Evaluating fleet effectiveness in tactical emergency response missions using a maximal covering formulation

ABSTRACT

This paper concerns the evaluation of alternative fleets of advanced special vessels, like coast guard or emergency response and rescue vessels. The paper proposes a mathematical programming formulation of the Fleet Deployment with Maximal Covering problem and combines analysis of this problem with tradespace exploration and epoch-era analysis. A solution of the mathematical program provides an optimal deployment plan for a given fleet in a given context. The objective function value provides a measure of effectiveness for the fleet alternative. By evaluating the effectiveness of a set of alternative fleets in several alternative scenarios using epoch-era analysis, we obtain strategic insights about dynamic trade-offs and provide decision support for fleet size and mix planning. The paper reconciles the use of mathematical programming for measurement of fleet effectiveness with a design of experiments approach to concept exploration under uncertainty. The results show that it is effective to use mathematical programming for planning horizons with less uncertainty, and account for strategic uncertainties using the epoch-era framework.

INTRODUCTION

Determination of fleet size and mix is an important strategic problem facing commercial and governmental organizations that operate ships (Christiansen, Fagerholt, Nygreen, & Ronen, 2007; Pantuso, Fagerholt, & Hvattum, 2014). For commercial fleets of transport vessels, the objective is to minimize costs or maximize profits, determined on basis of the demand for transportation services. For fleets of special vessels, and particularly non-commercial fleets, like coast guards and navies, determination of fleet mix is a *wicked problem* (Andrews, 2018; Rittel & Webber, 1973), with many opposing stakeholder interests and uncertainty with respect to both current and future expectations. This makes it necessary to formulate other *measures of effectiveness* for evaluation of alternative fleets (Gawande & Wheeler, 1999; Hootman & Whitcomb, 2005; Martens & Rempel, 2011; Rains, 1999), that consider the demand for functionality and capabilities beyond the individual ship design problem.

In defining measures of effectiveness for fleets, there is a need for considering the interactions between vessels constituting the fleet (Kujawski, 2014). For example, in an emergency preparedness setting, measures like area coverage and mission response time strongly depend on the relative location of assets. For coast guards, these measures must be traded against strategic military and civilian objectives: How much patrolling is required in an area to deter illegal activities? How many patrol days can we afford? Furthermore, it becomes a question of whether a given amount of functionality should be delivered by a smaller or larger number of assets. Can a smaller number of multi-functional assets replace a large fleet of smaller assets, and maintain the same effectiveness at a lower cost, or does such a solution add to the vulnerability of the fleet as a whole? For stakeholders to evaluate whether a given fleet renewal plan is appropriate, it is necessary to evaluate fleet effectiveness in the scenarios the fleet is likely to face.

Operations research suggests that mathematical models be used for optimizing fleet compositions, even under uncertainty (Pantuso et al., 2014). However, methods that use stochastic programming to account for uncertainty in fleet size and mix (Pantuso, Fagerholt, & Wallace, 2016) aim to find the optimal fleet, without explicitly exploring the effectiveness of alternative architectures under different scenarios. This paper makes an argument for a more extensive exploration of solution spaces, rather than optimizing the solution itself.

Exploration enables stakeholders to learn more about potential compromises before making decisions (Singer, Doerry, & Buckley, 2009) resonating with the "exploitation-exploration" trade-off (March, 1991). March (1991) argued that exploration, generating new knowledge about something, competes with exploitation, which is to use existing knowledge to make decisions, for scarce organizational resources. Both are necessary to achieve success, but we focus on *exploration* in this paper.

This paper proposes a mathematical model for fleet deployment based on the structure of a maximal covering problem to address one measure of effectiveness. We refer to the proposed model structure as the Fleet Deployment with Maximum Covering (FDMC) model. By solving this deployment model under an experimental design that defines both a set of alternative fleets, and several alternative scenarios, or epochs, we enable exploration of the solution space and obtain insights about trade-offs and compromises that can guide decision-makers when acquiring new vessels.

LITERATURE REVIEW

The literature review draws from three main sources of knowledge: First, naval design research has highlighted the need for novel measures of effectiveness to be investigated. Second, systems engineering research has made advances including methods like tradespace exploration and epoch-era analysis inspired by design of experiments and response surface methodology. Third, operations research has contributed to numerous mathematical programming models that can be used to measure fleet effectiveness.

A trend in naval design has been to move from ship design as the main problem to a holistic design approach aimed at improving fleet capabilities (Doerry & Fireman, 2009; Hootman & Whitcomb, 2005). This effort has included development of measures of effectiveness (Gawande & Wheeler, 1999; Hootman & Whitcomb, 2005) and wide use of set-based design and concept exploration methods in early design phases to generate knowledge before committing costly decisions (Martens & Rempel, 2011; Singer, Doerry, & Buckley, 2009). A problem that is especially persistent for naval design, is the highly political design environment (Koenig, Czapiewski, & Hootman, 2008).

Addressing the need to study the compromises that must be made in complex decision processes, including those faced by navies or other government organizations, Ross & Rhodes (2008) developed epoch-era analysis for studying dynamic system value sustainment. Epoch-era analysis applies an experimental design approach to developing scenarios, in which *epochs* represent short-term, static system contexts, and *eras* represent sequences of several epochs, capturing the long-term, dynamic development of system contexts. Epoch-era analysis is often used in combination with concept exploration, in which an experimental design approach is taken to define and evaluate a valid design space. Metrics based on analysis of the Pareto front are developed to evaluate long-term value robustness across multiple epochs and through eras (Fitzgerald, Ross, & Rhodes, 2011; Smaling & Weck, 2004). In this paper, an epoch is taken to correspond with a tactical planning horizon, for which the fleet deployment problem is solved. The epoch-era methodology has been applied to naval ship design (Schaffner, Ross, & Rhodes, 2014; Vascik, Ross, & Rhodes, 2016), and for special vessels in the offshore industry (Gaspar, Erikstad, & Ross, 2012; Pettersen et al., 2018). The problem complexity greatly increases on the fleet level, compared to the level of single ships, as the capabilities we are interested in emerge from the interaction between several vessels (Kujawski, 2014; Vascik, et al., 2016). In that sense, fleet effectiveness is a *latent* property to any single ship design problem, that can not explicitly be designed for (Pettersen, Erikstad, & Asbjørnslett, 2017). This makes it even more difficult to formulate evaluate performance, as needed for proper examination of alternative fleet structures.

Resource allocation models for emergency preparedness in operations research are a useful tool for evaluation of fleet effectiveness. Thorough reviews of that literature are given by Altay & Green (2006), Galindo & Batta (2013) and Simpson & Hancock (2009). Among common maritime emergency missions addressed, we find oil spill response (Belardo, Harrald, Wallace, & Ward, 1984; Garrett, Sharkey, Grabowski, & Wallace, 2017; Psaraftis & Ziogas, 1985; Srinivasa & Wilhelm, 1997), emergency towing (Assimizele, Royset, Bye, & Oppen, 2018), and search and rescue (Asiedu & Rempel, 2011; Brachner &

Hvattum, 2017; Karatas, Razi, & Gunal, 2017; Razi & Karatas, 2016). A few models have been suggested for allocation of assets owned by agencies like the U.S. Coast Guard (Wagner & Radovilsky, 2012). Resource allocation for oil spill recovery has been most extensively studied. For such problems, possible objectives are maximization of area coverage (Belardo et al., 1984; Verma, Gendreau, & Laporte, 2013), minimization of total costs associated with oil spills (Iakovou, Ip, Douligeris, & Korde, 1997; Psaraftis & Ziogas, 1985), or minimization of response time to oil spills (Srinivasa & Wilhelm, 1997). Verma et al. (2013) present a two-stage stochastic program with recourse to address the problem of oil spill response. The first stage optimizes the location of oil spill response equipment and facilities, while the second stage uses the event-specific details about the oil spill to make decisions in emergency response. They find that decisions regarding facility location and equipment stockpiling depends on the trade-off between expected environmental costs and the cost of investing in facilities and equipment.

Search and rescue has also seen increased attention (Brachner & Hvattum, 2017; Karatas et al., 2017; Pelot, Akbari, & Li, 2015; Razi & Karatas, 2016). Pelot et al. (2015) present several alternative extensions for a maximal covering location model for locating maritime search and rescue vessels. Extensions include considerations of workload capacity for vessels, stochastic vessel availability, and considerations of uncovered demand for response. Brachner & Hvattum (2017) combine helicopter routing for offshore personnel transport with a covering approach that ensures that emergency response units are located sufficiently close to helicopter routes. This approach is developed due to the need for novel emergency preparedness approaches for offshore personnel transport in the Arctic, where long distances constitute a significant vulnerability for emergency response. Search and rescue has also been considered in the context of the Mediterranean refugee crisis, where minimization of response time is an important objective, achieved by combining integer programming and discrete event simulation (Karatas et al., 2017; Razi & Karatas, 2016).

