Norwegian University of Science and Technology

Equity Risk Premium Estimation Models

A study of the effects of trading liquidity on traditional asset pricing models

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- uttak av masteroppgave

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Preface

This thesis constitutes the results of the 10th semester of my master degree program at the Norwegian University of Science and Technology (NTNU). My research was done in the spring of 2011, while I was attending the NTNU School of Entrepreneurship.

The thesis was supervised by Associate Professor Einar Belsom at the Department of Industrial Economics and Technology Management. I would like to thank him for providing me with the trust and opportunity of writing my master thesis on financial theory, despite of my academic profile.

I also wish to thank my parents for their support in this five-year period, as well as Associate Professor Sjur Westgaard at NTNU and Johannes A. Skjeltorp at Norges Bank for valuable input. Finally I wish to thank NHH Børsprosjektet for supplying me with additional data.

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Abstract

I ask whether added liquidity factors improve the ability of the Sharp-Lintner CAPM and the Fama French three-factor model to explain asset returns, ex-post, in the Norwegian stock market. Through cross-sectional and time-series regression tests, on both the original and the liquidity-augmented versions of the equity risk premium models, I search for a reversed liquidity premium in the period 2006-2011. I find that the liquidity factors, represented by the bid-ask spread and turnover, marginally improve the empirical ability of the models to explain asset prices and conclude that there is empirical support for a multidimensional liquidity premium. The implications of my results contradict flight-to-liquidity theory and suggest that different dimensions of liquidity are rewarded a premium in different stages of the business-cycle - offering liquidity based rationale for the size and value-effect.

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1. Introduction

"…asset pricing ignores the central fact that asset prices evolve in markets. Markets provide liquidity and price discovery, and I argue that asset pricing models need to be recast in broader terms to incorporate the transactions costs of liquidity and the risks of price discovery"

Maureen O'Hara. Presidential address to the American Financial Association, 2003

I ask whether the ability of the Sharp-Lintner CAPM and the Fama French three-factor model to explain asset returns, ex-post, is improved by accounting for a liquidity risk premium. The motivation behind this research is to make a contribution to the overall understanding of the equity risk premium.

The equity risk premium (ERP) has a large impact on portfolio allocation decisions, investments strategies and estimating the cost of capital (Mehra, 2008). Because the equity risk premium is the only connection between the abstract concept of risk and the concrete concept of capital return, it also has a large impact on valuation. This makes it a decisive factor in terms of how wealth is allocated across different asset classes and which securities we invest in (Damodaran, 2010). It plays a central role in how funds are managed and which projects managers choose to accept. A wrong estimation of the ERP could lead to pension funds allocating the wrong amount of capital in equity. It could also lead to managers accepting high risk projects that never should be done, or denying good projects that would have created value. It is not difficult to see that a better understanding of the ERP has importance that reach far beyond its academic relevance.

The Capital Asset Pricing Model (CAPM) of Sharpe (1964), Lintner (1965) and Black (1972) is arguably the most common way of estimating the ERP[1]. However, despite its simplicity and clever rationale, it has never performed as well empirically as hoped (Basu, 1977; Reinganum, 1980; Fama & MacBeth, 1973). The adjustment for firm size and price-to-book ratio, suggested by Fama and French, improved empirical results, but with a limited theoretical rationale (Fama & French, 1992; 2004). Size and value of firms are, at best, proxies for state variables – variables reflecting changes in consumption (Merton, 1973).

A common weakness of both equity risk premium models is that none of them necessarily account for the costs and risks related to owning illiquid assets. Gibson and Mougeot (2004, p.1) write: *"Typically, continuous-time arbitrage or equilibrium asset pricing models ignore liquidity since the cost and time required to transfer financial wealth into cash is assumed to be nil…"* The CAPM, as described by Sharp and Lintner, assume that the investor only holds a one-period portfolio - effectively removing the issue of liquidity in trading. The Fama French three-factor model does not account for trading directly. However, it does not rule out that size and value proxy for state variables and liquidity.

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[1] A survey by Graham and Harvey (2001) show that 74% of 392 US CFOs reliy on the CAPM when estimating he cost of equity.

There is support for a liquidity premium in both theoretical and empirical research. In theory, traders of illiquid asses will face higher opportunity costs (Grossman & Miller, 1987), higher risks of bankruptcy in recessions – due to lower funding abilities (Lustig & Chien, 2001; Liu, 2006), and information risk – as consequence of asymmetric information (O'Hara, 2003). There are also theoretical and empirical grounds for a premium to compensate a flight-to-liquidity in recessions (Acharya & Pedersen, 2005; Amihud, Mendelson, & Wood, 1990; Amihud, 2002). Empirical evidence supports the theoretical claims of a liquidity premium (Amihud & Mendelson, 1986; Datar, Naik, & Radcliffe, 1998; Brennan & Subrahmanyam, 1996; Chan & Haff, 2005), but struggle to connect cause and effect – due to the multidimensional liquidity risks.

Unlike other anomalies[2], the presence of a liquidity risk premium does not necessarily imply market inefficiency. Amihud writes: *"We emphasize that the spread effect [referring to the bid-ask spread effects due to illiquidity] is by no means an anomaly or an indication of market inefficiency; rather, it represents a rational response by an efficient market to the existence of the spread."* (Amihud & Mendelson, 1986, p. 224). If investors deserve a liquidity premium in addition to their risk premium, as a rational response to the existence of illiquidity, the CAPM may be unable to account for differences in assets liquidity. This may also be the case for the Fama French three-factor model, but the possibility of a correlation between liquidity and size (Amihud, 2002) – explaining the empirical performance of the three-factor mode, must be considered as well. My purpose is to decide whether a liquidity premium might be a missing piece of the puzzle – improving the empirical performance of the CAPM and providing theoretical support for the Fama French three-factor model.

Based on this rationale, I test the following hypotheses:

H1: The ability of the CAPM to explain expected returns, ex post, is improved by the addition of liquidity factors.

H2: The ability of the Fama French three-factor model to explain expected returns, ex post, is ***not*** *improved by the addition of liquidity factors.*

I test the hypotheses through OLS cross-sectional and time-series regressions on both liquidity augmented and original versions of the CAPM and the Fama French three-factor model. The models ability to explain returns are appreciated on basis of the size and significance of alphas, their adjusted R^2 and the significance of priced factors. However, since there is a risk that my liquidity-proxies not necessarily reflect liquidity; I also look for a rationale which is consistent with consumption-based asset pricing theory (Cochrane, 2001) before a final conclusion can be drawn. My methodical approach has three original contributions:

1. Instead of looking for positive premiums, in the form of average excess returns over a long period (Sharp, 1964; Amihud & Mendelson, 1986), I look at data from 2006-2011 in search of a negative premium in recessions – supporting a positive premium for the same factor when consumption is high.

[2] Like the momentum effect (Jegadeesh & Titman, 1993) and several calendar effects (Rozeff & Kinney, 1976; Keim, Brown, Kleidon, & Marsh, 1983; Ariel, A montly effect in stock returns, 1987)

2. Due to the multidimensional nature of liquidity, I use both the bid-ask spread, turnover and combinations of the two as proxy for liquidity in my regressions tests.
3. I look at the Norwegian stock market because it is less liquid than traditional U.S. data - amplifying any illiquidity-effects. This choice also reduces the probability of data snooping.

I will not quantify the size of the liquidity premium, because I expect to find a negative premium in the period and don't believe it is representative of the average excess return on illiquid assets.

I find that the bid-ask spread and turnover appear to be priced factors in both models, but only improved their ability to price assets marginally. My results contradict the flight-to-liquidity theory (Acharya & Pedersen, 2005; Amihud, Mendelson, & Wood, 1990) and suggest that different dimensions of liquidity are rewarded a premium in different stages of the business-cycle – providing a liquidity-based rationale for the size and value-effect (Fama & French, 1992).

In the next chapter I introduce the main rationale behind the CAPM and the three-factor model as special cases of the consumption based model (Cochrane, 2001). I will discuss their main weaknesses and rationalize the need for improvements before introducing both theoretical and empirical evidence in support of a liquidity risk premium. I will further describe the methodical framework for testing the equity risk pricing models and the choice of data. Finally I will present the results of the empirical tests, discuss them and conclude with what implications the results have for the greater understanding of the equity risk premium.

2 Theoretical background

In this chapter I rationalize the need for empirical research on the effects of trading liquidity on the equity risk premium.

2.1 Equity risk premium models

This section introduces the consumption-based model – of which both the CAPM and the Fama French three-factor model are special cases. This generic model provides a theoretical platform of which the liquidity augmented versions of the two equity risk premium models can be built. I also offer a review of the theoretical and empirical challenges of the CAPM and the Fama French model.

2.1.1 The consumption-based model

The consumption-based asset pricing model tries to understand the value of claims to uncertain payments (Cochrane, 2001). Although these uncertain payments have proven difficult to price, the consumption-based model is based on a basic equality: *The marginal utility of consuming a little less today, in order to buy more of an asset, must be equal to the marginal utility gain of consuming the profits of that asset at a later point*[3]. Whether investors should be expected to act this rationally and informed is debatable, but this is beyond the scope of this paper[4]. Based on the assumption above, asset prices can be calculated the following way:

$$P_i = E(mx_i) \tag{1}$$

P_i is the price of asset i, x_i is the expected cash flow from asset i and m is a factor that adjusts the cash flow according to what the marginal utility of cash is at the given time. Different equity risk premium models handle the m, also known as the stochastic discount factor, in different ways. The common denominator should be that it reflects the marginal investor's current utility of the cash flow.

What about the cash flow, x_i? It would be natural to expect asset prices to vary mostly with variations in cash flow expectations. This may however not be as important as first assumed. In his 2011 presidential address to the American Finance Association, John Cochrane claimed that: *"All price-dividend ratio volatility corresponds to variation in expected returns. None corresponds to variation in expected dividend growth, and none to 'rational bubbles.'"* (Cochrane, 2011, p 5). Although this is a strong statement, it can be argued that changes in expected cash flow are diversifiable – leaving pricing to changes in the discount factor alone.

The discount factor, m, representing the cost of capital, will not be diversifiable. Whether it is constant or varies with time, is however highly debatable (Brealey, Myers, & Allen, 2011). The

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[3] How to describe utility as a function of consumption is clearly debatable, but the standard assumption is that the function is concave – meaning that each additional dollar is valued, but a little less each time.

[4] My opinion is that the assumption of efficient markets is not rooted in the belief that markets always act efficiently, but that they are more efficient than inefficient and should be modeled thereafter. The market appears inefficient enough for there to be an industry trying to benefit from informed decisions, but this does not necessarily mean that the efficient market hypothesis is not a good model for the market in general. More on the discussion on market efficiency is summarized by Dimson (Dimson & Mussavian, 1998)

standard approach of assessing the discount factor was to look at historical averages (Sharp, 1964) – assuming it to be the best indicator of future expected returns[5]. Although the total average will be indicative of the expected discount factor[6], historical averages must be observed and learned from on the basis of their respective macro-regime as well[7].

Accepting the premise of Cochrane (2011), asset pricing is a function of changes in the discount rate, m, also called the expected rate of return or the equity risk premium (ERP). All equity risk premium models are based on different ways of assessing m. I will limit my scope to the factor pricing models, where the discount factor is a linear function of a set of proxies,

$$m = a + b_A f^A + \ b_B f^B + \ \dots \tag{2}$$

In this equation, f represent different risk factors and a and b are the factor loadings – the assets exposure to the given factor. Each factor should be a plausible proxy for changes in marginal utility, leaving researchers with the question of what causes changes in marginal utility. Finding proxies for changes in the marginal utility is however one of the more debated subjects in financial theory. Although the equity risk premium (ERP) is easily defined as; the expected return of common stock, minus the return of a risk-free investment in the same period (Brealey, Myers, & Allen, 2011), it's forward-looking nature makes it surprisingly difficult to measure. Nobel prizewinner Merton Miller (2000) writes:

"I still remember the teasing we financial economists, Harry Markowitz, William Sharpe, and I, had to put up with from the physicists and chemists in Stockholm when we conceded that the basic unit of our research, the expected rate of return, was not actually observable. I tried to tease back by reminding them of their neutrino—a particle with no mass whose presence was inferred only as a missing residual from the interactions of other particles. But that was eight years ago. In the meantime, the neutrino has been detected." (Miller, 2000, p 13)

Cochrane (2001, p 149) suggests that factors to be considered should have certain characteristics: *"…the essence of asset pricing is that there are specific states of the world in which investors are especially concerned that their portfolios not do badly. <…> The factors are variables that indicate that these "bad states" have occurred."* The rationale is that assets exposed to factors that cause them to underperform when marginal utility is low should be less attractive among traders. This causes a reduction in prices for all assets exposed to the factor. Because a reduction in price is independent of the asset's dividends, the asset will provide higher excess returns and thus a higher risk premium than similar stock, without exposure to the factors.

Research and intuition both agree that consumption is high when marginal utility is low (Cochrane, 2001). Factor models thus look for factors correlating with changes in consumption. There are several signs in support of a strong correlation between the volatility of macroeconomic factors like inflation, interest rates and expected GDP growth, and the ERP – confirming that such changes in

[5] Alternatives to historical methods are implied methods and surveys (Koller, Goedhart, & Williams, 2002; Graham & Harvey, 2008). These methods are not as influential and beyond the scope of this paper.

[6] Dimson et al. measures the ERP from 1900 – 2008 to 7.1% and other results vary from 3% - 8.5%. (Brealey, Myers, & Allen, 2011)

[7] The historical average discount factor would i.e. change significantly measured before and after 2008.

consumption might be the right place to look for factors (Lettau, Ludvigson, & Wachter, 2008; Brandt & Wang, 2002)[8] [9].

Based on this insight in the rationale behind the consumption-based model, I will now introduce the most common equity premium models, their assumptions and their empirical significance.

2.1.2 The CAPM

The CAPM claims that the only factor in equation 2 is the excess return on the mean-variance efficient market portfolio, less the risk-free rate (Markowitz, 1959; Tobin, 1958). Assets are priced only depending on their covariance with this market portfolio. In order to arrive at this conclusion, Markowitz made the following assumptions[10]:

- All investors are risk averse
- All investors use a portfolio that optimizes mean-variance
- All investors think only about this one-period investment
- All investors are rational
- All investors have the same information
- All trades are made without tax and transaction costs

These assumptions , in addition to Tobin's assumption of risk-free borrowing and lending finalized the capital asset pricing model, which states that each asset, i, has an expected excess return ($E(r_i) - r_f$) given by the following equation

$$E(r_i) - r_f = \beta \cdot (E(r_m) - r_f) \quad (3)$$

Where r_f is the risk free rate, $E(r_m)$ is the expected return on the market portfolio and β is the covariance between return on the asset and the market portfolio $E(r_m) - r_f$ and $E(r_i) - r_f$ can be described respectively as the excess return on the market (ERP_m) and the excess return on the asset i (ERP_i), leaving us with the CAPM on its factor from:

$$m = ERP_i = \beta \cdot (ERP_m) \quad (4)$$

Based on the factor- model in equation 2, it appears that the excess market return is a proxy for changes in marginal utility – consistent with the philosophy of the consumption-based model. It seems intuitive that high beta assets should provide higher returns. Due to their large co-variance with the market, their price will be low in recessions, when money is needed and high in bull markets, when money is in excess. From a marginal utility point of view, these assets are less

[8] However, growth of GDP or employment rate might come as a consequence of the stock market volatility and not vice versa. The accumulated value of all public stocks at OSEBX increased from 5% of the total BNP in 1980 to 90% in 2006 (Næs, Skjeltorp, & Ødegaard, 2007). An interesting example of this would be a probable change in macroeconomic volatility after the financial crisis in 2008, and not before.

[9] A collective change in risk aversion also changes the ERP (Damodaran, 2010). Most authors assume investors to be risk averse (Sharp, 1964; Pastor & Stambaugh, 2001), others risk neutral, (Miller E. , 197).

[10] Note that these are special assumptions of the CAPM, and not of the consumption-based model.

attractive and investors should receive a premium from owning them. Another question is however whether the market portfolio fully reflects the change in consumption and marginal utility[11].

Critique of the CAPM

The CAPM has been widely criticized for its assumptions. The market is naturally not completely efficient (De Bondt & Thaler, 1985), some investors will be risk seeking and irrational (Lakonishok, Shleifer, & Vishny, 1994) and there will be limitations for how much investors can lend or borrow (Fama & French, 2004). In addition, the CAPM is unclear about what the risk free-rate should be[12]. Despite the critique, it is important to realize that the object of the model is to simplify the world around us enough so that we can understand it. The question is therefore not whether the assumptions are always correct, but whether the model benefits from the assumptions.

The main problem of the CAPM is that it has been somewhat empirically disappointing. The first cross-sectional test, indicated there was a positive relation between average return and beta, but that it was too flat compared to what the CAPM predicted (Black, Jensen, & Scholes, 1972; Blume & Friend, 1973). The intercept was also found to be consistently larger than the risk free rate. Time-series regression tests showed the same effect and that the intercepts of high beta portfolios were negative, while the intercepts on low beta portfolios were positive. The less-restrictive Black-version of the CAPM (Black, Jensen, & Scholes, 1972), demanding only a positive slope by allowing unrestricted short selling of risky assets, managed restored some faith in the CAPM, but the claim that no other variables than the market beta should be able to explain excess returns, became difficult to defend empirically (Basu, 1977; Reinganum, 1980). This was the beginning of the ICAPM, Arbitrage pricing theory and the Fama French three-factor model.

