NTNU – Trondheim
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Science and Technology

Investments in the LNG Value Chain

A Multistage Stochastic Optimization Model
focusing on Floating Liquefaction Units

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Preface

This thesis is written as the concluding part of a master's degree in Industrial Economics and Technology Management with specialization in Managerial Economics and Operations Research at the Norwegian University of Science and Technology (NTNU). The thesis is a continuation of our project in the fall of 2011 with a similar topic.

We would like to thank our supervisor Henrik Andersson and our co-supervisors Ruud Egging and Marte Fodstad at NTNU and SINTEF for interesting discussions and much appreciated feedback during the work with the thesis. Moreover, we thank Vibeke Stærkebye Nørstebø at SINTEF for valuable contributions in an early stage of this thesis.

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Abstract

In this thesis, we have developed a strategic optimization model of investments in infrastructure in the LNG value chain. The focus is on floating LNG production units: when they are a viable solution and what value they add to the LNG value chain. First a deterministic model is presented with focus on describing the value chain, before it is expanded to a multistage stochastic model with uncertain field sizes and gas prices. The objective is to maximize expected discounted profits through optimal investments in infrastructure. A dataset based on a set of potential fields on the Norwegian continental shelf, with shipping of LNG to three markets in the Atlantic basin, is used to solve the model.

The results illustrate when FLNG units can add value to the value chain. They are used as a supplement to onshore processing plants; for example expanding peak capacity or to react to the resolution of uncertain parameters. The floating liquefaction option is especially attractive for fields located far from shore. We also find that the main reason for using FLNG units is their lower liquefaction costs, not the ability to move between fields. The stochastic version of the model results in solutions very similar to the solutions of the deterministic model, even though it is significantly harder to solve. Dantzig-Wolfe decomposition is implemented to reduce run times, but does not converge.

Sammendrag

I denne oppgaven har vi utviklet en strategisk optimeringsmodell for investeringer i infrastruktur i LNG verdikjeden. Fokuset er på flytende LNG-produksjonsenheter: Når de er en foretrukket løsning, og hvilken verdi de tilfører LNG verdikjeden. En deterministisk modell, med fokus på å beskrive verdikjeden presenteres, før den utvides til en flertrinns stokastisk modell med usikre feltstørrelser og gasspriser. Målet er å maksimere forventet nåverdijustert overskudd gjennom optimale investeringer i infrastruktur. Et datasett basert på et sett av potensielle felt på norsk sokkel, med frakt av LNG til tre markeder i Atlanterhavsbassenget, brukes til å løse modellen.

Resultatene illustrerer når FLNG enheter kan tilføre verdi til verdikjeden. De brukes som et supplement til landbaserte foredlingsanlegg, for eksempel ved å utvide toppkapasitet eller for å reagere på utfallene av usikre parametre. Flytende løsninger er spesielt attraktive for felt som ligger langt fra land. Vi finner også at den viktigste grunnen til å bruke FLNG enheter er deres lavere kostnader, ikke evnen til å flytte mellom felt. Den stokastiske og deterministiske versjonen av modellen resulterer i lignende løsninger, selv om den stokastiske er vesentlig vanskeligere å løse. Danzig-Wolfe dekomponering er implementert for å redusere kjøretiden, men konvergerer ikke.

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List of Abbreviations

bcf	-	billion cubic feet
bcm	-	billion cubic meters
bnBtu	-	billion British thermal units
EDEV	-	Expected result of the Dynamic Expected Value solution
EEV	-	Expected result of the Expected Value solution
EVPI	-	Expected Value of Perfect Information
FLNG	-	Floating Liquefied Natural Gas
FPSO	-	Floating Production, Storage and Offloading
FSRU	-	Floating Storage & Regasification Unit
LNG	-	Liquefied Natural Gas
LPG	-	Liquefied Petroleum Gas
MMBtu	-	Million British thermal units
mtpa	-	million tonnes per annum
RP	-	Recourse Problem
VSS	-	Value of Stochastic Solution
WS	-	Wait-and-See

Chapter 1

Introduction

Global energy demand is growing, and population and economic growth are the two most important drivers. Since 1900, the population has increased by a factor of 4, the real income with a factor of 25 and the primary energy consumption with a factor of 22.5. Industrialization, urbanization and motorization have been driving the modern energy consumption trends. Natural gas is the cleanest fossil fuel and is expected to give the highest percentual contribution to the total energy growth towards 2030. (BP, 2011)

The consumption of gas is increasing, and more and more gas is transported between markets to meet the demand. The two most important ways to transport natural gas is by pipelines or to cool the natural gas to its boiling point (about -160°C) and transport the liquefied natural gas (LNG) in specially built ships. Both options can transport large amounts of natural gas. The transportation cost is dependent on how the gas is transported and the distance between the production unit and the market, shown in Figure 1-1. It can be seen that LNG has the lowest unit cost for distances over approximately 3500 kilometers, and onshore pipelines for shorter distances.

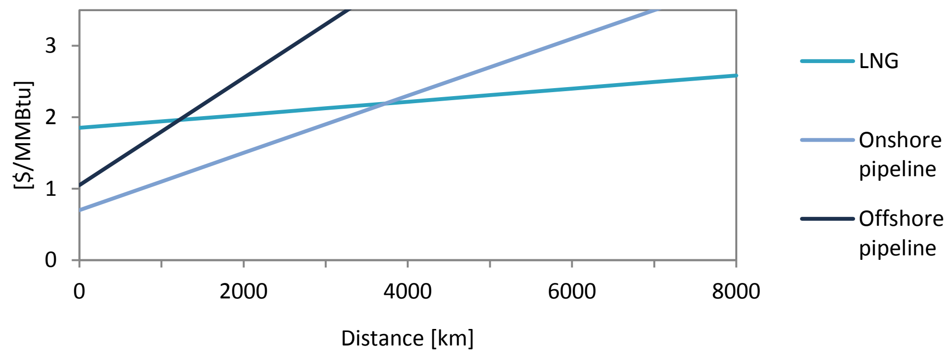

Figure 1-1: *Transportation cost of natural gas (Pettersen, 2011).*

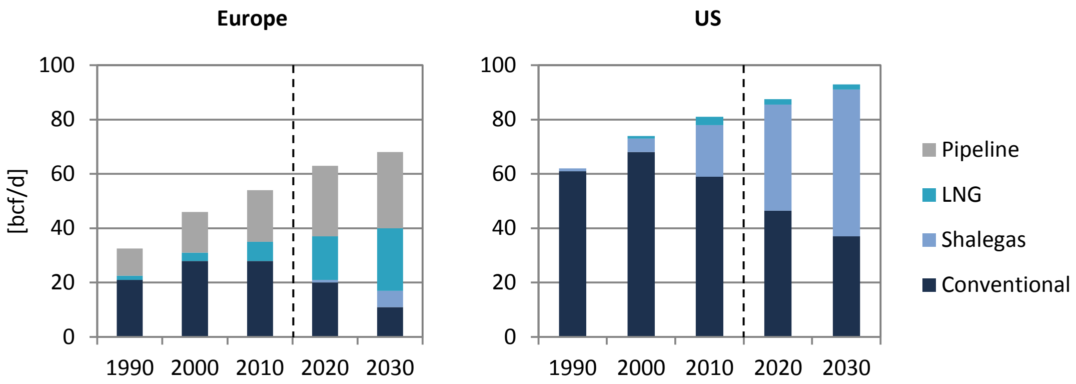

Figure 1-2: *Sources of gas supply for Europe (left) and US (right) (BP, 2011). Shalegas and conventional are domestic production, while Pipeline and LNG are net imports.*

The LNG supply in the world is expected to grow with an average rate of 4.4% per annum until 2030, more than twice as fast as the total gas production. The main demand drivers are Europe and non-OECD Asia. North America is expected to increase the domestic production to meet the demand. The historic and expected sources of gas supply between 1990 and 2030 for North America and Europe can be seen in Figure 1-2. It shows that both areas have increasing demand for natural gas. The domestic production is expected to decrease in Europe, and there will be need for a large growth in the net import, both from pipelines and LNG. The share of LNG of the net imported gas is expected to rise from 30% to 42%. (BP, 2011)

There are many factors to take into consideration when a natural gas field is developed. One factor is that there must be an appropriate location for a processing plant to be built, and it often takes many years to get the approval to use the land. A pipeline between the field and the plant must also be built, and the pipeline costs are proportional to the distance. Many fields today are located too far from shore to be profitable to develop. Such fields are called stranded fields, and are an untapped gas source. A possible way to exploit these reserves is to move the processing plant offshore, removing the need of pipelines and land areas. This is called a floating LNG (FLNG) production unit. At present no FLNG solutions exist, but Shell has decided to build one for the Prelude field in Australia.

Tools to support decisions like when and where to use onshore facilities or FLNG solutions are needed. Such a tool will contribute to the economic feasibility evaluation of investing in natural gas fields. This thesis examines when floating LNG production units are a viable solution, and what value they add to the LNG value chain. This is done through the construction of a mathematical model representing the whole value chain. The model selects investments in infrastructure for a strategic horizon, where floating production units is one of the options.

Chapters 2 and 3 give some background information on LNG production. First some general characteristics of the market are presented, before the value chain is explained from exploration to market. In Chapter 4 we give a short introduction to our problem, and a review of relevant literature is given in Chapter 5. The model is described in Chapter 6; first we discuss the main assumption, then a deterministic version is formulated before it is expanded to a stochastic version. In Chapter 7, Dantzig-Wolfe decomposition is introduced and an implementation for our stochastic problem is given. The models were solved on a few different datasets that are presented in Chapter 8 together with implementation details. A review and discussion of the results is found in Chapter 9, before some concluding remarks and possible expansions are presented in the last chapter.

Chapter 2

The LNG Market

The characteristics of the LNG market are affected by the high transportation costs of natural gas. Two ways of transporting the gas were presented in the introduction: Constructing pipelines and LNG transportation in specially built ships. Pipelines are only feasible over shorter distances, and both methods are very expensive and require large capital investments. At the same time, much of the available gas is located far away from the consumers, and the availability differs largely by region. This has led to the development of several gas markets, instead of a global market usually seen for internationally traded commodities. The Energy Charter Secretariat (2009) names four distinct markets: North America, UK, Continental Europe and Northeast Asia. These markets have different prices, regulation and reliance on contracts.

In addition to requiring large capital investments, gas projects have long lead times between project start and production. A large part of the investments must also be made in an early stage of a project. This has led to an industry historically dominated by risk sharing among the links in the value chain, hence by long term contracts. This reduces the risk for the producers, and makes it possible to get financing for the large investments. The long term contracts have usually linked the price to oil, since this historically has been an alternative source of energy.

There are, however, big regional differences. In markets with domestic supply, the government could regulate upstream prices; while in countries relying on imports, the prices were set in long term contracts negotiated between buyer and seller. The markets in Continental Europe and Japan have many long term contracts in effect today; these territories do not have a significant domestic gas supply. (Energy Charter Secretariat, 2009)

2.1 Recent Changes in the Market: Liberalization and Spot Trading

The gas markets in North America and UK have been restructured and liberalized. The gas prices are increasingly set in the market where LNG imports compete with gas from other sources; this is called gas-on-gas competition. Markets in Continental Europe and especially in Japan are on the other hand still dominated by long term contracts linked to oil prices. Pipelines between UK and Continental Europe are, however, expected to undermine oil-linked prices on the continent, and the European Union is pushing for liberalization. The growth in LNG production and regasification capacity in the world, deregulation opening up the market for more players and the price differences between markets open up opportunities for short-term trading. The LNG market has seen a growth in spot trading and short-term contracts over the last years. Figure 2-1 shows the global LNG trade, split in type of contracts. (Energy Charter Secretariat, 2009)

The changes in market have also affected the ownership structure of ships involved in the LNG trade. Historically, a ship was tied to a specific project and transported LNG exclusively between a production- and receiving facility, in an inflexible pairing. Recent arbitrage opportunities and high prices in the spot markets have caused the market to change, allowing more flexibility. More independent or specialized shipping companies are appearing. The resulting destination-flexible trade is important to increase gas competition in the world market. (Energy Charter Secretariat, 2009)

In a market with gas-on-gas competition, a buyer on oil-linked contracts is put in a difficult situation when spot gas prices fall below the oil-linked. These kinds of clauses are therefore used less and less in North America and UK. Instead, prices are linked to prices in the gas markets. Buyers can easily resell unwanted volumes,

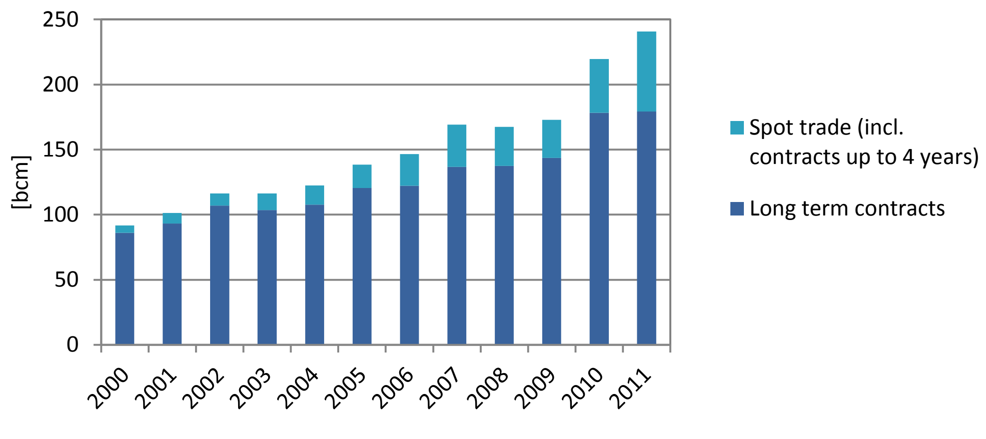

Figure 2-1: *The global LNG trade split in type of contracts (GIIGNL, 2011).*

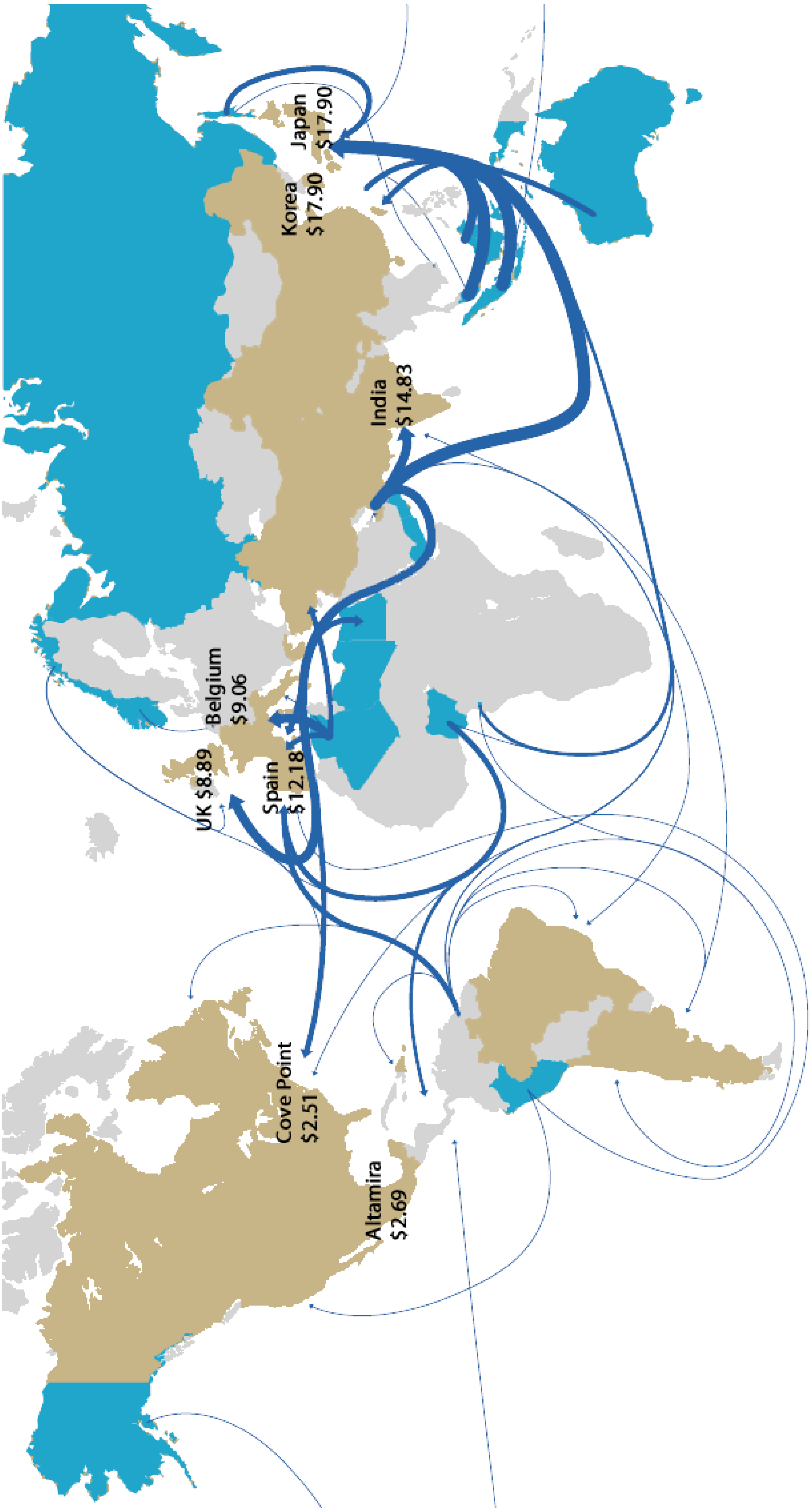

Figure 2-2: *International LNG trade in 2011 (GIIGNL, 2011) and June 2012 LNG prices (Waterborne Energy, Inc., 2012 citied in FERC, 2012b).*

and risk has migrated upstream to the producers. It is, however, also important to point out that most of the trade is still executed on long-term contracts. The high investment cost causes them to have an important role also in liberalized markets. (Energy Charter Secretariat, 2009)

The world LNG market changed dramatically towards the end of the 2000's. Earlier in the decade, the market was dominated by tight supply and increasing prices. The US was expected to become a large importer of LNG. However, demand was reduced significantly as an effect of the financial crises and growing production from unconventional gas reserves in the US. The world LNG supply also increased, as investments executed earlier in the decade became operational. Especially the large LNG exports from Qatar increased the supply and created surplus in the market. (Jensen, 2011). The last years have demonstrated that liquefaction of natural gas is an important method for connecting regional markets, and thus can contribute to international competition. It is, however, unclear if LNG has effectively influenced actual price levels. Jensen (2011) states that a unified world gas market still remains elusive. The world LNG prices still vary largely between geographical regions. Figure 2-2 shows the trade flows in 2011, and the current LNG prices in the world.

Chapter 3

The LNG Value Chain

In this chapter each stage in the value chain of LNG, from exploration to final consumer, is described in more detail. The presentation is based on offshore fields. The traditional value chain is divided into five steps that are covered in this chapter. Each of these links has different costs that contribute to the possible sales price of the gas. The value chain with an indication of associated costs for each stage is shown in Figure 3-1. When floating production units are used, production and liquefaction are merged into one link.

The calculation assumes a distance from production to market of about 11,300 km, which is roughly the distance from Nigeria to the Mexican gulf. The project would need about 12,000 trillion Btu of gas to support a 20 year contract. All of these costs varies largely from project to project, and are only given as an indication of how different parts of the value chain affect the total cost of LNG. In total the costs add up to $5.08 per MMBtu, with capital expenses (capex) of $10.5 billion. (Jensen, 2009)

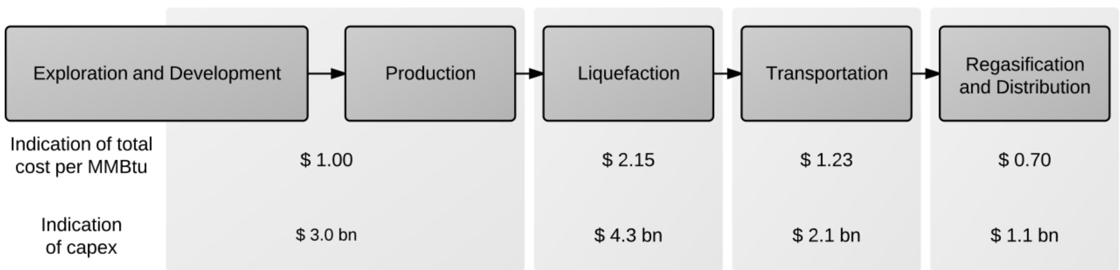

Figure 3-1: *The LNG value chain with indication of cost levels (Jensen, 2009).*

3.1 Exploration and Development

The first part of the LNG value chain is exploration and development of the gas fields. Exploration of natural gas usually begins by examining the geological characteristics of the surface in order to discover where it is likely to find hydrocarbons. Hydrocarbons are created from sediments exposed to high heat and pressure. Several geological factors affect the accumulation of hydrocarbons. Most of the production sites today are for example located on the west coasts. This is thought to be because of the sediments drifting eastwards due to the rotation of the world, and is called the *West Coast Effect*. (Pasternak, 2009)

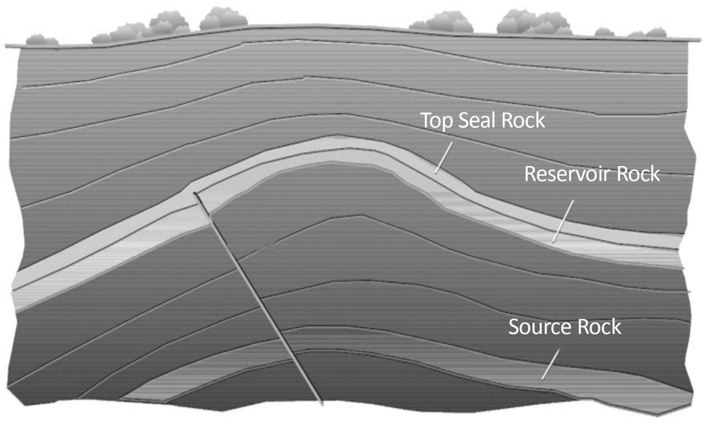

Figure 3-2: *Required layers of rocks for hydrocarbon accumulation. (Heriot Watt University, 2012)*

To have a commercial deposit of hydrocarbons, three geological conditions must be met: A *source rock* is needed, which are a type of rock containing sufficient quantities of organic matter to generate hydrocarbons. During generation of oil and gas, the volume increases creating fractions in the rock. The hydrocarbons migrate upwards to a new layer of rock, called *reservoir rock*. It is characterized of the capability to both store and transmit fluids. To ensure that the buoyant hydrocarbons are stored and concentrated into commercial amounts in the reservoir rock, a *trap* is required. It is an impermeable seal rock layer that does not allow fluids to flow through it. (Jahn et al., 1998). Figure 3-2 shows an illustration of the various layers.

To find and map the underground rock formations, a geophysicist uses a technology called seismology. The Earth's crust consists of various layers with different characteristics, and seismic waves react differently with each of the layers. After the waves are emitted, they will be reflected and measured at the surface. From these results, the geophysicist is able to make an underground map of what the layers consists of and at what depth they are located. Offshore, seismic waves are emitted from a ship and picked up by hydrophones towed behind the ship. The waves are created by a large air gun that releases a burst of compressed air under water. (NaturalGas.org, 2004-2011b)

3.1.1 Offshore Drilling

The only way to be sure that there is an accumulation of hydrocarbons is to drill an exploratory well. This is extremely costly and time consuming, and is only done on areas where data has indicated a high probability of finding hydrocarbons. During or after the drilling process, tests on the layers and rocks in the well are performed to get a true picture of the characteristics of the formations and monitor the drilling. This is called logging.

For offshore drilling, a platform has to be constructed. There are two main types of platforms; moveable and permanent. The characteristics of the well to be drilled, including the depth, are factors influencing which type of platform to use. A moveable platform is commonly used for exploration because it is cheaper to use for this purpose than a permanent platform. When a large field of hydrocarbons are found, it is often more economical to build a permanent platform for completion of the well, extraction and production. (NaturalGas.org, 2004-2011c)

3.1.2 Cost of Developing Gas Fields

The investment cost of establishing a production platform varies enormously from field to field. Many factors affect the costs. Examples are the depth where the gas is found, the size of the field, the quality of the gas, the pressure in the system and the distance from the liquefaction plant. (Lee, 2005)

3.2 Production: Extraction of the Gas

Once an economically viable gas field is found, a production platform is built. The development in the production rate over time forms a *production profile*.

3.2.1 Production Profile

The production profile for a gas field varies widely from field to field. Factors influencing the profile are among others: field size, driving force in the reservoir, composition of the gas and earth layers, depth of reservoir and technology. Production profiles have three phases: Build-up, plateau and decline. In the *build-up phase*, the production rate increases progressively as more production wells are drilled and the production rates in each well increases. In the *plateau phase*, the new wells will still have increasing production, while older wells start declining. A constant production rate is therefore maintained. The final phase is usually the longest, and the production rate will *decline* (Jahn et al., 1998). Different typical production profiles are shown in Figure 3-3. Production profiles a), b) and c) show different fields. Profile d) and e) represent the same field as c), but have a new process technology and improved recovery rate respectively.

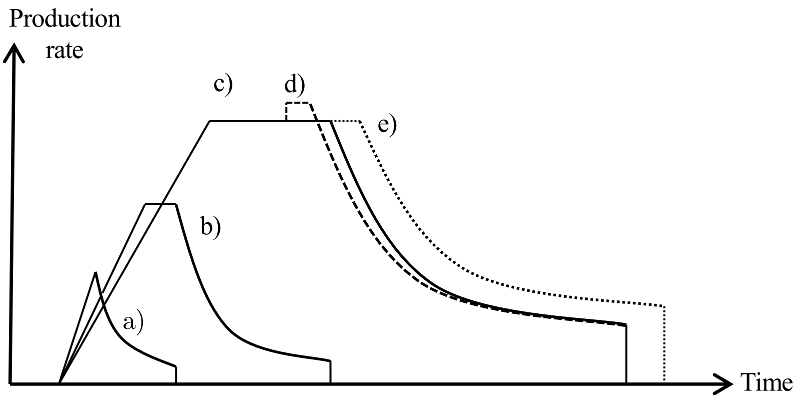

Figure 3-3: Various production profiles (Jahn et al., 1998).

3.2.2 Chemical Composition of Natural Gas

Natural gas is a mixture of various hydrocarbon gases. It primarily consists of methane, but the components and the amounts can vary largely. Table 3-1 gives an overview of a typical composition. Most natural gas contains some water too, and the amount varies between fields.

Table 3-1: Typical composition of natural gas (NaturalGas.org, 2004-2011a)

Chemical compound	Chemical formula	Portion
Methane	CH_4	70-90%
Ethane	C_2H_6	0-20%
Propane	C_3H_8	0-20%
Butane	C_4H_{10}	0-20%
Carbon dioxide	CO_2	0-8%
Oxygen	O_2	0-0.2%
Nitrogen	N_2	0-5%
Hydrogen sulfide	H_2S	0-5%
Rare gases	A, He, Ne, Xe	traces

3.2.3 Total Unit Cost of Production

The investment and operating cost of gas production varies widely from one field to another. It depends on many factors, inter alia the purity of the gas, the size of the platform and the technology used (Lee, 2005). Jensen (2009) uses a unit cost of $1.00 per MMBtu, which is an example in a wide range of possible costs.

3.3 Liquefaction and Storage

Liquefaction is the process where the natural gas is cooled down to its boiling point of approximately -160°C, at which it is transformed to a liquid state and then stored in tanks under atmospheric pressure. The natural gas contains other

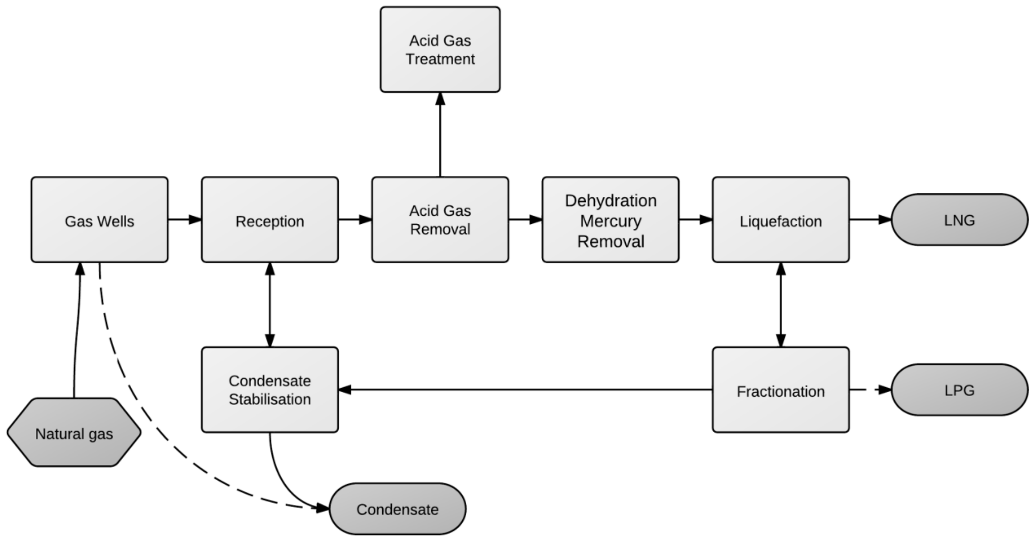

Figure 3-4: *Typical process from extraction to LNG (Shukri, 2004).*

components than methane, like condensates, acid gases, water, mercury and heavy hydrocarbons. These impurities can create plugs in the valves and pipelines, or may freeze during the liquefaction process, and must therefore be removed prior to the liquefaction. (GIIGNL, 2009). This purification can be done at the production platforms or as part of the liquefaction process. Figure 3-4 shows a typical process from the extraction until the LNG is produced. The process also produces another petroleum product, Liquefied Petroleum Gas (LPG), which mainly consists of propane and butane (Shukri, 2004).

From the gas wells, the feed gas goes through a purification process. First, any associated condensate is removed, the gas is metered and its pressure is controlled. Then the acid gases, water and mercury are removed. (Shukri, 2004). Mercury and acid gases, like hydrogen sulfide and carbon dioxide, causes corrosion, reduces heating value and may freeze and create solids in the freezing process. Water would also freeze during the liquefaction process, and is therefore removed by an adsorbent. (Pettersen, 2011). The higher the content of water, the higher the investment in dehydration equipment becomes (Lee, 2005). During liquefaction, heavier hydrocarbons are separated out. These are fractioned to recover ethane, propane and butane which are either reinjected in the liquidation process or exported as petroleum products. (Shukri, 2004)

3.3.1 Technical Aspects of the Liquefaction Process

The facility that transfers gas into a liquid is called a *train*. Each LNG plant consists of at least one train. The basic principle applied in the trains is to freeze the gas, using a substance called a refrigerant to transfer heat away from the natural gas. Key equipment in the liquefaction process are compressors to circulate the refrigerants, and heat exchangers to cool and liquefy the gas and to exchange heat between refrigerants. (Shukri, 2004)

Currently there are various processes available:

- Air Products and Chemicals Inc (APCI): C3-MR process
- ConocoPhillips: (optimized) Cascade process
- Shell: Double Mixed Refrigerant (DMR)
- Black & Veatch: PRICO Process, Single Mixed Refrigerant (single MR)
- Statoil/Linde: Mixed Fluid Cascade Process (MFCP)

The C3-MR process, developed by APCI, is the most used onshore liquefaction process, and has a high thermal efficiency. It consists of two main refrigerant cycles: The first is a precooling cycle which use propane and are able to cool the gas down to -40°C. The second cycle is a liquefaction and sub-cooling cycle which use a mixed refrigerant, containing nitrogen, methane, ethane and propane. (Shukri, 2004)

An offshore processing plant has other requirements than an onshore plant. The space is limited and the weather and waves make safety considerations different. The C3-MR and Cascade process are not suited for offshore plants, since they need large space for storage of the refrigerants. Chiu (2006) states that multiple expander process and nitrogen expander cycles are suitable. This includes the single MR, DMR and MFCP processes in the list above. (Chiu, 2006)

3.3.2 Liquefaction Costs

The liquefaction process is the most expensive part of the value chain. It constitutes 35-40% of the final cost of a unit of LNG. Between 1993 and 2003, the costs of liquefaction dropped from about $575 per ton to $200 per ton (about $4.99 to $1.74 per MMBtu) (Lee, 2005). The reason for the decline was the technology development which made it possible to increase the train sizes from the old standard of two million tonnes per annum (mtpa) to much larger trains, and the construction costs benefited from economics of scale. Because of the increased demand for LNG after 2003, the construction of liquefaction plants has increased. There are only a few constructors with the competence to develop the complex liquefaction plants, which has created a “bottleneck” and raised the prices (Jensen, 2007). Jensen (2009) has an estimated cost of $2.15 per MMBtu for liquefaction.

3.3.3 FLNG: Offshore Production, Liquefaction, Storage and Offloading

Onshore liquefaction facilities have several disadvantages. They have big environmental footprints, both on the ocean floor where the pipelines connect the field to the shore, and on land where the processing facilities are located. Getting access to land areas can be a time consuming process and the land can also be expensive. Some fields are furthermore located so far from shore that connecting them with a processing plant on land becomes too costly (stranded fields). All these factors contribute to an increase in the research on offshore liquefaction plants. (Chiu, 2006)

A floating production unit has several of the elements from the traditional value chain assembled on a ship: Production and liquefaction of the gas, storage and offloading of the LNG to the transporter ships. There are several challenges when building an FLNG. Every element of a conventional production plant needs to fit on an area roughly one quarter of the size. High towers cannot be used for stability reasons, and the whole process must stand the weather conditions (Chiu, 2006). There are, however, many advantages as well. Many analysts expect that FLNG units can be built faster and cheaper than onshore plants. They can for example be used to get an early cash flow from larger offshore gas fields, or used on currently flared or reinjected gas. The stranded fields can be served by a floating solution, and smaller fields can be developed since the FLNG unit can be moved once the field is depleted. (FLEX LNG, 2010)

Several companies have proposed different FLNG solutions. We have decided to present two main types here. The first one is Shell's solution for the Prelude Floating LNG project in Australia, which is expected to be finished in 2017. Shell made the final investment decision in May 2011. The ship will be the longest floating facility in the world, measuring 488 meters long, which is longer than four football fields. The width will be 74 meters. The weight when the FLNG is fully equipped and the storage tanks are full will be around 600,000 tonnes, roughly six times as much as the largest aircraft carrier. About 260,000 tonnes of the total weight will be steel. (Shell, 2011a)

The tanks can store up to 220,000 cubic meters of LNG, 90,000 cubic meters of LPG and 126,000 cubic meters of condensate. For the Prelude field, the FLNG unit can produce at least 5.3 mtpa (258 trillion Btu per year) of liquids. This production is divided into 3.6 mtpa of LNG, 0.4 mtpa of LPG and 1.3 mtpa of condensate. The LNG production is enough to cover Hong Kong's annual natural gas needs (Shell, 2011c). Figure 3-5 shows some illustrations of the Prelude FLNG.

The second type of FLNG units we present here is a smaller ship with a liquefaction capacity of 1 to 3 mtpa (about 49-147 trillion Btu per year). Both FLEX LNG and Höegh LNG have presented solutions of this type. An illustration of the two vessels is shown in Figure 3-6. The FLEX LNG producer has storage capacity of up to 185,000 cubic meters of LNG and up to 50,000 cubic meters of

Figure 3-5: *Illustrations of Prelude FLNG (Shell, 2011b)*

Figure 3-6: *Illustrations of the FLEX LNG producer (left) and Höegh LNG FPSO (right)*

LPG or condensate. FLEX argues that a smaller production solution has a lower cost per produced unit of gas than a larger unit, hence there are no economies of scale for FLNG solutions. Larger concepts do not fit in standard ship yards, while smaller ships can be built faster and more efficiently. They propose connecting two smaller FLNG units to a field, rather than one big unit (FLEX LNG, 2010). Höegh LNG FPSOs have sizes between 1 and 3 mtpa. They are developing FLNG units for the PNG project in Papua New Guinea, and the Tamar field outside of Israel. (Höegh LNG, 2012)

3.4 Transportation in LNG Carriers

Almost all LNG transportation is done by shipping from the liquefaction plants to regasification plants in different geographical areas. The gas is transported in specially built ships at around atmospheric pressure, but at a low enough temperature to keep the gas in a liquid form. In this state, the gas is about 600 times smaller than in a gaseous state, which is critical to make the shipping feasible. The very low temperature of LNG makes the shipping challenging. Insulation is needed to maintain the temperature for the whole trip, and all equipment must work at these low temperatures. (Statoil, 2011)

The fleet of LNG ships has grown rapidly over the last decade and consisted of 359 ships at the end of 2011. This expansion has slowed down the last few years, according to GIIGNL (2011). The order book for new LNG-ships at shipyards is, however, up from 20 at the end of 2010 to 59 in 2011. The tankers vary in capacity, but the majority is close to the average of about 145,000 cubic meters (3242 billion Btu). (GIIGNL, 2011). Figure 3-7b gives the breakdown of the number of vessels with different capacities. The energy volume of the average ships corresponds to around 1-1.4 terawatt-hours, which can cover the annual energy consumption of roughly 50,000 households in Norway (Statoil, 2011).

LNG tankers have a very long service life, longer than usual for conventional oil tankers. Even with very strict safety standards, ships operate for 20-40 years. (Tusiani and Shearer, 2007). Most of the LNG shipping fleet is, however, quite young with almost half of the ships being less than five years old (GIIGNL, 2011). Figure 3-7a shows the construction decades of the fleet at the end of year 2011.

3.4.1 Ship Design

Ships are usually classified by their tank design. The two dominating containment designs are Moss Rosenberg spherical containment system and Membrane systems. Moss Rosenberg has been the most common design historically, but there has been a shift towards more membrane systems in the recent years, and they now make up about 68% of all operating LNG ships. Figure 3-7c shows the share of each design in the current fleet. Of the ships under construction at the end of year 2011, 92% were designed with membrane systems. (GIIGNL, 2011)

The Moss design was introduced in 1971 and has been licensed to shipyards all over the world. The spherical design is robust and easy to inspect and repair. There is no problem with sloshing, and these ships can operate with partly filled tanks (Pettersen, 2011). The tanks are usually made from aluminum and are completely separated from the ship's hull. They are therefore not affected by stresses to the hull, and they are not essential for the hulls strength. A major disadvantage is that the spherical design gives a poor utilization of the hull. They also make the ships harder to maneuver and reduce visibility from the bridge. (North West Shelf Shipping, 2011)

The Membrane containment systems generally consist of two categories originally designed by two separate companies: GAZ Transport and Technigaz. These designs do not contain self-supporting tanks, but are instead built with a membrane that is added against the inner hull of the ship. The main advantage of these designs are that they utilize the hull space more efficiently, which also makes it easier to scale the design to larger ships. (North West Shelf Shipping, 2011)

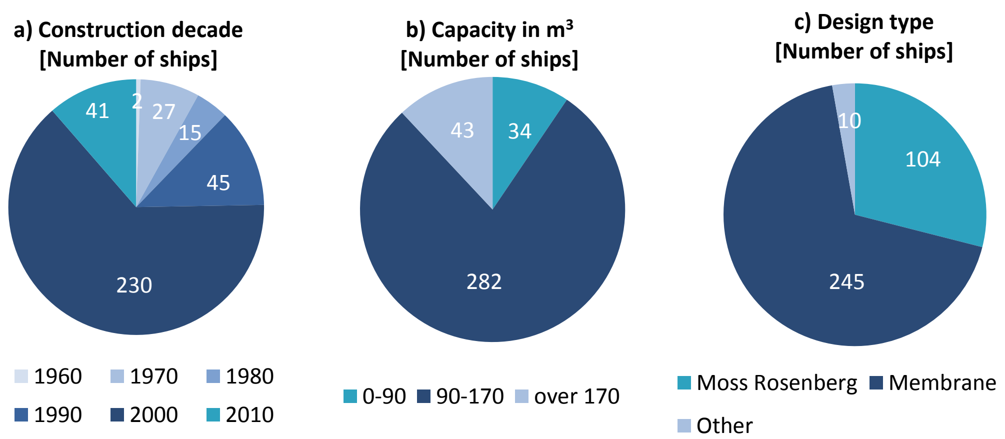

Figure 3-7: *Characteristics of the fleet at the end of 2011 (GIIGNL, 2011).*

Figure 3-8: *Illustration of Moss Rosenberg and membrane systems (North West Shelf Shipping, 2012; GlobalSecurity, 2012).*

3.4.2 Characteristics of the Operation

Cooldown is the process of cooling down the tanks before LNG is loaded, to minimize the vaporization of the LNG, and to avoid a thermal shock for the tanks. This is usually done by carefully filling some LNG into the tanks at the load port. Cooldown takes shorter time with the membrane system than with the spherical Moss design, since the spherical tanks have a larger mass. The process is very expensive and takes long time, so it is common to leave some LNG in the tanks for the return journey to keep the tanks cool; this is called heel. The tanks are usually only allowed to warm up for maintenance, since thermal cycles (cooling down and warming up) stresses the tanks. (Tusiani and Shearer, 2007)

Even though the insulation in the LNG tanks is very good, it is not perfect. The LNG is therefore boiling, and some of the liquid is returning to gas. To avoid increased pressure in the tanks, this gas has to be let out and is called *boil off*. Many tankers use this gas as fuel and get both environmentally friendly propulsion and save money on fuel. One notable exception is the new large (209,000-266,000 cubic meter, 4672-5947 billion Btu) ships for the Qatari projects that have onboard reliquefaction units that cool the gas again. This has not been feasible until recently because of the size and cost of these units. The boil off is usually at around 0.1 – 0.25% of the cargo capacity per day. The rate is higher in bad weather which causes sloshing in the tanks, and in old ships that have worse insulation. (Tusiani and Shearer, 2007)

The time needed to load or unload LNG ships largely depends on the cargo capacity. Both loading and unloading typically happens at a rate of about 10,000-12,000 cubic meters per hour, resulting in about fourteen hours to complete the process for average sized ships.

