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Age replacement policy in the case of no data: The effect of Weibull parameter estimation

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Abstract

Age replacement is a common maintenance policy when wear-out failures occur, and it is characterised by periodic replacement of components. Data on time to failure (TTF), often modelled with the Weibull function, are necessary for estimating optimal replacement intervals to minimise the total maintenance costs. In many cases, such as new components, new machines or new installations, no TTF data are available, so the Weibull parameters and optimal replacement interval cannot be estimated. To overcome this problem, these parameters can be assessed from the experience of the maintenance engineers and technicians. The aim of this study is investigating the relationship between the error in parameter estimation and additional maintenance costs related to this error. Analysis of variance (ANOVA) and multifactorial analysis are carried out for investigating the influence of these estimations on the final costs. Economic decision maps are introduced for supporting maintenance engineering in defining the maintenance policy with minimal additional cost in the case of no data being available. The analysis shows that, when no data are available, the application of the age replacement policy can result in a global savings of more than 50% compared with corrective maintenance.

Keywords: Age replacement policy, Weibull distribution, TTF, No data, Maintenance costs

1. Introduction

Production and logistic systems require various actions to maintain high performance levels during their lifecycles. They are affected by degradations and accidents caused by operational and environmental conditions. Maintenance actions are implemented for guaranteeing the availability and efficiency of the systems; however, maintenance is costly in terms of both resources and materials. Establishing an efficient maintenance strategy that considers the required resources and production plans is essential and has direct consequences for production efficiency and economic performance (Regattieri et al., 2010).

The concept of maintenance has evolved significantly over time due to contributions made by research in the field. Maintenance policies, such as failure-based maintenance, time-based maintenance, condition-based maintenance, design-out maintenance and detection-based maintenance (Waeyenbergh and Pintelon, 2004) have also been developed. Many studies have introduced various models for supporting practitioners in defining the most effective maintenance strategy. These are based on the availability of data, time to failure (TTF), time to repair, spare part cost, lost productivity cost and the cost of resources. In many cases (e.g. when the systems or components are new or when failures are not reported), data can be unavailable, with the consequence that previous models cannot be applied. Consequently, the most efficient maintenance action is difficult to define. This challenge is often linked to the complexity of industrial environments and problems collecting the correct data from the field. Thus, the definition and estimation of reliability, maintainability functions and related costs are extremely complicated; this means that the selection of the correct maintenance policy is similarly difficult. For these reasons, the most commonly applied maintenance policies are corrective (where TTF data are not needed) and calendar-based preventive approaches (with a predefined component replacement determined by the calendar date).

In terms of corrective maintenance policies, component/system failure determines the maintenance action that will 'correct' the breakdown and restart the system. If a time-based maintenance policy is implemented, TTF data are required for defining the failure distribution and correct time to perform the preventive action, instead of waiting for a corrective response after failure. The key point of this policy is the TTF, which is directly related to the distribution of failures and maintenance costs.

The Weibull distribution is one of the most popular probability functions used to describe a system's lifetime and failure events (Lawless, 1982; Hallinan, 1993). A Weibull distribution signals when time-based maintenance is suitable and when components are affected by degradation. It can be used for the estimation of the optimal maintenance interval and relative costs. The main Weibull parameters are β, which identifies the failure trend of the components, and the scale θ, which is related to the unit of time used for measuring TTF. Various methodologies have been used to estimate the Weibull parameters from TTF data, including the maximum likelihood estimation (MLE) and least squares estimation (LSE) approaches.

The availability of correct and complete data is a common assumption of maintenance model analysis, but often, this assumption does not reflect reality (Ross, 1996; Montanari et al., 1997a; 1997b). Censored data are often evident, as there is frequently a lack of data collection; in the existing literature, many studies have investigated the suitability of failure models in such cases. Reducing bias in the parameter estimation has also been explored (Zhang et al., 2006).

Generally, the consequences of errors in the estimation of reliability parameters for maintenance costs are unclear. This is especially true when no data are available, such as in the case of new components or systems, since data have an effect on the interval time of preventive maintenance, and consequently, on cost. A quantification of the additional cost of incorrect estimation needs to be investigated.

In this study, for the first time, the consequences of the incorrect estimation of reliability parameters are investigated in relation to the age replacement policy (ARP). Since no model is available for estimating Weibull parameters without TTF data, first, a simple procedure for assessing β and θ is introduced. Then, the additional costs of age replacement maintenance, based on the estimated parameters from no data availability, is calculated as a percentage of the cost of minimal age replacement maintenance in the case of complete data. Moreover, the savings when applying the ARP based on wrong reliability parameters have been analysed compared with the application of a corrective maintenance policy.

The errors in the estimation of mean TTF (MTTF) and the shape parameter β are included in the analysis. The corrective and replacement costs are also included. Statistical analyses in the form of a multifactorial analysis and one-way analysis of variance (ANOVA) test have been conducted to study the effects of various factors on maintenance costs and savings.

The paper is organised as follows: The next section presents the literature review, highlighting the research gap this paper covers. The third section presents the problems and mathematical models used for defining maintenance parameters. First, it deals with the Weibull distribution for failure events and statistical tools for estimating the MTTF and β in relation to no data. Then, the optimal replacement interval time is defined. In the fourth section, the cost estimation and its analysis are discussed. The additional costs related to incorrect estimations are developed. Multifactorial and ANOVA analyses are also carried out, varying the parameters of the maintenance models. In section 5, the results of the study are discussed along with some guidelines for practitioners. Finally, a simple case study shows the applicability of this approach and the effects on total yearly costs and savings from implementing an ARP when no data are available.

2. Literature Review

The corrective and preventive (time-based and condition-based) maintenance types are the most used maintenance strategies in industrial settings. A corrective maintenance policy is based on replacements carried out after failures occur; in contrast preventive maintenance aims to reduce system faults through preventive actions of replacement. A time-based policy is a highly common type of preventive maintenance. Barlow and Hunter (1960) present the two following types of time-based maintenance: ARP type I (ARP-I) is useful for maintaining simple equipment, while ARP-II is useful for large and complex systems. ARP-I performs preventive maintenance after a fixed period, called an interval period or interval time, of continued functioning without failure, or after failures, whichever occurs first. After the replacement, the interval period begins once again. This policy is suitable only when the components are subjected to wear-out failures, and it is assumed that the component is as good as new after maintenance (or replacement) is performed. ARP-II is executed after a fixed period of continued functioning and after failures. It is assumed that after each failure, only minimal repairs are made. In both cases, the optimum interval time is identified as the time that minimises the total costs, including preventive and corrective costs.

The first formulation of ARP-I, developed by Barlow and Hunter (1960), is a simple mathematical model based on the reliability function, $\boldsymbol{R(t)}$, of the component; average cost of preventive action, $\boldsymbol{C_p}$; and average cost of intervention after a failure, $\boldsymbol{C_f}$. Thus, the unit expected cost (UEC) at replacement time $\boldsymbol{t_p}$ is expressed by

$$\boldsymbol{UEC(t_p) = \frac{C_p R(t_p) + C_f[1 - R(t_p)]}{\int_0^{t_p} R(s)ds}}, \qquad \textbf{(1)}$$

where $\boldsymbol{R(t_p)}$ is obtained for the Weibull distribution function, with $\boldsymbol{\beta}$ as the shape parameter and $\boldsymbol{\theta}$ as the scale factor. This is described by

$$\boldsymbol{R}(\boldsymbol{t_p}) = \boldsymbol{exp}\left[-\left({}^{\boldsymbol{t_p}}/_{\boldsymbol{\theta}}\right)^{\boldsymbol{\beta}}\right]. \quad \textbf{(2)}$$

Note that the ARP-I is only applicable if there is a wearing condition, and $\boldsymbol{\beta}$ has to be higher than 1. In the other cases, the replacement interval time is at failure, as in the setting period ($\boldsymbol{\beta}$ < 1) or the failures are random and difficult to prevent ($\boldsymbol{\beta}$ = 1).

The optimal value of t_p, which minimises the $UEC(t_p)$, indicated by t_p^*, cannot be calculated with a closed-form equation. Faccio et al. (2014) develop an easy-to-use abacus for calculating the correct value of t_p^* and related $UEC(t_p^*)$, knowing the ratio between the failure and preventive costs (C_f/C_p) and shape and scale parameters (β and θ) of the Weibull distribution function.

More generally, Waeyenbergh et al. (2004) present a framework for discussing the application of various maintenance concepts, focusing on the two principal interventions, namely, corrective and preventive maintenance. Wang et al. (2006) present a study on evaluating different maintenance strategies, mainly corrective and preventive ones. They also identify the optimal mix of maintenance policies by applying an analytic hierarchical process method. In addition, Sheut et al. (1994) present a work focussed on corrective and preventive maintenance, underlining when corrective maintenance may be convenient.