2.1.3 Fama and French three-factor model

Based on the work of Basu (1977), Banz (1981), Reiganum (1980) and several others, Fama and French (1992) made the discovery that once historical data was adjusted for market capitalization and price-to-book ratio, the beta had little explanatory ability left. In order to explain the anomalies, other than calling them miss-estimations (Fama & French, 1988; Stambaugh, 1982; Roll, 1981)[13] or abandon the efficient market hypothesis (Lakonishok, Shleifer, & Vishny, 1994), Fama and French adjusted the assumptions of the CAPM. Inspired by the Intertemporal Capital Asset Pricing Model (ICAPM) (Merton, 1973), they rejected the assumption that the investors only think about their first investment. *"For example, the assumption that investors care only about the mean and variance of one-period portfolio returns is extreme"* (Fama and French, 2004, p. 37). They also abandoned the

[11] This problem is called Roll's critique (Roll, 1976), and argues that the use of a large index, only reflects changes of consumption in equity markets. In order to account for the changes in marginal utility, the market portfolio should contain bonds, commodities, real-estate and all other assets reflecting consumption. This portfolio would be virtually impossible to create.
[12] A lot of researchers use a 1 year US T-bill as the risk free rate. Koller et al. (Koller, Goedhart, & Wessels, Valuation, 2005) argue that, from a valuation point of view, is important that one uses the same year of maturation on the bonds or bills as once would use to discount the cash-flow.
[13] There has also been a debate on the subject of arithmetic vs geometric averaging. Some also choose to use a weighted average of the two, with the weight of geometric premium increasing with longer time series (Indro & Lee, 1997).

idea that the market is the only factor that explains return – allowing for market capitalization and price-to-book ratios as additional factors in the consumption-based factor model.

However, size and value of firms don't appear to be state variables, as neither size nor price-to-book ratios are likely to be indicators of changes in consumption or marginal utility. Fama and French explain this by appealing to Ross's arbitrage pricing theory (APT) (Ross, 1976). Arbitrage pricing theory uses the same principal as the generalized factor model of the consumption-based model - a "price of risk" – the risk premium, multiplied with the "how much of risk" – the beta. However, the arbitrage model does not suggest which factor the expected return might vary with and accept that the factors could proxy for a state variable not captured by the market beta. This rationale supported the Fama French three-factor mode, another special case of equation 2:

$$m = ERP_i = E(r_i) - r_f = \alpha_i + \beta_i\left[E(r_m) - r_f\right] + s_i E(SMB) + h_i E(VMG) \qquad (5)$$

Where SMB and VMG are factors representing the excess returns on small cap and value stock respectively, and s_i and h_i is the respective "beta" of the two factors representing the exposure of asset/portfolio to each factor.

The three-factor model has over time become a benchmark for asset pricing models. Fama and French agree that the main shortcoming of the model, from a theoretical perspective, is its empirical motivation. It has been argued that small companies run a larger risk of bankruptcy in recessions (Amihud, 2002) and that value companies are leveraged and pro-cyclical (Zhang, 2005), thus providing a consumption-based rationale for a risk premium. However, the authors themselves argue from a statistical point of view: *"In support of this claim, they [Fama & French, 1992] show that the returns on the stock for small firms covary more with one another than with returns on the stock of large firms and returns on high book-to-market (value) stock covary more with one another than with returns on low book-to-market (growth) stock."* (Fama & French, 2004, p 38). Regardless of this debate, the CAPM and the three-factor model are the main asset pricing models taught in corporate finance classes around the world.

Critique of the three-factor model

Although the three-factor model may be viewed as an improvement of the CAPM, due to better empirical performance, it has weaknesses. Several researchers have found other anomalies which are orthogonal to the market and unaffected by the size and value factors provided by Fama and French. Some find a momentum effect, implying continuing high returns on previous winners (Jegadeesh & Titman, 1993; Carhart, 1997) while others find an opposite effect (De Bondt & Thaler, 1985). Several seasonal anomalies have been found. Among them, that some months provide better returns than others (Rozeff & Kinney, 1976; Keim, Brown, Kleidon, & Marsh, 1983), some weeks of the month give higher returns (Ariel, 1987), some days of the week give higher returns (French, 1980), and finally that some hours of the day provide higher returns (Harris, 1986).

In addition to the landslide of anomalies, the spread between small cap and large cap has been reduced since its discovery in 1980 (Amihud, 2002). This could imply that the rewarded premium for size, or what size is a proxy for, varies with time[14]. Research also finds that over fifty percent of the premium is found in the month of January (Keim D. , 1982). Both of these observations leave room for a debate on whether size is a proxy for a state variable. Are the factors found by Fama and French unique, or coincidently the once found first? The price-to-book ratio as a factor is also criticized. Kothari, Shanken and Sloan argue that this factor will be biased by the fact that low price-to-book assets will historically have a lower tendency to go bankrupt – also known as survivorship bias (Kothari, Shanken, & Sloan, 1995). Even Eugene Fama, states in his 1991 article that the APT in combination with the ICAPM has provided researchers with a license to search for data that, ex-post, describe the cross section of average returns. Based on this statement, I find it important to search for properly rationalized factors in the further research on equity risk premium models.

[14] Given that the market is presumed to be efficient. If it is not efficient, this could be a result of the market adjusting for its previous neglect.

2.2 Liquidity Premium in theory and empirical research

I have observed that the three-factor model appears to be empirically better than the CAPM[15], but lack a clear theoretical motivation. I propose that including a liquidity premium in the equation will result in a better empirical performance of the CAPM or a more intuitive theoretical motivation of the three-factor model. In support of improved theoretical motivation, Amihud (2002) find that illiquidity strongly affects small firm stocks – implying liquidity could explain the size effect variations over time.

This section shows why liquidity should be a priced factor in the consumption-based factor models. I review both theoretical and empirical evidence in support of a multidimensional liquidity premium and show that a liquidity premium is a rational compensation for increased opportunity cost (Grossman & Miller, 1987), risk of flight-to-liquidity (Acharya & Pedersen, 2005) and higher risks of bankruptcy in recessions – due to lower funding abilities (Lustig & Chien, 2001; Liu, 2006).

Before introducing these costs, I define liquidity and consider whether the origin of liquidity itself could be the cause of a liquidity premium.

2.2.1 The theoretical evidence on the liquidity premium

Liquidity can affect asset prices in several ways. Assets may be affected by how sensitive they are to changes in the aggregated market trading liquidity (Pastor & Stambaugh, 2001; Chordia, Roll, & Subrahmanyam, 2000)[16], to changes in individual trading liquidity (Amihud & Mendelson, 1986; Grossman & Miller, 1987; Datar, Naik, & Radcliffe, 1998; Eleswarapu & Reinganum, 1993; O'Hara, 2003), how sensitive they are to the company's financial liquidity (Brunnermeier & Pedersen, 2009) and combinations these (Acharya & Pedersen, 2005). Although a connection between the company's financial liquidity and the trading liquidity has been found, my research will only be concerned with trading liquidity. Similarly, my contribution will be on how individual stock returns vary with its liquidity and not how it varies with the total liquidity in the market. This is illustrated in figure 1:

[15] In the favor of the CAPM it should be mentioned that the anomalies used in the Fama French model was discovered as a consequence of the CAPM.

[16] Pastor and Stambaugh (2001) found that expected stock returns are significantly higher for stocks with high sensitivity to market-wide liquidity than stocks with low sensitivity. Similar studies have supported the claim of a systematic liquidity risk (Gibson & Mougeot, 2004)

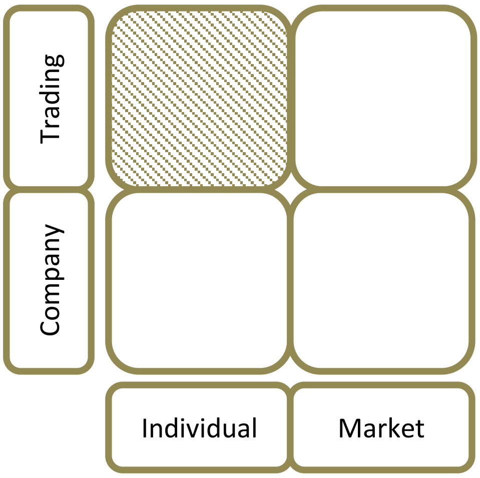

Figure 1: Different ways liquidity affects asset prices. I will focus on how the trading liquidity of individual assets explains asset returns as market in the figure.

Could the origin of liquidity rationalize a liquidity premium?

This section looks at the origin of liquidity and whether the origin itself is reason to expect a premium.

The seller and the buyer may agree on the "fundamental value" of an asset, but that does not necessarily imply that they find each other in the marketplace (O'Hara, 2003). Differences in liquidity stem from factors affecting the probability of these two traders meeting – like timing, accessibility and the number of traders. The issue of timing and accessibility has largely been improved by internet trading[17]. It is, however, difficult to explain why some assets appear to be more "popular" in an efficient market. The EMH would argue that there must be a rational reason why assets become illiquid, not just negligence[18].

O'Hara (2003) provides such a solution, arguing that some traders will, in periods, have better information, but that this changes over time: *"New information arrives, old information becomes stale"* (O'Hara, 2003, p 1351). This is not the same as saying that some traders always have arbitrage opportunities. The asymmetrical information would imply that no single market portfolio would be held. Rather, informed traders would have overweight of stock they believed to provide higher returns and underweight in underperforming stocks. The movement away from the market portfolio would also imply that the informed trader would bear idiosyncratic risk. The same could be said about the uninformed trader, also bearing idiosyncratic risk, believing assets to be mispriced. It is important to recognize that these asymmetries are not a sign of market inefficiencies, but rather an assumption that the markets can be efficient without everybody having the same information at all times.

[17] According to Amihud, as much as 11% of American traders traded primarily online in 1999. These figures were expected to quadruple already by 2003 (Amihud, 2000).

[18] Contrary to the suggestion of Amihud (2000) – stating that companies are "forgotten" by the market and can be made more liquid by increasing the flow of information from the company.

Despite the first impression, O'Hara's rationale does not imply that the uninformed investors are less intelligent or irrational[19]. O'Hara argues that the uninformed trader recognize the information risk and wants compensation for bearing it through increased returns on illiquid assets. The uninformed traders also lower their informational disadvantage by trading familiar assets of which they have access to information. Studies have shown that local investors tend to valuate local stocks higher (Coval & Moskowitz, 1999). This is because traders tend to buy assets which they have a secure flow of news from and previous experience with. Given the fact that 1/3 of the assets at OSEBX are owned by foreign investors[20], this would suggest that the liquid companies in Norway, are those in familiar industries which provide a good flow of information.

If some stocks are illiquid due to information risk, this alone suggests that a liquidity premium should be rewarded. However, the origin of illiquidity could also be related to a more simple argument concerning market capitalization. When large funds buy equity, only the largest companies have large enough market capitalization to provide sufficient volume, without getting control of the company[21]. Given their size and the fact that transaction cost represents a large portion of their total cost, this reduces the number of potential assets such funds can consider. The consequence of this would be that assets with large market cap would be more liquid and provide lower returns - coinciding with the size-effect. This explanation of the origin of liquidity does not necessarily imply that traders of illiquid assets should be rewarded a premium. To make the chaos complete, O'Hara (2003) even argues that liquidity might be a zero-sum game – due to transfer of liquidity between markets[22].

Due to this uncertainty about the origin of illiquidity, it is difficult to know whether traders of illiquid assets already deserve a premium due to information risk. However, in the next section, I will also consider whether liquidity cost exists, independently of its origin.

Liquidity costs

Regardless of the origin of differences in liquidity, it has costs that should be accounted for. The literature on the subject of a liquidity premium is vast and non-trivial. Research on microstructure theory point to a potential premium connected to liquidity caused by increased **transaction cost**, from trading illiquid assets (Amihud & Mendelson, 1986; Pastor & Stambaugh, 2001). These are cost related to price impact of block sales and appear unrelated to changes in marginal utility[23]. There is also a growing literature branch, arguing that liquidity affects the risk of holding an illiquid asset because they will underperform in recessions – **liquidity risk** (Lustig & Chien, 2001; Holmstrom & Triole, 2001; Liu, 2006). This branch argues that there might be a non-diversifiable risk of holding illiquid assets that is more market dependant. It appears there are two costs of different nature related to liquidity. I find support for this through Chordia, Roll and Subrahmanyam stating (2000, p. 6)*"… there are potentially two different channels by which trading costs influence asset pricing, one*

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[19] According to behavioral finance, there would be suspicions that the traders could be overconfident or affected by their beliefs when interpreting information (Shleifer & Summers, 1990).
[20] www.oslobors.no
[21] The Norwegian state pension fund has strict rules about not making any strategic impact on any company.
[22] I argue that even if it can be assumed the aggregated flow of liquidity to be constant, it will be the percentage change in trading volume that must be accounted for.
[23] Although they appear relatively independent, it is easily arguable that they will covary with the market.

static and one dynamic: a static channel influencing average trading costs and a dynamic channel influencing risk"[24].

In order to discuss the implications of this theory I will separate between increased transaction costs - caused by illiquid assets, and liquidity risk – caused by the cost of being illiquid in recessions[25]. The overall costs connected to liquidity are summarized in figure 2:

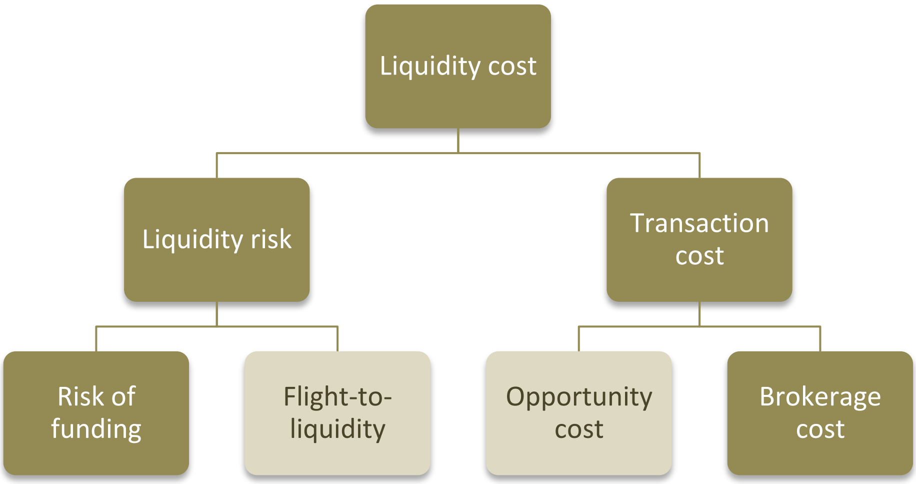

Figure 2: Theoretical costs related to illiquid assets. The left side consists of cost related to risks of holding illiquid assets in recessions. The right side consists of cost related to the increased transaction costs of illiquid assets, which are present also outside recessions.

In the following section I will explain the rationale behind the complete figure and provide theoretical support for a multidimensional liquidity risk premium. I will explain how all costs related to illiquid assets can be found in the risk of funding, risk of flight-to-liquidity and of an increased opportunity costs – as well as how the last two can be expressed as a market maker cost.

Increased transaction costs – consequences of market impact in imperfect markets

The increased transaction costs of illiquid assets are most easily understood through microstructure theory. Kyle identified three main characteristics of market liquidity; tightness, depth and resilience (Kyle, 1985). In the diagram in figure 3, first portrayed by Kerry in 2008[26], these three characteristics are portrayed.

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[24] Although Chordia, Roll and Subrahmanyam, refer to the commonality in liquidity in this quote, addressing individual stocks sensitivity to movements in aggregated market liquidity, they make a point in the direction that assets might have illiquidity risk in addition to its, static, transaction cost.

[25] The distinction between transaction costs as a function of illiquidity and liquidity risk is indicated by O'Hara (O'Hara, 2003).

[26] Found in article by Hibbert et al. (2009)

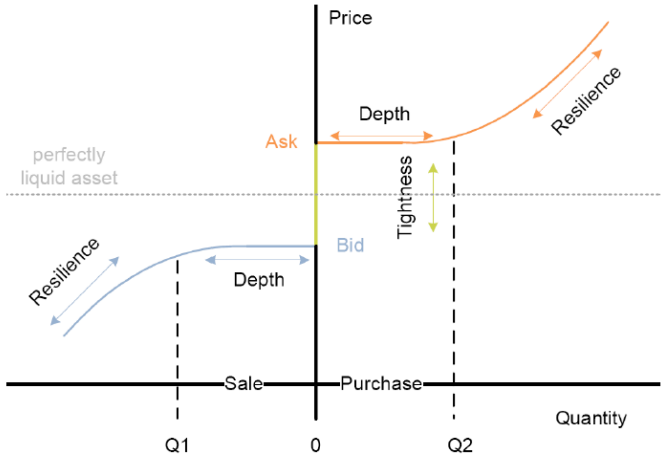

Figure 3: Dimensions of market liquidly; Depth, Tightness and Resilience.

Depth is the volume of trades possible without affecting the given price. Tightness refers to size of the bid-ask spread. Resilience is the speed with which prices recover from a random, uninformative shock. Due to their lower depth and higher spread and resilience, illiquid assets have higher transaction costs than their liquid counterparts. These costs are bid-ask spread costs – the cost of buying at a spread, rather than the current value, and the market impact cost – related to unnecessary movement of the price in block sales, due to low depth and high resilience (Keim & Madhavan, 1996)[27]. These increased costs are present, to some degree, independent of the changes in consumption – thus, not reflected in market beta. In equilibrium, these costs should be accounted for by a premium to motivate traders to buy illiquid assets, despite their increased costs.

However, Amihud argues that the increased transaction costs[28] are not necessarily unavoidable and can be reduced in two ways[29] – through increased brokerage costs and opportunity costs, as shown in figure 2. The first approach is to choose a market with higher commissions and fees. This will lead to higher **brokerage costs,** but trading there might results in lower market impact costs. Although such trading solutions marginally reduce transaction costs, this will be close to a zero-sum game, as brokers will receive most of the margin. **Opportunity cost** is as an alternative cost to market impact cost. Instead making a large impact on the assets price, an investor looking to do a block sale could stay patient and sell small parts of the asset at the time to traders willing to pay "full price". The cost is then related to the risk of price changes during the search for such traders, missing opportunities in other assets, or worse – the market discovers that a block sale is taking place and adjusts.

It could be argued that these costs are diversifiable. It appears there are upsides to holding illiquid assets as well, although not necessarily equal in size. An investor holding an illiquid stock could be

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[27] Larger block trades that have a large impact on the market could reflect information asymmetry (Easley, et al., 2002; O'Hara, 2003).

[28] Amihud calls these cost adverse selection costs because he argues that the bid-ask spread and the impact cost are related to information asymmetries, where uninformed traders must pay the increased transaction cost (Amihud, 2000).

