If the set of equations is too large and complex, simulation models can be used instead. Simulation models additionally enable the possibility of including almost any kind of strategic behavior, but this requires that the assumptions in the simulation are theoretically justified. In Otero-Novas et al. (2000) a simulation model is presented, where each firm maximizes its profit subject to technical constraints on thermal and hydro generating units. There is a dynamic procedure where each firm updates its strategy in an iterative process. The next paragraphs show a few interesting modeling aspects.

One pan-European electricity model is the large scale general non-linear model ELMOD developed by Leuthold et al. (2010). The model encompasses over 2,000 nodes and 3,000 power lines in Europe. The model maximizes social welfare, subject to technical constraints. The model has a wide area of applications, among others, Weigt et al. (2010) used ELMOD to study the alternative to building HVDC (High Voltage DC) cables from the North Sea coast to the demand centers in the south instead of traditional AC lines. They found the plan to be superior and suggested it would be more politically feasible. There are some drawbacks to the model though, all participants act perfectly competitive, and the uncertainty of wind and solar energy is disregarded. This has become more important in recent years, due to the EU renewables targets. A proof of concept for stochasticity of wind was done by Huppmann and Kunz (2011), where they showed that stochasticity of wind could impact the generation dispatch in the morning hours.

Vespucci et al. (2012) formulated a stochastic linear programming model to investigate the integration of wind and pump storage[1] in Italy. The focus was on the stochasticity of wind, and did not venture into grid related issues. They concluded that a stochastic formulation yielded higher profits compared to naively assuming the expected value, and that quantile regression described the wind uncertainty best.

Schill and Kemfert (2009) introduced the model ElStorM, which formulates a Cournot competition model with strategic pump storage. The model was applied to different cases in Germany, and found that strategic operators would under-utilize the storage facilities. The model did not take transmission, renewables or uncertainty into account, as the purpose of the study was to identify the effects of market power on pump storage utilization.

When reviewing the literature, we haven't located any models that capture both stochasticity of wind, pump storage, and different market power behavior of the participants. This report will therefore formulate such a model to study the consequences of changes in the power market. We will also apply a Dantzig-Wolfe decomposition scheme based on Fuller and Chung (2005), where each subproblem represents a scenario. This multi-stage DWD approach hasn't been shown in the literature on equilibrium models yet.

[1]Pump storage: Stores energy in the form of water. During time of low power prices, water is pumped into reservoirs, and is later used for power generation when the price is higher. See Section 3.2.5

2.2 Electricity Markets Fundamentals

This section intends to describe some aspects of the electricity market in Northern Europe. The section does not intend to give a complete picture, see Wangensteen (2007) or Kirschen and Strbac (2004) for further details. These books are also the main sources for this chapter when there are no references. The presented aspects are followed up in the model formulation.

2.2.1 Characteristics of Electricity Markets

Electricity markets have some important characteristics that differ from other commodity markets. First of all, generation and consumption are instant and need to be continuously in balance. It is necessary to transmit and sell the electricity immediately after production. Secondly, electricity cannot normally be stored in an economic manner. The only realistic exception is pump storage plants, where water is pumped into reservoirs for later power generation to exploit price differences.

The transmission of electricity is also subject to the physical laws of Ohm and Kirchoff. Ohm's law causes losses in the grid, while Kirchoff's laws govern the flow and is the source of some counterintuitive results; such that an increase in competition in one node can result in higher prices elsewhere and a lower total consumer surplus (Berry et al., 1999b).

The consumption of electricity also varies through the day (day/night), week (weekday/weekend) and season. Other important characteristics are that electricity cannot be traced back to the producer that actually generated the unit, and it is a homogeneous commodity that is essential to society. Every household and company is connected and relies on a stable connection.

These characteristics, coupled with the fact that the short run demand elasticity is very low and few users are subject to real time pricing, means that electricity markets are vulnerable to market power exploitation.

2.2.2 Roles and Participants

There are many actors involved in producing, trading on different levels and consuming. Each actor can have one or more roles. The composition of roles differs a bit between countries but is mainly:

- *Producers.* Produce power and sell it on the market.
- *Grid Companies.* Responsible for operation and maintenance of the electricity network.
- *Retailers.* Sells electricity to end-users. They are often generating companies or traders.
- *End-users/ Final customer.* Includes both industrial players and households. Their consumption is affected by the price.

- *Regulator.* Represents the political authorities, and ensures that laws and regulations are followed. They also make sure that public interests are taken into account.
- *Power Exchange (PX).* Also called the market operator. Receives bids for sales and offers of generation and matches these to settle quantities and prices. Controls the short-term physical spot market.
- *Transmission System Operator (TSO).* The TSO is responsible for the security of supply; hour-by-hour balance and for keeping sufficient capacity margins in the generating system. They also make sure the frequency and voltage is within an acceptable range.
- *Balance responsible entities.* Works out a balanced schedule each hour and keeps the real-time balance close to that schedule. Penalties are imposed if there are deviations from the schedule.
- *Traders and brokers.* Traders buy and sell on the electricity market. Includes most generating companies. Both physical and financial trade.

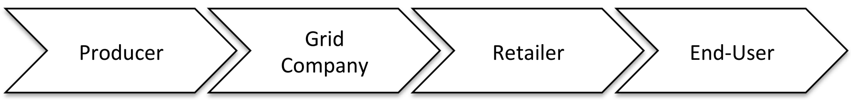

Figure 2.1: Key roles in a simplified electricity value chain

2.2.3 Market Design

Market design concerns the rules and practical arrangements regarding how the different participants operate. It includes for instance grid access, power trade and price settlement which are briefly explained in this section.

One possibility is to have a single buyer (SB). The buyer buys electricity from the generators and sells to the end-users. This gives a monopoly position to the single buyer and is therefore not preferred and not used in Northern Europe. In an efficient market all players need full access to the market. The players need both legal access and practical arrangements concerning metering and billing. This is the basis for the Third Party Access (TPA) principle. There are two types of TPA: rTPA (regulated) assumes there is a regulatory authority and that grid access is based on regulated prices. The nTPA (negotiated) assumes negotiated prices.

Electricity might be traded both on long-term and short-term contracts (Huisman et al., 2007). The long-term market consists of forward contracts for different delivery periods (e.g. from a few weeks to a year). Short-term contracts are traded on either the day-ahead or intra-day market. The day-ahead market is the electricity market for the following day. Electricity producers offer electricity on this market based on their ability to produce electricity for a specific period on the following day with separate prices each specific period. Intra-day markets involve trade of electricity delivered the same day, and often within 30 minutes.

The prices in the day-ahead market are settled in a price clearing process. There are mainly two categories; periodic clearing and continuous auction. In a periodic clearing, the clearing process is done in one operation. All the players bid and the power exchange (PX) collects the information through one or a few set of bids. The process is repeated with regular intervals, for instance once a day. The PX decides the quantity and price based on the intersection between the supply and demand curves. All players receive the same price, which should be equal to the short-term marginal cost in a competitive market. Periodic clearing is used by for e.g. Nord Pool, and other PXs in Europe. In a continuous auction the clearing process is done continuously. Bids, both sales and purchases are displayed on a market place. Buyers and sellers can pick the offers they like. The price for each transaction is stated in the bid (pay-as-bid price).

There are many power exchanges in Europe that handle trade between different actors; some are listed in Table 2.1.

Power Exchange	Coverage
Nord Pool Spot	Norway, Sweden, Denmark, Finland, Estonia
EPEX Spot	France, Germany, Austria, Switzerland
APX-ENDEX	Netherlands, United Kingdom
Belpex	Belgium

Table 2.1: Some important electricity spot markets (Rademaekers et al., 2008).

In recent years, there has been introduced a market coupling between some of the markets in Europe. This means that a buy order on one exchange may be fulfilled by a sell order on another exchange, subject to the capacity constraints between them. The Nordic region is coupled with the Central Western European region through the joint venture European Market Coupling Company (EMCC, 2011).

2.2.4 Takeaways

From this description, it is apparent that electricity supply and demand must be continuously in balance. For short-term modeling, it is also important to have hourly resolution to capture the variation in demand. Due to the liquid spot markets and the recent market coupling, the whole market can be modeled as a network with transmission constraints. The free market ensures that everyone is subject to the same market price. Consideration must be made to what level of physical detail is necessary on the transmission network, and what market form that should be modeled. The most relevant actors for a model are the Transmission System Operator that controls the flow on the power lines, the individual producers, and the end customer.

2.3 Equilibrium Modeling

This section explains the concept of equilibrium modeling and application on Cournot competition. An equilibrium problem can be used to find out how different actors behave strategically in a market. The basic idea is to simultaneously solve different actors' optimization problems within a system, and hopefully reach an equilibrium point that is unique and globally optimal. The concept of a market equilibrium is fundamental to economics, and Arrow and Debreu (1954) first proved the existence of an equilibrium in a competitive economy. Later work by among others Frank Jr. and Quandt (1963) and Szidarovszky and Yakowitz (1977) proved the existence of an equilibrium for the Cournot problem (Cournot, 1838). The existence of an equilibrium is a prerequisite for equilibrium programming. Important aspects to investigate are equilibrium problem classes, Cournot competition, KKT conditions, optimality conditions, and variational inequalities.

2.3.1 Electricity Market Modeling

There are many ways of modeling the electricity market. It is quite common to assume a perfect competitive market and maximize the social welfare, which is the sum of the producers' profit and the consumer surplus. Another option is to assume the market operates with imperfect competition and that firms are able to influence the electricity price. From microeconomics we know that the well known Cournot and Stackelberg models can be used in an oligopoly with market power. In the Stackelberg model one of the companies, the Stackelberg leader, sets its output before all other firms, and all the other players maximize their profit given the Stackelberg leader's decision. The Stackelberg leader has a first mover advantage (Pindyck and Rubinfeld, 2009)[2]. The Cournot model can be used if we assume there is no Stackelberg leader and all players decide their production level simultaneously. Both Cournot and Stackelberg competition can be formulated as equilibrium problems.

2.3.2 Equilibrium Problem Classes

There are different classes of equilibrium problems that are divided into one level or bilevel problems. Bilevel problems are described in Luo et al. (1996). In such problems the decisions are decided on two levels. The bilevel problems are further divided into mathematical programs with equilibrium constraints (MPEC) and equilibrium programs with equilibrium constraints (EPEC). In an MPEC the constraints itself are the result of an equilibrium problem and the objective function is a single firm's optimization problem. The Stackelberg game can be formulated as an MPEC, where one firm is dominant and the other firms makes decisions based on the Stackelberg leader's decision. An example of an EPEC is a multi-leader

[2]By announcing the capacity first creates a fait accompli. No matter what your competitors do, your output will be large. Your competitors must take your output as given and set a low level of output for itself, to not drive down the price so both they and the Stackelberg leader lose money.

follower game, where each leader is solving an MPEC. The objective function gives the solution to another equilibrium problem. According to Midthun (2007), the solution cannot be guaranteed to be unique.

One-level problems are described in Gabriel et al. (2012). They are also called Mixed Complementarity Problems (MCP), and can be classified according to whether or not they include equality constrains, and whether or not the constraints are linear (see Table 2.2). MCPs can have both equality and nonlinear constraints, while the Mixed Linear Complementarity Problems (MLCP) must have linear constraints. Nonlinear Complementarity Problems (NCP) have only inequality constraints, while in an Linear Complementarity Problem (LCP) the constraints must be linear. One application of MCPs is in solving Cournot problems which is demonstrated in the next sections.

	Non-linear	Linear
With equality constraints	MCP	MLCP
Only inequality constraints	NCP	LCP

Table 2.2: One-level equilibrium problems

2.3.3 Cournot Competition

Cournot competition is within the economic field of game theory. Game theory was formally described in von Neumann and Morgenstern (1944), and has later been developed and made applicable for different disciplines. Game theory is especially useful in describing behavior in markets with many players, where they act strategically. Each player makes decisions based on all the other players' decisions. For instance all players must decide how much they are going to produce in the next time period. The Cournot model is described in Pindyck and Rubinfeld (2009). There must be a fixed number of players, and they are not allowed to cooperate. The firms have market power, which means that the production level affects the price. All the players choose the quantities simultaneously of a homogeneous product. In addition all the players are behaving economically rationally and try to maximize their own profit.

The essence of the model is that the players treat the output level of their competitors as fixed when calculating their own production level. The following is a small example for two producers. Producer $p \in \{1, 2\}$ has the profit function π_p in a duopoly. $P(x)$ is the price, MC the marginal cost that is equal for both players, and INT and SLP that are the intercept and slope of the inverse demand function.

$$\pi_p = P(x_T) \times x_p - MC \times x_p \tag{2.1}$$

The inverse demand function is given by

$$P(x_T) = INT - SLP \times x_T \tag{2.2}$$

$$x_T = \sum_p x_p \tag{2.3}$$

To find player p's best choice of production, we set the derivative equal to 0:

$$\frac{d\pi_p}{dx_p} = \frac{dP(x_T)}{dx_p} \times x_p + P(x_T) - MC = 0 \tag{2.4}$$

We also have

$$\frac{dP}{dx_p} = \frac{dP(x_T)}{dx_T} \times \frac{dx_T}{dx_p} \tag{2.5}$$

$$\frac{dP(x_T)}{dx_T} = -SLP \tag{2.6}$$

In Cournot problems, $\frac{dx_T}{dx_p} = 1 \quad \forall p$ and we have:

$$\frac{d\pi_p}{dx_p} = INT - SLP \times x_T - SLP \times x_p - MC = 0 \tag{2.7}$$

Solving for actor 1 by substituting x_T we derive the best response function for producer 1, depending on the output decision of producer 2.

$$x_1 = \frac{INT - MC}{2 \times SLP} - \frac{x_2}{2} \tag{2.8}$$

By substituting player 2's best response function into player 1's, we obtain a equilibrium with the following prices and quantities:

$$x_1 = \frac{INT - MC}{3 \times SLP} \tag{2.9}$$

$$P = \frac{INT + 2 \times MC}{3} \tag{2.10}$$

By symmetry $x_2 = x_1$. Notice that price will be higher than the marginal cost, as we have imperfect competition. According to Pindyck and Rubinfeld (2009) a Nash-Equilibrium is a state in which each firm is doing the best it can given what its competitors are doing. With these outputs, none of the players have an incentive to deviate from the decided quantity and a Nash-Cournot equilibrium has been reached. In order for the solution to be unique, the demand and marginal cost functions have to be twice differentiable and the functions need to be concave (Szidarovszky and Yakowitz, 1977).

In this case it was a small problem and it was easy to solve the two optimization problems simultaneously. Both players established a best response function to the other player's decision (Pindyck and Rubinfeld, 2009), resulting in a small set of equations. Symmetry can be used on larger problems with more players if they are identical with the same cost characteristics. In reality they are different and there are other constraints, such as production and transfer limits, that make the problem too complex to solve directly. In practice a smarter method must be used, such as the MCP formulation (Gabriel et al., 2012), that rely on the KKT conditions of the different players.

2.3.4 Karush-Kuhn-Tucker Conditions

The Karush-Kuhn-Tucker (KKT) conditions can be derived using the Lagrangian function (Lundgren et al., 2010), and uses the concept complementarity. Complementarity is described in the literature and Gabriel et al. (2012) gives an excellent description in the context of equilibrium modeling. The general form of complementarity is to find a vector x that satisfies the complementarity condition $f(x)^T x = 0$. Then for each element i of the vector, either x_i or $f(x_i)$ must equal zero. This is written mathematically with the perpendicular operator ($\perp$). x and $f(x)$ are said to be complementary when $x \perp f(x)$.

The following optimization problem is defined.

$$
\begin{aligned}
\text{Min}_x \quad & f(x) \\
\text{s.t.} \quad & g_i(x) \geq 0, (\lambda_i) \quad \forall \quad i \in I \\
& h_j(x) = 0, (\mu_i) \quad \forall \quad j \in J \\
& x \geq 0
\end{aligned}
\tag{2.11}
$$

The KKT conditions, written with the complementarity notation, are then:

$$
\begin{aligned}
\nabla f(x) - \sum_i \lambda_i \nabla g_i(x)^T + \sum_j \mu_j \nabla h_j(x)^T = 0 \\
0 \leq \lambda_i \perp g_i(x) \geq 0 \\
h_j(x) = 0 \\
\mu_j - \text{free}
\end{aligned}
\tag{2.12}
$$

The partial derivative of the objective function is a non-negative linear combination of the constraint gradients, and defines dual feasibility together with $\lambda_i \geq 0$. The equations $g_i(x) \geq$ and $h_j(x) = 0$ define primal feasibility and states that in order for a point to be a candidate for a local minimum solution, it must be feasible. The perpendicular operator defines the complementarity and states which constraints are active in the solution point. If the dual variable of an inequality constraint is greater than zero, the constraint is active.

According to Lundgren et al. (2010), KKT conditions are necessary if some regularity conditions, also called Constraint Qualification (CQ) conditions are satisfied. We will not go into details about CQs but refer to Bazaraa et al. (2005) where several CQs are discussed. In order for the KKT conditions to also be sufficient for optimality, the problem must satisfy the KKT conditions and be convex. The solution will then be both a local and a global optimum.

The KKT conditions to a problem are therefore sufficient for optimality for a minimization problem if the objective function $f(x)$ is convex and the feasible solution space is convex. It is proven that function $f(x)$ is convex if and only if its Hessian matrix is positive semi definite. In order to have a unique solution, the problem must be strictly convex, which means that the Hessian must be positive definite (Lundgren et al., 2010).

2.3.5 Solving a Nash-Cournot Equilibrium Problem as an MCP

A Cournot problem cannot be solved isolated for each player, since the problem for one player depends on all the other players. This means that all the problems must be solved simultaneously. Since the KKT conditions for all the players must be satisfied in an equilibrium solution, the problem can be solved by aggregating the KKT conditions and linking them with a market-clearing condition, yielding an MCP. In other words a Cournot problem can be formulated and solved as an MCP, by deriving the KKT conditions of all the different actors' optimization problems (Gabriel et al., 2012). This approach is used by e.g. Hobbs (2001), Gabriel et al. (2005) and Egging et al. (2010). An advantage of the MCP formulation is that you can easily manipulate both the primal (physical) and the dual (price) variables (Gabriel et al., 2012).

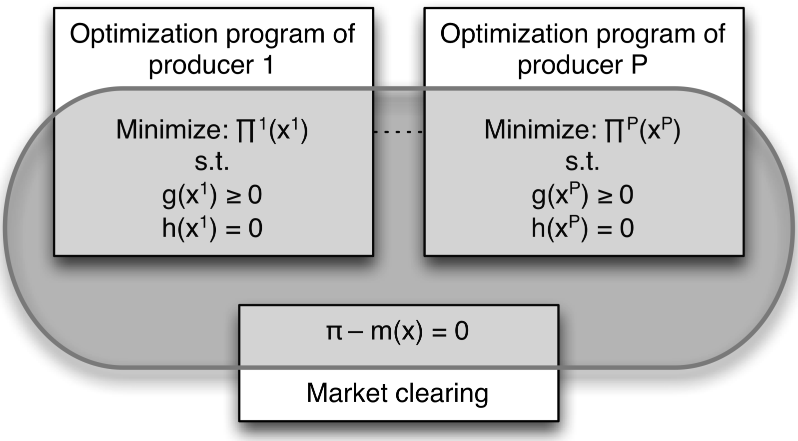

Figure 2.2: Model structure of an equilibrium problem

Summarized, the steps in solving a Nash-Cournot Equilibrium Problem are:

1. State each player's optimization problem
2. Check that the KKT conditions are necessary and sufficient
3. Find the KKT-conditions for each player
4. Merge the KKT-conditions to a large complementarity system

5. Add the market-clearing condition and other relevant constraints

6. Solve the aggregated system in one operation

2.3.6 Variational Inequalities

Variational inequalities (VI) is a mathematical framework for studying both optimization and equilibrium problems. Facchinei and Pang (2003) provides a comprehensive coverage of Variational Inequality theory and applications. If not otherwise noted, the theory presented in this section is from that book.

The Variational Inequality problem $VI(F, K)$ is to find a vector x^* in an Euclidean subspace K with a mapping $F : K \to \mathbb{R}^n$ such that

$$F(x^*) \cdot (x - x^*)^T \geq 0, \quad \forall x \in K \tag{2.13}$$

This can be interpreted geometrically that $F(x^*)$ needs to form an acute angle with any vector originating from the solution point x^* to any point in the feasible area K, see Figure 2.3[3]. Formally we say that $-F(x^*)$ must belong to the normal cone to K at x^*.

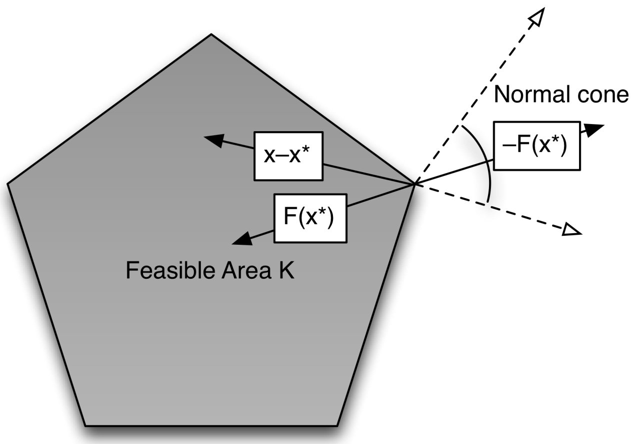

Figure 2.3: Graphical interpretation of a VI

The connection with complementarity problems arises when the feasible set K is a cone. The problem $CP(F, K)$ is to find a vector $x \in \mathbb{R}^n$ that satisfies

$$K \ni x \perp F(x) \in K^*$$

where K^* is the dual cone to K, which means that any vector in K^* forms a non-obtuse angle with any vector in K.

[3] As a reminder the dot product of two vectors is $\|x\| \times \|y\| \times \cos\theta$

A vector x solves the $VI(F, K)$ if and only if x solves $CP(F, K)$. This can be seen by inserting $x = 0$ and $x = 2x^*$ into the definition of the VI (equation 2.13), which together produce the result that $x^* F(x^*) = 0$, the complementarity condition. By definition, a vector x that solves $VI(F, K)$ fulfills $x \in K$. It is then only necessary to show that $F(x^*) \in K^*$ is satisfied. Since $x^* F(x^*) = 0$ we know that $xF(x^*) \geq 0$. This means that the vector $F(x^*)$ forms a non-obtuse angle with x, and since x is in K, we have the definition of the dual cone and $F(x^*) \in K^*$ is fulfilled. The converse can also be trivially seen.

There are more of these special relationships, and of particular interest is the fact that an Affine VI ($AVI(F, K)$) is equivalent to an MLCP. A VI is called affine if F is affine and K is a polyhedral set. Any solution to an AVI, with an augmented mapping $\tilde{F}$, must also be a solution to the MLCP. In fact, the equivalent MLCP is known as the KKT system of the VI.

Let $K \equiv \{x \in \mathbb{R}^n : g(x) \geq 0, h(x) = 0\}$. $AVI(F, K)$ only has a solution if there exists vectors λ and μ such that

$$0 = F(x) - \lambda \nabla g(x)^T + \mu \nabla h(x)^T$$

$$0 \leq \lambda \perp g(x) \geq 0$$

$$h(x) = 0$$

$$\mu - \text{free}$$

The corresponding augmented mapping $\tilde{F}$ would be

$$\tilde{F}(x, \lambda, \mu) \equiv \begin{pmatrix} F(x) - \lambda \nabla g(x)^T + \mu \nabla h(x)^T \\ g(x) \\ h(x) \end{pmatrix}$$

It is trivial to see that if we let $F(x)$ be the derivative of the objective function, we have the familiar KKT system as presented in Section 2.3.4.

For a multiple player Nash-Equilibrium problem, the individual KKT-systems can be aggregated together with a market clearing condition in the constraint set, to form an aggregate VI.

2.4 Market Power Representation

Market power is the ability of either seller or buyer to affect the price of a good (Pindyck and Rubinfeld, 2009). In a Cournot model, as shown in Section 2.3.3, firms use their market power to reduce output and raise prices to maximize their own profit. In a real market a firm can have less market power than a Cournot player, but still more market power than a price taker. Two common measures to assess the competition in a market are the Herfindahl-Hirschman Index (HHI) and the Lerner index (Wang et al., 2004).

The HHI is an indicator of the concentration in a market (Hirschman, 1964). The index is the sum of the squared market shares of each firm, and a figure above 1,800 is considered to be a concentrated market. As an example, the concentration among electricity producers in the EU is high, with an average HHI of 4,177 (Commission, 2011a). The indicator is commonly used by competition authorities when assessing mergers (Wang et al., 2004).

The Lerner index (Lerner, 1934) which measures the price-cost margins is another index used to evaluate market power. While the HHI focus on the market, the Lerner index focus on each firm's market power. The index describes the relationship between price elasticity and a profit maximizing firm's price margin. It can never be greater than one, which is the monopoly.

$$L = \frac{P - MC}{P} = -\frac{1}{E_d}$$

P is the price, MC is the firms' marginal cost and E_d is the demand elasticity. As an example, Wolak (2003) used the index to analyze the liberalization of the Californian power market, which experienced significantly higher prices and outages in 2000 and 2001.

2.4.1 Market Power Modeling

Market power has been modeled in different ways. Both equilibrium, linear and quadratic problems can capture aspects of market power. Including a mark-up is probably the simplest way to implement market power (Smeers, 2008). The mark-up is defined as the difference between the cost of a good and its market price. The easiest form of mark-up usage is to start with a perfect competition model and add a value to the pure competition price. Smeers (2008) argues that an exogenous mark-up is equally arbitrary as for instance conjectural variation explained in Section 2.4.2. Mark-ups are easy to implement in models, to interpret and to compare with observations. Steen and Salvanes (1999) is one example of mark-up usage. A parameter λ is used to adjust the market power, and the model is illustrated with an analysis of the salmon market. Mark-ups have been used for linear and quadratic programs, because they sufficiently capture market power aspects and are less complex than equilibrium problemsin many cases. Equilibrium problems have the advantage that they can model market power as it is described in microeconomics, such as Cournot competition. They also enable a practical way to model different market power configurations between perfect competition and

Cournot competition (Chyong and Hobbs, 2011). We therefore only discuss market power in equilibrium problems in the rest of this section.

According to de Haro et al. (2007), commonly used approaches to include market power in equlibrium models are:

- Supply function equilibrium (SFE, e.g. Anderson and Xu (2005))
- Cournot-based or Stackelberg-based equilibrium (e.g. Hobbs (2001))
- Conjectural variations-based equilibrium[4] (CVE)

Willems et al. (2009) compares Cournot and SFE, and suggests that Cournot, and therefore also CVE models are better fit for short-term analysis, since it is easier to include network and generation constraints. Cournot models are on the other hand highly sensitive to the demand elasticity, but using a contract factor for the amount of electricity sold in the forward market can make the outcome more realistic. SFE does not have this problem, but are confined to small problems (a few nodes), as it is difficult to calculate an equilibrium, and there may not even exist one (Berry et al., 1999a). SFE needs each firm's optimal supply function for a good to be matched to the aggregated supply and demand functions. It is computationally complex, and therefore not efficient in most cases and will not be further discussed. The CVE based equilibrium includes the Cournot model and is discussed in the next section.

2.4.2 Conjectural Variations Approach

The term conjectural variations refers to the assumption a firm makes about the reaction of other firms to its own decision (Figuières et al., 2004). Cournot, Bertrand and Stackelberg models can be interpreted as conjectural variations models (Carlton and Perloff, 2005). In the Cournot model, a firm assumes that the other firms will not be affected by its own output decision.

In Egging et al. (2010) a mixed conjectural variation approach is applied. A similar concept is also presented in Carlton and Perloff (2005). A market power factor $MP_p = \frac{d \sum_p sales_p}{dsales_p}$, the change in total sales when producer p changes his sales, is introduced to be able to adjust the market power. This method is deduced in the illustrative example in Section 2.4.3. Note that values between one and zero are a heuristic way of dealing with market power, where the extent of market power lies between perfect competition and Cournot competition (Egging et al., 2010). Firms can also have different levels of market power, called hybrid markets (Egging et al., 2010). Often some producers are dominant, while a competitive fringe acts as price takers.

García-Alcalde (2002) proposes another conjectural variations approach, which is further developed by de Haro et al. (2007). A firm must consider the competitors

[4] Conjectural Variations Approach is also known as Cournot conjecture approach, conjectural supply function approach, conjectural demand elasticity approach and price response approach. For more details on the differences please see Díaz et al. (2010) that conclude that the methods are equivalent

reactions when deciding the production, given by the competitors' individual supply and residual demand curves for a good. Each firm's marginal revenue depends on the firm's demand elasticity[5], and changing the elasticity results in different levels of market power. A method to estimate the elasticity based on the marginal cost is proposed. de Haro et al. (2007) provides an even more advanced method, advanced implicit estimation, where the firms' elasticities are found iteratively by comparing the model to real data. The method seems to give good results applied on real data.

Both methods increase the difference between producer p's marginal revenue (MR_p) and the price, as seen in equation (2.14) and (2.15). P is the price, E_p (negative) is the demand elasticity and $sales_p$ is the producer's sales. In the equilibrium, the marginal revenue equals the marginal cost, and the price is therefore higher than the marginal cost.

Marginal revenue with demand elasticity

$$MR_p = P(1 + \frac{1}{E_p}) \leq P \tag{2.14}$$

Marginal revenue with a market power factor

$$MR_p = P + \frac{dP}{d\sum_p sales_p} \times MP_p \times sales_p \leq P \tag{2.15}$$

To cover all aspects of market power is a difficult task. Thus all methods trying to represent market power has weaknesses. Pure Cournot equilibrium models are criticized to barely provide credible prices (García-Alcalde, 2002), being too high compared to the real market. In hybrid markets there are also examples where Cournot players are better off as price takers (Ulph and Folie, 1980). Cournot players assume the competitive fringe keeps their production unchanged and try to limit the production to increase the price. Ulph and Folie (1980) show that a problem arises when the fringe isn't meeting the Cournot players conjecture and start to produce more.

