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Spin Current Cross-Correlations as a Probe of Magnon Coherence

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Motivated by the important role of the normalized second-order coherence function, often called $g^{(2)}$, in the field of quantum optics, we propose a method to determine magnon coherence in solid-state devices. Namely, we show that the cross-correlations of pure spin currents injected by a ferromagnet into two metal leads, normalized by their dc value, replicate the behavior of $g^{(2)}$ when magnons are driven far from equilibrium. We consider two scenarios: driving by ferromagnetic resonance, which leads to the coherent occupation of a single mode, and driving by heating of the magnons, which leads to an excess of incoherent magnons. We find an enhanced normalized cross-correlation in the latter case, thereby demonstrating bunching of nonequilibrium thermal magnons due to their bosonic statistics. Our results contribute to the burgeoning field of quantum magnonics, which seeks to explore and exploit the quantum nature of magnons.

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Introduction.—In the early years of quantum mechanics, the drive towards demonstrating its classical limit led Schrödinger to examine a class of wave functions that replicates the classical dynamics of a harmonic oscillator [1]. In the language of second quantization, these states turned out to represent the eigenstates of the annihilation operator [2]. These have come to be known as "coherent states". Remarkably, the purely quantum phenomenon of Bose-Einstein (BE) condensation results in a condensate characterized by a coherent state wave function, comprising phenomena like superconductivity and superfluidity.

Characterizing and quantifying the "quantumness" of photonic states is one of the central themes in the field of quantum optics [3,4]. A measure particularly relevant for condensation phenomena is the so-called normalized second-order temporal coherence function, typically denoted by $g^{(2)}(\tau)$:

$$g^{(2)}(\tau) = \frac{\langle :\hat{I}(t)\hat{I}(t+\tau): \rangle}{\langle \hat{I}(t)\rangle\langle \hat{I}(t+\tau)\rangle}, \tag{1}$$

where $\hat{I}$ is the optical intensity magnitude operator, $::$ represents normal ordering of the photon ladder operators, and $\langle\,\rangle$ denotes the expectation value. For a coherent state (e.g., a laser), the numerator in Eq. (1) factorizes, and $g^{(2)}(\tau) = 1$. $g^{(2)}(\tau)$ also quantifies the relative probability of detecting two photons separated by a time lag τ. Specifically, $g^{(2)}(0)$ gives the rate of simultaneous detection of two photons in a given optical state. A single-mode thermal light field exhibits the so-called photon bunching effect, corresponding to $g^{(2)}(0) = 2$, which is indicative of the bosonic nature of the photons. The concept of bosonic bunching has been extended beyond quantum optics in more recent years; an analogue of $g^{(2)}(\tau)$ has been measured in a system of ultracold trapped atoms, demonstrating temporal atomic bunching and BE condensate coherence [5].

While quantum optics is now a mature field, similar advances have just begun to be made for bosonic excitations in magnets—magnons [6,7]. Recent advances [8] in manipulating and detecting these excitations have enabled exciting fundamental physics in magnetic systems with the potential for technological applications [9]. Observation of a nonequilibrium magnon BE condensate has been reported [6,10] wherein magnon coherence has been demonstrated by various optical techniques [11]. The same system seems to exhibit a superfluidlike flow of spin current providing further evidence towards these claims [12], which, however, are not uncontested [13] but are recently supported by new observations [14]. The existence of robust spin superfluidity with a different physical origin has been postulated in specific circumstances [15–18]. However, a direct signature of magnon coherence measurable in all-solid-state systems, where optical probes may not be feasible, is still lacking.

In this Letter, we give a proposal for the solid-state detection of magnon state coherence, adapting concepts from quantum optics to metal-magnet hybrid structures. We show that metallic contacts, such as those used in nonlocal magnon transport experiments [19], can play the role of coherence-sensitive "detectors" of magnon

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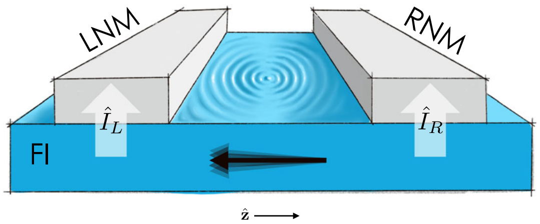

FIG. 1. Magnetic heterostucture, with ferromagnetic insulator (FI) and metallic leads (LNM and RNM). The arrow in the FI depicts its total spin. Spin currents $\hat{I}_L$ and $\hat{I}_R$ are injected into the LNM and RNM, respectively, when the FI is excited.

emission, in analogy with the photon detectors in quantum optical experiments. A key difference between quantum optical and spintronic systems lies in the fact that light, due to its long wavelength, weak interaction with surroundings, and ability to propagate in vacuum, exhibits coherence over large distances. This eliminates the need for the explicit inclusion of detectors in its description. In solid-state systems, we find it prudent to develop the coherence theory of magnons for a specific realization of detection integrated into the device structure. Nevertheless, the ideas developed herein are general and can be adapted to a different detection scheme. Furthermore, our findings are suggestive of employing heat currents as a probe into phonon coherences in an analogous manner.