3.4.3 Cost of Shipping

LNG shipping costs consist of investment, voyage and operating costs. Voyage costs are all costs specific to a trade route, like fuel or port fees, and operating costs consist of manning, insurance, maintenance, etc. Because of the low temperature of the cargo and the strict safety requirements, LNG tankers have very high investment cost. As the LNG market expanded over the first few years of this millennium, shipyards were able to benefit from economies of scale, and prices fell (Lee, 2005). In 1990 a new 125,000 cubic meter tanker was priced at around $260 million. Because of scale and increased competition from Korean and Japanese ship builders, the price fell to around $160 million for a 138,000 cubic meters big tanker in the beginning of this millennium (Tusiani and Shearer, 2007). Since 2000 the prices and sizes of tankers have increased, and an average size tanker cost about $200 million at the end of 2010. (Petroleum Economist, 2011).

LNG vessels were historically built to serve fixed routs on long term time charters. The spot freight rates are highly volatile; in 2010 they varied between $30,000 and $70,000 per day. Long term contracts mean that the ship owners can cover their investment costs without beeing affected by the volatility of a spot market (Petroleum Economist, 2011). The cost of chartering is mainly decided by the number of days of the voyage. A typical cost is about $55,000-$65,000 per day plus fuel costs for contracted trips and about double for short term chartering or in the spot market. Because the costs is mainly determined by the number of days, LNG projects closer to consuming markets will have a major cost advantage. (Lee, 2005).

3.5 Regasification, Storage and Distribution

When the ships arrive the destination, the LNG is unloaded at a regasification plant. The gas is transferred in a liquid form into storage tanks similar to the ones at the liquefaction plant, and then returned to a gaseous form through a regasification process. This process is not technically advanced, and has been done for many years (Lee, 2005). At the end of 2011, there were 89 regasification terminals worldwide, including 10 floating structures. Together they have a send-out capacity of 640 mtpa (31,104 trillion Btu per year), while the total consumption of LNG is about 241 mtpa. This gives an average utilization rate of about 38%, which means that there are big flexibilities in this part of the value chain. There are big regional discrepancies in the utilization with 40-50% in Europe and Asia, but only 5% in North America. This is because of large development in other gas sources (unconventional gas) in the region. Most of the regasification plants are located in the US, Europe and Japan (GIIGNL, 2011). Figure 3-9 shows how the LNG regasification capacity is split between the major geographical regions and the capacity in the top seven countries.

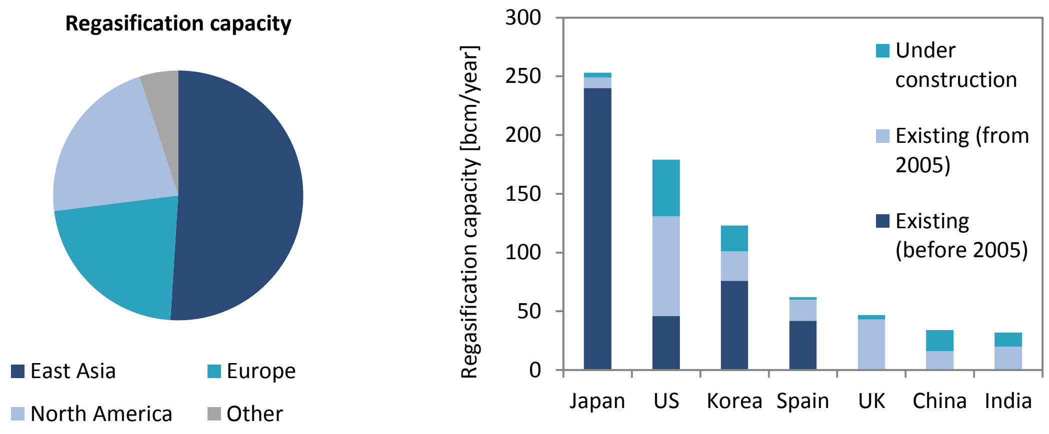

Figure 3-9: *Left) LNG regasification capacity by region (IGU, 2010). Right) Top seven holders of regasification capacity (IEA, 2010).*

3.5.1 Technical Aspects of the Regasification Process

Even though there are some variations between different facilities, most regasification processes go through the same stages. The unloading of the LNG is done through specially designed unloading arms, designed to avoid rupture due to ship movement. Sensors detect if the ship is moving out of normal operating area, stop the flow of LNG and decouple the ship and terminal. After unloading, the LNG is transferred to storage tanks, where it is kept in a liquid state. Next, the LNG is sent to vaporizers to return to a gaseous state. There are different solutions in different facilities. The main types used are:

- *Open Rack Vaporizers*: These vaporizers pump seawater through heat exchangers where it heats the LNG which vaporizes in the tubes.
- *Submerged Combustion Vaporizers*: Natural gas from the facility is burned to heat seawater. The LNG passes through tubes through a bath with this water. The heating of the water increases the efficiency of the heat transfer, but burn 1.2-1.5% of the processed gas.
- *Intermediate Fluid Vaporizer*: The gas goes through a heat exchange with an intermediate fluid, like propane. This fluid is then passed through a heat source like sea water.
- *Ambient Air Vaporizer*: This design use heat from an air flow and vaporize the gas through natural convection. Usually used at smaller installations, or terminals where seawater systems are unsuitable.

At the end, odor is added and the gas is metered before it is sent out through pipelines. The odor is added to make it easier to detect a leak; natural gas is odor- and colorless. (GIIGNL, 2009)

3.5.2 Offshore LNG Receiving Solutions

Traditionally, receiving terminals have been located onshore. But since much of the natural gas is used in densely populated areas, offshore terminals are an interesting alternative; they do not require expensive land. These terminals also allow for easier access for ships, and thereby reduced voyage times and no need to dredge inland waterways. The initial investment in the facility might, however, be higher and the access to distribution pipelines limited. The solution is also less established, and thereby associated with more uncertainty. There are several different offshore solutions; a brief overview is given below. Water depth is an important factor when deciding what type of structure to use. Deeper water and longer distance from shore means higher costs. But visibility from land is also an important factor. Locating a facility far away from shore may make it esthetically acceptable since it cannot be seen from land. (Center for Energy Economics, 2006)

Three types of offshore regasification solutions are: Fixed structures, Floating Storage and Regasification Units (FSRU), and Shuttle and Regasification Vessels (SRV). Figure 3-10 shows an example of each of these solutions. Fixed structures are facilities that stand on the seafloor. Often existing oil or gas facilities are converted to an offshore LNG receiving terminal. FSRUs are solutions for areas with larger water depth and consist of large ships (typically 350 to 400 meters long) which can be built at normal shipyards worldwide. The ships unload normal LNG tankers and store the LNG in tanks on the ships before it is vaporized and sent through a connection to the onshore pipeline system. (Center for Energy Economics, 2006)

Figure 3-10: *Three examples of offshore regasification units: a) Adriatic LNG terminal is the world's first gravity based LNG receiving structure. It is located off the coast of Italy, started operation in 2009, measures 47 by 88 meters and has a capacity of 8 bcm per year (about 10% of Italy's gas consumption). b) Golar Spirit is the first FSRU converted from an LNG carrier. It is located off the coast of Brazil, started regasification in 2009 and has a throughput capacity of 2.5 bcm per year. c) Illustration of Port Dolphin SRV. This vessel has a buoy connection under water.*

A different solution closely related to the FSRUs is a Shuttle and Regasification Vessel (SRV). In this solution, the LNG tankers used for shipping also operate as regasification units; the LNG is never unloaded in a liquid form, and no storage capacity is needed at the receiving terminal. The transportation ships are connected to the gas pipeline system, and unload vaporized gas from an onboard regasification unit. A drawback is that more ships are needed to operate a project, since one of ships needs to be at the receiving terminal for continuous delivery of gas. One such project is Port Dolphin SRV (see Figure 3-10). This system vaporizes the gas in a regasification unit on deck, and uses an underwater unloading buoy to transfer the gas to an onshore pipeline. (Port Dolphin Energy LLC, 2011)

3.5.3 Regasification Cost

There are several factors that influence the cost of regasification facilities. Land cost, needed storage capacity and local regulation are major factors. Sometimes it is cheaper to locate the facility in a remote area, and transport the gas to the consumers in longer pipelines. Land cost is also a reason why some facilities are built as floating units off the coast of the market. One estimate from The Gas Technology Institute (referenced in Lee, 2005) puts the cost of a new regasification plant in the US at around \$200-\$300 million, which translates to a price of about \$0.30-\$0.50 per MMBtu. This price can vary largely, but it is generally one of the cheapest links in the total value chain. (Lee, 2005)

Chapter 4

Introduction to the Problem

Based on the thesis description and the review of the LNG value chain, we define our problem. The purpose of this work is to develop a decision support tool to help answering our problem statement:

> ***When are floating LNG production units a more profitable solution than onshore facilities, and what value do they add to the LNG value chain?***

To answer when floating LNG solutions should be used, an optimal schedule of investment decisions, like which fields to develop and what processing solutions to use, should be found. Optimal is defined as the decisions that will lead to the greatest total expected profits within a given horizon. Since the different cash flows occur at different times, future cash flows are discounted to obtain the present value.

The problem covers a long horizon, typically several decades, which implies a focus on the strategic issues. The decisions are on a high level, including investment decisions and location of facilities. Many detailed characteristics are simplified or omitted to reduce the size of the problem and maintain computational tractability. Examples are: ship routing and allocation, maintenance schedules and production operation. Some tactical aspects like production quantity and where to sell the LNG are, however, included in the problem to illuminate which investment decisions to make.

The whole value chain, from field development to regasification, is incorporated in the problem. Fields need be connected to at least one floating or onshore processing facility before it can produce natural gas. Each onshore plant can process gas from several fields simultaneously, while floating units are connected to only one field at the time. Floating production units can be moved between fields, and the focus of the problem is on comparing the onshore and floating liquefaction solutions. Ships load LNG from the processing units and transport it to the markets. The gas markets are given as several aggregated demand points with potential contracts and spot demand. All potential investment options, including relevant properties like capacities and costs, are given as input to the problem. The recoverable amount of gas and future gas prices are uncertain, and their value is given as discrete scenarios with individual probabilities.

Infrastructure can be constructed at any time. After it is built, it can be closed down again, such that operational costs for infrastructure that is not used can be avoided. Parts of the value chain are already operational at the beginning of the planning horizon. Any existing fields may be at different stages of production.

In the next chapter we present literature related to our problem, before an optimization model will be formulated in Chapter 6.

Chapter 5

Literature Review

In this chapter, a review of existing literature related to our problem is given. The literature on oil and gas investments is extensive, so most of the models presented are from this field. Some investment models from other industries are also included. The literature can broadly be divided into three levels: Strategic, tactical and operational. Strategic level involves investment decisions and design of the value chain and typically has a horizon of several years or even decades. On the tactical level, medium term planning is performed and resources are allocated. Decisions from the strategic level constrain the choices on this level, but more details can usually be modeled due to the shorter horizon. Short range planning and scheduling is done on the operational level. Our problem is concerned with strategic decisions, but it is important to integrate tactical and operational perspectives into the strategic design. This chapter therefore contains mainly strategic models, but we also include some more tactical.

The discussion in this chapter is divided into different aspects that are relevant to our problem. Different approaches to handling investment decisions are presented in Section 5.1, before Section 5.2 covers methods for aggregating time periods. Some more technical aspects are reviewed in 5.3 Reservoir Modeling and Gas Transportation in Pipelines. Section 5.5 and 5.6 contains discussion on uncertainty; first a general discussion of how to handle uncertain cash flows is given, before stochastic programming is covered in more detail. In Section 5.7 we look at models of LNG production. The literature on this subject is limited and we therefore want to present them separately from the other discussion. At the end of the chapter, a short comment of how our problem fits in with the existing literature is given.

5.1 Investment Decisions

Our problem focuses on investment decisions in the LNG value chain. This section include existing literature about general investment decisions, contracts and different types of objective functions.

Aboudi et al. (1989) present an investment model of development of oil and gas fields and transportation systems. It is a deterministic multi-period investment model used to analyze different scenarios. The core of the model is a directed network representing potential projects. The model chooses which parts to develop and at what time. Each part of the network has a capacity, and the model also specifies the flow through the network throughout the horizon. The projects have time windows for investments and there are dependencies between projects, as some parts of the network may have to be developed before other parts. The user of the decision support tool can also specify a subset of the projects that are mutually excluding. Several sizes or variations on one project can thus be specified as different projects. The objective is to maximize the net present value of the future cash flows. They are the revenue from oil and gas sales minus investments, operating and transportation costs.

A model closely related to Aboudi et al. (1989) is presented in Nygreen et al. (1998). This model tries to solve a similar problem, and is also a model of investment planning in fields, but with focus on the Norwegian continental shelf. The Mixed Integer Programming (MIP) model has been continuously developed and had, at the time of publishing the article, been used by The Norwegian Petroleum Directorate for more than fifteen years. It has a strategic focus and assumes deterministic cash flows. Several potential projects are given; these can be investments in fields, processing or transportation elements. The projects can be started in one of several years, or never started at all. A field can be developed in several different ways. To accomplish this, each alternative is defined as a separate project, and a constraint ensures that only one of the alternatives can be developed. Constants define the amount of product that must be delivered to each node, either within an interval or not at all.

The investment models for the oil and gas supply chain share many similarities with supply chain investment models for other sectors. One example from the pharmaceutical industry is described in Papageorgiou et al. (2001). Like in Aboudi et al. (1989) and Nygreen et al. (1998), the objective is to maximize the discounted future profits through investments in potential production locations with a given maximum capacity. An extension is the existence of different types of plants depending on which pharmaceutical product they produce. The main variables are which plants to invest in, and what type, where and which amount of products to produce to cover the demand in different sales regions.

Another non-oil and gas model is described in Hugo et al. (2005). It is a strategic investment planner for hydrogen infrastructure, including manufacturing, storage and delivery of hydrogen to the consumer. The model describes a superstructure based on the supply chain, capturing all possible alternatives and interactions between the various supply chain components. The structure consists of four sets; energy resources, production facilities, transportation systems and distribution channels. The oil and gas investment models often focus on either strategic or tactical level, while Hugo et al. (2005) present a model where the optimal investment planning is performed in four levels; The first level is strategic supply chain design, where some combinations from the super structure are chosen as hydrogen supply chains. The second level consists of capacity expansion and planning shut-down of elements in the supply chain. The third level is production planning, and the fourth is a trade-off analysis to find an optimal compromise between different objectives.

Naraharisetti et al. (2008) have modeled a general, extensive investment model on supply chain redesign. The authors illustrate the impact of including possibilities to disinvest and/or relocate facilities, which will give an increase in profit for their example case study. The decisions in the model include facility location, relocation, investment, disinvestment, technology upgrade, production-allocation, distribution, supply contracts, capital generation etc. The disinvestments can only occur once per facility during the time horizon, and a facility cannot be partially disinvested.

Some articles have focus on particular aspects of the value chain. Jørnsten (1992) presents a decision support tool, where the only decisions are sequencing of investment in different potential fields. Both a deterministic model and a stochastic version, with uncertain demand, are presented. The production profile for each field is assumed known, such that once a field starts producing, the production is deterministic for the next several years. The investments and cost profiles, and oil and gas prices are also assumed known. For each field and each possible starting period, the net present value is calculated based on the known values. The objective function is to maximize the total net present value of profits.

Iyer et al. (1998) formulate a multi-period planning problem, integrating facility location, production planning and scheduling. This model does not look at the transportation of petroleum products after the production platform, but models the production in greater detail. Given a set of fields to develop, the model finds a schedule of well drilling and which well- and production-platforms to build among several potential options. It also selects production rates and how to connect wells, well platforms and the production platform. The objective function maximizes the discounted profit, which includes revenues from sale, fixed and variable investment cost, fixed drilling cost of wells and costs for moving the drilling rigs.

5.1.1 Contracts

The gas markets in Continental Europe and in Japan are, as mentioned in Section 2.1, still dominated by long-term contracts. The contracts provide guarantied sales volumes for the seller, and ensure a stable supply for the buyer. Jørnsten (1992) implements contracts in the investment model by constraining the quantity produced and sold to the market, to a lower limit for each time period. Another oil and gas model including contractual agreements is presented in Haugen (1996). The contracts specify quantities per year and a price structure for a given horizon. Economical punishments and rewards, to ensure some delivery profile for the future, are implemented by giving no price for quantum delivered above demand, and a linearly decreasing price function for deliveries below demand.

Naraharisetti et al. (2008) include contracts with material suppliers, specifying several prices based on purchase amount. An upper limit for purchases outside of contracts can also be specified. The contract durations are unknown and are treated as decision variables. Once a contract with a supplier begins, it must remain in effect for some minimum duration. In the model, contracts can exist in the initial state or be sealed during the decision period.

5.1.2 Multiple Performance Criteria in the Objective Function

Multiple performance criteria, like profit, environment or risk, can be used in the objective function. Maximum net present value of investment is the most common criteria in the models mentioned earlier in this section. Both Haugen (1996) and Nygreen et al. (1998) use this measurement, but do also have an alternative performance criterion. The first author uses minimization of the expected absolute deviation between given demand profiles and delivered quantities in the markets, while Nygreen et al. (1998) minimize a weighted sum of deviations from a given goal on production of resources. Hugo et al. (2005) present an objective function which includes both economic *and* environmental criteria. The outcome from the model is a set of optimal trade-offs of the often conflicting criteria.

Net present value is a non-linear function and is often implemented by calculating a parameter representing the reduction in value for each time period in advance. Aboudi et al. (1989) and Papageorgiou et al. (2001) find the total cash flow, including revenue and costs for each year, and then multiply them with the discount rate parameter.

5.2 Time Aggregation

Most of the articles described in this literature review use a finite number of discrete time periods of equal duration (e.g. Haugland et al., 1988, Papageorgiou et al., 2001, Naraharisetti et al., 2008). The computational burden increases with the number of time periods, and some articles describe methods to aggregate the time to reduce the computational burden and make the models more tractable.

Aboudi et al. (1989) present a model where the user can specify how the splitting of the time horizon is done. The input data will have different values depending on the aggregation of the time periods. User error when calculating the aggregated values can easily lead to errors in the input data. The model is therefore built such that the user enters the basic yearly data once, and the model will perform necessary aggregation of the data automatically. To be more readable, the results are disaggregated and presented yearly.

Iyer et al. (1998) present and demonstrate how to use a sequential decomposition algorithm to solve their multiperiod mixed-integer linear programming (MILP) model. The algorithm aggregates time periods and wells, to make the large model possible to solve. The aggregation leads to fewer details in the production profile, so the last step of the algorithm smooths the profiles. Examples in the article show that the calculation time is greatly reduced compared to solving the MILP model without the algorithm.

5.3 Reservoir Modeling

A central part of an analysis of a petroleum field is reservoir modeling. Many articles are written on how to handle this aspect. Haugland et al. (1988) describe a simple LP-model that finds the production profiles from each well which leads to maximum net present value. The production is described as a discrete function, where the life-time of the field is divided into a set of finite time periods. The production rate within each period is assumed constant and below a level determined by the pressure and platform capacity. For oil reservoirs, the pressure is estimated from Darcy's law of liquids. Pressure in gas reservoirs are given by non-linear equations that make the model hard to solve. The authors suggest that the equations can be linearized, but the representation may be too imprecise if the pressure varies over a large interval. By numerical estimations, Haugland et al. (1988) have found that if the price is constant, the production will decrease slowly over the time periods. If the price increases, the production will start in a later time period and then decrease slowly. This is consistent with the results of Hotelling's rule. It states that the most profitable extraction path of a non-renewable resource is dependent on the price development and the rate of interest. (Hotelling, 1931)

Sullivan (1988) looks at differences between representations of reservoir behavior in different models. He states that using equations describing the production behavior (implicit representation) gives great freedom in finding the best solution. The degree of realism achieved by this approach is, however, limited by computational resources. He proposes a representation using several alternative production profiles (explicit), such that the model can select one profile for each field. These predetermined fixed profiles limit the variations in investment and production choices. Sullivan also introduces variable profiles with interpolation between a minimum rate and maximum rate of a selected profile.

Production profiles are also a central part of the model in Nygreen et al. (1998). The model solves the uncertainty of the actual production profiles by making the capacity constraints less hard. They present two approaches: The first is to introduce surplus variables with penalties in the objective. The second is to allow the production to differ from the given profiles, like the interpolation method in Sullivan (1988). The author implements a method with the possibility to produce less than the specified profiles. Any surplus is saved, such that production can continue for additional periods as long as maximal production in the profile is not exceeded. The production is also constrained to be above a given ratio of the profile, to reduce variation in production. An exponential decline rate ensures the reduction of production when the cumulative production approaches the total reservoir size.

In Iyer et al. (1998), the flow and production rates from the fields are dependent on the pressure in the reservoir. The oil and gas flow rates are modeled as linear functions of the pressure. The model uses a piecewise linear approximation of the reservoir performance. The formulation involves a large number of binary variables which makes the problem hard to solve. A sequential decomposition algorithm is presented. It yields a good feasible solution and an upper bound, but it is not guaranteed to give an optimal solution.

Van den Heever and Grossmann (2000) present a multi-period mixed-integer non-linear programming model for offshore oil field infrastructure, where the decision variables determine the platforms and wells to install or drill as well as the drilling schedule. They suggest that approximations of the non-linear reservoir behavior by one or more linear constraints are not always realistic. The reservoirs often contain a substantial volume of gas and a single linear constraint will be too imprecise if the pressure varies over a large interval. Van den Heever and Grossmann (2000) incorporate the non-linear reservoir behavior directly into the formulation, and use an iterative aggregation/disaggregation algorithm to solve the problem. An extension of the model is given in Van den Heever et al. (2000). They add complex economics factors, like tariff, tax and royalty, to the model to improve the net present value of the projects. The computational burden of introducing this is high, and a specialized heuristic algorithm that relies on the concept of Lagrangean decomposition is presented in Van den Heever et al. (2001). The algorithm reduces the run time drastically.

Another optimization model for the planning of infrastructure in offshore oilfields is presented in Carvalho and Pinto (2006). The objective is to maximize the net present value of revenue and installation, drilling and connection costs. The mixed integer linear programming (MILP) model determines which platforms to build, their connection with wells and the timing of extraction and production rates. The authors are the first to incorporate reservoir dependent pressure, rather than field dependent pressure. They state that this will give a better representation of the problem, since a field can contain many reservoirs with different characteristics and thereby also different pressure patterns. To be able to handle larger datasets, the authors assume that the pressure of the reservoir declines linearly with the oil removal. They do not implement piecewise linear (e.g. Iyer et al., 1998) or non-linear (e.g. Van den Heever and Grossmann, 2000) reservoir behavior. Solving the MILP is computationally expensive and different solution techniques are applied to make it solvable.

5.4 Gas Transportation in Pipelines

Gas flow through pipelines can be modeled in many ways. One of the simplest is to neglect technological characteristics, like pressure variations and flow, and use a maximum capacity of the pipeline. An example of this implementation can be found in Gabriel and Smeers (2006). A more detailed model is given by letting the capacities vary with the components in the petroleum (Nygreen et al., 1998 and Ulstein et al., 2007). The model given in Nygreen et al. (1998) also has the possibility to build new pipelines to expand the existing capacity. A further extension is given in Ulstein et al. (2007) and Tomasgard et al. (2007), where pressure variations are used to describe the flow in long pipelines. The latter authors use the Weymouth equation, which is non-linear, to describe the flow as a function of input and output pressure. The equation is linearized through Taylor series expansions around a point which are represented by a fixed pressure in and a fixed pressure out of the pipeline. For relatively short pipelines, the pressure drop is neglected and the constraint is reduced to a simple maximum flow restriction.

5.5 Financial Aspects: Valuation of Uncertain Cash Flows

In our problem we want to select a set of projects or investments that maximize the profit. Investments lead to different cash flows over a period of time, and a key issue is how to value these cash flows; how to scale the flows occurring each year. There are many ways of evaluating investments. The most commonly used measure is Net Present Value (NPV), the sum of all discounted expected future cash flows from the investments. Under certainty this valuation measure works well: If a project has a positive NPV, the investor should invest in it. (Trigeorgis, 1996)

Trigeorgis (1996) points to two problems of using NPV. Firstly, *flexibility* is not captured by NPV. Managers often have options to react to the uncertain future, for example by expanding or ending projects. NPV relies on an expected scenario of future cash flows and allows no changes once a strategy is chosen. The second problem is that the cash flows are *uncertain*. The assumption of certain future cash flows rarely holds in reality. One approach to dealing with this uncertainty is the use of a risk adjusted discount rate. Since most investors are risk averse, projects with higher uncertainty are discounted more heavily. It is, however, hard to determine this risk adjusted rate, and the outcome of the model depends heavily on it. Bagajewicz (2008) examines models that maximize expected NPV and some simple alternatives like internal rate of return and return on investments. These methods are not presented here, but the conclusion is that none of the methods lead to the capital being utilized at the maximum profitability. An approach that considers the whole portfolio of investments is needed.

Orman and Duggan (1999) demonstrates how the idea of the capital asset pricing model (CAPM) can be used in selection of projects in upstream oil and gas companies. By using a portfolio approach and illuminating the relation between risk and return, managers can allocate capital more efficiently. The basic idea introduced by Markowitz (1952), builds on risk reduction through diversification. By choosing investments that are not correlated, an investor can reduce or eliminate the risk that is unique to an individual asset, and only be exposed to systematic risk, like for example inflation or events that affect the whole market. The group of investments with minimal risk for a given return level, is called an efficient portfolio. Walls (2004) note that without knowing which level of risk taking is appropriate for a specific firm, managers do not know which of the efficient portfolios that is best for the firm. He shows how integrating risk preference analysis with portfolio management can further improve decision process and firm performance. Trigeorgis (1996) points to some problems with utilizing CAPM: It can be hard to determine the risk of each project. Flexibility to change projects in the future might also lead to the risk changing over time. CAPM does not capture this and might lead to suboptimal results.

5.5.1 Representing Uncertainty using Real Options

Brennan and Schwartz (1985) argue that while traditional NPV approaches can be used in application with predictable prices, the highly uncertain nature of prices of natural resources commands a different approach. Natural resources, like coal, oil and gas, often require high investments and have uncertain output prices. The authors use a real options approach to value projects and also determine optimal polices for developing, managing and abandoning them. *Real options* are a tool for valuing projects. Future flexibility is viewed as financial options, and methods from mathematical finance are used to value these. This method does not rely on risk adjusted discount rates; instead it values the options by using underlying financial assets that are traded. The main advantage of real options is the ability to capture flexibility to adapt and change decisions in the future. Trigeorgis (1996) and Dixit and Pindyck (1994) describe the approach in detail.

Many authors have looked at how to value different types of options. McDonald and Siegel (1986) and Dixit and Pindyck (1994) look at investment timing and the option of waiting to invest. They show that real options approaches lead to higher profits than traditional NPV, when the decision maker has flexibility in the timing of investments. McDonald and Siegel (1985) look at the option of closing projects and Pindyck (1988) looks at capacity choice under uncertainty. Cortazar et al. (2001) present a model where real options are applied to investments in natural resources. It follows the work of Brennan and Schwartz (1985) and evaluates exploration investments under price and geological-technical uncertainty. Projects go through an exploration phase with several stages before entering production. Exploration investments can be stopped at each stage, development investments can be postponed, and production can be closed and reopened. The paper applies the model on a copper mine.

The ability to represent flexibility and link future uncertainty to current decisions makes real options an attractive tool, but there are some problems with practical applications of the method. Bowman an Moskowitz (2001) look at problems with how the pharmaceutical company Merck & Co applied real options on an investment in an R&D project, and Borison (2003) looks at limitations in several different real option models. They point at two major challenges of using real options. Firstly, the assumptions of the models being utilized might not fit the investment proposal; the analogy between financial options and real options are not perfect. Option valuation tools rely on a model of the underlying stock price with assumptions about the probability distribution of the development of this price. These models are built for the development of financial assets, and might be inappropriate for real options. The other problem is to determine the inputs of the model. Option pricing models need a traded underlying stock price or asset. For real options, a traded twin security has to be identified, since the project itself is not traded. The value of this twin security must be highly correlated with the value of the project. Finding an asset like this might be difficult or impossible for many practical applications. This critique does not mean that real options are unsuitable for all decisions, just that the method should be applied carefully; the assumptions might not fit every problem.

Wang and de Neufville (2004) divide real options into two categories: "Real options 'on' projects are financial options taken on technical things, treating technology itself as a 'black box'. Real options 'in' projects are options created by changing the actual design of the technical system". Options 'in' projects are often complex and interdependent, which makes it hard to determine input parameters for the model. They are also likely to be path dependent; their value depends on the history of the uncertain parameter. Stock option's prices, on the other hand, only rely on the current price. The authors suggest stochastic mixed integer programming as a tool for an implicit evaluation of the options instead of a market driven approach. We describe this method in the next section.

5.6 Stochastic Programming

Deterministic models assume that all properties of the future (like prices, demand, or capacities) are known. In reality this is rarely true. One approach to handle the uncertainty is *sensitivity analysis*. By solving the deterministic model with different datasets or scenarios, one can see the manner in which the solution changes when the input data changes; it is a way of assessing the robustness of a solution. But sensitivity analysis gives no answer to what to do if the solution is not robust. The obtained solutions are also adapted to only one particular scenario at the time. A solution that is not best in any one scenario, but fairly good in all, might give a better expected objective value. To find these solutions we need to consider all scenarios collectively, balancing the impact of each of them. (Higle, 2005)

In this section we first present stochastic programming, a method addressing the problem of representing uncertainty. Two methods of evaluating stochastic solutions are then reviewed before some examples of stochastic models in the literature is given.

5.6.1 Introduction to Stochastic Programming

Stochastic programming is a method for optimization under uncertainty and can be traced back to Dantzig (1955), which introduced the recourse model. The goal is to find a solution that considers the outcomes of random parameters with known distributions. Most models have discrete time periods corresponding to points in time where decisions are made. The model also contains several discrete stages, where a new stage represents a point in time when we get new information; some uncertainty is resolved. These stages do not have to correspond to time periods in the model; each stage could contain multiple time periods. Stochastic programs are called two-stage if they contain two stages or multi-stage if they have more than two. Some decisions must be fixed from the beginning of the horizon, while others can be delayed until after some of the uncertainty is resolved. The latter decision variables are called *recourse variables*, and are allowed to vary with the scenario. An important observation is that since uncertainty is resolved at the beginning of a stage, the model is deterministic within the stage. All decision variables within the stage can then be determined at the beginning of the stage.

The stochastic problem can be represented by a scenario tree where each branch corresponds to a resolution of some uncertainty. Each node on a path from the root to a leaf represents a time period in a scenario, and the path represents a scenario. Figure 5-1a shows an example of one such scenario tree. If we represent each scenario as an independent problem, we get an alternative, but equivalent, representation given in Figure 5-1b. Here we have four independent deterministic problems represented by a set of nodes (time periods) connected by arcs. The ellipsoids around some of the nodes indicate that decision variables in these nodes have to be identical; they are not allowed to vary with the scenario since the

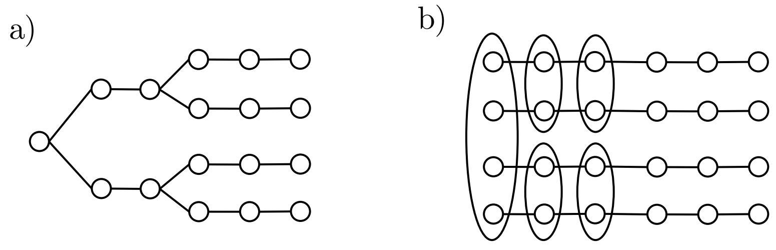

Figure 5-1: *Example of a stochastic decision tree. It illustrates six time periods, where uncertainty is resolved in period two and four, resulting in four scenarios.*

scenarios are indistinguishable in these time periods. The four independent problems are thus forced to be similar in the time periods prior to the uncertainty resolution. The constraints ensuring this are called non-anticipativity constraints. (Higle, 2005)

We can now formulate the stochastic program. An example of a two-stage problem is given below. We present a two-stage version to demonstrate the general principle; a multistage version is presented below. The formulation is called a stochastic *recourse problem* (RP).

$$\begin{aligned} \min \quad & cx + E[h(x,\tilde{\omega})] \\ s.t \quad & Ax \geq b \\ & x \geq 0 \end{aligned} \tag{A}$$

$$\begin{aligned} \text{where } h(x,\omega) = \quad & \min \quad g_\omega y \\ s.t. \quad & W_\omega y > r_\omega - T_\omega x \\ & y \geq 0 \end{aligned} \tag{B}$$

In this example x is the decision in the first stage; it is not dependent on the different scenarios. y on the other hand is decided after the uncertain variable $\tilde{\omega}$ is observed. Problem (B) is known as the second stage problem and tries to find an optimal solution given the resolution of the uncertainty and the decision in the first stage. The whole model (A) minimizes the first stage costs cx and the expectation of the costs from the second stage problem. This general recourse model can be implemented by using the deterministic equivalent formulation. Problem (B) is then included in (A) and a ω index is added to the y variable. The objective would then be $\min\ cx + \sum_{\omega\in\Omega} p_\omega g_\omega y_\omega$, where p_ω is the probability of each scenario. (Higle, 2005)

For multi-stage problems, uncertainty is resolved at several points in time. The first stage and recourse decisions are no longer denoted by separate variables, now x_ω

represent the decisions in scenario ω, and c_ω the objective function coefficients in scenario ω. The multi-stage problem can be expressed as:

$$\begin{aligned} \min \sum_{\omega \in \mathbf{\Omega}} p_\omega c_\omega x_\omega \\ s.t \quad x_\omega \in X(\omega) \quad \forall\, \omega \in \mathbf{\Omega} \\ \{x_\omega\}_{\omega \in \mathbf{\Omega}} \in \boldsymbol{\mathcal{N}} \end{aligned} \tag{C}$$

The scenario constraints are denoted by $X(\omega)$, the set of solutions that are feasible for scenario ω. $\boldsymbol{\mathcal{N}}$ is the set of non-anticipative solutions. The non-anticipativity constraints are not formulated explicitly. This formulation corresponds to the example in Figure 5-1b. The ellipsoids are groups of decisions that are equal in $\boldsymbol{\mathcal{N}}$. (Higle, 2005)

5.6.2 Evaluation of Stochastic Solutions

Implementing stochastic programs can be computationally expensive, and it is therefore interesting to look at what value this method adds to the solution. The following discussion assumes a maximization problem, in accordance with our problem. *Expected Value of Perfect Information* (EVPI) is one metric used to analyze a stochastic problem. It is defined as the difference between the objective value of the Wait-and-See (WS) solution and the objective value of the stochastic Recourse Problem (RP):

$$EVPI = WS - RP$$

WS is defined as the weighted average of the deterministic solution to each of the scenarios, where the probability of the scenario is the weight. WS is in other words the expected value of the objective function if we had perfect foresight. EVPI is therefore the amount a decision maker would be willing to pay for this perfect information; the expected increase in objective value. (Birge and Louveaux, 2011)

A second important metric is *Value of the Stochastic Solution* (VSS), which measures the added value of using a stochastic model, compared to a deterministic. For two-stage models, this is calculated by replacing all stochastic parameters in the model with their expected value (EV model). The solution obtained from this deterministic problem is called the expected value solution. By locking this solution on the first stage decisions and letting second stage variables vary with the scenarios, we can measure how this solution performs in each scenario. The weighted average (using probabilities of the scenarios) of these objective values is called the Expected result of the Expected Value solution (EEV). VSS can then be defined as:

$$VSS = RP - EEV$$

VSS is therefore the expected increase in objective function by using a stochastic program instead of a deterministic version with expected values for all uncertain parameters. (Birge and Louveaux, 2011)

Escudero et al. (2007) point out that there are complications to this way of calculating the EEV for multistage problems. The most obvious challenge is that there is no clear way to determine which variables from the EV solution to fix. A trivial solution would be to only fix the first stage decisions, similarly to the calculations for two-stage models. The decision variables for the following stages will then be able to adapt to the different scenarios when we measure how this solution performs in each scenario. All non-anticipativity constraints after the first stage are ignored. This method can lead to a paradox, since the EEV can be higher than the objective value of the RP model, giving a negative VSS. The stochastic model contains non-anticipativity constraints, which are removed when finding the EEV. This example shows that the EEV must be redefined. Escudero et al. (2007) present two approaches.

The first approach calculates the value of the stochastic solution for stage s, and EEV_S is introduced. It is similar to EEV, but differs in two ways. Firstly, instead of solving each scenario independently, the expected value solution is inserted in the stochastic model, RP. The non-anticipativity constraints are thus active, and a negative VSS is avoided. The other difference is that the EEV is calculated for each stage s. The EEV_S is defined as the optimal value of the stochastic model, RP, where the decision variables prior to stage s are fixed to the optimal solution of the expected value problem (EV). The value of stochastic solution for stage s is defined as the difference between the objective value of the stochastic problem and EEV_S:

$$VSS_s = RP - EEV_s$$

The second approach also calculates the value of stochastic solution for each stage s, but uses a dynamic approach to calculate the expected value solutions. Escudero et al. (2007) claim that this gives a more realistic value of the EV solution than the first method, since a deterministic model would be run multiple times. First, we need to introduce some new notation, illustrated in Figure 5-2. Each node in the scenario tree is associated with a scenario group $g \in \boldsymbol{G}$. Scenario groups are defined such that two scenarios belong to the same group in a given stage if they have the same realizations of the uncertain parameters up to that stage. $\boldsymbol{G_s}$ denotes the set of scenario groups in stage s. $\boldsymbol{\Omega_g}$ denotes the set of scenarios ω that are part of scenario group g. The last time period prior to the stage of scenario group g is denoted T_g^{PREV}. Note that each stage can contain several time periods.