Smeers (2008) claims that conjectural variations has no foundation in economics, that it is just a computational trick to calibrate the model. From an economic perspective, the conjecture, such as the MP factor, should represent a firm's belief regarding how its competitors will react if it changes its output. Lindh (1992) notes that this is not really the case in practice, but that the conjectural variation can be interpreted as a measure of the deviation against Cournot, or expectation of dynamic effects that are not modeled. The choice of market power representation is therefore a trade-off between a model with complete economical foundation and a model with credible results by using conjectural variations to calibrate the model. For more details about conjectural variations see Figuières et al. (2004)

[5] $MR_p = P(1 + \frac{1}{E_p})$, $E_p \approx \frac{P}{MC-P}$, MR_p is the marginal revenue, p is producer, P is price, E_p is the producer's demand elasticity (user-supplied parameter) and MC is the marginal cost of the most expensive generation technology producing.

2.4.3 Illustrative Example of Conjectural Variations Approach

To illustrate how market power is implemented in the model we look at the two cases, first a price taker and then a player exploiting his market power. We use the example in Section 2.3.3 as a starting point. $P(x_T)$ is price, MC is marginal cost, x_p is sales for producer p, and INT and SLP define the interception and slope of the inverse demand curve. In contrast to the Cournot competition example there are other players in this market. Equation 2.4 and 2.5 are combined to form the profit maximization equation:

$$\frac{d\pi_p}{dx_p} = \frac{dP}{dx_T} \times \frac{dx_T}{dx_p} \times x_p + P(x_T) - MC = 0 \tag{2.16}$$

If producer one is a price taker he can't influence the price, $\frac{dP(x_T)}{dx_1} = 0$, which means that the price equals marginal cost.

$$P = INT - SLP \times x_T = MC \tag{2.17}$$

If producer one exerts market power, $\frac{dP(x_T)}{dx_1} \neq 0$. We introduce the MP_p factor for each producer p, and defines it as the partial derivative of the total sales with respect to the producer's own sales (Egging et al., 2010). The MP_p factor describes how much the total sales increase if the player's sales is increasing. For a Cournot player $MP_p = 1$, because a Cournot player assumes the production of the other players are fixed when deciding the production.

$$MP_p = \frac{d\sum_p x_p}{dx_p} \tag{2.18}$$

This gives us

$$\frac{dP}{dx_1} = \frac{dP}{dx_T} \times \frac{dx_T}{dx_1} = -SLP \times MP_1 \tag{2.19}$$

$$\frac{d\pi_1}{dx_1} = -SLP \times MP_1 \times x_1 + P(x_T) - MC = 0 \tag{2.20}$$

$$\frac{d\pi_1}{dx_1} = -SLP \times MP_1 \times x_1 + (INT - SLP \times x_T) - MC = 0 \tag{2.21}$$

Player one's best response function

$$x_1 = \frac{INT - MC}{SLP \times (MP_1 + 1)} - \frac{x_T - x_1}{MP_1 + 1} \tag{2.22}$$

Producer one uses this equation to find his optimal price and production output. If player p is a Cournot player then $MP_p = 1$, and if the player is a price taker then $MP_p = 0$. The producers best response function in the Cournot example (equation 2.8) can easily be obtained from equation 2.22 by setting $MP_1 = 1$. If MP_p is between 0 and 1, then the player has some degree of market power. Table

2.3 outlines the different choices for the market power parameter. In special cases, the market power factor can also be greater than 1 (Carlton and Perloff, 2005). If two firms are colluding, the firm expects that the total output will change with more than one unit when he changes his production. If there are only two firms selling in the market, and $MP_p = 2$ they will produce the cartel output.

MP factor	Economical interpretation	Comment
$MP = 0$	Perfect competition	Could model with one agent
$0 < MP < 1$	No direct interpretation	Enables calibration of the model
$MP = 1$	Cournot competition	Often barely credible prices

Table 2.3: Interpretation of various MP factors

2.5 Stochastic Programming

Stochastic programming is about making decisions under uncertainty (Birge and Louveaux, 1997), and was first introduced in Dantzig (1955). For a detailed presentation of stochastic programming we refer to Ruszczyński and Shapiro (2003) and Kall and Mayer (2010).

The aim of stochastic programming is to find optimal decisions given uncertain information. Stochastic means that some parameters are uncertain and programming refers to the fact that various parts of the problem can be modeled as linear or nonlinear programs. Typically you face uncertain parameters, such as wind power production or demand, that can be reasonably described by stochastic processes in discrete time. When time passes, information about the uncertain parameters are revealed. Decisions are always made based on the information available at the time and on probabilistic information about the future (Eichhorn et al., 2010). The most basic form is the two-stage recourse problem, where you first take an initial decision that is best considering the probabilities of the future outcomes, before you are able to take a recourse decision when the information is revealed in the second time period. The general form is the multi-stage recourse problem, where the information is revealed in several stages. Higle (2005) provides a good introduction that is used as the basis for the next sections.

2.5.1 Scenario Trees

The two- and multi-stage problems can be described by a scenario tree, or event tree. The tree is a discrete description of the stochastic world, which often is an approximation of the real world, since it is not possible to represent the infinite possibilities of nature. An example of a tree is illustrated in Figure 2.4.

The structure in a scenario tree is based on stages, where new information is revealed in each stage. A stage may be comprised of one or more time periods, so that the uncertain parameters can be revealed for several time periods at once. In Figure 2.4 the scenario tree branches in each second time period, and each stage is therefore two time periods. An example would be that you learn how much wind power that will be produced in the next two hours, and the decision would be to decide on the hydro production to meet the demand. In each branching point the probability for each branch is defined. Only in the beginning of each stage a decision is made about the production output (Wallace, 2002). If there are many time periods in each stage, the decisions for all the time periods in the stage are decided simultaneously.

The nodes in the scenario tree are also called event nodes, and technically there would be one event node per stage as there is only one information event per stage. But, it is easier to grasp the underlying time structure when each node represents a time period, since the length of a stage can vary. All the nodes, except the leaf nodes, are connected to one or more child nodes and one parent node, and each path through the scenario tree defines one scenario. These concepts can be used to describe the scenario tree, either directly or through the non-anticipativity formulation. As we'll see later in Section 2.7, the non-anticipativity formulation

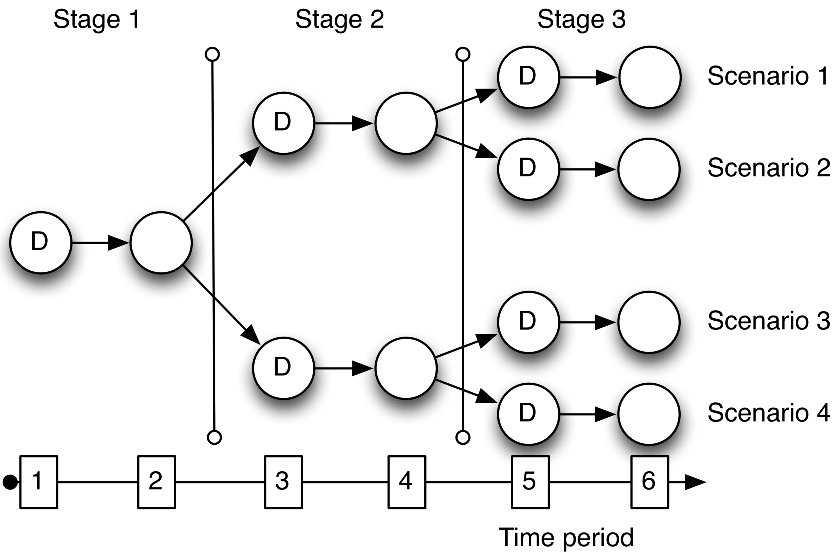

Figure 2.4: Scenario tree with three stages, decisions are made in nodes marked D.

enables a special problem structure which can be decomposed by scenarios.

2.5.2 Scenario Tree Formulation

The scenario tree formulation explicitly represents the information process as described above. The model can be formulated using the parent-child connections between the nodes, which separate the model into the time structure, and the probability of each node in the stage. Each parent node can have one or more child nodes and the probabilities can differ between the nodes. This can be exploited to make any scenario tree structure.

2.5.3 Non-Anticipativity Formulation

The alternative formulation method, the non-anticipativity formulation includes one formulated problem for all possible scenarios. The tree is flattened so that each scenario has its own separate variables in each stage. Nodes that have the same parent in the scenario tree are called sibling nodes. Non-anticipativity constraints are then added to the problem to ensure that the structure and information processes are correct. The non-anticipativity constraints ensure that the decisions in all sibling nodes are equal.

For each scenario $s \in S$, let C_s represent the objective function coefficients for each scenario, and let X_s denote the feasible set for each s. The probability for each scenario is $PROB_s$. B_n is the set of sibling nodes that correspond to the node

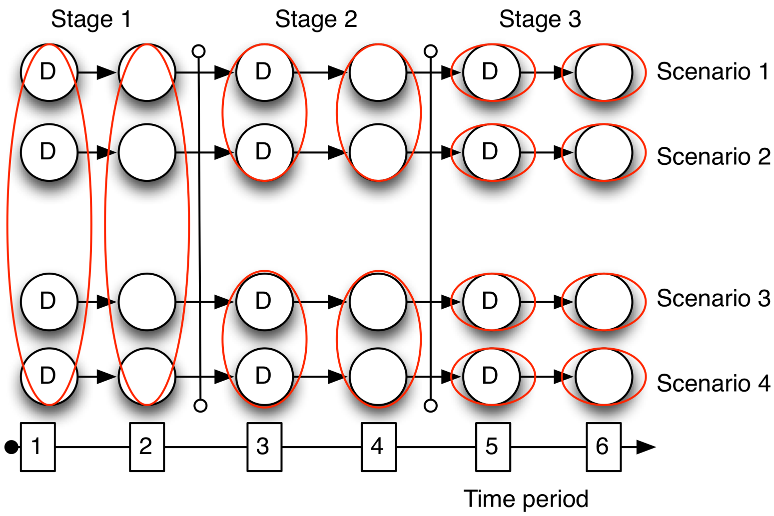

Figure 2.5: Non-anticipativity formulation. All paths through the scenario tree are explicit. The ellipsoids represent the non-anticipativity constraints.

in the scenario tree. x_{FIRST_n} is defined as the first scenario in the scenario tree node. A multi-stage problem could be formulated as:

$$\text{Min} \quad \sum_s PROB_s \times C_s \times x_s$$

$$\text{s.t} \quad x_s \in X_s \quad \forall s \in S$$

$$x_{FIRST_n} - x_n = 0 \quad \forall s \in B_n, n \in N$$

The last constraint is the non-anticipativity constraint. This constraint can also be modeled in other equivalent ways, such as comparing x_1 to x_2, x_2 to x_3 etc. In a multi-stage problem, there will be one set of non-anticipativity constraints per sibling group (B_n).

2.5.4 Scenario Reduction

To be able to solve a stochastic model in a reasonable time, it is often necessary to reduce the number of scenarios. Several methods exist to reduce an initial scenario tree to a smaller, while preserving the underlying probability structure. Dupačová et al. (2000) gives an overview and Eichhorn et al. (2010) is one implementation that is the basis for SCENRED2 in GAMS. The method in Eichhorn et al. (2010) uses the probability metric based approximations principle. The idea is to remove scenarios that are close or have very small probabilities. The method can also construct a scenario tree based on a finite number of individual scenarios that can be based on historical data, and the output is a usable scenario tree, see Gabriel et al. (2009) as an example.

2.6 Wind Forecasting

The topic of wind forecasting has become more important in recent years due to the increased wind generation capacity. Naturally wind forecasting methods have become an important field of study, with Giebel et al. (2011) providing a comprehensive overview of the current developments in short term wind prediction. This chapter will provide a brief overview of the principles behind wind forecasting, and how the forecast errors can be used to develop scenario trees for the forecast.

2.6.1 Methods

The simplest forecasting method is the persistence method, where you assume that the wind speed will stay at the current level. This method is typically very accurate for forecast horizons[6] up to six hours. After approximately 15 hours the climatological mean for the relevant region starts to be a better predictor (Lange and Focken, 2006). Nielsen et al. (1998) combined these two forecast methods into a new reference which other forecast methods are compared to.

Numerical Weather Prediction (NWP) is a method to forecast the wind speed that uses the physical laws of nature. This is done by extrapolating from a known state of the atmosphere, and calculating an average value of the wind speed inside a grid. A global model with a spatial resolution down to 25 km^2 is first used to find the global atmospheric developments, before a local model with spatial resolution down to 2.5 km^2 refines the predictions. NWP models are typically run on large clusters owned by national weather services due to the size and complexity of the models (Lange and Focken, 2006).

To forecast the wind power, most models take forecasts from NWP models as input. Most research goes into short term models (6 - 72 hours), as this is relevant for most day-ahead electricity markets. There are two types of approaches, physical or statistical models. Physical models try to find the local wind profile for the wind farm by first finding the wind speed at the hub height, and then taking into account the terrain characteristics at the site and other local properties. The local wind field is then converted to power output using the manufacturers' turbine curve or statistical data describing the relationship between measured wind speed and power output at the site (Monteiro et al., 2009).

Statistical models typically combine input from NWP models with real time measurements at the site, and transform the input directly into a wind power forecast. The statistical model can take many forms, from black box models such as neural networks to analytical models based on kernel regression. Generally the models have an autoregressive part to capture the persistence of the wind, while a meteorological part takes into account the NWP forecast (Monteiro et al., 2009).

The physical approach produces good results up to 72 hours, but is outperformed by persistence at short horizons. Statistical models are better at short time scales, and both methods can be successfully combined for an even better forecast (Monteiro et al., 2009).

[6] A forecast horizon is defined as how far in advance you predict the value. E.g. if you predict the wind speed at 18:00 at noon, the forecast horizon is 6 hours

2.6.2 Characteristics of the Wind Uncertainty

The errors in a wind *speed* forecast are normally distributed, but this does not apply to the errors in the wind *power* forecast. This is due to the non-linear relationship between the wind speed and the power output, known as the wind turbine power curve. The theoretical wind power output is proportional to the cube of the wind speed, but in practice you also have the cut-in and cut-out speeds where the turbine goes in and out of production. Figure 2.6 is an illustration of how the power output varies with the wind speed for a turbine (Lange and Focken, 2006).

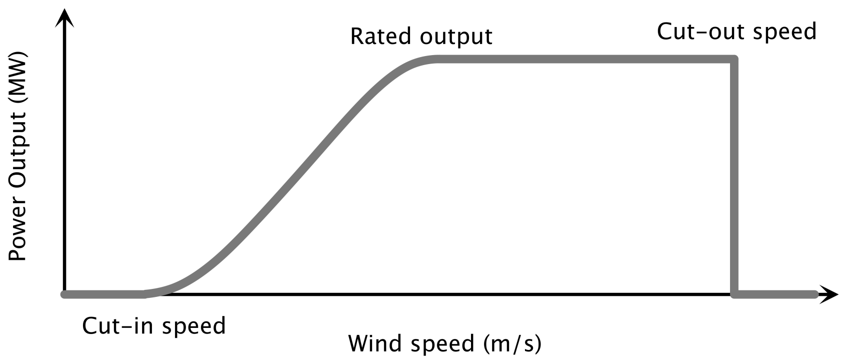

Figure 2.6: Illustration of a wind turbine power curve

In general the forecast error is smaller when the forecasted wind speed is either very low or very high (Lange and Focken, 2006). Statistical analysis done by Bludszuweit et al. (2008) show a conditional beta distribution of the forecast error, dependent on the forecast value and provide a good approximation.

The magnitude of the forecast error is also dependent on the area of the forecast region, and up to a point, the number of wind farm sites in the region (Lange and Focken, 2006). A common measurement of the forecast error is the Mean Absolute Error (MAE) defined as the difference between the measured production and the forecasted production. Porter and Rogers (2010) include an overview of the MAE for several power market regions in North America with day-ahead forecasts ranging between 3.3 to 11.5 %. The MAE depends on the characteristics of the region, so it is not possible to directly compare two different forecast methods used in two different regions.

2.6.3 Use of Forecasts in Stochastic Programming Models

The models mentioned above can not only be used to generate point forecasts, but also forecasts with confidence intervals or quartiles. The forecasts with uncertainty are valuable, but it is necessary to know something about the development of the uncertainty through the forecast to be able to use it in stochastic programming models.

Pinson et al. (2009) presented a general method to generate scenarios based on any type of probabilistic forecast where you know the inverse cumulative distribution function of the range of possible forecast values in a period. A set of prediction errors are generated by using the covariance of the historic errors. These are then transformed to a uniform distribution between 0 and 1 that is used as a random seed in the inverse cumulative distribution function of the probabilistic forecast.

If no information regarding the uncertainty of the forecast is available, Jaramillo et al. (2009) proposed a similar approach, but using the beta distribution as an approximation of the probabilistic forecast.

Both methods produce a range of independent scenarios that can be transformed into a scenario tree with a program such as SCENRED2.

2.7 Decomposition

Decomposition is a method to divide a problem into several smaller problems and solve them iteratively until a solution is found. It emerged because of the need to solve larger problems, since it can be faster and smaller problems require less memory (Fuller and Chung, 2008). Equilibrium programming in itself can be computationally demanding, especially stochastic formulations. Even though we have several orders of magnitude faster computers today than before, computation time is still a limiting factor in many situations. Additionally it may be easier to manage more complex models, since a team can have responsibility for a separate submodel (Fuller and Chung, 2005). Decomposition also enables parallel processing that can significantly speed up solution times, for instance by using a cluster. Another advantage of decomposition is to iteratively improve the solution. In other words, if an approximation is sufficient, a ”good enough” solution can be provided long before the model converges. In the rest of this section we will present some algorithms that can be used in equilibrium programming.

2.7.1 Dantzig-Wolfe Decomposition Technique

The Dantzig-Wolfe decomposition technique was presented by Dantzig and Wolfe (1960). The principle behind the algorithm is to separate the problem such that you get subproblems that are easier to solve. Many practical problems lead to very large models in terms of the number of constraints and variables. Dantzig-Wolfe decomposition takes advantage of a block angular structure of the constraint set, as seen in Figure 2.7. The complicating constraints that link the subproblems together are separated out in the master problem. The subproblems are then solved as sequence of smaller and easier problems, and the master problem chooses among the solutions and passes info regarding the complicating constraints to the subproblems. This enables the subproblems to produce better solutions. The method is illustrated in Figure 2.8, the algorithm is solved iteratively until the model converges. Conejo et al. (2006) provides an excellent introduction to Dantzig-Wolfe decomposition and Lundgren et al. (2010) shows the algorithm applied on primal problems.

2.7.2 Benders Decomposition Technique

Benders (1962) introduced the aptly named Benders decomposition (BD). While DWD separates out complicating constraints, BD moves complicating variables to decompose the problem by blocks. The block structure of such a problem is illustrated in Figure 2.9. These complicating variables can for instance be investment decisions, whether to open a factory or not and the non-complicating variables has to do with the production. If the complicating variables are fixed to given values (determined by a master problem), the rest of the problem decomposes by blocks, and can easily be solved separately. Based on the duals in the subproblems, a cutting plane is found and added to the master problem. Again, Conejo et al. (2006) provides a good and more detailed introduction to BD. It is generally

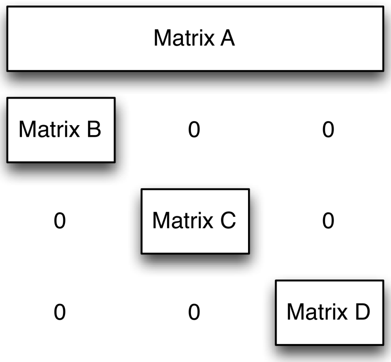

Figure 2.7: Block structure with complicating constraints (matrix A)

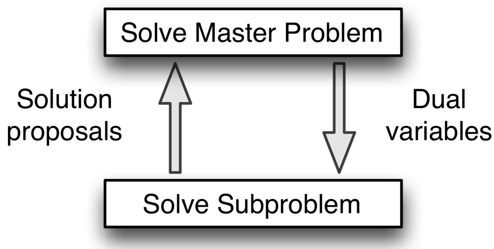

Figure 2.8: Illustration of Dantzig-Wolfe decomposition

most efficient if there are few complicating variables relative to the problem size. Figure 2.10 shows the relationship between the master and subproblem. The master problem determines the complicating variables and sends fixed variables to the subproblems in each iteration. Based on these variables the subproblems calculate new cuts.

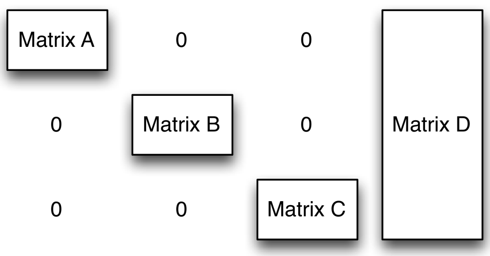

Figure 2.9: Block structure with complicating variables (matrix D)

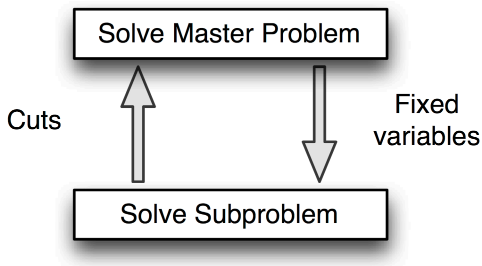

Figure 2.10: Illustration of Benders Decomposition

2.7.3 Lagrangian Relaxation Technique

Lagrangian relaxation (LR) is well described in Lundgren et al. (2010). The idea is to relax some constraints in the original formulation and consider them implicit through the objective function. Any infeasibility with respect to the relaxed constraints are penalized by the use of dual multipliers in the objective function. In contrast to BD and DWD, LR is therefore not separated into master and subproblems, but can be separated in different problems that can be solved separately. In order to use LR the problem structure must be similar to the block structure in Figure 2.7.

2.7.4 Relationship Among Benders, Dantzig-Wolfe and Lagrangian Relaxation

As shown in Lim (2010) there are relationships among Benders, Dantzig-Wolfe Decomposition and even Lagrangian relaxation. When solving a linear programming problem the techniques can be interpreted as equivalent procedures applied to different representations of the problem: Primal linear, dual linear and Lagrangian dual formulation. DWD applied on a primal problem is equivalent to using BD to solve the dual problem and implement a cutting plane method to solve its Lagrangian dual problem. The relationship is shown in Figure 2.11. This property is used by Gabriel and Fuller (2010) and Egging (2010) and others to develop Benders decomposition for MCP, based on DWD.

2.7.5 Other Decomposition Techniques

There are also other decomposition techniques. Worth mentioning is Simplicial decomposition, Cobweb decomposition and Partitionable decomposition (Çelebi, 2011).

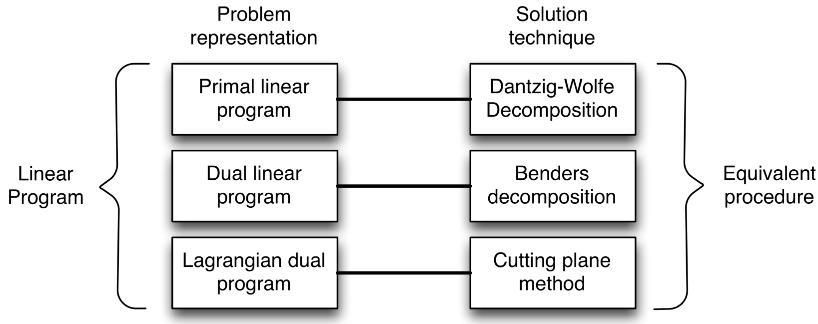

Figure 2.11: Relationship among DWD, BD and LR. From Lim (2010)

2.7.6 Dantzig-Wolfe Decomposition Techniques Applied on Equilibrium Problems

There has been some research into decomposition techniques applied to equilibrium problems. Dantzig-Wolfe decomposition was employed by Chung et al. (2006) for a class of equilibrium problems, and later expanded in the work of Fuller and Chung (2005) to be applicable to general VI-problems. Gabriel and Fuller (2010) further refined the algorithm to a Benders Decomposition algorithm, and applied it on a stochastic problem. Lagrangian methods have been used by among others He et al. (1999), and Auslender and Teboulle (2000).

The non-anticipativity constraints in the problem formulation in our thesis are well suited for DWD, such that the problem separates into one subproblem per scenario. Therefore we will only describe DWD further.

Overview

This section is based on Fuller and Chung (2005) and describes DWD applied on variational inequalities. The variational inequality, $VI(F, K)$, with mapping F and feasible region K, is solved by separating the problem into a master problem and several subproblems. The same method can be applied on MCP problems. In general, there can be one subproblem per commodity, country, scenario or similar. The subproblems are relaxations of $VI(F, K)$, where the complicating constraints are removed, while the master problem solves a restricted version of the VI. The master problem is restricted to convex combinations of the proposals from the subproblems and contains the complicating constraints. The dual variables of the complicating constraints are passed back to the subproblems in each iteration. The mapping in the subproblems is modified to include the dual variables. In each iteration the solutions from the subproblems are added to the set of solutions which the master problem uses to construct convex solutions. Figure 2.12 illustrates the relationship between the master problem and subproblem. The algorithm is repeated iteratively until a stopping criterion is satisfied.

The original $VI(F, K)$ consists of the mapping $F(z)$, variables $z \in \mathbb{R}^n$, and the

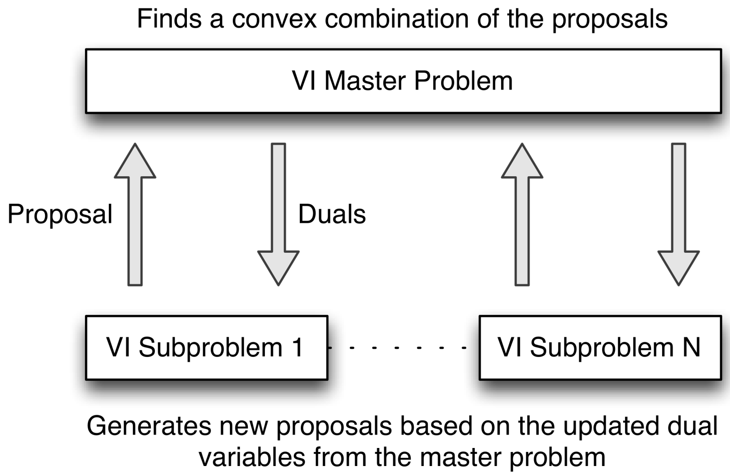

Figure 2.12: DWD on an Equilibrium Problem

feasible area $K = \{g(z) \geq 0, h(z) \geq 0\}$, where $g(z)$ are the easy constraints and $h(z)$ the complicating constraints. Both functions are concave and continuously differentiable. The VI is then to find $z^* \in K$ such that:

$F(z^*)(z - z^*)^T \geq 0$, for all $z \in K$.

The method takes advantage of the fact that the VI can be expressed in terms of the KKT-system, as seen in Section 2.3.6. β is then the dual variables to $h(z)$.

Subproblem

The complicating constraints $h(z)$ are removed in the subproblem, so that the reduced feasible area is $K' = g(z) \geq 0$. The mapping F is modified to take into account the information from the most recent master problem solution, namely the dual variables β^{k-1} and the gradients of $h(z)$ evaluated at the master problem solution z_m^{k-1}. The subproblem VI is then:

$Sub\text{-}VI^k(F - \nabla h(z_M^{k-1}\beta^{k-1}, K'))$: find $z_S^k \in K'$ such that

$(F - \nabla h(z_M^{k-1})\beta^{k-1})(z - z_S^k)^T \geq 0$ for all $z \in K'$

Master Problem

The master problem finds a convex combination of all the subproblem proposals. Z^k is the set of subproblem proposals $[z_S^1 \dots z_S^k]$, while $\lambda \in \mathbb{R}_+^k$ is the weight of the proposals. The master problem variable z_M^k can then be stated in terms of λ as $Z^k\lambda$. The feasible region is therefore $\Lambda^k = \{h(Z^k\lambda) \geq 0, e^k\lambda^T = 1\}$, where $e^k \in R^k$ is the unit vector. The mapping F is modifed to be in terms of λ, $H^k(\lambda) = F^T(Z^k\lambda)Z^k$. The master problem VI is then:

$Master\text{-}VI^k(H^k, \Lambda^k)$: find $\lambda^k \in \Lambda^k$ such that

$H^{kT}(\lambda^k)(\lambda - \lambda^k) \geq 0$ for all $\lambda^k \in \Lambda^k$.

Convergence Gap

The algorithm needs a stopping criterion. The suggested convergence gap in Fuller and Chung (2005) is:

$CG^k = (F(z_M^k) - \nabla h^T(z_M^k)\beta^k)^T(z_s^{k+1} - z_M^k)$

The algorithm stops when the CG is sufficiently small ($|CG| < \epsilon$) The convergence gap doesn't necessarily approach 0 monotonically. If $VI(F, K)$ is a linear program, the convergence gap equals the difference between the lower bound and upper bound provided by the subproblem and the master problem. Else, if the program is not linear, the convergence gap can be interpreted as the closeness to equilibrium.

Algorithm

The following algorithm is used to solve the decomposed problem:

- Solve the first subproblem
- Solve the first master problem
- Set the iteration counter = 1
- Loop:
  - Solve subproblem based on duals from the last master problem
  - Calculate the convergence gap, CG
  - If $|CG| > \epsilon$: Solve master problem, increment iteration counter
  - Else STOP

Feasibility of the Master Problem

It may be necessary to include artificial variables that ensure feasibility of the complicating constraints, if the subproblem produces proposals that are infeasible in the master problem. Instead of $h(z) \geq 0$, the master problem has $h(z) + a \geq 0$, where $a \geq 0$. The mapping is augmented to include costs on the artificial variables, so the artificial variables will be driven out of the problem. The cost of the artificial variables are determined by a penalty factor. It is essential that the penalty factor is big enough, but a too high value will result in scaling and numerical problems (Chung and Fuller, 2010). The penalty values are also bounds of the complicating constraints dual variables (Chung and Fuller, 2010).

