Throughput models for a dual-bay VLM Order Picking system under different configurations

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Abstract

Purpose Vertical Lift Module (VLM) is a parts-to-picker system for order picking of small products, which are stored into two columns of trays served by a lifting crane. A dual-bay VLM Order Picking (dual-bay VLM-OP) system is a particular solution where the operator works in parallel with the crane, allowing higher throughput performance. The present paper aims to define models for different operating configurations able to improve the total throughput of the dual-bay VLM-OP system.

Design/methodology/approach Analytical models are developed to estimate the throughput of a dual-bay VLM-OP. A deep evaluation has been carried out, considering different storage assignment policies and the sequencing retrieval of trays.

Findings A more accurate estimation of the throughput is demonstrated, compared to the application of previous models. Some use guidelines for practitioners and academics are derived from the analysis based on real data.

Originality/value Differing from previous contributions, these models include the acceleration/deceleration of the crane and the probability of storage and retrieve of each single tray.

This permits to apply these models to different storage assignment policies and to suggest when these policies can be profitably applied. They can model also the sequencing retrieval of trays.

Keywords: Automated Warehouses, Vertical Lift Module (VLM), Order Picking, Analytical Modelling, Throughput Performance Analysis.

1. Introduction and background

Order picking is one of the most time- and cost-consuming activities in a warehouse, often requiring the presence of human operators, who travel within the warehouse aisles to retrieve the items that are needed to fulfil the various orders of the customers (De Koster, Le-Duc and Roodbergen, 2007). Consequently, the travelling activity can represent up to 50% of the total picking time, as demonstrated by Tompkins et al. (2010). Moreover, this aspect can become even more critical when also small objects are stored in pallets, occupying a large amount of space (Azzi et al. 2014; Battini et al., 2014; Bartholdi and Hackman, 2017; Battini et al., 2018). Some examples of small objects order picking systems are: general e-commerce products, personal care and home care products, drugs and other healthcare products, small electronic devices etc.

In the case of a traditional picking low-level picker-to-parts warehouse, the items are stored on pallets that are positioned on the lower stocking locations of the shelves. The pickers usually use electric pallet trucks to move along the aisles and to transport one or more mixed pallets, composed of the items collected during their order picking activity (Battini et al., 2018). As per the authors' experience, and also confirmed by relevant scientific contributions (Bartholdi and Hackman, 2017; Franzke et al. 2017), in this type of warehouse, the average time per order line is typically from 60 to 120 s/line. The main part is due to travelling and searching activities (Tompkins et al., 2010; Dijkstra and Roodbergen, 2017). Moreover, the picking activity could be severe from an ergonomic perspective, especially when the operators are picking the last (so, farthest) items from the pallet (Calzavara et al., 2017; Calzavara et al. 2018b).

An alternative picker-to-parts solution for small objects picking is the creation of a dedicated storage area with small racks where the products are stocked in cartons or boxes (Battini et al., 2018). The main benefit is reducing the total space needed and, hence, the distances travelled, leading to a higher system throughput (Choe and Sharp, 1991; Tompkins and Smith, 1998; Caputo and Pelagagge, 2006; Battini et al., 2015a). Other solutions are the parts-to-picker ones, where automated system brings products to the operator, who works in a fixed picking station (Choe and Sharp, 1991; Tompkins and Smith, 1998).

The present paper focuses on a parts-to-picker solution based on Vertical Lift Modules, also called VLMs. A VLM consists of a storage column in which small products are stored in extractable trays. These trays are inserted and extracted by a powered device, which travels vertically between the front and the rear shelving of the column. It delivers and retrieves the specific tray in front of the picker, in the so-called picking bay, where the operator processes the picking order. The moving device is guided by an automated control system, which is usually interfaced with a software system, to set the correct order of trays retrieval. There are different alternatives of this automated system developed by various manufacturers (such as Modula, Spacesaver, Kardexremstar etc.); however, the most implemented one is like the one just described, in which there is one crane for each couple of storage columns, with a picking bay and an operator serving at least one VLM.

Such VLM solution represents an interesting combination of some of the benefits of parts-to-picker systems (Battini et al., 2016; Lenoble, Frein and Hammami, 2018, Calzavara et al., 2019). Indeed, a VLM warrants a small layout and high-volume utilization like vertical carousels, but it avoids the risk of damaging the stored products and it has no need to balance the loads inside each tray. This enables a consequent reduction of the distances travelled by the operators, with a modularity and a system throughput which are comparable to those of horizontal carousels, and with the security and the storage density of miniloads (Tompkins and Smith, 1998). Moreover, the employment of VLM solutions is also encouraged by the increasing attention that practitioners and researches are giving to the ergonomic working conditions of human operators (Grosse et al., 2015). In fact, in such systems the picker stands in front of the picking bay, without assuming postures that could lead to musculoskeletal issues (Cantor, 2008; Neumann and Medbo, 2010; Dukic, Opetuk and Lerher, 2015; Calzavara et al., 2018a). Another interesting aspect concerning VLMs is the potential reduction of picking errors: since the picker has in front of him just one tray at a time, the probability of making mistakes decreases (Battini et al., 2015b). Furthermore, there is the possibility of signalling the correct item to pick, for example with a system of lights or laser pointers. Finally, the specific structure of the VLM ensures safe storage of the products, preventing possible thefts or damages to goods.

However, traditional VLMs present some weaknesses as well, like the potential idle time for the picker who, once a pick is performed, has to await the storage of the current tray and the retrieval of the following one (Rosi et al., 2016). In this sense, the development of some recent VLM solutions is leading these systems to gain growing success in several warehouse applications. For example, an interesting VLM Order Picking solution, often called *dual-bay VLM* or *VLM with dual tray delivery*, presents the possibility of having two different picking places, and so the crane and the operator work in parallel. Thus, since the throughput depends on the performance of both the dual-bay VLM and of the operator, they are studied as a unique system, here called *dual-bay VLM Order Picking (dual-bay VLM-OP) system*, to take into account also the interdependency between the automated part and the human one.

Although some researches are available on the dimensioning and performance evaluation of carousel systems (Van Den Berg, 1996; Park, Park and Foley, 2003; Hassini, 2009), very few propose models for Vertical Lift Modules (Meller and Klote, 2004; Dukic, Opetuk and Lerher, 2015, Rosi et al., 2016, Lenoble, Frein and Hammami, 2018). The studies that have been developed so far specifically dealing with traditional Vertical Lift Modules with single bay are by Meller and Klote (2004) and by Rosi et al. (2016). Meller and Klote (2004) compare the productivity of a single-bay VLM with a horizontal carousel system, while Rosi et al. (2016) use a simulation-based approach to estimate the performance of single-bay VLM-OP with human operator. On the other hand, the research by Dukic, Opetuk and Lerher (2015) is exactly focused on dual-bay VLM-OP systems, proposing an analytical throughput model useful for dimensioning such storage solutions. In their researches, Meller and Klote (2004), Dukic, Opetuk and Lerher (2015) and Rosi et al. (2016) consider a random storage assignment policy. Moreover, they refer to the estimation of the dual command time, assuming an average vertical speed of the crane and considering the racks divided into discrete sections. In all these papers, the authors estimate the average expected time to perform only one line of the order. In case of multiple-lines orders, where more than one line has to be picked to complete an order (i.e. more than one Stock Keeping Unit, SKU, per order), they do not investigate the impact of the sequencing retrieval of the trays, assuming a random retrieval policy.

Lenoble, Frein and Hammami (2018) focus on the optimization of order batching, by developing a metaheuristic approach to find the optimal order batching with the objective of minimizing the total order completion time. Moreover, these authors consider a single-bay VLM, with a constant picking time and a constant time for the storage and the retrieval of the tray.

From the analysis of the existing contributions, it turns out an important research gap in the modelling of the operations of a dual-bay VLM-OP system. Consequently, the estimation of the throughput performance of this system has not been enough clearly defined yet, and, for now, there are no analytical models to support the warehouse managers in understanding the impact of different operating conditions, such as the storage policy and the sequencing retrievals of trays.

The main reasons are that the existing scientific contributions do not consider the real speed profile of the crane and they assume the storage racks are divided into few discrete sections. This leads to an inaccurate estimation of the system throughput, as it will be demonstrated in the next sections. More importantly, the applicability of these models is limited, since they do not estimate the cycle time in case of storage assignment policies, in which the SKUs are assigned based on their popularity or picking frequency. Moreover, the previous contributions do not investigate the impact of the sequencing retrieval of trays in order to perform a multiple-lines order in a dual-bay VLM-OP system.

