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Markus Löschenbrand

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Thesis for the Degree of
Philosophiae Doctor
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Modeling Competition under Large-Scale Storage

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"A smart machine will first consider which is more worth its while: to perform the given task or, instead, to figure some way out of it." - Stanislaw Lem

To the people of Norway...

Preface

Acknowledgements

I want to dedicate this page to all the people that supported me in the journey that my PhD studies were. First and foremost, I want to thank my main supervisor, Magnus Korpås. He believed in me and supported me actively on my path, but most importantly throughout the times were I myself was doubting my chances for success.

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Last but not least I want to thank all my friends, family and beloved in Austria and beyond that supported me throughout these years.

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Markus Löschenbrand, Trondheim, May, 2019

Abstract

Electricity shows similar characteristics to traditional commodity goods which led to the market mechanisms to trade electricity being developed in similar manner to markets for other commodities. Kirchhoff's laws, however, require a constant equilibrium state, as supply surplus or deficit is not physically possible in electricity systems. Thus, flexibility in ramping and startups/shutdowns of generation units is a key characteristic that is topic of vast literature on power systems. Competition models, however, have traditionally been focused on representative points in time. Transitions between those time periods have been either approximated or neglected, in order to reduce model complexity and allow for practical applications. The same goes for decisions in-between those time stages. However, as shown by examples such as the Bellmann equations, future decisions can and often will have implications on current periods.

In changing systems with decreasing prices and marginal cost, cost factors associated with discontinuous decisions will grow in importance. In electricity systems, these discontinuous decisions are mostly occupied with intertemporal decisions. Therefore, traditional models from game/equilibrium theory might not be fit for these future applications.

In this dissertation and the presented publications, a novelty in literature is presented: the state decisions of storing inventory and dispatching units are considered in single-level competitive games. This allows for previously ignored applications, such as assessing the strategic impact of dispatch decisions on market prices and electricity storage. In systems with decreasing shares of peak-units and increasing uncertainty, such models could prove key to assessing functionality of market designs and existence of market power.

Various other solution methods for equilibrium models beyond the traditional approach of deriving the Karush-Kuhn-Tucker conditions are described and successfully applied within the work of this dissertation. These include Nikaido-Isoda convergence algorithms and Gröbner basis formulations. Approximation techniques are used, either through analytical approaches or dynamically via metaheuristics.

Due to non-convexity in the presented interaction models, traditional views on the characteristics of Nash equilibria are reconsidered and redefined. Accurate mapping of these potential outcomes

might prove crucial in practical applications, where the results for individual players might vary depending on the equilibrium solution. Thus, analysis on multiple Nash equilibria was provided, in order to display the characteristics of the problem accurately.

Further, various applications were introduced. A focus on reserve markets/ancillary services was chosen based on the assumptions of flexible units being the key players in such.

Different, modular methodologies were proposed. This allows for using parts of the model individually and potentially combining them with parts of the other presented models.

Due to most large-scale storage being provided by hydropower, a focus on realistic examples from this field was chosen. This also resulted in analyzing the modeling of uncertainty, due to the strong dependency of hydropower on natural forces such as precipitation. Formulation of uncertainty in equilibrium models was chosen to be mainly focused on robust/(weighted) interior point methods.

These main findings and contributions are meant to contribute to future research on the topic of non-convex multistage games under storage. Due to the complexity/$\mathcal{NP}$-hardness of the problem, the presented methods - even though well performing - can be considered only a starting point for future studies on the here presented novel problem setup.

Contents

List of Figures

Nomenclature

	Indexes		
i	...	generation unit	
j	...	player/generation company	
t	...	period	
Th	...	thermal generation	
Hy	...	hydropower generation	
S	...	Supply	
D	...	Demand	
	Sets		
X	...	decision space	
I	...	set of all units	
	Variables		
x, y	...	decision vectors	
p	...	price	$[\frac{€}{MW}]$ or $[\frac{€}{MWh}]$
q	...	quantity	$[MW]$ or $[MWh]$
d	...	demand	$[MW]$ or $[MWh]$
r	...	reservoir storage	$[MWh]$ or $[mm^3]$
b	...	binary decision vector	$\mathbb{Z}^2$
u	...	auxiliary variable	
s	...	1) step variable	$\mathbb{Z}$
	...	2) spillage	$[MWh]$ or $[mm^3]$
	Dual Variables		
σ	...	equality constraint	
λ	...	inequality constraint	
	Parameters		
l	...	hydrological inflow	$[MWh]$ or $[mm^3]$
ξ	...	stochastic parameter	
	Constants		
M	...	large constant	
	Functions		

Π	...	profit function	[€]
h	...	equality constraint	
g	...	inequality constraint	
c	...	cost functions	[€]
$\mathscr{L}$	...	Lagrangian	
m	...	market clearing function	
ρ	...	price curve approximation	
	Sets		
X	...	feasible decision space	
G	...	Gröbner basis	

Abbreviations

Ch.	...	Chapter
DoR	...	Degree of Regulation
ENTSO-E	...	European Network of Transmission System Operators for Electricity
EPEC	...	Equilibrium Problem with Equilibrium Constraints
GHz	...	Giga-Hertz
ISO	...	Independent System Operator
KKT	...	Karush-Kuhn-Tucker
MCP	...	Mixed Complimentarity Problem
MILP	...	Mixed Integer Linear Program
MPEC	...	Mathematical Program with Equilibrium Constraints
Mm^3	...	Million Cubic Meter
MW	...	MegaWatt
MWh	...	MegaWattHours
TSO	...	Transmission System Operator

Chapter 1

Introduction

Electricity market liberalization/deregulation and the corresponding steps such as publicity of information and vertical disintegration of the power system imposed a range of new questions on the associated authorities. Questions such as *'which market type should be chosen?'* - e.g. pool or nodal auction, options markets, long-term or short-term contracts - and *'which mechanisms should be implemented to correct grid failures and who should bear the respective financial cost of compensation?'* proved difficult to decision makers having little experience with electricity as a commercial instead of a non-commercial public good [118]. In addition, various changes in generation portfolios due to changing resource prices for traditional sources of generation and lower investment cost for renewable generation have led to a still ongoing shift in power systems which has to be captured by the design of the respective markets [53].

On the surface, it might be intuitive to assume that a central electricity system planner acting as a 'benevolent dictator' focusing on maximizing social welfare would, in such a dynamically changing system, allow fulfillment of the societal goals that are secure supply, generation cost minimization and low emission levels. Traditionally, most power systems have been based on single-firm models. Even modern examples of 'benevolent monopolists/oligopolists' exist, an example being the Chinese power grid [155].

However, the assumption of a social-welfare maximizing central planner might not be as straightforward, as it is indicated that markets that allow for strategic behavior are able to increase the social welfare in the long term [116]. The rationale behind market liberalization can be found in that private, profit-maximizing market participants have more incentive to acquire information and to act as a driver for innovation as they are able (and due to competition: often forced) to invest short-term gains resulting from strategic bidding. In a system governed by expense reimbursement at short-term marginal cost level, there would be no room for investment and research, which could potentially lead to long-term stagnation. For example, a politically motivated central planner could cut short-term cost by reducing long term investments and research into more efficient means of generation.

Another liberalization-supporting aspect is given by systems with historically rooted cultural disparity. Integrating systems such as the European Union under central planning would already prove problematic on country-level. For example, Germany's transmission system alone is separated into four different control areas operated by four different TSOs. Even though close cooperation through ENTSO-E exists, TSOs still aim to act in national interests and as such full cooperation therefore not be considered a realistic option for trans-European electricity grid operation [153]. On the contrary, separated national grids would, in addition to denying positive effects on grid stability, deny consumers considerable surplus gains caused by interconnection [123].

Based on this, it is plausible to assume a power market composed of strategic agents with individual, not necessarily aligned goals (e.g. profit maximization or emission minimization/short-term or long-term focus) and individual traits (e.g. preferences for a certain generation portfolio such as 'phase out nuclear generation'). Due to the complexity of the financial side of power systems, it is reasonable to assume that certain agents are able to solidify their position on the market via the exercise of market power, either through sheer size or strategic utilization of their position in the grid - geographically[1] as well as characteristically[2] [16].

1.1 Motivation

The assumption of strategic decisions by participants of an electrical power system leads to unique problems not shared with other commodity markets. The reason herefore is that agents' goals do not necessarily align with the goals of system operators. To provide an example: system stability is not a natural goal of profit-making producers.

An example is given by Figure 1.1 which demonstrates a market price clearing between a weakly elastic demand represented by the demand side price function $p^D(q^D)$ and an ascending stack of price bids submitted by producers represented by the supply side price function $p^S(q^S)$.

Through unforeseen events the generation q_1 could not suffice to fulfill the clearing demand q^* or the generation q_2 could exceed the market clearing demand. In other commodity markets the outcome would be market prices of p_1 or p_2 which would result in losses in social welfare.

In electrical power systems however, Kirchhoff's laws do not allow for another outcome than supply equaling demand, i.e. the equilibrium state $p^* = q^*$. Furthermore, this condition has to hold continuously over time, as only short disparities can cause problems in the form of system frequency distortions. Thus, mechanisms have to be in place to correct the imbalances of the quantities $q^* - q_1$ or $q^* - q_2$.

[1] An example would be a generation firm 'artificially' separating an area/node in order to act as a price-maker in this node.

[2] An example would be a generation firm holding a monopoly on a certain type of generation, being able to define the prices for this specific form.

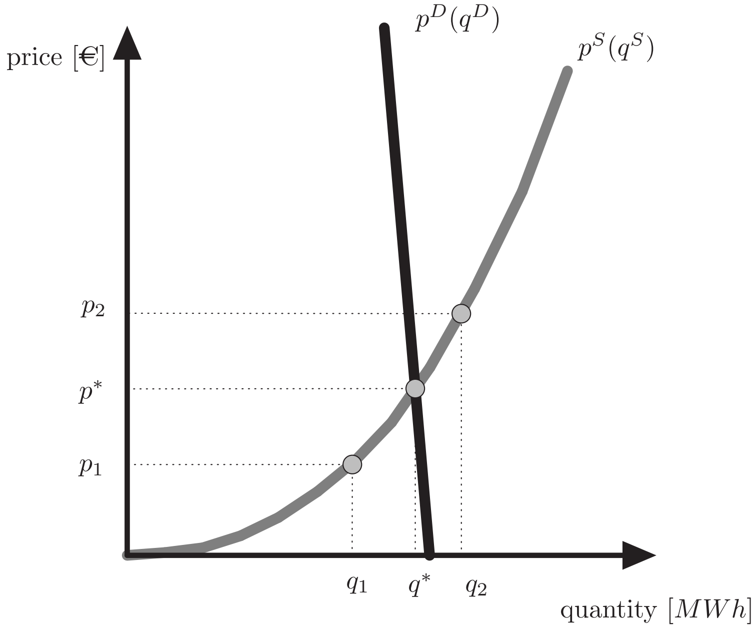


Figure 1.1: Demand and Supply in Electricity Markets

Ancillary services such as reserve markets and capacity requirements/markets are such mechanisms and are intended to support system stability. However, such additional services increase the complexity of the bidding process as more markets and remuneration systems have to be implemented.

Factors such as the increased share of uncertain renewable generation in generation portfolios do not point towards a reduction in system complexity due to a rise in importance of ancillary services. Paired with the rather high entry barriers for participation in electricity markets and strategic bidding being a inevitable component of such financial systems, market power and utilization of such deserves appropriate analysis. The importance of such analysis is given by that existence and utilization of market power effects, in the most severe case - firms behaving as monopolies or oligopolies - cannot necessarily be ruled out in reformed and deregulated markets [89, 119].

The here presented work intents to provide a new angle on the definition of market power and adequate tools to conduct future studies on the subject. The overall goal of this research is to strengthen the robustness of electrical systems to faults caused by misuse of financial tools[3] and to support the electrical system in fulfilling its public mission of providing a secure supply of emission-low electricity at low cost. Further, the developed tools would aim to analyze and prevent cases of market participants utilizing their size and impact in the power system in order to allocate their transactional risk onto the society [96].

The context of the presented work is aimed on hydropower optimization, stemming from it being the main source of generation in the Norwegian power system. Due to a scarcity of publications on price-making storage operators, or more general: dynamic problems of competition, analysis of price-effects of producers in such hydropower-dominated systems as the Norwegian grid have traditionally been neglected [142].

Thus, the proposed methods do not only offer practically applicable tools for systems with large shares of (hydropower) storage, but also fill a crucial gap in literature.

1.2 Organization of the Dissertation

The structure of the work is the following:

Ch. 2 presents an introduction to equilibrium models with a focus on electrical power markets. It presents various forms of competition such as Bertrand and Cournot Competition. In addition, various solution techniques to such models are listed: (linearized) Karush-Kuhn-Tucker conditions, Nikaido-Isoda function convergence algorithm and Gröbner basis reformulation with respective code snippets provided in the appendix of the dissertation.

Ch. 3 gives an introduction to the economic aspects of hydropower storage. It analyzes the formulation of uncertainty as well as the welfare impact of utilizing storage

[3] A famous example would be the impact of the trading decisions conducted by the Norwegian electricity trader Einar Aas, which caused a 110 Million € disparity on the Nordic NASDAQ market.

capacities. Further, the chapter aims to define the role of hydropower storage and the competitors of this form of electricity generation.

Ch. 4 shows the basic principles of hydropower dispatch and unit commitment. In addition, the chapter analyzes the impact of a price-maker assumption on unit commitment and the resulting multiplicity of equilibria.

Ch. 5 formulates the main research question and the core assumptions considered in the following publications yielded during the work on this dissertation. Further it introduces research requirements that were additionally imposed on the research work in order to increase range of applications of the research results.

Ch. 6 introduces the main literature that establishes the knowledge foundation of this work. It analyzes topics such as unit commitment, hydropower optimization, systems under multiple equilibria.

Ch. 7 introduces the publications that were a direct result from the work on this dissertation. The publications are introduced via an extended abstract and provided in their pre- or post-print format.

Ch. 8 introduces potential future applications of the presented concepts. The chapter discusses extensions for usage in electric power systems but also extends to applications in other energy or commodity markets and general operations research.

Ch. 9 concludes the findings and scientific contributions of the dissertation.

Chapter 2

Energy Market Equilibria

"Young man, in mathematics you don't understand things. You just get used to them." - John von Neumann

Similar to Reference [152], market optimization models can be classified by the number of participating firms, or more general - players, j (see Figure 2.1). The single firm model in Figure 2.1a shows a reaction of an observed player to an external market that can come in form of price or demand signals. The multi-firm case in Figure 2.1b recognizes the interactions between players which has the potential to influence those signals and thus other players' decisions. Utilizing this influence has the potential to prevent competing players in such a *'game'* from being able to conduct their optimal strategies that they would choose in the setup without competition that Figure 2.1a provides.

As Figure 2.1b provides a generalization of the system in Figure 2.1a, this work will focus mainly on multi firm systems. In accordance with Reference [65] the general market optimization problem of a single such player's decision in such a system can be formulated the following:

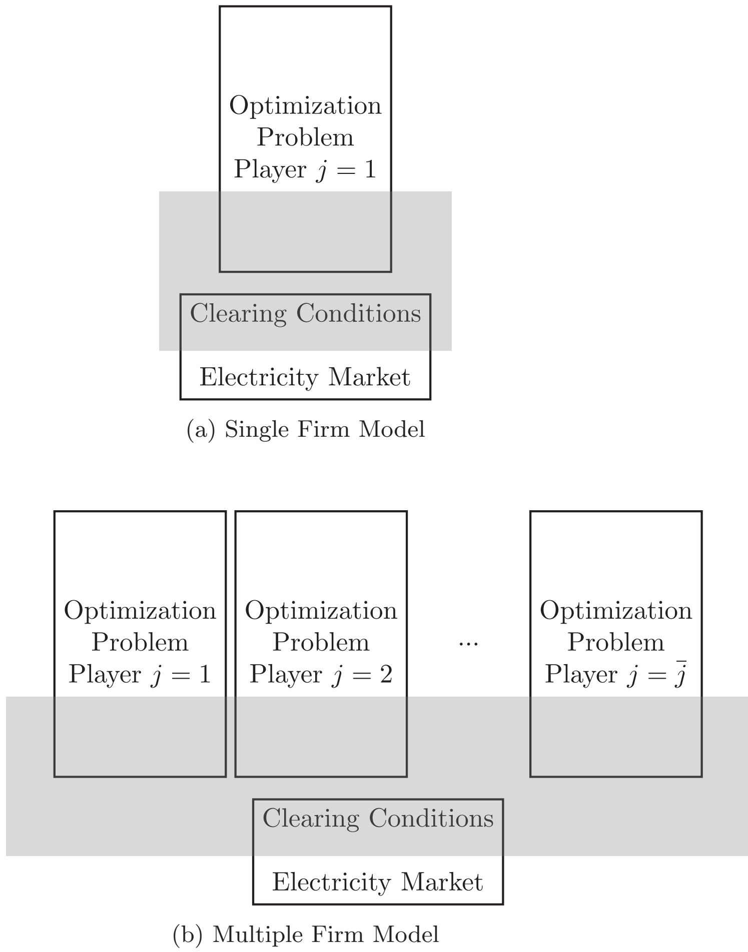
$$\max_{x_j} \Pi_j(x) \tag{2.1a}$$

$$\text{s.t.} \quad h_j(x) = 0 \tag{2.1b}$$

$$g_j(x) \leq 0 \tag{2.1c}$$

$$x_j \subseteq x \tag{2.1d}$$

In this formulation, Equation (2.1a) defines the profit function. The reason for the chosen formulation is the market focus of this work. In a more traditional approach, the objective function could also be denoted as a payoff-function. Equation (2.1b) defines the equality constraints and Equation (2.1c) the inequality constraints of a player. Equation (2.1d) expresses that a single player cannot necessarily make all decisions



(a) Single Firm Model

(b) Multiple Firm Model

Figure 2.1: Electricity Market Optimization Problems

in the market but also has its objective function and constraints influenced by other players' decisions x_{j2} where $j2 \neq j$.

In practical applications such as liberalized power markets, players are usually connected via their objective functions. For example, instead of a central planner assigning specific line flows to generation companies, a TSO decides on a transmission price on certain lines. Thus, this usually means that $h_j(x_j) \equiv h_j(x)$ and $g_j(x_j) \equiv g_j(x)$.

The section below will introduce several objective function formulations commonly used in wholesale energy markets. It has to be noted that collusion via cooperative games were intentionally kept out of this work. The reason herein is the focus of this work being on large-scale generation, specifically hydropower. This in turn means a focus on wholesale electricity markets that can generally considered to be competitive. As described above, this is the main incentive for countries to deregulate electricity markets [144]. This, however, does not mean that collaborating players is impossible in such environments. To provide an example - Reference [21] shows that tacit collusion conducted by oligopoly players can be captured by non-cooperative models.

2.1 Profit Function Formulation

Reference [44] categorizes different types of interactions traditionally utilized in power market applications. Generally, these model types can be defined by the type of decision that a firm has to make: price decision, i.e. $x_j = p_j$, or quantity decision, i.e. $x_j = q_j$. The interaction models, and thus the formulation of the profit functions, depend on the model users' assumption on the type of competition[1].

Under **Pure Competition** ("No Market Power), prices p are considered as exogenous parameters that cannot be influenced by the players. The profit function is that of a price-taker:

$$\Pi_j(q_j) = pq_j - c(q_j) \tag{2.2}$$

Under **Bertrand Competition** ("Game in Price"), Quantity bids are set depending on a price decision of a firm and fixed price bid assumptions of other firms denoted as p':

$$\Pi_j(p_j) = p_j q_j(p_j, p'_{j2} \forall j2 \neq j) - c\big(q_j(p_j, p'_{j2} \forall j2 \neq j)\big) \tag{2.3}$$

Cournot Competition ("Game in Quantity") presents the reverse - quantity decisions and assumptions on other firms' decisions q' are set by players:

$$\Pi_j(q_j) = p_j(q_j, q'_{j2} \forall j2 \neq j)q_j - c(q_j) \tag{2.4}$$

[1] It has to be noted that leader-follower games are considered out of scope for the here presented work.

General Conjectural Variations represent an interaction similar to Cournot Competitition. However, instead of fixed quantity bid assumptions the bids of other firms are considered to be dependent on(/function of) the bid of the deciding firm:

$$\Pi_j(q_j) = p_j(q_j, q'_{j2}(q_j)\forall j2 \neq j)q_j - c(q_j) \tag{2.5}$$

In **Supply Function Equilibrium** models, quantities of all players are considered supply functions of a players' price bid p_j that is conducted facing an (assumed) market price function p^{market}. This leads to the following profit function:

$$\Pi_j(p_j) = p^{\text{market}}(q_j(p_j), q'_{j2}(p_j)\forall j2 \neq j)q_j(p_j) - c\big(q_j(p_j)\big) \tag{2.6}$$

It can be observed that in general, the profit functions fulfill the definition of:

$$\begin{gathered} \textit{Profit Function} = \textit{Revenue Function} - \textit{Cost Function} \\ \text{or:} \\ \textit{Profit Function} = \textit{Price} \times \textit{Quantity} - \textit{Cost Function} \end{gathered}$$

Regardless and independent of the objective function, player problems might be reformulated as Lagrangian functions, also referred to in short as *Lagrangians*. This will be defined below.

2.2 The Lagrangian

It can be assumed that all elements of a players' decision vector x_j can be formulated as a set of variables x_i. In practical applications this means e.g. x_i being a bidding block of a price curve x_j submitted by a player to an energy market or x_i being the specific quantity bid of a generation company to a specific network node with x_j being the total generation of the firm. Further it shall be assumed that $\Pi_j(x) = \sum\limits_{i \in I_j} \Pi_i(x)$. Using this formulation allows expressing the Lagrangian function of Equation (2.1) similar to Reference [24] as:

$$\mathscr{L}_i(x_i, \sigma_i, \lambda_i) = -\Pi_i(x_i | x_{i2} \forall i2 \neq i) + \sigma_i h_i(x_i) + \lambda_i g_i(x_i) \quad \forall i \in I_j \tag{2.7}$$

Assuming the set of feasible decisions, i.e. the set of all decisions that do not breach any constraints, is defined as X_i in turn allows to establish the *Lagrangian Dual Problem* for a single decision x_i:

$$\begin{aligned} &\max_{\sigma_i, \lambda_i} \Big(\inf_{x_i \in X_i} \mathscr{L}_i(x_i, \sigma_i, \lambda_i) \Big) \\ \text{s.t.} \quad & \sigma_i \in \mathbb{R}^+ \\ & \lambda_i \in \mathbb{R} \end{aligned} \tag{2.8}$$

The so-called dual variables σ and λ are often also referred to as shadow-prices, or in other words: the value of "breaching those constraints". I.e. for generation capacity constraints it would be the answer to "what would be the monetary value of having an additional MW available in this particular time step?".

Assumed the optimal solution to the *primal problem* in Equation (2.1) is denoted by * and the optimal solution to this Lagrangian Dual Problem is denoted by ** allows to formulate *weak duality*:

$$\Pi_j(x^{**}) \geq \Pi_j(x^*) \tag{2.9}$$

This shows that the optimal objective value yielded by solving the Lagrangian Dual Problem provides an upper bound to the original profit function. The difference between those objectives is in literature referred to as *Duality Gap*. Having no duality gap, i.e. the solution of the primal problem is equal to the solution of the dual problem, is in literature referred to as *strong duality*:

$$\Pi_j(x^{**}) = \Pi_j(x^*) \tag{2.10}$$

For convex problems, this strong duality can in general (but as Reference [24] annotes: not always) be considered to hold.

Solution techniques that use this concept of Lagrangians to solve for equilibria will be presented below. First, however, a definition of Nash equilibria and their importance in power markets will be provided.

2.3 Generalized Nash Equilibrium

The Oxford dictionary defines an equilibrium as:

> *"A state in which opposing forces or influences are balanced."*

In interaction models, or 'games', these forces are the decisions conducted by players. These players might compete for limited, shared resources (e.g. generation quantities sold to consumers, transmission capacities) or work towards shared goals (e.g. firms in an oligopoly deciding to raise the prices by withdrawing quantity).

In competitive games, the concept of *Nash equilibra* originally proposed in Reference [117] describes a state of a system/market where no participants have any incentives to change their decisions. In (power) market terminology, this e.g. means that supply/generation and demand/used quantity is matched and thus the market can be considered 'cleared'.

Due to its notational and thematic similarities, the here provided mathematical definition is based on Reference [39]. A solution x^* provides a Nash equilibrium if for every participating player j the following condition holds for the feasible space X:

$$\Pi_j(x^*) = \max_{x_j \in X_j} \Pi_j(x_j | x^*_{j2} \forall j2 \neq j) \tag{2.11}$$

Such a solution might not be unique, but could also be non-existent or have a multiple and potentially infinite number of possible equilibrium-solutions [106]. As shown in e.g. Reference [128], multiple Nash equilibria can differ vastly in their characteristics such as profits and decisions. Therefore it can be assumed, that in general yielding a single numerical solution for a Nash equilibrium might paint an inaccurate picture of the outcome of a system if no information on the set of potential equilibria is available.

In practical applications such as power markets clearing conditions are usually applied to systems to support yielding equilibrium states [65]. Such conditions will be discussed below.

2.4 Market Clearing Conditions

Clearing conditions are constraints shared amongst the participants in a system. In electricity markets, a *cleared market* is often considered the equilibrium point where usage (demand) equals generation (supply) [152]:

$$d = \sum_j q_j \tag{2.12}$$

As discussed in Chapter 1, this is supported by the physical characteristics of electrical systems, as Kirchhoff's laws state that electrical systems are continuously in equilibrium states. Reference [132] applies a similar concept to other kinds of energy/-commodity markets. However, different systems might require adjustments to apply the concept. As an example, reference [66] uses time delays to account for the inventory in gas pipelines. Another potential clearing condition is the assumption of equilibrium prices, i.e. the price assumption of all bidders equaling to a market clearing price p^{MC}:

$$p^{\mathrm{MC}} = p_j \quad \forall j \tag{2.13}$$

Such an application is e.g. presented in Reference [33] where prices in $ provide a common basis to find equilibria for transmission systems under a CO2 emission permit market. As shown in Reference [65], these market clearing conditions can be added directly to systems, or in the form of (social-welfare-maximizing) system operators, such as TSOs or market operators [78].

In general, market clearing conditions are thus assumed to be equality constraints in the form of:

$$m(x) = 0 \tag{2.14}$$

Considering them as equalities has the advantage of "forcing" a market to reach an equilibrium state if possible.

Often, converging towards market clearing conditions can be enforced by assuming *complete information* on other players' decisions x'_{j2}:

$$x_{j2} \approx x'_{j2} \quad \forall j, j2 \neq j \tag{2.15}$$

In liberalized energy markets public data centers[2] established by the market operators support the validity of such an approach. Nonetheless, games of incomplete information - often referred to as 'Bayesian Games' after Reference [75] - are subject to ongoing research, see e.g. References [99, 111], but will be omitted here for the sake of simplicity.

2.5 Solving for Equilibria

Having established the problem formulation, chosen an appropriate profit function and established the market clearing conditions then requires application of fitting solution techniques to yield the Nash equilibrium/equilibria. Some techniques are introduced below. Appendix A provides compilable code for the respective techniques, on the basis of a problem from literature.

The here presented techniques do not provide a full overview of the spectrum of available techniques and intentionally does not list other solution methods such as '(Quasi)-Variational Inequalities' [65]. Instead it will introduce the techniques utilized in the later presented publications that represent the core work of this dissertation.

As frequent use in practical applications rely on the Karush-Kuhn-Tucker optimality conditions [152], the discussion will begin with their introduction.

2.5.1 Karush-Kuhn-Tucker Conditions

Assumed Π_i, h_i and g_i are differentiable allows the KKT optimality conditions to be formulated in short form as:

$$\frac{\partial \mathscr{L}_i(x_i,\sigma_i,\lambda_i)}{\partial x_i} = 0 \qquad \forall i \tag{2.16a}$$

$$h_i(x_i) = 0 \qquad \forall i \tag{2.16b}$$

$$0 \leq \lambda_i \perp g_i(x_i) \leq 0 \quad \forall i \tag{2.16c}$$

$$\sigma_i \in \mathbb{R}, \lambda_i \in \mathbb{R}^+ \qquad \forall i$$

$$m(x) = 0 \tag{2.16d}$$

Condition 2.16a states that in an optimal point the gradient of the Lagrangian has to be 0. Condition 2.16b shows the equality conditions that have to hold. Condition 2.16c is referred to as complementary slackness and ensures 'activation' of constraints whose boundaries are reached. Reference [17] offers additional optimality conditions for different problem settings (characteristics of the functions Π_i, h_i and g_i). For most practical applications however, convex primal problems are of importance. This means that for convex functions Π_i, g_i and affine h_i there exists no duality gap for primal and dual variables that fulfill these conditions [24].

Finding optimal solutions x^*, σ^* and λ^* that fulfill the KKT conditions is a Mixed Complimentarity Problem (MCP). Assuming appropriate clearing conditions (2.16d)

[2] Examples include the German 'Smard' Platform and the Scandinavian 'Nordpool'.

are included will result in an optimal solution to such an MCP being a Nash equilibrium. Finding such a stationary point however requires dealing with the non-linearity introduced by the slackness condition 2.16c. There exists commercial software applied to derive solutions to such problems, most notably the PATH solver [50]. However, and as shown below, it is also possible to use linear representations of the KKT conditions in order to yield equilibrium solutions.

