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# Moving from traditional to responsive mathematics classrooms: a proposition of an intervention model

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**ABSTRACT**

This article discusses how mathematics didactics can be inspired by and further developed through responsive pedagogy, understood as feedback directed at self-regulation and self-efficacy, in mathematics teaching. The authors explain the rationale behind an intervention model for improving mathematics teachers' feedback practices. The model is a developmental framework for intervention that is context dependent, rather than a fixed model for intervention. The overall aim with such an intervention is to establish a recursive feedback dialogue between teachers and students. Next, the backdrop for the participatory approach used in the development and implementation of such an intervention model is presented, emphasising the importance of researchers and teachers working closely together and engaging in mutual learning. The aim is to develop understanding and a shared language when discussing responsive pedagogy. Finally, the article explains the implementation process of the described model. The results of the implementation will be presented in forthcoming articles.

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Teacher learning; mathematics; responsive pedagogy; intervention model

## Introduction

Traditional beliefs about teaching and learning mathematics seem to have an impressive ability to survive. Two decades ago, in 'Why Teach Mathematics?', Ernest (1998) confronted several myths about learning mathematics. Among these were beliefs that learning mathematics depends on talent, and that mathematics is something unapproachable, difficult and abstract. Pehkonen (2003) highlighted studies in which he identified beliefs about mathematics in school to be solely about finding the correct answer, and that the teacher's role was to transfer mathematical knowledge and then ensure that the pupils had learnt this knowledge. These descriptions of attitudes towards mathematics teaching are, to a certain degree, somewhat caricatured and oversimplified, but nevertheless quite easily identified in mathematics classrooms around the world (Klette 2003; Mellin-Olsen 1996; Murphy 2015; Topphol 2012). This, by no means, suggests that mathematics teachers are more resistant to change than other teachers, but research shows that school mathematics, to a greater extent than other school subjects, has preserved recognisable teacher and student practice and interaction patterns for centuries (Clarke 1997; Eikrem et al. 2012).

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On several occasions, mathematics classrooms have been selected as a context for intervention-based research projects (e.g. Harel 2013). These include studies specifically related to mathematical topics (e.g. Stylianides and Stylianides 2014), as well as studies on more general educational issues where mathematics classrooms were chosen as the context for the study (e.g. Ottmar et al. 2015; Stanulis, Little, and Wibbens 2012; Wylie and Lyon 2015). There may be a number of reasons for focusing on the mathematics classroom as an arena for research. One may be related to the distinctive character of the subject of mathematics; another reason may involve the preserved patterns of teacher and student practice and interaction in school mathematics.

The purpose with this article is to present and discuss how mathematics didactics can be inspired and further developed through responsive pedagogy, understood as feedback directed at self-regulation and self-efficacy, in mathematics teaching. Further, we explain the rationale behind an intervention model for improving mathematics teachers' feedback practices. The model is a developmental framework for intervention that is context dependent, rather than a fixed model for intervention. We use the planning and organisation of the RespMath1 project as an example when discussing the theoretical foundation of the intervention model. The intervention emphasises teachers engaging in mutual learning with researchers, building on their own experiences of how to practise responsive pedagogy. Hence, the explicit issue discussed in this article is the rationale and design of an intervention model that supports mathematics teachers in moving from a traditional to a responsive mathematics classroom. Forthcoming articles report on the results from the implementation of the intervention model in the RespMath project.

The RespMath project (40 teachers, 10 principals and 7 researchers participated) examined how and to what extent teachers' practice of responsive pedagogy affects learning, understood as self-regulation, self-efficacy and achievements in mathematics. The context for the investigation was ninth-grade mathematics teachers and their mathematics classrooms. Responsive pedagogy is understood as a recursive dialogue between the student's internal feedback and external feedback provided by the teacher and peers (Smith et al. 2016). When learners are given an assignment, they start an internal dialogue with regard to what the assignment is about, how to go about it, and will they succeed in completing it. Responsive pedagogy is about how teachers tap into the learner's internal dialogue, and respond to it conducting a recursive dialogue. The explicit intention with teachers practising responsive pedagogy is to prepare for the learning society of tomorrow by educating independent self-regulated learners, to help learners develop belief in their own competence and ability successfully to complete assignments and meet challenges (self-efficacy), and to strengthen their overall self-concept (Smith et al. 2016).

