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Roar Grønhaug

Optimization Models and Methods for Industrial Supply Chains

Doctoral Thesis

NTNU
Norwegian University of Science and Technology
Thesis for the degree of philosophiae doctor
Faculty of Social Sciences and Technology Management
Department of Industrial Economics and Technology Management

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To Siri and Elisabeth

Acknowledgements

This thesis fulfils four years of intensive work with development of Operations Research models and corresponding solution methods for various supply chain applications. Although there has been a long, and sometimes lonely journey, there are several people that have provided me with valuable support and encouragement.

First of all, I would like to express my deepest gratitude to my supervisor, Professor Marielle Christiansen. It has been a great pleasure to work with you. You have been a true inspiration, and I have appreciated our discussions and your guidance through these years. Furthermore, I am very grateful for your availability for supervision in busy work-days and even in your vacations. I would also like to thank my colleagues at the department at NTNU, in particular Professor Kjetil Fagerholt who has been the project manager for the INSUMAR project, Jardar Andersen, Henrik Andersson, and Nina Linn Ulstein for interesting and rewarding collaboration.

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My special thanks go to my Siri and Elisabeth for your love and support. Siri, you have been a great encouragement and support for me during these years. Elisabeth, I am grateful for every day I am able to spend with you, watching you growing up. Finally, I would like to thank my parents for always encouraging me to follow my own path.

Roar Grønhaug

Trondheim, September 2008

Contents

Chapter 1

Introduction

This thesis delves into how Operations Research may offer managers in the industry valuable decision support for solving real-world problems. In particular, the thesis treats how optimization models can be developed for design and operation of international industrial supply chains. Furthermore, it focuses on supply chains where maritime transportation is a part of the supply chain. The thesis consists of four papers that are devoted to development of optimization models and methods for such supply chains. The research focus has been on real-world problems, dealing with optimization-based strategic and tactical decision support. The applications have been from the metal industry and from the liquefied natural gas, LNG, industry. The literature dealing with problems similar to the challenges faced by these industries is sparse. Hence, new and innovative models that can handle industry-specific issues were required. For instance, when working with strategic supply chain planning for the metal industry, we had to consider furnace conversions, post-production aspects, and the impact from the by-products, in addition to model the supply chain network. On the other hand, the tactical supply chain planning problem faced by the LNG business forced us to develop a new type of optimization models. This type of model has aspects that are new to the Operations Research society, in addition to some industry-specific considerations that have not been dealt with in the literature.

The research project leading to this thesis is a part of a larger research project at NTNU, integrated supply chain and maritime transportation planning, INSUMAR (INSUMAR, 2004). The idea behind INSUMAR was to collaborate with large international industry actors to analyze their supply chains, in particular supply chains where maritime transportation is important. Hence, the main objective of INSUMAR is to create the methodological basis for improved planning decisions for actors in global maritime transportation and their industry customers.

The first two papers in this thesis concern strategic network design for a global actor in the metal industry. Paper 1 describes how Operations Research supported the restructuring of this company. A strategic optimization model that handles the plant structure, product mix and product reallocation among the production plants, as well as the distribution of the finished products and power purchase is developed. In addition, the paper discusses the organizational implementation of the model and the usage of it. Paper 2 gives further insight to strategic supply chain modeling for industries where by-products are created from the production process. The paper focuses on the impact from by-products on the optimal supply chain configuration. It explores how the market developments for both the by-products and the main products affect important issues as plant closings and product reallocation. To achieve this knowledge a new optimization model, based on the optimization model from paper 1, is developed to handle such issues.

Papers 3 and 4 are concerned with tactical planning issues inspired from an actor in the global energy business. In the third paper we consider problems for combined production planning, inventory management, transportation planning, and sales management for this company. Maritime transportation is important in this supply chain as the production facilities and the distribution facilities are spread all over the world. Two optimization models are proposed for this problem, and the paper discusses the performance of these models. Then, in the fourth paper we develop a tailormade solution method, based on the branch-and-price-and-cut method, for one of the models proposed in paper 3. Using this solution method, we are able to solve larger instances than in paper 3, which is a matter of necessity to solve real-world problem instances.

The rest of the introduction is organized as follows. Section 1.1 presents the field of supply chain management and supply chain optimization. Then, Section 1.2 describes the purpose of the thesis and gives an introduction to the four papers. The contributions from the thesis to the society and the contributions from the author of this thesis to the papers are discussed in Section 1.3. Finally, some concluding remarks with some directions for future research are presented in Section 1.4.

1.1 Background

This section gives an introduction to relevant theory for the topics in the thesis. Section 1.1.1 presents definitions of supply chain and supply chain management. Then, Section 1.1.2 gives brief definitions of Operations Research and optimization. Finally, Section 1.1.3 discusses optimization in a supply chain perspective, and presents a few applications of strategic and tactical applications of supply chain optimization that are related to this thesis.

1.1.1 Supply Chain Management

Logistics and supply chain management originates from issues that mankind has been dealing with for thousands of years. Although the ideas are not new, the field of supply chain management did not receive attention before early in the 1980's (Croom et al., 2000). The terms logistics and supply chain are strongly related, and the term supply chain has a wider meaning than logistics. The term logistics typically refers to the organization of transportation and production within one organization. Furthermore, logistics management concerns the coordination of the flow of the goods through the organization. Christopher (2005) defines logistics as follows:

> Logistics is the process of strategically managing the procurement, movement and storage of materials, parts and finished inventory (and the related information flows) through the organization and its marketing channels in such a way that current and future profitability are maximized through the cost-effective fulfillment of orders (Christopher, 2005, p. 4).

The importance of logistics and logistics management have been known and studied by the military long before business companies recognized this importance (Ballou, 2004; Christopher, 2005). The term business logistics can be used to distinguish from the military usage of the term logistics (Ballou, 2004). While logistics concerns the flow through one organization, the supply chain typically refers to the flow through several organizations. Beamon (1998) gives the following definition of the supply chain:

> A supply chain is defined as the set of relationships among suppliers, manufacturers, distributors, and retailers that facilitates the transformation of raw materials into finished products (Beamon, 1998, p. 292).

This definition corresponds to the definition found on page 1 in Simchi-Levi et al. (2003). Furthermore, Simchi-Levi et al. (2003) also use the term logistics network for the supply chain. Note that even if the earlier terms as physical distributions, materials management, industrial logistics, and channel management, covers a broader scope than logistics, they lack the attempts to implement logistics beyond the borders of one individual company (Ballou, 2004, p. 5). A supply chain is not a set of stages that a product sequential traverses through from supplier via producer to end costumer. It is rather a network of interrelated stages that seeks to fulfill the demands from the end customers, and where there are multiple suppliers and suppliers for the suppliers (Christopher, 2005; Geunes and Pardalos, 2003).

The definition of supply chain is more unified than the definition of supply chain management (Mentzer et al., 2001). A reason for this is that supply chain management can be viewed as a management philosophy, a set of activities to implement a management philosophy, and a set of management processes. For instance, Christopher (2005) focuses on the relationship among the suppliers and customers in the supply chain in his definition of supply chain management. On the other hand, Simchi-Levi et al. (2003) focus on the set of approaches that are utilized to integrate the actors in the supply chain in their definition of the term. Mentzer et al. (2001) try to give a definition that unites and encompasses the numerous definitions of supply chain management that exist.

> Supply chain management is defined as the systemic, strategic coordination of the traditional business functions and the tactics across these business functions within a particular company and across business within the supply chain, for purposes of improving the long-term performance of the individual companies and the supply chain as a whole (Mentzer et al., 2001, p. 18).

The coordination and planning of the supply chain are carried out at the three traditional planning levels; the strategic, the tactical, and the operational (Simchi-Levi et al., 2003). Planning at the strategic level deals with issues that have long-term impact on the company, for instance the locations of the production plants and the distribution network design. Decisions at the tactical levels usually deal with planning horizons less than a year. For instance, such issues can concern inventory and transportation policies. Finally, at the operational level, the managers deal with day-to-day planning as lead time quotations and work-force planning.

1.1.2 Operations Research

Operations Research is a multidiscipline science that uses an analytical approach to help making better decisions. Operations Research is applied to problems that concern how to coordinate and improve the activities within organizations, for instance in manufacturing, transportation, construction, telecommunications, financial planning, health care, the military, and public services (Hillier and Lieberman, 2005). Although some of the techniques used have their origins several centuries ago, the birth of Operations Research is considered to be during World War II (Gauss and Assad, 2005; Hillier and Lieberman, 2005). More details about the history of Operations Research are presented by Gauss and Assad (2005) in their edited volume.

Optimization, or mathematical programming, is a vital part of Operations Research. In fact, this is the heaviest used discipline within Operations Research (Gupta, 1997; Lane et al., 1993). Optimization modeling is concerned with formulating a decision problem as a mathematical problem that either maximizes or minimizes the objective of the problem. A formal definition of optimization is found in Rardin (1998):

> Optimization models (also called mathematical programs) represent problem choices as decision variables and seek values that maximize or minimize objective functions of the decision variables subject to constraints on variable values expressing the limits on possible decision choices (Rardin, 1998, p. 4–5).

Today, many optimization models can be solved by commercial software with embedded optimization solvers. These solvers might fail when the problems are to large, i.e. large-scale problems, or when the structure of the problem is too complex. In such occurrences, there is a need for developing tailor-made solution methods.

1.1.3 Supply Chain Optimization

Research on supply chain optimization within the field of Operations Research focuses on developing optimization models and solution techniques for analyzing and improving parts of the supply chain. To be able to find the best configuration of the supply chain, a system wide perspective is needed. Supply chain design, coordination, and management are active research fields within the Operations Research community (Geunes and Pardalos, 2005; Kok and Graves, 2003; Tayur et al., 1999). The literature spans both pure theoretical contributions and contributions based on real problems for large corporations that have recognized a need to improve their competitive strength. In addition, optimization-based decision support systems are also extensively used when companies evaluate their supply chain (Ballou and Masters, 1999; Muriel and Simchi-Levi, 2003).

There has been active research on problems that fall into the field of supply chain optimization since the seventies, and there is a growing interest for such research from both the industry and the academia. Muriel and Simchi-Levi (2003) list several reasons for this growing interest in supply chain optimization. The first reason is the great savings that can be achieved by increasing the efficiency of the supply chain. Secondly, the restructuring of many companies has broken down traditional organizational barriers, and information technology provides better access to data from all components of the supply chain. The third reason Muriel and Simchi-Levi (2003) list is the deregulation of the transportation industry. Other reasons to the growing interest in supply chain optimization include among others; the recently advances in optimization techniques, and the growing number of practitioners who recognize the value of taking a global view on decisions problems (Geunes and Pardalos, 2003).

The focus in this thesis is strategic and tactical supply chain optimization. Furthermore, the thesis does not delve much into the aspect of uncertainty. Hence, the models developed are deterministic optimization models. This section will give a brief introduction to research related to the thesis, with emphasis on the modeling aspect. Some model types and some of the research related to these types of models are described. Section 1.1.3.1 presents a few examples of strategic supply chain optimization, while Section 1.1.3.2 describes some related research on optimization for tactical supply chain planning. More comprehensive literature reviews on supply chain optimization can be found in Melo et al. (2008), Geunes and Pardalos (2003), Goetschalckx et al. (2002), Beamon (1998), and Vidal and Goetschalckx (1997).

1.1.3.1 Strategic Supply Chain Optimization

At the strategic planning level the focus is to analyze different configurations of the supply chain that are hard to reverse in short-term and medium term planning. Supply chain optimization at this level typically refers to optimization models that handles more than one of the following aspects; facility location, distribution network design, and major investments production equipment. The models we present in this section are either single- or multi-period mixed integer optimization problems.

The combined facility location and distribution design model presented in Geoffrion and Graves (1974) is formulated as a a single-period mixed integer program for multi-commodity distribution design. The model determines the facility location, the flow of products from production sites, via distribution centers, to the end costumers. Moreover, Benders decomposition (Benders, 1963) was used to solve the problem (Geoffrion and Graves, 1974). This paper is recognized as the first important contribution to optimization based supply chain design (Geunes and Pardalos, 2003), even though this was a decade before the term supply chain management was invented.

A heavily cited application of supply chain optimization modeling is found in Arntzen et al. (1995). By using this model in strategic planning, Digital Equipment Corporation reported large savings in their operations. The background for redesigning their supply chain was to face the shift in demand from large mainframes to small computers in the computer industry. Their multi-period mixed integer program handles issues as the number of plants and distribution centers. Moreover, it handles the choice of capacity and technology at each plant, and how many plants that should be involved in producing a product. Finally, the optimization model included the effect from tax and transportation on the optimal supply chain design. The optimization model was used to redesign Digital Equipment Corporation's supply chain in the early nineties.

A more recent contribution to strategic supply chain optimization is found in Sha and Che (2006). They present a model formulation based on analytic hierarchy process for analyzing a supply chain. The objective of this model is to choose the optimal network configuration by finding suitable participant for partnership at all stages of the supply chain, e.g. suppliers, manufacturers and distributors. The proposed model is a multi-objective optimization problem which among other factors aims to minimize the number of corporations in the supply chain while maximizing the supply chain network's yield.

1.1.3.2 Tactical Supply Chain Optimization

The tactical planning level on supply chain optimization handles issues such as production planning, inventory management, transportation planning, sales planning, and contract evaluation. The first four of these issues are of interest for the tactical planning part of this thesis as the tactical planning model proposed in this thesis combines production planning, inventory management, transportation and sales management. Since such contributions are rare, this section describes some models that includes some of the issues of interest for this thesis.

Christiansen (1999) and Christiansen and Nygreen (2005) study an integrated inventory management and transportation planning problem for a company producing and consuming ammonia. The company produces and consumes the product, in addition to controlling the transportation of the product by ships. Christiansen (1999) propose an inventory routing problem formulation and proposes a solution method based on column generation for this problem. Furthermore, Christiansen and Nygreen (2005) expands this work to create a more robust planning model by introducing a set of soft constraints for the inventories. Another inventory routing problem can be found in Persson and Göthe-Lundgren (2005). They propose a model formulation that integrates shipment planning of petroleum products from refineries to depots, aggregated production scheduling at the refineries, and the inventory management at the refineries. To strengthen the model formulation, a set of valid inequalities is proposed, and the model is solved by use of column generation.

Inclusion of sales and marketing management are handled by Timpe and Kallrath (2000) and Ouhimmou et al. (2008). Timpe and Kallrath (2000) discuss production planning on a tactical level in process industry. Their application combines production, inventory management, and distribution and sales management. They do also discuss how to define the capacity in a production network with multiple production plants and products. A mixed integer programming model is proposed. Much of the focus is to formulate tight constraints for changeovers in the production, i.e. turnover between different modes of production. On the other hand, the objective of the optimization model proposed by Ouhimmou et al. (2008) is to find manufacturing and logistics policies for a furniture company that minimize the total costs while still maintaining a competitive level of service.

1.2 Purpose and Outline of Thesis

First, Section 1.2.1 discusses the purpose of the thesis. Then, Sections 1.2.2-1.2.5, present a brief summary of each paper.

1.2.1 Purpose of Thesis

The purpose is to develop new optimization models that can help managers in particular industries to design and plan their supply chain. The models are aimed to provide decision support either at a strategic or at a tactical level. For some of the models, we develop an advanced solution method to be able to solve real-world instances. The models are developed in close cooperation with large international companies realizing the need for improved decision support. Although we have developed models for real-world planning, they are generic and may be used in other contexts as well.

Four scientific papers are presented in this thesis. The first two papers are devoted to strategic supply chain design and how to develop appropriate optimization models for this purpose. Furthermore, they give insight in how to use optimization for top level management planning purposes. The last two papers deal with tactical supply chain planning. These papers give a methodological basis for how to handle hard-to solve integrated production and sales management, inventory management, and maritime transportation problems.

The papers have some differences regarding writing style, notation, and models syntaxes. These differences have come from the influence of different co-authors on the papers, and from inputs from the referees during the review process for each paper. On the other hand, the formatting of the references, the paper format, and the fonts in the papers have been standardized throughout the thesis. Hence, the formatting of the papers in the thesis differ from the formatting in the published papers.

1.2.2 Paper 1: Elkem Uses Optimization in Redesigning Its Supply Chain

This paper describes a strategic planning model for actors in the metal industry. A multi-period optimization model that captures the characteristics of this industry is developed to provide quantitative decision support for redesign of the supply chain.

In this context, the supply chain consists of suppliers, production plants, and customers. There are several production plants, i.e. smelting plants. The smelting plants differ in size and production technology. The production technology restricts the product range at each plant. It is possible to invest in improved production technology at the plants. In addition to the supply chain for the products produced there is a supply chain for energy, which is a major input factor in smelting metal. In fact, the company acts as both a buyer and seller in the energy market depending on the energy prices.

The optimization model addresses decisions regarding future plant structure, including possible plant closures and new plant acquisitions, power purchase and sales, product allocation among the plants, conversions of production technology at the furnaces, and investments in post-production equipment. The relationship to the suppliers and customers are handled through a set of contracts. There is a set of contracts for each product sold to the customers with a given volume and price. In addition, there are costs for transporting the finished products by ship to the customers. Most of the suppliers are tightly connected to the production plants; hence there are no supplier selections in this supply chain, except for energy sourcing. The sourcing of energy is handled through a combination of long-term contracts and spot purchases. Depending on the contract clauses, some of the energy can be sold in the spot market when profitable. Some of these contracts are long-term contracts linked to one or more plants, while other contracts are short-time contracts sold in the spot market.

This optimization model is a supply chain model as it handles the relationships between the company's plants, and it handles some of the relationships with the suppliers and customers. Hence, calling it a supply chain model fits well with the definition of a supply chain from Beamon (1998) and the definition of supply chain management given by Mentzer et al. (2001). The optimization model falls in the category of strategic supply chain applications as described in Simchi-Levi et al. (2003), since it deals with long-terms issues as the future plant structure.

The optimization model was developed in cooperation with an major actor in this industry, Elkem. Elkem's silicon division is the world's largest producer of silicon and ferrosilicon. The paper presents details in how this company used this optimization model to support their strategy process to solve challenging problems and to strengthen its position. Hence, the paper gives insight in how Operations Research models can be used as input to strategic decision making faced by top-level management.