2.5.2 Linearized Karush-Kuhn-Tucker Conditions

It is possible to apply reformulation on the KKT conditions (2.16) to transform the complementarity conditions into linear representations. This results in a Mixed Integer Linear Problem (MILP) that can be solved with a wider range of commercial software and methods than the previously presented problem. Some transformation techniques will be presented below.

The Fortuny-Amat Notation

Without the loss of generality, the slackness condition (2.16c) can be reformulated as:

$$\begin{aligned} g_i(x_i) \leq 0 & \quad \forall i \\ \lambda_i g_i(x_i) = 0 & \quad \forall i \\ \lambda_i \geq 0 & \quad \forall i \end{aligned} \tag{2.17}$$

As shown in Reference [64] this complementarity can be further reformulated as:

$$\begin{aligned} 0 \leq g_i(x_i) \leq M u_i & \quad \forall i \\ 0 \leq \lambda_i \leq M(1 - u_i) & \quad \forall i \\ u_i \in [0, 1] & \quad \forall i \end{aligned} \tag{2.18}$$

This reformulation is also known as the *'big M' formulation.* This highlights the importance of choosing an adequately high number for the constant M in order that an active constraint is not constrained below its maximum.

Standard Nonlinear Transformation

Reference [57] formulates an alternative linear reformulation for the slackness condition (2.16c), which can be defined as:

$$\begin{aligned} g_i(x_i) u_i \leq 0 & \quad \forall i \\ \lambda_i g_i(x_i) \leq 0 & \quad \forall i \\ \lambda_i \in \mathbb{R}^+, u_i \in \mathbb{R}^+ & \quad \forall i \end{aligned} \tag{2.19}$$

Compared to the previously introduced Fortuny-Amat notation, this formulation does not introduce integer variables. However, it increases the degree of the inequality constraint. At minimum it thus transforms a linear constraint into a quadratic constraint.

Reformulation as a Generalized Disjunctive Program

Reference [76] proposes a reformulation of the complementarity constraints into a *Generalized Disjunctive Program* by using logic operators. This would read:

$$
\begin{aligned}
&\begin{bmatrix} u_i \\ 0 = \lambda_i \\ 0 \leq -g_i(x_i) \end{bmatrix} \vee \begin{bmatrix} \neg u_i \\ 0 \leq \lambda_i \\ 0 = -g_i(x_i) \end{bmatrix} \quad \forall i \\
&\qquad u_i \in [\text{True}, \text{False}] \qquad\qquad \forall i
\end{aligned}
\tag{2.20}
$$

The advantage of using this reformulation of condition (2.16c) is that there exist a multitude of methods such as e.g. convex-hull reformulation, branch-and-bound and branch-and-cut that can be applied to solve such problems [37]. Furthermore, additional solution techniques for MPECS are e.g. presented in Reference [105] but shall not be discussed further here. Instead, other concepts to yield equilibria are introduced.

2.5.3 Nikaido-Isoda Convergence Algorithm

Originally presented in Reference [120], the *Nikaido-Isoda function* is an auxiliary function that sums the changes in profits for all players considering a single player j is the only player allowed to change strategies x_j to y_j. Mathematically this can be formulated as:

$$\Psi(x, y) = \sum_j \big(\Pi_j(y_j | x_{j2} \forall j2 \neq j) - \Pi_j(x)\big) \tag{2.21}$$

Assuming there exists a decision x^* that fulfills the definition of a Nash equilibrium from Equation (2.11) for all players j. This allows to formulate the value of a the Nikaido-Isoda function at such an equilibrium point:

$$\arg\max_{y \in X} \Psi(x^*, y) = \arg\max_{y \in X} \sum_j \big(\Pi_j(y_j | x^*_{j2} \forall j2 \neq j) - \Pi_j(x^*)\big) = 0 \tag{2.22}$$

Reference [93] describes that for a weakly convex-concave[3] Nikaido-Isoda function, a stepwise algorithm can be applied to converge towards such a Nash equilibrium. By choosing step sizes as a parameter $0 < u_s \leq 1$ for each step s, a convergence function for decisions x^s can be established:

$$x_i^{s+1} = (1 - u_s)x^s + u_s y_i^* \quad \forall i \tag{2.23a}$$

$$\text{where } \Psi(x^s, y^*) = \arg\max_{y \in X} \Psi(x^s, y) \tag{2.23b}$$

Assuming the algorithm has converged to an equilibrium point, thus fulfilling condition (2.22), results in no update in the decision, i.e. $x^{s+1} = x^s$. Reference [39] extends the concept of this algorithm to problems specific to electrical systems. By listing

[3] Convex functions are weakly convex functions that are continuous.

several practical examples it shows how the sub problem in Equation (2.23b) can be reformulated as a decision problem:

$$\begin{aligned} &\max_{y} \Psi(x^s, y) \\ \text{s.t.} \quad & h(y) = 0 \\ & g(y) \leq 0 \\ & m(y) = 0 \end{aligned} \tag{2.24}$$

Compared to the KKT conditions in Equation set (2.16), this problem has no complementarities and thus does not require similar linearization in order to be solved. In addition, the number of variables is smaller due to no dual problem having to be solved, which can be advantageous for larger scale problems. In contrast to this stands the disadvantage that for complex problems the amount of steps to converge might be high.

2.5.4 Gröbner Basis Reformulation

A Gröbner basis is a reformulation of a system of polynomial equations that generalizes *Gaussian elimination*, the *Euclidian algorithm* and the *Simplex Algorithm* [145]. This reformulation has several advantages:

- it allows to formulate replacement rules (i.e. shows a decision as a function of other decisions).
- it is more compact (and thus easier to solve for a numerical equilibrium).
- it is no approximation of the original problem (thus it includes all possible equilibrium outcomes).

The original method to derive Gröbner bases was presented in Reference [26]. Since the original publication and due to the computational complexity of the required operations, resource-efficient computation of such bases has become an active field of research itself [13, 56]. Therefore, methods to obtain these bases will be omitted in the here presented work. Instead, software implementations such as presented in Reference [109] will be applied to obtain a solution for the Gröbner basis $G(\cdot)$ where $\cdot$ denotes a polynomial system. Nonetheless, a numerical example should be provided to introduce the reader to the practical applications of Gröbner bases:

Numerical Example

Assumed be an equation system describing a Cournot market under elastic demand.
The price is denoted as a linear function p consisting of constants p^a and p^b.

Participating suppliers are bidding under the objective of profit maximization whereas no variable bounds such as capacity limits are imposed on them.
The system can thus be formulated the following:

$$\begin{aligned} p &= p^a - p^b \sum_j q_j \\ \Pi_j &= p * q_j - c_j(q_j) \quad \forall j \end{aligned} \tag{2.25}$$

Equivalently, this system can be reformulated as polynomial equations that allows derivation of the Gröbner basis:

$$G\begin{pmatrix} p - p^a - p^b \sum_j q_j = 0 & \\ p - \frac{\partial c_j(q_j)}{\partial q_j} & \forall j \end{pmatrix} \tag{2.26}$$

Assuming a game under uncertainty, where the intercept is $p^a = 100$ and the slope undefined $p^b =?$
The cost functions of the players are defined as $c_i(q_i) = 20 * q_i + 2 * q_i^2 \forall i = 1, 2$ and $c_i(q_i) = 10 * q_i + 1 * q_i^2 \forall i = 3$.
Equation (2.26) can be derived by e.g. applying Buchbergers algorithm and using graded reverse lexiographic order on the monomial ring that represents the variables [26]. This results in:

$$\text{Equation(2.26)} = \begin{Bmatrix} 2p^b q_3 - 5p^b + 2q_3 - 90, \\ p - 2q_3 - 10, \\ 2q_1 - q_3 + 5, \\ 2q_2 - q_3 + 5 \end{Bmatrix} \tag{2.27}$$

This formulation allows to define a distributionally robust replacement rule for the equilibrium quantity provided by the suppliers based on the uncertain slope of the price function:

$$\begin{aligned} q_1 = q_2 &= \tfrac{5p^b - 90}{8p^b + 4} - 2.5 \\ q_3 &= \tfrac{5p^b - 90}{4p^b + 2} \end{aligned} \tag{2.28}$$

This allows obtaining numerical equilibrium solutions,

$$\begin{gathered} \text{e.g. for } p^b = 4 \Rightarrow p = 32, q_1 = q_2 = 6, q3 = 22 \\ \text{or } p^b = 0 \Rightarrow p = 100, q_1 = q_2 = 20, q3 = 45. \end{gathered}$$

Using such polynomial formulations of decision problems to find multiple equilibria has been analyzed in literature previous to this work:

- Reference [43] applies Buchberger's algorithm to yield multiple Nash equilibria in games under disjunctive decisions.
- Reference [94] extends this concept to other areas such as a Bayesian Nash equilibria.
- Reference [162] uses a mixed-strategy formulation of a discrete decision space and applies it on bids in an electrical power market.
- Reference [74] applies an evaluation algorithm on a players' KKT conditions in order to derive the range of optimal points.

Latter method is extended on and described in the paper [EJOR] presented in chapter 7. For the sake of completeness, however, an initial discussion of the concept will be given here.

Applying Equation set (2.17) on the KKT conditions in Equation set (2.16) allows for the following reformulation:

$$G\begin{pmatrix} \frac{\partial \mathscr{L}_i(x_i,\sigma_i,\lambda_i)}{\partial x_i} = 0 & \forall i, \\ h_i(x_i) = 0 & \forall i, \\ \lambda_i g_i(x_i) = 0 & \forall i, \\ m(x) = 0 & \end{pmatrix} \tag{2.29a}$$

$$\begin{aligned} g_i(x_i) \leq 0 \quad & \forall i \\ \lambda_i \geq 0 \quad & \forall i \end{aligned} \tag{2.29b}$$

$$\sigma_i \in \mathbb{R}, \lambda_i \in \mathbb{R} \quad \forall i$$

As mentioned above, an optimal solution x^* to the KKT system in Equation set (2.16) will also be a solution to this reformulation, independent of the Gröbner basis G being applied on the equality conditions in Equation (2.29). However, this system can be reduced in a similar manner to Reference [74]:

1. find the Gröbner basis in Equation (2.29a).
2. apply the variable bounds in Equation set (2.29b) to cut the solution space.

The resulting Gröbner basis can appear in three different forms:

A an empty set.

B a set of polynomials with finite solutions.

C a set of polynomials with infinite solutions.

Case A shows that there is no equilibrium solution to the equality constraints. Case B can be considered solvable by algebra. Case C might yield a system of functions that can be solved via fixing certain parameters. In general, this infinite range of solutions will most often stem from homogeneous functions [80]. Publication [EJOR] demonstrates this by showing a non-symmetric decision problem with an uncertain factor that is set to different values to yield different equilibrium states.

An advantage of this approach is the unnecessity of approximation which results in an adequate representation of Nash equilibria. This shows its importance in problems with a potential range of equilibrium states where traditional methods might lose certain equilibria to approximations such as the cutting plane algorithm presented in Reference [128]. The main disadvantage of this technique is the solution times for the Gröbner basis. This issue is the subject of the core discussion in publication [EJOR].

As mentioned above, Appendix A provides a practical example and the corresponding, compilable code illustrating the here presented methods. The following chapter will aim to introduce the concept of dynamic problems, with the goal to extend the presented models to problems under the consideration of multiple time periods.

Chapter 3

Hydropower Optimization I - Storage

"The thermal stations where fossil minerals are burnt to produce steam have an intrinsic static feature at the individual level: the fuel is a flow that can be bought on upstream markets so that an increase in generation today does not burden future power generations. This is not true for hydro stations using water resource accumulated in dams: it is renewable only on a yearly basis but non renewable within the year so that any use of water to produce a kilowatt today is lost for tomorrows consumption." - Reference [41]

Generation of electrical energy through hydrological power has its' importance to power systems rooted in several characteristics [104]:

- hydropower provides the largest mean of renewable generation.
- hydropower plants provide a large share of the plants with the largest generation capacities globally.
- hydropower plants often show long life spans with investment time spans up to 100+ years.

Leaving other characteristics like environmental impacts[1] beside, the main focus of the work presented in this dissertation will be occupied with the following characteristic of hydropower generation:

- hydropower storage provides the largest wholesale storage medium for electrical generation.

[1]Positive impacts are e.g. the nearly CO2-neutral generation, negative impacts are e.g. the geological impact and the impact on local wildlife.

This is the basis for the research questions of this dissertation as well as this and the following chapter showing a focus on hydrological electricity storage.

Nonetheless, the later presented methods and models in this dissertation could equivalently be applied to other mediums of electricity storage or, in general, to systems under large-scale storage (e.g. manufacturing, logistics). However, the main practical application for price-making electricity storage is considered to be provided by large-scale hydropower generation. This is rooted in the potentially large financial gains of using storage to strategically shifting generation capacities to high-price periods, a concept referred to in literature as *peak skimming* [91, 144].

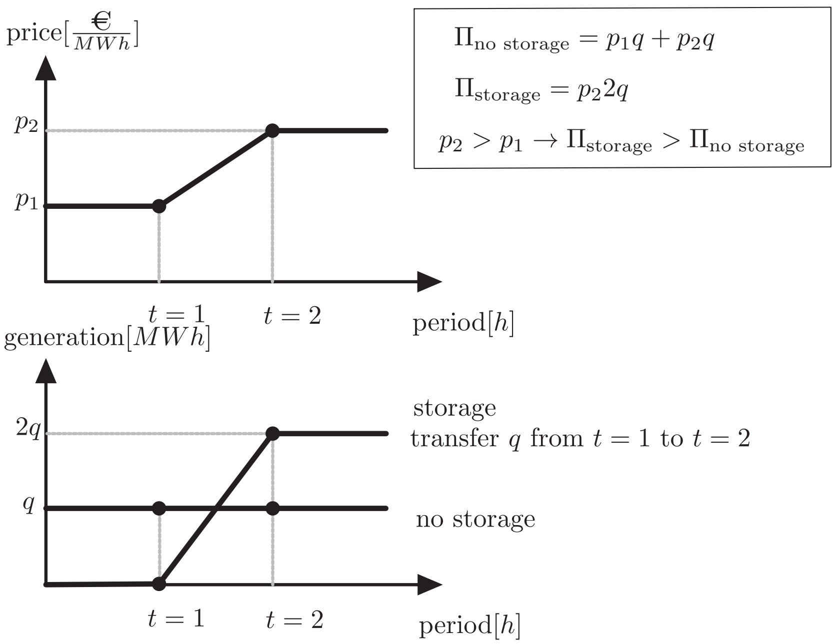


Figure 3.1: Increasing Profits through Peak Skimming

An example for the profit increase through holding inventory is given in Figure 3.1 which uses the example of a price-taking generator that is able to withhold available quantity from one period to use it for increased generation in the next. In principle, this market mechanic is similar to traditional price arbitrage, whereas instead of shifting quantity from market to market or player to player, it is here conducted from time stage to time stage [63].

For hydropower plants, the ability to conduct such arbitrage[2] is determined by the *Degree of Regulation* (DoR), which can be calculated the following:

$$DoR\ [\%] = \frac{storage\ capacity\ [MWh \text{ or } mm^3]}{\mathbb{E}[annual\ inflow][MWh \text{ or } mm^3]} \tag{3.1}$$

The DoR can range from $\approx 0\%$ to several $\times 100\%$ [97]. Based on this measure, the operation types of hydropower plants can be distinguished and defined as:

1. **Run-of-river**: low DoR
2. **Storage**: medium to high DoR
   (a) **No Pumping**: plants behave as suppliers
   (b) **Pumped Storage**: plants can behave as both wholesale customers and suppliers

However, even in units with low DoR, decisions on state variables (e.g. hydrological inventory) have to be made by the operators. Thus, traditional hydropower optimization shows a focus on planning models incorporating unit states, specifically dynamic programs [161].

The state constraints connecting the time periods t and $t+1$ for a single hydropower unit i can be generally formulated as:

$$r_{i,t+1} = r_{i,t} - q_{i,t} + l_{i,t} \tag{3.2}$$

However, it can be assumed that the hydrological inflow l cannot be foreseen exactly, as it is subject to uncertainties caused by precipitation, leading to a formulation based on uncertainty: $l(\xi)$. Further, a slack variable $s_{i,t} \in \mathbb{R}^+$ representing the spillage is often added in practical applications. This variable relaxes the equality condition in order to allow yielding a result in high inflow scenarios (otherwise, if the inflows exceed the maximum generation capacity, i.e. $l_{i,t} > \bar{q}_i$, the constraint could not be fulfilled.

Thus, a commonly used form of this state constraint can be formulated as:

$$r_{i,t+1} = r_{i,t} - q_{i,t} + l(\xi)_{i,t} - s_{i,t} \tag{3.3}$$

In multi-reservoir systems, inflows will be influenced by decisions made on upstream reservoirs and power stations. More water shedding will result in more river stream capacity and fill downstream reservoirs faster. However, in multi-reservoir systems, the displayed state transitions presented in Figure 3.2 can be assumed in similar manner [95]. In addition, it has to be noted that this state constraint should be adjusted for use

[2]Reference [54] discusses that this term might be misleading from a finance perspective. It might be argued that *time-stage-arbitrage* could be a more fitting definition. In this dissertation however, the term *arbitrage* will be used in order to highlight the strategic value of storage decisions.

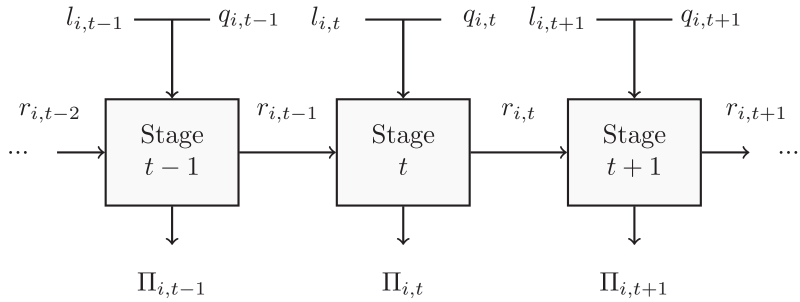


Figure 3.2: Sequential Decision Process

with pumped hydropower or other means of storage that allow active storage decisions in form of acting as a wholesale customer on the power market. As this, however, was not considered within the core scope of this dissertation, such a formulation is presented in Appendix B.

Considering there exists such a connection between the time periods, the Bellmann equation for the optimal profits under uncertainty in the previously introduced decision problem in Equation set (2.1) can be formulated as:

$$\Pi^*_{j,t}(\xi, x) = \left\{ \max_{x_{j,t}} \left(\Pi_{j,t}(\xi, x) + \Pi_{j,t+1}(\xi, x) \right) \middle| h_j(\xi, x) = 0, g_j(\xi, x) \leq 0 \right\} \tag{3.4}$$

It has to be mentioned that the optimal decision in the following time stage $\Pi_{j,t+1}(\xi, x)$ also incorporates $\Pi_{j,t+2}(\xi, x)$ and thus all following profits. Thus a single stage profit is therefore a potentially infinite sequence of profit functions. In practical applications this is usually solved by predetermining a finite time frame and assuming fixed end states for the last period of this time frame [124].

Current hydropower modeling trends show a focus on uncertainty, mainly assuming price-taking generators [142]. Such a simplifying assumption allows the formulation of more accurate technical specifications such as head-tail relations [59] and larger, more complex reservoir constellations [158] or additional improvements of the cutting plane algorithms [20] applied to deal with stochasticity. Reference [125] and based on this, Reference [136], introduced the basic principles of such a cutting plane framework named as *Stochastic Dual Dynamic Programming*, which can be considered the industry standard for price-taking hydropower producers (not to say that other approaches do not exist - see e.g. Reference [102] which applies *Approximate Dual Dynamic Programming*).

However, these principles will not be extended on in this work, as the underlying focus of this dissertation is on price-making (multi-player) models and not on un-

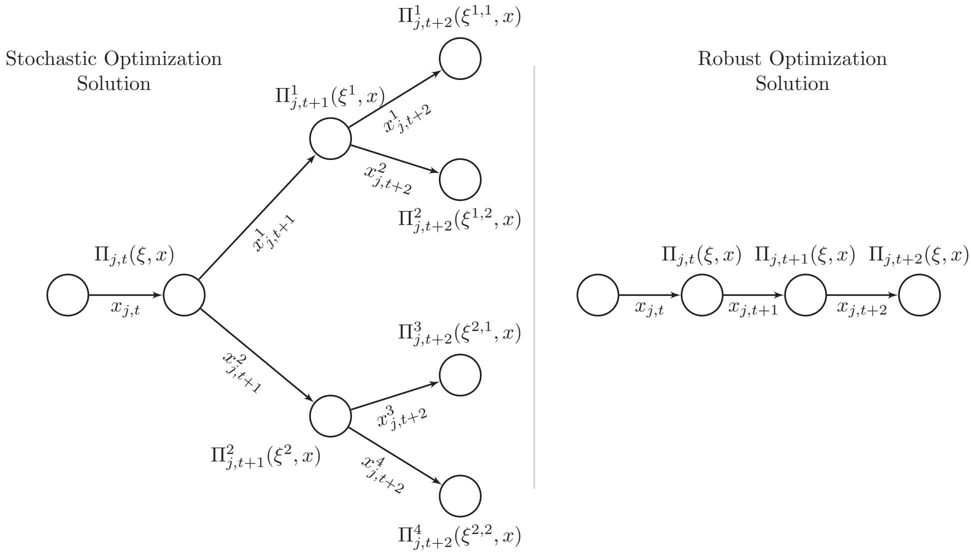


Figure 3.3: Comparison Stochastic and Robust Optimization

certainty models. Even though uncertainty will be considered in the later presented models, stochastic price clearing will be achieved by other techniques such as robust optimization [165] or a residual-minimizing approach based on Reference [25]. The reason for this is presented in Figure 3.3. Both stochastic programming and robust optimization methods yield single decisions for the current period. However, stochastic methods consider the possibility of future changes in decisions in form of branches [46], whereas robust optimization methods consider predetermined robust future decisions [165].

Nevertheless, those decisions will not be fixed for the future outcomes, neither in the case of stochastic programming nor for robust optimization. In both cases, dynamic programming requires recalculation of future decisions over a rolling time horizon conducted after a period has "passed"[3] and uncertainties of the period become "known". However, for robust methods there is an explicit solution for future periods that can be used as an assumption in the current stage for clearing markets for their equilibria instead of branches. This is important, as for a decision made in the current stage, equilibrium decisions for the future stages are required. These, however, cannot be yielded as long as there exists branching in the outcomes of those stages as it is the case with a stochastic solution.

This means, that the solution to the stochastic optimization problem can be con-

[3]It is also possible to conduct this via simulation of certain scenarios and does not necessarily require the model user to physically wait for additional data.

sidered similar to a set of pure strategies and the solution to the robust optimization problem similar to a mixed strategy [2]. Latter shows the favorable characteristic of supporting convergence to an equilibrium [9].

Further, technical constraints and extensive hydrological networks are also presented in simplified form in the publications presented in Chapter 7, as it can be assumed that there exist methods in literature to display larger systems in simplified form with little loss in accuracy [62, 151][4]. The case for that was the intend of notational simplicity, as adding additional complexities do not change the later presented core problem but instead simply expand notation and computational times.

As noted e.g. in References [111, 143] literature on hydropower lacks models analyzing the impact on the market and other players. Thus an important aspect of peak skimming might be ignored in traditional hydropower optimization: the possibility for storage operators to strategically create and deny emerging price peaks by exercising market power. Based on this it might be argued that traditional hydropower optimization methods as e.g. presented in References [139, 143] focus overly on uncertainty, similar to other means of renewable generation such as wind and solar [115]. However, it can be assumed that the accurate prediction of the impact of strategic storage decisions might gain in importance compared to accurate prediction of uncertain inflows the larger the DoR. Thus, large hydropower generators might neglect an important aspect of their strategy set by applying techniques more suitable for run-of-river plants compared to storage plants with a larger range of potential strategies. Based on this assumption that the methods and models developed during the work on this dissertation are aimed on (hydropower) storage units, a more in-depth analysis of their core characteristics is presented below.

3.1 The Role of Hydropower Storage in the Grid

In addition to the previously presented arbitrage, References [48, 54, 141] list several potential applications that can be attributed to (hydropower) storage:

- Electric Supply Capacity - utilities that require additional peak capacities might be able to utilize storage instead of purchasing from marginal units on the wholesale markets.
- Support Time-Of-Use Pricing Schemes - by flattening the price peaks the off-peak periods where time-of-use prices apply get smaller and make such pricing schemes more applicable.
- Reduce Demand Charges - wholesale customers could have their demand charges reduced by having the power draw reduced by storage units feeding into the grid.

[4] Which is also the topic of Publication [ENGIES] that was considered out of scope of this dissertation.

- Renewable Energy Time Shift - storage could assist renewable means of generation to bridge off-peak periods (or in case of pumped hydropower: consume excess supply).
- Ancillary Services:
  - Load Following (especially for pumped storage) - buying and releasing electrical loads within small time frames (minutes) in order to conduct arbitrage by being able to react fast.
  - Area Regulation - use storage to ensure that transfer flows between control areas are matched to scheduled flows.
  - Reserve Capacity Provision - providing *spinning* and *standing reserves*.
  - Transmission Support and Congestion Relief - with enough storage capacities plants can be used to reduce utilization of transmission lines and support stable performance in case of outages.
  - Improve Service Reliability - storage can e.g. support orderly shutdowns of units.

In general, it can be assumed that provision of such services leads to welfare gains [63, 140, 141]. This assumption can be considered similar to nodal arbitrage as e.g. presented in Reference [78], which leads to decreasing price differences. This goal of increasing consumer surplus by minimizing price variation and price peaks is also a main argument for opening balancing mechanisms to competition and associated publication of information [35]. However, it might be the case that such goals do not necessarily align with the strategies of storage providers under market power. This will be discussed below.

3.2 Welfare Effects of Storage

Withholding available arbitrage capacities and the resulting system welfare losses have been shown to be optimal strategies for profit-maximizing agents in electricity markets [23, 88]. Thus, for the comparable mechanism of time-stage-arbitrage allowed by holding reservoir inventories, similar effects can be expected.

However, generation firms neglect these effects under price-taker assumptions. This would result in price-taker models neglecting the influence of players using market power.

To demonstrate this, a Cournot model for a single generation company with a single hydropower unit will be utilized:

$$\max_{q,r} \sum_t p_t(q_t)q_t \tag{3.5a}$$
$$\text{s.t.} \quad r_{t+1} = r_t - q_t + l_t \quad \forall t \tag{3.5b}$$
$$q_t \geq 0 \quad \forall t \tag{3.5c}$$
$$0 \leq r_t \leq l_t \quad \forall t \tag{3.5d}$$

Objective function (3.5a) maximizes profits over all (finite) time periods. As presented above, state equation (3.5b) ensures reservoir consistency. Using the dual variable σ (in literature often also referred to as 'water value') for the state equation, the KKT conditions can be formulated in similar manner to the prior chapter:

$$\begin{array}{cc} \frac{\partial p_t(q_t)}{\partial q_t} q_t + p_t(q_t) + \sigma_t = 0 & \forall t \\ \sigma_t - \sigma_{t+1} = 0 & \forall t \\ r_{t+1} - r_t + q_t - l_t = 0 & \forall t \\ q_t \geq 0 & \forall t \\ 0 \leq r_t \leq l_t & \forall t \\ \sigma_t \in \mathbb{R} & \forall t \end{array} \tag{3.6}$$

These conditions can be solved for their equilibrium over a finite period $\bar{t}$, by assuming start and end reservoir values (e.g. $r_0 = 0$ and $r_{\bar{t}} = 0$). Using the 'bathtub'-representation from Reference [62], a two-period case with linear price curves (with increasing intercept) and equal inflows in each period can be displayed as in Figure 3.4.

Figure 3.4a shows the price gap under no utilized storage. The triangle defined by the two price clearing points and the intersection of the price curves is the lost welfare. In transmission models, where instead of two time periods two nodes are analyzed, this lost welfare is also referred to as 'congestion rent'.

Figure 3.4b describes the social welfare optimum which can be considered a variation of *Hotellings' rule* [81]. It shows no loss in social welfare due to even prices in both periods, assumed there is sufficient available inventory capacity.

In the case of a price-making monopolist, this optimum would not be reached, however. Instead, the storage operator would shed more load in period t in order to reach the profit maximum[5] in Figure 3.4c. This case again shows a loss in social welfare.

In contrast, a price-taking generator would face the problem presented in Figure 3.4d and thus shift all available inflow to the second period. It can be assumed that

[5]The given example has linear price curves of $p_t(q_t) = 4 - q_t$, $p_{t+1}(q_{t+1}) = 6 - q_{t+1}$; inflows of $l_t = l_{t+1} = 0$ and the condition of empty reservoirs, i.e. $r_{t+1} = 0$. This means that the generator profits are $(4 - 1 \times 2) \times 2 + (6 - 1 \times 4) \times 4 = 12$[€] for the social optimum case and $(4 - 1 \times 2.5) \times 2.5 + (6 - 1 \times 3.5) \times 3.5 = 12.5$[€] for a monopolist exercising market power.