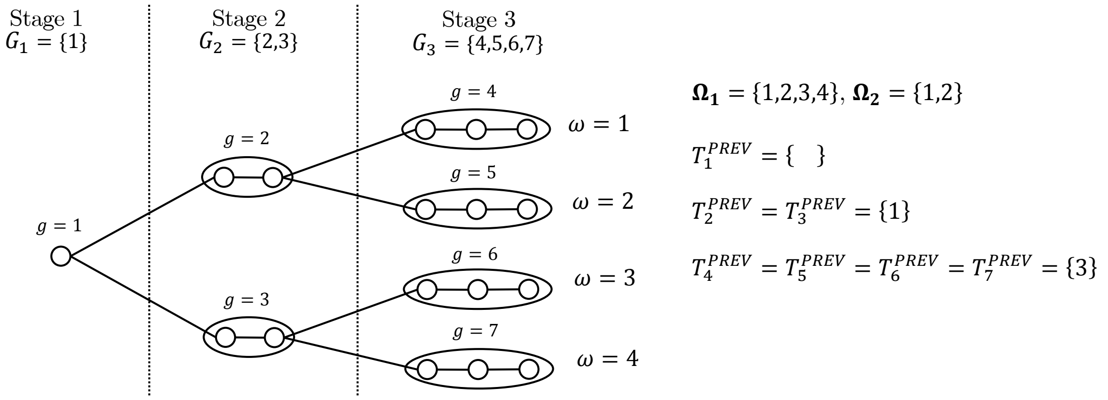

Figure 5-2: *Example of a multistage grouped scenario tree.*

The expected value of the uncertain parameters, $\bar{\xi}_g$, is calculated for each scenario group g. They are calculated as a weighted sum of the uncertain parameters for scenario ω, ξ_ω, weighted by the likelyhood of the scenario within the scenario group:

$$\bar{\xi}_g = \frac{\sum_{\omega \in \mathbf{\Omega}_g} \xi_\omega \pi_\omega}{\sum_{\omega \in \mathbf{\Omega}_g} \pi_\omega}$$

An expected value model EV_g, is solved for each scenario group g by replacing all uncertain parameters with the expected value, $\bar{\xi}_g$. All decision variables for previous stages are also fixed to the optimal solution obtained from the ancestor scenario group, α_g. The optimal objective value is defined as Z_g^{EV}, and the optimal variable values $u_{t,g}^{OPT}$. If $u_{t,g}$ represent any variable from scenario group g and time period t, the fixing constraints can be defined as:

$$u_{t,g} = u_{t,\alpha_g}^{OPT}, \qquad \forall\, t \leq T_g^{PREV}$$

The expected result in stage s of using the dynamic solution, $EDEV_S$, is defined as the sum of the optimal values for the scenario groups g within the stage s, multiplied with the probability of the scenario group.

$$EDEV_s = \sum_{g \in \mathbf{G}_s} Z_g^{EV} \sum_{\omega \in \mathbf{\Omega}_g} \pi_\omega$$

This value corresponds to the expected result of solving the deterministic model with expected values each time some uncertainty is resolved, and adjust the future decisions accordingly. The dynamic value of the stochastic solution is defined as the difference between the optimal objective of the stochastic model and the expected objective of using the dynamic solution of the average scenario for the last stage S:

$$VSS^D = RP - EDEV_S$$

We will in this paragraph illustrate how the EDEVs' are calculated for the two first stages in Figure 5-2. The EV_1 model is solved using the expected values of all the uncertain parameters. Note that no variables are fixed since scenario group 1 is the first stage and does not have any ancestor scenario groups. Optimal objective value Z_1^{EV} and optimal variable values $u_{t,1}^{OPT}$ are obtained. Only one scenario group is included in stage 1, and all scenarios are included in the scenario group, leading to $EDEV_1 = Z_1^{EV}$. Both scenario group 2 and scenario group 3 are included in stage 2, $\boldsymbol{G_2} = \{2,3\}$. First, we solve for scenario group 2, which includes scenario 1 and 2, $\boldsymbol{\Omega_2} = \{1,2\}$, and the ancestor scenario group is 1, $\alpha_2 = 1$. The EV_2 model is solved using the expected values of the random parameters, $\bar{\xi}_2$, and fixing variables for time period 1 to the optimal value of scenario group 1, $u_{1,2} = u_{1,1}^{OPT}$. The optimal objective value Z_2^{EV} and optimal variable values $u_{t,2}^{OPT}$ are obtained. Second, we solve for scenario group 3, which includes scenario 3 and 4, and the ancestor scenario group is also 1. EV_3 model is solved using $\bar{\xi}_3$ and fixing variables $u_{1,3} = u_{1,1}^{OPT}$. Optimal objective value Z_3^{EV} and optimal variable values $u_{t,3}^{OPT}$ are obtained. $EDEV_2$ is calculated by adding Z_2^{EV} multiplied with the probability for scenario 1 or 2 to Z_3^{EV} multiplied with the probability for scenario 3 or 4. Note that for all scenario groups in stage 3, the variables for both time periods 1, 2 and 3 are fixed to the optimal variable value for the ancestor node.

5.6.3 Stochastic Models in the Literature

Only the advantages of incorporating uncertainty into models have been covered so far. The main drawback is that the size and complexity of the model grows fast, and it might become unsolvable for many practical problems. The literature incorporating uncertainty was therefore initially limited, but many of the more recent models on oil and gas investments contain stochastic elements.

Sahinidis (2004) gives an extensive overview of work on problems incorporating uncertainty. This article also reviews other approaches than stochastic MIP, like *Fuzzy Programming* or *Stochastic Dynamic Programming*. One stochastic dynamic programming approach for valuing petroleum projects is described in Lund (2000). The focus is on the flexibility in the projects, and both market risk and reservoir uncertainty are considered. He finds a significant value of the flexibility and argues that this must be accounted for in future evaluation of development projects. The dynamic programming approach might be hard to apply when the state space and number of decision variables increases. The rest of this section will focus on stochastic integer programming.

Jonsbråten (1998) classifies uncertainty into two categories: exogenous and endogenous. Project *exogenous* uncertainty is resolved independent of the decisions in the project; the scenario tree is fixed. This could for example be demand or oil prices. Project *endogenous* uncertainty on the other hand depends on the project decisions; the scenario tree will change depending on the value of the decision variables. Reservoir size is an example, since the uncertainty is resolved when a field is explored or developed. Endogenous uncertainty is harder to model and solve, and most early work concerns exogenous uncertainty. Jonsbråten (1998) also presents a model of each type. His first model examines optimal development decisions of an oil field under oil price uncertainty using stochastic programming. The objective is to maximize expected NPV of the field, and the model contains continuous production variables and integer investment variables. The second model looks at optimal sequencing of oil wells when the reservoir properties are uncertain. Here, the resolution of the uncertainty depends on the decisions in the project. An implicit enumeration algorithm for solving the problem is also proposed.

Most of the literature focuses on exogenous uncertainty. Aseeri et al. (2004) expand the model of Iyer et al. (1998) adding uncertain oil prices and well productivity indexes. They analyze the financial risk and discuss how it can be handled. The model is solved using a sampling average algorithm. Khor et al. (2008) looks at tactical planning in a petrochemical refinery. Three uncertainties are considered simultaneously: Prices, demand and product yield from the processes. Plants and physical resources are assumed to be fixed and the objective is to find an optimal processing schedule. They investigate economical and operational risk through two-stage stochastic programming. Carneiro et al. (2010) optimize an investment portfolio in the oil supply chain under uncertainty. Two stage stochastic programming with fixed recourse (exogenous uncertainty) is used to find optimal investments when demand, supply and prices are uncertain. They show that the level of risk taken in projects highly affects the NPV in the objective function. A risk measure (Conditional Value at Risk) is therefore introduced to the model and constraints limit this risk, such that the overall economic risk in the solution is controlled. This work is an example of integration of financial engineering and supply chain management, and the authors highlight this as an exciting field for further research.

The work on endogenous uncertainty is more limited. Ahmed (2000) looks at how the probability distribution of uncertain events can be changed by decisions in the model. The problems are formulated as mixed integer linear programs. A different type of decision dependent uncertainty is investigated in Goel and Grossman (2004), where the objective is to find optimal investments of gas fields. They expand the model in Iyer et al. (1998) with uncertain gas reserves. The timing of the uncertainty resolution (and thereby also the scenario tree) is determined by the decisions in the model. All uncertainty about a gas reservoir is assumed to be resolved immediately when the investment in the field is made. To make the scenario tree dependent of the decisions, the non-anticipativity constraints are formulated with disjunctions that can be converted into a mixed integer linear program. This means that the constraints are not fixed, but change with the decision variables. This work was further expanded in Goel et al. (2006), Tarhan and Grossmann (2008), and Gupta and Grossmann (2011) by looking at more efficient solution approaches, non-linear reservoir models and allowing gradual resolution of the uncertainty.

5.7 LNG Specific Models

Most of the literature on LNG value chain modeling is written from a tactical perspective. Kuwahara et al. (1999) describe a model which optimizes the LNG value chain for the Brazilian Amazonas Region. Studies have concluded that LNG is the best fuel to generate electric power in the area, and the model minimizes the costs of investment and operating cost for an LNG supply chain. The output is optimal capacities of the liquefaction plant, vaporization plants, storage tanks, ships, the number of ships and trips per ship.

The focus in the ship routing literature has been on how to improve the maritime operations and less on the overall performance throughout the value chain. The LNG market is expected to turn towards more spot-trade, and thus become more flexible. This leads to a need of a more market oriented perspective in the models. The focus will turn the object function from cost minimizing of shipping to profit maximizing of the whole value chain (Wallace and Fleten, 2003). One of the first attempts to combine supply chain management and inventory routing in the LNG value chain was Grønhaug and Christiansen (2009), who called the problem LNG Inventory Routing Problem (LNG-IRP).

Andersson et al. (2010) present two planning problems related to transportation planning and inventory management within the LNG supply chain. Transportation decisions are closely integrated with the lower and upper limit on the inventory levels at both the liquefaction and regasification facilities. The article therefore focuses on a subset of the supply chain; storage after liquefaction, shipping and storage before regasification. The first problem describes the planning decisions for a producer, controlling one liquefaction plant and serving several regasification terminals. The challenge is to sequence and schedule voyages and to assign them to ships. The second problem is for a vertically integrated company, which controls several liquefaction plants and serves a number of regasification terminals. The model maximizes the revenue from the sales minus operating cost, by designing routes, schedules and determining sales volumes. The two models operate at a tactical and operational level respectively.

Fodstad et al. (2010) present a model covering a larger part of the LNG supply chain. More details around a wide range of contractual issues are included and trading in a spot market is allowed. The model, called LNGScheduler, makes it easier to evaluate the effect and values of contracts. It also shows that a vessel does not have to be fully loaded to be most profitable, while the “common knowledge” in the industry is to always fill ships to their maximum to reduce the transportation cost. Both the LNGScheduler and the model in Grønhaug and Christiansen (2009) are planning on the tactical level of the value chain.

Werner et al. (2012) present a stochastic model, called LNGPlanner, which supports planning on the strategic level of the LNG value chain; from liquefaction, via shipping and regasification, to the sales market. It covers investment and disinvestments in infrastructure and vessels, chartering vessels, timing of contract start dates, while considering uncertainty in prices. Werner et al. (2012) conclude that it is typically economically beneficial to operate a larger fleet when trading on the spot market. They also show that arbitrage opportunities between different geographical areas can be exploited when the fleet is larger than required, and furthermore makes it possible to deliver more on contracts before and after arbitrage opportunities occurs.

5.8 This Work

The literature on floating LNG units is limited, and mainly focuses on the technological challenges of building a floating production unit. We did not find any articles on decision support tools where FLNG units are a part of the LNG value chain, and the articles written about the LNG value chain are mainly written on a tactical level. Our problem includes strategic decisions for all the links in the LNG value chain, from the fields to the end-markets and with longer horizon than most of the articles in this review. It is therefore not possible to use the level of detail used in some of the more specific models. The focus will be on which natural gas fields to develop, and whether an onshore liquefaction or a floating production unit should be used. Flexibility is an important aspect in our problem, and the formulation must allow both investments (develop or build) and disinvestments (close or sell) in all elements of the infrastructure, while most of the articles in the review only consider investments.

Chapter 6

Model Formulation

In this chapter we formulate a strategic model based on the problem presented in Chapter 4. It is a mixed integer linear programming model describing the LNG value chain from gas production to sale at the demand points. There is an inherent tradeoff between how much details should be modeled, and the tractability of the model. The challenge is to define the right assumptions such that the model will be computationally tractable, while it still gives a realistic view of the reality. In the first section of this chapter, we present the main assumptions underlying our model. A deterministic version of our model is then presented in Section 6.2 to keep the focus on how the value chain is modeled, before we expand this formulation to a stochastic model in the last section.

6.1 Main Assumptions

The problem studied in this thesis is to find optimal investments in a long horizon, in order to maximize the expected discounted profits. This can be seen as selecting between potential projects, where each project represents an investment opportunity. To find the optimal investments, some more tactical aspects like the production and flow of products through the value chain must also be modeled. However, the same level of detail as a typical tactical model is not needed, because of the long planning horizon and the focus on investments decisions.

A set of potential fields, pipelines, processing facilities and demand points are given, and the model decides when to invest in different projects. Production (and sales) rates at each node in the production network are also found. One player is assumed to control the whole value chain. This is not necessarily how the real world works, but the focus of this thesis is not on modeling the different players of the LNG supply chain. In the remainder of our discussions, we refer to onshore processing plants as *plants* and floating production units as *FLNG units*.

There is a special focus on FLNG units and what value they add. This means that the emphasis is not on representing everything in the value chain, but to include the aspects that highlight the difference between FLNG units and other alternatives. The FLNG units are modeled separately from the onshore plants, and allowed to move between fields. This flexibility is an important advantage of FLNG production units compared to the traditional onshore plants. The model also allows for several other types of flexibility. Investments and disinvestments (closing down infrastructure) can be made in any time period, and production can be started and stopped. Capacity expansion is a type of flexibility that is only partially implemented in the model. An extra pipeline can for example be built between plants and fields, but plants cannot be expanded. This is further discussed in Section 10.2 Future Work.

6.1.1 Valuation of Uncertain Cash Flows

Oil and gas investments typically involve a high degree of uncertainty and big investments at an early stage of a project. The decision maker has to decide between investments with limited information about many aspects critical to the profitability of the project, like actual reserve sizes or sales prices. It is therefore important that our valuation of cash flows takes the uncertainty into account. The decision maker also has some flexibility; projects may for example be terminated or extended. This flexibility must also be represented in the model. In our discussion of literature in Chapter 5, real options were presented as a way of valuing flexibility when evaluating profitability under uncertainty. Our model does, however, contain complex and interdependent flexibilities in the projects, and finding a suitable traded twin asset is hard. Given the types of flexibility in our problem, we adopt a similar approach as Wang and de Neufville (2005); using stochastic programming to value investments with flexibility, instead of a market driven approach.

Net present value is used to value future cash flows. In combination with stochastic programming, this approach will value flexibility and make it possible to compare different cash flows occurring at different time periods. An important strength of using NPV is that it is well known and understood. Our decision support tool is thereby easier to use correctly, and the results easier to evaluate. The discount rate is assumed to be risk neutral in the sense that it is not adjusted for risks associated with specific projects. These uncertainties are handled through the scenario tree in the stochastic model. This assumption also leads to the assumption of a risk neutral decision maker. But it might still be hard to find an appropriate discount rate. The rate has to cover the needed return on capital of the company using the model, and is therefore left as an input parameter to the model.

6.1.2 Time: Discrete Periods of Variable Duration

The model is a discrete time model with several time periods. All values, like production rates or state of plants, are constant within each time period. All decisions, like fleet size, startup of fields or change in production, are made in the transition from one period to the next. By having many time periods of relatively short length, the real world can be represented more accurately; decisions can be made at more time points and processes are modeled in more detail. Increasing the number of time periods does, however, also increase the model size. To handle this trade off, time periods are allowed to have different durations. Decisions toward the end of the horizon are not actually going to be realized; they are a part of the model to give insight to what decisions to make today. There is also more available and reliable information about the near future, while events toward the end of the horizon are more uncertain. The first time periods are therefore shorter. The most important periods are thereby modeled in detail, while we still can solve the model with a long horizon. To implement this, all flows in the model are given as rates instead of absolute sizes, since rates are unaffected by the length of a period. The presentation in this chapter will not account for varying duration of time periods. This is done to make the mathematical formulation of the model more readable. More details on how the varying duration is implemented are given in Section 8.1.1.

Our investment problem has a long horizon, typically several decades. An important question is how to deal with events occurring after the end of the horizon. Excluding them might lead to suboptimal results. Future cash flows are discounted and the present value of costs and revenues far into the future are relatively low. Values towards the end of the horizon are also more uncertain, and the uncertainty increases as we get further into the future. The cash flows after the end of the horizon will therefore not significantly contribute to the net present value of the project and are, with one exception, excluded. Investment decisions towards the end of the horizon depend on events occurring after the horizon. An adjustment is made to avoid distortion towards the end of the horizon. This exception is further discussed in Section 6.2.6.

6.1.3 Investment- and Operational Costs

Cost modeling is an important part of our investment model. The cost of each potential infrastructure investment is divided in two parts: investment- and operational costs. The only exceptions are shipping and regasification costs which affect the onshore and offshore liquefaction solutions equally. Given the focus on the value of FLNG units, a variable unit cost will adequately describe these parts. In the remainder of this section, we further discuss investment costs and operational costs in that order.

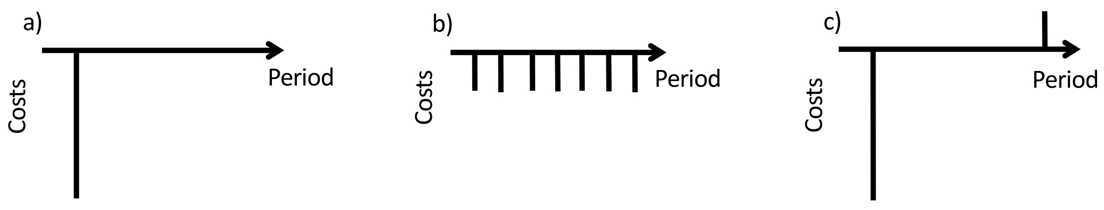

Figure 6-1: *Three ways of handling investment costs*

The LNG value chain is characterized by large *investment costs*. How these costs are distributed over time in the model, has a great impact on the investment decisions. Representing the cost as a big lump sum in the year of the investment, see Figure 6-1a, is intuitive, but leads to underinvestment close to the end of the time horizon. This, because the objective function includes the entire investment cost, but only the revenue from a few periods before the planning horizon ends; investments in the last time periods will not have a positive net present value. An alternative representation, shown in Figure 6-1b, that addresses this problem is to distribute the cost over the expected lifetime of the investment, for example as payments on a loan. Early work on our model did, however, show that this representation led to overinvestment towards the end of the horizon. The revenue of a field is typically large in the beginning and then declining when the production falls. The model pushed investments towards the end of the horizon such that only part of the investment cost would be included in the objective function; the whole decline phase of the field could be cut off.

We have chosen the representation shown in Figure 6-1c. All investment costs of an investment are converted into a single lump sum payment the year the investment decision is taken. In addition to this, an income is added for infrastructure that is still producing at the end of the horizon. This value represents the expected profit of the investments after the horizon and is calculated based on the remaining reserves. Solutions including investments toward the end of horizon are thereby incentivized by this value. Typical investment costs are not always a big lump sum, but rather several payments happening in different time periods. The costs do, however, generally occur mainly in the first years of a project, and we see little value of implementing a cost profile. The single lump sum can easily be calculated from a cost profile.

The operational cost is divided in two parts: A fixed yearly cost that is paid as long as the infrastructure is operational, and a variable cost that is dependent on the production rate. The variable cost is modeled as a single constant cost per unit produced. This gives the user the flexibility to model increasing or constant returns to scale, meaning that increasing the production leads to the same or decreasing

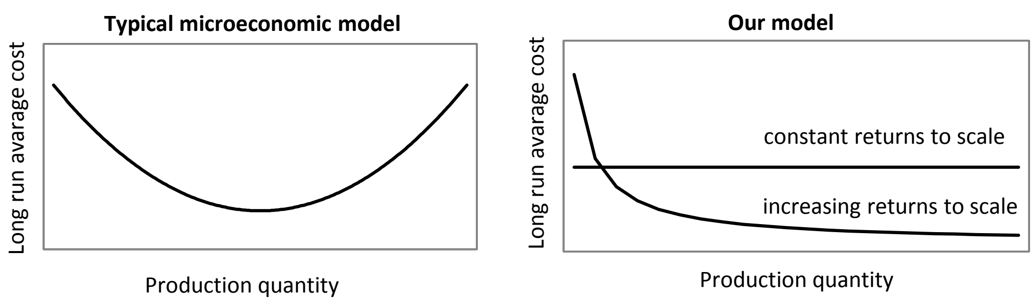

Figure 6-2: *Different approaches to modeling of operational costs*

average cost per unit. This could for example be due to some fixed costs or a learning effect. For very large production quantities, it is not reasonable that the average cost will continue to decrease and standard microeconomic theory often model decreasing returns to scale for higher production levels. The possible capacities of all infrastructure investments in our model are, however, limited to certain intervals. Within these limits, increasing or constant returns to scale is a reasonable assumption. Figure 6-2 compares a typical microeconomic model and our operational costs.

6.1.4 Production Modeling

Investors and field operators focus a significant amount of resources on reservoir assessment. Reservoir attributes, like field size and production profiles, have a big impact on profitability, which makes the choice of how to model them important. We want to represent the phases the production from fields usually go through: Build-up, plateau and decline. This can be achieved by defining a maximum production rate for each time period, forming a production profile, shown in Figure 6-3a. Production can take any value below the maximum, but if the model decides to produce less than the maximum rate, the surplus is assumed lost. A drawback of this approach is that to represent different extraction strategies, many potential production profiles must be specified. We also want the future maximum production to change depending on the chosen production levels. We have therefore chosen the method shown in Figure 6-3b. An equation limits the maximum rate contingent on the cumulative production, such that any surplus from production at a lower rate than maximum, results in an extended profile. The equations are described in more detail in Section 6.2.2.

The flow variables have a consistent unit in all links of the value chain. The unit used throughout the value chain is assumed to only include the components that are used in the LNG, not the impurities or other hydrocarbons that are removed in the early stages of the value chain. A typical well flow contains many components,

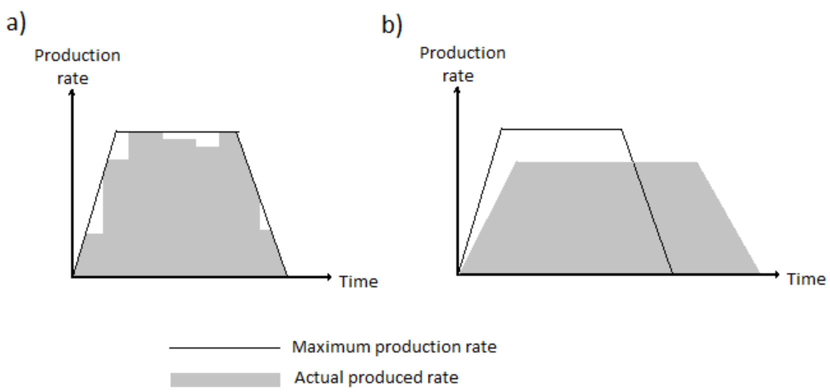

Figure 6-3: Different solutions on how to model the production profile

but the focus of this model is on the components that can be sold as natural gas. Any impurities that are costly to dispose of can be added to the variable cost. Components that increase the profitability of a given field, like LPG, can similarly be handled by subtracting this income from the costs of the field before the data is used in the model.

Several parts of the value chain also involve losses in the gas flow. Some gas is for example used in the compressors to maintain pressure in pipelines, or as part of the liquefaction process. When the LNG is transported, a small percentage boils off every day. These losses can also, like the impurities, be added as a variable cost, if the user wants to include them. Modeling these losses can be done by simply multiplying the flow through the infrastructure in question with a constant representing the percentage of gas that remains after the transportation or processing. They are not included in the model presented here to maintain focus on more significant aspects.

6.2 Deterministic Model

In this section, the deterministic model (D) is presented. The main components of the model are summarized in Figure 6-4, which shows the relationships between the elements in the value chain. Squared boxes represent processes, and the pipes and oval represent the transportation method between them. The first elements in the value chain are the fields, where the natural gas is extracted, before it is transported to a processing facility and converted to LNG. Two alternative processing methods are modeled: To transport the gas through pipelines to a processing plant on land, or to moor a floating processing unit (FLNG) to the sea floor over a field and then connect it to the field. From the processing facilities, the LNG is transported by ships to the demand points where the LNG is converted back to gas through regasification facilities. The following sections will present the various constraints and objective function of the model.

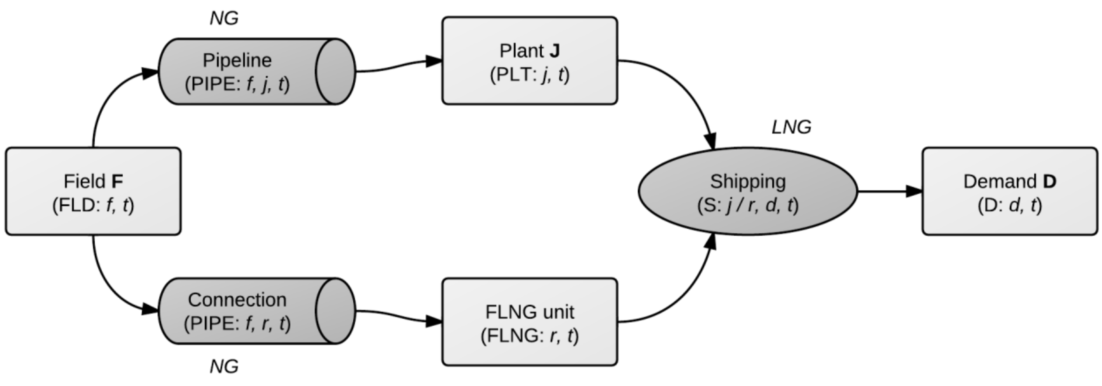

Figure 6-4: *Relationships between the elements in the LNG value chain*

Nomenclature

Sets and indexes

$a \in \boldsymbol{A}$	Contract group *a*
$d \in \boldsymbol{D}$	Demand point *d*
$f \in \boldsymbol{F}$	Field *f*
$j \in \boldsymbol{J}$	Potential onshore plants *j*
$k \in \boldsymbol{K}$	Potential contracts *k*
$k \in \boldsymbol{K}_a^{ALT}$	Set of disjunctive contracts
$r \in \boldsymbol{R}$	FLNG size *r*
$t, \tau \in \boldsymbol{T}$	Time period *t* and τ
$t \in \boldsymbol{G}_t^{FLD}$	Time gap of investment in fields. Set of time periods in which a field investment would be ready in period *t*
$t \in \boldsymbol{G}_t^{FLNG}$	Time gap of investments in FLNGs. Set of time periods in which an FLNG investment would be ready in period *t*
$t \in \boldsymbol{G}_t^{PIPE}$	Time gap of investments in pipelines. Set of time periods in which a pipeline investment would be ready in period *t*
$t \in \boldsymbol{G}_t^{PLT}$	Time gap of investments in plants. Set of time periods in which a plant investment would be ready in period *t*

Parameters

C^{CON}	Cost of connecting an FLNG unit to a field. Includes all switching costs
$C_f^{FLD\ INV}$	Investment cost of field *f*
$C_f^{FLD\ OPR\ FIX}$	Fixed operating cost of operating field *f*
$C_f^{FLD\ OPR\ VAR}$	Variable cost of operating field *f* per produced unit
$C_r^{FLNG\ INV}$	Investment cost of building a FLNG ship of size *r*
$C_r^{FLNG\ OPR\ FIX}$	Fixed cost of operating FLNG ship of size *r*
$C_r^{FLNG\ OPR\ VAR}$	Variable cost of operating FLNG ship of size *r* per produced unit
$C_{f,j}^{PIPE\ INV}$	Investment cost of a pipeline from field *f* to plant *j*
$C_{f,j}^{PIPE\ OPR\ FIX}$	Fixed cost of operating pipeline from field *f* to plant *j*
$C_{f,j}^{PIPE\ OPR\ VAR}$	Variable cost of operating a pipeline between field *f* and plant *j* per produced unit
$C_j^{PLT\ INV\ FIX}$	Fixed investment cost of plant *j*
$C_j^{PLT\ INV\ VAR}$	Variable investment cost of plant *j* dependent on processing capacity
$C_j^{PLT\ OPR\ FIX}$	Fixed operating cost of plant *j*
$C_j^{PLT\ OPR\ VAR}$	Variable operating cost of plant *j* per processed unit
C_d^{REG}	Variable regasification cost at demand point *d*
C^{SHIP}	Fixed cost per ship per period
$D_{d,t}$	Spot demand rate at demand point *d* in period *t*
E_t	Maximum investment in period *t*
F_f^{DEC}	Decline rate of field *f*
F^{INC}	Maximum increase in production in a field as a percentage of max production rate in the field
F_f^{MAX}	Max production rate of field *f*
F_f^{Q}	Total quantity of recoverable gas in field *f*
$H_{k,t}^{APL}$	Contract *k* is applicable (can be entered into) for period *t*. Each contract is only applicable for one period
$\underline{H}_{k,d,t}^{Q}$	Minimum yearly rate of gas delivered to demand point *d* in period *t* under contract *k*
$\overline{H}_{k,d,t}^{Q}$	Maximum yearly rate of gas delivered to demand point *d* in period *t* under contract *k*
I_f^{FLD}	Value per unit of gas remaining in field *f* at end of horizon if the field is producing in the last period
$I_{r,t}^{FLNG}$	Salvage value of an FLNG of size *r* in period *t*
N_t	Net present value of \$1, *t* periods into the future

$P_{k,t}^{CNT}$	Selling price of contract *k* per unit
$P_{d,t}^{SPOT}$	Spot selling price of LNG at demand point *d* in period *t* per unit
S^{CAP}	Transport capacity per ship
$S_{f,d}^{DIST\ F\rightarrow D}$	One way shipping time from field *f* to demand point *d*, in years
$S_{j,d}^{DIST\ J\rightarrow D}$	One way shipping time from plant *j* to demand point *d*, in years
$U_{f,t}^{FLD}$	Is 1 if field *f* can be developed in time period *t*, 0 otherwise
V_r^{FLNG}	Processing capacity per FLNG of size *r*
V^{PIPE}	Capacity of a pipeline
$\underline{V}_j^{PLT}$	Minimum processing capacity of plant *j*
$\overline{V}_j^{PLT}$	Maximum processing capacity of plant *j*

Variables

cf_t^{INC}	Cash flow from income in period *t*
cf_t^{INV}	Cash flow from investments in period *t*
cf_t^{OPR}	Cash flow from operations in period *t*
e_f	Remaining gas in field *f* at end of horizon. Set to zero if the field is not operational at the end of the horizon
EFP	Expected profit from fields after horizon
$g_{f,t}$	Cumulative production in field *f* up to period *t*
$q_{k,d,t}^{CNT}$	Rate of LNG delivered to demand point *d* in period *t* under contract *k*
$q_{f,j,t}^{F\rightarrow J}$	Rate of gas sent from field *f* to plant *j* in period *t*
$q_{f,r,d,t}^{FLNG}$	Rate of LNG sent from field *f* to demand point *d* through an FLNG of size *r* in period *t*
$q_{j,d,t}^{J\rightarrow D}$	Rate of LNG transported from plant *j* to demand point *d* in period *t*
$v_{j,t}^{PLT}$	Processing capacity of plant *j* that is being built in period *t*, 0 for all other *t*'s
$w_{f,r,t}^{CON}$	Is 1 if FLNG of size *r* is being disconnected from field *f* in period *t*, 0 otherwise
$w_{f,t}^{FLD}$	Is 1 if field *f* is being closed in period *t*, 0 otherwise
$w_{r,t}^{FLNG}$	Is 1 if FLNG of size *r* is sold in period *t*, 0 otherwise
$w_{f,j,t}^{PIPE}$	Is 1 if pipeline from field *f* to plant *j* is being closed in period *t*, 0 otherwise
$w_{j,t}^{PLT}$	Is 1 if plant *j* is being closed in period *t*, 0 otherwise
$x_{k,t}^{CNT}$	1 if contract *k* is sealed in period *t*, 0 otherwise
$x_{f,r,t}^{CON}$	Is 1 if a connection is being built between field *f* and an FLNG of size *r* in period *t*, 0 otherwise
$x_{f,t}^{FLD}$	Is 1 if field *f* is being developed in period *t*, 0 otherwise
$x_{r,t}^{FLNG}$	Number of FLNGs of size *r* being built in period *t*
$x_{f,j,t}^{PIPE}$	Is 1 if pipeline from field *f* to plant *j* is being built in period *t*, 0 otherwise
$x_{j,t}^{PLT}$	Is 1 if plant *j* is being built in period *t*, 0 otherwise
$y_{f,r,t}^{CON}$	Number of FLNGs of size *r* that field *f* is connected to in period *t*
$y_{f,t}^{FLD}$	Is 1 if field *f* can produce in period *t*, 0 otherwise
$y_{r,t}^{FLNG}$	Number of operational FLNGs of size *r* in period *t*
$y_{f,j,t}^{PIPE}$	Number of pipelines from field *f* to plant *j* can transport gas in period *t*
$y_{j,t}^{PLT}$	Is 1 if plant *j* of size *s* is operational in period *t*, 0 otherwise

6.2.1 Variables: Investment Decisions and Production Rates

The decision variables in the model can roughly be divided in two main groups: Investment- and production variables. The first group consist of the x, w and y variables, describing which infrastructure should be developed/constructed, closed/sold, and when the infrastructure is operational respectively. A superscript defines which element in the value chain they represent. The second group represent a simple long term production planning; how much to produce from each field in each time period and how to transport the gas to the end-market. These production rate variables are named q and describe the flow between different processes in the value chain.

A binary investment variable, x, is defined for each potential infrastructure investment and year. The only exception is for number of constructed FLNG units, $x_{r,t}^{FLNG}$, which is integer, since several FLNG units can be built each period. Constraints (D1) and (D2) ensure that fields and plants only are constructed once throughout the horizon. Pipelines and connections can, however, be built multiple times. In reality this pipeline expansion can represent an actual construction of a new pipeline, or an investment in a new compressor to increase the pressure, hence the capacity of the pipeline. The model is also allowed to connect FLNG units to the same field multiple times, since they can be moved between fields.

$$\sum_{t \in \boldsymbol{T}} x_{f,t}^{FLD} \leq 1, \qquad \forall\, f \in \boldsymbol{F} \tag{D1}$$

$$\sum_{t \in \boldsymbol{T}} x_{j,t}^{PLT} \leq 1, \qquad \forall\, j \in \boldsymbol{J} \tag{D2}$$

The binary w variables allow idle infrastructure to be closed down such that fixed operational cost, like maintenance, labor etc. do not have to be paid any more. Infrastructure cannot be reopened, so if the idle period is temporary, operational costs must be paid for the down time. Closing FLNG units also triggers a salvage value. Constraints (D3)-(D7) control the relationship between the investment variables. The state of the infrastructure, y, is constrained to match the construction and closing variables for each time period t.

$$y_{f,t-1}^{FLD} + \sum_{\tau \in \boldsymbol{G}_t^{FLD}} x_{f,\tau}^{FLD} - w_{f,t-1}^{FLD} = y_{f,t}^{FLD}\,, \qquad \forall\, f \in \boldsymbol{F}, t \in \boldsymbol{T} \tag{D3}$$

$$y_{f,j,t-1}^{PIPE} + \sum_{\tau \in \boldsymbol{G}_t^{PIPE}} x_{f,j,\tau}^{PIPE} - w_{f,j,t-1}^{PIPE} = y_{f,j,t}^{PIPE}\,, \qquad \forall\, f \in \boldsymbol{F}, j \in \boldsymbol{J}, t \in \boldsymbol{T} \tag{D4}$$

$$y_{j,t-1}^{PLT} + \sum_{\tau \in \boldsymbol{G}_t^{PLT}} x_{j,\tau}^{PLT} - w_{j,t-1}^{PLT} = y_{j,t}^{PLT}, \qquad \forall\, j \in \boldsymbol{J}, t \in \boldsymbol{T} \tag{D5}$$

$$y_{f,r,t}^{CON} + x_{f,r,t}^{CON} - w_{f,r,t}^{CON} = y_{f,r,t+1}^{CON}, \qquad \forall\, f \in \boldsymbol{F}, r \in \boldsymbol{R}, t \in \boldsymbol{T} \tag{D6}$$

$$y_{r,t-1}^{FLNG} + \sum_{\tau \in \boldsymbol{G}_t^{FLNG}} x_{r,\tau}^{FLNG} - w_{r,t-1}^{FLNG} = y_{r,t}^{FLNG}, \qquad \forall\, r \in \boldsymbol{R}, t \in \boldsymbol{T} \tag{D7}$$

The model supports a delay from the decision to invest in infrastructure until it becomes operational. This time gap is modeled by sets of time periods: $\boldsymbol{G_t^{FLD}}$, $\boldsymbol{G_t^{PIPE}}$, $\boldsymbol{G_t^{PLT}}$ and $\boldsymbol{G_t^{FLNG}}$. These are defined such that for example investments in a field in any of the time periods in $\boldsymbol{G_t^{FLD}}$ will be operational in period t. This is the least dense way to formulate the delay.

Gas field licenses usually contain an expiration date for the development of the field. The model therefore allows for limitations on what time periods the fields can be developed. Constraints (D8) limit development of field f to time periods specified by $U_{f,t}^{FLD}$:

$$x_{f,t}^{FLD} \leq U_{f,t}^{FLD}, \qquad \forall\, f \in \boldsymbol{F}, t \in \boldsymbol{T} \tag{D8}$$

All production and transportation variables are defined as yearly gas flow rates. They all have the same unit, measuring only deliverable LNG. Constraints (D9) enforce flow balance for each time period t, in each plant p:

$$\sum_{f \in \boldsymbol{F}} q_{f,j,t}^{F \to J} = \sum_{d \in \boldsymbol{D}} q_{j,d,t}^{J \to D}, \qquad \forall\, j \in \boldsymbol{J}, t \in \boldsymbol{T} \tag{D9}$$

6.2.2 Gas Production Modeling

Three equations with corresponding constraints limit the production rate in the fields: Maximum increase constraints (D10), maximum plateau rate constraints (D11) and decline constraints (D12); all shown in Figure 6-5.

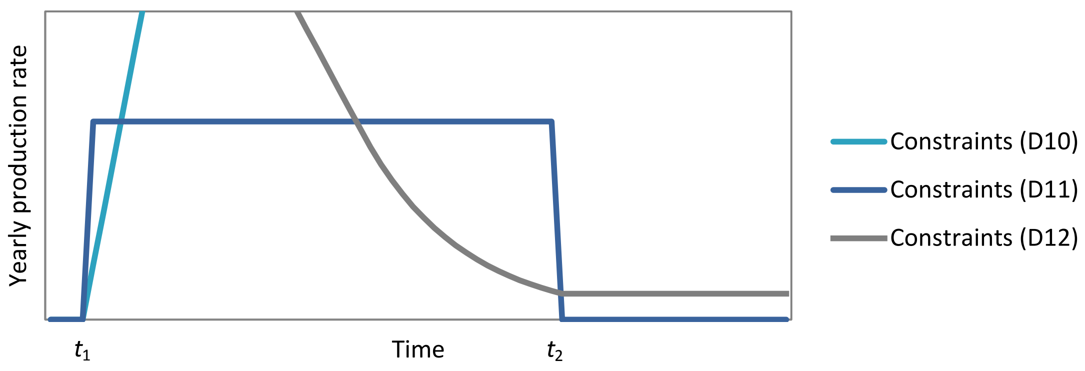

Figure 6-5: *Example of the maximum production expressed as the minimum of the constraints, given production start in t_1 and that the field is closed in t_2.*

The build-up phase is handled by Constraints (D10) which limit the increase in production, from one time period to the next, to a fixed volume for each field. The volume is given as a percentage of the maximum production rate:

$$\sum_{j \in \boldsymbol{J}} q_{f,j,t}^{F \to J} + \sum_{d \in \boldsymbol{D}} \sum_{r \in \boldsymbol{R}} q_{f,r,d,t}^{FLNG} \leq \left(\sum_{j \in \boldsymbol{J}} q_{f,j,t-1}^{F \to J} + \sum_{d \in \boldsymbol{D}} \sum_{r \in \boldsymbol{R}} q_{f,r,d,t-1}^{FLNG} \right) + F^{INC} F_f^{MAX} \qquad \forall f \in \boldsymbol{F}, t \in \boldsymbol{T} \tag{D10}$$

Nygreen et al. (1998) use minimum production levels to limit the amount of fluctuations in the production rates. We have not implemented this, since we want to allow temporary shutdown and restart of production. If production is temporarily shut down, it will take some time to increase the production rate to the maximum level again. This is also constrained by (D10), which ensures a gradually increasing production, and therefore also limits the amount of production fluctuations. The rate of increase in the build-up phase might differ from the increase after a temporary shutdown in the real world. Given our strategic long term focus, we have chosen to not capture this difference in the model.

The maximum production of field f in the plateau phase is defined by F_f^{MAX}. Constraints (D11) enforce this maximum rate for all time periods, and also ensure that a field only produces when it is operational. This is modeled by multiplying the maximum rate by $y_{f,t}^{FLD}$, which is 0 when the field is not operational.

$$\sum_{j \in \boldsymbol{J}} q_{f,j,t}^{F \to J} + \sum_{d \in \boldsymbol{D}} \sum_{r \in \boldsymbol{R}} q_{f,r,d,t}^{FLNG} \leq F_f^{MAX} y_{f,t}^{FLD}, \qquad \forall f \in \boldsymbol{F}, t \in \boldsymbol{T} \tag{D11}$$

After a certain amount of the gas is produced, the production rate declines due to e.g. reduced reservoir pressure. We follow the formulation in Nygreen et al. (1998), and assume an exponential decline rate, given by F_f^{DEC} for field f. Constraints (D12) enforce an exponential decline, once the cumulative production reaches a certain level:

$$\sum_{j\in \boldsymbol{J}} q_{f,j,t}^{F\to J} + \sum_{d\in \boldsymbol{D}} \sum_{r\in \boldsymbol{R}} q_{f,r,d,t}^{FLNG} \leq F_f^{DEC}\left(F_f^Q - g_{f,t}\right), \qquad \forall\ f \in \boldsymbol{F}, t \in \boldsymbol{T} \tag{D12}$$

Herein the cumulative production from field f up to, but not including, time period t is defined by:

$$g_{f,t} = \sum_{\tau=1..(t-1)} \left(\sum_{j\in \boldsymbol{J}} q_{f,j,\tau}^{F\to J} + \sum_{d\in \boldsymbol{D}} \sum_{r\in \boldsymbol{R}} q_{f,r,d,\tau}^{FLNG} \right), \qquad \forall\ f \in \boldsymbol{F}, t \in \boldsymbol{T}$$

This formulation will result in an extended production profile, if production is lower than the maximum rate. The model can represent a continuous specter of extraction strategies, while the number of binary variables is not increased.