As a consequence, the present paper aims to cover these research gaps by:

- developing more accurate mathematical models, where the acceleration/deceleration of the crane are considered and the stocking and retrieval of each tray are included;
- extending the application of these models to various operating configurations, where different storage assignment policies and the sequencing retrievals of the trays are taken into account;

- giving relevant guidelines to practitioners about how to improve the total throughput of a dual-bay VLM-OP system, based on these operating configurations.

The remainder of the paper is structured as follows. In the next section, the main scope and the focus of the paper are illustrated, in order to clarify how it covers the research gap highlighted from the analysis of the existing contributions. In the first part of the third section, the assumptions and the analysed system are described. Then, the analytical models, considering the acceleration/deceleration of the crane and the probability to store and retrieve each tray, are introduced and validated, comparing them to the already existing models. In Section 4, the models are applied to estimate the throughput of the system, also assuming different storage assignment policies. The formulations are extended in order to model also the sequencing retrievals of the trays in case of multiple-lines orders, as explained in Section 5. Here, the combined implementation of the previous operating configurations are investigated. Furthermore, all models are validated through a simulation based on real orders. The discussion of the results and some guidelines are included in the last two sections, about managerial implications and conclusions, together with the proposal of future researches that would be useful to extend these approaches.

2. Scope of the work

This paper studies an interesting alternative industrial solution involving dual-bay Vertical Lift Modules. Since a dual-bay VLM-OP system allows the picker to work in parallel to the system, the paper considers the possibility of employing such a storing system for the processing of picking orders of small objects.

To cover the research gap highlighted in the previous section, the first goal of this research is the development and validation of analytical models for the estimation of the dual-bay VLM cycle time. The acceleration and deceleration of the crane are introduced and the probability of storage and retrieval of each single tray is considered. The accuracy of these new models is evaluated and compared to the results obtained by using the existing models.

Moreover, in addition to the previous developed models, these formulations permit to support the warehousing managers by modelling also new operating configurations and by evaluating their impact on the productivity of the system.

The first configuration regards the possibility of adopting a CBS assignment policy, where the SKUs that are required more frequently are stocked in the trays closer to the picking bay. Generally, this could lead to shorter storage and retrieval cycle times and to a reduction of the total number of delivered trays, since there is a higher probability of picking different SKUs from the same tray. The accuracy of the developed models demonstrates their validity and applicability and they allow to estimate the impact of adopting different storage assignment policies.

Another way of improving the system throughput could be the sequencing retrievals of the trays in case of multiple-lines orders. In this configuration, the various lines of an order are reorganized in combined groups, according to the trays in which the SKUs are stocked. This approach reduces the number of trays retrievals and improves the system throughput. In this case, our developed models are extended in order to consider the probability to have more than one SKU to be picked from the same delivered tray and, thus, to estimate the effect of this configuration. This possible positive impact has already been studied by Pazour and Meller (2013); however, they refer to a carousel system served by a storage and retrieval machine.

Finally, after the validation of the models and the estimation of the impact of the CBS assignment policy and of the multiple-lines orders, some managerial insights are discussed based on the general results obtained using data from real cases.

3. Analytical models for cycle time estimation of dual-bay VLM-OP system

The operational logic of a dual-bay VLM-OP system can be described as follows, starting with one tray available in the first picking bay (initial point):

- Step 1: as long as the picker picks the items from the tray she/he has in front of her/him in the first picking bay, the crane is able to store the previous tray and to retrieve the following one in the second picking bay.
- Step 2: once the picker has picked all the items from the first tray, she/he can pick the subsequent items of the list from the tray already available in the second picking bay (she/he can execute immediately the next picking activity just after having completed the previous one). In the meantime, the crane can store the previous tray and deliver a new one to the first picking bay.
- Step 3: after the completion of step 2, the dual-bay VLM-OP system is in the same situation of the initial point (i.e. one tray available in the first picking bay); the operative cycle of steps 1 and 2 can be iterated as many times as the number of trays that have to be picked.

In the following paragraphs, the models for the estimation of the system cycle time are introduced. Table 1 reports the used notations.

Symbol	Description
H	dual-bay VLM height
v	dual-bay VLM I/E crane velocity
a	dual-bay VLM I/E crane acceleration/deceleration
$t_{p/d}$	Delay time to pick up or deposit a tray
i, j	Tray indices (1…M)
M	Total number of dual-bay VLM trays
p_i, p_j	Probability of extracting tray *i* (tray *j*)
h_i, h_j	Storage height of tray *i* (tray *j*)
t_{ij}	Crane travel time from tray location *i* to tray location *j*
t_{Ai}, t_{Bi}	Crane travel time from picking bay A (B) to tray location *i*

t_{jA}, t_{jB}	Crane travel time from tray location *j* to picking bay A (B)
L	Total number of lines per order
l_i	Number of lines of the order in tray *i*
p_T	Average picking time for one SKU
SA	Storage assignment policy (RS = Random Storage, CBS = Class-Based Storage)
$E[CT^{SA}]$	Expected cycle time under the SA policy
$E[DC^{SA}]$	Expected dual command time under SA policy.
$S[DC^{SA}]$	Standard deviation of the dual command time under SA policy
X^{SA}	Factor for occupancy problem
$VLMU$	Average utilization of dual-bay VLM (from simulation)
PU	Average utilization of operator (from simulation)
$E[R^{SA}(L,M)]$	Expected number of trays delivered for an order of L lines in case of dual-bay VLM with M trays
$E[DC^{SA}(L,M)]$	Expected dual command for an order of L lines in case of dual-bay VLM with M trays

Table 1. Notations.

Based on the description of the dual-bay VLM-OP system reported here above, some other detailed assumptions are here explained:

- There is one SKU per order line but there could be more than one item per SKU;
- The order has more than one line and in the standard operational logic the lines are ordered randomly (or following some basic conditions as alphabetical SKU code order);
- The products are always available, since we assume to have enough inventory to meet all the order requirements;
- Each SKU is stocked in one single tray and it has a dedicated space within it;

- p_T is the average time to pick the SKU from the trays and to perform other tasks like counting, weighing, stocking the items to new locations etc. It is assumed as the average time to pick all the items for that particular SKU.

Since a dual-bay VLM-OP system consists of a dual-bay VLM working in parallel with a picker, the resulting cycle time of the whole system $E[CT^{SA}]$ derives from the comparison between the time spent by the crane to perform a dual command $E[DC^{SA}]$ and the time spent by the picker to perform her/his activities p_T.

A general approach could consist in comparing the cumulative distribution function of the dual command to the picking time, as already discussed by Bozer and White (1990). Moreover, this problem can be simplified as reported by Bozer and White (1990), by considering that the dual command distribution function has a uniform distribution between $t_1 = E[DC^{SA}] - \sqrt{3}S[DC^{SA}]$ and $t_2 = E[DC^{SA}] + \sqrt{3}S[DC^{SA}]$, where $E[DC^{SA}]$ is the mean value and $S[DC^{SA}]$ the standard deviation. Therefore, the cycle time $E[CT^{SA}]$ can be estimated by using these formulas:

$$E[CT^{SA}] = \begin{cases} E[DC^{SA}], & 0 < p_T < t_1 \\ \frac{{p_T}^2 - 2p_T t_1 + {t_2}^2}{2(t_2 - t_1)} & t_1 \le p_T < t_2 \\ p_T & t_2 \le p_T < \infty \end{cases} \quad (1)$$

The accuracy of the estimation of $E[CT^{SA}]$ depends on the calculation of $E[DC^{SA}]$ and of the parameters t_1 and t_2 rather than on the distribution function used to model the picking time p_T.

In the next sections, the analytical models to estimate the expected dual command time are introduced and discussed, assuming a normal distribution function for the picking time, with an average value of p_T and a 20% of standard deviation, typical of manual activities. The acceleration and deceleration of the crane are considered in the travel time calculation. The probability of storage and retrieval of each single tray is taken into consideration in order to improve the accuracy of the estimation, under several operating configurations.