In a recent work, Qiu et al. (2017) propose a novel maintenance policy operating under a performance-based contract (PBC) context. The model they present aims at maximising the expected net revenue; they also compare their results with those of the optimal maintenance policies, demonstrating a relevant cost reduction. Yang et al. (2018) present a model that optimises the replacement interval and minimises the expected cost with failures in a random environment. Moreover, in their work, Qiu et al. (2019) study the optimal maintenance policy that minimises the expected cost per unit time of the system; they develop reliability and maintenance models for a single-unit system subject to hard failures under a random environment of external shocks considering, imperfect preventive maintenance actions. Other works are oriented towards joint preventive maintenance and quality targets, like that of He et al. (2019); they present a reliability-oriented optimisation model for joint preventive maintenance (PM) and process quality control with a time-between-events (TBE) control chart, where the effect of the manufacturing process on the product’s final reliability is considered to reduce the product reliability degradation originating from latent manufacturing defects. Chang (2018) presents a model that generalised the age replacement policy for a system which works at random time and considers random lead time for replacement delivery; the aim of his work is to determine an optimal schedule of age replacement that minimises the mean cost rate function of the system in a finite time horizon. Instead, Hajej *et al.* (2018) focused on the integration between production and maintenance goals, to define the optimal maintenance interval; they carried out a joint control policy is based on a stochastic production and maintenance planning problem with goals to determine the economic plan of production and the optimal maintenance strategy. Wang *et al.* (2018) presented an innovative work about preventive maintenance strategies for industrial equipment during successive usage-based lease contracts with consideration of a warranty period, from the lessor's perspective; they used an accelerated failure time model to capture the effect of usage rate. Other recent works are concentrated on the relation between productivity and the age replacement interval; Rezaei-Malek *et al.* (2018) proposed a robust possibilistic and multi-objective mixed-integer linear programming mathematical model to plan Preventive Maintenance activities for a serial multi-stage production system. Finally, Lai et al. (2018) presented an optimisation decision of economic production quantity model for an imperfect manufacturing system under hybrid maintenance policy; they carried out the optimal production quantity and maintenance interval during each production run minimising the expected average cost of the system.

Preventive maintenance requires data and cost estimation to define the optimal replacement interval time. Many authors have studied models and tools to define this interval, and in them, available data are employed to obtain the desired results (Ben Mabrouk et al., 2016). The main data required concern the reliability distribution of the component/system. Weibull distribution is one of the most common models used for describing failure time in a reliability analysis of complex systems (Hossain et al., 2003). It accurately describes the different phases of the life cycle of components, represented by the bathtub curve (initial, random and wear-out failure modes; Hisada at al., 2002). Especially, the shape parameter β indicates when the component is in the setting period ($\beta < 1$), random failure period ($\beta = 1$) or degradation period ($\beta > 1$) (Dedopoulos et al., 1998). Many authors describe the suitability of the Weibull distribution for reliability models and focus on the optimal methods for estimating its parameters. Murthy et al. (2004) present a study of various models derived from a two-parameter Weibull distribution, which aims to describe complex failure datasets using the Weibull probability plot. Pham et al. (2007) discuss the industrial cases of different failure models based on this distribution. They summarise some commonly known models and discuss their general properties, focussing on the fundamental relationship between the reliability function and its corresponding cumulative failure rate function.

Persona et al. (2009) investigate the use of the systemability function to model reliability under different operating and environmental conditions. In a more recent work, Zennaro et al. (2018) present a model in which a micro–downtime analysis is carried out, representing the reliability function with the Weibull distribution. Bala et al. (2018) present a three-parameter Weibull distribution approach used for analysing the Load-Haul-Dumper (LHD) datasets in underground mines, and the percentage reliability of each individual subsystem of LHD is estimated. Moreover, Jacobs et al. (2018) use the Weibull distribution model for predicting pump failure and applying a proactive and preventive maintenance strategy, while Nemati et al. (2019) present a work on the estimation of the failure rate of cables based on their age and a set of explanatory factors using a Weibull parametric proportional hazard model (PHM). Finally, Velásquez et al. (2019) use the Weibull distribution for estimating the main failures and their influence in a switchgears context.

In general, there are three steps in the empirical modelling of the data, including the Weibull distribution, an estimation of model parameters and model validation. Methods of estimating shape and scale parameters, such as MLE and LSE, which focus on the suitability of final models, are widely discussed in the literature. In their study, Genschel et al. (2010) discuss the different methods of estimating Weibull distribution parameters, such as the maximum likelihood and rank regression. Yavuz (2013) compared various regression methods for estimating Weibull shape parameters using a performance estimator based on the bias and mean square error criteria in Monte Carlo simulations. Xie et al. (2002) used a graphical estimation technique, based on the Weibull plot, to evaluate the parameters of a modified Weibull distribution. Knowing that the Weibull distribution has been widely used for modelling different phases of the lifecycle, an additive model is also explored, where the failure rate function is expressed as the sum of two failure rate functions in Weibull form. Ling et al. (1998) present a model based on the Kolmogorov–Smirnov test to evaluate the parameters.

In a recent work, de Assis et al. (2018) present a comparison between the use of two different models—the Weibull and q-Weibull reliability models—in the electrical power sector; they use the maximum likelihood method for parameter estimation. An et al. (2018) also use the maximum likelihood method to estimate Weibull parameters for their model, which aims to estimate railway track geometry conditions. Ramos et al. (2018)

present a study about five generalisations of the standard Weibull distribution to describe the lifetime of two important components of sugarcane harvesting machines, using maximum likelihood for parameter estimation.

Censored data have been extensively discussed in the literature. With censored data, the parameters of the failure distribution function are harder to investigate, and consequently, the optimal interval time is difficult to define. Regattieri et al. (2010) present a framework for the robustness of reliability estimation, considering the critical role of censored data. They estimate the distribution of reliability parameters for several critical components from the date of the initial installation. They discuss the effects of the amount of data, especially censored data, in calculating the robustness of reliability estimation and evaluation processes.

The sense of uncertainty and lack of information is a common trope in these studies, and it needs to be reconsidered. Red-Horse et al. (2004) use polynomial chaos expansion for investigating the correlation between failure model output and limited data input. The objective is determining a process for mapping uncertainties in the intervals, means, variances and other statistical measures of the model parameters. The researchers also aim to map uncertainties in similar statistical measures of the model outputs.

Coit et al. (1999) present an approach for the evaluation of censored data, and they explore whether the use of exponential distribution in modelling TTF when individual TTFs are not available can be dangerous. In this case, the distribution parameters can be estimated with the merged data. A hypothesis test is presented to test the suitability of the exponential distribution for a certain dataset composed of multiple merged data records. The test rejects the exponential distribution assumption when the data originate from a Weibull distribution, thereby demonstrating that this is a highly important result. The result shows that TTF data are always required to evaluate the suitability of specific TTF distributions.

Zhang (2006) focusses on the LSE method, proposing a bias correction approach linked to the sample size and censoring level. He demonstrates that bias correction is affected more by the censoring level than it is by the sample size. Economou et al. (2012) develop a model for reliability performance by studying underground water pipes. In another study, Montanari et al. (1997a; 1997b) compare various methods for estimating parameters of the two-parameter Weibull distribution of uncensored data. They study the six following techniques: maximum likelihood, least squares regression, the Jacqueline estimator, the Ross estimator, the White estimator and the Bain and Engelhardt estimator. They compare the simplicity and accuracy of these techniques in estimating the shape and scale parameters of the two-parameter Weibull distribution when applied to single censored data. Finally, Yang at al. (2003) propose an analysis of bias on MLE Weibull parameters. They demonstrate that bias increases as the degree of censorship increases and more people become involved.

Complete and correct data concerning failures and repair time are often unavailable, as industrial systems may be only recently installed or data may not have been collected in the past. Frequently, maintenance staff have to work with highly limited data or none at all. They often have to estimate maintenance parameters and replacement interval times.

Some works study uncertainty in data estimation using fuzzy input data, the bootstrap method and Monte Carlo simulation. Liu et al. (2010) propose a new technique for determining the membership functions of parameter estimates and the optimal preventive maintenance policy based on fuzzy reliability data. Liao et al. (2018) study the Remaining Useful Life (RUL) parameter related to condition-based maintenance; they use an Long Short-Term Memory-Feedforward Neural Network based on the bootstrap method for uncertainty prediction of RUL estimation. Instead, Yang et al. (2018) present the hazard rate curve analysis of the numerical control machine tool using the Markov chain Monte Carlo method to estimate the Weibull parameters. Sanchez et al. (2009) present a maintenance optimisation model based on lack of availability and cost criteria, considering epistemic uncertainty in the imperfect maintenance modelling; these researchers use a tolerance interval–based approach to address uncertainty in terms of the effectiveness parameter and imperfect maintenance model embedded in a multiple-objective genetic algorithm. These methods are complex, require a long time for their application and involve specified software and skills. Persona et al. (2010) and Sgarbossa et al. (2014) investigate the effect of operating and environmental condition on both types of ARP, adapting the typical Weibull reliability function with the systemability one.

From the literature, there is a lack of studies regarding the effect of the availability of failure data on the final maintenance costs. In this case, when no data about TTF are available, the reliability parameters must be estimated in an alternative way so that the ARP can be implemented. Therefore, the replacement interval times and maintenance costs are influenced by this estimation.