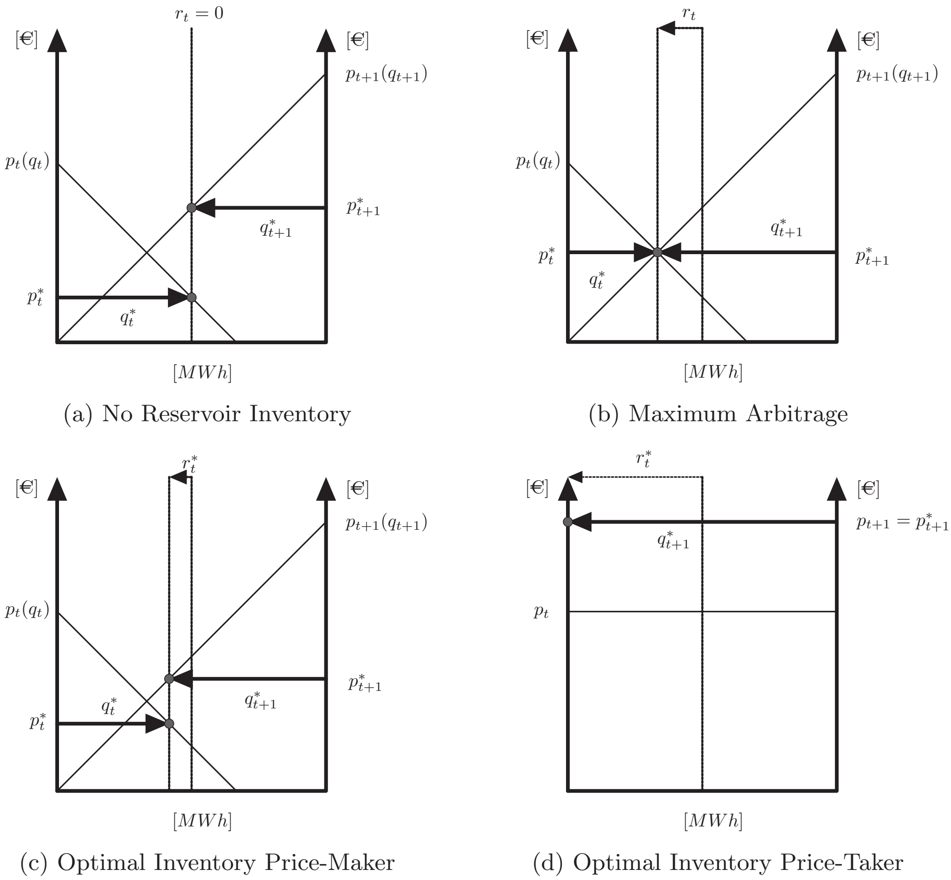


Figure 3.4: Two-Period Price-Maker Storage under Monopoly

if enough competition exists, so that individual generators have at best marginal influence on the price curves, price matching between time periods might be a valid representation of the outcome.

On the other hand, assuming price-taking behavior for price-making agents, as seems to be industry standard for scheduling models [142], might provide a skewed representation of reality, effectively approximating the monopolists' decision from Figure 3.4c with Figure 3.4d. This could prove especially problematic in regards to design of sensible system components such as ancillary services, where selection of participants is more rigorous and thus more excluding leading to a lower pool of competitors compared to other market forms [35]. Appendix C presents an analysis of the existence of the welfare gap caused by exercise of market power through storage operators.

3.3 The Competitors of Hydropower Storage in the Market

Hydropower shares comparable traits to other forms of renewable generation. Specifically, it shows greenhouse gas emissions in orders of one to two magnitudes lower compared to thermal generation [87], operating cost (and thus marginal cost) that are negligible low [58] and is usually associated with techniques for optimization under uncertainty [11, 142] similar to other forms of renewable generation such as wind and solar [115]. As mentioned above, behavior of hydropower plants might resemble traditional renewables in case of run-of-river plants, which offer a rather limited range of available storage decisions. Nonetheless, hydropower optimization for both low and high DoR orients itself at price levels of marginal units in the market, which are mostly provided by thermal plants [142, 158]. That is not to state that there exist no models analyzing the interactions of renewables and storage, however those formulations generally aim for modeling cooperation and not the competitive aspects of liberalized markets [48, 54]. The reasoning is given by the merit-order effect [124, 135]. It states that in a pool market participants base their bids on the units that provide the clearing prices, as those define the 'upper bound of acceptance'. Overstepping this boundary (i.e. bidding extra-marginal) means not being accepted, whereas in pay-as-bid markets understepping this boundary (i.e. bidding infra-marginal) results in losses of potential profits (also referred to as 'opportunity cost'). Thus, a popular way to incorporate renewables and other low-cost means of generation is by *'demand shift'* [144]. Figure 3.5 demonstrates this concept that is also applied on in some of the later presented publications. By using equivalent representations $\rho(q)$ of price curves $p(q)$ that have similar clearing points and slopes the auction can be formulated without the need to incorporate low cost generation.

After introducing (hydropower) storage decisions and their economic implications, the following chapter will focus on (hydropower) dispatch decisions.

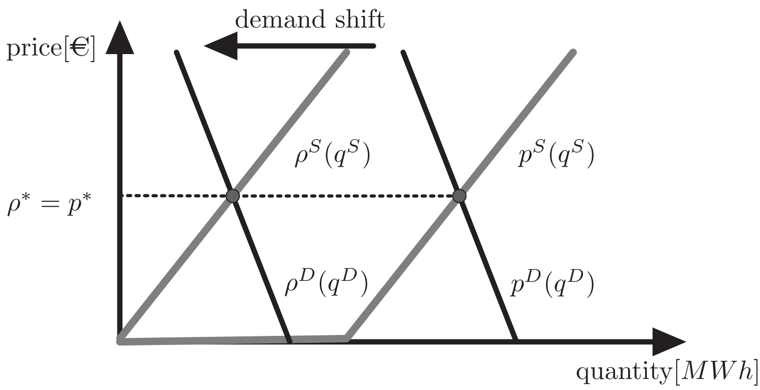


Figure 3.5: Equivalent Representations of Renewable Supply via Demand Shift

Chapter 4

Hydropower Optimization II - Dispatch

The previous chapter discussed the role of reservoir inventory as a state variable that connects current with future decisions. Historically, a different kind of state variable has been a major subject of research in energy system optimization. Initially occupied with thermal generation [12, 138], discontinuous (binary/integer) dispatch variables found recent applications in hydropower optimization [28, 126]. One reason for this trend might be given by the generally low variable cost of hydropower generation, so that even minor cost of starting generation units have significant impacts on the unit schedules [59]. Especially in markets with low prices this could mean that models that assume consistent generation limits over all future time periods (such as e.g. Reference [27] which is presented in Appendix A) might not reflect reality accurately. To demonstrate this, assumed be generation in a single unit i has lower and upper bounds:

$$\underline{q}_i \leq q_{i,t} \leq \bar{q}_i \quad \forall t \tag{4.1}$$

Considering binary schedules $b_{i,t} \in \mathbb{Z}^2$ allows reformulation of this constraint:

$$\underline{q}_i b_{i,t} \leq q_{i,t} \leq \bar{q}_i b_{i,t} \quad \forall t \tag{4.2}$$

Usually, this dispatch variable is included in the cost function and thus subsequently in the profit function. For the example of a price takers' profit function given in Equation (2.2), the formulation for a limited time frame and a player j holding a set of generation units I_j would look the following:

$$\Pi_j(q_j, b_j) = \sum_{i \in I_j} \sum_t p q_{i,t} - \sum_{i \in I_j} \sum_t c_i(q_{i,t}, b_{i,t}) \tag{4.3}$$

Several other types of constraints and adjustments can be added to such a model [83], e.g.:

- maximum/minimum on/off-time
- startup/shutdown time requirements
- reserve constraints (i.e. a minimum of active units)
- capacity markets (would adjust the profit function so that it includes remuneration for the binary variables)

As mentioned in the previous chapter, however, this dissertation does not aim to provide a literature synopsis on how to solve unit commitment problems. Various work is available on the techniques and methods [122] and would have to be extended to consider uncertainty to be able to cater applications in hydropower optimization. As this would be out of scope for the here presented work, further introduction to solution techniques is omitted.

Nonetheless, the later presented publications will consider unit commitment in simplified form. The applied methods, however, will be introduced within the specific publications and therefore not explicitly discussed here. For the here presented methods it suffices to assume that a method able to deal with the introduced capacity constraints from Equation (4.2) can also be considered to deal with the other additional constraints and model adjustments presented above.

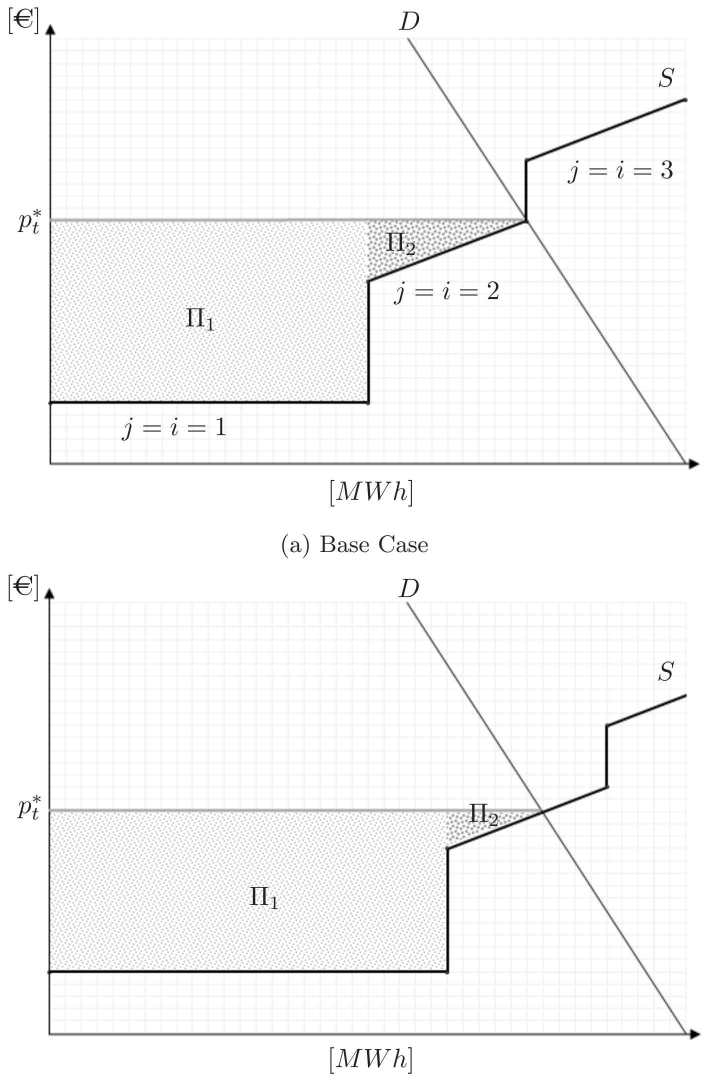
4.1 Unit Commitment and Price Takers

Now assumed be the market participants are able to influence prices, which can be considered a reasonable assumption in the light of the discussion of the previous chapters. This would mean a different profit function formulation compared to Equation (4.3). For the example of Cournot competition given in Equation (2.4), the profit function including binary decisions could be formulated as:

$$\Pi_j(q_j, b_j) = \sum_{i \in I_j} \sum_t p_j(\sum_{i \in I_j} q_{i,t}, \sum_{i2 \notin I_j} q'_{i2,t}) q_{i,t} - \sum_{i \in I_j} \sum_t c_i(q_{i,t}, b_{i,t}) \tag{4.4}$$

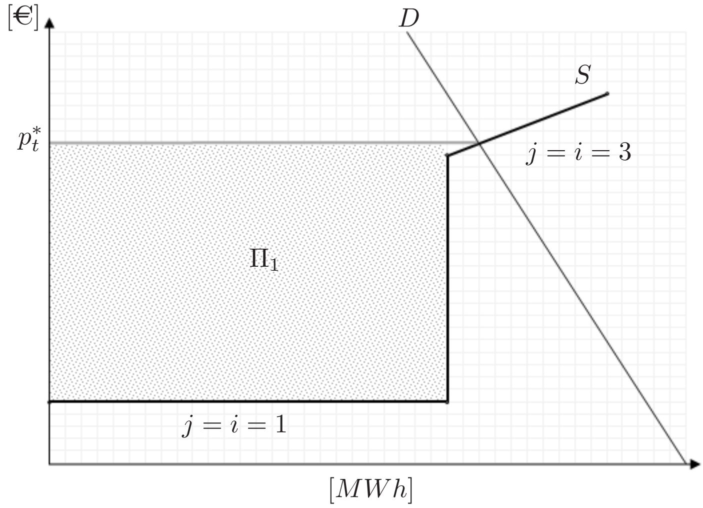
Whereas various techniques were introduced to deal with price-taking dispatch problems, this new formulation allows for a new range of strategies that are not considered in traditional unit commitment models.

One such example is given in Figure 4.1. Here a storage operator $j = 1$ and two thermal generators $j = 2, 3$ meet in a spot market under uniform price clearing. The base case in Figure 4.1a shows that the base loads are supplied by the storage operator $j = 1$, the marginal unit held by thermal generator $j = 2$ and thermal generator $j = 3$ holds an extra-marginal unit. However, it is assumed that $j = 3$ cannot shut the unit down in this period (e.g. is influenced by minimum up-times), whereas generator $j = 2$ could choose to shut this unit down but the profit outcome Π_2 is large enough to reimburse for the cost of keeping the unit running.



(b) Storage Decision by $j = 1$

Figure 4.1: Unit Dispatch and Competition



(c) Dispatch Decision by $j = 2$

Figure 4.1: Unit Dispatch and Competition

As described in the previous chapter, a storage decision from the previous period would extend the base load and decrease the clearing price. This is shown in Figure 4.1b. Such a storage decision and the resulting price drop, however, would also affect the profits of the other participants. In this case, if the profits of player $j = 2$ would not suffice to reimburse for the decision to keep the thermal unit running, a shutdown decision could be made.

Such a decision can also be interpreted as a player choosing to exit a market. The outcome is displayed in Figure 4.1c. In this case, a drastic increase in storage operator profits can be observed. This profit increase is a result of indirect influence on other players' state variables (here the dispatch decision of player $j = 2$) and can only be captured by models that incorporate competition and state variables (in this case both storage and dispatch decisions are required to portrait the decision).

As will be argued below, literature on such decision models can be considered scarce at best. This led to the formulation of the research focus of the scientific work presented in this dissertation and resulting publications such as Publication [POLICY], which discusses this principle in further detail, below.

4.2 Multiple Equilibria

Designing models of competition to be capable to incorporate state variables adds another complicating factor: the number of equilibria ranges from 0 to a finite or an

		$\boxed{j = 1}$	
		$b_1 = 1$	$b_1 = 0$
$\boxed{j = 2}$	$b_2 = 1$	$p(q_1^* + q_2^*)q_1^* < c_1^b$ $p(q_1^* + q_2^*)q_2^* < c_2^b$	$q_1^* = 0$ $p(q_2^*)q_2^* > c_2^b$
	$b_2 = 0$	$p(q_1^*)q_1^* > c_1^b$ $q_2^* = 0$	$q_1^* = 0$ $q_2^* = 0$

Figure 4.2: A Discrete Unit Dispatch Game

infinite number. Even though for most applications it might be possible to yield equilibria for mixed strategies [131], considering a discrete game as a continuous problem might not be a useful result.

Similar to Reference [9] a discrete two-player game under complete information might be considered here for the sake of illustration. Assumed are two players holding one generation unit each and with profit functions of $\Pi_j = p(\sum_{j2} q_{j2})q_j - c_j(b_j)$. Further, cost curves are assumed to be $c_j(b_j) = c_j^b b_j$ where b_j denotes the binary startup/shutdown decision and fixed cost component c_j^b is a constant.

The game is considered convex for fixed binary variables and can thus be solved similar to Chapter 2 for optimal generation quantities q_j^*. Figure 4.2 shows a potential payoff-matrix that could be established. As it can be observed, there are two cells that fulfill the requirements for a Nash equilibrium - i.e. $b_1 \neq b_2$. In case of both players supplying the market, both would operate under loss, whereas a single player supplying would result in that player profiting from participating in the market. For homogeneous players (i.e. similar plants), mixed strategies could result in outcomes where both players are having their plants shut down or running. Both of those outcomes would however not represent a Nash equilibrium of the original game and thus distort assumptions on e.g. the capacity available in the system. This highlights the importance of methodical selection and analysis of Nash equilibria, which the later presented publications show a strong focus on.

Chapter 5

Research Question and Main Assumptions

Based on the previously established principles and the discovered lack of literature, a **main research question** can be formulated:

> *How would a model need to be designed to accurately depict the decision process of a price-making storage operator?*

Based on the previously established theory, several **core assumptions and model requirements** were formulated:

Assumption 1	*As hydropower storage makes up the main part of large scale storage, the models are focused mainly on the core traits of (short-term) hydropower optimization models.*
Assumption 2	*The main competitor of large scale storage is thermal/-marginal units.*
Assumption 3	*Units are not assumed to be consistently participating over the entire time frame. Thus startup/shutdown decisions have to be included.*
Assumption 4	*Due to its importance in representing electricity pool markets, focus is laid on Cournot competition [16]. Nonetheless, applicability on other modes of competition should be given and possibility on applying the models beyond electricity, or generally, energy markets should be aimed for.*

Several **additional research requirements** were formulated:

Requirement 1	*Due to hydropower being the main medium for large scale storage, methods are focused on its main features.*
Requirement 2	*Multiple equilibria can be expected and thus have to be analyzed and appropriately dealt with as mixed strategies are considered unreliable for dispatch problems.*
Requirement 3	*A focus on reserve and capacity markets can be expected and models should be designed focused on these applications, but not specifically tailored for such.*
Requirement 4	*The case studies should be intended to demonstrate the basic capabilities instead of solving explicit practical questions. However, portability to problems from industry should exist.*

The next chapter intents to give an overview of the literature that covers the foundation of the later presented publications intended to answer the here presented research question.

Chapter 6

Background Literature

In general, most models from literature intending to deal with the topics of hydropower optimization or price-makers in electricity markets can be assigned to one of the two types(/directions) displayed in Figure 6.1. The publications yielded through the work on this dissertation, however, intent to follow the structure presented in Figure 6.2 that incorporates both views. To introduce the necessary background required to approach establishing such models and the necessary solution techniques, the most influential literature considered in the work on this dissertation will be introduced below.

Here it shall be recognized that the subject of the presented pieces of literature might not be assigned to a single classification and might also contribute to the other topics presented. To resolve this issue, the references were assigned to the section showing the best match to their main contributions.

6.1 Literature Reviews

The here presented literature reviews cover the basics of the different subjects considered in the later presented publications. As it will be shown below, literature on problems similar to Figure 6.2 is scarce and thus no existing literature reviews are available.

The bidding process of hydropower generators in liberalized markets are examined in Reference [142]. It lists several examples of models and categorizes them by time frame (immediate, short, medium, long term) and method classification. Even though it lists price-making hydropower operator models, it points out the lack in literature that approaches such problems.

Reference [152] exams competition in electricity markets by establishing the state-of-the-art of models with multiple players. Specifically it deals with equilibrium models, whereas a strong focus is put on Cournot models. Further model types and model

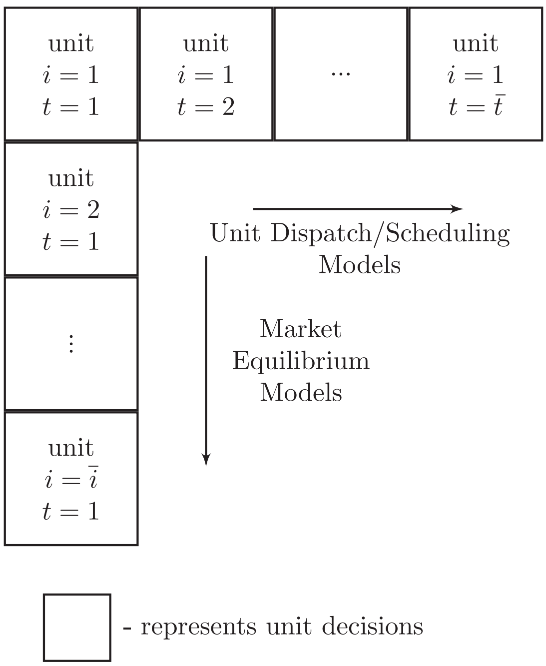


Figure 6.1: Models in Literature

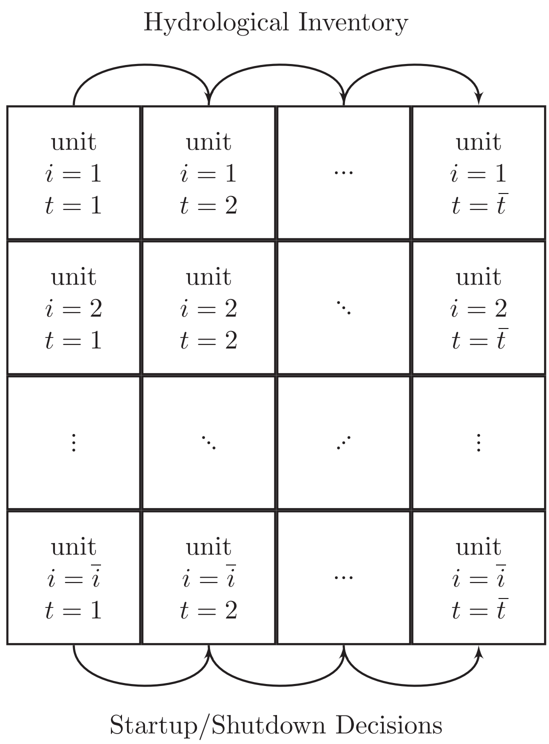


Figure 6.2: Proposed Model Concept

extensions such as capacity markets and transmission constraints are discussed as well.

A general overview of hydropower and its role in the European energy system is presented in Reference [68]. It further outlines the roles of plants with different DoRs in different markets. Similar to Chapter 3 it concludes with estimating a stronger role in ancillary services with rising DoR.

Reference [47] presents the basics of single-player hydro-thermal optimization, with a focus on stochastic optimization. It discusses the main components of such optimization models (e.g. cascading systems, thermal dispatch) and outlines the importance of state variables.

Another review of hydropower optimization models is presented by Reference [139]. The paper discusses the different technical specifications with a strong focus on head effects. In similar manner to Reference [47] it also discusses the importance of system states.

A classification of ancillary markets and services is provided by Reference [130]. It shows a strong focus on thermal generation. The reasoning for this can be found in such units generally providing marginal units and showing reaction times fitting for applications in provision of ancillary services.

In similar manner, Reference [69] provides an overview of markets for ancillary services. It lists a multitude of models aimed to deal with both models under competition and no competition. Further, a section is dedicated to the analysis of hydro-thermal optimization, whereas it focuses on price-taking models.

In Reference [95] an overview of hydropower optimization techniques is presented. The paper mainly focuses on models under hydrological networks, nonlinear objective functions through head-tail effects and the effects of uncertainty.

A literature review of applications of game theory in electricity systems is presented in Reference [108]. In addition to listing practical applications for both cooperative and non-cooperative games it also outlines the $\mathcal{NP}$-hard binary scheduling problems approached by the methods of unit commitment presented below.

6.2 'Traditional' Unit Commitment

The field of optimal unit commitment and dispatch has a wide range of literature available. The here presented publications are meant to only cover the surface of this field of research and are intended as an introduction to the basic characteristics of such models.

The basics of solution algorithms for dynamic models and optimal control is presented in References [18, 19]. Unit commitment models can be considered a subgroup of such models as presented in Figure 6.3.

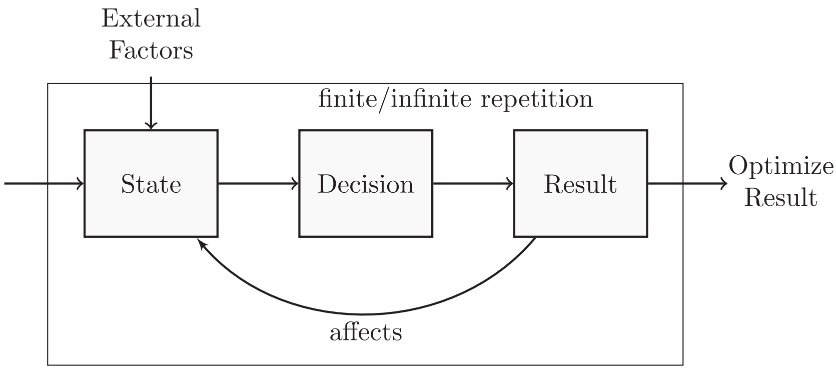


Figure 6.3: Optimal Control Problem

A basic problem formulation of deterministic unit commitment is presented in Reference [12]. Even though the solution technique might not be considered up-to-date to current methods, the core principles introduced are similar to modern and more complex unit commitment problems.

Chapter 5 of Reference [115] presents an extended example for a dispatch problem under uncertainty. In addition to various other generation forms it also discusses hydropower and its technical specifications, such as *'high ramping rates'* and *'virtually no minimum power outputs'*.

Ramping costs in a system under uncertainty are discussed in Reference [110]. It convexifies the ramping cost functions and uses deterministic equivalents of the chance constraints by applying their probability distributions.

6.3 Hydropower Unit Commitment/Price-taking Hydropower

Most of the sources presented within this section discuss unit commitment problems with focus on hydropower. The applied solution techniques are, as described previously, concerned with adequate modeling of dynamic generation/storage decisions and uncertainty. In addition to this, another important characteristic of such optimization problems is the topology of river networks connecting various reservoirs.

This can lead to reservoirs affecting each others' water levels through their shedding and storing decisions. Combined with e.g. non-linearities in turbine - water level efficiencies, such systems offer a high level of complexity, even without anticipating price influences. An example for a typical river basin in Southern Norway is given in

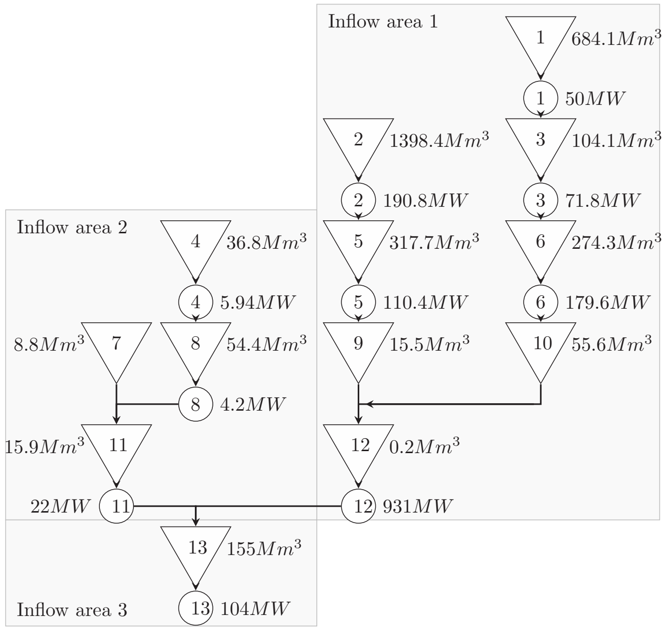


Figure 6.4: 'Sira Kvina' River Basin

Figure 6.4 (taken from [103]).

Unit commitment problems under uncertainty are solved in Reference [34] via applying robust optimization. In contrast to Reference [110] there is no convexification applied, thus a global optimum will not necessarily be obtained.

An analysis of the value of applying stochastic optimization on hydro-thermal optimization problems is given in Reference [134]. It utilizes reservoir state constraints in a similar manner as presented in Chapter 3.

The subject of hydropower unit commitment under uncertain demand is discussed in Reference [126]. It analyzes several approaches to minimize total startups and advises usage of rate-of-change cost approaches for similar problems.

Reference [36] lists several reasons for the importance of including startup cost for hydropower units: the loss of water during maintenance, wear and tear of the windings/the mechanical equipment, malfunctions in the control equipment, loss of water during the startup. The presented methodology provides a framework to solve the dispatch decisions for a single hydropower player with several reservoirs optimizing their unit commitment in a pool market.

A hydro-thermal dispatch problem under uncertainty is analyzed in Reference [121]. The paper considers reserve constraints and similar to Chapter 4 proposes two simplifications: negligible variable cost for hydropower and constant costs for binary variables. The resulting problem is solved via dynamic programming.

A special case of hydropower optimization is given by Reference [58]. The paper uses the case of a Nordic hydropower producer, which traditionally show a high share of hydropower generation related to the portfolio. It assesses the value of applying stochastic optimization compared to deterministic optimization and concludes that former approach is less sensitive to changes in the actual outcome.

Reference [61] analyzes dispatch decisions related to a single hydropower generator bidding on both day-ahead and balancing markets. It analyzes the methods of using linear representations of the binary values or fixing the binary decisions/i.e. dropping the binary variables entirely. Results indicate that the bidding decision on the balancing market is influenced (to an extend related to the size of the startup/shutdown cost) by the chosen representation of the dispatch decision. Further it is noted that for short-term decisions those dispatch decisions should not be relaxed.

The use of equivalent representations of multi-reservoir systems that have fewer or a single reservoir is discussed in Reference [137]. This paper and the work of Turgeon (see e.g. References [149, 150]) can be considered the basis of the decision to not consider multi-reservoir systems in the later presented publications. Instead, they consider river basins with single reservoirs, whereas the principles are assumed to be possibly extended to multi-reservoir systems in equal manner.