3.1 Expected dual command time with acceleration/deceleration considerations

The throughput model of a dual-bay VLM-OP system differs from the equations introduced by Dukic, Opetuk and Lerher (2015), because the acceleration/deceleration of the crane and the real probability of extracting each tray are considered. Moreover, the resulting model allows to analyze the performance of the system, considering different storage assignment policies and the application of sequencing trays retrievals.

In order to estimate the expected dual command time of the dual-bay VLM, the analytical formulation introduced here considers all the single trays of the dual-bay VLM and the delay time spent by the crane to accelerate and decelerate. This allows to point out the exact activities performed by the crane during its dual command execution, which is slightly different from what was proposed in previous researches (Meller and Klote, 2004; Dukic, Opetuk and Lerher, 2015), where the dual-bay VLM was divided only into discrete sections (Figure 1) and the delay time due to acceleration and deceleration was calculated as a fixed average time.

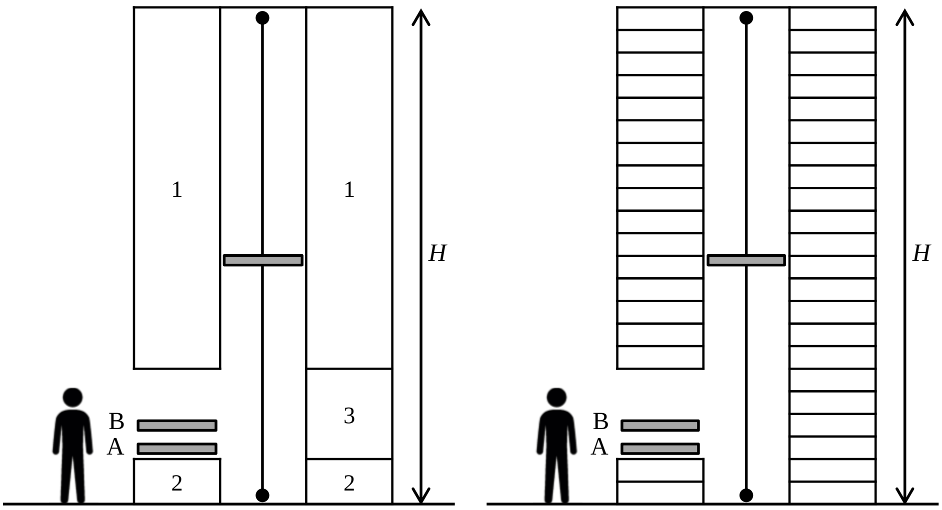


Figure 1. Comparison between discrete sections (left) and single tray (right) storage/retrieval assumptions.

Then, considering that the picker can pick from both picking positions A and B (Figure 1), the expected dual command time can be calculated as the average of the two expected dual command times from the two picking positions A and B, as follows:

$$E[DC^{SA}] = \frac{E[DC^{SA}]_A + E[DC^{SA}]_B}{2} \tag{2}$$

By knowing the probability of extracting each tray, the following factor is introduced to take into account that some SKUs can be in the same tray, according to the probability of extracting each tray *i*:

$$X^{SA} = [1 - \sum_{i=1}^{M} p_i{}^2 \cdot (1 - p_i)^3] \tag{3}$$

This factor is based on the multinomial probability distribution (Battini et al., 2015a), and its value depends on the number of trays M (i.e. on the dual-bay VLM height) and on the storage assignment policy. Other equations to consider the so-called "occupancy problem" can be found in Meller and Klote (2004), Dukic, Opetuk and Lerher (2015).

Thus, the equations to estimate the dual command time include the cases when two or more SKUs are picked from the same trays, together with the delay time to pick up and deposit a tray:

$$E[DC^{SA}]_A = X^{SA} \cdot \left[\sum_{i=1}^{M} \sum_{\substack{j=1 \\ j \neq i}}^{M} (t_{Ai} + t_{ij} + t_{jA}) \cdot p_i p_j + 4t_{p/d}\right] \tag{4}$$

$$E[DC^{SA}]_B = X^{SA} \cdot \left[\sum_{i=1}^{M} \sum_{\substack{j=1 \\ j \neq i}}^{M} (t_{Bi} + t_{ij} + t_{jB}) \cdot p_i p_j + 4t_{p/d}\right] \tag{5}$$

Furthermore, the travel time t_{ij} between two different positions or tray locations i and j has to take into account the acceleration/deceleration of the crane. In fact, the crane has a typical trapezoidal move profile, where it accelerates to the maximum velocity, it travels at this constant value and finally it decelerates to stop at the final position. In case the distance to be covered is short, the crane could be unable to reach the maximum velocity; then, it would move with a typical triangular move profile.

Under these conditions, whether or not the crane reaches its maximum speed, the travel time can be as follows (Azzi et al., 2011):

$$t_{ij} = \begin{cases} 2\sqrt{\frac{|h_j - h_i|}{a}}, & |h_j - h_i| \leq \frac{v^2}{a} \\ \frac{|h_j - h_i|}{v} + \frac{v}{a}, & |h_j - h_i| > \frac{v^2}{a} \end{cases} \quad (6)$$

The same equation can be used to calculate also the travel time from positions A and B to the tray locations i and j.

3.2 Testing models

To test the accuracy of the model, a RS assignment of the SKU in the dual-bay VLM is assumed. The expected dual command time $E[DC^{RS}]$ and the expected cycle time $E[CT^{RS}]$ are calculated, with different dual-bay VLM sizes and different values of the picker's picking time p_T.

The investigated values of p_T are: 0 s, 10 s, 20 s, 30 s and 40 s.

The case of $p_T = 0$ is introduced to estimate the real operating storage and retrieval times of the dual-bay VLM, its net performance, without being influenced by the operator's time.

On the other hand, the other values of p_T refer to different possible durations of the picking activity, according to the number and the kind of tasks the picker has to perform. For example, the picking activity can concern only the physical pick of the required items, or it can include also a search activity (if the stored items are similar to each other, or there are no signalling systems), or it can include the counting of the items to be picked, the confirmation of the pick etc.

The three different heights of the dual-bay VLM rack H that have been considered in the analysis are related to the different number of trays contained, each one having a unitary height of 0.35 m. These are: a dual-bay VLM with 30 trays (17 in the rear and 13 in the front) and a 5.95 m height, a dual-bay VLM with 40 trays (22 + 18) and a 7.7 m height and a dual-bay VLM with 60 trays (32 + 28) and an

11.2 m height. The vertical velocity of the crane is equal to 1 m/s and its acceleration/deceleration has been set to 2 m/s^2, corresponding to a delay time due to acceleration/deceleration per dual-bay VLM cycle equal to 2 s. The delay time to pick up or deposit a tray is equal to 4 s (Dukic, Opetuk and Lerher, 2015).

The results obtained by applying the developed analytical models are compared to the average value resulting from the simulation of a random generation of N=10,000 lines. The picking lines generation followed real order profiles. Each line n is referred to one SKU and, then, to one single tray. A discrete event simulation model has been used, in which the system performs the operational activities described in the introduction and in the flowchart of Figure 2, in order to retrieve sequentially all the trays containing the required SKUs. The simulation starts by assuming that in the two bays there are already two trays, chosen randomly, and ends once all the items of the N picking lines have been picked.