The solutions to resource allocation models like those reviewed here may constitute measures of effectiveness for fleet systems in an emergency response setting, which is what we will explore further in the paper.

METHODOLOGY

The methodology combines mathematical programming for tactical planning, the FDMC problem, with an epoch-era analysis approach used to explore strategic implications. The purpose of this combination of methods is to use optimization to *evaluate* the effectiveness of alternative solutions, in alternative contextual scenarios, and explore trade-offs between cost and effectiveness, rather than using optimization to *prescribe* solutions. Eventually, a good solution will be one that comes close to maximizing effectiveness across many scenarios, *epochs*.

Epoch-era analysis can be traced theoretically to design of experiments methodology (Box & Liu, 1999) and strategic scenario planning (Schoemaker, 1991). Epoch-era analysis works by eliciting several uncertain contextual factors that determine the state of the operating environment (Ross & Rhodes, 2008). These *factors* are called *epoch variables* and are discretized to an appropriate number of *levels*. A specific vector of epoch variables then forms an *epoch*, a possible static operating context for the system. Sequences of *epochs* constitute *eras*, long-term dynamic scenarios, in which effectiveness may fluctuate due to changes in the dynamic environment. As a method for strategic scenario planning, epoch-era analysis emphasizes a more qualitative and exploratory approach to decision-making under uncertainty, in which narratives are used to frame and construct possible futures. This qualitative approach is hence different from the use of stochastic programming to find optimal solutions to strategic decision problems (Owen & Daskin, 1998).

Note that a *tactical* planning horizon is normally defined as a period of one month to a year, whereas *strategic* planning horizons encompass at least one year (Christiansen et al., 2007). Hence, strategic decisions include decisions regarding the acquisition of new vessels and locating of vessel bases, while tactical decisions include deployment and routing of vessels (Christiansen et al., 2007; Pantuso et al., 2014). For this reason, we propose that the FDMC problem is solved for a set of parametrized system contexts, represented as epochs. An overview of epoch-era terminology, with relations to how tactical and strategic decisions are handled by the FDMC problem, is provided in Table 1.

Table 1: Describing the connection between mathematical programming and epoch-era analysis.

Concepts	**Scenario type**	**Planning horizon**	**Description of the concept**
Epoch	Static	Tactical	Solve a mathematical program (here the FDMC problem) to evaluate fleet effectiveness for each epoch.
Era	Dynamic	Strategic	Study the development of fleet effectiveness by comparing across epochs, or by creating sequences of epochs (eras).

The methodology can be described as follows:

1. Structure the problem and find an appropriate problem definition.
2. Develop the mathematical model for the tactical decision problem to be optimized, in this case, the FDMC problem.
   a. Develop the basic variant of the FDMC problem.
   b. Develop an ϵ-constraint variant of the FDMC problem for benchmarking alternatives against hypothetical "optimal" fleets, given a budgetary constraint.
3. Define epochs and eras.
4. Define fleet alternatives to evaluate.
5. Solve the FDMC problem:
   a. Solve the basic FDMC problem for all alternative fleets, for all epochs. The results are then represented in a cost-utility tradespace for each epoch. The utility is represented by the optimal objective function value for the FDMC problem presented in Section 5. The costs are determined directly from the fleet composition.
   b. Solve the ϵ-constraint version of the FDMC problem to generate an approximate Pareto front for all epochs.
6. Evaluate value robustness across epochs and eras using metrics from epoch-era analysis. We introduce metrics based on Pareto optimality for this, later in this section.

The objective values from each solution of the FDMC problem serve as a measure of effectiveness for proposed solutions to a corresponding fleet size and mix problem. Running the FDMC problem for multiple fleet alternatives, we obtain a tradespace documenting the cost-utility trade-offs (Ross & Hastings, 2005). The parameter input data is then varied to create different epochs, similar to design of experiments methodology (Box & Liu, 1999; Martens & Rempel, 2011). A value robust fleet alternative is one that obtains a stable, high objective function value across multiple epochs and eras, delivering stakeholder value in a diverse set of circumstances (Ross, Rhodes, & Hastings, 2008).

Measures commonly used in epoch-era analysis for evaluating how well the fleet alternatives sustain value over several epochs, include the Pareto trace which is essentially the frequency with which a given alternative is Pareto optimal, generalized to the fuzzy Pareto trace (Fitzgerald et al., 2011). "Fuzziness" in this context mainly serves as an approximation of what fleet alternatives can be considered sufficiently close to the Pareto front, to be included in an extended Pareto set. The fuzzy Pareto number is the smallest percentage k for which a design is within an acceptable range of the Pareto front. For example, an alternative with a 5 % fuzzy Pareto number will be within a 5% range of the Pareto front in terms of *both* cost and utility (Smaling & Weck, 2004). An approximation of the theoretical Pareto front is obtained by use of the ϵ-constraint method that repeatedly solves the extended variant of the FDMC problem with varying budgetary constraints (Paul, Lunday, & Nurre, 2017). A illustration of the fuzzy Pareto front concept is provided in Figure 1.

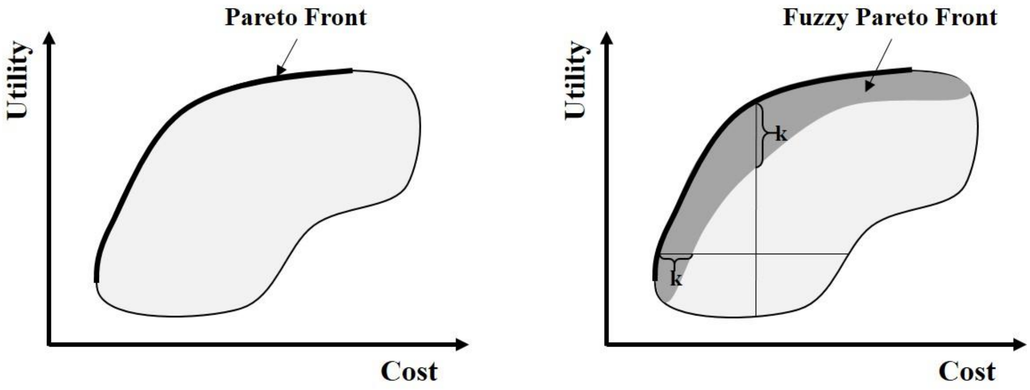

Figure 1: The fuzzy Pareto concept illustrated, with a k % fuzziness for a cost-utility tradespace (Pettersen et al., 2018).

Consequently, an alternative with a 5% fuzzy Pareto trace of 0.25, is within the 5% range of the Pareto front in 25% of the epochs. This concept is useful when there is substantial model uncertainty, or future uncertainty regarding system context or stakeholder needs, making Pareto optimality based on one model alone an insufficient criteria for eliminating alternatives in the search for a fleet structure.

The remainder of the paper will address the case study concerning a notional emergency response and rescue service.

PROBLEM DESCRIPTION

The case study concerns the deployment of a fleet of vessels for a hypothetical emergency response and rescue service, including tasks often performed by coast guards. The primary mission of the fleet is to perform patrols in a large geographical region, for example, bounded by national maritime boundaries. Patrols normally span tasks like controlling fishery activity, perform vessel inspections, and actively maintaining a presence in the region. Location of patrols is determined mostly by the fishery activity, to prevent overfishing. Beyond the baseline patrol mission, numerous additional stakeholders, both governmental and commercial, have an interest in being supported by the services the fleet can provide. For example, the fisheries, shipping companies, companies in the offshore oil and gas industries, and the public have an interest in the emergency preparedness offered by patrolling coast guard vessels. Strategic national interests, including military interests, are maintained by ensuring that an area is covered as well as possible by patrols. It may be difficult to properly separate the functions of the coast guard from the functions of a navy in many cases. We limit this study to the civilian functions serviced.

Among uncertain events that the fleet should respond to, are oil recovery operations, emergency towing operations, and search and rescue operations. Oil recovery operations will take place as a response to all types of oil spill events. These include possible vessel collisions or groundings, as well as blowouts from offshore wells. Oil spills due to grounding are sometimes attributed to a lack of available towing resources, in the case where a vessel loses navigational control. Emergency towing may therefore also be a concern here, possibly to assist dedicated tugboats. A final type of relevant emergency response operations is search and rescue missions. Search and rescue can be performed either by use of helicopters or by use of fast patrol craft launched from the emergency response and rescue vessel.

Several alternative objectives could be addressed by a mathematical programming approach to this situation, given that different aspects of the problem situation are emphasized. Some possible objectives are the minimization of operating costs, maximization of the number of patrol days per year, maximization of utilization, minimization of response time, and maximization of geographical coverage.

Limiting our approach, we define a deterministic, integer program in the next section, in which fleet deployment is done on basis of a single objective, which is to maximize geographical coverage of multiple missions. In principle, evaluating the same fleet against measures like minimized response time, maximized patrol time, and so on, could serve as interesting future additions to the work, accounting for the multiple objectives of the *actual* problem situation.