6.2.3 Construction and Capacities of Plants and Pipelines

A set of potential plant locations $\boldsymbol{J}$ is given in the input data. When a new plant is built, a key decision is to choose which capacity the plant should have. One way of modeling these capacities is to create a set of potential plant sizes, where the model chose one when the investment decision is made. The choice of potential sizes might, however, significantly affect the solution, especially if few potential alternatives are specified. Increasing the number of alternatives lead to many binary variables. We have instead modeled a continuous specter of plant sizes, which reduces the number of binary variables and increases the number of continuous variables. Let $v_{j,t}^{PLT}$ denote the constructed capacity of plant j, constructed in time period t. The t-index is added to avoid non-linearities in the objective function. Constraints (D13) limit the construction capacity to an interval defined by minimum capacity $\underline{V}_j^{PLT}$ and maximum capacity $\overline{V}_j^{PLT}$:

$$\underline{V}_j^{PLT} x_{j,t}^{PLT} \leq v_{j,t}^{PLT} \leq \overline{V}_j^{PLT} x_{j,t}^{PLT}, \qquad \forall\ j \in \boldsymbol{J}, t \in \boldsymbol{T} \tag{D13}$$

The constraint also forces $v_{j,t}^{PLT}$ to 0 for all other time periods than the investment period.

The production quantities are limited by the investment decisions. Constraints (D14) ensure that the flow into, and thereby also out of, each plant is 0 when it is not operational ($y_{j,t}^{PLT}$ is 0). This is a big-M formulation, where the maximum capacities are used as big-M.

$$\sum_{f \in \boldsymbol{F}} q_{f,j,t}^{F \to J} \leq \overline{V}_j^{PLT} \, y_{j,t}^{PLT} \,, \qquad \forall \, j \in \boldsymbol{J}, t \in \boldsymbol{T} \tag{D14}$$

The flow through plants and pipelines are limited to the maximum capacities, $v_{j,\tau}^{PLT}$ and V^{PIPE}, by Constraints (D15) and (D16) respectively. Constraints (D15) is formulated with the sum of the capacity variables for all time periods $\tau \in \boldsymbol{T}$, since $v_{j,\tau}^{PLT}$ is non-zero only for the time period when plant j is built. Constraints (D16) also ensure that the flow through a pipeline is 0 when it is not operational.

$$\sum_{f \in \boldsymbol{F}} q_{f,j,t}^{F \to J} \leq \sum_{\tau \in \boldsymbol{T}} v_{j,\tau}^{PLT} \,, \qquad \forall \, j \in \boldsymbol{J}, t \in \boldsymbol{T} \tag{D15}$$

$$q_{f,j,t}^{F \to J} \leq V^{PIPE} \, y_{f,j,t}^{PIPE} \,, \qquad \forall \, f \in \boldsymbol{F}, j \in \boldsymbol{J}, t \in \boldsymbol{T} \tag{D16}$$

6.2.4 Floating LNG Production Units

FLNG production units have some special characteristics that require them to be modeled separately from onshore plants. Instead of specifying each potential unit, like the plants were specified, the model is given a set of potential FLNG unit *sizes*. Several FLNG units of each size can be built or sold each period. They are not tied to a certain position; instead they can be moved between fields. This is formulated by having a variable, $y_{f,r,t}^{CON}$, that counts the number of FLNG units of size r that is connected to field f at any time period t. This formulation allows for several FLNG units to be connected to a field at the same time. Constraints (D17) ensure that the number of connections between fields and FLNG units does not exceed the number of available FLNG units, $y_{r,t}^{FLNG}$:

$$\sum_{f \in \boldsymbol{F}} y_{f,r,t}^{CON} \leq y_{r,t}^{FLNG}, \qquad \forall \, r \in \boldsymbol{R}, t \in \boldsymbol{T} \tag{D17}$$

Our formulation gives the model full flexibility to move the FLNG units at any time, by disconnecting a unit from one field and connect it to a new one. This flexibility is an advantage of the floating production solutions. The next constraints ensure that the flow of gas from a field through FLNG production units does not exceed the capacity of the FLNG units that are connected to that field:

$$\sum_{d\in \boldsymbol{D}} q_{f,r,d,t}^{FLNG} \leq V_r^{FLNG} y_{f,r,t}^{CON}, \qquad \forall\, f \in \boldsymbol{F}, t \in \boldsymbol{T}, r \in \boldsymbol{R} \tag{D18}$$

In addition to the fixed and variable cost of the FLNG unit, there is a cost of moving an FLNG from one field to another. This includes disconnection costs, transportation costs and new connection costs as well as lost production. These costs are aggregated to one cost in the model, called connection cost. It will ensure that the FLNG units are not moved around after only a short period of production at one field. This cost is part of the objective function presented in Section 6.2.6.

6.2.5 Contracts and Spot Demand

The majority of LNG is sold on long term contracts, even though the spot market is growing. A set of potential contracts, $k \in \boldsymbol{K}$, is given as input to the model, and the binary variable x_k^{CNT} is set to one if a contract is sealed. Each contract can include delivery to several demand points, d, and time periods, t. Quantum requirements are specified by lower and upper limits, $\underline{H}_{k,d,t}^{Q}$ and $\overline{H}_{k,d,t}^{Q}$ respectively. Constraints (D19) ensure that the delivered rates, $q_{k,d,t}^{CNT}$, are within these intervals, or zero if the contract is not sealed.

$$\underline{H}_{k,d,t}^{Q} x_k^{CNT} \leq q_{k,d,t}^{CNT} \leq \overline{H}_{k,d,t}^{Q} x_k^{CNT}, \qquad \forall\, k \in \boldsymbol{K}, d \in \boldsymbol{D}, t \in \boldsymbol{T} \tag{D19}$$

In reality, contracts often contain punishments for deviations from the delivery interval. This is, however, not the focus of our model; the advantage of including this is not big enough to justify the increased complexity.

Several variations of one contract may be defined as different contracts, and specified as mutually exclusive. Constraints (D20) ensure that at most one contract k within a contract group $a \in \boldsymbol{A}$ can be sealed. $\boldsymbol{K_a^{ALT}}$ is a subset of contracts that are mutually exclusive.

$$\sum_{k\in \boldsymbol{K_a^{ALT}}} x_k^{CNT} \leq 1, \qquad \forall\, a \in \boldsymbol{A} \tag{D20}$$

The demand for LNG on the spot market is defined as $D_{d,t}$ for demand point d in time period t. The delivery from the plants and the FLNGs constitutes the total delivery to demand point d. To get the spot delivery, the amount sold on contracts must be subtracted. Constraints (D21) ensure that the amount of LNG sold on the spot market does not exceed the spot demand.

$$\sum_{j\in \boldsymbol{J}} q_{j,d,t}^{J\to D} + \sum_{f\in \boldsymbol{F}} \sum_{r\in \boldsymbol{R}} q_{f,r,d,t}^{FLNG} - \sum_{k\in \boldsymbol{K}} q_{k,d,t}^{CNT} \leq D_{d,t}, \qquad \forall\, d \in \boldsymbol{D}, t \in \boldsymbol{T} \tag{D21}$$

Constraints (D22) ensure that the sale in the spot market is positive. A negative sales rate would correspond to exploiting arbitrage opportunities by buying gas in the spot market, and selling it on the contracts. If this behavior is wanted, Constraints (D22) can easily be removed.

$$\sum_{j \in \boldsymbol{J}} q_{j,d,t}^{J \to D} + \sum_{f \in \boldsymbol{F}} \sum_{r \in \boldsymbol{R}} q_{f,r,d,t}^{FLNG} - \sum_{k \in \boldsymbol{K}} q_{k,d,t}^{CNT} \geq 0, \qquad \forall\, d \in \boldsymbol{D}, t \in \boldsymbol{T} \tag{D22}$$

6.2.6 Incentivizing Investments toward the End of the Horizon

In Section 6.1.3, we discussed how to handle the end of the time horizon. Without some valuation of the final state, there will be no investments toward the end. This is because the investment costs are within the horizon and a part of the objective function, while the expected future income is not. To incentivize such investments, a value is calculated for remaining gas in fields that are producing at the end of the horizon. We assume that production is going to continue after the horizon, which is the behavior that best corresponds to the real world. The discounted Expected Future Profit from the producing fields is denoted EFP, and will be added to the objective function. It is important to point out that the goal is not to give an accurate valuation of the gas reserves. If that were the case, undeveloped fields had to be given a value as well. The motivation for including EFP is to make investment decisions toward the end of the horizon more realistic. To calculate EFP we define e_f, the remaining recoverable gas in field f at the end of the horizon. Constraints (D23) ensure that it cannot take a bigger value than what is actually remaining:

$$e_f \leq F_f^Q - g_{f,\mathrm{T}}, \qquad \forall\, f \in \boldsymbol{F} \tag{D23}$$

F_f^Q is the total recoverable gas, and the cumulative production, $g_{f,\mathrm{T}}$, is subtracted from this. e_f does not need a constraint on how low value it can take, since it will add a positive value to the objective function.

Only the fields that are developed and operational should be included in the calculation of EFP. If I_f^{FLD} is the expected profit from selling one unit of gas, we could set $EFP = I_f^{FLD} \cdot y_{f,T}^{FLD} \cdot e_f$ to value the remaining gas. There are, however, two problems with this expression. Firstly $y_{f,T}^{FLD}$ only ensures that a field is operational, not that it is producing. Using this variable could lead to the model investing in all fields at the end of the horizon, without investing in any connecting infrastructure like processing plants or pipelines. EFP would then be added to the objective function without having paid the full investment cost. We therefore check if there is production in the last period, which will ensure that investments have been made throughout the whole value chain. The second problem is that the expression is not linear. We linearize it by forcing e_f to zero for fields that are not producing in the last period:

$$e_f \leq F_f^Q \left(\sum_{j \in \boldsymbol{J}} q_{f,j,T}^{F \to J} + \sum_{d \in \boldsymbol{D}} \sum_{r \in \boldsymbol{R}} q_{f,r,d,T}^{FLNG} \right), \qquad \forall\, f \in \boldsymbol{F} \tag{D24}$$

This is a big-M implementation. The expression within the parentheses is the production in the last period, and the total amount of recoverable gas in field, F_f^Q, is used as a big-M. This constraint lets e_f take any value as long as the field is producing in the last period, or forces it to zero if there is no production. The constraints might also have a different effect. The model could extend production to the last period to let e_f take a value. This will, however, have minimal effect on the total discounted profit, and the actual production rates toward the end of the horizon are not going to be realized. We can now define EFP:

$$EFP = \sum_{f \in \boldsymbol{F}} I_f^{FLD} e_f$$

6.2.7 Objective Function

The objective of the model is to maximize total NPV adjusted profit for the whole horizon. The NPV function is non-linear, and a parameter representing the reduction in value of revenues and costs, is therefore calculated in advance. N_t is the present value of one dollar, t periods into the future:

$$N_t = \frac{1}{[1+r]^t}$$

Here r is the real required return on capital. Inflation must not be included since the costs and prices are given in 2012 dollars throughout the model. We can now define the objective function:

$$\max z = \sum_{t \in \boldsymbol{T}} N_t \left(cf_t^{INC} - cf_t^{INV} - cf_t^{OPR} \right) + N_T\, EFP$$

The first term is the sum of the cash flows occurring each period multiplied by the NPV parameter. cf_t^{INC}, cf_t^{INV} and cf_t^{OPR} are the cash flows from income, investments and operational expenses respectively. The second term is not an actual income, but the cash flow added for infrastructure that is still producing at the end of the horizon, EFP. Since this is a onetime value occurring at the end of the horizon it is discounted with N_T. The cash flows from investments are defined as:

$$
\begin{aligned}
cf_t^{INV} = &\sum_{f \in \boldsymbol{F}} C_f^{FLD\ INV} x_{f,t}^{FLD} \\
&+ \sum_{f \in \boldsymbol{F}} \sum_{j \in \boldsymbol{J}} C_{f,j}^{PIPE\ INV} x_{f,j,t}^{PIPE} \\
&+ \sum_{j \in \boldsymbol{J}} C_j^{PLT\ INV\ FIX} x_{j,t}^{PLT} + \sum_{j \in \boldsymbol{J}} C_j^{PLT\ INV\ VAR} v_{j,t}^{PLT} \\
&+ \sum_{r \in \boldsymbol{R}} C_r^{FLNG\ INV} x_{r,t}^{FLNG}
\end{aligned}
$$

The four lines represent lump sum investment costs in fields, pipelines, plants and FLNG production units respectively. The x variables are set to one for the construction year of different infrastructure and C parameters give the investment costs. Since onshore plants can be built within a continuous interval of capacities, the investment cost is split in two terms. The first term is a fixed base cost of constructing; this is similar to the investment cost of the other parts of the value chain. The second term describes the investment cost per unit of capacity. $v_{j,t}^{PLT}$ is the constructed capacity of plant j and will, as we described in Section 6.2.3, only be non-zero for the period the plant was constructed, t. This formulation allows representation of economies of scale for the plant investments. The operational expenses are defined as:

$$
\begin{aligned}
cf_t^{OPR} = &\sum_{f \in \boldsymbol{F}} C_f^{FLD\ OPR\ FIX} y_{f,t}^{FLD} + \sum_{f \in \boldsymbol{F}} C_f^{FLD\ OPR\ VAR} \left(\sum_{j \in \boldsymbol{J}} q_{f,j,t}^{F \to J} + \sum_{d \in \boldsymbol{D}} \sum_{r \in \boldsymbol{R}} q_{f,r,d,t}^{FLNG} \right) \\
&+ \sum_{f \in \boldsymbol{F}} \sum_{j \in \boldsymbol{J}} C_{f,j}^{PIPE\ OPR\ FIX} y_{f,j,t}^{PIPE} + \sum_{f \in \boldsymbol{F}} \sum_{j \in \boldsymbol{J}} C_{f,j}^{PIPE\ OPR\ VAR} q_{f,j,t}^{F \to J} \\
&+ \sum_{j \in \boldsymbol{J}} C_j^{PLT\ OPR\ FIX} y_{j,t}^{PLT} + \sum_{j \in \boldsymbol{J}} C_j^{PLT\ OPR\ VAR} \sum_{d \in \boldsymbol{D}} q_{j,d,t}^{J \to D} \\
&+ \sum_{f \in \boldsymbol{F}} \sum_{r \in \boldsymbol{R}} C^{CON} x_{f,r,t}^{CON} \\
&+ \sum_{r \in \boldsymbol{R}} C_r^{FLNG\ OPR\ FIX} y_{r,t}^{FLNG} + \sum_{r \in \boldsymbol{R}} C_r^{FLNG\ OPR\ VAR} \sum_{f \in \boldsymbol{F}} \sum_{d \in \boldsymbol{D}} q_{f,r,d,t}^{FLNG} \\
&+ \sum_{j \in \boldsymbol{J}} \sum_{d \in \boldsymbol{D}} \left(\frac{2 * S_{j,d}^{DIST\ J \to D}\ C^{SHIP}}{S^{CAP}} \right) q_{j,d,t}^{J \to D} \\
&\qquad + \sum_{f \in \boldsymbol{F}} \sum_{d \in \boldsymbol{D}} \sum_{r \in \boldsymbol{R}} \left(\frac{2 * S_{f,d}^{DIST\ F \to D}\ C^{SHIP}}{S^{CAP}} \right) q_{f,r,d,t}^{FLNG} \\
&+ \sum_{d \in \boldsymbol{D}} C_d^{REG} \left(\sum_{j \in \boldsymbol{J}} q_{j,d,t}^{J \to D} + \sum_{f \in \boldsymbol{F}} \sum_{r \in \boldsymbol{R}} q_{f,r,d,t}^{FLNG} \right)
\end{aligned}
$$

The first line in this expression represents the operational costs from fields. The first term is a fixed operating cost for every period the field is operational. The second term consists of the variable operating cost, $C_f^{FLD\ OPR\ VAR}$, which is multiplied by the total flow out of a field that period. Line two, three and five similarly describe the operational costs of pipelines, plants and FLNG production units. The flow of gas through an FLNG unit, $q_{f,r,d,t}^{FLNG}$, is indexed with the size of the unit, r, since FLNG units of different sizes are allowed to have different variable operating costs. The fourth line is a simple onetime payment for connecting an FLNG unit to a field.

Shipping costs are represented by the sixth line. The shipping fleet is assumed homogenous, and all ships have the same capacity. The costs of transporting one unit of LNG are found by multiplying the distance, S^{DIST}, between the production unit and the demand point by the fixed cost of a ship, and divide it by the capacity of each ship, S^{CAP}. The expressions are multiplied by two since S^{DIST} is defined as one way shipping times including loading. The return voyage and unloading takes roughly the same time. By multiplying this value with the LNG rate from the production units, the terms describe the shipping costs. This formulation assumes that the fleet can be expanded or reduced from period to period, and also allows fractional ships. This is, however, not an unreasonable assumption, since real voyages often start in one period and finishes in the next, which corresponds to fractional ships. Modeling a heterogeneous fleet or including routing of ships would not differentiate FLNG units from onshore plants; these costs are the same for both solutions. These aspects are therefore not prioritized in the model. Regasification costs are represented in the last term of the expression. We assume that regasification capacity can be rented for a given variable cost C_d^{REG}.

The third expression used in the objective function is the cash flow from income. It is defined as:

$$
\begin{aligned}
cf_t^{INC} = & \sum_{d \in \boldsymbol{D}} P_{d,t}^{SPOT} \left[\left(\sum_{j \in \boldsymbol{J}} q_{j,d,t}^{J \to D} + \sum_{f \in \boldsymbol{F}} \sum_{r \in \boldsymbol{R}} q_{f,r,d,t}^{FLNG} \right) - \sum_{k \in \boldsymbol{K}} q_{k,d,t}^{CNT} \right] \\
& + \sum_{d \in \boldsymbol{D}} \sum_{k \in \boldsymbol{K}} P_{k,t}^{CNT} q_{k,d,t}^{CNT} \\
& + \sum_{r \in \boldsymbol{R}} I_{r,t}^{FLNG} w_{r,t}^{FLNG}
\end{aligned}
$$

The first line is the income from spot sales. $P_{d,t}^{SPOT}$ is the spot price in demand point d which is multiplied by the gas available for spot sale. That amount is found by subtracting the quantity sold under contracts, $q_{k,d,t}^{CNT}$, from the total sellable gas. The second line is, similarly, the income from contract sales. The third line represents salvage values from sale of FLNG production units. The salvage value parameter has a t index such that it can vary with the year it is sold. Since the model only keeps track of how many FLNG units are operational in a given period, and does not have an index for each individual unit, the salvage value cannot depend on the actual age of the FLNG unit being sold.

There are several parts of the value chain that could have been modeled with a salvage value. Onshore plants do for example contain large amounts of steel that can be sold. But there are also many expenses associated with closing down gas infrastructure. The cost of dismantling and cleaning up might even exceed the salvage value of the structures. We have therefore not included any salvage value for most of the infrastructure. The only exception is FLNG production units. The reason for this is that since they are floating, they are easier to move and sell to other companies. A central part of this thesis is to look at the difference between onshore and floating solutions. Salvage is indeed an aspect that highlights this difference and we therefore include it for FLNG units.

One drawback of having a salvage value is that the model might sell all FLNG units at the end of the horizon to get the salvage value. This does not correspond with realistic behavior if the field is in the middle of its lifetime. But the EFP will to some degree prevent this from happening, and the income from salvage at the end of the horizon will be so heavily discounted that it does not significantly affect the total discounted profits.

The model also allows for setting a budget that limits the investments in each period. In most situations, companies have limited access to capital. E_t is defined as the maximum investment amount in period t. Constraints (D25) ensure that this limit is not exceeded.

$$cf_t^{INV} \leq E_t, \qquad \forall\, t \in \boldsymbol{T} \tag{D25}$$

6.2.8 Binary, Integer and Non-negativity Constraints

Constraints (D26) and (D27) ensure that the binary variables are assigned the value 0 or 1. The variables $y_{f,t}^{FLD}$ and $y_{j,t}^{PLT}$ are binary, but Constraints (D3) and (D5) respectively, in combination with (D26) and (D27), ensure that they take a binary value, and no extra constraints are needed.

$$x_{f,t}^{FLD},\, x_{f,j,t}^{PIPE},\, x_{j,t}^{PLT}, x_{f,r,t}^{CON}, x_{r,t}^{FLNG}, x_k^{CNT} \in \{0,1\}, \qquad \forall\, f \in \boldsymbol{F}, j \in \boldsymbol{J}, r \in \boldsymbol{R}, \text{t} \in \boldsymbol{T} \tag{D26}$$

$$w_{f,t}^{FLD}, w_{f,j,t}^{PIPE}, w_{j,t}^{PLT}, w_{f,r,t}^{CON}, w_{r,t}^{FLNG} \in \{0,1\}, \qquad \forall\, f \in \boldsymbol{F}, j \in \boldsymbol{J}, r \in \boldsymbol{R}, \text{t} \in \boldsymbol{T} \tag{D27}$$

Building and selling variables for FLNGs are forced to be integer by Constraints (D28). $y_{f,j,t}^{PIPE}$, $y_{f,r,t}^{CON}$ and $y_{r,t}^{FLNG}$ must also take integer values, but this is ensured by the relationship Constraints (D4), (D6) and (D7), and binary and integer Constraints (D26)-(D28), and do not need to be constrained.

$$x_{r,t}^{FLNG}, w_{r,t}^{FLNG} \in \mathbb{Z}, \qquad \forall\, r \in \boldsymbol{R}, \mathrm{t} \in \boldsymbol{T} \tag{D28}$$

Constraints (D29) force all variables determining the operational state of infrastructure to be non-negative, and thereby ensuring that no infrastructure is sold or closed before it is built.

$$y_{f,t}^{FLD}, y_{f,j,t}^{PIPE}, y_{j,t}^{PLT}, y_{f,r,t}^{CON}, y_{r,t}^{FLNG} \geq 0, \qquad \forall\, f \in \boldsymbol{F}, j \in \boldsymbol{J}, r \in \boldsymbol{R}, \mathrm{t} \in \boldsymbol{T} \tag{D29}$$

The natural gas is only allowed to flow in one direction, from the field to the demand point, through a plant or an FLNG. The production rates are defined positive in that direction, and Constraints (D30) enforce non-negativity.

$$q_{k,d,t}^{CNT}, q_{f,j,t}^{F\rightarrow J}, q_{j,d,t}^{J\rightarrow D}, q_{f,r,d,t}^{FLNG} \geq 0, \qquad \forall\, f \in \boldsymbol{F}, j \in \boldsymbol{J}, d \in \boldsymbol{D}, r \in \boldsymbol{R}, \mathrm{t} \in \boldsymbol{T} \tag{D30}$$

6.3 Expanding the Model to Incorporate Uncertainty

In this section, the deterministic model is expanded to a stochastic version. Uncertainties in some of the input parameters are represented in the model, such that the different realizations can be accounted for in the solution. In the stochastic model, different realizations of the uncertain parameters are represented as discrete scenarios. Each combination of parameter realizations constitutes a scenario. This means that the number of scenarios will increase quickly as the number of uncertain parameters increases. Two uncertain parameters with three realizations each will give $3^2 = 9$ scenarios, while ten parameters with three realizations give $3^{10} = 59049$ scenarios. Most of the parameters in the model are uncertain, but representing all of them is intractable. This raises the important question of which parameters should be modeled with uncertainty.

The focus of the model is to highlight differences between floating production units and onshore plants. Parameters that affect these options differently are therefore prioritized. Total amount of recoverable gas in the fields is one such parameter. Floating production units have greater ability to respond to uncertain field sizes since they can be moved between fields. This uncertainty is included to allow the model to value the flexibility of the floating production solutions. Another interesting parameter is gas sales price. It will affect both floating and onshore production, but it is also a highly uncertain parameter that has a great impact on the overall profitability of investments. This uncertainty is therefore interesting to include in the model. Several other parameters could also be included. The costs of different parts of the value chain are for example uncertain, especially for new technologies associated with floating liquefaction units. But to avoid too many scenarios, we have chosen to focus on field sizes and gas prices.

In the literature review, we covered the difference between decision dependent and decision independent uncertainty. The stochastic model presented in this chapter is based on decision independent uncertainty. The sales are assumed to not be big enough to affect the price in the market; we assume that we are price takers. Given this assumption, gas prices are a typical example of decision independent parameters. The gas price is determined in the market and evolves over time independently of the decisions taken in the model. The size of the gas fields are, on the other hand decision dependent. The timing of the resolution of the uncertainty relies on the decisions of the model. Information about the size of the field becomes available as an operator develops the field and produces the gas. This relationship could be modeled following the approach of Goel and Grossman (2004). But these models are very large and can be hard to solve. Given our long horizon we have instead chosen to model the field size as a decision independent parameter. The uncertainty is resolved at set points in time, independent of when the field is developed. These points in time could for example correspond to planned seismic surveys or other events that would increase the knowledge of the field size.

6.3.1 A Multistage Stochastic Model

Section 6.2 gives a detailed description of the deterministic model. The stochastic model (S) is an extension of this model, and only the differences between the models will be presented in this section. The full stochastic formulation can be found in Appendix C.

Each uncertain parameter is unknown in the beginning of the planning horizon. At certain time periods, this uncertainty is resolved, and the parameter takes one of several possible values, each corresponding to different scenarios. Our stochastic model is a multistage model, which means that the time horizon is divided into several stages. The resolution of one or more of the uncertain parameters marks the beginning of a new stage. Let $\omega \in \mathbf{\Omega}$ denote the discrete scenarios. An extra index, ω, is added to the uncertain parameters, such that they can take different values for different scenarios. The uncertain parameters in the model are field size $F_{f,\omega}^{Q}$, spot price $P_{d,t,\omega}^{SPOT}$ and contract price $P_{k,t,\omega}^{CNT}$. All other parameters are unchanged from the deterministic problem.

Decisions can be made at any of the time periods from the deterministic model. In the deterministic model, *one* decision plan for the entire horizon was found. In the stochastic model, the decisions must be adapted to the realizations of the different uncertain parameters; the decision variables are allowed to vary with the different scenarios. An index for scenario ω is added to every variable. All constraints from the deterministic problem will therefore also be scenario dependent, and the number of these constraints will be multiplied by the number of scenarios. To ensure that decisions taken in different scenarios are consistent with the information available in each time period, non-anticipativity constraints are added. They force the solutions in the different scenarios to be equal until the uncertainty is resolved and the solutions can be adapted to the individual scenario.

The non-anticipativity constraints can be formulated by forcing the variable in each scenario to be equal to the average of the variables in all the scenarios. This leads to Ω constraints per variable, where Ω is the number of scenarios. For binary variables, one constraint is sufficient, since the average of binary variables must be either 1 or 0. The disadvantage of the formulation is that it leads to a dense coefficient matrix; many variables are involved in each constraint. Dense coefficient matrixes require more mathematical operations when the model is solved. A single non-anticipativity constraint for binary variables would also result in weaker LP relaxations.

We have chosen a chained formulation, where each scenario is forced to be equal to the next one, and at most $\Omega - 1$ constraints are required for each variable. Non-anticipativity constraints are created for all variables that cannot be calculated from other variables. Let $u_{t,\omega}$ represent one such variable in period t and scenario ω. This general variable represents all building variables x, all closing variables w, all rates q and the plant capacity $v_{j,t,\omega}^{PLT}$. The general formulation for these constraints is given in:

$$u_{t,\omega} - u_{t,\omega+1} = 0, \quad \forall\ t \in \boldsymbol{T}, \omega \in \boldsymbol{\Omega} \mid \Phi_{t,\omega} = 1 \qquad \text{(S31)-(S46)}$$

Where $\Phi_{t,\omega}$ represents the scenario tree:

$$\Phi_{t,\omega} = \begin{cases} 1 & \text{if variables in time period t and scenario } \omega \text{ should} \\ & \text{equal the variables in scenario } \omega + 1 \\ 0 & \text{otherwise} \end{cases}$$

All variables not represented by $u_{t,\omega}$ can be calculated from the x, w, q and $v_{j,t,\omega}^{PLT}$ variables. They will therefore implicitly be forced to follow the scenario dependency given by the parameter $\Phi_{t,\omega}$, and no non-anticipativity constraints are needed for these variables.

The x variables for contracts do not have a time period index; this is not needed in the deterministic model. In the stochastic model, the non-anticipativity constraints ensure that the decision to seal a contract does not use foresight when selecting which contracts to seal. The decision period must therefore be defined explicitly. The parameter $H_{k,t}^{APL}$ is defined as 1 in the period the decision to seal a contract or not is taken, and 0 in all other periods. The non-anticipativity constraints for the contracts are defined as:

$$H_{k,t}^{APL} x_{k,\omega}^{CNT} - H_{k,t}^{APL} x_{k,\omega+1}^{CNT} = 0, \qquad \forall\, k \in \boldsymbol{K}, \boldsymbol{t} \in \boldsymbol{T}, \omega \in \boldsymbol{\Omega} \mid \Phi_{t,\omega} = 1 \tag{S36}$$

The amount of gas in a field is uncertain until it is resolved in a time period. The recoverable quantity will vary between the scenarios. It is important that the production up to the time period where the uncertainty is resolved does not exceed the quantity amount in the scenario with the smallest recoverable quantity in the fields. The decline rate constraints, (S12), make sure that the production in each scenario does not exceed the available amount. Since the non-anticipativity constraints ensure that the production in each scenario is equal until the uncertainty is resolved, none of the scenarios will produce more than the available amount in the scenario with smallest fields, before the uncertainty is resolved. Constraint (S12) will only be binding for the scenario with the smallest fields, until the uncertainty is resolved.

$$\sum_{j \in \boldsymbol{J}} q_{f,j,t,\omega}^{F \rightarrow J} + \sum_{d \in \boldsymbol{D}} \sum_{r \in \boldsymbol{R}} q_{f,r,d,t,\omega}^{FLNG} \leq F_f^{DEC} \left(F_{f,\omega}^{Q} - g_{f,t,\omega} \right), \ \ \forall f \in \boldsymbol{F}, t \in \boldsymbol{T}, \omega \in \boldsymbol{\Omega} \tag{S12}$$

The stochastic model maximizes the expected profit. All cash flows are scenario dependent and profit from each scenario is calculated in the same way as in the deterministic model. The expected profit in the stochastic model is the sum of the profit in each scenario multiplied with the probability π_ω of the scenario:

$$\max z = \sum_{\omega \in \boldsymbol{\Omega}} \pi_\omega \left[\sum_{t \in \boldsymbol{T}} N_t \left(cf_{t,\omega}^{INC} - cf_{t,\omega}^{INV} - cf_{t,\omega}^{OPR} \right) + N_T\, EFP_\omega \right]$$

Chapter 7

Dantzig-Wolfe Decomposition

The stochastic model presented in Chapter 6 involves many binary variables, and the number of variables grows quickly with the size of the input data. All decision variables and constraints depend on the scenarios, and the number of scenarios is exponential in the number of uncertain parameters. This is known as the *curse of dimensionality*. As the number of scenarios grows, the model fast becomes computationally intractable. We have therefore applied a decomposition method to the stochastic model to reduce the run time. In this chapter we first present the theory of the decomposition method, before we apply it to our model.

7.1 Theory

Dantzig-Wolfe decomposition is a principle introduced by Georg Dantzig and Phil Wolfe in Decomposition Principle for Linear Programs (Dantzig and Wolfe, 1960). It involves a reformulation of a large Linear Programming (LP) problem into a master- and sub-problem, to make it possible to solve larger problems. In the rest of this section we assume we have a LP problem. The method relies on a special structure in the constraints matrix, called a block angular structure. This is demonstrated in Figure 7-1, and means that the constraint matrix can be divided into blocks of non-zero elements. Each non-zero element in the matrix represents one block. Some of these blocks that only involve some columns are called *independent constraints,* while the blocks that connect the different columns are called the *connecting constraints*. If we ignore the connecting constraints, the problem would decompose into N independent problems.

$$\begin{bmatrix} A_1 & A_2 & A_3 & \cdots & A_N \\ D_1 & 0 & 0 & & 0 \\ 0 & D_2 & 0 & & 0 \\ 0 & 0 & D_3 & & 0 \\ \vdots & & & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & D_N \end{bmatrix}$$

Figure 7-1: *Illustration of a block angular structure where A_n are the coefficients of the connecting constraints, and D_n are the coefficients of the independent constraints.*

Let (P) be the original linear programming problem, defined as:

$$\max z = \sum_{k \in \boldsymbol{K}} \boldsymbol{c}_{\boldsymbol{k}}^{\boldsymbol{T}} \boldsymbol{x}_{\boldsymbol{k}}$$

$$s.t. \qquad \sum_{k \in \boldsymbol{K}} A_k \boldsymbol{x}_{\boldsymbol{k}} \leq \boldsymbol{b} \tag{P1}$$

$$D_k \boldsymbol{x}_{\boldsymbol{k}} \leq \boldsymbol{e}_{\boldsymbol{k}}, \qquad \forall\, k \in \boldsymbol{K} \tag{P2}$$

$$\boldsymbol{x}_{\boldsymbol{k}} \geq 0, \qquad \forall\, k \in \boldsymbol{K} \tag{P3}$$

Here $\boldsymbol{K}$ is the set of blocks in the constraints matrix, and we have divided the constraints into the connecting (P1) and the independent (P2). Note that $\boldsymbol{x}_{\boldsymbol{k}}$ are vectors since each block can contain multiple variables and constraints. We define $\boldsymbol{X}_{\boldsymbol{k}} = \{\boldsymbol{x}_{\boldsymbol{k}} \mid D_k \boldsymbol{x}_{\boldsymbol{k}} \leq \boldsymbol{e}_{\boldsymbol{k}}, \boldsymbol{x}_{\boldsymbol{k}} \geq 0\}$. If $D_k \boldsymbol{x}_{\boldsymbol{k}} \leq \boldsymbol{e}_{\boldsymbol{k}}$ and $\boldsymbol{x}_{\boldsymbol{k}} \geq 0$ defines a bounded area, $\boldsymbol{X}_{\boldsymbol{k}}$ is a convex set, since we have a linear problem. Dantzig-Wolfe decomposition also works for problems where the independent constraints define an unbounded area. This is done by using extreme rays, but will not be shown here. Each variable in the convex set $\boldsymbol{X}_{\boldsymbol{k}}$ can now be stated as a convex combination of the corner points, $\boldsymbol{x}_{\boldsymbol{k}}^{(\boldsymbol{i})}$, of the set:

$$\boldsymbol{x}_{\boldsymbol{k}} = \sum_{i \in \boldsymbol{I}} \boldsymbol{x}_{\boldsymbol{k}}^{(\boldsymbol{i})} \lambda_{k,i} \qquad where \quad \sum_{i \in \boldsymbol{I}} \lambda_{k,i} = 1\ , \lambda_{k,i} \geq 0, \qquad \forall\, i \in \boldsymbol{I}$$

$\lambda_{k,i}$ is defined as the weight of corner point i of sub-problem k. The original problem can now be reformulated to an equivalent formulation called the master problem (MP):

$$\max z = \sum_{k \in \boldsymbol{K}} \sum_{i \in \boldsymbol{I}} \boldsymbol{c}_{\boldsymbol{k}}^{\boldsymbol{T}} \boldsymbol{x}_{\boldsymbol{k}}^{(\boldsymbol{i})} \lambda_{k,i}$$

$$s.t. \qquad \sum_{k \in \boldsymbol{K}} \sum_{i \in \boldsymbol{I}} A_k \boldsymbol{x}_{\boldsymbol{k}}^{(\boldsymbol{i})} \lambda_{k,i} \leq \boldsymbol{b} \tag{MP1}$$

$$\sum_{i\in\boldsymbol{I}} \lambda_{k,i} = 1\,, \qquad \forall k \in \boldsymbol{K} \tag{MP2}$$

$$\lambda_{k,i} \geq 0, \qquad \forall i \in \boldsymbol{I}, k \in \boldsymbol{K} \tag{MP3}$$

Note that given all the extreme points of the sets $\boldsymbol{X_k}$, the weights are the only variables in this problem. This reformulation contains fewer constraints; all the independent constraints are removed from the problem. The number of variables may, however, be very large. Finding all the corner points, $\boldsymbol{x_k^{(i)}}$, may also be intractable.

The key to solving the problem above is to only include a subset of the variables (columns) in the model. The master problem is then called the restricted master problem (RMP). Variables are added iteratively until the optimal solution is found. This technique is called *delayed column generation*. To decide which columns to include in the next iteration, we look at the reduced costs of the columns. A sub-problem finds the columns with a positive reduced cost. These can improve the solution in the master problem (assuming it is a maximization problem). Let $\boldsymbol{\delta_k}$ be the dual variables of the connecting constraints (MP1) from solving the RMP, and γ_k be the dual variables of the convexity constraints (MP2). The reduced cost of a column will then be $\left(\boldsymbol{c_k^T} - \boldsymbol{\delta_k} A_k\right)\boldsymbol{x_k^{(i)}} - \gamma_k$. The sub-problem (SP) finds the columns with the largest reduced costs:

$$\max z_k = \left(\boldsymbol{c_k^T} - \boldsymbol{\delta_k} A_k\right)\boldsymbol{x_k} - \gamma_k$$

$$s.t. \qquad D_k \boldsymbol{x_k} \leq \boldsymbol{e_k} \tag{SP1}$$

$$\boldsymbol{x_k} \geq 0 \tag{SP2}$$

One such sub-problem is solved for each block, k, of independent constraints. They find new columns, which are included in the master problem if they have a positive reduced cost. The master problem is re-solved to obtain new dual variables, and these are sent to the sub-problems. The algorithm iterates until none of the sub-problems generates a column with positive reduced costs. The master problem has then found the optimal solution to the original problem. Note that the final solution to the decomposition always is optimal in the original problem, but the method may take long to converge and is often used to find a good, but not necessarily optimal, solution.

So far we have not stated how the algorithm should be initialized. The dual variables from the master problem are needed to solve the sub-problems. A feasible solution to the master problem is therefore needed to start the iterations. How this solution should be found depends on the specific problem instance.

7.2 Applying Dantzig-Wolfe Decomposition to the Stochastic Model

Dantzig and Madansky (1961) presented a solution method for two-stage stochastic programs using the decomposition principle of Dantzig and Wolfe (1960) to split the problem into manageable sub-problems. Our problem is a multistage stochastic program, but consists of several scenarios that are connected through the non-anticipativity constraints. If these constraints are relaxed, the problem decomposes to several independent problems; one for each scenario. This means that the stochastic model has the block angular structure that makes Dantzig-Wolfe decomposition effective.

Several parameters in the model could be chosen as the basis of a decomposition. Each field could for example constitute a separate sub-problem. The different sub-problems would then generate investment and production schedules while the master problem ensures that capacity and demand restrictions involving several fields are met. This would, however, lead to a big master problem, since many constraints connect different fields. Scenarios were chosen as the basis of the decomposition because only the non-anticipativity constraints link them. A smaller master problem will give a bigger effect of the decomposition. To better represent the uncertainty, it could also be interesting to increase the number of scenarios. With the scenarios as the basis of the decomposition, the run time of the model could grow slower as the number of scenarios is increased, than what is the case for the standard stochastic model (S).

The stochastic model is a mixed integer programming model while the Dantzig-Wolfe decomposition requires an LP model. One sub-problem might invest in an onshore solution while another might choose an FLNG unit. Combining these in the master problem is impossible. To invest 30% in onshore and 70% in an FLNG is not a feasible option. The decomposition method must consequently be adapted. The general approach of our adaptation is to apply Dantzig-Wolfe decomposition on an LP relaxation of our model. The solution from this first step is then modified to become a feasible solution of the original problem. By removing the binary and integer constraints we expand the feasible area of the model. The decomposition will therefore offer an upper bound for our original problem. The details of how this solution is modified are presented in the following sections.

7.3 Mathematical Formulation

In this section, we first present the master problem before the sub-problems are formulated. Each sub-problem represents a scenario and finds solutions to that scenario. The master problem will find the optimal convex combination of the solutions from the sub-problems and update the prices used in the objective function of the sub-problems.