The contributions from this paper are a successful combination of Operations Research, managerial economics, technical knowledge, and supply chain management. The originality of the paper is that it describes a project developing a mathematical model that optimized the company's supply chain and was used intensively in top-level strategic planning processes. Furthermore, the paper illustrates how we can exploit management knowledge and Operations Research models. The key challenges during the project was to make sure that we were answering the right questions to the right people in the organization, and to ensure a sufficiently level of detail in the model, for instance with respect to the level of aggregation and the time period resolution.

The paper is coauthored with Nina Linn Ulstein, Marielle Christiansen, Nick Magnussen, and Marius M. Solomon. Furthermore, it was published in *Interfaces* Vol. 36, No. 4, July-August 2006, pp. 314-325. The paper was also a finalist in the *EURO Excellence in Practice Award 2007* (Voß, 2007).

1.2.3 Paper 2: The Impacts of By-products on Optimal Supply Chain Design

This paper builds further on the work from paper 1, and here we introduce some new aspects on strategic supply chain planning. The paper addresses the issue of how the market developments for by-products from the production processes in the metal industry affect the optimal supply chain configuration. We present a new optimization model based on the model presented in paper 1. The optimization model handles the issues described in paper 1. In addition, the production and sales of the by-products are handled more exhaustive in this paper. The quality chosen of the by-product affect the quality of the main product and visa versa. In general, poorer quality on the main products will increase the quality of the by-product at a furnace. Moreover, the marked for by-products is modeled as a set of contracts where the solution of the optimization model decides the degree of fulfillment of these contracts. The computational testing of the optimization model shows that the prizing of the by-product influence the production of the main products at the plants. The paper presents extensive analyzes of how different prizing strategies of the by-product affect the overall profit for the company, and how these strategies affect optimal structure of the company.

The arguments for calling the optimization model from this paper a supply chain model are the same as for paper 1. In fact, this paper treats the supply chain for this industry in a more precise manner. Adding the by-products dimension into the problem adds complexity in deciding the optimal plant structure and product allocation, since the decision for main products and for the by-products are interdependent.

The contributions of this paper are a supply chain model in the metal industry that include the impacts from the by-products, and an optimization model that handles this problem for actors in the metal industry. Furthermore, the paper gives an thorough analyzes of how the prizing of the by-products affect the optimal supply chain configuration.

The paper is co-authored with Marielle Christiansen and was published in G. Hasle, K.-A Lie, and E. Quak (Eds.), 2007, Geometric Modelling, Numerical Simulation and Optimization: Applied Mathematics at SINTEF, pp. 497-520. Springer.

1.2.4 Paper 3: Supply Chain Optimization for the Liquefied Natural Gas Business

This paper introduces the *Liquefied Natural Gas Inventory Routing Problem, LNG-IRP*. The purpose of the LNG-IRP is to provide tactical decision support for actors in the Liquefied Natural Gas, LNG, business. The objective is to maximize the profit from the LNG supply chain where an actor controls the production, the maritime transportation, the consumption/sales management, and the storages at all facilities. The facilities are spread all over the world. Hence, maritime transportation is an important part of the supply chain. The LNG-IRP is a variant of the maritime inventory problem. For the maritime transportation part of the LNG-IRP, we have to decide the routing and scheduling of the LNG tankers, in addition to handle the inventories at the LNG tankers. The LNG-IRP has some complicating aspects compared with other maritime inventory problems, as the production and the consumption are variable and can change at any time period between predetermined lower and upper limits. Moreover, a constant rate of cargo evaporates from the LNG vessels during the maritime transportation, and a variable number of cargo tanks can be unloaded at the consumption ports.

Two different optimization models for the LNG-IRP are proposed, an arc flow formulation and a path flow formulation. The formulations are tested and compared on instances motivated from the real-world problem. The LNG-IRP is hard-to-solve. Hence, the instances tested were relative small. Both formulations provide good solutions to these instances, and it is hard to conclude the superior formulation.

The relationships to the suppliers and customers are handled through a set of contracts at both the production and distribution facilities. Since the optimization handles the relationship among the company's facilities, the ships, the suppliers, and customers, this paper as well presents a supply chain optimization model. This is in line with the definitions of a supply chain from Beamon (1998) and the definition of supply chain management by Mentzer et al. (2001). The plant structure is not an issue in this paper, the focus is on inventory management, transportation, production management, and sales management within a 1-3 months planning horizon. Hence, this supply chain optimization model carries out decision support at the tactical planning level according to Simchi-Levi et al. (2003) definitions.

The main contribution of this paper is to introduce a new type of real-world optimization problem within maritime transportation, the LNG-IRP. The paper presents two different model formulations for the LNG-IRP, one arc-flow formulation, and one path-flow formulation. In addition, the paper proposes an algorithm to enumerate all columns for the path-flow formulation.

The paper is coauthored with Marielle Christiansen and is accepted for publication in L. Bertazzi, J. v. Nunen, and M. Speranza (Eds.), *Innovations in Distribution Logistics, Lecture Notes in Economics and Mathematical Systems*, Springer.

1.2.5 Paper 4: A Branch-and-Price-and-Cut Method for a Liquefied Natural Gas Inventory Routing Problem

This paper continues the work from paper 3. The paper proposes a column generation approach to solve the Liquefied Natural Gas Inventory Routing Problem, LNG-IRP. The decomposition of the problem results in a master problem that controls the production and sales management, the port capacities, and the inventory levels at all facilities. In, addition it ensures that feasible combinations of routes and schedules are chosen for the LNG tankers. There is one subproblem for each LNG tanker that generates columns to the master problem. These subproblems correspond to longest path problems with complicating side problems, and are solved by dynamic programming. The column generation algorithm is embedded in a branch-and-price-and-cut method in order to generate integer solutions.

The contribution of this paper is a solution method to the LNG-IRP that is able to solve larger and more realistic instances than presented earlier. The objective of this paper is to present a branch-and-price-and-cut method for the LNG-IRP. This method relies on a decomposition of the LNG-IRP into a master problem that handles the inventory management and port capacity constraints, and hard-to-solve subproblems generating ship routes. The outcome is a tailormade column generation method with branching in both the master problem and the subproblems. In addition, some accelerating techniques are developed. In particular, a greedy heuristic is used for solving the subproblems as long as columns can be generated, before turning over to an exact algorithm. The proposed branch-and-price-and-cut method is tested on instances inspired by an actor in the LNG business.

The paper is co-authored with Marielle Christiansen, Guy Desaulniers, and Jacques Desrosiers. Furthermore, the paper has been submitted to *Transportation Science.*

1.3 Contributions

The thesis has contributions to both the research community and the industry. In this section these contributions are elaborated. This section is organized as follows. First, we discuss the contributions to the research community. Then, the corresponding contributions to the industry are presented. Finally, an overview of mine contributions to each of the papers that constitutes this thesis is given.

1.3.1 The Contributions to the Research Community

Details about the specific scientific contributions from each paper in this paper are given in Sections 1.2.2-1.2.5. In addition, the work on these papers have been presented at many different scientific conferences around the world and discussed with other researchers within the Operations Research society. Moreover, the work on the papers that is included in this thesis has also lead to my involvement in other projects that are not included in the thesis. Three of them will be described in the following. Firstly, Andersen et al. (2008) present a solution method for solving the service network design problem with asset management constraints based on column generation and branch-and-price. My contribution in this paper is mainly the basic elements of the column generation and the corresponding branch-and-bound algorithm. Secondly, Andersson et al. (2008) continue the research in papers 3 and 4 in this thesis. Andersson et al. (2008) deals with a similar supply chain as we did in paper 3 and 4, but where the actor does not control the production or the storage at the pickup ports. Instead, some pickup contracts with complex origin-destination clauses handles the interaction between the producer and the distributer of LNG. Finally, the work with Operations Research models on LNG issues lead into a research collaboration where we created an optimization based decision support tool for operational planning for unloading LNG from a new type of LNG vessels (Rokstad et al., 2008).

1.3.2 The Contributions to the Industry

All four papers describe decision problems that can be found in the real world. The strategic models in papers 1 and 2 have been heavily used by top-level management to support strategic planning. Figure 1.1 provides details of how our industry partner values this cooperation.

The development of the tactical optimization models in papers 3 and 4 has given valuable insight for our industrial partner in the complexity of such models and how to deal with such issues. Moreover, the problem we have tried to capture will increase in importance both for our industrial partner and the LNG business in general as well. Hence, we expect that the models developed or the extensions from them will be important contributions in future decision support applications.

1.3.3 The Contributions to the Papers

Table 1.1 lists the levels of contributions I have had to the four different papers that constitute this thesis. The table differentiates between *Intellectual Input*, *Implementation*, and *Writing*. Intellectual input refers to identification and formulation of the planning problem, development of the mathematical models and appropriate solution algorithms. Furthermore, implementation covers the data

Elkem

Your ref.:

Our ref.:

Date:
8 July 2004

Restructuring of the Elkem Silicon Division

The drop in demand for silicon metal in 2001 confirmed the need for a comprehensive analysis of the production structure for Elkem's silicon division. A restructuring process of this division has impact for 10 production plants worldwide and will bring a long both expansion and closings at the different plants. A condition for accomplishing such a major restructuring process is that our choices are well documented.

The optimization model we used in this process provides an unbiased decision support tool for planning our future plant structure. We have in cooperation with the research team and skilful colleagues in Elkem, established a restructuring plan that all parties within the company now have agreed upon. We expect significant benefits from it in the strategic planning for the silicon division.

We also have the opportunity to continually look at alternative solutions as technology opportunities and market demand patterns change. In addition, the work has opened up for the use of this kind of tools in other business units in Elkem. We intend to maintain the good cooperation with the research team, as they are capable to help us to address these important and hard decisions we are faced with as an industrial enterprise.

for Elkem ASA

Richard Aa
Chief Financial Officer

Elkem ASA
Corporate Headquarters

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Figure 1.1: Statement from Elkem's CFO

handling, coding and execution of the models, as well as analyzing the results. Finally, writing refers to the writing of the scientific papers. The levels of contributions ranges from 1 to 3, where 1 means some contribution, 2 stands for significant contribution, and 3 refers to major contribution.

Paper	Intellectual Input	Implementation	Writing
1	3	3	1
2	3	3	3
3	3	3	3
4	3	3	2

Table 1.1: The levels of contributions to the papers in the thesis

The idea for the work done in paper 1 originates from Nick Magnussen. Nick Magnussen and I were the main contributors to problem identification, problem formulation, and development of the mathematical model with inputs from Marielle Christiansen and Nina Linn Ulstein. I had the sole responsibility for the computer implementation of the model, while I cooperated closely with Nick Magnussen on the data analyzes for this project. The main contributor to the writing of this paper belongs to Nina Linn Ulstein, with some contributions from Marielle Christiansen, Marius M. Solomon, and me. I have been the main contributor to the problem identification, the problem formulation, and the development of mathematical models and solution methods on papers 2, 3, and 4 in this thesis. The models and the solution methods have been further developed in coordination with the other co-authors. The computer implementation and the data analyzes for these papers have been handled exclusively by me. I wrote the drafts for papers 2 and 3 which was further developed in cooperation with Marielle Christiansen, while the writing of paper 4 was been a joint effort between all co-authors.

1.4 Concluding Remarks

This thesis presents issues that are of importance for industries dealing with strategic and tactical planning. Moreover, it presents optimization models that are of importance for planning of global industrial supply chains. The optimization models and the corresponding solution methods presented have also several contributions to the research community.

Four papers devoted to this research area constitute this thesis. Papers 1 and 2 are concerned with strategic planning for an actor in the metal industry. The supply chain is modeled, and computational testing of real-world instances are presented. In addition, paper 1 discusses how such optimization models can be used for strategic planning in large companies. Papers 3 and 4 are devoted to tactical supply chain optimization for actors in the LNG business. The supply chain is modeled as a variant of the maritime inventory routing problem, solution algorithms are developed, and computational testing of these solution methods are given.

Although these papers give new understanding of optimization of industrial supply chains, there are still open issues for further research. The thesis does not focus on how to deal with uncertainty. Hence, working on stochastic programming formulations and their solution methods might be a research direction. Moreover, the LNG-IRP, from paper 3 and 4, is hard-to-solve for real-world instances. Hence, more work on solution methods for this problem is of interest. Finally, the level of vertical control differs among the actors in the LNG business. The LNG-IRP aims at supporting actors that control a major part of the supply chain, but other companies might have less or more control over their supply chain. Hence, more work on similar problems as the LNG-IRP should be of interest for both the LNG business and academia.

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Paper I

Nina Linn Ulstein, Marielle Christiansen, Roar Grønhaug,
Nick Magnussen, and Marius M. Solomon:

Elkem Uses Optimization in Redesigning Its Supply Chain

Interfaces, 2006, Vol 36, No 4, pp 314-325

Is not included due to copyright

Paper II

Marielle Christiansen and Roar Grønhaug:

The Impacts of By-Products for Optimal Supply Chain Design

A chapter in G. Hasle, K.-A. Lie, E. Quak (eds.): Geometric Modelling, Numerical Simulation, and Optimization, Springer Verlag, 2007, pp. 497-520

Chapter 3

The Impacts of By-Products for Optimal Supply Chain Design

Abstract:

During the last half decade, the metal industry has been in a harsh situation seeing their profit margins squeezed to an extreme. Many companies have been forced to close down furnaces and plants. To help a major metal producing company manage this process, we developed a strategic mixed integer programming model. The main decisions addressed by the model involve the future plant structure and production capacities, the production portfolio at each plant and the by-product production. Here, we present the underlying MIP-model and give computational results. In addition, we show how the valuable by-product production can have impact on the optimal supply chain design.

3.1 Introduction

The last decade has been characterized by an increasing globalization resulting in international groups of companies collaborating and merging. This has also been the case in the metal industry. Together with the global economy slowdown that started some years ago, this industry has needed to emphasize supply chain redesign. In this paper we focus on how to improve the efficiency of the supply chain network, and show how these decisions depend on the production of finished products and important by-product production.

There are very few contributions in the literature focusing on by-product production, but in general more and more corporations are realizing the importance of using mathematical programming models to support their supply chain decisions. Already in 1993, Ballou and Masters Ballou and Masters (1993) survey the use of commercial optimization software for location analysis in the USA. They find that about 51 % of the companies responding use commercial software to support their network design. Furthermore, Geoffrion and Powers Geoffrion and Powers (1995), in discussing the possible future development of OR tools for the design of production and distribution networks, have proved right in believing that supply chain will continue "to gain in scope and influence as a corporate function".

The metal company Elkem ASA is the largest supplier of silicon metal in the world. The difficult situation in this industry has resulted in a clear corporate objective to improve the efficiency of the supply chain and evaluate the product portfolio. To help Elkem in this process, a mathematical programming model was developed Ulstein et al. (2006). The decisions addressed by the model pertain to future plant structure including possible closures, new plant acquisitions and investments in production equipment. The silicon division has made extensive use of the model and its scenario analysis capabilities, resulting in important organizational benefits and an expected significant relative improvement in yearly revenue.

Silicon and ferrosilicon are produced by mixing different raw materials in large furnaces at high temperatures. From the furnaces, the molten metal goes through production to become different qualities of silicon and ferrosilicon metals. In addition, some smoke comes out of the furnaces during heating. This smoke used to be disposed as waste until it was discovered that the smoke could be collected by filtering. The resulting product is called microsilica, and today, microsilica is an important by-product from the production of silicon and ferrosilicon metals. The production process, furnace and raw materials have great impact on the quality of microsilica. Some plants produce high-quality microsilica, and the revenue from the microsilica production can be important for the further existence of the plant. In the existing strategic optimization model Ulstein et al. (2006) the impact of by-product on the supply chain is not considered. Elkem has expressed a need for an extension of that model to address this issue.

This paper presents a supply chain design in the metal working industry where by-product production is focused and gives a mathematical model of the problem. In addition, the aim is to show the effects on the plant structure and production mix by different conditions of by-product production.

The rest of this paper is organized as follows: In Section 3.2, we describe the supply chain design with emphasis on the by-product production. Comparisons to related studies in the literature are presented in Section 3.3. Both facility location and supply chain studies are addressed. Section 3.4 is dedicated to the description of the underlying mixed-integer programming model. We will also in this section point out some modeling challenges. Computational results are given and discussed in Section 3.5. Finally, some concluding remarks follow in Section 3.6.

3.2 Problem Description

Elkem is the largest producer of silicon metal in the world, and the western world's largest producer of ferrosilicon products. The silicon division has nine plants located in Canada, Iceland, Norway and USA. The division's main products are silicon and ferrosilicon metals in a wide range of qualities. Several by-products result from the smelting process when producing silicon and ferrosilicon metals. The most valuable by-product is silica fume, called microsilica. Elkem is also the market leader of microsilica with a market share of 50%.

The slowdown of the global economy that started in 2000 had a great impact on the metal production industry. Indeed, Elkem's silicon division experienced a serious weakening in the end market in 2001 and 2002, compared with 2000. For example, by the end of 2001, spot prices for silicon and ferrosilicon metals in the European market were reduced to half those seen in the mid-90s. This resulted in widespread restructuring in the entire industry as a number of companies were forced to close down furnaces and plants. Nevertheless, the global production capacity was still greater than market demand.

To ensure that Elkem could keep the position as the world's leading silicon manufacturer, its management evaluated the supply chain of the silicon division. By the end of 2001, Elkem had closed down five furnaces at four different plants. It became necessary to improve the supply chain and evaluate the product portfolio. Closing a production plant might be the best alternative if the plant is not competitive enough, although this decision can be expensive. Elkem must demolish the plant and rehabilitate the environment to the same condition it had before the plant was built. Moreover, the company has to compensate the redundant employees. Another alternative is to temporarily close down the plant. This is less expensive than a permanent closure, but it is not allowed for a long period of time.

Elkem has studied its supply chain for the silicon division since 2001. Ulstein et al. (2006) describes the development of an optimization package to support restructuring the silicon division. Although this optimization model has been of great value for Elkem in restructuring, further analyses have brought up the question of how the production and sale of by-products can affect the of supply chain. The model neither treats the production and the demand for the by-products explicitly, nor satisfactorily treats conversions between different furnace technologies. Because the impacts of plant location decisions are drastic, not only for stockholders' value, but also for the thousands of affected employees and their families, the management wants to ensure that they are making the right decisions. The management expresses a need to analyze if and how the decisions of by-product allocation between plants affect the supply chain design of the silicon division.