[29] Although Amihud only describe the different ways of handling the increased transaction cost, I assume that the alternative ways of avoiding market impact cost reduces the total costs. If they did not, the alternatives would not have any reason to exist – according to standard transaction cost theory (Coase, 1937).

offered a premium when the stock suddenly becomes attractive. The impact cost, referred to by Amihud, might be reversed in a scenario where traders wish to buy the asset. However, there is a difference between the need of buying an asset and the need for selling one (Chordia, Roll, & Subrahmanyam, 2000). If investors are to be modeled as rational, they would not buy an asset at any cost, there are however examples of solvency constraints forcing traders to sell at fire-sale prices (Lustig & Chien, 2001). This asymmetry cause increased opportunity costs, due to illiquidity, to be only partly diversifiable.

So far, the increased transaction costs appear to be different ways of accepting the static cost created by illiquid assets. These costs may vary slightly with changes in consumption and marginal utility, but are also founded in real monetary cost of the trader, unrelated to market beta. As I will explain in the following section, illiquid assets also have a cost dimension that is more related to the business-cycle.

Liquidity risk – consequences of exposure to illiquid assets in recessions

In the other end of figure 2 from the static transaction costs, I find theory pointing towards a liquidity risk related to changes in consumption and marginal utility. These are costs that are not diversifiable, and should ideally be explained by market beta. Lui argues that illiquid assets will have a hard time getting **company funding** when consumption is low (Liu, 2006). This theory coincides with theory stating that solvency constraints give rise to a liquidity risk (Lustig & Chien, 2001). This dimension of liquidity risk also has clear similarities to the risk of owning small companies in recessions (Amihud, 2002).

In addition to the risk of bankruptcy of illiquid companies, research also suggests a **flight-to-liquidity** when consumption is low and marginal utility is high (Acharya & Pedersen, 2005). This theory is based on the rationale that illiquid assets will underperform in recessions because this is a time when traders might need their savings to cover other costs[30]. In periods of low consumption, market illiquidity rises causing a decline in stock prices and a rise in expected returns. Due to the flight-to-liquidity, liquid stocks should decline less than illiquid stocks in these periods (Amihud, 2002). Amihud base this on findings from the market crash of October 1987, where liquid assets outperformed illiquid assets (Amihud, Mendelson, & Wood, 1990). Ideally, this dynamic part of the illiquidity cost should be accounted for by market beta of Sharp and Lintner – which could also be said about arguments used to validate the size and value factors as proxies for state variables (Fama & French, 1992). This completes figure 2. However, although it appears that there are four separate costs related to liquidity, they can in practice be viewed as two, given rational investors.

Up to now, I have assumed that all traders are equally suited to hold illiquid assets – accepting the opportunity cost as unavoidable. Amihud and Mendelson (1986), have another view on this. If I accept the assumption that markets should not be modeled as frictionless, making liquidity an issue investors must incorporate in their rationale, it is possible to argue that different investors' time-

[30] Chordia states the following about the liquidity crisis in 1998: "This event precipitated financial distress in certain highly leveraged trading firms which found themselves unable to liquidate some positions to pay lenders secured by other, seemingly unrelated positions" (Chordia, Roll, & Subrahmanyam, 2000)

frames may play an important role. Some investors might have a financial strength that allows them to invest with different demands to when realization of the assets must take place.

Assuming this is true, an alternative way of assessing the risk of being illiquid in a flight-to-liquidity scenario is the existence of market makers, known as the clientele effect. Amihud and Mendelson (1986, p. 225) write: *"The market makers bridge the time gaps between the arrivals of buyers and sellers to the market, absorb transitory excess demand or supply in the inventory positions, and are compensated by the spread."* If we accept that investors make decisions based on their investment horizon, Amihud and Mendelsons point seems intuitive. Grossmann and Miller (1987) explain market liquidity as a function of the demand for immediacy, where market makers supply immediacy to investors with limited investment horizons – an effect that should intuitively correlate with the business-cycle. A large need for immediacy will raise the premium of illiquid assets in order to provide incentives for market makers to provide the service of waiting for what Grossman describes as "the ultimate buyer"[31]. Gibson and Mougeot (2004) and Lustig and Chien (2001) all claim that the supply of market makers, is market dependent factors, changing with marginal utility.

As illustrated in figure 4, opportunity cost – as an alternative to market impact cost is of the same nature as the fire-sales costs in a flight-to-liquidity. They are a function of the number of market makers and the need for immediacy. A difference is however that the flight-to-liquidity risk is primarily a cost related to recessions, while the opportunity cost is a not necessarily related to recessions.[32].

Acerbi and Scandolo (2008) make an interesting point in light of the market maker framework. One should distinguish carefully between assets and value. They argue that assets do not have value until they are placed in a portfolio and the intentions of the investor are articulated in terms of a "liquidity policy". If all investors are rational, they would not risk increased opportunity cost and brokerage costs or a flight-to-liquidity, if another trader is in a better position to bear the same risk – minimizing the cost. If my assumptions are correct, the consequences of my rationale is that a possible liquidity premium only varies with changes in funding risk and changes in market maker costs.

[31] The ultimate buyer is the buyer that is willing to pay the price that includes the premium.
[32] The opportunity cost is naturally also considerably higher in recessions, but present at some level independent of macro-economy.

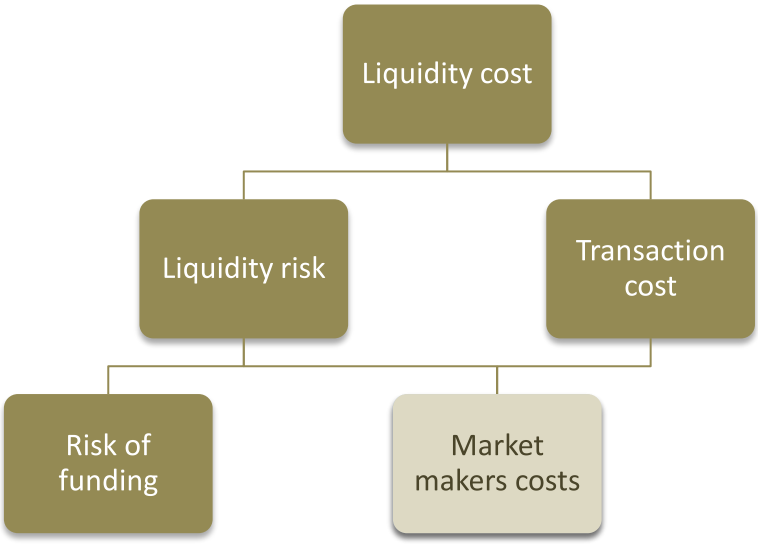

Figure 4: Theoretical costs related to illiquid assets - provided investors act rationally. If investors only trade assets with liquidity that match their financial strength, opportunity costs and flight-to-liquidity risk will be optimized because they are held by market makers. These costs should therefore be observed as market maker costs. The probability of bankruptcy due to limited funding opportunities for illiquid assets is also present and unaffected by the market makers ability to hold assets.

This finalizes my theoretical evidence in support of a liquidity premium. As shown in figure 4, the increased costs of illiquid assets can ideally be thought of as two theoretical costs:

- A liquidity risk related to funding of illiquid assets in recessions.
- The combined opportunity cost and risk of flight-to-liquidity risk faced by the market maker.

Depending on whether illiquidity has its origin in inefficient markets, information asymmetries or a market cap restrictions, there may also exist a cost related to information risk.

2.2.2 The empirical evidence of a liquidity premium

Before empirically evaluating whether my two liquidity costs are priced in equity markets, I will look at previous empirical findings on the nature of the liquidity premium.

Several empirical results from the U.S. stock market support a liquidity premium. Although it is nearly impossible to isolate a single factors ability to affect stock prices, an interesting experiment performed by Amihud and Lauterbach looked at changes in stock returns, when moved from an illiquid market to a liquid market (Amihud, 2000). They found that the change in liquidity corresponded to a premium of 5.5%[33]. This does not necessarily imply that all changes in trading liquidity are exogenous[34], but is one of the few results of tests where a large liquidity change has been observed "independently" of other factors.

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[33] The gradual movement from a market, where trades were done once a day, to a continuous market, improved liquidity significantly. Compared to similar experiments, connected to IPOs, these stocks were moved in "random" groups from one market to another, not reflecting any company specific news one might find in an IPO. One can however not rule out that the sample was affected by market effects in the given period.

[34] The same authors find evidence that i.e. improved flow of information from the company, increases liquidity.

In more regular regression tests of asset pricing models, both Amihud and Mendelson (1986) and Brennan and Subrahmanyam (1996) observe annual expected return as an increasing and concave function of the bid-ask spread. Similar tests from the U.S. market apply the stock turnover rate as a proxy for its liquidity (Datar, Naik, & Radcliffe, 1998). They found that turnover rate is significantly negatively related to stock returns - a result supporting the work of Amihud and Mendelson. Based on this research, Datar, Naik and Radcliffe suggest an annually liquidity premium around 4.5%. They did, however, not speculate about the possibility of a multidimensional premium nor find any results indicating that the size or value factors affect the significance of the turnover or bid-ask spread[35].

Other researchers have developed their own models in order to find empirical evidence of the liquidity premium; Keim and Madhaven (1996) find that the impact cost alone is in the area of 1.86 – 3.28%. These are pure transaction costs related to block sales. Archarya and Pedersen (2005) developed a liquidity augmented CAPM, adjusted for both market wide liquidity and asset liquidity. They found empirical evidence pointing towards an annual liquidity risk premium of 3.5% which they explain as a response to a strong positive correlation between illiquidity costs and market return – supporting the flight-to-liquidity theory. This is similar to the explanation Amihud, Mendelson and Wood, (1990) has for the excess returns on liquid assets in the October crash, 1987.

The contributions from less liquid markets are perhaps even more relevant for the study of the effects of trading liquidity on asset returns. Chan and Haff (2005) find support for their turnover augmented Fama French three-factor model based on the Australian stock market between 1990 and 1998. Similarly, Norwegian data show that a bid-ask spread augmented three-factor model improves the understanding of asset pricing in the Norwegian market (Næs, Skjeltorp, & Ødegaard, 2007).

A summary of the empirical results is found in table 1. It is important to recognize that most of these premiums, rationalized by the consumption-based model, are found by looking for average excess returns in historical data. This only tells half the story. Although positive average excess return over time suggests a premium, several of the authors conclude that the premium is paid due to underperformance when consumption is low – the flight-to-liquidity (Acharya & Pedersen, 2005; Amihud, 2002; Lustig & Chien, 2001). Although this is an intuitive suggestion, a premium found when marginal utility is low does not necessarily imply a negative premium when marginal utility is high. I will look further into this issue in my research.

[35] Datar, Naik and Radcliffe used data similar to the data Fama and French used in their important 1992 article.

Table 1: Summary of empirical evidence of a liquidity premium

Literature	Market	Time period	Proxy	Method	LP-estimate	Remarks
Amihud and Mendelson (1986)	NYSE	1961-1980	Bid-ask spread	OLS	4.5%	Cross-sectional
Eleswarapu and Reinganum (1993)	NYSE	1955-1979	Bid-ask spread	OLS	-	Cross-sectional
Brennan and Subrahmanyam (1996)	NYSE	1984-1991	Bid-ask spread	GLS	-	Time-series and cross-sectional
Keim and Madhavan (1996)	NYSE, AMEX and NASDAQ	1985-1992	Trade size	Regression on own model	1.86-3.28%[36]	Covers only impact cost
Datar, Naik and Radcliffe (1998)	NYSE	1962-1991	Turnover	GLS	4.5%	Cross-sectional
Acharya and Pedersen (2005)	NYSE	1962-1999	Cost of selling[37]	Regression on own model	3.5%[38]	Liquidity adjusted CAPM
Chan and Faff (2005)	ASX (Australia)	1989-1998	Turnover	GMM	-	Time-series
Næs, Skjeltorp and Ødegaard (2007)	OSEBX	1980-2005	Bid-ask spread	GMM	-	Time-series

[36] The article only reports research on the cost of impact through block sales.
[37] This is the ILLIQ measure used by Amihud (2002), incorporating how much the stock moves due to a small change in volume.
[38] Low liquidity stocks have 4.6% higher expected returns than high liquidity stocks. 3.5% is due to differences in expected liquidity, 1.1% is due to the liquidity risk premium

2.3 Hypotheses

Based on the discussion above, some expectation can be formulated about the impact of liquidity factors on the equity risk pricing models. I expect the augmented CAPM to perform better empirically and that liquidity factors can improve the theoretical motivation of the Fama French model. However, due to expected correlation between size and certain dimensions of liquidity, I do not expect improved empirical results for the augmented three-factor model. I do however believe that the size effect will be significantly reduced once it has been adjusted for a liquidity factor.

In order to quantify my expectations, I formulate the following hypotheses:

H1: The ability of the CAPM to explain expected returns, ex post, is improved by the addition of liquidity factors.

H2: The ability of the Fama French three-factor model to explain expected returns, ex post, is ***not*** *improved by the addition of liquidity factors.*

3 Data and Methodology

I have established that the standard asset pricing models are to a certain degree empirically unsatisfying and that both theory and empirical data support a liquidity premium for equity. The next step is to see whether the asset pricing models could be improved by the addition of liquidity factors. This chapter first discusses the choice of proxy for liquidity, before rationalizing the method and data chosen to test whether liquidity factors improve asset pricing models.

3.1 Methodology

3.1.1 The liquidity proxy

In order to test my hypotheses, the choice of liquidity proxy should ideally reflect the two central liquidity costs found in the theory – funding risk and the market makers costs of increased opportunity costs and flight-to-liquidity risk[39].

The original measurements the bid-ask spread, intuitively reflects the price impact and is used in several central publications (Amihud & Mendelson, 1986; Eleswarapu & Reinganum, 1993). It is calculated as

$$\frac{P_{Ask} - P_{Bid}}{\frac{1}{2}(P_{Ask} + P_{Bid})} \qquad (6)$$

where P_{Ask} is the asking price of the stock at close and P_{Bid} is the bidding price at close in the last day of each month. The rationale behind this liquidity proxy is that an increase in the number of traders will lead to a lower spread due to the increasing number of asking and bidding prices – improving the resolution.

The bid-ask spread has been criticized for being a noisy measure of illiquidity because block trades will happen outside the spread and small trades might happen within the spread (Lee, Mucklow, & Ready, 1993). Petersen and Fialkowski (1994) also suggest that there is a large difference between the posted spread and the effective spread in terms of what will be the actual transaction cost. Lee, Mucklow and Ready and Petersen and Fialkowski, are however mostly concerned with the actual reflected cost of the bid-ask spread, while I only ask whether the proxy indicates a particular dimension of liquidity.

Due to the critique of the bid-ask spread, Datar, Naik and Radcliffe (1998) use turnover as their proxy for liquidity. Turnover is the total trading volume of an asset in a period divided by the amount of the asset in circulation in that period and is more related to trading frequency. It has been criticized for mainly indicating price disagreement – thus to a larger degree reflecting firm specific uncertainty (Karpoff, 1987; Blume, Easley, & O'Hara, 1994). Despite these arguments, turnover also addresses the interest in the asset and its ability to attract funding – making it a good measure for a potential risk premium.

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[39] Amihud - one of the leading researchers on the relationship between liquidity and asset prices states that it is doubtful that there is one single measure that captures all its aspects (Amihud, 2002). Different proxies appear to account for the different dimensions of liquidity.

The bid-ask spread and turnover are only two of many possible liquidity proxies[40]. However, each of them captures the two dimensions of liquidity found in the previous chapter:

- The bid-ask spread is intuitively related to the increased transaction cost because it accounts for the increase in spread the trader have to pay for an illiquid asset. Based on the rationale of Amihud (2000), the increased transaction cost can, and should, be handled as opportunity cost – costs that are minimized in the hands of the proper market maker. Thus, it appears that the spread could be an indicator of the premium rewarded to the market makers exposed to opportunity costs or the risk of flight-to-liquidity.
- The turnover is also a good measure of the liquidity of an asset, but may be a better indicator of the number of traders interested in the asset, than the bid-ask spread. Based on this rationale, turnover would be a good measure of the premium rewarded to unattractive assets over attractive assets – a subject related to how easily a company can obtain funding though i.e. equity offerings in recessions.

I do not expect the two factors to be independent of each other. I.e. turnover will probably be a good indicator of how easy it is to sell an asset in a flight-to-liquidity scenario. I do however believe they are able to price assets differently on the basis of their exposure to the different kinds of liquidity risks. I therefore use both the bid-ask spread, turnover and combinations of the two as proxies for liquidity in my regression tests.

3.1.2 Empirical tests

In order to evaluate whether a liquidity factor would improve traditional asset pricing models, I perform both cross-sectional and time-series OLS regression tests[41] on both the two original and six liquidity-augmented versions of the CAPM and the Fama French three-factor model. The cross sectional tests are run across all assets in four different time periods and the five different time-series test are performed on portfolios sorted on their beta, market capitalization (size), price-to-book-ratio (value), bid-ask spread and turnover. A summary of these tests is shown in table 2 and 3.

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40 There are several others to be considered, but they are beyond the scope of this paper. Here are some: Unique roundtrip cost, return-to-volume measure (ILLIQ - the so called Amihud measure (Amihud, 2002)), number of zero-day trades, volatility in liquidity, trading speed (Liu, 2006) and several others (Hibbert, Kirchner, Kretzshmar, Li, & McNeil, 2009).

41 I use the ordinary least square (OLS) regression in this paper. Methods like GLS and GMM are improvements of this model, but since I will not draw any binary conclusions in this paper, I believe the OLS to be sufficient.

Table 2: Summary of cross-sectional tests. Eight models in four periods, across all assets.

<table>
<tr><th>Models</th><th>Periods</th><th>Assets</th></tr>
<tr><td rowspan="4">CAPM
CAPM + bid-ask spread factor
CAPM + turnover factor
CAPM + bid-ask spread factor and turnover factor

Three-factor model
Three-factor model + bid-ask spread factor
Three-factor model + turnover factor
Three-factor model + bid-ask spread factor and turnover factor</td><td>Pre crisis
2006 – 2008</td><td rowspan="4">All assets</td></tr>
<tr><td>Crisis
2008 – mid 2009</td></tr>
<tr><td>Post crisis
mid 2009 – 2011</td></tr>
<tr><td>Entire period
2006 – 2011</td></tr>
</table>

Table 3: Summary of time-series tests. Eight models in one period, on 25 portfolios.