## Responsive pedagogy meets mathematics didactics

Smith et al. (2016) explain responsive pedagogy in relation to three main concepts: formative feedback, self-regulation and self-efficacy. Zimmerman (2000) refers to self-regulation as self-generated thoughts, feelings and behaviours that are oriented towards attaining goals, according to which there are three cyclic phases:

(1) Forethought: the reflective processes that precede action.
(2) Performance/volitional control: the self-monitoring taking place while working on a task.
(3) Self-reflection: assessment processes taking place when the task is completed.

The research literature documents a positive relationship between self-regulation and achievement, which is also seen over a long-term perspective. Zimmerman (2000) and Boud (2014) claim that education should strive to empower students to become independent and self-regulated learners.

Self-efficacy is about students' belief in their ability to organise and execute the course of action needed to learn and perform behaviours at designated levels (Bandura 1995). A person's experience of self-efficacy differs in relation to physical and social contexts (Bandura 1995; Zimmerman 2000). Thus, a student could experience different levels of self-efficacy in different subjects. According to Bandura (1997), there are three factors related to self-efficacy that determine human agency at different levels in different contexts (Bandura 1997):

(1) Personal factors such as cognition.
(2) Affect and biological events.
(3) Behaviour and environmental influences.

Regarding self-regulation, several studies document a close relationship between self-efficacy and academic achievement (van Dinther, Dochy, and Segers 2010), as well as between self-efficacy and self-regulation (Zimmerman 2000). Productive formative feedback requires an educational encounter between teacher and students, focusing on responding to students' learning through a feedback dialogue. Black and Wiliam (2009) argue that this involves exploiting moments of contingency during teaching processes. This means that student learning changes direction depending on assessment of student feedback to the teacher, and the teacher's response to the students' questions, misconceptions and actions. In other words, there is deviation from the teaching plan when needed. This means, that for teachers, it is paradoxically important to be prepared for the unprepared to be able to improvise their teaching and set themselves apart from detailed teaching plans when students report back on their own learning needs (Sawyer 2011). Butler and Winne (1995) illustrate the interplay between self-regulation and feedback. Their basic assumption is that the way a student perceives a task is strongly connected to his/her prior domain, strategy knowledge and multiple motivational beliefs (self-efficacy). The self-regulated learner monitors his/her own learning process and progress according to personal goals and criteria (Smith et al. 2016). Self-assessment is considered to be an inherent feature of self-regulation (Andrade 2010). Smith et al. (2016) emphasise that the challenge for the teacher is how to elicit the student's internal feedback, that is, his/her self-assessment, and how to make use of this in forming the self-regulation process to enhance learning. Thus, working on teachers' feedback skills should be a central part of an intervention focusing on teacher learning for a more responsive pedagogy-based mathematics classroom.

### *Teaching and learning mathematics characterised by dialogue and interaction*

The distinctive character of mathematics is recognised by its striving for generality (Kahane 1998). Generalisation, and thereby demands for a proof that shows that the general result counts for all special cases, is a basic feature of mathematics. This feature is challenged on a regular basis; by that we mean that mathematics' striving for generality is constantly challenged through the desire for specific solutions or answers. Often students want their teacher to tell them exactly what they are supposed to do when facing a mathematical problem. They are waiting for a fixed algorithm or approach to solve the problem, perhaps with tacit reference to one or several of the myths identified by Ernest (1998).

The mathematics teacher has to pave the way for students' development of understanding that specificity can never replace generality, and that the general will always contain the specific. In the mathematics classroom, it is therefore important to make students direct their own knowledge-based understanding towards generality. At the same time, the students' focus on specificity should be maintained and strengthened. The teacher has to keep in mind that students have different strengths and challenges regarding the learning of mathematics. Therefore, the mathematics teacher needs to pay attention to the students' own perspectives and their beliefs about learning mathematics, to identify and exploit emerging moments of contingency (Black and Wiliam 2009). The quality of the teacher's feedback practice can provide a necessary bridge for the students between specificity and generality.