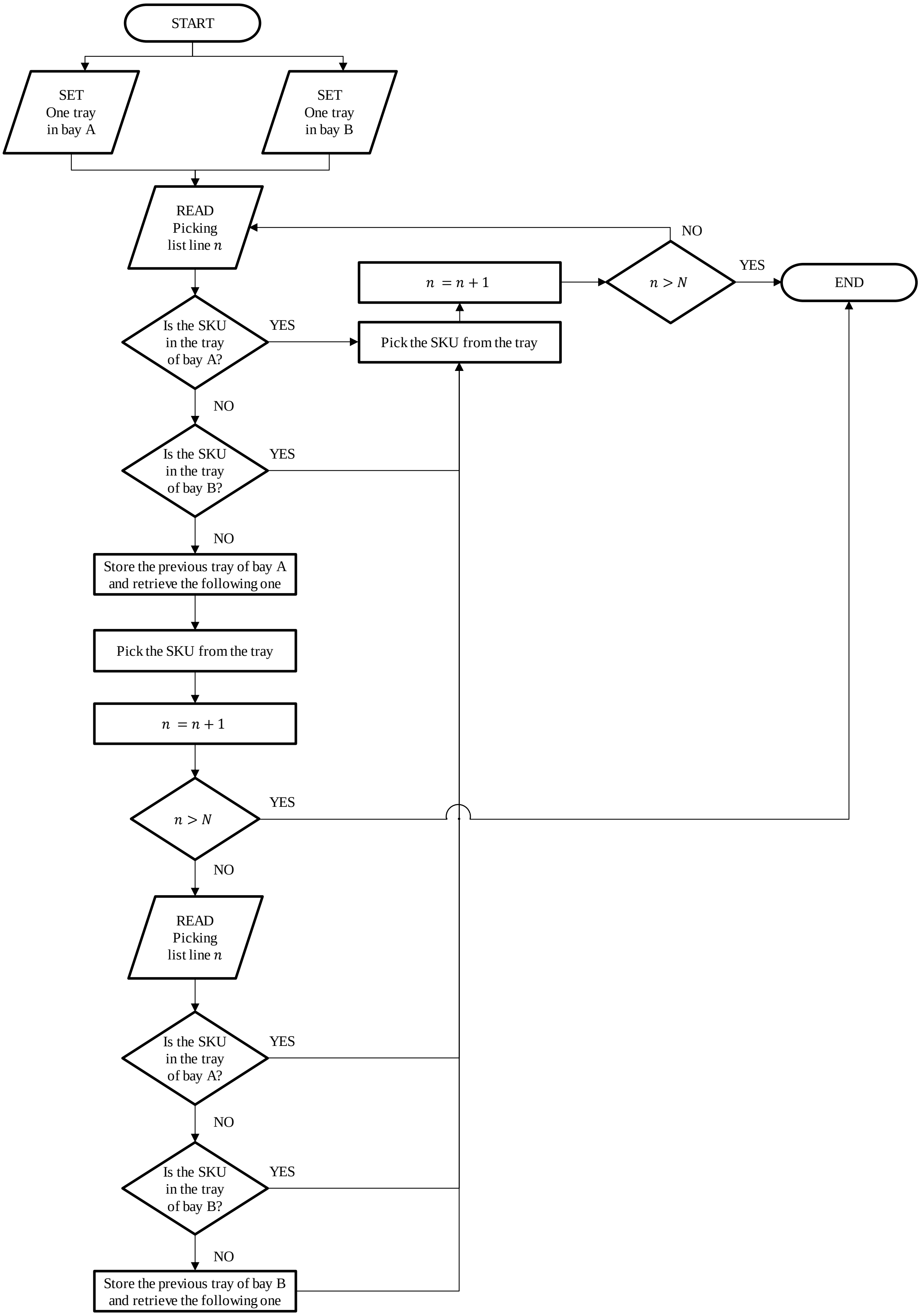


Figure 2. Flowchart of the simulation model.

In the simulation, for each step of the operational logic defined at the beginning of Section 3, the time spent by the operator to pick all the items of the required SKU is compared with the time spent by the crane in storing the previous tray and in retrieving the tray containing the subsequent SKU to be picked. The overall duration of the step depends on the maximum value of these two times (operator and crane). The expected cycle time $E[CT^{RS}]$ is calculated as the average value of the time spent for each step for executing the 10,000 lines.

Differing from Dukic, Opetuk and Lerher's (2015) approach, in which acceleration and deceleration of the crane are introduced as a constant value of delay time for each storage and retrieval trip equal to 2 s, in the present simulation model the real speed profile of the crane is considered. Indeed, the expected dual command times of the dual-bay VLM, represented in Table 2, are slightly lower than those estimated by using the equation introduced by Dukic, Opetuk and Lerher (2015).

Since the previous analysis of $E[DC^{RS}]$ (Table 2) has demonstrated the higher accuracy of the model with acceleration/deceleration consideration, the same table includes the evaluation of $E[CT^{RS}]$ only by applying this more precise model. As it can be seen, the estimated errors calculated comparing the values resulting from the application of the analytical model to the ones obtained from the simulation runs are very low also in this case.

		Analytical Model			Simulation Model			Error		
H [m]		**5.95**	**7.70**	**11.20**	**5.95**	**7.70**	**11.20**	**5.95**	**7.70**	**11.20**
		$E[DC^{RS}]$ $[s]$								
without acc/dec		24.78	27.02	31.50	21.90	24.52	29.44	13.1%	10.2%	7.0%
with acc/dec		21.91	24.47	29.39				0.0%	-0.2%	-0.2%
		$E[CT^{RS}]$ $[s]$								
p_T	**0**	21.91	24.47	29.39	21.90	24.52	29.44	0.0%	-0.2%	-0.2%
	10	21.91	24.52	29.45	22.24	24.76	29.59	-1.5%	-1.0%	-0.5%
	20	23.74	25.47	29.62	23.48	25.57	30.02	1.1%	-0.4%	-1.3%
	30	30.17	30.69	32.83	30.30	30.69	32.75	-0.4%	0.0%	0.2%
	40	40.00	40.00	40.07	40.18	40.19	40.45	-0.4%	-0.5%	-0.9%

Table 2. $E[DC^{RS}]$ and $E[CT^{RS}]$ comparison between analytical models and simulation results

Figure 3 depicts the average utilization of the dual-bay VLM $VLMU$ and of the picker PU, for different picking times per SKU and several heights of the rack H, resulting from the simulation analysis. Looking at this figure, it can be noticed that for low values of the picking time (in this case, less than 30 s), the cycle time is mainly affected by the performance of the dual-bay VLM. On the other hand, for picking times greater than 30 s the picker becomes the bottleneck of the entire system, influencing its throughput. In fact, in Figure 3, the bottleneck of the system is the side (picker or dual-bay VLM) with the higher value of utilization.

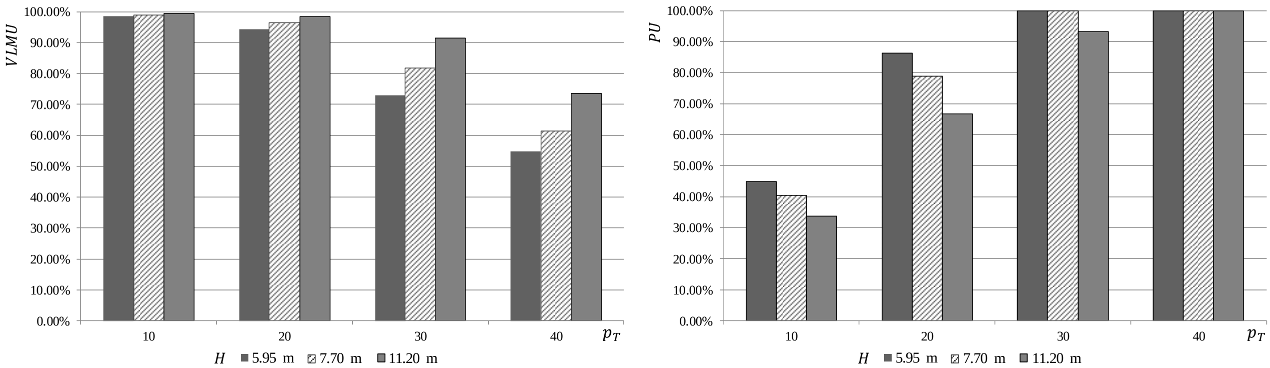


Figure 3. Simulation analysis: $VLMU$ and PU varying picker's picking time per tray and dual-bay VLM height.

Starting from this analysis, two possible approaches are proposed to improve the performance of the whole dual-bay VLM-OP system:

1. A storage assignment policy on the dual-bay VLM, focused on SKU demand frequency, in order to understand whether the dual-bay VLM operating cycle time can be reduced, and
2. Sequencing retrievals of the trays, which consider the ordering of the order lines according to the various trays that are needed.

Both are presented in the following sections with analytical models.

4 Dual-bay VLM storage assignment policy: application of the developed analytical models

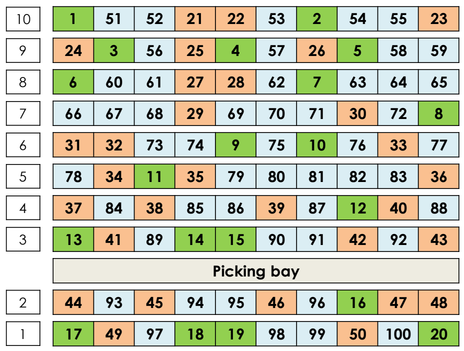
As introduced in the previous paragraph, a possible way to increase the dual-bay VLM-OP system throughput is to consider the possible advantages deriving from the application of a CBS assignment strategy with respect to a RS one (Petersen, Aase and Heiser, 2004; Dijkstra and Roodbergen, 2017).