In a previous study, Sgarbossa et al. (2018) investigate the effects of unavailable failure data on total maintenance costs. They present tables about the percentage additional cost due to the shape parameter estimation, evaluating the effect of a wrong estimation. Moreover, a numerical case study is presented.

In contrast to the previous research, the present paper carries out a complete analysis of all the parameters as the scale and shape parameters of the Weibull distribution function and MTTF; they are estimated using graphics to support practitioners during their assessment. The impact of the error on these estimations is then calculated and analysed via new and more complete graphs (e.g. the additional cost of the ARP is defined with estimated parameters compared with the optimal one); in addition, ANOVA and multifactorial analysis are carried out. Furthermore, since the corrective maintenance policy is generally applied when no data are available, the savings obtained using the ARP-I policy based on estimated parameters are calculated. General discussions and guidelines have been included in terms of how to define the maintenance policy and which parameters should be used as inputs in the decision-making process.

3. ARP-I: Optimal Replacement Interval Time with No Data

Notations	
β	Shape parameter of the Weibull distribution
θ	Scale parameter of the Weibull distribution
$MTTF$	Mean time to failure (MTTF), calculated as the average of the time to failure (TTF) when data are available
$\lambda(t)$	Hazard function of the analysed component (in this paper, it is modelled with a Weibull distribution)
$f(t)$	Probability distribution function of the analysed component (in this paper, it is modelled with a Weibull distribution)
$R(t)$	Reliability function of the analysed component (in this paper, it is modelled with a Weibull distribution)
$\hat{\beta}$	Estimated shape parameter of the Weibull distribution, based on the experience of the maintenance technicians and engineers
$\hat{\theta}$	Estimated scale parameter of the Weibull distribution, based on the experience of the maintenance technicians and engineers
$\widehat{MTTF}$	Estimated MTTF, based on the experience of the maintenance technicians and engineers
$\hat{R}(t)$	Reliability function of the analysed component (using the estimated parameters ($\hat{\beta}$, $\hat{\theta}$))

t_p	Replacement time
t_p^*	Optimal replacement time
$\widehat{t_p^*}$	Optimal replacement time calculated when the reliability parameters are estimated ($\hat{\beta}$, $\hat{\theta}$, $\widehat{MTTF}$)
C_p	Average cost of preventive maintenance action at planned time t_p
C_f	Average cost of corrective maintenance intervention after a failure
$UEC(t_p)$	Unit expected cost (UEC) of the age replacement policy (ARP) with planned time t_p
$UEC(t_p^*)$	Minimal UEC of the ARP with planned time t_p^*
$\widehat{UEC}(t_p)$	UEC of the ARP with planned time t_p when the reliability $\hat{R}(t)$ is calculated using the estimated parameters ($\hat{\beta}$, $\hat{\theta}$)
$UEC(\widehat{t_p^*})$	UEC of the ARP with planned time $\widehat{t_p^*}$ when component has the actual reliability parameters (β, θ).
ΔUEC	Percentage additional cost in ARP-I using the estimated parameters
$\overline{\Delta UEC}$	Absolute additional cost in ARP-I using the estimated parameters

Often, the data necessary for estimating the reliability parameters (TTF) are unavailable or inaccurate (due to new machines or the absence of data collection); thus, it is necessary to use an alternative method for assessing the survival functions.

The MTTF is a common reliability parameter that can easily be estimated from experience or manuals and used to construct reliability parameters when TTF data are unavailable. However, this is insufficient for properly implementing the ARP-I, since it represents only the average value of the failure distribution function. As shown in the previous section, it is necessary to have a complete reliability distribution function, such as the Weibull function. In fact, the shape parameter $\boldsymbol{\beta}$ has a significant impact on the optimal ARP-I estimation (Faccio et al., 2014), while the parameter $\boldsymbol{\theta}$ is just a scale factor of the problem.

The following figures can be used by practitioners to estimate the reliability parameters. Figure 1 shows the hazard function $\boldsymbol{\lambda(t)}$, which expresses the failure rate of the component, varying the value of $\boldsymbol{\beta}$. Different curves denote the different degradation rates.

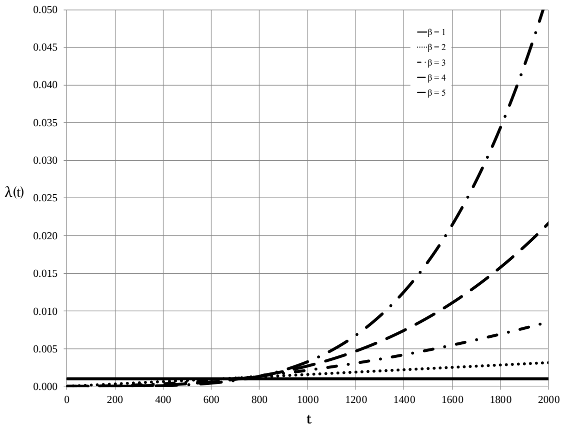

Figure 1: Estimation of $\boldsymbol{\lambda(t)}$ with varying $\boldsymbol{\beta}$.

The same can be done for the estimation of the TTF distribution ***f(t)***, which expresses the failure rate of the component without considering previous failures and the reliability function ***R(t)*** of the component, as Figures 2 and 3 illustrate.

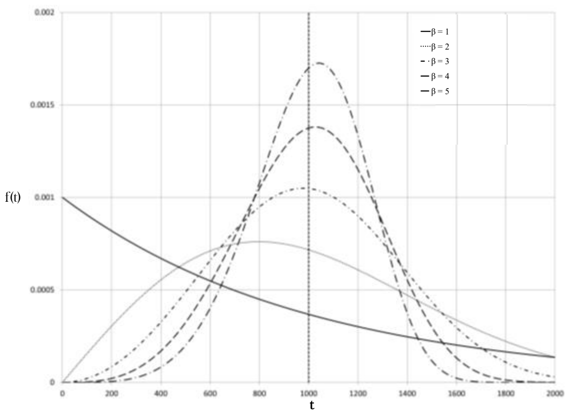

Figure 2: Estimation of $\boldsymbol{f(t)}$ with varying $\boldsymbol{\beta}$.

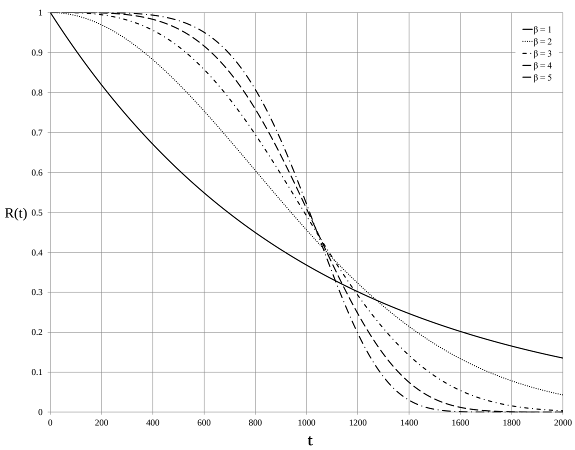

Figure 3: Estimation of $\boldsymbol{R(t)}$ with varying $\boldsymbol{\beta}$.

In this case, the MTTF is fixed to 1,000 units of time (e.g. days, hours, minutes, etc.). The maintenance engineers can scale these figures to their specific case, characterised by the estimated MTTF (indicated by $\widehat{\boldsymbol{MTTF}}$). They can then estimate the shape parameter, assuming the best curve for the component, based on their experiences. We call this $\widehat{\boldsymbol{\beta}}$ to differentiate it from the real value of $\boldsymbol{\beta}$.

Once $\widehat{\boldsymbol{\beta}}$ and $\widehat{\boldsymbol{MTTF}}$ are defined, the scale parameter $\boldsymbol{\theta}$ can be estimated from the graph, as shown in Figure 4. Analogously to the previous terms, it is called $\widehat{\boldsymbol{\theta}}$.

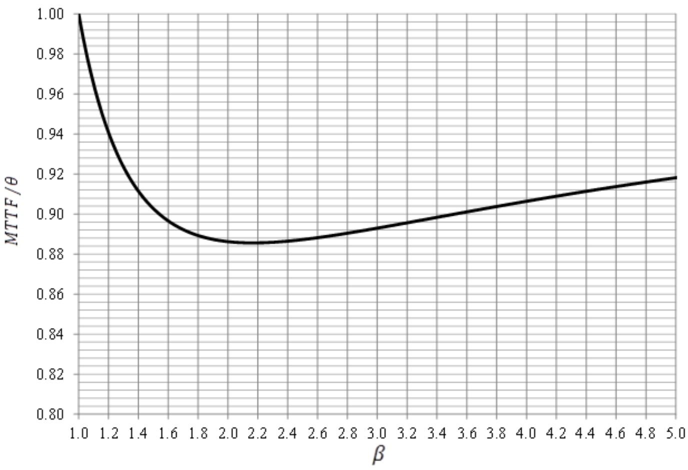

Figure 4: *MTTF/θ* as a function of the Weibull shape parameter.