Sfard (1998) distinguishes between the metaphors *acquisition* and *participation* as alternative learning theory representations. She points to a learning environment that gives priority to explanation, instruction, activity and exploration and at the same time emphasises dimensions such as sharing and students' influence on choice of teaching approaches as elements of successful learning. Sfard (1998) argues that the *acquisition* metaphor represents the fundamental interpretation of knowledge as accumulation and revision of concepts, which are 'combined to form ever richer cognitive structures' (5) where learning is based on acquisition either by development or construction. This means that knowledge may be transferred to some extent to new contexts or shared with others. The *participation* metaphor represents a fundamental interpretation of participative action as a condition for learning, that is, an action where a continuous learning process related to the context frames the ongoing learning activity or activities. The learning can then be seen as the socialisation process of the student into 'a certain community' (6), from being an apprentice to becoming a vital contributor in the community of learners. In light of responsive pedagogy, this basis for the teaching of mathematics utilises the power of feedback (Hattie and Timperley 2007), stimulates the student's internal feedback and expectations, and builds cultures for external feedback processes involving both the teacher, and students and their peers. In other words, responsive pedagogy promotes the use of formative feedback processes in the classroom, the stimulation of students' self-regulation development, and the establishment of students' positive thoughts about their self-efficacy in mathematics.

A responsive pedagogy approach to teaching the subject of mathematics demands that students' participation is a strongly prioritised part of what goes on in the classroom. In accordance with Dweck's (2008) emphasis on a willingness to step out of one's comfort zone and see the value of struggle, setbacks and criticism, an openness to questions, alternative approaches and joint analysis of the content to be learnt must be emphasised. This demands that the students become visible in explaining and understanding mathematics and that they are given the opportunity to organise and explain the mathematical content to be learnt. Consequently, an intervention model that includes an encounter between responsive pedagogy and mathematics didactics needs to emphasise teachers' attention to increased student participation in the mathematics classroom. Motivated students who plan and explain mathematical content, orally and in writing for peers or others, experience a strengthened emphasis of participation in the mathematics classroom. Subsequently, it can be assumed that students develop a clearer impression of their own self-regulation and self-efficacy regarding mathematics.

Within the social constructivist perspective, the interaction of social and cultural norms influences the construction of knowledge. The students' actions represent key elements for learning and offer a communicative basis that paves the way for their experience of increased impact on the learning of mathematics. This influence is recognised through a change in their participation in learning activities. The social constructivist perspective views social factors such as collaboration and conversation as more than a frame around the learning situation. They are important in creating a classroom climate that is suitable for learning mathematics (Cobb 2007). Cobb and Yackel (1996) refer to two perspectives related to a social constructivist approach, namely the social perspective and the psychological perspective. With the myths from Ernest (1998) and Pehkonen (2003) in mind, as well as the possibility of giving priority to both acquisition and participation (Sfard 1998), the mathematics classroom contains social norms for what can be said and done and ideas about one's own and others' roles in the classroom. A process-oriented mathematics classroom with a climate for discussion, cooperative work, feedback dialogue between student and teacher, and students' genuine experience of influence differs from a product-oriented mathematics classroom focusing on individual work and silence on the premises. This aligns with Dweck's (2008) identification of a growth mindset and a fixed mindset, and which of these two the teacher wants the students to develop. Hence, the impressions of what represents a good mathematical activity and what constitutes a good question, differ from classroom to classroom.

In summary, the aim of the intervention model presented below is to strengthen mathematics teachers' feedback practice focusing on formative feedback, self-regulation and self-efficacy. The intervention model in the RespMath project addresses Sfard's (1998) suggestion of a combination of teaching perspectives, heeds Cobb and Yackel's (1996) awareness of differences between mathematics classrooms, and is supported by Dweck's (2008) theory on mindsets. Teaching and learning mathematics based on dialogue and interaction demands that the teacher fosters students' involvement in what happens in the classroom. Hence, this article argues that the key to develop teachers' attention to feedback which focuses on students' self-regulation and self-efficacy in the mathematics classroom, is student participation, time and space available for discussion, alternative teaching approaches, and a climate that promotes understanding, curiosity and interest.

