The Relationship Between Prices and Inflow in Hydroelectric Scheduling

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Abstract— **We analyze the relationship between the system price in the Nordic electricity market and inflow to three different Norwegian power producers. Price and inflow are the main drivers of uncertainty in the hydroelectric scheduling problem and in principle their relationship needs to be captured for ensuring a high quality in the planning process. Our experiments indicate that regarding the value of revenues, the influence of the relationship between price and inflow is relatively minor. For the three power plants we studied, the difference in expected revenues from dependent and independent modeling of price and inflow is from 2-4%, on a two-year horizon.**

Keywords-reservoir operations; electricity markets; stochastic programming; state space models

I. INTRODUCTION

Scheduling the release of water from reservoirs in order to generate power must be done under uncertainty of electricity prices and inflow or runoff from upstream precipitation. In power systems with a significant share of hydropower, there is a relationship between the local inflow and the market price: both weather patterns and price patterns are regional, so local inflow is correlated with regional inflow, which is a significant factor explaining prices. In theory, hydroelectric scheduling should be performed respecting such relationships, i.e. planning models should include joint stochastic modeling of prices and inflow. In practice, this is challenging, and this paper contributes to the scheduling literature by investigating the effect of capturing such relationships versus ignoring them. As it turns out, for the three power plants we investigate, the effect is in the order of 2-4%, measuring the expected revenue on a two-year horizon, i.e. on a medium to long term scheduling scale. On the other hand, capturing the price-inflow correlation is important in risk management studies, since the variance of revenues is very much affected by the price-inflow correlation.

Hydroelectric scheduling is reviewed in [1] and later in [2], which focuses on implementations and with an emphasis on the US context. [3] reviews stochastic programming models in energy, and [4] the same, focusing on short term bidding and scheduling. Of particular interest to us is a situation where market power is of minor concern, and where the power plant owners perform the operations at their discretion. In short, the context of the problems we discuss is most relevant for the Scandinavian electricity market. [5] and [6] explain how scheduling is performed in practice using decision support models that includes stochastic models for price and inflow.

We develop models for inflow and spot price based on the factor models in [6]. In addition we explore an alternative based on state space modeling [7]. An analysis of the correlation between price and inflow is also conducted, and we suggest a set of intuitive approaches to match inflow and price. Fan scenarios are generated based on the factor and state space models, and these are subsequently reduced to scenario trees. A stochastic optimization model is then finally solved as a deterministic equivalent using linear programming [8].

The remainder of the article is organized as follows. Chapter II explains the data, Chapter III presents the price and inflow models and their estimation, and Chapter IV explains the results. Chapter V concludes.

II. DATA

A. Data context: The Nordic Power Market

Nord Pool ASA is the Nordic power exchange and consists of both a physical and a financial market. Nord Pool’s role in the power market is to provide a wholesale marketplace for electricity, where the electricity is traded between generators and users (such as industry) or distribution. The consumer market consists of electricity distributors who sell power to consumers. The differences between spot and consumer prices are due to different distribution models in the Nordic countries. It is the most liquid marketplace for electricity in Europe and accounts for 63 % of the total value of the Nordic regions power consumption.

The market for physical contracts is organized by Nord Pool Spot AS. This is an auction based market that trades electrical power contracts for each hour the following day. The physical market forms the basis for all trading in the Nordic power market, and sets the reference price in the financial market. Players at the physical market need to have an agreement with Nord Pool in advance in order to place bids. Bids from the individual producers are prepared and submitted to Nord Pool before 12.00, consisting of tables with the amount of energy wanted bought or sold at different market prices for the coming day. Market clearing is calculated from the total demand and sales bids for every hour which then constitute next day’s spot prices for the respective hours.