Based on this set of reliability factors, $\widehat{\boldsymbol{\beta}}$, $\widehat{\boldsymbol{\theta}}$, $\widehat{\boldsymbol{MTTF}}$ and the cost parameters $\boldsymbol{C_f}$ and $\boldsymbol{C_p}$, assessed with traditional accounting methodologies, the optimal time to replacement $\widehat{\boldsymbol{t_p^*}}$ is estimated using the graphs available in Faccio et al. (2014).

4. Cost Estimation and Analysis

The main objective of this paper is evaluating the impact of the wrong estimation of Weibull parameters ($\widehat{\boldsymbol{\beta}}$, $\widehat{\boldsymbol{\theta}}$ and $\widehat{\boldsymbol{MTTF}}$) on the UEC. Thus, the equation below permits the calculation of total annual costs when the time to replacement $\widehat{\boldsymbol{t_p^*}}$ is calculated using $\widehat{\boldsymbol{\beta}}$, $\widehat{\boldsymbol{\theta}}$ and $\widehat{\boldsymbol{MTTF}}$, but the component has a real reliability function described by $\boldsymbol{\beta}$, $\boldsymbol{\theta}$ and $\boldsymbol{MTTF}$.

Using $\widehat{\boldsymbol{\beta}}$, $\widehat{\boldsymbol{\theta}}$ and $\widehat{\boldsymbol{MTTF}}$, based on the estimated input parameters, the UEC is expressed by

$$\widehat{\boldsymbol{UEC}}(\boldsymbol{t}) = \frac{\boldsymbol{C_p}\widehat{\boldsymbol{R}}(\boldsymbol{t}) + \boldsymbol{C_f}[\mathbf{1} - \widehat{\boldsymbol{R}}(\boldsymbol{t})]}{\int_0^t \widehat{\boldsymbol{R}}(\boldsymbol{s})\boldsymbol{ds}}, \qquad \textbf{(4)}$$

where

$$\widehat{\boldsymbol{R}}(\boldsymbol{t}) = \boldsymbol{exp}\left[-\left({}^{\boldsymbol{t}}/_{\widehat{\boldsymbol{\theta}}}\right)^{\widehat{\boldsymbol{\beta}}}\right]. \quad \textbf{(5)}$$

Once $\widehat{\boldsymbol{t}_{\boldsymbol{p}}^{*}}$ is defined as the optimal maintenance interval time that minimises $\widehat{\boldsymbol{UEC}}(\boldsymbol{t})$, it is applied to the systems that have real $\boldsymbol{\beta}$, $\boldsymbol{\theta}$ and $\boldsymbol{MTTF}$. Consequently, the real $\boldsymbol{UEC}(\boldsymbol{t})$ is

$$\boldsymbol{UEC}\left(\widehat{\boldsymbol{t}_{\boldsymbol{p}}^{*}}\right) = \frac{\boldsymbol{C_p R}\left(\widehat{\boldsymbol{t}_{\boldsymbol{p}}^{*}}\right) + \boldsymbol{C_f}\left[\boldsymbol{1} - \boldsymbol{R}\left(\widehat{\boldsymbol{t}_{\boldsymbol{p}}^{*}}\right)\right]}{\int_{\boldsymbol{0}}^{\widehat{\boldsymbol{t}_{\boldsymbol{p}}^{*}}} \boldsymbol{R}(\boldsymbol{s})\boldsymbol{ds}}, \quad \textbf{\textit{(6)}}$$

where

$$\boldsymbol{R}\left(\widehat{\boldsymbol{t}_{\boldsymbol{p}}^{*}}\right) = \boldsymbol{exp}\left[-\left({}^{\widehat{\boldsymbol{t}_{\boldsymbol{p}}^{*}}}/_{\boldsymbol{\theta}}\right)^{\boldsymbol{\beta}}\right]. \quad \textbf{(7)}$$

For evaluating the impact on optimal costs, the additional cost function $\Delta\boldsymbol{UEC}$ can be introduced as follows:

$$\Delta\boldsymbol{UEC} = \boldsymbol{1} - \frac{\boldsymbol{UEC}\left(\widehat{\boldsymbol{t}_{\boldsymbol{p}}^{*}}\right)}{\boldsymbol{UEC}\left(\boldsymbol{t}_{\boldsymbol{p}}^{*}\right)}. \quad \textbf{(8)}$$

This equation allows for an evaluation of the impact of the estimation error on the Weibull parameters. It represents the percentage of additional cost paid by the system in applying the ARP-I, using $\widehat{\boldsymbol{t}_{\boldsymbol{p}}^{*}}$, based on the estimated reliability parameters. It is calculated by comparing the UEC in $\widehat{\boldsymbol{t}_{\boldsymbol{p}}^{*}}$ under the real reliability function, with the optimal $\boldsymbol{UEC}\left(\boldsymbol{t}_{\boldsymbol{p}}^{*}\right)$ and the real $\boldsymbol{\beta}$ and $\boldsymbol{\theta}$ parameters.

The total cost function $\boldsymbol{UEC}\left(\boldsymbol{t}_{\boldsymbol{p}}^{*}\right)$ is based on the following equation:

$$\boldsymbol{UEC}\left(\boldsymbol{t}_{\boldsymbol{p}}^{*}\right) = \frac{\boldsymbol{C_p R}\left(\boldsymbol{t}_{\boldsymbol{p}}^{*}\right) + \boldsymbol{C_f}\left[\boldsymbol{1} - \boldsymbol{R}\left(\boldsymbol{t}_{\boldsymbol{p}}^{*}\right)\right]}{\int_{\boldsymbol{0}}^{\boldsymbol{t}_{\boldsymbol{p}}^{*}} \boldsymbol{R}(\boldsymbol{s})\boldsymbol{ds}}, \quad \textbf{(9)}$$

where

$$\boldsymbol{R}\left(\boldsymbol{t}_{\boldsymbol{p}}^{*}\right) = \boldsymbol{exp}\left[-\left({}^{\boldsymbol{t}_{\boldsymbol{p}}^{*}}/_{\boldsymbol{\theta}}\right)^{\boldsymbol{\beta}}\right]. \quad \textbf{(10)}$$

4.1 Multifactorial and ANOVA analyses

Based on Faccio et al. (2014), the UEC depends on the ratio $\boldsymbol{C_f}/\boldsymbol{C_p}$ and shape parameter $\boldsymbol{\beta}$, while the MTTF must have a scale effect on the total cost. In this section, multifactorial and ANOVA analyses (using MINITAB ®) are conducted to investigate the impact of these factors on the additional UEC ($\boldsymbol{\Delta UEC}$). In fact, there is no closed formula for calculating the optimal replacement time in case of ARP-I. A set of different scenarios has been created by varying the values of $\boldsymbol{\beta}$ and $\widehat{\boldsymbol{\beta}}$ from 1.5 to 5 (with steps of 0.5); $\widehat{\boldsymbol{MTTF}}$ and $\boldsymbol{MTTF}$ from 800 to 1200 (every 100); and the cost ratio $\boldsymbol{C_f/C_p}$ from to 5 to 10, 25, 50 and 100.

4.1.1 Impact of the estimated shape parameter $\boldsymbol{\beta}$

In the first analysis, the impact of the estimation of $\boldsymbol{\beta}$ is jointly investigated with the ratio $\boldsymbol{C_f/C_p}$ for the fixed value of MTTF = 1000 units of time. **MTTF and the parameter $\boldsymbol{\theta}$ are scale factors of the model, they affect just the absolute value of UEC and not the additional cost in percentage. The following example supports this statement: MTTF = 1,000 hours gave the same results of MTTF = 60,000 minutes.** Figure 5 presents the main effects plot of the $\boldsymbol{\beta}$, $\widehat{\boldsymbol{\beta}}$ and $\boldsymbol{C_f/C_p}$values on $\boldsymbol{\Delta UEC}$. It emerges that the impact of the estimated parameter $\widehat{\boldsymbol{\beta}}$ on the UEC is low ($\boldsymbol{\Delta UEC}$ < 0.1; i.e. up to 10% additional cost) when $\widehat{\boldsymbol{\beta}}$ is higher than 2.5 (high values) or lower than 3.5. Moreover, this impact increases as the $\boldsymbol{C_f/C_p}$ ratio increases (and consequently, failure costs are much higher than preventive ones). In contrast, Figure 6 presents the Pareto chart plot of the standardised effects of $\boldsymbol{\beta}$, $\widehat{\boldsymbol{\beta}}$ and $\boldsymbol{C_f/C_p}$ on $\boldsymbol{\Delta UEC}$. The most influential factors in terms of the response are the combination of $\boldsymbol{\beta}$ and $\widehat{\boldsymbol{\beta}}$, then the $\boldsymbol{C_f/C_p}$ ratio. This confirms the findings of the previous plot.

