NTNU – Trondheim
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Science and Technology

Priced Liquidity Risk Factors at the Oslo Stock Exchange

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Submission date: June 2012
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3. Masteroppgave

Oppstartsdato **16. jan 2012**	Innleveringsfrist **11. jun 2012**
Oppgavens (foreløpige) tittel **Priced Liquidity Risk Factors on Oslo Stock Exchange**	
Oppgavetekst/Problembeskrivelse Investigate to what degree different factors drive equity risk premiums on Oslo Stock Exchange, and develop improved model for equity risk pricing in the Norwegian market. Main contents: 1. Review and discussion of theoretical and empirical literature related to equity risk pricing. 2. Formulation of testable hypothesis, discussion of data, and analysis of data with the intention of gaining new insights regarding risk pricing of common stock listed on Oslo Stock Exchange. 3. Overall assessment of the implications of the empirical study.	
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Originalen lagres i NTNUs elektroniske arkiv. Kopi av avtalen sendes til instituttet og studenten.

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Trondheim, 23.01.2012
Sted og dato

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Hovedveileder

Originalen lagres i NTNUs elektroniske arkiv. Kopi av avtalen sendes til instituttet og studenten.

Preface

This study is conducted as our Master's thesis at the department of Industrial Economics and Technology Management at the Norwegian University of Science and Technology (NTNU). We have specialized in Empirical Finance. Jerkø has technical specialization in Heat and Energy Processes, while Morken has specialized in Production and Quality Engineering. The technical specializations have not been applied directly in this study, but have contributed to our understanding of mathematics and statistics.

The reason for our choice of topic is that liquidity appeared to be an interesting topic for further research when we wrote a thesis about asset pricing theories during the fall of 2011.

We would especially like to thank our teaching supervisor, Einar Belsom, for academic support, review and constructive feedback. Thanks should also be rewarded to NHH Børsprosjektet for providing us with data.

Abstract

We examine how liquidity risk influences stock returns at the Oslo Stock Exchange by investigating if differences between returns are related to liquidity, and how liquidity should be measured. A wide range of distinct liquidity measures is studied, and the measures which best express liquidity risk are combined to a multifactor model. We use a multi-perspective approach to select and compare measures, and perform Fama-MacBeth regressions to evaluate the performance of factor combinations. We find liquidity risk to be priced. Turnover is found to be the liquidity measure that best captures liquidity risk, and trade-based measures are found to be more important than order-based measures. Our multifactor model consisting of amortized spread, trading volume, turnover in shares and the market factor seem to perform better than the capital asset pricing model (CAPM) empirically, and can be used as an alternative to the CAPM for practical applications. Both common and non-common variances of measures are important to express liquidity risk, and we suggest that asset pricing models should include several liquidity measures.

Sammendrag

I dette studiet undersøker vi hvordan likviditetsrisiko påvirker aksjeavkastningen på Oslo Børs ved å studere om forskjellene i avkastning har sammenheng med likviditet, samt hvordan likviditet bør måles. Vi undersøker et bredt spekter av likviditetsmål og kombinerer de målene som best forklarer likviditetsrisiko til en flerfaktormodell. Vi anvender en multiperspektivtilnærming for å velge mål og sammenlikne disse, og bruker Fama-MacBeth-regresjon til å evaluere hvordan faktorkombinasjonene gjør det empirisk. Vi finner at likviditet er priset. Omsetning ser ut til å være det likviditetsmålet som best fanger opp likviditetsrisiko, og handelsbaserte mål fremstår som viktigere enn ordrebaserte mål. Vår flerfaktormodell, bestående av amortisert spread, handlet volum, omsetning i antall aksjer og markedsfaktoren, ser ut til å gjøre det bedre empirisk enn kapitalverdimodellen (CAPM), og kan dermed være et alternativ til CAPM i praktiske applikasjoner. Både felles og individuell varians til likviditetsmålene er viktig for å uttrykke likviditetsrisiko, og vi mener derfor at en prisingsmodell bør inneholde flere likviditetsmål.

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1 Introduction

The mystery of differences in risk premium between stocks has been a major topic of financial research since the 1960s. There exist several competing theories regarding which risk that should be priced, and divergent perceptions of which asset pricing models that best explain the risk. Liquidity risk is found to be an important contribution to explain equity premiums, as less liquid stocks on average have higher returns, while the downside risk also is higher for these stocks (Amihud, 2002). Many liquidity-augmented models are found to perform empirically better than traditional asset pricing models. A possible reason is that liquidity models relax restrictive assumptions of the traditional models and contain a more extensive part of the risk picture.

There are many unanswered questions regarding liquidity, because it has several sources, is multi-dimensional, and affects stock returns in several ways. No common practice has been found for which liquidity measures to apply and how to best combine these to models. Our motivation for focusing on liquidity risk is the many unanswered questions regarding the impact of liquidity on equity premiums, and we pursue new solutions to how liquidity risk can be expressed in asset pricing models.

In order to investigate the effects of liquidity, we find it particularly interesting to study the Oslo Stock Exchange. Many rather illiquid stocks are listed here, although it is an open and well-functioning marketplace. We suppose that liquidity can play an even more important role at the Oslo Stock Exchange compared to more liquid markets, as the U.S. market. New insights about liquidity risk might therefore be acquired by studying the Norwegian market.

To grasp the complexity of liquidity risk we analyze a broad specter of liquidity measures. We are interested in how the various aspects of liquidity contribute to express the total liquidity effect, and will therefore use multiple liquidity factors directly in our model. This is a new way of developing liquidity models. Most other research include only one liquidity measure, like Acharya and Pedersen (2005), Amihud and Mendelson (1986) and Sadka (2003), or only include the common variance between a selection of liquidity factors, as done by Korajczyk and Sadka (2008), Chen (2005) and Chollete, Næs and Skjeltorp (2006; 2007; 2008). Selection of variables to include in a model is an important part of our study. As far as we have seen, this has not been the focus of other liquidity studies, and there exist no common methods or selection criteria. In order to achieve a solid foundation for the selection, we use a multi-perspective approach by performing several analyses.

Our contribution is an examination of whether liquidity risk is priced, how it should be measured, and how to include it in models. An extensive review of existing theoretical and empirical findings is provided as a foundation for our perspectives. For the 13 liquidity measures in our study, we examine the cross-sectional and time series variation of the measures, and the correlation between these. We perform cross-sectional stepwise regressions and Fama-MacBeth regressions on several factor combinations in order to find the optimal model. Furthermore, our model is compared to the CAPM by examining the fit of the regressions and prediction performance.

The rest of the thesis is organized as follows. Sections 2 and 3 discuss the theories of asset pricing and liquidity risk. Section 4 presents our data set, and section 5 discusses the methods applied in the analyses. Section 6 presents and discusses our empirical results. In section 7 we discuss the totality of our findings in light of theory and empirical findings of others. We conclude in section 8.

2 Asset Pricing

This section shortly introduces asset pricing concepts in order to examine which aspects that contribute to explain the differences in stock prices. In addition, issues of traditional asset pricing will be discussed and the context for liquidity models will be provided.

2.1 Asset Pricing Models

To explain the prices of stocks is complicated, as the future payoff is stochastic and unexpected events influence returns. As most investors are risk averse, an equity risk premium is required as compensation for bearing this risk. In order to understand variations in return between stocks, the expected excess return of assets can be expressed by factor models, given by

$$E(R_i) = \sum_j \beta_{i,j} \cdot F_j$$

where R_i is the excess return, F_j is the risk factor j and $\beta_{i,j}$ is the sensitivity of asset i to risk factor j. The most commonly applied factor models are summarized in Table 1. None of these models succeeds to describe the variations in return accurately, as the risk associated with equity investments makes it impossible to predict returns perfectly.

As investors worry about variations in their total wealth and consumption rather than variations in the value of each single stock in their portfolio, risk should only be priced if it is systematic. The systematic risk of stocks can be defined by the correlation with the return on the stock market, as in the capital asset pricing model (CAPM) by Sharpe (1964), Lintner (1965) and Mossin (1966). However, investors also diversify their holdings across other asset classes like for instance bonds, real estate, private equity and derivatives, as well as stocks from national and international markets. The systematic risk of stocks should therefore also be considered in relation to these asset classes.

The ICAPM by Merton (1973) and the consumption CAPM by Lucas (1978) and Breeden (1979) can be considered improvements of the CAPM, as the systematic risk factors in these models are related to variations in the consumption and wealth opportunities of investors, not only to the value of their equity holdings. The conditional CAPM by Jagannathan and Wang (1996) also takes into account the changes in investment opportunities by including the systematic risk of changes in the correlation between asset and market return.

In contrast to the equilibrium models mentioned above, for which the systematic risk is directly linked to the correlation between the asset and measures of wealth or consumption, the models based on arbitrage pricing theory (APT) (Ross, 1973) relate the systematic risk factors to return more indirectly. This approach focuses less on identifying the underlying risk factors, and rather concentrates on stock properties that could be considered symptoms of these underlying risks. Fama and French (1992) include firm-specific factors, while the macroeconomic models in the tradition of Chen, Roll and Ross (1986) include various macroeconomic risk factors. For these models, the most important selection criterion for variables is how well the factors contribute to explain differences in return between stocks.

Table 1: The most commonly applied asset pricing models

Model	Risk factors	Equation
CAPM	Market risk	$E(R_i) = R_f + \beta_{iM}[E(R_M) - R_f]$ $\beta_{iM} = \frac{cov(R_i, R_M)}{var(R_M)}$
ICAPM	Many possibilities: Market risk Interest rate level risk Interest rate volatility risk January factor Sharpe ratio Inflation	$E(R_i) = R_f + \beta_{iM}[E(R_M) - R_f + \beta_{i0}[E(R_0) - R_f] + \cdots$ $\beta_{iM} = \frac{cov(R_i, R_M)}{var(R_M)}$ $\beta_{i0} = \frac{cov(R_i, R_0)}{var(R_0)}$
Consumption CAPM	Consumption risk	$E(R_i) = R_f + \beta_{iC}\left[E(R_C) - R_f\right]$ $\beta_{ic} = \frac{cov(R_i,\ R_C)}{var(R_C)}$
Conditional CAPM	Market risk Beta instability risk	$E\left(R_{i,t}\right) = E\left(R_{f,t-1}\right) + E\left(\beta_{iM,t-1}\right) * E\left(R_{M,t-1} - R_{f,t-1}\right) + cov(R_{M,t-1}, \beta_{iM,t-1})$ $\beta_{iM,t-1} = \frac{cov(R_{i,t}, R_{M,t} \mid I_{t-1})}{var(R_{M,t} \mid I_{t-1})}$
Macroeconomic models	Many possibilities: Industrial production Inflation Default premium Term structure Business cycle Income level Residual market risk	$E(R_i) = \alpha_i + \beta_{i1}F_1 + \beta_{i2}F_2 + \cdots + \beta_{ik}F_k + \varepsilon_i$
Fama-French model	Market risk Market capitalization Book-to-market ratio Extensions: Momentum Liquidity	$E(R_i) = R_f + a_i + \beta_{iM}[E(R_M) - R_f] + \beta_{iS}SMB + \beta_{iH}HML + \varepsilon_i$

2.2 Issues of Asset Pricing

Asset pricing models have introduced many factors that relate the return of assets to systematic risk. However, there is still room for improvements of the CAPM and the other models.

The CAPM has been criticized for its restrictive assumptions and poor empirical performance (Merton, 1973). The assumptions of no market frictions, riskless borrowing and one period investment decisions can be reasons for the CAPM not to explain returns sufficiently (Jensen, 1972). There are also problems related to finding the correct input variables, like a good market return proxy (Roll, 1977). However, as the CAPM is easy to interpret and apply, it remains one of the most used models both for asset pricing purposes and as a reference model to assess the performance of other models.

The CAPM has also been criticized for only including one risk factor. According to Cochrane (1999) it is widely recognized that there are multiple sources of risk that give rise to high returns. As such, it is natural to look in the direction of multifactor models in order to improve the CAPM. It seems difficult to find one common factor that can capture all the relevant systematic risk, as various risk aspects affect security returns in different ways. Statistically, a model explains equally much or more of the return when another factor is added. However, adding insignificant factors give negligible improvements and can lead to statistical issues if the factors are correlated. Including several factors also make the models more complex and less attractive for practical applications. However, we still find multifactor models to be superior to single-factor models.

Most new models have less restrictive assumptions and include more risk factors than the CAPM. However, the equilibrium models relax only a few of the CAPM assumptions, so they still are quite restrictive. The ICAPM and the macroeconomic models have been criticized for not clearly defining the risk factors, and the consumption CAPM has poor empirical performance. One of the main issues regarding the Fama-French model is the lack of economic rationale of the factors (Kothari, Shanken, & Sloan, 1995; MacKinlay, 1995). Nevertheless, the Fama-French model tends to perform better empirically than the CAPM.

Liquidity models can resolve many of the issues related to traditional asset pricing models. Pastor and Stambaugh (2001) mention shifts in market liquidity as a way of including the changes in the investor opportunity set. Amihud, Mendelson and Pedersen (2006) discuss how liquidity models do not assume a perfect market, and take market frictions and aspects of trading into account. Liquidity models do not assume investors to be price takers, as price impact is a part of the liquidity risk. Including liquidity risk can also be seen to relax assumptions of homogenous expectations and single period investment decisions.

If the factors in arbitrage pricing models are symptoms of underlying systematic risk, it would be interesting to understand what this risk is, and rather include it directly in models. Liquidity risk could be the underlying risk of for instance the factors of the Fama-French model and the momentum factors, as Archarya and Pedersen (2005), Liu (2006) and Sadka (2003) find these factors to correlate with liquidity factors. The contribution of liquidity risk to explain stock returns will be further discussed in the next section.

3 Liquidity

Liquidity is defined in this section, and theoretical and empirical evidence of the liquidity risk premium is presented. We also discuss how to measure liquidity, and how various liquidity models perform empirically.

3.1 Definitions of Liquidity

The term liquidity is used for trading liquidity in this study, which refers to how easily an asset can be traded. Liquidity is a complex concept that can be defined in several ways. One widely applied definition states that liquidity is *"an ability to trade large quantities quickly at low cost with little price impact"* (Chollete, Næs, & Skjeltorp, 2007, p. 6). This definition includes all four dimensions of liquidity, namely depth, immediacy, width and resiliency. Other definitions often focus on one dimension. The definition by Aitken and Comerton-Forde (2003, p. 45) focuses on width by defining liquidity as *"the ability to convert shares into cash (and converse) at the lowest transaction costs"*. Amihud (2002, p. 33) rather focuses on the resiliency dimension in his definition, by stating that *"illiquidity reflects the impact of order flow on price – the discount that a seller concedes or the premium that a buyer pays when executing a market order – that results from adverse selection costs and inventory costs"*.

All these definitions express important aspects of liquidity. We however prefer the definition by Chollete et al. (2007), as the various aspects of liquidity should be included.

3.2 Sources of Illiquidity

Liquidity premiums depend on fundamental liquidity characteristics of assets and market conditions. How liquidity affects a security's expected return depends on the sources of illiquidity present and to what extent investors require compensation for bearing the associated costs. The sources of illiquidity which will be discussed are transaction costs, asymmetric information and search frictions.

Exogenous **transaction costs** relate to the price of trading assets. Market frictions are present in real markets, and influence stock prices. Therefore, these frictions should be accounted for in asset pricing. Amihud and Mendelson (1986) study the effects of transaction costs on stock prices, and find assets with higher bid-ask spreads to yield higher returns. Also, it can suddenly become costly to trade due to time variations in transaction costs (Amihud et al., 2006). Furthermore, liquidity shocks can force investors to liquidate their positions, and holding periods are therefore uncertain. The impact of transaction costs is thereby also uncertain, as transaction costs are depreciated over the holding period. As such, investors cannot be certain about the future transaction costs that will incur at the time of sale, and fluctuations in transaction costs represent a systematic risk.

Due to clientele effects, transaction costs lead to market segmentation, since long-term investors hold relatively more illiquid assets than short-term investors. As investors can choose to avoid securities with high associated transaction costs, also long-term investors would prefer assets with low transaction costs if returns were the same. However, Amihud and Mendelson (1986) find that expected return is an increasing and concave function of transaction costs, and investors with longer expected holding periods can obtain a liquidity premium that exceeds the expected transaction costs by holding high spread stocks (Amihud et al., 2006). Long-term investors are not exposed to transaction costs as frequently as short-term investors, and can depreciate the expected costs over a longer holding period.

Asymmetric information occurs when one of the trading parts possesses private information relevant to the trade that the other part does not, which results in a trading loss for the uninformed part (Amihud et al., 2006). Relevant information for the trading decision can be company specific information, information about future trades, or information about future market prospects. Asymmetric information can be considered a systematic risk, as uninformed investors will always have a disadvantage relative to the informed investors. This effect cannot be removed by diversification, as the uninformed investors can never be certain of the proper weights of each stock to hold since they do not have the correct expectations regarding risk and return. This is supported by O'Hara (2003), who finds that investors hold different portfolios according to what information they possess. The uninformed investor might also become aware of the situation and choose not to trade, which will reduce the liquidity of the market.

Brennan and Subrahmanyam (1996), Easley, Hvidkjaer, and O'Hara (2002) and O'Hara (2003) all find evidence in support of a liquidity premium associated with information costs. Brennan and Subrahmanyam (1996) find that investors possessing private information create illiquidity costs for investors without this information, as stocks become illiquid due to information asymmetry. Easley et al. (2002) and O'Hara (2003) discover that assets with a larger fraction of private information have higher risk premiums due to the risk of information-based trading.

Search frictions relate to the situation when buyers or sellers are not immediately available when an investor wants to execute a transaction. This creates a tradeoff for the investor between immediate execution of a less attractive trade and searching for a better trade opportunity, and thus imposes search costs (Amihud et al., 2006). Furthermore, this situation results in an opportunity cost for the investor, as the investor might not be able to carry through the desired transactions due to the absence of trading opponents. Search frictions as a source of illiquidity is supported by Weill (2008), who find cross-sectional variations in stock returns to be entirely caused by cross-sectional variations in the number of tradeable shares, as a higher number of tradeable shares is related to lower search frictions and higher liquidity. Search frictions also vary over time according to changes in the market conditions, and can be considered systematic risk. When market liquidity decreases, search frictions increase, since it becomes more costly to trade simultaneously as it becomes more challenging to find a trading opponent.

As discussed above, the sources of illiquidity lead to differences in the absolute level of liquidity between assets and to differences in how assets are affected by systematic fluctuations in liquidity. Investors will as such require a premium for holding assets influenced by these sources of illiquidity.

3.3 The Liquidity Risk Premium

Liquidity appears to influence returns both due to differences between stocks in absolute level of liquidity and due to systematic fluctuations in liquidity. How these aspects of liquidity risk are related to equity risk premiums, and how they are related to each other, will be discussed.

3.3.1 Level of Liquidity

The level of liquidity of assets may affect expected returns on assets. Amihud and Mendelson (1986) find that securities with high liquidity costs in general have higher returns than securities with low liquidity costs. Brennan and Subrahmanyam (1996) and Chalmers and Kadlec (1998) also find asset prices to reflect liquidity levels. However, Næs and Skjeltorp (2006) question whether these studies have risk-adjusted the returns properly, as the proposed relation between liquidity costs and return might be due to measurement errors of the asset's risk.

3.3.2 Systematic Fluctuations in Liquidity

Investors require compensation for being exposed to systematic fluctuations in liquidity. Sadka (2003) finds liquidity to vary across stocks and over time, and suggests that liquidity risk associated with systematic fluctuations in liquidity is priced.

Amihud et al. (2006) argue that investors require a liquidity premium due to time-variations in liquidity costs because fluctuations in liquidity influence the volatility of asset prices. Pastor and Stambaugh (2001) find stocks with returns which are more sensitive to fluctuations in market-wide liquidity to provide higher expected returns, also after controlling for sensitivities to market return and size, value, and momentum factors. Acharya and Pedersen (2005) identify three forms of liquidity risk that supports the existence of a liquidity premium, namely the covariance of the security's liquidity with the market return, the covariance of the security's liquidity with the market liquidity, and the covariance of the security's return with the market liquidity. Also, market liquidity is found to be a leading indicator of the state of the economy, as the market, and in particular illiquid stocks, become less liquid prior to market downturns (Næs, Skjeltorp, & Ødegaard, 2011).

3.3.3 Relation Between Level of and Systematic Fluctuations in Liquidity

Level of liquidity seems related to systematic fluctuations in liquidity, as stocks with low absolute levels of liquidity tend to have the largest reduction in liquidity during recessions. This phenomenon is referred to as flight to liquidity, and is supported by e.g. Amihud (2002), Vayanos (2004) and Acharya and Pedersen (2005). The level of liquidity in the cross-section of stock returns is as such an indication of how exposed stocks are to systematic liquidity risk. Therefore, both level of liquidity and systematic fluctuations in liquidity appear to contribute to the existence of a liquidity premium.

Vayanos (2004) also finds flight to liquidity to be related to flight to quality, as investors prefer safer, and not only more liquid, investments during recessions. Næs et al. (2011) also find evidence of flight to quality during recessions, as investors' holdings in stocks which are assumed to perform particularly poorly during economic downturns decrease when the market liquidity worsens. Flight to liquidity and flight to quality often appear together, as risky assets also tend to be less liquid. These phenomena result in acceleration of the poor market situation, as investors liquidate equity positions or invest in more liquid assets.

3.3.4 Evidence Against the Existence of a Liquidity Premium

Some studies suggest that liquidity risk is not priced. Transaction costs are often small, and detecting liquidity effects among the noise in asset returns is complicated. Some studies are criticized for overemphasizing the impact of transaction costs, as this will have greater impact on asset returns when the holding period over which the transaction costs are amortized is shorter (Chalmers & Kadlec, 1998).