All the players at the exchange are linked to certain areas in the Nordic region depending on their geographical location. The different power producers have to report their buy/sales bids in the area where they are connected to the grid. In each area a unique spot price is developed as a reference price for the whole area. The reason why we get different prices in different areas is due to transmission restrictions, which prevents transfer of enough electric power as is demanded from an unconstrained market. These bottlenecks in the system generally create higher prices in deficit areas, and lower prices in surplus areas. The total system price, which also serves as the reference price in the financial market, is calculated without considering congestions and is the average of the 24 spot prices calculated for the respective day.

B. Data context: Hydropower scheduling

The objective of power generation scheduling can be defined as maximizing the value of operational cash flows subject to the physical constraints including reservoir balance, and reservoir and flow bounds. The water flows to the reservoirs at no cost and the variable cost of hydro production is very low, and is ignorable in this context. However, the amount of water available is limited and uncertain, and so the water has an opportunity cost. Production of 1 MWh more in the present period prevents the production of 1 MWh in a later period when prices might be higher. The marginal cost of the water, also known as the water value, is therefore dependent on both reservoir volume and production capacity in own system, inflow expectations to the reservoir and future spot market prices.

The Nordic power system has roughly 50% hydropower, and its scheduling is usually separated into a long, seasonal and short term planning [5]. Long term scheduling has a weekly resolution and a horizon of up to 5 years. For all but the very largest producers, a price taker assumption is employed, and prices are considered exogenous. Seasonal and short term scheduling employ separate models which contain more technically detailed relationships of the physical system. We focus on medium to long-term in this paper.

In systems with a large share of hydro power, such as the Nordic, the inflow variations are one of the main drivers of uncertainty. The expected annual hydro generation is 119 TWh in a normal year, but actual generation may vary between 95 TWh and 140 TWh, depending on precipitation to the water reservoirs. By microeconomic reasoning, prices and inflow should be negatively correlated, suggesting that in a year with above average inflow to the reservoirs, the price of electricity should be lower than the average price level.

Intuitively the correlation between system price and inflow is stronger on an aggregated national level, than between the inflow of a certain power plant and the system price. This because it is the total inflow of water to the power plants that decide if we will have an electric energy surplus or if we need to import energy, use rationing or other energy sources with a higher marginal cost, resulting on average in higher prices. This picture is of course a bit simple since we have reservoirs in the Nordic system that can store water over several years from wet periods to dry. Other factors such as temperature and climate also affect the total demand for energy and further complicate the relationship. Some correlation does however exist between temperature and precipitation. Wet winters are typically warmer than normal, and vice versa, [10]. Thus during cold winters with little precipitation, demand for electricity is high and further stimulate higher prices.

Supply-side reasoning and empirical observations also imply that a high degree of precipitation one day does not affect electricity spot prices as much if it is followed by several days of drought. Low inflows to hydro reservoirs are positively correlated with high market prices and low temperatures (resulting in higher end-user sales), [11]. For hydro producers the correlations between accumulated inflow for the whole season or year and market prices are much more important than the correlations between weekly inflow and market prices because of storage capacity. It is the accumulated precipitation over a period of time that affects prices, and for example less than expected inflow over a long period of time will typically result in higher electricity prices, all else held constant. [12] suggests using a 26 week aggregation of inflow data when modeling the relationship between inflow and price. Direct correlations between inflow at local power plants and system prices can however be justified noticing that high inflow at individual power plants is probably strongly correlated with high inflow at an aggregated national level, and so indirectly correlated with the system price.

C. Inflow data Producer 1

The inflow series of power plant 1 consists of weekly observations in the period 1990(1) – 2006(52). It shows a predictable seasonal pattern with a mean annual inflow of 99.3 GWh. The abnormally high observation in week 34 in 2002 is due to the correction of an observation error and will be addressed later when modeling the inflow. The plant has a reservoir size of 177.4 Mm3, a maximum production capacity of 28 MW and an average energy equivalent of 0.67kWh/m3. The plant has a capacity factor of 49 % and a degree of regulation of 1.22, i.e. the reservoir can store more water than a year's worth of average inflow. This makes this plant relatively flexible in terms of power generation. Fig. 1 displays the historic inflow characteristics for this power plant.