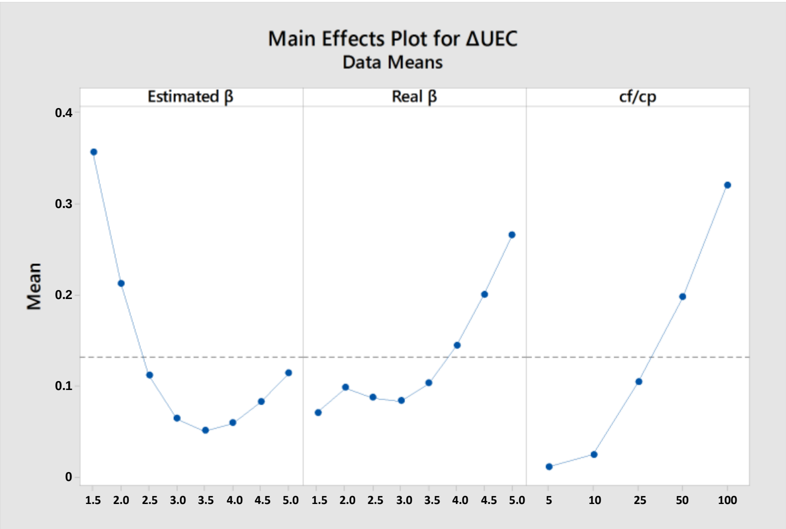

Figure 5: Main effects plot for $\boldsymbol{\Delta UEC}$, fixing MTTF = 1000 units of time.

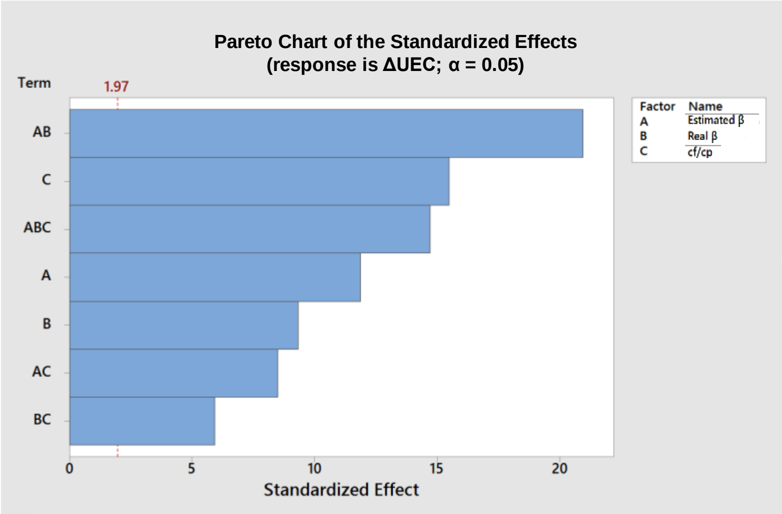

Figure 6: Pareto chart of the standardised effects on ΔUEC, fixing MTTF = 1000 units of time

4.1.2 Impact of the estimated MTTF parameter

The analysis concerning the interaction of the $\boldsymbol{\beta}$, $\widehat{\boldsymbol{MTTF}}$, $\boldsymbol{MTTF}$ and $\boldsymbol{C_f/C_p}$ parameters is presented below. Figure 7 shows the main effects plot for these factors on $\Delta\boldsymbol{UEC}$. It is evident that, as the shape parameter ($\boldsymbol{\beta}$) grows, the degradation increases, and the impact of the MTTF estimation becomes more relevant. If MTTF is overestimated (i.e. higher than 1,000), the impact is higher. The plot shows that the $\boldsymbol{C_f/C_p}$ ratio does not have a relevant impact on $\Delta\boldsymbol{UEC}$. This underlines that it is more relevant to accurately estimate the shape parameters than it is to do so for the scale parameters. Figure 8 presents the main effect plots, where the most influential factors are the combination of $\widehat{\boldsymbol{MTTF}}$ and MTTF, followed by the combination of $\boldsymbol{\beta}$, $\widehat{\boldsymbol{MTTF}}$ and MTTF, and finally, $\boldsymbol{\beta}$ and MTTF.

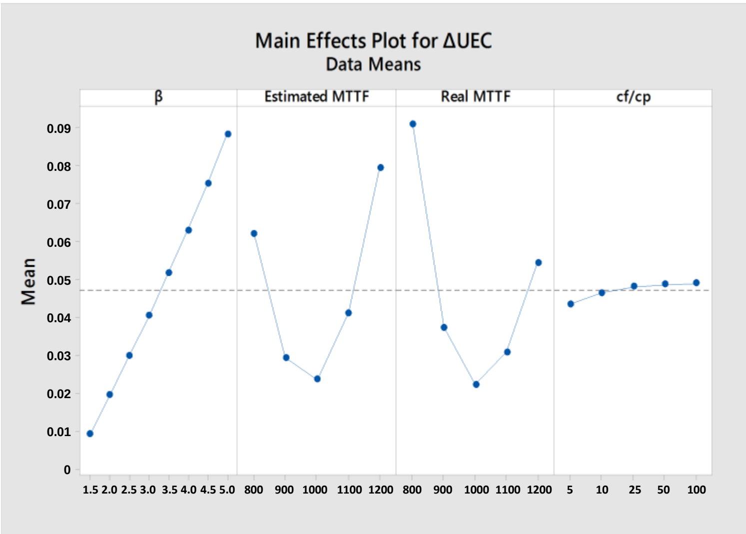

Figure 7: Main effects plot for ΔUEC, in case of estimated MTTF.

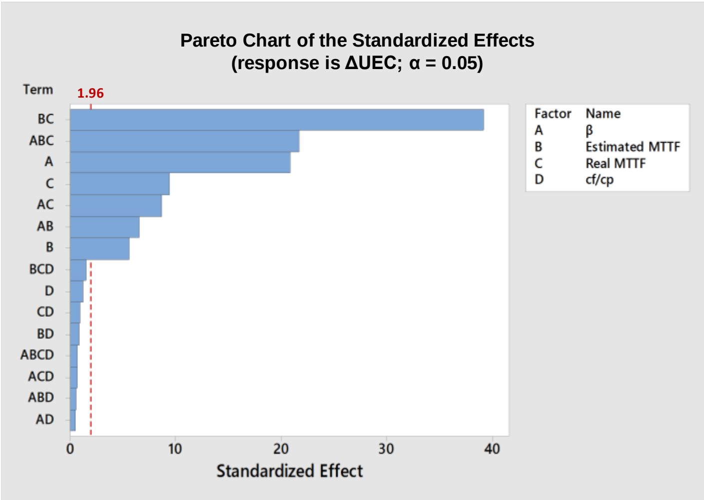

Figure 8: Pareto chart of the standardised effects on ΔUEC, in case of estimated MTTF.

4.2 What if $\hat{\beta}$ is wrong?

Following the outcomes of the previous analysis, the impact of $\boldsymbol{\beta}$ on the parameter estimation of ΔUEC is now presented. Figure 9 shows useful maps of ΔUEC (expressed in %) combining estimated $\hat{\boldsymbol{\beta}}$ with real $\boldsymbol{\beta}$ values. These graphs were created based on a fixed MTTF (1,000 units of time), fixed scale parameters and varying C_f/C_p ratios.

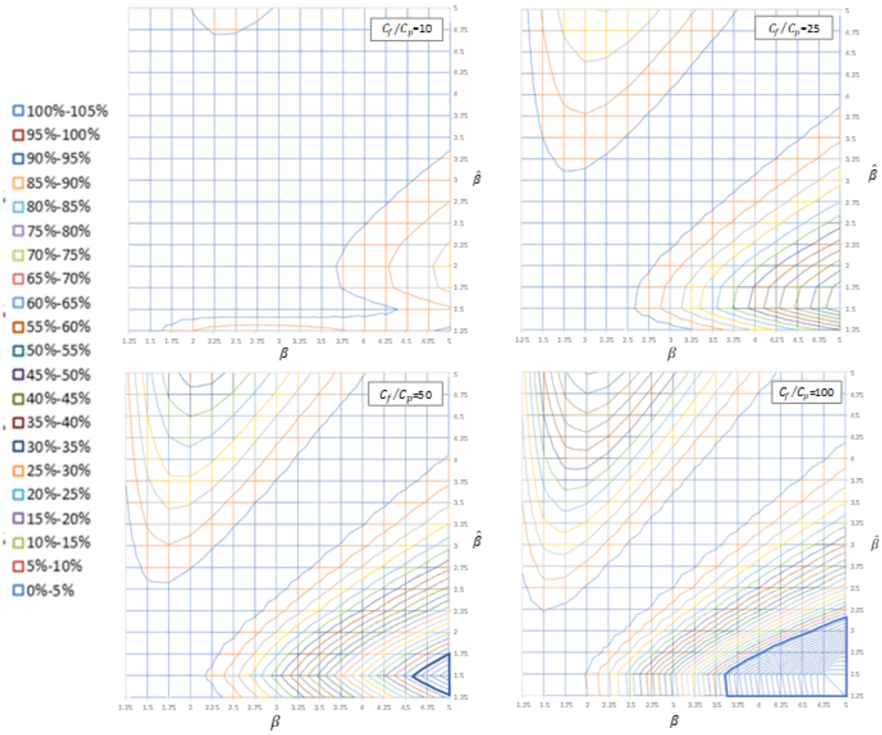

Figure 9. Estimated $\boldsymbol{\beta}$ impact (%) on ΔUEC; the areas delimited in the two graphs in the bottom include curves for values greater than 105% (every 5%).