Convergence and Uniqueness of the Solution

According to Fuller and Chung (2005) the algorithm will converge under the following assumptions.

- K is bounded
- Each component of $h(z)$ and $g(z)$ is concave and continuously differentiable
- F is continuous
- Subproblem and master problem are feasible at each iteration
- F satisfies property A

Property A is satisfied under either of the following constraints:

- $F(z)$ is strictly monotone
- $F(z) = [G(q), \nabla c(x)]^T$, where $z = [q, x]^T$ and $G(q)$ is strictly monotone and $c(x)$ is a convex function.

If $CG^k \geq 0$, and property A is satisfied then the master solution solves $VI(F, K)$.

The solution to $VI(F, K)$ is unique if the mapping F is strictly monotone. If the mapping can be divided into $G(q)$ and $c(x)$, as in property A, and $G(q)$ is strictly monotone, the solution is unique in q and the scalar value $c(x)$. If additionally $c(x)$ is strictly convex, the solution is unique in both q and x.

Chapter 3

Stochastic Power Market Model

Our Stochastic Power Market Model is presented in this chapter. The purpose of the model is to study the short-term intra-day spot market in Northern Europe, and this sets the scope for what is included in the model. The model is divided into three parts; each producer's profit maximization problem, the transmission system operator's (TSO) profit maximization problem and the market-clearing conditions that link the problems together.

Each country or area is represented with a node in a network. In each node there can be both consumers and producers. The consumers are represented by a linear inverse demand function in each node. The producers generate electricity and can sell electricity in the node for the market price in that node. They may also sell their power in other nodes, but they must then pay a transmission fee to the TSO. The market price is determined based on available supply and demand in each node, as a power exchange. The producers can have different production technologies, and they can have pump storage.

The TSO ensures that the flow on each power line adheres to the transmission capacity. If the power line is fully utilized by the producers, the TSO receives a congestion rent.

A producer's optimization problem does not only depend on his own decision, but also on the other producers and the TSO. A market clearing condition on the line capacity is used to link the problems together, by combining the producers' flow into one common flow on each power line.

3.1 Definitions

This section contains notations, sets, indices, parameters and variables used in the model. All variables are non-negative unless noted otherwise.

3.1.1 Notations

There are a few notations used to make the equations more readable. Both indexes and sets are defined in the model. These are matched such that time period i belongs to the set of time periods I. To simplify the notation, the sets are not shown explicitly, as seen in equation (3.1).

$$\sum_{i} sales_{n,p,i,s} \equiv \sum_{i \in I} sales_{n,p,i,s} \tag{3.1}$$

Each sum symbol can also sum over more than one set.

$$\sum_{n,p} q_{n,p,t,i,s} \equiv \sum_{n} \sum_{p} q_{n,p,t,i,s} \tag{3.2}$$

3.1.2 Sets, Indices, Parameters and Variables

For consistency, all parameters and sets are in capital letters, variables and indices are in lower case, and dual variables are in greek letters.

Sets and Indices	
I	Set of time periods i
L	Set of power lines l
L_n^+	Set of power lines going into n
L_n^-	Set of power lines going out of n
N	Set of nodes n
P	Set of producers p
S	Set of scenarios s
T	Set of generation technologies t

Parameters	
AFC_l	Second order term of the regulated fee on power line l
$AMC_{n,p,t}$	Second order term of marginal generation cost for producer p in node n for technology t
$AV_{n,t,i,s}$	Availability factor for node n for technology t in time period i for scenario s
FC_l	Regulated fee on power line l
$FIRST_{s,i}$	The first scenario in the same event node as scenario s in time period i
$GCAP_{n,p,t}$	Maximum generation capacity in node n for producer p and technology t
$INT_{n,i}$	Intercept of the inverse demand curve in node n in time period i
$LAST_{s,i}$	The last scenario in the same event node as scenario s in time period i
MC_t	First order term of the marginal generation cost for technology t
$MP_{n,p}$	Market power in node n for producer p
$PROB_s$	Probability of scenario s
RC_t	Ramping cost of technology t
RU_t	Maximum ramping up for technology t between two time periods
$SLP_{n,i}$	Negative slope of the inverse demand curve in node n in time period i
$TCAP_l$	Maximum transmission capacity on power line l

Primal Variables	
$flow_{l,i,s}$ (free)	Total transmission flow on line l in period i and scenario s
$f^{+}_{l,p,i,s}$	Negative transmission flow on line l by producer p in time period i and scenario s
$f^{-}_{l,p,i,s}$	Positive transmission flow on line l by producer p in time period i and scenario s
$price_{n,i,s}$	Price in node n in time period i and scenario s
$q_{n,p,t,i,s}$	Production in node n of technology t by producer p in time period i and scenario s
$qi_{n,p,t,i,s}$	Increase in production between time period i and $i-1$ of technology t, producer p in node n and scenario s
$qp_{n,p,i,s}$	Power stored in pump power plants in node n, by producer p in time period i and scenario s
$sales_{n,p,i,s}$	Amount sold in node n, by producer p in time period i and scenario s

Dual Variables	
$\alpha_{n,p,t,i,s}$	Dual to the production limit constraint
$\beta_{n,p,t,i,s}$	Dual to the ramping constraint
$\delta_{n,p,t,i,s}$ (free)	Dual to the non-anticipativity constraint
$\epsilon^{+}_{l,i,s}$	Dual to the total positive transmission capacity constraint
$\epsilon^{-}_{l,i,s}$	Dual to the total negative transmission capacity constraint
$\gamma_{n,p,i,s}$ (free)	Dual to the node balance constraint
$\phi_{n,p,t,i,s}$	Dual to the ramping limitation constraint
$\psi_{n,p,i,s}$	Dual to the installed pump power constraint
$\tau_{l,i,s}$ (free)	Dual to the market clearing condition. Transmission fee on power line l in period i and scenario s

3.2 Producer's Optimization Problem

3.2.1 Producer's Objective Function

The objective of the producer is to maximize his profit by deciding production q, sales $sales$ and pump storage qp in each node, and flows $f^{\pm}$ on each power line. Equation (3.3) represents all the components of the profit for producer p.

$$\begin{aligned}\pi_p = \sum_s PROB_s \times (&\text{Sales revenue}_{p,s} \\ &+\text{Pump storage revenue}_{p,s} \\ &-\text{Production cost}_{p,s} \\ &-\text{Pump storage cost}_{p,s} \\ &-\text{Transmission cost}_{p,s} \\ &-\text{Ramping cost}_{p,s})\end{aligned} \tag{3.3}$$

The sales revenue for producer p is the aggregated sales in all nodes and time periods times the price in the respective node, period and scenario.

$$\text{Sales revenue}_{p,s} = \sum_{n,i} (price_{n,i,s} \times sales_{n,p,i,s}) \tag{3.4}$$

The market price is represented by a linear inverse demand curve for each node n and time period i, where $INT_{n,i}$ and $SLP_{n,i}$ are the interception and negative slope respectively. The net sales to consumers are equal to the gross sales in the node less the amount bought by pump storage, since the demand curve represents the demand of regular customers.

$$price_{n,i,s} = INT_{n,i} - SLP_{n,i} \times \sum_p (sales_{n,p,i,s} - qp_{n,p,i,s}) \tag{3.5}$$

The pump storage revenue is the water value of hydropower (MC_{hydro}) corrected for the efficiency, EF, of the pump storage plant times the stored power $qp_{n,p,i,s}$, aggregated for all nodes n and time periods i.

$$\text{Pump storage revenue}_{p,s} = \sum_{n,i} MC_{hydro} \times EF \times qp_{n,p,i,s} \tag{3.6}$$

There are four cost elements. The production cost for producer p is the quantity generated by each technology t in time period i, scenario s and node n, $q_{n,p,t,i,s}$ times the marginal cost of the specific technology, aggregated for all nodes technologies and time periods. A second degree term, $AMC_{n,p,t}$ is included for a slightly increasing marginal cost. This makes the function strictly convex, which will make the production unique in the optimal solution.

$$\text{Production cost}_{p,s} = \sum_{n,t,i} (MC_t + AMC_{n,p,t} \times q_{n,p,t,i,s}) \times q_{n,p,t,i,s} \tag{3.7}$$

The pump storage cost is the cost to buy the electricity qp that is stored in a time period. This is equal to the price in that node.

$$\text{Pump storage cost}_{p,s} = \sum_{n,i} price_{n,i,s} \times qp_{n,p,i,s} \tag{3.8}$$

The next cost element is the transmission cost. The transmission fee $\tau_{l,i,s}$ is paid for each MWh transferred on the transmission line l. The transmission fee is defined as the cost paid for transporting power in the defined positive direction on the line. $f^{+}_{l,p,i,s}$ and $f^{-}_{l,p,i,s}$ define producer p's positive and negative flow on power line l in time period i and scenario s. A regulated fee of FC_l and the second order term AFC_l may also be included. The second order term has no physical interpretation, but can be included to get unique flows.

$$\begin{aligned}\text{Transmission cost}_{p,s} = \sum_{l,i} \Big[& \Big(\tau_{l,i,s} + FC_l + AFC_l \times f^{+}_{l,p,i,s}\Big) \times f^{+}_{l,p,i,s} \\ & + \Big(-\tau_{l,i,s} + FC_l + AFC_l \times f^{-}_{l,p,i,s}\Big) \times f^{-}_{l,p,i,s}\Big]\end{aligned} \tag{3.9}$$

The last cost element is the ramping cost. If producer p increases production in a node with technology t he incurs a ramping cost RC_t depending on the technology t. The ramping variable $qi_{n,p,t,i,s}$ shows how much the production with technology t in node n in time period i and scenario s has increased from the previous time period.

$$\text{Ramping cost}_{p,s} = \sum_{n,t,i} (RC_t \times qi_{n,p,t,i,s}) \tag{3.10}$$

3.2.2 Node Balance

Each producer has to maintain a balance in each node in the transmission network. The sum of all flows in and out of the node, production and sales must be equal to zero in each node. This fulfills Kirchoff's current law (Section 2.2.1) and allows the producer to generate and sell in different nodes. In each node n and in all time periods i and scenarios s the producer p can generate power with technology t, $q_{n,p,t,i,s}$, if he has one or more power plants in that node. He may store power, $qp_{n,p,i,s}$, if he has a pump storage plant in that node. In all nodes he can sell power, $sales_{n,p,i,s}$, by either producing in that node, or importing power on the power lines. Sales include electricity sold to households and industry, but also electricity sold to pump storage plants. L^{+}_{n} and L^{-}_{n} define the set of all the power lines l that go in or out of the respective node n. The flows are directional and in pairs so a producer p's positive flow in line l in time period i is $f^{+}_{l,p,i,s}$ and the negative flow on that line is $f^{-}_{l,p,i,s}$. The producer can transport and sell power in all nodes as long as there is available transmission capacity between the production and sales node. Figure 3.1 illustrates the node balance in one node.

$$\begin{aligned} &\sum_{t} q_{n,p,t,i,s} + \sum_{l \in L_n^+} (f^+_{l,p,i,s} - f^-_{l,p,i,s}) \\ &- sales_{n,p,i,s} - \sum_{l \in L_n^-} (f^+_{l,p,i,s} - f^-_{l,p,i,s}) = 0 \quad \forall \quad n,p,i,s \end{aligned} \tag{3.11}$$

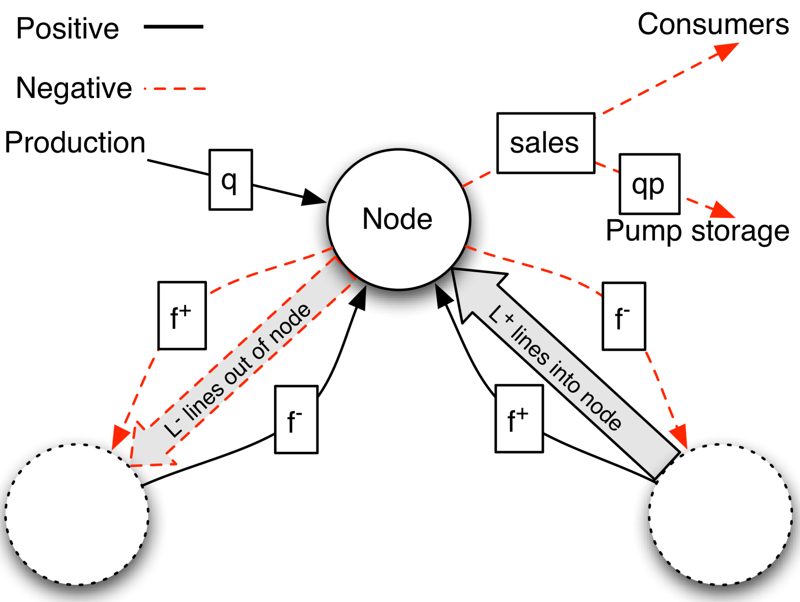

Figure 3.1: Illustration of the node balance

Since qp is not explicitly in the node balances, the formulation requires that the data set is well formed. It is possible to get pumping when there are no sales in the node if $INT_{n,i} < MC_{hydro} \times EF$. For any reasonable data sets, this will not be a problem.

The node balance formulation is flexible and can easily be changed to as many or as few nodes as required by increasing or reducing the set of nodes N and necessary parameters. Nodes can also act as transfer nodes without any production or demand, or be isolated without any exchange with other nodes.

3.2.3 Generation Capacities

The produced power $q_{n,p,t,i,s}$ in each time period i in scenario s is limited by the available generation capacity $GCAP_{n,p,t}$ for each producer p in node n with technology t. The technology, time and node specific availability factor $AV_{n,t,i,s}$ is most important for the wind and solar power plants, where the weather conditions determine the maximum output on a continuous basis. But it can also be used to reflect maintenance or outages for conventional technologies.

$$q_{n,p,t,i,s} - AV_{n,t,i,s} \times GCAP_{n,p,t} \leq 0 \quad \forall \quad n, p, t, i, s \tag{3.12}$$

3.2.4 Production Ramping

In each time period each producer can change the production according to demand. There are physical constraints on how fast different technologies can increase the electricity output, and that increase can incur a cost. The ramping constraint (3.14) limits the producer from increasing the production too rapidly. The ramping limit is decided by a technology specific ramping factor RU_t in percent of the total production capacity $GCAP_{n,p,t}$ in the node. As in Traber and Kemfert (2009), we don't include shut down costs in our model, because the cost of ramping up is significantly higher.

The increase in production for technology t in node n and time period i and scenario s, $qi_{n,p,t,i,s}$, is the difference between the producer p's current and previous period's production (3.13). $qi_{n,p,t,i,s}$ is defined as non-negative, and will therefore be zero when the production ramps down.

$$q_{n,p,t,i,s} - q_{n,p,t,i-1,s} \leq qi_{n,p,t,i,s} \quad \forall \quad n, p, t, (i > 1), s \tag{3.13}$$

$$RU_t \times GCAP_{n,p,t} - qi_{n,p,t,i,s} \geq 0 \quad \forall \quad n, p, t, i, s \tag{3.14}$$

3.2.5 Pump Storage Plants

Some hydropower plants have the ability to pump water back into a reservoir, to take advantage of arbitrage opportunities. When the power price is low, you can buy power and pump water back into the reservoir for later use. When the price is high, you can use the stored water to produce power and sell it in the market.

The pump storage capacity in the model is defined as a percentage, $IN_{n,p}$, of the hydro generation capacity for producer p and node n. The maximum power bought is then limited by (3.15). The model does not track the reservoir level for the pump storage, which means that the pump storage plants in the model must be connected to a large or seasonal reservoir.

$$qp_{n,p,i,s} - IN_{n,p} \times GCAP_{n,p,hydro} \leq 0 \quad \forall \quad n, p, i, s \tag{3.15}$$

Since the model does not consider pump storage as a separate technology from hydro, we have assumed that the market price when the power eventually is sold, is equal to the water value. This will underestimate the earning potential as the market price could be higher, but it is not an unreasonable assumption if the node is dominated by hydropower.

3.2.6 Non-Anticipativity

The non-anticipativity constraint makes sure the production in the same event node is equal in all scenarios. The $FIRST_{s,i}$ and $LAST_{s,i}$ parameters defines what scenarios are the first and last scenario within an event node. E.g. if we have 4 scenarios we would compare we would compare scenario 1 with 2, 1 with 3 and 1 with 4 in the first period.

$$q_{n,p,t,i,s} - q_{n,p,t,i,FIRST_{s,i}} = 0 \quad \forall \quad n, p, t, i, s \tag{3.16}$$

It is sufficient to only enforce non-anticipativity on q if the data set results in a unique solution, as the node balance connects q with the other variables. If not, the other primal variables must also have non-anticipativity enforced.

3.2.7 Market Power

Each producer may exert market power in one or several nodes. The market power factor allows us to differentiate between different producers' market power, and is described in Chapter 2.3. In the model the market power factor for producer p in node n, $MP_{p,n}$ is the partial derivative of the total sales in the specific node n in a time period i and scenario s with respect to producer p's sale in that node and time period.

$$MP_{p,n} = \frac{\partial \sum_p sales_{n,p,i,s}}{\partial sales_{n,p,i,s}} \tag{3.17}$$

The market power factor emerges in the derivation of the KKT conditions.

3.3 TSO's Optimization Problem

The transmission system operator (TSO) is modeled after the congestion management based on area pricing principle (Wangensteen, 2007), where the TSO receives a congestion rent equal to the price difference between two areas if the power line is at full capacity. In the model the TSO maximizes the transmission tariff $\tau_{l,i,s}$ times the flow per time period and line. His only decision variables are the flows, $flow_{l,i,s}$, which he must keep within the transmission capacity constraints (3.19) and (3.20). The former constraint is binding for a positive flow, the latter for a negative flow.

$$\Pi_{TSO} = \sum_{l,i,s} (PROB_s \times \tau_{l,i,s} \times flow_{l,i,s}) \tag{3.18}$$

$$flow_{l,i,s} - TCAP_l \leq 0 \quad \forall \quad l, i, s \tag{3.19}$$

$$-flow_{l,i,s} - TCAP_l \leq 0 \quad \forall \quad l, i, s \tag{3.20}$$

The TSO may have costs that are covered by the regulated fee, FC_l. These costs are paid by the producers to the TSO, and are therefore canceled out in the TSO's profit function. The model ignores losses and the effects of Kirchoff's voltage law. Additionally there may be restrictions on how fast the flow on a power line may change, due to physical characteristics and overall system security concerns (ENTSO-E, 2010), but this has not been included in the model.

3.4 Market-Clearing Condition

The market-clearing conditions bind the producers' optimization problems and the TSO's optimization problem together. The conditions make sure the aggregated flow on one line are equal to the TSO's flows; $flow_{l,i,s}$. $\tau_{l,i,s}$ are the dual variables, and are the congestion charges for transporting one unit on line l in time period i and scenario s. The congestion charge is the value producers would value to send one more unit on each specific power line. Since the transmission fee is included in the producers' optimization problems, this makes sure that the producers that value the flow the most gets the capacity.

Market-clearing condition ($\tau_{l,i,s} -$ free)

$$\sum_p (f^+_{l,p,i,s} - f^-_{l,p,i,s}) - flow_{l,i,s} = 0 \quad \forall \quad l, i, s \tag{3.21}$$

3.5 Optimality and Feasibility

As explained in Chapter 2.3, the KKT conditions are sufficient for the problem to be optimal if the problem is convex and necessary if a constraint qualifications (CQ) hold. All the constraints in our problem are linear or affine, and the feasible area is therefore convex. To have a convex problem the objective function of the producers and the TSO must also be convex for a minimization problem. The TSO's problem is linear, which means that it is also convex. The objective functions of the producers are convex if $SLP_{n,i}$, $AMC_{n,p,t}$ and AFC_l are non-negative.

CQs guarantee that every KKT point provides an optimal solution. The CQs from Bazaraa et al. (2005) hold for our model. All constraints are affine inequalities or linear equalities and thus the feasible region is a convex polyhedral. With a feasible region defined by linear constraints, KKT are necessary for optimal solutions, independent of the objective function.

The problem will not necessarily have a unique solution, it depends on the data set. $AMC_{n,p,t}$, $MP_{a,n}$ and AFC_l must be non-negative and results in unique production, sales and individual flow. As a consequence, the change in production and the aggregated flow will also be unique. The pump storage volume is not necessarily unique, if there are several producers in a node and the price is equal the revenue they gain by pumping they are indifferent about who should pump.

3.6 Solving the Problem

The presented maximization problems must be solved simultaneously since all the problems influence on all the other problems' solution. To be able to solve all the problems, each maximization problem is first converted into a minimization problem with only less-than or equality constraints and then converted to an MCP as described in Chapter 2.3. The KKT conditions for all the producers' problems are aggregated together with the TSO's KKT conditions and the market-clearing condition. The full mathematical formulation can be found in appendix A. As we know from Section 2.3.6, the problem can also be formulated as a VI-problem. The formulation with the mapping and the constraint set of the VI-problem can be found in appendix C.

3.7 Model with Scenario Tree Formulation

The model formulation can also be expressed in terms of a scenario tree formulation instead of the non-anticipativity constraints. In the non-anticipativity formulation the variables are defined for all scenarios s and time periods i, while in the scenario tree formulation variables are defined for the scenario tree nodes e. The full scenario tree formulation can be found in appendix B.

3.8 Karush Kuhn Tucher Conditions of the Stochastic Power Market Model

The KKT conditions are derived from the stochastic non-anticipativity problem formulation. n^+ and n^- refer to the node where power line l starts and stops.

Producer

$$\begin{aligned} 0 \leq q_{n,p,t,i,s} \perp & PROB_s \times (MC_t + 2 \times AMC_{n,p,t} \times q_{n,p,t,i,s}) \\ & -\gamma_{n,p,i,s} + \alpha_{n,p,t,i,s} + \beta_{n,p,t,i,s|i>1} - \beta_{n,p,t,i+1,s|i<|I|} \\ & +\delta_{n,p,t,i,s|s\geq FIRST_{s,i}} - \sum_{s|s\leq LAST_{s,i}} \delta_{n,p,t,i,s=FIRST_{s,i}} \geq 0 \end{aligned} \tag{3.22}$$

$$\begin{aligned} 0 \leq sales_{n,p,i,s} \perp & PROB_s \times \left(-INT_{n,i} + SLP_{n,i} \times \sum_p (sales_{n,p,i,s} - qp_{n,p,i,s}) \right) \\ & +PROB_s \times MP_{p,n} \times SLP_{n,i} \times sales_{n,p,i,s} \\ & +\gamma_{n,p,i,s} \geq 0 \end{aligned} \tag{3.23}$$

$$\begin{aligned} 0 \leq qp_{n,p,i,s} \perp & PROB_s \times (-MC_{hydro} \times EF) \\ & +PROB_s \times \left(INT_{n,i} + SLP_{n,i} \times \sum_p (sales_{n,p,i,s} - qp_{n,p,i,s}) \right) \\ & +\psi_{n,p,t,i,s} \geq 0 \end{aligned} \tag{3.24}$$

$$\begin{aligned} 0 \leq f^+_{l,p,i,s} \perp & PROB_s \times \left(\tau_{l,i,s} + FC_l + 2 \times AFC_l \times f^+_{l,p,i,s} \right) \\ & -\gamma_{n^+,p,i,s} + \gamma_{n^-,p,i,s} \geq 0 \end{aligned} \tag{3.25}$$

$$\begin{aligned} 0 \leq f^-_{l,p,i,s} \perp & PROB_s \times \left(-\tau_{l,i,s} + FC_l + 2 \times AFC_l \times f^-_{l,p,i,s} \right) \\ & +\gamma_{n^+,p,i,s} - \gamma_{n^-,p,i,s} \geq 0 \end{aligned} \tag{3.26}$$

$$0 \leq qi_{n,p,t,i,s} \perp PROB_s \times RC_t - \beta_{n,p,t,i,s} + \phi_{n,p,t,i,s} \geq 0 \tag{3.27}$$

$$\begin{aligned} & \sum_t q_{n,p,t,i,s} + \sum_{l\in L^+_n} (f^+_{l,p,i,s} - f^-_{l,p,i,s}) \\ & -sales_{n,p,i,s} - \sum_{l\in L^-_n} (f^+_{l,p,i,s} - f^-_{l,p,i,s}) = 0 \\ & (\gamma_{n,p,i,s} - \text{free}) \end{aligned} \tag{3.28}$$

$$0 \leq \alpha_{n,p,t,i,s} \perp AV_{t,i,s} \times GCAP_{n,p,t} - q_{n,p,t,i,s} \geq 0 \tag{3.29}$$

$$0 \leq \beta_{n,p,t,i,s} \perp qi_{n,p,t,i,s} - q_{n,p,t,i,s} + q_{n,p,t,i-1,s} \geq 0 \tag{3.30}$$

$$0 \leq \phi_{n,p,t,i,s} \perp RU_t \times GCAP_{n,p,t} - qi_{n,p,t,i,s} \geq 0 \tag{3.31}$$

$$0 \leq \psi_{n,p,s} \perp IN_{n,p} \times GCAP_{n,p,hydro} - qp_{n,p,i,s} \geq 0 \tag{3.32}$$

$$\begin{gathered} q_{n,p,t,i,FIRST_{s,i}} - q_{n,p,t,i,s} = 0 \\ (\delta_{n,p,t,i,s} - \text{free}) \end{gathered} \tag{3.33}$$

Transmission System Operator

$$0 \leq flow_{l,i,s} \perp -PROB_s \times \tau_{l,i,s} + \epsilon^{+}_{l,i,s} - \epsilon^{-}_{l,i,s} \geq 0 \tag{3.34}$$

$$0 \leq \epsilon^{+}_{l,i,s} \perp TCAP_l - flow_{l,i,s} \geq 0 \tag{3.35}$$

$$0 \leq \epsilon^{-}_{l,i,s} \perp TCAP_l + flow_{l,i,s} \geq 0 \tag{3.36}$$

Market-Clearing

$$\begin{gathered} flow_{l,i,s} - \sum_{p} (f^{+}_{l,p,i,s} - f^{-}_{l,p,i,s}) \geq 0 \\ (\tau_{l,i,s} - \text{ free}) \end{gathered} \tag{3.37}$$

Chapter 4

Decomposed Stochastic Power Market Model

This section presents our Dantzig-Wolfe decomposition of the Stochastic Power Market Model done on the non-anticipativity constraints, based on the theory presented in Section 2.7. The algorithm is presented and solved as an MCP, as this is how the model is implemented. First the algorithm will be explained, then the master problem and subproblem formulations.

4.1 Algorithm

Figure 4.1 shows the main algorithm in solving the decomposed problem. The algorithm starts by generating the first set of proposals by the subproblems. The subproblems are linked to the master problem through a variable, $\delta_{n,p,t,i}$, in each iteration. $\delta_{n,p,t,i}$ are the dual variables to the non-anticipativity constraints. In the first iteration $\delta_{n,p,t,i} = 0$. The result of each subproblem is saved in a vector that contains the generated proposals. The master problem proposes a set of weights, $\lambda_{k,s}$, for each subproblem solution that provide the current best solution. $\delta_{n,p,t,i}$ are recorded and sent to the next subproblem. This completes the first iteration.

In the next iteration, the subproblems use the new information from the dual variables $\delta_{n,p,t,i}$ to generate new proposals. The convergence gap is then checked to see if the algorithm has completed. If not, the master problem generates a new set of weightings $\lambda_{k,s}$ and updated dual variables δ. This procedure loops until the convergence gap is reached.

The algorithm uses one weight for each subproblem solution, instead of one weight for the set of subproblem proposals in an iteration. This is important because the extra flexibility in combining proposals from different subproblems differently can speed up the convergence and reduce the number of iterations (Chung et al., 2006). The cost of using more weights is an increase in the number of variables and therefore also the size of the master problem (Chung and Fuller, 2010).

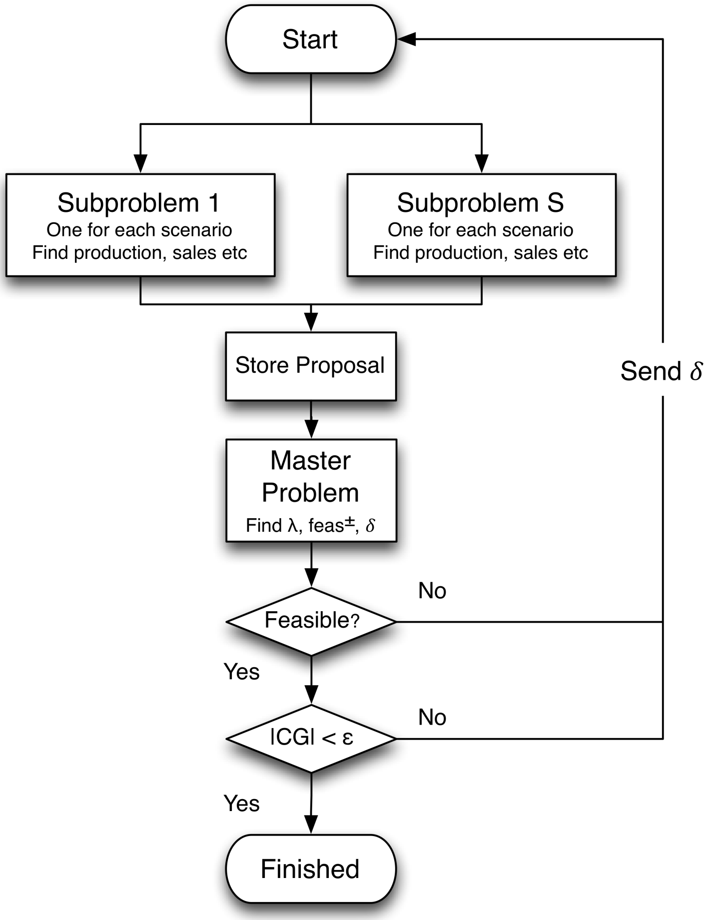

Figure 4.1: Problem specific DWD algorithm

4.2 The Master Problem

The master problem receives solutions from the subproblems, and finds the best solution by taking a convex combination of all the proposed solutions for a scenario. The master problem then passes on the dual variables of the non-anticipativity constraints, so the subproblem can propose better solutions. The subproblems propose solutions that are infeasible in the master problem, since the non-anticipativity constraints are relaxed. The artificial variables ($feas^{+}_{n,p,t,i,s}/feas^{-}_{n,p,t,i,s}$) are introduced in the non-anticipativity constraints in the master problem, to ensure that the master problem is feasible. An associated penalty parameter penalizes solutions with artificial variables so they eventually are removed from the master problem solution.