Constantinides (1986) finds the risk premium due to transaction costs to be very small, and therefore does not consider it important to account for transaction costs in asset pricing. Constantinides assumes a relatively long holding period, and argues that investors reduce the frequency and volume of trades when transaction costs become large, and that bid-ask spreads only have a second order impact on asset returns. However, Sadka (2003) criticizes this approach for assuming constant transaction costs and that investors can freely choose when to trade, which is not the reality in financial markets.

Eleswarapu and Reinganum (1993) find a positive liquidity premium to exist only in January, and relates liquidity effects to the January effect. According to this study, the connection between equity premium and liquidity risk therefore seems questionable.

Despite this criticism of liquidity risk, the majority of research on liquidity finds evidence in support of a liquidity premium.

3.4 Liquidity Measures

It will further be discussed how to capture the total effect of liquidity on stock returns. There are numerous liquidity measures, which are calculated from either trade or order data, and capture different dimensions of liquidity.

3.4.1 Dimensions of Liquidity

Liquidity can be considered to have four dimensions, namely width, depth, immediacy and resiliency (Chollete, Næs, & Skjeltorp, 2006). **Width** refers to the transaction costs of securities, often expressed by the spread. For a trade to be executed, the buyer and seller have to meet at a common price. If an investor wants to sell immediately, he/she has to accept the bid-price, and if the investor wants to buy immediately, he/she has to accept the ask-price. Therefore, high spreads indicate that it is costly to trade.

Depth relates to how large quantities of a security that can be traded at a given price without affecting the price substantially (Chollete et al., 2007), and can be expressed by the volumes of trades or orders. Investors want to be able to sell or buy large quantities of shares, and less liquid securities are often traded in smaller quantities.

Immediacy is concerned with the time it takes to carry through a transaction. Some shares are traded rarely, and it can therefore take time from the investor places an order until the transaction is executed. How often transactions or offers take place is thus an indication of how liquid the stock is.

Resiliency, or price impact, refers to how much the price changes per volume traded. Resiliency relates to width, as the price change can be a cost the investor has to pay in order to meet the price offered by other investors (Næs, Skjeltorp, & Ødegaard, 2008).

The dimensions of liquidity relate to the various sources of liquidity. The width dimension captures transaction costs, as the spread is an indication of the cost investors have to pay in order to trade. Asymmetric information in the market leads to less trading activity by uninformed investors, which affects the depth and immediacy dimensions because stocks are traded in smaller amounts and at lower frequency. Search frictions make it more time consuming for investors to trade, and thereby affect the immediacy dimension. The different liquidity dimensions are therefore not only ways of categorizing liquidity measures, we also consider them to reflect various sources of liquidity. It is however ambiguous which sources each dimension is associated with, as the illiquidity sources most likely lead to lower liquidity according to more than one dimension.

The perceived liquidity of an asset varies depending on which of the dimensions one focuses on, and an asset is not necessarily liquid according to one dimension even if it is liquid according to another. For instance, an asset can be liquid in terms of being frequently traded, but might still be traded in small quantities and therefore also have illiquid characteristics. In general, the liquidity measures of different dimensions are highly correlated, and the most liquid stocks are liquid according to all the dimensions (Chollete et al., 2006). Even if the liquidity dimensions theoretically explain different aspects of the liquidity concept, they can explain the same variation in returns. Many researchers find the common variance between liquidity measures to capture the most important part of the underlying liquidity risk, and argue that the unique variance of each measure does not contain valuable information to explain returns (e.g. J. Chen, 2005; Chollete et al., 2006; Chollete et al., 2007; Chollete, Næs, & Skjeltorp, 2008; Korajczyk & Sadka, 2008). If there exists an underlying liquidity risk, it can thus be sufficient only to use one measure in order to express the total effect of liquidity.

However, as none of the liquidity dimensions are perfectly correlated, they all explain some parts of the variance in returns that the other dimensions do not. It therefore seems likely that the non-common variance of liquidity measures also contributes to explain asset returns, and that information is lost when only the commonality between factors is used to express liquidity risk. This is supported by Hasbrouck and Seppi (2001), who find that a common factor based on several liquidity measures is not sufficient to explain asset returns. Amihud (2002) also supports this, by claiming that several liquidity factors must be combined in order to capture the totality of the multidimensional liquidity risk.

3.4.2 Trade- and Order-Based Measures

Another distinction is between trade- and order-based liquidity measures. Trade-based measures are based on information about executed trades, while order-based measures express the available liquidity for potential trades, and are based on information about orders placed in the market (Chollete et al., 2007).

According to Aitken and Comerton-Forde (2003) and Chollete et al. (2007), order-based measures are best to empirically predict time variations in return, since they are based on the available liquidity at a certain point of time instead of the ex post trading activity. On the contrary, Chollete et al. (2006) find trade-based measures to be most relevant. A possible reason for this is that the order data can be strongly influenced by noise, as investors can place orders without the intention of trading at the current prices. An example of this is the frequent offers from stock trading algorithms, which makes many offers that only last for an extremely short time, and the offers disturb the data when analyzing trading opportunities. Also, the computation of many of order-based measures requires high frequency intraday data that can be problematic to attain and analyze, while the trade-based measures can be calculated more easily from daily data. Aitken and Comerton-Forde (2003) and Chollete et al. (2007) find low correlation between trade- and order-based measures, and therefore emphasize the importance of including measures from both cathegories.

3.4.3 Liquidity Proxies

As liquidity has many aspects, we consider it important for a liquidity model to cover several of the dimensions, and both the trade- and order-based aspects. However, the complexity of liquidity risk complicates the unification to a single liquidity measure that successfully captures all of these aspects, while including many measures also is difficult as the correlation between the measures is high. Therefore, an important discussion is which of the measures that best capture each dimension. The liquidity measures included in our analysis are shown in Table 2, and the calculation of the measures can be found in Appendix 1. Most of the liquidity measures presented are trade-based, since we use daily data and available order data was limited.

Table 2: Overview of liquidity measures

Dimension	Width	Depth	Immediacy	Resiliency	Other
Trade-based	Amortized spread	Trading volume Value	Turnover (shares) Turnover (NOK) Zero trade ratio	Amihud measure Liquidity ratio Amivest measure	Liu measure Size
Order-based	Absolute spread Relative spread Amortized spread				

The measures included from the width dimension are absolute spread, relative spread, and amortized spread. Absolute spread is the difference between the quoted ask- and bid-prices, while relative spread is the absolute spread divided by the midpoint between the quoted ask- and bid-prices, in order to express the liquidity measure relative to the stock price. The amortized spread measure by Chalmers and Kadlec (1998) is relative spread multiplied by turnover, as the trading costs imposed on an investor depend on the trading frequency. By taking the holding period of the average positions in the stock into account, the measure reflects the realized trading costs of an average investor in that stock. It can thereby be considered a relative spread adjusted for clientele effects. Absolute and relative spread are order-based measures, while amortized spread is both order- and trade-based. From a theoretical standpoint we consider relative spread to be a better measure than absolute spread, as absolute spread will be higher for stocks with higher share price, even if share price should not affect liquidity. There is no clear answer to which of amortized and relative spread that is the most relevant measure, as both capture the trading costs and are frequently used in the literature.

Trading volume and value are depth measures expressing the quantity of shares traded. Trading volume is the total amount of shares traded in number of shares, while value is the amount of shares traded in NOK. Both measures are trade-based. These measures are quite similar and highly correlated (Aitken & Comerton-Forde, 2003). Trading volume seems to be most frequently used. However, we consider value to be a better liquidity measure. If two stocks are traded at the same daily amount in NOK, we suppose the trading quantities should be considered equal even though the share price of the two stocks are different.

To seize the immediacy dimension we use turnover in number of shares, turnover in NOK and the zero trade ratio. All these measures are trade-based. The zero trade ratio is calculated as the ratio of the trading days without trade in the security to the total number of trading days in each month, similar to the measure by Bekaert, Harvey, and Lundblad (2007). The turnover measures are related to the trading volume measures, as all these measures are associated with the quantities of shares traded. The turnover measures can therefore also be considered to capture some aspects of the depth dimension. Turnover in number of shares appears to be commonly used in the literature, while turnover in NOK (or other currencies) is not frequently used. The turnover measures are likely to be highly correlated, as they are calculated based on the same data. The zero trade ratio most likely provides different information than the other measures, but has the limitation of the ratio being equal for all shares that are traded the same number of days in a month, even if some of the stocks are traded more frequently during the day. However, as we do not have information about the number of trades, we consider the zero trade ratio to be the best available alternative.

To capture the resiliency dimension we include the Amihud measure (Amihud, 2002), the liquidity ratio (Chollete et al., 2006) and the Amivest measure (Kerry Cooper, Groth, & Avera, 1985). All these measures are trade-based. The Amihud measure is calculated as the daily absolute return divided by the volume traded in NOK, and expresses the price change per volume traded. The liquidity ratio and the Amivest measure are both versions of the inverse of the Amihud measure, and express the volume of shares required to move the share price by one percentage. The measures are closely related, and we perceive them to be equally applicable. According to Amihud (2002), the Amihud measure provides a more direct measure of the price impact than the Amivest measure. However, we consider the price change per volume change and the volume change per price change to be equally intuitive.

Some of the measures included cannot be categorized according to one particular dimension. The Liu measure is standardized turnover adjusted for zero daily trading volumes. According to Liu (2006), the measure captures the immediacy, width and depth dimensions. However, as the measure is based on turnover and trading frequency information, we consider it mainly to be an immediacy measure. The measure consists of one part related to the number of days without trade and one part related to turnover, which is divided by a deflator. The deflator determines the weighting of these two parts, and if both the deflator and turnover are high, the Liu measure will be almost equal to the zero trade ratio. Conversely, if the deflator is low, the measure will be similar to the inverse of turnover.

We also include the size measure by Fama and French (1992), defined as the market capitalization of each security. Size is not a pure liquidity measure, but as e.g. Acharya and Pedersen (2005) find it to be correlated to liquidity measures, it could indirectly provide information about liquidity. Also, as Amihud (2002) finds the market illiquidity effects to be stronger for small firms in economic downturns, liquidity risk could be the cause of the size effect.

Of the liquidity measures discussed, high values of the measures indicate lower liquidity for the spread measures, the zero trade ratio, the Amihud measure and the Liu measure, while it indicates higher liquidity for the others. Therefore, high values of the liquidity measures have a different interpretation depending on the measure being considered.

3.5 Liquidity Models

In addition to different liquidity measures being used, different methods are applied to combine these to models.

In order to account for liquidity effects, Amihud and Mendelson (1986) and Sadka (2003) add a liquidity measure directly to the CAPM or the Fama-French model. In the liquidity-adjusted CAPM by Acharya and Pedersen (2005), the liquidity measure is rather incorporated as a liquidity beta calculated as the sum of three betas, representing distinct forms of liquidity risk.

Another method frequently used is factor analysis, in which a set of different liquidity measures are grouped to common liquidity factors. Among others, Hasbrouck and Seppi (2001), Eckbo and Norli (2002), J. Chen (2005), Chollete et al. (2006; 2007; 2008), and Korajczyk and Sadka (2008) apply factor analysis, and add one or more of the common factors to the CAPM or the Fama-French model. A common factor can also be used as liquidity proxy in the model by Acharya and Pedersen. Liu (2006) rather constructs a liquidity factor by algebraically combining several measures in order to capture multiple dimensions of liquidity, and adds the factor to the CAPM.

Amihud and Mendelson, Sadka, Acharya and Pedersen, and Liu all find models that incorporate liquidity effects to better explain cross-sectional returns than the CAPM or the Fama-French model. Except from Hasbrouck and Seppi (2001), the results from factor analysis also prove that the liquidity-adjusted models outperform the CAPM and the Fama-French model. Results from these studies suggest that liquidity risk is priced, and that adding liquidity to asset pricing models increases the ability to explain returns. However, there is no definitive answer to how liquidity optimally should be incorporated, as the liquidity models apparently perform well for most of the methods applied.

4 Data

Based on the theoretical findings about asset pricing and liquidity, it seems important to find a way to incorporate liquidity in asset pricing. The following chapters will investigate the impact of liquidity risk at the Oslo Stock Exchange (OSE). This section describes the dataset used in the analysis, and explains the computation of the liquidity factors.

4.1 The Oslo Stock Exchange

The OSE was founded in 1918, and was by the end of 2010 among the 30 largest regulated equity markets in the world by market capitalization (World Federation of Exchanges), with 239 listed companies. At the OSE a few large companies constitute most of the value in terms of market capitalization and generate most of the trading activity. At the end of 2010 the five largest companies in terms of market value represented 61% of the total market value at the exchange, and accounted for approximately 44% of the total turnover value (Oslo Børs). Due to these conditions, the majority of the stocks listed on the OSE are likely to be relatively illiquid.

Another characteristic of the OSE is the co-movement of the stock prices and the oil price due to the importance of oil-related industries to the Norwegian economy, which represents a distinction between the OSE and other stock exchanges. However, the oil price is found not to be a priced risk factor at the OSE (Næs, Skjeltorp, & Ødegaard, 2007).

4.2 The Data Sample

The data used in this study is obtained from the database NHH Børsprosjektet, which contains financial data from the OSE. Our data sample consists of daily data from January 2000 until December 2010. We chose not to include data prior to year 2000 mainly due to the changes in the market structure that occurred as a result of the introduction of the new electronic trading system (ASTS) in 1999. The introduction of the new system could have changed the liquidity situation in the market, and the market conditions before the system was introduced might not be representative for current conditions. A more consistent set of data is also achieved by limiting the sample period, as some relevant data from earlier years is unavailable.

4.3 Data Filtering

In order to avoid noise occurring due to mergers, listing and delisting, we have chosen to limit the number of companies to those that have been listed the entire sample period. This reduces our sample to include 86 securities. Other research limit the sample even further by removing the most rarely traded securities (e.g. Chollete et al., 2006). However, we have decided not to make further reductions, since we believe the less liquid securities should be included in a liquidity study. By excluding too many securities we would end up investigating variations in liquidity among the most liquid stocks, which is not the intention of our study.

Removing securities that are not listed for the whole sample period also provides a balanced panel of data. A balanced panel gives equal weighting of the time periods and provides a better basis for comparing the impact of liquidity across stocks, as the same selection of stocks is used through the entire sample period. In addition, R^2 as a measure of goodness of fit can only be calculated for regressions based on complete data series.

Removing stocks that are not listed for the entire sample period may lead to survivorship bias. This might result in a skewed dataset since only the most successful stocks are included, and their return may be above average. However, the implications are assumed to be limited in this study, as the selection criterion does not relate directly to liquidity, and less liquid securities have equal chances of being represented in the reduced sample. Finally, the relatively short sample period contributes to limit survivorship bias.

4.4 Structuring of the Data

We have used a daily dataset in our study. Liquidity factors calculated from intraday data are thus not considered. By computing monthly measures from the daily data we obtain a larger set of measures to study, since some liquidity measures, such as the Liu measure, must be calculated over a longer time period. Using monthly measures also provide us with a more consistent dataset, since illiquid stocks that are not traded every day would lack data points if daily measures were used. Using monthly measures thus enables us to keep the less liquid securities while maintaining complete data series. However, with monthly measures we end up with fewer data points. Fluctuations in the dataset will decrease, and the accuracy of our results might be somewhat reduced. On the other hand, noise in the dataset will also decrease. We regard the implications of decrease in accuracy to be limited, and we consider the benefits of less noise in addition to be able to analyze more liquidity measures and obtain a balanced panel as relatively more advantageous.

Monthly measures are computed by programming in Excel VBA. Monthly spread measures are calculated as the monthly average of the spread for those days the security is traded. Trading volume, value, turnover (shares), turnover (NOK), the Amihud measure and the liquidity ratio are calculated daily and thereby summed over the month. The zero trade ratio, the Amivest measure and the Liu measure are calculated from monthly data. Size is the market capitalization the last day each month.

The market return is represented by the return of the OSEAX. A value-weighted index of all the stocks listed at the OSE provides a better representation of the Norwegian equity market, compared to the OSEBX, which only includes the stocks with the largest market capitalization. The difference in return between these indices is however small, and the choice of index is not likely to affect the results considerably. Monthly returns of the stocks and the OSEAX are calculated as arithmetic returns. Stock prices adjusted for dividends and corporate events are used in the calculations in order to remove disturbances.

4.5 Descriptive Statistics of the Liquidity Measures

The Amivest measure, trading volume, value and size show a substantial relative range in the measures compared to return (Appendix A2). There are some outliers with very high values, which can have great impact on the regression line. In addition, the effects of differences in the measures are expected to have greater influence on return for stocks with low values of the measures than differences of the same magnitude will have for stocks with high values of the measures.

As there is a substantial range in the observations and a non-linear relationship between return and the variables, logarithmic transformation is applied to obtain a more linear relationship between the factors and return. Appendix A2.1 provides an overview of the factors plotted with return before and after the transformation. As Table 3 shows, the logarithmic transformation makes the skewness and the kurtosis of the distributions of the measures closer to the skewness and the kurtosis of the normal distribution. This indicates that a better approximation of the relationship between return and the measures is obtained than with a direct linear relationship.

Table 3: Logarithmic transformation of variables

	Skewness		Excess kurtosis	
Variables	**Without LN**	**With LN**	**Without LN**	**With LN**
Trading volume	16,2	-0,27	408	-0,43
Value	9,7	-0,04	141	-0,51
Amivest measure	31,1	0,02	1217	-0,16
Size	7,6	0,16	79	-0,36

Table 4 presents descriptive statistics for all the liquidity measures used in our analysis. The statistics are calculated based on all data points. The table shows great variations in the means of the variables, and some of the measures have a few extreme max and min values. It can be seen that skewness and kurtosis are relatively high for the spread measures, the turnover measures and the Amihud measure. However, logarithmic transformation was not done for these factors, as they appeared to be more linearly related to return than the four transformed factors.

Table 4: Descriptive statistics

Variables	**Mean**	**Median**	**Max**	**Min**	**Standard deviation**	**Skewness**	**Excess kurtosis**
Absolute spread	7,3	1,10	1301	0,00	36,32	16,4	396
Relative spread	0,048	0,021	1,5	0,00	0,080	5,4	51
Amortized spread	0,00008	0,00002	0,030	0,00	0,00058	33,0	1388
Trading volume	12,9	13,12	22,1	1,61	3,54	-0,27	-0,43
Value	16,6	16,69	24,7	3,69	3,12	-0,04	-0,51
Turnover (shares)	0,1	0,018	6,4	0,00	0,175	14,1	369
Turnover (NOK)	4,1	0,874	197	0,00	10,812	6,85	67
Zero trade ratio	0,2	0,048	1,000	0,00	0,326	1,03	-0,47
Amihud measure	0,3	0,047	91,7	0,00	1,725	28,9	1169
Liquidity ratio	138	108	2214	0,00	137	2,26	13
Amivest measure	15,1	15,09	26,0	4,99	3,14	0,02	-0,16
Liu measure	4,9	0,955	22,6	0,00	6,60	1,06	-0,42
Size	20,7	20,69	26,4	14,4	1,89	0,16	-0,36

5 Methodology

This section discusses the methods we apply to test if liquidity factors are priced, to select measures for a liquidity model, and to test the performance of the chosen model. In order to achieve a broader perspective on the implications of liquidity, various sub-analyses are performed.

5.1 Empirical Analysis of the Factors

It is examined if liquidity is priced through considering if level of liquidity and returns of securities are related. High and low liquidity portfolios are constructed for each measure. The high liquidity portfolios contain the 10% most liquid securities, while the low liquidity portfolios consist of the 10% least liquid securities. The stocks included are equally weighted in the portfolios. The portfolios are rebalanced at the end of each year, since we expect liquidity characteristics to change over time. For the measures to be priced, the low liquidity portfolios should provide returns in excess of the high liquidity portfolios.

Fama-MacBeth regressions (Fama & MacBeth, 1973) are done for each liquidity measure in combination with the market factor, to evaluate whether the different factors contribute to explain returns adjusted for market risk. The regression method will be discussed below.

Next, we investigate if systematic fluctuations in liquidity are priced by comparing a time series of each liquidity measure averaged for all securities with the price development of the OSEAX, and by analyzing the correlation between the time series of the measures and return.

5.2 Selection of Factors

The analyses described above consider each liquidity measure separately. However, the measures might overlap with other liquidity factors. In order to decide which factors to include in an asset-pricing model, we perform a stepwise ordinary least squares (OLS) regression in OxMetrics to determine the factors that in combination best contribute to explain returns. In the stepwise regression the least significant factor is eliminated before rerunning the model until only significant factors at a 5% level remain, simultaneously as R^2 for the model is maximized. For each time period we regress the model:

$$R_{i,t} = \alpha_t + \sum_{k=1}^{N} \left(\beta_{k,t} \cdot F_{i,k,t}\right) + \varepsilon_{i,t}$$

where α_t is the intercept, $\beta_{k,t}$ is the parameter that quantifies the effect that factor $F_{i,k,t}$ has on the return $R_{i,t}$ for security i at time t, and $\varepsilon_{i,t}$ is the residual.

We do the stepwise regression cross-sectional, since the objective is to explain variations in return between stocks rather than time-variations in return. We do cross-sectional regressions for each year and for ten randomly chosen months to see which factors that occur most frequently. The outcome of the stepwise regression provides an indication of possible factor combinations.

Furthermore, we test factor combinations by regression in order to find the optimal model. Ideally, all possible model combinations with the original 13 measures should be regressed. However, it would require analysis of approximately 6 billion factor combinations (faculty of 13), and we therefore limit the factors to include before we perform the Fama-MacBeth regression. The selection of factors is based on the outcome of the aforementioned analyses. We start by considering the results from the empirical analysis of each factor to determine which measures that according to the portfolio analysis appear to be priced by the level of liquidity, that best contribute to explain returns in combination with the market factor, and that appear to be priced due to systematic fluctuations in liquidity. Next, the results from the stepwise regressions are considered in relation to the results from the analysis of each measure. Through considering which factors that perform well in all of these analyses, we choose a set of measures that stand out to be the most relevant to add to a model.