<table>
<tr><th>Models</th><th>Periods</th><th>Assets</th></tr>
<tr><td rowspan="5">CAPM
CAPM + bid-ask spread factor
CAPM + turnover factor
CAPM + bid-ask spread factor and turnover factor

Three-factor model
Three-factor model + bid-ask spread factor
Three-factor model + turnover factor
Three-factor model + bid-ask spread factor and turnover factor</td><td rowspan="5">Entire period
2006 – 2011</td><td>5 beta portfolios</td></tr>
<tr><td>5 size portfolios</td></tr>
<tr><td>5 value portfolios</td></tr>
<tr><td>5 bid-ask spread portfolios</td></tr>
<tr><td>5 turnover portfolios</td></tr>
</table>

In theory, the model that works best has intercepts equal to zero in all regressions and the cross-sectional test has a slope equal to the excess market return in the period. This approach is widely recognized as the standard method of testing asset pricing models[42]. I evaluate the factors ability to improve the model on the basis of the significance of the intercepts, the R^2, and the significance of

[42] (Black, Jensen, & Scholes, 1972; Fama & French, 1992; Fama & MacBeth, 1973; Chan & Haff, 2005; Pastor & Stambaugh, 2001; Datar, Naik, & Radcliffe, 1998; Gibson & Mougeot, 2004; Chordia, Roll, & Subrahmanyam, 2000)

the factors. In order to validate my results further, I also demand that they have an intuitive rationale – founded in financial theory.

An important assumption of such historical regressions is that ex-post returns equals ex-ante returns, meaning that the historical realized returns are the expected returns. I also assume that the betas of the respective factors are constant in the entire estimation period[43]. In order to perform the OLS test, I also need to assume that the returns are drawn from a normally, identical and independent distribution (n.i.i.d). The weakest of the assumptions is probably that the distribution is identical in each draw, given that my regression is done on an intertemporal model. This problem will however be reduced by the limited timeframe, giving reason to believe that all data belongs within the same macro-regime. On the other hand, the financial crisis in 2008 probably changed the distribution.

3.1.3 Time-series regression

The OLS time-series tests are done to observe how different portfolios were explained by the models over time. This test is based on the methodical work of Fama and French (Fama & French, 1993), originally developed by Jensen, Black and Scholes (Black, Jensen, & Scholes, 1972). Monthly excess returns on a variety of portfolios are regressed against the right hand variables of the CAPM and the Fama French three-factor model. The two models are evaluated both with and without additional liquidity factors in order to observe changes caused by the additional independent variable. Five orthogonal proxy factors are constructed: ERP – representing the excess market return (R_m) over the risk-free rate (r_f), SMB – representing the excess return on small companies over large companies, VMG – representing the excess return on low price-to-book companies over high price-to-book companies, WMN – representing the excess return on companies with wide bid-ask spread over companies with narrow bid-ask spread and HML – representing the excess return on companies with low turnover over companies with high turnover. Time-series regressions are performed on the following equations:

CAPM:

$$E(r_{it}) - r_f = \alpha_{it} + \beta_i ERP_{it} \tag{7}$$

CAPM w/Bid-ask spread:

$$E(r_{it}) - r_f = \alpha_{it} + \beta_i \, ERP_{it} + w_i WMN_{it} \tag{8}$$

CAPM w/turnover:

$$E(r_{it}) - r_f = \alpha_{it} + \beta_i \, ERP_{it} + h_i HML_{it} \tag{9}$$

CAPM w/Bid-ask spread and turnover:

$$E(r_{it}) - r_f = \alpha_{it} + \beta_i \, ERP_{it} + w_i WMN_{it} + h_i HML_{it} \tag{10}$$

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[43] Not only is this debatable, as companies change over periods – changing their exposure to the factor, but it also causes a large debate on how they should be estimated.

Fama French three-three factor model:

$$E(r_{it}) - r_f = \alpha_{it} + \beta_i \, ERP_{it} + s_i SMB_{it} + v_i VMG_{it} \quad (11)$$

Fama French three-three factor model w/Bid-ask spread:

$$E(r_{it}) - r_f = \alpha_{it} + \beta_i \, ERP_{it} + s_i SMB_{it} + v_i VMG_{it} + w_i \, WMN_{it} \quad (12)$$

Fama French three-three factor model w/turnover:

$$E(r_{it}) - r_f = \alpha_{it} + \beta_i \, ERP_{it} + s_i SMB_{it} + v_i VMG_{it} + h_i \, HML_{it} \quad (13)$$

Fama French three-three factor model w/Bid-ask spread and turnover:

$$E(r_{it}) - r_f = \alpha_{it} + \beta_i \, ERP_{it} + s_i SMB_{it} + v_i VMG_{it} + w_i \, WMN_{it} + h_i \, HML_{it} \quad (14)$$

Where $E(r_{it})$ is the expected return on asset or portfolio i, r_f is the risk free rate, α_{it} is the intercept, β_i is the market beta, s_i is the size-beta, v_i is the value-beta, w_i is the bid-ask spread-beta, and h_i is the turnover-beta

The results of each regression are five betas representing the factor loadings to each of the factors, ERP, SMB, VMG, WMN and HML. Significance-tests are done on all betas, in the form of p-values. The main interest is however the alpha variations for the different models. As stated by Black, Jensen and Scholes (1972) the capital asset pricing models must provide zero alphas for all portfolios in order to explain the market prices. It is not sufficient to have the average of alphas equal to zero. Because of this, it is not sufficient to look at individual p-values to make sure that the alphas are all zero.

A common way of assuring al the alphas are zero is the GRS-test, developed by Gibbons, Ross and Shanken (1989). This test assumes no heteroskedasticity and no autocorrelation as well as n.i.i.d in the residuals (Cochrane, 2001). The GRS-test tests the joint significance of the alphas across the different portfolios and provides a test statistic J, which tests the null hypothesis that all alphas equals zero.

$$J = \frac{T-N-1}{N} \left(1 + \frac{\mu_m^2}{\sigma_m^2}\right)^{-1} \alpha' \, \Sigma^{-1} \alpha \quad (15)$$

T is the number of months, N is the number of asset/portfolios tested, μ_m and σ_m is the average excess return and standard deviation on the market portfolio proxy, α is the vector of the Nx1 alphas from the regression and ∑ is the T x N matrix with the residuals from the same regression. I further calculate p-values for the F-distributed J statistics.

Factor construction

In order to properly test different factors' ability to price assets, I need to secure that the factors are orthogonal to each other – meaning that the factors are constructed so that they are not biased by changes in other factors (Fama & French, 1992). I.e. if the SME factor is created by subtracting the difference between the excess returns on small cap less the excess return on large cap, the method makes sure both portfolios have a 50/50 percentage of value and growth stock, so that changes in returns are independent of the value factor.

The companies are first divided into two groups based on their respective market capitalization[44], each group is further divided in two according to their price-to-book ratio, then their bid-ask spread and finally their turnover. The portfolios are rebalanced on a yearly basis. From this process, sixteen orthogonal portfolios are made – SVWH, SVWL, SVNH, SVNL, SGWH, SGWL, SGNH, SGNL, BVWH, BVWL, BVNH, BVNL, BGWH, BGWL, BGNH, BGNL. The factors are then calculated in the following fashion:

$$\mathbf{SMB} = \frac{1}{8}\ (\mathrm{SVWH} + \mathrm{SVWL} + \mathrm{SVNH} + \mathrm{SVNL} + \mathrm{SGWH} + \mathrm{SGWL} + \mathrm{SGNH} + \mathrm{SGNL}) - \frac{1}{8}(\ \mathrm{BVWH} + \mathrm{BVWL} + \mathrm{BVNH} + \ \mathrm{BVNL} + \mathrm{BGWH} + \mathrm{BGWL} + \ \mathrm{BGNH} + \ \mathrm{BGNL})$$

$$\mathbf{VMG} = \frac{1}{8}\ (\mathrm{SVWH} + \mathrm{SVWL} + \mathrm{SVNH} + \mathrm{SVNL} + \mathrm{BVWH} + \mathrm{BVWL} + \mathrm{BVNH} + \ \mathrm{BVNL}) - \frac{1}{8}(\ \mathrm{BGWH} + \mathrm{BGWL} + \ \mathrm{BGNH} + \ \mathrm{BGNL} + \mathrm{SGWH} + \mathrm{SGWL} + \mathrm{SGNH} + \mathrm{SGNL})$$

$$\mathbf{WMN} = \frac{1}{8}\ (\mathrm{BGWH} + \mathrm{BGWL} + \mathrm{SVWH} + \mathrm{SVWL} + \mathrm{BVWH} + \mathrm{BVWL} + \mathrm{SGWH} + \mathrm{SGWL}) - \frac{1}{8}(\ \mathrm{SVNH} + \mathrm{SVNL} + \ \mathrm{BGNH} + \ \mathrm{BGNL} + \mathrm{BVNH} + \ \mathrm{BVNL} + \mathrm{SGNH} + \mathrm{SGNL})$$

$$\mathbf{HML} = \frac{1}{8}\ (\mathrm{SGNH} + \mathrm{SGWH} + \mathrm{BGNH} + \mathrm{BGWH} + \mathrm{SVWH} + \mathrm{SVNH} + \mathrm{BVWH} + \mathrm{BVNH}) - \frac{1}{8}(\ \mathrm{BGWL} + \ \mathrm{BGNL} + \mathrm{SGWL} + \mathrm{SGNL} + \mathrm{SVWL} + \mathrm{SVNL} + \mathrm{BVWL} + \ \mathrm{BVNL})$$

Portfolio construction

Both the CAPM and the three-factor models should, in theory, work just as well on portfolios as on single assets. If any of the models are unable to account for the returns of portfolios sorted by a common factor, this suggests that the model has flaws. Based on this, five portfolios[45] are made, for

[44] Although it is somewhat more common to divide each factor into three levels - high, medium and low, I find it necessary to divide into two levels - high and low due to a limited dataset.
[45] Other authors created up to ten portfolios; however, limited sample size could create misleading returns.

each of the factors used in the liquidity augmented versions of the CAPM and the three factor model; beta, market capitalization, price-to-book ratio, bid-ask spread and turnover[46].

Five, equally weighted beta portfolios are made on the basis of their average 12-month beta and re-balanced yearly. The individual betas are calculated based on monthly returns on the individual assets and the use of OSEBX as market portfolio[47]. Returns are calculated as a percentage change in the value of one NOK invested in that portfolio during month t.

The five size portfolios are created on the basis of their average market capitalization in the month of December the previous year and rebalanced each year. Similarly to the size factor, monthly average price-to-book data is used as guidelines for the creation of five value portfolios. The companies with low price-to-book ratio are named value stocks, while the companies with high price-to-book ratio are named growth stocks. Beta portfolios are adjusted based on their 12-month average beta, while the size portfolios and the price-to-book ratios are adjusted according to their respective average values in the month of December. This is because an investor would use a data series stretching over some months when assessing different company beta because it is a statistical result in need of a time-series. When assessing market capitalization and price-to-book ratio, averaged yearly data provide poorer information of the stock's current status and does not reflect recent price changes that would be relevant in the investment choice.

Liquidity portfolios are also constructed on the basis of the companies' relative bid-ask spread and turnover. For each of the two liquidity proxy's, five portfolios are made. The portfolios are rebalanced each year. The issue of the rebalancing is not as straight forward as with the other factors because it is not obvious whether a 12 month average or the average of December is more representative. The average over the last month of the previous year would of course be a more updated measure of the stocks liquidity, but might be biased by whether or not the company reported their yearly figures in that month or other company specific factors. I therefore use a rebalancing strategy based on the previous 12 months[48].

3.1.4 Cross-sectional regression

In addition to a time-series test, I also perform a cross-sectional test on all eight equity risk premium models. The object of this test is to observe how well the models priced the factors across all assets in given periods. In order to avoid non-independent cross sectional residuals, I use the Fama and MacBeth improved two-pass method (Fama & MacBeth, 1973). This two-pass test calculates the different factor loadings for each asset through a time-series regression in the first pass. In the second pass, asset returns are regressed monthly against their respective betas, obtained in the first pass. Finally, the intercept and slopes are averaged. This way of performing regressions gets around

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[46] A graphical presentation of the portfolios that shows the performance of a strategy of holding and rebalancing the equally weighted average of the highest beta portfolio rather than holding and rebalancing the lowest beta portfolio, are provided in the appendix.

[47] The choice of proxy for the market portfolio has been a heavily debated (Roll, 1976), but research on the subject supports that the main index will be a sufficient proxy (Levy & Roll, 2010)

[48] With the exception of the initial portfolio, that was based on the average of January 2006 due to absence of earlier data.

the error term correlation problem, without changing the cross sectional estimates. The monthly cross-sectional regression tests are run on equation 7, 8, 9, 10, 11, 12, 13 and 14.

The regression results in T alphas, T ERP-factors, T SMB-factors, T VMG-factors, T WMN-factors and T HML-factor, where T is the number of months. Average factors are calculated on the basis of these results. If the model is valid, the incept alpha should be zero and the slopes should equal the observed risk premium for each factor[49]. The alpha can be evaluated based on a Fama MacBeth adjusted t-statistic calculated as:

$$t(\alpha_i) = \frac{avg(\alpha_i)}{(\frac{std(\alpha_i)}{\sqrt{T}})} \qquad (16)$$

Where $avg(\alpha_i)$ and $std(\alpha_i)$ are the average and standard deviation of all alphas.

In my entire cross sectional tests, I choose not to create portfolios because it might bias the results in some direction. Although individual assets might create a lot of noise, the models should in theory be able to price them this way.

3.2 Data

In order to best evaluate my hypotheses, I use data from the Norwegian stock exchange, OSEBX[50] in the period of January 2006 to February 2011[51]. Although this is a limited period, in terms of obtaining a sufficient time series, I find several upsides of using short term data in the light of my research. (1) The near term data have a better relevance and predictability than data averaged over several macro-economic regimes. (2) Due to the rise of internet trading and information access, the influence of liquidity in asset pricing must be studied with post year 2000 – data, in order to be representative of how liquidity is priced today. (3) Data in the period provides the opportunity to observe changes in the premium caused by the financial crisis in 2008.

The rationale behind including the financial crisis in 2008 in my data set can be criticized for being genuinely unwise. If changes in asset prices are mainly to due to changes in the equity risk premium (Cochrane, 2011), I should expect a noisy result from my cross-sectional regression test due to the macro-economic changes in the observed period. However, I believe this could be an advantage. Given that the risk premium change over time and reflect the market macro-economic view, this period offers an unique opportunity to study how these premiums change in periods where there is a common understanding of the macro-economic picture. By dividing the cross-sectional regression in three – before, during and after the financial crisis[52][53], I get front row seats to observe changes in the

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[49] This would mean the market excess returns in the period would be the same as the average ERP for the CAPM test.

[50] Companies are included from different indexes with different demands to liquidity; OBX, OB Match and OB Standard in order to create a representative selection.

[51] Through Reuter's Ecowin and Amadeus 2.0 (Thanks to the kindness of "Børsprosjektet" at NHH in Bergen)

[52] Before (2006-2008), during (2008 – mid-2009) and after (mid-2009 – 2011)

[53] An alternative to dividing in periods according to the return on the market portfolio, I could also have used a more inductive approach – following the main changes in the WMN- and HML-factors. This approach would probably provide me with more significant liquidity results, but at the cost of understanding its relation to the movement of the market.

premium in the given period. Unlike similar tests of equity risk premium models, I ask whether the reversed premium is found in times when marginal utility is high. This is the opposite of the standard approach of looking for a positive premium, in the form of average excess returns over time. Although equilibrium theory demands an opposite and equal cost to findings of these average excess returns, this does not necessarily imply a negative premium in recessions. There might be costs of trading or economic benefits that is not reflected in market returns. Based on this rationale, it is equally relevant to search for consumption-based factors that are rewarded a negative premium in recessions.

In order to avoid changes due to mergers, listing or delisting, that might dominate my results I choose to only look at firms that were listed in the entire period. This provided me with 92 companies' monthly data points, in a five year period. Although the focus on obtaining a dataset without delisting might suggest a survivorship bias, the short term dataset limits such bias.

Data from the OSEBX is chosen because the stock exchange is illiquid and unexplored, relative to the U.S stock exchanges, where most data sets are gathered from. I choose this because effects of illiquidity will be even more prominent if the market itself is small and illiquid assets hardly got traded at all. A byproduct of this is also a lower probability of data snooping (Chan & Haff, 2005; Lo & MacKinlay, 1990). A potential weakness of using Norwegian data from this period is that the financial crisis did not have the same impact on the Norwegian economy as other economies. However, the ramifications of the 2008–crisis were too large to not affect the Norwegian stock exchange in a similar way.

The standard choice of risk-free rate is usually a one month US Treasury bill (Fama & French, 1996). The best approximation of this in the Norwegian market is the efficient NIBOR interest received in one month[54]. This is the rate of return Norwegian traders can expect to receive from a risk-free investment. Since the data obtained are listed in 12-month returns, I calculate one month returns with equation 26:

$$r_{monthly} = \left(1 + r_{yearly}\right)^{\frac{1}{12}} - 1 \qquad (17)$$

where r_{montly} is the monthly interest rate and r_{yearly} is the yearly interest rate.

3.3 Evaluation of method

My econometric approach of answering my hypotheses is the natural choice both because the models are statistical of nature and because it enables me to compare my results against other research. However, this statistical approach will not explain the human aspect of the risk premium to the same degree as other methods, such as interviews or surveys. These methods reflect the trader's expectations, which the risk premium is all about. They also reduce the joint hypothesis problem

[54] This data is retrieved from www.norgesbank.no

related to the statistical approach[55]. However, interviews and surveys might not be representative of the complete market – a feature better achieved by the statistical models.