The rest of the section is organized as follows: In Section 3.2.1 we will discuss Elkem's supply chain, while the production process is described in Section 3.2.2.

3.2.1 Elkem's Supply Chain

These products have a complex production process, and reach the customers at the end of a global supply chain. The supply chain is illustrated in Figure 3.1. Raw materials are sourced from mines in Europe, Asia and South America. Because Elkem has either a long-term relationship with or ownership of its suppliers, smelting plants have few raw materials suppliers to choose among.

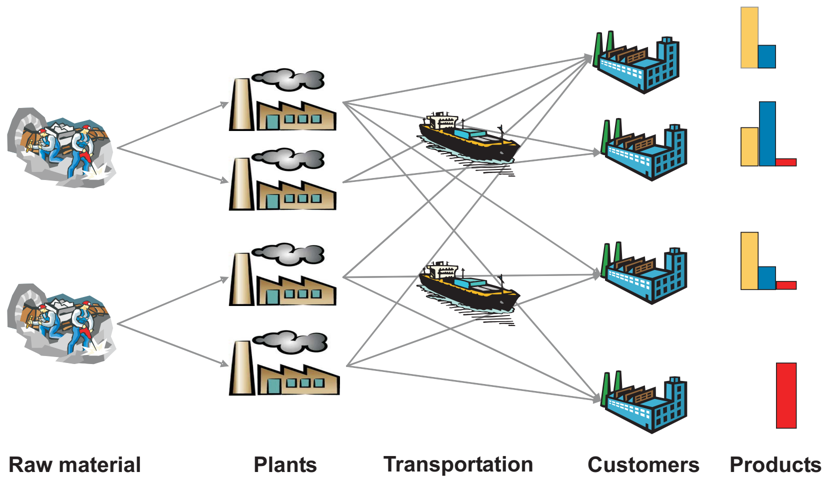

Figure 3.1: Elkem's supply chain for silicon, ferrosilicon and microsilica. Raw materials are transported from coal and coke mines to the production plants where the silicon and ferrosilicon metals are produced. The finished products are transported mainly at sea to the end-customers. The bars at right illustrate different product mixes of silicon and ferrosilicon metals sent to end-customers

Smelting plants have different costs due to economies of scale. Elkem plants differ substantial in size with production output ranging from 25 to 115 thousand metric tons per year. Production capacity is mostly limited by the number and capacity of furnaces at the plants. Each furnace operates at a relatively fixed production level close to its' maximum capacity. The crew size to operate a furnace is similar for small and large furnaces. Thus, small furnaces are more expensive to operate per ton of product output. Nevertheless, they are still necessary as some products can only be made on small furnaces. Maintenance costs and reinvestments depend on the state of the equipment in terms of age and previous restoration work.

Elkem's main customers are located in Europe, the United States and Japan. For ferrosilicon products there are a large number of medium-sized customers, while for both silicon metal and microsilica products there are a few large- and many small-sized customers. Elkem is particularly capable of producing high quality silicon metal. For example, the company supplies 50% of the total demand for silicon to the highly exacting Japanese electronics industry. The finished products are usually transported by ship to the customers. Microsilica and special grades of silicon metal are shipped in containers while standard products are transported in bulk. Because the production facilities are far away from most of the customers, transportation costs are considerable: for standard ferrosilicon products transportation costs can be up to 15% of the sales prices.

3.2.2 Elkem's Production Process

To identify components assembled into a product assembly industries use a bill-of-materials which is often referred to as a recipe in the process industries. A recipe states the amount of each raw material required to produce a finished product. Raw materials purchase costs, inbound transportation costs, and handling costs are often denoted recipe cost. The production costs depend on the products; in general it is twice as expensive to produce silicon metal as ferrosilicon metal. The costs related to recipe and energy are the major cost components. Recipe costs make up 35-50% of the sales prices depending on plant and product, while energy costs can be up to 25% of the sales prices. At each plant the raw material is fed into the furnace, and heated by electric power until it smelts at 1800° C before it is tapped into big ladles. To increase quality, the metal can be refined in the ladles by adding slag-forming elements and blowing gas through the metal. After this, the refined metal is cast into ingots. Alternately, when it is not necessary to refine the metal, the molten metal is cast directly from the furnace into ingots. Furthermore, the ingots are crushed into pebbles in grinding machines. Finally, the finished products are transported to the customers by either bulk- or container freight in bags. The production process is illustrated in Figure 3.2.

Microsilica was earlier considered as waste. As a consequence of strict Norwegian environmental laws, Elkem was forced to clean the smoke from production. This led to research on how to use microsilica as an additive in other products. Elkem has developed several different application areas for microsilica, and offers a range of different qualities for sale. For example, microsilica is used as an additive in concrete, fiber cement and refractory materials. Microsilica is generated from intermediate products in the smelting process. The purity, or quality, of microsilica is dependent on the main product the furnace is producing. Microsilica is collected by filtering smoke from the furnace. Production of 1 ton of main product generates 0.15-0.5 ton of microsilica, depending on the furnace character-

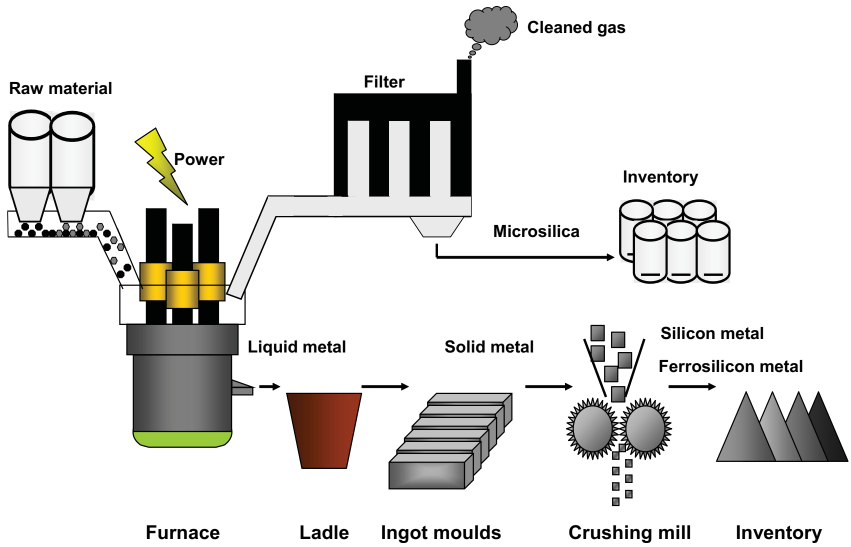

Figure 3.2: Silicon, ferrosilicon and microsilica production. Raw materials are fed into a furnace where they melt and transform to liquid metal. The liquid metal is poured into big ladles before it goes through several post-smelting activities. Finally, the finished product is stored. Microsilica is collected by filtering the smoke from the furnace and packed in bags before it is stored

istics and the purity of the main product. Because microsilica from this filtering has low density, it is often densified before it is packed into bags. Some plants offer microsilica mixed with water to slurry, which can be transported by tankers to the customers. Because the plants are obligated to clean the smoke from the production of ferrosilicon and silicon metals, the marginal cost of producing microsilica is very low. The price obtained for microsilica depends heavily on its quality. There is a high demand for low-quality microsilica, but the demand is limited for the best quality. Since there are several suppliers of low-quality microsilica, the prices are not attractive. On the other hand it is hard to produce high-quality microsilica, so there are few suppliers of high-quality microsilica. Elkem, one of the few suppliers capable of producing this quality, can obtain relatively high prices for it.

The throughput of silicon and ferrosilicon from a plant is limited by furnaces' capacity, in addition to refining and casting capacities. Because building new furnaces is very expensive, this is seldom an alternative, but a closed furnace might be reopened at a reasonable cost. Expanding the refining or grinding capacities is also possible to increase the plant's total throughput.

There are several different furnace technologies which put limitations on the product range. For instance, a furnace producing high quality silicon metal needs electrodes which do not liberate iron to the liquid metal. Iron from the electrodes will contaminate the silicon metal, resulting in lower purity. Converting a furnace to another technology costs 10 – 20 million USD. Furthermore, conversion of a furnace to better technology will also lead to higher operating costs, although it will also lead to more flexibility by increasing the furnace's product range. The choice of furnace technology will also affect the production of by-products.

Each Elkem plant consumes enormous quantities of energy: the furnaces at Elkem's plants consume between 10 and 45 MW. Earlier, the Norwegian Government supported industrial development by awarding good electricity contracts to plants. Many of those contracts are due to expire in the near future. However Elkem can often negotiate new first-rate contracts with local providers, and there is a range of contract agreements. Some contracts state that Elkem can only buy energy for use in metal production, while others allow Elkem to buy specified amounts and to sell any surplus electricity in the spot market. A third type of contract is called take-or-pay; meaning that Elkem must pay a given amount for the energy even if they do not use it. In a market with high energy prices and low metal prices, it is sometimes profitable to close down production temporarily and sell the energy instead.

3.3 Related Literature

In this section we will discuss the theory, optimization models and real cases presented in the literature that are related to the supply chain modeling at Elkem's silicon division. The rest of the section is organized as follows: First, in Section 3.3.1 we address facility location models, while Section 3.3.2 is devoted to supply chain models. Finally, in Section 3.3.3 we will discuss supply chain planning in the process industry.

3.3.1 Facility Location Models

The operations research literature on facility location and supply chain design is extensive. Problem formulations and solution methods for multi-commodity facility location problems have been published since the early seventies. One of the earliest implementation of strategic facility design is given by Geoffrion and Graves Geoffrion and Graves (1974). They present a single-period mixed integer linear program for multi-commodity distribution design. Warszawski Warszawski (1973) presents model formulations for both the single-period multi-commodity location problem and the multistage location problem with several time periods. A more recent study of the multi-period location problem is presented by Canel and Das Canel and Das (1999). They present a multi-period location problem with profit maximization as the objective. The demand for different products can be served by different plants, although the model can choose to not fill all demand. The decisions to be made are where to locate facilities and when to invest in them; furthermore, the problem formulation also allows plant closing at no cost. Although most of the facility location literature assumes linear costs and revenues, this assumption might not always be realistic due to economies of scale. Harkness and ReVelle Harkness and ReVelle (2003) discuss several model formulations for the facility location problem with convex production costs.

Choice of capacity and technology can often influence the decision of where to locate manufacturing facilities; for example the decision between the numbers of production plants versus their sizes. Verter Verter (2002) presents an integrated model for simultaneously solving the facility location, capacity acquisition and technology selection problems. Although the author considers technology selection he does not include conversion of existing equipment. Verter and Dasci Verter and Dasci (2002) have expanded this work to include both product-dedicated and flexible technology alternatives. Because flexible technology alternative can manufacture the entire product range, it offers economics of scale. Both models are single-period multi-commodity optimization models. When the plant locations are predetermined, the problems from Verter (2002) and Verter and Dasci (2002) reduce to the plant loading problem. This problem is concerned with the issue of plant product mix, sourcing and market allocation in the context of a firm's supply chain for production and distribution. A model formulation of this problem with fixed facility costs and concave production costs is given by Cohen and Moon Cohen and Moon (1991). Corporations that want to evaluate their locations usually already have an existing network of facilities. The reverse location problem handles such problems where the main objective is to improve this network efficiently within a limited budget Zhang et al. (2000).

3.3.2 Supply Chain Models

The increased emphasis on improving and optimizing the supply chain, from both the business strategists and logistics practitioners, has given an opportunity and motivation for the operations research community to develop mathematical programming models for supply chain optimization Shapiro et al. (1993). Such modeling techniques are well suited to support decision making for integrated planning in the corporations' supply chain. Shapiro, Singhal and Wagner Shapiro et al. (1993) describe a mixed integer program for strategic redesign of the supply chain for a large corporation which had just recently acquired another company. The main issue in their model is to decide which plants to operate and which plants to close. The model also considers plant expansion, and deployment and utilization of equipment among the operative plants. An implementation of a decision support system for supply chain management at tactical level is given by Brown, Graves and Honczarenko Brown et al. (1987). They have developed a mixed integer program designed for facility selection, equipment location and utilization, manufacturing and distribution of products for the biscuit producer Nabisco.

Global supply chains have a number of issues that make them different from single-country supply chains Cohen et al. (1989). Some of these differences are taxes and duties, currency exchange rate fluctuation, market penetration strategies, product design differences between countries, and handling of multinational supply chain as global system to achieve economics of scale. Cohen, Fisher and Jaikumar Cohen et al. (1989) present an optimization model which has incorporated many of these international issues. The model is a mixed integer program where the objective is to maximize the total after-tax profits for the corporation. One of the main contributions of that paper is the inclusion of vendor supply contracts. A well documented application of global strategic supply chain optimization is the redesign of Digital Equipment Corporation in the 1990's Arntzen et al. (1995). The rapid changes in the computer industry in the beginning of the nineties lead to a shift in demand from large central mainframes to networks of smaller, less expensive computers. To face this challenge Digital developed a model called Global Supply Chain Model. The goal was to evaluate and redesign their worldwide manufacturing and distribution strategy. The decision support system was designed to handle questions like how many plants and distribution centers are needed, which capacity and technology should be acquired at each plant, how many plants should be involved in producing a product, and how do tax and transportation affect the optimal supply chain design.

Yan, Yu and Cheng Yan et al. (2003) present a multi-commodity and multi-echelon single period strategic supply chain model. The model's objective is to minimize total cost and they include bill-of-material considerations in the form of logical constraints. The bill-of-materials constraints are used to express the relationship between product, suppliers and producers.

Activity-based costing (ABC) is a valuable tool in determining product and customer costs for managerial decision support Shapiro (1999). ABC is a descriptive tool for allocation of costs to different cost drivers. Shapiro Shapiro (1999) discusses synergies between mathematical programming models for strategic supply chain design and ABC. The author argues that ABC analysis can be a valuable tool for developing much of the necessary cost data needed for a strategic optimization model, moreover accurate product and consumer costs can only be extracted from an optimal solution of an optimization model.

3.3.3 Supply Chain Planning in the Process Industry

Recently, there have been many contributions on optimization in forestry in the literature, which from a modeling point of view have some similarities with modeling the silicon metal industry. Carlsson and Rönnqvist Carlsson and Ronnqvist (2005) give an overview of supply chain optimization projects in the Swedish forest industry, while Bredström et al. Bredstrom et al. (2004) present a tactical decision support tool for short-term planning of daily supply chain decisions, developed for a major manufacturer of market pulp. Another model for strategic and tactical supply chain planning in the paper industry is given by Philpott and Everett Philpott and Everett (2001). For a general overview of supply chain models for strategic, tactical and operational levels in forestry, see Rönnqvist Rönnqvist (2003). By-products from forestry can be used as bio-energy fuel by conversion of by-products from sawmills and forest residue to forest fuel. Gunnarsson, Rönnqvist and Lundgren Gunnarsson et al. (2004) consider strategic analysis and tactical planning for a forest fuel supply chain.

There is some supply chain optimization literature in the smelting industry. Sinha et al. Sinha et al. (1995) describe the development of a mixed integer program for strategic decision support and planning at Tata Steel. The mathematical program was developed to give decision support for optimal allocation of steel-making capacity in order to maximize profit. The model considers the use of scarce power, operating hours, capacity of processes and upper and lower bounds on demand. Timpe and Kallrath Timpe and Kallrath (2000) discuss production planning on a tactical level in process industry. They apply a rolling horizon principle and use time periods with different lengths. Much of the focus is on turnover between different mode of operation, e.g. between production of different products. Furthermore, Wolsey Wolsey (1997) presents a more general framework for modeling production changeover in planning models.

Ulstein et al. Ulstein et al. (2006) discuss supply chain redesign in the silicon metal industry. They have developed a strategic optimization model which determinates optimal plant structure for Elkem. The main features include plant acquisitions, closings and expansions. The model also takes into consideration the product mix at each plant and decisions regarding the total product portfolio at Elkem's silicon division.

3.4 The Strategic Planning Model

The strategic planning problem described in Section 3.2 will be formulated as a deterministic problem where the objective function is to maximize the net present value of the future incomes and costs. All incomes and cost data given are discounted and an appropriate discount rate is used. Here we will present one model, but to facilitate the presentation we have split the model into three parts. First, we present the supply chain network in Section 3.4.1. Then in Section 3.4.2, the conditions related to the production of silicon and ferrosilicon products are described. Furthermore, the by-product constraints are presented in Section 3.4.3, and in Section 3.4.4 the overall objective function is given. Finally, in Section 3.4.5 we discuss some of the challenges we faced during the modeling process.

We present here a simplified version of the strategic planning model for Elkem. Some of the model features are left out to enhance the readability and understanding of the model. The features mentioned in the text that are not included in this formulation are: different types of electricity contracts and the separate investments in refining and casting equipment; instead we just consider a general type of equipment. Trade barriers and duties are omitted as well.

The notation is based on the use of lower-case letters to represent subscripts and decision variables, and capital letters to represent constants. Normally, the variables and constraints are only defined for some combination of subscripts. To limit the level of details in the mathematical formulation, we introduce only the necessary sets in the model.

3.4.1 The Supply Chain

The main decisions for the company regarding the supply chain are the operating status of the plants during the planning horizon. An existing plant can either be operating during the entire planning horizon or be closed down in any of the time periods. Due to the seriousness of a closure decision, the company cannot re-open a plant which has been closed down. However, the company can invest in new plants during the planning horizon.

Another important decision for the supply chain is the allocation to customers of products produced at particular plants. The supply chain from the raw material vendors to the plants is not included in the model, because for each plant the raw material vendors are given and there is no reason to include this issue into the model.

First, we describe the indices, sets, parameters and variables for the supply chain part of the problem. Figure 3.3 gives an overview of the variables in the supply chain.