7.3.1 Master Problem

In general, the restricted master problem objective is defined as $\sum_{k\in\boldsymbol{K},i\in\boldsymbol{I}} \boldsymbol{c}_{\boldsymbol{k}}^{\boldsymbol{T}}\boldsymbol{x}_{\boldsymbol{k}}^{(\boldsymbol{i})}\lambda_{k,i}$. In our problem, $\boldsymbol{c}_{\boldsymbol{k}}^{\boldsymbol{T}}\boldsymbol{x}_{\boldsymbol{k}}^{(\boldsymbol{i})}$ is in fact the profit in scenario k from iteration i of the sub-problem multiplied by the probability of that scenario. In the description of the stochastic model, the scenarios were represented by ω. This notation is also used in the following formulation. If $\lambda_{\omega,i}$ is the weight of column i from scenario ω, π_ω is the probability of the scenario and $Profit_{\omega,i}$ is the profit from scenario ω using solution i, the objective function of the restricted master problem (SRMP) becomes:

$$\max \text{z} = \sum_{\omega\in\boldsymbol{\Omega}} \pi_\omega \left[\sum_{i\in\boldsymbol{I}} Profit_{\omega,i} \cdot \lambda_{\omega,i}\right]$$

This function will find the optimal combination of weights from the different solutions of the sub-problems. The master problem also contains the non-anticipativity constraints. The constraints are implemented in the same way as in the stochastic problem. The variables in time periods before the resolution of an uncertain parameter are forced to be equal in a chain; each scenario is constrained to be equal to the next one. This will give at most $\omega - 1$ non-anticipativity constraints for each variable in each time period. In time periods closer to the end of the horizon, fewer variables will be constrained, since more of the uncertainty is resolved. In the formulation of the constraints, the variables from the stochastic model are replaced by the solution variables from the sub-problems multiplied by the weight. We do not state all the non-anticipativity constraints here, instead we use a general formulation, similar to the one used in the description of the stochastic model, see Section 6.3. Let $u_{t,\omega,i}$ be solution i of a variable from scenario ω. The non-anticipativity constraints are then defined as:

$$\sum_{i\in\boldsymbol{I}} u_{t,\omega,i} \cdot \lambda_{\omega,i} - \sum_{i\in\boldsymbol{I}} u_{t,\omega+1,i} \cdot \lambda_{\omega+1,i} = 0 \quad \forall\, t \in \boldsymbol{T}, \omega \in \boldsymbol{\Omega} \mid \Phi_{t,\omega} = 1 \qquad \text{(SRMP1}[\omega])$$

The master problem contains one such constraint for each variable that depends on the scenario in the stochastic model. This includes all building variables x, all closing variables w, all rates q and plant capacity built $v_{j,t,\omega}^{PLT}$. Note that the only variables in the master problem are the weights $\lambda_{\omega,i}$. The $u_{t,\omega,i}$ is a solution from the sub-problem and will be a parameter in the master problem. Constraints (SRMP1[ω]) contain both variables from scenario ω and $\omega + 1$. The variables from each scenario ω, are therefore part of two constraints, with coefficient +1 in (SRMP1[ω]) and coefficient -1 in (SRMP1[$\omega + 1$]).

In addition to the non-anticipativity constraints, convexity constraints are also needed. These constraints will ensure that a convex combination of the solutions from the sub-problems is chosen:

$$\sum_{i \in \boldsymbol{I}} \lambda_{\omega,i} = 1 , \qquad \forall\, \omega \in \boldsymbol{\Omega} \tag{SRMP2}$$

$$\lambda_{\omega,i} \geq 0, \qquad \forall i \in I, \omega \in \boldsymbol{\Omega} \tag{SRMP3}$$

From the solution of the master problem, we can calculate the dual costs of the constraints. Let δ_ω be the dual cost of constraints (SRMP1[ω]) and γ_ω be the dual cost of (SRMP2). These are the prices that are sent to the sub-problems in each iteration.

7.3.2 Sub-Problems

The formulation of the sub-problems are based on (SP) in our discussion of Dantzig-Wolfe decomposition. The sub-problems are in essence the deterministic version of our problem with a different objective function. The uncertain parameters are replaced with the realization of that parameter in the scenario the sub-problem represents. The objective is to find the column with the greatest reduced cost, generally defined as $(\boldsymbol{c_k^T x_k} - \boldsymbol{\delta_k} A_k \boldsymbol{x_k} - \gamma_k)$, where $c_k^T x_k$ is the objective function in the deterministic model. We can then formulate the objective of sub-problem ω (SSP[ω]):

$$\max z_\omega = \pi_\omega \left[\sum_{t \in \boldsymbol{T}} N_t \left(cf_{t,\omega}^{INC} - cf_{t,\omega}^{INV} - cf_{t,\omega}^{OPR} \right) + N_T\, EFP_\omega \right] - \delta_\omega u_{t,\omega,i} + \delta_{\omega+1} u_{t,\omega+1,i} - \gamma_\omega$$

The first term in the objective function is the objective of the deterministic model, multiplied by the probability of the scenario. The second and third terms are the dual costs from the master problem. Since each scenario is part of two non-anticipativity constraints, we need to include the dual cost from both constraints (SRMP1[ω]) and (SRMP1[$\omega + 1$]). They have different coefficients since the coefficients are different in the master problem. Note that the formulation above still uses the general variable $u_{t,\omega,i}$. Each variable which depends on the scenario will be part of this objective function. The last term is the dual cost of the convexity constraint from the master problem.

The constraints of the sub-problem are the same as in the original formulation of the deterministic model. The constraints that contain an uncertain parameter will use the realization of that parameter in the scenario the sub-problem represents. Dantzig-Wolfe decomposition does not require the sub-problems to be LP programs. The binary and integer restrictions of the deterministic model are therefore also included in this implementation of Dantzig-Wolfe.

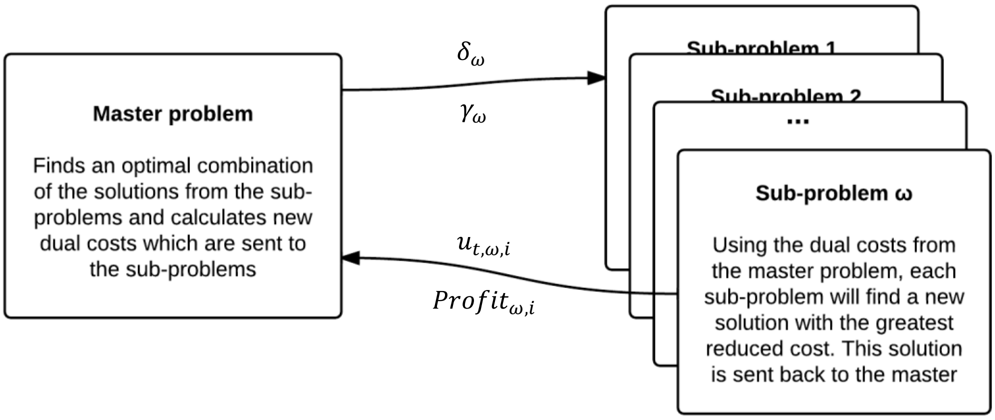

Figure 7-2: *Illustration of the flow of information between the master and sub-problems.*

Figure 7-2 summarizes the exchange of information between the master and the sub-problems. Note that $Profit_{\omega,i}$ does not equal the objective value of the sub-problem. It is the expected profit of the solution generated by the sub-problem. This can be calculated by using the original objective function from the deterministic model.

7.4 Implementation of the Algorithm

Our implementation of the decomposition algorithm goes through three phases. In the first phase, the problem is set up, and a starting point for the algorithm is found. The iterations of the algorithm are performed in the second phase. This phase gives an upper bound to the objective value of the stochastic model (S), since the Dantzig-Wolfe algorithm runs on an LP relaxation of our stochastic model. The last phase modifies the solution from the second phase, to find a feasible solution to the MIP stochastic model. The discussion in this section is organized according to these phases.

In the discussion so far, we have assumed that we have a feasible solution to the master problem, to initialize the iterations. This is needed to calculate the first set of dual costs. There are several ways to find a feasible starting point to our problem. A straight forward approach is to set all variables to 0. This is, however, not a feasible solution in all cases. If a contract with a minimum delivery is sealed in the initial state, the model will not be able to fulfill this requirement. The choice of an initial solution also has an impact on the time the algorithm needs to converge in phase 2. Setting all variables to 0 is probably not a solution close to the optimal one, and is therefore most likely a poor starting point.

A different approach is to use a solution from one of the scenarios, and add it to all the others. To guarantee that this will yield a feasible solution, we need a scenario that has the lowest amount of gas in all fields at the same time. The solution, and production schedule, from this scenario will be feasible for all other scenarios, since they have more or equal amount of gas in the fields. The uncertain prices will only affect the profitability of each scenario; they will not render any scenario infeasible. This approach might not be suitable for all problem instances, but if a scenario with the lowest amount of gas in all fields at the same time exist, it should provide a good starting point for the iterations. In *phase 1* each scenario is solved once and added to the master problem. The solution from the scenario with smallest field sizes is also added as a solution to all the different scenarios in the master problem. This provides a feasible starting point for the iterations.

The main iterations in *phase 2* follow the description in the previous section. The algorithm iterates until no sub-problem generates a column with a reduced cost. This means that none of the sub-problems have a positive objective value. The solution of the master problem after phase 2 provides an upper bound on the optimal solution, since it is an LP relaxation. This upper bound can be tighter, having a lower value, than the bound provided by the standard Mosel algorithms. The solution from phase 2 can therefore itself be useful for proving that a solution is close to optimum. Note that the Dantzig-Wolfe algorithm must run until it finds the optimal solution to give a upper bound. If the iterations are stopped earlier, we find a lower bound of an upper bound, which means that we have no information about the solution of the original problem.

In *phase 3* we adapt the solution from the Dantzig-Wolfe decomposition to make it feasible. This is done by re-solving the master problem with the added constraints of binary and integer variables. Let $u_{t,\omega,i}$ represent a solution of a binary variable from iteration i of sub-problem ω. The added constraints for this variable would then be:

$$\sum_{i \in \boldsymbol{I}} u_{t,\omega,i} \cdot \lambda_{\omega,i} \in \{0,1\}, \qquad \forall\, t \in \boldsymbol{T}, \omega \in \boldsymbol{\Omega} \tag{SRMP4}$$

One such constraint is added for each binary variable in the stochastic model. Integer constraints are formulated in a similar manner. This adaptation is not guaranteed to give an optimal solution to our stochastic problem. It might, however, give good solutions that can be proven to be within a certain percentage of optimum by using the upper bound.

Figure 7-3 summarizes the flow of the algorithm. It is implemented in Xpress using two files: *DW_master.mos* and *DW_sub.mos*. *DW_master.mos* contains the master problem, and the controlling logic. It initializes the algorithm, and starts the sub-models. *DW_sub.mos* only runs the sub-problem and sends the results back to *DW_master.mos.* The *mmjobs* module is used to send controller messages between the models, and the different solutions and prices are sent between the models using shared memory.

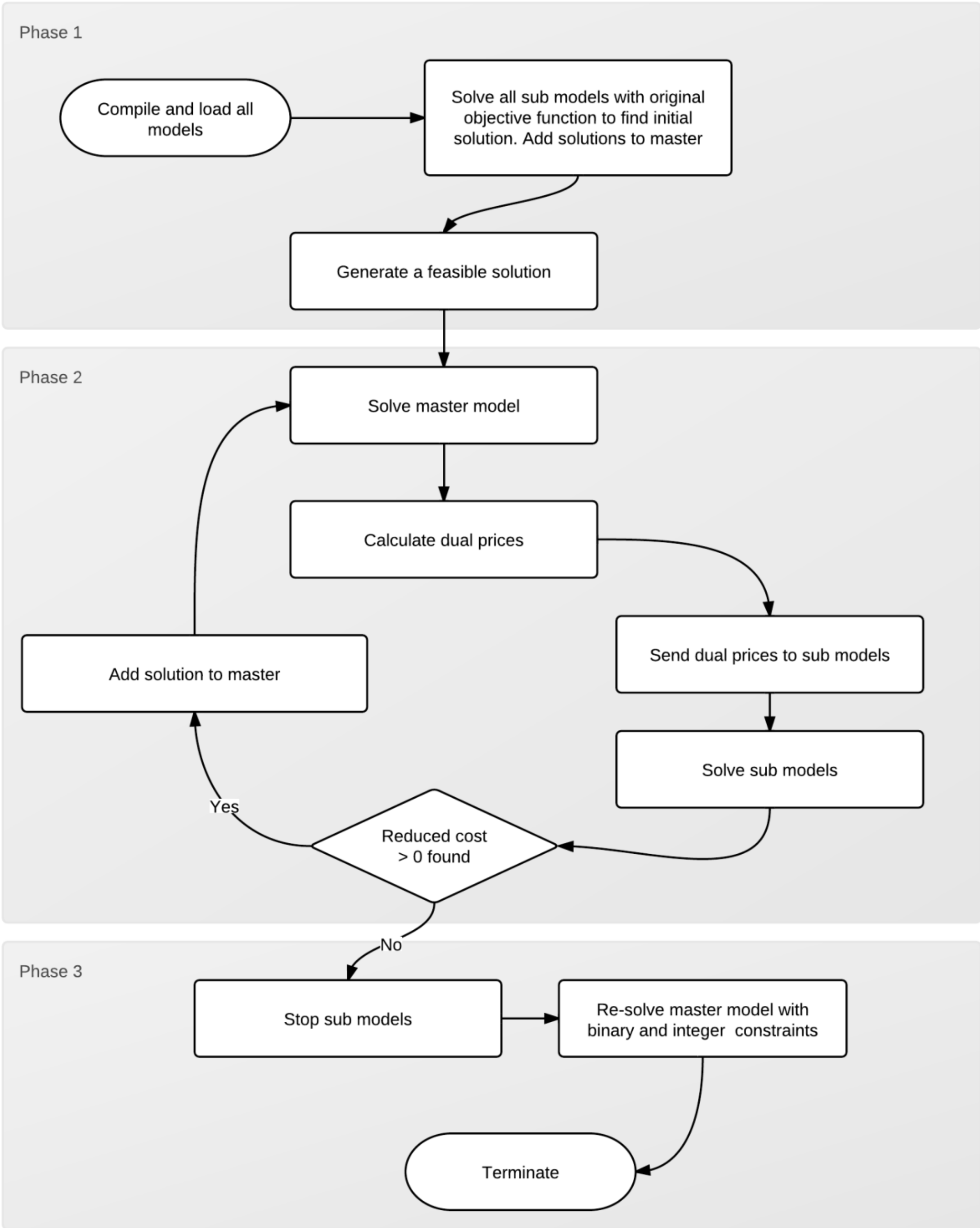

Figure 7-3: *Flow chart of our implementation of the Dantzig-Wolfe algorithm.*

The Dantzig-Wolfe algorithm can be adjusted in several ways to make it converge faster. The adjustments might have different effects on different problem instances, so there are no fixed optimal settings for our model. The first adjustment is to not solve the sub-models to optimality, but to return to the master once a solution with reduced cost is found. Any solution with a positive reduced cost can improve the master problem solution, and solving the sub-models can take long. It might therefore be effective to iterate faster without necessarily finding the biggest reduced cost each time. A variation of this is to set a time limit on the sub-models. A good reduced cost and solution can then be found without having to wait for the optimal one. The motivation for this approach is that it often takes very long to move from a nearly optimal solution to the optimal one, and this is the approach we implemented on our sub-models.

Another variation is to stop the execution of the sub-models and iterate once a certain share of the sub-models have found a solution. Some instances of the coefficients in the sub-models might be very hard to solve, and when a solution is found to some of the models, new dual prices can be calculated for all the sub-problems. The algorithm can then iterate without always having to wait for the slowest sub-model.

Chapter 8

Implementation

The two models described in Chapter 6 were implemented in Xpress IVE optimization suite using the Xpress-Mosel programming language. In this chapter, we first present some details of how the models were implemented. The second section describes the implementation of two methods used to evaluate the solutions from the stochastic model. The datasets used as input to the models are presented in last section in this chapter. The full datasets and Mosel source code are available in the attachments to the thesis.

8.1 Implementation of the Mathematical Models

The formulation from Chapter 6 allows some initial infrastructure to be present at the start of the horizon. No investment cost is paid for these parts of the value chain, since they are built before the decision period of the model. This is implemented by expanding the set of time periods; an extra period, 0, is added to the beginning. In the code, two sets are specified:

```
TimePeriodsInitial  := 0 .. nTimePeriods;
TimePeriods         := 1 .. nTimePeriods;
```

`nTimePeriods` is the number of time periods in the model. `TimePeriods` is used in the objective function and only include the periods we actually make decisions about. The added period 0 is included in `TimePeriodsInitial`. This set is used in some constraints and for all x, y and $v_{j,t}^{PLT}$ variables. All initial infrastructures are specified in the same way as potential infrastructure, but their construction variables, x, and their operational status, y, are set to 1 in period 0. Since time period 0 is not included in the objective function it will not contribute to any cost. The decision variables of all the potential infrastructures are all set to 0 for time period 0. All constraints that include one of the x variables or $v_{j,t}^{PLT}$ are updated to incorporate time period 0. Constraints (1) and (2) in the deterministic model (D) and in the stochastic model (S) ensure that initial infrastructure is not built again.

The production modeling must also account for existing fields. The total amount of recoverable gas, F_f^Q, is defined as the *remaining* recoverable gas for the existing fields. The decline constraints will then work in the same way as for potential fields. Adjusting the different field parameters now allows representation of fields that are past their build-up phase and already have entered plateau- or decline-phase.

8.1.1 Variable Period Durations

In Section 6.1.2 we argued that variable duration of periods is an effective way of handling the trade-off between accurately representing the real world, while also reducing the number of variables. In this section, the implementation for the stochastic model (S) is presented, the deterministic model (D) is updated similarly. Varying period length is implemented by introducing a parameter, Θ_t, describing the number of years in each period. This parameter is used in the objective function and some of the constraints. Since all flows in the model are given as rates instead of volumes, most of the model can stay unchanged. The cumulative production must be updated such that it sums over the yearly rate times the number of years in each period instead of just summing over the yearly rates. Constraints (S12) which are limiting the decline phase of the fields must be updated accordingly:

$$\Theta_t \cdot \left(\sum_{j \in \boldsymbol{J}} q_{f,j,t,\omega}^{F \to J} + \sum_{d \in \boldsymbol{D}} \sum_{r \in \boldsymbol{R}} q_{f,r,d,t,\omega}^{FLNG} \right) \leq F_f^{DEC} \cdot \left(F_{f,\omega}^{Q} - g_{f,t,\omega} \right), \qquad \forall f \in \boldsymbol{F}, t \in \boldsymbol{T}, \omega \in \boldsymbol{\Omega} \tag{S12}$$

The other updated constraints are Constraints (S10) which limits the amount of increase in production each year. They must be updated so the increase takes the number of years into account:

$$\sum_{j \in \boldsymbol{J}} q_{f,j,t,\omega}^{F \to J} + \sum_{d \in \boldsymbol{D}} \sum_{r \in \boldsymbol{R}} q_{f,r,d,t,\omega}^{FLNG} \leq \left(\sum_{j \in \boldsymbol{J}} q_{f,j,t-1,\omega}^{F \to J} + \sum_{d \in \boldsymbol{D}} \sum_{r \in \boldsymbol{R}} q_{f,r,d,t-1,\omega}^{FLNG} \right) + F^{INC} \cdot F_f^{MAX} \cdot \Theta_t, \qquad \forall f \in \boldsymbol{F}, t \in \boldsymbol{T}, \omega \in \boldsymbol{\Omega} \tag{S10}$$

The aggregation of time periods, and changes to Constraints (S10) and (S12), results in a slightly changed production profile. The production cannot be constrained with the same resolution and the production profile forms a stepwise curve.

In the objective function, the sale rates and operational costs each year must be multiplied by the number of years in that period. The only operational cost which is not affected by the change is the cost of connecting an FLNG unit to a field. The varying period length must also be taken into account when specifying the input data to the model. Only parameters that are dependent on the time period must be adjusted before they are used in the model. We have in general chosen to use the average of the yearly values for input parameters that span several years. This includes net present value constants, demand rates, prices and contract rates. The model also supports a time gap between investment decisions and when the infrastructure is becoming operational. The formulation already allows for time periods of different length, but they must be taken into account when specifying the gap parameters, $\boldsymbol{G_t}$.

8.1.2 Special Ordered Sets of Type One (SOS1)

SOS1 are sets of variables where at most one of the variables can take a non-zero value. In mixed integer programming problems, specifying this relationship between variables can help improve the run time. The branch-and-bound algorithm can branch on sets instead of individual variables, reducing the size of the search tree.

The models presented in Chapter 6 contain some sets with this property. Fields and plants can only be built once throughout the whole horizon of the model. Constraints (1) and (2) ensure this, and were implemented using SOS1 sets on start-period variables of each potential field and plant. Similarly, each field and plant can only be closed once during the horizon. SOS1 sets were used on the end-period variables as well.

8.2 Evaluation of the Stochastic Solution

Two metrics for evaluating the solution from the stochastic model (S) are Expected Value of Perfect Information (EVPI) and Value of Stochastic Solution (VSS). A description of how these are implemented is given below.

8.2.1 Expected Value of Perfect Information

EVPI is defined as the difference between the objective value of the Wait-and-See (WS) solution, and the stochastic model (S). The WS solution is found by solving the problem, once for each scenario, and calculate the expected profit from all these solutions. This is implemented with two models, a master model which contains the control logic and a sub-model which solves the problem, once for each scenario. Both uses the same input data file as the stochastic model. We denote the optimal objective value as Z and the optimal decisions as u^{OPT} in the following pseudo-code, which describes the calculation procedure.

```
Step 1:   Master model sets scenario ω = 1, and starts the sub-model
          for this scenario.
Step 2:   Sub-model solves the deterministic problem, using the
          parameter realizations of scenario ω. Optimal objective
          value Z and solution u^OPT are obtained.
Step 3:   The master model assign a scenario index to the objective
          value Z_ω and the optimal solution u_ω^OPT. Scenario ω is
          redefined: ω ≔ ω + 1
Step 4:   If ω ≤ the number of scenarios, go to step 2.
Step 5:   Calculate WS solution, WS = ∑_ω Z_ω π_ω .
```

8.2.2 Dynamic Value of Stochastic Solution, VSS[D]

Escudero et al. (2007) argue that using the dynamic solution of the expected problem can give a good representation of the expected value solution since it gives updated information to the model. We therefore calculate the $EDEV_s$ in accordance with Escudero et al. (2007), see Section 5.6.2.

The optimal value Z_g^{EV} for scenario group g is calculated by replacing all uncertain parameters with the expected values for scenarios included in the scenario group, and fixing the decision variables for previous stages to the optimal solution from the ancestor scenario group. $EDEV_s$ is defined as the sum of the optimal values for scenario groups g within the stage s, multiplied with the probability of the scenario group.

$$EDEV_s = \sum_{g \in \boldsymbol{G_s}} Z_g^{EV} \sum_{\omega \in \boldsymbol{\Omega_g}} \pi_\omega$$

The uncertain parameters in our model are spot prices, contract prices and field sizes. The expected value of these parameters in scenario group g are calculated as the weighted average of the uncertain parameters in the scenarios included in the scenario group, $\omega \in \boldsymbol{\Omega_g}$, weighted by the probability of the scenario π_ω.

$$\bar{P}_{d,t,g}^{SPOT} = \frac{\sum_{\omega \in \boldsymbol{\Omega_g}} P_{d,t,\omega}^{SPOT} \cdot \pi_\omega}{\sum_{\omega \in \boldsymbol{\Omega_g}} \pi_\omega} \qquad \forall\, d \in \boldsymbol{D}, t \in \boldsymbol{T}, g \in \boldsymbol{G}$$

$$\bar{P}_{k,t,g}^{CNT} = \frac{\sum_{\omega \in \boldsymbol{\Omega_g}} P_{k,t,\omega}^{CNT} \cdot \pi_\omega}{\sum_{\omega \in \boldsymbol{\Omega_g}} \pi_\omega} \qquad \forall\, k \in \boldsymbol{K}, t \in \boldsymbol{T}, g \in \boldsymbol{G}$$

$$\bar{F}_{f,g}^{Q} = \frac{\sum_{\omega \in \boldsymbol{\Omega_g}} F_{f,\omega}^{Q} \cdot \pi_\omega}{\sum_{\omega \in \boldsymbol{\Omega_g}} \pi_\omega} \qquad \forall\, f \in \boldsymbol{F}, g \in \boldsymbol{G}$$

$EDEV_s$ are calculated by implementing two models; a master model which contains the control logic and a sub-model which solves the problem, once for each scenario group. Both models use the same input data as the stochastic model in addition to a VSS specific data input file with information about scenario groups, stages and the relationship between them. The following pseudo-code describes the procedure of calculating the $EDEV_s$.

Step 1: Master model sets scenario group $g = 1$, and starts the sub model for this scenario group.

Step 2: If $g > 1$, Optimal solution for the ancestor scenario group, α_g, is defined as $u_t^{FIX} := u_{t,\alpha_g}^{OPT}$. The sub-model is started and decision variables for time periods up to the last time period in the ancestor node, T_g^{PREV}, for scenario group g are fixed; $u_t := u_t^{FIX}, \forall t \leq T_g^{PREV}$.

Step 3: The sub-model calculates the expected value of the uncertain parameters for scenario group g, and solves the expected value problem. Optimal objective value Z^{EV} and solution u_t^{OPT} is obtained.

Step 4: The master model assign a scenario group index to the objective value Z_g^{EV} and the optimal solution $u_{t,g}^{OPT}$. Scenario group g is redefined; $g := g + 1$.

Step 5: If $g \leq$ the number of scenario groups, go to step 2.

Step 6: Calculate $EDEV_s$ of each stage s: $\sum_{g \in \boldsymbol{G_s}} Z_g^{EV} \sum_{\omega \in \boldsymbol{\Omega_g}} \pi_\omega$.

One problem with this method is that it in some cases will lead to infeasible problems. Field size is one of the recourse parameters in our model, and when expected sizes are used to solve the model, the optimal solution might be to produce more gas than the field actually contains. The problem might then be infeasible for some scenarios if production variables from earlier stages are fixed at a higher total level than the available amount of gas in that scenario. One way to handle this challenge is to fix only the investment variables and not the production variables. When less variables are fixed, the $EDEV_s$ will get the same or higher value than when all variables are fixed.

8.3 Presentation of Datasets

In this section, we present the datasets used to solve the model. Some of the parameters in the stochastic model are uncertain and are presented with all realizations. When solving the deterministic version of the model, the expected values of uncertain parameters are used. The model is tested with one main dataset called the base case, and several variations to see how varying input data affects the results. The base case is presented in detail before we go through variations toward the end of the chapter.

The base case is based on fields located in Northern Norway. The problem consists of six fields and three onshore plant locations. The fields and plants are presented in Figure 8-1. Parts of the infrastructure are already built, and the model is solved to find the optimal investments in a set of potential expansions of the operation. Liquefaction plant (2) is operational at the beginning of the planning period and has a capacity of 180 trillion Btu per year. It is located at Melkøya, and field (1) is connected to the plant through a pipeline. The rest of the fields and plants are potential investments. The actual Snøhvit field, Melkøya plant and pipeline are also shown in the figure for reference. The light blue squares marks all announced licensing blocks by November 11, 2011.

The LNG is transported to one of three demand points in the Atlantic basin; UK, the east coast of US or the south west of Europe. A discount rate of 8% is used in the calculation of the net present value. Inflation is not included in this rate since we assume constant costs and all prices and costs are given in 2012-dollars.

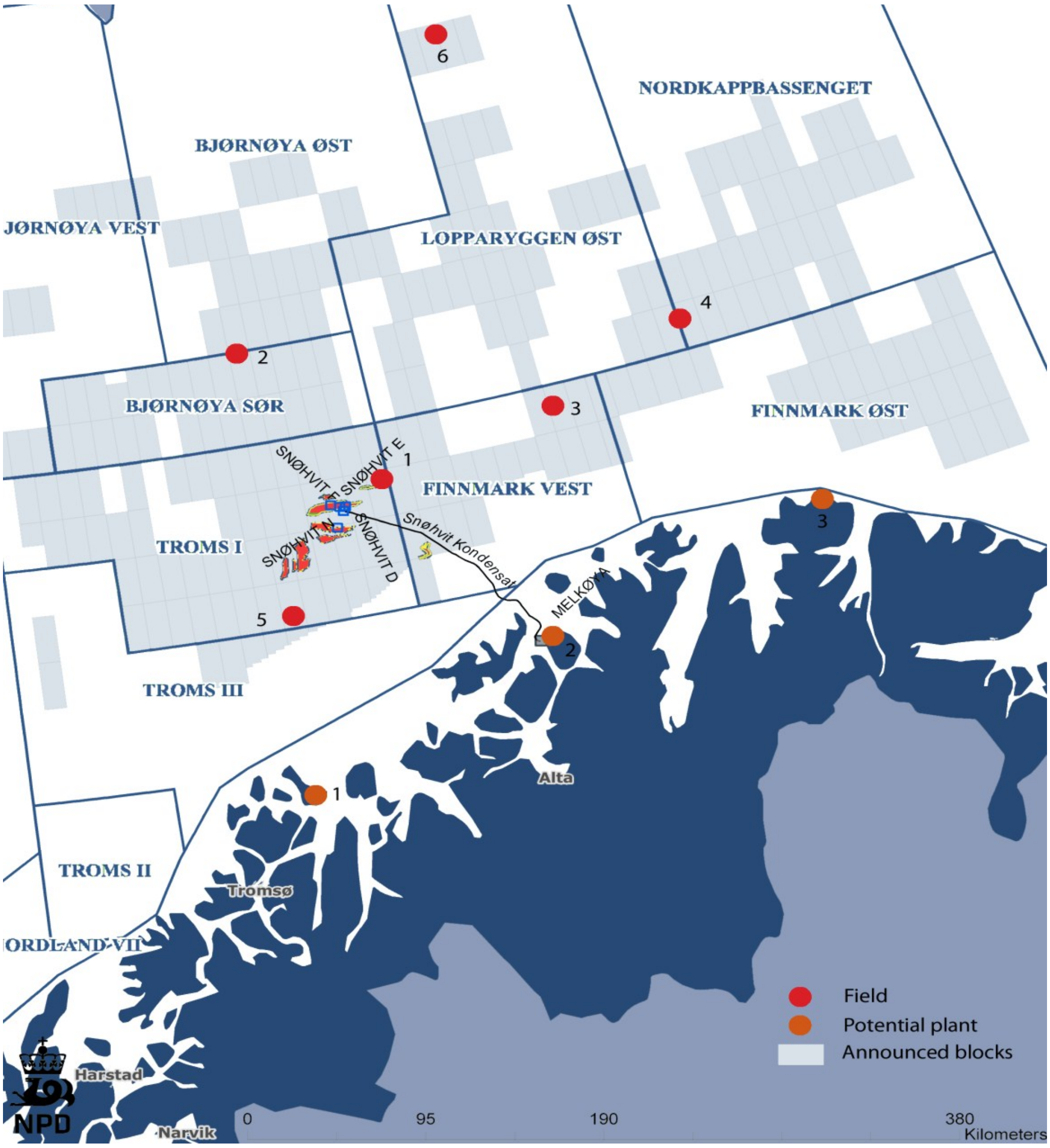

Figure 8-1: *Map of infrastructure (based on map from NPD, 2012)*

The horizon is divided into 15 time periods. The time periods consist of different numbers of years, such that the near future can be modeled with more detail. The first five periods represent one year each, the following four periods contain two years and the next three periods consist of three years. The last three periods have four, four and five years respectively. This gives a horizon of 35 years in total. With the 8% discount rate, all values in the last period are multiplied by 0.079, which are the average of the net present value constants for the years in the last period. This illustrates that any cash flows occurring after the horizon would have had minimal effect of the objective value if they were included. In this section, we present the input data with a yearly resolution. Before it is used in the model, average values are calculated for periods with more than one year.

The delay from the decision to invest in infrastructure to it becomes operational is set to 5 years for fields, 2 years for pipelines and 3 years for plants and FLNG units. This time gap will significantly affect the profitability of different investments, since it pushes the cash flow from sales further away from the year investment costs are paid. The NPV of the investment will thereby decrease.

8.3.1 Selection of Scenarios

Selection of scenarios has a great impact on the outcome of the model. Increasing the number of scenarios means that more uncertainty can be captured. But computational constraints limit the number of scenarios that can be included. Our formulation allows uncertain field sizes and gas prices. The limit on the number of scenarios means that there is a choice between representing the uncertainty in all the fields and prices with a few possible outcomes each, or selecting some of the uncertain parameters and representing them with more possible outcomes. Since the interplay between the fields is important in our model, we have chosen to represent uncertainty in multiple fields, but have few possible outcomes for each parameter. When the amount of gas is lower than expected in one field, and higher in another, the FLNG units can be moved between the fields.

The size of four fields and the spot gas prices are used as uncertain parameters in the model. Field sizes and gas prices are assumed not to be correlated, and we must therefore include scenarios for all combinations of the uncertain parameters. Each of the parameters is given a high and a low outcome, resulting in a total of 32 scenarios. Information is resolved at four points in time, and the dataset thereby has five stages. The first stage lasts for four periods before the size of field 2 is resolved in period 5. The size of field 3 and 4 are resolved in period 7. Since two parameters are resolved in the same period, each node in period 6 has four children. In period 9, the size of field 5 is resolved, and the prices in period 10. The last stage consists of period 10 to 15. Because of the varying period length, this last stage represents 22 of the total 35 years in the model. The scenario tree is shown in Figure 8-2.

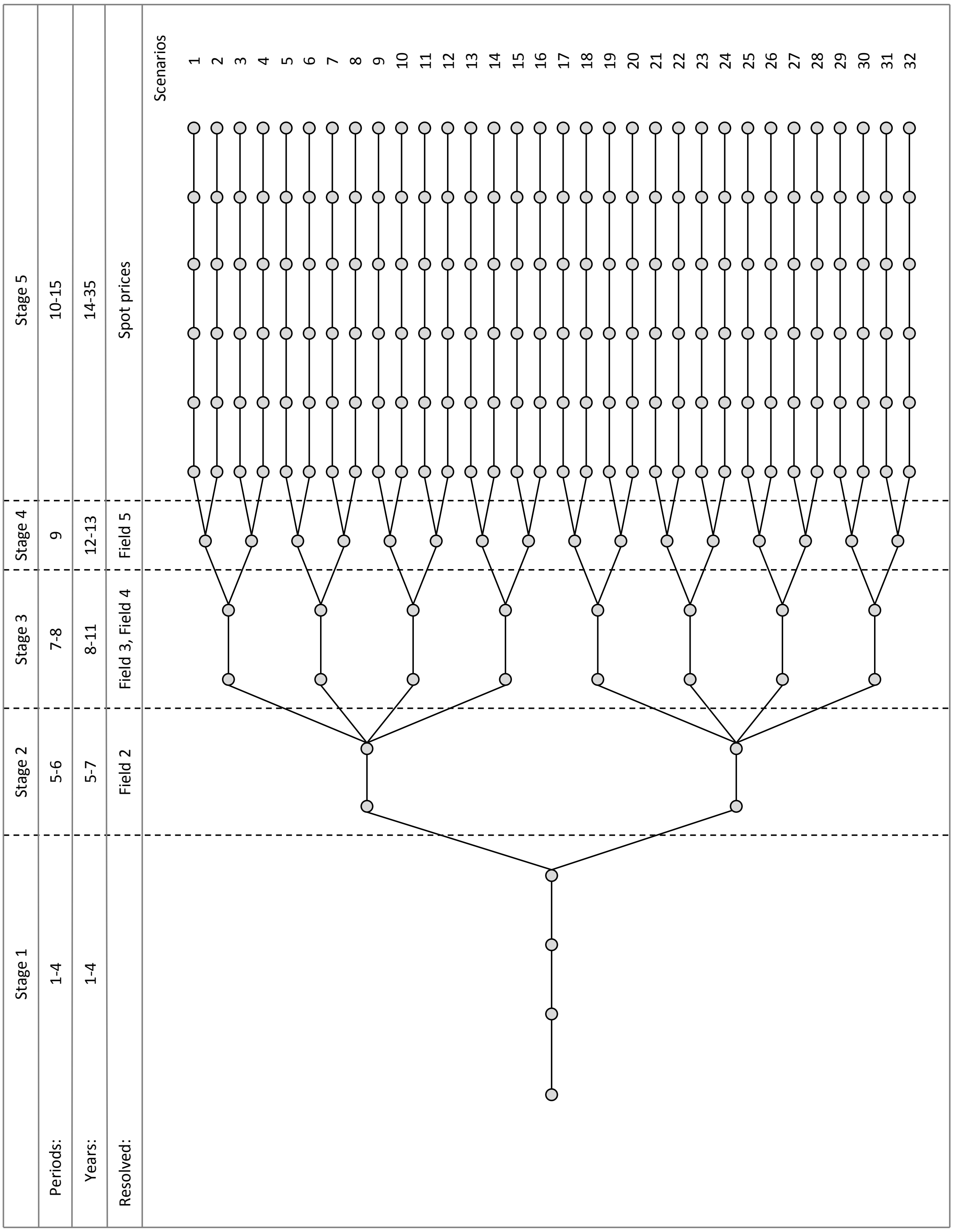

Figure 8-2: *The scenario tree used in the base case. Each path from the root to a leaf node represents a scenario.*

8.3.2 Field Production Profiles

Production profiles are modeled by a set of parameters describing total size of the field, maximal increase, production capacity and decline. The parameters are given values based on actual production of three gas fields outside the Norwegian coast: Ekofisk, Frigg and Heimdal. They are chosen since they are mature, and data exists on their production profiles, shown in Figure 8-3a. Snøhvit has only produced gas for five years, but is added in the figure as a reference, since it is the only Norwegian field connected to a liquefaction plant. The expected recoverable amount of gas in Snøhvit is 5733 trillion Btu, while Ekofisk, Frigg and Heimdal have 5644, 4148 and 1670 trillion Btu of recoverable gas respectively. Total amount of gas in field 2, field 3, field 4 and field 5 are uncertain, and varies between the scenarios. The average size of each field f for all scenarios are set to F_f^Q = {5700, 1600, 800, 2000, 3000, 500} trillion Btu. The sizes of the uncertain fields are set to 20% higher or lower in the high and low scenario respectively.

Figure 8-3b illustrates the production profiles of fields 1-6, given expected field sizes and production at the maximum possible rate in all time periods. If less is produced, the life time of the field will increase. Field 1 has a production profile based on production from Ekofisk. Total amount of recoverable gas is set to approximately the same, and the production capacity is 200 trillion Btu per year. Heimdal is the basis for field 2, 4 and 5. The production capacities are approximately equal, with 130, 100 and 120 trillion Btu per year respectively. The duration of the plateau-phase is increasing with field size and steeper decline rate. Field 3 and 6 are the smallest fields, and are characterized by short or non-existing plateau-phases.

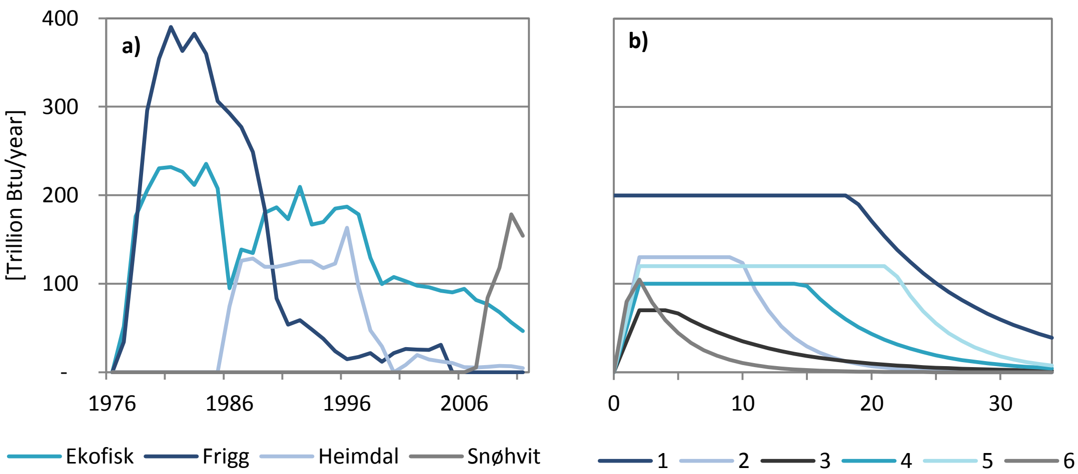

__Figure 8-3:__ a) Actual production profiles for four gas fields located outside the Norwegian Coast (NPD, 2012). b) Production profiles for fields 1-6, given maximal production in all time periods.

8.3.3 Capacities of Pipelines, Plants, FLNG units and Ships

FLNG units can be built with two different capacities, 1.7 and 3.5 mtpa (86.6 and 170.1 trillion Btu per year). These capacities are similar to the FLNG solutions from FLEX and Shell Prelude respectively (EP, 2009). Onshore plants can be built with a capacity between 2 and 10 mtpa (97.2 and 486.0 trillion Btu per year).

A pipeline can only be built between a field and an onshore plant. The pipeline capacity is set to 200 trillion Btu per year, which corresponds to the highest production rate of any of the fields. This value might be lower than a typical pipeline, but higher capacities lead to more slack in the variables and makes the model harder to solve. The model assumes a homogenous fleet of ships, with a capacity of 160,000 standard cubic meters (3577 billion Btu).

8.3.4 Investment- and Operational Costs

Finding accurate costs to use in the dataset is hard. Costs of different parts of the value chain vary hugely between projects, and also over time. According to IHS' Upstream Capital Cost Index (UCCI), the investment costs in upstream LNG projects rose by 94% from 2005 to 2011. The index tracks the costs in a diversified portfolio of LNG projects (IHS, 2012). The costs used in our dataset are typical examples of cost levels.