4.2.1 Definitions

The solution of the variables from the subproblems are denoted in capital letters (e.g. Q, SALES).

Sets	
K	Set of proposals k from the subproblems

Parameters	
$F^{+}_{l,p,i,s,k}$	Positive flow in proposal k
$F^{-}_{l,p,i,s,k}$	Negative flow in proposal k
$FLOW_{l,i,s,k}$	Total flow in proposal k
$PENALTY_{n,p,t,i,s}$	Penalty for infeasibility in the non-anticipativity constraints
$Q_{n,p,t,i,s,k}$	Production in proposal k
$QI_{n,p,t,i,s,k}$	Production increase in proposal k
$QP_{n,p,s,k}$	Pump stored power in proposal k
$SALES_{n,p,i,s,k}$	Sales in proposal k
$TAU_{l,i,s,k}$	Transmission fee in proposal k

Primal variables	
$feas^{+}_{n,p,t,i}$	Positive artificial variable in the non-anticipativity constraints
$feas^{-}_{n,p,t,i}$	Negative artificial variable in the non-anticipativity constraints
$\lambda_{k,s}$	Weight on proposal k for scenario s

Dual variables	
$\Delta_{n,p,t,i,s}$	Dual variable to the non-anticipativity constraint in the master problem
σ_s	Dual to the convexity criterion

4.2.2 Formulation

The master problem consists of the stationarity conditions for $\lambda_{k,s}$ and the feasibility-variables, together with the non-anticipativity constraints and the convexity.

Stationarity for Lambda

The stationarity constraint for $\lambda_{k,s}$ has been divided into different parts for clarity. The aggregate sum is perpendicular ($\perp$) to $\lambda_{k,s} \geq 0 \quad \forall s$. Some of the terms related to the flows cancel out when the master problem is derived. The redundant terms are found in Appendix D.

Production

$$0 \leq \lambda_{k,s} \perp \sum_{n,p,t,i} (PROB_s \times (MC_t + 2 \times AMC_{n,p,t} \times \sum_{k} (\lambda_{k,s} \times Q_{n,p,t,i,s,k})) \times Q_{n,p,t,i,s,k}) \tag{4.1a}$$

Sales

$$+ \sum_{n,p,i}(PROB_s \times (-INT_{i,n} + SLP_{i,n} \times \sum_{p,k} \lambda_{k,s} \times (SALES_{n,p,i,s,k} - QP_{n,p,i,s,k})) \times SALES_{n,p,i,s,k}) + \sum_{n,p,i}(PROB_s \times MP_{p,n} \times SLP_{i,n} \times \sum_{k}(\lambda_{k,s} \times SALES_{n,p,i,s,k}) \times SALES_{n,p,i,s,k}) \tag{4.1b}$$

Pump storage

$$+ \sum_{n,p,i}(PROB_s \times (-MC_{hydro} \times EF + (INT_{i,n} - SLP_{i,n} \times \sum_{p,k} \lambda_{k,s} \times (SALES_{n,p,i,s,k} - QP_{n,p,i,s,k}))) \times QP_{n,p,i,s,k}) \tag{4.1c}$$

Positive flow

$$+ \sum_{l,p,i}(PROB_s \times (FC_l + 2 \times AFC_l \times \sum_{k}(\lambda_{k,s} \times F^{+}_{l,p,i,s,k})) \times F^{+}_{l,p,i,s,k}) \tag{4.1d}$$

Negative flow

$$+ \sum_{l,p,i}(PROB_s \times (FC_l + 2 \times AFC_l \times \sum_{k}(\lambda_{k,s} \times F^{-}_{l,p,i,s,k})) \times F^{-}_{l,p,i,s,k}) \tag{4.1e}$$

Production increase

$$+ \sum_{n,p,t,i} (PROB_s \times RC_t \times QI_{n,p,t,i,s,k}) \tag{4.1f}$$

Non-anticipativity constraint

$$+ \sum_{n,p,t,i} \left(-Q_{n,p,t,i,FIRST_{s,i},k} + Q_{n,p,t,i,s,k}\right) \times \Delta_{n,p,t,i,s} \tag{4.1g}$$

Convexity criterion

$$+\sigma_s \geq 0 \tag{4.1h}$$

Stationarity for Feasibility

$$0 \leq feas^{+}_{n,p,t,i,s} \perp PROB_s \times PENALTY_{n,p,t,i,s} - \Delta_{n,p,t,i,s} \geq 0 \tag{4.2}$$

$$0 \leq feas^{-}_{n,p,t,i,s} \perp PROB_s \times PENALTY_{n,p,t,i,s} + \Delta_{n,p,t,i,s} \geq 0 \tag{4.3}$$

Convexity Criterion

$$1 - \sum_k \lambda_{k,s} = 0 \quad \perp \sigma_s - free \tag{4.4}$$

Non-Anticipativity

$$\begin{aligned} &\sum_k \lambda_{k,s} \left(Q_{n,p,t,i,FIRST_{s,i},k} - Q_{n,p,t,i,s,k} \right) \\ &\qquad + feas^{+}_{n,p,t,i,s} - feas^{-}_{n,p,t,i,s} = 0 \quad \perp \Delta_{n,p,t,i,s} - free \end{aligned} \tag{4.5}$$

4.3 The Subproblem

There is one subproblem for each scenario s, since we decompose on the non-anticipativity constraints. Consequently, the variables in the subproblems only depend on the time period i, and not the scenario s. The master problem makes sure that the variables in the subproblem fulfill the non-anticipativity constraints by passing along the dual variables Δ.

4.3.1 Formulation

The subproblem formulation is the same as the Stochastic Power Market Model in Chapter 3, except that the non-anticipativity constraints are removed. The formulation can be found in Appendix E.

The KKT conditions are as before, except that the production stationarity condition is changed to accommodate the dual variables to the non-anticipativity constraints from the master problem, $\Delta_{n,p,t,i,s}$. $\Delta_{n,p,t,i,s}$ from the master problem are passed to the subproblem as $\delta_{n,p,t,i}$. If scenario s is defined as the first scenario ($s = FIRST_{s,i}$) then:

$\delta_{n,p,t,i} = -\sum_{FIRST_{s,i} < s \leq LAST_{s,i}} \Delta_{n,p,t,i,s}$

If scenario s is not the first scenario, then:

$\delta_{n,p,t,i} = \Delta_{n,p,t,i,s}$

The modified KKT condition for q is given by:

$$\begin{aligned} 0 \leq q_{n,p,t,i} \perp & PROB_s \times (MC_t + 2 \times AMC_{n,p,t} \times q_{n,p,t,i}) \\ & - \gamma_{n,p,i} + \alpha_{n,p,t,i} + \beta_{n,p,t,i|i>1} - \beta_{n,p,t,i+1|i<|I|} + \delta_{n,p,t,i} \geq 0 \end{aligned} \tag{4.6}$$

4.4 Convergence and Uniqueness

The convergence criterion explained in Section 2.7.6 are satisfied, and the algorithm will converge. The convergence gap can be found in appendix F.

The uniqueness of the solution is dependent on the data set. If MP, AMC, and AFC are non-negative for all relevant producers and power lines, the relevant mappings are strictly monotone and strictly convex and the solution is unique in production, sales and the producers flow. By definition qi, $flow$ and τ are also unique. qp is not necessarily unique since the mapping is not strictly convex.

4.5 Alternative Model Versions

We implemented two slightly different models. The first model, as presented above, produces solutions that start out being infeasible in the original problem. However, we know that the penalty parameter bounds the dual variable Δ of the non-anticipativity constraint. This property can be used to control the algorithm and avoid large fluctuations in the subproblems and too big changes between the proposals.

The second model always produces feasible solutions, since it doesn't include the artificial variables ($feas^{+}_{n,p,t,i,s}/feas^{-}_{n,p,t,i,s}$). An initial feasible solution must be constructed in the first iteration. One approach we use is to take the average solution from a run of the first subproblem iteration. The dual variables are not bounded in this version, and the approach may use several iterations before the master problem decides to use the new proposals.

4.6 Techniques to Improve the Computational Performance

We experienced early that the algorithm worked fine for smaller problems but had problems with bigger data sets. This led us to consider different methods to improve the convergence and solution time, outlined in Table 4.1.

In the second model version we have to generate a feasible starting solution, but this can also be done in the first version. This can provide a better starting point for the algorithm. The generated starting point is based on the average of the productions in the first subproblems.

A second technique was to fix some technologies either to full capacity or to zero. Due to the characteristics of our problem, we can be quite certain that technologies with low marginal cost, such as solar, wind and run-of-river power generation, will be fully utilized and can be considered fixed. Similarly, if a producer doesn't use a technology in the first iteration, we force the capacity to zero until the algorithm reaches feasibility or after a certain number of iterations. We did this because at the start of the algorithm there may be large swings in the production that cause technologies that otherwise wouldn't be included to be profitable. The swings slow down the convergence of the algorithm.

Method	Description
Feasible start solution	Provide a starting point
Fixing technologies	Fixing technologies that can reasonably be assumed to be zero or at full capacity
Proposal removal	Remove proposals not used in the last k iterations
Non-anticipativity slack	Accept solutions that are slightly infeasible
Penalty adjustments	Various methods to control the penalty
Step length reduction	Limit the step length by modifying δ in the subproblem

Table 4.1: Overview over techniques to improve the solution time

The master problem is increasing in size each iteration, which makes the problem larger and more complex. Eventually we experienced that GAMS did not solve the master problem, it was reported being infeasible. Considering that the master problem solutions found in the former iterations are feasible in the current master problem, this is not correct. To alleviate this and make the master problem smaller, we check when a proposal was last included in the master problem solution and remove it if it hasn't been used recently. This reduces the size of the master problem and the model is generated faster in GAMS and solved faster.

A small slack value is included in each non-anticipativity constraint, since it is sufficient that two variables are considered equal if the difference is marginal. The detail level is defined by the upper and lower limit of the slack variables, which is 0.01 in this thesis. As a result, the algorithm doesn't need to adjust nearly equal variables and therefore solves faster.

The convergence is also highly sensitive on the penalty value. Little research has been done to find good values for the penalty. Watson et al. (2008) explores different techniques within progressive hedging and suggests different strategies to adjust the penalty values on a normal constrained optimization problem. The article suggests that adjusted penalty values give much better results than fixed values.

Similarly we try to limit the fluctuations in the model by reducing δ in the subproblems. By reducing this value, the step length in each iteration is smaller.

Chapter 5

Data set

To study the European power market today and in 2020, we have chosen to include Norway, Sweden, Denmark, Germany, the Netherlands and Great Britain with each country as their own node. These countries were chosen based on the strategic connections between the countries and the interesting generation mix. Norway and Sweden have a lot of hydropower, while Denmark and Germany have a lot of wind power. The Netherlands has less wind power, but is strategically important because of the market's closeness to Germany and sub-sea cables to Great Britain and Norway. Great Britain does not have so much wind power yet, but has future plans of considerable wind power production.

The model has a short-term perspective with a time horizon of 24 hours with one hour resolution. This allows us to study the intra-day spot market and how the price and dispatch changes with variable wind production.

The underlying data set that is used for the different case studies is presented in the following sections. We will refer to the "today" situation as 2010 in the results, since the latest comprehensive generation capacity data set we were able to obtain was from year-end 2010.

5.1 Generation Capacity

The generation capacity data was obtained from a combination of EURELECTRIC[1] and NREAPs[2]. Eurelectric was used for the generation data in 2010 and the conventional generation in 2020. The 20-20-20 goals for the generation capacity of bio, solar and wind power in 2020 were obtained from the NREAPs. Technologies with negligible capacity in a country were removed. The run-of-river capacity was assumed to be 20 % of the total hydro capacity in Great Britain, Norway and Sweden. Generation capacity was split on each producer using the market share

[1] The Union of the Electric Industry: `http://eurelectric.org`

[2] National Renewable Energy Action Plans: `http://ec.europa.eu/energy/renewables/transparency_platform/action_plan_en.htm`

numbers from the ADAM project[3]. Since the market share data did not include renewables, this was allocated to the fringe. The market share data results in a fringe with a leading market share in each country, which is not really consistent with the economic definition. But since the formulation will treat them as price takers if the market power factor, MP, is set to zero, there would be no difference from a case where we divided the fringe into many smaller actors. The aggregate per-country capacities are presented in Figure 5.1 and 5.2.

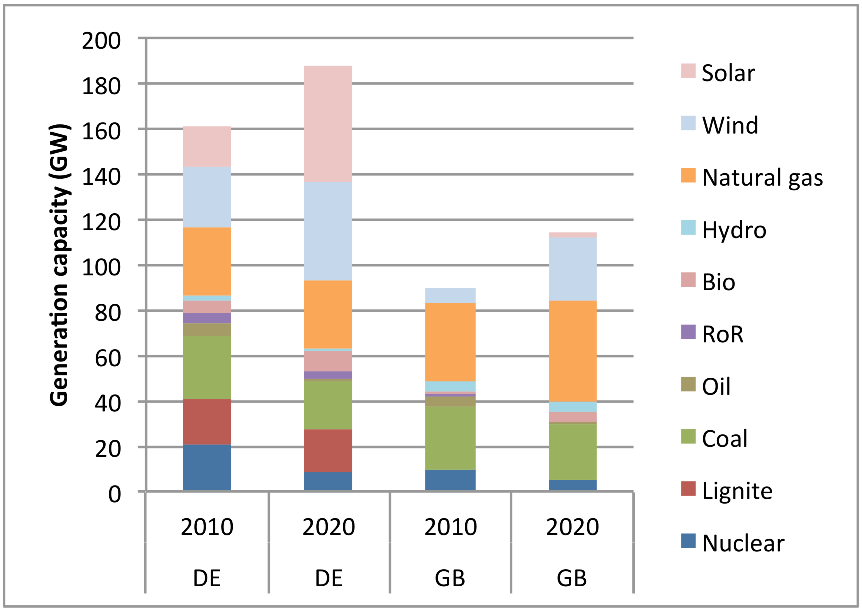

Figure 5.1: Generation capacity in Germany and Great Britain

In the model, the available capacity is corrected for plant maintenance and outages. Using data from Eurelectric, we set the factor to 0.85 for thermal generation technologies. For hydropower we set the value to 1, since the actual production will depend on the water value. In run-of-river power plants the factor represents the amount of flow in the river, we set it to 0.5. For solar the factor represents the weather conditions. The solar production profile is based on a German autumn day[4] from the EEX transparency platform, with a peak of 0.53 at 13:00.

Pump storage is disregarded outside of Scandinavia, as the plants usually operate in the balancing market due to their very small reservoirs (a couple of hours) (EEX, 2011) and our model formulation does not support any limits on how much you can pump in aggregate. In Norway we include the 320 MW Saurdal Pump Storage Plant for Statkraft. The efficiency factor for the pump storage plants, EF is set to 75%, which is in the range of what a modern pump storage plant achieves (Leuthold et al., 2010).

[3]Project supporting the EU's climate policy: `http://www.adamproject.eu/`
[4]Solar production based on the production the 28th of October 2011

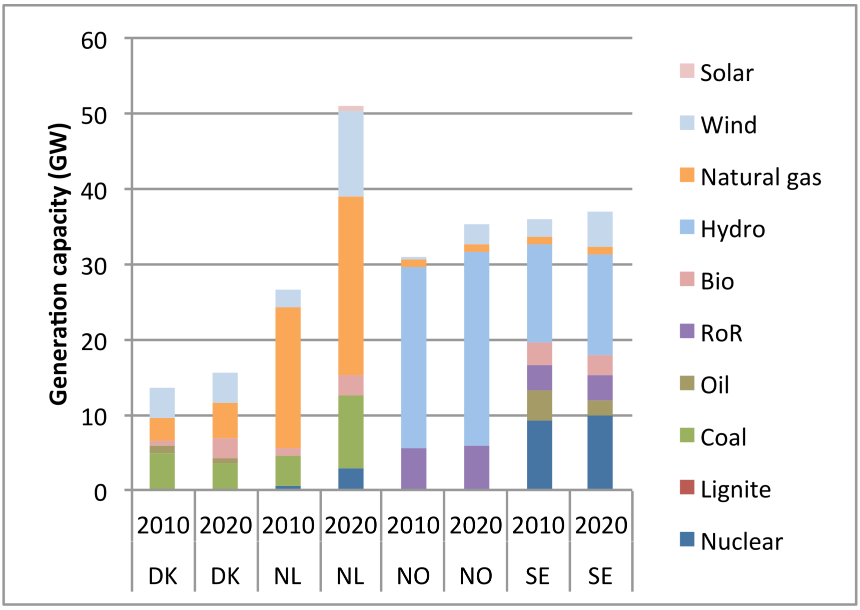

Figure 5.2: Generation capacity in Denmark, the Netherlands, Norway and Sweden.

5.2 Electricity Demand

We calculated the inverse demand curve for each hour by collecting demand and prices from a weekday in October 2011 (27th), since it was difficult to obtain historic data for the Netherlands and Great Britan. Table 5.1 contains the source of the price and demand data. We estimated the load curve for the Netherlands based on the consumption in Great Britain, due to lack of data[5].

Country	Price	Demand
DE	EEX.com	EEX.com
DK	Nordpoolspot.com	Nordpoolspot.com
GB	APX-ENDEX.com	BMReports.com
NL	APX-ENDEX.com	Scaled to GB load curve
NO	Nordpoolspot.com	Nordpoolspot.com
SE	Nordpoolspot.com	Nordpoolspot.com

Table 5.1: Sources for price and demand data

The data set is illustrated in Figure 5.3 and 5.4. For the 2020 case, Eurelectric projections were used to scale the demand as seen in Table 5.2. The demand increases by 5-16 % in all countries, except in Germany that decreases by 5 %.

[5]The Dutch load curve is the same as the British load curve, but is scaled based on maximum demand that specific day in the two countries.

Country	Demand Change
DE	-5 %
DK	9 %
GB	12 %
NL	16 %
SE	5 %
NO	12 %

Table 5.2: Change in demand from 2010 to 2020

Lijesen (2007) gives an overview of different elasticity estimates, and finds that the real time elasticity in the Netherlands is -0.0043 for spot market customers, or -0.029 if all customers are assumed to behave as the spot market customers. This is very inelastic, which means that big changes in the price would not affect the demand that much. Producers with even just a little market power could easily exploit this to gain a windfall profit, but this would not be sustainable in the long term. Regulators would intervene, and customers would switch to alternative sources or install their own diesel generators. We have therefore decided on an elasticity of -0.20 and assumed that the customers have adjusted to the new market condition. This is comparable to the elasticities used in Leuthold et al. (2010) and Green (2007) of -0.25 and -0.15, and several studies cited in Lijesen (2007).

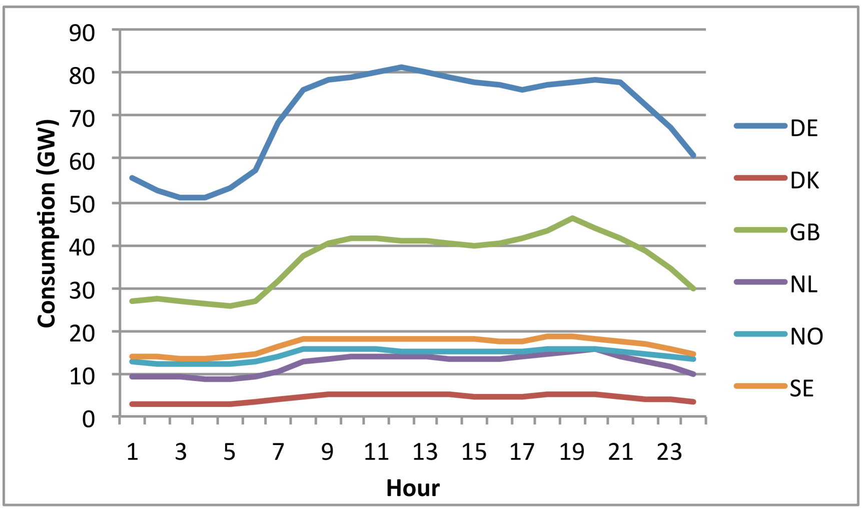

Figure 5.3: Reference consumption 2010

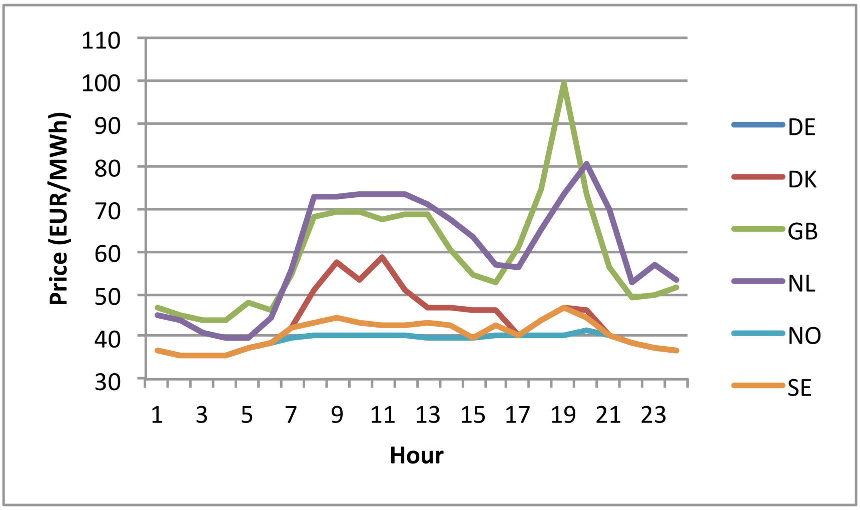

Figure 5.4: Reference prices

5.3 Transmission Capacities and Costs

Transmission capacities are modeled based on the maximum net transfer capacity values and data on the subsea cables (ENTSO-E, 2011). The capacities are illustrated in Figure 5.5.

For the situation in 2020, we referred to the ENTSO-E Ten Year Development Plan[6]. The projects are outlined in Table 5.3, we additionally redefined a planned cable from Belgium to Great Britain to start from the Netherlands.

Line	Capacity (MW)	Project
NO-SE	1,200	South West Link
NO-DK	700	Skagerak 4
DK-DE	500	Midterm increase
DE-NL	1,500	New double circuit 400 kV line
NO-NL	700	NorNed 2
NL-GB	1,000	Nemo project
NO-DE	1,400	Nord.Link
NO-GB	1,400	New link to GB

Table 5.3: New transmission lines in 2020

The fixed costs, FC_l are set to zero, but we include the increasing AFC_l cost term to get unique flows. The value is set so that a producer pays 1 EUR/MWh if he uses the full capacity of a power line.

[6]Available at `https://www.entsoe.eu/index.php?id=232`

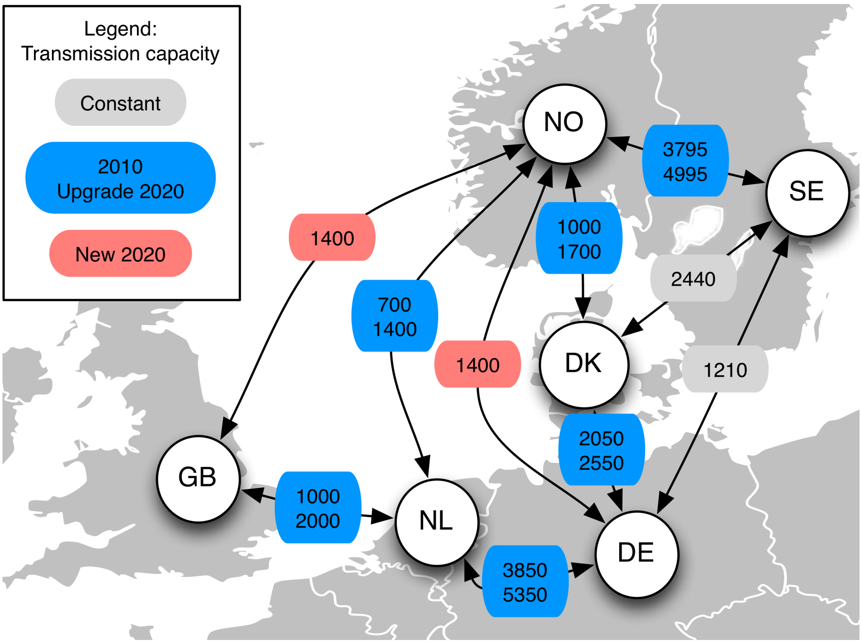

Figure 5.5: Transmission topology in MW

5.4 Marginal and Ramping Costs

Marginal and ramping costs were calculated using data on typical efficiencies, ramping fuel requirements and start-up depreciation from Traber and Kemfert (2009). Using updated fuel prices and a carbon price of 10 EUR/ton we calculated the costs in Table 5.4. We set the marginal costs equal to zero for run of river, solar and wind power production, since the extra production does not cost the producer anything.

The marginal cost of bio power production is more uncertain, as there are many different types of plants, both in size, feedstock, and technology. We chose 60 EUR/MWh as the marginal cost, which is just above the lowest level of the German feed-in tariff[7] for bio power at 58.9 EUR/MWh (BMU, 2012). Similarly, the ramping cost was set at 80 EUR/MWh, which is comparable to coal power plants.

For hydropower we set the marginal cost equal to the water value, which is the value of one extra unit of water in the reservoir. It would be possible to have a separate model to calculate the water values and feed this into the model, but this is beyond the scope of this thesis. Trötscher and Korpås (2008) notes that in principle the water value should be constant throughout the year, but deviations from the expected reservoir levels cause the water value to fluctuate. The

[7] A feed-in tariff is a fixed price that a power producer is paid for the electricity to cover the higher costs associated with renewable energy sources

production in Norway is dominated by regulated hydropower, and it can therefore be expected that the average price corresponds to the water value. The average spot price in Nordpool's NO1 price area in 2011 and until May 2012 was 46.4 and 35.3 EUR/MWh respectively, therefore it seems reasonable to choose 40 EUR/MWh as the water value in our data set.

Technology	Marginal Cost (EUR/MWh)	Ramping Cost (EUR/MWh)	Ramping limit (1/h)
Solar	0	0	100 %
Wind	0	0	100 %
Bio	60	80	50 %
Hydro	40	0	100 %
RoR	0	0	100 %
Oil	148	66	50 %
Gas	48	40	50 %
Coal	36	82	20 %
Lignite	30	66	20 %
Nuclear	7	37	20 %

Table 5.4: Marginal and ramping costs, and ramping limits

Individual ramping rates for a power plant may be very high, Wangensteen (2007) reports that individual thermal plants may ramp 5-10% of the nominal capacity per minute. De Jonghe et al. (2011) mentions that this may be true for a single power plant, but that for an aggregate system you would never see such behavior. Minimum run times and load factors indicate that the system as a whole is not able to ramp as fast. Together with data from Maddaloni et al. (2009) and Traber and Kemfert (2009) we have used the ramping rates in Table 5.4.

For the prices in 2020, we assume that the real prices stay the same. It can be expected that the average efficiency of each technology increases, but it would also not be surprising if the fuel costs increase above the general inflation due to strong demand from emerging markets. The carbon price is also assumed to stay constant in real terms, although that may change depending on the economic situation in Europe and the speed of deployment of renewables. This would especially have an effect if the price rises above 33 EUR/ton, which would make natural gas production cheaper than coal in our data set.

The marginal generation costs also have a second order term, $AMC_{n,p,t}$. We included the slightly increasing cost term AMC to get unique solutions. These are set per producer, node and technology, and correspond to 5 % increase in the marginal cost if the production is at full capacity.

5.5 Market Power

The market power factors used are given in Table 5.5. We defined the fringe to have a very small level of market power to get unique sales, the impact on the resulting prices is negligible. The factor is based on the discussion in Section 7.2 of the results.

MP factor	Description
0.001	The producer has no market power and belongs to the fringe
0.4	The producer has some degree of market power

Table 5.5: Market power factors

5.6 Wind Power

We use the same wind forecast for every node in our model. Nordpool offers data on production and forecasts of wind power production in Denmark, but this was not available for the other countries. We lose some dynamics in the system because of this, since different areas may experience different wind levels and have different levels of forecast uncertainty. The forecast for Denmark is issued at 17:00 for the 24 hours the following day. The forecast does not contain any information regarding the uncertainty, but we use the scenario generation method of Jaramillo et al. (2009) where this information is not needed.

We used data for 2011 and until May 4th 2012, in total 485 data points for each hourly forecast error. Data for 2010 was also available, but it was unreliable and contained errors. We normalized the data to the maximum of the actual production since Denmark has had a relatively stable wind generation capacity in recent years. A histogram of the production and error of the forecast is presented in Figure 5.6 and 5.7. The wind production is skewed with a mean availability factor of 0.277 and a median of 0.207. The lower and upper quartiles are 0.084 and 0.423 respectively. The error has a slightly positive bias in our sample with a mean of 0.0045, while the median is 0.001.

The data also shows a slightly increasing forecast error through the forecast horizon. This is illustrated by the mean absolute error (MAE) in Figure 5.8.

We generated four different forecasts, with the first three corresponding to the lower, median and upper quartile of the historic data, called the Low, Median and High forecast. The Dip forecast corresponds to an actual wind power profile in Denmark from January 2012, which has a large input of wind at night, followed by a decrease. The forecasts are illustrated in Figure 5.9.