Normalized spin current cross-correlation.—We consider a ferromagnetic insulator (FI) in contact with two nonmagnetic left and right metal (LNM and RNM) leads (e.g., see Fig. 1), into which spin current is injected and can be measured via the inverse spin Hall effect [8]. We define the normalized spin current cross-correlation $c^{(2)}(\tau)$:

$$c^{(2)}(\tau) \equiv \frac{\frac{1}{2}\langle\{\hat{I}_L(t), \hat{I}_R(t+\tau)\}\rangle}{\langle\hat{I}_L(t)\rangle\langle\hat{I}_R(t+\tau)\rangle}, \tag{2}$$

where $\hat{I}_l$ with $l = L, R$ are operators corresponding to the spin currents injected into LNM and RNM, respectively, and $\{\cdots\}$ denotes an anticommutator. A key feature of this measure, distinguishing it from $g^{(2)}(\tau)$, is that the spin currents may be positive or negative. In this sense, LNM and RNM at finite temperatures may be considered as "nonideal detectors" which may also emit magnons back into the FI, in addition to absorbing (detecting) them from it. The optical detectors, in contrast, are designed to predominantly absorb, and hence detect, photons. Thus, the positivity of intensity in Eq. (1) is maintained in this case.

In the present work, we evaluate $c^{(2)} \equiv c^{(2)}(0)$ considering a single magnon mode driven into (i) a coherent state and (ii) a thermal state with temperature higher than the metal leads. We find that at sufficiently large drives, $c^{(2)}$ emulates the behavior expected from $g^{(2)} \equiv g^{(2)}(0)$ (Fig. 2), i.e., it approaches 1 and 2 for cases (i) and (ii) respectively, demonstrating it to be a valid coherence measure. The need for strong driving of the FI arises because magnon detection via the metal leads is rendered nonideal by the processes in which the metal leads emit magnons into the FI. At sufficiently large drives, quantified by the temperature of the metal leads, magnon absorption by the metal lead detectors dominates over emission, and the detection process is efficient and similar to its optical counterpart.

Electron-magnet coupling.—We consider the FI to be axially symmetric around the z direction. Accordingly, we take the equilibrium macrospin to be oriented in the $-\mathbf{z}$ direction, so that excitations thereof carry spin in the $+\mathbf{z}$ direction. For simplicity, in order to contrast coherent and incoherent spin excitations of the FI, we allow for small angle dynamics of the macrospin $\mathbf{S}$ only, neglecting micromagnetic degrees of freedom. Such excitations are conveniently parametrized by the Holstein-Primakoff transformation:

$$\hat{S}_z \equiv \hat{\phi}^\dagger\hat{\phi} - S, \qquad \hat{S}_- \equiv \hat{S}_x - i\hat{S}_y = \sqrt{2S - \hat{\phi}^\dagger\hat{\phi}}\hat{\phi}, \tag{3}$$

where S is the integer macrospin of the FI, and $\hat{\phi}$ and $\hat{\phi}^\dagger$ are, respectively, magnon annihilation and creation operators, subject to the bosonic commutation relations $[\hat{\phi}, \hat{\phi}^\dagger] = 1$. The operator $\hat{\phi}$ has two components:

$$\hat{\phi} = \hat{\varphi} + \varphi. \tag{4}$$

The first term, $\hat{\varphi}$, corresponds to incoherent fluctuations, with $\langle\hat{\varphi}\rangle \equiv 0$, while the second, φ, is a c-number corresponding to a coherent magnon. Only φ contributes to the ensemble-averaged transverse dynamics. Assuming small amplitudes for φ and $\hat{\varphi}$, we obtain $\langle S_x\rangle = \sqrt{2S}\mathrm{Re}[\varphi]$ and $\langle S_y\rangle = -\sqrt{2S}\,\mathrm{Im}[\varphi]$.