Here, four cases are presented, varying the ratio between corrective and preventive costs as $\boldsymbol{C_f/C_p}$ values equal to 10, 25, 50 and 100. In the first case ($\boldsymbol{C_f/C_p}$ = 10), the two costs have highly similar values; if $\boldsymbol{\beta}$ is underestimated, as $\boldsymbol{\hat{\beta}}$ is equal to 2 and the real value is 4.5, the percentage added costs due to this error is between 10% and 15%. In contrast, if $\boldsymbol{\beta}$ is overestimated, as $\boldsymbol{\hat{\beta}}$ is equal to 4.5 and the real value is 2, the percentage added costs due to this error is between 0% and 5%. In the second case ($\boldsymbol{C_f/C_p}$ = 25), the ratio between the two maintenance costs is increasing, as the failures cost is becoming more relevant. If $\boldsymbol{\beta}$ is underestimated (as $\boldsymbol{\hat{\beta}}$ = 2 while $\boldsymbol{\beta}$ = 4.5), the impact on $\Delta\boldsymbol{UEC}$ is between 40% and 45%, while in the case of overestimation (as $\boldsymbol{\hat{\beta}}$ = 4.5 while $\boldsymbol{\beta}$ = 2), the impact is lower, between 15% and 20%. The same consideration can be adopted in the last case ($\boldsymbol{C_f/C_p}$ = 100), where distances between the corrective and maintenance costs are relevant. If the shape parameter is underestimated (as $\boldsymbol{\hat{\beta}}$ = 2 while $\boldsymbol{\beta}$ = 4.5), the impact on total costs is between 100% and 105%, while in the opposite case (as $\boldsymbol{\hat{\beta}}$ = 4.5 while $\boldsymbol{\beta}$ = 2), the impact is lower, between 50% and 55%.

In conclusion, it is evident that overestimating $\boldsymbol{\beta}$ has a lower impact than underestimating it does. In all the cases discussed, the impact is always half that of the opposite. Finally, as the ratio between the two maintenance costs increases ($\boldsymbol{C_f/C_p}$), the impact on $\Delta\boldsymbol{UEC}$ also increases. Thus, it is highly important to find the estimation of the scale parameter when the failure cost is higher than the preventive cost is.

4.3 What if $\widehat{\boldsymbol{MTTF}}$ is wrong?

The impact of estimation errors in MTTF on $\Delta\boldsymbol{UEC}$ can now be discussed. In this case, we consider $\boldsymbol{C_f/C_p}$ and the scale parameter values to be fixed. We vary the $\boldsymbol{\beta}$ parameter, as we observe in the multifactorial analysis that varying $\boldsymbol{C_f/C_p}$ has no relevant impact on $\Delta\boldsymbol{UEC}$ (see section 4.1.2, Figure. 7). Figure 10 presents useful graphs concerning $\Delta\boldsymbol{UEC}$ and the combined estimated $\widehat{\boldsymbol{MTTF}}$ and real MTTF values.

Here, four cases are presented, varying the $\boldsymbol{\beta}$ value as 2, 3, 4 and 5. In the first case ($\boldsymbol{\beta}$ = 2), if MTTF is underestimated, as $\widehat{\boldsymbol{MTTF}}$ is 850 units of time while the real value of MTTF is 1,150 h, the impact on $\Delta\boldsymbol{UEC}$ is between 0% and 5%. If MTTF is overestimated (e.g. $\widehat{\boldsymbol{MTTF}}$ **=** 1,150 units of time and MTTF = 850 units of time), the percentage impact of $\Delta\boldsymbol{UEC}$ is between 0% and 5%. In the second case ($\boldsymbol{\beta}$ = 3), if MTTF is bigger than the estimation (e.g. $\widehat{\boldsymbol{MTTF}}$ = 850 units of time and MTTF = 1,150 units of time), the impact on the total cost is between 5% and 10%, while in the opposite case (e.g. $\widehat{\boldsymbol{MTTF}}$ = 1,150 units of time and MTTF = 850 units of time), the impact is higher, at between 10% and 15%. The last case is when $\boldsymbol{\beta}$ is equal to 5; as before, when MTTF is underestimated (e.g. $\widehat{\boldsymbol{MTTF}}$ = 850 units of time and MTTF = 1,150 units of time), the impact of $\Delta \boldsymbol{UEC}$ is between 10% and 15%. If the MTTF is overestimated ($\widehat{\boldsymbol{MTTF}}$ = 1,150 units of time and MTTF = 850 units of time), the final impact is between 25% and 30%. In this case, three factors can be taken into consideration. Clearly, the smaller the difference between $\widehat{\boldsymbol{MTTF}}$ and MTTF, the better, as the estimation is representative of the reality. In addition, underestimating the mean TTF is generally better than overestimating it, as corrective maintenance costs are higher. Finally, by increasing the shape parameter, the impact on $\boldsymbol{UEC(t)}$ generally increases as degradation increases and failures occur more frequently.

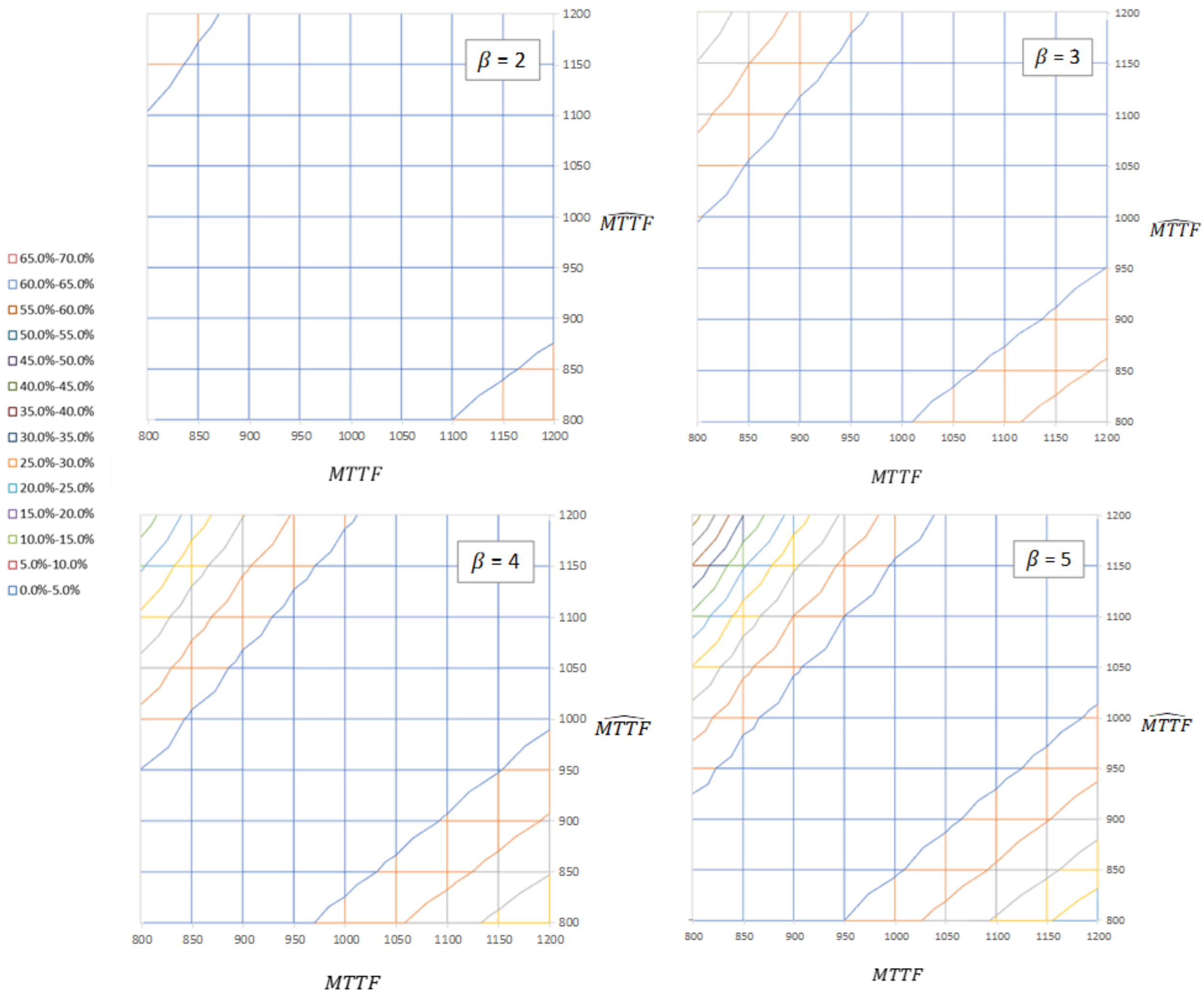

Figure 10. Estimated MTTF impact (%) on $\Delta \boldsymbol{UEC}$.