One of the major concerns of both cross-sectional and time-series regressions is the portfolio creation in combination with the calculation of the betas. Jensen, Black and Scholes (1972) and Fama and MacBeth (1973) originally developed a method for creating portfolios, calculating the betas, and estimating portfolio returns, at different times. If the portfolios are created at the same time that the betas are estimated and returns are calculated, high beta portfolios tend to have positive measurement errors. However, this is a trade-off I made on order to keep my period short.

The goal of my portfolio approach is to reveal potential weaknesses of the CAPM and the Fama French three-factor model, by improving them with liquidity factors. However, I suspect that this approach mainly should be used to test models, and not create new ones. Although I provide support for whether a liquidity factor improves the models ability to price assets, I do not fully test the augmented models. A real test of the models ability to price assets would be to price single assets or testing dummy-portfolios on the liquidity augmented versions of the models.

[55] When testing the asset pricing model, one cannot be certain whether a bad of good result is due to the model, or the assumptions made about market efficiency and rational investors

4 Results and Discussion

4.1 Results

In this section I introduce relevant findings from my empirical tests. I first discuss the main result, related to the hypotheses, and then other findings, substantiating the main results.

4.1.1 Main result

I find that the bid-ask spread and turnover factor explains asset returns (Table 4) and that none of the liquidity factors improve the alphas or adjusted R^2 noticeably in either model (Table 5 and 6).

In general, all results are affected by the market factors ability to explain returns in the period. The CAPM has only one portfolio with significant non-zero alphas at 95% significance level, in all regressions tests. The GRS-tests thus validate the CAPM as fully capable to price assets. Table 5 and 6, show an example of the marginal improvement of alphas and adjusted R^2 provided by adding significant bid-ask spread-betas. Since the additional factors provide only marginal contribution to alphas and R^2, the significance and ability to price portfolios in a given direction is my only indication of whether a liquidity factor is priced.

Table 4, summarizes whether the different regressions test provided three or more significant liquidity factors that price portfolios in a monotonic direction. A green circle indicates the results support the hypothesis, a red cross indicates the results are unsupportive of the hypothesis and a line indicated the results are inconclusive or noisy. I observe that the liquidity factors are able to explain the beta portfolios and liquidity portfolios, but are unable to explain the size and value portfolios. These observations are similar on both the liquidity augmented versions of the CAPM and the liquidity augmented versions of the Fama French model. The fact that I observe significant factors both in the cross-sectional and time-series test means that the two factors to able to explain returns both over time and across assets. Although both liquidity proxies are able to explain returns on portfolios created according to their respective characteristics, the bid-ask spread is able to explain beta portfolios as well (table 5 and 6).

Table 4: Summary of time-series and cross-sectional regressions tests.

Cross-sectional	Total	Pre-crisis	Crisis	Post-crisis
H1: Improved CAPM	-	X	O	X
H2: Unimproved Fama French	-	O	X	O

Time-series	Beta	Size	Value	Spread	Turnover
H1: Improved CAPM	O	X	X	O	O
H2: Unimproved Fama French	X	O	O	X	X

Table 5 (left) Results of time-series regression test on beta sorted portfolios (low to high beta) in CAPM.

Table 6 (right): Results of time-series regression test on beta sorted portfolios (low to high beta) on bid-ask spread and turnover augmented version of the CAPM.

Beta	R^2	α	β	R^2	α	β	l	w
Portfolio 1	0,30	0,00	0,40	0,33	-0,10	0,56	0,28	0,37
p-value		*0,99*	*0,00*		*0,85*	*0,00*	*0,16*	*0,05*
Portfolio 2	0,75	-0,39	0,65	0,78	-0,40	0,69	-0,08	0,24
p-value		*0,23*	*0,00*		*0,22*	*0,00*	*0,47*	*0,03*
Portfolio 3	0,86	-0,19	0,93	0,87	-0,33	1,03	0,12	0,31
p-value		*0,57*	*0,00*		*0,31*	*0,00*	*0,30*	*0,01*
Portfolio 4	0,93	0,24	1,17	0,93	0,25	1,15	0,12	-0,19
p-value		*0,40*	*0,00*		*0,37*	*0,00*	*0,23*	*0,05*
Portfolio 5	0,85	0,19	1,86	0,88	0,46	1,54	-0,54	-0,77
p-value		*0,77*	*0,00*		*0,47*	*0,00*	*0,02*	*0,00*
W	**0,01**			**0,01**				
p	**1,00**			**1,00**				

4.1.2 Other findings

I find that a positive liquidity premium and a negative value premium are rewarded in recessions.

I find several indications of a positive liquidity premium in 2008 in combination with a negative value premium in the same period. Figure 5, shows how both liquidity factors increase during the market collapse, while the value factor falls at the same time. Additional support is also found in the correlation matrix (table 7) where the bid-ask spread and turnover factors are negatively correlated with the market, while the value factor is positively correlated. The cross-sectional results from the crisis support that the bid-ask spread and value factors explain returns, but the turnover is insignificant (table 8). Despite its insignificance, I observe that the turnover factor makes the alphas less significant.

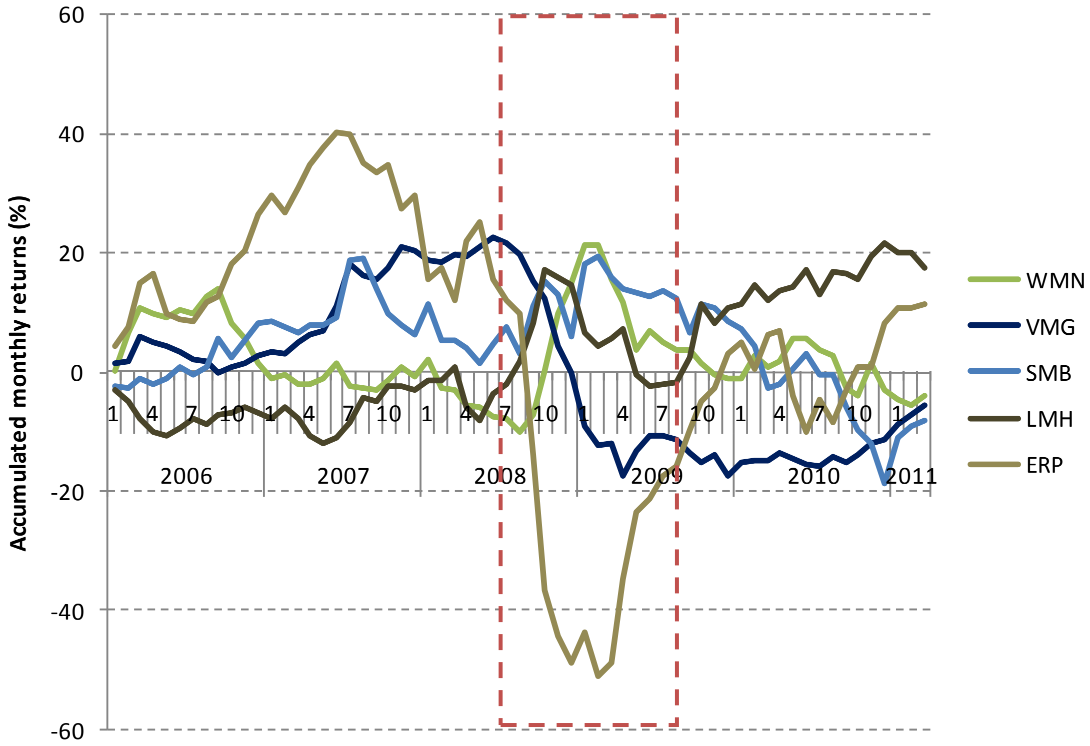


Figure 5: Accumulated monthly returns (%) of factors in the period 2006-2011.

Table 7: Correlation between the factors in the period 2006-2011.

	Rm - Rf	VMG	SMB	LMH	WMN
Rm - Rf	1				
VMG	0,24	1			
SMB	-0,35	-0,09	1		
LMH	-0,51	-0,07	0,11	1	
WMN	-0,49	-0,34	0,25	0,09	1

Table 8: Results from cross-sectional regression from 2008 - mid 2009 on all models. FF represent the Fama French three-factor model, WMG represent a bid-ask spread augmented model, LMH represent a turnover-augmented model and 2LIQ represents a model that contains both liquidity proxies.

Crisis (2008)	alpha	ERP	WMN	LMH	SMB	VMG
CAPM	-2,62	-2,70				
p-value	*0,28*	*0,51*				
CAPM + WMG	-4,02	-1,28	2,96			
p-value	*0,17*	*0,78*	*0,04*			
CAPM + LMH	-1,67	-3,68		-1,59		
p-value	*0,44*	*0,33*		*0,43*		
CAPM + 2LIQ	-2,90	-2,43	3,02	-1,80		
p-value	*0,25*	*0,56*	*0,04*	*0,40*		
FF	-2,66	-2,67			-0,67	-2,68
p-value	*0,24*	*0,51*			*0,76*	*0,03*
FF + WMN	-3,44	-1,87	2,67		-0,87	-2,22
p-value	*0,22*	*0,68*	*0,04*		*0,70*	*0,04*
FF + LMH	-1,66	-3,68		-1,34	-0,47	-2,48
p-value	*0,37*	*0,31*		*0,50*	*0,83*	*0,04*
FF + 2LIQ	-2,27	-3,07	2,77	-1,50	-0,61	-2,20
p-value	*0,33*	*0,44*	*0,04*	*0,48*	*0,78*	*0,05*

I find a large, negative size premium in the post-crisis regressions.

The cross-sectional regression of the period after 2008 shows a negative and significant size factor (table 9). This finding is supported by the time-series regression. In table 10, I observe that the size factor explains the size portfolios over time better than the beta, which is reasonably flat. Finally, the correlation matrix in table 7, indicate a negative correlation between size and the market in the period.

Table 9: Results from cross-sectional regression from mid 2009 - 2011 on all models. FF represent the Fama French three-factor model, WMG represent a bid-ask spread augmented model, LMH represent a turnover-augmented model and 2LIQ represents a model that contains both liquidity proxies.

Post-crisis	alpha	ERP	WMN	LMH	SMB	VMG
CAPM	0,28	2,43				
p-value	*0,76*	*0,13*				
CAPM + WMG	0,38	2,32	-0,76			
p-value	*0,64*	*0,11*	*0,39*			
CAPM + LMH	-0,11	2,83		0,32		
p-value	*0,91*	*0,09*		*0,64*		
CAPM + 2LIQ	-0,21	2,93	-0,79	0,30		
p-value	*0,82*	*0,07*	*0,37*	*0,63*		
FF	0,58	2,12			-1,54	0,10
p-value	*0,51*	*0,17*			*0,06*	*0,88*
FF + WMN	0,63	2,07	-0,67		-1,53	0,07
p-value	*0,44*	*0,14*	*0,45*		*0,06*	*0,91*
FF + LMH	0,15	2,56		0,40	-1,63	0,02
p-value	*0,87*	*0,12*		*0,57*	*0,05*	*0,98*
FF + 2LIQ	0,06	2,65	-0,72	0,38	-1,65	0,06
p-value	*0,95*	*0,09*	*0,41*	*0,55*	*0,04*	*0,93*

Table 10: Results of time-series regression on size portfolios (small to large size) on bid-ask spread augmented Fama French three-factor model.

Size	R^2	α	β	v	s	l	w
Portfolio 1	0,75	0,74	0,85	-0,04	0,55	-0,56	0,01
p-value		*0,14*	*0,00*	*0,85*	*0,00*	*0,00*	*0,94*
Portfolio 2	0,84	-0,36	1,19	-0,04	0,41	0,44	0,13
p-value		*0,37*	*0,00*	*0,79*	*0,00*	*0,00*	*0,34*
Portfolio 3	0,85	0,26	0,99	0,13	-0,09	0,07	0,02
p-value		*0,50*	*0,00*	*0,40*	*0,38*	*0,64*	*0,88*
Portfolio 4	0,86	-0,02	0,95	-0,02	-0,27	0,02	0,15
p-value		*0,96*	*0,00*	*0,91*	*0,01*	*0,88*	*0,23*
Portfolio 5	0,94	-0,56	1,02	-0,03	-0,54	0,03	-0,29
p-value		*0,05*	*0,00*	*0,80*	*0,00*	*0,75*	*0,01*
W	**0,38**						
p	**0,86**						

I find that the size factor and the bid-ask spread factor explains returns on assets differently, despite their positive correlation.

I observe that both the portfolios of small cap with narrow bid-ask spread and the portfolio with small cap and wide bid-ask spread outperforms the two similar portfolios with large market capitalization (figure 6). However, in the market collapse, returns depend on whether assets have a wide or narrow bid-ask spread. I observe that the illiquid portfolios outperform the liquid portfolios in the recession, independently of market capitalization. In support of this finding, I also find that both spread and size explain asset returns in the-time series regression on the beta portfolio (table 11). They appear to price assets in the same direction without reducing each other.

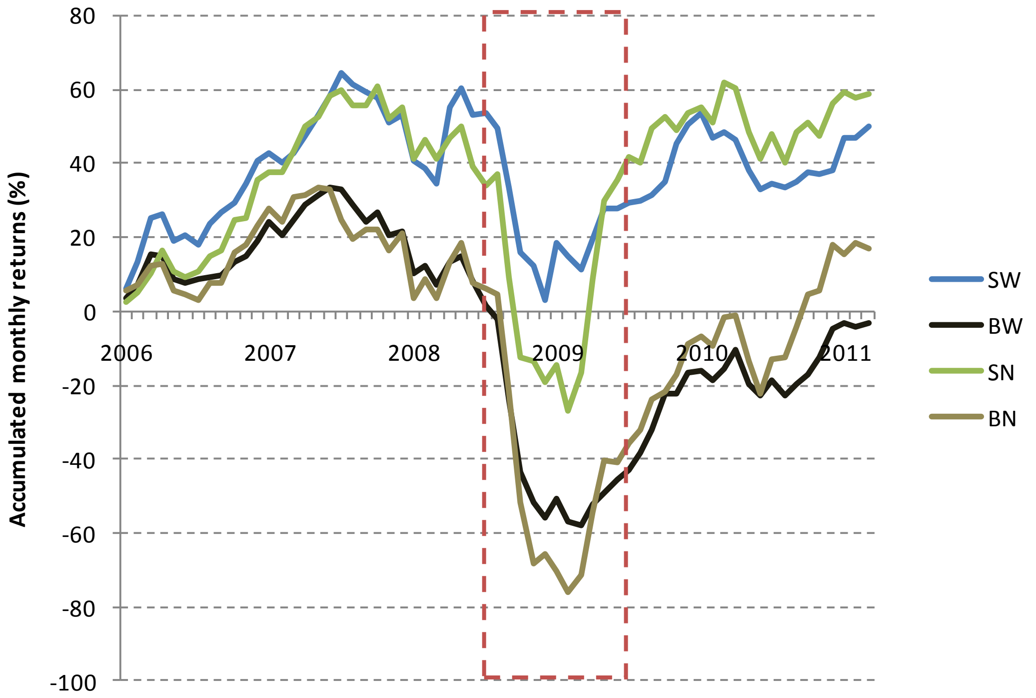


Figure 6: Accumulated monthly returns (%) on orthogonal portfolios based on size and spread.

Table 11: Results of time-series regression on beta portfolios (high to low beta) on bid-ask spread augmented Fama French three-factor model.

Beta	R^2	α	β	v	s	w
Portfolio	0,45	0,07	0,55	0,09	0,51	0,27
p-value		*0,88*	*0,00*	*0,64*	*0,00*	*0,11*
Portfolio	0,77	-0,41	0,72	-0,22	0,01	0,20
p-value		*0,19*	*0,00*	*0,08*	*0,92*	*0,07*
Portfolio	0,87	-0,18	1,02	-0,13	0,12	0,25
p-value		*0,55*	*0,00*	*0,30*	*0,18*	*0,03*
Portfolio	0,94	0,21	1,08	0,03	-0,21	-0,19
p-value		*0,41*	*0,00*	*0,78*	*0,00*	*0,04*
Portfolio	0,88	0,17	1,62	0,27	-0,44	-0,54
p-value		*0,78*	*0,00*	*0,26*	*0,01*	*0,01*
W	**0,15**					
p	**0,98**					

4.2 Discussion

My main result can be interpreted in several ways. One way is to conclude that the two significant liquidity proxies are statistical deviations caused by a volatile period in the stock market. They do not appear to have the same impact on the models as the size and value factor, and the GRS-tests and alpha results suggest that the Sharp-Lintner CAPM is sufficient to explain all asset returns. Several factors are also insignificant, showing no ability to explain returns on either portfolio. Had the only goal of these asset pricing models been empirical performance, I would have supported the CAPM – based on my results from this period. However, the spread factor, WMN, appears to price both beta, spread and turnover portfolios in a given direction. Although somewhat less impressive, the ability to price portfolios in a simple direction is also showed by the turnover factor, LMH, on the turnover portfolios. These results are inconsistent with both the Black (1972) and the Sharp-Lintner (1964) version of the CAPM because they indicate that beta alone is insufficient to explain asset returns.

Another way of interpreting the results is by recognizing the factors as significant, but reject them as priced factors because their ability to improve the CAPM and the Fama French three-factor model is negligible – making the significant results unimportant in practice. However, the fact that I find significant results, indicating that the market cares about liquidity when pricing assets, point towards that both the CAPM and the Fama French three-factor model are unable to fully appreciate liquidity. Although the impact on accuracy of the models is somewhat disappointing, the significant factors suggest that our understanding of asset pricing is not complete without adjusting for liquidity. My initial conclusion, based only on the empirical findings, is that the added liquidity factors improve both models.

Although the factors were created in order to reflect different dimensions of liquidity, there is no guarantee that the significant factors of my results reflect liquidity. This is why there is little reward in finding significant factors without a proper theoretical rationale. I will therefore try to explain my significant results in the light of the consumption-based model and my other findings.