The evolution of the heterostructure spin dynamics is governed by the Hamiltonian $\mathcal{H} = \mathcal{H}_m + \mathcal{H}_e + \mathcal{H}_J$. The uncoupled FI magnon Hamiltonian is given by $\mathcal{H}_m = E_m\hat{\phi}^\dagger\hat{\phi}$, with E_m as the magnon gap or, equivalently, $\hbar$ times the ferromagnetic resonance frequency. The uncoupled normal-metal electron Hamiltonian is $\mathcal{H}_e = \sum_{l=L,R}\mathcal{H}_l$, where $\mathcal{H}_l = \sum_{k\sigma}\epsilon_k\hat{b}^\dagger_{lk\sigma}\hat{b}_{lk\sigma}$, with $\hat{b}_{lk\sigma}$ as a σ-spin annihilation operator for an electron in the $l = L, R$ lead with quantum number k [20]. Magnons in the FI and the spins of electrons in the normal metals are coupled by exchange at the metal-magnet interfaces, which is captured by hopping of spin between magnons and electron-hole excitations in the normal metals: $\mathcal{H}_J = \sum_K J_K\hat{\phi}^\dagger(\hat{L}_K + \hat{R}_K) + \mathrm{H.c.}$ [21]. Here J_K is the effective exchange interaction for both interfaces, while $\hat{L}^\dagger_K = \hat{b}^\dagger_{Lk\uparrow}\hat{b}_{L\tilde{k}\downarrow}$ and $\hat{R}^\dagger_K = \hat{b}^\dagger_{Rk\uparrow}\hat{b}_{R\tilde{k}\downarrow}$ are creation operators for up-electron–down-hole pairs in the left and right leads, respectively, with $K = k, \tilde{k}$ as a collective index.

The coupling $\mathcal{H}_J$ gives rise to spin currents through the left and right interfaces. The operator expression for each of these currents is obtained from the Heisenberg equation of motion for the electron spin density $\hat{s}_l = \sum_K (\hat{b}^\dagger_{lk\uparrow}\hat{b}_{lk\uparrow} - \hat{b}^\dagger_{lk\downarrow}\hat{b}_{lk\downarrow})$ in the respective lead. The current in the left lead is

$$\hat{I}_L = \frac{i}{2}[\hat{\mathcal{H}}_J, \hat{s}_l] = \sum_K (\hat{L}^\dagger_K - \hat{L}_K).$$

Similarly, the current in the right lead is $\hat{I}_R = \sum_K (\hat{R}^\dagger_K - \hat{R}_K)$.

In equilibrium, the average currents $I_L = \langle \hat{I}_L \rangle$ and $I_R = \langle \hat{I}_R \rangle$ vanish. The exchange Hamiltonian also gives rise to correlations between LNM and RNM electrons. To zeroth order in $\hat{\mathcal{H}}_J$, such correlations are destroyed by coupling to the environment, and the equilibrium density averages are unaffected by the exchange coupling. Thus, for LNM and RNM electrons in equilibrium, $\langle \hat{b}^\dagger_{lk\sigma}\hat{b}_{l'k'\sigma'} \rangle = \delta_{ll'}\delta_{kk'}\delta_{\sigma\sigma'} n_F(\epsilon_k)$ where $n_F(\epsilon) = 1/(1 + e^{(\epsilon-\epsilon_F)/T})$ is the Fermi-Dirac distribution with common electronic temperature T and Fermi energy $\epsilon_F \gg T$. As phase coherent dynamics vanish in equilibrium, the z component of the incoherent spin density under equilibrium conditions is $\langle \hat{\varphi}^\dagger \hat{\varphi} \rangle = N = N_i$, with $N_i = 1/(e^{E_m/T_m} - 1)$ as the Bose-Einstein distribution for magnon energy E_m and temperature T_m.