Indices

c	customer
i	production plant
p	product
q	by-product quality
t	time period

Sets

I^N	the set of candidate plants

Parameters

CCL_{it}		cost of closing plant i in period t
CIP_{it}		cost of investing at plant i in period t
COP_{it}		cost of operating plant i in period t
CT_{ipct}		unit transportation cost for product p from plant i to customer c in period t
CMT_{iqct}		unit transportation cost for by-product quality q from plant i to customer c in period t
F_{pct}		fixed contract volume of product p for customer c in period t
FN_{ipct}		fixed contract volume for product p at potential plant i for customer c in period t
MD_{qct}		demand for by-product quality q for customer c in period t
$P0_i$	=	1, if plant i is operating at the beginning of the planning period, and 0 otherwise

PM_{qct}	unit sale price for by-product quality q to customer c in period t
PS_{pct}	unit sale price for product p to customer c in period t
S_{pct}	spot volume of product p for customer c in period t
SN_{ipct}	spot volume of product p at new plant i for customer c in period t

Variables

pin_{it}	=	1, if there is an investment at plant i in period t, and 0 otherwise
po_{it}	=	1, if plant i is operating in time period t, and 0 otherwise
xs_{ipct}		quantity of product p produced at plant i and sold to customer c in period t
ys_{iqct}		quantity of by-product quality q produced at plant i and sold to customer c in period t
zsc		the objective function related to the supply chain network

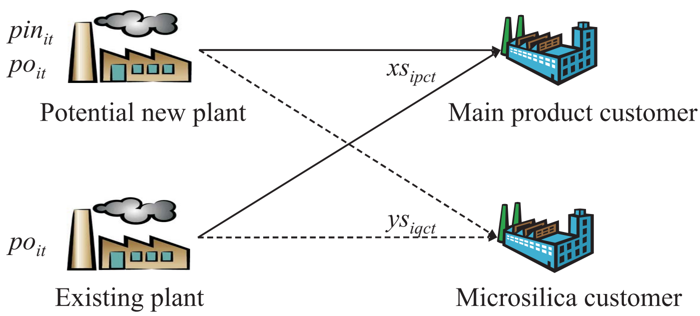

Figure 3.3: Variables in the supply chain. Solid-drawn lines indicate transportation of silicon and ferrosilicon between plants and customers while the dashed lines indicate transportation of microsilica between plants and customers

Model Formulation

The part of the model regarding the supply chain can then be written as:

$$
\begin{aligned}
\max\ zsc \quad &= \sum_i \sum_p \sum_c \sum_t \left(PS_{pct} - CT_{ipct}\right) xs_{ipct} \\
&+ \sum_i \sum_q \sum_c \sum_t \left(PM_{qct} - CMT_{iqct}\right) ys_{iqct} \\
&- \sum_i \sum_t COP_{it}\, po_{it} - \sum_i \sum_t CIP_{it}\, pin_{it} \\
&- \sum_i \sum_t CCL_{it} \left(po_{i(t-1)} - po_{it} + pin_{it}\right) \qquad (3.1)
\end{aligned}
$$

Subject to:

$$po_{it} = P0_i, \qquad \forall i, t = 0, \qquad (3.2)$$

$$po_{it} - po_{i(t-1)} - pin_{it} \leq 0, \qquad \forall i, t, \qquad (3.3)$$

$$\sum_i xs_{ipct} - \sum_{i \in I^N} \sum_{\tau \leq t} FN_{ipct}\, pin_{i\tau} \geq F_{pct}, \qquad \forall p, c, t, \qquad (3.4)$$

$$\sum_i xs_{ipct} - \sum_{i \in I^N} \sum_{\tau \leq t} \left(SN_{ipct} + FN_{ipct}\right) pin_{i\tau} \leq S_{pct} + F_{pct}, \qquad \forall p, c, t, \qquad (3.5)$$

$$\sum_i ys_{iqct} \leq MD_{qct}, \qquad \forall q, c, t, \qquad (3.6)$$

$$xs_{ipct}, ys_{iqct} \geq 0, \qquad \forall i, p, q, c, t, \qquad (3.7)$$

$$pin_{it}, po_{it} \in \{0, 1\}, \qquad \forall i, t. \qquad (3.8)$$

The objective function (3.1) expresses the profit from selling products less the associated costs. The variable costs include variable operating costs at the plants and transportation costs from the plants to customers. The fixed costs include the costs to operate plants and potential investment and closure costs at plants. Constraints (3.2) represent the initial operating status of existing and potential plants. Constraints (3.3) impose that a plant can only operate if the plant was operating in the previous time period or if the company has invested in the plant at the beginning of the time period. The company cannot re-open a plant which has been closed down. Constraints (3.4) make sure that the sales to a customer in a given time period have to be at least equal to the amount required by the fixed order contract. Constraints (3.5) require that sales to a customer in a given time period cannot exceed the sum of fixed and spot orders. In constraints (3.6), we ensure that the amount of by-products sold to a customer do not exceed the demand. Finally, constraints (3.7) and (3.8) handle the non-negativity and binary requirements.

3.4.2 The Production

Production at each plant starts with a given recipe for each product type and electricity as input to the furnaces. The smelt from the furnaces is constrained by furnace and equipment capacities. Furnaces operating with one furnace technology can be converted to another technology producing different products. Capacity can be increased by equipment investment. Figure 3.4 gives an overview of the variables in the production process.

The indices, sets, parameters and variables not defined in Section 3.4.1, will be defined in the following.

Indices

d	furnace technology
e	equipment
f	furnace

Sets

P^e	the set of products that require production equipment e
P^d	the set of products that can be produced with furnace technology d

Parameters

$CCO_{ifdd\prime t}$	cost of converting furnace f at plant i from technology d to technology $d\prime$ in period t
CE_{iet}	cost of using equipment e at plant i in period t
CEL_{it}	unit cost of buying spot electricity for plant i in period t
CIE_{iet}	cost of investing in equipment e at plant i in period t
COF_{ift}	cost of operating furnace f at plant i in period t
CR_{ifpt}	recipe costs for producing product p in furnace f at plant i in period t
EC_{iet}	capacity of equipment e at plant i in period t
ECI_{iet}	capacity increase from investing in equipment e at plant i in period t
ELC_{ifpt}	amount of electricity consumed to produce one unit of product p in furnace f at plant i in period t
ELF_{it}	amount of electricity stipulated in the fixed contract for plant i in period t

$FCAP_{ifdpt}$		capacity of furnace f with furnace technology d at plant i for the producing product p in period t
$F0_{ifd}$	=	1, if plant i has a furnace f with furnace technology d at the beginning of the planning period, and 0 otherwise
PEL_{it}		unit sale price for electricity from plant i in period t

Variables

ein_{iet}	=	1, if an investment in equipment e at plant i is made at the beginning of time period t, and 0 otherwise
elb_{it}		amount of electricity bought in the spot market for plant i in period t
els_{it}		amount of electricity sold in the spot market from plant i in period t
$fclos_{ifdt}$	=	1, if furnace f with furnace technology d at plant i is closed down in period t, and 0 otherwise
$fconv_{ifdd\prime t}$	=	1, if furnace f with furnace technology d is converting to furnace technology $d\prime$ at plant i in period t, and 0 otherwise
fo_{ifdt}	=	1, if furnace f with furnace technology d at plant i is operating in period t, and 0 otherwise
xm_{ifdpt}		quantity of smelted product p tapped from furnace f with furnace technology d at plant i in period t
zpp		the objective function related to the production process

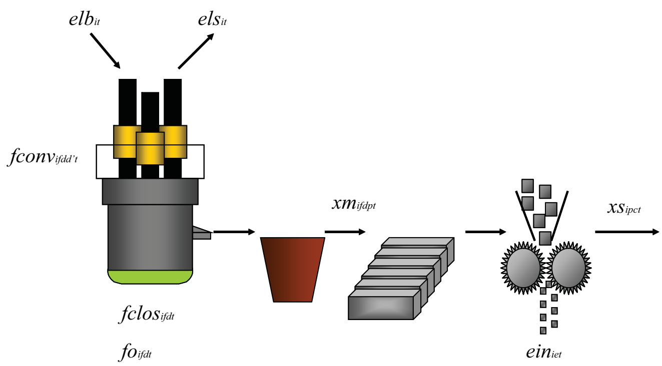

Figure 3.4: Variables in the production process

Model Formulation

The part of the model regarding the production can then be written as:

$$\begin{aligned}
\max zpp \;=\; & \sum_i \sum_t PEL_{it}\, els_{it} \\
& - \sum_i \sum_f \sum_d \sum_p \sum_t CR_{ifpt}\, xm_{ifdpt} - \sum_i \sum_t CEL_{it}\, elb_{it} \\
& - \sum_i \sum_f \sum_d \sum_t COF_{ift}\, fo_{ifdt} \\
& - \sum_i \sum_f \sum_d \sum_{d'} \sum_t CCO_{ifdd't}\, fconv_{ifdd't} \\
& - \sum_i \sum_f \sum_d \sum_e \sum_{p \in P^e} \sum_t CE_{iet}\, xm_{ifdpt} \\
& - \sum_i \sum_e \sum_t CIE_{iet}\, ein_{iet} \qquad (3.9)
\end{aligned}$$

Subject to:

$$po_{it} - \sum_d fo_{ifdt} \geq 0, \qquad \forall i, f, t, \qquad (3.10)$$

$$\sum_d fo_{ifdt} + \sum_d fclos_{ifdt} = 1, \qquad \forall i, f, t, \qquad (3.11)$$

$$\begin{aligned}
& fo_{ifdt} + fclos_{ifdt} \\
& + \sum_{d'} fconv_{ifdd't} - \sum_{d'} fconv_{ifd'dt} \\
& = F0_{ifd}|_{t=1} + (fo_{ifd(t-1)} + fclos_{ifd(t-1)})|_{t>1}, \qquad \forall i, f, d, t, \qquad (3.12)
\end{aligned}$$

$$fo_{ifdt} - \sum_p \frac{1}{FCAP_{ifdpt}} xm_{ifdpt} \geq 0, \qquad \forall i, f, d, t, \qquad (3.13)$$

$$\begin{aligned}
& \sum_f \sum_d \sum_{p \in P^e} xm_{ifdpt} - \sum_{\tau \leq t} ECI_{iet}\, ein_{ie\tau} \\
& \leq EC_{iet}, \qquad \forall i, e, t, \qquad (3.14)
\end{aligned}$$

$$\begin{aligned}
& \sum_f \sum_d \sum_p ELC_{ifpt} xm_{ifdpt} - elb_{it} + els_{it} \\
& \leq ELF_{it}, \qquad \forall i, t, \qquad (3.15)
\end{aligned}$$

$$\sum_{f}\sum_{d} xm_{ifdpt} - \sum_{c} xs_{ipct} = 0, \qquad \forall i, p, t, \tag{3.16}$$

$$xm_{ifdpt} \geq 0, \qquad \forall i, f, d, p \in P^d, t, \tag{3.17}$$

$$elb_{it}, els_{it} \geq 0, \qquad \forall i, t, \tag{3.18}$$

$$ein_{iet}, fclos_{ifdt}, fconv_{ifdd't}, fo_{ifdt} \in \{0,1\}, \qquad \forall i, e, f, d, d', t. \tag{3.19}$$

The objective function (3.9) expresses the profit from selling electricity less the associated costs. Variable costs include recipe costs, electricity costs and costs for using the equipment. The fixed costs include the costs of operating the furnaces, converting a furnace from one technology to another and investment costs in refining and casting equipment. Constraints (3.10) ensure that furnaces at a plant can only operate if the plant is in operation. Constraints (3.11) impose that each furnace is either operating or closed down in a time period. The status of each furnace in each time period is calculated in constraints (3.12), and they concern both the furnace technology and operation status. In each time period the furnace technology should be known, even if the furnace is closed down. From one period to the next, a furnace can operate or be closed with the same furnace technology. Alternately, the furnace can convert to another furnace technology. These furnace technology conservation constraints are given in (3.12). Constraints (3.13) state that the amount of all products produced in a furnace cannot exceed its capacity. Constraints (3.14) force total output not to exceed the total capacity of any equipment used. The capacity of a type of equipment in a given time period is equal to its initial capacity and the capacity added by investments in all earlier time periods. Constraints (3.15) restrict total electricity consumed or sold to that obtained from fixed contracts or purchased on the spot market. The mass balance constraints (3.16) describe the relationship between the smelted products and the sales products transported to the customers. Finally, constraints (3.17), (3.18) and (3.19) handle the non-negativity and binary requirements not declared in Section 3.4.1.

3.4.3 By-Product Production

The smelt from the furnaces is split in a given ratio between main products and by-products. The marginal costs of producing the by-product are very low and are therefore ignored in the model. Figure 3.5 gives an overview of the variables in the production of the by-product.

The indices, sets, parameters and variables not defined in Sections 3.4.1 and 3.4.2, will be defined in the following.

Sets

W the set containing the lowest quality of the by-product

Parameters

Y_{ifpq} the portion of by-product with quality q generated due to production of product p at plant i and furnace f

$YT_{qq\prime}$ = 1 if quality $q\prime$ can be transferred and sold as quality q,and 0 otherwise. For each quality $q\prime$ just one transfer is possible

Variables

ym_{iqt} quantity of by-product of quality q produced at plant i in time period t

y^{+}_{ipt} quantity of by-product of quality q that is in surplus and can be sold as a lower quality at plant i in time period t

yw_{iqt} quantity of by-product of the lowest quality q that is in surplus and must be considered as waste at plant i in time period t

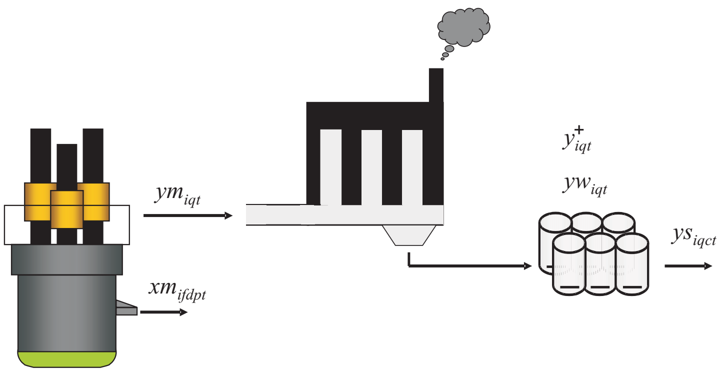

Figure 3.5: Variables in the production of the by-product

Model Formulation

The profit from the by-product production is included in the objective function term for the supply chain network. Because the marginal costs of producing the by-product are ignored in the model, we have no contribution to the objective function from the by-product production. The constraints regarding the by-product production can then be written as:

$$ym_{iqt} - \sum_{f}\sum_{d}\sum_{p} Y_{ifpq}\, xm_{ifdpt} = 0, \qquad \forall i, q, t, \tag{3.20}$$

$$ym_{iqt} + \sum_{q'} YT_{qq'}\, y^{+}_{iq't} - \sum_{q'} YT_{q'q}\, y^{+}_{iqt}$$
$$-yw_{iqt} - \sum_{c} ys_{iqct} = 0, \qquad \forall i, q, t, \tag{3.21}$$

$$ym_{iqt}, y^{+}_{iqt} \geq 0, \qquad \forall i, q, t, \tag{3.22}$$

$$yw_{iqt} \geq 0, \qquad \forall i, q \in W, t. \tag{3.23}$$

The amount of by-products generated in the furnaces depends on the products produced and the particular furnace, and this dependency is described in constraints (3.20). The mass balance of the by-product produced and sold is given by constraints (3.21). The amount by-product sold of a particular quality is equal to the amount produced adjusted for by-products sold as a lower quality than produced. With a surplus of the lowest quality, this amount of by-product is wasted. Finally, constraints (3.22) and (3.23) handle the non-negativity requirements not declared in Sections 3.4.1 and 3.4.2.

3.4.4 The Objective Function

The resulting objective function consists of the terms from the supply chain network and the production process and becomes:

$$\max\ z = zsc + zpp \tag{3.24}$$

3.4.5 Some Modeling Challenges

The first and perhaps the most important challenge in the modeling is to find a level of detail which corresponds to the decision problem at hand. For instance, it was always clear that furnace operations had to be modeled, but it soon became evident that other major equipment types also had to be included. Furthermore, given the size of the problem, we had to aggregate products of similar type into groups and customers into clusters. The aggregation of time into time periods was implemented so the user can easily change the aggregation. This enables the user to trade-off run-time and detail output of data in each run.

Another challenge was extracting and breaking down economic information about the operations of each plant. Prior to the development of the optimization tool, top managers were accustomed to comparing more aggregated economic information. The mathematical model made it necessary to derive the costs with the financial directors at the plants in a way that disaggregated the costs of operating plants into product specific activity costs per equipment and fixed and variable maintenance and administration costs.

3.5 Computational Results

An advanced version of the optimization model has been implemented and solved using Xpress-MP version 14 Dash Optimization (2002) and run on a computer with a 3GHz processor and 1GB RAM. Furthermore, the model has been tested with real data from Elkem's silicon division. Xpress-MP takes less then 30 seconds to find optimal solutions for the optimization model with 5 time periods.

Some of the plants in this division have fixed contracts and/or are producing very specialized products that other plants cannot produce. Because the future operating status of these plants does not affect the rest of the plants, they are not considered in this optimization model. As a result, the model has been tested for six plants using a rolling horizon of 11 years. Furthermore, the time periods are not equal in length. We have split the time horizon into 5 distinct time periods where the first three periods are 1 year long and the last time period is 6 years long. The choice of splitting up the planning horizon in different length of each time period is mainly due to the fact that decisions early in the planning horizon are more critical for the company.

The development and the sales of silicon and ferrosilicon metals are carried out by the silicon division itself. On the other hand, a separate business unit in Elkem has been responsible for the development, marketing and sales of the various microsilica products. This business unit is called Elkem Materials. Because the production and sales of microsilica are handled by two separate units in Elkem, there have been some discussions regarding the level of the transfer prices for microsilica. These transfer prices are currently 2/3 of the market prices. Elkem's corporate management has expressed a need for a model that can assure that the plant structure is optimal and is not affected by the chosen level of transfer prices for microsilica.

The rest of the section is organized as follows: In Section 3.5.1 we will discuss the optimal operational status for the plants. Furthermore, in Section 3.5.2 we will examine furnace conversions and optimal furnace technology for the plants, and finally, in Section 3.5.3 product allocations between plants will be discussed.

3.5.1 Operating Status of the Plants

Table 3.1 shows the transfer prices between Elkem's silicon division and Elkem Materials. A price level of 100% equals the current level of the transfer prices for microsilica, while a price level of 150% is the current market price. A price level of

0% means that the silicon division does not get any revenues from the microsilica sales. The expected net present value of the cash flow for the different scenario is standardized to be 100 for scenario F, when the transfer prices are held at the current level. The net present value increases considerably when the microsilica prices increase. The reason for this is that there are no marginal costs for the microsilica production and therefore the sale of microsilica will only give positive contribution to the profit. As we can see from Table 3.1, the operating status of plant 1 is dependent on the price obtained for microsilica. If the microsilica prices are cut to 60% of the current transfer prices or lower, it is optimal to close plant 1.