In order to combine the selected measures to models, the correlation between the measures is assessed. Highly correlated factors represent an issue for model selection, because multicollinearity can lead to unstable estimates of the coefficients and low t-values of the correlated factors, even if the model performs well overall (Dielman, 2001). There is no exact limit to how large the correlation between the factors can be before this seriously affects the results (Wooldridge, 2009). Dielman (2001) suggests that the limit is a correlation of 0.5. Other methods for identifying the issue, like for example variance inflation factors (VIFs), can also be used. However, excluding a variable because it is too highly correlated to another might result in loss of valuable information, since the part of the variation that is not correlated to the other factors might also contribute to explain returns. In addition to avoid including highly correlated factors, we do not include factors from the same liquidity dimension, as these are likely to contain much of the same information.

We construct multifactor models by adding liquidity measures directly together with market betas. The models are as such liquidity-augmented versions of the CAPM. Other approaches, like factor analysis (e.g. Chollete et al., 2008), can also be applied to construct factors, or the measures can be combined algebraically to a multidimensional factor like Liu (2006) does. However, as the intention of our study is to decide which of the distinct liquidity measures that are relevant for asset pricing purposes, we want to measure the contribution of these measures separately rather than combining the measures to common factors.

An alternative approach is to add the factors directly as in the Fama-French method (Fama & French, 1993), where factors are constructed based on the correlation between the return of stocks and the return of portfolios with high exposure to the factor effects. However, with 13 factors, it would be complicated to isolate the effects from each liquidity measure in order to rank the portfolios, which is required to construct the factors. Also, there are a limited number of stocks listed on the OSE, and the portfolios would as such have consisted of only a few stocks when several factors are used.

The liquidity measures could additionally be included as liquidity betas in line with the method by Acharya and Pedersen (2005). This method would also be complicated to apply with 13 factors, as calculation of three betas is required for each factor.

5.3 Regression Method

In order to evaluate the models, we use the Fama-MacBeth regression method (Fama & MacBeth, 1973). This method is also applied by e.g. Fama and French (1992) and Miller and Scholes (1982) with firm-specific factors.

In the first step of the regression, we estimate the market beta based on a time-series regression for each security. We estimate the beta for each month, t, based on regressions of the 24 previous months. The regression model is:

$$R_{i,t} = \alpha_{i,t} + \beta_{Marketfactor,i,t} \cdot R_{M,t} + \varepsilon_{i,t}$$

where $R_{i,t}$ is the excess return of security i, $\alpha_{i,t}$ is the intercept, $\beta_{Marketfactor,i,t}$ is the market beta, $R_{M,t}$ is the excess market return, and $\varepsilon_{i,t}$ is the error term. Thereafter, we use the market beta as input to the cross-sectional regression together with the liquidity factors added directly. The cross-sectional regression below is performed for each month:

$$R_{i,t} = \alpha_t + \beta_{Marketbeta,t} \cdot \beta_{Marketfactor,i,t} + \sum_{k=1}^{N} \left(\beta_{k,t} \cdot F_{i,k,t}\right) + \varepsilon_{i,t}$$

where $R_{i,t}$ is the return of security i at time t, α_t is the intercept, $\beta_{Marketbeta,t}$ is the risk premium associated with the market beta, $\beta_{Marketfactor,i,t}$, estimated in the time-series regression. $\beta_{k,t}$ is the risk premium associated with factor $F_{i,k,t}$ and $\varepsilon_{i,t}$ is the residual. The regressions are computed in Matlab with a generalized linear model regression (glmfit) under the assumption of a normal distribution.

In order to determine which of the models that performs the best, the models are evaluated according to their relative goodness of fit measured by R^2 and adjusted R^2, the significance level of the factors, and the significance level of the intercept. As these statistics might give diverging suggestions to which factor combination that is optimal, we do an overall evaluation. We do not use any exact significance limit, as we cannot find theoretical support for this. We rather compare the significance of the factors.

Fama-MacBeth regression is however not the only possible method for testing factor combinations. As an alternative to the static OLS approach we could have used a dynamic panel data method like the generalized method of moments (GMM), in which time-series and cross-sectional regressions are estimated simultaneously. Panel data methods could have provided different results, but are more complicated than the OLS method. Since OLS is the traditional approach for performing regressions in asset pricing literature (Næs et al., 2007), we regard the Fama-MacBeth method to be sufficient for our analysis.

Another possibility is to estimate all the liquidity factors with time-series regressions and add these estimates to the cross-sectional regression in the same manner as we do for the market betas, rather than adding the liquidity factors directly in the cross-sectional step. This would be more similar to panel data methods. However, we did not choose this approach, as the factors appeared to be less significant compared to when we added the factors directly (Appendix A2.2).

5.3.1 Estimation of Market Betas

In order to obtain optimal beta approximations, we perform test regressions in which the betas are assumed to be constant for the entire sample period, constant for two-year intervals, and time-varying betas calculated based on the past two years. Results from the various estimation techniques can be found in Appendix A2.2. The results from the regression with constant betas for the entire period are not very promising, which supports that betas change trough time. The model with betas calculated for two-year intervals performs the best. This result is not surprising, as the betas are estimated based on a time period that includes data from later time periods than the period the beta is estimated for. As the model should be usable for predicting future returns, information about future time periods cannot be used for estimation. We therefore apply time-varying betas calculated from the past 24 months of data. This estimation technique is supported by Lewellen and Nagel (2006), who argue that betas can be treated as constant for short time intervals.

By estimating the betas from the past 24 months, an acceptable number of data points are included in the estimation. Shortening the time period will result in less accurate estimates, while calculating betas based on longer time series is not appropriate since betas vary over time. We could alternatively have used daily data to estimate the betas in order to obtain more data points. However, returns will be zero for days without trade, which results in underestimation of the betas.

5.4 Testing the Model

When the optimal model is selected, we test its performance against the CAPM. We consider the CAPM to be a natural choice of reference model, as the CAPM is the most established model in asset pricing, and most new models are compared to it. Another widely applied reference model is the Fama-French model. We have chosen not to use this model, as our objective is to test if liquidity risk contributes to explain cross-sectional variations in return. Then, testing our model against the CAPM is more relevant, since our liquidity model can add valuable information even if it is outperformed by the Fama-French model. We will not compare our model to any of the other asset pricing models presented in Section 2.1, since they are rarely used for comparison purposes.

To assess the statistical performance of the liquidity model compared to the CAPM, and to examine whether the regression results are stable through time, we also consider regression results from sub-periods. The evaluation of the regression results is based on the same criteria as those applied to select the optimal model. We also test whether the OLS assumptions hold.

Finally, we compare the forecasting ability of our liquidity model to the CAPM. We find it interesting to examine predictive power, as prediction is a main purpose of asset pricing models. As the models are cross-sectional, we predict the return of each security in each time period based on the average factor loadings from the past 12 months of cross-sectional regressions. To compare the models, the absolute deviations and the squared deviations from the true returns are analyzed for our model and the CAPM.

6 Results

This section presents and discusses results from our analyses. The first part considers the liquidity measures separately, while the second and the third part discuss how the liquidity measures perform when combined to multifactor models.

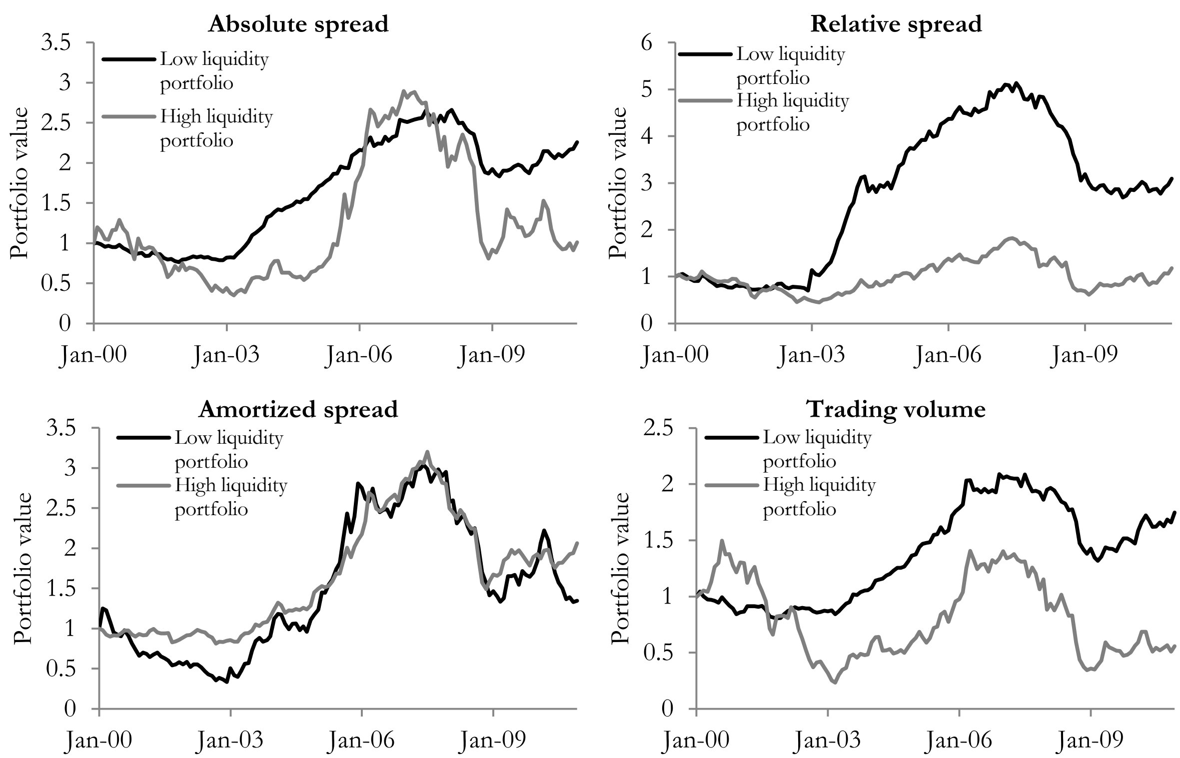
6.1 Priced Liquidity Risk

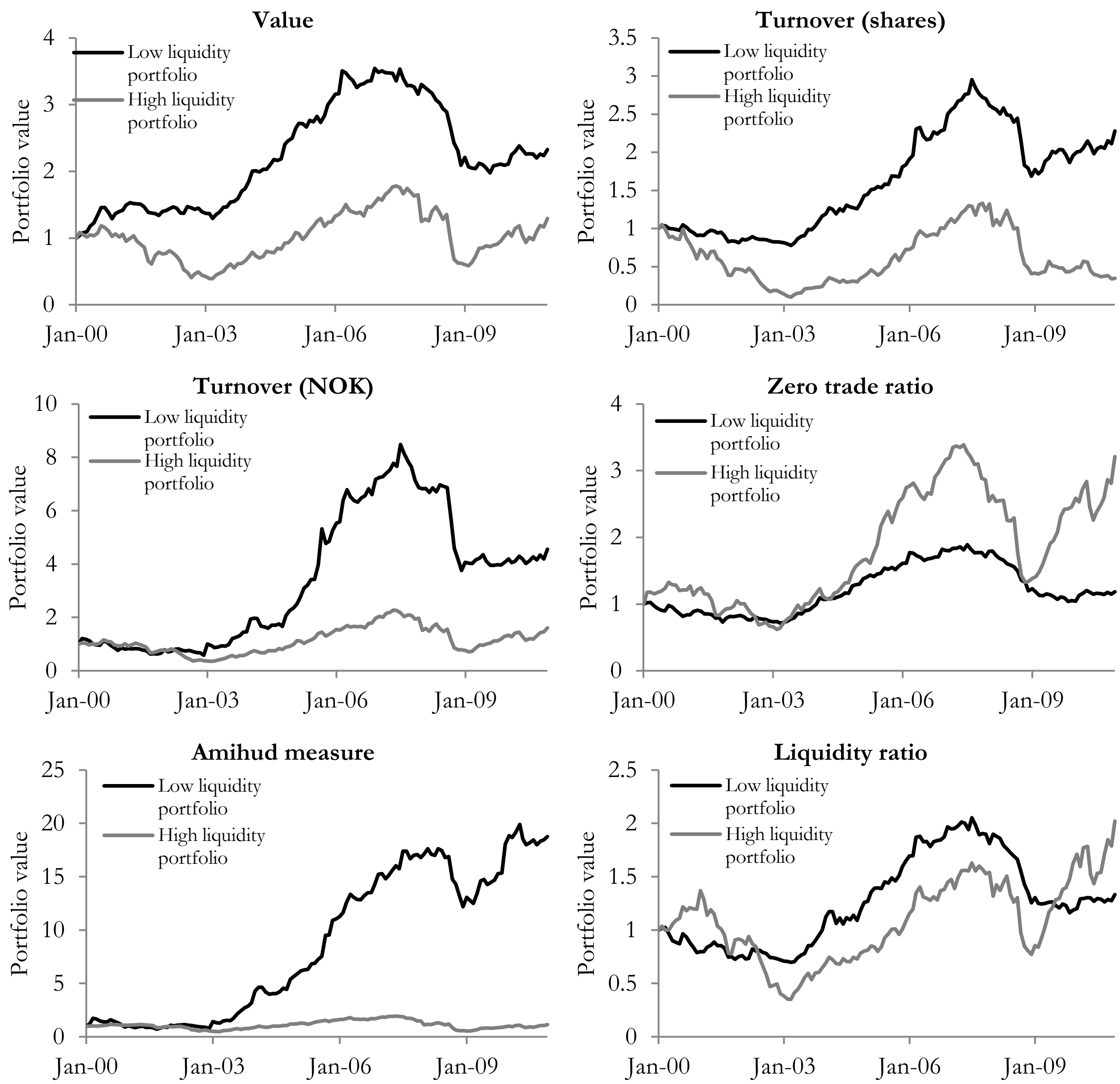
To examine if liquidity is priced, we consider the cross-sectional variations in return for assets with different levels of liquidity, to what extent each factor contributes to describe returns when added to the CAPM, and how the variation is over time for the liquidity factors compared to the variation in market returns.

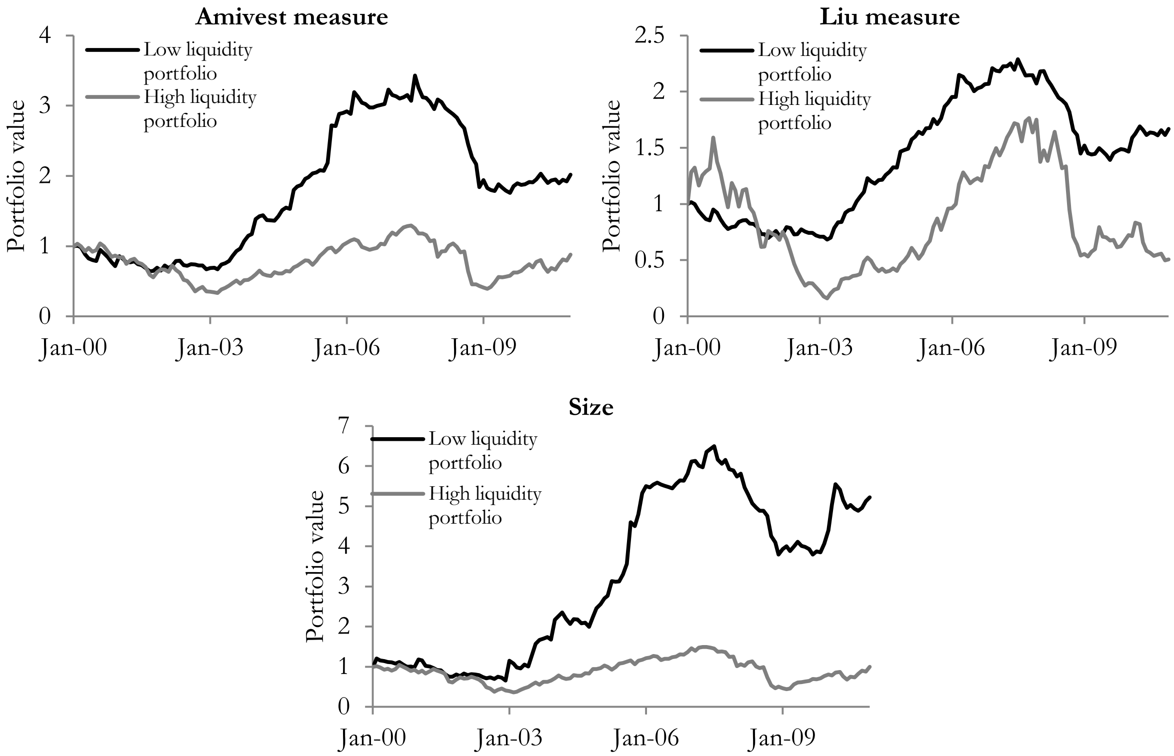
6.1.1 Cross-Sectional Differences in Liquidity

The relation between returns and cross-sectional variations in the liquidity measures is examined by considering the differences in return of portfolios with different levels of liquidity. The development over time in value of the extreme portfolios is plotted in Figure 1.

Figure 1: Performance over time of liquidity measure portfolios




Value
Turnover (shares)
Turnover (NOK)
Zero trade ratio
Amihud measure
Liquidity ratio
Low liquidity portfolio
High liquidity portfolio
Portfolio value
Jan-00
Jan-03
Jan-06
Jan-09



The plots show that among the width measures, the low liquidity portfolio performs the best for relative spread. For amortized spread and absolute spread, the difference in portfolio performance is small. Both depth measures have a clear difference in portfolio performance. Of the immediacy measures, the turnover measures show cross-sectional difference in return. Surprisingly, the zero trade ratio has the highest return for the high liquidity portfolio. For this measure, the portfolio with high liquidity outperforms the low liquidity portfolio in the periods of high market return, while the low liquidity portfolio has the best performance during recessions. This is opposite of the development of most of the other measures. Of the resiliency measures, the low liquidity portfolio performs the best for the Amivest and the Amihud measure, while the liquidity ratio portfolios have approximately equal returns. For the Liu measure and size, the less liquid portfolios have the highest return.

Based on these results, absolute spread, amortized spread,the zero trade ratio and the liquidity ratio seem to be less promising liquidity measures. However, there could be cross-sectional differences in the measures without the extreme portfolios being priced. These results in isolation do therefore not give a clear answer to, but rather provides an indication of, which of the measures that are priced.

As the low liquidity portfolios outperform the high liquidity portfolios for most of the measures, the differences in level of liquidity between stocks appear to be priced, and less liquid stocks are associated with a return premium. The gap between the portfolios increases when the market return is high for relative spread, value, turnover (NOK), the Amihud measure, the Amivest measure and size. This indicates that return of less liquid stocks are the most affected by recessions, since these stocks decline the most in value during market recessions.

Another interpretation of higher portfolio gaps in upturns is that differences between stocks in liquidity level is related to differences in market betas, as less liquid stocks have higher fluctuations in returns, and these fluctuations are related to fluctuations in market return. Therefore, returns should be adjusted for market risk. To quantify cross-sectional differences in return captured by each liquidity measure adjusted for market risk, cross-sectional regressions with each liquidity measure and the market beta are performed. The results are presented in Table 5 (and in further detail in Appendix A3.1).

Table 5: Fama-MacBeth regressions with each liquidity measure and the market factor

Model (R^2)	**Average statistics**	**Intercept**	**Liquidity factor**	**Market factor**
CAPM	beta	0.0115		-0.0009
(5%)	p-value	0.17		0.08
CAPM + Absolute spread	beta	0.0113	0.0000	-0.0006
(6%)	p-value	0.07	0.32	0.00
CAPM + Relative spread	beta	0.0171	-0.1043	-0.0040
(7%)	p-value	0.08	0.21	0.11
CAPM + Amortized spread	beta	0.0073	74.83	-0.0002
(11%)	p-value	0.10	0.08	0.07
CAPM + Trading volume	beta	-0.0197	0.0029	-0.0102
(9%)	p-value	0.20	0.10	0.16
CAPM + Value	beta	-0.0540	0.0044	-0.0137
(8%)	p-value	0.18	0.13	0.15
CAPM + Turnover (shares)	beta	0.0087	0.0449	-0.0052
(13%)	p-value	0.08	0.03	0.12
CAPM + Turnover (NOK)	beta	0.010	0.001	-0.0033
(7%)	p-value	0.08	0.22	0.09
CAPM + Zero trade ratio	beta	0.0203	-0.0234	-0.0070
(8%)	p-value	0.05	0.18	0.15
CAPM + Amihud measure	beta	0.0106	0.0077	-0.0008
(6%)	p-value	0.09	0.18	0.09
CAPM + Liquidity ratio	beta	0.0114	0.0000	-0.0024
(7%)	p-value	0.11	0.24	0.11
CAPM + Amivest measure	beta	0.0201	-0.0005	-0.0013
(4%)	p-value	0.15	0.20	0.09
CAPM + Liu measure	beta	0.0207	-0.0012	-0.0075
(5%)	p-value	0.05	0.18	0.14
CAPM + Size	beta	-0.0632	0.0037	-0.0058
(8%)	p-value	0.18	0.17	0.10

R^2 is averaged over months with complete data

The results in Table 5 show that turnover (shares) is the only significant factor at a 5% level, while amortized spread and trading volume are significant at a 10% level. All factors except absolute spread are significant at a 25% level. This indicates a statistical relation between the measures and return. However, noise makes it difficult to be certain that all the factors really have an impact. It is interesting to discover that amortized spread and the zero trade ratio contribute to explain returns in the cross-sectional regression, even though the measures do not appear to be priced according to Figure 1.