In order to estimate the expected dual command time with CBS policy $E[DC^{CBS}]$, the travel time to store and retrieve each single tray has to be calculated as described by Equation (2) introduced in the previous section. In fact, the models already presented in literature cannot be applied since they assume to have discrete sections of the racks. This assumption does not allow to study the CBS policy at a detailed level (each single tray); therefore, it is not possible to estimate the throughput properly.

Then, the comparison here studied and proposed is between RS and CBS per tray (Figure 4), analysing the possible interactions between the dual-bay VLM and the picker. In case of RS, all the SKUs have proper dedicated space in the trays (usually with pods or bins) but they are stored randomly in the different trays (one SKU-one tray and one tray-more SKUs), without considering their picking frequency. On the other hand, in case of CBS, the SKUs are stocked based on their popularity, the A-class products are stored in the trays that are closer to the picking bay, the B-class products are in an intermediate position, while the C-class products are in the furthest trays. In this analysis, knowing the dedicated space necessary for each SKUs, items are stocked sequentially from the most picked to the less picked ones, respectively in the closest tray to the farthest one.

For the CBS per tray, three different possible curves are considered: 20/30, 20/40, 20/50 (Bender, 1981). Other curves (for example 20/60, 20/70 or more) are not taken into account in this study. In fact, in case there are very high-required SKUs, it would be more appropriate and efficient to stock them in different storage systems, like shelves for small parts or directly on pallets in low-level order picking systems. Thus, it is proposed to limit the application of the dual-bay VLM-OP system only to medium- and low-required SKUs, which have similar picking frequencies, as in the case of the 20/30, 20/40 or 20/50 curves (Figure 3). Another important assumption of the model introduced here is that, for now, the dual-bay VLM replenishment cycles are not considered in the system throughput calculation. In fact, it is considered that these are performed in another work shift, as already assumed in some similar previous researches (Meller and Klote, 2004; Dukic, Opetuk and Lerher, 2015).

Random Storage (RS)

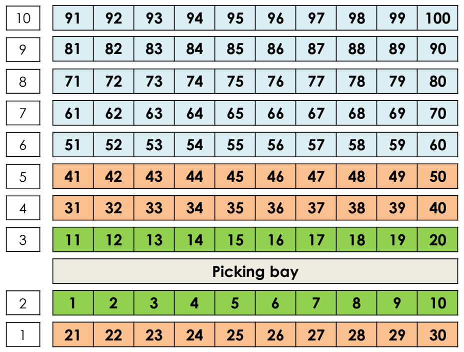


VLM

SKU Code	Tray	Bay
1	10	A
5	9	B
47	2	A
10	6	B
18	1	A
88	4	B
27	8	A
13	3	B
19	1	A
16	2	B
43	3	A

Order Picking list
Total retrieved trays: 11

Class-Based Storage (CBS)



VLM

SKU Code	Tray	Bay
1	**2**	**A**
5	**2**	**A**
47	5	B
10	**2**	**A**
18	3	B
88	9	A
27	1	B
13	**3**	**A**
19	**3**	**A**
16	**3**	**A**
43	5	B

Order Picking list
Total retrieved trays: 7

CBS curves

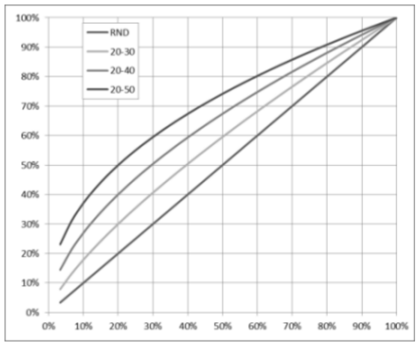


Figure 4. Comparison of different storage assignment strategies: random storage and class-based storage.

In order to properly compare these two storage assignment policies, the definition of analytical models to estimate the cycle time of the dual-bay VLM-OP system for both scenarios is needed.

The equations defined in Section 3 are useful to estimate the travel time of the crane also in case of different storage assignment policies. For example, with CBS, the probability of extracting the trays is not uniformly distributed among them. Moreover, it is also possible to consider the case in which in the currently delivered tray there are also SKUs that have to be picked subsequently to the second dual command, for instance, in the further following two lines. This situation is particularly frequent in the case of CBS storage assignment. In fact, in case of CBS storage, it could happen that different A-class products are picked sequentially from the same tray and, hence, from the same bay. Therefore, they do not require further tray retrievals. Two possible cases are reported for trays 2 and 3 in Figure

4. Tray 3, in bay A, is needed for picking 3 different SKUs one after the other, while tray 2 stays in bay A for SKUs 1 and 5, that are in sequence, but also for SKU 10, which is in the same tray but after SKU 47.

Table 3 shows a comparison of the results obtained for $E[DC^{SA}]$ by applying Equation (2) and the simulation approach, for different storage assignment policies and different dual-bay VLM heights.

First of all, these results show that the introduced analytical model for $E[DC^{SA}]$ estimation turns out to be very accurate. In fact, the errors of estimation of the dual command times obtained by using the analytical model with respect to the values of the simulation are very low.

Moreover, further analysing the results, it can be seen that the introduction of the CBS assignment policy leads to a reduction of the expected dual command time of the dual-bay VLM, compared to the RS: in case of RS the dual command time always corresponds to the highest value.

This dual command time decreases when the SKUs are stored considering CBS, and the global percentage reduction is about 5% in the case of the 20/30 curve and reaching about 15% for the SKUs with a 20/50 curve. This is a consequence of stocking the most frequently picked items in the trays that are closer to the dual bay. Furthermore, the reduction is also due to the lower value of X^{SA}, which is the factor of the occupancy problem. Since the most picked items are stocked in the same trays, it has an impact on the number of moved trays, reducing the required movements.

As reported in both Table 3 and Figure 5, the variation of the height of the dual-bay VLM causes a small difference in this percentage reduction. This reduction turns out to increase when increasing the dual-bay VLM height. In fact, the travel time is composed by constant terms, such as the delay time for picking up or for depositing a tray or for acceleration and deceleration, and by a term depending on the travelled distance. In case of a higher dual-bay VLM, the latter term is more relevant and, as a consequence, the CBS assignment policy has a higher impact on the dual command time, since the crane mostly performs its movements in the lower levels of the racks, closer to the dual-bay area.

		Analytical Model			Simulation Model			Error		
H [m]		**5.95**	**7.70**	**11.20**	**5.95**	**7.70**	**11.20**	**5.95**	**7.70**	**11.20**
SA		$E[DC^{SA}]$ $[s]$								
RS		21.91	24.47	29.39	21.90	24.52	29.44	0.0%	-0.2%	-0.2%
CBS 20-30		21.26	23.61	28.07	21.13	23.57	27.98	0.6%	0.2%	0.3%
CBS 20-40		20.26	22.40	26.43	20.13	22.39	26.38	0.7%	0.0%	0.2%
CBS 20-50		18.93	20.86	24.45	18.48	20.57	24.28	2.5%	1.4%	0.7%
SA	**p_T**	$E[CT^{SA}]$ $[s]$								
CBS 20-30	**0**	21.26	23.61	28.07	21.13	23.57	27.98	0.6%	0.2%	0.3%
	10	21.14	23.58	27.99	21.54	23.84	28.16	-1.8%	-1.1%	-0.6%
	20	23.39	24.89	28.49	23.02	24.84	28.74	1.6%	0.2%	-0.9%
	30	30.12	30.52	32.35	30.26	30.58	32.26	-0.5%	-0.2%	0.3%
	40	40.00	40.00	40.02	40.16	40.19	40.38	-0.4%	-0.5%	-0.9%
CBS 20-40	**0**	20.26	22.40	26.43	20.13	22.39	26.38	0.6%	0.0%	0.2%
	10	20.21	22.40	26.39	20.66	22.76	26.65	-2.2%	-1.6%	-1.0%
	20	23.09	24.38	27.46	22.56	24.12	27.50	2.4%	1.1%	-0.2%
	30	30.10	30.44	31.97	30.23	30.49	31.85	-0.4%	-0.2%	0.4%
	40	40.00	40.00	40.01	40.16	40.18	40.34	-0.4%	-0.5%	-0.8%
CBS 20-50	**0**	18.93	20.86	24.45	18.48	20.57	24.28	2.4%	1.4%	0.7%
	10	19.05	20.84	24.34	19.35	21.22	24.75	-1.5%	-1.8%	-1.7%
	20	22.81	23.91	26.47	22.05	23.33	26.17	3.4%	2.5%	1.1%
	30	30.12	30.44	31.69	30.19	30.40	31.45	-0.2%	0.1%	0.7%
	40	40.00	40.00	40.00	40.14	40.16	40.28	-0.3%	-0.4%	-0.7%

Table 3. $E[DC^{SA}]$ and $E[CT^{SA}]$ comparison between analytical model and simulation results, for different storage assignment policies *SA*, dual-bay VLM heights *H* and picking time p_T.