The plant operating status is much more robust for increases in the microsilica prices than for price reductions if the demand is held at a constant level. Indeed, the microsilica prices must increase up to a level of 800% before it is optimal to let plant 4 stay open in one period. The analyses show that plant 6 and plant 4 should close down in all scenarios. In fact, if management decides to keep plant 6 open in the entire planning horizon, the expected total profit for the silicon division will cut by 10 %.

The sales department at Elkem Materials is constantly developing the end-market for microsilica. As a consequence, the market for high quality microsilica is likely to increase during the next decade. A linear annual growth of 20% in demand is a well-supported assumption. Table 3.2 shows the optimal operating status for the plants under these conditions. The plant operating status is very robust for changes in microsilica demand, and a higher demand for microsilica does not affect the future plant status.

3.5.2 Furnace Technology Conversions

To produce high-quality microsilica, a special furnace technology has to be installed at the smelting plants. Such furnace conversion is expensive, and will only be considered if Elkem can sell this high-quality microsilica with a positive profit. Moreover, plants that are only capable of producing ferrosilicon products have to convert their furnaces in order to produce silicon products if profitable. These conversions are even more expensive than converting furnaces to produce high-quality microsilica.

Table 3.3 lists the optimal furnace technology at each open plant in scenario A-I. The table shows that the price level of microsilica affects the product range at Elkem's smelting plants. Today, only plant 2 is capable of producing the finest microsilica quality. On the other hand, plant 3 has a lower cost than plant 2, and Elkem can increase its profitability by reallocating the high quality microsilica products to plant 3 in all scenarios except in scenario A. Although it is profitable to reallocate this microsilica quality from plant 2, it is still optimal

Table 3.1: Optimal plant operating status for different microsilica price scenarios. A price level of 100% reflect the current level of transfer prices for microsilica and a price level of 150% is the current market prices for microsilica. MS price = microsilica price, NPV = net present value

Scenario	MS Price	NPV	Plant 1	Plant 2	Plant 3	Plant 4	Plant 5	Plant 6
A	0 %	59	Closed	Open	Open	Closed	Open	Closed
B	20 %	65	Closed	Open	Open	Closed	Open	Closed
C	40 %	73	Closed	Open	Open	Closed	Open	Closed
D	60 %	80	Closed	Open	Open	Closed	Open	Closed
E	80 %	90	Open	Open	Open	Closed	Open	Closed
F	100 %	100	Open	Open	Open	Closed	Open	Closed
G	150 %	126	Open	Open	Open	Closed	Open	Closed
H	500 %	305	Open	Open	Open	Closed	Open	Closed
I	800 %	463	Open	Open	Open	1 period	Open	Closed

Table 3.2: Optimal plant operating status with 20% linear annual growth in microsilica demand. MS = microsilica

Scenario	MS Price	MS Demand	NPV	Plant 1	Plant 2	Plant 3	Plant 4	Plant 5	Plant 6
J	60 %	120 %	83	Closed	Open	Open	Closed	Open	Closed
K	80 %	120 %	93	Open	Open	Open	Closed	Open	Closed
L	100 %	120 %	105	Open	Open	Open	Closed	Open	Closed
M	150 %	120 %	133	Open	Open	Open	Closed	Open	Closed
N	200 %	120 %	161	Open	Open	Open	Closed	Open	Closed

to leave plant 2 open producing silicon metal. Both plants 1 and 5 are today producing ferrosilicon products, but by converting the furnaces, the plants can also produce silicon products. A conversion of furnaces at plant 1 is profitable as long as the microsilica prices do not reduce to below 30% of the current transfer price. Furthermore, if the microsilica price is 30% of the current transfer price or lower, it is more profitable to close plant 1 and instead convert plant 5 to handle the demand for silicon products. Note that plant 5 is open in all of these scenarios, but the product mix at the plant is dependent on the price obtained for the by-products.

Table 3.3: Optimal furnace technology for the plants. FeSi = Ferrosilicon products, Si = Silicon products, hqMS = high quality microsilica, * = plant is open first period and closed in the other periods

Scenario	Plant 1	Plant 2	Plant 3	Plant 4	Plant 5	Plant 6
A	Closed	Si, hqMS	Si	Closed	Si, FeSi	Closed
B	Closed	Si, hqMS	Si, FeSi, hqMS	Closed	Si, FeSi	Closed
C	Closed	Si, hqMS	Si, FeSi, hqMS	Closed	Si, FeSi	Closed
D	Closed	Si, hqMS	Si, FeSi, hqMS	Closed	Si, FeSi	Closed
E	FeSi, Si	Si, hqMS	Si, FeSi, hqMS	Closed	FeSi	Closed
F	FeSi, Si	Si, hqMS	Si, FeSi, hqMS	Closed	FeSi	Closed
G	FeSi, Si	Si, hqMS	Si, FeSi, hqMS	Closed	FeSi	Closed
H	FeSi, Si	Si, hqMS	Si, FeSi, hqMS	Closed	FeSi	Closed
I	FeSi, Si	Si, hqMS	Si, FeSi, hqMS	Si*	FeSi	Closed

3.5.3 Product Allocation Between Plants

As a consequence of furnace technology conversions and plant closings, the portfolio of the main products will also change. The optimal overall product mix when the silicon division obtains prices at the current transfer rate for the microsilica production (scenario F) is 65% ferrosilicon products and 35% silicon products. On the other hand, if there are no revenues from the microsilica production (scenario A), some of the ferrosilicon contracts will no longer be attractive and Elkem should reduce this production to a level of 47% of the main products from the silicon division. This means that even if Elkem chooses to close down a smelting plant producing silicon products when the microsilica market collapses, Elkem should also convert furnaces at other plants to secure the total production of silicon metal. The reason for this is that the silicon products have higher margins than the ferrosilicon products.

Another issue is how the product mix at the plants depend on the microsilica prices. Plant 5 is today a ferrosilicon plant, and as long as plant 1 is open, plant 5 will continue to produce ferrosilicon products. On the other hand, when it is optimal to close plant 1, Elkem should convert one furnace to produce silicon metal at plant 5. When there are no revenues from microsilica production (scenario A), the optimal product mix for plant 5 is 25% silicon metal and 75% ferrosilicon metal.

The optimal production mix changes as a consequence of reduced production volume for ferrosilicon metal when the microsilica transfer prices are cut down. The reason for this is that the sales revenues from selling microsilica with no marginal production costs contribute to a positive margin for the low-margin products. If the revenues from microsilica sales are removed, these low-margin products will not be profitable anymore, and the plants producing these low margin products will have to close down. Note that neither the optimal product mix nor optimal production volume is affected by the current microsilica transfer price rate.

3.6 Concluding Remarks

In this paper we have presented a mixed integer programming model for strategic supply chain optimization. The model is tailor-made for actors in the metal industry, and handles the production and sales of by-products in addition to the production and sales of the main products. Moreover, the model optimizes the supply chain design and allocates the production of both main products and by-products at the different plants. It also considers plant and furnace closings, plant acquisitions, capacity extensions, and technology conversions.

The model has been tested with real data from Elkem, the largest producer of silicon metal in the world and the western world's largest producer of ferrosilicon products. The testing of the model for different scenarios shows that the production and sales of by-products influence the optimal supply chain configuration. In fact, for a smelting plant the sales of by-products can make the difference between negative and positive profit. As a consequence, the sales of by-products may help the plants to survive in a harsh situation. Furthermore, the sales of by-products influence the product mix at the smelting plants and the total corporate product portfolio.

The optimization model described in this paper is an extension of the model described in Ulstein et al. (2006), and the optimization tool from Ulstein et al. (2006) is used in Elkem today. Due to the importance of the strategic restructuring process, Elkem employed optimization experts from our research modeling team to support further model developments and use. These optimization experts have triggered the extension of the model presented in this paper. The new version of the model is about to be implemented in the company.

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Paper III

Roar Grønhaug and Marielle Christiansen:

Supply Chain Optimization for the Liquefied Natural Gas Business

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Chapter 4

Supply Chain Optimization for the Liquefied Natural Gas Business

Abstract:
The importance of natural gas as an energy source is increasing. Natural gas has traditionally been transported in pipelines, but ships are more efficient for transportation over long distances. When the gas is cooled down to liquid state it is called *liquefied natural gas* (LNG). The LNG supply chain consists of exploration, extraction, liquefaction, transportation, storage and regasification. Maritime transportation is a vital part of the LNG supply chain, and LNG is transported in special designed ships, LNG tankers. The demand for LNG tankers has increased considerably as the entire LNG industry continues to see strong growth. Hence, there is a great potential and need for optimization based decision support to manage the LNG fleet, liquefaction plants, and regasification terminals in this business.
Here, we are studying the LNG supply chain in close cooperation with a worldwide actor within the LNG business. This actor is responsible for the LNG supply chain management except the exploration and extraction.
We describe the real planning problem and present both an arc-flow and a path-flow model of the problem. Both models are tested and compared on instances motivated from the real-world problem. It is a very complex problem, so only small instances can be solved to optimality by these solution approaches.

4.1 Introduction

Worldwide, there are large reserves of natural gas. Several existing gas producers are increasing their production capacity and new sources are explored. However, in some of these areas there are no significant markets (for instance North Africa, West Africa, South America, The Caribbean, The Middle East, Indonesia, Malaysia and Northwestern Australia). Some of the natural gas is liquefied at these locations for shipping to areas far away where usage of natural gas exceeds indigenous production. Such markets include Japan, Taiwan, Korea, Europe and the U.S. The transformation process from gas to liquefied natural gas (LNG) is done by cooling down the gas at atmospheric pressure at a temperature of $-260°$F ($-162°$C) before loading it into special designed tank ships, LNG tankers. By liquefying the natural gas into LNG the volume is reduced by a factor of 610

(EIA, 2003). The reduction in volume makes transportation and storage more efficient. In addition, LNG offers greater trade flexibility than pipeline transport, allowing cargoes of natural gas to be delivered where the need is greatest and the commercial terms are most competitive.

Natural gas as an energy source is of increasing importance as the world's demand for natural gas is expected to increase by 70% between 2002 and 2025 (EIA, 2005). Hence, the demand for LNG tankers is increasing. In 2007 there were 220 LNG tankers in operation, and 35 LNG tankers were scheduled for delivery in 2007. Furthermore, by 2015 the number of LNG tankers in operation will almost double to 400 (IEA, 2007). As a consequence of the increasing market for LNG, the supply chain management has become more complex and the need for decision support has become even more evident. We consider a real tactical supply chain optimization problem for LNG including the production volumes, liquefaction, transportation, storage, regasification and sale volumes. Suez Energy International (SEI) is a global energy actor and is facing such a planning problem. The company is involved within most of the LNG supply chain except exploration and extraction, and is using a number of liquefaction plants and regasification terminals throughout the world. For the company's activity, the LNG can be considered a single product. The natural gas is cooled down at the liquefaction plants, stored at given pick-up ports, and transported at sea by LNG tankers to inventories at delivery ports before regasification. Inventory storage capacities are given at all ports. The production and consumption volumes are variable at all terminals. The transportation at sea is carried out with SEI's own heterogeneous fleet of LNG tankers. The hold at the LNG tanker is separated into several cargo tanks. It is assumed that an LNG tanker is always fully loaded when it leaves the pick-up port, but it is possible to unload a variable number of cargo tanks at each regasification terminal. In fact, the LNG is at boiling state in the cargo tanks. Thus, some of the LNG evaporates during a voyage. Hence the term boil-off. This gas is used as fuel. The planning problem is to maximize the profit by designing routes and schedules for the fleet, including determining the production and consumption volumes at all terminals, without exceeding the ship capacities and the inventory limits of the storages. We call this problem the LNG inventory routing problem (LNG-IRP).

Maritime transport optimization is a well established field of research within transportation planning with reviews in Ronen (1983, 1993), Christiansen et al. (2004) and Christiansen et al. (2007). Though the attention to maritime transportation has been limited compared to other modes of transportation, we have witnessed an accelerating amount of research in the literature during the last decade and the interest in these types of problems is increasing.

In maritime transportation, usually large quantities are loaded and unloaded at each port call (ship visit at port). Both the (un)loading and transportation are time consuming and expensive. Thus, the potential is great if the planning of the transportation and the inventory management at each end of a sailing leg is integrated. In practice, we can find several maritime supply chains where one of the actors has the responsibility for both the transportation and the inventory management. For instance, Christiansen (1999) studies such a problem for a company producing and consuming ammonia. Here the company both produces and consumes the product and is controlling the fleet of ships. Furthermore, Al-Khayyal and Hwang (2007) consider a maritime inventory routing problem with multiple chemical and oil products. These products have to be transported in separated compartments on board the ship and stored in separate storages at the ports. Moreover, Persson and Göthe-Lundgren (2005) study a planning problem that integrates both the shipment planning of petroleum products from refineries to depots, and the production scheduling at the refineries. More maritime inventory routing problems are referred in Christiansen and Fagerholt (2007).

However, no research on LNG-IRP is reported in the literature as far as we know. With increased focus on this type of problems in the industry, we expect several contributions in near future.

The purpose of this paper is to introduce a new type of problem within maritime transportation and provide two types of formulations for the same problem. Moreover, it will contribute to increased knowledge about the LNG supply chain from an OR point of view.

The rest of the paper is organized as follows: Section 4.2 gives some insights into the LNG industry and describes the real planning problem considered. The problem is formulated as an arc-flow model in Section 4.3, while Section 4.4 is devoted to the path-flow model. Computational results on small instances of the real planning problem are reported in Section 4.5. Finally, concluding remarks and future research follow in Section 4.6.

4.2 Description of a Real LNG Supply Chain Planning Problem

Suez Energy International (SEI) is a global energy actor. The company is a subsidiary of the international conglomerate Suez and together with its sister company, Suez Energy Europe (SEE) has the responsibilities of maintaining Suez' energy operations. SEI are involved within most of the liquefied natural gas (LNG) supply chain except exploration, extraction, and transportation to end-customers. Hence, the company is involved in liquefaction, transportation, storage and regasification of LNG. In addition, the company can influence the amount produced at the liquefaction plants and sold at the regasification terminals. Figure 4.1 shows the LNG supply chain and highlights the considered parts of the chain.

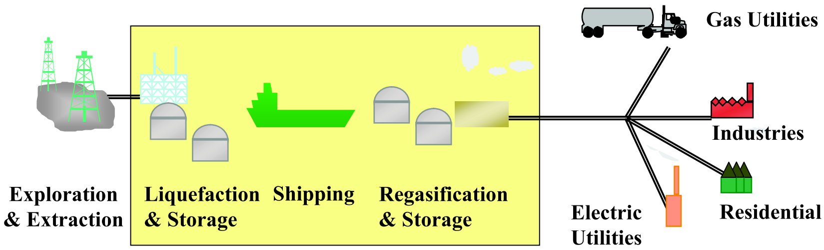

Figure 4.1: The LNG supply chain

SEI is engaged in LNG supply chain planning at all levels, ranging from strategic decisions as determining the fleet size and mix, acquisition of plants and terminals, long-term contracts, to operational planning like determining the speed of each LNG tanker. In this paper, we consider the tactical supply chain planning problem and the typical planning horizon spans two to four months.

At the liquefaction plants the natural gas is cooled down to liquid state. Then, from ports located close to the liquefaction plants, the LNG is transported in special purpose vessels to ports close to storages and regasification terminals. Here, the LNG is converted from the liquefied state to the gaseous state, ready to be moved to the final destination through the natural gas pipeline system.

SEI controls two regasification terminals located in Zeebrugge, Belgium and Boston, USA. It has also 10% equity participation in a liquefaction plant located in Trinidad and Tobago. In addition, the company uses third-party facilities for pick-up and deliveries in other parts of the world. SEI currently purchases and distributes approximately 8 million tons of LNG per year from Algeria, Quatar, Trinidad and Tobago. The LNG operations are continuously increasing. For instance, a sales and purchase contract for 2.5 million tons of LNG per year was signed with Yemen LNG in August 2005. This contractual supply is expected to begin in 2009, and has 20 years duration.

Due to the expected increase in activity, SEI sees the need for an advanced decision support tool to coordinate and manage the fleet, and the inventories at both liquefaction and regasification terminals. For that reason, inventory management considerations are included at all ports. A reality consisting of 10 liquefaction plants and 10 regasification terminals is not impossible to imagine in the future. The number of spot cargoes in the LNG business is still limited, but we will see an increase in this activity in the future. In order to limit the size of the model and the level of details, we have disregarded possible spot trade in this paper.

The production of LNG at the liquefaction plants is normally at a maximum level. However, it is possible to regulate the production within certain limits. At each plant there exists given capacities of the storages. Production costs are dependent on volume and plant.

At the other end of the supply chain, the gas is unloaded from the LNG tankers and stored in storages with specified capacities. SEI's customers are governments, industrial corporations, the service industry, and residential users throughout the world. The sales contracts include fixed contracts where the agreed volume cannot be violated, contracts with lower and upper limits on quantities to deliver, and short term contracts which should be satisfied only if profitable. From this contract structure, we assume that for each port, we can specify upper and lower limits of demand for gas and an associated revenue per day. In reality, the consumption rate varies from day to day.

Extraction of natural gas, and hence the production of LNG, takes place all over the world. Natural gas is transported by either pipe lines or by ship to the customers. The main flows of natural gas and LNG in the world are shown in Figure 4.2. Hence, the ports associated to the LNG liquefaction plants and regasification terminals are placed all over the world.