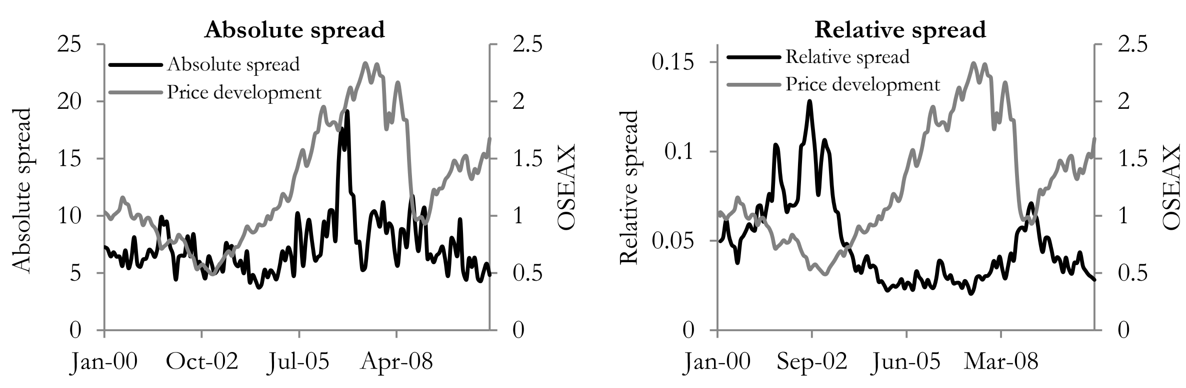
The low p-values of the market factor indicate that this factor is important for explaining returns, and therefore should be included in asset pricing models. R^2 and adjusted R^2 are improved by adding liquidity for most measures. As the liquidity factors also have high significance, it is likely that they capture variations in the cross-section of returns that the market factor alone does not. Therefore, the results indicate that liquidity risk is priced. The intercept has low p-values in all the regressions, which is a sign of the market factor combined with a single liquidity factor not succeeding to capture all the variation in return. Consequently, an optimal model should include multiple factors in addition to the market factor.

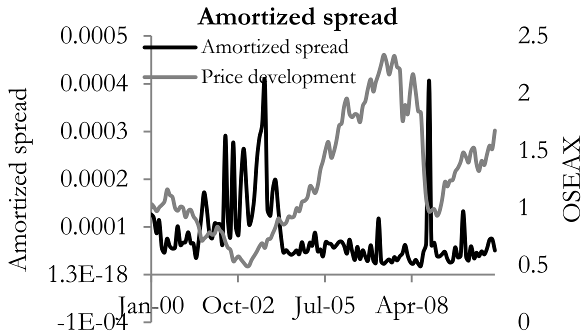
Some of the average factor coefficients have opposite signs in the regressions compared to what is suggested by the portfolio results. However, the sign of the coefficients are not stable over time, so this does not necessarily indicate that more liquid stocks have higher return over time in normal market conditions. Our sample period includes several recessions, in which the sign of the coefficients are likely to be opposite than in upturns. Also, the market betas are highly correlated to many of the liquidity factors (Appendix A3.1), which can result in unstable coefficient estimates.

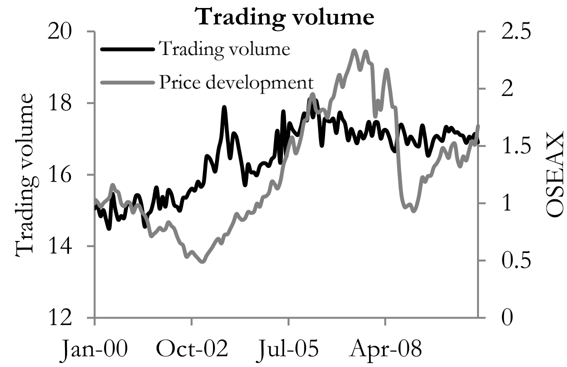
6.1.2 Time-Variations in Liquidity

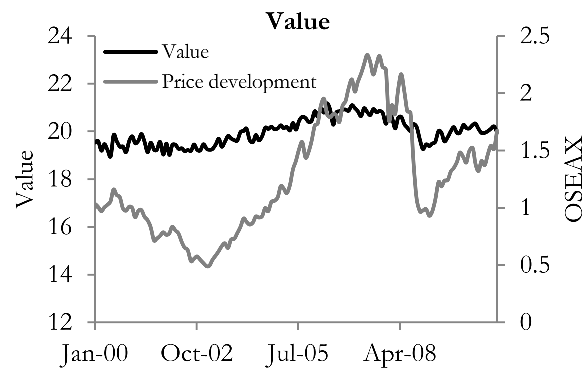
The relation between the development of the liquidity measures and market return over time is presented in Figure 2. This relation is relevant to decide if level of liquidity is priced, as the risk factors should be systematic in order to be priced. If the average liquidity level is low when the market return is low, the risk factors appear to be systematic.

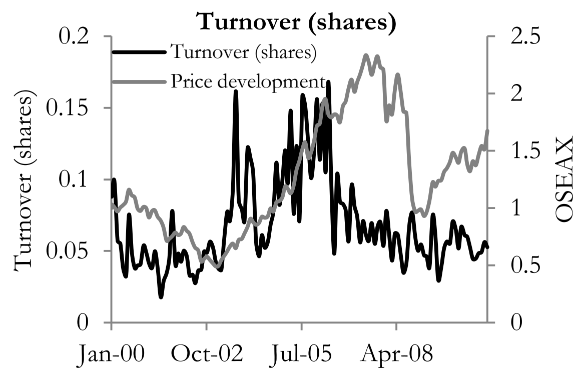
Figure 2: Time variation in market averages of liquidity measures and return

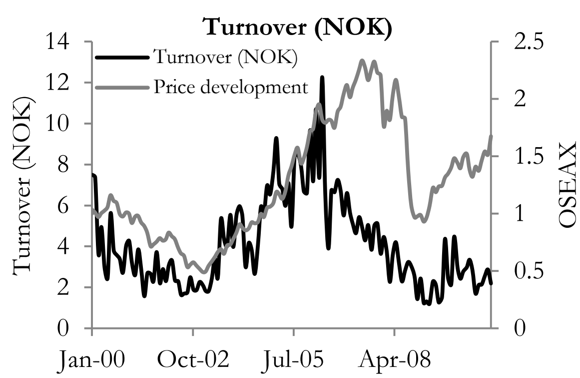


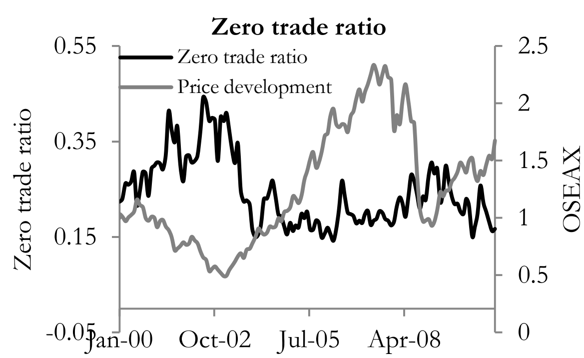

Amortized spread
Amortized spread
Price development
Amortized spread
OSEAX
0.0005
0.0004
0.0003
0.0002
0.0001
1.3E-18
-1E-04
2.5
2
1.5
1
0.5
0
Jan-00
Oct-02
Jul-05
Apr-08

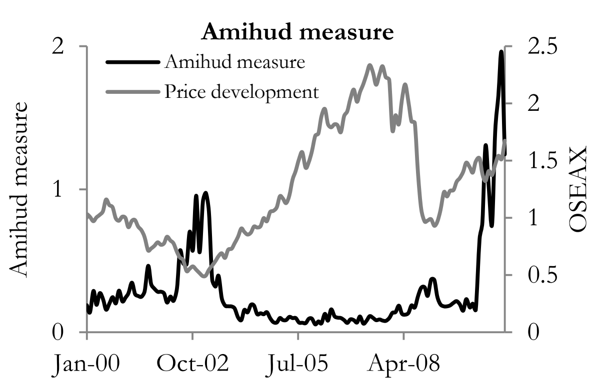

Trading volume
Trading volume
Price development
Trading volume
OSEAX
20
18
16
14
12
2.5
2
1.5
1
0.5
0
Jan-00
Oct-02
Jul-05
Apr-08

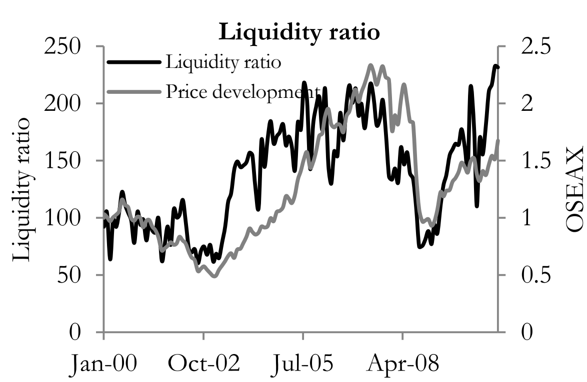

Value
Value
Price development
Value
OSEAX
24
22
20
18
16
14
12
2.5
2
1.5
1
0.5
0
Jan-00
Oct-02
Jul-05
Apr-08

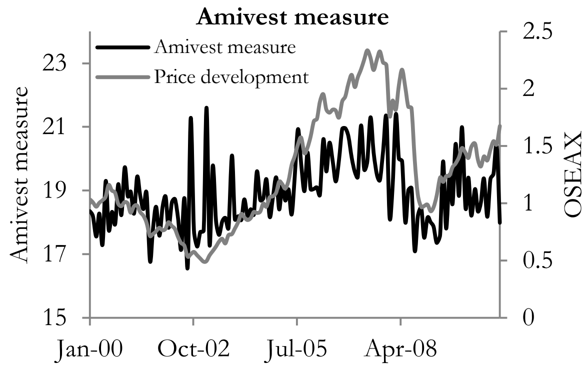

Turnover (shares)
Turnover (shares)
Price development
Turnover (shares)
OSEAX
0.2
0.15
0.1
0.05
0
2.5
2
1.5
1
0.5
0
Jan-00
Oct-02
Jul-05
Apr-08

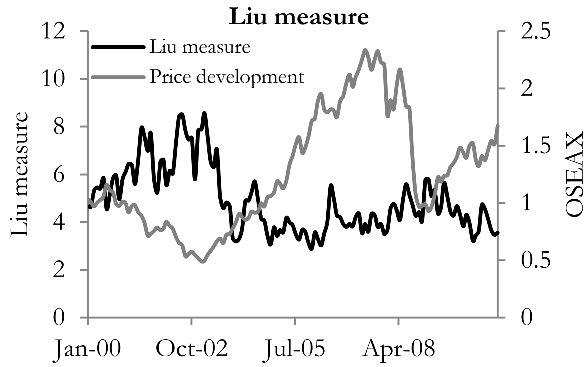

Turnover (NOK)
Turnover (NOK)
Price development
Turnover (NOK)
OSEAX
14
12
10
8
6
4
2
0
2.5
2
1.5
1
0.5
0
Jan-00
Oct-02
Jul-05
Apr-08

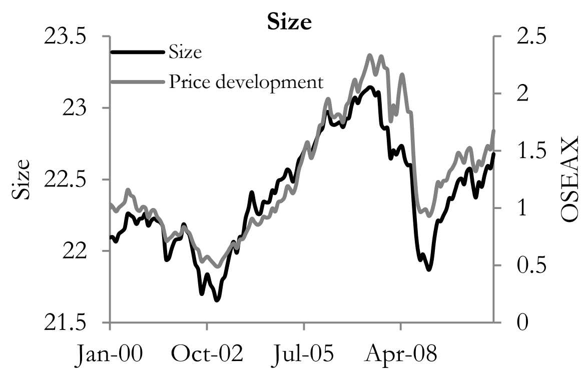

Zero trade ratio
Zero trade ratio
Price development
Zero trade ratio
OSEAX
0.55
0.35
0.15
-0.05
2.5
2
1.5
1
0.5
0
Jan-00
Oct-02
Jul-05
Apr-08


Amihud measure
Amihud measure
Price development
Amihud measure
OSEAX
2
1
0
2.5
2
1.5
1
0.5
0
Jan-00
Oct-02
Jul-05
Apr-08


Liquidity ratio
Liquidity ratio
Price development
Liquidity ratio
OSEAX
250
200
150
100
50
0
2.5
2
1.5
1
0.5
0
Jan-00
Oct-02
Jul-05
Apr-08







As previously discussed (in section 3.4.3), high values of the measures indicate high liquidity for some of the measures while the opposite is true for others. Higher values of the spread measures imply that the market is less liquid. Relative and amortized spread seem systematic, as the spread is high when the market value is low in Figure 2. Absolute spread is highest when the market price is high, and does therefore not seem systematically related to return. Trading volume, value, turnover (shares) and turnover (NOK) are higher when the market return is higher and seem to have systematic characteristics. Higher zero trade ratio indicates a less liquid market, and the development over time of the measure is opposite of the market return, which suggests that this risk factor is systematic.

Of the resiliency measures, the Amihud measure should be negatively related to return, while the others should be positively related to return. The liquidity ratio seems to follow the price development well. So do also the Amihud and the Amivest measure, with exception of a few time intervals. The Amihud measure is high in 2010 when the market is rising, and the Amivest measure is high in 2002 when the market is at its lowest, which contradicts that liquidity is low during recessions. The Liu measure appears to be systematic, as it is oppositely related to return. Size is positively related to return and appears to follow the development of the market value. This is not surprising, as the average market capitalization of all stocks is the same as the value-weighted average price. Thus, size should rather be considered in the cross-section of stocks.

When considering the performance of the measures together, the overall impression is that liquidity risk is systematic. However, as the evaluation based on the time series plots is rather qualitative, we also examine the correlation between market return and the measures over time. As can be seen from the last row in Table 6, the correlation between the market return and the liquidity measures tends to be low. However, return is positively correlated to liquidity for all the measures, which is a clear indication of liquidity to be systematic.

Table 6: Correlation matrix for time series of average liquidity measures

	Absolute spread	Relative spread	Amortized spread	Trading volume	Value	Turnover (shares)	Turnover (NOK)	Zero trade ratio	Amihud measure	Liquidity ratio	Amivest measure	Liu measure	Size
Absolute spread													
Relative spread	-0.1												
Amortized spread	-0.1	0.5											
Trading volume	0.3	-0.6	-0.2										
Value	0.4	-0.8	-0.4	0.8									
Turnover (shares)	0.1	-0.5	0.1	0.5	0.6								
Turnover (NOK)	0.2	-0.6	-0.1	0.4	0.6	0.9							
Zero trade ratio	-0.1	0.9	0.3	-0.7	-0.7	-0.6	-0.6						
Amihud measure	-0.3	0.3	0.1	-0.1	-0.3	-0.3	-0.5	0.2					
Liquidity ratio	0.1	-0.8	-0.4	0.7	0.8	0.5	0.5	-0.8	-0.1				
Amivest measure	0.3	-0.4	-0.3	0.4	0.6	0.2	0.3	-0.3	-0.1	0.5			
Liu measure	-0.1	0.9	0.3	-0.7	-0.7	-0.6	-0.6	1.0	0.2	-0.8	-0.3		
Size	0.4	-0.8	-0.5	0.6	0.9	0.4	0.6	-0.7	-0.3	0.8	0.6	-0.7	
OSEAX return	-0.2	-0.3	-0.1	0.2	0.1	0.2	0.2	-0.2	-0.1	0.4	0.1	-0.2	0.2

Correlation larger than |0.3| and |0.7| is marked

In addition to being correlated to market return, liquidity measures are correlated to each other. This is not surprising, as all the measures are proxies the market liquidity. Relative spread, value, the zero trade ratio, the liquidity ratio, the Liu measure and size are the most correlated, both to other liquidity measures and to market return. Therefore, these measures seem to capture the effects of some underlying systematic liquidity risk. However, this does not imply that the other liquidity measures do not capture systematic variation in returns, as their correlation to return is only marginally lower.

The time series plots and the correlation with return show that fluctuations in market liquidity are related to fluctuations in market return. Also, less liquid stocks have the strongest decrease in return during market downturns, as previously mentioned (Figure 1). An interpretation of this is that level of liquidity is related to systematic risk. This implies that investors have to take the risk of lower performance in poor market conditions to earn the high average spread between the portfolios. Our empirical investigation of liquidity at the OSE thereby shows that level of liquidity and systematic fluctuations in liquidity are related to differences in return, and that there is a relation between these two aspects. Liquidity risk thereby seems to be priced.

6.2 Selection of Liquidity Measures

As liquidity seems to be priced, we further want to decide how liquidity risk best can be expressed by liquidity measures. Results from stepwise cross-sectional regressions and the correlation between the measures in cross-sections are examined. In addition, the results from the previously presented analyses of priced liquidity measures are considered. The factors that perform best are thereafter combined to models, which are tested with Fama-MacBeth regressions in order to determine which model that is optimal.

6.2.1 Combining Measures

Results from stepwise cross-sectional regressions for annual averages of the liquidity measures and for random months are presented in Table 7 (and in Appendix A3.2).

Table 7: Stepwise regression results from cross-sectional regressions

	Number of times included in model	
Variable	**Annual averages**	**10 random months**
Absolute spread	0	1
Relative spread	1	5
Amortized spread	4	4
Trading volume	3	5
Value	6	7
Turnover (shares)	4	2
Turnover (NOK)	3	3
Zero trade ratio	3	4
Amihud measure	3	1
Liquidity ratio	0	2
Amivest measure	2	7
Liu measure	2	3
Size	1	2

The results show that relative spread, amortized spread, all the depth measures, all the immediacy measures, and the Amivest measure are included in the regression model relatively often. They therefore contribute to explain cross-sectional differences in returns in combination with other measures. Some factors might be included less frequently because explain differences in returns less accurate than other measures. They might however also be excluded because they are highly correlated to other measures, and therefore are less significant in combination with these.

As displayed in Table 8 and 9, many of the factors have correlation above 70%, and multicollinearity problems would most likely be extensive if all the factors were included in the same model. Absolute spread, amortized spread, the turnover measures and the Amihud measure appear to be less correlated to other measures, and can most likely be included in a model without causing multicollinearity issues. However, the highly correlated measures might be considered the most important to include, because the reason why they are highly correlated might be that they all capture aspects of a common underlying liquidity risk. Including many of these measures in the same model is thereby most likely not optimal, but some of them should still be included in the final model.

Table 8: Average correlation of annual cross-sectional measures

	Absolute spread	Relative spread	Amortized spread	Trading volume	Value	Turnover (shares)	Turnover (NOK)	Zero trade ratio	Amihud measure	Lilquidity ratio	Amivest measure	Liu measure	Size
Absolute spread													
Relative spread	0.3												
Amortized spread	0.1	0.3											
Trading volume	-0.4	-0.6	0.1										
Value	-0.3	-0.7	-0.1	0.9									
Turnover (shares)	-0.1	-0.3	0.4	0.6	0.5								
Turnover (NOK)	0.0	-0.3	0.0	0.3	0.6	0.4							
Zero trade ratio	0.4	0.8	0.2	-0.8	-0.8	-0.4	-0.3						
Amihud measure	0.1	0.6	0.2	-0.3	-0.5	-0.2	-0.3	0.4					
Lilquidity ratio	-0.3	-0.7	-0.2	0.7	0.9	0.4	0.5	-0.7	-0.5				
Amivest measure	-0.3	-0.7	-0.2	0.7	0.9	0.4	0.5	-0.7	-0.6	0.8			
Liu measure	0.4	0.8	0.2	-0.8	-0.8	-0.4	-0.3	1.0	0.4	-0.7	-0.7		
Size	-0.2	-0.6	-0.4	0.5	0.8	0.2	0.5	-0.6	-0.5	0.8	0.8	-0.6	
Return	0.0	-0.1	0.0	0.0	0.1	0.0	0.1	-0.1	-0.1	0.0	0.1	-0.1	0.1

Correlation larger than |0.3| and |0.7| is marked

Table 9: Average correlation of cross-sectional measures for random months

	Absolute spread	Relative spread	Amortized spread	Trading volume	Value	Turnover (shares)	Turnover (NOK)	Zero trade ratio	Amihud measure	Lilquidity ratio	Amivest measure	Liu measure	Size
Absolute spread													
Relative spread	0.4												
Amortized spread	0.1	0.1											
Trading volume	-0.4	-0.6	0.2										
Value	-0.3	-0.7	0.0	0.9									
Turnover (shares)	-0.1	-0.3	0.5	0.6	0.5								
Turnover (NOK)	0.1	-0.2	0.1	0.3	0.5	0.4							
Zero trade ratio	0.4	0.7	0.1	-0.8	-0.8	-0.4	-0.2						
Amihud measure	0.1	0.5	0.1	-0.3	-0.4	-0.2	-0.2	0.3					
Lilquidity ratio	-0.2	-0.5	-0.1	0.7	0.8	0.3	0.4	-0.7	-0.4				
Amivest measure	-0.3	-0.7	-0.1	0.8	0.9	0.4	0.4	-0.7	-0.4	0.7			
Liu measure	0.4	0.7	0.1	-0.7	-0.7	-0.3	-0.2	1.0	0.3	-0.6	-0.6		
Size	-0.2	-0.5	-0.3	0.5	0.8	0.1	0.5	-0.6	-0.4	0.7	0.7	-0.6	
Return	0.0	0.0	0.1	0.0	0.1	0.0	0.1	0.0	0.0	0.0	0.0	0.0	0.1

Correlation larger than |0.3| and |0.7| is marked

The correlation is high between some measures from the same dimension, but also between measures from different dimensions. Relative spread is for instance not very correlated to the other spread measures, but highly correlated to value, the zero trade ratio, the Amivest measure and the Liu measure. The high correlation between trading volume and value, and the Amivest measure and the liquidity ratio, shows that the correlation also is high between measures from the same dimension. As the correlation is high both within and between dimensions, we perceive the dimensions to be better for theoretical than for statistical categorization.

The correlation between the order-based spread measures is low, while the relative spread is highly correlated to trade-based measures. Whether measures are trade-based or order-based thereby neither seems to be related to the correlation between measures. It is also worth mentioning that the correlation between the Liu measure and the zero trade ratio is approximately one in all the correlation tables. This is most likely due to a high deflator in the Liu measure, as explained in Section 3.3.3.