FLEET DEPLOYMENT WITH MAXIMAL COVERING

Modeling assumptions

This section presents the Fleet Deployment model with Maximum Covering (FDMC). A heterogeneous fleet of service vessels (Coast Guard, emergency response and rescue vessels, or similar services) shall be deployed to a larger geographical region. The fleet is heterogeneous in terms of maximum speed and functionality. The operating region is divided into smaller areas to which patrols are assigned. These patrol areas represent the primary operating environment for each vessel. We will consider each patrol area as a node in the model reflecting the "mean" patrol location of the vessel. The patrol areas to which vessels can be assigned are represented as nodes in Figure 2. In the figure, dashed lines indicate the border between the patrol areas.

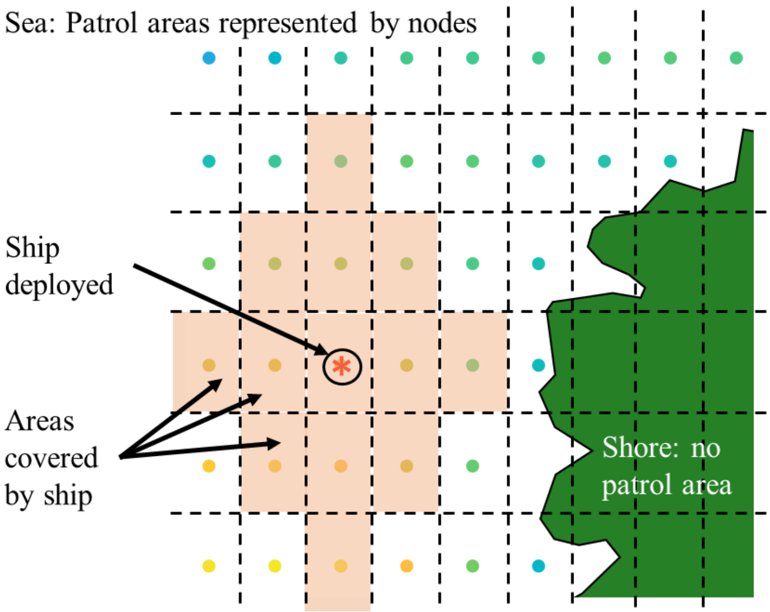

Figure 2: Representing the patrol areas as nodes. Dashed lines indicate the border between patrol areas. Dots indicate nodes with demand for some response mission. Lighter nodes are valued more. The red 'star' indicates an area with a patrol requirement. The circle indicates that a ship is deployed to that patrol area. The colored patrol areas are considered covered by the deployed vessel.

Besides the nodes that symbolize patrol areas, there is a risk that sudden-onset emergency response needs are triggered at other nodes. These emergency response needs are addressed by vessels performing missions to resolve the emergencies. This is conditional on (i) vessel functionality and (ii) the ability of the vessel to respond within an acceptable time. First, vessel-mission compatibility is treated as a binary criterion, meaning that vessels are not differentiated in terms of *how well* they meet a demand, as long as they are compatible. Second, response time thresholds are used as binary criteria that define whether a vessel is close enough to a node to cover it. In Figure 2, the colored patrol areas are hence within the response time threshold for a vessel located at the location marked 'Ship deployed'.

Given that the fleet of vessels must be deployed to primary patrol areas, the objective of the model is to optimize the covering of nodes where emergency response may be needed, by vessels assigned to patrol area nodes. The importance of covering the nodes where emergencies may occur is assumed known and set to reflect static measures of the risk of emergency in that node location.

The mathematical formulation of the FDMC problem is an extension of the maximal covering problem. For a generic formulation of maximal covering problems in the facility location context, see Owen & Daskin (1998). Several extensions are made compared with the maximal covering problem in its basic form, beyond those discussed above. First, a common critique of covering problems is that nodes are either covered or not, as determined by some defined critical distance determining whether a node is within the relevant response time threshold. Hence, covering models normally do not reflect that nodes can be partially covered (Drezner, Drezner, & Goldstein, 2010; Karasakal & Karasakal, 2004). Here this problem is handled by introducing several *layers* to the response time threshold. Second, our model reflects vessel interaction effects by considering that the marginal contribution to the objective diminishes when an increasing number of vessels cover a node. This is handled introducing *batches* that are associated with a specific contribution to the objective. Preserving model linearity, our model hence assumes that piecewise linear functions sufficiently well approximate non-linear phenomena like the demand for node coverage. This approach is inspired by a linearization presented by Pantuso, Fagerholt, & Wallace (2016), though for another problem.

Notation for the mathematical model

The sets and indices used in the program are:

i, j	nodes, ie. operating areas. The nodes to which vessel are deployed for patrol are denoted j.
v	vessel types.
m	mission types.
k	batches in the piecewise constant function.
r	layers for acceptable response time thresholds.
N	set of all nodes.
N^P	set of nodes with a patrol requirement, $N^P \subset N$.
N_{ivmr}	set of nodes within acceptable response time for vessels v responding to mission of type m at node i, within response time threshold layer r.
V	set of vessels.
V_i	set of vessels that can be assigned to a patrol at node i, $V_i \subset V$.
M	set of missions.
K	set of batches for diminishing return on coverage.
R	set of layers for acceptable response time thresholds.

The following parameters are used in the formulation:

P_v	number of vessels of type v available in the given fleet.
H_{imkr}	demand for functionality for mission m at node i at batch k, within response time threshold layer r.
H_i^P	required number of vessels at node i.
L_{imkr}	maximum number of vessels that provide coverage for mission m at node i in batch k, within response time threshold layer r, i.e. batch size.
A_{ivm}	binary parameters defining the compatibility at node i between vessel v and mission m.
D_{ij}	distance between nodes i and j.
T_{mr}	acceptable response time for mission m, in response time threshold layer r.
S_v	maximum speed of vessel v.

Finally, the variables included in the model are:

x_{iv}	integer variable describing the number of vessels of type v assigned to node i.
y_{imkr}	integer variable describing the coverage of mission m at node i in batch k, within response time threshold layer r.

Preprocessing of model data

Before introducing the mathematical program, some explanation of the preprocessing is required. The preprocessing generates data for the mathematical program, including subsets constraining the solution space. The set of vessels V_i includes all vessels that can operate at node i, considering whether a vessel can meet the physical operating environment at a node or not. Further, the set of nodes N_{ivmr} defines whether a vessel v located at node j can contribute to mission m, given a time threshold layer r. This depends on two vessel characteristics, the maximum speed S_v and the compatibility A_{ivm} between vessel v and mission m at node i. Additionally, the distance D_{ij} between the node j representing assigned patrol area and the node i at which response is needed, and the acceptable response time T_{mr}. Hence, node j is included in N_{ivmr}, according to the criteria in Equation (1).

$$j \in N_{ivmr} \mid \left(\frac{D_{ij}}{S_v} \leq T_{mr}\right) \cap (A_{ivm} = 1), \qquad i, j \in N, v \in V_i, m \in M, r \in R \tag{1}$$

To ensure linearity in the mathematical program, we approximate non-linear phenomena using piecewise constant functions, following Pantuso et al. (2016). First, the layers for acceptable response time thresholds r, can be interpreted from Figure 3. Figure 3 illustrates the dependency of the contribution to the objective (H_{imkr}), on the calculated response time $\frac{D_{ij}}{S_v}$ for vessel v deployed to node j, responding to mission m at node i, given an acceptable response time threshold T_{mr}. When $\frac{D_{ij}}{S_v} > T_{m2}$ the contribution towards the objective is therefore 0. No subset N_{ivmr} will include node j in such a case. The example in Figure 3 considers only two layers r of the response time threshold, making the covering more gradual. This will correspond to valuing “fast” response, in the case where $r = 1$, or “slow” response, in the case where $r = 2$.

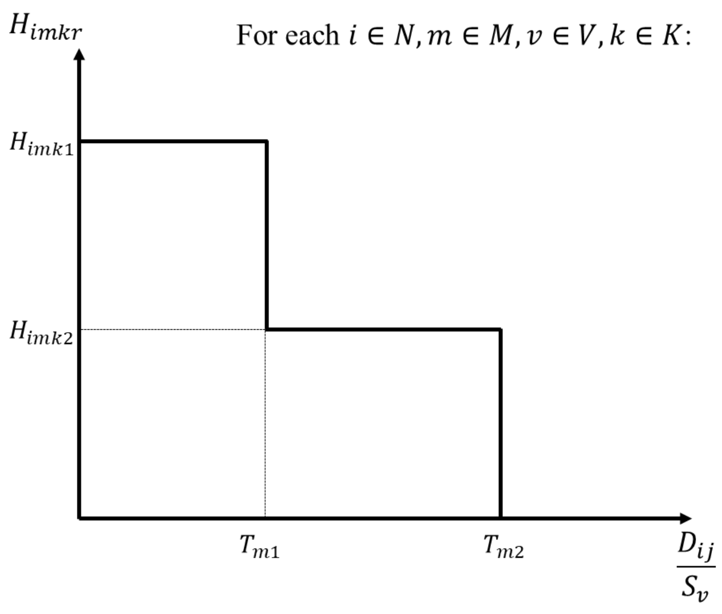

Figure 3: Influence on H_{imkr} of the response time of a vessel v deployed to a node j towards a mission of type m at node i.