## Participatory design approach

In the RespMath project, the conservative acquisition-based perspective on how to organise the mathematics classroom was challenged. When designing the intervention, we decided to start where the teachers are, to avoid the possibility that they would find the suggested practices meaningless and impossible to implement in their contexts. Darling-Hammond and McLaughlin (2011) note that competence develops in what can be called informal work-based learning arenas such as experience-based learning, rather than in organised courses and education beyond qualification offered from 'experts'. However, not all kinds of experience lead to learning. The adaptation of impressions where the connections between theory and practice are reflected upon is necessary (Krawec and Montague 2014). If teachers have the possibility to influence the way competence development should be organised, in addition to its content and direction, to make it match their actual needs, then learning outcomes tend to be better than if the framework and content are decided externally (Timperley et al. 2007). Teachers' professional learning is embedded in professional development opportunities that have the features of 'learning communities', in which teachers learn from and with their colleagues in communities of practice (Wenger 1998). Through collaborative, ongoing reflective practice, teachers' professional learning moves teachers from isolation to interaction and personal engagement in communities in which they can analyse and improve their own practice (Shulman 2004). We therefore claim that the best way to promote interaction between participants in the mathematics classroom, and to improve classroom practices further, is through involving teachers in embodying developmental processes in their own classroom practices (Smith and Engelsen 2012; Timperley et al. 2007). Such processes need to be initiated by each teacher with his/her current classroom practice as the starting point when participating in such an intervention.

Because the intervention was supposed to involve teachers suggesting change in some way, the planning in the RespMath project paid close attention to the participating teachers' professional development. This development aimed to be sustained, ongoing and include participant-driven enquiry, reflection and experimentation (Darling-Hammond and McLaughlin 2011). Based on a thorough review of the literature on teachers' professional learning, Timperley et al. (2007) provide research-based arguments for combining and balancing teacher engagement and external experts in teachers' professional development. An important success factor is to emphasise that teachers must have the opportunity to participate in a community of learners, and to experience having an impact on the developmental process provided by the intervention.

Among a number of studies that combine the use of intervention and a focus on the development of teacher practice in mathematics, Swafford et al. (1999) report on a three-year project that combined teachers' formal learning through education beyond qualification, work-based courses, reflective writing and discussion groups in an intervention programme. The aim of Swafford et al.'s study was to support teachers' professional competence in mathematics. Their findings show that a combination of theoretical knowledge, collaboration and reflection contributed to increased professional competence if the teachers had enough time for the necessary change processes to take place. Kiemer et al. (2015) applied a video-based teacher professional development intervention on productive classroom discourse to influence students' motivation to learn mathematics and science. The intervention led teachers to provide a significant increase in constructive feedback and a decrease in simple feedback. Moreover, as a result of the change in feedback, pre- and post-tests showed that students significantly increased their perceived autonomy, competence and intrinsic learning motivation. Furthermore, an intervention study by Wylie and Lyon (2015) examined the implementation of formative assessment. Over a two-year span, the researchers conducted workshops with all the participating teachers and a teacher representative ('teacher leader') from each school. In addition, the teacher leaders facilitated monthly school-based learning community meetings where teachers had the opportunity to reflect on and obtain feedback on their formative assessment practice. These reflections deepened their knowledge of formative assessment strategies and provided the grounds for planning the implementation of formative assessment during the next month. An earlier study on the same issue by Simon and Schifter (1991) focused on student learning. Their intervention study was designed to stimulate teachers' development of a constructivist view of learning to serve as a basis for their instructional decision-making. The results indicate that the intervention influenced teachers' beliefs about learning, which in turn affected the decisions they made in the classroom. Core elements in their intervention were the construction of meaning-based lessons for the teachers followed by group discussions among teachers facilitated by a research group member, and learning through facilitated discussions of videotaped interviews and follow-up of each teacher in his/her teaching by an external facilitator. At the same time, the teachers also attended workshops where they shared implementation efforts, analysis of student learning and misconceptions, and together planned new mathematics lessons.