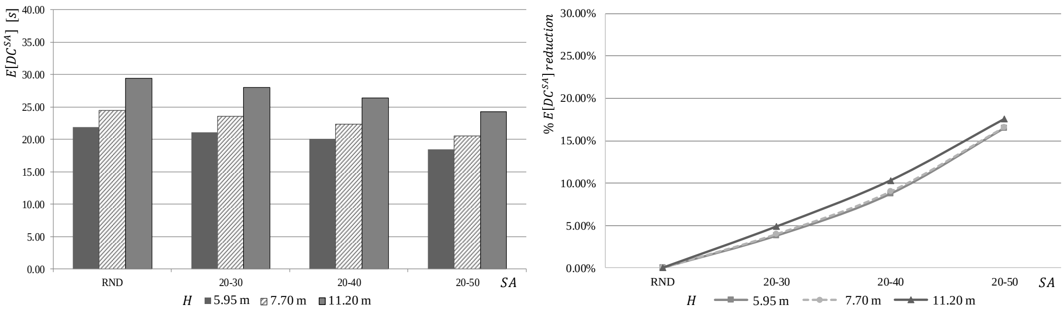


Figure 5. Left: $E[DC^{SA}]$ comparison for different storage assignment policies and dual-bay VLM heights. Right: $E[DC^{SA}]$ percentage reduction.

Table 3 reports the results, both of the analytical model and of the simulation one, in terms of $E[CT^{SA}]$, hence by considering the whole dual-bay VLM-OP system, composed by the dual-bay VLM and the picker, for different values of picking time p_T (10, 20, 30, 40 s). The application of such an approach, with the introduction of a CBS assignment policy within the dual-bay VLM trays, leads to interesting benefits when the picking time p_T is low (10 s and 20 s), and, hence, until the dual-bay VLM is the bottleneck. On the other hand, by extending the analysis of the system to higher values of picking times (30 s and 40 s), with the picker becoming the bottleneck, the effect of the CBS assignment policy has a lower impact (Figure 6).

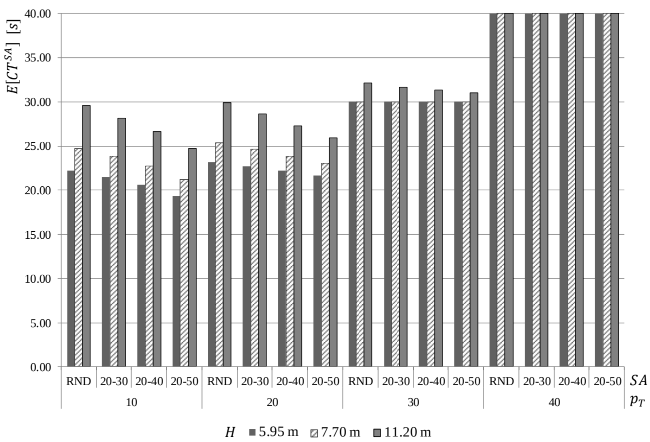


Figure 6. $E[CT^{SA}]$ comparison for different storage assignment policies *SA*, dual-bay VLM heights H and picking time p_T.

5 Sequencing retrievals: extended analytical models

In the case of multiple-lines order, the dual-bay VLM can perform sequenced retrievals of the trays, thanks to the ordering of the order lines based on the required trays. Hence, it can be derived that, during the processing of the entire group of lines in one order, each tray is delivered only one time, with a consequent lower overall number of delivered and retrieved trays (Figure 7).

Random retrievals – RS policy

SKU Code	Tray	Bay
1	10	A
5	9	B
47	2	A
10	6	B
18	1	A
88	4	B
27	8	A
13	3	B
19	1	A
16	2	B
43	3	A

Order Picking list
Total retrieved trays: 11

Random retrievals – CBS policy

SKU Code	Tray	Bay
1	2	A
5	2	A
47	5	B
10	2	A
18	3	B
88	9	A
27	1	B
13	3	A
19	3	A
16	3	A
43	5	B

Order Picking list
Total retrieved trays: 7

Sequenced retrievals – CBS policy

<table>
<tr><th>SKU Code</th><th>Tray</th><th>Bay</th></tr>
<tr><td>1</td><td rowspan="3">2</td><td rowspan="3">A</td></tr>
<tr><td>5</td></tr>
<tr><td>10</td></tr>
<tr><td>47</td><td rowspan="2">5</td><td rowspan="2">B</td></tr>
<tr><td>43</td></tr>
<tr><td>18</td><td rowspan="4">3</td><td rowspan="4">A</td></tr>
<tr><td>13</td></tr>
<tr><td>19</td></tr>
<tr><td>16</td></tr>
<tr><td>88</td><td>9</td><td>B</td></tr>
<tr><td>27</td><td>1</td><td>A</td></tr>
</table>

Order Picking list
Total retrieved trays: 5

Figure 7. Different approaches for processing order picking lists: random vs. sequenced retrievals of trays.

In such a situation, the expected dual command time $E[DC^{SA}(L,M)]$ is defined as follows:

$$E[DC^{SA}(L,M)] = \frac{E[R^{SA}(L,M)]}{L} \cdot \ E[DC^{RS}] \tag{7}$$

where $E[R^{SA}(L,M)]$ is the expected number of trays delivered for an order of L lines. In order to estimate this expected number of trays, it is possible to consider the multinomial probability distribution (Battini et al, 2015a). This probability distribution calculates the probability of having a certain combination of trays extracted k, knowing the dimension of the order L and all the various probabilities p_i, referring to the extraction of each single tray i:

$$P_k = \frac{L!}{\prod_{i=1}^{M} l_i!} \prod_{i=1}^{M} p_i^{l_i} \tag{8}$$

where L corresponds to the sum of all the numbers of picking lines per tray l_i

$$L = \sum_{i=1}^{M} l_i \quad (9)$$

Then, by introducing a parameter x_i as:

$$x_i = \begin{cases} 1 & if\ l_i \neq 0 \\ 0 & if\ l_i = 0 \end{cases} \quad (10)$$

and defining

$$X_k = \sum_{i=1}^{M} x_i \quad (11)$$

as the total number of trays delivered for the combination k, it is possible to estimate the expected number of trays delivered for an order of L lines with

$$E[R^{SA}(L, M)] = \sum_{k=1}^{K} X_k \cdot P_k \quad (12)$$

where K is the total number of possible combinations of trays extracted k, calculated as:

$$K = \binom{M + L + 1}{L} \quad (13)$$

Here, the total number of different combinations K depends on the total number of trays of the dual-bay VLM M and on the number of lines in the order L and it can be calculated with Equation (13). For example, considering M=30, for L=2, K=528; for L=5, K=376,992, while K=1,121,099,408 if L=10. Then, since K can be very high, in this case the analysis of the impact of L has been carried out using only the results obtained by the simulations, reported in Table 4. However, the validation of the analytical model has been performed in the same way, with a small but representative set of data, reported in Table 5.

	$E[R^{SA}(L,15)]/L$								
	Analytical Model			**Simulation**			**Error**		
SA \ L	**3**	**5**	**7**	**3**	**5**	**7**	**3**	**5**	**7**
RND	0.935	0.875	0.821	0.929	0.873	0.826	0.6%	0.3%	-0.7%
20-30	0.929	0.866	0.808	0.920	0.863	0.808	1.0%	0.3%	0.1%
20-40	0.910	0.835	0.772	0.901	0.837	0.781	0.9%	-0.3%	-1.1%
20-50	0.874	0.782	0.713	0.858	0.783	0.715	1.8%	0.0%	-0.3%

Table 4. $E[R^{SA}(L,15)]/L$ comparison between analytical model and simulation results, for different storage assignment policies and different L.