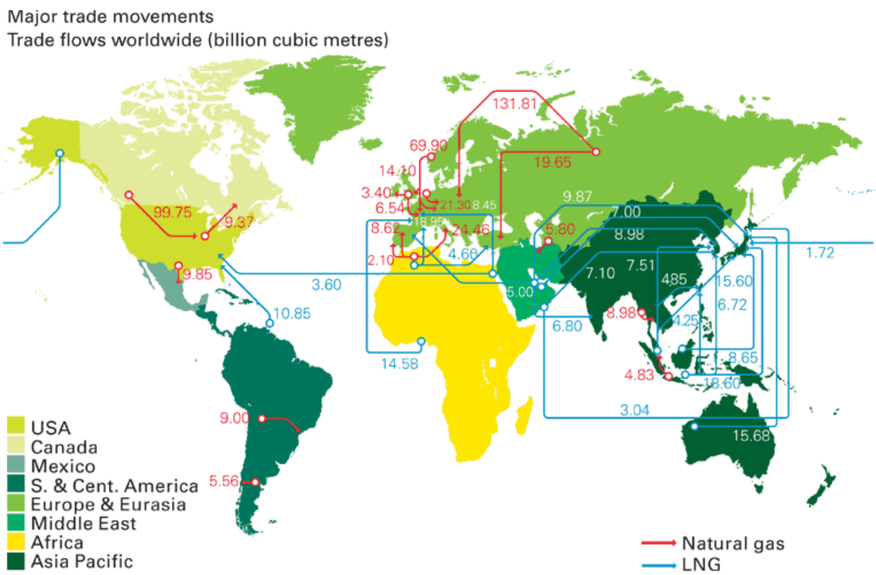

Figure 4.2: The main flows of natural gas and LNG in the world. Figure from BP (2007)

The LNG is transported by a fleet of LNG tankers controlled by SEI or by the associated company Suez Energy Europe. This fleet consists currently of 6 LNG tankers which they either have ownership over, or have chartered on long-term agreements. However, this number of LNG tankers will increase with increased activity. The LNG tankers have different cost structure, load capacity and specific ship characteristics. The hold of an LNG tanker is separated into several cargo tanks. Since the LNG is at boiling state in the cargo tanks, some of the cargo evaporates each day. This is called boil-off. Each day, the amount of boil-off in each cargo tank is a constant rate of the cargo capacity in the tank. Usually, the boil-off is used as fuel. It is the cargo itself that keeps the tanks cool, so if a cargo tank runs empty, the temperature will gradually increase. It is costly and time consuming to recool the cargo tanks before loading. Thus, there should always be some LNG left in the cargo tanks to keep them cool until (re)loading starts. Then, only a safety level should be left in the cargo tanks. No boil-off is assumed for the active tanks during loading and unloading in a port, while boil-off is considered for the tanks not affected at delivery ports. The loading and unloading of a ship are assumed to take one time period (one day) independent of the quantity loaded.

Successive calls at liquefaction plants are not relevant to consider. Furthermore, we can disregard the safety level from the calculations if we reduce the tank capacities with an appropriate safety level. Hence, when we speak about an 'empty cargo tank', there is a safety level of cargo left in the tank. In an optimal plan, an LNG tanker always arrives at a pick-up port and starts loading in the moment all the cargo tanks are empty and departs from the port fully loaded. However, at the delivery ports it is possible to unload partially. This means that several regasification terminals might be called in sequence, and a maximum number of successive delivery ports is given. In practice, this number is two. Due to sloshing problems for some types of LNG tankers, it is assumed that it is impossible to unload partial cargo tanks. Thus, a number of full cargo tanks adjusted for future boil-off until the next call to a liquefaction plant, must be unloaded at each delivery port. Figure 4.3 shows four snapshots of a voyage for an LNG tanker containing four cargo tanks. In Figure 4.3a), the LNG tanker leaves the liquefaction plant fully loaded and the storage there is in one of its extreme situation; empty. The LNG tanker sails to a regasification terminal, and it has to arrive this terminal before the storage is empty. In Figure 4.3b), we see that some of the gas has evaporated while sailing. The LNG tanker can then unload one or several of its cargo tanks. In this example, all tanks have been unloaded in one regasification terminal. Then, the LNG tanker returns in Figure 4.3d) to the same liquefaction plant and the LNG tanker is just empty when it arrives the port.

The sailing time from one port to another is calculated based on the speed of the LNG tanker and the distance, but does not depend on the load aboard. There

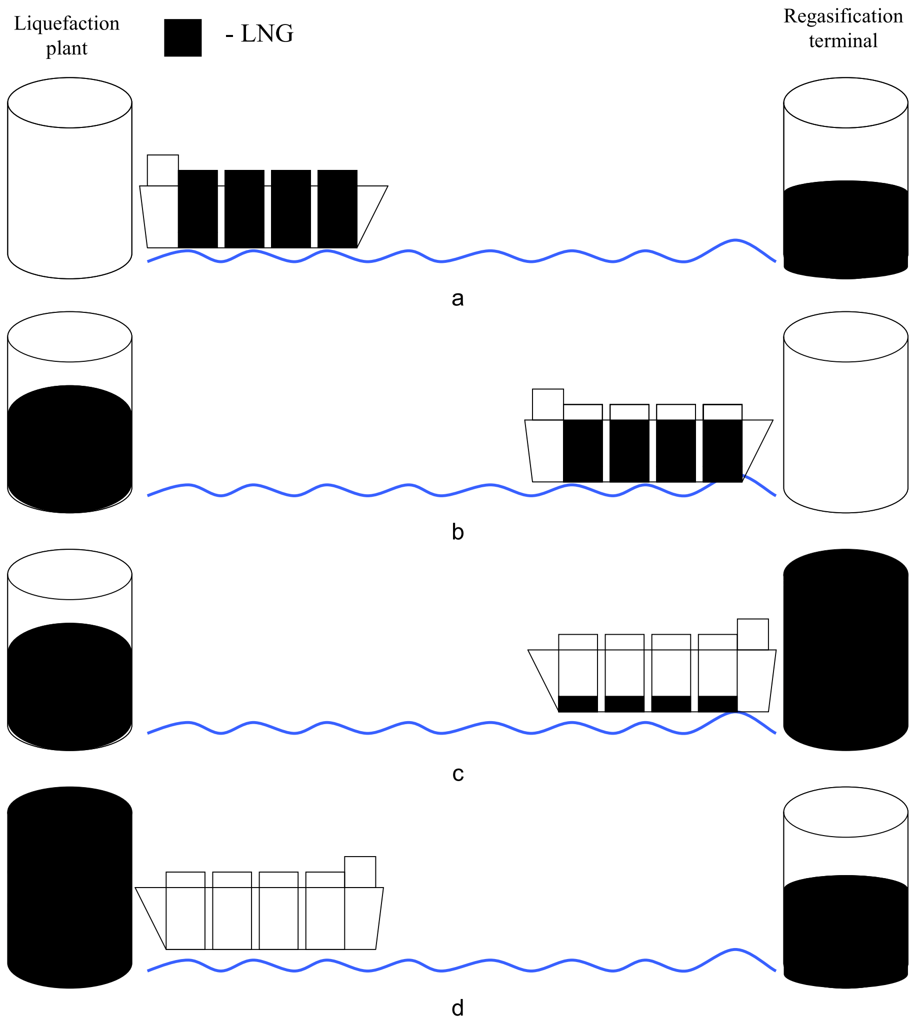

Figure 4.3: LNG tanker inventory

might also be several paths between two ports with different time consumptions and costs. For instance, this gives the possibility to use the Suez Canal or to sail around Africa.

The berth capacity of the ports is limited. Thus, a maximum number of LNG tankers can visit each port in each time period. However, it is possible to wait outside a port before loading and unloading, and the maximum number of waiting days outside each port is given. Normal boil-off is assumed during such waiting days. In contrast, no boil-off is assumed for the time periods from the last port call in a ship route until the end of the planning horizon. There is no natural depot for the LNG tankers. The initial position of an LNG tanker may be at a port or a point at sea. Furthermore, the LNG taker might be empty or loaded, and there is a set of first port call candidates in its route. Since there is no depot for the LNG tankers, there is no requirement for a specified position for any ship at the end of the planning horizon. In fact, the LNG tanker will end their route in one of the ports in the planning problem.

The LNG tankers are very specialized tank ships without any other area of application. In the short-term, there is no option to change the fleet size. The ship costs consist of several components. The fixed costs are the time charter rates which exist for all ships, while the variable costs consist of port and canal fees, and bunker oil costs.

In contrast to pickup and delivery vehicle routing problems (Desaulniers et al., 2002), the number of calls to a port is not known, the quantity loaded or unloaded at each call is unknown and finally, there exist no pickup and delivery pairs. The LNG-inventory routing problem (LNG-IRP) aims at maximizing the profit by designing ship routes and schedules for the fleet in the planning period. Furthermore, the problem consists of deciding the production volumes of LNG, and determining the level of demand fulfillment. Finally, feasible inventory levels at both port types and load aboard the LNG tankers regarding the ship capacity and boil-off must be ensured.

4.3 Arc-Flow Formulation

This section describes the arc-flow formulation of the LNG-IRP. First, in Section 4.3.1 we introduce the network and describe the ship routing and scheduling constraints for the problem. Then, in Section 4.3.2 we present the constraints representing the ship inventory management. Section 4.3.3 is devoted to the activities at the ports, including the port inventory management. Finally, the objective function is addressed in Section 4.3.4.

The notation is based on the use of lower-case letters to represent decision variables and indices, while capital letters represent sets, constants and any constant superscripts.

4.3.1 Ship Routing and Scheduling

In the mathematical description of the problem, let $\mathcal{N}$ be the set of physical ports indexed by i. This set consists of pick-up ports $\mathcal{N}^P$ and delivery ports $\mathcal{N}^D$. Further, let $\mathcal{V}$, indexed by v, represent the heterogeneous fleet of ships (LNG tankers) available for routing and scheduling. Then, the set $\mathcal{N}_v^{PD}$ denotes all ports feasible for ship v (except its origin and destination node). Furthermore, $(\mathcal{N}_v, \mathcal{A}_v)$ is the total network associated with a specific ship v. Here, $\mathcal{N}_v = \mathcal{N}_v^{PD} \cup \{o(v), d(v)\}$ is the set of ports that ship v can visit, and $o(v)$ and $d(v)$ are the (artificial) origin node and (artificial) destination node, respectively. The set $\mathcal{A}_v$ contains all feasible arcs for ship v, which is a subset of $\{i \in \mathcal{N}_v\} \times \{i \in \mathcal{N}_v\}$. This set will be calculated based on capacity, time and inventory constraints, and other restrictions such as those based on precedence of pick-up and delivery nodes. From these calculations, we can extract the sets $\mathcal{N}_v^P = \mathcal{N}^P \cap \mathcal{N}_v$ and $\mathcal{N}_v^D = \mathcal{N}^D \cap \mathcal{N}_v$ consisting of pick-up and delivery nodes that ship v may call, respectively.

The length of the planning horizon is given by the parameter T^{MX}. Moreover, $\mathcal{T}$ denotes the set of time periods, $\mathcal{T} = \left\{1, 2, \ldots, T^{MX}\right\}$, which is indexed by t. Let the parameter T_{ijv} represent the sailing time on arc (i, j) for ship v. Sailing on arc (i, i) is considered waiting outside port i, $T_{iiv} = 1$. The maximum number of time periods a ship can wait outside a port before loading or unloading is denoted T^W. Cargo handling, i.e. loading and unloading, is assumed to take one time period. To ease the representation, the cargo handling at port i is assumed to take place during the first time period on the sailing on arc $(i, j), i \neq j$. Each ship has a number of cargo tanks, W_v^{MX}, where the set of cargo tanks on each ship is given by $\mathcal{W}_v$ and w is the corresponding index.

The binary flow variable x_{ijvt}, $(i, j) \in \mathcal{A}_v$, $v \in \mathcal{V}$, $t \in \mathcal{T}$ serves two purposes; sailing between two ports and waiting outside a port. If the variable equals 1 and $i = j$, ship v waits one time period outside port i. On the other side, when $i \neq j$ and $x_{ijvt} = 1$, ship v either loads or unloads at port i in time period t before it immediately starts sailing toward port j. The decision to load or unload a cargo tank is handled by the binary variable z_{iwvt}, $i \in \mathcal{N}_v^{PD}$, $w \in \mathcal{W}_v$, $v \in \mathcal{V}$, $t \in \mathcal{T}$, which equals 1 if ship v decides to load or unload cargo tank w in port i during time period t. Furthermore, the binary variable u_{ivt}, $i \in \mathcal{N}_v^{PD}$, $v \in \mathcal{V}$, $t \in \mathcal{T}$ equals 1 if ship v loads or unloads any cargo tank in port i during time period t.

In order to increase the readability of the arc-flow model, we eliminate the possibility of several paths between two nodes. However, this can easily be included in the model by introducing an additional index for the paths on the flow variable.

Then, the routing and scheduling part of the arc-flow LNG-IRP formulation is as follows:

$$\sum_{i \in \mathcal{N}_v^{PD}} x_{jivt} - \sum_{i \in \mathcal{N}_v^{PD} | t > T_{ijv}} x_{ijv(t-T_{ijv})} = 0, \quad \forall j \in \mathcal{N}_v^{PD}, v \in \mathcal{V}, t \in \mathcal{T}, \quad (4.1)$$

$$\sum_{j \in \mathcal{N}_v} x_{o(v)jvt} = 1, \quad \forall v \in \mathcal{V}, t = 1, \quad (4.2)$$

$$\sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{N}_v} x_{id(v)vt} = 1, \quad \forall v \in \mathcal{V}. \quad (4.3)$$

Constraints (4.1)-(4.3) describe the flow on the route used by ship v. The first sailing from ship v's origin node, $o(v)$, is handled by constraints (4.2), while constraints (4.3) give the end conditions for ship v, i.e. the ship must end its route in the destination node $d(v)$.

$$\sum_{j \in \mathcal{N}_v | j \neq i} x_{ijvt} - u_{ivt} = 0, \quad \forall i \in \mathcal{N}_v^{PD}, v \in \mathcal{V}, t \in \mathcal{T}, \quad (4.4)$$

$$x_{iivt} u_{ivt} = 0, \quad \forall i \in \mathcal{N}_v^{PD}, v \in \mathcal{V}, t \in \mathcal{T}, \quad (4.5)$$

$$\sum_{\tau = t | \tau \leq T^{MX} - T^W - 2\min_j \{T_{ijv}\}}^{t + T^W + 2\min_j \{T_{ijv}\}} x_{iiv\tau} \leq T^W, \quad \forall i \in \mathcal{N}_v^{PD}, v \in \mathcal{V}, t \in \mathcal{T}, \quad (4.6)$$

$$x_{ijvt} \left(\sum_{w \in \mathcal{W}_v} z_{iwvt} + \sum_{\tau = t | \tau \leq T^{MX} - T_{ijv}}^{t + T^W} \sum_{w \in \mathcal{W}_v} z_{jwv(\tau + T_{ijv})} - W_v^{MX} \right) = 0,$$
$$\forall i \neq j, i \in \mathcal{N}_v^D, j \in \mathcal{N}_v^D, (i,j) \in \mathcal{A}_v, v \in \mathcal{V}, t \in \mathcal{T}. \quad (4.7)$$

In constraints (4.4) we describe the connection between the cargo handling and the sailing. If a ship starts sailing between two ports, it must either load or unload depending on the type of port. Moreover, constraints (4.5) state that a ship cannot wait outside a port, i.e. traverses on an arc (i,i), in the same time period it loads or unloads. Constraints (4.6) limit the number of waiting days outside port i for ship v. Furthermore, constraints (4.7) assure that if a ship calls two consecutive delivery ports, all cargo tanks must unload at these ports.

$$z_{iwvt} \in \{0,1\}, \quad \forall i \in \mathcal{N}_v^{PD}, w \in \mathcal{W}_v, v \in \mathcal{V}, t \in \mathcal{T}, \quad (4.8)$$

$$u_{ivt} \in \{0,1\}, \quad \forall i \in \mathcal{N}_v^{PD}, v \in \mathcal{V}, t \in \mathcal{T}, \quad (4.9)$$

$$x_{ijvt} \in \{0,1\}, \quad \forall (i,j) \in \mathcal{A}_v, v \in \mathcal{V}, t \in \mathcal{T}. \quad (4.10)$$

Finally, the formulation involves binary requirements (4.8)-(4.10).

A possible route through the network is illustrated in Figure 4.4. The ship starts from its origin node in time period 1. The ship is either at sea and spends one time period to reach pick-up port i or it waits outside port i in this time period. After the arrival at port i in time period 2 the ship loads its cargo tanks in this time period, before it sails to delivery port j. The sailing time between i and j is three time periods. When the ship has unloaded some of its cargo tanks in port j in time period 6, the ship starts sailing towards delivery port k in time period 7. Here, the ship unloads the rest of the cargo tanks and starts sailing towards port i. The ship arrives port i in time period 14, but waits one day outside the port, before the ship loads all its cargo tanks and sails towards the destination node.

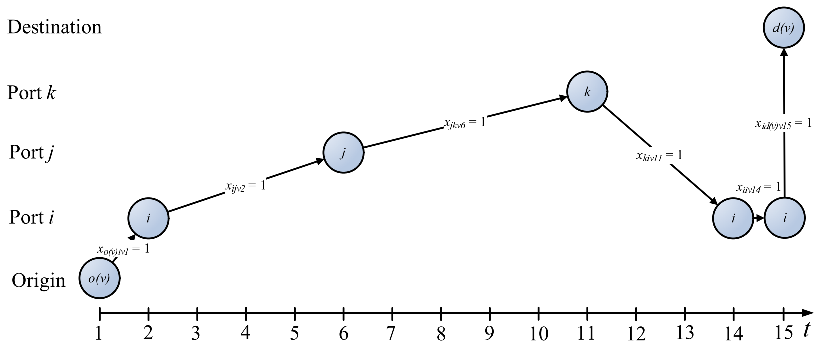

Figure 4.4: Illustration of a possible route through the network for ship v

4.3.2 Ship Inventory Management

In order to describe the ship inventory management part of the LNG-IRP, we need the following additional notation.

The capacity of cargo tank w on ship v is given by L_{wv}. There is no loading or unloading at the origin node $o(v)$, but all cargo tanks have an initial load at the beginning of the planning horizon, L^O_{wv}. Furthermore, the parameter I_i, equals -1 if port i is a pick-up port, and 1 if the port is an delivery port. The boil-off parameter B^F_{wv}, states the amount of cargo evaporating in each time period. A *duty* is a journey which starts when a ship either loads all its cargo tanks in a pick-up port or leaves the origin node, and the duty ends immediately before the ship starts loading at the next call at a pick-up port or when the ship reaches the destination node. We can for ship v calculate upper and lower bounds on the total sailing time including waiting for the duties with visiting delivery port i. These upper and lower bounds are denoted T_{iv}^{DMN} and T_{iv}^{DMX}, respectively.