4.4 Savings analysis

This section analyses the savings obtained when applying ARP-I based on estimated reliability parameters. The UEC with the estimated parameters $\boldsymbol{UEC}(\widehat{\boldsymbol{t}_{\boldsymbol{p}}^{*}})$ is compared with the UEC when only a corrective maintenance policy is applied, calculated as $\boldsymbol{UEC}(\infty) = \frac{\boldsymbol{C_f}}{\boldsymbol{MTTF}}$.

In this analysis, if no data are available, the corrective maintenance is the most applied policy, as presented in Kelly et al. (1997) and Kaio et al. (1978). If it is known that there is some degradation phenomenon in the system, ARP-I could be implemented. However, in this case, it would be interesting to understand the extent of the obtainable savings. The analysis is divided into two parts. It first considers the effects when the MTTF parameter is fixed, and then, it focusses on the case when the shape parameter is fixed.

4.4.1 Analysis of the impact on savings of the estimated shape parameter $\boldsymbol{\beta}$

Figure 11 presents the savings, expressed in percentage of $\boldsymbol{UEC}(\infty)$ and with varied estimated values of $\boldsymbol{\beta}$. For each $\boldsymbol{\beta}$, the mean value of savings (blue circle), median (straight line), first and third quartiles (blue box) and possible minimum and maximum values of savings are represented. It is easy to understand that the maximum value of the bars corresponds to the case of applying ARP-I in the optimal interval time $\boldsymbol{t_p^*}$.

If we consider the first case, $\boldsymbol{C_f/C_p}$ = 10, we can observe that the mean value of the savings does not change significantly when varying the estimated $\boldsymbol{\beta}$ value (roughly 60%). The first quartile is over 50% for all β values estimated; this means that 75% of cases have a savings of more than 50%. Considering that the maximum value of savings is 75%, the convenience of implementing a ARP-I is high in most cases.

When $\boldsymbol{C_f/C_p}$ increases, we see that the mean value of saving increases ($\boldsymbol{C_f/C_p}$ = 25 is 70%; $\boldsymbol{C_f/C_p}$ = 50 is 80%; $\boldsymbol{C_f/C_p}$ = 100 is 85%), and the first quartile value increases (from 60% to 70%). In general, the higher the $\boldsymbol{C_f/C_p}$ value, the better, as the mean savings and first quartile values also increase. Moreover, it is evident from these graphs that the $\boldsymbol{\beta}$ estimation does not influence the savings.

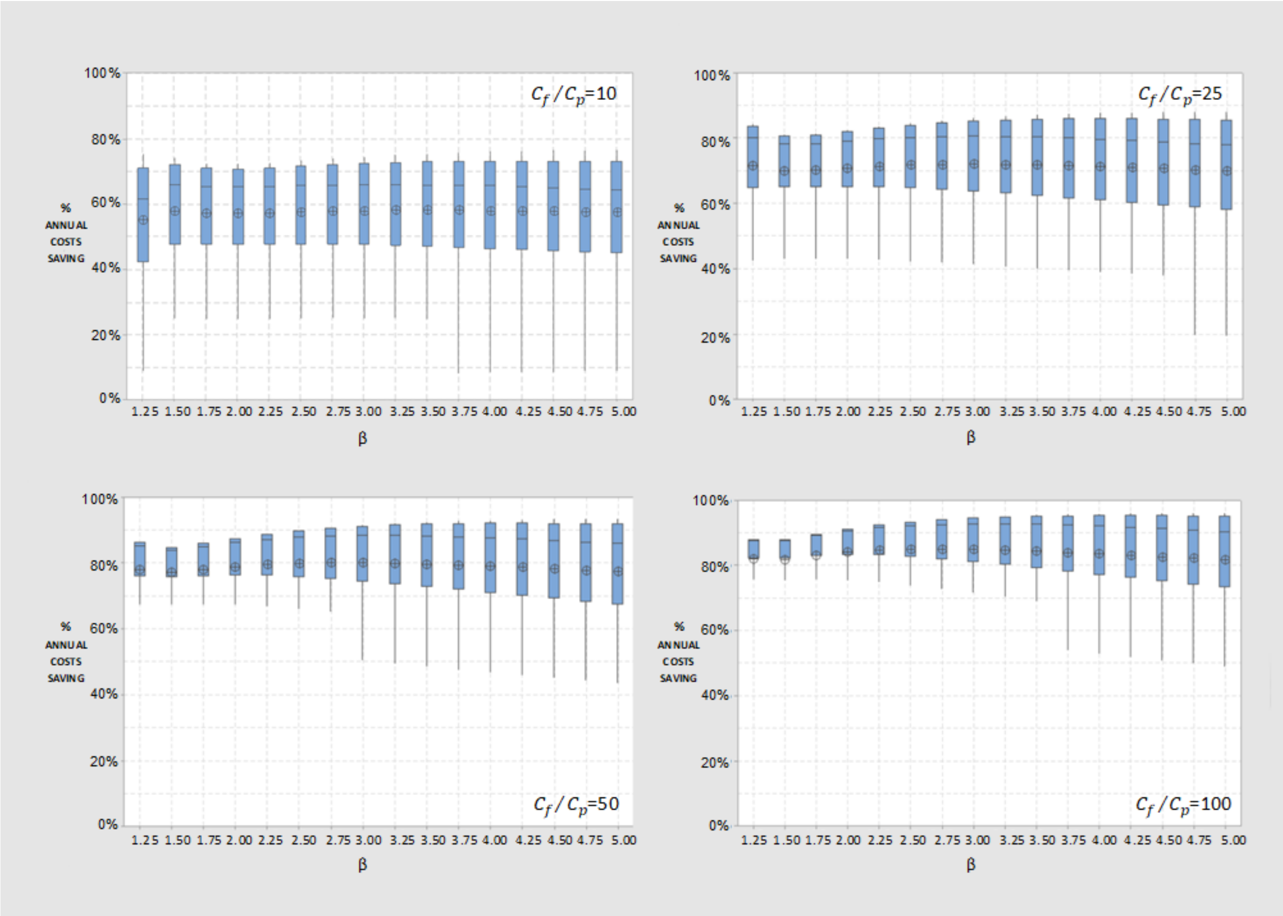

Figure 11: Percentage of saving of total annual costs, with a varying $\boldsymbol{\beta}$ estimation, for different cases of $\boldsymbol{C_f/C_p}$.

4.4.2 Analysis of the impact on savings of the estimated MTTF

This section presents the savings when varying the estimated values of the MTTF (Figure 12). For each MTTF value, the mean value of savings (blue circle), median (straight line), first and third quartiles (blue box) and possible minimum and maximum value of savings are reported.

If we consider the case $\boldsymbol{\beta}$ = 2, we observe that the mean value of savings does not change with varying MTTF values; it is around 65%. Increasing the $\boldsymbol{\beta}$ values sees the mean savings increase but not in a uniform way. In general, the first quartile value is extremely high, approaching the maximum value (i.e. 75% of cases have extremely high savings). Finally, the savings are higher when $\boldsymbol{C_f/C_p}$ increases, as the failure costs increase concomitantly.