Based on my theoretical research, I predicted that recessions will cause a flight-to-liquidity (Acharya & Pedersen, 2005; Amihud, Mendelson, & Wood, 1990; Amihud, 2002) and increased opportunity costs (Grossman & Miller, 1987) as well as lower ability to fund companies – increasing the probability of bankruptcy among illiquid firms (Lustig & Chien, 2001; Liu, 2006). Archarya and Pedersen (2005), Amihud (2002), Amihud, Mendelson and Wood (1990) and Lustig and Chien (2001) all suggest that these risks will cause illiquid assets to provide lower returns when consumption is low and a premium when consumption is high – consistent with utility theory (Cochrane, 2001). Inspired by Amihud and Mendelson (1986) and Datar, Naik and Radcliffe (1996), I also suggested that the bid-ask spread would be a good proxy for the premium rewarded to market makers and that the turnover would be a good proxy for the risk premium related to the increased risk of bankruptcy of illiquid companies in recessions. According to Amihud (2002), I also had reason to expect a correlation between the risk of bankruptcy and the size effect. Based on these rationales, I expected both the bid-ask spread and the turnover factor to be negative in 2008 – consistent with the findings from the October 1987 crash (Amihud, Mendelson, & Wood, 1990).

Contrary to my beliefs, the results of the cross-sectional regressions in 2008, the time-series regressions and the correlation matrix suggest that stocks with wide bid-ask spreads, provide higher excess returns when consumption is low. Provided that the positive premium is not a result of my limited sample size – causing some companies extreme performance to dominate the average returns, I find no evidence of a flight-to-liquidity in recessions as Acharya and Pedersen (2005) suggests.

The results appear inconsistent with both intuition and literature. In theory, I should have observed a reversed premium as a natural response to the findings of a positive premium in bull markets (Brennan & Subrahmanyam, 1996; Amihud & Mendelson, 1986; Datar, Naik, & Radcliffe, 1998; Keim & Madhavan, 1996). However, I find little research actually controlling whether this negative premium takes place in recessions. Although equilibrium theory demands an opposite and equally large cost (Cochrane, 2001), this does not imply that it is found in the form of lower excess returns. Perhaps researchers have, wrongfully, assumed that the observed liquidity premium from turnover and bid-ask spread implicate a harder punishment of illiquid assets when markets plummet. My results suggest they might have. However, a positive liquidity premium in market crashes is inconsistent with the results Amihud found in the 1987 October crash (Amihud, Mendelson, & Wood, 1990) – which indicated that liquid stock outperformed the illiquid stocks in the collapse and was explained as a flight-to-liquidity.

The main difference between my results and the results of Amihud, Mendelson and Wood is that no macro-economic recessions followed the crisis of October 1987. This suggests that the events of October 1987 were more likely a case of panic in the marketplace, rather than a miss-estimation of underlying growth or changes in consumption. The authors themselves argue that this was the first time the stock market tested liquidity limitations, and that the cause of the crash was realization of the illiquidity of the market place. One explanation could therefore be that the 1987- crash was a correction, where illiquid assets had, over time, received to high premium due to the increase in market liquidity. My results suggest that the market has learned and that the events of 2008 were not a case of flight-to-liquidity, but a more rational response to the state of the economy.

Nevertheless, I recognize the results of Amihud, Mendelson and Wood as one of the most important weaknesses of my results. In support of their case, it could be argued that my Norwegian dataset is biased by the fact that this market is heavily weighted in favor of the energy sector (Næs, Skjeltorp, & Ødegaard, 2007). If liquidity is priced differently in different sectors, my results would not account for an overweight of certain sectors. It could for example be argued that oil companies are more capital intensive and more dependent on the possibility of equity offerings. This would suggest that the OSEBX is a poor sample to draw generic conclusions from. Although this is a weakness that should be adjusted for in further research, it is somewhat reduced by my choice of using equally weighted company returns.

A possible explanation of the absence of the flight-to-liquidity could be that market makers hold illiquid stock because they don't have a liquidation need, even in recessions. Assuming that stocks were sold at fire-sale prices in the fall of 2008, this may not have been the case for stocks held by long-term investors. As discussed in the theory chapter; if investors are expected to be rational, they will not own illiquid assets which they are forced to sell at fire-sale prices in recessions. My results indicate that owners of illiquid assets are able, and willing, to hold their assets through recessions. Due to the fact that the separation between liquid an illiquid assets is not a binary state, this balance is continuous. However, my findings suggest that traders, on average, act rationally and decide the level of liquidity of their assets based on their own ability to outlast recessions.

The theory about market makers with long term investment horizons sounds plausible. However, if a market maker liquidity premium is paid in recessions, equilibrium theory states that the premium has an equally large cost. In theory, this could be average negative returns in the years before the collapse, but I don't find any significant negative premium in the other cross-sectional periods. This could be a result of my limited dataset, unable to observe these small negative premiums over time. However, a positive bid-ask spread premium has been observed in longer time series – also in the Norwegian market (Næs, Skjeltorp, & Ødegaard, 2007). A more plausible explanation is therefore that the costs faced by market makers are the increased transaction costs and opportunity costs of illiquid assets faced also outside recessions. These costs would not be reflected in excess returns, but would still be real costs for the market makers. Alternatively, the market makers could be rewarded the premium in recessions for facing information risk (O'Hara, 2003; Easley, Hvidkjær, & O'Hara, 2002) – caused by asymmetrical access to information in the marketplace.

I find support for the market maker theory as an interesting byproduct of the tests on the Fama French factors. As mentioned in the theory chapter, the rationale behind the value effect is that companies with low price-to-book ratios are leveraged and cyclical – causing them to be vulnerable to a fall in consumption (Zhang, 2005). However, from a liquidity point-of-view, companies with high price-to-book ratios are growth companies that might be held by investors with long term intentions. This theory coincides with the large negative correlation between the spread factor and the value factor. However, the two factors are able to co-exist without reducing one another, suggesting that neither factor is a perfect measure of the flight-to-liquidity risk and opportunity costs or that both also explain other risks.

What about the risk of bankruptcy? So far I have discussed the results of the bid-ask spread, neglecting that the turnover factor remains insignificant in the cross-sectional sample. Due to the disappointing turnover results, I will not be able to consider whether it might explain bankruptcy risk. My suggested explanation on how market makers avoid fire-sales in recessions might explain the wrongful assumption about the flight-to-liquidity, but does not explain the fact that some companies will underperform as a consequence of lowered ability to attract funding. My theory of balancing the observed positive illiquidity premium observed in 2008 with opportunity costs also leaves the premium observed in bull markets by Amihud and Mendelson (1986), Datar, Naik and Radcliffe (1998), and Brennan and Subrahmanyam (1996), unexplained. The missing negative premium, intended to compensate for bankruptcy risk, may be a result of my small sample size or that the Norwegian market was not affected as hard as other markets by the credit crunch. I do, however, suggest another explanation.

Although the collapse was in 2008, the economic consequences of the crisis were first noticeable after the crisis. Based on this rationale, my expectation of a negative turnover premium in 2008 might be premature. Perhaps I should expect to find lower returns on illiquid stocks in the post-2008 period. Although none of the liquidity-factors are significant in this period, the results of the cross-sectional tests show a large negative size factor in the post-2008 regression.

Consistent with Amihud (2002) – one of the arguments behind the size effect is that small companies run a larger risk of bankruptcy in recessions and should be rewarded a premium as they, on average, have lower returns when marginal utility is high and consumption is low. In the light of my earlier speculations about the multiple dimensions of liquidity, this could be interpreted as size being a proxy for a dimension of liquidity, not accounted for by the bid-ask spread. This is also consistent with the co-existence of size and spread found in tests performed on the Norwegian stock market (Næs, Skjeltorp, & Ødegaard, 2007). This explanation of the size-effect coincides with market efficiency because the premium is paid as a rational compensation for an illiquidity costs. An interesting byproduct of this theory could be an explanation of the weakening of the size-effect observed since its discovery in 1980 – as it would correlate with the increase in internet trading from around this period (Amihud, 2000).

If the size factor accounts for the illiquidity risk of being unable to perform equity offerings in recessions and the spread factor represent the market maker premium of holding illiquid assets due to opportunity costs, this leaves the problem of explaining the strong correlation between the spread and the size factor. Both the time-series tests of the spread portfolios and the correlation matrix suggest that these factors explain similar effects. However, the result of the two factors plotted orthogonal to each other (figure 6) show a large spread between the returns on small and large companies, occurring independently of the spread. Although this might suggest that the spread is a proxy for the size effect, I find that the outperformance of portfolios with wide spread occurs independently of the market cap in recessions. This support my theory that the spread and the size factor have different ability to explain asset returns. These results also support my suggestion of the multidimensional liquidity risks and the several proxies are needed to fully explain the effects of liquidity.

5 Conclusion

The results of my research support O'Hara in her claim that asset pricing models need to be cast in broader terms to incorporate liquidity costs. I find significant liquidity factors that provide improved insight in the pricing of assets for both the CAPM and the Fama French three-factor model – confirming the first and rejecting the second hypothesis on the basis of my empirical results. Although the models' abilities to explain returns are only marginally improved, the byproducts of my findings could have large implications for our understanding of asset pricing. My results suggest that liquidity risk is multidimensional and that several factors explain several risks. However, some factors appear to price certain risks better than others.

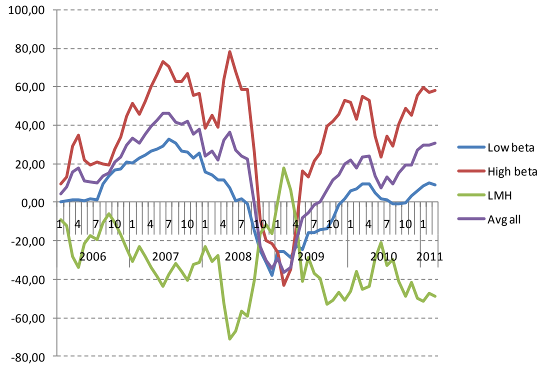
I explain the positive excess returns on stocks with high price-to-book ratios and wide bid-ask spread as a premium paid to market makers, holding illiquid assets through recessions – adding a liquidity based rationale behind the value effect (Reinganum, 1980; Fama & French, 1992). I balance this premium with the liquidity cost of the market maker in the shape of opportunity cost and transaction cost, not reflected in returns. The market marker premium is thus the ability to outperform the market in recessions – contradicting the flight-to-liquidity theory (Acharya & Pedersen, 2005; Amihud, 2002).

Further, my results suggest that company market cap is a better indicator of a funding risk premium than turnover and the bid-ask spread. The risk of lower funding ability in recessions leads to a cost in the period after the collapse, when the economy struggles and illiquid companies go bankrupt. This cost is well captured by the market capitalization of companies and the bid-ask spread in bull markets (Basu, 1977; Fama & French, 1992; Amihud & Mendelson, 1986), but according to my observation better explained by the size factor, alone, post-recessions.

Although my liquidity factors don't appear to be the holy grail of asset pricing models, they have provided a contribution to the understanding of how liquidity affects asset pricing. Further research should look into the positive premium awarded to market makers in recessions by assessing how different liquidity factors performed in other crisis. I also recommend researchers to look for a correlation between value and liquidity, as well as size and liquidity, across different sectors. Finally, I recommend my approach of looking for reversed premiums in recessions, regardless of the factor in question, as it provides insight beyond average returns in long term time-series.

Appendix

An overview of the portfolios returns over the period



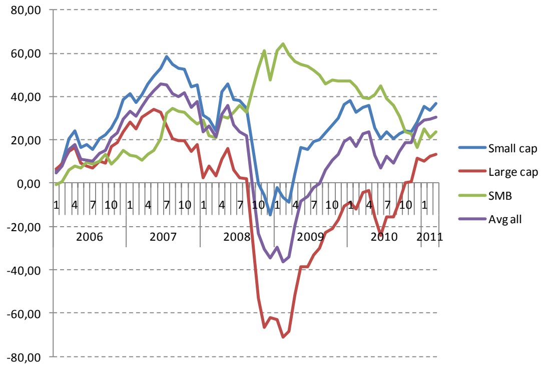
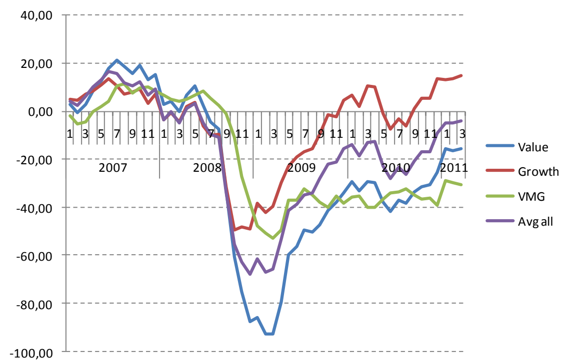


Figure 7 (left): Accumulated monthly return from equally weighted portfolios with high and low beta respectfully.

Figure 8 (right): Accumulated simple average of equally weighted portfolios with high and low size, name respectively large cap and small cap



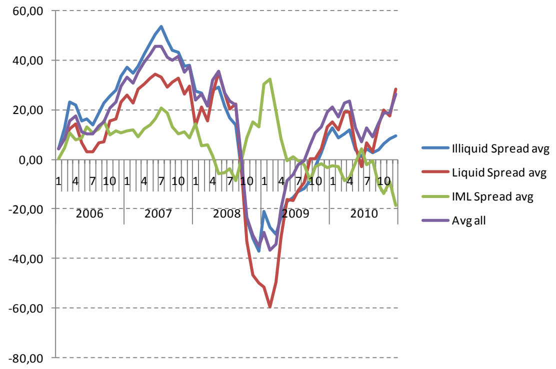
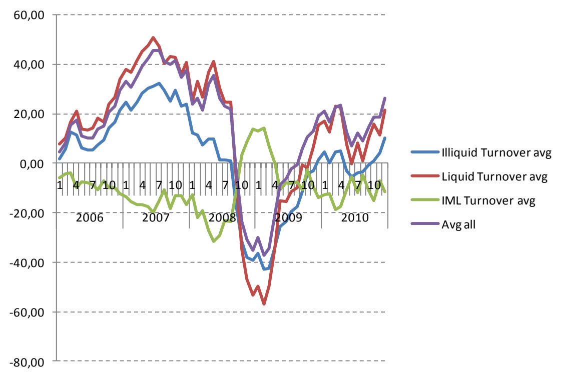
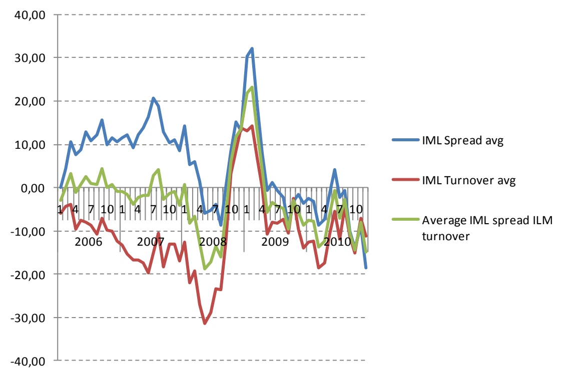


Figure 9 (left): Accumulated simple average of equally weighted portfolios with low and high price-to-book ratio, named respectively Value and Growth.

Figure 10 (right):Accumulated simple average of equally weighted portfolios with high and low bid-ask spread, name respectively Illiquid Spread avg and Liquid Spread avg.





Figur 11 (left): Accumulated simple average of equally weighted portfolios with high and low turnover, name respectively Illiquid Turnover avg and Liquid Turnover avg.

Figure 12 (right): Accumulated difference in holding illiquid and liquid portfolios if stock, where portfolios are rebalanced yearly based on the previous 12 month bid ask spread and turnover. The data is from the period 2006 – 2010.

Results of cross-sectional regression tests

Table 12: Cross- sectional tests. Entire period 2006-2011

2006-2011	alpha	ERP	WMN	IML	SMB	VMG
CAPM	-0,44	0,93				
p-value	*0,52*	*0,40*				
CAPM + WMG	-0,74	1,24	0,28			
p-value	*0,32*	*0,28*	*0,60*			
CAPM + IML	-0,47	0,97		-0,15		
p-value	*0,46*	*0,37*		*0,78*		
CAPM + 2LIQ	-0,89	1,39	0,27	-0,22		
p-value	*0,20*	*0,21*	*0,61*	*0,69*		
FF	-0,30	0,80			-0,77	-0,17
p-value	*0,64*	*0,46*			*0,19*	*0,67*
FF + WMN	-0,61	1,11	0,38		-0,85	0,01
p-value	*0,39*	*0,33*	*0,46*		*0,16*	*0,99*
FF + IML	-0,34	0,83		-0,09	-0,78	-0,18
p-value	*0,57*	*0,43*		*0,86*	*0,18*	*0,66*
FF + 2LIQ	-0,74	1,24	0,37	-0,20	-0,88	0,00
p-value	*0,27*	*0,26*	*0,47*	*0,71*	*0,14*	*0,99*

Table 13: Cross-sectional test. Pre-crisis 2006-2008

Pre-crisis	alpha	ERP	WMN	LMH	SMB	VMG
CAPM	0,09	1,50				
p-value	*0,92*	*0,09*				
CAPM + WMG	0,00	1,59	-0,21			
p-value	*1,00*	*0,08*	*0,78*			
CAPM + LMH	-0,15	1,75		0,21		
p-value	*0,88*	*0,05*		*0,72*		
CAPM + 2LIQ	-0,43	2,02	-0,23	0,16		
p-value	*0,64*	*0,03*	*0,76*	*0,79*		
FF	0,15	1,43			-0,03	1,00
p-value	*0,87*	*0,13*			*0,96*	*0,06*
FF + WMN	-0,26	1,86	0,14		-0,14	1,24
p-value	*0,76*	*0,06*	*0,87*		*0,83*	*0,04*
FF + LMH	-0,07	1,67		0,13	-0,08	0,95
p-value	*0,94*	*0,09*		*0,82*	*0,90*	*0,06*
FF + 2LIQ	-0,67	2,28	0,11	-0,03	-0,23	1,23
p-value	*0,50*	*0,03*	*0,91*	*0,95*	*0,72*	*0,04*

Table 14: Cross-sectional test. Crisis 2008 – mid 2009.