To account for vessel interaction effects represented by diminishing marginal utility of additional coverage of a node, we need to linearize this non-linear phenomenon: The addition to the objective function of an additional vessel covering mission m at node i is dependent on the number of vessels already covering mission m at node i. A limited number of vessels, given by the batch size L_{imkr} can contribute with a value H_{imkr} to the objective, at each batch k. If there are diminishing returns for each additional resource, the batch size is $L_{imkr} = 1$. The formulation of the concave dependency between the objective and the number of vessels assigned is linearized using Equation (2), modelled using a geometric series representing the diminishing returns on resources.

$$H_{imkr} = H_{imr} \sum_{\lambda=1}^{k} \alpha_m^{\lambda-1}, i \in N, m \in M, k \in K, r \in R \quad (2)$$

In Equation (2), H_{imr} is the basic contribution associated with the first batch $k = 1$, and α_m is a parameter between 0 and 1. If $\alpha_m = 0$, then coverage of a node i is counted only for the first vessel that covers the node, and no additional coverage will contribute to the objective. If $\alpha_m = 1$, there are no diminishing returns of increasing the number of vessels covering the node. The relationship between the contribution to the objective function and coverage is illustrated in Figure 4, with $\alpha_m = 0.5$. Here, it is assumed that $y_{imr} = \sum_{k \in K} y_{imkr}$.

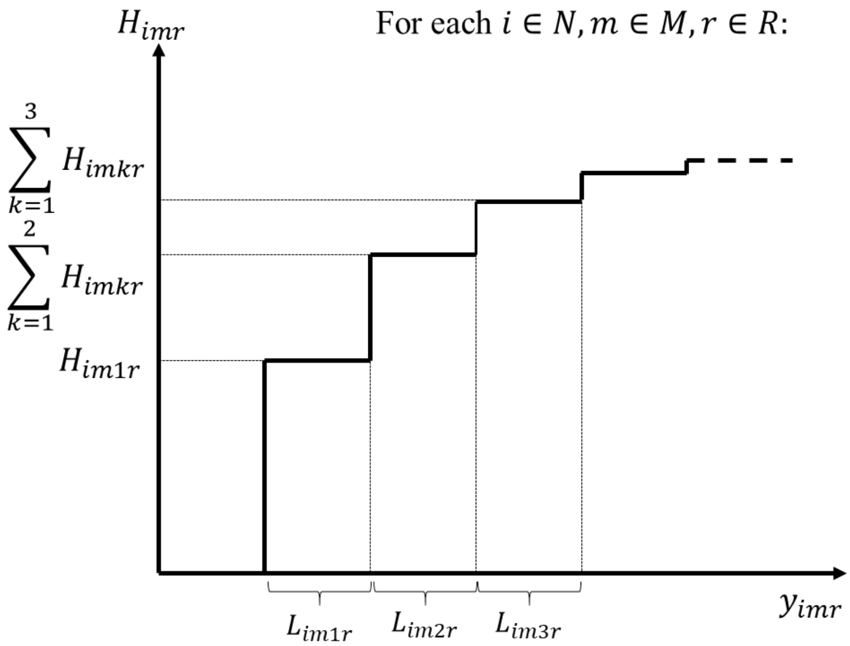

Figure 4: Modelling the dependency of H_{imkr} on the number of vessels y_{imr} covering mission m at node i for response threshold r.

Mathematical formulation

We here present the mathematical program for the FDMC problem, which is used to optimize the deployment of given fleet alternatives. We hence analyze the value of a given fleet, by optimizing its deployment. The model formulation is presented below:

$$\max \quad \sum_{i\in N}\sum_{m\in M}\sum_{k\in K}\sum_{r\in R} H_{imkr} y_{imkr} \tag{3}$$

$$\text{s. t.} \quad \sum_{v\in V_i} x_{iv} \geq H_i^P, \qquad i \in N^P \tag{4}$$

$$\sum_{k\in K} y_{imkr} - \sum_{v\in V_i}\sum_{j\in N_{ivmr}} x_{jv} \leq 0, \qquad i \in N, m \in M, r \in R \tag{5}$$

$$\sum_{i\in N} x_{iv} \leq P_v,\ v \in V \tag{6}$$

$$y_{imkr} \leq L_{imkr},\ i \in N, m \in M, k \in K\backslash\{\bar{K}\}, r \in R \tag{7}$$

$$x_{iv} \in \mathbb{Z}^+, i \in N,\ v \in V \tag{8}$$

$$y_{imkr} \in \mathbb{Z}^+,\ i \in N, m \in M, k \in K, r \in R \tag{9}$$

In the FDMC problem described above, the objective is to maximize the amount of demand covered (3). Constraints (4) ensure that the patrol requirement for nodes $i \in N^P$ is met by assigning a sufficient number of vessels v to node i. Constraints (5) ensure that mission m is only covered by vessels v assigned to nodes within acceptable response time layer r, hence ensuring that node $j \in N_{ivmr}$. Constraints (6) restrict the number of vessels of type v that can be deployed to the number of vessels available of type v in the fleet. Constraints (7) limit the number of vessels v that can contribute to the objective function by covering mission m at node i, at each batch up to $k \in \{1,..,K-1\}$, at response time threshold layer r, to the batch size L_{imkr}. This ensures that the most valuable batch is filled first. In the case where $k = K$, the contribution is very small, ensuring that little additional value results from additional coverage. Constraints (8) – (9) limit the variables to taking integer values.

ϵ-constraint method

The mathematical program above only presents us with the optimal deployment of a given fleet alternative. Hence, in the form presented above it does not account for cost-benefit trade-offs. The main purpose of using the ϵ-constraint method is to provide a benchmark Pareto set of fleet alternatives deployed through the use of the basic FDMC model, given the cost-benefit trade-off. The ϵ-constraint variant of the problem essentially solves the problem when we include decisions regarding fleet size and mix, given multiple levels of a budgetary constraint. For a previous example of the ϵ-constraint method applied to a maximal covering problem for emergency response, see Paul et al. (2017). In the ϵ-constraint variant of the FDMC model, we allow the model to select the number and types of vessels to include. The Pareto front is found by running the model with some modifications. Constraints (6) is replaced by Constraints (10). The ϵ-constraints (11) and Constraints (12) on the number of vessels of each type that can be added to the fleet, are added to the model.

$$\sum_{i\in N} x_{iv} \leq z_v,\ v \in V \tag{10}$$

$$\sum_{v\in V} C_v\, z_v \leq \epsilon \tag{11}$$

$$z_v \leq B_v, v \in V \quad (12)$$

Here, z_v is the variable that describes the number of vessels of type v included in the fleet. Notice that z_v replaces the parameter P_v, and that now the number of vessels in the fleet is a decision variable. Constraints (11) provide budgetary constraints on investment in vessels to a budget ϵ. C_v denotes the investment costs for vessel type v, which are given in Table 2. By systematically rerunning the model with different levels chosen for the available budget ϵ, we obtain a set of Pareto optimal solutions. To be consistent with the random sampling of fleet alternatives, the total available number of each ship type v to invest in, B_v, is set to 4, meaning that $B_v = 4$, in Constraints (12). Finally, Constraints (13) state that the number of vessels to invest in, is an integer:

$$z_v \in \mathbb{Z}^+, v \in V \quad (13)$$

CASE STUDY

Ship data

Here we present the input data for the FDMC problem. Missions are to be addressed by a given fleet composed of vessels with various capabilities. Vessels are described by their maximum speed, some important characteristics that define if a vessel will be able to address specific missions, and the capital expenditures assumed for each vessel. The expenditure is not considered in the basic FDMC formulation but is used to provide insight into the cost-benefit trade-off when exploring alternatives. Hence, capital expenditures are used for C_v in the ϵ-constraint variant of the FDMC problem. Vessel data is presented in Table 2. The technical specifications in Table 2 are derived from vessels operated by the Norwegian Coast Guard (Forsvaret, 2018). The capital expenditure is based on estimates according to a simplified cost model presented by Buland (2017). In addition, for vessels with helicopter capabilities, estimated unit costs of 40 mUSD per helicopter are included in the estimates given in Table 2. This assumes that there is one helicopter per vessel, neglecting that there may exist a related helicopter assignment problem in which helicopters are assigned to vessels where they are most needed.

Table 2: Description of vessel types, in terms of functionality and equipment.