We generated 5,000 scenarios for each of the four wind forecasts. Figure 5.10 illustrates the wind development for 25 scenarios of the Median forecast.

To be able to solve the model in a reasonable time, we use the scenario reduction tool SCENRED2 in GAMS to reduce the initial set of 5,000 scenarios to a scenario

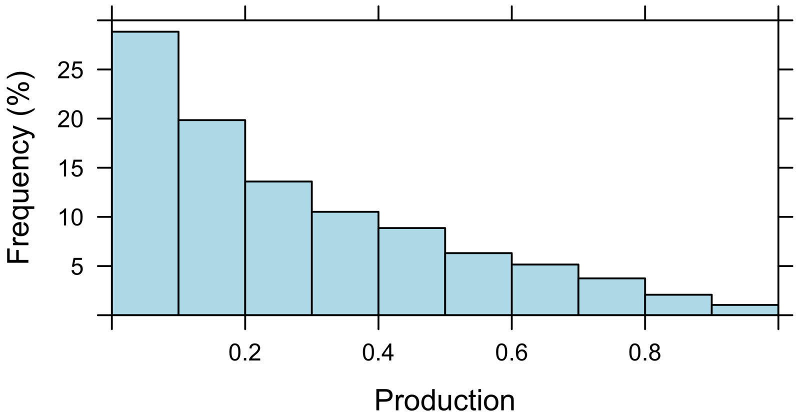

Figure 5.6: Histogram of the wind power production in Denmark

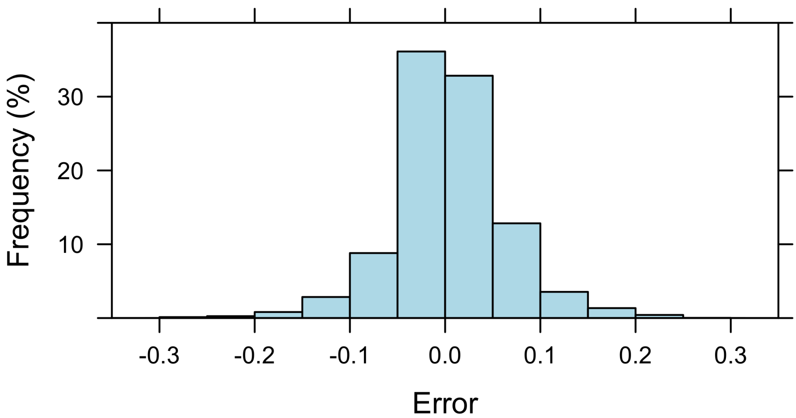

Figure 5.7: Histogram of the wind forecast error in Denmark

tree with 25 scenarios and approximately 350 nodes. A typical model then solves in 3-4 hours. An example of the branching tree for the Median wind forecast can be seen in Figure 5.11 and the wind development in Figure 5.12. The reduced constructed scenario tree has a lower MAE than the originally generated scenarios. Considering that our wind data is for Denmark, and the MAE goes down when you aggregate over a large area, we don't consider this a problem, but the impact will be considered in the results (Section 7.3).

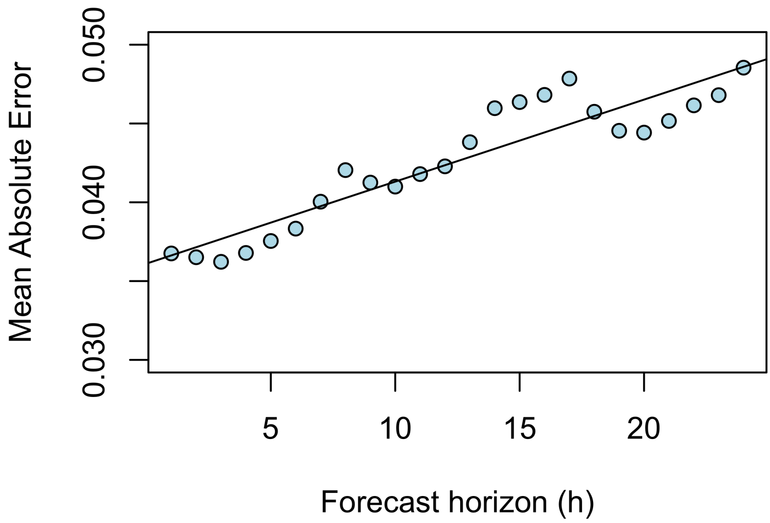

Figure 5.8: Development of the wind forecast error in Denmark

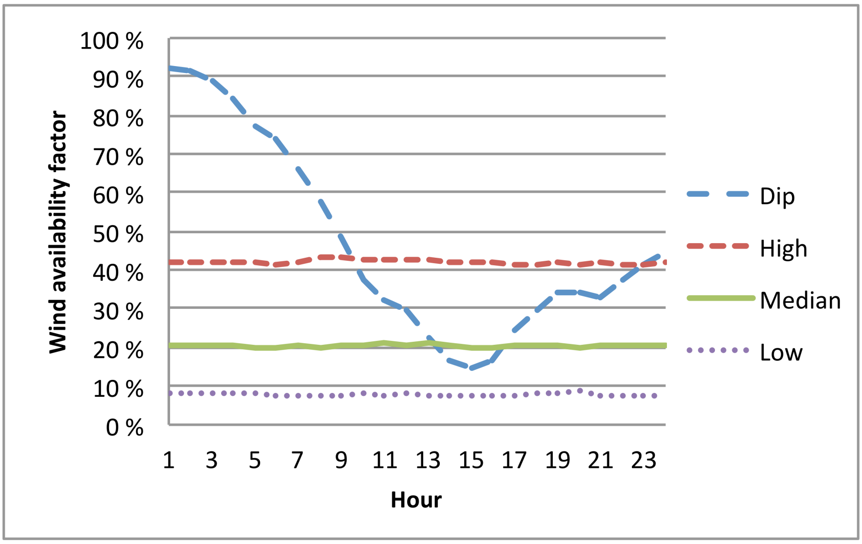

Figure 5.9: Mean value of the wind forecasts

5.6.1 Issues with the Scenario Generation Algorithm

There were some issues with our scenario generation implementation. An analysis of the forecast error in the scenarios showed that the MAE was approximately one third of what was found in the historic data set. The mean of the scenarios was also lower/higher than the supplied forecast, depending on the forecast of the

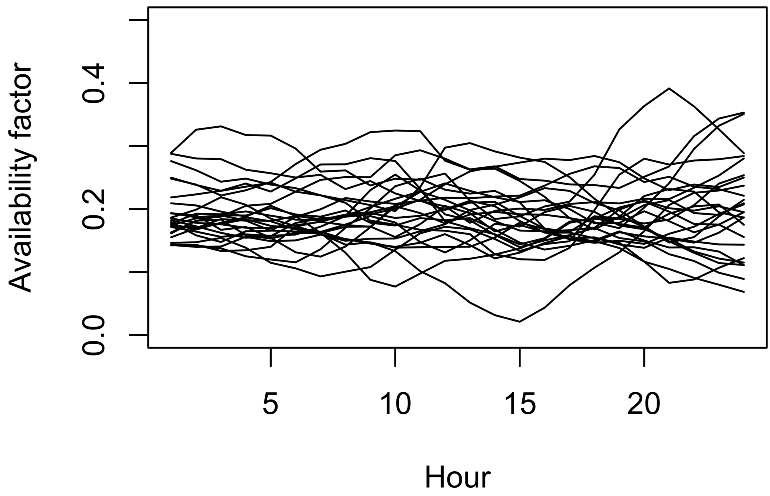

Figure 5.10: 25 typical scenarios generated in R for the Median forecast

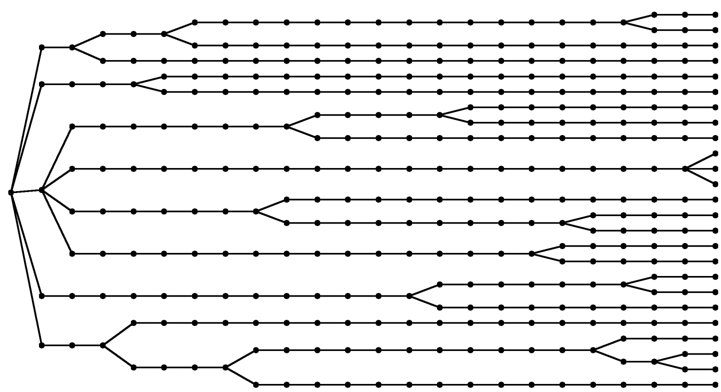

Figure 5.11: Branching tree of the constructed scenario tree for the Median forecast

availability factor being lower/higher than 0.5. This arises from the fact that the implemented method uses the Beta-distribution to represent the behavior of the wind power production. By multiplying the covariance matrix of the historic errors, that is the basis for the forecast errors, we were able to get scenarios that were more in line with the behavior in the source data. For our purposes the generated scenarios will be adequate, and the method produces scenarios that take into account that the error in the forecast in the next period depends on the value in the previous period, instead of being random.

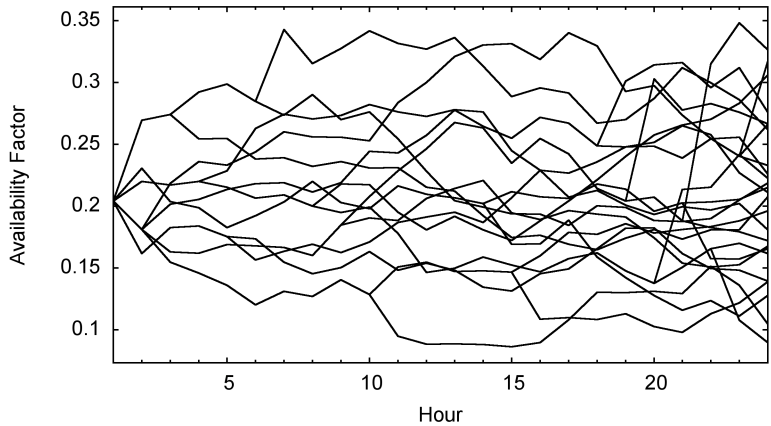

Figure 5.12: Wind development in the constructed scenario tree for the Median forecast

For our Median wind forecast, we had to multiply the covariance matrix of the forecast errors with 6 to get approximately the same error distribution as seen in Figure 5.13. The match with the MAE distribution differs slightly through the forecast horizon, but is between 0.01 and -0.005.

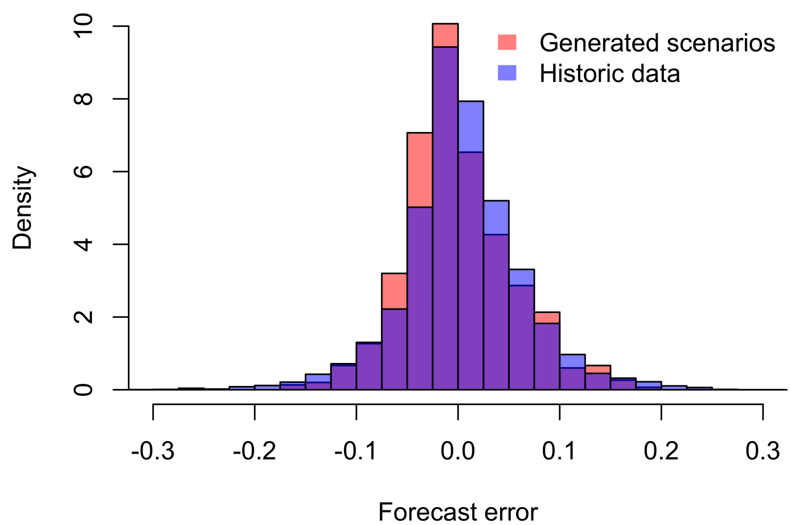

Figure 5.13: Error distribution in the historic source data and generated scenarios

5.7 Assessment of the Data Quality

We believe that we have compiled a realistic and representative data set, but there are of course a few weaknesses that are worth pointing out. The generation capacity data is most vulnerable to the market share numbers and to the projections in 2020. Since the big producers have no wind, solar or bio-powered generation in the data set, they have less market share and power than they should. The transmission capacity figures are good, but of course the 2020 data are projections. We don't consider the projections to be a problem for the generation and transmission capacity, as they are a case study of the situation in 2020.

Our demand data has two weaknesses. First, it is only based on one day, and secondly because of the elasticity assumption. Nothing indicates that the day was an outlier, but the point elasticity will be on the price/demand intersection since we have a linear specification of the inverse demand curve. The marginal costs are dependent on the efficiency assumptions we have used, while the ramping costs also have the issues of start-up fuel requirements and increased depreciation of the equipment. In reality, different plants have different characteristics, while we model them as just one type. For our study purposes, we believe our numbers are of sufficient quality to give representative results.

The 2020 projections of the prices are more uncertain however, and one big source of uncertainty is the carbon price. Currently there is a debate if the EU should tighten the emission cap in the EU Emission Trading Scheme, or institute a price floor as Great Britain is planning to do from 2013 (Clark, 2012). Since carbon allowances in the scheme can be saved for future years ("banked"), we believe that the price is heavily dependent on the rate of economic recovery in the EU after the ongoing sovereign debt crisis. This concern was shared by professional analysts in a recent article in the Financial Times (Clark and Blas, 2012). The fuel prices are of course uncertain as well, but they are more dependent on the world economy, not just the EU. The carbon price will not only affect the electricity price, but also the relative dispatch between coal and natural gas. We will therefore perform a sensitivity analysis on the carbon price.

It is impossible to model all the possible outcomes of how the wind may develop through the forecast period, but our four wind forecasts should be able to highlight some important differences between the cases.

Chapter 6

Implementation

6.1 Software

GAMS is a high level modeling system, which makes it easy to formulate MCP models that are sent to a solver for solution. GAMS IDE 23.8.2 was used to write the code, and PATH 4.7.02 as the solver. PATH is based on a generalization of Newton's method and a linearization based on Lemke's method (Ferris and Munson, 2000). Some important experiences with GAMS and the PATH solver are described in Appendix G.

Integration between GAMS and Excel is very helpful in two ways; input and output. The input data is imported from Excel sheets, which makes it easy to switch between different data sets. The solution can easily be exported to pivot tables in Excel for analysis. For details about interaction with Excel see Appendix G.

The scenario generation algorithm of Jaramillo et al. (2009) was implemented in R, a language and environment for statistical computing.

6.2 Hardware

The program was run on a desktop with the following specifications:

- CPU: Intel Core i5 3570K (3.4 GHz quad-core)
- Memory: 8 GB

By monitoring the solver process in Resource Monitor, it was apparent that the CPU was the limiting factor, as the solver is not multi-threaded. GAMS do have facilities handing of problems to solvers in parallel, that could have been used on the decomposed problem, but this was not used.

6.3 Solution Time

The solution time of the non-anticipativity formulation (NA) and scenario tree formulations (T) are shown Table 6.1. Both models are solved as MCPs. The time includes the time needed for reading and processing data, generating models, communication with the solvers, and the solver time. The non-anticipativity formulation takes considerable more time to solve compared to the scenario tree formulation. This is not directly linked to the number of variables, as the tree formulation is able to solve larger problems. This may be indicate that the non-anticipativity constraints are difficult to solve, since the rest of the formulation is largely the same. The VI formulation also uses PATH as the solver, after it has been automatically reformulated with JAMS in GAMS. Additionally the mappings must be adjusted, since GAMS doesn't consider equations that are zero to exist. Both the reformulation and adjusted mappings increase the number of constraints, and the solution time goes up. As a consequence, the MCP-tree formulation was used to produce the results in Chapter 7.

Model	Scenarios	Time periods	Variables	Solution time
MCP-T	8	12	39,198	0:01:02
MCP-NA	8	12	86,595	8:15:47
MCP-T	10	24	139,176	0:25:31
MCP-NA	10	24	215,019	>72:00:00
MCP-T	25	24	305,806	2:49:50
MCP-NA	25	24	541,864	>72:00:00

Table 6.1: Solution time for different model versions and problem sizes

Chapter 7

Results

This chapter contains the results of some case studies done with our model. We start by presenting some overall model results for the power market in 2010 and 2020 with emphasis on the role of different generation technologies. Then we will investigate the effect of market power, before we study the impact of different levels of wind forecast uncertainty. Then we study the robustness of the power market by comparing the effect of different wind power forecasts on prices, generation dispatch and consumption. At last we will perform a sensitivity analysis with focus on how different carbon prices and feed-in tariffs for bio power generation affect the results.

7.1 Power Market in 2010 and 2020

The result of a run of the model on the base case in 2010 with the Median wind forecast produces the generation dispatch in Figure 7.1. As expected, nuclear, lignite, run-of-river and coal generation is responsible for the base load. Hydro generation takes some of the demand increase during the day, but it is mostly natural gas generation that ramps up. At the end of the day, coal generation is ramping down. This is a result of producers not having to take into account the subsequent ramping cost the following day. Bio-fueled generation is not used at all, which is not realistic given the feed-in tariff in Germany. Oil generation is also absent from the generation dispatch. In 2020 (Figure 7.2) the situation is mostly the same, but with significantly higher wind and solar production that offset some of the reduction in nuclear capacity.

The system price[1] decreases from 51.58 EUR/MWh in 2010 to 49.54 EUR/MWh in 2020. In Figure 7.3 we can see that this is attributed to a dip in the price during the day, even though the prices increase during the night. The benefit is mostly seen in Germany and the Netherlands, where the prices decrease by 3.8 and 5.1 EUR/MWh respectively as seen in Table 7.1. In Denmark and Norway we see a modest increase in the prices, and overall there is a convergence of the prices in 2020.

[1] The aggregate volume-weighted price for all nodes

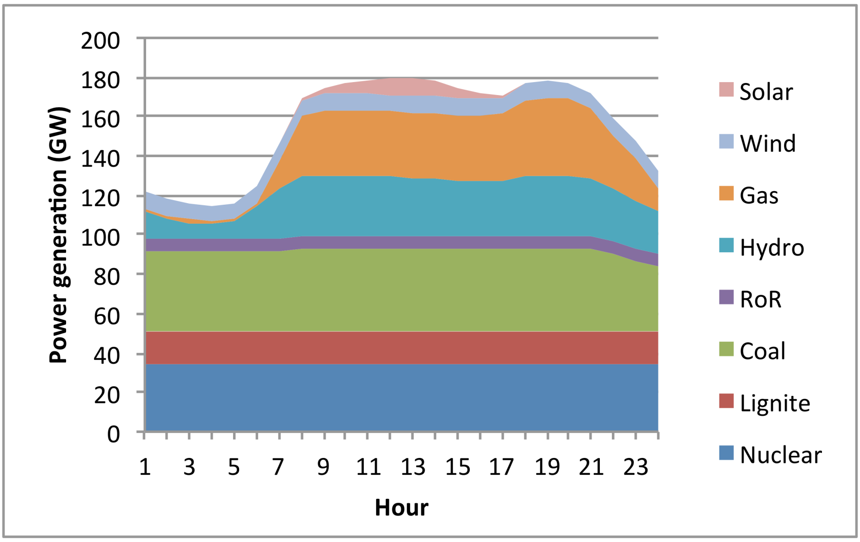

Figure 7.1: Aggregate generation dispatch in 2010

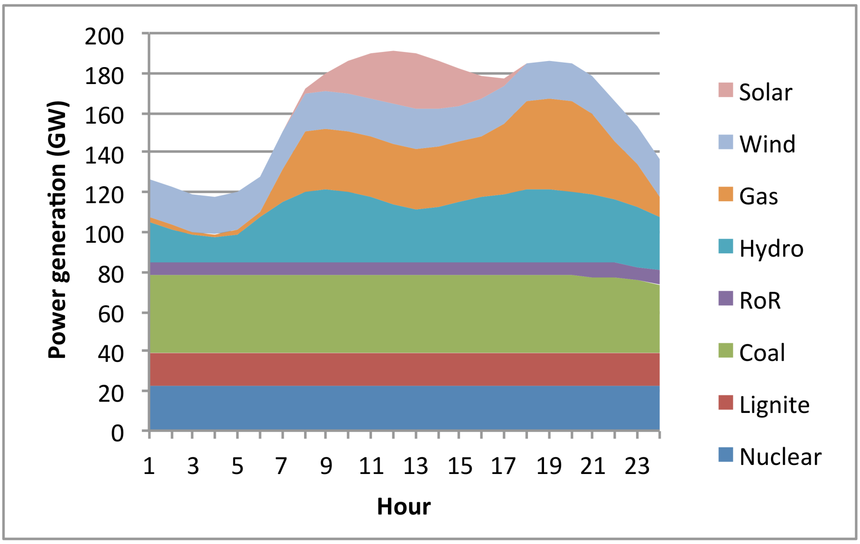

Figure 7.2: Aggregate generation dispatch in 2020

The price dip can be explained by the increased solar production in Germany. In the results, Germany has a generation surplus during the modeled day in 2020 in contrast to 2010, and is exporting power to its neighboring countries as seen in Figure 7.4. Before the lines from Scandinavia to Germany was exporting at full capacity, while in 2020 Germany is the one exporting. Germany also has a price

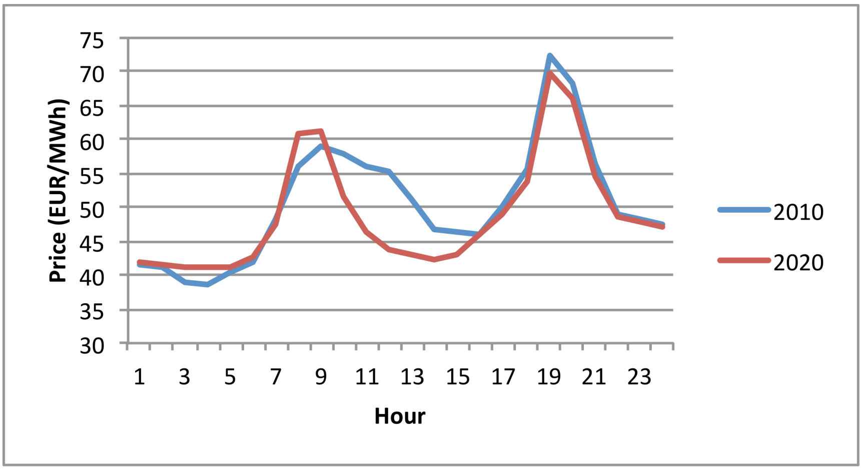

Figure 7.3: Hourly system price

Node	2010 (EUR/MWh)	2020 (EUR/MWh)	Difference (EUR/MWh)
DE	55.74	51.91	−3.82
DK	44.99	46.48	1.48
GB	49.84	49.91	0.07
NL	56.03	50.91	−5.12
NO	41.75	43.65	1.90
SE	44.34	44.30	−0.04
SYS	51.58	49.54	−2.04

Table 7.1: Area prices in 2010 and 2020

increase during the night in 2020, which causes an increase in the system price. A reasonable assumption would be to think this was caused by the increased interconnection at night, but Germany is exporting less power at night in 2020 compared to 2010. One reason is that Germany has lost 20 GW of baseload capacity, and this is further lowered since Eon does not utilize 2.3 GW of their spare coal-fired capacity. As a result, the market price in Germany during the night is around 41 EUR/MWh. Although this is lower than the marginal cost of natural gas production, the fringe chooses to operate 2 GW of natural gas production where the loss during the night is offset by the saved ramping cost.

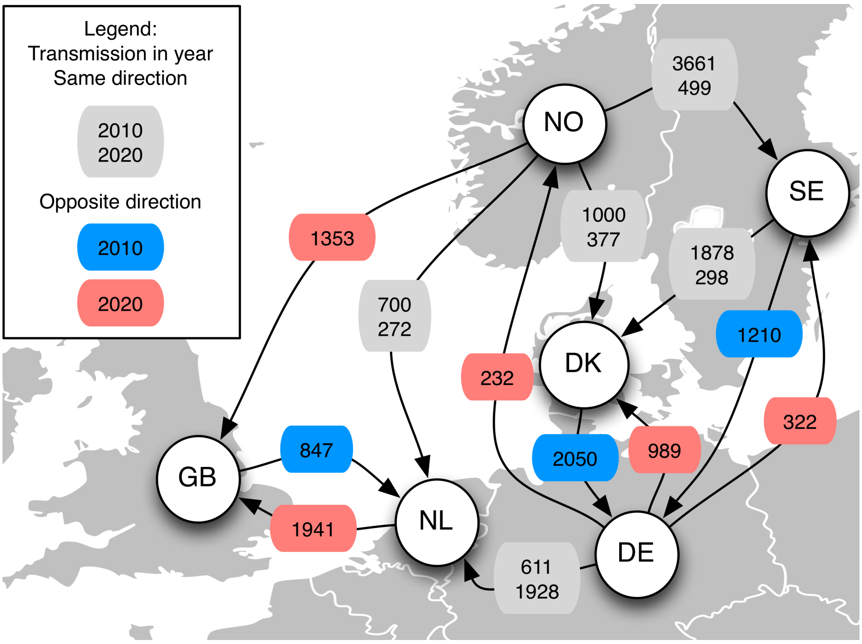

Figure 7.4: Comparison of the transmission in 2010 and 2020 at 13:00

In the overall transmission utilization rates in Table 7.2, it can be seen that the utilization goes down on most of the lines in 2020 compared to 2010. This is most instances due to the increased capacity on the lines, and the spare capacity is one of the reasons why the area prices, in Table 7.1, converged more. Of the new power lines, the line from Norway to Great Britain is the one that is used the most, although the average price difference is larger between Norway and Germany.

Transmission line	2010	2020
DE-NL	36 %	48 %
DK-DE	92 %	63 %
NL-GB	73 %	60 %
NO-DE	N/A	73 %
NO-DK	88 %	69 %
NO-GB	N/A	90 %
NO-NL	92 %	72 %
NO-SE	75 %	41 %
SE-DE	94 %	68 %
SE-DK	68 %	56 %

Table 7.2: Transmission utilization rates Median forecast

The consumption in the results increased in 2020 due to the higher demand for electricity, with a total of 4.1 %. Compared to the reference Eurelectric projections that we used for the demand, consumers in Germany and the Netherlands has a relative increase of their consumption by 0.9 and 1.7 percentage points. This increase comes during the day and the evening. Consumers in Denmark and Norway decrease their consumption by 0.7 and 1.1 percentage points relative to the projections. Great Britain and Sweden show insignificant differences.

7.2 Effect of Market Power

A central part of our model is the ability to represent market power based on the conjectural variations approach. To determine a proper level of the MP parameter, we ran the model for six different market power factors from 0 to 1 for the biggest players, while the competitive fringe has no market power. This was done using our data set for 2010 with the Median wind forecast without any uncertainty.

The most important question to answer is if the producers' profit are increasing with MP or not. An analysis of the profits in Figure 7.5 shows that Eon is most profitable when the market power factor is 0.4, while it is 0.6 for RWE. The other producers have increasing profit with a higher MP factor. Eon and RWE try to lower their production, but the decrease is replaced by generation from the fringe.

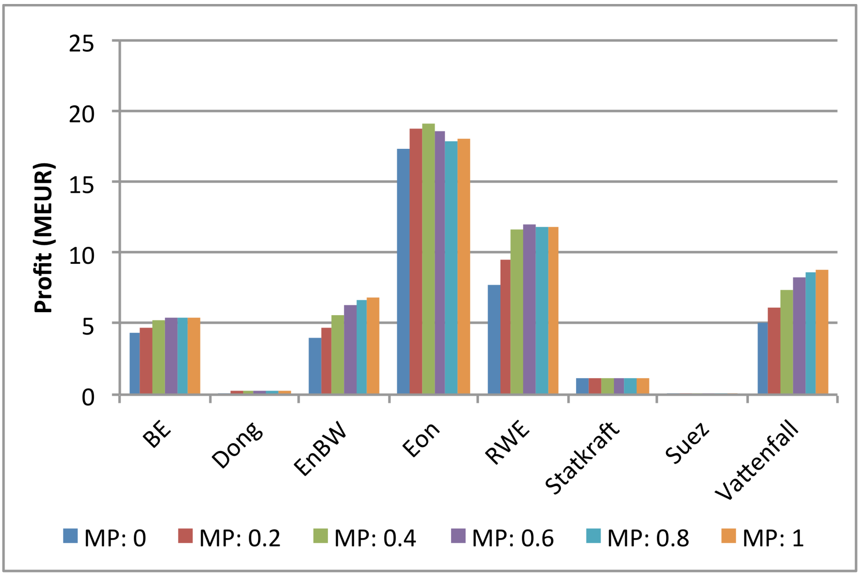

Figure 7.5: Profit of the producers with varying market power factor

The prices are generally lower in our model runs compared to the reference prices, except for Norway and Sweden where the prices are a bit higher. This may be due to the fact that we run the model with 2010 generation capacity data, but we compare with prices collected in 2011. Our costs assumptions also affect the results. Figure 7.6 shows the prices in Germany and the Netherlands for selected MP levels, the prices for MP levels 0 and 0.2 are much lower than the reference, particularly during the night. Looking at the sales in Figure 7.7 reveals the same trend, the sales for the MP levels 0 and 0.2 is too high during the night relative to the reference sales, while 0.4 and above are more in line with the reference.