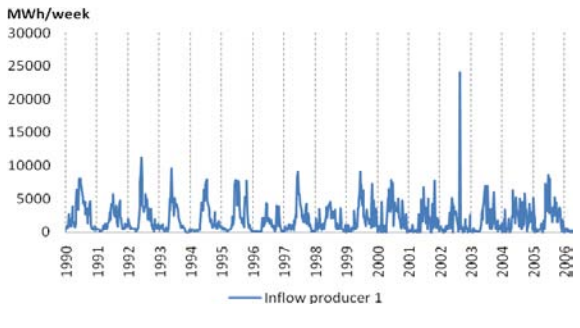

Figure 1. Inflow data producer 1. The figure shows the time series of inflow for power producer 1 on a weekly resolution over the period from 1990 to 2006.

D. *Inflow data Producer 2*

The inflow series of power plant 2 consists of weekly observations in the period 1990(1) – 2006(52). The time series show a very seasonally dependent inflow, and has a mean annual inflow of 275.3 GWh. The plant has a reservoir size of 204 Mm3, a maximum production capacity of 68 MW and an average energy equivalent of 1.25kWh/m3. The plant has a utilization factor of 59 % and a degree of regulation of 1.67, which is the highest among the plants considered in this paper. Fig. 2 displays the historic inflow data for this power plant.

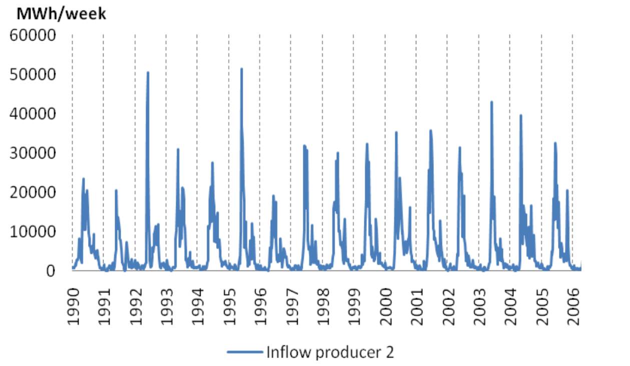

Figure 2. Inflow data producer 2. The figure shows the historic inflow for power producer 2 on a weekly resolution over the period from 1990 to 2006.

E. *Inflow data Producer 3*

The inflow series of power plant 3 consists of weekly observations in the period 2000-2006. The time series has a low degree of seasonal dependence but the highest average annual inflow of 1 247.3 GWh. The plant has a reservoir size of 869.4 Mm3, a maximum production capacity of 210 MW and an average energy equivalent of 1.46 kWh/m3. The plant has a utilization factor of 47 % and a degree of regulation of 0.7. Fig. 3 displays the historic inflow data for this power plant.

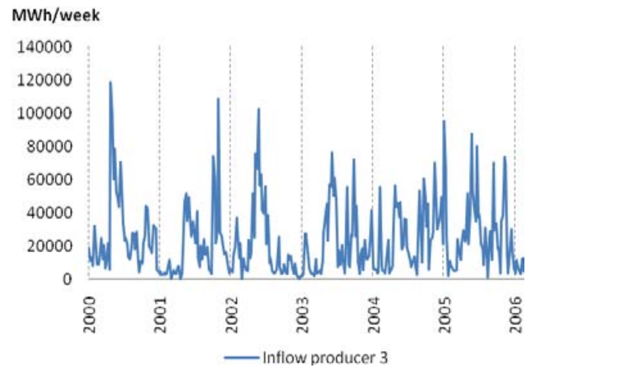

Figure 3. Inflow data producer 3. The figure shows the historic inflow for power producer 3 on a weekly resolution over the period from 1990 to 2006.

F. *Electricity price data*

The time series of system prices consists of weekly observations from the period 1993(week 1) – 2006(week 52), obtained from Nord Pool's FTP server. System prices are used instead of local area prices to simplify the analysis.