Design parameters	**Vessel types $v \in V$**							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Max speed [kn]	20	21	23	25	18	28	22	16
Oil recovery [m^3]	0	1000	500	0	1000	0	500	1000
Bollard pull [tons]	50	50	70	70	150	50	100	100
Helicopter	0	0	1	1	0	1	1	1
Small boats	1	1	1	2	2	3	2	2
Arctic capabilities	0	0	0	0	0	1	1	1
Capital expenditures [mUSD]	27	38	74	78	38	117	95	92

Of these, the maximum speed for each vessel is used directly, as S_v determines N_{ivmr} in accordance with Equation (1). The cost data is used to compare the investment costs for alternative fleets, to find compromises between costs and the objective function maximizing coverage. Based on the other data in Table 2, we infer what missions each ship will be able to perform, and map from vessel form to function. This materializes in the compatibility matrix A_{ivm}, for which input data is presented in Table 3. All vessels can be assigned to the basic mission which is patrol, subject to the constraint that some operating areas require Arctic capabilities. Only vessels with Arctic capabilities can operate in areas at 74° or further north.

Table 3: Compatibility between the vessel type v, and missions m.

Missions, $m \in M$	**Vessel types, $v \in V$**							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Patrol (baseline mission)	1	1	1	1	1	1	1	1
Oil recovery (500 m^3) ($m = 1$)	0	1	1	0	1	0	1	1
Oil recovery (1000 m^3) ($m = 2$)	0	1	0	0	1	0	0	1
Towing missions (small) ($m = 3$)	1	1	1	1	1	1	1	1
Towing missions (large) ($m = 4$)	0	0	0	0	1	0	1	1
Search and rescue (Boat support) ($m = 5$)	0	0	1	1	1	1	1	1
Search and rescue (Heli support) ($m = 6$)	0	0	1	1	0	1	1	1

Epoch-independent parameters

The operating region to which the vessels will be assigned is represented as a set of nodes with associated coordinates. We set the limitations of the operating region to roughly overlap with the Norwegian Sea, as well as parts of the North Sea and the Barents Sea, represented by the coordinates between 55° and 80° North, and between 10° West and 30° East. The grid of nodes is generated with a step length of 0.5°, yielding a total number of 4131 nodes.

In addition to defining the node coordinates, the model requires criteria for acceptable response times, which are then be used to generate N_{ivmr}, in correspondence with Equation (8). The maximum acceptable response times are set according to Table 4. Note that two response times are provided for each mission, a "fast" and a "slow" response, in accordance with Figure 3.

Table 4: Maximum acceptable response times for missions (in hours).

Missions, $m \in M$	T_{m1}	T_{m2}
Oil recovery (500 m^3) ($m = 1$)	6	10
Oil recovery (1000 m^3) ($m = 2$)	6	10
Towing missions (small) ($m = 3$)	6	10
Towing missions (large) ($m = 4$)	6	10
Search and rescue (Boat support) ($m = 5$)	3	5
Search and rescue (Heli support) ($m = 6$)	3	5

Note that for $m = 6$, the critical time limit is set in accordance with helicopter response times. Hence, when setting N_{ivmr}, we do not use the expression in Equation (8) directly. Rather, for this mission, we replace the ship speed given in Table 2, with the official maximum speed of an NH 90 helicopter, which is approximately 160 knots (NH Industries, 2018).

Epoch-dependent parameters

Several parameters in the mathematical program represent things that will be subject to change within a strategic planning horizon. This particularly concerns the parameters describing the required patrol areas H_i^P, and the importance of covering a specific node, H_{imkr}. For all practical purposes, context-dependent parameters constitute what could be considered as *epoch variables*. A static contextual scenario, or an *epoch* (Ross & Rhodes, 2008), can then be represented by a specific configuration of these two sets of parameters.

The parameters can be tuned such that they reflect the approximated risk levels associated with certain events at different locations. Input for the demand for node coverage can hence draw on results from accident analyses, risk analyses, and estimates of ship traffic densities based on automatic identification system (AIS) data. For possible means to generate these inputs, see Kristiansen (2005) for an overview of techniques for risk assessment for maritime transport, Rausand (2011) for a review of measures to quantify risk to people, useful for quantification of the importance of search and rescue, and Kontovas, Psaraftis, & Ventikos (2010) for an example of an empirical analysis of oil spill cost data.

In this paper, we develop illustrative scenarios by assigning coverage demands H_{imkr}. H_{imkr} could be varied according to the development of the demand of node coverage, as perceived by the key stakeholders, or tuned to reflect the stakeholder belief about current or future importance of response. An illustration of one possible scenario, which corresponds to one *epoch*, is presented in Figure 5. Coordinates in this operational environment are assigned values according to whether they correspond to high traffic density locations, or are close to offshore oil and gas installations or fishing fields. The locations are approximated based on visual inspections of traffic density maps based on AIS data from MarineTraffic.com (2018).

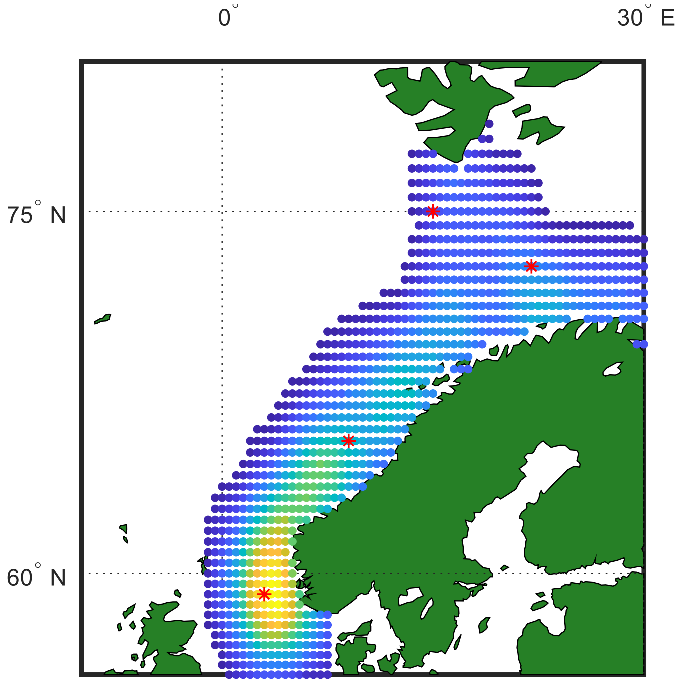

Figure 5: Possible scenario (data for Epoch 1 - "Baseline"). Hollow circles indicate ship deployments to a node. Dots indicate nodes with demand. Lighter nodes are valued more. Red 'stars' indicate nodes with required patrol presence.

Qualitative descriptions of four epochs the system can face are given in Table 5. Input data for all epochs are presented in Table 6, connected to the type of maritime activity generating risks. Data is given for the "smaller" types of missions, for the similar mission type of a larger complexity, H_{imkr} is given twice the value of the smaller, similar mission. A "center location" is given, indicating the center of the area where there is a risk of an event of some type. Any node within a specified distance D_{MAX} is assigned a positive H_{imkr}, decreasing linearly until the distance between the nodes i and i' exceeds D_{MAX}. Demand for patrols at certain locations vary according to the epochs as presented in Table 7.

Table 5: Quantitative descriptions of epochs.

Epoch	Description
1 - "Baseline"	The situation at the time of initial deployment.
2 - "Decommissioned"	A complete shutdown of oil and gas production, and increase in fishing.
3 - "Short sea shipping"	Increase in the coastal marine traffic activity overall.
4 - "Arctic boom"	Increase in Northern Sea Route traffic, fishing, and oil and gas activity in the Arctic region.

Table 6: Input data for demand for node coverage for all missions, for all epochs.

Type of area	Center location [lat., long.]	H_{im11} in the epochs {Epoch 1, Epoch 2, Epoch 3, Epoch 4}					
		$m = 1$	$m = 2$	$m = 3$	$m = 4$	$m = 5$	$m = 6$
Traffic lane	Inner – south	{7, 7, 15, 7}	2x H_{i111}	{7, 7, 15, 7}	2x H_{i311}	{7, 7, 15, 7}	2x H_{i511}
	Outer – south	{5, 5, 12, 7}		{5, 5, 12, 7}		{5, 5, 12, 7}	
	North	{5, 5, 12, 15}		{5, 5, 12, 15}		{5, 5, 12, 15}	
Oil fields	[56.5°, 3.0°]	{18, 0, 18, 18}		{10, 0, 10, 10}		{15, 0, 15, 15}	
	[58.5°, 2.0°]	{18, 0, 18, 18}		{10, 0, 10, 10}		{15, 0, 15, 15}	
	[59.5°, 2.0°]	{18, 0, 18, 18}		{10, 0, 10, 10}		{15, 0, 15, 15}	
	[61.5°, 2.0°]	{18, 0, 18, 18}		{10, 0, 10, 10}		{15, 0, 15, 15}	
	[65,0°, 7.0°]	{18, 0, 18, 25}		{10, 0, 10, 20}		{15, 0, 15, 25}	
	[72.0°, 22.5°]	{18, 0, 18, 35}		{10, 0, 10, 20}		{15, 0, 15, 25}	
Fishing fields	[72.0°, 16.0°]	{0, 0, 0, 0}		{8, 8, 8, 12}		{15, 15, 15, 18}	
	[74.0°, 20.0°]	{0, 0, 0, 0}		{8, 8, 8, 12}		{15, 15, 15, 18}	
	[76.0°, 16.5°]	{0, 0, 0, 0}		{8, 8, 8, 12}		{15, 15, 15, 18}	

Table 7: Vessels required at nodes with patrol demand H_i^P (nodes given in [latitude, longitude]).