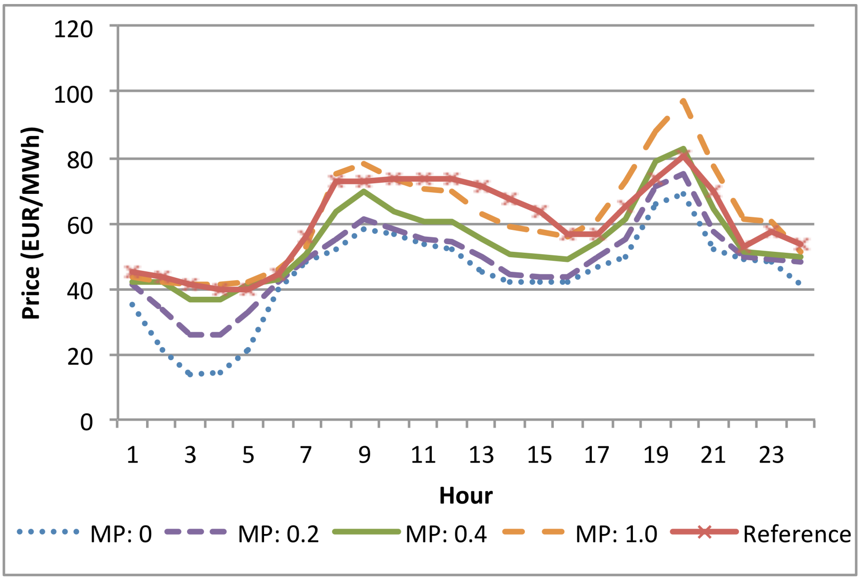

Figure 7.6: Price in Germany and the Netherlands with varying market power

The profit results indicate that it is only Eon and RWE that are large enough to impact the prices on their own, the definition of market power. But, running the model with only Eon and RWE exerting market power results in too low prices and too high sales compared to figures above. Since the electricity market is quite inelastic, it makes sense to put some weight on matching the modeled sales with the reference. This illustrates some of the problems with the conjectural variations approach, and is a result of too much generation capacity controlled by producers without market power. Based on the fact that Eon has the highest profit with a 0.4 market power factor, and that especially the sales are too high with an MP level of 0 or 0.2, we choose to model all the explicitly included producers with 0.4 as the MP factor.

As a result, the system price increases by 6.81 EUR/MWh from 44.75 EUR/MWh to 51.56 EUR/MWh between the perfectly competitive case and the model with the chosen MP factors. The price increase is largest in Germany where the price increases from 45.59 EUR/MWh to 55.71 EUR/MWh. There are many large producers in Germany, and they are able to increase the price especially during the night. Norway sees almost no increase in the price, going from 41.17 to 41.76 EUR/MWh, due to a large fringe with spare hydropower capacity.

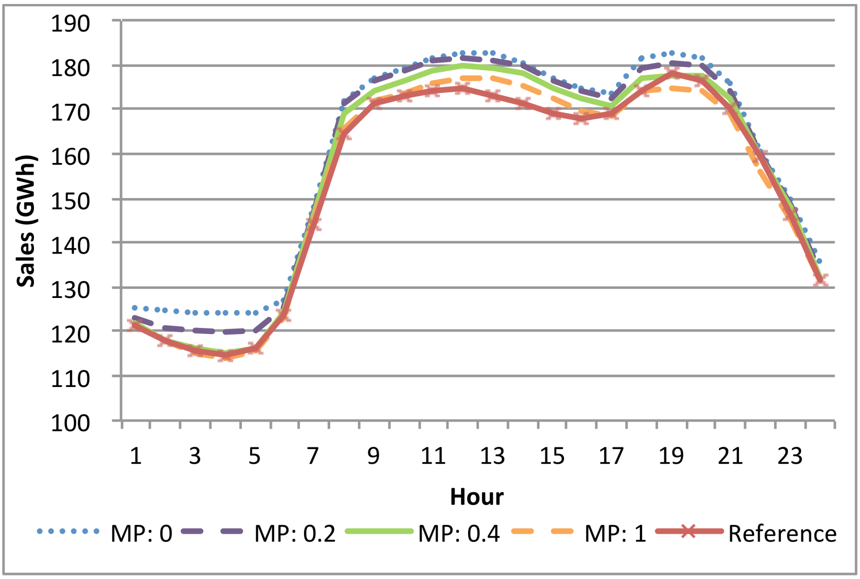

Figure 7.7: Aggregate sales in the model with varying market power

7.3 Effect of Wind Uncertainty

To study the effect of including stochastic wind in the model, we checked the 2020 case with the High wind level. We generated forecasts that had different levels of mean average error (MAE), as seen in Table 7.3.

Covariance matrix	MAE (%)
Unmodified	3.0–5.3
Double	5.5–7.8
Triple	7.8–13.3
Quadruple	7.4–18.6

Table 7.3: MAE of modified scenarios

The first aspect to study is how do the expected prices compare to the equivalent deterministic prices? The system price turned out to be lower in all the cases, although not by much. The expected price in the Unmodified case was 6 cents lower per MWh compared to the equivalent deterministic result (47.53 vs. 47.59 EUR/MWh), while in the Triple case was 13 cents lower (47.45 vs. 47.58 EUR/MWh). The area prices, in Table 7.4 showed slightly larger deviations, with Denmark and Great Britain as outliers. The prices in Denmark increase with the stochastic wind, while they get lower in Great Britain. The price changes in individual countries may be due to a shift in where the production is generated, Denmark has an average of 2.5 % and 8.7 % increase in domestic coal production in the Unmodified and Quadruple case accordingly, which would raise the marginal costs due to the quadratic cost term and may drive the price upwards. The shift in Great Britain seems to stem from lower prices at night, with resulting higher consumption, that bring down the area price.

Country	Unmodified (EUR/MWh)	Double (EUR/MWh)	Triple (EUR/MWh)	Quadruple (EUR/MWh)
DE	0.01	0.00	−0.02	0.12
DK	0.23	0.33	0.43	0.70
GB	−0.15	−0.19	−0.34	−0.35
NL	−0.09	−0.12	−0.22	−0.11
NO	−0.02	0.03	0.07	0.05
SE	−0.02	0.04	0.08	0.06

Table 7.4: Change in area prices compared to equivalent deterministic problem

The slightly increased system prices indicates that the production has increased slightly to be able to handle the uncertain wind. Looking closer at the production reveals that the producers are dispatching more natural gas on average at the expense of coal and hydropower production. In the Unmodified case natural gas dispatch increases by 1.7 %, while coal and hydro go down with 0.4 % and 0.2 % respectively. The changes are larger in the Quadruple case, where natural gas production increases by 4.15 %, while coal and hydro production both decrease by 0.9 %. The hourly absolute change for the Quadruple case can be seen in Figure 7.8.

7.4 Robustness of the Power Market

We ran the model with the four different wind forecasts from the data set to check the robustness of the power market. The first three forecasts correspond to the quartiles of the historic data set. The fourth forecast was chosen to see how the system handles a large influx of wind at night, before it falls during daytime. We'll first present the results on the Median case, before we compare with the others. We consider the robustness to be the system's ability to remain unaffected when the wind power is uncertain, and will therefore study the price volatility in a case and the difference between the price between the different forecasts. The production variability is also interesting, since it shows what technologies are responsible for balancing the uncertain wind.

7.4.1 Median

Figure 7.9 shows the price volatility, as the standard deviation of the price, for the Median case in 2010. The result shows that Norway, and to some degree Great Britain has almost zero variability in the price. Germany, Denmark, Sweden and

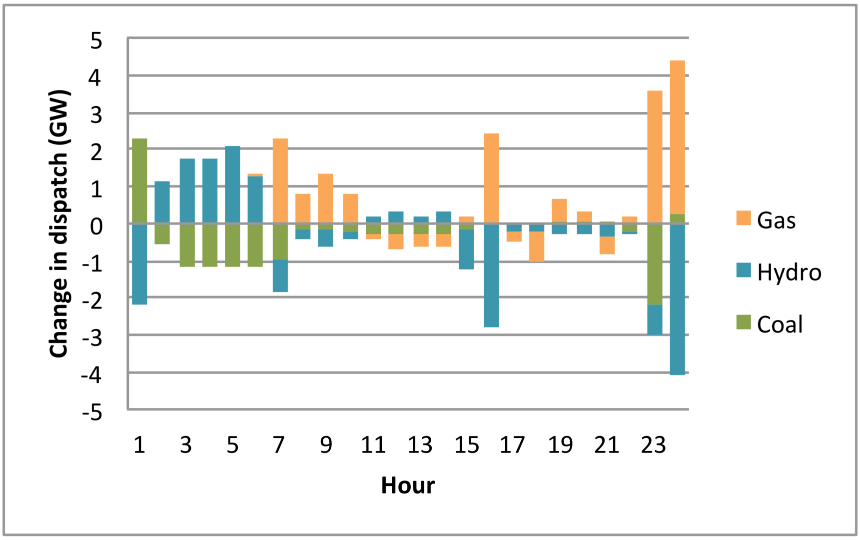

Figure 7.8: Change in the dispatch for the Quadruple case compared to the deterministic equivalent.

the Netherlands show a higher degree of variability. In 2020 the picture changes (Figure 7.9), the variability almost vanishes at night and the middle of the day, but with large peaks around the morning and the early evening hours. This can partly be caused by the additional transmission lines with lower utilization rates in 2020 that increase the flexibility of the system. The additional solar power during the middle of the day also frees up capacity. That the peaks are higher are understandable from the fact that the growth in wind power capacity has exceeded other technologies, and hence the aggregate uncertainty is higher.

Coal, natural gas and hydro generation are responsible for balancing the stochastic wind in 2010, as seen in Figure 7.11. The balancing done by hydropower is predominantly in Norway, while German producers operate most of the balancing done by coal power generation. Producers in both Germany and Great Britain contribute to the balancing with their natural gas production. In 2020 the situation changes, coal generation is not used as much as a balancing source, as it is operating close to capacity. Increased interconnection with the continent allows Norway and Sweden to increase their balancing contribution from hydropower during the night and the middle of the day. Natural gas production ramps up in the morning with producers in Germany, Great Britain and the Netherlands contributing an almost equal share to the balancing of the stochastic wind.

7.4.2 Low, High and Dip

The price volatility for the other forecasts does not show any conclusive differences. Low and High show the same overall development as the Median forecast in 2020

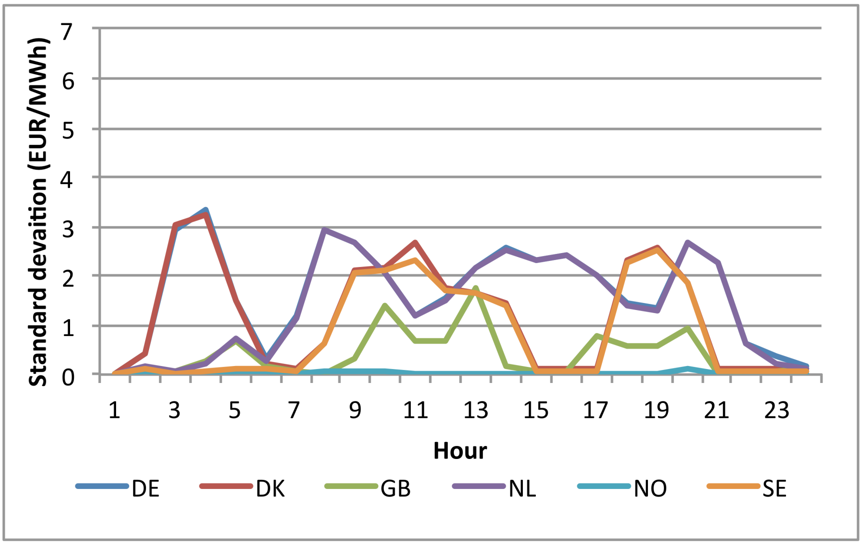

Figure 7.9: Volatility of the price in 2010 Median

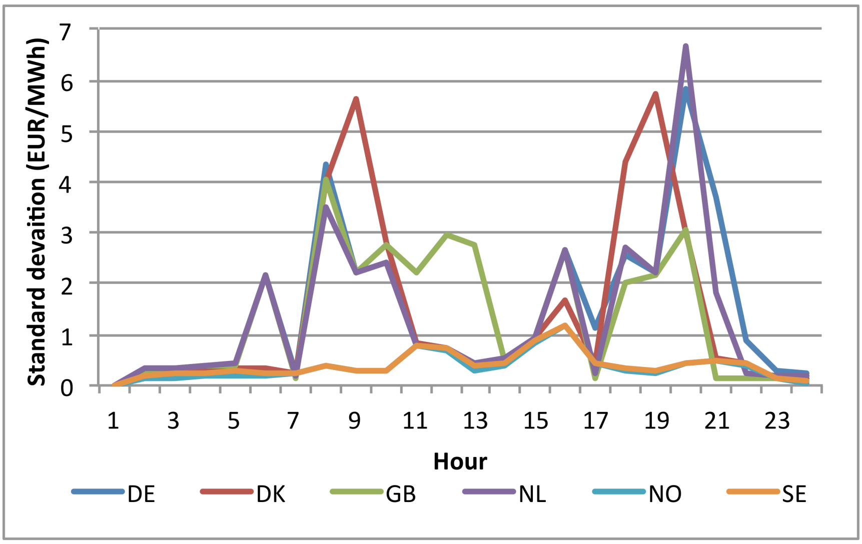

Figure 7.10: Volatility of the price in 2020 Median

in Figure 7.10, except that Germany, Great Britain, and the Netherlands has an extra spike during the night with the High forecast. It is therefore more interesting to see how stable the prices are, Figure 7.13 shows the hourly system price for the different wind forecasts in 2020. The price resulting from the Low forecast resembles the price in the Median forecast, but with higher peaks in the morning

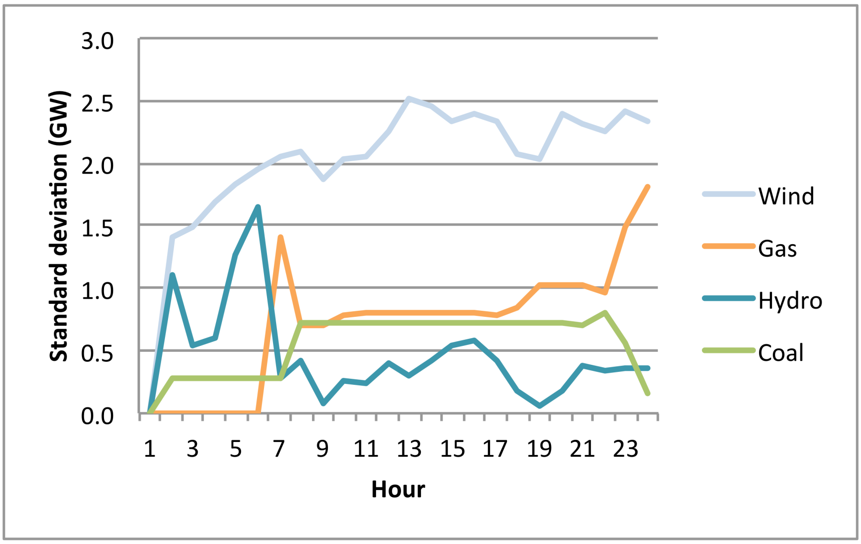

Figure 7.11: Standard deviation of the production in 2010 Median

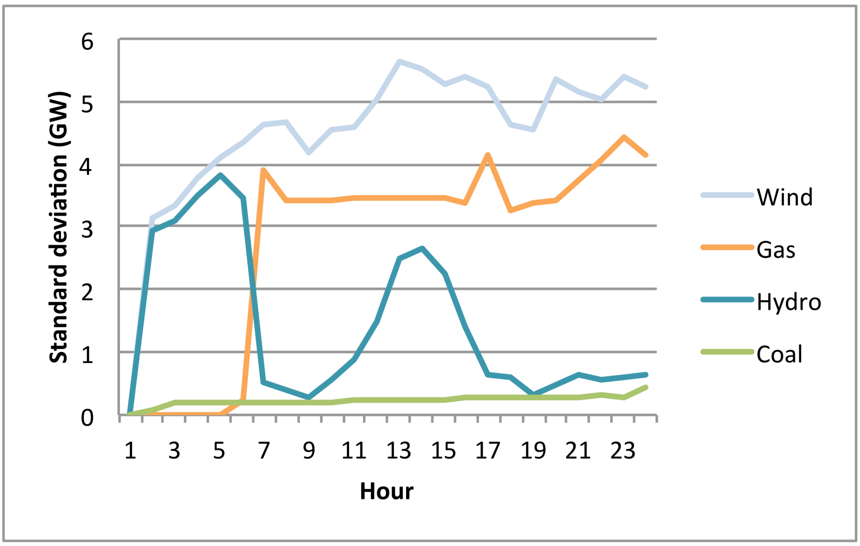

Figure 7.12: Standard deviation of the production in 2020 Median

and evening. The price resulting from the High forecast deviates somewhat from the Median forecast during the night, but is otherwise similar. The Dip forecast shows quite diverging prices compared to the other forecasts. The price is much lower during the night, but higher during the middle of the day. The lower price at night is due to to the large amount of wind production, and Statkraft operate their pump storage plant in Norway. But, because of the high wind feed-in at night, there is not as much base load capacity running and the prices stay high during the middle of the day.

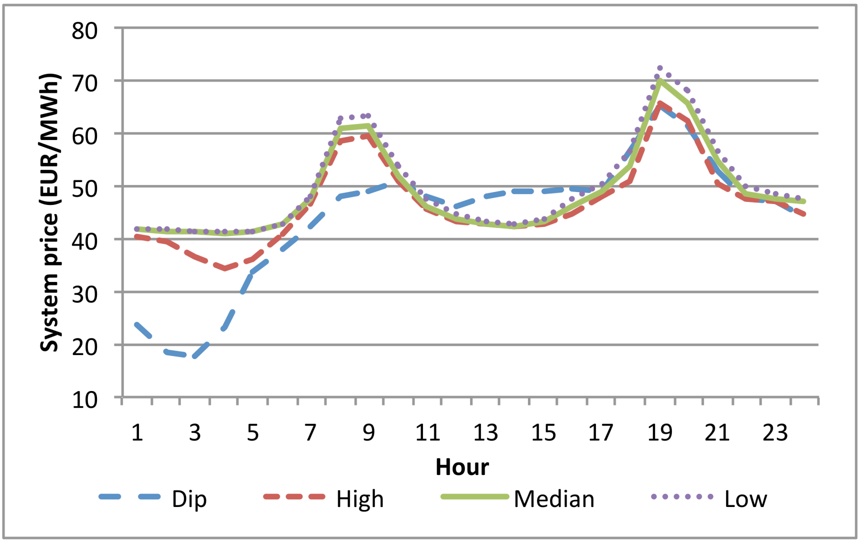

Figure 7.13: Hourly system price for different wind forecasts in 2020

A look at the production variability reveals that coal producers are contributing to the balancing of the stochastic wind in the High forecast, in contrast to the Median and Low forecasts. In the Dip case even lignite producers step in to counter the wind variability. The producers are able to do this because they operate with spare capacity.

Overall the 24-hour aggregate consumption in 2020 does not vary much among the scenarios. The aggregate sales in the Low and High forecast are within one percentage point of the sales in the Median forecast. The sales in the Dip forecast is higher, at up to two percentage points. But, looking at the sales on a hourly basis reveals that there are big differences. The Low forecast sales is relatively stable, but the High forecast shows consumption differences of around 4 % at 04:00 in Germany, Great Britain and the Netherlands. This difference is even more pronounced in the Dip forecast, with a total increase of 11.6 % at 03:00. It can be questioned whether this consumption increase is possible in response to a high wind feed-in, and whether it rather should be lower. This is affected by our elasticity assumption, and could indicate that the elasticity is too high for this hour of the day. A more inelastic demand curve would lead to lower prices for the same change in sales.

7.5 Sensitivity Analysis

We performed sensitivity analysis on the model to see how the results change with varying assumptions. In the assessment of the data set in Section 5.7 we mentioned that we considered the carbon price assumption to be a source of uncertainty. Bio power was also absent from our results, so we will check what effect this had on the dispatch and prices. Other dimensions that could be interesting to study but are not included in this thesis are the demand elasticity, the water value, and different cases of generation and transmission capacity.

7.5.1 Carbon Price

To check the carbon price sensitivity, we run the model with a real carbon price of 20 and 30 EUR/ton, compared to the 10 EUR/ton carbon price in the above results. The adjusted marginal and ramping costs for the fossil fuels are in Table 7.5 and 7.6.

	Carbon price scenario		
Technology	10 EUR/ton (EUR/MWh)	20 EUR/ton (EUR/MWh)	30 EUR/ton (EUR/MWh)
Lignite	30	40	51
Coal	36	45	53
Gas	48	51	55
Oil	148	155	162

Table 7.5: MC_t in the carbon price sensitivity analysis

	Carbon price scenario		
Technology	10 EUR/ton (EUR/MWh)	20 EUR/ton (EUR/MWh)	30 EUR/ton (EUR/MWh)
Lignite	66	90	115
Coal	82	103	124
Gas	40	42	44
Oil	66	69	72

Table 7.6: RC_t in the carbon price sensitivity analysis

The production for the affected technologies are presented in Figure 7.14. In general, coal production decreases, while natural gas and hydropower increase. German production is the most affected by the increased carbon price, where the coal generation is more than halved between the 10 and 30 EUR/ton scenario. The dirtier lignite-fired generation is not reduced until the carbon price reaches 30 EUR/ton, as it is still considerable cheaper than regular coal.

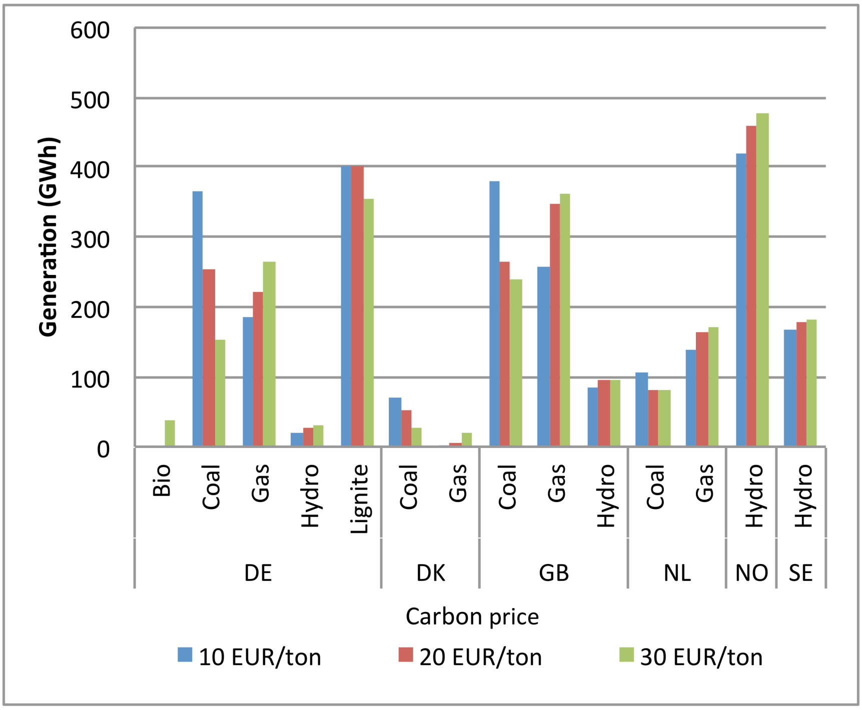

Figure 7.14: Generation of selected technologies against the carbon price in 2020

The increased cost of fossil-fueled generation leads to an increased price in the system, the area and system prices can be seen in Figure 7.15. Generally, a 10 EUR/ton increase in the carbon price leads to a 3.5 EUR/MWh increase in the electricity price. The highest increases are seen on the continent, while Norway and Sweden experience more modest increases. The utilization of the hydropower plants increase from 53 % to 58 % in Sweden, and 68 to 78 % in Norway, with the highest increase seen when going to 20 EUR/ton as the transmission lines from Scandinavia are reaching capacity. These utilization rates are very high, and would indicate that the water value has increased, leading to higher prices in Norway and Sweden.

There are no significant differences in the price volatility that are worth mentioning, but the production variability changes. Natural gas generation becomes important to balance the stochastic wind at night, at the expense of hydropower. Particularly in Germany there are some interesting changes, the 30 EUR/ton scenario is illustrated in Figure 7.16. Coal-fired generation loses importance as a contributor to the balancing, with lignite power replacing regular coal in the 30 EUR/scenario. This can be explained by the fact that the marginal costs are almost the same, but the ramping cost of lignite-fired generation is cheaper. In the 30 EUR/ton scenario, German producers also use bio power production to balance the uncertain wind.

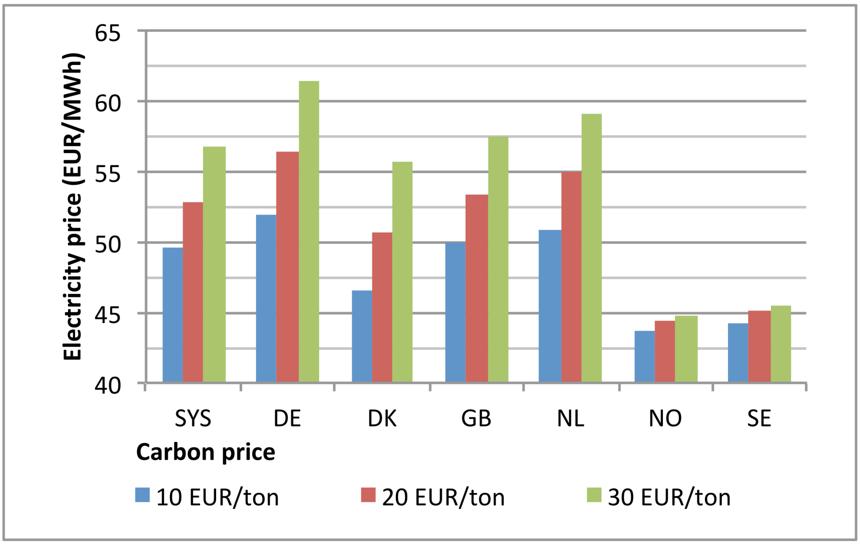

Figure 7.15: System and area price for different carbon price scenarios in 2020

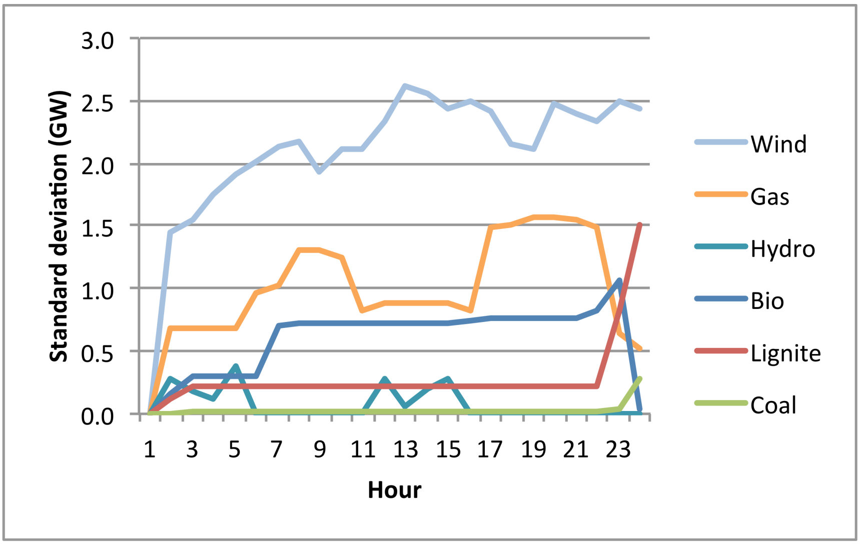

Figure 7.16: Production variability in Germany for the 30 EUR/ton carbon price scenario

In conclusion, a higher carbon price will lead to higher prices, generally 3.5 EUR/MWh per 10 EUR/ton increase in the carbon price. As a result, the water value may increase as well. Coal production is hit first, before the dirtier, but cheaper, lignite production is phased out. The robustness measured as the price volatility does not show any significant change, but the Natural Gas generation does become more important for balancing the stochastic wind.

7.5.2 Bio Power

It is not realistic that our results showed no dispatch of bio power, but the market price did not get high enough for the production to be profitable. In reality, bio power is supported by various subsidies. Feed-in tariffs are used in Germany, while Great Britain and Sweden use a quota system with renewable energy certificates[2]. These two schemes lead to different incentives. A German producer will not care about the market price, but rather produce as much as he can during the whole day. A British or Swedish producer will take into account the market price and produce when the market price plus the certificate price is above the marginal cost. In 2012, Germany also introduced a new optional premium tariff[3] to align the incentives of the renewable producers with the rest of the market (BMU, 2011).

Our formulation does not explicitly include modeling of support schemes, but the feed-in tariff can be represented by setting the marginal cost to zero, and the premium tariff by subtracting the premium from the marginal cost. To check the generation from bio when all the generation is subject to a feed-in tariff support scheme, we set the marginal cost equal to 0 and the availability factor equal to 2/3, which was the availability factor of German bio power plants in 2010.

As a result, Coal-fired generation goes down by 6 % in 2010 and 2020, natural gas generation is reduced by 11 % in 2010 and 29 % in 2020. Hydro generation is also affected, with a decrease of 11 % in 2010 and 15 % in 2020. Overall the system price decreases by approximately 1.5 EUR/MWh with the inclusion of bio power. The price volatility is slightly lower, and the overall production variability shows the same behavior as the case without dispatch of bio power.

[2]See `http://res-legal.eu` for a comprehensive overview of European renewable electricity support schemes

[3]A premium tariff a fixed price premium in addition to the market price

Chapter 8

Efficiency of the Decomposed Model

The decomposed model will give the same results as the non-decomposed stochastic model when solving the same data sets. In this section we will therefore focus on the properties and efficiency of the solution algorithm. We use a reduced data set consisting of Norway and Germany with a 1,400 MW cable between them. The scenario tree has three hours and four scenarios (branches in each time period). This results in a subproblem with 771 variables and a master problem that starts with 3,396 variables. The corresponding non-decomposed model has 3,091 variables and solves in 1.5 seconds.

8.1 Convergence

To measure the convergence of the algorithm, we used several measures as outlined in Table 8.1, and ran the algorithm with a fixed penalty of 20.