$E[R^{SA}(L,M)]/L$		H		
SA	***L***	**5.95**	**7.7**	**11.2**
RND	10	0.8599	0.8963	0.9283
	20	0.7424	0.7982	0.8603
	30	0.6425	0.7129	0.7996
	40	0.5594	0.6386	0.7379
	50	0.4914	0.5739	0.6854
20-30	10	0.8493	0.8856	0.9205
	20	0.7279	0.7871	0.8491
	30	0.6288	0.7035	0.7831
	40	0.5481	0.6283	0.7232
	50	0.48	0.5654	0.6698
20-40	10	0.8166	0.8585	0.8955
	20	0.6871	0.748	0.8118
	30	0.5912	0.6629	0.7396
	40	0.5161	0.5923	0.6815
	50	0.4558	0.5348	0.6279
20-50	10	0.7566	0.7941	0.8425
	20	0.6268	0.6812	0.7458
	30	0.5392	0.5996	0.6783
	40	0.473	0.537	0.6209
	50	0.4209	0.4863	0.5731

Table 5. $E[R^{SA}(L,M)]/L$ calculation through simulative approach, varying storage assignment policy *SA*, dual-bay VLM height H, number of lines per order L.

Figure 8 reports the effect of the number of lines per order on the expected dual command times, varying L and the CBS assignment policy *SA*. As already done in the previous approach, we firstly assume $p_T = 0$, in order to understand the net performance of the dual-bay VLM and then to compare it to the picker activities, as shown in the following step.

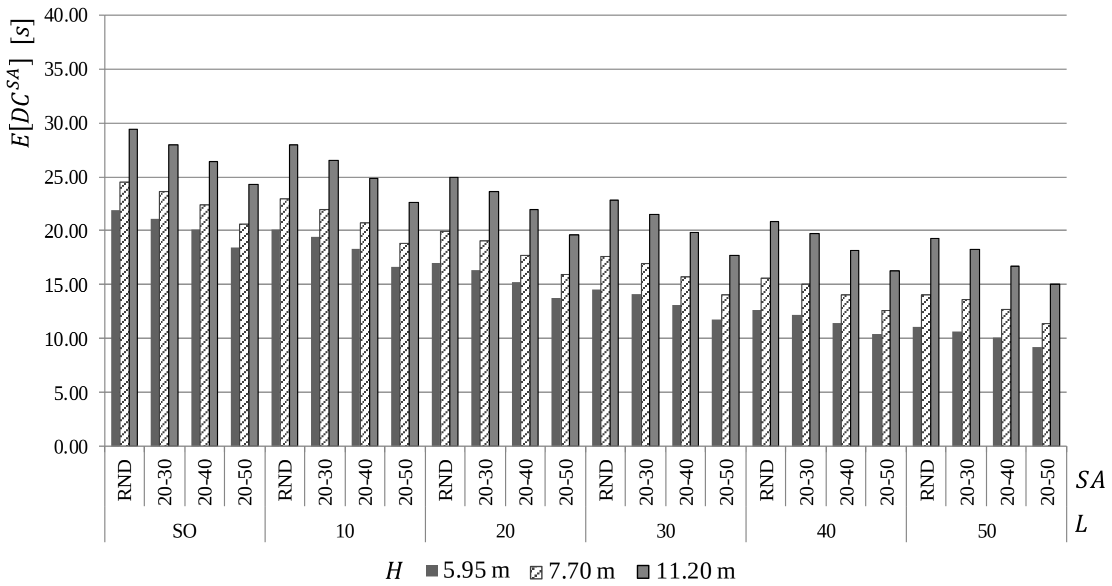


Figure 8. $E[DC^{SA}]$ comparison for different storage assignment policies *SA*, dual-bay VLM heights H and number of lines per order L.

Figure 9 shows the values of $E[DC^{SA}]$ and the percentage reduction, obtained by fixing the height of the dual-bay VLM and by varying the frequency curves. In this case, the effect of the number of lines per order is affecting the overall results in terms of $E[DC^{SA}]$, both for the absolute values and for the percentage ones, while the impact of the *CBS* seems to be limited. In fact, it can be noticed that the $E[DC^{SA}]$ strongly decreases by increasing the number of lines contained in the order, while the resulting $E[DC^{SA}]$, obtained by varying the CBS and by considering the same number L, are very similar to each other.

This is mainly due to the fact the sequencing retrievals have a greater impact than the CBS assignment policy: in fact, it permits to reduce the travelled distance and, then, the CBS assignment policy has a very low effect.

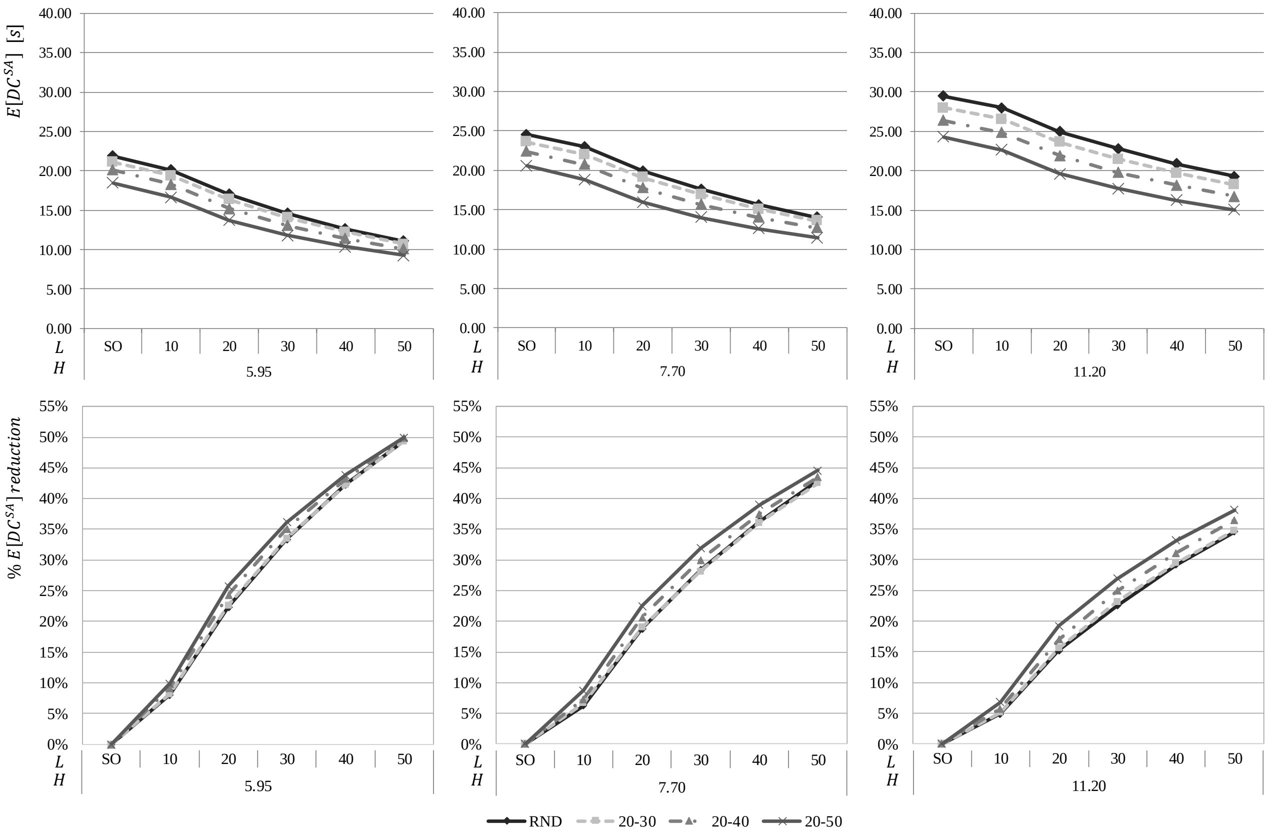


Figure 9. $E[DC^{SA}]$ results and percentage reduction for different storage assignment policies SA, dual-bay VLM heights H and number of lines per order L.