The common denominator in the four intervention projects outlined above is the aim of promoting teachers' professional development to improve students' learning by situating the teachers' collaborative learning activities in their day-to-day routine work. Thereby, change in teaching practice becomes a collective rather than an individual responsibility (Pedder and Darleen Opfer 2013). Lee and Wiliam (2005) remind the research community that the design structures of teacher professional development should also be sufficiently strong to afford a flexibility that takes into account the differences between teachers. To illustrate one of the pitfalls that might occur, Krawec and Montague (2014), in an intervention study on improving students' problem-solving in mathematics, concluded that a design without emphasis on teachers' both formal and informal professional development sets the teachers up to fail. Based on feedback from the teachers in the study, they state that lack of support throughout an intervention will influence the teachers' confidence, since they will not have been enough involved in the development of the intervention to feel at ease with the instructional methodology core in the intervention. Research indicates that teachers' professional development depends on explicit time, real possibilities for participation from the teacher and active leadership, combined with collaborative learning arenas with possibilities for reflection on theoretical knowledge connected to practical experiences (Timperley et al. 2007). Such conditions are naturally not restricted to mathematics teachers. Several studies support the view that effective teacher competence development continues for a long time in communities of practice (Darling-Hammond 2013), and in intervention projects that are not particularly related to school mathematics (Lee and Wiliam 2005; Stanulis, Little, and Wibbens 2012). However, in a meta-analysis on intervention studies at primary and secondary school level focusing on self-regulated learning among students, Dignath and Büttner (2008) found that interventions attained higher effects when conducted in the scope of mathematics than in other school subjects. Hence, on occasions the distinctive character of mathematics becomes an impact factor in itself (Murphy 2015), and it may be necessary to take this into account when planning an intervention involving mathematics and mathematics teachers.

## Intervention design, model and rationale in the RespMath project

When planning the intervention for the RespMath project, based on the rationales presented above, the RespMath project group defined three aspects as crucial conditions for an influential implementation in the classroom:

- involving the participating teachers in designing the intervention;
- RespMath project group members' direct involvement during the intervention;
- sufficient intervention time span to foster progress.

### *Involving the participating teachers in designing the intervention*

Hoekstra et al. (2009) assert that four activities tend to be important for learning: learning through experiments, through research on personal practice, through exchange of ideas with colleagues and finally through practical experiences. Furthermore, studies by Timperley (2011) and Darling-Hammond (2013) show that teachers consider the classroom as an important learning arena for themselves. Kelchtermans and Ballet (2002) claim that if teachers are to learn from personal experiences in formal as well as informal contexts, the content of the competence development should be anchored in the teachers' personal needs.

The RespMath project group wanted the participating teachers and the researchers to develop the intervention design together, in a way that would work in accordance with the participatory design approach for teacher learning outlined above, emphasising the importance of researchers and teachers working closely together and engaging in mutual learning. Stenhouse (1983) regards theory and practice as intertwined. Accordingly, he warns against a process where researchers are responsible for the development of models and theories, while teachers instrumentally put them into practice. Stenhouse argues that the consequence of such development would be less professional teaching, and research is separated from the context it was supposed to influence. Another argument for involving participating teachers in the development of the content of the intervention, including the planning, organising and implementation, is given in the account by Zeichner and Klehr (1999) of teachers as researching practitioners. They show that teachers develop a more profound confidence in their own professional identity, and accordingly they regard themselves as learners rather than being fully educated; they see themselves as career-long learners. Furthermore, Zeichner and Klehr (1999) show that teachers should have the possibility to access relevant theoretical knowledge to be included as real members of the research community, and that it is necessary to include time for theoretical discussions and practical experiences as a substantial part of the research process.

Based on the results and experiences provided by previous research on teacher learning through interventions, the RespMath group emphasised that both formal and informal learning processes should be part of the intervention in the project. The formal learning processes, in which both the researchers and teachers participated, were organised as project meetings that consisted of lectures and group work. The teachers faced challenges students might experience in responsive pedagogy-based mathematics teaching. This included for instance how to provide feedback and how to discuss going about solving a mathematical problem. The teachers then implemented ideas they found useful in their own practice. These learning contributions represent a holistic view on teacher learning. Experiences that teachers gained in their own classrooms in between the formal learning activities, during their everyday practice, were examples of informal learning.