The field investment cost is one of the costs which differ most between different sources. In some cases it might even be negative or close to zero, if gas is currently flared or reinjected. CBI puts a typical investment at around $2 billion, quoting Wood Mckenzie and Deutche Bank as sources (CBI, 2011). We have chosen values within the range of $1 billion to $2.4 billion for the fields in this dataset. Pipeline cost is usually measured in $ per inch diameter per km distance, and for offshore pipelines, this cost is typically around $25,000-$40,000. Steel is the major component of this cost (Chandra, 2006). A cost of $30,000 per inch per km, and a diameter of 30 inches is used in the base case. This gives a pipeline cost of $900,000 per km for connecting the fields to onshore liquefaction plants. The operating expenditures for pipelines are set to 2% of the capital expenditures regardless of the actual flow through the pipe.

All liquefaction costs are a very important input parameter, given the focus on FLNG units in this thesis. Facts Global Energy (FGE) and FLEX LNG both have presented estimated capital costs of different existing liquefaction projects (FGE, 2010; FLEX LNG, 2010). Based on these estimations we have done a linear regression with the cost of a capacity of 0 mtpa set to zero. Figure 8-4 shows the capacity and cost of the different projects, and our linear approximation. Some of the projects have different estimated costs for the same project. In these cases the average of the estimates is used.

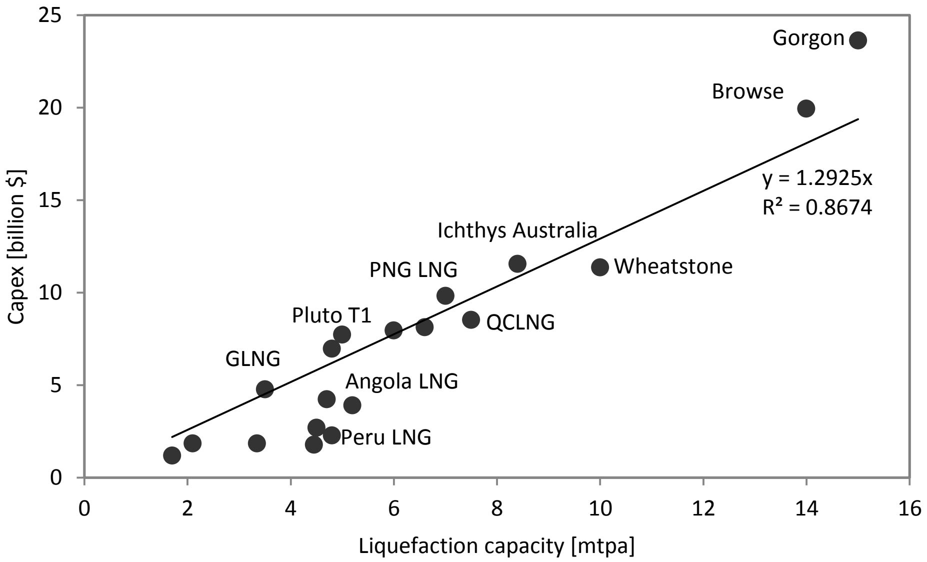

Figure 8-4: *Linear approximation of the capital costs for onshore liquefaction facilities.*

The regression gives a variable investment cost for onshore plants of $26.59 per MMBtu of yearly liquefaction capacity. Building extremely small and cheap plants would be unrealistic since the costs are not linear in the plant capacity when approaching a capacity of zero. The minimum plant capacity is, however, set to 2 mtpa, which means that this is not a problem. The capital expenditures of FLNG units are based on Energy and Power's case study of different FLNG solutions (EP, 2009). They estimate the capex of a small (1.7 mtpa) and large (3.5 mtpa) FLNG unit to $1.56 billion and $4.2 billion respectively. These are the investment costs used in the base case. This case study is also the basis for the operating expenditures.

Shipping costs consist of capital, voyage and operating costs. Where the voyage costs are all costs specific to a trade route, like fuel or port fees, and operating costs are manning, insurance, maintenance, etc. Capital expenditures have been highly volatile the last few years, and the fuel price has also varied a lot. The Petroleum Economist describes the level of these different costs. (Petroleum Economist, 2011). The cost in the base case is based on long term contracts. The ship owner then covers the various components of the shipping cost, and the charterer pays a daily rate. Based on the costs in The Petroleum Economist, $130,000 is used as a daily rate, including fuel cost. This corresponds to a yearly cost of $47.5 million per carrier. Further are the regasification cost is based on Jensen (2009), and set to $0.7 per MMBtu.

8.3.5 Three Demand Points in the Atlantic Basin

Demand is represented by three aggregated points in the Atlantic: Falmouth in UK, Philadelphia in the US and Algeciras in southern Spain. Shipping distances between to the markets are calculated using Portworld's online voyage calculator (Portworld, 2012). The time is assumed to be symmetric, so there is no difference in the time if the ship is traveling to or from the demand point. The shipping times are calculated assuming a speed of 20 knots, and we use Tromsø as the origin for all plants and fields. The distance between the different facilities is so short that it does not affect the shipping time significantly. One way shipping time for demand points Falmouth, Philadelphia and Algeciras are calculated to 4.1, 8.4 and 6.1 days respectively, including 1 day for either loading or unloading.

The future of LNG demand and prices are highly uncertain. To try to forecast these values is beyond the scope of this project. Total LNG demand is based on the forecast of natural gas demand in the *GAS scenario* from IEA (2011). Europe gets a larger proportion of its gas through LNG imports compared to US. In Europe we assume that the LNG share increases from 14% to 36% the next 35 years, based on *BP Energy Outlook 2030* (BP, 2011). Of this, the amount that is delivered to Falmouth and Algeciras is based on the regasification capacity of their countries compared to the total regasification capacity in Europe (IGU, 2010). In the US, an increasing domestic production will limit the LNG demand. We assume an LNG share in the interval from 1.5% to 3.8%. (BP, 2011). The demand points Falmouth, Philadelphia and Algeciras are from now on referred to as UK, US and Spain respectively.

We define six potential contracts with delivery to one demand point each. The delivery amount and duration are based on LNG contracts in force in 2011 (GIIGNL, 2011). The terms of the first two contracts are to deliver between 20 and 30 trillion Btu each year to UK. They start in period 9 and 5 respectively and have durations of 19 and 9 years. Contract three starts in period 5, and is a large contract with delivery between 100 and 170 trillion Btu each year to the US. Contract four also delivers to US, but with lower yearly delivery of 20-40 trillion Btu. Contracts five and six deliver LNG to Spain, with start in period 1 and 8 respectively. The yearly quantum are 30-50 trillion for contract five and 40-70 trillion Btu for contract six.

8.3.6 Spot LNG Prices

The prices in our model are based on the assumptions in IEA (2011). These price paths are shown in Figure 8-5a, together with a logarithmic trend line fitted to the data. The expected prices in the base case start at the LNG prices of May 2012 (Waterborne Energy, Inc., 2012, citied in FERC, 2012a). The development over time follows the logarithmic trend line shown in Figure 8-5a. After year 2027, the prices are split in a high and low path, corresponding to the two price scenarios.

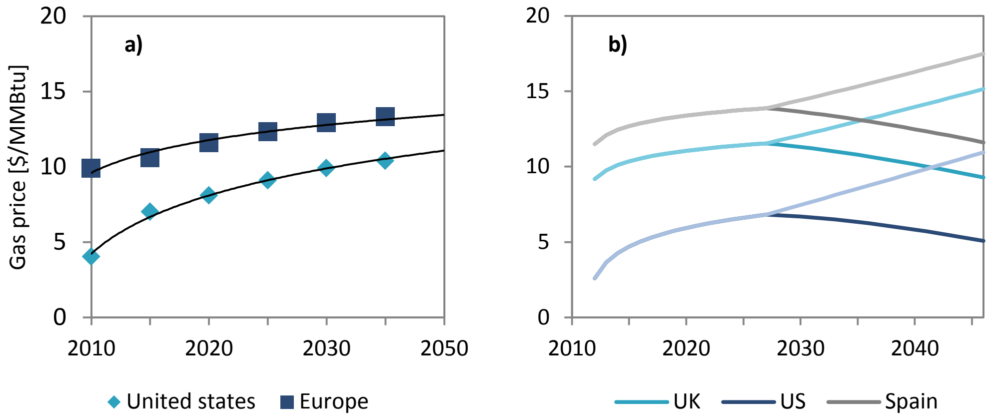

Figure 8-5: *a) Price forecast in US and Europe by IEA (2011) with logarithmic trend line. b) Prices used in the base case.*

The full price paths are shown in Figure 8-5b. The prices in 2012 are $9.17, $2.59 and $11.50 per MMBtu for UK, US and Spain respectively.

8.3.7 Dataset Variations

The stochastic model was also solved with some variations in the dataset. The use of FLNG units is restricted in *noFLNG* and *noMoving*, to see how this affects the solution. The rest of the datasets are variations in the sizes of some of the core sets in the model. Table 8-1 summarizes the different datasets and how they vary from the base case.

Table 8-1: *Summary of datasets.*

Name	Difference from base case
noFLNG	The FLNG option is excluded from the model
noMoving	Can build FLNGs, but not move them between fields
f5	Field 6 is removed
f7	An extra field: field 7 is added to the dataset
s16	One uncertain parameter, size of field 5, is made deterministic
s64	Size of field 6 made uncertain, resulting in 64 scenarios
t10	Reduced to 10 time periods. Still 35 year horizon
t20	Expanded to 20 time periods. Still 35 year horizon

Chapter 9

Results and Discussion

In this chapter, we first present some problem statistics and run times in Computational Results. In the second section, the solution of the deterministic model, which corresponds to the expected value solution of the stochastic model, is presented and analyzed. Results from the stochastic model are given in the third section.

9.1 Computational Results

Expanding the deterministic model to a stochastic version increased the model size significantly. Figure 9-1 shows the number of variables and constraints of the two models for the base case. Both models solve the exact same problem with the same number of fields, time periods, etc.; the only difference is the uncertain prices and amount of recoverable gas in the stochastic model. The large number of variables and constraints illustrates how stochastic models quickly become intractable. The reason for the increased number of restrictions relative to the number of variables in the stochastic model is the non-anticipativity constraints. Before the Xpress solver starts solving the problem, the model is *presolved*. This process reduced the problem size significantly. The deterministic model was reduced to 992 restrictions (60% reduction) and 2022 variables (40%), while the stochastic model was reduced to 29,070 restrictions (76%) and 40,699 variables (62%).

The model was run on an HP ProLiant DL165 G6 blade server with two six core AMD Opteron 2431 2.4 GHz processors and 24GB of physical memory. Table 9-1 gives a summary of the results from solving the different datasets. The best bound and objective values are the results from the Branch and Bound (BB) tree search. *Gap* is defined as the percentual difference between the best available objective value and the best bound, and is given in the gap column. The run times of the different data instances were all limited to 24 hours (86,400 seconds). None of the

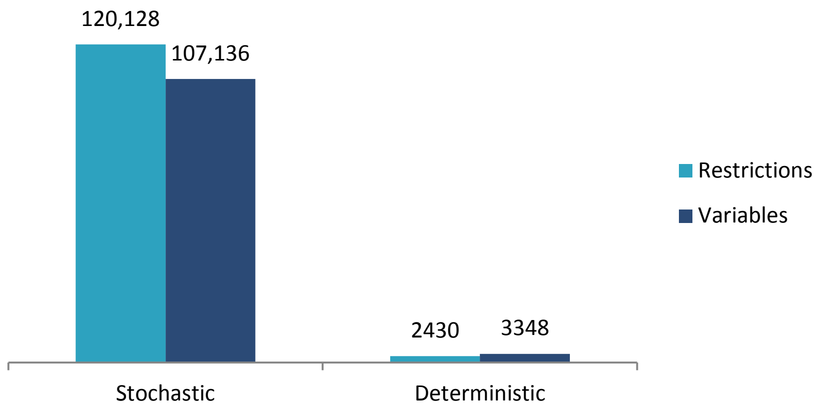

Figure 9-1: *Size of the deterministic and stochastic problems solving the base case.*

stochastic datasets were solved to optimality within this time limit. The number of nodes in the branch and bound tree search is given in the BB tree nodes column. The gap of the *noFLNG* set is interestingly higher than the gap of the *basecase* and *noMoving*, even though the number of nodes is much higher.

Table 9-1: *Key results from solving the different datasets.*

	Branch and bound					
Name	BB tree size [nodes]	Best Bound [1000 $]	Best Solution [1000 $]	Gap	Solutions	Time [s]
basecase	56 077	27 810 008	25 180 714	10.44 %	32	86 400
noFLNG	560 000	27 394 148	23 835 250	14.93 %	12	86 400
noMoving	8 623	27 830 284	24 929 542	11.64 %	69	86 400
deterministic	101 035	25 546 678	25 544 136	0.01 %	26	712
f5	49 000	27 364 072	25 041 914	9.27 %	55	86 400
f7	15 000	30 282 592	26 438 485	14.54 %	79	86 400
s16	217 000	27 610 786	25 314 401	9.07 %	48	86 400
s64	1 000	27 771 111	23 631 554	17.52 %	11	86 400
t10	210 000	21 541 799	19 785 812	8.87 %	60	86 400
t20	14 000	25 031 081	21 461 253	16.63 %	36	86 400

The model was solved with different sizes of some of the key sets in the model, to see how the run time was affected. Figure 9-2 summarizes the gap after solving the different datasets for 24 hours. Small and large sets correspond to removing/adding: one field, five time periods or one uncertain parameter. The gap is lower with smaller datasets and larger when the datasets are expanded. The difference is, however, bigger when the size of the datasets is increased, than when they are decreased. These results indicate that the run time of the model grows faster than linearly in the size of the input sets. Expanding the input data beyond

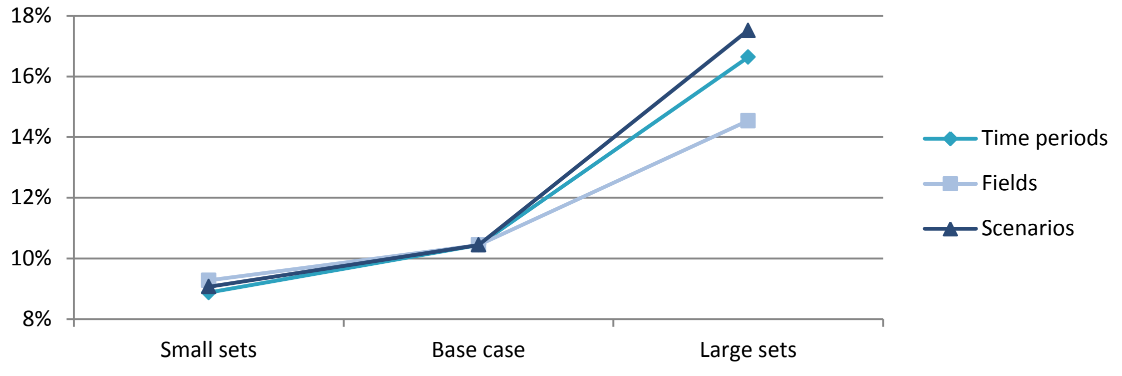

Figure 9-2: *Change in gap after solving the model for 24 hour with different set sizes.*

the size of the base case makes the problem hard to solve. When solving the model with large datasets, we also observed that the bounds converged slower towards the end of the run. This indicates that it may take extremely long time to reduce the gap to acceptable levels.

The gap for the base case is reduced fast in the first few seconds, before the bounds converge more slowly. The model found a solution with about 12% gap after six hours, but the progress after this was slow. Figure 9-3 shows the convergence of the upper bound and the best solution of the stochastic model, solving the base case. The number of solutions is also shown, using the secondary axis. We see that the best solution increases when a new solution is found, while the bound decreases more continuously.

The Wait-and-See (WS) solution for the base case resulted in an objective value of 25,461,921. The WS model is similar to the stochastic model, but the non-anticipativity constraints have been ignored. The resulting WS solution is, in fact, an upper bound on the solution of the stochastic model, since the feasible area has

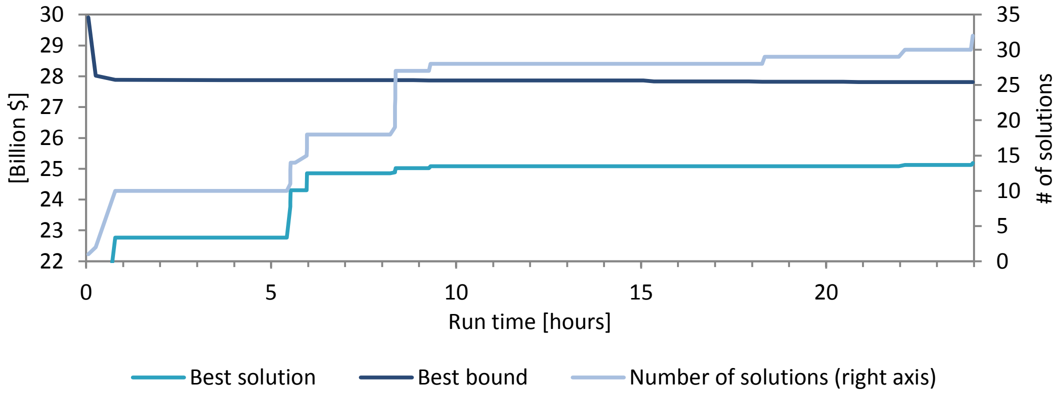

Figure 9-3: *Convergence of upper bound and best solution of the base case.*

been expanded. This upper bound is better than the one calculated by Xpress, and proves that the solution is within 1.12% of the optimal solution, which generally is within acceptable levels for practical applications. Approximations and uncertainties in input data represent a greater impact on the optimum than a small optimality gap.

9.1.1 Dantzig-Wolfe Decomposition

The Dantzig-Wolfe decomposition was run on the base case to see if a better solution or bound could be found. The implementation was tested on a small scale dataset where it found the optimal solution, but the calculations where slower than the standard Xpress solver. This is not surprising, since decompositions can result in longer solution times on small-scale problems due to additional overhead. The decomposition does, however, converge slowly on our full problem as well. After solving the base case for 168 hours (7 days), the model found an objective value of 23,727,800, which is well below the results we obtained using the standard Xpress solver. Since the Dantzig-Wolfe calculations use the dual prices of the constraints, Presolve had to be turned off. Presolve removes redundant constraints in the problem to solve it more efficiently. Earlier in this chapter we presented the large effect (76% reduction in constraints) of the Presolve on the problem size, and this difference might partly explain the poor performance of our Dantzig-Wolfe implementation. The large amount of non-anticipativity constraints in the master problem, and thereby large amount of dual variables, might be another cause of the slow convergence.

The Dantzig-Wolfe algorithm can also be used to find an upper bound before it is finished, by taking the objective value of the master problem in any iteration, and adding the reduced costs from the sub problems. The resulting upper bound was, however, far worse than the bounds from the Xpress solver or WS calculations. Hence, our implementation of Dantzig-Wolfe decomposition did not provide any useful results.

9.2 Expected Value Solution

This section contains a presentation of the results from solving the base case using the deterministic model; the solution corresponds to the expected value solution to the base case of the stochastic model. The base case returns a total NPV-adjusted profit of $25.5 billion. Figure 9-4 summarizes all the investment decisions over the time horizon of 35 years. Note that fields, pipelines and FLNGs have a gap between when the investment decisions are taken (shown in the figure) and when the elements become operational. This gap can, because of the aggregation of time periods, not be enforced on a yearly resolution after year 5. Field 1 is developed and connected to plant 2 prior to the time horizon; they constitute the existing infrastructure.

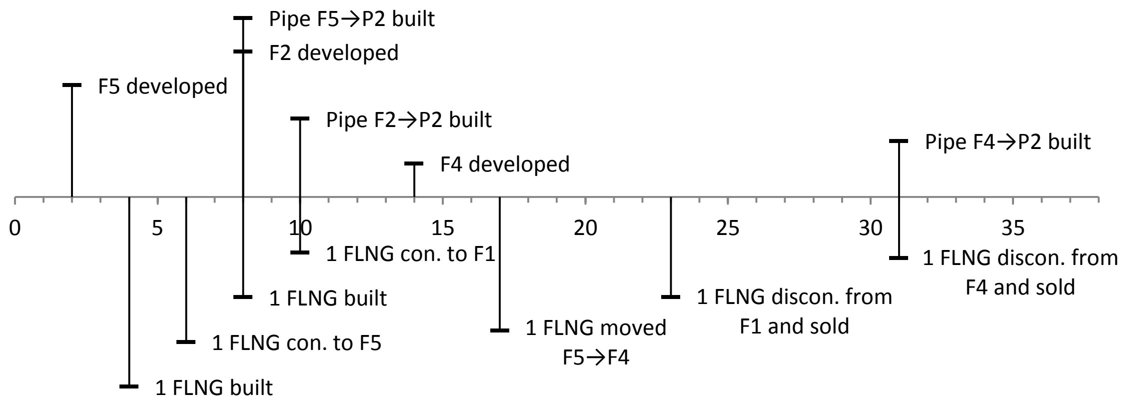

Figure 9-4: *Timeline of infrastructure investments in the base case, shown with yearly resolution. Fields are denoted with an* F *and plants with* P*.*

Three of the six fields are developed throughout the horizon: Field 2, 4 and 5. This means that these fields contribute positively to the net present value. The capacity of plant 2 is not high enough to process the gas from all the developed fields, and additional liquefaction facilities are needed. FLNG units have lower costs than plants, and two FLNG units are built.

Field 1, 4 and 5 are connected to both plant 2 and an FLNG unit, while field 2 is connected to plant 2 only. The decision to develop field 5 is taken in year 2, and an FLNG unit is connected to the field once it becomes operational. Two years later, a pipeline to plant 2 is also built. The FLNG is moved to field 4 at the end of year 17, and processes gas from that field until the end of year 30 when it is disconnected and sold. At that time, plant 2 has spare capacity and field 4 is connected to the plant. Field 1 is connected to plant 2 prior to the horizon. At the end of year 9, it is also connected to an FLNG, which is disconnected and sold at the end of year 22. An interesting observation is that the FLNG units are not used as the only liquefaction solution on any of the fields. They are instead used to supplement the onshore plant to increase the capacity at the peak of the fields' production. This behavior might be linked to our dataset where we have many fields in a small geographical area, and thereby relatively low cost of connecting the fields to the onshore plant.

The production profiles for the fields are given in Figure 9-5, and are split in type of liquefaction solution. We clearly see that the production from each field enters the decline-phase. The discounting of future cash flows make production in the early years more profitable, and pushes production towards the start of the planning period. The model does, however, choose to spread the investments and production over the whole horizon. The needed peak capacity, and then also investment costs, of the infrastructure is thus reduced.

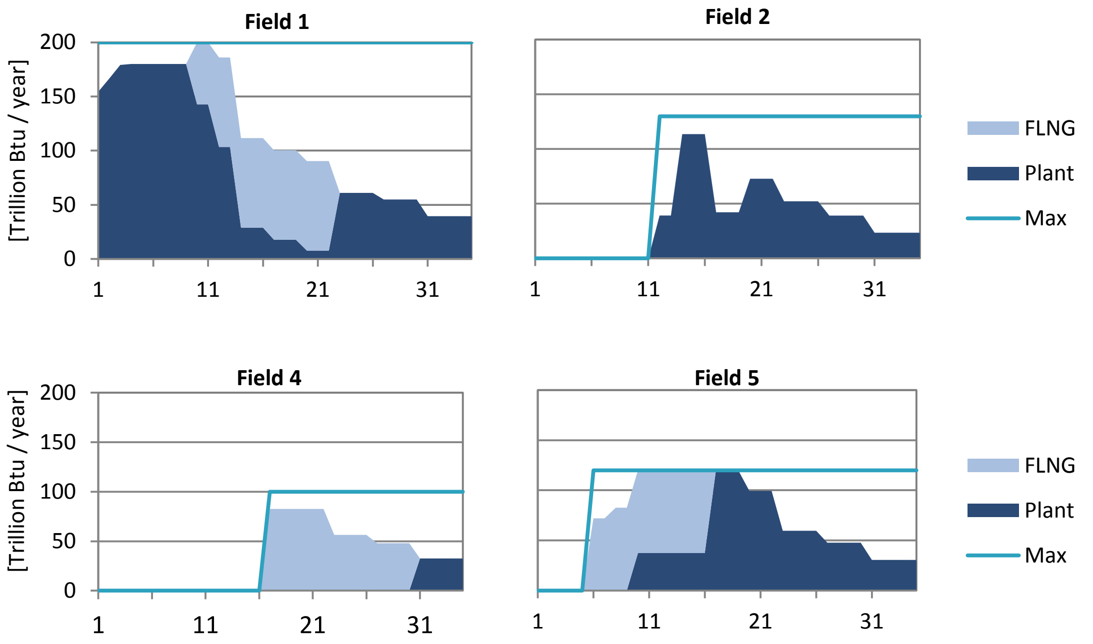

Figure 9-5: *Production profiles for the developed fields; 1, 2, 4 and 5. The production is split in type of liquefaction facility.*

Field 3 and 6 are never developed. This is reasonable since they are the smallest fields and has the highest total cost per unit gas produced. Field 6 is also located far from shore, so connecting it to the onshore plant is very expensive.

Figure 9-6 shows the capacity and gas flow through plant 2. The flow is broken down in which fields the gas is coming from. The amount of processed gas is at the capacity limit of the plant for most of the horizon. The production for all fields is reduced towards the end of the horizon, since they have all reached the decline-phase. Plant 2 then has spare capacity, even though all FLNGs are sold and the plant is the only processing facility left.

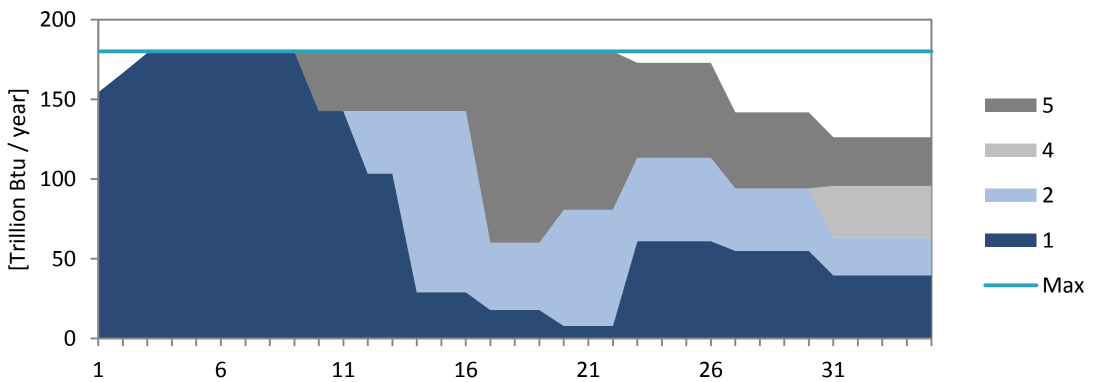

Figure 9-6: *Plant 2 throughput and capacity. The flow through the plant is split in originating fields.*

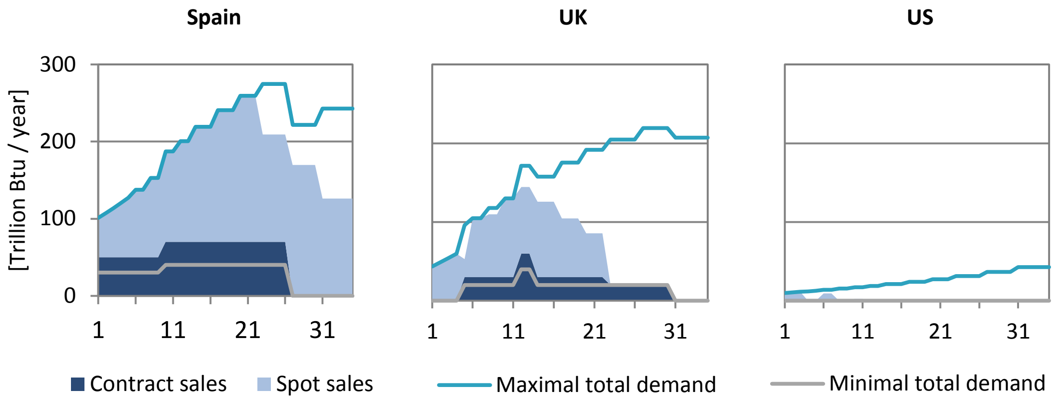

Figure 9-7: *LNG delivered to Spain, UK and US, split into spot and contract sale. The lines show the minimum and maximum demand, included spot demand and sealed contract demand.*

Four contracts are sealed; two with delivery to Spain and two to UK. These are the contracts that have higher prices than the spot price in their respective demand points. The deterministic model has no uncertainty, hence only contracts with higher prices than the spot market is sealed. The spot price is highest in Spain in all periods, and even higher than the contract price for UK. Selling LNG to Spain is most profitable, even though the shipping distance to Spain is longer than to UK. Total demand, including spot and contracts, in Spain is therefore covered first, then UK and then US. LNG is sold to the US only in periods where the production exceeds the total demand for both Spain and UK. The contract deliveries to the UK are at the maximum level until year 23, when spare spot demand arises in Spain. Since it is more profitable to sell gas to Spain, the minimum quantum is delivered to UK for the rest of the contract period, and the rest is sold to Spain. Figure 9-7 shows the delivered amount of LNG to each of the demand points, split by LNG sold on contract and spot. The minimum and maximum deliveries are given by the lines. Minimum deliveries are a result of the sealed contracts.

Figure 9-8 shows the total cost of the produced gas from the onshore liquefaction plant and the floating production units. Investment costs are divided by the total amount of gas flowing through the infrastructure, to find the costs per produced unit. The investment cost of field 1, plant 2 and the pipeline connecting them are included to make the onshore and offshore solution comparable, even though the investments are made prior to the horizon. The two liquefaction solution processes gas from different fields, still, the same average field cost is used for both solutions. This is because the field costs are not affected by the choice of liquefaction solution. The costs in this calculation are not discounted.

Field development and liquefaction contribute most to the overall costs. The onshore solution gives a lower unit cost than the floating solution. This is reasonable since the utilization of plant 2 is higher than the utilization of the floating solution, with 92% and 88% respectively. It is, however, important to point out that the floating solution is used precisely because this leads to the highest profits; a new onshore plant would have led to a higher cost per unit. The gas is sold with an average profit of $9.02 per MMBtu. Most of the gas is delivered to Spain, and well into the planning horizon.

The utilization rates are in general very high, and are a result of the decision to distribute investments over the horizon and avoid high peaks in the production. The production becomes more profitable, since the fixed operating costs can be divided by a higher average production. The high utilization rates are also an effect of choices made in the modeling. The model is allowed to close down infrastructure at no cost, and can adjust the number of operational FLNG units when less liquefaction capacity is needed.

To benchmark the results from the model, the costs were compared to the estimated costs from three different sources: Jensen (2009), Nexant (2009) and EP (2009). The comparison is shown in Figure 9-8. Different assumptions are made in the different sources, so the values are not totally comparable, but we see that our solution is in the same order of magnitude as the other sources. Jensen (2009) assumes a shipping distance that are approximately 2.5 times longer than the distance between Tromsø and Spain, which explains the much higher shipping cost. Nexant (2009) assumes that the liquefaction is on a large floating unit (6 mtpa), with a shipping distance in the same order of magnitude as ours. EP (2009) divide investment costs by the maximum potential production over 15 years, while we use the actual production over the whole 35-year horizon. Investment cost are thereby not divided by the same amount of gas, which might explain the difference in average unit cost.

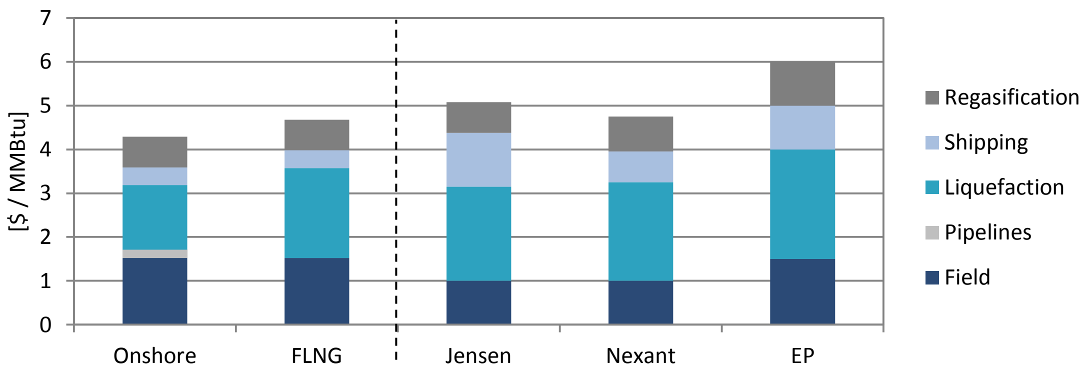

Figure 9-8: *Unit costs of elements in the LNG value chain. Left) Unit costs for the base case. Right) Unit costs from Jensen (2009), Nexant (2009) and EP (2009).*

9.3 Solution to the Stochastic Model

The results from the stochastic model are presented in this section. It is important to note that the optimal solution to the stochastic problem not was found. Some non-optimal decisions might therefore exist, even though the presented solution is close to optimum.

The results in the different scenarios are in general very similar. The total discounted revenue and costs of the different scenarios are shown in Figure 9-9. Even though there are significant differences in field sizes and gas prices, the different scenarios result in relatively similar profits. One important cause of this outcome is that decisions and uncertain parameters are equal until the uncertainty is resolved. The first stages will be very similar across the different scenarios. The early years will also contribute more to the overall profits, since the later years are more heavily discounted. The price scenarios do for example have the same prices for the first 16 years, which also are the years that constitute the majority of the total discounted revenue. The differences between the scenarios are on the other hand significant, even though the expected profits are relatively similar. The profit in scenario 1 is for example more than $3 billion higher than in scenario 32.

The expected profit from the stochastic solution is slightly lower than in the deterministic solution. The deterministic model finds optimal decisions for only one expected future, and does not have to take alternative scenarios into account. The deterministic solution is not necessarily feasible for the stochastic model, since it can involve production of more gas than what is available in some of the scenarios. The higher expected profit in the deterministic model does not mean that it finds a better solution. The difference only shows that the calculation of expected profits in the deterministic model takes less information into account.

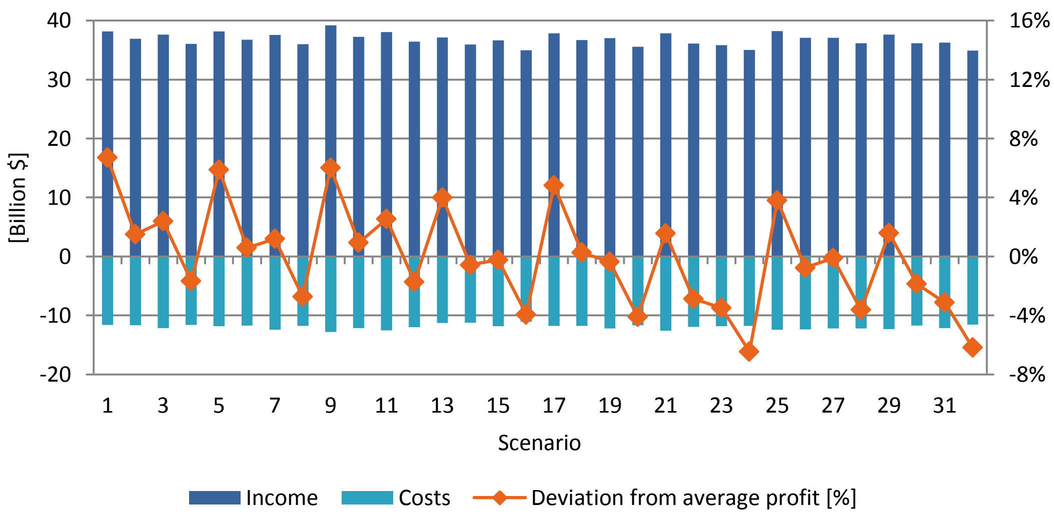

Figure 9-9: *Total discounted revenue and costs in the different scenarios.*

Also the *decisions* in the stochastic solution are similar to the ones from the deterministic solution. The investment decisions are identical for the first stage; Field 5 is developed in period 2 and an FLNG is built in period 4. These decisions must be equal for all scenarios since year 1 to 5 is in the first stage. A stochastic model takes all scenarios into account when the optimal first stage decisions are found, and usually results in better solutions than the deterministic model can provide. This is, however, not the case in the solutions of the base case.

For later stages, the decisions diverge from the deterministic solution, and also vary between the scenarios. Field 2 and 4 are built between period 8 and 22, with different decisions in the different scenarios. Field 6 is, in contrast to the deterministic solution, developed in eight of the scenarios. All of these scenarios have high gas prices, which makes field 6 profitable. When field 6 is developed, it is done in year 20 of 35, which is in the last stage. Field 6 is not profitable in many of the scenarios, and the model waits until after all uncertainty is resolved before the decision on whether to invest in field 6 or not is taken.

Production rates from the fields vary between the scenarios. This is expected since different scenarios have different amount of recoverable gas, and the decline phase will come earlier in some scenarios. Figure 9-10 shows the production profile for the different scenarios for field 1, 2, 4 and 5. Field 1 is already operating at the start of the horizon, and does not have an uncertain recoverable amount of gas. The production profile is almost identical for all scenarios. In the other fields, a larger discrepancy between the scenarios can be observed. The year when the field's uncertainty is resolved is shown in Figure 9-10 as a dotted line. Note that the

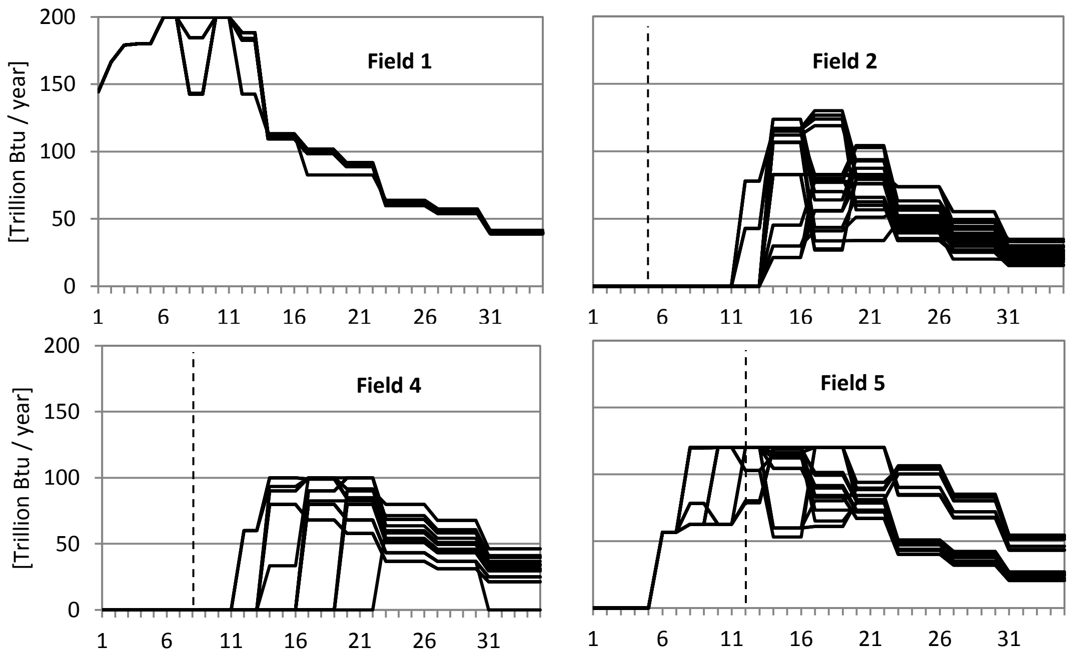

Figure 9-10: *Production rate for Field 1, 2, 4 and 5 in the different scenarios. The dotted line represent the year the uncertainty in the respective fields are resolved.*

investment decision must be taken 5 years before production start because of the investment gap. The investment decision in both field 2 and 4 is taken after the uncertainty is resolved, while field 5 is developed before. The gap between investment and production in field 4 is shorter than 5 years, because several years are aggregated in one period, and the constraint cannot be enforced on a yearly resolution after year 5.

No plants are built in any of the scenarios. Two FLNG units are built in all scenarios, except scenario 9 which utilizes three FLNG units. There are differences in how long they operate, and which fields they are connected to. The first FLNG unit becomes operational in year 7 in all scenarios, which is when field 5 can start producing. The liquefaction capacity of plant 2 is not big enough to process the gas from both field 1 and 5. All FLNG units are built in the smallest available size, which has the lowest liquefaction cost per unit of production.

The FLNG units are connected to fields and moved around to adjust for the different amounts of recoverable gas in the different scenarios. The interplay between the different fields does, however, mean that it can be difficult to see why the FLNG units are connected to specific fields. Field 6 is connected to an FLNG unit in all the scenarios it is developed, which is an expected result of the long distance from field 6 to the shore. In general, the FLNG units are moved well into the planning horizon. They are often connected to fields after the uncertainty is resolved. The existing onshore plant provides a base liquefaction capacity, while FLNG units are used to react to the uncertain field sizes.

Contract 1, 2, 5 and 6 are sealed in all scenarios. These contracts have a price between the spot prices in the low and high price scenarios. Expected profits are thereby reduced in the high spot price scenarios, and increased in low spot price scenarios by sealing these contracts. The expected spot price of all the scenarios combined is, however, lower than the prices in the sealed contracts. Since the model is risk neutral, the contracts are sealed.