Reference [30] discusses the constraints and challenges faced by profit-maximizing hydropower generators. It discusses non-linearities such as head-tail dependencies and discreteness of unit dispatch variables. The approach applies piece-wise linear penalty functions and, based on a short-term time frame, assumes a deterministic setup to formulate a MILP.

6.4 Competitive Games in (Electric) Power Systems

As indicated by the publication chart presented in Figure 6.5, over recent years analysis of competition efficiency has risen in power system. This can be related to liberalization in global power markets.

As such, multi-player models with single firms not necessarily being aligned with optimizing social welfare in a system have been arising and have been utilized to discuss the effects of competition distortions such as market power and information asymmetry.

Reference [79] provides an early work on potential formulations of multi-player models in electricity markets. It utilizes a convergence algorithm that stepwise updates the player decisions with the goal of ending up in a Nash equilibrium.

Another early work on the topic is provided by Reference [71]. The paper discusses the opportunity for oligopolstic generators to use tacit collusion in order to create

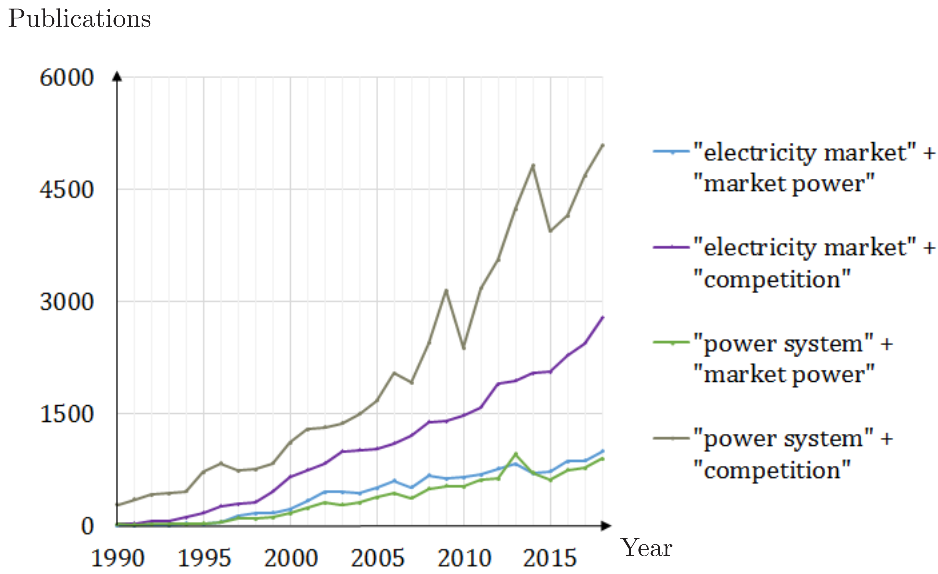


Figure 6.5: Publication History for selected Keywords [49]

artificial price spikes. Instead of formulating a specific solution model like the previous source, this source uses analysis methods from non-cooperative game theory and supports the results with empirical observations.

A more general solution model for such *Mathematical Programs with Complimentarity Constraints* (MPEC) setups was presented in Reference [78], which analyzes the case of price-making firms participating in a multi-nodal pool market. It further introduces the concept of nodal arbitrage to power systems.

In addition, Reference [160] extends the model formulations presented above by formulating a search algorithm that is intended to solve such MPECs.

Such MPEC formulations were succesfully applied on a two-stage (forward-spot market) framework in Reference [163]. The resulting *Equilibrium Problem with Equilibrium Constraints* (EPEC) is solved similar to References [79, 160] - by applying a convergence algorithm on the KKT conditions.

A compendium of formulations of such EPEC models and methods to yield stationary points (equilibria) is provided by Reference [146]. Even though the focus of such models lies on Stackelberg competition, as the previously presented paper shows, EPECs are not necessarily limited to such applications.

Another compendium to approach such EPEC problems can be found in Reference [98] which aims to provide solution techniques for multi-leader-multi-follower games.

Reference [156] shows a different application of the here presented models. It applies the previously introduced techniques from non-cooperative game theory to solve a cooperative game with differing objectives - minimizing generation costs and emissions simultaneously. It proposes a cutting plane algorithm to find the Nash equilibrium within a Pareto set. An extension to this concept is given by Reference [157], which presents an outer approximation algorithm to reformulate a non-convex dispatch problem as a convex problem.

Another practical application of competition models is provided by Reference [129]. It models the implications of market power on firms participating in energy and reserve markets. Compared to the single-firm price-taker models presented in the previous section, binary dispatch decisions are considered predetermined parameters.

A wide range of competition models for applications in (single-period and thus state-less) energy markets is provided by Reference [65]. A focus on Nash equilibria in Cournot competition can be observed, which supports the assumption of such a competition model being a reasonable representation of electricity pool market competition.

Reference [45] provides a multi-period formulation. However, the model does not incorporate decisions on state-variables but instead formulates the states as a result of the market clearing results.

Reference [66] establishes a multi-period framework under storage to model equilibria in gas markets. However, the infrastructure (pipelines and storage) operators are considered price-takers in the presented model. In addition, storage operators are relaxed similar to Reference [129] - by assuming predetermined states.

Even though less literature can be found on these topics, other approaches on deriving electricity market equilibria have been presented:

- *Nikaido-Isoda convergence algorithms* (see Appendix A.2) as introduced by Reference [93] and applied e.g. by Reference [39].
- *Supply function equilibria* as introduced by Reference [92] and applied e.g. in References [10, 21].

6.5 Price-making Hydropower

In traditional power system scheduling, optimization of decisions to minimize the deviation from the optimum point of the cost function is a core aspect. In hydropower, low marginal cost and flexibility in ramping times and frequencies shift the focus towards another aspect: the maximization of utility of a deplorable good - water. As such, accurate representation of uncertainty related to hydrological inflows and prices has become the main focus of literature on hydropower optimization.

Applying price-maker assumptions on such models has however proven difficult. This might be a result of the hurdle presented in Chapter 3: stochastic methods might complicate finding equilibria. Or, as in most systems, hydropower does not provide large enough capacities to have a measurable impact on the markets. However, liberalized markets with large generation portfolio shares of hydropower capacities exist in e.g. Austria, Norway, Switzerland and Brazil. This and recent trends in large-scale battery storage technologies underlines the necessity of models analyzing the perspective of price-making storage providers.

Reference [5] provides an analysis of a similar problem outside of the field of power systems: exercising market power through holding inventory on goods. The author notes that with appropriate storage space, a supplier could choose to withhold capacity until time periods where it holds enough market share to act as a monopolist. This is similar to the principle of arbitrage as shown above, but takes into account supply shortages. Due to vast network sizes such effects might be rare on energy system level, but might still appear on local level.

A two-unit hydro-thermal game under different modes of competition is introduced in Reference [41], ranging from a welfare maximizing social planner to a profit maximizing monopolist. The paper supports the assumption of price-arbitrage being, in general, a profit- and welfare- maximizing choice of strategy. Additional findings are made. For example, it is shown that hydropower generators participating in a market are able to generate positive effects for the consumer surplus even if generation decisions are not optimal, i.e. are not profit-maximizing, since their marginal cost are negligible low.

By embedding an equilibrium convergence algorithm in a dynamic program, Reference [154] attempts to formulate hydro-thermal competition between firms holding several generation units. Even though reservoir levels are considered, they only impact the level variation over the total time frame instead of affecting inventory in individual periods.

The model presented in Reference [27] is introduced in detail in Appendix A of this dissertation. It presents a multi-firm multi-period Cournot model, similar to many of the approaches presented in Reference [65]. Even though the inflow formulation does not consider periodical inflows nor uncertainty, the proposed model provides an important starting point that many of the publications presented in the following chapter are based on.

Built on the approach presented in Reference [27], Reference [60] establishes a stochastic game formulation with a focus on risk hedging. It analyzes the utilization of market power by large scale producers in the Nordic electricity markets. The model is formulated as a two-stage stochastic Cournot model, whereas in the first stage both output and contract decisions are made and in the second stage only output decisions are made whilst contracts continue. The paper concludes in that large hydropower producers do not show capabilities to exercise market power, whereas large thermal producers do.

Reference [42] establishes a Cournot model for pumped hydropower storage in hydro-thermal competition similar to Reference [41]. Instead of a state constraint it applies a similar concept to Reference [27]: a capacity constraint over several (here: two) periods. To prevent the possibility of pumping backwards in time, infinite slack costs are utilized. The findings point out that a monopolist holding thermal and pumped hydro generation might even decide to generate via thermal generation to raise prices whilst at the same time pumping into their hydropower plants by purchasing from the market.

An economical analysis of the welfare effects of storage is presented in Reference [140]. It demonstrates that ownership of storage units affects the optimal strategies. Due to consumers and producers aiming to maximize their surplus (e.g. minimize the cost they pay or maximize the profits they receive), the usage is not only limited solely to the arbitrage-profits that a pure storage operator would aim to maximize.

Reference [133] analyzes pumped hydropower storage on the German electricity market. It applies a Cournot model to show a similar outcome to Reference [140] - different actors (e.g. consumers, strategic wholesalers) utilize storage technology differently. In similar manner they find that perfect competition supports the highest utilization of storage capacities. This and the previous paper thus demonstrate that the meaning of 'optimal storage strategy' is influenced by the environment of the power system such as level of competition and role of the agent holding the storage unit.

By using the structure of the previously introduced Nikaido-Isoda convergence algorithm, Reference [114] implements a hydro-thermal Cournot model under network constraints. Holding inventory is not realized via states but dispatched via an *'active set method'*. In short this can be described as: 1.) calculate equilibria of each period individually, → 2.) find periods which would show the highest profit increases and shift there, → 3.) if no shift possible or necessary: converged, else back to 1.

Reference [3] nests a (decentralized or centralized) dispatch model formulated as an MPEC in a market. The result is a bi-level EPEC, that is in turn transformed back into a single-level problem by finding analytic solutions to the lower-level problems. An equilibrium is then found by applying the additional condition of minimizing spillage. Similar to the publications presented in the next chapter, this model is only applicable for period 1 and has to be applied in a rolling time horizon to calculate future outcomes.

In chapter 11 of Reference [63] the case of a monopolistic hydropower storage provider is analyzed. The conclusion drawn is that a storage provider holding market power actively withholds capacity to raise prices. This outcome is similar to traditional monopoly models in e.g. Cournot competition. As the book notices, in the case of hydropower this could result in a generator choosing to spill water instead of supplying the market with additional generation.

In addition to in-depth literature reviews on electricity market models with price-making players and hydropower bidding, Reference [143] provides a model to approach hydro-thermal competition. The proposed method uses the approach of a *potential*

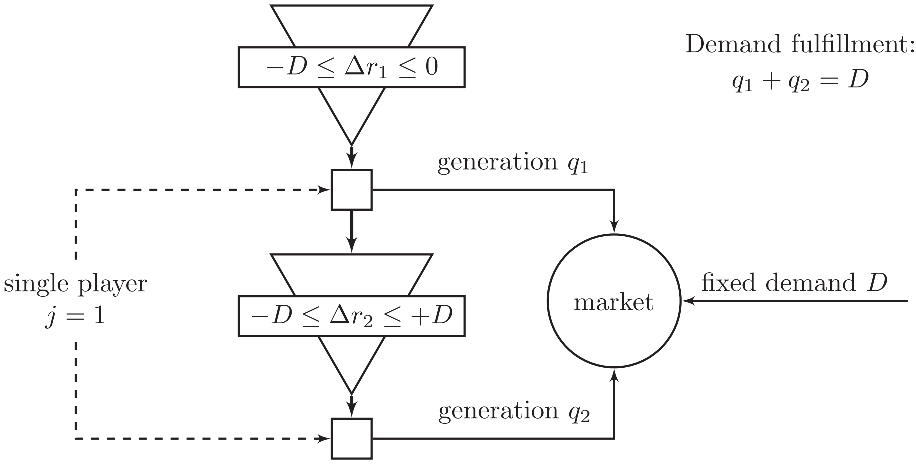


Figure 6.6: Multiple Equilibria in Hydropower Systems

game (i.e. using an interpolated joint revenue function instead of individual player revenues). Further it discusses the existence of multiple equilibria and outlines a framework that helps decision makers on which equilibrium to choose.

Reference [127] models hydro-thermal competition under uncertainty and aims to provide a tool to diagnose strategic behavior by generators. The applied framework is based on the assumptions that hydropower agents tend to bid prices above the levels that a water-conserving agent would aim for. Therefore, the model solves the problem of a social planner and then individually adjusts the player decisions based on their risk attitudes. This is conducted by solving a number of individual agents and applying bounds on the total decision space of all players through complementarity constraints.

A model that utilizes two techniques - Bayesian and Robust Nash equilibria - to solve hydropower-dominated systems under uncertainty is introduced in Reference [111]. It uses various techniques as proposed above, such as applying slackness to force duality and using the Fortuny-Amat formulation to solve the complementarity conditions. The paper further discusses a search algorithm based on integer cuts aimed on finding multiple equilibria. Such multiple (Nash) equilibria are further discussed below.

6.6 Multiple (Nash) Equilibria

Several literature sources such as e.g. References [111, 143] highlight the importance of finding and mapping several equilibria that provide (potential) solutions to a system.

Reference [111] also emphasizes another common characteristic of systems with a range of possible equilibria:

> *"Practitioners typically are satisfied when a single Nash equilibrium is found, even in cases in which multiple (or even infinite) Nash equilibria exist."* - Reference [143]

Contrary to this, Publication [ENERGY] presented below shows an analysis of the practical importance of finding and evaluating various equilibria in its appendix.

Another example is given by Figure 6.6 which shows the example of multiple equilibria in hydropower systems. Even though the generation level have no impact on the market clearing constraint (demand fulfillment), the changes in reservoir levels (Δr_i) show a range of potential equilibrium outcomes that varies depending on the choice of generation.

Assume:

- a single player $j = 1$

holding two homogeneous generation units $i = 1, 2$

- a clearing condition for fixed demand:

$$D_t = q_{1,t} + q_{2,t} \quad \forall t = 1, 2$$

- price is too low to compensate for running both units,

i.e.: $b_{1,t} \neq b_{2,t} \quad \forall t = 1, 2$

This results in four potential equilibrium solutions:

$t = 1$				$t = 2$			
$i = 1$		$i = 2$		$i = 1$		$i = 2$	
$q_{1,1}$	$b_{1,1}$	$q_{2,1}$	$b_{2,1}$	$q_{1,2}$	$b_{1,2}$	$q_{2,2}$	$b_{2,2}$
D_1	1	0	0	0	0	D_2	1
0	0	D_1	1	D_2	1	0	0
D_1	1	0	0	D_2	1	0	0
0	0	D_1	1	0	0	D_2	1

Figure 6.7: Multiple Equilibria under Unit Commitment

In similar manner, Figure 6.7 displays multiple equilibria in a unit commitment problem. Assuming homogeneous cost functions and all additional runtime constraints (e.g. maximum/minimum runtime) are fulfilled during the two periods, all of those four potential equilibria might be viable. However, the results itself could have implications on future/not considered time periods, as it would be possible to e.g. not be able to supply with both units in $t = 3$ if the maximum runtime of a unit is reached after running the unit for two periods (which would render equilibria 3 and 4 problematic).

These examples illustrate the necessity to consider multiple equilibria in the context of price-making hydropower and multi-player dispatch models.

A game between electricity producers under supply-function competition is shown in Reference [40]. To solve the problem for multiple Nash equilibria, a linear sub problem is nested in a nonlinear problem which has its equilibria searched for via a heuristic approach. Instead of binary unit dispatch, in this model non-convexity is caused by line flows that are either strictly below, equal or above 0. However, the scalability of such problems and the resulting 'curse of dimensionality' are not further discussed.

Reference [43] analyzes the use of polynomials to solve discrete games. It focuses mainly on solution methods to polynomial systems, such as the previously presented Gröbner basis approach. Limited scalability is discussed swiftly, as the author proposes a heuristic based on Taylor series (or a heuristic search provided by the *Gambit* software package) to deal with larger model sizes.

Scalability of such concepts is analyzed in Reference [9]. The paper further discusses several forms and applications of discrete games and discusses a mixed strategy approach to solve for multiple equilibria.

The work presented in Reference [128] shows a strong focus on scalability and subsequently on practical applications. Apart from proposing linearization of discrete variables and the Fortuny-Amat transformation, the paper also suggests using a heuristic algorithm adding cutting planes to find multiple equilibria (which in turn again uses the Fortuny-Amat notation to become a linear problem).

Transformation of the equilibrium conditions into polynomial functions is proposed in Reference [162]. In addition to formulating multiple equilibria, the paper discusses how to select adequate solutions out of the set of Nash equilibria in order to be able to support practical applications. Regarding complexity it notes an exponential growth in solution paths with respect to the size of the game but does not propose specific techniques or heuristics to approach this issue.

References [6, 7] introduce a model that formulates the problem of a system operator (optimal demand) as an equilibrium set that is then overlapped by the best response function of generators (optimal bids) to yield the Nash equilibria.

With a similar application in mind, Reference [77] formulates a technique to yield extremal Nash equilibria (i.e. the stationary points minimizing/maximizing dispatch cost in a power system). This is conducted by applying linearization techniques on quadratic equations and solving a number of linear problems iterated for binary variables that determine the social cost of dispatch. The paper also highlights the satisfying computational performance of the method, which might be attributed to the applied linearization techniques.

6.7 Equilibria under Dispatch

Most of the literature on games presented above considers convexity of individual player problems. The choices to either use convex formulations or approximate them as such might be given by the complimentarity formulation transforming the multi-player model to a non-convex problem that requires additional techniques, such as the introduced transformation methods, to converge to equilibrium points. However, in certain cases, approximations might not be valid or - in case of the problems analyzing dispatch - might eliminate the fundamental issue from the problem analysis. Models analyzing such problems are presented here.

Reference [8] formulates a sequential day-ahead and reserve market framework with a period for potential adjustments within the market stages. Even though dispatch is assumed, the non-convex problem is transformed via assuming fixed unit commitment schedules and the resulting problem solved via applying the Fortuny-Amat notation as discussed before. Uncertainty is considered in simplified form via sampling distributions and optimizing under expected values.

A game under uncertain curtailment of wind power plants is formulated in Reference [112]. The model realizes generators with binary ramping decisions that are allowed to exercise market power in a leader-follower game. The case study observes the leaders (generators) withholding capacity to maximize profits, affecting the dispatch costs and the prices in the system the follower (market operator) sets. It also discusses the (potential) range of multiple Nash equilibria by applying the method proposed in Reference [77].

The model presented in Reference [38] introduces an adjusted version of a Cournot-model for a hydro-thermal system. It applies an iterative approach to continuously update supply and demand curve slopes and eventually reach an equilibrium point. This allows formulation of individual player problems as MILPs and thus is able to incorporate binary startup/shutdowns and respective capacity limits.

Reference [67] illustrates how in discontinuous Nash-Cournot games a tradeoff between complementarity and integrality exists. It proposes a relaxation of the integrality constraint within the KKT conditions as a continuous formulation. This is done by adding a continuous slack variable to the integer variables that allows departing from the integer conditions. This slack variable is then in turn minimized as part of the objective function of the MILP formulation (via the Fortuny-Amat notation) of the KKT conditions. This allows fulfilling the complementarity conditions whilst simultaneously minimizing the distance to fulfilling the integer conditions.

The curse of dimensionality when considering binary decisions in power market games is discussed in Reference [86]. It approaches the problem by calculating the optimal binary variables from the perspective of each single player in the game forming a best-response function. The topic of multiple equilibria is discussed and the assumption is made, that a market operator selects the - from its own perspective - 'most optimal' Nash equilibrium .

Reference [70] outlines the problem of marginal cost bidding not considering traits such as reactive support and black start capability. The paper approximates the non-convex cost function with unit commitment decisions as its convex lower envelope, i.e. the convex hull. This convex formulation allows applicability on large-scale applications as it removes the curse of dimensionality. Compared to previous references, however, it does not allow to derive explicit startup and shutdown decisions and in turn cannot be applied to analyze problems as demonstrated in Chapter 4: influencing other players' dispatch decision via usage of market power or finding multiple equilibria under dispatch.

Similar to the previous reference, Reference [82] formulates a convex-hull problem approximation. A market problem considering startups/shutdowns, ramping constraints and ancillary services is reformulated via using the convex hull of a units feasible set and each units cost function respectively. After solving the problem via this approach, the duality gap is calculated as a measure of fitness of the computed results.

An approach different to binary schedules is presented in Reference [113], where state transitions in generators between periods is modeled via continuous ramp-rates. The problem is solved via applying conjectural variations (i.e. effectively modeling a single players output as a function of another players output) for a single-level and bi-level problem (where the ramp rates have to be determined prior to market clearing). It concludes that in a two-level approach which requires inflexible ramp-rates, i.e. ramp-rates stated ahead of market clearing, such as applied by the New York City ISO, might be contradicting to the goals of regulation. It states that a bi-level problem would create an additional layer of strategic behavior, that might not appear in single-level problems as e.g. on the flexible Nordic markets. Further, the paper discusses various methods to yield multiple Nash equilibria.

Reference [100] compares different approaches such as convex-hull reformulation and discusses pricing schemes such as uplift pricing (remunerating suppliers for inconsistencies between central and individual unit dispatch) in their capabilities to address the profit differences between the individual player's MI(L)P solutions and the results under marginal cost pricing. The model formulated in the paper aims to provide a minimum uplift for the players to cover losses but offer little leverage for achieving additional profits through gaming.

An algorithm to derive an equilibrium for a Cournot problem with integer variables is established in Reference [148]. However, the applied search algorithm does not consider capacity constraints for the variables and thus might show weaknesses in practical applications.

Reference [72] models a thermal-renewable system with a focus on gas power plants. It considers startup and shutdown decisions of those units on the basis of the associated cost components being the primary component in the operators' cost minimization (i.e. its unit scheduling). To embed these individual operator scheduling decisions in a Cournot market, a bi-level game based on best-response functions is established and solved similar to traditional KKT systems.

6.8 Ancillary Service Market Games

This section lists work which analyzes the impact of strategic players in ancillary service markets.

In Reference [15] the authors analyze a model for hydropower arbitrage that considers multiple market forms. In the presented deterministic model hydropower generates profits nearly exclusively on the reserve markets. This arguably highlights the importance of modeling the traits defining such a market (uncertainty and dispatch decisions) accordingly.

Reserve market bids are modeled via a Bayesian approach (not to be confused with the previously described Bayesian game) that assumes probability distributions for other members of the game apart of the analyzed player in Reference [147]. This player is considered to be bidding against a time-series of stepwise merit-orders that are linearized. The results indicate an incentive for price-makers to withhold capacity on reserve markets.

The model formulated in Reference [90] uses supply functions to establish a game under symmetrical information (marginal cost functions are known among participants). It solves a bidding problem for players participating in the German spot and reserve markets which are maximizing expected profits under marginal cost bids. Similar to the previously presented paper it focuses on the merit order and concludes that there is an incentive for the players to focus on providing reserve capacities.

6.9 Robust Optimization

For reasons mentioned in Chapter 3, robust optimization was the chosen approach to implement uncertainty in the later presented publications (as it is comparable to a mixed strategy approach over the scenarios). The methods applied were based on the following publications.

Reference [165] formulates a column-and-constraint generation algorithm as a robust optimization alternative to traditional cutting-plane methods such as Benders' decomposition.

Based on this work, Reference [164] extends the application of this algorithm to bi-level mixed integer problems under complete recourse.

Reference [25] proposes an adjustment to the KKT conditions that allows incorporating uncertainty distributionally robust via a minimum residual approach. Even though it shows a focus on price-taking suppliers in electricity markets, the concept might equivalently be applied on any (commodity) markets and competition formats.

Chapter 7

List of Publications

The presented publications resulting directly from the work on this dissertation are, in chronologically descending order:

POLICY	Markus Löschenbrand, Xiaomei Cheng, Magnus Korpås, "Exercising Market Power under Marginal Cost Bidding: the Future Development of European Power Market", [submitted to], *Energy Policy*, vol. -, no. -, pp. -, 2019
EJOR	Markus Löschenbrand, "Finding Multiple Equilibria in Power Systems via Machine Learning-supported Gröbner Bases," [submitted to] *European Journal of Operational Research*, vol. -, no. -, pp. -, 2018
ENERGY	Markus Löschenbrand, Wei Wei, Feng Liu, "Hydro-thermal power market equilibrium with price-making hydropower producers," [published in] *Energy*, vol. 164, no. 1, pp. 377 - 389, 2018
ITRANS	Markus Löschenbrand, Magnus Korpås, "Multiple Nash Equilibria in Electricity Markets with price-making Hydrothermal Producers," [published in] *IEEE Transactions on Power Systems*, vol. 34, no. 1, pp. 422 - 431, 2019
IAEE	Markus Löschenbrand, "Market Power in a Hydro-Thermal System under Uncertainty," [published in] proceedings of the *IAEE International Conference*, Groningen, 2018
EEM	Markus Löschenbrand, Magnus Korpås, Marte Fodstad, "Market Power in Hydro-Thermal Systems with Marginal Cost Bidding," [published in] proceedings of the *International Conference on the European Energy Market*, Łódź, 2018

POWERT Markus Löschenbrand, Hossein Farahmand, Magnus Korpås, "Impact of Inertial Response Requirements on a Multi Area Renewable Network," [published in] proceedings of *IEEE Powertech*, Manchester, 2017

The above presented publications were considered sufficient to answer the imposed research questions. Therefore, the following publications are results from the work on this dissertation but were chosen to not be further presented in this dissertation:

SEST Elise Tveita, Markus Löschenbrand, Sigurd Bjarghov, Hossein Farahmand, "Comparison of cost allocation strategies among prosumers and consumers in a cooperative game," [published in] proceedings of the *International Conference on Smart Energy Systems and Technologies*, Sevilla, 2018

ENGIES Markus Löschenbrand, Magnus Korpås, "Hydro Power Reservoir Aggregation via Genetic Algorithms," [published in] *Energies*, vol. 10, no. 12, 2017

PMAPS Markus Löschenbrand, Magnus Korpås, "An Agent Based Model of a Frequency Activated Electricity Reserve Market," [published in] proceedings of the *International Conference on Probabilistic Methods Applied to Power Systems*, Beijing, 2016

Figure 7.1 presents the developmental flow of the work during this dissertation visually. The following pages are dedicated to introduction of the publications.

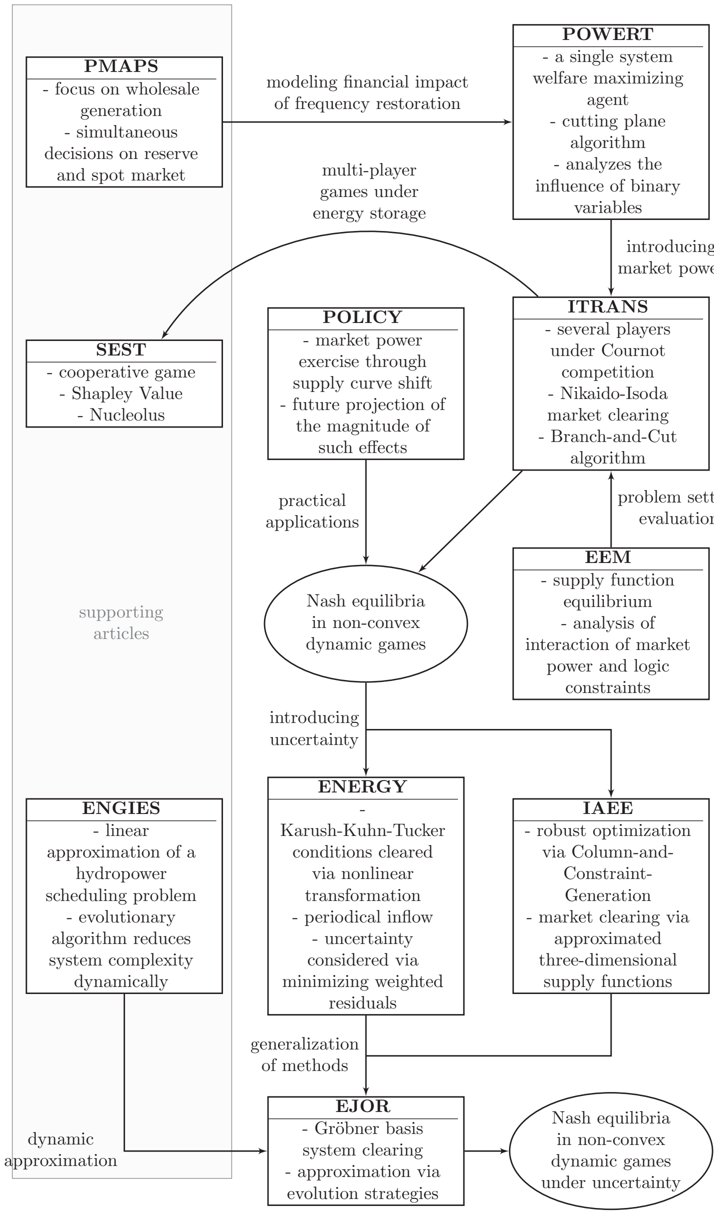


Figure 7.1: Publication Chart

7.1 Exercising Market Power under Marginal Cost Bidding: the Future Development of European Power Markets

This work is currently under review in *Energy Policy*.