Figure 10 reports the values of $E[CT^{SA}]$, obtained through the comparison of the $E[DC^{SA}]$ of the VLM with the picking time p_T of the picker, for different storage assignment policies SA, dual-bay VLM heights H, number of lines per order L and picking times p_T. It is interesting to see how, also in this case, the bottleneck of the entire system for low values of picking time is again the dual-bay VLM. On the other hand, for a picking time greater than 30 s, it is clear that the expected cycle time $E[CT^{SA}]$ is always the same, even when varying the CBS assignment policy and the number of lines per order. Only in the case of a dual-bay VLM height of 11.2 m, the cycle time is affected by the dual-bay VLM performance also for p_T=30 s.

Finally, it can be derived that the operational mode with a sequenced retrieval of the trays has a relevant impact on the performance of the dual-bay VLM and, consequently, of the entire system for low values of picking time, leading, for example, to a reduction of the cycle time from 5% to 10% in the case of a group of 10 lines and to a reduction from 35% to 50% in the case of a set of 50 lines.

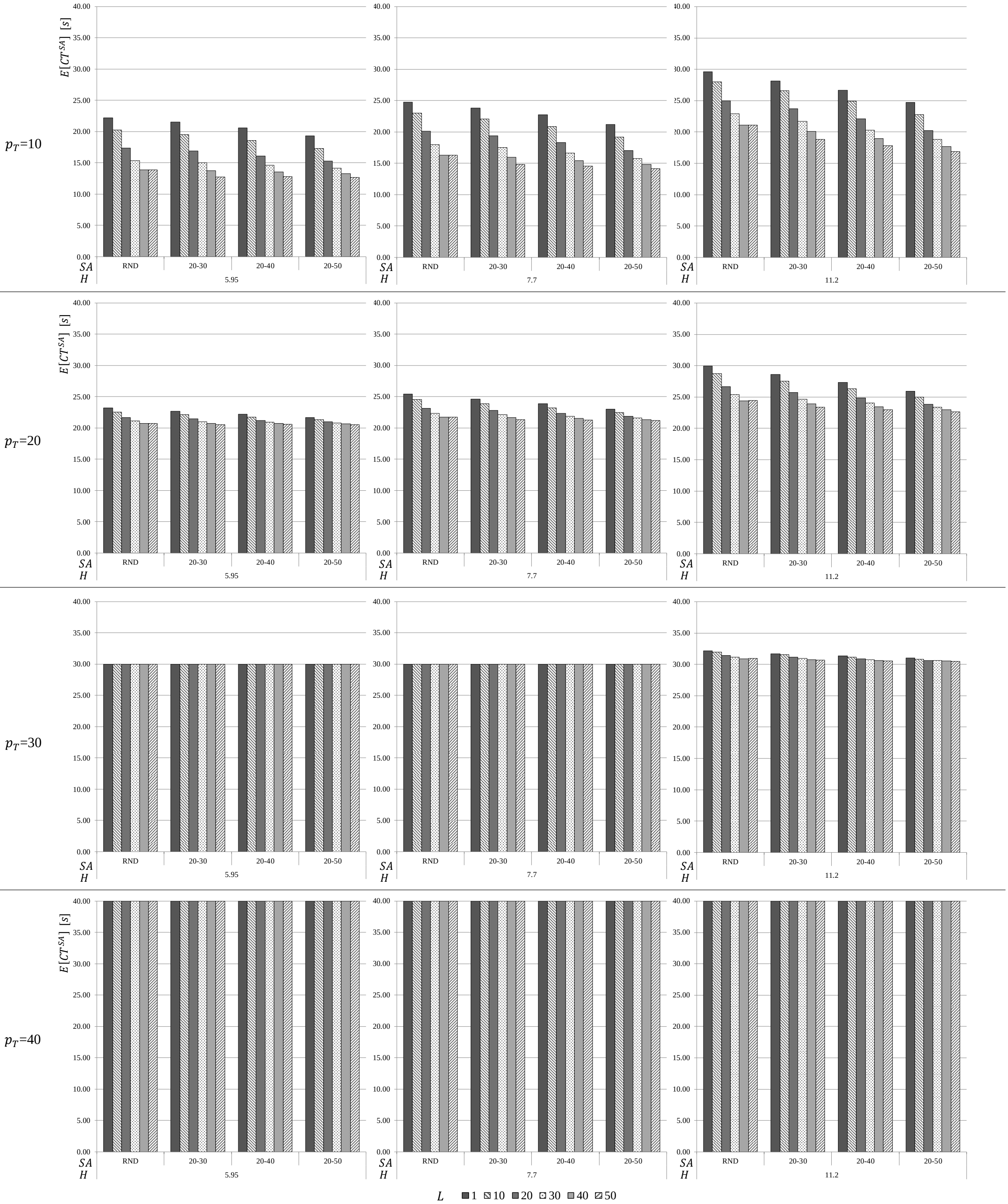


Figure 10. $E[CT^{SA}]$ comparison for different storage assignment policies SA, dual-bay VLM heights H, number of lines per order L and picking times p_T.

6 Managerial implications

As described in the introduction, the dual-bay VLM-OP system is an alternative solution for small objects order picking, typical of some e-commerce companies or some distribution centres for personal care or home care products. The main issue the warehousing managers have to face when they employ this kind of system is to understand how to manage them and to improve their performance in order to be competitive and cost-effective.

The introduction of these models and the throughput performance analysis explained in the previous sections allow to highlight some important managerial implications on how dual-bay VLM-OP systems perform and which are the most relevant decision variables the practitioners can use to better manage their operating configurations.

It has been clearly stated that the global throughput are strongly dependent on both dual-bay VLM and operator performance.

In particular, if the picking times p_T are higher than the dual-bay VLM dual command time, the bottleneck is the operator, so any improvement on the dual-bay VLM is not actually needed.

On the other hand, it has been demonstrated that the investigated operating configurations are effective when the picker's picking time is, on average, lower than the dual-bay VLM dual command time.

The whole analysis of the applied configurations has demonstrated that CBS assignment policy leads to an interesting effect in terms of reducing the dual-bay VLM dual command time, while sequencing retrievals bring benefits in terms of reducing trays extractions and, hence, in terms of decreasing the system cycle time.

Moreover, the results have demonstrated that the application of CBS brings some benefits in terms of improving the system throughput, but with the need for and, thus, the subsequent activity of, proper positioning of the SKUs within the trays. On the other hand, the sequencing retrievals strategy leads to interesting improvements in terms of tray retrieval time, with a very low implementation effort. Finally, it turns out that the simultaneous application of sequencing retrievals and CBS has a low synergistic effect, as shown in Figure 9 and Figure 10.

7 Conclusions and future researches

This work has presented a study concerning the estimation of the performance of dual-bay Vertical Lift Modules Order Picking (dual-bay VLM-OP) systems. In this paper, it is studied how this storage solution is intended to create advantage by increasing the picking throughput.

In particular, it has been investigated the case when the dual-bay VLM is the bottleneck of the system, so when the time the operator spends to pick the products is low. This is quite common for these kind of systems and it reflects the case of orders with small quantities of product required per line.

The possible actions proposed here to improve the throughput of the system concern the implementation of a CBS assignment policy within the trays and the employ of the dual-bay VLM executing the sequencing retrievals of the trays.

After a first description of the system and of its possible operating context, analytical models to estimate the dual command times have been proposed. They are the first models that consider acceleration/deceleration of the crane into the estimation of the dual command time. Furthermore, the consideration of the probability of storage/retrieval of each single tray and the combinations of more SKUs per tray have permitted to apply the models to different operating configurations, such as different storage assignment policies and sequenced retrievals.

The models have been validated through an in-depth comparison of their results with those of a simulative approach, varying different factors and parameters. The accuracy of the models is promising.

Future researches on this topic could extend the results obtained in this paper by introducing a deeper analysis of the pickers' activities and times, in order to understand how to further improve the overall system performance, when the operator is the bottleneck of the system. For example, it would be interesting to investigate the possible impact of order batching and of the subsequent sorting. Moreover, it would be interesting to add a formulation to consider the activity of refilling the dual-bay VLMs, in particular as far as its management, frequency, and the number of products that have to be refilled are concerned. Finally, it would be interesting to extend this work by introducing a preliminary step, with the aim of understanding which SKUs are more suitable to be stored and picked with the dual-bay VLM-OP system rather than a traditional picking system, according, for example, to the SKUs physical characteristics (volume, weight) and picking frequency (Battini et al., 2018).

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