Driving the magnet.—We assume the normal metals to be ideal heat and spin sinks, so that the electrons there are described by equilibrium conditions discussed above. We consider two methods of directly driving the FI. First, ferromagnetic resonance (FMR) can coherently excite magnetic dynamics. Under the influence of a microwave field with frequency ω, a coherent excitation described by $\varphi(t) = \varphi e^{i(\omega t+\theta)}$ is created. Here the U(1) symmetry of the FI is explicitly broken, as the precessional phase $\omega t + \theta$ is determined by the applied microwave. As a second driving scheme, incoherent magnetic dynamics can be excited beyond equilibrium by heating the FI to a temperature T_m higher than that of the metal leads, for example by heating with a laser pulse. Thus, in the presence of one or both types of drives, the nonequilibrium magnon density, to zeroth order in $\hat{H}_J$ becomes $\langle \hat{\varphi}^\dagger \hat{\varphi} \rangle = N = N_c + N_i$, where $N_c = \varphi^* \varphi$. The steady-state current resulting from either type of drive has been calculated to second order in the exchange coefficient J_K [21]:

$$\langle \hat{I}_L \rangle = \langle \hat{I}_R \rangle = I_L = I_R = 2\pi D^2 J^2 E_m (N - N_{\rm NM}), \quad (5)$$

where $N_{\rm NM} = 1/(e^{E_m/T} - 1)$ is the Bose-Einstein distribution function describing electron-hole excitations in the leads. The Fermi-surface averaged square of the exchange interaction is given by $J^2 \equiv \sum_K |J_K|^2 \delta(\epsilon_k - \epsilon_F) \times \delta(\epsilon_{\tilde{k}} - \epsilon_F)/D^2$, with D as the electronic density of states at the Fermi energy ϵ_F in the metal leads. Thus we see that it is not possible to distinguish the coherent and incoherent magnons from the spin currents I_L and I_R alone, since these depend on the total number of magnons $N = N_c + N_i$. Instead, we turn to the cross-correlations of the spin currents.

Spin current cross-correlations.—We now investigate the current-current cross-correlator $C(\tau) = \frac{1}{2}\langle \{\hat{I}_L(t), \hat{I}_R(t+\tau)\} \rangle$ to obtain $c^{(2)}(\tau)$ in Eq. (2). One can see directly how $C(\tau)$ encodes information about magnon fluctuations. When the magnons are completely coherent ($\hat{\phi} = \varphi$) and magnon fluctuations can be neglected, the coupling Hamiltonian $\mathcal{H}_J$ does not give rise to correlations between left and right electrons. Here, $C(\tau) = \langle \hat{I}_L(t) \rangle \langle \hat{I}_R(t+\tau) \rangle$ factorizes, yielding $c^{(2)}(\tau) = 1$. If, however, incoherent fluctuations of the magnon operators are taken into account, then cross-correlations give rise to

$$c^{(2)}(\tau) = 1 + \Delta c(\tau), \quad (6)$$

where the correction $\Delta c(\tau)$ comes from the Wick decomposition of magnon correlators. As discussed above, we focus on equal-time ($\tau = 0$) correlations in the steady state, abbreviating $C \equiv C(0)$ and $\Delta c(0) \equiv \Delta c$.

The current-current cross-correlation function is calculated perturbatively in J [22]. Such an approach is consistent with the idea that the metallic leads act as weak probes of the magnet, thereby preserving a well-defined notion of a magnon rather than an entangled magnon–electron-hole excitation. Relegating the detailed calculation to the Supplemental Material [24], we summarize our results here.

Because the lowest nonvanishing contribution to $\mathcal{C}$ is fourth order in J, the lowest order terms (which we consider here) each contain four magnon operators, or two factors of the magnon density. There are therefore three types of terms: those $\sim N_c^2$, those $\sim N_i^2$, and those $\sim N_i N_c$, which we denote as C_{cc}, C_{ii}, and C_{ic}, respectively. One thus has

$$C = C_{cc} + C_{ii} + C_{ic} \quad (7)$$

where $C_{ii} = C_{ii}^{(\times)} + C_{ii}^{(\parallel)}$ and $C_{ic} = C_{ic}^{(\times)} + C_{ic}^{(\parallel)}$, with the superscripts $(\times)$ and $(\parallel)$ respectively denoting diagrams that cross and do not cross the FI (see Supplemental Material [24]). As shown in the Supplemental Material [24], one finds $C_{cc} = I_L^{(c)} I_R^{(c)}$, $C_{ii}^{(\parallel)} = I_L^{(i)} I_R^{(i)}$, and $C_{ic}^{(\parallel)} = I_L^{(c)} I_R^{(i)} + I_L^{(i)} I_R^{(c)}$, where $I_l^{(i)} = 2\pi D^2 J^2 E_m (N_i - N_{\rm NM})$ and $I_l^{(c)} = 2\pi D^2 J^2 E_m N_c$ are the incoherent and coherent contributions to the $l = L, R$ spin current $I_l = I_l^{(i)} + I_l^{(c)}$. The sum of C_{cc} and the uncrossed terms is thus equal to the product of the currents: $C_{cc} + C_{ii}^{(\parallel)} + C_{ic}^{(\parallel)} = I_L I_R$. One thus obtains $\Delta c = (C_{ii}^{(\times)} + C_{ic}^{(\times)})/I_L I_R$, so that $c^{(2)}$ is obtained directly by evaluating the crossed diagrams.