9.3.1 Restricting the Use of FLNG units

The stochastic model was solved for two versions of the base case where the use of FLNG units was restricted. The first was without the possibility to construct FLNG units. The second was without possibility to move the FLNG after it was connected to a field. None of the solutions solved to optimality, but the WS model was solved for both cases, and provided an adequate upper bound. The optimal interval for these cases and the base case is given in Figure 9-11.

The best objective value without FLNG units is about 5.3% ($1.3 billion) lower than the solution including FLNGs, which gives us an indication of the value of the FLNG solution for the entire value chain. When FLNG units cannot be built,

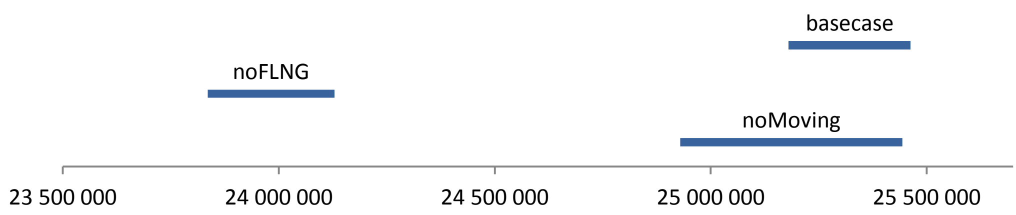

Figure 9-11: *Interval in which the optimal objective values are for the problems: basecase, noFLNG and noMoving. The lower bound is the best solutions from the solving the stochastic models. The upper bound is provided by the Wait-and-See solutions.*

plant 1 and, in some scenarios plant 3, are built instead. They are both built with a capacity of about 100 to 120 trillion Btu per year, which is close to the minimum. Plant 1 is built in year 4 in all scenarios, while plant 3 is built after some of the uncertainty is resolved, and generally in scenarios where the total amount of recoverable gas is high. Plant 3 is hence used to adjust the capacity as the uncertainty is resolved, similarly to the way some of the FLNG units were used in the stochastic model. More fields are also built compared to the general stochastic model. Field 6 is developed in 25 scenarios, and field 3 in 7. The fields are often connected to more than one plant. A possible reason behind this behavior is that the pipelines are relatively cheap, and connecting the field to multiple plants gives flexibility to adjust to the uncertain field sizes; building redundant pipelines is a cheap insurance against variable recoverable amounts.

To further analyze the value of FLNG units, the stochastic model was also solved with FLNG units, but without the option of moving them. This resulted in an objective value of $24.9 billion (1.0% lower than the base case). Figure 9-11 shows that the optimal objective value interval of *noMoving* overlaps with the interval of the base case. We can therefore not conclude that *noMoving* gives a worse solution than the base case. The objective value is, however, higher (4.6%) than the objective value with no FLNGs. The result shows that the value of moving FLNG units is very small and that the value of the FLNG units in the base case mainly derives from the lower liquefaction costs. It is, however, important to note that this result is dependent on the dataset. All fields are located within a relatively small geographical area in our dataset. The ability to move the FLNG units might have a much larger value if the fields are spread over a larger area.

9.3.2 VSS^D & EVPI

Expected Value of Perfect Information was defined as the difference between the objective value of the Wait-and-See (WS) solution and the stochastic solution. Without the optimal objective value of the stochastic problem we cannot find the EVPI exactly. Using the best solution we found gives an EVPI of 281,208 (1.12% of base case objective value). The real EVPI is the same or lower than this value because the optimal stochastic objective value must be the same or higher than the value we found. This result means that knowing the outcomes of the uncertain parameters gives little value in the base case.

The EDEV calculations resulted in a higher objective value than the best solution from solving the base case with the stochastic model. This corresponds to a negative VSS^D, which is impossible. The reason is that the stochastic model did not solve to optimality. The $EDEV_S$ value is well below the upper bound (UB) on the objective value. The optimal stochastic solution must have an equal or higher objective value than the $EDEV_S$, since the $EDEV_S$ solution is also feasible in the stochastic model. We have therefore not found the optimal solution to the stochastic model. The EDEV calculations are in effect a heuristic for the base case. Figure 9-12 shows the $EDEV_S$ and WS objective values together with the LB and UB found when solving the stochastic model. The optimal stochastic solution is between the $EDEV_S$ and WS solutions, and the VSS^D must thus be relatively low. The low VSS^D means that the stochastic model gives little value over a deterministic one. It is again important to note that this result might be different for different datasets.

A possible reason for both the low VSS^D and EVPI is that the model exhibits a large amount of flexibility. Typical stochastic models involves decisions in the first stage that constraints the options in the recourse stages. If early decisions do not restrict the future significantly, the $EDEV_S$ and WS will be close to the optimal stochastic solution. Both the stochastic model and the EDEV calculations are able to adapt fairly quickly to the changing environment. The model can for example close down infrastructure at no cost, and move FLNG units. The flexibility causes a lower value of planning ahead, and of knowing the outcome of uncertainty. Some decisions would on the other hand be expected to restrict the future to a larger degree. Sealing contracts is for example a decision that benefits from foresight into the future.

Figure 9-12: *The different objective values from solving the base case. The lower and upper bound are the results from the standard Xpress solver.*

Chapter 10

Concluding Remarks

This thesis develops an optimization model of investments in infrastructure in the LNG value chain to answer when floating LNG production units are a viable solution, and find out what value they add to the LNG value chain. A review of the literature found no articles on decision support tools where FLNG units are a part of the LNG value chain, and the articles written about LNG are mainly written on a tactical level. Our model describes the whole value chain at a strategic level. A deterministic version is formulated with a focus on modeling the LNG value chain, before it is expanded to a multistage stochastic formulation with uncertain field sizes and gas prices. The objective is to maximize expected discounted profits through optimal investments in infrastructure. The datasets used to solve the model are based on a set of potential fields on the Norwegian continental shelf with shipping of LNG to three markets in the Atlantic basin.

10.1 Conclusion

The solution of both the deterministic and stochastic model involves several FLNG units, in combination with the excising onshore plant. Several fields are developed and often connected to both FLNG units and an onshore plant. The results of the model illustrate when FLNG units can add value to the value chain. The FLNG units are not used as a separate standalone solution, but rather a supplement to the onshore processing plant; the capacity is expanded in periods at the peak of a field's production. In the stochastic model we also observed that the FLNG units were used to react to the resolution of uncertain parameters. The floating liquefaction option is especially attractive for fields located far from shore, like for example field 6, which used an FLNG unit in all scenarios it was developed. Another reason for the use of FLNG units is the relatively small size. They can process smaller flows, and investments in plants with excess capacity are avoided.

It is hard to quantify the exact value of the FLNG units. Solving the model without the FLNG option resulted in significantly lower total expected profits (5.3%). This number is dependent on the input data and the specifics of each potential project, but shows that floating production units can provide significant cost savings. The model might also overlook some of the potential of the floating production units because the fields are located close together and can be connected to several plants. If the fields where more geographically separated, only FLNG units could react to uncertain field sizes by processing gas from other fields. The results also show that the value of FLNG units mainly derives from the lower liquefaction costs, not their ability to move and the flexibility it provides. Restricting the model to no moving still resulted in investments in several FLNG units, and the reduction in expected profits was significantly lower than when the model was solved without the FLNG option (1.0% compared to 5.3% reduction).

Expanding the deterministic model to a stochastic version increased the problem size significantly. The standard Xpress solver resulted in a large gap of 10.44%, and it is unlikely that the dataset can be expanded significantly beyond the size used in the base case. Our implementation of Dantzig-Wolfe decomposition, to find more efficient solution approaches, did not converge and proved to give worse run times than the standard Xpress solver. The Wait-and-See solution interestingly provided a better upper bound, and proved that the solution of the stochastic model is close to optimum (1.12% gap). Both the Expected Value of Perfect Information and the Value of the Stochastic Solution turned out to be very small for the base case. A possible explanation might be that the deterministic and stochastic models are able to adapt fairly quickly to the changing environment. This flexibility causes a lower value of planning ahead, and of knowing the outcome of uncertainty. If early decisions do not restrict the future significantly, the $EDEV_S$ and WS will be close to the optimal stochastic solution.

10.2 Future Work

There are three major areas of potential future work. The first is to solve the model with different *input data*. The base case gives almost no added value of using the stochastic model over the deterministic model. It would be interesting to look into the results of the models when the fields are distributed over a larger area, such that onshore plants cannot react to uncertain field sizes by processing gas from other fields. The second is to reduce the *run time* to make the model able to solve bigger problems. Other decomposition methods can for example be applied on the model to see if they perform better than the Dantzig-Wolfe decomposition. The last area is to *expand the model* to represent more details of the LNG value chain. This area contains several aspects and is covered in the rest of this section.

The current models assume a risk-neutral user. The expected profit is maximized, regardless of the risk involved. To include a risk measure in the model, like for example Conditional Value-at-Risk (CVaR), might give a more balanced solution. The model could be expanded by specifying a maximum level of risk and add this as a constraint, or to include the risk as a cost in the objective function.

The uncertainty in the stochastic model is assumed to resolve at given time periods. The reality is much more complex. The estimation of field sizes has higher certainty after geological surveys are performed, and after the fields are developed and production has started. To make the resolution of uncertainty decision dependent would give a better representation of reality. A challenge is that this formulation creates a larger model which often is harder to solve.

The current formulation can also be expanded by allowing capacity expansion of liquefaction plants. The capacity of a plant can in reality often be expanded by adding new liquefaction trains. This would add flexibility to the plants, creating an alternative flexibility option to the floating production units.

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Appendix A

Approximate Conversion Table

The following tables were used when converting between different units in the thesis.

Table A-1: *Conversion table*

	1 billion cubic metres NG	1 billion cubic feet NG	1 million tonnes oil equivalent	1 million tonnes LNG	1 trillion British thermal units	1 million barrels oil equivalent
1 billion cubic metres NG	1	35.3	0.9	0.740	35.7	6.6
1 billion cubic feet NG	0.028	1	0.025	0.021	1.01	0.19
1 million tonnes oil equivalent	1.11	39.2	1	0.820	39.7	7.33
1 million tonnes LNG	1.36	48	1.22	1	48.6	8.97
1 trillion British thermal units	0.028	0.99	0.025	0.021	1	0.18
1 million barrels oil equivalent	0.15	5.35	0.14	0.110	5.41	1

Source: BP Statistical Review of World Energy June 2011

Table A-2: *Conversion table, LNG specific*

	1 million tonnes LNG	1 billion cubic metres LNG
1 billion cubic metres LNG	0.46	1
1 million tonnes LNG	1	2.17

Source: Sempra LNG [http://sempralng.com/Files/pdf/LNG_ CardLrg102904.pdf]

Appendix B

Deterministic Model

Nomenclature

Sets and indexes

$a \in \boldsymbol{A}$	Contract group *a*
$d \in \boldsymbol{D}$	Demand point *d*
$f \in \boldsymbol{F}$	Field *f*
$j \in \boldsymbol{J}$	Potential onshore plants *j*
$k \in \boldsymbol{K}$	Potential contracts *k*
$k \in \boldsymbol{K}_a^{ALT}$	Set of alternative contracts
$r \in \boldsymbol{R}$	FLNG size *r*
$t, \tau \in \boldsymbol{T}$	Time period *t* and *τ*
$t \in \boldsymbol{G}_t^{FLD}$	Time gap of investment in fields. Set of time periods in which a field investment would be ready in period *t*
$t \in \boldsymbol{G}_t^{FLNG}$	Time gap of investments in FLNGs. Set of time periods in which an FLNG investment would be ready in period *t*
$t \in \boldsymbol{G}_t^{PIPE}$	Time gap of investments in pipelines. Set of time periods in which a pipeline investment would be ready in period *t*
$t \in \boldsymbol{G}_t^{PLT}$	Time gap of investments in plants. Set of time periods in which a plant investment would be ready in period *t*

Parameters

C^{CON}	Cost of connecting an FLNG unit to a field. Includes all switching costs
$C_f^{FLD\ INV}$	Investment cost of field *f*
$C_f^{FLD\ OPR\ FIX}$	Fixed operating cost of operating field *f*
$C_f^{FLD\ OPR\ VAR}$	Variable cost of operating field *f* per produced unit
$C_r^{FLNG\ INV}$	Investment cost of building a FLNG ship of size *r*
$C_r^{FLNG\ OPR\ FIX}$	Fixed cost of operating FLNG ship of size *r*
$C_r^{FLNG\ OPR\ VAR}$	Variable cost of operating FLNG ship of size *r* per produced unit
$C_{f,j}^{PIPE\ INV}$	Investment cost of a pipeline from field *f* to plant *j*
$C_{f,j}^{PIPE\ OPR\ FIX}$	Fixed cost of operating pipeline from field *f* to plant *j*
$C_{f,j}^{PIPE\ OPR\ VAR}$	Variable cost of operating a pipeline between field *f* and plant *j* per produced unit
$C_j^{PLT\ INV\ FIX}$	Fixed investment cost of plant *j*
$C_j^{PLT\ INV\ VAR}$	Variable investment cost of plant *j* dependent on processing capacity
$C_j^{PLT\ OPR\ FIX}$	Fixed operating cost of plant *j*
$C_j^{PLT\ OPR\ VAR}$	Variable operating cost of plant *j* per processed unit
C_d^{REG}	Variable regasification cost at demand point *d*
C^{SHIP}	Fixed cost per ship per period
$D_{d,t}$	Spot demand rate at demand point *d* in period *t*
E_t	Maximum investment in period *t*
F_f^{DEC}	Decline rate of field *f*
F^{INC}	Maximum increase in production in a field as a percentage of max production rate in the field
F_f^{MAX}	Max production rate of field *f*
F_f^{Q}	Total quantity of recoverable gas in field *f*
$\underline{H}_{k,d,t}^{Q}$	Minimum yearly rate of gas delivered to demand point *d* in period *t* under contract *k*
$\overline{H}_{k,d,t}^{Q}$	Maximum yearly rate of gas delivered to demand point *d* in period *t* under contract *k*
I_f^{FLD}	Value per unit of gas remaining in field *f* at end of horizon if the field is producing in the last period
$I_{r,t}^{FLNG}$	Salvage value of an FLNG of size *r* in period *t*
N_t	Net present value of \$1, *t* periods into the future
$P_{k,t}^{CNT}$	Selling price of contract *k* per unit

$P_{d,t}^{SPOT}$	Spot selling price of LNG at demand point *d* in period *t* per unit
S^{CAP}	Transport capacity per ship
$S_{f,d}^{DIST\ F \to D}$	One way shipping time from field *f* to demand point *d*, in years
$S_{j,d}^{DIST\ J \to D}$	One way shipping time from plant *j* to demand point *d*, in years
$U_{f,t}^{FLD}$	Is 1 if field *f* can be developed in time period *t*, 0 otherwise
V_r^{FLNG}	Processing capacity per FLNG of size *r*
V^{PIPE}	Capacity of a pipeline
$\underline{V}_j^{PLT}$	Minimum processing capacity of plant *j*
$\overline{V}_j^{PLT}$	Maximum processing capacity of plant *j*
Θ_t	Number of years in time period *t*
Variables	
cf_t^{INC}	Cash flow from income in period *t*
cf_t^{INV}	Cash flow from investments in period *t*
cf_t^{OPR}	Cash flow from operations in period *t*
e_f	Remaining gas in field *f* at end of horizon. Set to zero if the field is not operational at the end of the horizon
EFP	Expected profit from fields after horizon
$g_{f,t}$	Cumulative production in field *f* up to period *t*
$q_{k,d,t}^{CNT}$	Rate of LNG delivered to demand point *d* in period *t* under contract *k*
$q_{f,j,t}^{F \to J}$	Rate of gas sent from field *f* to plant *j* in period *t*
$q_{f,r,d,t}^{FLNG}$	Rate of LNG sent from field *f* to demand point *d* through an FLNG of size *r* in period *t*

$q_{j,d,t}^{J \to D}$	Rate of LNG transported from plant *j* to demand point *d* in period *t*
$v_{j,t}^{PLT}$	Processing capacity of plant *j* that is being built in period *t*, 0 for all other *t*'s
$w_{f,r,t}^{CON}$	Is 1 if FLNG of size *r* is being disconnected from field *f* in period *t*, 0 otherwise
$w_{f,t}^{FLD}$	Is 1 if field *f* is being closed in period *t*, 0 otherwise
$w_{r,t}^{FLNG}$	Is 1 if FLNG of size *r* is sold in period *t*, 0 otherwise
$w_{f,j,t}^{PIPE}$	Is 1 if pipeline from field *f* to plant *j* is being closed in period *t*, 0 otherwise
$w_{j,t}^{PLT}$	Is 1 if plant *j* is being closed in period *t*, 0 otherwise
x_k^{CNT}	1 if contract *k* is sealed in period *t*, 0 otherwise
$x_{f,r,t}^{CON}$	Is 1 if a connection is being built between field *f* and an FLNG of size *r* in period *t*, 0 otherwise
$x_{f,t}^{FLD}$	Is 1 if field *f* is being developed in period *t*, 0 otherwise
$x_{r,t}^{FLNG}$	Number of FLNGs of size *r* being built in period *t*
$x_{f,j,t}^{PIPE}$	Is 1 if pipeline from field *f* to plant *j* is being built in period *t*, 0 otherwise
$x_{j,t}^{PLT}$	Is 1 if plant *j* is being built in period *t*, 0 otherwise
$y_{f,r,t}^{CON}$	Number of FLNGs of size *r* that field *f* is connected to in period *t*
$y_{f,t}^{FLD}$	Is 1 if field *f* can produce in period *t*, 0 otherwise
$y_{r,t}^{FLNG}$	Number of operational FLNGs of size *r* in period *t*
$y_{f,j,t}^{PIPE}$	Number of pipelines from field *f* to plant *j* can transport gas in period *t*
$y_{j,t}^{PLT}$	Is 1 if plant *j* of size *s* is operational in period *t*, 0 otherwise

Objective function

The objective function is a sum of the discounted cash flows from income, investment cost and operational cost; and expected future profit from operating fields:

$$\max z = \sum_{t\in \boldsymbol{T}} N_t\,(cf_t^{INC} - cf_t^{INV} - cf_t^{OPR}) + N_T\,EFP$$

Cash flows in each time period of investment cost for fields, pipelines, plants and FLNGS:

$$\begin{aligned} cf_t^{INV} = &\sum_{f\in \boldsymbol{F}} C_f^{FLD\ INV} x_{f,t}^{FLD} \\ &+\sum_{f\in \boldsymbol{F}}\sum_{j\in \boldsymbol{J}} C_{f,j}^{PIPE\ INV} x_{f,j,t}^{PIPE} \\ &+\sum_{j\in \boldsymbol{J}} C_j^{PLT\ INV\ FIX} x_{j,t}^{PLT} + \sum_{j\in \boldsymbol{J}} C_j^{PLT\ INV\ VAR} v_{j,t}^{PLT} \\ &+\sum_{r\in \boldsymbol{R}} C_r^{FLNG\ INV} x_{r,t}^{FLNG} \end{aligned}$$

Cash flow in each time period of operational cost for fields, pipelines, plants, connection between a field and an FLNG, FLNG, shipping and regasification facilities:

$$\begin{aligned} cf_t^{OPR} = &\,\Theta_t \sum_{f\in \boldsymbol{F}} C_f^{FLD\ OPR\ FIX} y_{f,t}^{FLD} \\ &+\Theta_t \sum_{f\in \boldsymbol{F}} C_f^{FLD\ OPR\ VAR} \left(\sum_{j\in \boldsymbol{J}} q_{f,j,t}^{F\to J} + \sum_{d\in \boldsymbol{D}}\sum_{r\in \boldsymbol{R}} q_{f,r,d,t}^{FLNG} \right) \\ &+\Theta_t \sum_{f\in \boldsymbol{F}}\sum_{j\in \boldsymbol{J}} C_{f,j}^{PIPE\ OPR\ FIX} y_{f,j,t}^{PIPE} + \Theta_t \sum_{f\in \boldsymbol{F}}\sum_{j\in \boldsymbol{J}} C_{f,j}^{PIPE\ OPR\ VAR} q_{f,j,t}^{F\to J} \\ &+\Theta_t \sum_{j\in \boldsymbol{J}} C_j^{PLT\ OPR\ FIX} y_{j,t}^{PLT} + \Theta_t \sum_{j\in \boldsymbol{J}} C_j^{PLT\ OPR\ VAR} \sum_{d\in \boldsymbol{D}} q_{j,d,t}^{J\to D} \\ &+\sum_{f\in \boldsymbol{F}}\sum_{r\in \boldsymbol{R}} C^{CON} x_{f,r,t}^{CON} \\ &+\Theta_t \sum_{r\in \boldsymbol{R}} C_r^{FLNG\ OPR\ FIX} y_{r,t}^{FLNG} + \Theta_t \sum_{r\in \boldsymbol{R}} C_r^{FLNG\ OPR\ VAR} \sum_{f\in \boldsymbol{F}}\sum_{d\in \boldsymbol{D}} q_{f,r,d,t}^{FLNG} \end{aligned}$$

$$+\Theta_t \sum_{j\in \boldsymbol{J}} \sum_{d\in \boldsymbol{D}} \left(\frac{2 * S_{j,d}^{DIST\ J\rightarrow D}\ C^{SHIP}}{S^{CAP}} \right) q_{j,d,t}^{J\rightarrow D}$$

$$+ \Theta_t \sum_{f\in \boldsymbol{F}} \sum_{d\in \boldsymbol{D}} \sum_{r\in \boldsymbol{R}} \left(\frac{2 * S_{f,d}^{DIST\ F\rightarrow D}\ C^{SHIP}}{S^{CAP}} \right) q_{f,r,d,t}^{FLNG}$$

$$+\Theta_t \sum_{d\in \boldsymbol{D}} C_d^{REG} \left(\sum_{j\in \boldsymbol{J}} q_{j,d,t}^{J\rightarrow D} + \sum_{f\in \boldsymbol{F}} \sum_{r\in \boldsymbol{R}} q_{f,r,d,t}^{FLNG} \right)$$

Cash flows in each time period of income from sales on spot market, contract sales and salvage value of FLNGs:

$$cf_t^{INC} = \sum_{d\in \boldsymbol{D}} P_{d,t}^{SPOT} \Theta_t \left[\left(\sum_{j\in \boldsymbol{J}} q_{j,d,t}^{J\rightarrow D} + \sum_{f\in \boldsymbol{F}} \sum_{r\in \boldsymbol{R}} q_{f,r,d,t}^{FLNG} \right) - \sum_{k\in \boldsymbol{K}} q_{k,d,t}^{CNT} \right]$$

$$+ \Theta_t \sum_{d\in \boldsymbol{D}} \sum_{k\in \boldsymbol{K}} P_{k,t}^{CNT} q_{k,d,t}^{CNT}$$

$$+ \sum_{r\in \boldsymbol{R}} I_{r,t}^{FLNG} w_{r,t}^{FLNG}$$

Expected future profit of the producing fields:

$$EFP = \sum_{f\in \boldsymbol{F}} I_f^{FLD} e_f$$

Constraints

Each field and plant can at maximum be developed / built once:

$$\sum_{t\in \boldsymbol{T}} x_{f,t}^{FLD} \leq 1, \qquad \forall\, f \in \boldsymbol{F} \tag{D1}$$

$$\sum_{t\in \boldsymbol{T}} x_{j,t}^{PLT} \leq 1, \qquad \forall\, j \in \boldsymbol{J} \tag{D2}$$

Constraints describing the relationship between developing/building variables, closing/selling variables and is-operational variables for fields, pipelines, plants, FLNGs and the connection between fields and FLNGs:

$$y_{f,t-1}^{FLD} + \sum_{\tau\in\, \boldsymbol{G}_t^{FLD}} x_{f,\tau}^{FLD} - w_{f,t-1}^{FLD} = y_{f,t}^{FLD}\,, \qquad \forall\, f \in \boldsymbol{F}, t \in \boldsymbol{T} \tag{D3}$$

$$y_{f,j,t-1}^{PIPE} + \sum_{\tau \in \boldsymbol{G}_t^{PIPE}} x_{f,j,\tau}^{PIPE} - w_{f,j,t-1}^{PIPE} = y_{f,j,t}^{PIPE}, \qquad \forall f \in \boldsymbol{F}, j \in \boldsymbol{J}, t \in \boldsymbol{T} \tag{D4}$$

$$y_{j,t-1}^{PLT} + \sum_{\tau \in \boldsymbol{G}_t^{PLT}} x_{j,\tau}^{PLT} - w_{j,t-1}^{PLT} = y_{j,t}^{PLT}, \qquad \forall j \in \boldsymbol{J}, t \in \boldsymbol{T} \tag{D5}$$

$$y_{f,r,t}^{CON} + x_{f,r,t}^{CON} - w_{f,r,t}^{CON} = y_{f,r,t+1}^{CON}, \qquad \forall f \in \boldsymbol{F}, r \in \boldsymbol{R}, t \in \boldsymbol{T} \tag{D6}$$

$$y_{r,t-1}^{FLNG} + \sum_{\tau \in \boldsymbol{G}_t^{FLNG}} x_{r,\tau}^{FLNG} - w_{r,t-1}^{FLNG} = y_{r,t}^{FLNG}, \qquad \forall r \in \boldsymbol{R}, t \in \boldsymbol{T} \tag{D7}$$

Fields can only be developed in certain time periods:

$$x_{f,t}^{FLD} \leq U_{f,t}^{FLD}, \qquad \forall f \in \boldsymbol{F}, t \in \boldsymbol{T} \tag{D8}$$

The same rate of gas must be transported from a plant as the rate into the plant in all time periods:

$$\sum_{f \in \boldsymbol{F}} q_{f,j,t}^{F \to J} = \sum_{d \in \boldsymbol{D}} q_{j,d,t}^{J \to D}, \qquad \forall j \in \boldsymbol{J}, t \in \boldsymbol{T} \tag{D9}$$

The increase in the total production rate of gas from a field from one time period to the next must be equal to or lower than the maximum increase:

$$\begin{aligned} &\sum_{j \in \boldsymbol{J}} q_{f,j,t}^{F \to J} + \sum_{d \in \boldsymbol{D}} \sum_{r \in \boldsymbol{R}} q_{f,r,d,t}^{FLNG} \\ &\qquad \leq \left(\sum_{j \in \boldsymbol{J}} q_{f,j,t-1}^{F \to J} + \sum_{d \in \boldsymbol{D}} \sum_{r \in \boldsymbol{R}} q_{f,r,d,t-1}^{FLNG} \right) \\ &\qquad\quad + \Theta_t F^{INC} F_f^{MAX}, \qquad \forall f \in \boldsymbol{F}, t \in \boldsymbol{T} \end{aligned} \tag{D10}$$

The total production rate of gas from a field must at all times be at most the maximal rate, given that the field is operational:

$$\sum_{j \in \boldsymbol{J}} q_{f,j,t}^{F \to J} + \sum_{d \in \boldsymbol{D}} \sum_{r \in \boldsymbol{R}} q_{f,r,d,t}^{FLNG} \leq F_f^{MAX} y_{f,t}^{FLD}, \qquad \forall f \in \boldsymbol{F}, t \in \boldsymbol{T} \tag{D11}$$

The total production rate of gas from a field must in each time period be equal to or lower than the remaining gas in the period multiplied with a decline constant:

$$\Theta_t \left(\sum_{j \in \boldsymbol{J}} q_{f,j,t}^{F \to J} + \sum_{d \in \boldsymbol{D}} \sum_{r \in \boldsymbol{R}} q_{f,r,d,t}^{FLNG} \right) \leq F_f^{DEC} (F_f^Q - g_{f,t}), \qquad \forall\ f \in \boldsymbol{F}, t \in \boldsymbol{T} \tag{D12}$$

Where the cumulative production is:

$$g_{f,t} = \sum_{\tau = 1..(t-1)} \Theta_\tau \left(\sum_{j \in \boldsymbol{J}} q_{f,j,\tau}^{F \to J} + \sum_{d \in \boldsymbol{D}} \sum_{r \in \boldsymbol{R}} q_{f,r,d,\tau}^{FLNG} \right), \qquad \forall\ f \in \boldsymbol{F}, t \in \boldsymbol{T}$$

The capacity of a plant is decided in the period when the plant is build, and must lie within an interval:

$$\underline{V}_j^{PLT} x_{j,t}^{PLT} \leq v_{j,t}^{PLT} \leq \overline{V}_j^{PLT} x_{j,t}^{PLT}, \qquad \forall\, j \in \boldsymbol{J}, t \in \boldsymbol{T} \tag{D13}$$

Flow through a plant only when it is operational:

$$\sum_{f \in \boldsymbol{F}} q_{f,j,t}^{F \to J} \leq \overline{V}_j^{PLT} y_{j,t}^{PLT}, \qquad \forall\, j \in \boldsymbol{J}, t \in \boldsymbol{T} \tag{D14}$$

Flow through a plant cannot exceed the capacity built:

$$\sum_{f \in \boldsymbol{F}} q_{f,j,t}^{F \to J} \leq \sum_{\tau \in \boldsymbol{T}} v_{j,\tau}^{PLT}, \qquad \forall\, j \in \boldsymbol{J}, t \in \boldsymbol{T} \tag{D15}$$

Capacity restriction of the pipelines:

$$q_{f,j,t}^{F \to J} \leq V^{PIPE} y_{f,j,t}^{PIPE}, \qquad \forall\, f \in \boldsymbol{F}, j \in \boldsymbol{J}, t \in \boldsymbol{T} \tag{D16}$$

The number of connections between fields and FLNGs cannot exceed the number of available FLNGs:

$$\sum_{f \in \boldsymbol{F}} y_{f,r,t}^{CON} \leq y_{r,t}^{FLNG}, \qquad \forall\, r \in \boldsymbol{R}, t \in \boldsymbol{T} \tag{D17}$$

The production rate from each field through FLNG(s) cannot exceed the capacity of the connected FLNGs:

$$\sum_{d \in \boldsymbol{D}} q_{f,r,d,t}^{FLNG} \leq V_r^{FLNG} y_{f,r,t}^{CON}, \qquad \forall\, f \in \boldsymbol{F}, t \in \boldsymbol{T}, r \in \boldsymbol{R} \tag{D18}$$

The rate of sold LNG on a sealed contract must lie within an interval. If the contract is not sealed, the sold amount is set to zero:

$$\underline{H}^{Q}_{k,d,t} x^{CNT}_{k} \leq q^{CNT}_{k,d,t} \leq \overline{H}^{Q}_{k,d,t} x^{CNT}_{k}, \qquad \forall\, k \in \boldsymbol{K}, d \in \boldsymbol{D}, t \in \boldsymbol{T} \tag{D19}$$

At most one contract can be sealed within a group of mutually exclusive contracts:

$$\sum_{k \in \boldsymbol{K}^{ALT}_{\boldsymbol{a}}} \sum_{t \in \boldsymbol{T}} x^{CNT}_{k} \leq 1, \qquad \forall\, a \in \boldsymbol{A} \tag{D20}$$

The rate of gas sold on the spot market cannot exceed the demand, and must at least be zero to prevent trading between contract and spot markets:

$$\sum_{j \in \boldsymbol{J}} q^{J \to D}_{j,d,t} + \sum_{f \in \boldsymbol{F}} \sum_{r \in \boldsymbol{R}} q^{FLNG}_{f,r,d,t} - \sum_{k \in \boldsymbol{K}} q^{CNT}_{k,d,t} \leq D_{d,t}, \qquad \forall\, d \in \boldsymbol{D}, t \in \boldsymbol{T} \tag{D21}$$

$$\sum_{j \in \boldsymbol{J}} q^{J \to D}_{j,d,t} + \sum_{f \in \boldsymbol{F}} \sum_{r \in \boldsymbol{R}} q^{FLNG}_{f,r,d,t} - \sum_{k \in \boldsymbol{K}} q^{CNT}_{k,d,t} \geq 0, \qquad \forall\, d \in \boldsymbol{D}, t \in \boldsymbol{T} \tag{D22}$$

Remaining gas variable for a field can only take a value if there is production in the last time period, and never higher than the actual remaining gas in the field:

$$e_f \leq F^{Q}_{f} - g_{f,\mathrm{T}}, \qquad \forall\, f \in \boldsymbol{F} \tag{D23}$$

$$e_f \leq F^{Q}_{f} \left(\sum_{j \in \boldsymbol{J}} q^{F \to J}_{f,j,T} + \sum_{d \in \boldsymbol{D}} \sum_{r \in \boldsymbol{R}} q^{FLNG}_{f,r,d,T} \right), \qquad \forall\, f \in \boldsymbol{F} \tag{D24}$$

The investment costs for each period must lie below a budget limit:

$$cf^{INV}_{t} \leq E_t, \qquad \forall\, t \in \boldsymbol{T} \tag{D25}$$

Binary, integer and non-negativity constraints:

$$x_{f,t}^{FLD},\ x_{f,j,t}^{PIPE},\ x_{j,t}^{PLT}, x_{f,r,t}^{CON}, x_{r,t}^{FLNG}, x_{k}^{CNT} \ \in \{0,1\}, \qquad \forall\, f \in \boldsymbol{F}, j \in \boldsymbol{J}, r \in \boldsymbol{R}, \mathrm{t}\ \in \boldsymbol{T} \tag{D26}$$

$$w_{f,t}^{FLD}, w_{f,j,t}^{PIPE}, w_{j,t}^{PLT}, w_{f,r,t}^{CON}, w_{r,t}^{FLNG} \in \{0,1\}, \qquad \forall\, f \in \boldsymbol{F}, j \in \boldsymbol{J}, r \in \boldsymbol{R}, \mathrm{t}\ \in \boldsymbol{T} \tag{D27}$$

$$x_{r,t}^{FLNG}, w_{r,t}^{FLNG} \ \in \mathbb{Z}, \qquad \forall\, r \in \boldsymbol{R}, \mathrm{t}\ \in \boldsymbol{T} \tag{D28}$$

$$y_{f,t}^{FLD},\ y_{f,j,t}^{PIPE},\ y_{j,t}^{PLT}, y_{f,r,t}^{CON}, y_{r,t}^{FLNG} \geq 0, \qquad \forall\, f \in \boldsymbol{F}, j \in \boldsymbol{J}, r \in \boldsymbol{R}, \mathrm{t}\ \in \boldsymbol{T} \tag{D29}$$

$$q_{k,d,t}^{CNT}, q_{f,j,t}^{F\to J}, q_{j,d,t}^{J\to D}, q_{f,r,d,t}^{FLNG} \ \geq 0, \qquad \forall\, f \in \boldsymbol{F}, j \in \boldsymbol{J}, d \in \boldsymbol{D}, r \in \boldsymbol{R}, \mathrm{t} \in \boldsymbol{T} \tag{D30}$$

Appendix C

Stochastic Model

Nomenclature

Sets and indexes

$a \in \boldsymbol{A}$	Contract group *a*
$b \in \boldsymbol{B}$	Scenario group *b*
$d \in \boldsymbol{D}$	Demand point *d*
$f \in \boldsymbol{F}$	Field *f*
$j \in \boldsymbol{J}$	Potential onshore plants *j*
$k \in \boldsymbol{K}$	Potential contracts *k*
$k \in \boldsymbol{K}_{a}^{ALT}$	Set of alternative contracts
$r \in \boldsymbol{R}$	FLNG size *r*
$t, \tau \in \boldsymbol{T}$	Time period *t* and *τ*
$t \in \boldsymbol{G}_{t}^{FLD}$	Time gap of investment in fields. Set of time periods in which a field investment would be ready in period *t*
$t \in \boldsymbol{G}_{t}^{FLNG}$	Time gap of investments in FLNGs. Set of time periods in which an FLNG investment would be ready in period *t*
$t \in \boldsymbol{G}_{t}^{PIPE}$	Time gap of investments in pipelines. Set of time periods in which a pipeline investment would be ready in period *t*
$t \in \boldsymbol{G}_{t}^{PLT}$	Time gap of investments in plants. Set of time periods in which a plant investment would be ready in period *t*
$\omega \in \boldsymbol{\Omega}$	Scenario *ω*

Parameters

C^{CON}	Cost of connecting an FLNG unit to a field. Includes all switching costs
$C_{f}^{FLD\ INV}$	Investment cost of field *f*
$C_{f}^{FLD\ OPR\ FIX}$	Fixed operating cost of operating field *f*
$C_{f}^{FLD\ OPR\ VAR}$	Variable cost of operating field *f* per produced unit
$C_{r}^{FLNG\ INV}$	Investment cost of building a FLNG ship of size *r*
$C_{r}^{FLNG\ OPR\ FIX}$	Fixed cost of operating FLNG ship of size *r*
$C_{r}^{FLNG\ OPR\ VAR}$	Variable cost of operating FLNG ship of size *r* per produced unit
$C_{f,j}^{PIPE\ INV}$	Investment cost of a pipeline from field *f* to plant *j*
$C_{f,j}^{PIPE\ OPR\ FIX}$	Fixed cost of operating pipeline from field *f* to plant *j*
$C_{f,j}^{PIPE\ OPR\ VAR}$	Variable cost of operating a pipeline between field *f* and plant *j* per produced unit
$C_{j}^{PLT\ INV\ FIX}$	Fixed investment cost of plant *j*
$C_{j}^{PLT\ INV\ VAR}$	Variable investment cost of plant *j* dependent on processing capacity
$C_{j}^{PLT\ OPR\ FIX}$	Fixed operating cost of plant *j*
$C_{j}^{PLT\ OPR\ VAR}$	Variable operating cost of plant *j* per processed unit
C_{d}^{REG}	Variable regasification cost at demand point *d*
C^{SHIP}	Fixed cost per ship per period
$D_{d,t}$	Spot demand rate at demand point *d* in period *t*
E_{t}	Maximum investment period *t*
F_{f}^{DEC}	Decline rate of field *f*
F^{INC}	Maximum increase in production in a field as a percentage of max production rate in the field
F_{f}^{MAX}	Max production rate of field *f*
$F_{f,\omega}^{Q}$	Total quantity of recoverable gas in field *f* in scenario *ω*
$H_{k,t}^{APL}$	Contract k can be sealed in period t. Each contract is only applicable for one period
$\underline{H}_{k,d,t}^{Q}$	Minimum yearly rate of gas delivered to demand point *d* in period *t* under contract *k*

$\overline{H}^{Q}_{k,d,t}$	Maximum yearly rate of gas delivered to demand point *d* in period *t* under contract *k*
I^{FLD}_{f}	Value per unit of gas remaining in field *f* at end of horizon if the field is producing in the last period
$I^{FLNG}_{r,t}$	Salvage value of an FLNG of size *r* in period *t*
N_t	Net present value of \$1, *t* periods into the future
$P^{CNT}_{k,t,\omega}$	Selling price of contract *k* per unit in scenario ω
$P^{SPOT}_{d,t,\omega}$	Spot selling price of LNG at demand point *d* in period *t* per unit in scenario ω
S^{CAP}	Transport capacity per ship
$S^{DIST\ F\rightarrow D}_{f,d}$	One way shipping time from field *f* to demand point *d*, in years
$S^{DIST\ J\rightarrow D}_{j,d}$	One way shipping time from plant *j* to demand point *d*, in years
$U^{FLD}_{f,t}$	Is 1 if field *f* can be developed in time period *t*, 0 otherwise
V^{FLNG}_{r}	Processing capacity per FLNG of size *r*
V^{PIPE}	Capacity of a pipeline
$\underline{V}^{PLT}_{j}$	Minimum processing capacity of plant *j*
$\overline{V}^{PLT}_{j}$	Maximum processing capacity of plant *j*
π_ω	Probability of scenario ω
$\Phi_{t,\omega}$	Is 1 if variables in time period *t* and scenario ω should equal the variables in scenario $\omega + 1$
Θ_t	Number of years in time period *t*

Variables

$cf^{INC}_{t,\omega}$	Cash flow from income in period *t* in scenario ω
$cf^{INV}_{t,\omega}$	Cash flow from investments in period *t* in scenario ω
$cf^{OPR}_{t,\omega}$	Cash flow from operations in period *t* in scenario ω
$e_{f,\omega}$	Remaining gas in field *f* in scenario ω at end of horizon. Set to zero if the field is not operational at the end of the horizon
EFP_ω	Expected profit from fields after horizon in scenario ω
$g_{f,t,\omega}$	Cumulative production in field *f* up to period *t* in scenario ω
$q^{CNT}_{k,d,t,\omega}$	Rate of LNG delivered to demand point *d* in period *t* under contract *k* in scenario ω
$q^{F\rightarrow J}_{f,j,t,\omega}$	Rate of gas sent from field *f* to plant *j* in period *t* in scenario ω
$q^{FLNG}_{f,r,d,t,\omega}$	Rate of LNG sent from field *f* to demand point *d* through an FLNG of size *r* in period *t* in scenario ω
$q^{J\rightarrow D}_{j,d,t,\omega}$	Rate of LNG transported from plant *j* to demand point *d* in period *t* in scenario ω
$v^{PLT}_{j,t,\omega}$	Processing capacity of plant *j* that is being built in period *t* in scenario ω, 0 for all other *t*'s
$w^{CON}_{f,r,t,\omega}$	Is 1 if FLNG of size *r* is being disconnected from field *f* in period *t* in scenario ω, 0 otherwise
$w^{FLD}_{f,t,\omega}$	Is 1 if field *f* is being closed in period *t* in scenario ω, 0 otherwise
$w^{FLNG}_{r,t,\omega}$	Is 1 if FLNG of size *r* is sold in period *t* in scenario ω, 0 otherwise
$w^{PIPE}_{f,j,t,\omega}$	Is 1 if pipeline from field *f* to plant *j* is being closed in period *t* in scenario ω, 0 otherwise
$w^{PLT}_{j,t,\omega}$	Is 1 if plant *j* is being closed in period *t* in scenario ω, 0 otherwise