The load in cargo tank w at ship v at the end of time period t is measured by the continuous variable l_{wvt}, $w \in \mathcal{W}_v$, $v \in \mathcal{V}, t \in \mathcal{T}$. Note the initial condition $l_{wv0} = L_{wv}^{O}$. Finally, the continuous variable q_{iwvt}, $i \in \mathcal{N}_v^{PD}$, $w \in \mathcal{W}_v$, $v \in \mathcal{V}$, $t \in \mathcal{T}$ measures the amount of cargo loaded into or unloaded from cargo tank w at ship v in time period t.

$$- l_{wvt} + l_{wv(t-1)} + B_{wv}^{F} \left(\sum_{i \in \mathcal{N}_v^{PD}} z_{iwvt} + \sum_{i \in \mathcal{N}_v} \sum_{\tau=1}^{t-1} x_{id(v)v\tau} \right) - \sum_{i \in \mathcal{N}_v^{PD}} I_i q_{iwvt} = B_{wv}^{F}, \forall w \in \mathcal{W}_v, v \in \mathcal{V}, t \in \mathcal{T}, \quad (4.11)$$

$$W_v^{MX} u_{ivt} - \sum_{w \in \mathcal{W}_v} z_{iwvt} = 0, \quad \forall i \in \mathcal{N}_v^{P}, v \in \mathcal{V}, t \in \mathcal{T}, \quad (4.12)$$

$$q_{iwvt} - L_{wv} z_{iwvt} = 0, \quad \forall i \in \mathcal{N}_v^{P}, w \in \mathcal{W}_v, v \in \mathcal{V}, t \in \mathcal{T}, \quad (4.13)$$

$$u_{ivt} - z_{iwvt} \geq 0, \quad \forall i \in \mathcal{N}_v^{D}, w \in \mathcal{W}_v, v \in \mathcal{V}, t \in \mathcal{T}, \quad (4.14)$$

$$u_{ivt} - \sum_{w \in \mathcal{W}_v} z_{iwvt} \leq 0, \quad \forall i \in \mathcal{N}_v^{D}, v \in \mathcal{V}, t \in \mathcal{T}, \quad (4.15)$$

$$q_{iwvt} - \left(L_{wv} - B_{wv}^{F} T_{iv}^{DMX}\right) z_{iwvt} \geq 0, \quad \forall i \in \mathcal{N}_v^{D}, w \in \mathcal{W}_v, v \in \mathcal{V}, t \in \mathcal{T}, \quad (4.16)$$

$$q_{iwvt} - \left(L_{wv} - B_{wv}^{F} T_{iv}^{DMN}\right) z_{iwvt} \leq 0, \quad \forall i \in \mathcal{N}_v^{D}, w \in \mathcal{W}_v, v \in \mathcal{V}, t \in \mathcal{T}, \quad (4.17)$$

$$0 \leq l_{wvt} \leq L_{wv}, \quad \forall w \in W_v, v \in \mathcal{V}, t \in \mathcal{T}. \quad (4.18)$$

The inventory balance on the ships are handled by constraints (4.11), which calculate the volume of LNG in each cargo tank on each ship in every time period. The amount of LNG in a cargo tank on a ship decreases at sea at an amount B_{wv}^{F} in each time period, although there is no boil-off from a cargo tank that is being loaded or unloaded. In addition, there is no boil-off from the cargo tanks on a ship at the destination node. An illustration on how the variables and parameters affect the ship inventory is given in Figure 4.5. Constraints (4.12)-(4.13) ensure that all cargo tanks are fully loaded at pick-up ports, when a loading starts. On the other hand, constraints (4.14)-(4.15) ensure that if a ship is unloading at least one cargo tank is unloaded, and vice versa. Moreover, constraints (4.16) give lower limits to the amount of LNG unloaded at the delivery ports, where the amount of cargo not delivered must be less or equal to the accumulated boil-off during the longest possible trip between two pick-up ports. The boil-off rate is usually a small percentage. Hence, constraints (4.16) allow only one unloading of a cargo tank before it is reloaded. Furthermore, constraints (4.16) in combination with constraints (4.7) limit the ships to sail to maximum two consecutive delivery ports. The upper limits on the unloading volumes at the delivery ports are given by (4.17). Finally, bounds on the ship inventory variable are described in constraints (4.18).

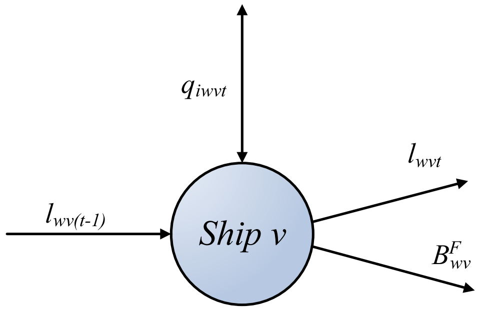

Figure 4.5: Inventory balance for the ships

If a ship ends its route in an delivery port, the cargo tanks that have been unloaded should have sufficient LNG left aboard to reach a pick-up port. This amount is represented by the parameter L^E_{wv}. The constraints required for handling the end conditions for the delivery ports are as follows:

$$x_{id(v)vt} z_{iwvt} \left(l_{wvt} - L^E_{wv} \right) = 0, \qquad \forall i \in \mathcal{N}^D_v, w \in \mathcal{W}_v, v \in \mathcal{V}, t \in \mathcal{T}, \tag{4.19}$$

$$\sum_{\tau=t}^{t+T^W} x_{id(v)v\tau} \sum_{j \in \mathcal{N}^D_v} x_{jiv(t-T_{ijv})} \left(l_{wvt} - L^E_{wv} \right) = 0, \qquad \forall i \in \mathcal{N}^D_v, w \in \mathcal{W}_v, v \in \mathcal{V}, t \in \mathcal{T}. \tag{4.20}$$

Constraints (4.19) ensure that if there is an unloading immediately before the destination node, the cargo left in these cargo tanks at the unloading node should equal the parameter L^E_{wv}. Constraints (4.20) have similar purpose when a ship calls two consecutive delivery ports before the destination node. If a ship calls two consecutive delivery ports, all cargo tanks must have been unloaded when leaving the second delivery port. Thus, there is no need for including z_{iwvt} or z_{jwvt} in constraints (4.20).

4.3.2.1 Linearization of Constraints

Constraints (4.19)-(4.20) are nonlinear when we relax the binary requirements of the variables. In this section we linearize those constraints.

$$l_{wvt} - L^E_{wv} x_{id(v)vt} \geq 0, \qquad \forall i \in \mathcal{N}^D_v, w \in \mathcal{W}_v, v \in \mathcal{V}, t \in \mathcal{T}, \tag{4.21}$$

$$l_{wvt} + L_{wv}\left(x_{id(v)vt} + z_{iwvt}\right) \leq 2L_{wv} + L^E_{wv}, \quad \forall i \in \mathcal{N}^D_v, w \in \mathcal{W}_v, v \in \mathcal{V}, t \in \mathcal{T}. \tag{4.22}$$

Constraints (4.21) assure that all ships should have at least L^E_{wv} of LNG aboard each cargo tank when they reach the destination node. Furthermore, constraints (4.22) are bounding when $x_{id(v)vt} z_{iwvt} = 1$. When $x_{id(v)vt} z_{iwvt} = 1$, i.e. when a ship unloads a cargo tank before sailing to the destination node, constraints (4.22) limits the cargo aboard that cargo tank to be less or equal than L_{wv}. Hence, constraints (4.21)-(4.22), can be regarded as linearized reformulations of constraints (4.19).

$$l_{wv\left(t+T_{id(v)v}\right)} + L_{wv}\left(\sum_{\tau=t}^{t+T^W} x_{id(v)v\tau} + \sum_{j \in \mathcal{N}^D_v} x_{jivt}\right) \leq 2L_{wv} + L^E_{wv},$$
$$\forall i \in \mathcal{N}^D_v, w \in \mathcal{W}_v, v \in \mathcal{V}, t \in \mathcal{T}. \tag{4.23}$$

When a ship unloads at two consecutive delivery ports before sailing to the destination node, there should be exact L^E_{wv} of LNG left in the cargo tanks. Constraints (4.23) are bounding when a ship calls consecutive delivery ports before sailing to the destination node. Then, the cargo left in each cargo tank should be equal or less than L^E_{wv}. Hence, we can use constraints (4.23) in combination with constraints (4.21) to linearize constraints (4.20).

4.3.3 Port Operations and Inventory Management

Here, the constraints handling both the port operations and the inventory management at the ports are presented. We need to introduce the following additional parameters and variables:

The daily production and sales of LNG at the ports have to be within a given interval $\left[\underline{Y}_{it}, \overline{Y}_{it}\right]$, which can change from one time period to another. Furthermore, the inventory levels at the ports should be within the upper and lower limits $\left[\underline{S}_i, \overline{S}_i\right]$. The maximum number of ships at a port in a time period is given by the parameter N^{CAP}_i.

The sales and production of LNG are given by the continuous variable y_{it}, $i \in \mathcal{N}$, $t \in \mathcal{T}$, while the continuous variable s_{it}, $i \in \mathcal{N}$, $t \in \mathcal{T}$ represents the inventory level at the liquefaction plants and regasification terminals in the different time periods. Note that s_{i0} represents the initial inventory.

$$s_{it} - s_{i(t-1)} - \sum_{v\in\mathcal{V}}\sum_{w\in\mathcal{W}_v} I_i q_{iwvt} + I_i y_{it} = 0, \qquad \forall i \in \mathcal{N}, t \in \mathcal{T}, \qquad (4.24)$$

$$\sum_{v\in\mathcal{V}} u_{ivt} \leq N_i^{CAP}, \qquad \forall i \in \mathcal{N}, t \in \mathcal{T}, \qquad (4.25)$$

$$\underline{S}_i \leq s_{it} \leq \overline{S}_i, \qquad \forall i \in \mathcal{N}, t \in \mathcal{T}, \qquad (4.26)$$

$$\underline{Y}_{it} \leq y_{it} \leq \overline{Y}_{it}, \qquad \forall i \in \mathcal{N}, t \in \mathcal{T}. \qquad (4.27)$$

The port inventory balances are given by constraints (4.24), and are further illustrated in Figure 4.6. Constraints (4.25) ensure that the port capacity in the number of ships in each time period is not exceeded. The upper and lower bounds for the variables are given in constraints (4.26)-(4.27).

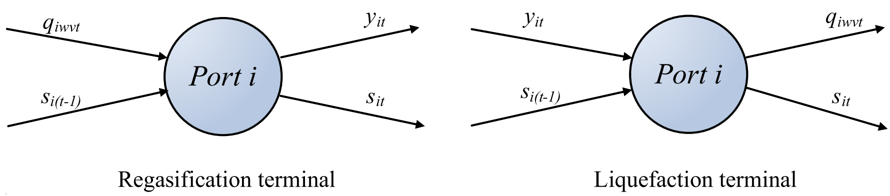

Figure 4.6: Inventory balance at regasification and liquefaction terminals

4.3.4 Objective Function

Finally, we can present the objective function for the arc-flow LNG-IRP formulation. We need to introduce the following revenue and cost parameters.

The parameter REV_{it} represents the unit revenue in each time period for selling LNG to the customers at the delivery ports, while $COST_{it}$ is the unit cost from producing LNG at the pick-up ports. Finally, C_{ijv} is the transportation cost, i.e. the cost of traversing arc (i,j) for ship v. The transportation cost parameter is a compound cost parameter, consisting of daily operating costs for ship v, port fees at port i, and any canal fees.

$$\max \sum_{i\in\mathcal{N}^D}\sum_{t\in\mathcal{T}} REV_{it} y_{it} - \sum_{i\in\mathcal{N}^P}\sum_{t\in\mathcal{T}} COST_{it} y_{it} - \sum_{(i,j)\in\mathcal{A}_v}\sum_{v\in\mathcal{V}}\sum_{t\in\mathcal{T}} C_{ijv} x_{ijvt} \qquad (4.28)$$

The objective function (4.28) maximizes total profit of selling LNG to end-customers while minimizing the costs of production and transportation.

4.4 Path-Flow Formulation

In the arc-flow formulation, the routes including arrivals times and load quantities are constructed based on the values on the variables, while these routes are enumerated a priori and feed into the path-flow formulation. The model is presented in Section 4.4.1, while the algorithm for enumerating the paths is described in Section 4.4.2.

4.4.1 The Model

In the path-flow formulation of the LNG-IRP, a route $r \in \mathcal{R}_v$ contains the geographical route and the schedule with information about the arrival times for all port calls for ship v in the planning horizon. In addition, the route contains information about the quantities loaded and unloaded at the liquefaction plants and regasification terminals, respectively.

The parameter Z_{ivtr} equals 1 if ship v calls port i in time period t on route r and 0 otherwise, while the corresponding (un)loading volume is given by the parameter Q_{ivtr}. The cost of sailing route r for ship v is given by the parameter C_{vr}. The cost parameters are composed of ship operation cost, port fees, and canal fees.

The binary variable λ_{vr}, $v \in \mathcal{V}$, $r \in \mathcal{R}_v$ is the ship route variable, and equals 1 if ship v chooses to sail route r, and 0 otherwise.

Then, the path-flow formulation of the LNG-IRP can be modeled as follows:

$$\max \sum_{i \in \mathcal{N}^D} \sum_{t \in \mathcal{T}} REV_{it} y_{it} - \sum_{i \in \mathcal{N}^P} \sum_{t \in \mathcal{T}} COST_{it} y_{it} - \sum_{v \in \mathcal{V}} \sum_{r \in \mathcal{R}_v} C_{vr} \lambda_{vr}, \tag{4.29}$$

subject to

$$s_{it} - s_{i(t-1)} - \sum_{v \in \mathcal{V}} \sum_{r \in \mathcal{R}_v} I_i Q_{ivtr} \lambda_{vr} + I_i y_{it} = 0, \qquad \forall i \in \mathcal{N}, t \in \mathcal{T}, \tag{4.30}$$

$$\sum_{v \in \mathcal{V}} \sum_{r \in \mathcal{R}_v} Z_{ivtr} \lambda_{vr} \leq N_i^{CAP}, \qquad \forall i \in \mathcal{N}, t \in \mathcal{T}, \tag{4.31}$$

$$\underline{S}_i \leq s_{it} \leq \overline{S}_i, \qquad \forall i \in \mathcal{N}, t \in \mathcal{T}, \tag{4.32}$$

$$\underline{Y}_{it} \leq y_{it} \leq \overline{Y}_{it}, \qquad \forall i \in \mathcal{N}, t \in \mathcal{T}, \tag{4.33}$$

$$\sum_{r \in \mathcal{R}_v} \lambda_{vr} = 1, \qquad \forall v \in \mathcal{V}, \tag{4.34}$$

$$\lambda_{vr} \in \{0, 1\}, \qquad \forall v \in \mathcal{V}, r \in \mathcal{R}_v. \tag{4.35}$$

The objective function (4.29) maximizes the profit from the LNG activities. The inventory balances are given in constraints (4.30), while the port capacity constraints are given by (4.31). The bounds in constraints (4.32)-(4.33) are identical to constraints (4.26)-(4.27). Constraints (4.34) are the convexity constraints, limiting the ships to sail exactly one route. Finally, the formulation involves binary requirements (4.35) on the ship route variables λ_{vr}.

4.4.2 Path Enumeration Algorithm

Here, we present an algorithm for complete enumeration of all possible paths, called routes, for the path-flow LNG-IRP formulation. By use of a recursive algorithm we can identify all possible routes with information regarding the geographical route, the arrival times and the number of cargo tanks loaded and unloaded at all ports. When a route has been found, the algorithm must identify all the duties within the routes. These duties are in fact distinct subpaths in the route. The algorithm must calculate the aggregated boil-off from each cargo tank during the duties in order to calculate the exact amount of cargo unloaded at the delivery ports. This can be expressed mathematically if we introduce some new notation. Let Δ_{vr}, indexed by d, denote the set of duties for ship v on route r. The aggregated boil-off in cargo tank w on ship v on duty d on route r is given by B^{FA}_{wvrd}. Furthermore, $X_{ijvtrd} = 1$ if arc (i, j) is traversed by ship v starting in time period t on duty d on route r, and 0 otherwise. Moreover, $Z^{\Delta}_{iwvtrd} = 1$ if ship v loads or unloads cargo tank w in time period t on duty d on route r, and 0 otherwise.

$$B^{FA}_{wvrd} = B^{F}_{wv} \left(\sum_{(i,j) \in \mathcal{A}_v} \sum_{t \in \mathcal{T}} X_{ijvtrd} T_{ijv} - \sum_{i \in \mathcal{N}^{PD}_v} \sum_{t \in \mathcal{T}} Z^{\Delta}_{iwvtrd} \right),$$
$$\forall w \in \mathcal{W}_v, v \in \mathcal{V}, r \in \mathcal{R}_v, d \in \Delta_{vr}, \tag{4.36}$$

$$Q_{ivtr} = \begin{cases} \sum_{w \in \mathcal{W}_v} \sum_{d \in \Delta_{vr}} Z^{\Delta}_{iwvtrd} L_{wv}, \forall i \in \mathcal{N}^P_v, v \in \mathcal{V}, t \in \mathcal{T}, r \in \mathcal{R}_v, \\ \sum_{w \in \mathcal{W}_v} \sum_{d \in \Delta_{vr}} Z^{\Delta}_{iwvtrd} \left(L_{wv} - B^{FA}_{wvrd} \right), \forall i \in \mathcal{N}^D_v, v \in \mathcal{V}, t \in \mathcal{T}, r \in \mathcal{R}_v, \end{cases} \tag{4.37}$$

$$Z_{ivtr} \geq Z^{\Delta}_{iwvtrd}, \forall i \in \mathcal{N}_v, w \in \mathcal{W}_v, v \in \mathcal{V}, t \in \mathcal{T}, r \in \mathcal{R}_v, d \in \Delta_{vr}. \tag{4.38}$$

With equations (4.36)-(4.38) we can calculate the input from the path enumeration algorithm to the path-flow model. In equations (4.36) we calculate the aggregated boil-off during a duty. The aggregated boil-off is used as input when calculating the amount of cargo loaded and unloaded at the ports in equations (4.37). Finally, equations (4.38) gives the relation between the cargo tank (un)loading parameter on a duty and the ship call parameter.