The convergence gap is the most used measure in the literature, and is included in appendix F. As we know from Section 2.7.6, the convergence gap represents the closeness to equilibrium. The convergence gap compared to the total profit of the producers, $Error = \frac{|CG|}{\text{Total profit}}$, is used to determine when the solution has converged. As can be seen in Figure 8.1 and 8.2, both measures decrease with the iterations, but not monotonically. In our test case we use $Error < 10^{-6}$ as the stopping criterion.

The next measure is how much the production variables q change from one iteration to the next in the master problem, and is shown in Figure 8.2. The change from one iteration to the next is quite small, which indicates slow convergence of the algorithm.

The last two measures depend on a known optimal solution, and are therefore only possible to use on small problems which can be solved with the non-decomposed model. The first is the sum of the deviations absolute values, while the other is the maximum deviation from the optimal solution. As we see in Figure

Method	Description
Convergence gap	As described in the literature
Error	CG divided by the total profit
Total deviation last iteration	Sum of the absolute value deviations of the production q from the last iteration
Total deviation optimal	Sum of the absolute value deviations of the production q from the optimal solution
Max deviation optimal	Maximum deviation of the production q from the optimal solution

Table 8.1: Different methods to measure the convergence

8.1, the solution starts out being quite good, but it is not until the end that the solution get significantly better.

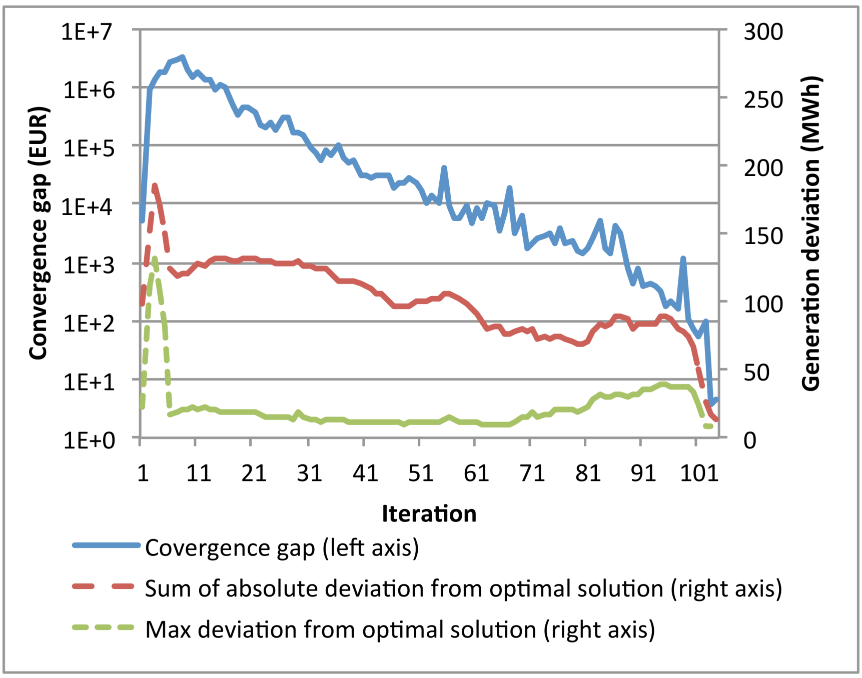

Figure 8.1: Convergence gap and deviation of the production from optimal solution

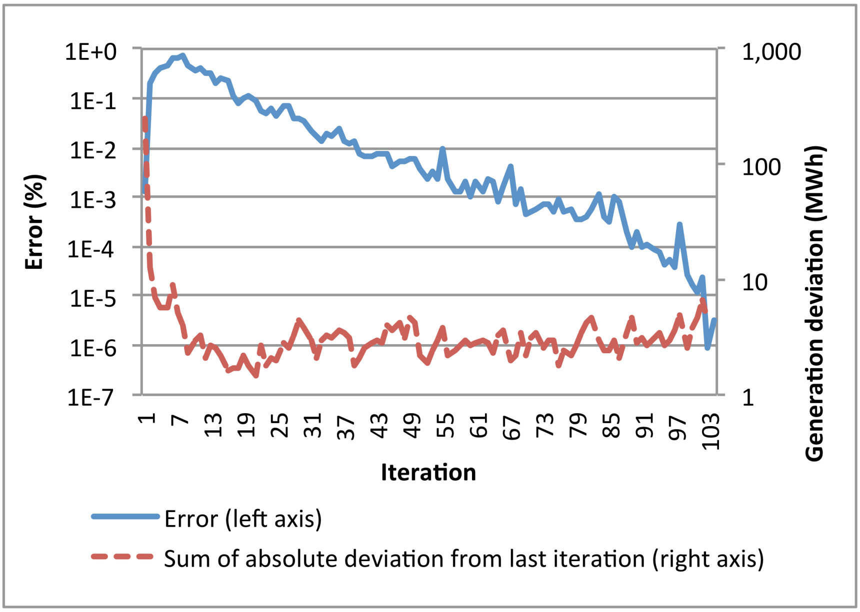

Figure 8.2: Error and deviation of the production from last iteration

8.2 Choosing the Penalty Values

As described in Section 4.2, artificial variables are included to ensure feasibility in the non-anticipativity constraints in the first iterations. Since the penalty value is a bound on the associated dual variable, it must be high enough to ensure that the problem can become feasible. In Figure 8.3 the model is run with different penalty values. It is apparent that the higher the penalty, the faster the problem becomes feasible, as the inclusion of artifical variables is penalized more. If the penalty is too low, e.g. 0.1, the problem never becomes feasible because the added value from violating the non-anticipativity constraints is higher than the penalty.

The next question is what other effects the penalty values have. Figure 8.4 shows the convergence gap for different penalty values, and it is clear that the speed of convergence is highly sensitive to the penalty value. A too low value will never become feasible, while a too high value takes a very long time to converge. It is quite clear that the best fixed penalty is 1 in this example.

Until now we have assumed that the penalty value must be fixed. This must not necessarily be the case. The penalty values can be adjusted with the iterations and be different for each variable. A reasonable method we propose is to start with a small initial value on all penalties, and increase the penalty value in each iteration until the corresponding artificial variable is zero. If the corresponding artificial variables is zero, the penalty value can be decreased, but it increases if the artificial variable is introduced again. A run with a start penalty of 1, increase of 20 % and decrease of 10 % is shown in Figure 8.4. The number of iterations is

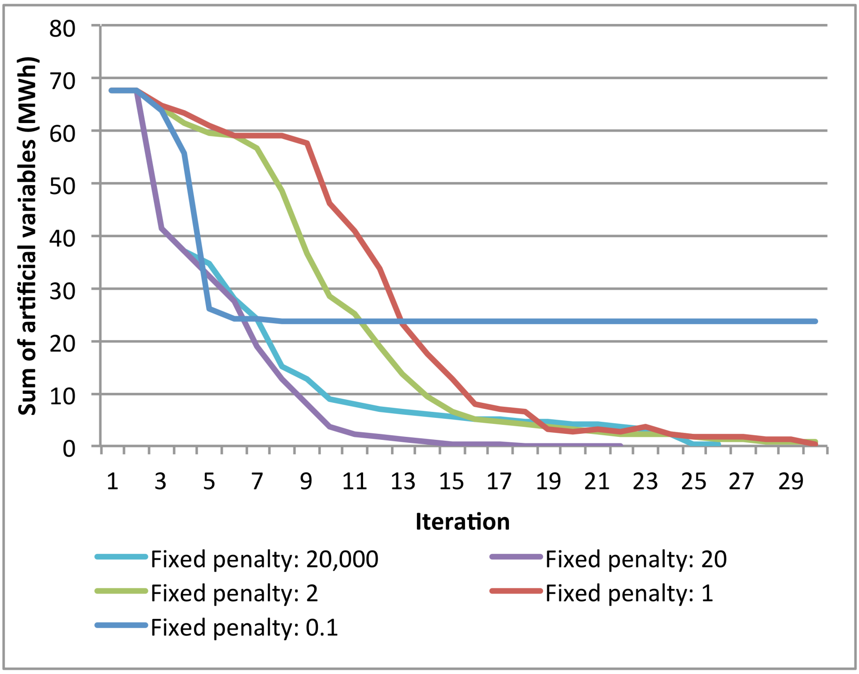

Figure 8.3: Sum of artificial variables in each iteration for different penalty values

in this case drastically reduced to 17.

It is also possible to run the model version without the artificial variables, if a feasible starting point is provided. Figure 8.4 also shows the convergence gap for this version, and although it convergences, it is significantly slower than the other methods.

8.3 Solution Time Improvement

As we outlined in Section 4.6 there are also other methods to improve the performance. The most important improvements are the removal of unused proposals in the master problem and fixing technologies. The step length reduction in each iteration also showed improved results.

Table 8.2 shows the results of proposal removal and fixing technologies on the model with artificial variables. A fixed penalty of 1 is considered the benchmark. Fixing variables for technologies, that can reasonably be assumed to be zero or at full capacity, reduces the number of iterations slightly while the solution time reduction is more significant.

The removal of unused proposals that haven't been used in the last 10 iterations significantly speeds up the process. The total iteration number until convergence is nearly the same, but the solution time is much faster. This can be explained by

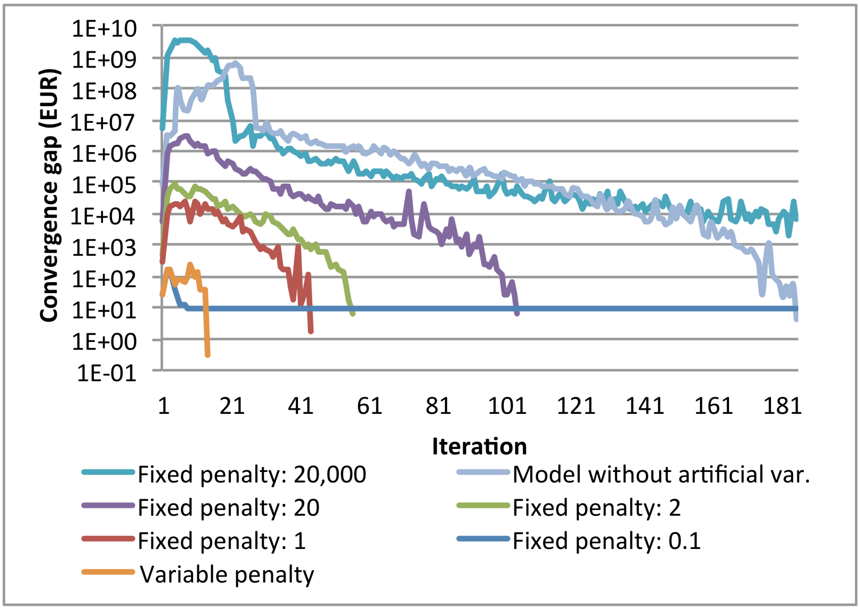

Figure 8.4: Comparison of convergence gap for different penalty values

Technique	Optimal (iterations)	Time (min:sec)
Fixed penalty: 1	43	1:22
Fixed variables	40	1:05
Removal of unused proposals	43	0:55
Fixed variables and removal of unused proposals	40	1:03

Table 8.2: Effect of various techniques on the solution time

the fact that the size of the master problem is reduced and is easier to solve. The number of iterations is not affected in this case, but can increase if the algorithm has to regenerate removed proposals. The effects seen in this small case may be greater in a bigger problem.

The step length reduction was applied on the model version without artificial variables. This method reduces the dual variables of the NA-constraints, Δ, sent from the master problem to the subproblems. As a result, there are less fluctuations which reduces the number of iterations. By dividing δ by two, we reduced the number of iterations from 185 to 115 and the solution time from 9:15 to 5:29 minutes.

8.4 Complexity of the Master Problem

Figure 8.5 shows the number of variables in the master problem for the first hundred iterations of a problem which solves after approximately 100 iterations. The graph shows two cases, with and without removal of unused proposals after ten iterations. In the first case, the problem size is continously increasing as four stationarity constraints for $\lambda_{k,s}$ are added for each iteration. In the second case the master problem seems to converge to a certain size, as previous proposals are removed at the same rate new ones are added. The density[1] of the problem is presented in Figure 8.6 and the problem with proposal removal is less dense, which might indicate that the problem is less complex and easier for PATH to solve.

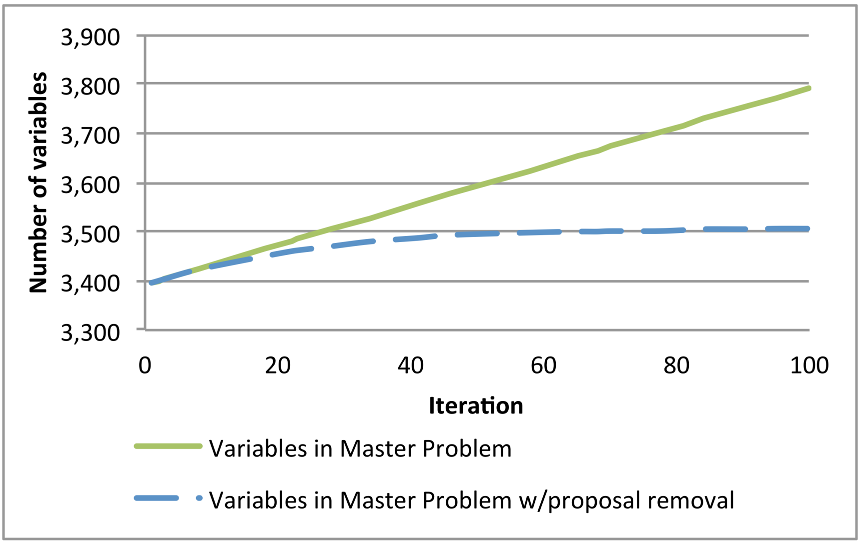

Figure 8.5: Variables in the master problem

This is important for two reasons. First of all, the solver PATH is unable to solve too large master problems. By removing proposals that are not used, it is possible to run the algorithm on bigger problems.

Secondly, GAMS must generate each iteration's master problem. This can take a significant amount of time on a large problem. Thus the effect of the proposal removal is greater than it would be if another program was used. GAMS is not optimized for data processing or model generation and processes the code line by line and executes the instructions. Since these overhead parts are repeated it is important that it is as efficient as possible. In other programming languages, like C++, the code is compiled and can handle repetitive steps faster and you may implement a procedure that enlarges the current model each iteration, instead of generating it again from scratch.

[1]Percentage share of non-zero elements

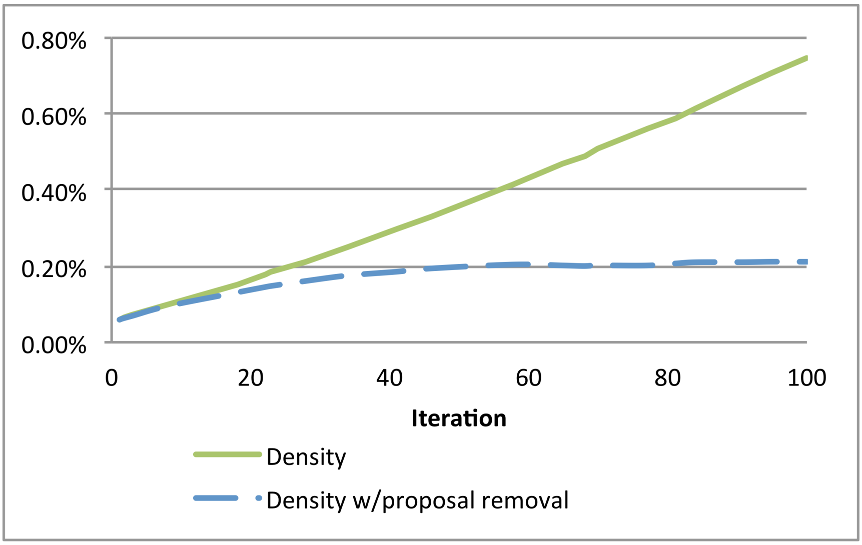

Figure 8.6: Density of the master problem

8.5 Starting Point for the Regular Model

One possible usage of the decomposed model is to produce a starting point for the non-decomposed model to reduce the overall solution time. We constructed a test data set with 86,995 variables in the non-decomposed model, consisting of the normal 2010 data set, but with a scenario tree with 4 stages, 12 time periods and 8 scenarios.

We used the model without artificial variables, because we think it provides a better start solution since it is feasible in the original problem. The decomposed model was only able to run five iterations before the master problem became infeasible. The solution changed only marginally from iteration 1 to 5, so the result is very similar to only one iteration being run.

The solution time can be seen in Table 8.3. The time with the constructed starting point includes the time to run the algorithm. Equilibrium problems are sensitive to their starting point and as we can see, a good starting point can significantly reduce the solution time.

Solution strategy	Solution time (hours:min)
No initial solution	8:15
Constructed starting point	3:19

Table 8.3: Solution time comparison for constructed starting point

Chapter 9

Discussion

This chapter will discuss the limitations and possibilities in the presented power market model and the decomposition scheme.

9.1 Comparison of Formulations

In this thesis we have presented two stochastic power market model versions; The scenario tree formulation and the non-anticipativity formulation. The non-anticipativity formulation is also presented as a VI-model and a decomposed MCP model. All these models give the same result and most of the equations are the same. However there are differences on how fast the PATH solver solves the different formulation variants as seen in Table 6.1. We experienced that the scenario tree formulation was the fastest to solve, probably because it has fewer variables and the problem size is smaller. The VI formulation included in Appendix C is very similar to the MCP formulation except that it does not explicitly include any dual variables in the mapping (corresponding to the stationarity constraint in the MCP problem). The constraints can therefore easily be changed in the formulation without affecting the mapping. On the other hand changing the constraints in an MCP formulation affect the stationarity constraints and it is therefore more time consuming. We therefore recommend using VI-formulation to develop the code and convert it to MCP formulation when the model is used on bigger problems.

9.2 Model Limitations

Our power market model has some limitations that are worth mentioning.

The market is represented by a intra-day spot market for electricity. This is sufficient to gain an overall understanding of the market, but a better representation would be to model the day-ahead market along with the balancing market. A bilevel multi-stage model (see Section 2.3.2) can potentially be used to formulate such a problem. The upper level problem can be the day-ahead market, where the price is settled in a price clearing process, while the lower level problem represents the balancing market. A disadvantage of a bilevel problem is that it is computationally demanding.

The transmission network is highly simplified, in that we only require that there must be a node balance and that the electricity transmitted on a line is within an upper limit. In reality there are losses in the power grid, and electric power flows are governed by Kirchoff's voltage law that restricts the transmission possibilities. Additionally, cables are in real life subject to ramping conditions that are not included in the model. Since we were interested in running the model on a country level, it did not seem as necessary to include these additional characteristics.

The ramping restrictions are applied in aggregate on a producer's generation capacity for a technology, although they are applicable to a single plant. This was necessary because the binary variables that would be required for unit commitment models are not possible in an MCP formulation.

Pump storage is only applicable to seasonal storage plants with a high capacity, since we don't have any restrictions on how much you can pump. Since there are several GW of pump storage plants operating in the balancing market in Great Britain and Germany, our formulation is less flexible.

Our linear demand and market power specification is also sensitive to the price-demand elasticity and the conjecture about the market power factor, and care must be taken when choosing the value of these two factors, as they interact with each other.

However, as we demonstrate in our results chapter, the model gives valuable insight in the dynamics of generation, electricity flows and prices in the modeled countries under various market developments.

9.3 Validity of the Results

Our data set contains some assumptions that may affect the validity of the results.

Our marginal costs are similar for each technology, in reality they are more granular and dependent on the age and type of power plant. For instance, a simple natural gas turbine has a much lower efficiency compared to a combined cycle natural gas plant.

The water value is in reality specific to each individual reservoir, as the reservoirs have different capacities. A reservoir close to capacity may produce at full power, or else the water will be wasted. Additionally, a wrong water value may cause the model to produce too little or too much compared to the actual reservoir levels.

We have also assumed that the internal power grid in each country is able to handle the full capacity at the interconnections and within the country. This is not exactly right. For instance are Norway, Denmark and Sweden comprised of several price areas due to internal congestion. Additionally, the transmission capacities in the data set are based on the maximum net transfer capacity, but the capacity often differs depending on the direction of the power flow. This overestimates the flexibility in the system.

The model is run with only demand and price data for an autumn weekday, and as we know the consumption changes with the season. Run-of-river and solar power plants also have different capacities depending on the day and season. It would be important to take into account these aspects if the model is used for investment decisions.

Our wind data, based on historic data from Denmark, was used as the wind profiles of the other countries as well. We lose some dynamics because of this, since the wind profiles would be different for each node. This was mainly done because of time and data availability constraints.

9.4 Dantzig-Wolfe Decomposition

9.4.1 Infeasible Master Problem

The DWD algorithm applied on smaller problems works fine, but is unsolvable with bigger data sets. The master problem becomes too big and the solver has problems to find an optimal solution. The solution from the former iteration should always be a feasible solution in the master problem of the current iteration, but even so we sometimes end up with infeasible master problems in GAMS. One of the issues we have looked at is numerical problems and insufficient scaling. GAMS automatically scales the variables and the equations by default, but according to McCarl (2011) this is not always sufficient for complex problems. Thus manually scaling must be used based on an understanding of the model structure. According to McCarl (2011), all values in the constraint matrix should ideally be in the range between 0.1 and 100. We have not been able to achieve this for our problem.

As far as we know from the literature, nobody has applied DWD on bigger equilibrium problems than this thesis. Fuller and Chung (2005), Chung et al. (2006), Fuller and Chung (2008) and Chung and Fuller (2010) only demonstrate the algorithm on a tiny data set with approximately 100 primal variables, and the solution times are negligible. But Cabero et al. (2010) and Gabriel and Fuller (2010) apply the related Benders decomposition technique, based on Fuller and Chung (2005), on full equilibrium problems with 77,842 and 560,000 variables respectively. These are that are too big to be solved by PATH, but the decomposition technique solves the problems. In addition to the number of variables, the complexity of the problem is also important to assess. Because our problem is a multi-stage rather than a two-stage problem, the problem is considered much harder to solve than for instance the model in Gabriel and Fuller (2010). The complexity affects the solution time, and probably also the limit of how big problems GAMS can solve.

Our model didn't solve for larger problems where potentially the decomposed algorithm solved faster than the non-decomposed model, because of solver problems. Still we see some aspects that tell us something about what kind of problems that gain on decomposition methods.

Our strategy was to relax the non-anticipativity constraints to decompose along the scenarios. With one scenario per subproblem, a large problem could easily be solved in parallel in a cluster environment. But, there are many NA-constrains, approximately the same number as there are production variables. As a result, the master problem is quite big. A key factor to providing an efficient algorithm must be to limit the size of the master problem. If we return to the non-decomposed problem, we recognize several other ways to split the structure into subproblems. It can be split by producers and the market clearing condition is part of the master problem, or it can be split by country. The lines' transmission fees would be determined by the master problem. This would hypothetically reduce the size of the master problem, but would also limit the number of subproblems that can be run in parallel. In the latter strategy a BD approach could have been used. It is also important to note that the subproblems are solved multiple times and therefore should be relatively easy to solve, it might be that one subproblem for each country is too comprehensive.

9.4.2 Algorithm Improvements

The proposals from the subproblems in the first iteration give quite good results, and the constructed initial proposal even better. In the literature (e.g. Tebboth (2001)) there is a frequently reported problem that the DWD has a "tail" of nearly optimal solutions. Since the solution after the first iteration is quite good, we may be in the tail where the convergence rate is slow.

In this master's thesis we have shown that the model is sensitive to the penalty value. Little research on this value exists, none within equilibrium problems. Watson et al. (2008) claims to be the first to discuss penalty on non-anticipativity constraints. We suggest more research should be done on this topic in order to be able to apply decomposition efficiently. We propose that a variable penalty, both increasing and decreasing, can be used in order to reduce fluctuations and solution time. In our thesis we also successfully used step length reduction, which reduces the transferred dual variable from the master to the subproblem, that could be further assessed.

In our procedure we fix production variables that have a small marginal value, to reduce the number of variables in the problem. Even more variables have the potential of being fixed during the solution time. If production variables stay constant for a certain number of iterations it might be sufficient to fix the variable.

The algorithm can also be improved by approximating the master problem. Both Çelebi (2011) and Chung and Fuller (2010) suggest such approaches and show promising results compared to solving the original master problem.

9.4.3 Potential Gain from Parallelization

In the literature parallelization is often discussed when decomposition techniques on equilibrium problems are presented, e.g. in Fuller and Chung (2005), Çelebi (2011), Gabriel and Fuller (2010), but seldom implemented. Parallelization would probably speed up the solution time for the subproblems, but in our problem it is the master problem that uses most of the solution time and causes problems in PATH. Parallel processing will not resolve the master problem infeasibility issues, and we have therefore not focused on parallelization in this thesis.

9.5 Further Work

As we have seen, our proposed model has both weaknesses and potential for further development. We focus on stochastic production technologies, but the demand can also be considered uncertain and modeled with the same principle. Properly representing different support schemes should also be considered, as particularly the bio power generation is very sensitive to this assumption. The representation of pump storage plants can also be improved by including a reservoir for the small plants operating in the balancing market.

The presented model can be used to analyze new cases. A particular interesting subject to investigate would be the effect of stochastic solar power. Recently solar production in Germany met half of the demand at noon (Kirschbaum, 2012), having a significant impact on the market. Solar power's uncertainty could be analyzed with the same techniques as we have used on wind power.

Further research should be made on finding out what causes the master problem to become unsolvable and if it is possible to overcome the numerical problems. When it comes to the algorithm we suggest more research into the adjustment of the penalty value with Watson et al. (2008) as a starting point, and approximating the master problem (see Çelebi (2011) and Chung and Fuller (2010)).

Chapter 10

Conclusion

In this master's thesis we propose a multi-period stochastic short-term power market equilibrium model which can be used to analyze the power market today and in the future. The model is formulated as a Mixed Linear Complementarity Problem, where all the producers and the transmission system operator simultaneously solve their optimization problems and reach a Nash-Equilibrium. The problem formulation enables a practical way to model different market power configurations by using the conjectural variations approach. The model is unique in its combination of market power representation, grid representation, ramping costs and possibility of stochastic generation technologies. We have formulated two variants of the stochastic model, the first uses non-anticipativity constraints to represent the scenario tree, while the second uses an explicit scenario tree formulation. The models are implemented in GAMS and solved with the PATH solver both as an MCP and VI problem.

We apply the model on the Northern European power market in 2010 and 2020, modeling the intra-day spot market over 24 hours with one hour granularity. The 2020 case corresponds to industry projections and the fulfillment of the EU's 20-20-20 goals. In the results we see a clear distinction between two types of technologies, those included in the base load and those responsible for balancing the uncertain wind. Hydropower and natural gas-fired generation are used to balance the variable wind power in 2010 and 2020, while coal-fired generation contributes in 2010. The price in Germany and the Netherlands will probably decrease towards 2020 due to the higher share of renewables, while Norway and Denmark are projected to face higher prices than in 2010 as the transmission system is built out and the market becomes more integrated.

We also study the effect on profit, prices and sales for different market power factors. The profit is not always higher with an increasing market power factor in a hybrid market, and through analysis we conclude that $MP = 0.4$ is the best value for the explicitly included producers.

Next, we investigate the effect of including the wind uncertainty. The prices are slightly lower than the equivalent deterministic case, but more importantly there is an increase of natural gas dispatch to handle the uncertain wind. The wind uncertainty also affects the robustness of the power market. Measured as the price volatility the robustness is good in Norway and Great Britain in 2010, while the volatility is higher in Germany, Denmark, Sweden and the Netherlands. In 2020 the prices fluctuate mostly in the morning and evening, but not as much during the rest of the day and night when the system takes advantage of increased transmission capacity.

Our sensitivity analysis shows that a higher carbon price will result in a higher production by hydro and natural gas-fired power plants. As a consequence, the system price increases by approximately 3.5 EUR/MWh per 10 EUR/ton increase in the carbon price. Modeling biomass-based power generation with a feed-in tariff assumption significantly reduces the production of natural gas and hydropower, and to some degree coal, compared to our regular model results that had no dispatch of biomass-based generation.

A Dantzig-Wolfe decomposition approach for solving the model is also presented, where we decompose on the non-anticipativity constraints. We have shown that the method and the algorithm work conceptually. Unfortunately we are not able to solve as large problems as the non-decomposed model, and a smaller data set is used to demonstrate the algorithm. Using this smaller data set, we demonstrate significant algorithmic improvements.

By properly adjusting the penalty on the artificial variables we are able to drastically reduce the number of iterations, from 183 iterations when no artificial variables are included in the model, to 43. We also show the impact of a method where the penalty values are adjusted in each iteration according to the corresponding artificial variables. The number of iterations is then further reduced to 17. By removing subproblem proposals that haven't been used for several iterations, we are able to reduce the size of the master problem, and as a consequence the solution time decreases.

Although the presented model does not include all aspects of the power market, we have demonstrated that it can be used to give valuable insight into the development of the power market. Further work can still be done, particularly further research of the Dantzig-Wolfe algorithm.