7.1.1 Extended Abstract

This paper discusses the exercise of market power conducted by producers bidding under system marginal or unit marginal cost in energy markets under pay-as-bid or uniform pricing. It discusses two effects to conduct 'supply curve manipulation' with the goal of increasing supplier welfare under completely inelastic demand:

- Strategic Unit Commitment - removing *marginal capacity* in order to increase market clearing prices.
- Strategic Capacity Displacement - removing *infra-marginal capacity* in order to increase market clearing prices.

Further, the paper discusses traditional measures for market power such as the *Herfindahl-Hirschman Index* and explores the effects of a supply curve shift on such measures. The paper uses a numerical example to demonstrate that such measures might show a skewed picture of real market power, if base capacities are included.

It concludes in addition, that the traditional definition of supply-sided market power (i.e. "monopolists/oligopolists bid above marginal cost") does not hold for the proposed mechanism. This is proven by a complementarity model that emulates a market clearing. By nesting such a model in an optimization model of optimal capacity withdrawal, players are able to increase their profits.

The paper continues to use projections of future merit orders and supply elasticities in the Northern European power system to demonstrate that such effects can be expected to increase in future. Based on these models, a future growth in market power in systems under higher CO_2 emission cost and larger generation portfolio shares of renewables can be projected. It is further discussed, that the effects might increase on local levels where congested transmission lines might lead to steep increases of market power and higher amplitudes of such supply curve manipulation effects.

Exercising Market Power under Marginal Cost Bidding: the Future Development of European Power Markets

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Abstract

This paper introduces an alternate representation of market power and analyzes how this concept gains importance in liberalized electricity markets with higher renewable shares and higher CO_2 emission cost. In the proposed concept, market participants are able to lever their capacities and exercise market power whilst bidding at unit marginal (in markets with uniform prices) or system marginal cost (in markets with pay-as-bid pricing). The paper also shows how traditional measures such as the Herfindahl-Hirschman Index fail to capture such mechanisms without adjustment. A game-theoretical model and example case highlights how this theoretical effect can be utilized in practice by bidders with different supply elasticities that are bidding against an inelastic market demand. In addition, analysis of future projections of installed capacities in the European power system are analyzed and suggest an increased leverage to utilize the presented mechanism.

Keywords: Electricity Market, Renewables, Supply Functions, Game Theory, Europe

1. Introduction

Even though efforts in market liberalization have been generally focused on preventing arising market power, effects on market power asymmetries are screened for and observed regularly in electricity markets. In general, however, it is assumed that modern deregulated electricity markets on the whole have shown behavior similar to perfect competition, but display small scale/localized market power effects caused by e.g. congestion of distribution lines [1].

Such effects have to be taken into consideration regarding changing power systems, which are currently undergoing a global shift towards a higher generation portfolio share of intermittent capacities.

Several studies have approached this topic of market power in changing power systems. It has been shown that in systems under market power, benefits of exercising such are distributed favoring conventional units rather than intermittent generation [2]. Further have price impacts of emission taxation programs been discussed and found to affect thermal units disproportionally to traditional base load units [3].

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In addition, it has been shown previously that the emissions of the marginal units in electricity system determine the impact of emission prices on electricity prices [4, 5]. Compared to these models that assume market power in form of elastic demand functions and analyze the shift of plants within the merit order, this paper analyzes non-elastic demand functions and the increase in slope of the merit order curve. Thus, and as demonstrated below, the effects displayed in this paper are often less or not detected by traditional measures of market power.

Further, it seems intuitive that effects on connected factors such as electricity prices should be considered regarding emission pricing in equal manner. Implications on those systems require careful monitoring, especially considering the European Union has chosen a 'Learning-by-doing' approach with their implementation of emission trading and related effects [6]. Even though initial phases of the establishment of the market have been conducted, long-term implications by implementing emission schemes still have yet to come in effect. Thus, this paper aims to analyze the long-term impact of discouraging emissions in changing power systems under increasing shares of renewables financially, especially focusing on the effects on competition in the electricity market. Such analysis could provide essential in securing the power system against clustering of market power that allows utilities to distribute their risks onto other stakeholders in the system [7].

Indication has been found, that emission allowances positively impact the generator side profits, though not necessarily all generation firms, even if distributed through auctions [8]. In addition, similar effects can be observed for the market power of the generation firms, whereas previous studies have been conducted on electrical systems under price-elasticity [9].

In addition to markets for electric power, market power within emission markets such as the European Union's Emission Trading System (ETS) have been analyzed previously as well. This is particularly important considering that participants in emission markets do not have the incentive to minimize emissions but rather maximize their own profits, giving them incentives to inflate emission prices [10].

The work presented below, however, focuses on market power under inelastic demand curves, which can be considered a fitting assumption for electricity markets [11].

In the presented work, the definition of market power in the context of dynamic electricity markets will be analyzed. The authors show, that even under short-run inelastic prices, that were previously assumed to be a cause of market power influences [12], exercise of market power in order to increase supply side welfare is a possibility.

Further, the future importance of the presented mechanism is analyzed. It is estimated that through decreasing share of thermal units in the system, a survival-of-the-fittest effect might be observed. As such, the thermal generation units remaining in operation after portfolio restructuring could show a disproportionate potential to impact market prices through gaming.

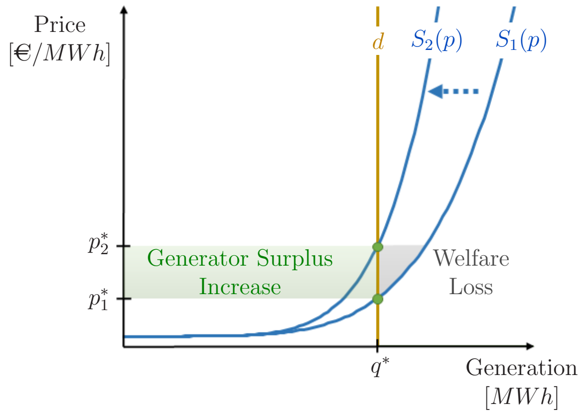
Such market power of thermal generators has been in discussion since the beginning of market deregulation [13], as the periods with the highest thermal generation portfolio shares preceded market liberalization. Modern studies aim to assess the impact of restructuring on the scheduling and generation optimization of electrical power systems. To provide an example, Ref. [14] studies an increase in intermittent generation capacities and the result on market power and ramping of generation units. Approaches to accurately model the impact of dynamic decisions on power system equilibria has been emerging only recently. Ref. [15] presents a model to describe discontinuous decisions in dynamic games between thermal producers and solves it without approximations for those discontinuous decisions. Ref. [16] describes a similar problem setup, with the addition of storage between time periods. Ref [17] describes a game under uncertainty, storage and consideration of ramping. Ref. [18] provides a game under uncertainty that also solves a scheduling problem within a single-level framework.

As available literature only recently approaching modeling this topic of discontinuous dynamic decisions in power markets, assessment of the impact of such decisions has been scarce at best [19]. This is the research gap this paper aims to close.

The authors of this work aim to contextualize the importance of applying dynamic gaming in strategic bidding regarding the changes within the European power system. As mentioned previously, the presented mechanism shows a novelty of application under non-elastic demand and also does not fulfill traditional criteria of market power. In addition, the importance of such effects is clearly highlighted by the projected generation portfolio of the European power system. As a result and as a starting point for future research, the presented mechanisms have to be observed and analyzed as well as additional work on both the practical implications as well as the model techniques might be advised.

	Nomenclature	
	Index	
g	generator	
$*$	equilibrium point	
	Set	
G	generation units/firms	
	Variables	
p	market price	[€/MWh]
q	quantity	[MWh]
Δq	withheld capacity	[MW]
	Parameters	
$\underline{q}$,$\bar{q}$	generation capacities	[MW]
d	demand	[MWh]
	Functions	
S	supply curve	[MWh]
C	cost function	[€]
ϵ	supply elasticity	
$\mathscr{L}^S$,$\mathscr{L}^D$	supply and demand side Lagrangian	
	Dual Variables	
$\underline{\mu}$,$\bar{\mu}$	shadow price of generation	
γ	shadow price of marginal bidding	
σ	shadow price of demand fulfillment	

Figure 1: Impact of Supply Curve changes on Market Clearing



2. Definition of Market Power

In this work, an exercise of market power by generation firms through exerting supply curve shifts is assumed [20, 21]. The core principle is illustrated in Fig.1, which shows the impact of such exercise of market power on an energy market under entirely non-elastic demand, where a supply-curve shift results in higher market prices. The resulting demand-side losses are given by the sum of the increase in generator surplus and the welfare losses. Latter are lost entirely, with the supply-side gains only being the generator surplus increase.

The supply curves are given by a merit order[1] of generation units participating in the market, whereas the units defining the clearing price (indicated with a $*$) are referred to as 'marginal units'. The units to the left of the demand can be referred to as 'infra-marginal units' and the units to the right as 'extra-marginal' [11].

In general, such supply curve shifts are usually observed by market operators. The traditional definition of exercising market power is the observation of *Price Bid* $>$ *Marginal Revenue* or *Price Bid* $>$ *Marginal Cost*. In Fig.1 this would be represented by a producer in the merit order of curve $S_1(p)$ bidding at price $p_2 > \frac{\partial C_g(q_g)}{\partial q_g}$ instead of price $p_1 = \frac{\partial C_g(q_g)}{\partial q_g}$. Assumed several participants in the market would act in similar manner, a new supply curve $S_2(p)$ could be established, which has the participants exert market power via bidding above their respective marginal cost.

Based the publicity of historical (cost) data[2] and market power measures such as price-cost margins [22] little to no indication of existence of market power via such bidding above marginal cost can be found in liberalized electricity systems such as in Northern Europe [23].

However, such an arguably narrow definition of market power could potentially

[1]The submitted price bids/cost curves in ascending order.

[2]An example for such is the online platform of the oldest liberalized power exchange Nordpool.

Figure 2: Conducting a Supply Curve shift through Unit Rescheduling

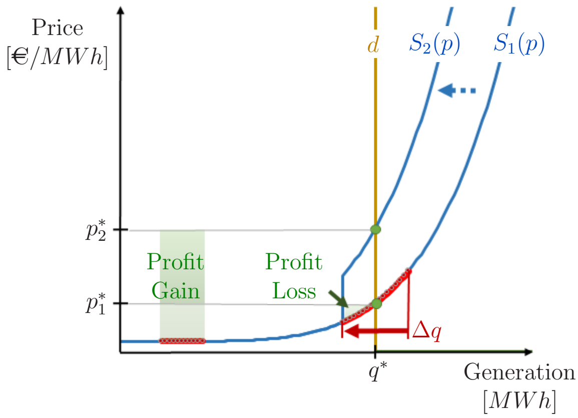
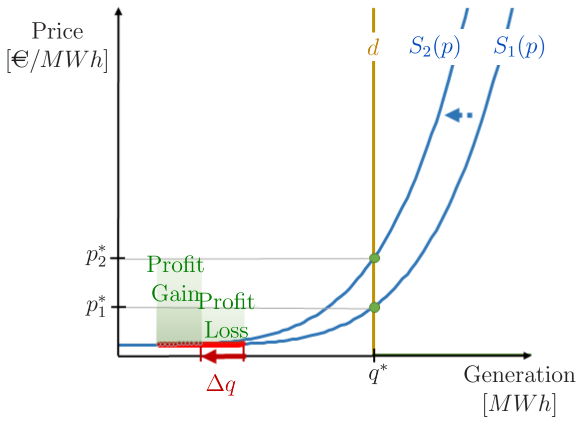


Figure 3: Conducting a Supply Curve shift through Capacity Withholding



lose validity in electricity markets of growing complexity. In traditional electricity market analysis, strategic bidding to obtain occasional profits by remuneration are conducted by suppliers utilizing renewable generation [24]. This is generally accepted as a requirement to assure investments into generation capacities and research. However, with renewable generation having close to no variable cost associated, every conducted bid would thus represent a bid above marginal cost and thus fulfill the criteria of market power exercise.

In addition, more complex systems with ancillary service markets, capacity and energy markets with different bidding/provision time frames and various clearing modes (options markets/intra-day markets/spot markets) and pricing details (uniform or pay-as-bid) may provide more room for such strategic bidding. A supplier recognizing their capabilities to exercise such could thus utilize these complexities and the difficulty for power market operators to differentiate between strategic bidding and exercise of market power.

This paper will analyze two mechanisms that provide levers to exercise such market power in order to increase generation firm profits in the short term:

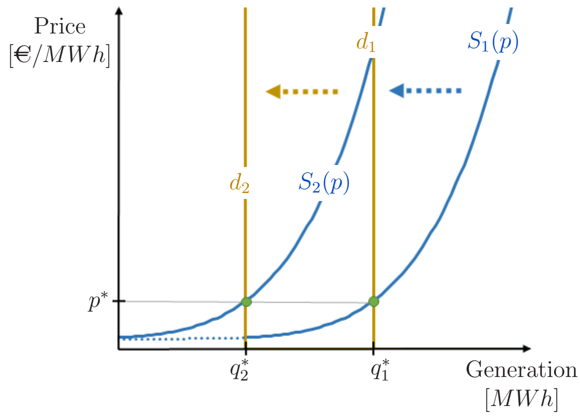
1. **Strategic Unit Commitment**. This is demonstrated in Fig.2, where a generation firm holding two units (indicated in red) is able to conduct a Supply Curve shift by withdrawing a quantity of Δq provided by the marginal unit from the market. This allows increasing profits (indicated in green) for the infra-marginal unit held by the generator whilst suffering a comparably minor loss in profits.
2. **Strategic Capacity Displacement**. This is shown in Fig.3, where a generation firm holding an infra-marginal unit (indicated in red) is able to conduct a Supply Curve shift by withdrawing quantity Δq (in this example: 50% of the available generation of the infra-marginal unit) from the market. This can be conducted through traditional exercise of market power, i.e. withholding available capacity. In addition, a similar effect is possible through storage operators transferring inventory to other time periods [25] or conventional generation units operating in multi-market/-area systems deploying in other market forms or areas [26]. This would not require any available generation capacity to be withheld and thus not fulfill the traditional definition of market power abuse.

Approach 1 is based on the assumption that strategic scheduling would allow the indirect withdrawal of capacity in certain selected time periods and thus artificially raise peak prices. This would be possible through strategically timed ramping of units aiming to exercise market power.

However, a similar effect could be created by Approach 2. A generation firm being able to participate in several market areas, i.e. a firm being both generator and arbitrageur, would be able to withdraw capacity in nodes where it could be convenient to increase prices.

Another form of such arbitrage would be withholding capacity by storing available capacity [27], i.e. conduct arbitrage over time periods. Such arbitrage would withhold capacity from an area or period important to a profit-maximizer but not directly withhold it, as the available capacity is not lost (as in Approach 1) but sold at another location or time.

Figure 4: Alternative Representation via Demand Shift



Nonetheless the approach, applying such mechanics strategically would require flexibility in the generation units or in other words, low required transition times in dispatch and scheduling.

Models analyzing strategic games between flexible and inflexible suppliers have been presented previously. This paper therefore does not intend to discuss these concepts further, as there is sufficient literature available discussing modeling techniques for approach 1 [28, 29, 15], approach 2 [26, 30, 25, 17] or both approaches combined [31, 16, 18].

However, this paper intends to discuss the future potential for conducting such supply curve shifts (i.e. *supply curve manipulation*), especially in the context of the European power system. The reason for this is given by the European mainland providing the largest liberalized power market in the world, making strategic bidding and related market power an important topic of research.

3. Supply Elasticity as a Driver for Market Power

As presented above, traditional measures of market power might not apply equivalently to power systems. Based on the previously introduced principles it can be argued that the pool of units able to exercise market power is significantly smaller than the total installed capacity. This is a result of the strict separation in base and peak load units, an effect that also allows for formulating equivalent supply function representations via a concept referred to in literature as *'demand shift'* [11]. As displayed in Fig.4, a supply-demand crossing can be equivalently represented via removing base load capacities (symbolized by the dotted blue line) from the merit-order (i.e. from the supply curve) and in parallel adjusting the demand by a similar generation level. The result of removing these base load capacities is a similar price and a similar (though adjusted) generation level. However, in Fig.4, the new representation of $S_2(p)$ shows, compared to $S_1(p)$, a smaller range of observed generators in the market clearing cross. This resulting market clearing model

Table 2: Numerical Example for Herfindahl-Hirschman Index under Demand Shift

firm	$\bar{q}_g^{\text{peak}}[MW]$	$\bar{q}_g^{\text{base}}[MW]$	total market share [%]	peakers' market share [%]	HHI:
$g=1$	15	15	20	30	
$g=2$	15	15	20	30	total: 2000
$g=3$	10	20	20	20	peakers: 2400
$g=4$	5	25	20	10	
$g=5$	5	25	20	10	

has similar characteristics to the original with the only difference being the starting point at the x-axis now being the total capacity of the removed base load units instead of 0 MWh.

Even though the clearing price p^* stays on a similar level, results for traditional measures of market power yield different results for such an alternative representation. An example of such a measure is the Herfindahl-Hirschman Index (HHI) that analyzes market shares [32]. This measure for *total* (i.e. peak+base) and *peak capacity only* can be calculated the following:

$$\begin{aligned} \text{HHI}_{\text{total}} &= \sum_{g \in G^{\text{total}}} \Big(\frac{100\bar{q}_g}{\sum_{g_2 \in G^{\text{total}}} \bar{q}_{g_2}}\Big)^2 \\ \text{HHI}_{\text{peak}} &= \sum_{g \in G^{\text{peakers}}} \Big(\frac{100\bar{q}_g}{\sum_{g_2 \in G^{\text{peakers}}} \bar{q}_{g_2}}\Big)^2 \end{aligned} \quad (1)$$

A generation firm owning little but flexible peak capacities (i.e. units that have fast ramping times such as coal power) might have stronger leverage exercising its market power than a firm holding large inflexible base load capacities (i.e. intermittent production or nuclear power). A numerical example showing how the consideration of base load capacity shifts the observed market power is provided in Table 2.

To highlight this assumption, the price elasticity of supply, or in short *'supply elasticity'*, might be analyzed. This measure is defined by the change in quantity and the related price effect by such a quantity change:

$$\epsilon(q) = \frac{\partial S(q)}{\partial q}\frac{q}{S(q)} \quad (2)$$

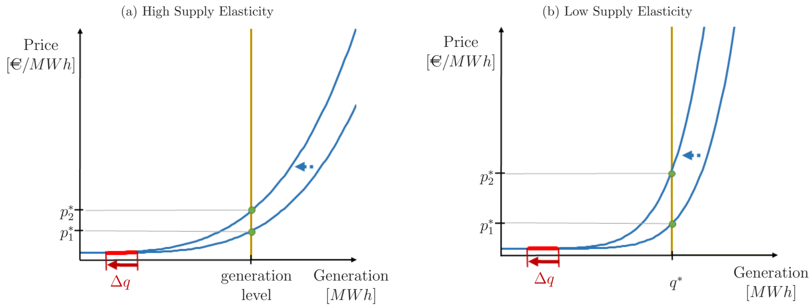
A lower *extra-marginal price elasticity of supply* (i.e. $\epsilon(q^*)$ at $q^* = d$) would thus mean a larger market clearing price impact of conducting a Supply Curve shift via exercising market power.

This is illustrated by the examples given in Fig.5. Fig.5a shows a high and Fig.5b a low Supply Elasticity for marginal and extra-marginal units and inflexible demand. The result is a different price effect by a Supply Curve shift as a result of withdrawing a similarly sized base load capacity (indicated in red).

Based on this, it can be expected that measures increasing (extra-marginal) price elasticity of supply will lead to more effective market power of flexible peak units.

In addition, one considerable factor might be *information asymmetry*: extra-marginal bids are mostly not published, leading to system participants potentially not being able to determine their own market power. However, for markets with transparent data platforms this factor can be insignificantly minor, as competitors

Figure 5: Extra-marginal Price Elasticity of Supply ($\epsilon(q^*)$)



are able to retrieve such information by assumptions based on historical data. In addition, it can also be assumed that a market participant will likely be aware of the network topologies and surrounding competitors, as locations, sizes and types of power plants are public information.

In previous studies, demand elasticity has been regarded as a driver for market power [26, 2]. Our presented novel principle however allows market power to be exercised under inelastic demands. This principle will further be demonstrated by a competition model in the following section.

4. A Game-Theoretical Framework

The model presented in this section will be based on the assumption that the generators either participate in a market with uniform prices or are in a pay-as-bid market with enough public information to bid (approximately) at the price level of the system marginal units[3]. This assumption can be considered valid due to the previously introduced mechanisms where most power markets are build on the existence of a gap between marginal cost curves and market prices in the short term that is meant to reimburse generators for long term investments [11].

For the sake of simplicity, the model will not consider network flows, different market types, nodes/areas and is restricted to a single time period. A single energy market is considered, where a number of generation companies each ($\forall g$) aims to maximize their profits whilst staying within their generation capacity limits:

$$\begin{aligned} &\max_{q_g} p\, q_g - C_g(q_g) \\ \text{s.t.} \quad &\underline{q}_g \leq q_g \leq \bar{q}_g \qquad (\underline{\mu}_g, \bar{\mu}_g) \end{aligned} \tag{3}$$

Based on these individual generator decisions, a market operator clears the system for a minimum price. This market operator has to consider that the participants have to be paid their marginal costs $\frac{\partial C_g(q_g)}{\partial q_g}$ at minimum, as otherwise they will not choose to supply the market (as they would be operating at a loss):

[3]Bids below system marginal cost in a pay-as-bid scheme would still allow for a similar effect but were excluded for the sake of presentational simplicity.

$$\begin{aligned} &\min_p \sum_g p\, q_g \\ \text{s.t.}\quad & p \geq \tfrac{\partial C_g(q_g)}{\partial q_g} \quad \forall g \quad (\gamma_g) \end{aligned} \tag{4}$$

The market is cleared to an equilibrium by matching supply to demand. In the here presented example, completely inelastic demand is considered:

$$\sum_g q_g = d \qquad (\sigma) \tag{5}$$

This system can be reformulated for an equilibrium by deriving its' Karush-Kuhn-Tucker (KKT) conditions which are shown in Appendix A [26, 33].

Disregarding its simplicity, the presented model allows for adding a characteristic novel in literature that displays profit-maximizing generators being able to exercise market power in a market with *marginal-cost bidding* and *no demand elasticity*. In classical literature, both of these traits were considered detrimental to the ability of such market power exercise [34].

In the here presented model, however, such producers exercising market power (denoted as G^{ex}) are emulated via a bi-level profit-maximization problem:

$$\begin{array}{rll} & \max\limits_{\Delta q_g \in G^{\text{ex}}} \sum\limits_{g \in G^{\text{ex}}} p\, q_g - C_g(q_g) & \\ \text{s.t.} & \text{Eq.(A.1)} & \\ \text{Eq.(A.1c)} \rightarrow & 0 \leq \bar{\mu}_g \perp q_g - \bar{q}_g + \Delta q_g \leq 0 & \forall g \in G^{\text{ex}} \\ & \Delta q_g \leq \bar{q}_g - \underline{q}_g & \forall \in G^{\text{ex}} \end{array} \tag{6}$$

The lower level problem is the market clearing formulated as the KKT conditions in Eqs.(A.1). The upper level problem is the profit maximization of all units that choose to collude and cause a supply curve shift as presented in Fig.3: through a capacity reduction by a quantity of Δq.

The model shows that collusion between several players is not even a necessity to exercise market power in such a system. Table 3 displays this by the example of a generator providing a system-marginal unit. By withdrawing available generation this player increases the market prices and subsequently increases the cost for demand fulfillment (i.e. decreasing the demand surplus) whilst increasing the profits of all generators (i.e. increasing the supply surplus). By increasing the elasticity of the supply functions (from Table 3a to Table 3b), the increase in supply surplus rises from 24% to 27.5%. This enforces the assumption from the previous section: a higher price elasticity of supply leads to a higher reward for the exercise of market power.

Based on this, the following chapter will analyze the changes in supply elasticity in the European power market. This is meant to give an estimate of the severity of exercising the previously introduced supply curve manipulation.

5. Future Projection of Price Elasticity of Supply

Fig.6 shows the estimated future changes in the European power generation [35]. Classifying hydropower, gas, oil and solids as peak load (a rather conservative approach, as hydropower includes run-of-river plants which closely resembles

Table 3: Exercise of Market Power at Marginal Cost Bidding

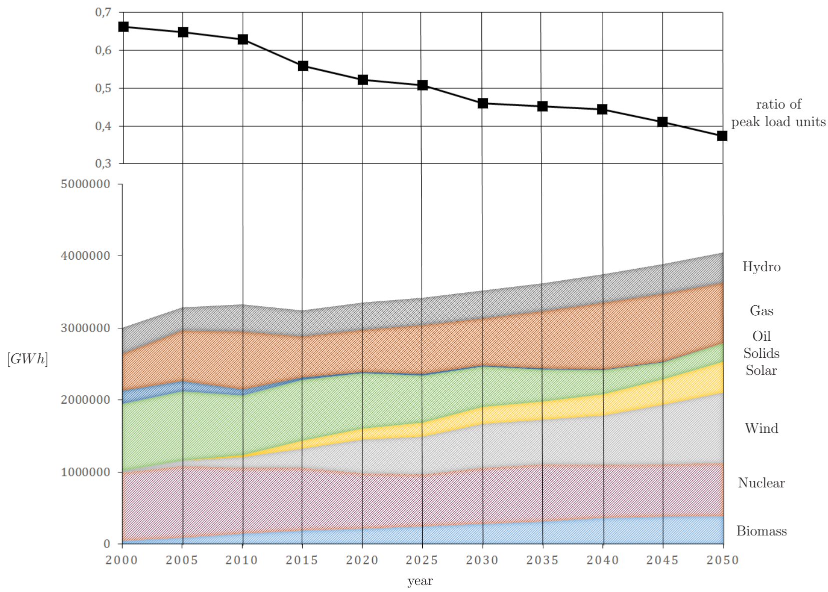
(a) Case 1: Example Case

generators	$g = 1$	$g = 2$	$g = 3$	$g = 4$	market operator	
$C_g(q_g) =$	$0q_1$	$7q_2 + 0.02q_2^2$	$5q_3 + 0.015q_3^2$	$5q_4 + 0.01q_4^2$		
$[\underline{q}_g, \bar{q}_g] =$	$[10, 200]$	$[150, 850]$	$[100, 800]$	$[50, 350]$	$d =$	1700
no quantity withheld:						
$q_g^* =$	200	464.3	685.7	350		
$\frac{\partial C_g(q_g^*)}{\partial q_g^*} =$	0	25.6	25.6	12	$p^* =$	25,6
$p^*\, q_g^* - C_g(q_g^*) =$	5114.3	4311.2	7053.1	5975	$\sum_g p^*\, q_g^* =$	43471.4
colllusion between generators $g = 1, 2$						
$q_g^* =$	200	350	800	350		
$\frac{\partial C_g(q_g^*)}{\partial q_g^*} =$	0	21	29	12	$p^* =$	29
$p^*\, q_g^* - C_g(q_g^*) =$	5800	5250	9600	7175	$\sum_g p^*\, q_g^* =$	49300

(b) Case 2: Higher Supply Elasticity

generators	$g = 1$	$g = 2$	$g = 3$	$g = 4$	market operator	
$C_g(q_g) =$	$0q_1$	$7q_2 + 0.04q_2^2$	$5q_3 + 0.03q_3^2$	$5q_4 + 0.02q_4^2$		
$[\underline{q}_g, \bar{q}_g] =$	$[10, 200]$	$[150, 850]$	$[100, 800]$	$[50, 350]$	$d =$	1700
no quantity withheld:						
$q_g^* =$	200	478.6	671.4	350		
$\frac{\partial C_g(q_g^*)}{\partial q_g^*} =$	0	45.3	45.3	19	$p^* =$	45.3
$p^*\, q_g^* - C_g(q_g^*) =$	9057.1	9161.2	13524.5	11650	$\sum_g p^*\, q_g^* =$	76985.7
colllusion between generators $g = 1, 2$						
$q_g^* =$	200	350	800	350		
$\frac{\partial C_g(q_g^*)}{\partial q_g^*} =$	0	34	53	19	$p^* =$	53
$p^*\, q_g^* - C_g(q_g^*) =$	10600	11200	19200	14350	$\sum_g p^*\, q_g^* =$	90100

Figure 6: European Electricity Portfolio



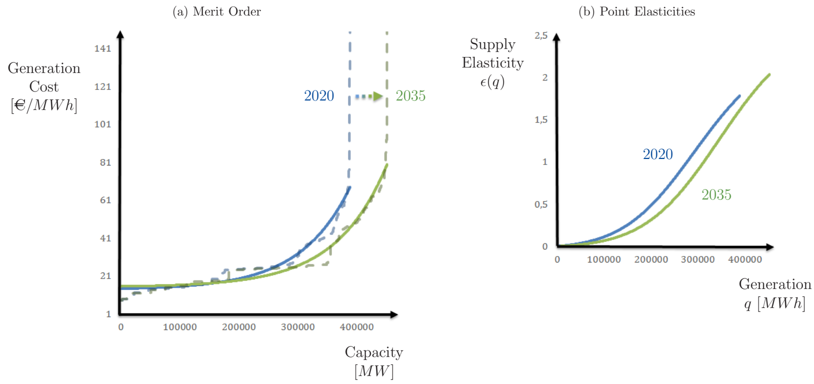
the characteristics of traditional renewables such as wind and solar) allows to illustrate the projected decreasing ratio of peak to base load units which shows a clear downward trend.