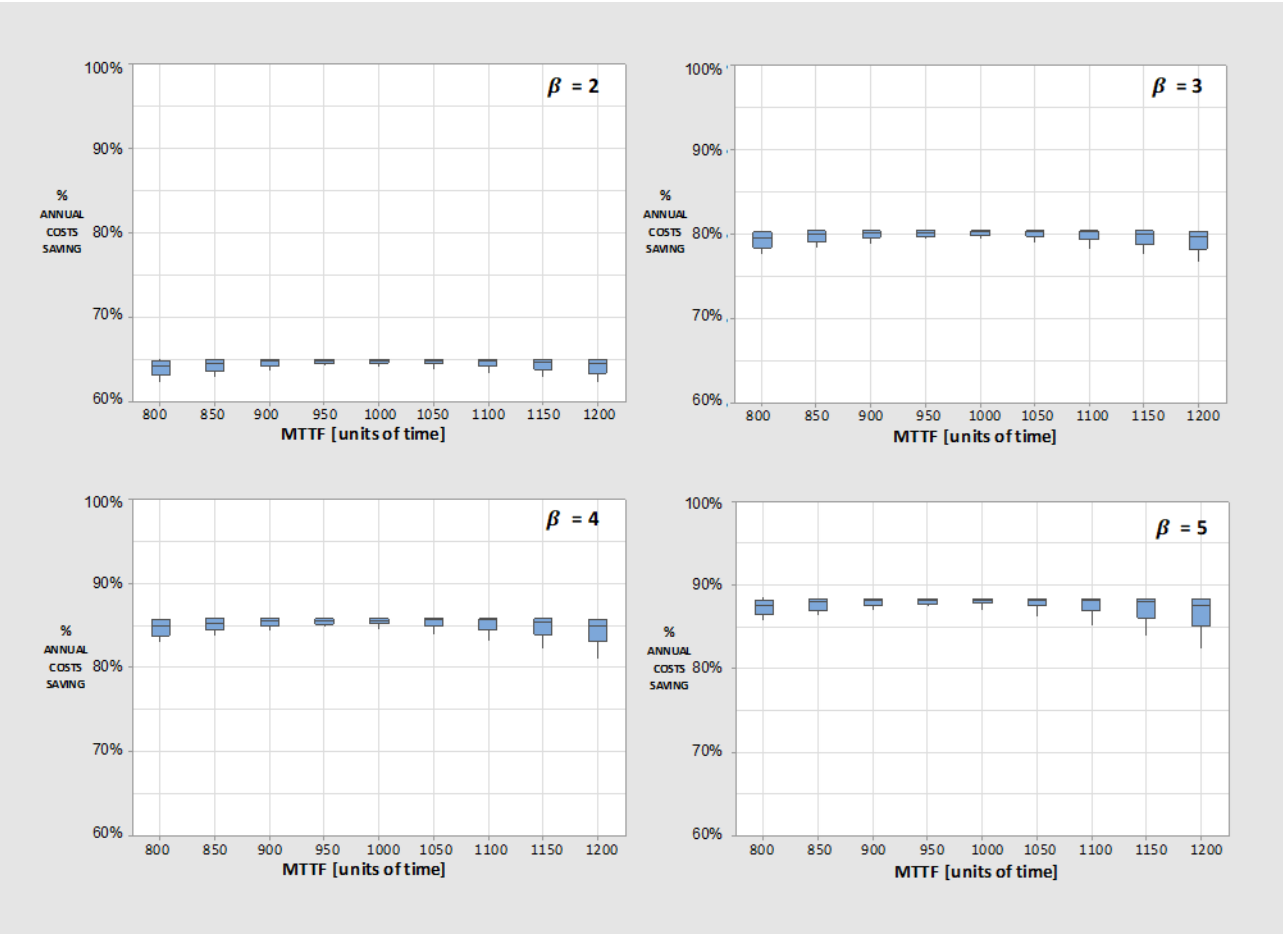

Figure 12: Percentage of savings of total annual costs with a varying MTTF estimation for different cases of $\boldsymbol{C_f/C_p}$.

5 Case Study

In the present case study, the critical components of a sheet metal cutting machine are analysed to define an ARP-I. Especially, the case study focusses on the shear blade and gearbox for a robot. The company maintenance policy provides the substitution of the blade when lack of sharpening results in the degradation of the cut quality, while the gearbox is replaced after failure. The estimation of the useful life parameters was performed based on the historical data obtained from the company information system, or if available, according to the data supplied by the manufacturer (see Table 1). Using this case, we compare the real yearly costs of the ARP-I with the ones the company would have incurred if the data were not available and the reliability parameters were estimated.

Let us assume $\boldsymbol{MTTF} = \widehat{\boldsymbol{MTTF}}$; indeed, as reported in section 4.4.2, the overestimation or underestimation of this term has a negligible impact on the overall maintenance costs. The following costs are applied to the shear blade: $\boldsymbol{C_f}$ = 5,000€ and $\boldsymbol{C_p}$ = 500€, and thus, ${}^{\boldsymbol{C_f}}/_{\boldsymbol{C_p}}$ = 10. For the gearbox, however, $\boldsymbol{C_f}$ = 4,200€, $\boldsymbol{C_p}$ = 300€ and ${}^{\boldsymbol{C_f}}/_{\boldsymbol{C_p}}$ = 14. We define $\overline{\boldsymbol{\Delta UEC}}$ as the additional cost as an absolute value, as follows:

$$\overline{\Delta \mathrm{UEC}} = \mathrm{UEC}(\hat{\mathrm{t}}_{\mathrm{p}}^{*}) - \mathrm{UEC}(\mathrm{t}_{\mathrm{p}}^{*}) \tag{11}$$

and

$$\overline{\mathrm{S}} = \mathrm{UEC}(\infty) - \mathrm{UEC}(\hat{\mathrm{t}}_{\mathrm{p}}^{*}), \tag{12}$$

$$\overline{S^{*}} = \mathrm{UEC}(\infty) - \mathrm{UEC}(\mathrm{t}_{\mathrm{p}}^{*}). \tag{13}$$

These are the savings obtained when applying the ARP-I based on the estimated reliability parameters and the savings obtained with real reliability parameters.

Table 1 shows the values in bold related to the real reliability parameters and maintenance costs. Then, for each component, assuming $\boldsymbol{MTTF = \widehat{MTTF}}$, two scenarios are simulated with different reliability parameters to elucidate the impact of this estimation on the final yearly cost.

Asset	$\boldsymbol{\beta}$	$\boldsymbol{\theta}$	$\boldsymbol{MTTF = \widehat{MTTF}}$ **[day]**	$\boldsymbol{t_p^*}$ **[day]**	$\boldsymbol{UEC(t_p^*)}$ **[€/year]**	$\boldsymbol{UEC(\infty)}$ **[€/year]**	$\overline{\boldsymbol{\Delta UEC}}$ **[€/year]**	$\boldsymbol{\Delta UEC}$	$\overline{\boldsymbol{S^*}}$ **[€/year]**	$\overline{\mathrm{S}}$ **[€/year]**
Shear Blade	**4**	**66.2**	60	**29**	**5,525**	20,000	-	-	**14,475 (72.4%)**	
	3	67.2		26	5,619		94	1.7 %	-	14,381 (71.9%)
	2	67.7		23	5,914		389	7 %	-	14,086 (70.4%)
Gearbox	**2**	**270.8**	240	**76**	**1,930**	4,200	-	-	**2,270 (54.1%)**	
	3	268.8		91	1,961		31	1.6%	-	2,239 (53.3%)
	4	264.8		106	2,033		103	5.3%	-	2,167 (51.6%)

Table 1: Cost and Savings Analysis for the Case Study

In the case of the shear blade, $\boldsymbol{\beta}$ has been underestimated; the effect is a reduction of the replacement time and increase in total costs. This effect becomes greater as the distance from the real parameters increases. The same trend is exhibited in the case of the gearbox, for which the value of $\boldsymbol{\beta}$ has been overestimated instead. However, the absolute values of additional costs are extremely low and almost negligible, and in any case, the savings obtained with the ARP-I compared to corrective maintenance are always extremely high. The advantage derived from the application of this strategy is clear, even with no data available.

6 Conclusion

Production and logistics systems are subject to failures and degradation due to their lifecycles. Using failure data (as TTF) makes it possible to prevent and avoid failures through the application of maintenance policies. The most common maintenance policies involve preventive and corrective maintenance, but they require accurate estimations of reliability parameters.

In this paper, the unavailability of TTF data was investigated, especially in terms of their impact on the cost of the maintenance policy. To evaluate the impact of parameter estimation on optimal costs (parameters derived from real data), the additional cost function $\Delta\boldsymbol{UEC}$ was introduced and analysed. Both the $\boldsymbol{\beta}$ and $\boldsymbol{MTTF}$ parameters were found to have effects on total annual costs. It was also evident that overestimating $\boldsymbol{\beta}$ has a lower impact by about half than underestimating it does. As the ratio between the two maintenance costs increases ($\boldsymbol{C_f/C_p}$), the impact on $\Delta\boldsymbol{UEC}$ also increases. Underestimating the MTTF is generally better than overestimating it, as corrective maintenance costs are higher. Finally, when increasing the shape parameter, the impact on $\Delta\boldsymbol{UEC}$ generally increases as the degradation increases and failures occur more frequently.

From the savings analysis, it is clear that applying an ARP-I based on the estimated parameters is always better than applying a corrective policy. Clearly, it is necessary for the component under analysis to be subject to wearing effects. The results show that the savings are always higher than 50% in 75% of cases, without any specific impact related to the $\boldsymbol{\beta}$ value. The savings increase as $\boldsymbol{C_f/C_p}$ increases, as do the failure costs. The MTTF estimation influence is lower and more stable with values of $\boldsymbol{\beta}$ between 2 and 3, so setting an initial value of the scale parameter in this range is recommended. **Since the unit expected cost depends only on shape parameter $\boldsymbol{\beta}$ and cost ratio $\boldsymbol{C_f/C_p}$ (Faccio et al., 2014), these savings are valid for all the**

cases. Moreover, most of the cases that can be presented in real industrial case have been simulated, varying parameter $\boldsymbol{\beta}$ from 1.5 to 5 (with steps of 0.5) and the cost ratio $\boldsymbol{C_f/C_p}$ with these values 5, 10, 25, 50 and 100. Finally, MTTF and the parameter $\boldsymbol{\theta}$ are considered only as scale factors of the model and they affect just on the absolute value of the additional cost and saving as demonstrated in the case study, but they do not affect the percentage.

In general, it is evident that a more detailed study of maintenance parameters using real data is needed when the $\boldsymbol{C_f/C_p}$ ratio is extremely low and the two costs are highly similar. In this case, the percentage of additional cost is more relevant, so the final savings could be extremely low. For values of $\boldsymbol{C_f/C_p}$ lower than 10, the savings could also be negative, so the corrective maintenance could outperform the preventive type. However, in general, as we consider degradation cases (β > 1), the ARP-I is recommended as being able to return high savings in most cases.

In conclusion, this study presented analyses to elucidate the economic benefits or added costs in the case of the ARP-I application with no data. In this field of study, future research could be carried out on other maintenance policies, especially in relation to predictive or condition-based maintenance.

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