Next, let us consider the cross-correlations under various scenarios. First, in equilibrium, $N = N_{\rm NM}$, and the currents I_L and I_R vanish, as does coherent dynamics ($\varphi = 0$), so $C_{cc} = C_{ic} = 0$. At finite temperature, however, incoherent magnons are excited by thermal fluctuations. While the uncrossed diagrams vanish ($C_{ii}^{(\|)} = I_L I_R = 0$), the crossed diagrams $C_{ii}^{(\times)}$ do not, reflecting correlations between fluctuations of the left and right currents. Thus, in equilibrium, the normalized correlation coefficient $c^{(2)} = 1 + \Delta c$ diverges.

Second, consider heating of the FI. Under heating, a spin current flows from the FI to the normal-metal leads. All of the coherent terms, which are $\propto N_c$, are zero, leaving $C = C_{ii} = C_{ii}^{(\times)} + C_{ii}^{(\|)}$. As can be seen in the left-hand side of Fig. 2, as T_m increases from T, $I_L = I_R$ becomes nonzero, decreasing $c^{(2)}$ from infinity. As shown in the Supplemental Material [24], when T_m is sufficiently large that $N_m \gg N$, one finds after some work that $C_{ii}^{(\times)} \to I_L I_R$, and therefore $c^{(2)} \to 2$. Under strong incoherent driving, thermal magnons dominate the spin currents and their correlations, and $c^{(2)} \sim \langle \hat{\varphi}^\dagger \hat{\varphi}^\dagger \hat{\varphi} \hat{\varphi} \rangle / |\hat{\varphi}^\dagger \hat{\varphi}|^2 = 2$, reflecting bunching of thermal magnons [25]. Note also that because of *quantum* fluctuations, even as the metal leads temperature T is reduced to zero, a sufficiently large bias $\Delta T = T_m - T$ is required in order to observe thermal magnon bunching, i.e., $c^{(2)} = 2$.

Last, consider driving by FMR. Here, only coherent magnons are excited by the external field, and the spin currents $I_L = I_L^{(c)} = I_R = I_R^{(c)}$ are determined by the FMR power. Because the incoherent spin currents are zero ($N_i = N_{\rm NM}$), $C_{ii}^{(\|)} = C_{ic}^{(\|)} = 0$, but the crossed terms $C_{ii}^{(\times)}$ and $C_{ic}^{(\times)}$ survive, reflecting correlations that arise from spin cotunneling. At sufficiently large FMR power, however, $N_c \gg N_i$, and $C_{cc} = I_L I_R \gg C_{ii}^{(\times)}, C_{ic}^{(\times)}$, and therefore $\Delta c \to 0$, so $c^{(2)} \to 1$. Thus, at sufficiently large FMR power, the spin current correlations are dominated by the coherent magnons, and $c^{(2)} \approx g^{(2)} = \langle \varphi^* \varphi^* \varphi \varphi \rangle / |\varphi^* \varphi|^2 = 1$. Note that even as $T \to 0$, $c^{(2)}$ is not necessarily equal to 1 at finite N_c. As with temperature biasing, *quantum* fluctuations of magnons and electron-hole pairs require that additionally $N_c \gg 1$ in order for $c^{(2)}$ to saturate at 1.

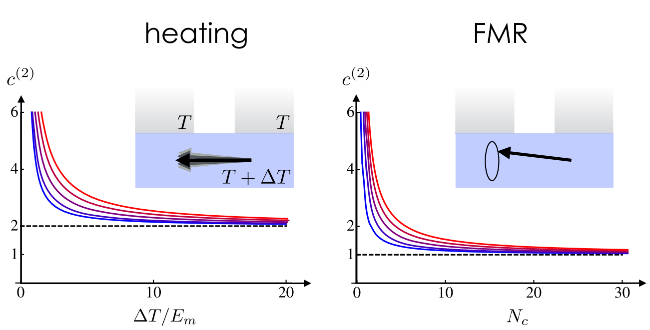

FIG. 2. Normalized spin-current cross-correlation $c^{(2)}$ for FI driven via heating (left) and FMR (right). The coherent mode number $N_c = (\chi h)^2$ is assumed to be driven by a circularly polarized FMR field h, with χ as the susceptibility. Solid lines, blue to red, represent electronic temperatures $T/E_m = 0.01$, 2.5, 5, 7.5, 10.