The correlation between liquidity measures and return is lower in the cross-section of stocks than for the time series of market averages (Table 6). Also, liquidity is positively related to return for some measures, while negatively related for others. This can be explained by less liquid stocks being likely to have the lowest returns in recessions, while they perform above average in upturns. The average correlation is therefore influenced by the market conditions that dominate the time period investigated, and will be close to zero on average.

6.2.2 Model Selection

In order to decide what liquidity measures that best capture the liquidity effects and should be included in a model, we consider the totality of the analyses presented so far. An overview of the results obtained is provided in Table 10.

Table 10: Overview of the empirical performance of each liquidity measure

Liquidity measure	Portfolio performance	Fama-MacBeth regression	Relation to return	Stepwise regression	Include in models?
Width measures					
Absolute spread	-	-	-	-	No
Relative spread	+	-	+	+	Yes
Amortized spread	-	+	+	+	Yes
Depth measures					
Trading volume	+	+	+	+	Yes
Value	+	-	+	+	Yes
Immediacy measures					
Turnover (shares)	+	+	+	+	Yes
Turnover (NOK)	+	-	+	+	No
Zero trade ratio	-	-	+	+	No
Resiliency measures					
Amihud measure	+	-	-	-	No
Liquidity ratio	-	-	+	-	No
Amivest measure	+	-	+	+	Yes
Other measures					
Liu measure	+	-	+	-	No
Size	+	-	+	-	Yes

We consider relative and amortized spread to be the most relevant spread measures, and they are potential candidates for a model. Absolute spread is not selected as it does not have priced portfolios, has low relation to return, has the lowest significance in the Fama-MacBeth regression and is rarely included in the stepwise regression models. Amortized spread has the advantage of low correlation with other measures, but the portfolio performance is poor. Relative spread on the other hand is less significant in the Fama MacBeth regression. Both depth measures are potential candidates, as they perform well in all the analyses. Trading volume performs slightly better in the Fama-MacBeth regression with a p-value of 10%, while the p-value of the value factor is 13%.

Of the immediacy measures, the zero trade ratio does not perform well in the analyses of priced liquidity factors, but is included frequently in the stepwise regressions. Turnover (shares) has a p-value of 3% in the Fama-MacBeth regression, and therefore performs substantially better than turnover (NOK) with a p-value of 22%. As everything else is quite equal and the correlation between these measures is high, we exclude turnover (NOK) from further analyses. The Amivest measure seems to be the best resiliency measure as it is the only measure to perform well when it comes to relation to price, portfolio return and the stepwise regression. However, all the resiliency measures have high p-values of around 20% in the Fama-MacBeth regression.

The Liu measure does not perform well in the tests and is not included in further analyses. Size is included mainly because we consider it is interesting to include as it is used in the Fama-French model. By including size we can examine whether it captures the same variations in return as the liquidity measures do.

Based on these considerations, we proceed with relative spread, amortized spread, trading volume, value, turnover (shares), the Amivest measure and size. All the liquidity dimensions and both trade- and order-based measures are thereby represented. In addition to the liquidity factors, we also include the market factor in the models.

Many of the liquidity factors we use for factor combinations are highly correlated, and all seven factors are therefore not included in the same model. Results from regressions of factor combinations are summarized in Table 11 (and in further detail in Appendix A3.3). Value, the Amivest measure and size are all more than 70% correlated to each other. We will therefore not test factor combinations with more than one of these three factors. Value and the Amivest measure are also more than 70% correlated to trading volume. As turnover (shares) is the most significant in the regression with the market factor, and is not highly correlated to the other factors, we include this factor in many of the models. Also, we include only one of the spread measures in each model, as we find it theoretically best to avoid including factors from the same dimension.

Table 11: Fama-MacBeth regression results of factor combinations

Model (R^2)	**Average statistics**	**Intercept**	**Relative spread**	**Amortized spread**	**Trading volume**	**Value**	**Turnover (shares)**	**Amivest measure**	**Size**	**Market factor**
CAPM	beta	0.0115								-0.0009
(5%)	p-value	0.17								0.08
Model 1	beta	-0.0471	0.0662			0.0037	0.0113			-0.0132
(20%)	p-value	0.26	0.24			0.22	0.04			0.16
Model 2	beta	0.0658	-0.2092				0.0481	1.53		-0.0053
(22%)	p-value	0.09	0.17				0.02	0.12		0.13
Model 3	beta	-0.0782	0.0107				0.0426		0.0043	-0.0103
(17%)	p-value	0.22	0.27				0.03		0.21	0.14
Model 4	beta	-0.0024		28.82	0.0010		0.0445			-0.0088
(21%)	p-value	0.26		0.18	0.19		0.11			0.17
Model 5	beta	0.0078		27.49			0.0547			-0.0050
(16%)	p-value	0.09		0.17			0.06			0.12
Model 6	beta	-0.0160		65.07	0.0023					-0.0083
(19%)	p-value	0.22		0.09	0.14					0.17
Model 7	beta	-0.0993		90.77			0.0229		0.0052	-0.0100
(18%)	p-value	0.19		0.19			0.07		0.18	0.13
Model 8	beta	-0.0764					0.0426		0.0043	-0.0107
(16%)	p-value	0.18					0.03		0.17	0.13
Model 9	beta	0.0003			0.0009		0.0430			-0.0087
(19%)	p-value	0.26			0.19		0.06			0.16

R^2 is averaged over months with complete data

The results from the Fama-MacBeth regressions provide no clear answer to which model that is optimal. It is important that the explanatory power is high, and model 1, 2 and 4 with high values of R^2 and adjusted R^2 are favored. It is also desirable that all the factors have high significance in order to select a model where all the factors contribute to explain returns. Turnover (shares) has low p-values in almost all the models, and we therefore include this measure in the final model. The significance of all the other factors is between 12% and 27% when they are included in combination with turnover (shares). However, only including turnover and the market factor is not a very good option, as this results in high significance of the regression intercept. Model 1 and 3 have higher p-values than 20% for some of the factors, and are therefore not considered to be good models. Model 2 and 5 also have significant intercepts at a 9% level, and therefore seem less optimal.

The remaining models, model 4, 6, 7, 8 and 9, all have one significant factor at an 11% level, and the other factors are significant at a 20% level. They are thereby all good candidates. Model 4 has the highest R^2 and the highest p-value of the intercept, and is therefore chosen as the optimal model. The factors in model 4 are the three most significant factors in the regressions with only one factor in Table 5, and also perform well in the other analyses of our study.

It is interesting to observe that relative spread is not very significant alone or in combination with other factors, as it appears to be one of the most used liquidity measures in other studies. Conversely, the commonly used measure turnover performs very well in our study. Size is not included in the final model. The factor has a p-value between 17% and 21%, so it does not perform much worse in model combinations than some of the factors that are selected. However, R^2 is relatively low for the models that include size. Other factors which are highly correlated with size are also excluded. Based on this, we conclude that some of the liquidity factors seem to be better than size for explaining returns.

The best model, model 4, consists of the market factor, amortized spread, trading volume, and turnover (shares). We name it the MATT-model, because of the first letters of the factors. We find our model to be representative for how a multifactor liquidity model performs, and will test it in further detail in the next section.

6.3 Performance of Our Liquidity Model

In order to evaluate the relative performance of our model, it is compared to the performance of the CAPM. We consider both statistical and predictive performance.

6.3.1 Statistical Performance

Both on average over the whole sample period and for each year, our liquidity model performs better than the CAPM according to R^2, as Table 14 shows. R^2 is however not an entirely fair way of comparing models with a different number of factors. As our model includes the market factor in addition to other factors, it will always explain at least as much of the return as a model with only the market factor. However, the MATT-model is also better according to adjusted R^2. The p-values of an F-test of the overall fit of the model show that the factors contribute to explain variations in return of the MATT-model with 87% certainty, while it is only 68% certain that the market factor alone contributes to explain variations in returns. Overall, the MATT-model therefore explains returns better than the CAPM.

The significance of the intercept is quite equal for the two models, as shown in table 12. The market factor in the CAPM is significant at an 8% level. This is better than the MATT-model, in which the factors have p-values between 10% and 20%. As the market factor in the CAPM is more significant than the liquidity factors in the MATT-model, the additional factors in the MATT-model do not seem to contribute as much to explain returns as the market factor does. It also seems like the performance of the market factor is more stable than the performance of the liquidity factors. This could partly be caused by high correlation between the liquidity factors. However, the turnover measure is more significant than the market factor in the MATT-model. As the liquidity measures are correlated to the market factor, the liquidity model could possibly also explain returns well without the market factor. Since R^2 is not affected by multicollinearity, we consider it as a more important evaluation criterion than the significance of the factors.

By considering the average regression results for each year in Table 12 and Table 14, it seems like the MATT-model has the best performance in the first four years, where the significance of the factors is higher than for the market factor in the CAPM. The CAPM has significant factors at a 10% level in 4 out of 9 time periods. The intercept is not significant for any of the models in any of the years at a 10% level. In general, the results observed for the different time periods are much of the same as the average results over the whole sample period, as the MATT-model explain more of the return and have less significant factors for almost all the years. The results therefore seem to be quite stable over time, and are consistent in both upturns and recessions.

Table 12: Regression results for each year

		CAPM		MATT				
Year	**Average statistics**	**Intercept**	**Market factor**	**Intercept**	**Amortized spread**	**Trading volume**	**Turnover (shares)**	**Market factor**
Average	beta	0.011	-0.0009	-0.0024	28.82	0.0010	0.0445	-0.0088
	p-value	0.17	0.08	0.26	0.18	0.19	0.11	0.17
2002	beta	0.000	-0.0363	0.0074	-25.34	-0.0019	-0.1625	-0.0114
	p-value	0.12	0.00	0.29	0.06	0.23	0.14	0.02
2003	beta	0.036	0.0216	0.0190	-33.11	0.0027	0.1504	-0.0034
	p-value	0.15	0.11	0.17	0.10	0.09	0.08	0.07
2004	beta	0.027	0.0041	0.0156	-59.86	0.0007	0.1306	-0.0055
	p-value	0.16	0.09	0.23	0.20	0.15	0.02	0.18
2005	beta	0.036	0.0037	0.0079	135.48	0.0012	0.1380	-0.0091
	p-value	0.14	0.16	0.16	0.11	0.11	0.01	0.24
2006	beta	0.022	-0.0043	-0.0051	169.23	0.0017	0.0464	-0.0131
	p-value	0.11	0.12	0.34	0.19	0.33	0.21	0.12
2007	beta	-0.003	0.0068	-0.0105	197.83	0.0004	0.0808	-0.0049
	p-value	0.13	0.14	0.28	0.25	0.20	0.18	0.25
2008	beta	-0.026	-0.0470	-0.0190	-68.50	-0.0024	0.0249	-0.0194
	p-value	0.23	0.02	0.30	0.31	0.20	0.25	0.23
2009	beta	0.003	0.0384	-0.0443	-44.36	0.0068	-0.0459	-0.0111
	p-value	0.26	0.22	0.25	0.23	0.18	0.19	0.31
2010	beta	0.006	0.0023	0.0080	-16.53	0.0001	0.0202	-0.0017
	p-value	0.25	0.05	0.30	0.18	0.25	0.17	0.20

Table 13: Regression results for different market conditions

		CAPM		MATT				
Year	**Average statistics**	**Intercept**	**Market factor**	**Intercept**	**Amortized spread**	**Trading volume**	**Turnover (shares)**	**Market factor**
Upturn, before Oct 07	beta	0.0220	-0.0010	0.0062	62.17	0.0009	0.0689	-0.0088
	p-value	0.14	0.08	0.25	0.14	0.18	0.07	0.13
Recession, Nov 07 - Jan 09	beta	-0.0250	-0.0331	-0.0174	-18.51	-0.0021	0.0241	-0.0119
	p-value	0.20	0.03	0.29	0.30	0.20	0.26	0.24
Upturn, after Feb 09	beta	0.0037	0.0207	-0.0187	-40.35	0.0035	-0.0155	-0.0070
	p-value	0.25	0.11	0.27	0.21	0.21	0.17	0.25

Table 14: Overall fit of the models

Year	Average	2002	2003	2004	2005	2006	2007	2008	2009	2010
R^2										
CAPM	5 %	11 %	4 %	4 %	2 %	6 %	5 %	5 %	2 %	6 %
MATT	23 %	28 %	27 %	24 %	36 %	20 %	21 %	14 %	16 %	21 %
Adjusted R^2										
CAPM	3 %	9 %	3 %	3 %	0 %	4 %	3 %	4 %	1 %	5 %
MATT	17 %	22 %	21 %	18 %	31 %	14 %	16 %	8 %	10 %	15 %
P-values from F-test										
CAPM	0.32	0.16	0.21	0.32	0.45	0.38	0.33	0.34	0.38	0.28
MATT	0.13	0.09	0.02	0.07	0.09	0.24	0.13	0.27	0.16	0.14

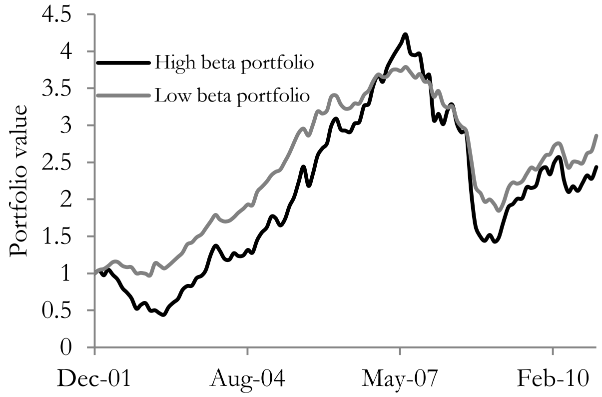
Based on a subset of the data with 59 companies to avoid blank cells in calculations

The market factor in the CAPM is negative on average, which implies that over the whole sample period, stocks with low market betas have performed better than high beta stocks. This result is unexpected, as it is theoretically more reasonable that investing in high risk stocks is rewarded with higher return. However, when the regression results are divided into upturns and recessions in Table 13, it becomes apparent that the coefficients in the CAPM tend to be positive in upturns and negative during recessions. As betas express how exposed stocks are to market risk, this is reasonable.

Figure 3 shows the price development of a low beta portfolio with the 50% lowest betas stocks, and a high beta portfolio with the 50% highest beta stocks. The portfolios are not rebalanced. The reason for the low beta portfolio to perform better over the whole sample period, is that the market downturns in 2002 and 2008 had an extensive negative impact on the return. The average negative market factor might not be representative in general.

Some issues of the CAPM is that beta estimates vary substantially through time, and betas calculated from historical data do not seem to give optimal estimates, as discussed in section 5.3.1. The low explanatory power of the CAPM is likely to be a consequence of difficulties in estimating betas, as the average historical correlation with the market factor will not be the same as the future correlation because fundamental characteristics of the companies change. However, the exposure to systematic fluctuations in return and the beta of the assets are related, as Figure 3 shows.

Figure 3: Price development of beta portfolios



In the MATT-model regressions, amortized spread, trading volume and turnover (shares) have positive average coefficients for most years, except for some of the years with poor market performance. As higher trading volume and turnover imply higher liquidity, this was not expected. Higher amortized spread implies decrease in liquidity, so this result is in line with theory and our previous findings. Also, the market factor has negative coefficients for all the years in the MATT-model. One possible explanation of these results is that the correlation between the measures is high, which can affect the coefficient estimates. Even though we excluded factor combinations with high correlation between the measures, the correlation between turnover (shares), trading volume and the market factor is around 0.5, which still is relatively high.

6.3.2 Statistical Issues

In order to evaluate the models, we also test if the five OLS assumptions hold, and as such if the model provides accurate estimations. If the assumptions are violated, it can be necessary to adjust the model. The test results are included in the regression outputs in Appendix A3.3 and A3.4.

First it is tested if the expected values of the residuals are zero. The expected value of the residuals in the MATT-model and the CAPM is 0.00, so this assumption holds for both models. Furthermore, the Breuch-Pagan and the White's test for heteroscedacity are performed. The p-values from both these tests are higher than 0.05 for both models, and heteroscedacity is therefore not a problem.

A Jarque-Bera test is performed to check if the residuals are normally distributed. The low p-values for both models imply that this assumption is violated. To overcome this problem, transformation of the variables could have been performed. However, this is a quite usual problem in asset pricing modeling because returns are not actually normally distributed, and the violation of this assumption is not considered to have serious consequences (Brooks, 2008). We therefore do not adjust for this.

We also test if the variables are non-stochastic. As the p-values are 0.00 for all of the factors in the MATT-model and the CAPM, this assumption holds. The Durbin-Watson autocorrelation test is performed on a subset of the data which only include the stocks with complete data series in order to avoid blank datapoints. The p-values are 0.49 for both models, and we therefore conclude that autocorrelation is not an issue.

6.3.3 Prediction Performance

The ability of the models to predict future returns is finally assessed. The absolute and squared deviations from true returns are calculated for each security and each month. As the results in Table 15 show, the CAPM performs better than the MATT-model on average, both with regard to mean absolute deviations and mean squared deviations. The CAPM also performs better for most of the sub-periods. It therefore seems like the CAPM predicts returns more accurately, even though our model explains returns better statistically.

Table 15: Prediction results

	Mean absolute deviations		Mean squared deviations	
Year	**MATT**	**CAPM**	**MATT**	**CAPM**
2003	0.109	0.105	0.026	0.025
2004	0.117	0.112	0.061	0.056
2005	0.082	0.085	0.020	0.022
2006	0.069	0.069	0.010	0.010
2007	0.065	0.065	0.010	0.008
2008	0.102	0.099	0.022	0.020
2009	0.110	0.107	0.026	0.025
2010	0.086	0.085	0.015	0.015
Average	0.0926	0.0908	0.024	0.022

Table 16: Hypothesis tests of equal means of deviations

	Mean absolute deviations	Mean squared deviations
P-value	0.47	0.82

H_0: $\mu_{MATT} - \mu_{CAPM} = 0$, H_a: $\mu_{MATT} - \mu_{CAPM} \neq 0$

In order to examine if there is a difference in the means of the deviations of the MATT-model and the CAPM, we test if the means of the prediction deviations really are different, as shown in Table 16. The hypothesis of equal means cannot be rejected for the absolute deviations or the mean squared errors, as the two-sided test provides high p-values. We can therefore not be certain that the CAPM actually predicts returns better than the MATT model, as the prediction results of the CAPM are only marginally better.

The overall evaluation of the MATT-model is ambiguous, as it explains more of the cross-sectional variations in return than the CAPM according to the statistical fit of the model. However, the market factor of the CAPM is more significant than the other factors in the MATT-model, and the liquidity factors are therefore not superior to the market factor. Also, the CAPM performs marginally better for prediction. As most research evaluates models solely based on statistical fit, we conclude that our liquidity model performs better than the CAPM for explaining stock returns.

7 Discussion

In this section, we consider the relationship between our results and the theoretical and empirical findings of others, and discuss our results in broader terms.

7.1 Is Liquidity Risk Priced?

The first question we want to answer is whether or not liquidity risk is priced. Our results give strong indications of less liquid stocks having higher returns on average, which concurs with most other research. We find differences in level of liquidity of stocks to be related to systematic fluctuations in liquidity, which suggests that liquidity risk is priced. This relation can be seen from our results, as less liquid stocks perform better in upturns and worse in recessions than liquid stocks. Also, the liquidity measures have high significance when the effect is adjusted for market risk. Due to the relation between level of and systematic fluctuations in liquidity, we find a cross-sectional model to be appropriate for capturing liquidity risk.

The observation of less liquid stocks to decline more in value during recessions can be caused by flight to liquidity. When the market conditions worsen, investors find it less attractive to hold less liquid stocks. The demand of these stocks declines, which leads to decrease in both price and liquidity of these assets. We therefore find it plausible that flight to liquidity effects exist, consistent with findings of Amihud (2002), Vayanos (2004) and Acharya and Pedersen (2005). The flight to quality effect is also plausible according to our results, as the equity market as a whole becomes less liquid during recessions. This could be caused by investors selling stocks in downturns to move to safer investments, like for instance bonds.

7.2 How Should Liquidity be Measured?

We examine which of the liquidity measures that best describe returns from many different perspectives. We find several measures from each of the dimensions to be priced, and we do not find evidence of any dimension to explain returns better than the others. This is in line with theory, as none of the dimensions seems to be superior theoretically. Empirical studies conducted by others come to divergent conclusions regarding whether order- or trade-based measures are better for explaining returns. Our results support that trade-based measures perform better. Of the pure order-based measures, absolute spread does not perform well in any of the analyses, and relative spread performs worse than the trade-based measures. It should however be noted that our study does not include many order-based measures, and other studies might have examined this issue more extensively.

According to our results, turnover in shares is the measure that contributes the most to explain variation in returns. In our liquidity model, this is the most significant of the measures on average and in most of the sub-periods. It also performs well according to the other analyses.

How liquidity best can be implemented in models is also important to discuss. Should the total variation of each measure be included, or only the common variation shared by several factors? Many studies include only one measure or combine several measures by factor analysis. We find evidence both in the direction of a common factor and in the direction of several distinct factors being relevant. In time series of market averages, the correlation between the liquidity measures and return is highest for the liquidity measures that correlate the most to other measures. This is a sign of the commonality of the measures to be most closely related to returns. Also, among the measures which perform adequately well for being included in model combinations, five out of seven are highly correlated to other measures. However, the two most significant factors in the regression with the market factor and one liquidity variable are not very correlated to any of the other factors. As such, even if we find indications of commonality in liquidity to be priced, we also find aspects of the non-common variance in liquidity to be important for explaining returns.