Epoch	[75.0°, 15.0°]	[73.0°, 22.0°]	[66.0°, 9.0°]	[59.0°, 3.0°]
1 - "Baseline"	1	2	2	2
2 - "Decommissioned"	1	1	2	2
3 - "Short sea shipping"	1	2	2	3
4 - "Arctic boom"	2	4	2	2

Furthermore, to reflect that we consider gradual covering to a limited extent, the number of layers is set to $R = 2$. This means that we divide coverage into considering both "fast" and "slower" responses. The value of a slow response, is set to 40% of a fast response.

The number of batches, to account for the diminishing return on additional covering, is set to $K = 5$, where the capacity of each batch, $L_{imkr} = 1$. The parameter for Equation (2) is $\alpha = 0.5$. Note that this indicates that $H_{im5r} = 0.03125H_{imr}$, which means that the contribution from additional resources assigned after the fourth vessel covers a mission at a node, is very small.

COMPUTATIONAL STUDY

The FDMC problem was implemented in Mosel and solved with the commercial solver Xpress MP, with pre- and postprocessing in MATLAB. Two variants of the model were run. First, the model of the basic FDMC problem is run. Second, we run the variant that includes the ϵ-constraint method. The first variant tests fleet alternatives that may or may not be Pareto optimal in all four epochs. The second variant generates the Pareto optimal set of fleet alternatives to benchmark the former set.

Experiments with notional fleet alternatives were done for the four epochs defined in Table 5, setting a maximum time per model run to 300 seconds. 30 fleet alternatives were generated randomly using Latin hypercube sampling (McKay, Beckman, & Conover, 1979), allowing a maximum of 4 of the same ship type in the fleet. The purpose of this is to explore how the sampled alternatives will perform, with no specific search for solutions that are "optimal", given the chosen model, and the current context (combination of input parameters). This corresponds with a central argument for concept exploration, namely to reduce the reliance on modeling assumption when generating alternatives, and gain an understanding of what drives the value of the dominated part of the design space. Improvements of the experimental design could be to evaluate a fully enumerated space of fleet alternatives (Ross & Hastings, 2005), or to select an experimental design specifically intended for evaluating systems-of-systems and capturing interaction effects (Kujawski, 2014). However, Latin hypercube sampling provides a set of fleet alternatives that represents the real variability in the solution space quite well (McKay, et al., 1979).

The 30 randomly sampled fleet alternatives tested are presented in Table 8. In addition to running the model for these alternative fleets, the problem variant using the ϵ-constraint method is run to generate the Pareto front.

Table 8: Fleet alternatives evaluated.

Fleet no.	**Number of vessels of each type v**								**Total no. of vessels**
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	
1	2	1	2	0	3	2	2	4	16
2	2	2	2	2	1	3	1	3	16
3	1	0	3	2	1	3	3	1	14
4	2	2	1	2	1	2	4	0	14
5	0	4	0	0	2	1	3	0	10
7	1	1	2	3	1	2	3	0	13
8	3	1	2	0	1	4	2	2	15
9	0	4	1	2	3	4	2	2	18
10	1	0	4	2	0	3	0	4	14
11	3	4	4	2	2	1	1	3	20
12	2	3	2	3	3	3	2	3	21
13	4	3	3	1	4	1	1	1	18
14	3	4	2	1	3	3	1	1	18
15	1	0	1	3	2	0	0	3	10
16	4	3	0	4	3	3	3	2	22
17	2	2	1	1	2	0	4	2	14
18	4	2	3	1	2	4	3	1	20
19	2	1	1	0	0	1	0	2	7
20	2	0	1	4	3	1	2	4	17
21	0	1	4	3	0	2	2	2	14
22	3	2	2	3	1	3	1	0	15
23	1	2	3	3	4	1	1	1	16
24	1	2	0	2	4	4	4	2	19
25	2	3	1	1	4	0	1	3	15
26	0	1	3	1	0	2	2	1	10
27	4	2	3	4	1	2	3	2	21
28	3	3	0	4	2	2	2	3	19
29	1	1	3	3	2	0	4	4	18
30	3	3	2	1	2	2	3	3	19

Results

Results from runs of the FDMC model were obtained for all 30 fleet alternatives in all four epochs, and costs were quantified. Hence, the mathematical program was evaluated in total 120 times. Again, note that only the *deployment* of the fleet is determined by the model, while the fleet configuration is considered as parameters, and given as input at this stage. Additionally, the ϵ-constraint method produced a Pareto front for benchmarking the design space of 30 alternative fleets in each epoch. In the ϵ-constraint variant, the fleet configuration is also decided by the model, to provide an upper bound for the objective function value, at each given budgetary constraint. The cost-utility tradespace for Epoch 1 – “Baseline”, is shown in Figure 6.

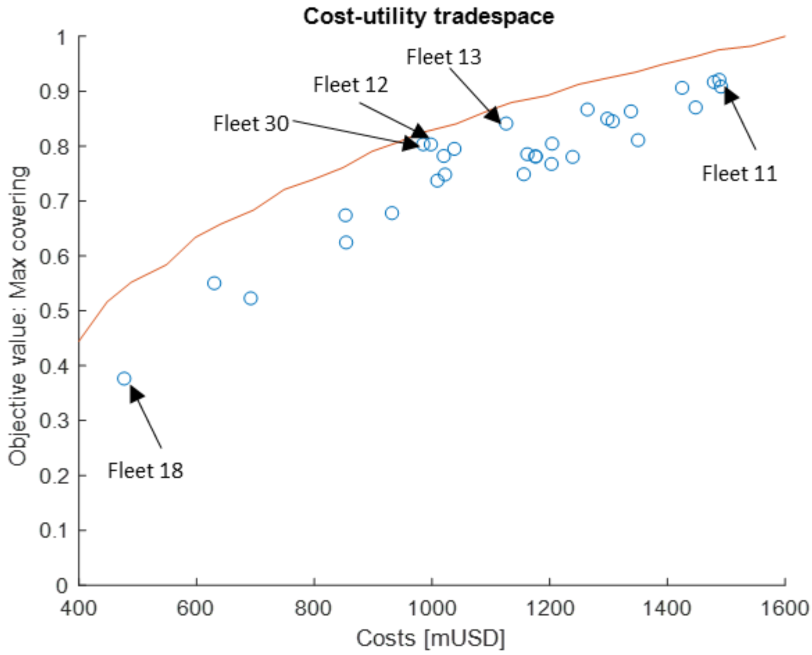

Figure 6: Cost-benefit tradespace for Epoch 1 – “Baseline”, showing the 30 fleet alternatives generated by random sampling, and the Pareto front found using the ϵ-constraint method. Some of the fleet alternatives are highlighted.

The costs and objective values for all fleet alternatives in all epochs are presented in Table 9. The optimal deployment in each epoch was found within the time limit of 300 seconds for nearly all tested fleet alternatives. For the remaining alternatives, we accepted solutions with negligible optimality gaps of less than 0.1 %. The objective function values have all been unit-normalized. We use the optimal objective function value obtained by the ϵ-constraint method at the maximum budgetary constraint, as the maximum of the normalization scale.

Table 9: Optimal objective function value, and costs for the fleet alternatives, given the different epochs.

Fleet no.	**Objective function value in the epochs, normalized** Epoch 1	**Epoch 2**	**Epoch 3**	**Epoch 4**	**Costs [mUSD]**
1	0.75	0.72	0.74	0.69	1156
2	0.80	0.78	0.80	0.75	1204
3	0.78	0.73	0.77	0.70	1176
4	0.75	0.69	0.74	0.66	1022
5	0.55	0.41	0.53	0.38	630
6	0.74	0.67	0.73	0.63	1009
7	0.79	0.75	0.78	0.72	1162
8	0.86	0.85	0.86	0.83	1338
9	0.77	0.72	0.76	0.69	1203
10	0.87	0.86	0.86	0.84	1264
11	0.91	0.90	0.91	0.90	1491
12	0.80	0.77	0.80	0.75	998
13	0.84	0.82	0.83	0.80	1126
14	0.52	0.40	0.50	0.36	692
15	0.92	0.92	0.92	0.91	1488
16	0.68	0.62	0.67	0.58	932
17	0.91	0.90	0.90	0.89	1425
18	0.38	0.00	0.34	0.00	477
19	0.78	0.75	0.77	0.73	1239
20	0.78	0.72	0.77	0.69	1176
21	0.80	0.74	0.79	0.71	1038
22	0.78	0.74	0.77	0.71	1020
23	0.87	0.86	0.87	0.85	1448
24	0.67	0.63	0.66	0.59	853
25	0.62	0.48	0.61	0.45	854
26	0.92	0.91	0.91	0.91	1479
27	0.85	0.83	0.84	0.82	1298
28	0.81	0.79	0.80	0.76	1350
29	0.85	0.83	0.84	0.81	1307
30	0.80	0.75	0.80	0.72	985

Table 10 shows the estimated fuzzy Pareto number for each fleet alternative in each epoch. Subsequently, the fuzzy Pareto trace at 5 % and 10 % fuzziness for each fleet alternative in each epoch, is also included. The reference Pareto set used to obtain the fuzzy Pareto numbers and the Pareto traces was found using the ϵ-constraint method.