Wholesale effects of such merit-order changes have been discussed in the Italian [36], German [37] and Slovak [38] wholesale electricity markets. These studies find significant decreases in electricity wholesale prices attributed to those generation portfolio shifts. In addition and aligned with the assumptions imposed in the previous sections, these studies show that remuneration of flexibility in production (which is generally associated with peak producers) might increase. Such an increasing spread between base and peak prices would lead to sinking elasticities around the marginal units (as shown in Fig.5).

Another aspect is provided by the expected future increase of CO_2 emission cost [39, 3]. Such an increase will lead to an increase in generation cost which the end customers will have to compensate for through higher marginal cost. This will have an effect on the merit order and decrease supply elasticity whilst increasing the market power of the system participants.

Fig.7 illustrates this by the projected development of the merit order of the Northern European Power System (Germany, Denmark, Sweden, Norway, Finland, Netherlands)[39]. As seen in Fig.7b, the fitted exponential functions (Fig.7a: continuous lines) of the projections of the generation portfolios of the years 2020 and 2035 (Fig.7a: dotted lines) showed lower point elasticities for similar quantities. This supports the previous assumptions on changes in elasticity. The effect might be traced back to the two previously mentioned factors: an increase in CO_2 emission

Figure 7: Prognosed Change in the Northern European Power System



cost and a reduction of thermal units in the generation portfolios.

In addition can be observed that the elasticities vary greatly depending on location. To provide an example, as it can be observed in Fig.8, North-Eastern Germany shows point elasticities ranging between 0 to 0.05 depending on capacity utilization. This suggests, that specific areas might be more prone to market power exercise via supply curve manipulation. This is aligned with the assumptions of Ref.[1], that market power increases on the local levels.

6. Discussion

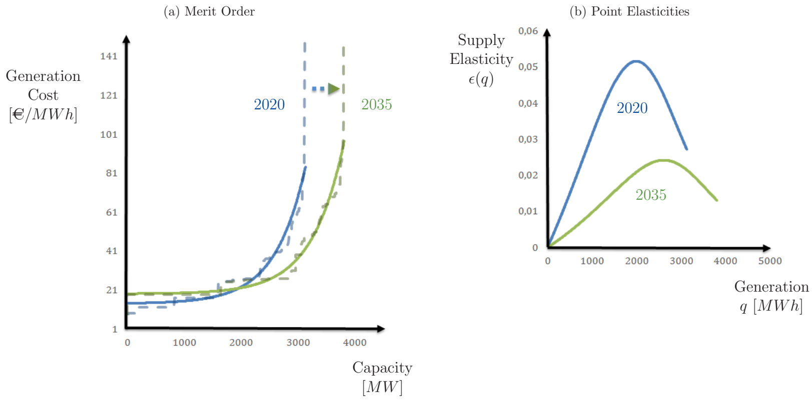
The here presented work introduces a new form of market power exercise: manipulation of supply curves. Through capacitating infra-marginal units, the system marginal price is shifted and participants in such commodity markets are able to yield a higher surplus. This comes at the cost of a loss in consumer surplus and subsequently system welfare. The novelty in this view is that producers are not required to bid above system marginal cost in pay-as-bid markets and unit marginal cost in uniform markets.

Due to this, the traditional definition of market power (i.e. bidding above marginal cost) might not necessarily apply, and traditional measures such as the Herfindahl-Hirschman Index might misrepresent reality. This comes as a result of a rising share of intermittent generation and reduced competition between the remaining flexible units in the system. These, mostly thermal, units are also facing increased marginal cost due to increasing CO_2 emission cost and thus will have an increased leverage in such a market power exercise.

For sake of generality, the paper does not specify the proposed CO_2 emission scheme. However, popular mechanisms such as carbon taxes or emission trading schemes can be expected to raise generation cost for the system-marginal thermal units and thus also increase market prices [40].

The increased lever yielded by such an effect is discussed by a game-theoretic model and an analysis of supply curve elasticities. Both the supplied case study

Figure 8: Prognosed Change in North-Eastern Germany



and data from the European power markets enforce the notion that the presented mechanism of 'supply curve manipulation' might rise in importance in future power systems under liberalized markets.

Further, increasing granularity indicates an increase in this effect. This can be expected, as competition on smaller scales (e.g. zones separated by congested lines) can only be expected to increase compared to the total system level. Thus, further research on this matter is suggested by the authors of this paper, in order to support power system decision makers to detect and screen for the mechanisms introduced in this paper.

Appendix A. Karush-Kuhn-Tucker conditions

The Karush-Kuhn-Tucker conditions for Eqs. (3), (4) and (5) can be formulated the following:

$$\frac{\partial \mathscr{L}^S}{\partial q_g} = -p + \frac{\partial C_g(q_g)}{\partial q_g} - \underline{\mu}_g + \bar{\mu}_g + \frac{\partial^2 C_g(q_g)}{\partial^2 q_g}\gamma_g + \sigma \quad \forall g \tag{A.1a}$$

$$0 \leq \underline{\mu}_g \perp \underline{q}_g - q_g \leq 0 \quad \forall g \tag{A.1b}$$

$$0 \leq \bar{\mu}_g \perp q_g - \bar{q}_g \leq 0 \quad \forall g \tag{A.1c}$$

$$\frac{\partial \mathscr{L}^D}{\partial q_g} = \sum_g q_g - \sum_g \gamma_g = 0 \tag{A.1d}$$

$$0 \leq \gamma_g \perp \frac{\partial C_g(q_g)}{\partial q_g} - p \leq 0 \quad \forall g \tag{A.1e}$$

$$\sum_g q_g - d \tag{A.1f}$$

$$\underline{\mu}_g, \bar{\mu}_g, \gamma_g \in \mathbb{R}^+ \quad \forall g \tag{A.1g}$$

$$\sigma \in \mathbb{R} \tag{A.1h}$$

Assuming convex cost functions C allows the problem to fulfill Slater's condition [41], which assures no duality gap. This means that an optimal equilibrium solution q^*,p^* can be yielded as long as the model parameters render the problem feasible.

[1] Fridolfsson, S.-o. & Tangera, T. P. Market power in the Nordic electricity wholesale market : A survey of the empirical evidence. *Energy Policy* **37**, 3681–3692 (2009).

[2] Twomey, P. & Neuhoff, K. Wind power and market power in competitive markets. *Energy Policy* **38**, 3198–3210 (2010). URL http://dx.doi.org/10.1016/j.enpol.2009.07.031.

[3] Rocchi, P., Serrano, M. & Roca, J. The reform of the European energy tax directive : Exploring potential economic impacts in the EU27. *Energy Policy* **75**, 341–353 (2014). URL http://dx.doi.org/10.1016/j.enpol.2014.09.022.

[4] Chen, Y., Sijm, J., Hobbs, B. F. & Lise, W. Implications of CO2emissions trading for short-run electricity market outcomes in northwest Europe. *Journal of Regulatory Economics* **34**, 251–281 (2008).

[5] Lise, W., Sijm, J. & Hobbs, B. F. The Impact of the EU ETS on Prices, Profits and Emissions in the Power Sector: Simulation Results with the COMPETES EU20 Model. *Environmental and Resource Economics* **47**, 23–44 (2010).

[6] Parker, L. CRS Report for Congress The Barcelona Process : The European Union's Emissions Trading System /EU-ETS). Tech. Rep. (2006).

[7] Larsen, E. R., van Ackere, A. & Osorio, S. Can electricity companies be too big to fail? *Energy Policy* **119**, 696–703 (2018).

[8] Burtraw, D., Palmer, K., Bharvirkar, R. & Paul, A. The effect on asset values of the allocation of carbon dioxide emission allowances. *Electricity Journal* **15**, 51–62 (2002).

[9] Zhao, J., Hobbs, B. F. & Pang, J.-S. Long-Run Equilibrium Modeling of Emissions Allowance Allocation Systems in Electric Power Markets. *Operations Research* **58**, 529–548 (2010). URL http://pubsonline.informs.org/doi/abs/10.1287/opre.1090.0771.

[10] Hintermann, B. Market Power in Emission Permit Markets: Theory and Evidence from the EU ETS. *Environmental and Resource Economics* **66**, 89–112 (2017).

[11] Stoft, S. *Power System Economics - Designing Markets for Electricity* (Wiley, New York, 2002).

[12] Borenstein, S. Understanding Competitive Pricing and Market Power in Wholesale Electricity Markets. *The Electricity Journal* **13**, 49–57 (2000).

[13] Newbery, D. M. Privatisation and liberalisation of network utilities. *European Economic Review* **41**, 357–383 (1997).

[14] Traber, T. & Kemfert, C. Gone with the wind? - Electricity market prices and incentives to invest in thermal power plants under increasing wind energy supply. *Energy Economics* **33**, 249–256 (2011). URL http://dx.doi.org/10.1016/j.eneco.2010.07.002. arXiv:1006.5371v1.

[15] Huppmann, D. & Siddiqui, S. An exact solution method for binary equilibrium problems with compensation and the power market uplift problem. *European Journal of Operational Research* **266**, 622–638 (2018). 1504.05894.

[16] Löschenbrand, M. & Korpas, M. Multiple Nash Equilibria in Electricity Markets with price-making Hydrothermal Producers. *IEEE Transactions on Power Systems* **34**, 422–431 (2019).

[17] Moiseeva, E. & Hesamzadeh, M. R. Bayesian and Robust Nash Equilibria in Hydro-Dominated Systems under Uncertainty. *IEEE Transactions on Sustainable Energy* **-**, 1–12 (2017).

[18] Löschenbrand, M., Wei, W. & Liu, F. Hydro-thermal power market equilibrium with price-making hydropower producers. *Energy* **164**, 377–389 (2018).

[19] Munoz, F. D., Wogrin, S., Oren, S. S. & Hobbs, B. F. Economic Inefficiencies of Cost-Based Electricity Market Designs (2017).

[20] Kirschen, D. Demand-Side View of Electricity Markets. *IEEE Transactions on Power Systems* **18**, 520–527 (2003).

[21] Löschenbrand, M., Korpås, M. & Fodstad, M. Market power in hydro-thermal systems with marginal cost bidding. In *International Conference on the European Energy Market, EEM*, 1–5 (2018).

[22] Borenstein, S., Bushnell, J., Kahn, E. & Stoft, S. Market power in California electricity markets. *Utilities Policy* **5**, 219 – 236 (1995).

[23] Hjalmarsson, E. Nord Pool: A power market without market power (2000). URL http://gupea.ub.gu.se/handle/2077/2838.

[24] Steeger, G., Barroso, L. A. & Rebennack, S. Optimal Bidding Strategies for Hydro-Electric Producers: A Literature Survey. *IEEE Transactions on Power Systems* **29**, 1758–1766 (2014).

[25] Førsund, F. R. *Hydropower Economics* (Springer, 2015), 2 edn.

[26] Hobbs, B. F. Linear Complementarity Models of Nash Cournot Competition in Bilateral and POOLCO Power Markets. *IEEE Transactions on Power Systems* **16**, 194–202 (2001).

[27] Bushnell, J. A Mixed Complementarity Model of Hydrothermal Electricity Competition in the Western United States. *Operations Research* **51**, 80–93 (2003).

[28] Gabriel, S. A., Siddiqui, S. A., Conejo, A. J. & Ruiz, C. Solving Discretely-Constrained Nash-Cournot Games with an Application to Power Markets. *Networks and Spatial Economics* **13**, 307–326 (2013).

[29] Moiseeva, E., Wogrin, S. & Hesamzadeh, M. R. Generation flexibility in ramp rates: Strategic behavior and lessons for electricity market design. *European Journal of Operational Research* **261**, 755–771 (2017).

[30] Molina, J. P., Zolezzi, J. M., Contreras, J., Rudnick, H. & Reveco, M. J. Nash-Cournot Equilibria in Hydrothermal Electricity Markets. *IEEE Transactions on Power Systems* **26**, 1089–1101 (2011).

[31] Contreras, J., Candiles, O., De La Fuente, J. I. & Gomez, T. A cobweb bidding model for competitive electricity markets. *IEEE Transactions on Power Systems* **17**, 148–153 (2002).

[32] Ventosa, M., Baíllo, Á., Ramos, A. & Rivier, M. Electricity market modeling trends. *Energy Policy* **33**, 897–913 (2005).

[33] Gabriel, S. A., Conejo, A. J., Fuller, J. D., Hobbs, B. F. & Ruiz, C. *Complementarity Modeling in Energy Markets* (Springer, New York, 2013).

[34] Berry, C. A., Hobbs, B. F., Meroney, W. A., O'Neill, R. P. & Stewart, W. R. Understanding how market power can arise in network competition: A game theoretic approach. *Utilities Policy* **8**, 139–158 (1999).

[35] European Commission. EU Reference Scenario 2016 - Energy, Transport and GHG Emissions - Trends to 2050. Tech. Rep. (2016).

[36] Clò, S., Cataldi, A. & Zoppoli, P. The merit-order effect in the Italian power market: The impact of solar and wind generation on national wholesale electricity prices. *Energy Policy* **77**, 79–88 (2015). URL `http://dx.doi.org/10.1016/j.enpol.2014.11.038`.

[37] Benhmad, F. & Percebois, J. Photovoltaic and wind power feed-in impact on electricity prices : The case of Germany. *Energy Policy* **119**, 317–326 (2018). URL `https://doi.org/10.1016/j.enpol.2018.04.042`.

[38] Janda, K. Slovak electricity market and the price merit order effect of photovoltaics. *Energy Policy* **122**, 551–562 (2018). URL `https://doi.org/10.1016/j.enpol.2018.07.021`.

[39] Sintef Energy Research. Balance management in multinational power markets - WP2: documentation and analysis of present costs. Tech. Rep. (2008).

[40] Chiu, F. P., Kuo, H. I., Chen, C. C. & Hsu, C. S. The energy price equivalence of carbon taxes and emissions trading-Theory and evidence. *Applied Energy* **160**, 164–171 (2015). URL `http://dx.doi.org/10.1016/j.apenergy.2015.09.022`.

[41] Boyd, S. & Vandenberghe, L. *Convex Optimization* (Cambridge University Press, Cambridge, 2009), 7 edn.

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7.2 Finding Multiple Equilibria in Power Systems via Machine Learning-supported Gröbner Bases

This work is currently under review in the *European Journal of Operational Research.*

7.2.1 Extended Abstract

This paper extends the principle of applying Gröbner basis calculations on optimization problems on KKT conditions to find multiple extreme points in multi-player systems. It discusses the advantages of doing this compared to other market/system clearing approaches such as using linear approximations of the complementarity constraints and applying MI(L)P solvers. These advantages are:

1. solutions incorporate all potential equilibum solutions.

2. possibility to display asymmetric problems (more decision variables than equations are possible).

3. non-convex player problems are a possibility.

The first advantage stems from that a Gröbner basis is only a reformulation of the original problem and not an approximation. This could save a model user complicated and potentially error-prone problem analysis that are a necessity for approaches that yield only single equilibria. Such analysis would e.g. come in the form of branching a tree for the complementarity constraints or - in the simplest case - solving the problem repeatedly with different variable starting values for the mathematical solver software/method.

The second advantage gives a model user the option to incorporate homogeneous functions. This means that instead of e.g. setting explicit numerical parameters for uncertainties such as price elasticity of demand, scenario outcomes or available resources, these parameters can be formulated as an external variable. The resulting Nash equilibrium will then come in the form of a function of those external variables instead of explicit numerical values. This gives the possibility to e.g. formulate distributional robust problems or conduct dynamic sensitivity analysis by turning a parameter of an original symmetric problem into a variable.

The third advantage stems from that Gröbner bases are derived by removing the variable limitations (such as e.g. for a decision x there is $x \in \mathbb{Z}^+$) before solving the problem for its equilibria. Later, those limitations are applied again to restrict the solution set. This allows to consider non-convexity without applying any additional methods such as branch-and-bound, bi-level problem formulations or cutting-plane algorithms.

Furthermore, the paper proposes a dynamic approximation scheme for the Karush-Kuhn-Tucker conditions, based on multiple regression conducted through evolutionary strategies. This approximation allows fitting more computationally complex player problems into polynomial form, fit for application in Gröbner basis derivation.

A case study on price-making hydropower players is presented, that considers unit commitment, head-tail relations, periodic inflows and both energy and capacity market bids. Via the Nikaido-Isoda function as a fitness measure, a total profit difference of around 5% to a Nash Equilibrium situation is assumed. The case study is further extended by making inflows uncertain under unknown distributions. In this case, a dynamic sensitivity analysis on the market clearing prices is conducted.

Due to the methods' general formulation, applicability beyond the power sector and beyond market applications can be assumed, with problems such as production and storage decisions and routing problems under inventory holding.

Finding Multiple Nash Equilibria via Machine Learning-supported Gröbner Bases

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Abstract

This paper demonstrates a new approach to yield all potential equilibria in a system with multiple actors by computing the Gröbner basis of the Karush-Kuhn-Tucker conditions. Further, it discusses the advantages of choosing this method over traditional numerical approaches.

In addition, it provides a concept that applies machine learning, specifically evolution strategies, to approximate utility functions. This is done in order to remove the requirement for solving the dual problem of the system and thus allows to scale the equilibrium computation method to larger, more complex problem setups. This is demonstrated by a novel case study on non-convex players in form of hydropower generators operating simultaneously on spot and reserve markets whilst considering startups and shutdowns, periodical inflows and non-linear water conversion head effects. The model is further extended by considering uncertainty under unknown distributions and analyzes this case via dynamic sensitivity analysis.

Keywords: Game Theory, OR in energy, Evolutionary computations, Non-linear Programming

1. Introduction

Restructuring of power systems results in changing challenges for its' stakeholders. Such include new regulations and market types as well as changes in generation portfolios. Similar to other commodity markets, interaction in such systems has traditionally been modeled as games between units operating at or close to the marginal price of the system[1, 2]. However, indication exists that such marginal players might be influenced by infra-marginal players with lower[3] or infinitesimally low cost functions[4]. This influence can result in a rise of new problems for system and market designers, such as increasing CO_2 emissions due to wind power plant operators aiming to minimize their 'spillage' and therefore offering negative bids, causing marginal units to react by changing dispatch and schedules[5]. Few models on such non-convex decisions have been proposed in literature[6, 7] and solved for multiple equilibria[8, 9, 4, 10]. However, existing literature focuses exclusively on non-convexity in form of binary variables. Even though an important cause of such non-convexity in power systems, the approaches might be limited to such applications and not be

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applicable to other highly non-linear problems[1].

Thus, in practical applications and with the aim to cover large scale systems, traditional approximation techniques are applied in order to ensure problem convexification[11, 12]. Applied on systems with multiple participants with individual objectives, such convex approximations might result at a cost of model accuracy. To attest this, approaches allowing mixed strategies have been shown to be able to derive multiple Nash equilibria[13], but might show too large of distances to exact solutions to be applied for practical decision making[7].

Based on this lack of literature, this paper proposes a machine-learning based approach to dynamically approximate decision problems as polynomials. Such polynomials have been used successfully in previous literature to yield multiple equilibria[14, 13]. The approach in this paper is based on the concept of Gröbner bases[15], which provide an equivalence system to a set of polynomials. It has been shown prior that this reformulation (not to be mistaken with an approximation) can successfully be applied on single-player problems[16] and multi-player games with a limited number of given decisions[17, 14] in order to solve these algebraically. The concept proposed below offers a formulation based on Karush-Kuhn-Tucker conditions that allows the players a range of continuous (and therefore unlimited) decisions and a similar range of potential equilibria. Further, it shows that by applying machine-learning, the main issue of Gröbner bases - scalability - can be mitigated to a level that allows users to apply modern Gröbner basis computation algorithms such as Refs.[18, 19] to present-day problem setups. In comparison to previous Nash approximation algorithms which approximate stationary points[20, 21], the proposed methodology provides an exact algebraic reformulation of the set of all stationary points fulfilling the conditions of a Nash equilibrium in a game between dynamically approximated agents.

In addition to displaying all Nash equilibria, this paper shows additional advantages of applying Gröbner bases such as solving games with variable inputs, essentially extending the application area of equilibrium models to an entirely new class of problems even beyond the demonstrated applications in power systems.

General Nomenclature

Indices:	
j	player
k	element of the decision vector
Decision Variables:	
$Y_j \in Y$	decisions of player j
$\widetilde{Y}$	approximated decisions of player j

[1] An example of such is provided by the combination of non-linear efficiency curves and binary schedules in the later presented case study.

X	"global" system/market decisions
σ,λ	dual variables
Parameters:	
θ	polynomial constants
Functions:	
U	utility function
f	payoff function
h	equality constraint
g	inequality constraint
m	clearing constraint
p	polynomial function
error	error function
ψ	Nikaido Isoda function

2. System/Market Clearing via Gröbner Bases

Originally presented in the PhD Thesis of Bruno Buchberger and subsequently named after his advisor, Gröbner bases allow simplification of equation systems via establishing representative polynomials[15]. The derivation is a generalization of three concepts: Gaussian elimination, the Euclidean algorithm and the Simplex Algorithm[22]. This does not only allow to derive 'solution rules' e.g. in the form of "in the optimal solution the factor 1 has to be equivalent to 10 times factor 2" but also to define the number/range of real(/complex) solutions. Latter provides a particularly important question in economics regarding multiplicity of market equilibria. Ref.[14] approaches this by demonstrating potential applications to find equilibria e.g. in games with a limited number of discontinuous decisions or a game under known utility functions. Instead of fully known polynomial curves as assumed by Ref.[14] in practical applications such utility functions come in the form of player-specific optimization problems themselves that can be formulated as e.g.:

$$\begin{aligned} & U_j(X) = \max_{Y_j} f_j(Y_j|X) \\ \text{s.t.} \quad & h_j(Y_j, X) = 0 \\ & g_j(Y_j, X) \leq 0 \end{aligned} \tag{1}$$

To provide an example, market decision X could be the total supply on a market and Y the individual players' demand or supply bids. As shown in Ref.[2] such problems can be converted to systems of equalities via establishing the Karush-Kuhn-Tucker (KKT) conditions to find the optimal solution for a player's decisions denoted by $*$:

$$U_j(X) = f_j(Y_j^*|X) \tag{2a}$$

$$\nabla f_j(Y_j^*|X) - \sigma_j^T \nabla h_j(Y_j^*, X) - \lambda_j^T \nabla g_j(Y_j^*, X) = 0 \tag{2b}$$

$$h_j(Y_j^*, X) = 0 \tag{2c}$$

$$\lambda_j^T g_j(Y_j^*, X) = 0 \tag{2d}$$

$$g_j(Y_j^*, X) \leq 0 \tag{2e}$$

$$\sigma_j \in [\mathbb{R}], \lambda_j \in [\mathbb{R}^+] \tag{2f}$$

Ref.[16] solves such a problem for a single player by first deriving the Gröbner basis for the Lagrangian (2b), the equality constraints (2c) and the active inequality constraints (2d). Subsequently, the solutions that breach the bounds of the variables are removed from the set of polynomial solutions that is the Gröbner basis. This concerns specifically the inequality constraint (2e), constraint (2f) and all limitations of the player decisions such as e.g. $Y_j \in [\mathbb{Z}]$. Without loss of generality, compared to single player optimization problems, problems with multiple Nash equilibria can be solved similarly by applying market or - in broader sense - system clearing conditions:

$$\begin{array}{cc} m(X, Y^*) = 0 & \\ \nabla f_j(Y_j^*|X) - \sigma_j^T \nabla h_j(Y_j^*, X) - \lambda_j^T \nabla g_j(Y_j^*, X) = 0 & \forall j \\ h_j(Y_j^*, X) = 0 & \forall j \\ \lambda_j^T g_j(Y_j^*, X) = 0 & \forall j \\ g_j(Y_j^*, X) \leq 0 & \forall j \\ \sigma \in [\mathbb{R}], \lambda \in [\mathbb{R}^+] & \end{array} \tag{3}$$

Such clearing conditions m can appear in various forms and are intended to connect the individual players decision to form the game (which in this case is formulated as a complementarity problem) and to 'force' an equilibrium amongst the players or - in other words - ensure a 'cleared' market[2]. The example presented in the case study below is that of "the quantity provided to the power market equals to the sum of quantities provided by other players". This corresponds to the assumption of complete information.

In theory, such a system can be solved in similar manner to Ref.[16]. However, solving primal and dual variables for all players might, even in simple problems, lead to an increase in degrees of freedom that imposes too high of a computational complexity to allow for practical applications. This matter will be discussed further below.

As computation of Gröbner bases is an active and established field of research, various algebraic solver implementations exist in high-level programming languages (such as Python, Matlab, Maple, Julia)[17]. In the later presented case study, the implemented algorithm to derive the Gröbner basis of the case study problem was the 'F5B' algorithm[19], adapted from the 'Polynomials Manipulation Module' of the Python library SymPy[23].

3. Advantages and Disadvantages of 'Gröbner Basis Clearing'

As mentioned above and suggested by the title of this paper, one important application of Gröbner bases lies in the possibility to yield all (Nash) equilibria[14]. Opposed to traditional solution techniques that either yield single stationary points and analytically describe the range of solutions (e.g. Refs.[1, 2]) or techniques that apply search algorithms (e.g. Refs.[24, 4]) equilibria in form of a Gröbner basis are an exact representation of the range of all equilibria in the original system[17].

As such, Gröbner bases allow for a polynomial set instead of explicitly numerical form. This is an important characteristic considering potential homogeneity of players or their characteristics. As an example, having two renewable generators with marginal cost of approximately 0 compete in an electricity market would, in traditional market clearing, require an initial assumption on the decisions of one of the players to overcome the homogeneity of degree zero and yield an equilibrium solution[25]. As the provided example in the appendix shows, this is not the case for Gröbner bases - they allow formulation of multiple equilibria without the need of additional analysis.

Another aspect favoring a Gröbner basis equilibrium is the unnecessity of symmetrical primal-dual problems. Examples such as Ref.[1] highlight the requirement of such symmetry in traditional solution techniques which otherwise will not be able to terminate and deliver results. As shown in the example in the appendix, equilibria formulated via Gröbner bases do not require all inputs to be defined as parameters, but allow for inputs to be variables (and thus have no duals assigned). This allows for a range of new, unexplored possibilities such as sensitivity analysis of Nash equilibria, which the case study below illustrates.

Despite the Gröbner basis providing an exact representation, most practical applications of such rely on adequate approximation or simplification techniques(see e.g. [26]). The reason for this, as stated by Ref.[16], is in that computing Gröbner bases for even small practical problems might be too complex to solve with finite resources[27]. This is still the case for modern and more advanced algorithms than the original developed by Buchberger[28].

To deal with this 'curse of dimensionality', the algorithm below aims to tackle this utility function approximation dynamically, a task which has been previously conducted manually.

4. Approximation of Utility Functions

Establishing utility function representations of player problems in order to apply equilibrium analysis has been applied successfully in practical applications. To provide an example, in power markets *Conjectured Supply Functions* as e.g. described in Ref.[11] express market quantity as global decisions X and single player price bids as player-specific decisions Y_j, with the supply offered by other players to the market being expressed in the form of a (linear) utility function $U_j(X)$.

In this paper, however, all players are modeled via utility functions. A more general formulation that, contrary to Ref.[11] does not require assumptions on the type of game, can therefore be established. Thus, let it be assumed that for every element in the approximation of the decision vector of a single player $y_j^k \in \widetilde{Y}_j$ exists a representative polynomial:

$$y_j^k = p(\theta^k, X) \tag{4}$$

Assuming the approximated optimal decisions are (nearly) equivalent to the optimal decisions, i.e. $\widetilde{Y}^* \approx Y^*$, allows reformulating Eq.(3) as:

$$\begin{aligned} m(X, \widetilde{Y}^*) &= 0 \\ y_j^{k*} - p(\theta^k, X) &= 0 \quad \forall j, k \end{aligned} \tag{5}$$

This reformulation has two advantages to the previous formulation. This is due to being expressed without dual variables. First, it shows lower complexity as instead of individually formulating Lagrangians and related constraints it requires a single utility function approximation (per player decision y_j^k). Second, it does not require additional bounds on the dual variables which also simplifies sorting out non-viable solutions. This becomes more apparent when the full polynomial approximation for the player problem in Eq.(1) (or Eq.(2)) is formulated:

$$\begin{aligned} U_j(X) &\approx f_j(\widetilde{Y}^*|X) \\ y_j^{k*} &= p(\theta^k, X) \qquad \forall k \\ y_j^{k*} &\in \widetilde{Y}_j^* \end{aligned} \tag{6}$$

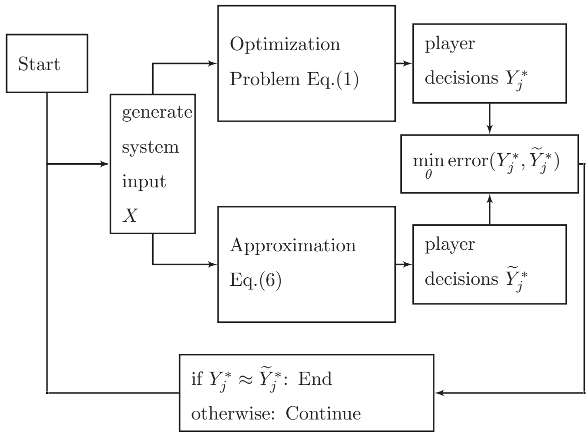
It also shows that selecting the right representative polynomial function is of equal importance as adjusting the constants in order to approach the desired outcome of $U_j(X) = f_j(\widetilde{Y}^*|X)$, or similarly: $Y^* = \widetilde{Y}^*$. The method selected in the later presented case will be introduced below.