Crisis (2008)	alpha	ERP	WMN	LMH	SMB	VMG
CAPM	-2,62	-2,70				
p-value	*0,28*	*0,51*				
CAPM + WMG	-4,02	-1,28	2,96			
p-value	*0,17*	*0,78*	*0,04*			
CAPM + LMH	-1,67	-3,68		-1,59		
p-value	*0,44*	*0,33*		*0,43*		
CAPM + 2LIQ	-2,90	-2,43	3,02	-1,80		
p-value	*0,25*	*0,56*	*0,04*	*0,40*		
FF	-2,66	-2,67			-0,67	-2,68
p-value	*0,24*	*0,51*			*0,76*	*0,03*
FF + WMN	-3,44	-1,87	2,67		-0,87	-2,22
p-value	*0,22*	*0,68*	*0,04*		*0,70*	*0,04*
FF + LMH	-1,66	-3,68		-1,34	-0,47	-2,48
p-value	*0,37*	*0,31*		*0,50*	*0,83*	*0,04*
FF + 2LIQ	-2,27	-3,07	2,77	-1,50	-0,61	-2,20
p-value	*0,33*	*0,44*	*0,04*	*0,48*	*0,78*	*0,05*

Table 15: Cross-sectional test. Post-crisis mid 2009-2011

Post-crisis	alpha	ERP	WMN	LMH	SMB	VMG
CAPM	0,28	2,43				
p-value	*0,76*	*0,13*				
CAPM + WMG	0,38	2,32	-0,76			
p-value	*0,64*	*0,11*	*0,39*			
CAPM + LMH	-0,11	2,83		0,32		
p-value	*0,91*	*0,09*		*0,64*		
CAPM + 2LIQ	-0,21	2,93	-0,79	0,30		
p-value	*0,82*	*0,07*	*0,37*	*0,63*		
FF	0,58	2,12			-1,54	0,10
p-value	*0,51*	*0,17*			*0,06*	*0,88*
FF + WMN	0,63	2,07	-0,67		-1,53	0,07
p-value	*0,44*	*0,14*	*0,45*		*0,06*	*0,91*
FF + LMH	0,15	2,56		0,40	-1,63	0,02
p-value	*0,87*	*0,12*		*0,57*	0,05	*0,98*
FF + 2LIQ	0,06	2,65	-0,72	0,38	-1,65	0,06
p-value	*0,95*	*0,09*	*0,41*	*0,55*	*0,04*	*0,93*

Results of time-series regression tests

Beta

Table 16: Time-series regression, beta portfolios, CAPM

Beta	R^2	α	β
Portfolio 1	0,30	0,00	0,40
p-value		*0,99*	*0,00*
Portfolio 2	0,75	-0,39	0,65
p-value		*0,23*	*0,00*
Portfolio 3	0,86	-0,19	0,93
p-value		*0,57*	*0,00*
Portfolio 4	0,93	0,24	1,17
p-value		*0,40*	*0,00*
Portfolio 5	0,85	0,19	1,86
p-value		*0,77*	*0,00*
W	**0,01**		
p	**1,00**		

Table 17: Time-series regression, beta portfolios, CAPM with turnover factor

Beta	R^2	α	β	l
Portfolio	0,29	-0,08	0,45	0,19
p-value		*0,89*	*0,00*	*0,32*
Portfolio	0,76	-0,38	0,62	-0,14
p-value		*0,26*	*0,00*	*0,23*
Portfolio	0,86	-0,30	0,94	0,05
p-value		*0,38*	*0,00*	*0,45*
Portfolio	0,93	0,00	1,21	0,17
p-value		*0,41*	*0,00*	*0,11*
Portfolio	0,86	0,40	1,77	-0,36
p-value		*0,56*	*0,00*	*0,15*
W	**0,52**			
p	**0,76**			

Table 18: Time-series regression, beta portfolios, CAPM with bid-ask spread factor

Beta	R^2	α	β	w
Portfolio 1	0,32	0,01	0,48	0,30
p-value		*0,98*	*0,00*	*0,09*
Portfolio 2	0,77	-0,39	0,71	0,25
p-value		*0,22*	*0,00*	*0,02*
Portfolio 3	0,87	-0,18	1,00	0,29
p-value		*0,56*	*0,00*	*0,01*
Portfolio 4	0,93	0,23	1,11	-0,22
p-value		*0,40*	*0,00*	*0,02*
Portfolio 5	0,87	0,18	1,70	-0,64
p-value		*0,77*	*0,00*	*0,00*
W	**0,03**			
p	**1,00**			

Table 19: Time-series regression, beta portfolios, CAPM with turnover factor and bid-ask spread factor

Beta	R^2	α	β	l	w
Portfolio 1	0,33	-0,10	0,56	0,28	0,37
p-value		*0,85*	*0,00*	*0,16*	*0,05*
Portfolio 2	0,78	-0,40	0,69	-0,08	0,24
p-value		*0,22*	*0,00*	*0,47*	*0,03*
Portfolio 3	0,87	-0,33	1,03	0,12	0,31
p-value		*0,31*	*0,00*	*0,30*	*0,01*
Portfolio 4	0,93	0,25	1,15	0,12	-0,19
p-value		*0,37*	*0,00*	*0,23*	*0,05*
Portfolio 5	0,88	0,46	1,54	-0,54	-0,77
p-value		*0,47*	*0,00*	*0,02*	*0,00*
W	**0,01**				
p	**1,00**				

Table 20: Time-series regression, beta portfolios, Fama French three-factor model

Beta	R^2	α	β	v	s
Portfolio 1	0,43	0,06	0,50	0,01	0,53
p-value		*0,90*	*0,00*	*0,97*	*0,00*
Portfolio 2	0,76	-0,42	0,68	-0,28	0,02
p-value		*0,18*	*0,00*	*0,03*	*0,78*
Portfolio 3	0,86	-0,19	0,97	-0,21	0,13
p-value		*0,54*	*0,00*	*0,10*	*0,13*
Portfolio 4	0,94	0,22	1,12	0,09	-0,23
p-value		*0,40*	*0,00*	*0,41*	*0,00*
Portfolio 5	0,87	0,19	1,73	0,43	-0,48
p-value		*0,75*	*0,00*	*0,08*	*0,01*
W	**0,13**				
p	**0,98**				

Table 21: Time-series regression, beta portfolios, Fama French three-factor model with bid-ask spread factor

Beta	R^2	α	β	v	s	w
Portfolio 1	0,45	0,07	0,55	0,09	0,51	0,27
p-value		*0,88*	*0,00*	*0,64*	*0,00*	*0,11*
Portfolio 2	0,77	-0,41	0,72	-0,22	0,01	0,20
p-value		*0,19*	*0,00*	*0,08*	*0,92*	*0,07*
Portfolio 3	0,87	-0,18	1,02	-0,13	0,12	0,25
p-value		*0,55*	*0,00*	*0,30*	*0,18*	*0,03*
Portfolio 4	0,94	0,21	1,08	0,03	-0,21	-0,19
p-value		*0,41*	*0,00*	*0,78*	*0,00*	*0,04*
Portfolio 5	0,88	0,17	1,62	0,27	-0,44	-0,54
p-value		*0,78*	*0,00*	*0,26*	*0,01*	*0,01*
W	**0,15**					
p	**0,98**					

Table 22: Time-series regression, beta portfolios, Fama French three-factor model with turnover factor

Beta	R^2	α	β	v	s	l
Portfolio	0,45	0,07	0,57	0,02	0,58	0,25
p-value		*0,88*	*0,00*	*0,92*	*0,00*	*0,15*
Portfolio	0,78	-0,44	0,65	-0,29	0,00	-0,12
p-value		*0,17*	*0,00*	*0,02*	*0,97*	*0,29*
Portfolio	0,87	-0,33	0,99	-0,24	0,13	0,08
p-value		*0,33*	*0,00*	*0,07*	*0,18*	*0,52*
Portfolio	0,94	0,20	1,15	0,08	-0,22	0,14
p-value		*0,46*	*0,00*	*0,43*	*0,01*	*0,16*
Portfolio	0,88	0,38	1,61	0,47	-0,51	-0,44
p-value		*0,55*	*0,00*	*0,06*	*0,01*	*0,06*
W	**0,51**					
p	**0,77**					

Table 23: Time-series regression, beta portfolios, Fama French three-factor model with turnover factor and bid ask spread factor

Beta	R^2	α	β	v	s	l	w
Portfolio	0,48	0,06	0,66	0,11	0,56	0,32	0,34
p-value		*0,89*	*0,00*	*0,55*	*0,00*	*0,07*	*0,05*
Portfolio	0,79	-0,45	0,70	-0,23	-0,01	-0,08	0,19
p-value		*0,16*	*0,00*	*0,07*	*0,91*	*0,48*	*0,09*
Portfolio	0,88	-0,33	1,06	-0,16	0,11	0,13	0,26
p-value		*0,30*	*0,00*	*0,21*	*0,23*	*0,26*	*0,02*
Portfolio	0,94	0,21	1,11	0,04	-0,21	0,11	-0,16
p-value		*0,45*	*0,00*	*0,73*	*0,01*	*0,28*	*0,10*
Portfolio	0,90	0,40	1,44	0,28	-0,47	-0,58	-0,66
p-value		*0,50*	*0,00*	*0,24*	*0,01*	*0,01*	*0,00*
W	**0,12**						
p	**0,99**						

Size

Table 22: Time-series regression, size portfolios, CAPM

Size	R^2	α	β
Portfolio	0,64	0,44	0,88
p-value		*0,44*	*0,00*
Portfolio	0,78	-0,29	0,97
p-value		*0,51*	*0,00*
Portfolio	0,86	0,26	1,00
p-value		*0,46*	*0,00*
Portfolio	0,85	0,08	0,96
p-value		*0,81*	*0,00*
Portfolio	0,87	-0,45	1,18
p-value		*0,25*	*0,00*
W	**0,12**		
p	**0,99**		

Table 23: Time-series regression, size portfolios, CAPM with bid-ask spread factor

Size	R^2	α	β	w
Portfolio	0,64	0,45	0,92	0,18
p-value		*0,43*	*0,00*	*0,35*
Portfolio	0,78	-0,28	0,99	0,11
p-value		*0,52*	*0,00*	*0,45*
Portfolio	0,85	0,26	0,99	-0,03
p-value		*0,46*	*0,00*	*0,78*
Portfolio	0,85	0,08	0,99	0,12
p-value		*0,81*	*0,00*	*0,29*
Portfolio	0,89	-0,46	1,09	-0,35
p-value		*0,21*	*0,00*	*0,01*
W	**1,64**			
p	**0,17**			

Table 24: Time-series regression, size portfolios, CAPM with turnover factor

Size	R^2	α	β	l
Portfolio	0,69	0,61	0,72	-0,63
p-value		*0,27*	*0,00*	*0,00*
Portfolio	0,80	-0,44	1,05	0,36
p-value		*0,31*	*0,00*	*0,02*
Portfolio	0,86	0,25	1,02	0,08
p-value		*0,50*	*0,00*	*0,11*
Portfolio	0,85	0,00	0,97	0,01
p-value		*0,86*	*0,00*	*0,93*
Portfolio	0,88	-0,44	1,22	0,15
p-value		*0,27*	*0,00*	*0,28*
W	**4,40**			
p	**0,00**			

Table 25: Time-series regression, size portfolios, CAPM with bid-ask spread factor and turnover factor

Size	R^2	α	β	l	w
Portfolio	0,69	0,61	0,75	-0,61	0,08
p-value		*0,27*	*0,00*	*0,00*	*0,68*
Portfolio	0,81	-0,45	1,11	0,41	0,18
p-value		*0,30*	*0,00*	*0,01*	*0,21*
Portfolio	0,85	0,25	1,01	0,07	-0,02
p-value		*0,50*	*0,00*	*0,59*	*0,89*
Portfolio	0,85	0,06	1,00	0,04	0,13
p-value		*0,88*	*0,00*	*0,76*	*0,31*
Portfolio	0,89	-0,41	1,12	0,08	-0,33
p-value		*0,27*	*0,00*	*0,58*	*0,01*
W	**0,20**				
p	**0,96**				

Table 25: Time-series regression, size portfolios, Fama French three-factor model

Size	R^2	α	β	v	s
Portfolio	0,72	0,49	1,00	-0,09	0,60
p-value		*0,33*	*0,00*	*0,65*	*0,00*
Portfolio	0,81	-0,26	1,04	-0,06	0,37
p-value		*0,53*	*0,00*	*0,69*	*0,00*
Portfolio	0,86	0,26	0,96	0,12	-0,11
p-value		*0,45*	*0,00*	*0,38*	*0,28*
Portfolio	0,86	0,05	0,92	-0,04	-0,24
p-value		*0,87*	*0,00*	*0,75*	*0,01*
Portfolio	0,93	-0,50	1,07	0,07	-0,56
p-value		*0,09*	*0,00*	*0,55*	*0,00*
W	**0,23**				
p	**0,95**				

Table 27: Time-series regression, size portfolios, Fama French three-factor model with bid-ask spread factor

Size	R^2	α	β	v	s	w
Portfolio	0,71	0,50	1,02	-0,06	0,59	0,11
p-value		*0,33*	*0,00*	*0,78*	*0,00*	*0,54*
Portfolio	0,81	-0,25	1,05	-0,04	0,36	0,07
p-value		*0,54*	*0,00*	*0,79*	*0,00*	*0,65*
Portfolio	0,85	0,26	0,97	0,12	-0,11	0,00
p-value		*0,46*	*0,00*	*0,40*	*0,28*	*0,97*
Portfolio	0,87	0,06	0,95	0,00	-0,25	0,15
p-value		*0,86*	*0,00*	*0,97*	*0,01*	*0,20*
Portfolio	0,94	-0,51	1,01	-0,02	-0,54	-0,30
p-value		*0,06*	*0,00*	*0,84*	*0,00*	*0,00*
W	**0,16**					
p	**0,98**					

Table 26: Time-series regression, size portfolios, Fama French three-factor model with turnover

Size	R^2	α	β	v	s	l
Portfolio	0,76	0,74	0,85	-0,04	0,55	-0,56
p-value		*0,14*	*0,00*	*0,83*	*0,00*	*0,00*
Portfolio	0,84	-0,36	1,15	-0,08	0,42	0,41
p-value		*0,37*	*0,00*	*0,60*	*0,00*	*0,00*
Portfolio	0,85	0,26	0,98	0,12	-0,09	0,06
p-value		*0,50*	*0,00*	*0,40*	*0,38*	*0,65*
Portfolio	0,86	-0,01	0,92	-0,06	-0,26	-0,01
p-value		*0,97*	*0,00*	*0,66*	*0,01*	*0,92*
Portfolio	0,93	-0,56	1,09	0,05	-0,56	0,09
p-value		*0,06*	*0,00*	*0,64*	*0,00*	*0,39*
W	**3,69**					
p	**0,01**					

Table 27: Time-series regression, size portfolios, Fama French three-factor model with turnover factor and bid-ask spread factor

Size	R^2	α	β	v	s	l	w
Portfolio 1	0,75	0,74	0,85	-0,04	0,55	-0,56	0,01
p-value		*0,14*	*0,00*	*0,85*	*0,00*	*0,00*	*0,94*
Portfolio 2	0,84	-0,36	1,19	-0,04	0,41	0,44	0,13
p-value		*0,37*	*0,00*	*0,79*	*0,00*	*0,00*	*0,34*
Portfolio 3	0,85	0,26	0,99	0,13	-0,09	0,07	0,02
p-value		*0,50*	*0,00*	*0,40*	*0,38*	*0,64*	*0,88*
Portfolio 4	0,86	-0,02	0,95	-0,02	-0,27	0,02	0,15
p-value		*0,96*	*0,00*	*0,91*	*0,01*	*0,88*	*0,23*
Portfolio 5	0,94	-0,56	1,02	-0,03	-0,54	0,03	-0,29
p-value		*0,05*	*0,00*	*0,80*	*0,00*	*0,75*	*0,01*
W	**0,38**						
p	**0,86**						

Value

Table 28: Time-series regression, value portfolios, CAPM

Value	R^2	α	β
Portfolio	0,86	-0,36	1,10
p-value		*0,44*	*0,00*
Portfolio	0,82	-0,24	0,91
p-value		*0,58*	*0,00*
Portfolio	0,88	-0,14	1,01
p-value		*0,71*	*0,00*
Portfolio	0,88	-0,14	1,01
p-value		*0,71*	*0,00*
Portfolio	0,78	0,12	0,90
p-value		*0,81*	*0,00*
W	**0,16**		
p	**0,98**		

Table 29: Time-series regression, value portfolios, CAPM with bid-ask spread factor

Value	R^2	α	β	w
Portfolio	0,87	-0,44	1,00	-0,38
p-value		*0,32*	*0,00*	*0,02*
Portfolio	0,83	-0,30	0,84	-0,26
p-value		*0,49*	*0,00*	*0,09*
Portfolio	0,88	-0,12	1,04	0,12
p-value		*0,76*	*0,00*	*0,39*
Portfolio	0,84	0,11	0,88	0,12
p-value		*0,77*	*0,00*	*0,40*
Portfolio	0,80	0,21	1,00	0,41
p-value		*0,66*	*0,00*	*0,02*
W	**0,21**			
p	**0,96**			

Table 30: Time-series regression, value portfolios, CAPM with turnover factor

Value	R^2	α	β	l
Portfolio	0,87	-0,59	1,11	0,09
p-value		*0,21*	*0,00*	*0,54*
Portfolio	0,82	-0,32	0,92	0,08
p-value		*0,50*	*0,00*	*0,59*
Portfolio	0,89	-0,24	1,00	-0,03
p-value		*0,54*	*0,00*	*0,12*
Portfolio	0,84	0,00	0,84	-0,07
p-value		*0,55*	*0,00*	*0,59*
Portfolio	0,78	0,15	0,92	0,07
p-value		*0,77*	*0,00*	*0,69*
W	**0,03**			
p	**1,00**			

Table 31: Time-series regression, value portfolios, CAPM with turnover factor and bid-ask spread factor

Value	R^2	α	β	w	l
Portfolio	0,88	-0,62	1,00	0,02	-0,36
p-value		*0,16*	*0,00*	*0,87*	*0,02*
Portfolio	0,83	-0,34	0,85	0,03	-0,25
p-value		*0,46*	*0,00*	*0,82*	*0,12*
Portfolio	0,89	-0,23	1,04	0,00	0,13
p-value		*0,56*	*0,00*	*0,98*	*0,37*
Portfolio	0,84	0,25	0,87	-0,05	0,10
p-value		*0,54*	*0,00*	*0,70*	*0,50*
Portfolio	0,80	0,20	1,05	0,15	0,43
p-value		*0,70*	*0,00*	*0,37*	*0,02*
W	**0,49**				
p	**0,78**				