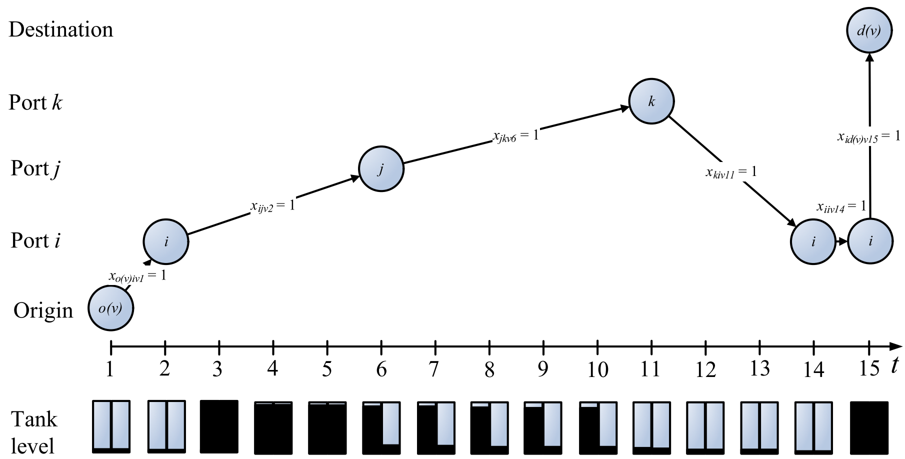

Figure 4.7: Illustration of a possible route through the network for ship v and the corresponding cargo tank levels

Figure 4.7 illustrates a possible route through the network for a ship with two cargo tanks. The ship loads both its cargo tanks in time period 2. Then it unloads one cargo tank in time period 6 and the other one in time period 11. Some LNG is left in the cargo tanks for the boil-off after the tanks have been unloaded. This boil-off keeps the cargo tanks cool and is also used as fuel until the cargo tanks are reloaded in time period 15.

The pseudocode for the path enumeration algorithm is described in Algorithm 1 and Algorithm 2. Here, Algorithm 1 initializes the path enumeration algorithm, while the recursive part of the algorithm which searches for all paths is given in Algorithm 2. To illustrate how the algorithm works we use the example from Figure 4.7. The path enumeration algorithm will first identify the complete route, then it will identify the three duties in the network. The first duty starts in the origin node and ends in port i in time period 2. The second one starts in time period 2 when the ship leaves port i, and ends in time period 15 when the ship once again will leave port i. Note that the waiting day outside port i in time period 14 is considered belonging to the second duty. Finally, the third duty both starts and ends in time period 15, as it covers the sailing from port i to the destination node $d(v)$.

Algorithm 1 initializePath

```
for all v ∈ V do
    r = 0, the route numbering starts from 0 for each ship
    N^LIST = ∅, the list of nodes in the route
    call createPath(r, o(v), N^LIST)
end for
```

Algorithm 2 createPath(r, i, N^{INLIST})

```
N^LIST = N^INLIST ∪ {i}
for all j|(i,j) ∈ A_v do
    if j ≠ d(v) then
        call createPath(r, j, N^LIST)
    else
        N^LIST = N^LIST ∪ {j}
        Identify all duties in N^LIST and create the set Δ_vr and the
          tables X_ijvtrd, Z_ivtr, and Z^Δ_iwvtrd
        Calculate the boil-off, B^FA_wvrd, in each duty with equations (4.36)
        Calculate the quantity loaded or unloaded, Q_ivtr, with equations (4.37)
        r = r + 1
    end if
end for
```

For instance, if the cargo tanks have a capacity of 75 000 m^3 each and the boil-off is 115 m^3, we can calculate the aggregated boil-off and the amount of cargo unloaded at the ports. The aggregated boil-off for cargo tank 1 on the second duty is then calculated based on a 13 time period long duty, less one time period for loading and one for unloading: $B^{FA}_{1vr2} = 115\,(13-2) = 1\ 265$ m^3. Note that the loading in time period 15 belongs to the third duty. Then, if cargo tank 1 is unloaded at port j in time period 6, 73 735 m^3 will be unloaded there. Since the two cargo tanks are identical, the second tank will unload 73 735 m^3 at port k in time period 11.

4.5 Computational Results

The purpose of the computational study is to evaluate the two proposed model formulations of the LNG-IRP, the arc-flow formulation presented in (4.1)-(4.28), and the path-flow formulation presented in (4.29)-(4.35). The models have been solved by use of XPRESS Optimizer v 17.1 on a computer with a 3 GHz processor and 8 GB RAM running on Rock Cluster v 4.2.1 operating system. Furthermore, the path enumeration algorithm presented in Section 4.4.2 has been programmed in C++ and solved on the same architecture.

We have created 21 instances motivated by the real planning problem faced by Suez Energy International. An overview of these instances is presented in Table 4.1. In this table the number of ships, cargo tanks, ports, and time periods are given for each instance. The number of pick-up and delivery ports are given in parenthesis behind the total number of ports. The number of ships ranges from 1 to 5 depending on the instance, while the number of ports is between 3 and 6. Keep in mind that each port may have ship calls several times during the planning period. Furthermore, the instances have 30, 45, or 60 time periods. To be able to test how the time horizon affect the solution time, some instances share most characteristics, but have different number of time periods. For instance, instance 1-3 have the same physical network and similar upper and lower bounds on production and sales, and 30, 45, and 60 time periods respectively. The other instances can also be grouped similarly (4-6, 7-9, ...).

The number of rows and columns for the instances for both formulations are presented in Table 4.2. Note that the number of integer columns for the path-flow formulation is the number of routes for the ships. This number gives some indication of the complexity of the problems, and how the instances escalate with the number of time periods. As we can see from the table, the path-flow formulation scales very poorly with respect to the number of time periods. Furthermore, for instances #9 and #15 we were not able to enumerate all routes as the optimizer ran out of memory during the enumeration process. For the other instances the number of routes spans from 426 for instance #10 to more than 1.5 million for instance #21.

In Table 4.3 the solution times for each instance are reported. For both formulations, the table reports the time to solve the linear relaxation, first integer solution, best integer solution, and the total solution time. All these solution times are measured starting from the moment the optimizer starts solving the problem, after the complete matrix has been fed into the solver. Maximum running time for the instances is 10 hours. Thus, if the search is not completed within the time limit, the search is interrupted. In addition, the table reports the time needed for the path enumeration algorithm to enumerate all routes and feed them into the solver for the path-flow formulation.

The LNG-IRP is hard to solve. In general, both formulations solve the minor instances efficiently, while both formulations have problems with solving the instances with longer planning horizon to optimality. The path-flow formulation is the most efficient formulation with respect to total solution time for 7 of the

Table 4.1: Instance overview

#	Ships	Tanks	Ports (P,D)	Time Periods
1	1	2	4(1,3)	30
2	1	2	4(1,3)	45
3	1	2	4(1,3)	60
4	2	2	3(1,2)	30
5	2	2	3(1,2)	45
6	2	2	3(1,2)	60
7	2	2	4(2,2)	30
8	2	2	4(2,2)	45
9	2	2	4(2,2)	60
10	2	1	5(2,3)	30
11	2	1	5(2,3)	45
12	2	1	5(2,3)	60
13	2	2	5(2,3)	30
14	2	2	5(2,3)	45
15	2	2	5(2,3)	60
16	3	1	4(2,2)	30
17	3	1	4(2,2)	45
18	3	1	4(2,2)	60
19	5	1	6(3,3)	30
20	5	1	6(3,3)	45
21	5	1	6(3,3)	60

instances, while the arc-flow performs better for 7 when we disregard 2 instances where we only have test results from the arc-flow formulation. The path-flow formulation suffers from its poor scaling capacity, which leads to large solution time even for the LP relaxation. For instance, solving the LP relaxation for instance #3 with the path-flow formulation is almost 2 hours, while the corresponding solution time for the arc-flow formulation is 0 seconds. As a consequence of the long solution time for the path-flow formulation's LP relaxation, the time to find the first integer solution is longer than for the arc-flow formulation for 11 of the instances, and faster for only 3 of the instances.

The solution values and the MIP-gaps are presented in Table 4.4. The MIP-gap is defined as |MIP*-Bound*| /Bound*, where Bound* is the best bound on the solution from the branch-and-bound procedure and MIP* is the best (mixed) integer solution. For the instances that were not solved to optimality, the MIP-gaps are in the interval $[2.9\%, 42.9\%]$. One of the reasons for the large MIP-gaps is the poor linear relaxation for the LNG-IRP. If we define the LP-gap as

Table 4.2: Dimensions of the instances

	Arc-Flow			Path-Flow		
#	Rows	Columns	Int Columns	Rows	Columns	Int Columns
1	2 072	1 411	871	243	1 791	1 551
2	3 107	2 116	1 306	363	63 101	62 741
3	4 142	2 821	1 741	483	2 534 165	2 533 685
4	2 764	2 044	1 384	184	1 110	930
5	4 144	3 064	2 074	274	28 348	28 078
6	5 524	4 084	2 764	364	849 858	849 498
7	3 424	2 764	1 924	244	9 018	8 778
8	5 134	4 144	2 884	364	461 293	460 933
9	6 844	5 524	3 844	-	-	-
10	3 544	2 102	1 502	304	728	426
11	5 314	3 152	2 252	454	6 883	6 431
12	7 084	4 202	3 002	604	93 325	92 723
13	4 564	2 702	1 802	304	4 010	3 710
14	6 844	4 052	2 702	452	239 208	238 758
15	9 124	5 402	3 602	-	-	-
16	3 846	3 036	2 346	245	1 339	1 096
17	5 766	4 551	3 516	365	20 863	20 500
18	7 686	6 066	4 686	485	363 802	363 319
19	9 670	7 480	6 070	367	2 774	2 414
20	14 500	11 215	9 100	547	65 695	65 155
21	19 330	14 950	12 130	727	1 561 996	1 561 276

the percentage deviation between the best MIP solution and the LP solution, |MIP*-LP*| /LP*, the average LP-gap is 19, 4% for both model formulations. This indicates that for some of the instances where we did not manage to prove the optimal solution, the best integer solution found might be near optimal or even the optimal solution.

Looking at the quality of the first integer solutions found during branch-and-bound, we can see that for 11 of the instances the optimizer finds better solutions for the path-flow formulation than for the arc-flow formulation. Also the arc-flow formulation finds better integer solutions for 6 of the instances when we disregard instances #9 and #15. The path-flow formulation's linear relaxations are tighter than the corresponding linear relaxations for the arc-flow formulation for all instances except instances #10 and #13. We would expect that the path-flow formulation would have best linear relaxation for all instances. However, when solving these instances without presolve applied, the conclusion alters. With the

Table 4.3: Solution times. All measures in sec. LP - linear relaxation, MIP [1] - first MIP solution, MIP* - best MIP solution, Total - total solution time, Enum = time to enumerate the routes

	Arc-Flow				Path-Flow				
#	LP	MIP^1	MIP*	Total	LP	MIP^1	MIP*	Total	Enum
1	0	0	1	2	0	0	0	0	0
2	0	6	6	63	5	12	12	27	1
3	0	31	7 548	7 548	7 131	8 992	23 540	36 000	56
4	0	0	2	120	0	0	0	1	0
5	0	3	7 105	36 000	1	6	186	423	0
6	0	6	23 706	36 000	140	378	32 096	36 000	19
7	0	1	1	1	0	0	1	43	0
8	0	1	3	4	27	43	180	1 761	8
9	0	155	454	456	-	-	-	-	-
10	0	0	0	0	0	0	0	0	0
11	0	0	3	15	0	4	196	973	0
12	0	7	27	56	15	70	33 605	36 000	2
13	0	0	21	22	0	0	1	5	0
14	0	0	2 693	3 797	67	237	1 162	36 000	4
15	2	63	33 093	36 000	-	-	-	-	-
16	0	1	42	55	0	0	13	14	0
17	0	0	225	9 217	0	0	206	13 625	0
18	0	0	3 354	36 000	35	223	35 896	36 000	7
19	0	1	17	172	0	0	3	39	0
20	0	11	7 476	36 000	3	13	3 074	36 000	1
21	1	41	12 214	36 000	321	8 724	20 492	36 000	32

optimizer's presolve turned off, the LP solutions for instances #10 and #13 are 2 433.7 for the arc-flow formulation and 2 403.1 for the path-flow formulation.

4.6 Concluding Remarks

In this paper we have introduced a new type of optimization problems, the *liquefied natural gas inventory routing problem*, LNG-IRP. This problem deals with managing the supply chain for the liquefied natural gas (LNG) business at a tactical planning level. Here, one actor controls the supply chain from liquefaction to sales, where both the production and sales levels are variable and may change from day to day. Furthermore, ship routing and scheduling of specialized ships

Table 4.4: Solution values and MIP-gaps for the instances

	Arc-Flow				Path-Flow			
#	LP	MIP[1]	MIP*	MIP-gap	LP	MIP[1]	MIP*	MIP-gap
1	1 036.9	807.0	887.0	0.0 %	1 024.7	887.0	887.0	0.0 %
2	1 504.8	1 314.7	1 314.7	0.0 %	1 470.4	1 314.7	1 314.7	0.0 %
3	1 940.0	1 354.7	1 533.0	0.0 %	1 896.6	1 133.8	1 331.5	28.8 %
4	782.7	490.7	681.0	0.0 %	760.9	537.0	681.0	0.0 %
5	1 082.1	507.8	940.8	2.9 %	1 044.6	688.5	940.8	0.0 %
6	1 375.3	429.2	1 122.6	15.2 %	1 326.8	804.2	1 095.3	16.9 %
7	1 313.6	992.1	1 018.6	0.0 %	1 262.5	905.0	1 018.6	0.0 %
8	1 838.2	1 142.2	1 492.6	0.0 %	1 761.3	1 166.2	1 492.6	0.0 %
9	2 346.9	1 243.8	1 784.2	0.0 %	-	-	-	-
10	2 329.7	2 170.0	2 170.0	0.0 %	2 396.1	2 002.0	2 170.0	0.0 %
11	3 371.3	2 231.9	2 544.8	0.0 %	3 350.2	1 996.8	2 544.8	0.0 %
12	3 944.5	2 247.8	2 920.3	0.0 %	3 892.7	2 403.2	2 745.6	26.6 %
13	2 394.5	1 742.5	2 170.0	0.0 %	2 396.2	2 068.3	2 170.0	0.0 %
14	3 462.3	1 659.0	2 591.0	0.0 %	3 352.6	2 259.9	2 591.0	15.2 %
15	4 421.3	1 997.1	2 882.3	18.8 %	-	-	-	-
16	1 290.5	867.1	1 118.0	0.0 %	1 287.3	1 085.7	1 118.0	0.0 %
17	1 778.4	1 291.3	1 435.4	0.0 %	1 771.2	1 291.3	1 435.4	0.0 %
18	2 244.9	1 243.0	1 579.7	16.6 %	2 228.2	953.2	1 579.7	27.6 %
19	1 910.9	1 530.8	1 697.6	0.0 %	1 909.4	1 180.8	1 697.6	0.0 %
20	2 498.4	947.4	2 032.5	6.0 %	2 483.1	1 304.0	2 011.3	16.0 %
21	3 026.6	910.3	2 062.3	25.1 %	2 995.7	1 090.4	1 684.5	42.9 %

(LNG tankers) are important parts of this supply chain. The problem is more complicated than many other maritime inventory routing problems, as it deals with variable rates of production and consumption. Moreover, the ship routing and scheduling are also more complicated, as the ships' cargo tanks should not run dry at sea, as they have to deal with a constant rate of boil-off. In addition, the ships load all their cargo tanks at the pick-up ports, and unload a discrete number of cargo tanks at the delivery ports.

We have proposed two formulations of the LNG-IRP; an arc-flow and a path-flow formulation. In the path-flow formulation, a path represents a possible geographical route and schedule for a ship during the entire planning horizon. In addition, the path handles the ship inventory management, the boil-off from the cargo tanks, and the amount of cargo loaded and unloaded at the pick-up and delivery ports. Moreover, we have presented an algorithm for complete enumeration of the columns in the path-flow model.

Both model formulations have been tested on instances motivated by a real planning problem. Both formulations provide good solutions to the test instances presented, although none of them were able to solve the largest instances to optimum. From the limited number of instances, it is hard to conclude which formulation is superior. The path-flow formulation was able to solve more instances faster to optimum than the arc-flow formulation, while the arc-flow formulation finds the first integer solution faster than the path-flow formulation. Furthermore, the path-flow formulation suffers from poor scaling capabilities. Hence, the optimizer ran out of memory when we tried to enumerate the columns for two of the instances.

The LNG-IRP is hard to solve. Thus, none of the proposed formulations where able to verify the optimal solutions for all instances presented. To be able to solve these, and even larger instances, more research is required. Solution approaches based on both exact methods and heuristics may be appropriate for solving larger instances of the LNG-IRP. Within exact methods, column generation seems like a particularly interesting alternative since we will be able to work with a subset of the columns for the problems. Moreover, the LP-gap for the LNG-IRP is poor. Thus, development of valid inequalities might also be valuable.

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Paper IV

Roar Grønhaug, Marielle Christiansen, Guy Desaulniers, and Jacques Desrosiers:

A Branch-and-Price-and-Cut Method for a Liquefied Natural Gas Inventory Routing Problem

Submitted to Transportation Science

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5.6 Concluding Remarks

In this paper we have presented a solution method for the *liquefied natural gas inventory routing problem* (LNG-IRP). The problem consists of designing routes and schedules for a heterogeneous fleet of LNG ships, while simultaneously managing the production and sales of LNG and handling the inventory levels at the different locations in each time period. LNG-IRP is more complex than Ítraditional maritime inventory routing problems because of the complicating side constraints for the routing of the ships, and due to the variable production and sales at the ports that must be chosen in each time period.

We have solved the LNG-IRP with a branch-and-price-and-cut method. The solution method is promising. It was able to find good integer solutions to most of the instances tested. Furthermore, the solution method gives better results than earlier reported in Grønhaug and Christiansen (2008). The LNG-IRP is hard to solve. Thus, the proposed solution method was not able to verify if the optimal solution was found for all instances tested within 10 hours of computational time.

Several directions are possible for further research within a column generation framework. A different decomposition of the LNG-IRP resulting in another master problem and subproblems might improve the running time. Moreover, developing valid inequalities is also an interesting research direction as valid inequalities will tighten the linear relaxation helping the solution method to close the MIP gap.

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