As the commonality between the liquidity measures does not capture all the variation that seems to be priced, it will not be optimal to use only a common factor in a model. Combining the measures into one measure as Liu (2006) does, also seems to result in loss of important information. The Liu measure is highly correlated to measures in all dimensions, and one could therefore claim that the measure explains much of the return captured by other measures. However, many other measures are also highly correlated to measures in all dimensions, so this is not unique for the Liu measure. Also, the Liu measure has a relatively low significance in the regression with the market factor. Therefore, we do not find the construction of the Liu measure to be optimal for capturing several dimensions.

We find it important for a liquidity model to include several measures mainly because of better statistical performance, and not due to theoretical considerations. As the correlation between measures in different dimensions is as high as the correlation between measures within the same dimensions in our study, including several dimensions in order to capture the totality of liquidity risk does not seem like the optimal decision from a statistical point of view. It seems more relevant to include several uncorrelated measures, as they contribute to explain a broader part of the variations in return, even though they might cover the same dimension.

Other studies find trade-based measures to have low correlation with order-based measures, and emphasize the importance of including measures from both categories in order to cover the total variation. However, we find the order-based measure relative spread to have high correlation to certain trade-based measures, while it is not very correlated to the other order-based measures. Absolute spread and amortized spread have low correlation to all other measures, also to the other order-based measures. Based on this, it does not seem like lack of correlation between the two categories is a reason for including both trade- and order-based measures in a model.

Despite our findings of correlation between measures not being dependent on dimensions, our model includes factors from three different dimensions and covers both trade- and order-based measures. The reason for this is purely incidental, as we did not select measures according to dimensions. The only exception is that we avoided combining relative and amortized spread in the factor combinations, even if these have low correlation. However, any model with these two measures would most likely not have been optimal, as the relative spread is not particularly significant in the other regressions.

7.3 Does Our Liquidity Model Perform Better than the CAPM?

Our model appears to describe more of the variation in asset returns at the OSE than the CAPM. As we find our model to be an improvement of the CAPM, our results regarding empirical performance of liquidity models are in line with the findings of others. We have not tested our model against other liquidity models. Hence, we cannot comment on the performance of our model relative to other models. However, by comparing the performance of our model to the CAPM with only the Liu measure added, our model seems better than the liquidity-augmented model by Liu.

The implications of the factors included in our model can be evaluated to gain insight into which effects our model captures. As our model contains amortized spread, trading volume and turnover in shares in addition to the market factor, it covers one measure which is both order-based and trade based, and two trade-based measures. The model should therefore theoretically capture liquidity both in terms of how the available trading opportunities in the market are, and in terms of how shares are traded.

Our model also covers three out of four dimensions, namely width, depth and immediacy. Resiliency is not covered. However, as this relates to price movement per volume traded, it can potentially be captured by trading costs in combination with volume. As the resiliency measures are highly correlated to measures in these dimensions, this seems plausible.

An interpretation of amortized spread occurring in the optimal model is that trading costs are important to investors. In particular, investors seem concerned about the costs in relation to how long they should hold the stocks, as the amortized spread performs better than the relative spread. Amortized spread is related to transaction costs and clientele effects as sources of illiquidity, as amortized spread expresses the costs of trading adjusted for the length of the holding period.

As trading volume is also included, it is important for investors to be able to trade stocks in large quantities. One reason for this might be that institutional investors, who often trade large amounts, account for the majority of the volume. It is interesting that trading volume performs better than value, as trading volume in NOK intuitively seem more relevant than volume in shares, as discussed in Section 3.4.3. However, these measures are highly correlated, and the difference in performance is minor.

An economic rationale of the turnover measure being the best measure for explaining returns is the importance of being able to sell the stocks when desired by the investors. The frequency of trades seem to be of even greater importance for investors than for instance transaction costs, volumes traded and price impact. The turnover measure is related to search frictions, as it takes longer time to trade and is more demanding to find a trading opponent when stocks are traded rarely. It is also related to information asymmetry, as private information might lead to fewer trades, which reduces the turnover of shares.

By including these measures, our model relaxes several of the CAPM assumptions. The assumption of no market frictions and no transaction costs are relaxed by including amortized spread. As trading volume and amortized spread together are related to price impact, the assumption of investors being price takers is relaxed. As asymmetric information is accounted for by including turnover and trading volume, homogenous expectations of investors are not assumed. With less restrictive assumptions and more aspects influencing asset prices covered, it seems reasonable that our liquidity model performs better than the CAPM.

7.4 Methodological Choices

In light of our results, we can further discuss our choices of methodology. The methods used to determine whether or not liquidity is priced are relatively qualitative, and a disadvantage of this could be lack of absolute evaluation criteria. Consequently, we have conducted several distinct analyses to be able to base our conclusions on a solid foundation. As all the results point in direction of liquidity being priced, we consider our results to be reliable.

Another potential weakness is that we do not test all possible factor combinations because testing that many models would have been very demanding. By testing more factor combinations, we could possibly have found other combinations of measures to be optimal. Also, more absolute evaluation criteria could have made the selection process easier. However, the selection of factors for the models is based on several analyses. As there is no clear limit for how significant factors should be in order to contribute to explain returns, an overall evaluation of the models seems like an appropriate alternative. The MATT-model is not distinctly better than the other models tested, which supports that other factor combinations possibly could perform equally good as the one we end up using. However, even if we did not test the optimal model, the evaluation of the performance of our selected model compared to the CAPM still provides information about how multifactor liquidity models perform.

The empirical performance of our model is not clearly better than the performance of the CAPM, as the statistical assessment and the prediction performance give diverging results. Using other regression methods could potentially have reduced the estimation errors and provided clearer results to the statistical evaluation, but it would most likely not have improved the results substantially.

As prediction performance is not frequently used for evaluating asset pricing models, these results seem less important than the statistical performance. It seems logical that the model which explains returns the most accurate also is the optimal model for prediction. However, our results contradict this, as the CAPM performs marginally better for prediction than the MATT-model. By testing the prediction performance of several factor combinations, we might find a model that is better for this purpose. Predictive power should be considered as a criterion, because prediction is an important application of asset pricing models. However, as prediction deviations are high, the choice of sample period used for prediction might influence the results extensively. Also, unbiased prediction should provide the same result as the statistical fit to which model that is optimal.

8 Conclusion

Our results show that liquidity contributes to explain equity risk premiums. We find cross-sectional variations in liquidity to be priced at the OSE, which implies that asset pricing should focus on including liquidity. Also, it should be in the interest of companies to increase the liquidity of their shares, since this leads to lower required return by investors. To accomplish this, companies can for instance increase transparency through making financial information available, improving investor relations and providing earnings outlooks.

Our main finding is that both common and non-common variance between different liquidity measures contribute to explain returns. We therefore conclude that multiple liquidity factors are required to express the complexity of liquidity risk. An implication of this is that the commonly applied method of including only the common variance of measures leads to loss of meaningful information. Also, liquidity does not seem to be one underlying property of stocks, but rather a combination of several distinct liquidity aspects. Both trade- and order-based measures, and measures representing all the dimensions, appear to be priced. Trade-based measures appear to be more important than order-based measures, while none of the dimensions seems superior to the others. As correlation between the measures from different categories is high, we do not regard it necessary to include measures from all dimensions and both trade- and order-based measures in models.

Our liquidity model consists of the market factor, amortized spread, trading volume, and turnover in shares, where turnover is found to be the measure which best captures liquidity risk. The model seems to perform better than the CAPM statistically. Consequently, we consider it important to add liquidity factors to the CAPM in order to capture more aspects of the risk conditions, and relax assumptions to make the model fit reality better. We find our model to be less complicated to understand and apply than other liquidity models, and therefore consider our findings relevant for investment applications.

There is still much research required regarding how to best define and measure liquidity and how to incorporate liquidity in asset pricing models. It is important for further research to establish evaluation criteria for comparing the performance of various liquidity models, and a common practice regarding what liquidity measures to use and how to combine these should be sought. We have not seen other studies that test the predictive performance of models. Prediction should be a criterion for evaluating models, as a main purpose of asset pricing models is to predict future returns.

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10 Appendix

Appendix 1 – Calculation of the Liquidity Measures

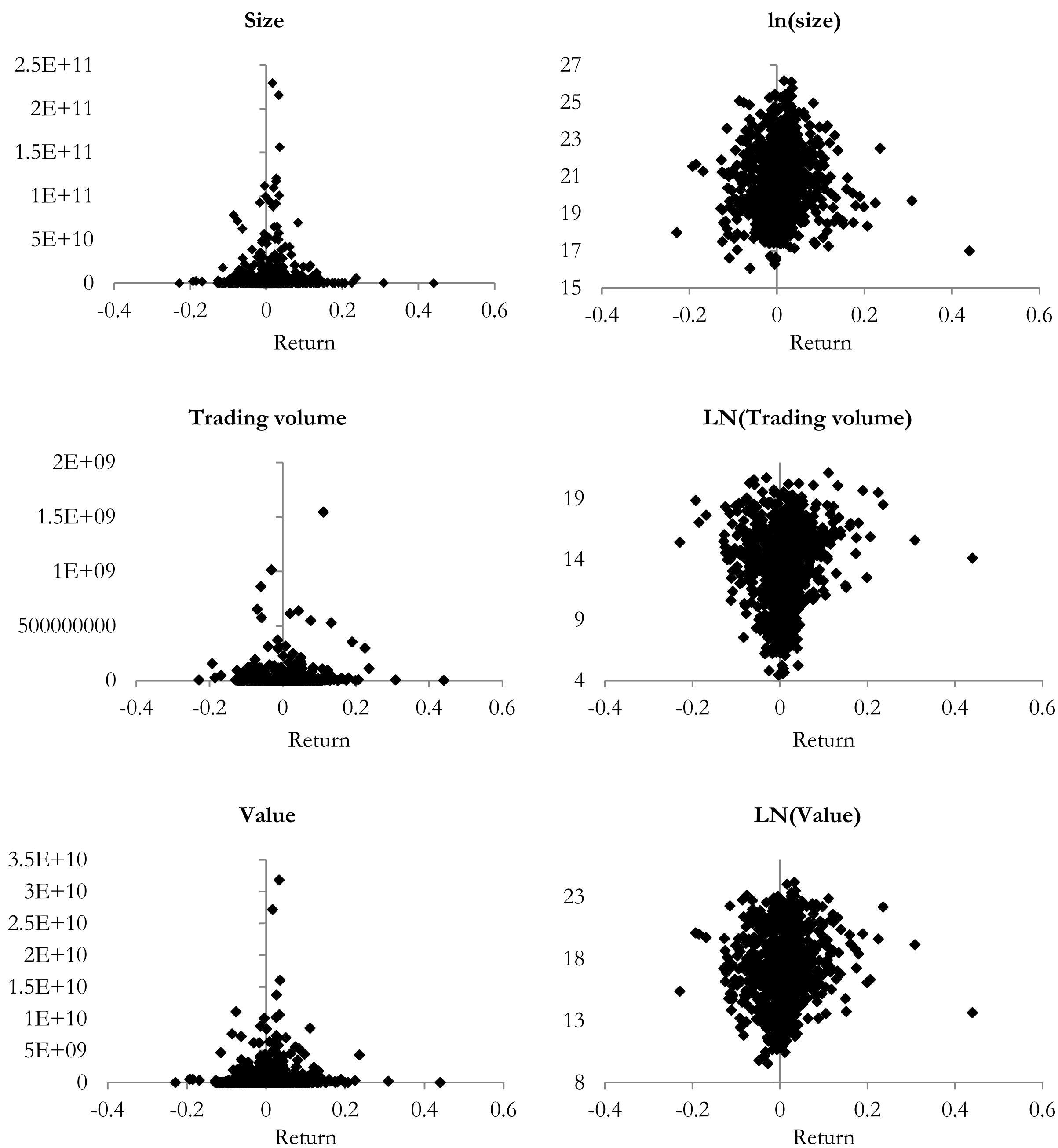
Liquidity measure	Calculation
Absolute spread	$p^a - p^b$ p^a is the ask price, p^b is the bid price
Relative spread	$\frac{p^a - p^b}{(p^a + p^b)/2}$ Absolute spread/bid-ask midpoint
Amortized spread	$\frac{p^a - p^b}{(p^a + p^b)/2} * Turnover$ Relative spread * Turnover
Trading volume	Number of shares traded in a day
Value	Number of shares traded in a day * Share price
Turnover (shares)	Trading volume/Number of shares outstanding
Turnover (NOK)	Turnover * Share price
Zero trade ratio	Number of days without trade/Total number of trading days Calculated for each month
Amihud measure	$ILR_{i,T} = 1/D_T \sum_{t=1}^{T} \frac{\lvert R_{i,T} \rvert}{VOL_{i,T}}$ D_T is the number of trading days within a time window T, $\lvert R_{i,T} \rvert$ is the absolute return on day T for security i, $VOL_{i,T}$ is the trading volume (in units of currency) on day t. Standard to multiply this estimate by 10^6 for practical purposes.
Liquidity ratio	$\frac{log(V)}{\lvert r \rvert * 100}$ V is the total daily share volume, r is the daily return
Amivest measure	$\frac{\sum P_i V_i}{\sum \lvert \% \Delta P_i \rvert}$ P_i is the daily closing price, V_i is the daily share volume and $\sum \lvert \% \Delta P_i \rvert$ is the sum of the absolute % price changes
Liu measure	$LM_x = \left[\#\, zero\ daily\ volumes\ in\ prior\ x\ months + \frac{1/(x - month\ turnover)}{Deflator} \right] \times \frac{21x}{NoTD}$ X = the number of months the measure is calculated for, x-month turnover = sum of daily turnover (shares) over the prior x months, $NoTD$ is the total number of trading days in the market over the prior x months We use x=1 month and then the deflator is 480 000 (Liu, 2006)
Market factor	Correlation between security return and the market return
Size	Number of shares outstanding * Share price

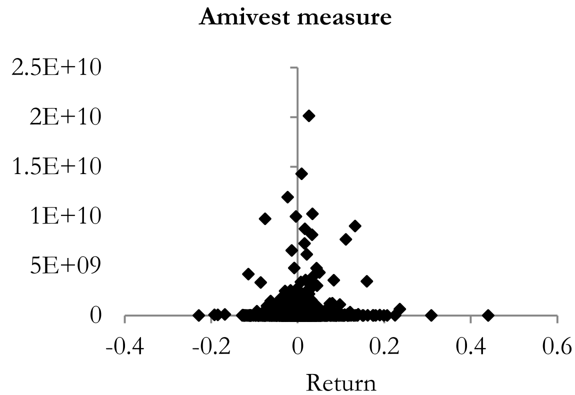
Appendix 2 – Data and Methodology

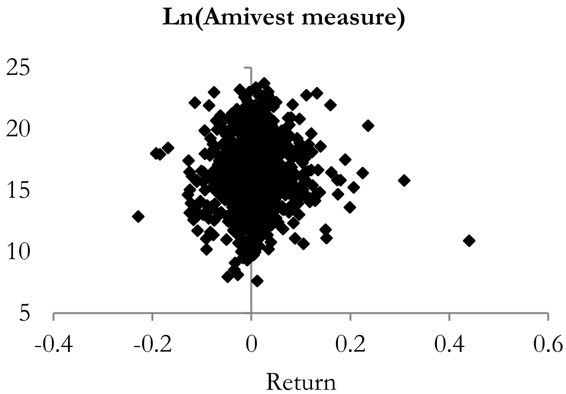
A2.1 Logarithmic Transformation of Variables

The following panel shows scatter plots with the distribution of data points with and without logarithmic transformation of the liquidity measures size, trading volume, value and Amivest measure.

Logarithmic transformation of liquidity variables




Amivest measure
2.5E+10
2E+10
1.5E+10
1E+10
5E+09
0
-0.4
-0.2
0
0.2
0.4
0.6
Return


Ln(Amivest measure)
25
20
15
10
5
-0.4
-0.2
0
0.2
0.4
0.6
Return

A2.2 Regression Methodology Tests

The following tables present regression tests with different ways of beta estimation for the CAPM.

CAPM - constant beta all years

Time series regression	**Average**	**# Significant variables (t>1.66)**
Betas		
Constant	0,00	
OSEAX return	0,77	
Absolute value of t-values		
Constant	0,82	9
OSEAX return	5,58	80
R^2	0,20	
Adj R^2	0,20	
Test results		
E(u)	0,00	
BPK	0,51	
W	0,49	
JBresid	0,04	
rescorrOSEAX_return	0,00	

CAPM - constant beta for 2 years intervals

Time series regression	**Average**	**# Significant variables (t>1.66)**
Betas		
Constant	0,00	
OSEAX return	0,78	
Absolute value of t-values		
Constant	0,85	9,4
OSEAX return	2,43	54,4
R^2	0,22	
Adj R^2	0,18	
Test results		
E(u)	0,00	
BPK	0,42	
W	0,49	
JBresid	0,26	
rescorrOSEAX_return	0,00	

Cross sectional regression	**Average**	**# Significant variables (t>1.66)**
Betas		
Constant	0,00	
Relative spread	1,00	
Absolute value of t-values		
Constant	0,89	19
Relative spread	1,21	36
R^2	0,09	
Adj R^2	0,08	
Test results		
E(u)	0,00	
BPK	-	
W	-	
JBresid	0,03	
rescorrOSEAX_return	0,00	

Cross sectional regression	**Average**	**# Significant variables (t>1.66)**
Betas		
Constant	0,00	
OSEAX return	1,00	
Absolute value of t-values		
Constant	1,31	37
OSEAX return	2,26	73
R^2	0,18	
Adj R^2	0,17	
Test results		
E(u)	0,00	
BPK	-	
W	-	
JBresid	0,05	
rescorrOSEAX_return	0,00	

CAPM - time varying beta

Time series regression	Average	# Significant variables (t>1.66)
Betas		
Constant	0,00	
OSEAX return	0,76	
Absolute value of t-values		
Constant	0,86	0
OSEAX return	2,54	66
R^2	0,23	
Adj R^2	0,22	
Test results		
E(u)	0,00	
BPK	0,45	
W	0,49	
JBresid	0,26	

Cross sectional regression	Average	# Significant variables (t>1.66)
Betas		
Constant	0,01	
OSEAX return	0,00	
Absolute value of t-values		
Constant	1,30	33
OSEAX return	1,83	47
R^2	0,05	
Adj R^2	0,04	
Test results		
E(u)	0,34	
BPK	0,36	
W	0,03	
JBresid	0,00	
rescorrMarketfactor	0,00	

The following tables present different ways of estimating betas for a test model that includes the market factor and the liquidity factors of relative spread, value, the Amivest measure and size. The liquidity measures are estimated in both time series and cross-sectional regression and only in cross-sectional regression. In addition, the betas are calculated in different ways.

Test model - constant beta all years

Time series regression	Average	# Significant variables (t>1.66)
Betas		
Constant	-0,37	
Relative spread	0,09	
Value	0,03	
Amivest measure	-0,02	
Size	0,01	
OSEAX return	0,79	
Absolute value of t-values		
Constant	1,02	18
Relative spread	0,98	14
Value	2,15	49
Amivest measure	2,05	54
Size	0,96	20
OSEAX return	5,44	79
R^2	0,31	
Adj R^2	0,28	
Test results		
E(u)	0,00	
BPK	0,01	
W	0,00	
JBresid	0,07	
rescorrRelative_spread	0,00	
rescorrValue	0,00	
rescorrAmivest_measure	0,00	
rescorrSize	0,00	
rescorrOSEAX_return	0,00	

Test model - constant beta for 2 years intervals

Time series regression	Average	# Significant variables (t>1.66)
Betas		
Constant	-3,78	
Relative spread	3,22	
Value	0,03	
Amivest measure	-0,02	
Size	0,18	
OSEAX return	0,75	
Absolute value of t-values		
Constant	1,26	26,2
Relative spread	0,91	12,2
Value	1,27	26,4
Amivest measure	1,17	21,2
Size	1,24	25
OSEAX return	2,17	47,6
R^2	0,47	
Adj R^2	0,32	
Test results		
E(u)	0,00	
BPK	0,25	
W	0,35	
JBresid	0,33	
rescorrRelative_spread	0,00	
rescorrValue	0,00	
rescorrAmivest_measure	0,00	
rescorrSize	0,00	
rescorrOSEAX_return	0,00	

Cross sectional regression	Average	# Significant variables (t>1.66)
Betas		
Constant	0,00	
Relative spread	0,02	
Value	0,00	
Amivest measure	0,37	
Size	-0,05	
OSEAX return	0,41	
Absolute value of t-values		
Constant	1,11	26
Relative spread	1,09	29

Cross sectional regression	Average	# Significant variables (t>1.66)
Betas		
Constant	0,00	
Relative spread	0,00	
Value	0,00	
Amivest measure	0,04	
Size	-0,43	
OSEAX return	-0,03	
Absolute value of t-values		
Constant	1,31	44
Relative spread	0,87	17

Value	1,10	21
Amivest measure	1,14	31
Size	1,16	31
OSEAX return	1,13	30
R^2	0,18	
Adj R^2	0,13	
Test results		
E(u)	0,00	
BPK	0,25	
W	0,29	
JBresid	0,05	
rescorrRelative_spread	0,00	
rescorrValue	0,00	
rescorrAmivest_measure	0,00	
rescorrSize	0,00	
rescorrOSEAX_return	0,00	

Value	1,06	25
Amivest measure	0,97	22
Size	1,16	28
OSEAX return	0,88	18
R^2	0,22	
Adj R^2	0,17	
Test results		
E(u)	0,00	
BPK	0,22	
W	0,24	
JBresid	0,04	
rescorrRelative_spread	0,00	
rescorrValue	0,00	
rescorrAmivest_measure	0,00	
rescorrSize	0,00	
rescorrOSEAX_return		