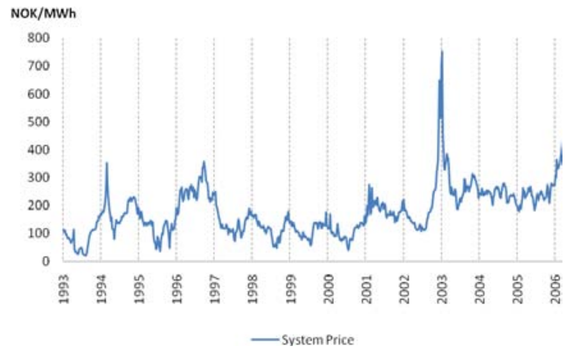

Figure 4. Electricity price data, weekly, 1993-2006

The time series shows signs of a seasonal trend with relatively lower prices during the filling season in summer compared to the depletion season in winter. The trace also indicates an increasing trend. Certain extreme years can be noticed in the period around 2002/2003 and in 2006, which both were considered as very dry periods with little precipitation.

Prices are controlled by supply and demand and show seasonal, weekly and daily variations. In the Nordic market prices are usually higher during winter since a lot of the precipitation comes as snow, and the demand for electricity for heating and lighting is higher.

Electricity prices also display mean reversion. This means that if we get shocks in the electricity price, for instance supply or demand shocks, the price tends to revert back to a long run equilibrium level. This and the previous characteristics of the electricity prices will be considered when the electricity spot price is modeled later in this paper.

G. *Correlation between inflow and price*

Theory suggests negative correlation between system price and inflow at a national level, but for a local power producer we need to examine if there is any correlation between local inflow and the system price. The correlation between local inflow and price and the 26 week aggregate local inflow and price is given in the table below.

TABLE I. ESTIMATED PARAMETERS FOR THE SYSTEM PRICE – INFLOW CORRELATION. AGGREGATE INFLOW IS 26 WEEK MOVING SUM. THE TIME SERIES IS ALSO DIVIDED IN TWO TO EXAMINE STABILITY.

	System Price		
	Total	1st half	Last half
Inflow 1	-0,227	-0,400	-0,233
Aggregate infl. 1	-0,109	-0,182	-0,146
Inflow 2	-0,208	-0,315	-0,247
Aggregate infl. 2	-0,126	-0,166	-0,239
Inflow 3	-0,217	-0,327	-0,490
Aggregate infl. 3	-0,448	-0,654	-0,474

In the first two power plants the weekly inflow is stronger correlated with the system price than the aggregated inflow. For power plant 3 however the correlation between the system price and the 26 weeks aggregate inflow is -0.45, about twice as large as for the weekly inflow. This is puzzling, but might be associated with low degree of seasonality in the inflow series of power plant 3.

III. MODELS FOR PRICE AND INFLOW

This paper uses two classes of models when modeling future inflow and price series, namely stochastic factor models and state space models (STAMP). The stochastic models used in this paper are mainly based on the ones described in [7].

All the one and two factor models used in this paper to explain the stochastic processes for price and inflow contain a deterministic part. This part attempts to explain predictable components of the time series such as level and seasonality. The level is modeled as a constant and the seasonality is captured in a sinusoidal function. The deterministic function takes the following form:

$$f(t) = \alpha + \gamma \cos\left((t+\tau)\frac{2\pi}{52} \right) \quad (1)$$

where t is measured in weeks, and hence the cosine function tries to capture annual seasonality. The parameters α, γ and τ are estimated as shown below.