$x_{k,\omega}^{CNT}$	1 if contract *k* is sealed in period *t* in scenario ω, 0 otherwise
$x_{f,r,t,\omega}^{CON}$	Is 1 if a connection is being built between field *f* and an FLNG of size *r* in period *t* in scenario ω, 0 otherwise
$x_{f,t,\omega}^{FLD}$	Is 1 if field *f* is being developed in period *t* in scenario ω, 0 otherwise
$x_{r,t,\omega}^{FLNG}$	Number of FLNGs of size *r* being built in period *t* in scenario ω
$x_{f,j,t,\omega}^{PIPE}$	Is 1 if pipeline from field *f* to plant *j* is being built in period *t* in scenario ω, 0 otherwise
$x_{j,t,\omega}^{PLT}$	Is 1 if plant *j* is being built in period *t* in scenario ω, 0 otherwise

$y_{f,r,t,\omega}^{CON}$	Number of FLNGs of size *r* that field *f* is connected to in period *t* in scenario ω
$y_{f,t,\omega}^{FLD}$	Is 1 if field *f* can produce in period *t* in scenario ω, 0 otherwise
$y_{r,t,\omega}^{FLNG}$	Number of operational FLNGs of size *r* in period *t* in scenario ω
$y_{f,j,t,\omega}^{PIPE}$	Number of pipelines from field *f* to plant *j* can transport gas in period *t* in scenario ω
$y_{j,t,\omega}^{PLT}$	Is 1 if plant *j* of size *s* is operational in period *t* in scenario ω, 0 otherwise

Objective function

The objective function is a sum of the expected discounted cash flows from income, investment cost and operational cost, and expected future profit from operating fields for all scenarios:

$$\max z = \sum_{\omega \in \boldsymbol{\Omega}} \pi_\omega \left[\sum_{t \in \boldsymbol{T}} N_t \left(cf_{t,\omega}^{INC} - cf_{t,\omega}^{INV} - cf_{t,\omega}^{OPR} \right) + N_T \, EFP_\omega \right]$$

Cash flows in each time period and scenario of investment cost for fields, pipelines, plants and FLNGS:

$$\begin{aligned} cf_{t,\omega}^{INV} &= \sum_{f \in \boldsymbol{F}} C_f^{FLD\,INV} x_{f,t,\omega}^{FLD} \\ &+ \sum_{f \in \boldsymbol{F}} \sum_{j \in \boldsymbol{J}} C_{f,j}^{PIPE\,INV} x_{f,j,t,\omega}^{PIPE} \\ &+ \sum_{j \in \boldsymbol{J}} C_j^{PLT\,INV\,FIX} x_{j,t,\omega}^{PLT} + \sum_{j \in \boldsymbol{J}} C_j^{PLT\,INV\,VAR} v_{j,t,\omega}^{PLT} \\ &+ \sum_{r \in \boldsymbol{R}} C_r^{FLNG\,INV} x_{r,t,\omega}^{FLNG} \end{aligned}$$

Cash flow in each time period of operational cost for fields, pipelines, plants, connection between a field and an FLNG, FLNG, shipping and regasification facilities:

$$cf_{t,\omega}^{OPR} = \Theta_t \sum_{f\in \boldsymbol{F}} C_f^{FLD\ OPR\ FIX} y_{f,t,\omega}^{FLD}$$

$$+ \Theta_t \sum_{f\in \boldsymbol{F}} C_f^{FLD\ OPR\ VAR} \left(\sum_{j\in \boldsymbol{J}} q_{f,j,t,\omega}^{F\to J} + \sum_{d\in \boldsymbol{D}} \sum_{r\in \boldsymbol{R}} q_{f,r,d,t,\omega}^{FLNG} \right)$$

$$+ \Theta_t \sum_{f\in \boldsymbol{F}} \sum_{j\in \boldsymbol{J}} C_{f,j}^{PIPE\ OPR\ FIX} y_{f,j,t,\omega}^{PIPE} + \Theta_t \sum_{f\in \boldsymbol{F}} \sum_{j\in \boldsymbol{J}} C_{f,j}^{PIPE\ OPR\ VAR} q_{f,j,t,\omega}^{F\to J}$$

$$+ \Theta_t \sum_{j\in \boldsymbol{J}} C_j^{PLT\ OPR\ FIX} y_{j,t,\omega}^{PLT} + \Theta_t \sum_{j\in \boldsymbol{J}} C_j^{PLT\ OPR\ VAR} \sum_{d\in \boldsymbol{D}} q_{j,d,t,\omega}^{J\to D}$$

$$+ \sum_{f\in \boldsymbol{F}} \sum_{r\in \boldsymbol{R}} C^{CON} x_{f,r,t,\omega}^{CON}$$

$$+ \Theta_t \sum_{r\in \boldsymbol{R}} C_r^{FLNG\ OPR\ FIX} y_{r,t,\omega}^{FLNG} + \Theta_t \sum_{r\in \boldsymbol{R}} C_r^{FLNG\ OPR\ VAR} \sum_{f\in \boldsymbol{F}} \sum_{d\in \boldsymbol{D}} q_{f,r,d,t,\omega}^{FLNG}$$

$$+ \Theta_t \sum_{j\in \boldsymbol{J}} \sum_{d\in \boldsymbol{D}} \left(\frac{2 * S_{j,d}^{DIST\ J\to D}\ C^{SHIP}}{S^{CAP}} \right) q_{j,d,t,\omega}^{J\to D}$$

$$+ \Theta_t \sum_{f\in \boldsymbol{F}} \sum_{d\in \boldsymbol{D}} \sum_{r\in \boldsymbol{R}} \left(\frac{2 * S_{f,d}^{DIST\ F\to D}\ C^{SHIP}}{S^{CAP}} \right) q_{f,r,d,t,\omega}^{FLNG}$$

$$+ \Theta_t \sum_{d\in \boldsymbol{D}} C_d^{REG} \left(\sum_{j\in \boldsymbol{J}} q_{j,d,t,\omega}^{J\to D} + \sum_{f\in \boldsymbol{F}} \sum_{r\in \boldsymbol{R}} q_{f,r,d,t,\omega}^{FLNG} \right)$$

Cash flows in each time period and scenario of income from sales on spot market, contract sales and salvage value of FLNGs:

$$cf_{t,\omega}^{INC} = \sum_{d\in \boldsymbol{D}} P_{d,t,\omega}^{SPOT} \Theta_t \left[\left(\sum_{j\in \boldsymbol{J}} q_{j,d,t,\omega}^{J\to D} + \sum_{f\in \boldsymbol{F}} \sum_{r\in \boldsymbol{R}} q_{f,r,d,t,\omega}^{FLNG} \right) - \sum_{k\in \boldsymbol{K}} q_{k,d,t,\omega}^{CNT} \right]$$

$$+ \Theta_t \sum_{d\in \boldsymbol{D}} \sum_{k\in \boldsymbol{K}} P_{k,t,\omega}^{CNT} q_{k,d,t,\omega}^{CNT}$$

$$+ \sum_{r\in \boldsymbol{R}} I_{r,t}^{FLNG} w_{r,t,\omega}^{FLNG}$$

Expected future profit in each scenario of the producing fields:

$$EFP_\omega = \sum_{f\in \boldsymbol{F}} I_f^{FLD} e_{f,\omega}, \qquad \forall\ \omega \in \boldsymbol{\Omega}$$

Constraints

Each field and plant can at maximum be developed / built once:

$$\sum_{t\in \boldsymbol{T}} x^{FLD}_{f,t,\omega} \leq 1, \qquad \forall\, f \in \boldsymbol{F}, \omega \in \boldsymbol{\Omega} \tag{S1}$$

$$\sum_{t\in \boldsymbol{T}} x^{PLT}_{j,t,\omega} \leq 1, \qquad \forall\, j \in \boldsymbol{J}, \omega \in \boldsymbol{\Omega} \tag{S2}$$

Constraints describing the relationship between developing/building variables, closing/selling variables and is operational variables for fields, pipelines, plants, FLNGs and the connection between fields and FLNGs:

$$y^{FLD}_{f,t-1,\omega} + \sum_{\tau\in \boldsymbol{G}^{FLD}_t} x^{FLD}_{f,\tau,\omega} - w^{FLD}_{f,t-1,\omega} = y^{FLD}_{f,t,\omega}\,, \qquad \forall\, f \in \boldsymbol{F}, t \in \boldsymbol{T}, \omega \in \boldsymbol{\Omega} \tag{S3}$$

$$y^{PIPE}_{f,j,t-1,\omega} + \sum_{\tau\in \boldsymbol{G}^{PIPE}_t} x^{PIPE}_{f,j,\tau,\omega} - w^{PIPE}_{f,j,t-1,\omega} = y^{PIPE}_{f,j,t,\omega}\,, \qquad \forall\, f \in \boldsymbol{F}, j \in \boldsymbol{J}, t \in \boldsymbol{T}, \omega \in \boldsymbol{\Omega} \tag{S4}$$

$$y^{PLT}_{j,t-1,\omega} + \sum_{\tau\in \boldsymbol{G}^{PLT}_t} x^{PLT}_{j,\tau,\omega} - w^{PLT}_{j,t-1,\omega} = y^{PLT}_{j,t,\omega}\,, \qquad \forall\, j \in \boldsymbol{J}, t \in \boldsymbol{T}, \omega \in \boldsymbol{\Omega} \tag{S5}$$

$$y^{CON}_{f,r,t,\omega} + x^{CON}_{f,r,t,\omega} - w^{CON}_{f,r,t,\omega} = y^{CON}_{f,r,t+1,\omega}\,, \qquad \forall\, f \in \boldsymbol{F}, r \in \boldsymbol{R}, t \in \boldsymbol{T}, \omega \in \boldsymbol{\Omega} \tag{S6}$$

$$y^{FLNG}_{r,t-1,\omega} + \sum_{\tau\in \boldsymbol{G}^{FLNG}_t} x^{FLNG}_{r,\tau,\omega} - w^{FLNG}_{r,t-1,\omega} = y^{FLNG}_{r,t,\omega}\,, \qquad \forall\, r \in \boldsymbol{R}, t \in \boldsymbol{T}, \omega \in \boldsymbol{\Omega} \tag{S7}$$

Fields can only be developed in certain time periods:

$$x^{FLD}_{f,t,\omega} \leq U^{FLD}_{f,t}\,, \qquad \forall\, f \in \boldsymbol{F}, t \in \boldsymbol{T}, \omega \in \boldsymbol{\Omega} \tag{S8}$$

The same rate of gas must be transported from a plant as the rate into the plant in all time periods:

$$\sum_{f\in \boldsymbol{F}} q^{F\to J}_{f,j,t,\omega} = \sum_{d\in \boldsymbol{D}} q^{J\to D}_{j,d,t,\omega}\,, \qquad \forall\, j \in \boldsymbol{J}, t \in \boldsymbol{T}, \omega \in \boldsymbol{\Omega} \tag{S9}$$

The increase in the total production rate of gas from a field from one time period to the next must be equal to or lower than the maximum increase in rate:

$$\sum_{j\in\boldsymbol{J}} q_{f,j,t,\omega}^{F\to J} + \sum_{d\in\boldsymbol{D}}\sum_{r\in\boldsymbol{R}} q_{f,r,d,t,\omega}^{FLNG} \leq \left(\sum_{j\in\boldsymbol{J}} q_{f,j,t-1,\omega}^{F\to J} + \sum_{d\in\boldsymbol{D}}\sum_{r\in\boldsymbol{R}} q_{f,r,d,t-1,\omega}^{FLNG}\right) + \Theta_t F^{INC} F_f^{MAX}, \qquad \forall\, f\in\boldsymbol{F}, t\in\boldsymbol{T}, \omega\in\boldsymbol{\Omega} \tag{S10}$$

The total production rate of gas from a field must at all times be at most the maximal rate, given that the field is operational:

$$\sum_{j\in\boldsymbol{J}} q_{f,j,t,\omega}^{F\to J} + \sum_{d\in\boldsymbol{D}}\sum_{r\in\boldsymbol{R}} q_{f,r,d,t,\omega}^{FLNG} \leq F_f^{MAX} y_{f,t,\omega}^{FLD}, \qquad \forall\, f\in\boldsymbol{F}, t\in\boldsymbol{T}, \omega\in\boldsymbol{\Omega} \tag{S11}$$

The total production rate of gas from a field must in each time period be equal to or lower than the remaining gas in the period multiplied with a decline constant:

$$\Theta_t\left(\sum_{j\in\boldsymbol{J}} q_{f,j,t,\omega}^{F\to J} + \sum_{d\in\boldsymbol{D}}\sum_{r\in\boldsymbol{R}} q_{f,r,d,t,\omega}^{FLNG}\right) \leq F_f^{DEC}\left(F_{f,\omega}^{Q} - g_{f,t,\omega}\right), \qquad \forall\, f\in\boldsymbol{F}, t\in\boldsymbol{T}, \omega\in\boldsymbol{\Omega} \tag{S12}$$

Where the cumulative production is:

$$g_{f,t,\omega} = \sum_{\tau=1..(t-1)} \Theta_\tau\left(\sum_{j\in\boldsymbol{J}} q_{f,j,\tau,\omega}^{F\to J} + \sum_{d\in\boldsymbol{D}}\sum_{r\in\boldsymbol{R}} q_{f,r,d,\tau,\omega}^{FLNG}\right), \forall\, f\in\boldsymbol{F}, t\in\boldsymbol{T}, \omega\in\boldsymbol{\Omega}$$

The capacity of a plant is decided in the period when the plant is build, and must lie within an interval:

$$\underline{V}_j^{PLT} x_{j,t,\omega}^{PLT} \leq v_{j,t,\omega}^{PLT} \leq \overline{V}_j^{PLT} x_{j,t,\omega}^{PLT}, \qquad \forall\, j\in\boldsymbol{J}, t\in\boldsymbol{T}, \omega\in\boldsymbol{\Omega} \tag{S13}$$

Flow through a plant only when it is operational:

$$\sum_{f\in\boldsymbol{F}} q_{f,j,t,\omega}^{F\to J} \leq \overline{V}_j^{PLT} y_{j,t,\omega}^{PLT}, \qquad \forall\, j\in\boldsymbol{J}, t\in\boldsymbol{T}, \omega\in\boldsymbol{\Omega} \tag{S14}$$

Flow through a plant cannot exceed the capacity built:

$$\sum_{f \in \boldsymbol{F}} q_{f,j,t,\omega}^{F \to J} \leq \sum_{\tau \in \boldsymbol{T}} v_{j,\tau,\omega}^{PLT}, \qquad \forall\, j \in \boldsymbol{J}, t \in \boldsymbol{T}, \omega \in \boldsymbol{\Omega} \tag{S15}$$

Capacity restriction on the pipelines:

$$q_{f,j,t,\omega}^{F \to J} \leq V^{PIPE}\, y_{f,j,t,\omega}^{PIPE}, \qquad \forall\, f \in \boldsymbol{F}, j \in \boldsymbol{J}, t \in \boldsymbol{T}, \omega \in \boldsymbol{\Omega} \tag{S16}$$

The number of connections between fields and FLNGs cannot exceed the number of available FLNGs:

$$\sum_{f \in \boldsymbol{F}} y_{f,r,t,\omega}^{CON} \leq y_{r,t,\omega}^{FLNG}, \qquad \forall\, r \in \boldsymbol{R}, t \in \boldsymbol{T}, \omega \in \boldsymbol{\Omega} \tag{S17}$$

The production rate from each field through FLNG(s) cannot exceed the capacity of the connected FLNGs:

$$\sum_{d \in \boldsymbol{D}} q_{f,r,d,t,\omega}^{FLNG} \leq V_r^{FLNG} y_{f,r,t,\omega}^{CON}, \qquad \forall\, f \in \boldsymbol{F}, t \in \boldsymbol{T}, r \in \boldsymbol{R}, \omega \in \boldsymbol{\Omega} \tag{S18}$$

The rate of sold LNG on a sealed contract must lie within an interval. If the contract is not sealed, the sold amount is set to zero:

$$\underline{H}_{k,d,t}^{Q} x_{k,\omega}^{CNT} \leq q_{k,d,t,\omega}^{CNT} \leq \overline{H}_{k,d,t}^{Q} x_{k,\omega}^{CNT}, \qquad \forall\, k \in \boldsymbol{K}, d \in \boldsymbol{D}, t \in \boldsymbol{T}, \omega \in \boldsymbol{\Omega} \tag{S19}$$

At most one contract can be sealed within a group of mutually exclusive contracts:

$$\sum_{k \in \boldsymbol{K}_{\boldsymbol{a}}^{ALT}} \sum_{t \in \boldsymbol{T}} x_{k,\omega}^{CNT} \leq 1, \qquad \forall\, a \in \boldsymbol{A}, \omega \in \boldsymbol{\Omega} \tag{S20}$$

The rate of gas sold on the spot market cannot exceed the demand, and must at least be zero to prevent trading between contracts and spot markets:

$$\sum_{j \in \boldsymbol{J}} q_{j,d,t,\omega}^{J \to D} + \sum_{f \in \boldsymbol{F}} \sum_{r \in \boldsymbol{R}} q_{f,r,d,t,\omega}^{FLNG} - \sum_{k \in \boldsymbol{K}} q_{k,d,t,\omega}^{CNT} \leq D_{d,t}, \quad \forall\, d \in \boldsymbol{D}, t \in \boldsymbol{T}, \omega \in \boldsymbol{\Omega} \tag{S21}$$

$$\sum_{j\in\boldsymbol{J}} q_{j,d,t,\omega}^{J\to D} + \sum_{f\in\boldsymbol{F}}\sum_{r\in\boldsymbol{R}} q_{f,r,d,t,\omega}^{FLNG} - \sum_{k\in\boldsymbol{K}} q_{k,d,t,\omega}^{CNT} \geq 0, \qquad \forall\, d \in \boldsymbol{D}, t \in \boldsymbol{T}, \omega \in \boldsymbol{\Omega} \tag{S22}$$

Remaining gas variable for a field can only take a value if there is production in the last time period, and never higher than the actual remaining gas in the field:

$$e_{f,\omega} \leq F_{f,\omega}^{Q} - g_{f,\mathrm{T},\omega}, \qquad \forall\, f \in \boldsymbol{F}, \omega \in \boldsymbol{\Omega} \tag{S23}$$

$$e_{f,\omega} \leq F_{f,\omega}^{Q}\left(\sum_{j\in\boldsymbol{J}} q_{f,j,T,\omega}^{F\to J} + \sum_{d\in\boldsymbol{D}}\sum_{r\in\boldsymbol{R}} q_{f,r,d,T,\omega}^{FLNG}\right), \qquad \forall\, f \in \boldsymbol{F}, \omega \in \boldsymbol{\Omega} \tag{S24}$$

The investment costs for each period must lie below a budget limit:

$$cf_{t,\omega}^{INV} \leq E_t, \qquad \forall\, t \in \boldsymbol{T}, \omega \in \boldsymbol{\Omega} \tag{S25}$$

Binary, integer and non-negativity constraints:

$$x_{f,t,\omega}^{FLD}, x_{f,j,t,\omega}^{PIPE}, x_{j,t,\omega}^{PLT}, x_{f,r,t,\omega}^{CON}, x_{r,t,\omega}^{FLNG}, x_{k,\omega}^{CNT} \in \{0,1\}, \quad \forall\, f \in \boldsymbol{F}, j \in \boldsymbol{J}, r \in \boldsymbol{R}, \mathrm{t} \in \boldsymbol{T}, \omega \in \boldsymbol{\Omega} \tag{S26}$$

$$w_{f,t,\omega}^{FLD}, w_{f,j,t,\omega}^{PIPE}, w_{j,t,\omega}^{PLT}, w_{f,r,t,\omega}^{CON}, w_{r,t,\omega}^{FLNG} \in \{0,1\}, \quad \forall\, f \in \boldsymbol{F}, j \in \boldsymbol{J}, r \in \boldsymbol{R}, \mathrm{t} \in \boldsymbol{T}, \omega \in \boldsymbol{\Omega} \tag{S27}$$

$$x_{r,t,\omega}^{FLNG}, w_{r,t,\omega}^{FLNG} \in \mathbb{Z}, \qquad \forall\, r \in \boldsymbol{R}, \mathrm{t} \in \boldsymbol{T}, \omega \in \boldsymbol{\Omega} \tag{S28}$$

$$y_{f,t,\omega}^{FLD}, y_{f,j,t,\omega}^{PIPE}, y_{j,t,\omega}^{PLT}, y_{f,r,t,\omega}^{CON}, y_{r,t,\omega}^{FLNG} \geq 0, \qquad \forall\, f \in \boldsymbol{F}, j \in \boldsymbol{J}, r \in \boldsymbol{R}, \mathrm{t} \in \boldsymbol{T}, \omega \in \boldsymbol{\Omega} \tag{S29}$$

$$q_{k,d,t,\omega}^{CNT}, q_{f,j,t,\omega}^{F\to J}, q_{j,d,t,\omega}^{J\to D}, q_{f,r,d,t,\omega}^{FLNG} \geq 0, \quad \forall\, f \in \boldsymbol{F}, j \in \boldsymbol{J}, d \in \boldsymbol{D}, r \in \boldsymbol{R}, \mathrm{t} \in \boldsymbol{T}, \omega \in \boldsymbol{\Omega} \tag{S30}$$

Non-anticipativity constraints:

$$x_{f,t,\omega}^{FLD} - x_{f,t,\omega+1}^{FLD} = 0, \qquad \forall\, f \in \boldsymbol{F}, t \in \boldsymbol{T}, \omega \in \boldsymbol{\Omega} \mid \Phi_{t,\omega} = 1 \tag{S31}$$

$$x_{f,j,t,\omega}^{PIPE} - x_{f,j,t,\omega+1}^{PIPE} = 0, \qquad \forall\, f \in \boldsymbol{F}, j \in \boldsymbol{J}, \boldsymbol{t} \in \boldsymbol{T}, \omega \in \boldsymbol{\Omega} \mid \Phi_{t,\omega} = 1 \tag{S32}$$

$$x^{PLT}_{j,t,\omega} - x^{PLT}_{j,t,\omega+1} = 0, \qquad \forall\, j \in \boldsymbol{J}, \boldsymbol{t} \in \boldsymbol{T}, \omega \in \boldsymbol{\Omega} \mid \Phi_{t,\omega} = 1 \tag{S33}$$

$$x^{CON}_{f,r,t,\omega} - x^{CON}_{f,r,t,\omega+1} = 0, \qquad \forall\, f \in \boldsymbol{F}, r \in \boldsymbol{R}, \boldsymbol{t} \in \boldsymbol{T}, \omega \in \boldsymbol{\Omega} \mid \Phi_{t,\omega} = 1 \tag{S34}$$

$$x^{FLNG}_{r,t,\omega} - x^{FLNG}_{r,t,\omega+1} = 0, \qquad \forall\, r \in \boldsymbol{R}, \boldsymbol{t} \in \boldsymbol{T}, \omega \in \boldsymbol{\Omega} \mid \Phi_{t,\omega} = 1 \tag{S35}$$

$$H^{APL}_{k,t} x^{CNT}_{k,\omega} - H^{APL}_{k,t} x^{CNT}_{k,\omega+1} = 0, \qquad \forall\, k \in \boldsymbol{K}, \boldsymbol{t} \in \boldsymbol{T}, \omega \in \boldsymbol{\Omega} \mid \Phi_{t,\omega} = 1 \tag{S36}$$

$$v^{PLT}_{j,t,\omega} - v^{PLT}_{j,t,\omega+1} = 0, \qquad \forall\, j \in \boldsymbol{J}, \boldsymbol{t} \in \boldsymbol{T}, \omega \in \boldsymbol{\Omega} \mid \Phi_{t,\omega} = 1 \tag{S37}$$

$$w^{FLD}_{f,t,\omega} - w^{FLD}_{f,t,\omega+1} = 0, \qquad \forall\, f \in \boldsymbol{F}, t \in \boldsymbol{T}, \omega \in \boldsymbol{\Omega} \mid \Phi_{t,\omega} = 1 \tag{S38}$$

$$w^{PIPE}_{f,j,t,\omega} - w^{PIPE}_{f,j,t,\omega+1} = 0, \qquad \forall\, f \in \boldsymbol{F}, j \in \boldsymbol{J}, \boldsymbol{t} \in \boldsymbol{T}, \omega \in \boldsymbol{\Omega} \mid \Phi_{t,\omega} = 1 \tag{S39}$$

$$w^{PLT}_{j,t,\omega} - w^{PLT}_{j,t,\omega+1} = 0, \qquad \forall\, j \in \boldsymbol{J}, \boldsymbol{t} \in \boldsymbol{T}, \omega \in \boldsymbol{\Omega} \mid \Phi_{t,\omega} = 1 \tag{S40}$$

$$w^{CON}_{f,r,t,\omega} - w^{CON}_{f,r,t,\omega+1} = 0, \qquad \forall\, f \in \boldsymbol{F}, r \in \boldsymbol{R}, \boldsymbol{t} \in \boldsymbol{T}, \omega \in \boldsymbol{\Omega} \mid \Phi_{t,\omega} = 1 \tag{S41}$$

$$w^{FLNG}_{r,t,\omega} - w^{FLNG}_{r,t,\omega+1} = 0, \qquad \forall\, r \in \boldsymbol{R}, \boldsymbol{t} \in \boldsymbol{T}, \omega \in \boldsymbol{\Omega} \mid \Phi_{t,\omega} = 1 \tag{S42}$$

$$q^{F\to J}_{f,j,t,\omega} - q^{F\to J}_{f,j,t,\omega+1} = 0, \qquad \forall\, f \in \boldsymbol{F}, j \in \boldsymbol{J}, \boldsymbol{t} \in \boldsymbol{T}, \omega \in \boldsymbol{\Omega} \mid \Phi_{t,\omega} = 1 \tag{S43}$$

$$q^{J\to D}_{j,d,t,\omega} - q^{J\to D}_{j,d,t,\omega+1} = 0, \qquad \forall\, j \in \boldsymbol{J}, d \in \boldsymbol{D}, \boldsymbol{t} \in \boldsymbol{T}, \omega \in \boldsymbol{\Omega} \mid \Phi_{t,\omega} = 1 \tag{S44}$$

$$\begin{aligned} &q^{FLNG}_{f,r,d,t,\omega} - q^{FLNG}_{f,r,d,t,\omega+1} = 0, \\ &\qquad \forall\, f \in \boldsymbol{F}, r \in \boldsymbol{R}, d \in \boldsymbol{D}, \boldsymbol{t} \in \boldsymbol{T}, \omega \in \boldsymbol{\Omega} \mid \Phi_{t,\omega} = 1 \end{aligned} \tag{S45}$$

$$q^{CNT}_{k,d,t,\omega} - q^{CNT}_{k,d,t,\omega+1} = 0, \qquad \forall\, k \in \boldsymbol{K}, d \in \boldsymbol{D}, \boldsymbol{t} \in \boldsymbol{T}, \omega \in \boldsymbol{\Omega} \mid \Phi_{t,\omega} = 1 \tag{S46}$$

Appendix D

Base Case Dataset

The base case dataset is presented in this appendix. A discount rate of 8% was used in all calculations, and investments each period was limited to $4,500,000,000.

Table D-1: *Size of the different sets in the base case*

Set	Size
Time Periods	15
Fields	6
Plants	3
Demand Points	3
Flng Sizes	2
Contracts	6
Contract Groups	6
Scenarios	32

All 32 scenarios are equally likely with a probability of 0.03125. The planning period is divided into 15 time periods. They represent different number of years: Period 1-5 represent 1 year each, 5-9 represent 2 years, 10-12 represent 3 years, 13-14 represent 4 years, and period 15 represent 5 years.

Fields

The properties of the different fields are presented in Table D-2 and Table D-3. In scenarios where field sizes were uncertain, the amount of recoverable gas was adjusted with +/- 20% for high and low scenarios. The model also allows for restricting the development period of the fields. All fields were unrestricted, except field 2 which could only be developed in period 1-8. The gap from investment decision to the field is operational is set to 5 years.

Table D-2: *Field production characteristics*

Field	Total size [bnBtu]	Decline rate	Max production rate [bnBtu/year]	Increase percentage	Remaining gas value [1000$/bnBtu]
1	5,700,000	0.1	200,000	30 %	1.00
2	1,600,000	0.25	130,000	30 %	1.00
3	800,000	0.12	70,000	30 %	1.00
4	2,000,000	0.15	100,000	30 %	1.00
5	3,000,000	0.2	120,000	30 %	1.00
6	500,000	0.25	70,000	30 %	1.00

Table D-3: *Field costs*

Field number:	Capex [1000$]	Fixed Opex [1000$/year]	Variable Opex [1000$/bnBtu]
1	1,800,000	50,000	0.2
2	1,000,000	50,000	0.2
3	2,400,000	50,000	0.2
4	1,600,000	50,000	0.2
5	1,600,000	50,000	0.2
6	1,800,000	50,000	0.2

Pipelines and onshore liquefaction plants

The datasets contains three plant locations, which has the same characteristics, summarized in Table D-4. The onshore plants have a gap of 3 years from investment decision until they are operational.

Table D-4: *Onshore liquefaction plants*

Fixed Capex [1000$]	0
Variable Capex [1000$/bnBtu capacity]	26.59465
Fixed Opex [1000$/year]	100,000
Variable Opex [1000$ /bnBtu]	0.17
Min. construction capacity [bnBtu]	97,200
Max. construction capacity [bnBtu]	486,000

The pipeline construction cost is proportional to the length of the pipeline, and is presented in Table D-5. The costs correspond to $900,000 per km. The operating expenditures of the pipelines are set to 2% of the capital expenditures presented in Table 0-5, and the investment gap is set to two years. Each pipeline has a capacity of 200,000 billion Btu per year. This is low, but corresponds to the maximum rate from any of the fields. The flow can thereby not be higher, and the model gets tighter constraints.

Table D-5: *Pipeline capex [1000$]*

		Plant		
		1	**2**	**3**
Field	**1**	207,720	140,400	256,320
	2	286,200	258,840	350,280
	3	284,760	149,400	166,680
	4	371,520	218,880	142,560
	5	115,920	150,480	314,640
	6	493,920	394,560	371,880

FLNG units

The FLNG units can be built in two available sizes, both are shown in Table D-6. The salvage value of both sizes is set to $300,000,000 and the cost of connecting an FLNG unit to a field is $100,000,000. The investment gap is 3 years.

Table D-6: *Properties of the FLNG units*

Type	Capacity [bnBtu/year]	Capex [1000$]	Fixed Opex [1000$/year]	Variable Opex [1000$/bnBtu]
1	82,620	1,560,000	50,000	0.17
2	170,100	4,200,000	70,000	0.17

Shipping

The distance to the three demand points are shown in Table D-7. The same shipping distances were used for all fields and plants, since they are all located relatively close. The calculations assume a shipping speed of 20 knots, and loading and offloading of 1 day. The one way voyage time include 1 day for ether loading or offloading. Each ship has a capacity of 3576.96 bnBtu (160,000 m^3), and a yearly cost of $47,450,000. The regasification cost at all three demand points is set to $0.7 per MMBtu.

Table D-7: *Shipping distances*

Destination	Distance [nautic miles]	One way voyage [years]
UK (Falmouth)	1509	0.011353
US (Philidelphia)	3560	0.023059
Spain (Algeciras)	2443	0.016684

Demand and spot prices

The spot sales prices and demand for the different demand points are given in Table D-8 and Table D-9. The chosen demand points are Falmouth in UK, Philiadelphia in US and Algeciras in Spain.

Table D-8: *Prices [1000$/bnBtu]*

	High scenario			Low scenario		
Period	**UK**	**US**	**Spain**	**UK**	**US**	**Spain**
1	9.17	2.59	11.50	9.17	2.59	11.50
2	9.76	3.65	12.09	9.76	3.65	12.09
3	10.11	4.27	12.44	10.11	4.27	12.44
4	10.36	4.70	12.69	10.36	4.70	12.69
5	10.55	5.04	12.88	10.55	5.04	12.88
6	10.77	5.44	13.10	10.77	5.44	13.10
7	11.00	5.85	13.33	11.00	5.85	13.33
8	11.18	6.17	13.51	11.18	6.17	13.51
9	11.33	6.44	13.66	11.33	6.44	13.66
10	11.49	6.72	13.82	11.49	6.72	13.82
11	11.90	7.25	14.23	11.39	6.74	13.72
12	12.44	7.89	14.77	11.11	6.57	13.44
13	13.09	8.65	15.42	10.73	6.28	13.06
14	13.86	9.52	16.19	10.22	5.88	12.55
15	14.75	10.50	17.08	9.58	5.34	11.91

Table D-9: *Demand [bnBtu/year]*

Period	**UK**	**US**	**Spain**
1	43,483	9,926	50,980
2	48,768	10,598	57,177
3	54,236	11,290	63,587
4	59,888	12,001	70,214
5	65,729	12,733	77,061
6	74,874	13,873	87,784
7	87,725	15,463	102,850
8	100,176	17,141	117,448
9	111,528	18,907	130,757
10	127,461	21,250	149,437
11	145,560	24,253	170,657
12	161,706	27,478	189,586
13	174,780	31,543	204,915
14	189,250	36,588	221,879
15	207,303	42,822	243,044

Contracts

The model contains 6 contracts, where each contract contains delivery to only one demand point. Contract 1 and 2 is to UK, 3 and 4 is to US and 5 and 6 is to Spain.

Table D-10: *Price of different contracts [1000$/bnBtu]*

Time period	Contract 1	Contract 2	Contract 3	Contract 4	Contract 5	Contract 6
1	10.09	10.09	2.46	2.46	12.65	12.65
2	10.74	10.74	3.46	3.46	13.30	13.30
3	11.12	11.12	4.05	4.05	13.69	13.69
4	11.39	11.39	4.47	4.47	13.96	13.96
5	11.60	11.60	4.79	4.79	14.17	14.17
6	11.85	11.85	5.17	5.17	14.41	14.41
7	12.10	12.10	5.56	5.56	14.66	14.66
8	12.30	12.30	5.86	5.86	14.86	14.86
9	12.47	12.47	6.12	6.12	15.03	15.03
10	12.64	12.64	6.38	6.38	15.20	15.20
11	12.81	12.81	6.65	6.65	15.37	15.37
12	12.95	12.95	6.87	6.87	15.52	15.52
13	13.10	13.10	7.09	7.09	15.66	15.66
14	13.24	13.24	7.31	7.31	15.81	15.81
15	13.38	13.38	7.52	7.52	15.94	15.94

Table D-11: Minimum delivery on the different contracts

	Minimum delivery [bnBtu/year]					
Period	**1**	**2**	**3**	**4**	**5**	**6**
1	0	0	0	0	30,000	0
2	0	0	0	0	30,000	0
3	0	0	0	0	30,000	0
4	0	0	0	0	30,000	0
5	0	20,000	100,000	0	30,000	0
6	0	20,000	100,000	0	30,000	0
7	0	20,000	100,000	20,000	30,000	0
8	0	20,000	100,000	20,000	0	40,000
9	20,000	20,000	100,000	20,000	0	40,000
10	20,000	0	0	20,000	0	40,000
11	20,000	0	0	20,000	0	40,000
12	20,000	0	0	20,000	0	40,000
13	20,000	0	0	20,000	0	40,000
14	20,000	0	0	20,000	0	0
15		0	0	0	0	0

Table D-12: Maximum delivery on the different contracts

	Maximum delivery [bnBtu/year]					
Period	**1**	**2**	**3**	**4**	**5**	**6**
1	0	0	0	0	50,000	0
2	0	0	0	0	50,000	0
3	0	0	0	0	50,000	0
4	0	0	0	0	50,000	0
5	0	30,000	170,000	0	50,000	0
6	0	30,000	170,000	0	50,000	0
7	0	30,000	170,000	40,000	50,000	0
8	0	30,000	170,000	40,000	0	70,000
9	30,000	30,000	170,000	40,000	0	70,000
10	30,000	0	0	40,000	0	70,000
11	30,000	0	0	40,000	0	70,000
12	30,000	0	0	40,000	0	70,000
13	30,000	0	0	40,000	0	70,000
14	30,000	0	0	40,000	0	0
15	0	0	0	0	0	0

Appendix E

Guide to Attachments

The Mosel source code and input datasets are included as digital attachments. The different datasets are located in individual subfolders, and a parameter in the models, `PathPrefix`, decides which dataset to run.

An excel workbook for displaying the solution of the stochastic and deterministic models, is also included. The models will write their output two a text-file. The mosel model called "Output_writer.mos", takes this text-file as input and write the results to the excel file. Figure E-1 shows the dataflow between the different files.

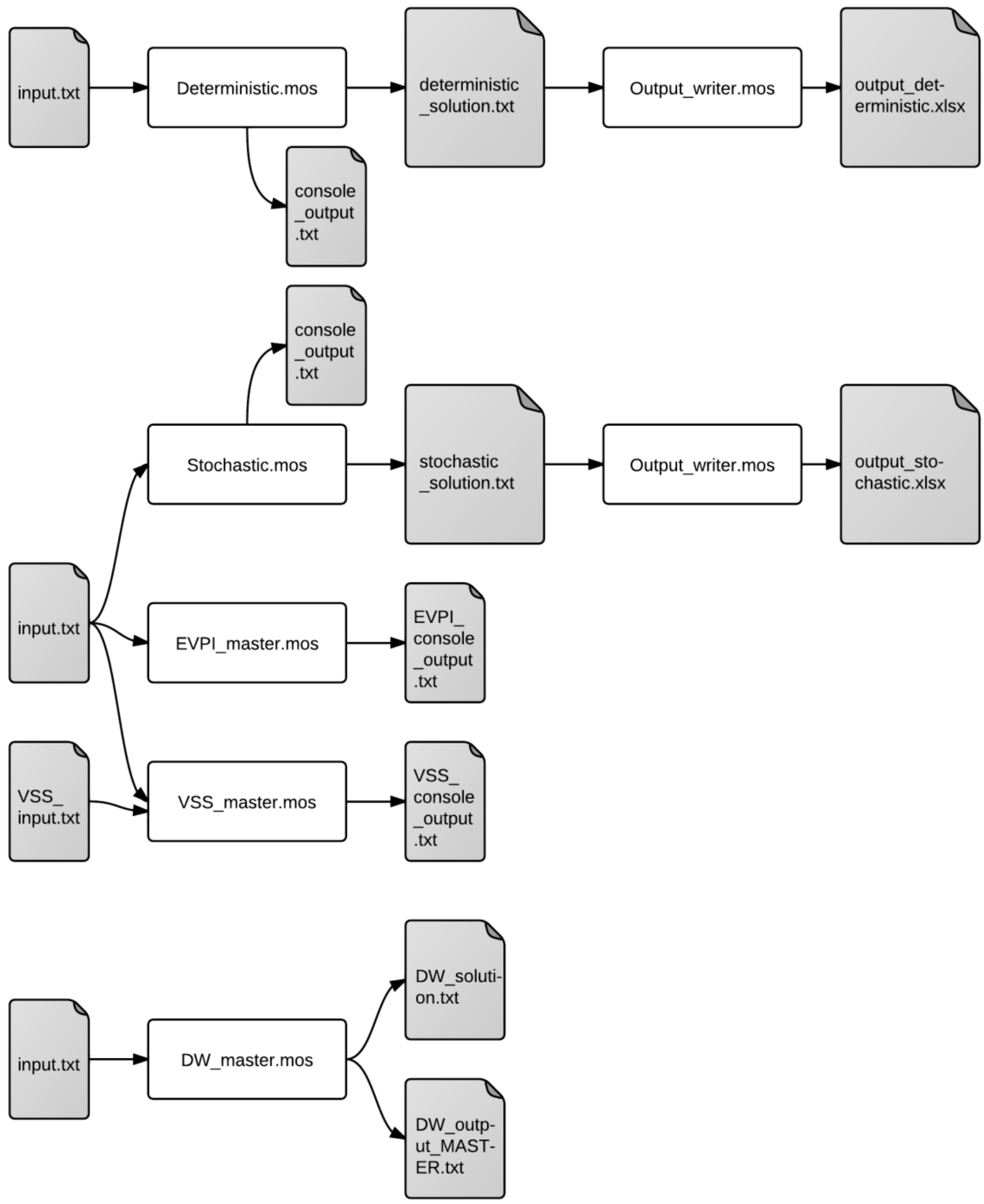

Figure E-1: *Flow of information between the different files in the attachments*