Test model - only cross sectional regression

Cross sectional regression	Average	# Significant variables (t>1.66)
Betas		
Constant	-0,08	
Relative spread	0,03	
Value	0,02	
Amivest measure	-0,02	
Size	0,00	
Absolute value of t-values		
Constant	1,05	19
Relative spread	0,93	15
Value	4,02	68
Amivest measure	3,72	70
Size	1,30	30
R^2	0,24	
Adj R^2	0,20	
Test results		
E(u)	0,00	
BPK	0,03	
W	0,02	
JBresid	0,03	
rescorrRelative_spread	0,00	
rescorrValue	0,00	
rescorrAmivest_measure	0,00	
rescorrSize	0,00	

Test model - beta in time series, other factors cross-sectional

Time series regression	Average	# Significant variables (t>1.66)
Betas		
Constant	0,00	
OSEAX return	0,77	
Absolute value of t-values		
Constant	0,82	9
OSEAX return	5,58	80
R^2	0,20	
Adj R^2	0,20	
Test results		
E(u)	0,00	
BPK	0,51	
W	0,49	
JBresid	0,04	

Cross sectional regression	Average	# Significant variables (t>1.66)
Betas		
Constant	-0,08	
Relative spread	0,03	
Value	0,02	
Amivest measure	-0,02	
Size	0,00	
OSEAX return	-0,03	
Absolute value of t-values		
Constant	1,05	32
Relative spread	0,88	18
Value	3,30	95
Amivest measure	3,53	101
Size	1,24	41
OSEAX return	1,20	37
R^2	0,26	
Adj R^2	0,21	
Test results		
E(u)	0,00	
BPK	0,04	
W	0,02	
JBresid	0,03	
rescorrRelative_spread	0,00	
rescorrValue	0,00	
rescorrAmivest_measure	0,00	
rescorrSize	0,00	
rescorrOSEAX_return	0,00	

Appendix 3 – Results

A3.1 Fama-MacBeth Regressions with Each Liquidity Measure and the Market Factor

Step 1: Time series regression - For estimation of market betas for all the cross-sectional regressions

Time series regression	Average	# Significant variables (t>1.66)
Betas		
Constant	0.00	
OSEAX return	0.76	
Absolute value of t-values		
Constant	0.86	0
OSEAX return	2.54	66
R^2	0.23	
Adj R^2	0.22	
Test results		
E(u)	0.00	
BPK	0.45	
W	0.49	
JBresid	0.26	

Step 2 – Cross-sectional regressions

Absolute spread

Cross sectional regression	Average	# Sign variables (t>1.66)
Betas		
Constant	0.01	
Absolute spread	0.00	
Marketbeta	0.00	
Absolute value of t-values		
Constant	1.89	54
Absolute spread	0.65	6
Marketbeta	1.75	46
R^2	0.06	
Adj R^2	0.04	
Test results		
E(u)	0.00	
BPK	0.44	
W	0.48	
JBresid	0.03	
rescorrFactor	0.00	
rescorrMarketfactor	0.00	

Relative spread

Cross sectional regression	Average	# Sign variables (t>1.66)
Betas		
Constant	0.02	
Relative spread	-0.10	
Marketbeta	0.00	
Absolute value of t-values		
Constant	1.77	52
Relative spread	1.11	25
Marketbeta	1.58	41
R^2	0.07	
Adj R^2	0.05	
Test results		
E(u)	0.00	
BPK	0.36	
W	0.42	
JBresid	0.03	
rescorrFactor	0.00	
rescorrMarketfactor	0.00	

Amortized spread		
Cross sectional regression	**Average**	**# Sign variables (t>1.66)**
Betas		
Constant	0.01	
Amortized spread	74.83	
Marketbeta	0.00	
Absolute value of t-values		
Constant	1.67	47
Amortized spread	1.83	44
Marketbeta	1.86	50
R^2	0.11	
Adj R^2	0.09	
Test results		
E(u)	0.00	
BPK	0.28	
W	0.19	
JBresid	0.03	
rescorrFactor	0.00	
rescorrMarketfactor	0.00	

Trading volume		
Cross sectional regression	**Average**	**# Sign variables (t>1.66)**
Betas		
Constant	-0.02	
Trading volume	0.00	
Marketbeta	-0.01	
Absolute value of t-values		
Constant	1.15	26
Trading volume	1.65	48
Marketbeta	1.37	32
R^2	0.09	
Adj R^2	0.07	
Test results		
E(u)	0.00	
BPK	0.16	
W	0.20	
JBresid	0.04	
rescorrFactor	0.00	
rescorrMarketfactor	0.00	

Value		
Cross sectional regression	**Average**	**# Sign variables (t>1.66)**
Betas		
Constant	-0.05	
Value	0.00	
Marketbeta	-0.01	
Absolute value of t-values		
Constant	1.25	28
Value	1.49	40
Marketbeta	1.37	34
R^2	0.08	
Adj R^2	0.06	
Test results		
E(u)	0.00	
BPK	0.33	
W	0.36	
JBresid	0.03	
rescorrFactor	0.00	
rescorrMarketfactor	0.00	

Turnover (shares)		
Cross sectional regression	**Average**	**# Sign variables (t>1.66)**
Betas		
Constant	0.01	
Turnover (shares)	0.04	
Marketbeta	-0.01	
Absolute value of t-values		
Constant	1.79	52
Turnover (shares)	2.36	63
Marketbeta	1.57	38
R^2	0.13	
Adj R^2	0.11	
Test results		
E(u)	0.00	
BPK	0.20	
W	0.20	
JBresid	0.03	
rescorrFactor	0.00	
rescorrMarketfactor	0.00	

Turnover (NOK)		
Cross sectional regression	**Average**	**# Sign variables (t>1.66)**
Betas		
Constant	0.01	
Turnover (NOK)	0.00	
Marketbeta	0.00	
Absolute value of t-values		
Constant	1.79	51
Turnover (NOK)	1.09	21
Marketbeta	1.74	43
R^2	0.07	
Adj R^2	0.05	
Test results		
E(u)	0.00	
BPK	0.43	
W	0.45	
JBresid	0.03	
rescorrFactor	0.00	
rescorrMarketfactor	0.00	

Zero trade ratio		
Cross sectional regression	**Average**	**# Sign variables (t>1.66)**
Betas		
Constant	0.02	
Zero trade ratio	-0.02	
Marketbeta	-0.01	
Absolute value of t-values		
Constant	2.08	57
Zero trade ratio	1.27	33
Marketbeta	1.42	39
R^2	0.08	
Adj R^2	0.06	
Test results		
E(u)	0.00	
BPK	0.27	
W	0.37	
JBresid	0.03	
rescorrFactor	0.00	
rescorrMarketfactor	0.00	

Amihud measure		
Cross sectional regression	**Average**	**# Sign variables (t>1.66)**
Betas		
Constant	0.01	
Amihud measure	0.01	
Marketbeta	0.00	
Absolute value of t-values		
Constant	1.70	49
Amihud measure	1.26	23
Marketbeta	1.73	40
R^2	0.06	
Adj R^2	0.03	
Test results		
E(u)	0.00	
BPK	0.35	
W	0.42	
JBresid	0.04	
rescorrFactor	0.00	
rescorrMarketfactor	0.00	

Liquidity ratio		
Cross sectional regression	**Average**	**# Sign variables (t>1.66)**
Betas		
Constant	0.01	
Liquidity ratio	0.00	
Marketbeta	0.00	
Absolute value of t-values		
Constant	1.59	41
Liquidity ratio	1.00	20
Marketbeta	1.61	41
R^2	0.07	
Adj R^2	0.05	
Test results		
E(u)	0.00	
BPK	0.35	
W	0.38	
JBresid	0.03	
rescorrFactor	0.00	
rescorrMarketfactor	0.00	

Amivest measure		
Cross sectional regression	**Average**	**# Sign variables (t>1.66)**
Betas		
Constant	0.02	
Amivest measure	0.00	
Marketbeta	0.00	
Absolute value of t-values		
Constant	1.39	37
Amivest measure	1.16	26
Marketbeta	1.76	41
R^2	0.04	
Adj R^2	0.02	
Test results		
E(u)	0.00	
BPK	0.27	
W	0.31	
JBresid	0.04	
rescorrFactor	0.00	
rescorrMarketfactor	0.00	

Liu measure		
Cross sectional regression	**Average**	**# Sign variables (t>1.66)**
Betas		
Constant	0.02	
Liu measure	0.00	
Marketbeta	-0.01	
Absolute value of t-values		
Constant	2.05	56
Liu measure	1.26	31
Marketbeta	1.44	39
R^2	0.05	
Adj R^2	0.02	
Test results		
E(u)	0.00	
BPK	0.29	
W	0.39	
JBresid	0.03	
rescorrFactor	0.00	
rescorrMarketfactor	0.00	

Size		
Cross sectional regression	**Average**	**# Sign variables (t>1.66)**
Betas		
Constant	-0.06	
Size	0.00	
Marketbeta	-0.01	
Absolute value of t-values		
Constant	1.26	31
Size	1.30	32
Marketbeta	1.65	41
R^2	0.08	
Adj R^2	0.06	
Test results		
E(u)	0.00	
BPK	0.30	
W	0.34	
JBresid	0.02	
rescorrFactor	0.00	
rescorrMarketfactor	0.00	

Correlation between the market factor and the liquidity factors:

Liquidity measure	Average cross-sectional correlation to market factor
Absolute spread	-0.22
Relative spread	-0.35
Amortized spread	0.00
Trading volume	0.58
Value	0.51
Turnover (shares)	0.34
Turnover (NOK)	0.17
Zero trade ratio	-0.51
Amihud measure	-0.24
Liquidity ratio	0.41
Amivest measure	0.43
Liu measure	-0.51
Size	0.34

A3.2 Model Selection

The following tables show the regression outputs from model selection in OxMetrics.

Cross-sectional regression - random selected months				
Month	**Factors selected**	**Coefficients**	**T-values**	**R^2**
Aug-00	Relative spread	0,95	2,38	0,54
	Value	0,05	6,03	
	Amivest measure	-0,05	-5,26	
	Turnover (shares)	0,46	3,56	
	Constant	-0,16	-1,50	
Dec-00	Constant	-0,87	-5,64	0,33
	Amortized spread	241,93	2,58	
	Trading volume	-0,03	-4,97	
	Size	0,06	6,18	
Oct-01	-			
Jun-02	Relative spread	-0,21	-2,40	0,54
	Amortized spread	13,71	2,27	
	Value	-0,03	-2,63	
	Zero trade ratio	0,28	2,40	
	Amivest measure	0,04	4,63	
	OSEBX	1,71	1,84	
	Liu measure	-0,01	-1,70	
	Turnover (shares)	-0,87	-3,10	
Apr-04	Constant	0,36	1,75	0,46
	Relative spread	-0,48	-1,79	
	Amortized spread	-489,34	-2,50	
	Trading volume	-0,02	-1,99	
	Value	0,01	0,66	
	Turnover (NOK)	0,00	-0,88	

	Zero trade ratio	-62,50	-0,43	
	Amihud measure	0,04	0,86	
	Liquidity ratio	0,00	2,72	
	Amivest measure	0,01	2,37	
	Size	-0,03	-1,99	
	Liu measure	2,98	0,43	
Jan-05	Absolute spread	0,00	-2,83	0,41
	Relative spread	1,19	2,30	
	Trading volume	-0,01	-2,25	
	Value	0,03	3,01	
	Zero trade ratio	-0,21	-3,46	
	Liquidity ratio	0,00	-1,90	
	Amivest measure	-0,02	-3,86	
	Turnover (shares)	0,23	4,07	
	Constant	0,08	0,71	
Mar-06	Relative spread	-0,83	-1,67	0,46
	Amortized spread	427,49	3,67	
	Value	0,07	5,31	
	Amivest measure	-0,07	-5,51	
	Constant	-0,04	-0,36	
Mar-07	Turnover (NOK)	0,00	3,11	0,35
	Zero trade ratio	-89,37	-4,52	
	OSEAX	0,69	3,37	
	Liu measure	4,25	4,52	
Dec-09	Trading volume	0,01	1,30	0,45
	Value	0,06	4,22	
	Turnover (NOK)	0,00	-4,68	
	Amivest measure	-0,06	-6,53	
	Turnover (shares)	-0,33	-5,17	
	Constant	-0,21	-2,47	
Feb-10	-			
May-10	Trading volume	-0,01	-3,58	0,58
	Value	-0,03	-4,87	
	Amivest measure	0,04	7,75	
	Constant	-0,01	-0,39	

Cross-sectional regression - annual averages				
Year	**Factors selected**	**Coefficients**	**T-values**	**R^2**
2000	Constant	-0,10	-1,86	0,21
	Turnover (shares)	0,44	3,58	
	Turnover (NOK)	0,00	-1,35	
	Trading volume	-0,01	-2,44	
	Value	0,01	2,40	
2001	Amihud measure	-0,02	-2,25	0,06
	Constant	-0,01	-1,27	
2002	Amortized spread	-24,20	-1,17	0,41
	Turnover (shares)	-0,45	-4,97	
	Turnover (NOK)	0,00	1,75	
	Amihud measure	-0,01	-1,84	
	Liu measure	0,00	1,24	
	Amivest measure	0,00	0,08	
	Constant	-0,02	-0,41	
2003	Value	0,03	4,02	0,19
	Amivest measure	-0,03	-4,38	
	Constant	0,03	0,67	
2004	Zero trade ratio	0,00	2,07	0,05
	Constant	-0,02	-0,88	
2005	Amortized spread	409,01	7,70	0,56
	Amihud measure	0,16	4,52	
	Liu measure	0,00	-2,60	
	Trading volume	-0,01	-2,58	
	Zero trade ratio	0,01	3,22	
	Constant	-0,07	-1,46	
2006	Amortized spread	152,28	2,95	0,13
	Zero trade ratio	-0,02	-1,59	
	Size	0,00	0,86	
	Constant	-0,03	-0,53	
2007	-			
2008	Constant	-0,06	-12,30	0,27
	Amortized spread	27,61	2,20	
	Turnover (shares)	-0,11	-2,47	
	Turnover (NOK)	0,00	2,23	
	Zero trade ratio	0,03	2,54	
2009	Turnover (shares)	-0,17	-3,54	0,29
	Zero trade ratio	-0,02	-0,85	
	OSEAX	-1,88	-1,53	
	Value	0,01	2,77	
2010	Amortized spread	-71,12	-2,92	0,33
	Trading volume	-0,01	-3,11	
	Value	0,01	3,61	
	Constant	-0,06	-2,56	

A3.3 Fama-MacBeth Regression of Models

Step 1: Time series regression - For estimation of market betas for all the cross-sectional regressions

Time series regression	**Average**	**# Significant variables (t>1.66)**
Betas		
Constant	0.00	
OSEAX return	0.76	
Absolute value of t-values		
Constant	0.86	0
OSEAX return	2.54	66
R^2	0.23	
Adj R^2	0.22	
Test results		
E(u)	0.00	
BPK	0.45	
W	0.49	
JBresid	0.26	

Step 2: Cross-sectional regressions

CAPM		
Cross sectional regression	**Average**	**# Significant variables (t>1.66)**
Betas		
Constant	0.01	
OSEAX return	0.00	
Absolute value of t-values		
Constant	1.30	33
OSEAX return	1.83	47
R^2	0.05	
Adj R^2	0.04	
Test results		
E(u)	0.00	
BPK	0.34	
W	0.36	
JBresid	0.03	
rescorrOSEAX_return	0.00	

Model 1		
Cross sectional regression	**Average**	**# Significant variables (t>1.66)**
Betas		
Constant	-0.05	
Relative spread	0.07	
Value	0.00	
Turnover (shares)	0.01	
OSEAX return	-0.01	
Absolute value of t-values		
Constant	0.94	14
Relative spread	0.99	22
Value	1.10	23
Turnover (shares)	2.15	50
OSEAX return	1.36	33
R^2	0.20	
Adj R^2	0.16	
Test results		
E(u)	0.00	
BPK	0.22	
W	0.15	
JBresid	0.04	
rescorrFactor1	0.00	
rescorrFactor2	0.00	
rescorrFactor3	0.00	
rescorrMarketfactor	0.00	

Model 2		
Cross sectional regression	**Average**	**# Significant variables (t>1.66)**
Betas		
Constant	0.07	
Relative spread	-0.21	
Turnover (shares)	0.05	
Amivest measure	0.00	
OSEAX return	-0.01	
Absolute value of t-values		
Constant	1.75	51
Relative spread	1.31	37
Turnover (shares)	2.52	59
Amivest measure	1.53	45
OSEAX return	1.50	36
R^2	0.22	
Adj R^2	0.19	
Test results		
E(u)	0.00	
BPK	0.11	
W	0.09	
JBresid	0.05	
rescorrFactor1	0.00	
rescorrFactor2	0.00	
rescorrFactor3	0.00	
rescorrMarketfactor	0.00	

Model 3		
Cross sectional regression	**Average**	**# Significant variables (t>1.66)**
Betas		
Constant	-0.08	
Relative spread	0.01	
Turnover (shares)	0.04	
Size	0.00	
OSEAX return	-0.01	
Absolute value of t-values		
Constant	1.07	28
Relative spread	0.88	15
Turnover (shares)	2.31	59
Size	1.13	27
OSEAX return	1.44	33
R^2	0.17	
Adj R^2	0.13	
Test results		
E(u)	0.00	
BPK	0.22	
W	0.16	
JBresid	0.04	
rescorrFactor1	0.00	
rescorrFactor2	0.00	
rescorrFactor3	0.00	
rescorrMarketfactor	0.00	

Model 4		
Cross sectional regression	**Average**	**# Significant variables (t>1.66)**
Betas		
Constant	0.00	
Amortized spread	28.82	
Trading volume	0.00	
Turnover (shares)	0.04	
OSEAX return	-0.01	
Absolute value of t-values		
Constant	0.93	19
Amortized spread	1.28	30
Trading volume	1.22	31
Turnover (shares)	1.61	39
OSEAX return	1.32	31
R^2	0.21	
Adj R^2	0.17	
Test results		
E(u)	0.00	
BPK	0.19	
W	0.14	
JBresid	0.04	
rescorrFactor1	0.00	

Model 5		
Cross sectional regression	**Average**	**# Significant variables (t>1.66)**
Betas		
Constant	0.01	
Amortized spread	27.49	
Turnover shares	0.05	
Market factor	-0.01	
Absolute value of t-values		
Constant	1.71	48
Amortized spread	1.32	33
Turnover shares	1.93	48
Market factor	1.53	38
R^2	0.16	
Adj R^2	0.13	
Test results		
E(u)	0.00	
BPK	0.24	
W	0.18	
JBresid	0.03	
rescorrFactor1	0.00	
rescorrFactor2	0.00	
rescorrMarketfactor	0.00	

rescorrFactor2	0.00
rescorrFactor3	0.00
rescorrMarketfactor	0.00

Model 6		
Cross sectional regression	**Average**	**# Significant variables (t>1.66)**
Betas		
Constant	-0.02	
Amortized spread	65.07	
Trading volume	0.00	
Market factor	-0.01	
Absolute value of t-values		
Constant	1.10	21
Amortized spread	1.70	43
Trading volume	1.46	41
Market factor	1.31	30
R^2	0.15	
Adj R^2	0.11	
Test results		
E(u)	0.00	
BPK	0.18	
W	0.15	
JBresid	0.04	
rescorrFactor1	0.00	
rescorrFactor2	0.00	
rescorrMarketfactor	0.00	

Model 7		
Cross sectional regression	**Average**	**# Significant variables (t>1.66)**
Betas		
Constant	-0.10	
Amortized spread	90.77	
Turnover shares	0.02	
Size	0.01	
Market factor	-0.01	
Absolute value of t-values		
Constant	1.20	30
Amortized spread	1.23	27
Turnover shares	1.86	48
Size	1.24	31
Market factor	1.48	34
R^2	0.18	
Adj R^2	0.15	
Test results		
E(u)	0.00	
BPK	0.22	
W	0.18	
JBresid	0.03	
rescorrFactor1	0.00	
rescorrFactor2	0.00	
rescorrFactor3	0.00	
rescorrMarketfactor	0.00	

Model 8		
Cross sectional regression	**Average**	**# Significant variables (t>1.66)**
Betas		
Constant	-0.08	
Turnover shares	0.04	
Size	0.00	
Market factor	-0.01	
Absolute value of t-values		
Constant	1.24	30
Turnover shares	2.34	61
Size	1.29	30
Market factor	1.50	35
R^2	0.16	
Adj R^2	0.13	
Test results		
E(u)	0.00	
BPK	0.20	
W	0.19	
JBresid	0.03	

Model 9		
Cross sectional regression	**Average**	**# Significant variables (t>1.66)**
Betas		
Constant	0.00	
Turnover shares	0.04	
Trading volume	0.00	
Market factor	-0.01	
Absolute value of t-values		
Constant	0.93	16
Turnover shares	1.96	47
Trading volume	1.22	32
Market factor	1.34	33
R^2	0.19	
Adj R^2	0.16	
Test results		
E(u)	0.00	
BPK	0.16	
W	0.19	
JBresid	0.04	

rescorrFactor1	0.00
rescorrFactor2	0.00
rescorrMarketfactor	0.00

rescorrFactor1	0.00
rescorrFactor2	0.00
rescorrMarketfactor	0.00

A3.4 Fama-MacBeth Regressions of Time Intervals

Model 4 - 2002	
Time series regression	**Average**
Betas	
Constant	0.00
OSEAX return	0.87
Absolute value of t-values	
Constant	0.77
OSEAX return	2.26
R^2	0.20
Adj R^2	0.19
Test results	
E(u)	0.00
BPK	0.45
W	0.49
JBresid	0.28

Model 4 - 2003	
Time series regression	**Average**
Betas	
Constant	0.01
OSEAX return	0.88
Absolute value of t-values	
Constant	0.87
OSEAX return	2.68
R^2	0.25
Adj R^2	0.24
Test results	
E(u)	0.00
BPK	0.46
W	0.51
JBresid	0.25