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Appendix A

Complete Formulation

Sets, Indices, Parameters and Variables are found in Section 3.1.2

A.1 Producers

Producer p's profit

$$
\begin{aligned}
\text{Min} \quad PROB_s \times [\\
& - \sum_{n,i,s} (INT_{n,i,s} - SLP_{n,i,s} \times \sum_{p} (sales_{n,p,i,s} - qp_{n,p,i,s})) \times sales_{n,p,i,s} \\
& + \sum_{n,t,i,s} (MC_t + AMC_{n,p,t} \times q_{n,p,t,i,s}) \times q_{n,p,t,i,s} \\
& + \sum_{l,i,s} (\tau_{l,i,s} + FC_l + AFC_l \times f^{+}_{l,p,i,s}) \times f^{+}_{l,p,i,s} \\
& + \sum_{l,i,s} (-\tau_{l,i,s} + FC_l + AFC_l \times f^{-}_{l,p,i,s}) \times f^{-}_{l,p,i,s} \\
& + \sum_{n,i,s} ((-MC_{hydro} \times EF) \times qp_{n,p,i,s}) \\
& + \sum_{n,i,s} (INT_{n,i,s} - SLP_{n,i,s} \times \sum_{p} (sales_{n,p,i,s} - qp_{n,p,i,s})) \times qp_{n,p,i,s} \\
& + \sum_{n,t,i,s} RC_t \times qi_{n,p,t,i,s}]
\end{aligned}
\tag{A.1}
$$

Mass balance ($\gamma_{n,p,i,s}$ – free)

$$\begin{aligned}&-\sum_{t} q_{n,p,t,i,s} - \sum_{l \in L_n^+} (f^+_{l,p,i,s} - f^-_{l,p,i,s}) \\ &+ sales_{n,p,i,s} + \sum_{l \in L_n^-} (f^+_{l,p,i,s} - f^-_{l,p,i,s}) = 0 \quad \forall \quad n,p,i,s\end{aligned} \tag{A.2}$$

Production capacity ($\alpha_{n,p,t,i,s} \geq 0$)

$$q_{n,p,t,i,s} - AV_{t,i,s} \times GCAP_{n,p,t} \leq 0 \quad \forall \quad n,p,t,i,s \tag{A.3}$$

Ramping ($\beta_{n,p,t,i,s} \geq 0$)

$$q_{n,p,t,i,s} - q_{n,p,t,i-1,s} - qi_{n,p,t,i,s} \leq 0 \quad \forall \quad n,p,t,i \neq 1,s \tag{A.4}$$

Ramping limit ($\phi_{n,p,t,i,s} \geq 0$)

$$qi_{n,p,t,i,s} - RU_t \times GCAP_{n,p,t} \leq 0 \quad \forall \quad n,p,t,i,s \tag{A.5}$$

Installed pump storage capacity ($\psi_{n,p,i,s} \geq 0$)

$$qp_{n,p,i,s} - IN_{n,p} \times GCAP_{n,p,hydro} \leq 0 \quad \forall \quad n,p,i,s \tag{A.6}$$

Non-anticipativity ($\delta_{n,p,t,i,s}$ – free)

$$q_{n,p,t,i,s} - q_{n,p,t,i,FIRST(s,i)} = 0 \quad \forall \quad n,p,t,i,s \neq FIRST_{s,i} \tag{A.7}$$

A.2 Transmission System Operator

System operator's profit

$$\text{Min} - \Pi_{TSO} = -\sum_{l,i,s} (PROB_s \times \tau_{l,i,s} \times flow_{l,i,s}) \tag{A.8}$$

Positive transmission capacity ($\epsilon^+_{l,i,s} \geq 0$)

$$flow_{l,i,s} - TCAP_l \leq 0 \quad \forall \quad l,i,s \tag{A.9}$$

Negative transmission capacity ($\epsilon^-_{l,i,s} \geq 0$)

$$-flow_{l,i,s} - TCAP_l \leq 0 \quad \forall \quad l,i,s \tag{A.10}$$

A.3 Market-Clearing Conditions

Market-clearing condition ($\tau_{l,i,s}$ – free)

$$\sum_{p} (f^+_{l,p,i,s} - f^-_{l,p,i,s}) - flow_{l,i,s} \leq 0 \quad \forall \quad l,i,s \tag{A.11}$$

Appendix B

Power Market Model with Scenario Tree Formulation

This appendix includes the full mathematical formulation and the KKT conditions for the power market model with scenario tree formulation. The parent-child connections between the event nodes can be defined by a set AN_e, which is the ancestor node of scenario tree node e. Each parent node can have one or many child nodes. This can be exploited to make any scenario tree structure. All scenario tree nodes are linked to exactly one time period by the set TP_e. The set defines the time period i of event node e. There is a probability for each scenario node to occur. The probability for node e, $PROB_e$, can be changed according to the scenario structure.

B.1 Definitions

B.1.1 Sets and Indices

Sets and Indices	
E	Set of event nodes e
AN_e	Ancestor node of event node e
TP_e	Time period i of event node e

B.2 Stochastic Formulation

B.2.1 Producers

Expected profit

$$\begin{aligned}\text{Min} - \Pi_p = \sum_e PROB_e \times \Big[\\ & - \sum_n (INT_{n,TP_e} - SLP_{n,TP_e} \times (\sum_p sales_{n,p,e} - qp_{n,p,e})) \times sales_{n,p,e} \\ & + \sum_n (INT_{n,TP_e} - SLP_{n,TP_e} \times (\sum_p sales_{n,p,e} - qp_{n,p,e})) \times qp_{n,p,e} \\ & - \sum_n (MC_{hydro} \times EF \times qp_{n,p,e}) \\ & + \sum_{n,t} (MC_t + AMC_{n,p,t} \times q_{n,p,t,e}) \times q_{n,p,t,e} \\ & + \sum_l (\tau_{l,s} \times (f^+_{l,p,e} - f^-_{l,p,e})) \\ & + \sum_{n,t} (RC_t \times qi_{n,p,t,e}) \Big] \quad \forall \quad p\end{aligned} \tag{B.1}$$

Node balance for each player ($\gamma_{n,p,e}$ – free)

$$\begin{aligned}&\sum_t q_{n,p,t,e} + \sum_{l \in L_n^+} (f^+_{l,p,e} - f^-_{l,p,e}) \\ &-sales_{n,p,e} - \sum_{l \in L_n^+} (f^+_{l,p,e} - f^-_{l,p,e}) = 0 \quad \forall \quad n, p, e\end{aligned} \tag{B.2}$$

Production limitation ($\alpha_{n,p,t,e} \geq 0$)

$$q_{n,p,t,e} - AV_{t,e} \times GCAP_{n,p,t} \leq 0 \quad \forall \quad n, p, t, e \tag{B.3}$$

Ramping ($\beta_{n,p,t,e} \geq 0$)

$$q_{n,p,t,e} - q_{n,p,t,AN_e} - qi_{n,p,t,e} \leq 0 \quad \forall \quad n, p, t, e \tag{B.4}$$

Ramping limit ($\phi_{n,p,t,e} \geq 0$)

$$qi_{n,p,t,e} - RU_t \times GCAP_{n,p,t} \leq 0 \quad \forall \quad n, p, t, e \tag{B.5}$$

Installed pump power ($\psi_{n,p,e} \geq 0$)

$$qp_{n,p,e} - IN_{n,p} \times GCAP_{n,p,hydro} \leq 0 \quad \forall \quad n, p, e \tag{B.6}$$

B.2.2 Transmission System Operator

Expected profit

$$\text{Min} - \Pi_{TSO} = -\sum_{l,e} PROB_e \times \tau_{l,e} \times flow_{l,e} \tag{B.7}$$

Transmission limitation positive ($\epsilon^+_{l,e} \geq 0$)

$$flow_{l,e} - TCAP_l \leq 0 \quad \forall \quad l, e \tag{B.8}$$

Transmission limitation negative($\epsilon^-_{l,e} \geq 0$)

$$-TCAP_l - flow_{l,e} \leq 0 \quad \forall \quad l, e \tag{B.9}$$

B.2.3 Market-Clearing Condition

($\tau_{l,s}$ − free)

$$\sum_p (f^+_{l,p,e} - f^-_{l,p,e}) - flow_{l,e} = 0 \quad \forall \quad l, e \tag{B.10}$$

B.3 KKT conditions

The KKT conditions are derived from the stochastic problem formulation.

B.3.1 Producers

$$0 \leq q_{n,p,t,e} \perp PROB_e \times MC_t + \gamma_{n,p,e} + \alpha_{n,p,t,e} + \beta_{n,p,t,e} - \beta_{n,p,t,AN_e} \geq 0 \tag{B.11}$$

$$\begin{aligned} 0 \leq sales_{n,p,e} \perp & PROB_e \times \left[-\sum_{i \in P_{i,e}} INT_{n,i} + \sum_{i \in P_{i,e}} SLP_{n,i} \times \sum_p (sales_{n,p,e} - qp_{n,p,e}) \right] \\ & + PROB_e \times \left[MP_{p,n} \times \sum_{i \in P_{i,e}} SLP_{n,i} \times sales_{n,p,e} \right] - \gamma_{n,p,e} = 0 \end{aligned} \tag{B.12}$$

$$0 \leq f^+_{l,p,e} \perp PROB_e \times \tau_{l,s} + \gamma_{n^+,p,s} - \gamma_{n^-,p,e} \geq 0 \tag{B.13}$$

$$0 \leq f^-_{l,p,e} \perp -PROB_e \times \tau_{l,e} - \gamma_{n^+,p,e} + \gamma_{n^-,p,e} \geq 0 \tag{B.14}$$

$$0 \leq qi_{n,p,t,e} \perp PROB_e \times RC_t - \beta_{n,p,t,e} + \phi_{n,p,t,e} \geq 0 \tag{B.15}$$

$$0 \leq qp_{n,p,e} \perp -PROB_e \times MC_{hydro} \times EF + \psi_{n,p,e} \geq 0 \tag{B.16}$$

$$\begin{aligned} &\sum_{t} q_{n,p,t,e} + \sum_{l \in L_n^+} (f_{l,p,e}^+ - f_{l,p,e}^-) \\ &-sales_{n,p,e} - \sum_{l \in L_n^-} (f_{l,p,e}^+ - f_{l,p,e}^-) = 0 \\ &(\gamma_{n,p,e} - \text{free}) \end{aligned} \tag{B.17}$$

$$0 \leq \alpha_{n,p,t,e} \perp AV_{t,e}^{stochastic} \times GCAP_{n,p,t} - q_{n,p,t,e} \geq 0 \tag{B.18}$$

$$0 \leq \beta_{n,p,t,e} \perp qi_{n,p,t,e} - q_{n,p,t,e} + q_{n,p,t,AN_e} \geq 0 \tag{B.19}$$

$$0 \leq \phi_{n,p,t,e} \perp RU_t \times GCAP_{n,p,t} - qi_{n,p,t,e} \geq 0 \tag{B.20}$$

$$0 \leq \psi_{n,p,e} \perp IN_{n,p} \times GCAP_{n,p,hydro} - qp_{n,p,e} \geq 0 \tag{B.21}$$

B.3.2 Transmission System Operator

$$\begin{aligned} -PROB_s \times \tau_{l,e} + \epsilon_{l,e}^+ - \epsilon_{l,e}^- = 0 \\ (flow_{l,e} - \text{free}) \end{aligned} \tag{B.22}$$

$$0 \leq \epsilon_{l,e}^+ \perp TCAP_l - flow_{l,e} \geq 0 \tag{B.23}$$

$$0 \leq \epsilon_{l,e}^- \perp TCAP_l + flow_{l,e} \geq 0 \tag{B.24}$$

B.3.3 Market-Clearing

$$\begin{aligned} &flow_{l,e} - \sum_{p} (f_{l,p,e}^+ - f_{l,p,e}^-) = 0 \\ &(\tau_{l,e} - \text{free}) \end{aligned} \tag{B.25}$$

Appendix C

VI-formulation

The VI-formulation is presented below. It uses the same variables and parameters as the MCP-formulation.

C.1 Mapping

The mapping for the VI is given by the derivative of the objective function for each decision variable and the mapping between τ and the market clearing condition.

C.1.1 Producer

$$F(q_{n,p,t,i,s}) = PROB_s \times MC_t + 2 \times AMC_{n,p,t} \times q_{n,p,t,i,s} \tag{C.1}$$

$$\begin{aligned} F(sales_{n,p,t,i,s}) =& PROB_s \times (-INT_{n,i} + SLP_{n,i} \times \sum_p (sales_{n,p,i,s} - qp_{n,p,i,s}) \\ &+MP_{p,n} \times SLP_{n,i} \times sales_{n,p,t,i,s}) \end{aligned} \tag{C.2}$$

$$\begin{aligned} F(qp_{n,p,i,s}) = PROB_s \times (-MC_{hydro} \times EF \\ +(INT_{n,i} - SLP_{n,i} \times \sum_p (sales_{n,p,i,s} - qp_{n,p,i,s}))) \end{aligned} \tag{C.3}$$

$$F(f^+_{l,p,i,s}) = PROB_s \times (\tau_{l,i,s} + FC_l + 2 \times AFC_l \times f^+_{l,p,i,s}) \tag{C.4}$$

$$F(f^-_{l,p,i,s}) = PROB_s \times (-\tau_{l,i,s} + FC_l + 2 \times AFC_l \times f^-_{l,p,i,s}) \tag{C.5}$$

$$F(qi_{n,p,t,i,s}) = PROB_s \times RC_t \tag{C.6}$$

C.1.2 Transmission System Operator

$$F(flow_{l,i,s}) = -PROB_s \times \tau_{l,i,s} \tag{C.7}$$

C.1.3 Market clearing

$$F(\tau_{l,i,s}) = flow_{l,i,s} - \sum_p \left(f^+_{l,p,i,s} - f^-_{l,p,i,s} \right) \tag{C.8}$$

C.2 Constraint Set

The constraint set K consists off all the original constraints in the MCP-formulation.

Mass balance

$$\begin{aligned} &-\sum_t q_{n,p,t,i,s} - \sum_{l \in L_n^+} (f^+_{l,p,i,s} - f^-_{l,p,i,s}) \\ &+ sales_{n,p,i,s} + \sum_{l \in L_n^-} (f^+_{l,p,i,s} - f^-_{l,p,i,s}) = 0 \quad \forall \quad n, p, i, s \end{aligned} \tag{C.9}$$

Production capacity

$$q_{n,p,t,i,s} - AV_{t,i,s} \times GCAP_{n,p,t} \leq 0 \quad \forall \quad n, p, t, i, s \tag{C.10}$$

Ramping

$$q_{n,p,t,i,s} - q_{n,p,t,i-1,s} - qi_{n,p,t,i,s} \leq 0 \quad \forall \quad n, p, t, i \neq 1, s \tag{C.11}$$

Ramping limit

$$qi_{n,p,t,i,s} - RU_t \times GCAP_{n,p,t} \leq 0 \quad \forall \quad n, p, t, i, s \tag{C.12}$$

Installed pump storage capacity

$$qp_{n,p,i,s} - IN_{n,p} \times GCAP_{n,p,hydro} \leq 0 \quad \forall \quad n, p, i, s \tag{C.13}$$

Non-anticipativity

$$q_{n,p,t,i,s} - q_{n,p,t,i,FIRST_{s,i}} = 0 \quad \forall \quad n, p, t, i, s \tag{C.14}$$

Positive transmission capacity

$$flow_{l,i,s} - TCAP_l \leq 0 \quad \forall \quad l, i, s \tag{C.15}$$

Negative transmission capacity

$$-flow_{l,i,s} - TCAP_l \leq 0 \quad \forall \quad l, i, s \tag{C.16}$$

Appendix D

Extended Master Problem Terms

Because of the identity of the market clearing condition, some of the terms in lambda's stationarity condition cancel out. These terms can be found in the this appendix.

Positive flow

$$\begin{aligned}&+\sum_{l,p,i}(PROB_s \times \sum_{k}(\lambda_{k,s} \times TAU_{l,i,s,k} \\ &\quad +FC_l + 2 \times AFC_l \times \sum_{k}(\lambda_{k,s} \times F^{+}_{l,p,i,s,k})) \times F^{+}_{l,p,i,s,k})\end{aligned} \tag{D.1}$$

Negative flow

$$\begin{aligned}&+\sum_{l,p,i}(PROB_s \times \sum_{k}(\lambda_{k,s} \times (-TAU_{l,i,s,k}) \\ &\quad +FC_l + 2 \times AFC_l \times \sum_{k}(\lambda_{k,s} \times F^{-}_{l,p,i,s,k})) \times F^{-}_{l,p,i,s,k})\end{aligned} \tag{D.2}$$

Total flow

$$+\sum_{l,i}(-PROB_s \times \sum_{k}(\lambda_{k,s} \times TAU_{l,i,s,k}) \times FLOW_{l,i,s,k}) \tag{D.3}$$

Appendix E

DWD Subproblem

E.1 Producer

$$
\begin{aligned}
0 \le q_{n,p,t,i} \perp & PROB_s \times (MC_t + 2 \times AMC_{n,p,t} \times q_{n,p,t,i}) \\
& -\gamma_{n,p,i} + \alpha_{n,p,t,i} + \beta_{n,p,t,i|i>1} - \beta_{n,p,t,i+1|i<|I|} + \delta_{n,p,t,i} \ge 0
\end{aligned} \tag{E.1}
$$

$$
\begin{aligned}
0 \le sales_{n,p,i} \perp & PROB_s \times (-INT_{n,i} + SLP_{n,i} \times \sum_p (sales_{n,p,i} - qp_{n,p,i}) \\
& +MP_{p,n} \times SLP_{n,i} \times sales_{n,p,i}) + \gamma_{n,p,i} \ge 0
\end{aligned} \tag{E.2}
$$

$$
\begin{aligned}
0 \le qp_{n,p,i} \perp & PROB_s \times (-MC_{hydro} \times EF \\
& +(INT_{n,i} - SLP_{n,i} \times \sum_p (sales_{n,p,i} - qp_{n,p,i}))) + \psi_{n,p,t,i} \ge 0
\end{aligned} \tag{E.3}
$$

$$
\begin{aligned}
0 \le f^+_{l,p,i} \perp & PROB_s \times (\tau_{l,i} + FC_l + 2 \times AFC_l \times f^+_{l,p,i}) \\
& -\gamma_{n^+,p,i} + \gamma_{n^-,p,i} \ge 0
\end{aligned} \tag{E.4}
$$

$$
\begin{aligned}
0 \le f^-_{l,p,i} \perp & PROB_s \times (-\tau_{l,i} + FC_l + 2 \times AFC_l \times f^-_{l,p,i}) \\
& +\gamma_{n^+,p,i} - \gamma_{n^-,p,i} \ge 0
\end{aligned} \tag{E.5}
$$

$$
0 \le qi_{n,p,t,i} \perp PROB_s \times RC_t - \beta_{n,p,t,i} + \phi_{n,p,t,i} \ge 0 \tag{E.6}
$$

$$
\begin{aligned}
& \sum_t q_{n,p,t,i} + \sum_{l \in L^+_n} (f^+_{l,p,i} - f^-_{l,p,i}) \\
& -sales_{n,p,i} - \sum_{l \in L^-_n} (f^+_{l,p,i} - f^-_{l,p,i}) = 0 \\
& (\gamma_{n,p,i} - \text{free})
\end{aligned} \tag{E.7}
$$

$$0 \leq \alpha_{n,p,t,i} \perp AV_{t,i} \times GCAP_{n,p,t} - q_{n,p,t,i} \geq 0 \tag{E.8}$$

$$0 \leq \beta_{n,p,t,i} \perp qi_{n,p,t,i} - q_{n,p,t,i} + q_{n,p,t,i-1} \geq 0 \tag{E.9}$$

$$0 \leq \phi_{n,p,t,i} \perp RU_t \times GCAP_{n,p,t} - qi_{n,p,t,i} \geq 0 \tag{E.10}$$

$$0 \leq \psi_{n,p} \perp IN_{n,p} \times GCAP_{n,p,hydro} - qp_{n,p,i} \geq 0 \tag{E.11}$$

E.2 Transmission System Operator

$$0 \leq flow_{l,i} \perp -PROB_s \times \tau_{l,i} + \epsilon^{+}_{l,i} - \epsilon^{-}_{l,i} \geq 0 \tag{E.12}$$

$$0 \leq \epsilon^{+}_{l,i} \perp TCAP_l - flow_{l,i} \geq 0 \tag{E.13}$$

$$0 \leq \epsilon^{-}_{l,i} \perp TCAP_l + flow_{l,i} \geq 0 \tag{E.14}$$

E.3 Market-Clearing

$$\begin{aligned} flow_{l,i} - \sum_{p}(f^{+}_{l,p,i} - f^{-}_{l,p,i}) = 0 \\ (\tau_{l,i} - free) \end{aligned} \tag{E.15}$$

Appendix F

Convergence Gap

The convergence gap, based on the description in Section 2.7.6, for the current iteration is:

$$CG_k =$$

Production

$$\sum_{n,p,t,i,s} (PROB_s \times (MC_t + 2 \times AMC_{n,p,t} \times \sum_k (\lambda_{k,s} \times Q_{n,p,t,i,s,k}) + \delta_{n,p,t,i,s}) \times (Q_{n,p,t,i,s,k} - \sum_k (\lambda_{k,s} \times Q_{n,p,t,i,s,k}))) \tag{F.1}$$

Sales

$$+ \sum_{n,p,i,s} (PROB_s \times (-INT_{i,n} + SLP_{i,n} \times \sum_{p,k} (\lambda_{k,s} \times (SALES_{n,p,i,s,k} - QP_{n,p,i,s,k}))) \times (SALES_{n,p,i,s,k} - \sum_k (\lambda_{k,s} \times SALES_{n,p,i,s,k}))) + \sum_{n,p,i,s} (PROB_s \times MP_{p,n} \times SLP_{i,n} \times \sum_k (\lambda_{k,s} \times SALES_{n,p,i,s,k}) \times (SALES_{n,p,i,s,k} - \sum_k (\lambda_{k,s} \times SALES_{n,p,i,s,k}))) \tag{F.2}$$

Pump storage

$$
\begin{aligned}
+\sum_{n,p,i,s} (PROB_s \times (-MC_{hydro} \times EF \\
+INT_{i,n} - SLP_{i,n} \times \sum_{p,k}(\lambda_{k,s} \times (SALES_{n,p,i,s,k} - QP_{n,p,i,s,k}))) \\
\times(QP_{n,p,i,s,k} - \sum_{k}(\lambda_{k,s} \times QP_{n,p,i,s,k})))
\end{aligned}
\tag{F.3}
$$

Production increase

$$
+\sum_{n,p,t,i,s} \left[PROB_s \times RC_t \times \left(QI_{n,p,t,i,s,k} - \sum_{k} (\lambda_{k,s} \times QI_{n,p,t,i,s,k}) \right) \right] \tag{F.4}
$$

Positive flow

$$
\begin{aligned}
+\sum_{l,p,i,s} (PROB_s \times (FC_l + 2 \times AFC_l \times \sum_{k}(\lambda_{k,s} \times F^{+}_{l,p,i,s,k})) \\
\times(F^{+}_{l,p,i,s,k} - \sum_{k}(\lambda_{k,s} \times F^{+}_{l,p,i,s,k})))
\end{aligned}
\tag{F.5}
$$

Negative flow

$$
\begin{aligned}
+\sum_{l,p,i,s} (PROB_s \times (FC_l + 2 \times AFC_l \times \sum_{k}(\lambda_{k,s} \times F^{-}_{l,p,i,s,k})) \\
\times(F^{-}_{l,p,i,s,k} - \sum_{k}(\lambda_{k,s} \times F^{-}_{l,p,i,s,k})))
\end{aligned}
\tag{F.6}
$$

Appendix G

GAMS in Practice

The following appendix contains experiences from using GAMS.

G.1 Import from Excel

When solving large realistic cases, the amount of data is significant and a model is often run with different data sets to compare. To explicitly write the data sets into GAMS is time consuming and not practical. Data handling is often performed in Excel when gathering data and direct import from Excel sheets was needed. GDXXRW described in McCarl[1] was used to read excel spreadsheets using GDX (GAMS data exchange) files.

This is an example of how $INT_{n,i}$, the intercept of the inverse demand curve, is imported from Excel. The parameter is declared, and loaded. The intercept is in the file dataset.xlsx in sheet int, in row and column A1:H300.

```
Parameter INT(i,n);
$onecho > import.txt
par=INT rng=int!A1:H300 cdim=1 rdim=1
* More parameters could be added on new lines
$offecho
$call GDXXRW dataset.xlsx @import.txt
$gdxin dataset.gdx
$load INT
```

G.2 Export to Excel

It is essential to be able to easily handle and compare results. GDXXRW could be used to perform a data dump, but we wanted to get all the data directly into pivot tables in Excel. Pivot tables enable easy analysis of the results, and we could

[1] http://www.gams.com/mccarl/mccarlhtml/

directly see the results in various graphs after each model run. To enable this Rutherford's pivot data exporter[2] was used.

G.3 Pitfalls

GAMS has some pitfalls, and to the modelers frustration some of these problems only apply in difficult (singular or near singular) cases. This means that you can develop and test a small problem, and when you are scaling the problem or have tighter restrictions, problems occur. The problem is how you should order an equation, since $a - b = 0$ is not the same as $b - a = 0$ in all cases in GAMS.

The following example is from the manual.

$$\min_x (x-1)^2, \text{where } x \in [0,2]$$

The first order optimality conditions are

$$0 \leq x \leq 2 \perp 2(x-1)$$

This equation has the solution $x = 1$. If this in written as d_f.. 0 =e= 2*(x-1), the solver is handed the problem

$$0 \leq x \leq 2 \perp -2(x-1)$$

This problem has solutions of $x = 0, 1, 2$ which is obviously wrong. Most of the time this difference is irrelevant because the equation is internally substituted into another equation in the problem. But when the problem is more difficult to solve a perturbation is applied, and if the equation is in the wrong order, the perturbation is in the wrong direction, and the problem gets even harder to solve.

In some cases we suspected that the PATH solver didn't find a solution if the node balance was in the wrong order. A trick we used to get it right was to treat the equality constraint as if it was a less than or equal constraint when deriving the KKT conditions. Then we modified it so it became a greater than or equal constraint in the KKT conditions before switching back to a equality constraint.

G.4 Solution Time Improvement

Option files[3] can be used to decrease the solution time. Try to find the right options with a small model before trying to solve larger models. Pay attention to the solver each time it restarts. In each restart the solver tries new options. By noticing the best options for the specific problem the solution time will be significant smaller for larger problems. Smaller stochastic problems were sometimes solved twice as fast with the right option file.

[2] `http://www.mpsge.org/pivotdata.htm`

[3] Our option file: crash_method=none, crash_perturb=0, lemke_start_type=slack, nms_initial_reference_factor=10, nms_memory_size=2, nms_mstep_frequency=1, proximal_perturbation=0

Another smart trick is to give the solver some start values. Especially if the same model is run with slightly different values, for example different water values. Since the solver among other techniques uses Newton's method, the start values makes a huge impact. A value with all the values are created and loaded with the following commands:

option savepoint = 1;

Execute_loadpoint 'Model-name_p'

A normal way of improving the solution time is to remove and fix variables, but we experienced that fixing variables didn't result in a much smaller solution time.

NTNU
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MASTERKONTRAKT
- uttak av masteroppgave

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Oppstartsdato **16. jan 2012**	Innleveringsfrist **11. jun 2012**
Oppgavens (foreløpige) tittel **European Power Market Model**	
Oppgavetekst/Problembeskrivelse Present a model of the European power market with emphasis on prices, generation dispatch, consumption and transmission. Focus areas: * Study the power market today and in future scenarios * Study the role of different technologies in the power market * Study the robustness of the system by including stochastic variables * Consider different market power representations * Collect data to the model and implement it Since the model and dataset is large, we will consider decomposition techniques and/or other ways to reduce the solution time.	
Hovedveileder ved institutt **Professor Asgeir Tomasgard**	Medveileder(e) ved institutt **Rudolf Gerardus Egging**
Merknader **1 uke ekstra p.g.a påske.**	

4. Underskrift

Student: Jeg erklærer herved at jeg har satt meg inn i gjeldende bestemmelser for mastergradsstudiet og at jeg oppfyller kravene for adgang til å påbegynne oppgaven, herunder eventuelle praksiskrav.

Partene er gjort kjent med avtalens vilkår, samt kapitlene i studiehåndboken om generelle regler og aktuell studieplan for masterstudiet.

Trondheim 16/1-2012
....
Sted og dato

....
Student

....
Hovedveileder

Originalen lagres i NTNUs elektroniske arkiv. Kopi av avtalen sendes til instituttet og studenten.

NTNU
Det skapende universitet

MASTERKONTRAKT

- uttak av masteroppgave

1. Studentens personalia

Etternavn, fornavn **Samseth, Eivind**	Fødselsdato **12. mar 1987**
E-post **eisa01@gmail.com**	Telefon **47056615**

2. Studieopplysninger

Fakultet **Fakultet for Samfunnsvitenskap og teknologiledelse**	
Institutt **Institutt for industriell økonomi og teknologiledelse**	
Studieprogram **Industriell økonomi og teknologiledelse**	Hovedprofil **Anvendt økonomi og optimering**

3. Masteroppgave

Oppstartsdato **16. jan 2012**	Innleveringsfrist **11. jun 2012**
Oppgavens (foreløpige) tittel **European Power Market Model**	
Oppgavetekst/Problembeskrivelse Present a model of the European power market with emphasis on prices, generation dispatch, consumption and transmission. Focus areas: * Study the power market today and in future scenarios * Study the role of different technologies in the power market * Study the robustness of the system by including stochastic variables * Consider different market power representations * Collect data to the model and implement it Since the model and dataset is large, we will consider decomposition techniques and/or other ways to reduce the solution time.	
Hovedveileder ved institutt **Professor Asgeir Tomasgard**	Medveileder(e) ved institutt **Rudolf Gerardus Egging**
Merknader **1 uke ekstra p.g.a påske.**	

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Trondheim 16/1-2012

Sted og dato

Student **Hovedveileder**

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SAMARBEIDSKONTRAKT

1. Studenter i samarbeidsgruppen

Etternavn, fornavn	Fødselsdato
Haga, Geir Anders	**12. apr 1986**
Etternavn, fornavn	Fødselsdato
Samseth, Eivind	**12. mar 1987**

2. Hovedveileder

Etternavn, fornavn	Institutt
Tomasgard, Asgeir	**Institutt for industriell økonomi og teknologiledelse**

3. Masteroppgave

Oppgavens (foreløpige) tittel
European Power Market Model

4. Bedømmelse

Kandidatene skal ha *individuell* bedømmelse ☐
Kandidatene skal ha *felles* bedømmelse ☒

Trondheim 16/1-2012
Sted og dato

Geir Anders Haga
Geir Anders Haga

Hovedveileder

Eivind Samseth
Eivind Samseth

Originalen oppbevares på instituttet.