### *RespMath project group members' direct involvement during the intervention*

The RespMath project also found support in the research literature for offering support from an external facilitator in the teachers' planning and implementation of the intervention parts situated in their classrooms. Ponte et al. (2004) expected that through the intervention they planned, teachers' involvement could contribute to their personal professional development. Their findings showed that this knowledge was mainly developed at a practical action level. To develop knowledge and gain a deeper understanding of the reasons for the actions or the values that guided the actions, it was necessary to obtain support from an external facilitator. The facilitator's role was to assist the learning process in ways that could contribute to the teachers' explanations and understanding of practice. Ponte et al. (2004) made a distinction between external teaching support and support from an external facilitator and claimed that the latter was more effective. The external facilitator asked questions that could promote reflection instead of simply providing explanations. In the RespMath project intervention, the participating schools were clustered into five regions, and it was decided that each of the regions should have a member of the RespMath group as their external facilitator ('regional contact'). The regional contact followed the regional work, arranged regional workshops in the schools and visited the schools during the intervention.

### *Sufficient intervention time span to foster progress*

The intervention time span is also part of intervention models that focus on teachers' professional competence development. For instance, Swafford et al. (1999) concluded that the combination of theoretical knowledge, collaboration and reflection contributed to increased professional competence if the teachers had sufficient time for the necessary change processes required. The intervention in the RespMath project was scheduled to be carried out within a seven-month period (October 2016 to April 2017). Influenced by Swafford et al. (1999), the RespMath group organised the meeting points and intervals of classroom implementation in a way that promoted the teachers' formal and informal learning processes without challenging the teachers' need for time to implement change processes.

To sum up, central research findings indicate that teachers' competence development (teacher learning) depends on explicit time, real possibilities for participation from the teacher, active leadership and collaborative learning arenas with possibilities for reflection on theoretical knowledge connected to practical experiences. The RespMath group made

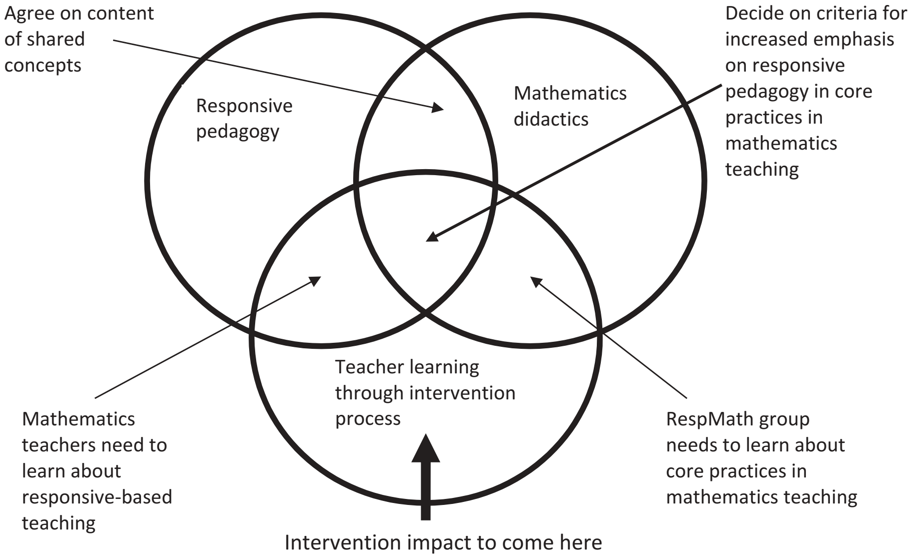


Figure 1. The intervention planning process.

these impact factors part of the intervention design and explicitly included the participating teachers in the identification of core practices related to the teaching of mathematics and in the identification of criteria related to teachers' feedback practice, students' self-regulation and students' self-efficacy for strengthened student motivation. Research supports that effective competence development continues for a long time in communities of practice (Darling-Hammond 2013). Hence, to foster an environment for teacher learning, the RespMath project made an effort to establish a learning community on an intersubjective level between the teachers involved and the RespMath group members. Both groups learnt from each other before the actual intervention in the classrooms could begin, and this process is illustrated in Figure 1.

The learning community was established through meetings between the RespMath group, principals and teachers participating in the project. This is described briefly in the next section.