5. Polynomial Fitting

Choosing the form of the polynomial - specifically the number of monomials as well as the degree of those - has to be a preliminary decision by the user of the model. As mentioned above, however, due to the computational limitations of Gröbner basis derivations, complexity of polynomials has to decrease with increasing player number and/or number of decisions. Assuming an adequately selected polynomial function exists, the remaining problem is that of polynomial fitting, where the problem for a single player can be expressed as in Fig.1. It has to be noted that for the subsequent method the form (e.g. linear/non-linear) of the optimization problem (1) is not of particular importance, but it requires the assumption that there exist (commercial) solvers/techniques to yield replicable, preferably global results[2].

[2]Thus, convex problems would be an advantage. However, this is not a requirement as the following case study shows.

Figure 1: Polynomial Approximation of Player j



As Ref.[29] illustrates there exists a wide variety of techniques that can be utilized for polynomial fitting. In the here presented paper *Evolution Strategies* were the selected method[30]. There are several reasons for this decision: no requirement for back-propagation allows faster computation and thus a larger amount of input scenarios whereas scalability with the possibility of parallelization allows for similar performance in small and large problems. In addition, the chosen algorithm showed high robustness to overfitting of functions, which can be considered a desirable trait since it compensates for model users overestimating the complexity of the player problems.

Similarly, various other techniques from the field of reinforcement learning could be applied on the problem[31, 29] and exploration of such offer a starting point for future research on the topic.

The selected error function to solve the parameter fitting problem was *Mean Squared Error*. The performance of the applied technique will be demonstrated by the results of the following case study and the algorithm is presented in pseudo-code in the appendix.

6. Evaluation of Nash Equilibria

Considering Jensen's inequality it can be expected that the approximated polynomial utility function performs either similar or worse compared to the original problem. In order to make viable predictions on system behavior, evaluation of the stability of the derived approximated equilibrium is therefore of significant importance.

Formulated first in Ref.[32] the Nikaido-Isoda function shows a distance measure to a Nash equilibrium. Various examples from literature use this function to approach a Nash equilibrium stepwise by decreasing this distance[33, 3, 4] based on an algorithm originally presented in Ref.[34]. In similar manner, the Nikaido-Isoda function can be applied to assess the stability of an existing equilibrium solution. To do so, the Nikaido-Isoda function in general form can be established as:

$$\psi(Y^*, \widetilde{Y}^*) = \sum_j \left(f_j(Y_j^*|X^*) - f_j(\widetilde{Y}_j^*|X^*) \right) \tag{7}$$

In words, this function defines the total incentive of players to deviate from a given solution. A value of 0 would indicate a Nash equilibrium and an adequately low value of ≈ 0 can be considered approximate to an equilibrium state.

To calculate this, first the (approximated) optimal system decisions X^* are derived via computing the Gröbner basis of the approximated problem in Eq.(5). Subsequently, this approximated system solution can be applied to the individual player problems in Eq.(1) which, as stated before, can be assumed to be possible to be solved for Y_j^*. This consequently allows derivation of the term $f_j(Y_j^*|X^*)$ and allows to yield a numerical result for the Nikaido-Isoda function ψ. To support comparability, the results of this Nikaido-Isoda function in the case study below will be expressed as $\frac{\psi(Y^*,\widetilde{Y}^*)}{\sum_j f_j(\widetilde{Y}_j^*|X^*)}$[%] instead of $\psi(Y^*, \widetilde{Y}^*)$[€].

Case Study Nomenclature

Indexes:		
$j \in J$	players	
$t \in T$	time period	[h]
System Variables:		
x^{en}	energy price	[€$/MWh$]
x^{cap}	reserve capacity price	[€$/MW$]
Player Variables:		
y^{en}	energy generated	[MWh]
y^{cap}	reserves supplied	[MW]
y^{st}	unit started/shut down	[binary]
Function:		
r	reservoir level	[mm^3]
η	water consumption	[mm^3/MWh]
c^{st}	runtime cost	[€]
Parameters:		
$\underline{y}^{\text{en}}$,$\bar{y}^{\text{en}}$	generation capacity limits	[MW]

$\underline{r}$,$\bar{r}$	reservoir limits	$[mm^3]$
l	periodical inflow	$[mm^3]$
a^η	water conversion base level	$[mm^3/MWh]$
b^η,c^η	water conversion head influence	$[1/MWh]$,$[1/MWh^2]$
a^{en},a^{cap}	price function constant	[€]
b^{en},b^{cap}	price function elasticity	[€$/MW(h)$]
External Variables:		
ξ	uncertain factor	$\xi \in \mathbb{R}$

7. Case Study

To present the proposed method, a case study on hydropower producers competing in a Cournot Game is introduced below. It has to be noted that the presented study aims to showcase the capabilities of the presented framework and thus provides a simplified[3] - even though realistic - example.

The chosen case study aims to combine two recently developing fields of research: integer-constrained games in systems of electrical power[6, 7] and price-making hydropower storage operators[35, 36]. In contrast to previously introduced studies attempting this task[4, 10], the proposed case study includes capacity markets, non-linear (quadratic) head-dependencies and inflow uncertainty, thus itself presenting a novelty in literature.

The deterministic, non-convex problem of a hydropower player j holding a single reservoir is formulated the following:

$$\max_{y_j^{\text{en}},y_j^{\text{cap}},y_j^{\text{st}}} \sum_t x^{\text{en},t} y_j^{\text{en},t} + \sum_t x^{\text{cap},t} y_j^{\text{cap},t} - \sum_t c_j^{\text{st}} y_j^{\text{st},t} \tag{8a}$$

$$\text{s.t.} \quad y_j^{\text{st},t} \underline{y}^{\text{en}} \leq y_j^{\text{en},t} \leq y_j^{\text{st},t} \bar{y}^{\text{en}} \quad \forall t \tag{8b}$$

$$y_j^{\text{en},t} - y_j^{\text{st},t} \underline{y}^{\text{en}} \geq y_j^{\text{cap},t} \leq y_j^{\text{st},t} \bar{y}^{\text{en}} - y_j^{\text{en},t} \quad \forall t \tag{8c}$$

$$\underline{r}_j \leq r_j^t \leq \bar{r}_j \quad \forall t \tag{8d}$$

$$\begin{aligned} \eta_j^1 &= a_j^\eta \\ \eta_j^t &= a_j^\eta + b_j^\eta \left(\frac{r_j^{t-1} - \underline{r}_j}{\underline{r}_j} - \frac{\bar{r}_j - r_j^{t-1}}{\bar{r}_j}\right) \\ &\quad + c_j^\eta \left(\frac{r_j^{t-1} - \underline{r}_j}{\underline{r}_j} - \frac{\bar{r}_j - r_j^{t-1}}{\bar{r}_j}\right)^2 \quad \forall t > 1 \end{aligned} \tag{8e}$$

$$r_j^t = \sum_{t2=1}^{t} l_j^{t2} - \sum_{t2=1}^{t} \eta_j^{t2} y_j^{\text{en},t2} \quad \forall t \tag{8f}$$

$$y_j^{\text{en},t} \in \mathbb{R}, y_j^{\text{cap},t} \in \mathbb{R}^+, y_j^{\text{st,t}} \in [0,1] \quad \forall t \tag{8g}$$

The optimization objective (8a) describes the profit maximization operating simultaneously on an

[3] Yet, in degrees of freedom and thus in computational complexity, comparable to previous studies on similar topics[6, 7, 4, 10].

Table 1: Case Study Parameters

	$\underline{y}^{\text{en}}$	$\bar{y}^{\text{en}}$	c^{st}	a^{η}	b^{η}	c^{η}	$\underline{r}$	$\bar{r}$
$j=1$	10	80	100	0.75	0.05	-0.01	50	100
$j=2$	5	60	100	0.75	0.05	-0.01	25	50
	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$			
a^{en}	49	47	50	49	50			
b^{en}	0.025	0.07	0.085	0.075	0.09			
a^{cap}	43	46	44	47	42			
b^{cap}	0.005	0.012	0.08	0.05	0.004			
l_1	75	40	5	20	30			
l_2	40	15	25	10	5			

energy and a reserve capacity market (whereas up- and down-regulation is bid simultaneously). For the sake of simplicity runtime cost instead of specific startup/shutdown cost are assumed and down-/up-time limits are neglected. In similar manner the simplification of assuming equal clearing periods for both markets describes that both decisions are made simultaneously. Thus for a capacity market clearing of e.g. $24h$ and an energy market of $1h$, both outputs are held constant for $24h$. Constraints (8b) and (8c) respectively describe the limits for energy and capacity provided, whereas the average down-regulation consumed is considered equal to the average up-regulation. Therefore any consumption of reserves is omitted. Reservoir capacities are described in constraint (8d), reservoir levels at the end of the total time frame are to be decided by the players. Eq.(8e) formulates the water conversion, which includes the quadratic head dependencies of the waterways[37, 38]. Eq.(8f) describes the reservoir state equation. This cubic equation hints that the problem might be of non-convex nature and the set of possible integer decisions as shown in Eq.(8g) confirm that this is in fact the case. However, for a limited time frame T it can be safely assumed that it is possible to use methods, such as the here applied mathematical solver *Couenne*[39], that (approximately) yield a global optimum for the problem of a single player. Similar to conventional Cournot models this case study uses linear price functions to clear the markets[2], which can be formulated in form of polynomial sets:

$$\begin{aligned} m(\{x^{\text{en}}\}, \widetilde{Y}^*) &= \{x^{\text{en},t} - \big(a^{\text{en},t} - b^{\text{en},t} \sum_j y_j^{\text{en},t}\big) \forall t\} \\ m(\{x^{\text{cap}}\}, \widetilde{Y}^*) &= \{x^{\text{cap},t} - \big(a^{\text{cap},t} - b^{\text{cap},t} \sum_j y_j^{\text{cap},t}\big) \forall t\} \end{aligned} \tag{9}$$

Tab.1 presents the parameters used in this case study. To assist improving the convergence rate of the utility function approximations, initial assumptions on the outcome of the system decisions can aid in establishing better training samples for X. In the here presented case study, this was

Figure 2: Random Price Sample Distribution Example

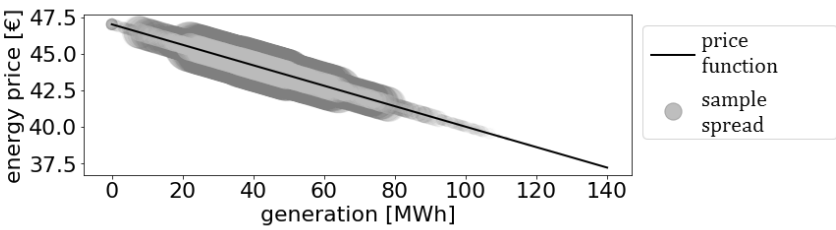
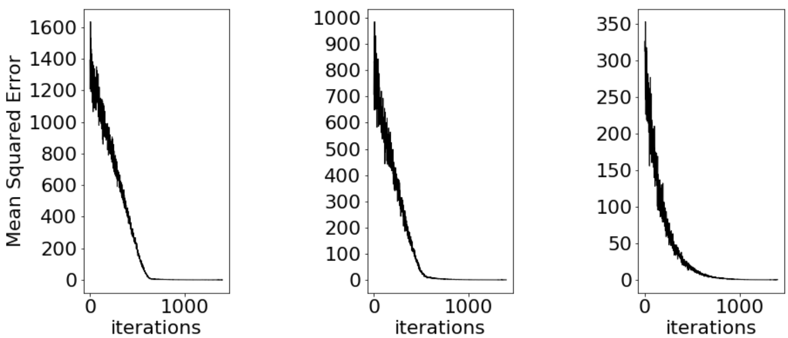


Figure 3: Convergence Results for player $j = 1$ in period $t = 1$

(a) Energy Price (b) Reserve Price (c) Unit Schedule



conducted via applying Poisson distributions[4] on the price functions. Fig.2 shows an example for such a distribution laid over the linear price function. Fig.3 shows some example convergence rates for the algorithm from Fig.1, with each iteration (simultaneously for all variables of a player) performing < 1sec at an Intel i7-5600|2.6Ghz. Thus the chosen polynomials of degree 1 were considered a sufficient representation. Another positive aspect of the proposed clearing algorithm is demonstrated by the binary scheduling variable y^{st}: as it has no effect on the market clearing variables it can therefore be omitted from calculating the Gröbner basis further simplifying its' complexity (nonetheless, those binary decisions can change based on the market clearing equilibrium results). In the applied simulation of 1200 iterations per player, the Gröbner basis yielded *a single Nash equilibrium* with a Nikaido-Isoda distance $\approx 4.1\%$.

The equilibrium decision results for player 1 can be found in Fig.4. The graphs denoted as 'approximated' are the results obtained by the Gröbner basis (i.e. $\widetilde{Y}_j^*$ for X^*), the graphs denoted as 'solved' are the individual optimization problem results using the market prices obtained from

[4]This is a subjective choice by the model user. In this case it was chosen due to the perception that absolutely no generation was considered unlikely and that the probability had to decrease going towards maximum capacity.

Figure 4: Market Equilibrium Player Decision Results

(a) Generation Player $j = 1$ (b) Reserve Player $j = 1$

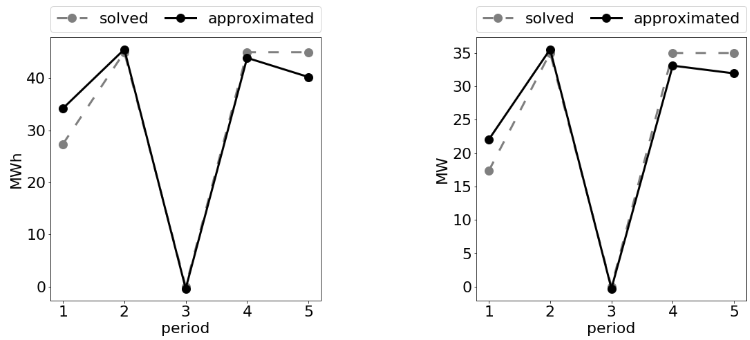


Table 2: Stochastic Inflows

	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$
l_1	-1.5ξ	$+3\xi$	$+1.2\xi$	-0.25ξ	$+0.25\xi$
l_2	$+2.5\xi$	-0.25ξ	-0.075ξ	$+0.35\xi$	$+0.1\xi$

the approximated market clearing (i.e. Y_j^* for X^*). The graphs illustrate that the player has little or no incentive to deviate from the energy and capacity market decisions in most of the 10 market clearing stages and no incentive to deviate from the obtained binary dispatch decision.

Uncertain Inflows

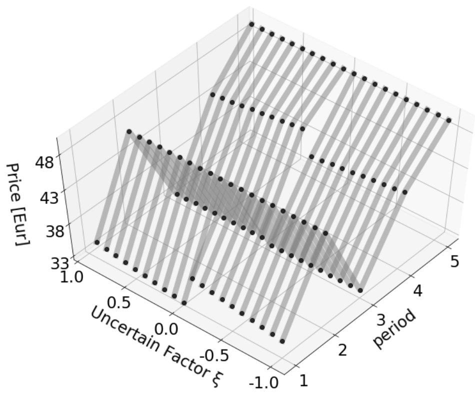
In addition and as discussed in Refs.[40, 10], individual players might be influenced by uncertainty. Correct modeling of such uncertainty could prove crucial in renewable-dominated games, where multiple equilibria caused by uncertainty could, from the perspective of a decision maker, distort the value of obtaining only a single equilibrium.

To consider this aspect in the presented case, Tab.2 adds the example of an external variable to the previously presented case. It be assumed here that there exists an uncertain factor $-1 \leq \xi \leq 1$ with no known distribution[5], that influences the inflows into the two considered reservoirs. This problem can be solved in similar manner as above, with the difference that the result is an infinite number of Nash equilibria. Due to the lack of space, the Gröbner bases will be omitted, though an excerpt will be presented: one of the polynomials in the basis reads $\xi - 0.390y_2^{\mathrm{en},5} + 17.667$ which can be reformulated as the replacement rule $\frac{\xi+17.667}{0.3897} = y_2^{\mathrm{en},5}$. Due to the graded reverse

[5]Thus a uniform distribution is used to train the utilization function approximation. It can be assumed, that for some problems, assumptions on the distribution of this variable might indirectly influence the outcome. However, the problem is still of inherently more robust nature as techniques that require variable distributions as direct input.

Figure 5: Equilibrium Price Curves under Uncertainty

(a) Energy Price Equilibria



(b) Capacity Price Equilibria

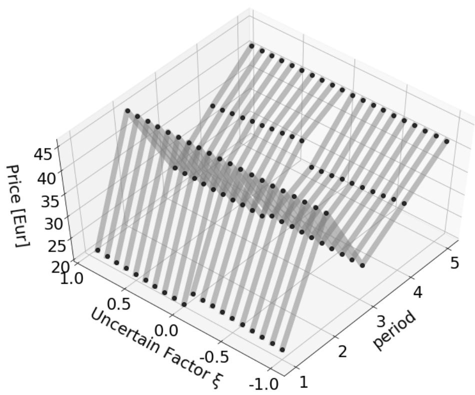


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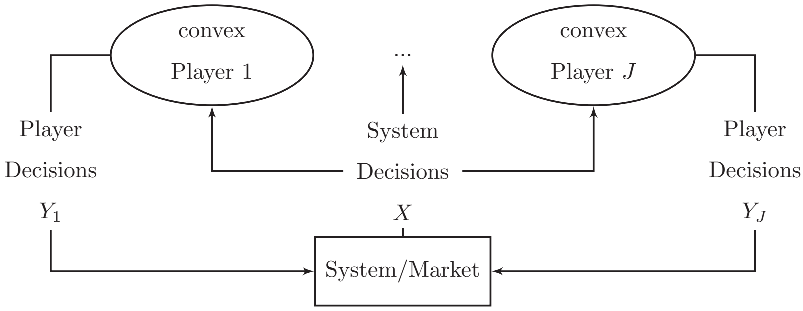
Figure 6: Traditional System/Market Clearing



lexicographic order applied on the monomial ring, the rest of the polynomials in the resulting basis is described in functions of $y_2^{\mathrm{en},5}$, for example one polynomial defining the capacity clearing price in the first period reads $x^{\mathrm{cap},1} - 0.787y_2^{\mathrm{en},5} - 13.522$.

Thus, the resulting Gröbner basis and thus the set of Nash equilibria is scalable by changing the value of the uncertain factor ξ. Knowing the limits of this factor, it is possible to dynamically yield numerical results for different uncertain outcomes. Fig.5 shows this by plotting the equilibrium prices for different stochastic outcomes. The Nikaido-Isoda function ranged between 9% to 11.5% depending on the value of ξ, reinforcing the intuitive assumption that more system variables might decrease the quality of the utility function approximation[6].

8. Conclusion

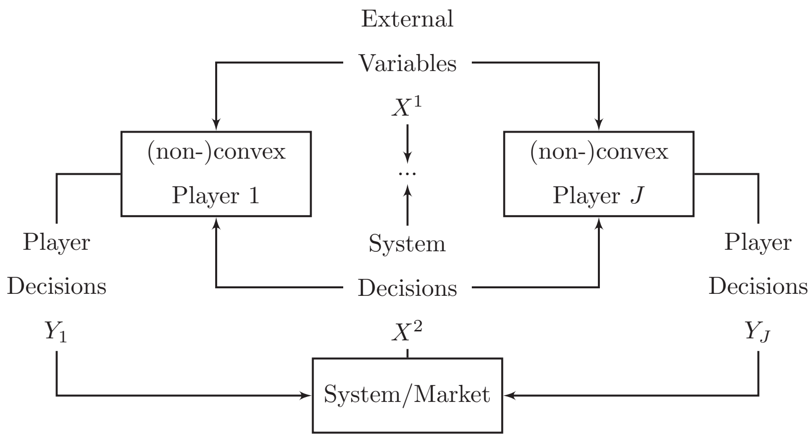
This paper extends the concept of finding multiple Nash equilibria to continuous decisions, a first in literature. Further, it proposes a machine-learning framework to train agents to replicate the single player results. These agents are then interconnected via market/system clearing constraints and the equilibria are formulated as a Gröbner basis. This approach allows application of such a solution algorithm to practical problems, as - even in modern algorithms - time-efficient computation of Gröbner bases is still a strong practical constraint.

The paper shows how to implement traditional problems from game theory, as presented in Fig.6, within the new framework and compares it to traditional solution methods such as derivation of Karush-Kuhn-Tucker conditions.

In addition, it presents a set of new problems (presented in Fig.7) with limited discussion in literature and proposes an approach based on the Nikaido-Isoda function to assess the quality of

[6] Yet, it can be assumed that the accuracy would increase by establishing several different approximations, e.g. split the training sets and resulting Gröbner bases into $-1 \leq \xi \leq 0$ and $0 \leq \xi \leq 1$. However, for the sake of presentational simplicity this is omitted from the presented case study.

Figure 7: Gröbner Basis Clearing



obtained results.

To demonstrate the capabilities of the framework, a novel case study for two competing hydropower players with each utilizing a non-linear multi-market/multi-period optimization model is solved. It shows how the proposed methodology allows for a wide variety of novel applications which are not restricted on power systems: sensitivity analysis of Nash equilibria, finding multiple equilibria in non-convex games and solving non-symmetric complementarity systems (i.e. games with inputs that are held variable instead of being fixed model parameters).

The concept promises applicability to solve arising questions in modern power system games which include equilibria under consideration of unit scheduling, power system clearing under uncertainty with unknown distributions and existence of multiple equilibria for systems under optimal control and competition,

This might prove important in future applications such as electricity systems with growing uncertainty and lower marginal costs of units which can be caused by a growing number of renewable means of generation and sinking share of flexible thermal units. Nonetheless, the presented model and techniques might be applied to other fields that show similar problem settings and are not explicitly discussed in this paper.

[1] B. F. Hobbs, Linear Complementarity Models of Nash Cournot Competition in Bilateral and POOLCO Power Markets, IEEE Transactions on Power Systems 16 (2) (2001) 194–202.

[2] S. A. Gabriel, A. J. Conejo, J. D. Fuller, B. F. Hobbs, C. Ruiz, Complementarity Modeling in Energy Markets, Springer, New York, 2013.

[3] J. P. Molina, J. M. Zolezzi, J. Contreras, H. Rudnick, M. J. Reveco, Nash-Cournot Equilibria

in Hydrothermal Electricity Markets, IEEE Transactions on Power Systems 26 (3) (2011) 1089–1101.

[4] M. Löschenbrand, M. Korpas, Multiple Nash Equilibria in Electricity Markets with price-making Hydrothermal Producers, IEEE Transactions on Power Systems (in press) (2018) 1–10. doi:10.1109/TPWRS.2018.2858574.

[5] L. Deng, B. F. Hobbs, P. Renson, What is the Cost of Negative Bidding by Wind? A Unit Commitment Analysis of Cost and Emissions, IEEE Transactions on Power Systems 30 (4) (2015) 1805–1814. doi:10.1109/TPWRS.2014.2356514.

[6] S. A. Gabriel, S. A. Siddiqui, A. J. Conejo, C. Ruiz, Solving Discretely-Constrained Nash-Cournot Games with an Application to Power Markets, Networks and Spatial Economics 13 (3) (2013) 307–326. doi:10.1007/s11067-012-9182-2.

[7] D. Huppmann, S. Siddiqui, An exact solution method for binary equilibrium problems with compensation and the power market uplift problem, European Journal of Operational Research 266 (2) (2018) 622–638. arXiv:1504.05894, doi:10.1016/j.ejor.2017.09.032.

[8] E. Moiseeva, M. R. Hesamzadeh, D. R. Biggar, Exercise of Market Power on Ramp Rate in Wind-Integrated Power Systems, IEEE Transactions on Power Systems 30 (3) (2015) 1614–1623. doi:10.1109/TPWRS.2014.2356255.

[9] E. Moiseeva, S. Wogrin, M. R. Hesamzadeh, Generation flexibility in ramp rates: Strategic behavior and lessons for electricity market design, European Journal of Operational Research 261 (2) (2017) 755–771. doi:10.1016/j.ejor.2017.02.028.
URL http://dx.doi.org/10.1016/j.ejor.2017.02.028

[10] M. Löschenbrand, W. Wei, F. Liu, Hydro-thermal power market equilibrium with price-making hydropower producers, Energy 164 (2018) 377–389. doi:10.1016/j.energy.2018.08.162.

[11] C. J. Day, B. F. Hobbs, J.-S. Pang, Oligopolistic Competition in Power Networks: A Conjectured Supply Function Approach, University of California Energy Institute, 2002.

[12] W. Wei, J. Wang, S. Mei, Convexification of the Nash Bargaining Based Environmental-Economic Dispatch, IEEE Transactions on Power Systems 31 (6) (2016) 5208–5209.

[13] Y. Yang, Y. Zhang, F. Li, H. Chen, Computing all Nash equilibria of multiplayer games in electricity markets by solving polynomial equations, IEEE Transactions on Power Systems 27 (1) (2012) 81–91. doi:10.1109/TPWRS.2011.2159815.

[14] F. Kubler, K. Schmedders, Tackling Multiplicity of Equilibria with Gröbner Bases, Operations Research 58 (4-part-2) (2010) 1037–1050. doi:10.1287/opre.1100.0819.

[15] B. Buchberger, Bruno Buchberger's PhD thesis 1965: An algorithm for finding the basis elements of the residue class ring of a zero dimensional polynomial ideal, Journal of Symbolic Computation 41 (3-4) (2006) 475–511. doi:10.1016/j.jsc.2005.09.007.

[16] K. Hagglof, P. O. Lindberg, L. Svensson, Computing Global Minima to Polynomial Optimization Problems Using Groebner Bases, Journal of Global Optimization (1995) 115–125doi:10.1007/BF01097057.

[17] R. S. Datta, Finding all Nash equilibria of a finite game using polynomial algebra, Economic Theory 42 (1) (2009) 55–96. arXiv:0612462, doi:10.1007/s00199-009-0447-z.

[18] J.-C. Faugére, A new efficient algorithm for computing Gröbner bases (F4), Journal of Pure and Applied Algebra 139 (1999) 61–88.

[19] J. C. Faugère, A new efficient algorithm for computing Gröbner bases without reduction to zero (F5), Proceedings of the 2002 international symposium on Symbolic and algebraic computation - ISSAC '02 - (-) (2002) 75–83. doi:10.1145/780506.780516.
URL http://portal.acm.org/citation.cfm?doid=780506.780516

[20] H. Tsaknakis, P. G. Spirakis, An optimization approach for approximate Nash equilibria, in: International Workshop on Web and Internet Economics, Vol. 5, 2007, pp. 42 – 56. doi:10.1080/15427951.2008.10129172.

[21] W. Grauberger, A. Kimms, Computing pure Nash equilibria in network revenue management games, European Journal of Operational Research 237 (3) (2014) 1008 – 1020. doi:10.1007/s00291-018-0507-5.
URL http://dx.doi.org/10.1016/j.ejor.2014.02.045

[22] B. Sturmfels, What is a Gröbner Basis, Notices-American Mathematical Society 52 (10) (2005) 2–3.

[23] A. Meurer, C. P. Smith, M. Paprocki, O. Certik, S. B. Kirpichev, SymPy: symbolic computing in Python (2017).
URL https://doi.org/10.7717/peerj-cs.103

[24] D. Pozo, J. Contreras, Finding Multiple Nash Equilibria in Pool-Based Markets : A Stochastic EPEC Approach, IEEE Transactions on Power Systems 26 (3) (2011) 1744–1752.

[25] N. Hosoe, K. Gasawa, H. Hashimoto, Textbook of Computable General Equilibrium Modelling, Springer, 2010.

[26] F. Castro, J. Gago, I. Hartillo, J. Puerto, J. M. Ucha, An algebraic approach to integer portfolio problems, European Journal of Operational Research 210 (3) (2011) 647–659. arXiv:1004.0905, doi:10.1016/j.ejor.2010.11.007.
URL http://dx.doi.org/10.1016/j.ejor.2010.11.007

[27] E. W. Mayr, Some complexity results for polynomial ideals, Journal of Complexity 13 (3) (1997) 303–325. doi:10.1006/jcom.1997.0447.

[28] M. Bardet, J. C. Faugère, B. Salvy, On the complexity of the F5 Gröbner basis algorithm, Journal of Symbolic Computation 70 (2015) 49–70. doi:10.1016/j.jsc.2014.09.025.

[29] K. Arulkumaran, M. P. Deisenroth, M. Brundage, A. A. Bharath, A Brief Survey of Deep Reinforcement Learning, IEEE Signal Processing Magazin, arxiv extended version (2017) 1–16arXiv:1708.05866, doi:10.1109/MSP.2017.2743240.

[30] T. Salimans, J. Ho, X. Chen, S. Sidor, I. Sutskever, Evolution Strategies as a Scalable Alternative to Reinforcement Learning, arXiv (2017) 1–13arXiv:1703.03864, doi:10.1.1.51.6328.

[31] V. Mnih, K. Kavukcuoglu, D. Silver, A. A. Rusu, J. Veness, M. G. Bellemare, A. Graves, M. Riedmiller, A. K. Fidjeland, G. Ostrovski, S. Petersen, C. Beattie, A. Sadik, I. Antonoglou, H. King, D. Kumaran, D. Wierstra, S. Legg, D. Hassabis, Human-level control through deep reinforcement learning, IJCAI International Joint Conference on Artificial Intelligence 518 (2016) 2315–2321. arXiv:1604.03986, doi:10.1038/nature14236.