Cross sectional regression	**Average**	**# Significant variables (t>1.66)**
Betas		
Constant	0.01	
Amortized spread	-25.34	
Trading volume	0.00	
Turnover (shares)	-0.16	
OSEAX return	-0.01	
Absolute value of t-values		
Constant	0.79	1
Amortized spread	1.96	6
Trading volume	1.06	3
Turnover (shares)	1.46	3
OSEAX return	2.39	8
R^2	-	
Adj R^2	-	
Test results		
E(u)	-	
BPK	0.17	
W	0.17	
JBresid	0.07	
rescorrFactor1	-	
rescorrFactor2	-	
rescorrFactor3	-	
rescorrMarketfactor	-	

Cross sectional regression	**Average**	**# Significant variables (t>1.66)**
Betas		
Constant	0.02	
Amortized spread	-33.11	
Trading volume	0.00	
Turnover (shares)	0.15	
OSEAX return	0.00	
Absolute value of t-values		
Constant	1.31	4
Amortized spread	1.69	5
Trading volume	1.73	5
Turnover (shares)	1.83	5
OSEAX return	1.87	5
R^2	0.28	
Adj R^2	0.24	
Tests		
E(u)	0.00	
BPK	0.26	
W	0.14	
JBresid	0.00	
rescorrFactor1	0.00	
rescorrFactor2	0.00	
rescorrFactor3	0.00	
rescorrMarketfactor	0.00	

Model 4 - 2004		
Time series regression	**Average**	
Betas		
Constant	0.01	
OSEAX return	0.91	
Absolute value of t-values		
Constant	0.86	
OSEAX return	2.81	
R^2	0.26	
Adj R^2	0.26	
Test results		
E(u)	0.00	
BPK	0.44	
W	0.50	
JBresid	0.26	

Model 4 - 2005		
Time series regression	**Average**	
Betas		
Constant	0.01	
OSEAX return	0.84	
Absolute value of t-values		
Constant	1.00	
OSEAX return	2.30	
R^2	0.20	
Adj R^2	0.20	
Test results		
E(u)	0.00	
BPK	0.41	
W	0.44	
JBresid	0.26	

Cross sectional regression	**Average**	**# Significant variables (t>1.66)**
Betas		
Constant	0.02	
Amortized spread	-59.86	
Trading volume	0.00	
Turnover (shares)	0.13	
OSEAX return	-0.01	
Absolute value of t-values		
Constant	1.05	4
Amortized spread	1.18	4
Trading volume	1.41	6
Turnover (shares)	2.41	8
OSEAX return	1.27	3
R^2	0.21	
Adj R^2	0.17	
Test results		
E(u)	0.00	
BPK	0.04	
W	0.05	
JBresid	0.00	
rescorrFactor1	0.00	
rescorrFactor2	0.00	
rescorrFactor3	0.00	
rescorrMarketfactor	0.00	

Cross sectional regression	**Average**	**# Significant variables (t>1.66)**
Betas		
Constant	0.01	
Amortized spread	135.48	
Trading volume	0.00	
Turnover (shares)	0.14	
OSEAX return	-0.01	
Absolute value of t-values		
Constant	1.37	4
Amortized spread	1.58	4
Trading volume	1.59	5
Turnover (shares)	2.97	7
OSEAX return	1.01	1
R^2	0.35	
Adj R^2	0.32	
Tests		
E(u)	0.00	
BPK	0.12	
W	0.03	
JBresid	0.00	
rescorrFactor1	0.00	
rescorrFactor2	0.00	
rescorrFactor3	0.00	
rescorrMarketfactor	0.00	

Model 4 - 2006	
Time series regression	**Average**
Betas	
Constant	0.01
OSEAX return	0.74
Absolute value of t-values	
Constant	0.88
OSEAX return	2.21
R^2	0.19
Adj R^2	0.18
Test results	
E(u)	0.00
BPK	0.42
W	0.47
JBresid	0.25

Model 4 - 2007	
Time series regression	**Average**
Betas	
Constant	0.01
OSEAX return	0.64
Absolute value of t-values	
Constant	0.80
OSEAX return	1.96
R^2	0.17
Adj R^2	0.16
Test results	
E(u)	0.00
BPK	0.45
W	0.50
JBresid	0.23

Cross sectional regression	Average	# Significant variables (t>1.66)
Betas		
Constant	-0.01	
Amortized spread	169.23	
Trading volume	0.00	
Turnover (shares)	0.05	
OSEAX return	-0.01	
Absolute value of t-values		
Constant	0.54	0
Amortized spread	1.23	4
Trading volume	0.59	1
Turnover (shares)	1.13	3
OSEAX return	1.56	4
R^2	0.21	
Adj R^2	0.17	
Tests		
E(u)	0.00	
BPK	0.21	
W	0.13	
JBresid	0.08	
rescorrFactor1	0.00	
rescorrFactor2	0.00	
rescorrFactor3	0.00	
rescorrMarketfactor	0.00	

Cross sectional regression	Average	# Significant variables (t>1.66)
Betas		
Constant	-0.01	
Amortized spread	197.83	
Trading volume	0.00	
Turnover (shares)	0.08	
OSEAX return	0.00	
Absolute value of t-values		
Constant	0.85	1
Amortized spread	0.95	2
Trading volume	1.15	2
Turnover (shares)	1.26	3
OSEAX return	0.95	2
R^2	0.14	
Adj R^2	0.10	
Tests		
E(u)	0.00	
BPK	0.21	
W	0.09	
JBresid	0.01	
rescorrFactor1	0.00	
rescorrFactor2	0.00	
rescorrFactor3	0.00	
rescorrMarketfactor	0.00	

Model 4 - 2008	
Time series regression	**Average**
Betas	
Constant	0.00
OSEAX return	0.55
Absolute value of t-values	
Constant	0.97
OSEAX return	2.61
R^2	0.24
Adj R^2	0.23
Test results	
E(u)	0.00
BPK	0.48
W	0.51
JBresid	0.26

Model 4 - 2009	
Time series regression	**Average**
Betas	
Constant	-0.01
OSEAX return	0.66
Absolute value of t-values	
Constant	0.85
OSEAX return	3.11
R^2	0.29
Adj R^2	0.29
Test results	
E(u)	0.00
BPK	0.45
W	0.49
JBresid	0.26

Cross sectional regression	Average	# Significant variables (t>1.66)
Betas		
Constant	-0.02	
Amortized spread	-68.50	
Trading volume	0.00	
Turnover (shares)	0.02	
OSEAX return	-0.02	
Absolute value of t-values		
Constant	0.73	1
Amortized spread	0.69	0
Trading volume	1.18	4
Turnover (shares)	0.95	3
OSEAX return	1.05	4
R^2	0.10	
Adj R^2	0.06	
Tests		
E(u)	0.00	
BPK	0.19	
W	0.26	
JBresid	0.12	
rescorrFactor1	0.00	
rescorrFactor2	0.00	
rescorrFactor3	0.00	
rescorrMarketfactor	0.00	

Cross sectional regression	Average	# Significant variables (t>1.66)
Betas		
Constant	-0.04	
Amortized spread	-44.36	
Trading volume	0.01	
Turnover (shares)	-0.05	
OSEAX return	-0.01	
Absolute value of t-values		
Constant	0.95	3
Amortized spread	1.03	2
Trading volume	1.25	3
Turnover (shares)	1.20	3
OSEAX return	0.70	1
R^2	0.15	
Adj R^2	0.10	
Tests		
E(u)	0.00	
BPK	0.25	
W	0.29	
JBresid	0.07	
rescorrFactor1	0.00	
rescorrFactor2	0.00	
rescorrFactor3	0.00	
rescorrMarketfactor	0.00	

Model 4 - 2010	
Time series regression	**Average**
Betas	
Constant	0.00
OSEAX return	0.75
Absolute value of t-values	
Constant	0.75
OSEAX return	2.89
R^2	0.27
Adj R^2	0.26
Test results	
E(u)	0.00
BPK	0.45
W	0.50
JBresid	0.28

Cross sectional regression	**Average**	**# Significant variables (t>1.66)**
Betas		
Constant	0.01	
Amortized spread	-16.53	
Trading volume	0.00	
Turnover (shares)	0.02	
OSEAX return	0.00	
Absolute value of t-values		
Constant	0.75	1
Amortized spread	1.26	3
Trading volume	0.97	2
Turnover (shares)	1.30	4
OSEAX return	1.15	3
R^2	0.10	
Adj R^2	0.06	
Tests		
E(u)	0.00	
BPK	0.28	
W	0.12	
JBresid	0.04	
rescorrFactor1	0.00	
rescorrFactor2	0.00	
rescorrFactor3	0.00	
rescorrMarketfactor	0.00	

Model 4 - Upturn: Before October 2007

Time series regression	Average
Betas	
Constant	0.01
OSEAX return	0.82
Absolute value of t-values	
Constant	0.86
OSEAX return	2.39
R^2	0.21
Adj R^2	0.21
Test results	
E(u)	0.00
BPK	0.44
W	0.48
JBresid	0.26

Model 4 -Recession: November 2007 - January 2009

Time series regression	Average
Betas	
Constant	0.00
OSEAX return	0.56
Absolute value of t-values	
Constant	-0.29
OSEAX return	2.53
R^2	0.23
Adj R^2	0.22
Test results	
E(u)	0.00
BPK	0.48
W	0.50
JBresid	0.26

Cross sectional regression	Average
Betas	
Constant	0.01
Amortized spread	62.17
Trading volume	0.00
Turnover (shares)	0.07
OSEAX return	-0.01
Absolute value of t-values	
Constant	0.97
Amortized spread	1.44
Trading volume	1.25
Turnover (shares)	1.87
OSEAX return	1.50
R^2	0.23
Adj R^2	0.19
Test results	
E(u)	0.00
BPK	0.17
W	0.10
JBresid	0.03
rescorrFactor1	0.00
rescorrFactor2	0.00
rescorrFactor3	0.00
rescorrMarketfactor	0.00

Cross sectional regression	Average
Betas	
Constant	-0.02
Amortized spread	-18.51
Trading volume	0.00
Turnover (shares)	0.02
OSEAX return	-0.01
Absolute value of t-values	
Constant	0.81
Amortized spread	0.77
Trading volume	1.18
Turnover (shares)	0.93
OSEAX return	1.02
R^2	0.11
Adj R^2	0.06
Tests	
E(u)	0.00
BPK	0.19
W	0.26
JBresid	0.10
rescorrFactor1	0.00
rescorrFactor2	0.00
rescorrFactor3	0.00
rescorrMarketfactor	0.00

Model 4 -Upturn: From February 2009	
Time series regression	**Average**
Betas	
Constant	-0.01
OSEAX return	0.71
Absolute value of t-values	
Constant	-0.27
OSEAX return	2.99
R^2	0.28
Adj R^2	0.28
Test results	
E(u)	0.00
BPK	0.45
W	0.49
JBresid	0.27

Cross sectional regression	**Average**
Betas	
Constant	-0.02
Amortized spread	-40.35
Trading volume	0.00
Turnover (shares)	-0.02
OSEAX return	-0.01
Absolute value of t-values	
Constant	0.88
Amortized spread	1.13
Trading volume	1.15
Turnover (shares)	1.30
OSEAX return	0.96
R^2	0.12
Adj R^2	0.08
Test results	
E(u)	0.00
BPK	0.26
W	0.18
JBresid	0.06
rescorrFactor1	0.00
rescorrFactor2	0.00
rescorrFactor3	0.00
rescorrMarketfactor	0.00

CAPM - 2002	
Time series regression	**Average**
Betas	
Constant	0.00
OSEAX return	0.87
Absolute value of t-values	
Constant	0.77
OSEAX return	2.26
R^2	0.20
Adj R^2	0.19
Test results	
E(u)	0.00
BPK	0.45
W	0.49
JBresid	0.28

CAPM - 2003	
Time series regression	**Average**
Betas	
Constant	0.01
OSEAX return	0.88
Absolute value of t-values	
Constant	0.87
OSEAX return	2.68
R^2	0.25
Adj R^2	0.24
Test results	
E(u)	0.00
BPK	0.46
W	0.51
JBresid	0.25

Cross sectional regression	**Average**	**# Significant variables (t>1.66)**
Betas		
Constant	0.00	
OSEAX return	-0.04	
Absolute value of t-values		
Constant	1.53	5
OSEAX return	3.38	9
R^2	0.13	
Adj R^2	0.12	
Test results		
E(u)	0.00	
BPK	0.15	
W	0.16	
JBresid	0.04	
rescorrMarketfactor	0.00	

Cross sectional regression	**Average**	**# Significant variables (t>1.66)**
Betas		
Constant	0.04	
OSEAX return	0.02	
Absolute value of t-values		
Constant	1.38	0
OSEAX return	1.61	0
R^2	0.04	
Adj R^2	0.03	
Test results		
E(u)	0.00	
BPK	0.37	
W	0.35	
JBresid	0.00	
rescorrMarketfactor	0.00	

CAPM - 2004	
Time series regression	**Average**
Betas	
Constant	0.01
OSEAX return	0.91
Absolute value of t-values	
Constant	0.86
OSEAX return	2.81
R^2	0.26
Adj R^2	0.26
Test results	
E(u)	0.00
BPK	0.44
W	0.50
JBresid	0.26

CAPM - 2005	
Time series regression	**Average**
Betas	
Constant	0.01
OSEAX return	0.84
Absolute value of t-values	
Constant	1.00
OSEAX return	2.30
R^2	0.20
Adj R^2	0.20
Test results	
E(u)	0.00
BPK	0.41
W	0.44
JBresid	0.26

Cross sectional regression	**Average**	**# Significant variables (t>1.66)**
Betas		
Constant	0.03	
OSEAX return	0.004	
Absolute value of t-values		
Constant	1.35	3
OSEAX return	1.74	6
R^2	0.05	
Adj R^2	0.03	
Test results		
E(u)	0.00	
BPK	0.21	
W	0.23	
JBresid	0.00	
rescorrMarketfactor	0.00	

Cross sectional regression	**Average**	**# Significant variables (t>1.66)**
Betas		
Constant	0.04	
OSEAX return	0.004	
Absolute value of t-values		
Constant	1.42	3
OSEAX return	1.37	3
R^2	0.03	
Adj R^2	0.02	
Test results		
E(u)	0.00	
BPK	0.40	
W	0.34	
JBresid	0.00	
rescorrMarketfactor	0.00	

CAPM - 2006	
Time series regression	**Average**
Betas	
Constant	0.01
OSEAX return	0.74
Absolute value of t-values	
Constant	0.88
OSEAX return	2.21
R^2	0.19
Adj R^2	0.18
Test results	
E(u)	0.00
BPK	0.42
W	0.47
JBresid	0.25

CAPM - 2007	
Time series regression	**Average**
Betas	
Constant	0.01
OSEAX return	0.64
Absolute value of t-values	
Constant	0.80
OSEAX return	1.96
R^2	0.17
Adj R^2	0.16
Test results	
E(u)	0.00
BPK	0.45
W	0.50
JBresid	0.23

Cross sectional regression	**Average**	**# Significant variables (t>1.66)**
Betas		
Constant	0.02	
OSEAX return	-0.004	
Absolute value of t-values		
Constant	1.62	6
OSEAX return	1.54	2
R^2	0.04	
Adj R^2	0.03	
Test results		
E(u)	0.00	
BPK	0.32	
W	0.37	
JBresid	0.04	
rescorrMarketfactor	0.00	

Cross sectional regression	**Average**	**# Significant variables (t>1.66)**
Betas		
Constant	0.00	
OSEAX return	0.01	
Absolute value of t-values		
Constant	1.49	4
OSEAX return	1.45	6
R^2	0.03	
Adj R^2	0.02	
Test results		
E(u)	0.00	
BPK	0.53	
W	0.55	
JBresid	0.01	
rescorrMarketfactor	0.00	

CAPM - 2008	
Time series regression	**Average**
Betas	
Constant	0.00
OSEAX return	0.55
Absolute value of t-values	
Constant	0.97
OSEAX return	2.61
R^2	0.24
Adj R^2	0.23
Test results	
E(u)	0.00
BPK	0.48
W	0.51
JBresid	0.26

CAPM - 2009	
Time series regression	**Average**
Betas	
Constant	-0.01
OSEAX return	0.66
Absolute value of t-values	
Constant	0.85
OSEAX return	3.11
R^2	0.29
Adj R^2	0.29
Test results	
E(u)	0.00
BPK	0.45
W	0.49
JBresid	0.26

Cross sectional regression	**Average**	**# Significant variables (t>1.66)**
Betas		
Constant	-0.03	
OSEAX return	-0.05	
Absolute value of t-values		
Constant	1.04	3
OSEAX return	2.41	6
R^2	0.08	
Adj R^2	0.07	
Test results		
E(u)	0.00	
BPK	0.36	
W	0.36	
JBresid	0.13	
rescorrMarketfactor	0.00	

Cross sectional regression	**Average**	**# Significant variables (t>1.66)**
Betas		
Constant	0.00	
OSEAX return	0.04	
Absolute value of t-values		
Constant	0.92	1
OSEAX return	1.08	2
R^2	0.02	
Adj R^2	0.01	
Test results		
E(u)	0.00	
BPK	0.23	
W	0.29	
JBresid	0.06	
rescorrMarketfactor	0.00	

CAPM - 2010	
Time series regression	**Average**
Betas	
Constant	0.00
OSEAX return	0.75
Absolute value of t-values	
Constant	0.75
OSEAX return	2.89
R^2	0.27
Adj R^2	0.26
Test results	
E(u)	0.00
BPK	0.45
W	0.50
JBresid	0.28

Cross sectional regression	**Average**	**# Significant variables (t>1.66)**
Betas		
Constant	0.01	
OSEAX return	0.00	
Absolute value of t-values		
Constant	0.96	3
OSEAX return	2.02	7
R^2	0.06	
Adj R^2	0.05	
Test results		
E(u)	0.00	
BPK	0.51	
W	0.59	
JBresid	0.00	
rescorrMarketfactor	0.00	

CAPM - Upturn: Before October 2007	
Time series regression	**Average**
Betas	
Constant	0.01
OSEAX return	0.82
Absolute value of t-values	
Constant	0.86
OSEAX return	2.39
R^2	0.21
Adj R^2	0.21
Test results	
E(u)	0.00
BPK	0.44
W	0.48
JBresid	0.26

CAPM -Recession: November 2007 - January 2009	
Time series regression	**Average**
Betas	
Constant	0.00
OSEAX return	0.56
Absolute value of t-values	
Constant	-0.29
OSEAX return	2.53
R^2	0.23
Adj R^2	0.22
Test results	
E(u)	0.00
BPK	0.48
W	0.50
JBresid	0.26

Cross sectional regression	Average
Betas	
Constant	0.02
OSEAX return	0.00
Absolute value of t-values	
Constant	1.44
OSEAX return	1.81
R^2	0.05
Adj R^2	0.04
Test results	
E(u)	0.00
BPK	0.33
W	0.33
JBresid	0.01
rescorrMarketfactor	0.00

Cross sectional regression	Average
Betas	
Constant	-0.03
Constant	-0.03
Absolute value of t-values	
Constant	1.18
OSEAX return	2.26
R^2	0.08
Adj R^2	0.07
Test results	
E(u)	0.00
BPK	0.35
W	0.36
JBresid	0.11
rescorrMarketfactor	0.00

CAPM -Upturn: From February 2009

Time series regression	**Average**
Betas	
Constant	-0.01
OSEAX return	0.71
Absolute value of t-values	
Constant	-0.27
OSEAX return	2.99
R^2	0.28
Adj R^2	0.28
Test results	
E(u)	0.00
BPK	0.45
W	0.49
JBresid	0.27

Cross sectional regression	**Average**
Betas	
Constant	0.00
Constant	0.02
Absolute value of t-values	
Constant	0.96
OSEAX return	1.61
R^2	0.04
Adj R^2	0.03
Test results	
E(u)	0.00
BPK	0.37
W	0.44
JBresid	0.03
rescorrMarketfactor	0.00

For the discussions regarding R^2, adjusted R^2, F-tests and autocorrelation of the models, we have run Model 4 and CAPM only for the companies without blanks. The results are given below:

Model 4 - Only companies without blanks

Time series regression	Average	# Sign var (>1.66)
Betas		
Constant	0.00	
OSEAX return	0.88	
Absolute value of t-values		
Constant	0.87	0
OSEAX return	2.90	53
R^2	0.27	
Adj R^2	0.27	
Test results		
E(u)	0.00	
BPK	0.44	
W	0.48	
DWresid	0.48	
JBresid	0.28	

Cross sectional regression	Average	# Sign var (>1.66)
Betas		
Constant	0.01	
Amortized spread	155.27	
Trading volume	0.00	
Turnover (shares)	0.00	
OSEAX return	0.00	
Absolute value of t-values		
Constant	0.97	21
Amortized spread	1.64	35
Trading volume	1.05	20
Turnover (shares)	1.47	36
OSEAX return	1.28	32
R^2	0.23	
Adj R^2	0.17	
F-test	0.13	
Test results		
E(u)	0.00	
BPK	0.22	
W	0.15	
DWresid	0.49	
JBresid	0.11	
rescorrRelative_spread	0.00	
rescorrValue	0.00	
rescorrAmivest_measure	0.00	
rescorrOSEAX_return	0.00	

CAPM - Only companies without blanks

Time series regression	Average	# Sign var (>1.66)
Betas		
Constant	0.00	
OSEAX return	0.88	
Absolute value of t-values		
Constant	0.87	0
OSEAX return	2.90	53
R^2	0.27	
Adj R^2	0.27	
Test results		
E(u)	0.00	
BPK	0.44	
W	0.48	
DWresid	0.48	
JBresid	0.28	

Cross sectional regression	Average	# Sign var (>1.66)
Betas		
Constant	0.01	
OSEAX return	0.00	
Absolute value of t-values		
Constant	1.81	52
OSEAX return	1.46	36
R^2	0.05	
Adj R^2	0.03	
F-test	0.32	
Test results		
E(u)	0.00	
BPK	0.41	
W	0.42	
DWresid	0.49	
JBresid	0.07	