A. One factor model for inflow

The inflow represented by A_t is modeled as the sum of two components. The first being the predictable deterministic function presented in eq. (1) and the second a mean-reverting stochastic process.

$$A_t = f(t) + X_t \quad (2)$$

The stochastic term, X_t, follows the stochastic process given by

$$\mathrm{dX_t} = -\kappa \mathrm{X_t dt} + \sigma \mathrm{dZ} \quad (3)$$

where κ > 0, $X(0) = x_0$, and dZ represents an increment to a standard Brownian motion. This is a mean-reverting process/Ornstein-Uhlenbeck process with a zero long run mean and a speed of adjustment of κ. Eq. (2) and (3) can be rewritten as

$$d(A_t - f(t)) = \kappa(f(t) - A_t)dt + \sigma dZ \quad (4)$$

showing the mean reverting nature of the process. As A_t deviates from the deterministic part, $f(t)$, it is pulled back at a rate that is proportional to the deviation, and the speed of reversion is given by the mean reverting factor κ.

The distribution of A_t conditional on x_0 is normal with mean and variance equal to (using $x_0 = A_0 - f(0)$):

$$E_0(A_t) = E(P_t / X_0) = f(t) + (P_0 - f(0))e^{-\kappa t} \quad (5)$$

$$Var_0(P_t) = Var(P_t / X_0) = \frac{\sigma^2}{2\kappa}(1 - e^{-2\kappa t}), \quad \kappa > 0 \quad (6)$$

TABLE II. ESTIMATED PARAMETERS FOR THE INFLOW MODELS.

Power station	A	γ	τ	κ
1 (week 1-18)	1009	-546	-16,1	998
1 (week 18-40)	1185	3016	-32,5	65,72
1 (week 40-52)	1603	-1256	20,9	45,38
2 (week 1-18)	1235	483	-13,6	989
2 (week 18-40)	-1183	16131	-32,5	26,32
2 (week 40-52)	2172	-3436	19,9	37,73
3 (week 1-18)	45935	-39734	-14,1	73,23
3 (week 18-40)	44449	27337	-32,2	65,36
3 (week 40-52)	12196	-15039	19,7	61,79

B. Two factor model for price

A two factor model based on [2] is also used to describe the price behavior in which the one factor model is expanded with an additional stochastic term. The stochastic price behavior of the spot price is modeled with one short-term mean reverting component and one long-term equilibrium price level component in the equation below:

$$P_t = f(t) + X_t + \varepsilon_t \quad (7)$$

where

$$dX_t = -\kappa X_t dt + \sigma_x dZ_x \quad (8)$$

$$d\varepsilon_t = \mu_\varepsilon dt + \sigma_\varepsilon dZ_\varepsilon \quad (9)$$

$$dZ_x dZ_\varepsilon = \rho dt \quad (10)$$

The stochastic term X_t is the short run component which follows a mean reverting Ornstein-Uhlenbeck process, and εt is the long term equilibrium and follows an arithmetic Brownian motion. The two stochastic processes (dZ_x and dZ_ε) are correlated through eq. (10).

TABLE III. ESTIMATED PARAMETERS FOR THE TWO FACTOR SPOT PRICE MODEL

α*	κ	μ_ε*	α	γ	τ
-51,5	0,0321	-0,0082	151,47	25,179	-2,1137

C. STAMP model for inflow and price

In this paper we will also model inflow and price time series using state space methodology. The state space methodology is described in [8]. The following general model has been used in this paper as a starting point for the analysis:

$$
\begin{aligned}
& y_t = \mu_t + \gamma_{1,t} + \beta_t x_t + \lambda_t w_t + \varepsilon_t && \varepsilon_t \sim NID(0,\sigma_\varepsilon^2) \\
& \mu_{t+1} = \mu_t + v_t + \xi_t && \xi_t \sim NID(0,\sigma_\xi^2) \\
& v_{t+1} = v_t + \varsigma_t && \varsigma_t \sim NID(0,\sigma_\varsigma^2) \\
& \beta_{t+1} = \beta_t + \tau_t && \tau_t \sim NID(0,\sigma_\tau^2) \\
& \lambda_{t+1} = \lambda_t + \rho_t && \rho_t \sim NID(0,\sigma_\rho^2) \\
& \gamma_{1,t+1} = -\gamma_{1,t} - \gamma_{2,t} - \gamma_{3,t} - \ldots\ldots - \gamma_{51,t} + \omega_t && \omega_t \sim NID(0,\sigma_\omega^2) \\
& \gamma_{2,t+1} = \gamma_{1,t} \\
& \gamma_{3,t+1} = \gamma_{2,t} \\
& \quad \ldots \\
& \gamma_{51,t+1} = \gamma_{50,t}
\end{aligned} \tag{11}
$$