Measurement of cross-correlations.—In the present device geometry, the spin current cross-correlation function C can be obtained from the spin current fluctuations $\mathcal{S}(\tau) \equiv \frac{1}{2} \langle \{ \delta \hat{I}_L(t), \delta \hat{I}_R(t+\tau) \} \rangle$, where $\delta \hat{I}_l \equiv \hat{I}_l - \langle \hat{I}_l \rangle$. In turn, $\mathcal{S}(\tau)$ is obtained by Fourier transforming the power spectral density $\mathcal{S}(\omega)$. It is straightforward to see that $C = I_L I_R + \mathcal{S}(\tau = 0)$, so $\Delta c = \mathcal{S}(\tau = 0)/I_L I_R$. It should be noted that in an actual measurement, the detector bandwidth sets the lower limit on τ for which $\mathcal{S}(\tau)$ can be measured. Thus, $\tau \to 0$ corresponds to τ approximately approaching the inverse of the detector bandwidth.

One might wonder why the spin current noise cannot alone be used to infer the coherence of the magnon state. The problem is that for both heating of the FI and FMR, $\mathcal{S}(0)$ increases with driving. In the former case, when the temperature difference between the FI and leads is large enough, $\mathcal{S}(0) = C_{ii}^{(\times)} \sim N_i^2$, while $I_L I_R \sim N_i^2$, so that their ratio Δc saturates at constant value. In the latter case, the noise $\mathcal{S}(0)$ grows with $C_{ic}^{(\times)} \sim N_c$, but because $I_L I_R \sim N_c^2$, $\Delta c \sim 1/N_c$ approaches zero with increasing FMR power. Thus, in order to distinguish the cross-correlations of coherent magnons from those of incoherent magnons, it is expedient to use the normalized quantity $c^{(2)}$, rather than simply the noise $\mathcal{S}$.

Discussion.—We have focused on a simple model to capture the essential physics of magnon bunching; we now comment on some of the approximations, along with their validity, employed in our analysis. First, we have neglected spatial dependence of cross-correlations on the assumption that the metal contacts are closer than the magnon coherence length (which for noninteracting magnons, is the thermal wavelength). If the contacts are further apart, a more general calculation, including finite wavelength thermal modes, should be carried out. By analogy with $g^{(2)}$, however, one may expect $c^{(2)}$ for thermal magnons to decay from 2 at distances smaller than the coherence length to 1 at larger distances; for coherent magnons, $c^{(2)}$ should remain equal to 1 [5], thus preventing one from distinguishing incoherent from coherent magnons if the contacts are not close enough. A distance-dependent study would, however, provide an avenue for measuring the magnon coherence length. If, on the other hand, the FI is too thin or the contacts too close, tunneling of electrons across the FI becomes possible, giving rise to a spin-polarized charge current that distorts the magnonic signal.

Second, the actual observation of pure spin currents and their fluctuations require conversion to measurable charge currents by, for example, the inverse spin Hall effect; this process may introduce additional charge current noise that is convoluted with that of cross-correlations. The conversion of spin to charge currents may also be complicated by interfacial spin-orbit coupling, which can spoil spin conservation; if interfacial spin-orbit coupling renormalizes effective transport coefficients and reduces the steady-state magnon efficiency [26], then in principle our dimensionless $c^{(2)}$ is unaffected. However, because in practice this could lead to a weaker signal and because inelastic spin-orbit scattering at the interface may convolute with the cross-correlations, it would be prudent to focus on materials (e.g., yttrium-iron-garnet–platinum heterostructures) in which such effects are thought to be minimal [27].

Third, we have considered a macrospin model, wherein higher energy magnons are gapped out. While this may be a reasonable assumption for small structures at low temperatures, clearly as the FI is heated, micromagnetic modes must be taken into account. Last, we have neglected magnon-magnon interactions, which arise through additional, nonlinear terms in the magnon Hamiltonian. For a single magnon mode with contacts in close proximity, such terms serve only to renormalize the magnon gap and thus do not qualitatively change our results. When magnons are allowed to occupy a spectrum of modes with different energies, however, inelastic magnon scattering can facilitate interesting interplays between coherent and incoherent magnons [28], which might be observed via $c^{(2)}$.

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