Table 10: Fuzzy Pareto number and fuzzy Pareto trace for all fleet alternatives in all epochs (Inf. = Infeasible).

Fleet no.	Fuzzy Pareto number (FPN) in the epochs				Fuzzy Pareto trace at		
	Epoch 1	Epoch 2	Epoch 3	Epoch 4	5 %	10 %	20 %
1	17.5 %	20.1 %	18.2 %	23.0 %	0	0	0.5
2	10.9 %	12.9 %	11.2 %	14.7 %	0	0	1
3	12.6 %	18.4 %	13.1 %	21.1 %	0	0	0.75
4	10.1 %	15.1 %	10.6 %	17.1 %	0	0	1
5	15.2 %	26.8 %	16.7 %	25.1 %	0	0	0.5
6	11.7 %	19.0 %	12.1 %	22.1 %	0	0	0.75
7	12.0 %	15.1 %	12.7 %	17.1 %	0	0	1
8	6.8 %	7.2 %	7.2 %	7.8 %	0	1	1
9	16.2 %	21.6 %	16.6 %	24.4 %	0	0	0.5
10	5.3 %	4.4 %	5.5 %	4.6 %	0.5	1	1
11	7.4 %	7.4 %	7.5 %	8.2 %	0	1	1
12	2.6 %	2.5 %	2.4 %	2.8 %	1	1	1
13	2.8 %	2.6 %	2.8 %	2.6 %	1	1	1
14	25.8 %	40.0 %	28.4 %	42.9 %	0	0	0
15	5.9 %	5.7 %	5.9 %	6.1 %	0	1	1
16	16.6 %	17.9 %	16.9 %	19.4 %	0	0	1
17	4.7 %	4.9 %	4.5 %	5.3 %	0.75	1	1
18	37.1 %	Inf.	47.2 %	Inf.	0	0	0
19	14.3 %	16.2 %	14.8 %	18.4 %	0	0	1
20	12.5 %	19.2 %	13.1 %	22.2 %	0	0	0.75
21	5.7 %	7.4 %	5.7 %	8.6 %	0	1	1
22	5.3 %	6.6 %	5.3 %	7.9 %	0	1	1
23	10.7 %	11.5 %	11.0 %	12.2 %	0	0	1
24	12.9 %	13.1 %	12.8 %	14.5 %	0	0	1
25	21.8 %	47.0 %	22.5 %	51.4 %	0	0	0
26	5.1 %	4.7 %	5.2 %	4.8 %	0.5	1	1
27	8.5 %	8.8 %	9.0 %	9.4 %	0	1	1
28	15.3 %	18.2 %	16.0 %	21.2 %	0	0	0.75
29	9.1 %	9.6 %	9.7 %	10.2 %	0	0.75	1
30	2.5 %	5.0 %	2.4 %	6.2 %	0.5	1	1

In Epoch 1, four sampled fleet alternatives have an FPN below 5 %, meaning that they are within 5 % of the Pareto front. Similarly, 13 fleet alternatives have an FPN below 10% in Epoch 1, meaning that these are within 10 % of the Pareto front. Two fleet alternatives are always within 5 % of the Pareto front, as shown by the fuzzy Pareto trace metric indicating robustness against change in parameter values. These alternatives are Fleet 12 and Fleet 13, which are fleet mixes in the medium price range. Fleet 12 consists of an evenly spread number of vessels, across all vessel types. Fleet 13 includes a majority of inexpensive vessel types like Vessel 1, Vessel 2, and Vessel 5.

The deployment solutions that were accepted for Fleet 12 for each scenario, are presented in Table 11. The corresponding maps for these scenarios are shown for Fleet 12 in Figure 7 - 10. These results reveal that multiple vessels are never deployed to the same node unless this is required in accordance with Table 7. A major reason for this is the diminishing returns associated with deploying several vessels to the same location. Further, we see that there is a general tendency to locate vessels in the southern parts of the operating area, where there is most demand. Unsurprisingly, there is a move towards northern locations in Epoch 4, as there is more demand in northern areas. The deployment in epochs with decreasing oil and gas activity (Epoch 2), and increasing short sea shipping (Epoch 3), are quite similar, although not identical. Compared to Epoch 2, Epoch 3 sees a slight increase in the deployment to northern areas.

Table 11: Number of vessels in Fleet 12 deployed to each node, for each epoch.

	Number of vessels assigned in each epoch			
Coordinates [lat., long.]	**Epoch 1**	**Epoch 2**	**Epoch 3**	**Epoch 4**
[57°, 2.5°]	1	0	0	0
[57°, 5.5°]	0	0	1	0
[57°, 6°]	0	1	0	0
[57.5°, 4°]	0	0	1	0
[57.5°, 4.5°]	1	1	0	0
[59°, 3°]	2	2	2	2
[60.5°, 1.5°]	1	0	0	0
[61°, 3°]	0	0	1	0
[61°, 3.5°]	0	1	0	0
[61.5°, 3°]	1	0	0	1
[63°, 4°]	1	1	1	0
[63.5°, 5°]	0	1	0	0
[64°, 5.5°]	0	0	1	1
[64.5°, 5.5°]	1	0	0	0
[65°, 6.5°]	0	1	0	1
[65°, 7°]	1	0	1	0
[66°, 9°]	2	2	2	2
[67.5°, 11°]	0	0	1	0
[68°, 11.5°]	0	1	0	0
[68°, 12°]	1	0	0	0
[68.5°, 12.5°]	0	0	1	1
[69°, 13°]	0	1	0	0
[69°, 13.5°]	1	0	0	0
[69.5°, 14°]	0	0	1	1
[70.5°, 16.5°]	1	1	0	0
[70.5°, 17°]	0	0	1	0
[71°, 17°]	0	0	0	1
[72°, 23.5°]	1	0	1	0
[72°, 25°]	0	0	1	1
[72°, 28.5°]	0	0	0	1
[73°, 22°]	2	3	2	4
[75°, 15°]	1	2	1	2

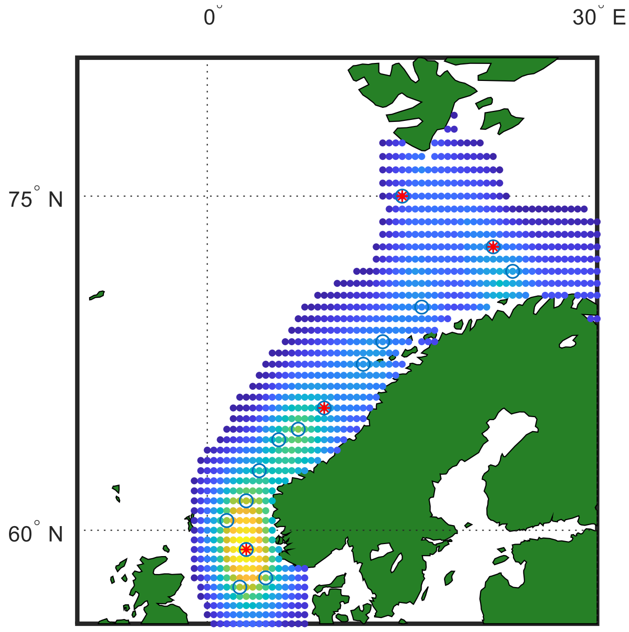

Figure 7: Fleet 12 deployed in Epoch 1 - “Baseline”. Hollow circles indicate ship deployments to nodes. Dots indicate nodes with demand. Lighter nodes are valued more. Red 'stars' indicate nodes with required patrol presence.