## The RespMath intervention model: content and organisation

### *Step 1: Preparatory meeting with representatives from the schools*

To start the involvement of the schools in designing the intervention, the RespMath project group invited the 10 participating schools' 'team contact teachers' (TCs: a contact representative for the project from each of the 10 participating schools) to a meeting six months before the planned start of the intervention. The aim of this meeting was to establish a platform for the intervention, bridging responsive pedagogy and the teachers' pragmatic approach, as well as to discuss and confirm content, contact and practical issues regarding the intervention. During the meeting, the RespMath group informed the TCs about the project and the basic structures of the concept of responsive pedagogy.

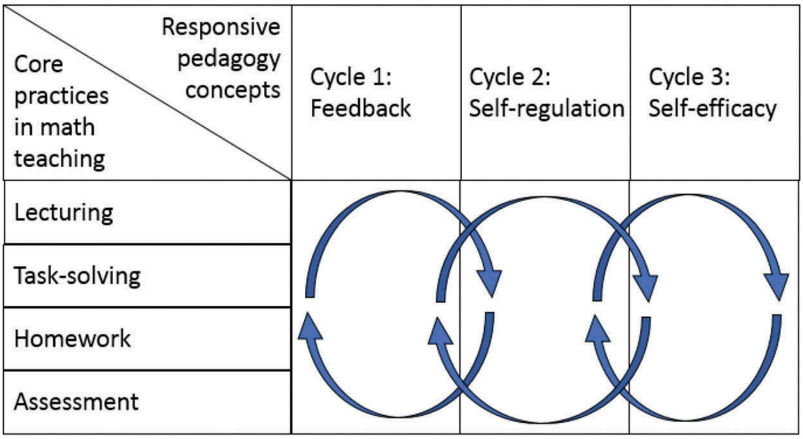


**Figure 2.** Main emphasis for each intervention cycle in the RespMath project.

After discussions and negotiations, the TCs decided on what they found to be the four most significant core practices in the ninth-grade mathematics classroom.

Based on the TCs' decisions on core practices, a matrix showing the materialisation of focused concepts from responsive pedagogy and mathematics teaching within three intervention cycles was developed comprising lecturing, task-solving, homework and assessment (see Figure 2).

Figure 2 illustrates that the teachers and researchers would focus on one responsive pedagogy concept in each cycle, related to four identified core practices. Experiences from Cycle 1 would be carried through to the work in Cycle 2, and experiences from Cycles 1 and 2 be carried through to the work in Cycle 3, in accordance with the definition of responsive pedagogy by Smith et al. (2016).

### *Step 2: Developing criteria for productive responsive pedagogy in mathematics classrooms and a structure for regional organised activities*

The first project meeting involving all 40 project teachers and the RespMath group (introducing Cycle 1) was dedicated to joint development of criteria for productive feedback in relation to the core practices in mathematics teaching. The active involvement of the teachers in this process was important to underpin and strengthen the common teacher learning aspect of the project. A common set of criteria was also important in order to create a manageable research design, where 10 different schools (cases) with different school cultures and practices were involved. An intervention design based on project meetings, regional organised seminars and locally initiated activities at each school would necessarily lead the schools to go in different directions. The operationalisation of the idea behind the intervention was implemented in diverse ways in the 40 classrooms. However, within the five regions, we expected a higher degree of similarity in the teachers' practices, coloured by the approach adopted by the regional contact from the RespMath group working within the specific region. To strengthen the common structure, the teachers and research group decided on an overall cycle structure for the regional meetings, as seen in Figure 3.