Here y_t is the dependent variable (price) that we are modeling. This model have stochastic level (μ), trend (v), explanatory variables(x), intervention variables(w) and seasonal parameters (γ), all of which are normally and independently distributed with zero mean and the respective variances given above. The models are estimated in the software OxMetrics using iterative procedures based on Kalman filtering. In order to capture the correlation between inflow and price, the inflow is used as an explanatory variable describing the price. This way the correlation is directly modeled when estimating the parameters in the model. Both inflow and a 26 week aggregation of inflow is used in the model and compared out of sample to see if any contain superior explanatory power.

When creating scenario trees from these models a time series for inflow is first simulated based on the inflow model. Then a price series is simulated using the inflow series as an explanatory variable. This way we introduce stochasticity in both series and the correlation is introduced through the coefficient of the explanatory variable. With a negative coefficient a simulated “high” inflow series will therefore tend to create a “low” price series since the high value of inflow will reduce the price series by an amount related to the coefficient above.

IV. ANALYSIS AND RESULTS

Unfortunately, using weekly correlations between the error terms in our models to correlate different scenarios for price and inflow is not suitable, as our analysis indicates the correlation is not stable over time. This was done plotting time series of rolling window correlations for the error terms, using 26 and 40 week window sizes.

A. Matching price and inflow scenarios

When performing simulations the random numbers drawn for each week does not have a constant correlation and so they could be drawn independently, much as is done in existing methods [5], and the scenarios then arranged afterwards based on some sort of matching rule. Descriptions of a few of these methods are given below together with certain intuitive drawbacks.

B. Method 1

A simple and very intuitive form of such a rule is to calculate the average of the price and inflow for the different series, and match the price series with the highest average price with the lowest average inflow series. A problem with this method is that since we are simulating more than two years ahead with this procedure we could end up drawing two scenarios with high inflow and price the first year and then low inflow and low price the second year, and matching these two together since their average match. The method also simplifies the relationship by assuming perfect negative correlation.

Only this method has been implemented; the methods 2-4 below are simply ideas.

C. Method 2

The problem described in the model above can be solved by simulating price and inflow series for one year at the time and then putting them together to form a longer time series. This way we can use the same procedure above but matching the scenarios for every year and not based on the whole series. Which series that follows each other every year should be random, since there is no reason why a very dry year should have a higher tendency of being followed by another dry year. One could however argue for some kind of correlation between the years since we do have reservoirs that can hold water for more than a year and so a very wet year could tend to reduce prices in the following if it turns out to be a dry year. It is also possible to further separate the year into periods such as winter/summer or the three periods used in this paper for modeling inflow based on the factor model.

D. Method 3

The two methods above are very strict in terms of their matching, and a year with high inflow does not necessary need to be a year with low prices. If we look at nationally aggregated inflow this is more correct, but for a local power plant this does not necessary have to be true. Occurrences of high inflow and high prices could occur if national aggregate inflow is low that year. In order to capture more of this randomness we could separate either every year as in method 2 or the whole set as in method 1 into equal sized groups of series based on their average inflow and price level. Each year could then be separated into e.g. 5 equal groups (extreme, high, normal, low, dry) based on their average inflow and price, and the different price scenarios be matched randomly with scenarios from the corresponding inflow group.