Table 32: Time-series regression, value portfolios, Fama French three-factor model

Value	R^2	α	β	v	s
Portfolio	0,92	-0,14	1,09	0,68	0,24
p-value		*0,70*	*0,00*	*0,00*	*0,01*
Portfolio	0,85	-0,10	0,92	0,36	0,20
p-value		*0,81*	*0,00*	*0,02*	*0,06*
Portfolio	0,89	-0,23	1,01	-0,27	-0,11
p-value		*0,53*	*0,00*	*0,05*	*0,27*
Portfolio	0,85	-0,02	0,83	-0,22	-0,17
p-value		*0,95*	*0,00*	*0,10*	*0,08*
Portfolio	0,83	0,03	0,96	-0,67	0,07
p-value		*0,94*	*0,00*	*0,00*	*0,54*
W	**0,04**				
p	**1,00**				

Table 33: Time-series regression, value portfolios, Fama French three-factor model with bid-ask spread factor

Value	R^2	α	β	v	s	w
Portfolio	0,92	-0,18	1,05	0,62	0,25	-0,17
p-value		*0,60*	*0,00*	*0,00*	*0,01*	*0,20*
Portfolio	0,85	-0,14	0,88	0,30	0,21	-0,17
p-value		*0,73*	*0,00*	*0,06*	*0,05*	*0,28*
Portfolio	0,89	-0,22	1,02	-0,26	-0,11	0,03
p-value		*0,55*	*0,00*	*0,08*	*0,27*	*0,83*
Portfolio	0,85	-0,01	0,85	-0,20	-0,18	0,05
p-value		*0,98*	*0,00*	*0,16*	*0,08*	*0,71*
Portfolio	0,84	0,08	1,01	-0,60	0,06	0,19
p-value		*0,85*	*0,00*	*0,00*	*0,58*	*0,25*
W	**0,07**					
p	**1,00**					

Table 34: Time-series regression, value portfolios, Fama French three-factor model with turnover factor

Value	R^2	α	β	v	s	l
Portfolio	0,92	-0,27	1,09	0,65	0,20	0,06
p-value		*0,47*	*0,00*	*0,00*	*0,04*	*0,64*
Portfolio	0,85	-0,07	0,94	0,36	0,22	0,07
p-value		*0,87*	*0,00*	*0,02*	*0,06*	*0,60*
Portfolio	0,90	-0,44	0,99	-0,31	-0,16	-0,02
p-value		*0,25*	*0,00*	*0,02*	*0,10*	*0,89*
Portfolio	0,85	0,08	0,82	-0,20	-0,17	-0,07
p-value		*0,84*	*0,00*	*0,15*	*0,11*	*0,58*
Portfolio	0,84	0,00	1,01	-0,67	0,10	0,14
p-value		*1,00*	*0,00*	*0,00*	*0,41*	*0,34*
W	**0,27**					
p	**0,93**					

Table 35: Time-series regression, value portfolios, Fama French three-factor model with turnover factor and bid-ask spread factor

Value	R^2	α	β	v	s	l	w
Portfolio 1	0,92	-0,30	1,05	0,59	0,20	0,03	-0,16
p-value		*0,42*	*0,00*	*0,00*	*0,04*	*0,80*	*0,23*
Portfolio 2	0,85	-0,10	0,90	0,31	0,22	0,05	-0,16
p-value		*0,81*	*0,00*	*0,06*	*0,05*	*0,73*	*0,33*
Portfolio 3	0,90	-0,43	1,00	-0,30	-0,16	-0,01	0,03
p-value		*0,26*	*0,00*	*0,04*	*0,11*	*0,92*	*0,83*
Portfolio 4	0,85	0,09	0,83	-0,19	-0,17	-0,07	0,04
p-value		*0,83*	*0,00*	*0,21*	*0,12*	*0,62*	*0,79*
Portfolio 5	0,84	0,04	1,07	-0,59	0,09	0,18	0,22
p-value		*0,92*	*0,00*	*0,00*	*0,43*	*0,24*	*0,19*
W	**0,17**						
p	**0,97**						

Bid-ask spread

Table 36: Time-series regression bid-ask spread portfolios, CAPM

Spread	R^2	α	β
Portfolio 1	0,72	-0,19	0,73
p-value		*0,64*	*0,00*
Portfolio 2	0,83	-0,27	0,88
p-value		*0,45*	*0,00*
Portfolio 3	0,83	0,57	1,01
p-value		*0,17*	*0,00*
Portfolio 4	0,89	-0,36	1,24
p-value		*0,36*	*0,00*
Portfolio 5	0,87	0,29	1,18
p-value		*0,48*	*0,00*
W	**0,14**		
p	**0,98**		

Table 37: Time-series regression, bid-ask spread portfolios, CAPM with bid-ask spread factor

Spread	R^2	α	β	w
Portfolio 1	0,78	-0,18	0,85	0,51
p-value		*0,62*	*0,00*	*0,00*
Portfolio 2	0,84	-0,27	0,92	0,17
p-value		*0,45*	*0,00*	*0,15*
Portfolio 3	0,83	0,57	1,02	0,06
p-value		*0,17*	*0,00*	*0,64*
Portfolio 4	0,90	-0,36	1,16	-0,32
p-value		*0,33*	*0,00*	*0,01*
Portfolio 5	0,90	0,28	1,06	-0,51
p-value		*0,43*	*0,00*	*0,00*
W	**0,27**			
p	**0,93**			

Table 38: Time-series regression, bid-ask spread portfolios, CAPM with turnover factor

Spread	R^2	α	β	l
Portfolio 1	0,73	-0,08	0,66	-0,28
p-value		*0,84*	*0,00*	*0,06*
Portfolio 2	0,84	-0,36	0,93	0,23
p-value		*0,31*	*0,00*	*0,07*
Portfolio 3	0,83	0,62	0,97	-0,13
p-value		*0,14*	*0,00*	*0,00*
Portfolio 4	0,89	0,00	1,26	0,10
p-value		*0,32*	*0,00*	*0,49*
Portfolio 5	0,87	0,25	1,20	0,10
p-value		*0,54*	*0,00*	*0,50*
W	**3,94**			
p	**0,00**			

Table 39: Time-series regression, bid-ask spread portfolios, CAPM with turnover factor and bid-ask spread factor

Spread	R^2	α	β	l	w
Portfolio 1	0,78	-0,12	0,80	-0,17	0,48
p-value		*0,75*	*0,00*	*0,21*	*0,00*
Portfolio 2	0,85	-0,38	1,00	0,28	0,23
p-value		*0,27*	*0,00*	*0,03*	*0,05*
Portfolio 3	0,83	0,62	0,99	-0,12	0,04
p-value		*0,14*	*0,00*	*0,43*	*0,78*
Portfolio 4	0,90	-0,37	1,17	0,02	-0,31
p-value		*0,33*	*0,00*	*0,87*	*0,02*
Portfolio 5	0,90	0,29	1,05	-0,02	-0,52
p-value		*0,42*	*0,00*	*0,85*	*0,00*
W	**0,05**				
p	**1,00**				

Table 40: Time-series regression, bid-ask spread portfolios, Fama French three-factor model

Spread	R^2	α	β	v	s
Portfolio 1	0,79	-0,03	0,80	0,14	0,46
p-value		*0,93*	*0,00*	*0,30*	*0,00*
Portfolio 2	0,87	-0,17	0,95	-0,01	0,36
p-value		*0,60*	*0,00*	*0,95*	*0,00*
Portfolio 3	0,85	0,48	1,04	-0,42	-0,03
p-value		*0,23*	*0,00*	*0,01*	*0,81*
Portfolio 4	0,91	-0,39	1,16	0,27	-0,31
p-value		*0,27*	*0,00*	*0,06*	*0,00*
Portfolio 5	0,93	0,13	1,07	0,03	-0,57
p-value		*0,68*	*0,00*	*0,83*	*0,00*
W	**0,07**				
p	**1,00**				

Table 41: Time-series regression, bid-ask spread portfolios, Fama French three-factor model with bid-ask spread factor

Spread	R^2	α	β	v	s	w
Portfolio 1	0,86	0,00	0,91	0,30	0,42	0,53
p-value		*1,00*	*0,00*	*0,01*	*0,00*	*0,00*
Portfolio 2	0,87	-0,16	0,97	0,03	0,35	0,14
p-value		*0,62*	*0,00*	*0,78*	*0,00*	*0,21*
Portfolio 3	0,84	0,48	1,03	-0,43	-0,02	-0,03
p-value		*0,23*	*0,00*	*0,01*	*0,83*	*0,83*
Portfolio 4	0,92	-0,40	1,11	0,19	-0,29	-0,24
p-value		*0,24*	*0,00*	*0,16*	*0,00*	*0,05*
Portfolio 5	0,95	0,10	0,97	-0,12	-0,53	-0,48
p-value		*0,68*	*0,00*	*0,24*	*0,00*	*0,00*
W	**0,00**					
p	**1,00**					

Table 42: Time-series regression, bid-ask spread portfolios, Fama French three-factor model with turnover factor

Spread	R^2	α	β	v	s	l
Portfolio 1	0,80	0,07	0,74	0,16	0,45	-0,24
p-value		*0,85*	*0,00*	*0,23*	*0,00*	*0,06*
Portfolio 2	0,88	-0,27	1,02	-0,03	0,38	0,27
p-value		*0,38*	*0,00*	*0,81*	*0,00*	*0,02*
Portfolio 3	0,84	0,52	1,01	-0,41	-0,03	-0,11
p-value		*0,20*	*0,00*	*0,01*	*0,76*	*0,46*
Portfolio 4	0,91	-0,41	1,17	0,26	-0,31	0,05
p-value		*0,26*	*0,00*	*0,06*	*0,00*	*0,72*
Portfolio 5	0,93	0,12	1,08	0,02	-0,57	0,04
p-value		*0,72*	*0,00*	*0,85*	*0,00*	*0,75*
W	**0,13**					
p	**0,99**					

Table 43: Time-series regression, bid-ask spread portfolios, Fama French three-factor model with turnover factor and bid-ask spread factor

Spread	R^2	α	β	v	s	l	w
Portfolio 1	0,86	0,05	0,86	0,31	0,41	-0,14	0,50
p-value		*0,86*	*0,00*	*0,01*	*0,00*	*0,20*	*0,00*
Portfolio 2	0,88	-0,28	1,07	0,03	0,36	0,31	0,20
p-value		*0,36*	*0,00*	*0,81*	*0,00*	*0,01*	*0,06*
Portfolio 3	0,84	0,52	1,00	-0,43	-0,03	-0,12	-0,05
p-value		*0,20*	*0,00*	*0,01*	*0,79*	*0,43*	*0,71*
Portfolio 4	0,91	-0,40	1,11	0,19	-0,29	0,00	-0,24
p-value		*0,25*	*0,00*	*0,17*	*0,00*	*0,97*	*0,06*
Portfolio 5	0,95	0,13	0,95	-0,12	-0,54	-0,07	-0,49
p-value		*0,62*	*0,00*	*0,25*	*0,00*	*0,48*	*0,00*
W	**0,36**						
p	**0,88**						

Turnover

Table 44: Time-series regression, turnover portfolios, CAPM

Turnover	R^2	α	β
Portfolio 1	0,75	-0,35	0,68
p-value		*0,33*	*0,00*
Portfolio 2	0,85	0,08	0,77
p-value		*0,79*	*0,00*
Portfolio 3	0,84	-0,32	0,99
p-value		*0,41*	*0,00*
Portfolio 4	0,83	0,62	1,22
p-value		*0,21*	*0,00*
Portfolio 5	0,87	-0,08	1,28
p-value		*0,86*	*0,00*
W	**0,52**		
p	**0,76**		

Table 45: Time-series regression, turnover portfolios, CAPM with bid-ask spread factor

Turnover	R^2	α	β	w
Portfolio 1	0,75	-0,35	0,72	0,15
p-value		*0,33*	*0,00*	*0,20*
Portfolio 2	0,85	0,08	0,80	0,10
p-value		*0,79*	*0,00*	*0,33*
Portfolio 3	0,86	-0,31	1,08	0,37
p-value		*0,39*	*0,00*	*0,00*
Portfolio 4	0,83	0,62	1,21	-0,05
p-value		*0,22*	*0,00*	*0,76*
Portfolio 5	0,90	-0,09	1,15	-0,54
p-value		*0,82*	*0,00*	*0,00*
W	**1,40**			
p	**0,24**			

Table 46: Time-series regression, turnover portfolios, CAPM with turnover

Turnover	R^2	α	β	l
Portfolio 1	0,76	-0,45	0,74	0,25
p-value		*0,20*	*0,00*	*0,05*
Portfolio 2	0,87	-0,05	0,85	0,32
p-value		*0,86*	*0,00*	*0,00*
Portfolio 3	0,85	-0,40	1,04	0,21
p-value		*0,30*	*0,00*	*0,97*
Portfolio 4	0,86	0,00	1,09	-0,55
p-value		*0,07*	*0,00*	*0,00*
Portfolio 5	0,87	0,00	1,23	-0,20
p-value		*1,00*	*0,00*	*0,22*
W	**2,89**			
p	**0,02**			

Table 47: Time-series regression, turnover portfolios, CAPM with turnover factor and bid-ask spread factor

Turnover	R^2	α	β	l	w
Portfolio 1	0,77	-0,47	0,81	0,30	0,22
p-value		*0,18*	*0,00*	*0,02*	*0,07*
Portfolio 2	0,88	-0,06	0,90	0,36	0,17
p-value		*0,82*	*0,00*	*0,00*	*0,07*
Portfolio 3	0,87	-0,44	1,17	0,31	0,43
p-value		*0,22*	*0,00*	*0,02*	*0,00*
Portfolio 4	0,86	0,85	1,04	-0,59	-0,17
p-value		*0,07*	*0,00*	*0,00*	*0,28*
Portfolio 5	0,91	0,04	1,05	-0,34	-0,61
p-value		*0,91*	*0,00*	*0,02*	*0,00*
W	**0,49**				
p	**0,78**				

Table 48: Time-series regression, turnover portfolios, Fama French three-factor model

Turnover	R^2	α	β	v	s
Portfolio 1	0,74	-0,30	0,68	0,13	0,07
p-value		*0,40*	*0,00*	*0,35*	*0,50*
Portfolio 2	0,85	0,08	0,79	-0,07	0,06
p-value		*0,79*	*0,00*	*0,53*	*0,48*
Portfolio 3	0,87	-0,24	1,07	-0,12	0,36
p-value		*0,49*	*0,00*	*0,38*	*0,00*
Portfolio 4	0,83	0,59	1,22	-0,08	-0,03
p-value		*0,24*	*0,00*	*0,67*	*0,82*
Portfolio 5	0,90	-0,17	1,18	0,16	-0,44
p-value		*0,67*	*0,00*	*0,29*	*0,00*
W	**0,32**				
p	**0,90**				

Table 49: Time-series regression, turnover portfolios, Fama French three-factor model with bid-ask spread factor

Turnover	R^2	α	β	v	s	w
Portfolio 1	0,75	-0,29	0,72	0,19	0,05	0,19
p-value		*0,41*	*0,00*	*0,19*	*0,60*	*0,13*
Portfolio 2	0,85	0,08	0,81	-0,05	0,05	0,08
p-value		*0,78*	*0,00*	*0,69*	*0,53*	*0,45*
Portfolio 3	0,88	-0,23	1,14	-0,02	0,33	0,32
p-value		*0,50*	*0,00*	*0,86*	*0,00*	*0,01*
Portfolio 4	0,82	0,59	1,21	-0,10	-0,03	-0,07
p-value		*0,25*	*0,00*	*0,61*	*0,85*	*0,69*
Portfolio 5	0,92	-0,20	1,08	0,01	-0,40	-0,49
p-value		*0,58*	*0,00*	*0,92*	*0,00*	*0,00*
W	**0,56**					
p	**0,73**					

Table 50: Time-series regression, turnover portfolios, Fama French three-factor model with turnover factor

Turnover	R^2	α	β	v	s	l
Portfolio 1	0,76	-0,40	0,75	0,11	0,09	0,25
p-value		*0,26*	*0,00*	*0,41*	*0,39*	*0,05*
Portfolio 2	0,87	-0,05	0,88	-0,10	0,08	0,33
p-value		*0,85*	*0,00*	*0,36*	*0,29*	*0,00*
Portfolio 3	0,88	-0,35	1,14	-0,14	0,37	0,26
p-value		*0,32*	*0,00*	*0,29*	*0,00*	*0,04*
Portfolio 4	0,85	0,81	1,08	-0,04	-0,07	-0,55
p-value		*0,09*	*0,00*	*0,82*	*0,61*	*0,00*
Portfolio 5	0,90	-0,07	1,11	0,18	-0,46	-0,26
p-value		*0,85*	*0,00*	*0,23*	*0,00*	*0,07*
W	**8,18**					
p	**0,00**					

Table 51: Time-series regression, turnover portfolios, Fama French three-factor model with turnover factor and bid-ask spread factor

Turnover	R^2	α	β	v	s	l	w
Portfolio 1	0,77	-0,41	0,81	0,18	0,07	0,31	0,25
p-value		*0,24*	*0,00*	*0,18*	*0,48*	*0,02*	*0,04*
Portfolio 2	0,88	-0,06	0,92	-0,05	0,07	0,36	0,15
p-value		*0,84*	*0,00*	*0,62*	*0,36*	*0,00*	*0,12*
Portfolio 3	0,90	-0,36	1,24	-0,03	0,35	0,34	0,39
p-value		*0,27*	*0,00*	*0,82*	*0,00*	*0,00*	*0,00*
Portfolio 4	0,85	0,81	1,03	-0,09	-0,05	-0,59	-0,19
p-value		*0,09*	*0,00*	*0,61*	*0,68*	*0,00*	*0,26*
Portfolio 5	0,93	-0,06	0,97	0,02	-0,42	-0,38	-0,57
p-value		*0,86*	*0,00*	*0,88*	*0,00*	*0,00*	*0,00*
W	**1,10**						
p	**0,37**						

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