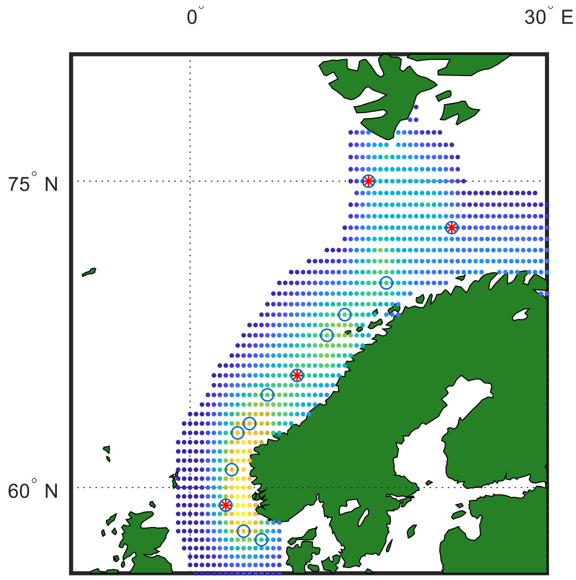

Figure 8: Fleet 12 deployed in Epoch 2 – “Decommissioned”. Hollow circles indicate ship deployments to nodes. Dots indicate nodes with demand. Lighter nodes are valued more. Red 'stars' indicate nodes with required patrol presence.

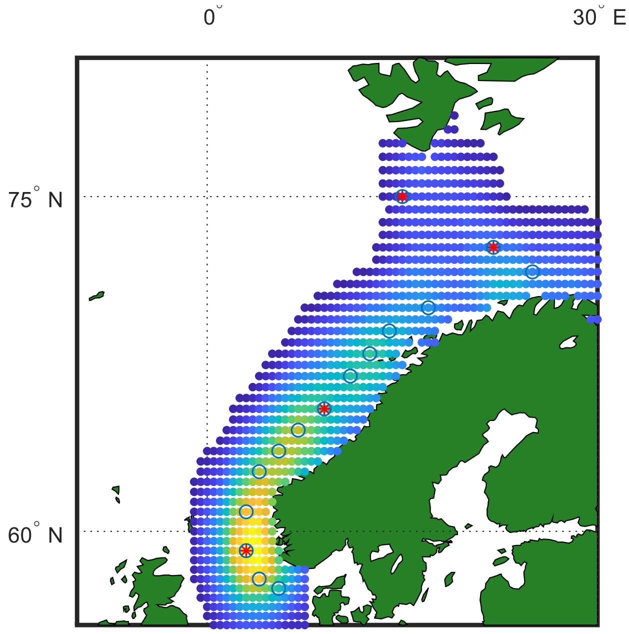

Figure 9: Fleet 12 deployed in Epoch 3 – “Short sea shipping”. Hollow circles indicate ship deployments to nodes. Dots indicate nodes with demand. Lighter nodes are valued more. Red 'stars' indicate nodes with required patrol presence.

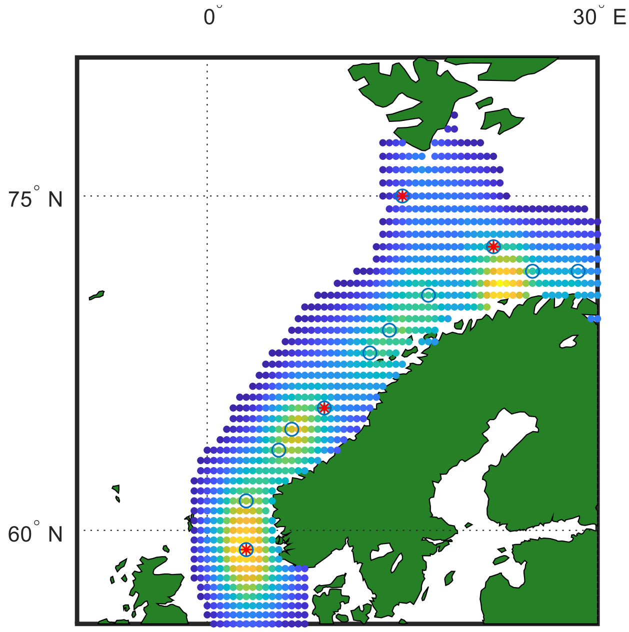

Figure 10: Fleet 12 deployed in Epoch 4 - "Arctic boom”. Hollow circles indicate ship deployments to nodes. Dots indicate nodes with demand. Lighter nodes are valued more. Red 'stars' indicate nodes with required patrol presence.

DISCUSSION

We have proposed a division of roles for decision support methodology in fleet planning by suggesting that mathematical programming mainly is used as a tool for evaluating alternatives in a deployment setting. In this case, the mathematical program provides a measure of fleet effectiveness. To explore the effectiveness of the alternatives across multiple scenarios and support strategic decision-making, including vessel acquisition and fleet renewal decisions, the FDMC model was solved for an experimental design that considers 30 fleet alternatives and 4 epochs.

The results of the case study by themselves have limited validity beyond an early design phase. Reasons for this include the use of partially illustrative model input data and some unrealistic modeling assumptions. However, important insights are extracted from this study, with respect to the exploration of alternative solutions to fleet size and mix in emergency preparedness and coast guard settings:

- First, moving beyond ship design to consider *measures of effectiveness* on the fleet level, rather than ship performance, requires tools that account for relative ship positioning, and other deployment insights.
- Second, the resource allocation literature from operations research provides a rich starting point for use of such techniques, where the o*ptimal deployment scheme* can be used as a *measure of effectiveness* for a set of fleet alternatives to be used for *evaluation* of alternatives.
- Third, rather than stochastic programming for the strategic decisions, we emphasize scenario planning in the slightly more qualitative sense (Schoemaker, 1991). This is accomplished via epoch-era analysis.
- Fourth, epoch-era analysis has largely neglected exploration of fleet level planning. Epoch-era analysis has, to the knowledge of the authors, not previously been combined with optimization of fleet deployment.

Beyond these four highlighted insights, the choice of the model objective function in the current approach needs to be addressed. Importantly, only one measure of effectiveness was considered. The reasoning behind selecting the maximal covering objective for the FDMC model is that it is a well-defined measure of effectiveness that values the fleet effectiveness, given that certain functional requirements and patrol mission requirements are met. The weighting of the importance of covering nodes can be adjusted on basis of risk analyses and automatic identification system data. The latter is a topic under intense current research and could contribute to improved ship design and fleet planning through use in the deployment setting.

Even though the FDMC model applies some weighting between alternative missions that need to be covered at different nodes, the alternatives evaluated by the current approach should also be tested against other objectives. The measured fuzzy Pareto trace after such an exercise could reveal additional insights as to how robust a fleet is in settings dictated by other objectives. This would contribute to truly scrutinizing the alternatives, and get an improved understanding of the compromises that decision-makers actually face when deciding what ships to invest in. Examples of such objectives include minimization of response time, maximization of important ship capabilities, and maximization of the available patrol days. Alternatively, taking a holistic approach, objectives that are aimed at improving the integrability of ships within the overall fleet could be employed, for example with respect to commonalities in spare parts or maintenance schemes (Doerry & Fireman, 2009).

In the current approach, aspects of response time minimization and maximization of ship capabilities are both present. Response time is directly used in the preprocessing to generate the subset of nodes to which a vessel located at another node could respond. Ship capabilities are valued through differentiating whether vessels can address certain response mission types. Still, explicitly considering these as objectives could provide greatly differing results. A likely result would be that a minimize response time objective would greatly emphasize maximum ship speed as an important vessel attribute while maximizing ship capabilities would likely provide favorable results for fleets with more technically advanced vessels. Ship capability objectives could probably be addressed without resorting to methods from operations research, with more focus on common ship-level performance indicators. Detailed exploration of the trade-offs between ship-level and fleet-level design characteristics could be an interesting scope for further studies and could influence acquisitions.

The discussion of the correctness of the chosen objectives for this study justifies why we chose to randomly sample fleet alternatives, rather than attempting to optimize the selection of fleet alternatives, e.g. as done by the ϵ-constraint method that we use to generate a Pareto front for benchmarking purposes. It is difficult to be sure that the selected decision support model most correctly frames the problem, and most adequately reflects stakeholder interests. This is particularly true in the politically constrained design environment of coast guards and other organizations that address maritime emergency preparedness. Hence, optimization of decisions with a strategic planning horizon should be done with caution only, and supplemented with more exploratory approaches like epoch-era analysis.

Rather than attempt to optimize the fleet size and mix, we show that certain fleet alternatives are close to Pareto optimal in several scenarios, through the epoch-era analysis. This suggests that these solutions will provide adequate robustness towards changes in the demand for coverage. The focus on deployment also means that we can apply the proposed FDMC model to optimize the deployment of existing fleets, possibly providing new insights to how the utilization of existing resources can be improved.

CONCLUSION

This paper documents the Fleet Deployment with Maximal Covering (FDMC) problem, for evaluation of alternative fleet structures in emergency preparedness. The main thesis of the paper is that tactical fleet deployment models can be adopted from operations research to provide insights into fleet size and mix decisions, at an early-stage in a fleet planning process. Coupled with epoch-era analysis, the FDMC problem allows evaluation of a central measure of effectiveness for fleet architectures, across multiple scenarios. A continuation of the study should incorporate additional objectives like minimization of response time to emergencies, and other objectives that improve our understanding of what constitutes a good fleet.

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