E. Method 4

Another procedure that could be attempted is to calculate average price and inflow for the series either for each year or the whole period and then assigning probability distributions to each series. Series with a high average price/inflow would be given a distribution with a high probability of sampling a high number. The price series would then be arranged in a descending order based on one sampling from the probability function of the individual price series. The opposite would be done for the inflow series and the series would then be matched based on the new order. As an example, the price series which drew the highest number from its probability distribution would be matched with the inflow series that drew the smallest number etc. A high probability of sampling a high number is given for series with high average value. The series are then arranged based on one sampling for each series based on their respective distribution. This way high inflow series will tend to be matched with low inflow series but there is still some randomness in how the scenarios are arranged. This would be a time consuming procedure when simulating many scenarios and also include the problem of assigning correct probabilities.

These decision rules should be adjusted to each producer since they might have different correlations with the national aggregated inflow. A producer with strong positive correlation between local inflow and the system price would tend to have high inflow in dry years when the prices are high and so the use of method 1 and 2 for instance would be wrong. They should therefore be adjusted to fit the specific case.

When running the optimization in this paper we have used method 1 to match the scenarios for the factor models for simplicity. The Matlab algorithm is however constructed in such a way that it is easy to include new matching rules/algorithms.

F. *Simulation and optimization scheme*

The flow chart of the computation process used in this paper is given in the figure below. It lists the programs used in the boxes together with a short descriptions of the tasks performed in the respective programs.

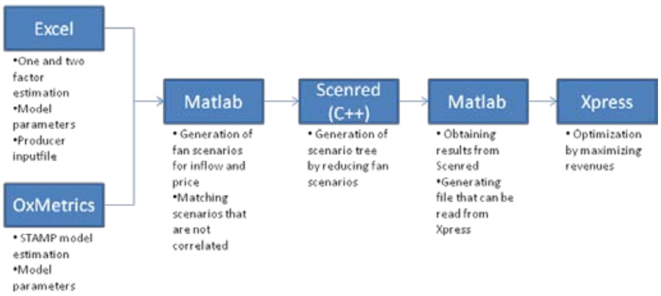

Figure 5. Flow chart of computation process. The figure displays a flow chart of the underlying computation process used in this paper. Each box represent a software program and a description of how the software is used is described.

The different models used to describe the dynamics of the inflow and price series are estimated in Excel for the one- and two-factor models from [2], and in OxMetrics for the state space models. The model parameters and all the input from the different producers are gathered in an Excel input file. The input file is read by Matlab, and the parameters from the different models are used to simulate price and inflow series resulting in fan scenarios. The inflow and price series are then matched together based on a matching rule described in chapter 6, except from the STAMP models where price and inflow are already linked. Matlab then generates a text file that can be read by Scenred [13], which reduces the amount of scenarios and creates a scenario tree. The output file is then read by Matlab and a file that can be read by Xpress is generated containing the scenarios from the scenario tree and all the relevant input parameters from the individual producers. A deterministic equivalent of the stochastic programming problem is then run in Xpress MP. The optimization model used is described by [9].

G. *Results*

When generating the fan scenarios by Monte Carlo simulation, 122 weeks are simulated. The initial stages have weekly resolution, but we aggregate the stages into monthly and then quarterly lengths. For all the analysis conducted, 1000 fan scenarios are generated and this is repeated 20 times. The factor model gives expected revenues (standard deviations) of 102 (1.24), 231 (2.33) and 101 (2.03), for power plants 1, 2 and 3 respectively. Numbers are in million NOK. Method 1 above is employed, giving in some way stronger correlation than is realistic. When the same runs are performed with unmatched scenarios, the results are 105 (6.21), 240 (2.50) and 103 (1.92) million NOK for the three power plants respectively. The difference in the value of revenues are 2.9 %, 3.9 % and 2.0 % respectively, so pretty modest.

V. CONCLUSIONS

Although price and inflow is related both theoretically and empirically, joint modeling of these factors as stochastic processes is challenging. Somewhat surprisingly, we find that such joint modeling has modest value with respect to expected revenues in a stochastic programming model for three Norwegian hydropower plants. As current methods in practical use in the industry relies on separate modeling of price and inflow, our results may indicate that new modeling efforts should concentrate on other aspects of the hydroelectric scheduling problem.

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