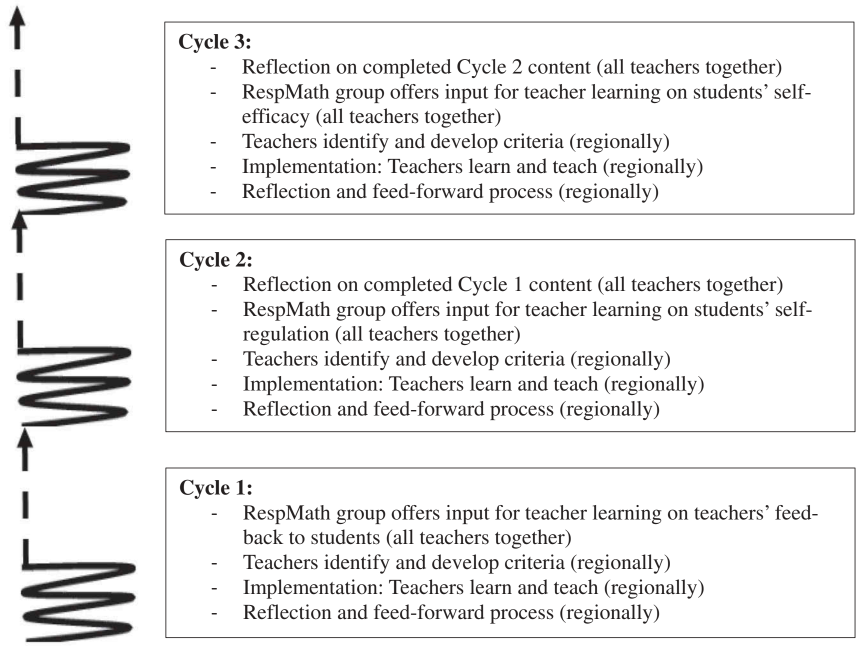


**Figure 3.** Intervention cycle structure for regional planning.

The structure displayed in Figure 3 shows the structure for the three intervention cycles. However, regarding content in the regional meetings (one meeting in each cycle), the detailed planning was left to the regional contact to decide together with the teachers in the region. To start each intervention cycle, all teachers were invited to join the research group in a two-day project meeting. These formal learning processes included both lectures and group work experiences related to the theme for the cycle at hand, and time for exchanging and discussing experiences from the previous cycle across schools and regions. These common project meetings provided opportunities for regional adjustment of interpretations of how responsive pedagogy might be enhanced in the identified core practices. A one-day project meeting with all the teachers after Cycle 3 is completed is also part of the intervention model, in order to reflect together on Cycle 3 content, and to close the intervention of the RespMath project.

## Conclusions

Classroom intervention can be chosen as a research tool to observe change and as a development tool for participants to develop a sustained change of practice. The learning that takes place is mutual, as both participants and researchers learn by planning, organising, implementing and noticing eventual changes. Research on classroom intervention (e.g. Swafford et al. 1999; Wylie and Lyon 2015) shows not only that achieving these goals requires a high level of engagement by all parties, but also that the introduced changes of practice have a more promising chance of being sustained (e.g. Lee and Wiliam 2005). Our claim is that the intervention planned and organised within the RespMath project provides an example of how a joint effort involving teachers, school leaders and researchers for teacher learning is recommended to be designed. The project found guidance for its intervention frame in previous research (e.g. Caroline and Lyon 2015). Teachers make changes in their teaching practice because they want to develop their own teaching, for the benefit of their students. In other words, for the mathematics teacher to become motivated for change processes in his/her own practice, as researchers we cannot simply state that the existing practice is not good enough, and that it ought to change. However, as researchers and collaborative partners, we can contribute in paving the way for improvement. The teacher can be encouraged to identify what may be changed and offered the opportunity to change it.

The main idea of the RespMath project is to identify and build bridges between theories on responsive pedagogy and mathematics didactics and practice through the joint efforts of teachers and researchers. The purpose of the project is to develop a more responsive pedagogy-based environment in the mathematics classroom, and to examine the impact on student learning. A main goal from the very beginning of the project has been to empower the involvement, influence and interest of the participating mathematics teachers. In the current article we have elaborated on the rationales behind an intervention model prior to describing the model. Our claim is that any intervention aimed at creating an encounter between responsive pedagogy and mathematics didactics needs to be characterised by a participatory approach in the planning and implementation. The intervention model draws on theoretical support presented in this article. Previous intervention-based research on teacher learning and teacher practice supports the argument that the proposed model is both viable and sustainable in developing teachers' beliefs and competence in practising responsive teaching in the mathematics classroom. Further research on the implementation of the suggested model is not only needed, some of it has also been conducted, and will be presented in forthcoming articles.

## Note

1. http://prosjektsider.hsh.no/respmath

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