[32] H. Nikaido, K. Isoda, Note on non-cooperative convex game, Pacific Journal of Mathematics 5 (5) (1955) 807–815.

[33] J. Contreras, M. Klusch, J. B. Krawczyk, Numerical Solutions to Nash Cournot Equilibria in Coupled Constraint Electricity Markets, IEEE Transactions on Power Systems 19 (1) (2004) 195–206.

[34] J. B. Krawczyk, S. Uryasev, Relaxation algorithms to find Nash equilibria with economic applications, Environmental Modeling and Assessment 5 (2000) 63–73.

[35] J. Bushnell, A Mixed Complementarity Model of Hydrothermal Electricity Competition in the Western United States, Operations Research 51 (1) (2003) 80–93. doi:10.1287/opre.51.1.80.12800.

[36] J. R. Cruise, L. Flatley, S. Zachary, Impact of storage competition on energy markets, European Journal of Operational Research 269 (3) (2018) 998–1012. arXiv:1606.05361, doi:10.1016/j.ejor.2018.02.036.
URL https://doi.org/10.1016/j.ejor.2018.02.036

[37] F. R. Førsund, Hydropower Economics, 2nd Edition, Springer, Oslo, 2015. doi:10.1007/978-1-4899-7519-5.

[38] S. Séguin, S. E. Fleten, P. Côté, A. Pichler, C. Audet, Stochastic short-term hydropower planning with inflow scenario trees, European Journal of Operational Research 259 (3) (2017) 1156–1168. doi:10.1016/j.ejor.2016.11.028.

[39] P. Belotti, T. Berthold, P. Bonami, S. Cafieri, F. Margot, C. Megaw, S. Vigerske, A. Wächter, Couenne, an exact solver for nonconvex MINLPs (2015).
URL `https://projects.coin-or.org/Couenne/wiki`

[40] E. Moiseeva, M. R. Hesamzadeh, Bayesian and Robust Nash Equilibria in Hydro-Dominated Systems under Uncertainty, IEEE Transactions on Sustainable Energy - (-) (2017) 1–12. `doi:10.1109/TSTE.2017.2762086`.

An illustrative Example

(Please note that the following section is meant as a demonstration of the capabilities of using 'Gröbner Clearing' and not necessarily intended to show a realistic case study.)

Assumed be generators that supply an energy market with a quantity generated $y_j^{\text{gen}}[MWh]$ at a price $y_j^{\text{price}}[€/MWh]$. The generators show a marginal cost of $\text{MC}_j[€]$, generation capacity limits of $\underline{y}_j^{\text{gen}}[MWh]$ and $\bar{y}_j^{\text{gen}}[MWh]$ and a maximum price bid of $\bar{y}_j^{\text{price}}[€/MWh]$. Assuming uniform prices, the problem can be formulated the following:

$$\begin{aligned} \max_{y_j^{\text{price}}, y_j^{\text{gen}}} \; & y_j^{\text{price}} y_j^{\text{gen}} - \text{MC}_j y_j^{\text{gen}} \\ & \underline{y}_j^{\text{gen}} \leq y_j^{\text{gen}} \leq \bar{y}_j^{\text{gen}} \qquad (\underline{\lambda}_j^{\text{gen}}, \bar{\lambda}_j^{\text{gen}}) \\ & 0 \leq y_j^{\text{price}} \leq \bar{y}_j^{\text{price}} \qquad (\underline{\lambda}_j^{\text{price}}, \bar{\lambda}_j^{\text{price}}) \end{aligned} \tag{.1a}$$

$$y_{j1}^{\text{price}} = y_{j2}^{\text{price}} \quad \forall j1, j2 \neq j1 \tag{.1b}$$

Here, Eq.(.1a) formulates a single player's quadratic problem and Eq.(.1b) the market clearing condition. Assumed be an example of two homogeneous players e.g. two wind farms with $\underline{y}_j^{\text{gen}} = 0MWh$, $\bar{y}_j^{\text{gen}} = 100MWh$ and no marginal cost (i.e. $\text{MC}_j = 0€$). For the sake of computational ability, finite bounds on the maximum bids have to be assumed: $\bar{y}_j^{\text{price}} = 9999€/MWh$. As described in Ref.[25] traditional equilibrium modeling would require an initial assumption on one of the players quantities or prices. In this example taking one of the extreme points (thus setting one of the player decisions to their maximum or minimum) and solving the problem would yield a single Nash equilibrium. This however might show a distorted picture of the real outcome as the following procedure will show.

By establishing the KKT conditions similar to Eq.(3) and applying the 'F5B' algorithm from Ref.[19], the Gröbner basis can be derived:

Gröbner basis $= \Big\{ \underline{\lambda}_1^{\text{price}} (y_2^{\text{gen}})^2 - 100 \underline{\lambda}_1^{\text{price}} y_2^{\text{gen}} + y_1^{\text{gen}} (y_2^{\text{gen}})^2 - 100 y_1^{\text{gen}} y_2^{\text{gen}}$, $(\underline{\lambda}_1^{\text{price}})^2 + 100 \underline{\lambda}_1^{\text{price}} - (y_1^{\text{gen}})^2 + 100 y_1^{\text{gen}}$, $\underline{\lambda}_1^{\text{price}} \underline{\lambda}_2^{\text{price}} + \underline{\lambda}_1^{\text{price}} y_2^{\text{gen}}$, $(\underline{\lambda}_2^{\text{price}})^2 + 100 \underline{\lambda}_2^{\text{price}} - (y_2^{\text{gen}})^2 + 100 y_2^{\text{gen}}$, $\underline{\lambda}_1^{\text{price}} y_1^{\text{gen}} - 100 \underline{\lambda}_1^{\text{price}} + (y_1^{\text{gen}})^2 - 100 y_1^{\text{gen}}$, $-\underline{\lambda}_1^{\text{price}} y_2^{\text{gen}} + \underline{\lambda}_2^{\text{price}} y_1^{\text{gen}}$, $\underline{\lambda}_2^{\text{price}} y_2^{\text{gen}} - 100 \underline{\lambda}_2^{\text{price}} + (y_2^{\text{gen}})^2 - 100 y_2^{\text{gen}}$, $\underline{\lambda}_1^{\text{price}} y_2^{\text{price}}$, $\underline{\lambda}_2^{\text{price}} y_2^{\text{price}}$, $-9999 \underline{\lambda}_1^{\text{price}} + y_1^{\text{gen}} y_2^{\text{price}} - 9999 y_1^{\text{gen}}$, $-9999 \underline{\lambda}_2^{\text{price}} + y_2^{\text{gen}} y_2^{\text{price}} - 9999 y_2^{\text{gen}}$, $100 \bar{\lambda}_1^{\text{gen}} - 9999 \underline{\lambda}_1^{\text{price}} - 9999 y_1^{\text{gen}}$, $100 \underline{\lambda}_1^{\text{gen}} - 9999 \underline{\lambda}_1^{\text{price}} - 9999 y_1^{\text{gen}} + 100 y_2^{\text{price}}$, $\bar{\lambda}_1^{\text{price}} - \underline{\lambda}_1^{\text{price}} - y_1^{\text{gen}}$, Table .3: Number of Nash Equilibria (# NE) for different Cases

MC_1	$\underline{y}_1^{\text{gen}}$	$\bar{y}_1^{\text{gen}}$	$\bar{y}_1^{\text{price}}$	MC_2	$\underline{y}_2^{\text{gen}}$	$\bar{y}_2^{\text{gen}}$	$\bar{y}_2^{\text{price}}$	# NE
0	50	100	9999	0	0	100	9999	1
10	50	100	9999	0	50	50	9999	2
10	50	100	9998	0	50	50	9999	0
2	0	100	9999	0	0	100	9999	1

$100\bar{\lambda}_2^{\text{gen}} - 9999\underline{\lambda}_2^{\text{price}} - 9999y_2^{\text{gen}}$, $100\underline{\lambda}_2^{\text{gen}} - 9999\underline{\lambda}_2^{\text{price}} - 9999y_2^{\text{gen}} + 100y_2^{\text{price}}$, $\bar{\lambda}_2^{\text{price}} - \underline{\lambda}_2^{\text{price}} - y_2^{\text{gen}}$, $y_1^{\text{price}} - y_2^{\text{price}}\Big\}$

As shown in Ref.[15], this system of polynomials is no simplification of the original problem but an exact representation. Compared to the original problem however, the Gröbner basis can be solved algebraically as a system of 18 equality equations with 12 variables. The results are two Nash equilibria with primal variable results of either $\{y_1^{\text{gen}} = y_2^{\text{gen}} = 0, y_1^{\text{price}} = y_2^{\text{price}} = 0\}$ or $\{y_1^{\text{gen}} = y_2^{\text{gen}} = 100, y_1^{\text{price}} = y_2^{\text{price}} = 9999\}$. As with all multiple equilibria, there is no statement about the likelihood of the outcomes. Thus finding both potential outcomes is of importance, especially considering their highly different characteristics (full generation/no generation).

This example shows, that calculation of Gröbner bases can replace time-consuming and potentially error-prone problem analysis in order to obtain Nash equilibria in such systems. This gains in importance regarding analysis of non-homogeneous agents. For example, in the here provided problem, the second Nash equilibrium (maximum production) vanishes, if one agent assumes a smaller price cap than 9999€$/MWh$. On the other hand, raising minimum production for any of the involved agents above $0MWh$ eliminates the first Nash equilibrium. Latter is presented with several other examples that show different potential Nash equilibria in Tab..3.

As discussed in the text above, using Gröbner bases also allows variable inputs, i.e. non-symmetric complementarity problems. An example could be given for the initial case presented before (homogeneous agents, no marginal cost) and with - as common for wind power plants - non-specified output capacities (usually influenced by uncertainty). Assumed that there is no information on the distribution of this uncertain parameter, the Gröbner basis with $\bar{y}_j^{\text{gen}}$ considered a decision variable can be found. With traditional solution techniques this would not be solvable for an equilibrium, as the number of decision variables exceeds the numbers of equations in the KKT conditions (i.e. a non-symmetric problem, due to a lack of dual variables). Nonetheless, it is possible to formulate the Gröbner basis, which comes in the form of 18 equations and 14 variables. This system can be solved to obtain a number of 4 Nash equilibria.

For example, two equilibria show that $y_j^{\text{gen}} = \bar{y}_j^{\text{gen}}$ (for each player $j = 1, 2$ respectively). Thus, instead of having definite numerical results as with symmetric primal-dual problems, the result is the relation between decision variables (i.e. a replacement rule). Therefore it bears practical application as it allows decision makers to analyze the impact of changing one parameter of the system - i.e. perform sensitivity analysis - without any recalculation of the equilibrium points.

Polynomial Fitting via Evolutionary Algorithms

This section presents the algorithm from Ref.[30] applied on polynomial fitting. It is extended with feature-rescaling, simulated annealing and a rounding mechanism to enhance performance of the Gröbner basis computation. The goal of the proposed algorithm is the minimization of the *error* function via adjusting the θ parameters of the chosen polynomial function. The algorithm in pseudo-code reads the following:

Algorithm 1. *Polynomial Fitting*

define *population Pop, noise standard deviation* σ*, learning rate* α*, annealing factors* a^σ*,* a^α*, maximum steps* $steps^{max}$*, minimum* θ^{min}*;*
define *polynomial functions* $p(\cdot)$*;*
define *scaling factors s for X (with the aim of holding* $0 \leq s_v x_v \leq 1 \forall x \in X, v \in |X|$*);*
init. $\theta_v^k = \boldsymbol{U}(0,1) \forall k \in |Y|, v \in |X|$*;*
init. $steps = 0$*;*
for $steps \leq steps^{max}$ ***do***
  solve *Eq.*(1) $\forall j$ *to receive* Y_j^**;*
  calc. X^* *and scale* $x_v := s_v x_v^* \forall x \in X, v \in |X|$*;*
  sample *step direction* $N_{pop,v}^k := \boldsymbol{N}(0,1) \forall k \in |Y|, pop \in Pop, v \in |X|$*;*
  calc. *steps* $\vartheta_{pop,v}^k = \theta^k + \sigma n_{pop,v}^k \forall k \in |Y|, pop \in Pop, v \in |X|$*;*
  calc. *approximation* $\tilde{y}_{j,pop,v}^{*,k} = p(\vartheta_{pop,v}^k, X) \forall k \in |Y|, pop \in Pop, v \in |X|$*;*
  calc. $error_{pop,v}^k(y_j^{*,k}, \tilde{y}_{j,pop,v}^{*,k}) \forall k \in |Y|, pop \in Pop, v \in |X|$*;*
  calc. *mean* $\mu_v^{error,k}$ *and standard deviation* $\sigma_v^{error,k}$ $\forall k \in |Y|, v \in |X|$*;*
  for $k \in |Y|$ ***do***
    for $v \in |X|$ ***do***
      calc. $A_v^k = \frac{error_v^k - \mu_v^{error}}{\sigma_v^{error,k}}$*;*
      update $\theta_v^k := \theta_v^k + \frac{\alpha}{\sigma Pop}\big((N_v^k)^\intercal \cdot A_v^k\big)$
      if $abs(\theta_v^k) < \theta^{min}$ ***then***
        update $\theta_v^k := 0$*;*
      end if
    end for
  end for
  update $\sigma := a^\sigma \sigma$*,* $\alpha := a^\alpha \alpha$*;*
  update $steps := steps + 1$*;*
end for
return *rescaled polynomial* $\tilde{y}_j^k = p(\theta^k, X \otimes s) \forall k \in |Y|$

Similar to traditional machine-learning problems, parameter selection might require trial and error. As mentioned above, the same holds for $p(\cdot)$, as an increase in degrees of freedom (more players and/or decision variables) require a decrease in complexity of their representative polynomials.

Optionally, the maximum step size might be replaced for target error values.

The parameter $\theta^{\min}$ defines which values are rounded to 0, as a variable denoted with an infinitesimally small θ^k value would still add to the computation times of the Gröbner basis, but cannot be expected to provide significantly to the quality of the result.

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7.3 Hydro-thermal power market equilibrium with price-making hydropower producers

This work has been published in *Energy*.

7.3.1 Extended Abstract

This paper formulates a multi-period hydro-thermal Cournot game under consideration of storage and unit commitment decisions.

The basic problem is formulated as a profit-maximization of a mixed portfolio player with thermal units under generation capacity constraints and hydropower units under generation and reservoir capacity constraints. Inflows are considered periodical and uncertain. The problem is solved similar to other Cournot problems, by assuming symmetric information on price curves, an assumption made based on the availability of public databases for historical data of liberalized electricity markets.

The storage problem is solved via decomposition of the periodical inflows. Instead of considering single reservoir decisions from a period into the next, the proposed solution technique transforms this decision vector into a decision matrix which considers the storage decision from each observed period into each other. This decomposes the original equilibrium problem into a number of sub-problems equal to the considered time periods, which can be solved to convergence via a backwards pass algorithm.

The binary decisions are reformulated into a set of preselected potential schedules, giving the option to preselect only viable schedules. A player could thus sort out schedules that do not fulfill conditions such as minimum/maximum uptimes or ramping times. This decreases computational complexity and allows for consideration of larger problems without suffering information loss. The resulting discrete game is then solved via iteration of the available schedules, whereas each sub-problem can be solved via taking the KKT-conditions for fixed scheduling decisions.

In addition to the proposed modeling techniques, an adjustment to the Karush-Kuhn-Tucker conditions is proposed that aims to establish robustness under uncertainty. The formulation is similar to a mixed-strategy game and extends current literature by a) allowing for distributions of uncertainty and b) extending the framework to the Lagrangians, whereas the original model presented in literature was implemented for complimentarity conditions.

A case study on the Scandinavian spot/intraday market is presented in the paper. This case study analyzes the impact of hydropower on the marginal cost of energy and capacity and observes an impact on the optimal schedules of the players. It shows a positive welfare impact of adding hydropower capacities, which are caused by both capacity and energy prices. However, there is a distinct profit loss observed for thermal units in the case of additional storage capacities. These results can be considered intuitive and are intended to provide a proof of concept for the proposed modeling techniques.

Hydro-thermal Power Market Equilibrium with Price-Making Hydropower Producers

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Abstract

This paper formulates an electricity market dominated by price-making hydro-thermal generation. Generation companies optimize their unit commitment, scheduling and bidding decisions simultaneously as a Mixed Integer Programming problem and participate in a market under quantity competition, giving rise to a discontinuous Nash-Cournot game. Both hydropower and thermal units are considered as price-makers. The market equilibrium under uncertainty is computed via time stage decomposition and nesting of a Continuous Nash game into the original Discontinuous Nash game that can be solved via a search algorithm. To highlight applicability of the proposed framework, a case study on the Scandinavian power market is designed and suggests positive welfare effects of large scale storage, whereas the implications on scheduling of conventional units are subsequently discussed. Reformulation allows computationally efficient scaling of the problem and possible extensions to allow large scale applications are discussed.

Keywords: Hydropower, Hydro-Thermal, Cournot game, Nash equilibrium, discrete game, electricity market

1. Introduction

1.1. Background

Larger integration of renewable resources increases the challenges on liberal electricity markets. Such means of generation are, compared to conventional forms of generation, characterized by their low cost curves and uncertain capacity profiles. Higher shares of renewable generation could thus lead to increased supply side volatility as well as increased gaps between peak and base prices. Those effects will be eventually carried financially by the end consumer and, in interconnected systems, might spread to otherwise unaffected nodes or areas[1]. Applying flexible means of production mitigates this issue by applying the principle of 'peak skimming'[2], where a producer strategically schedules generation for the periods showing the highest market prices. Such flexible generation can come in form of conventional plants or energy storage, whereas hydropower plants provide the most

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prevalent large-scale application for latter. Despite their negligibly small cost curves, hydropower units with large enough storage capacities [1] compete with conventional generation for peak loads rather than for base loads with other means of renewable generation[3]. This paper addressed the issue of an electricity market dominated by hydro-thermal generation as price makers. Differing from the existing works on this topic, in this paper the hydropower producers simultaneously decide their unit commitment and scheduling strategies under uncertainty.

Table 1: Model Feature Comparison

Ref	***Hydropower***	***Thermal power***	***Uncertainty***	***Multiple periods***	***Multiple players***	***Price-makers***	***Non-convex players***
[1]	✓	✓			✓		
[3]	✓	✓	✓	✓	✓	✓	
[4]		✓			✓	✓	
[5]		✓		✓	✓	✓	
[6]	✓	✓		✓	✓	✓	
[7]	✓		✓	✓	✓	✓	
[8]	✓	✓		✓			✓
[9]	✓	✓	✓	✓	✓		
[10]	✓			✓	✓	✓	
[11]	✓	✓		✓	✓	✓	
[12]	✓	✓		✓	✓	✓	✓
[13]	✓		✓	✓	✓	✓	
[14]	✓	✓		✓	✓	✓	
[15]	✓	✓				✓	✓
[16]	✓			✓			✓
[17]	✓	✓	✓	✓	✓		✓
[18]			✓		✓	✓	
[19]					✓	✓	
[20]	✓	✓	✓	✓	✓		✓
[*]	✓	✓	✓	✓	✓	✓	✓

[*] refers to the model proposed in this paper

1.2. Related Works

In the literature, there are a multitude of examples given for analyzing the strategic aspects of conventional means of generation[21]. In Ref.[4] nodal prices were

[1]In relation to their generation capacities, as a reservoir with large storage capacity and smaller output capacity has higher flexibility regarding the time stages it chooses to feed into the system.

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derived through modeling transmission system operators and market operators as players participating in a *Cournot competition*. Ref.[22] shows electricity market applications of modeling market clearing through *supply function equilibria*, subsequently deriving *Nash equilibria* on the base of the cost functions of market participants. Ref.[23] extends the concept of Stackelberg games to multi-leader games and solves it through an *Equilibrium Problem with Equilibrium Constraints*(EPEC) formulation. In the model presented in Ref.[5], demand sided players are presented as strategic entities in a pool market.

Not considering technical specifications such as nonlinear efficiency curves, hydropower shows two prominent characteristics that differentiate it from conventional generation and make the above presented methodologies hard or impossible to apply: negligibly small generation cost functions and uncertain, period-transferable capacity in form of hydrological inventory. As a result, contrary to their conventional counterparts, bidding models for hydropower units generally consider price-taker approaches[24], leading to models for hydrothermal competition usually strictly separating between exercise of market power by conventional plants and efficient unit commitment by hydrological plants. Game-theoretical applications that focus on market clearing are found in Ref.[6, 7], whereas applications that focus on unit commitment are given in Ref.[8, 9].

There is indication that anticipation of price-making storage operators can impact market outcomes[10]. A few examples of literature analyze this topic: Ref.[11] describes a Cournot market clearing based on a Nash equilibium convergence algorithm through the Nikaido-Isoda function that has an active set method applied to stepwise converge to an equilibrium with optimal storage. Ref.[12] extends this concept to a deterministic multi-nodal Mixed-Integer market clearing problem and finds the optimal unit schedules via branch-and-cut. In Ref.[13] two different approaches are considered to model Nash equilibria in hydro-thermal systems. As the system is hydro-dominated focus is put on modeling uncertainty. Ref.[14] implements hydrological storage through a capacity constraint connecting time stages and thus otherwise individual models into a single market clearing model under Cournot competition. Ref.[15] focuses on the scheduling decisions of the thermal plants, ignoring inventory transfers through hydropower reservoirs and solving a series of deterministic Mixed Integer Problems to converge towards a balance in supply and demand to represent a cleared market. Despite the listed approaches, no literature is found on a problem as shown in Fig.1: hydropower producers participating in markets with changing access (on/off) for marginal (thermal) units, i.e. a game with a time-dynamic set of participants. The reason herefore, as e.g. listed in Ref.[25] is the difficulty of dealing with the duality gap created by such integer decisions. The model presented in this paper however aims to connect exercise of market power in hydrothermal systems with optimal scheduling and unit commitment, which has historically been focus of cost minimization[26]. This competitive market is formulated as a Discrete game[27] and can be solved by commercial solvers for its (potentially multiple) Nash equilibria, raising computational efficiency through reformulation[28].

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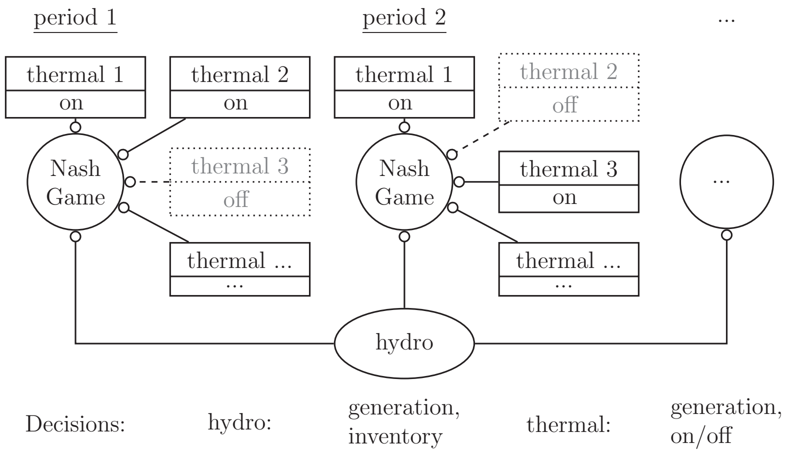
Figure 1: Multi Period Hydro-Thermal Game



1.3. Contributions

In the here presented paper the hydropower companies optimize their unit commitment and bidding strategies simultaneously under the consideration of uncertainty. The salient features of this paper are summarized as follows.

* **Combination of scheduling and price-maker bidding:** the presented model provides a novel tool for both generation companies and system operators to analyze the impact of storage capacities onto the network, thus incorporating the change of market power over time.
* **Model compactness:** the nested problem of a continuous game under uncertainty shows strong computational efficiency and thus has the potential to itself be used as an analytic tool (without solving the scheduling problem).
* **Practical applicability:** future applications of the presented model have a wide range. For example this could include analysis of capacity mechanisms or the impact of maintenance and involuntary down-times, refinement of hydropower-bidding through ability to model price impact, and analysis of the interactions of strategic scheduling and strategic storage. The here presented base model might be extended by additional dimensions (e.g. more nodes, market types) and constraints (e.g. reserve provision) in a similar manner to traditional equilibrium models[21] to enable it to cope with real-world problems.

A direct comparison to models from literature can be found in Tab.1.

1.4. Organization

The rest of this paper is organized as follows. In Section 2, the hydro-thermal model under consideration of uncertainty, periodic inflow and binary unit commitment decisions is formulated. Section 3 specifies the solution techniques used to

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yield what is later defined as 'Nash tuples'. Case studies are presented in Section 4 with discussions on welfare effects. Section 5 concludes the paper.

	Nomenclature	
J	players	
I_j	generation units of player j	
I_j^{Th}	thermal units of player j	
I_j^{Hy}	hydrological units of player j	
T	time periods	
Ξ	scenarios	
N	predefined schedules	
Φ	equilibrium tuples	
	Indexes	
j	player	$j \in J$
i	generation unit	$i \in I_j$
t	time period	[h]
s	inflow source period	[h]
t_2	inflow sink period	[h]
	Variables	
$q_{i,t}$	generation level	[MWh]
$b_{i,t}$	scheduling variable	[binary]
$q_{i,t}^s$	production from source period s used in period t	[MWh]
$q'_{i,t}$	generation decisions of other players $i \notin I_j$	[MWh]
n_i	selected schedule	$n_i \in \mathbb{R}^+$
	Parameters	
ξ	stochastic parameter	$\xi \in \Xi$
$\underline{q}_i, \bar{q}_i$	generation capacities	[MW]
$\bar{R}_i$	reservoir capacities	[MW]
P^ξ	scenario probability	[%]
k	convergence parameter	$k \in \mathbb{R}^+$
$c_{\text{fix}}, c_{\text{var}}^a, c_{\text{var}}^b$	cost function parameters	
	Functions	
$\Pi_{i,t}$	profit function	[€]
Q_i^s	inflow in hour s	[MWh/h]
$p_{j,t}$	price expectation of player j	[$\frac{€}{\text{MWh}}$]
$c_{i,t}$	cost function of unit i	[€]
d_t	demand function	[MWh]
$b_{i,t}$	scheduling function	[binary]
	Dual variables	
$\underline{\delta}_{i,t}, \bar{\delta}_{i,t}$	generation capacities	
σ_i	reservoir inflow	
$\gamma_{i,t}$	split representation	
$\psi_{i,t}$	reservoir capacity	
$\mu_{i,t}$	non-negativity	
$\omega_{i,t}^{p,\xi}$, $\omega_i^{Q,\xi}$	stochastic - deterministic gap	

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2. Model

2.1. Hydro-Thermal Generation

We assume a single area with a single pool market and competition in quantity. All supply sided participants, further referred to as generation companies or players, aim to solve a Mixed Integer profit maximization problem in the form of:

$$\max_{q_{i,t}, q^s_{i,t}, b_{i,t}} \quad \Pi_j = \sum_{i \in I_j} \sum_{t \in T} \Pi_{i,t}(\xi, q_{i,t}, b_{i,t}) \tag{1a}$$

$$\text{s.t.} \quad \underline{q}_i b_{i,t} \leq q_{i,t} \leq \bar{q}_i b_{i,t} \qquad \forall i \in I_j, t \in T \tag{1b}$$

$$\sum_{t=s}^{\max(T)} q^s_{i,t} \leq Q^s_i(\xi) \qquad \forall i \in I^{Hy}_j, s \in T \tag{1c}$$

$$\sum_{s=1}^{t} q^s_{i,t} = q_{i,t} \qquad \forall i \in I^{Hy}_j, t \in T \tag{1d}$$

$$\sum_{s=1}^{t-1} \sum_{t_2=t}^{\max(T)} q^s_{i,t_2} \leq \bar{R}_i \qquad \forall i \in I^{Hy}_j, (t > 1) \in T \tag{1e}$$

$$q_{i,t} \in \mathbb{R}, q^s_{i,t} \in \mathbb{R}^+, b_{i,t} \in [0,1]$$

Objective function Eq.(1a) incorporates all generation units owned by the player. The profit function of a single generation unit in a single time period, as shown below in Eq.(2), depends on a stochastic parameter representing uncertainty, as well as the chosen levels of generation and the scheduling variables. Each player might hold both thermal and hydropower units. The generation capacities given in Eq.(1b) depend on the unit schedules that define if the units are able to supply between their given minimum and maximum generation limits in a certain period. This depends on if the respective unit is running (i.e. $b_{i,t} = 1$) or shut down ($b_{i,t} = 0$). The inflow consistency constraint Eq.(1c) ensures that the hydropower units only use their given inflows, whereas s indicates the source period in which the inflow arrives at the reservoir. Decision variable $q^s_{i,t}$ represents how much of inflow from a source period is used for generation in period t. Subsequently, Eq.(1d) ensures that the total generation of those units matches this split representation. Physical capacities of reservoirs are considered in Eq.(1e): transfers from a source period $s < t$ into a sink period $t_2 \geq t$ count to the total inventory in period t which cannot exceed the upper limit of the reservoir. The reason why it is conducted over $(t > 1) \in T$ periods is that for a number of $\max(T)$ periods there are a number of $\max(T)-1$ inventory transfers between periods. For the sake of simplicity and similar to Ref.[14], no (mandatory) end reservoir levels are assumed. Starting reservoir levels are determined by the inflow in period 1.

The profit functions of the players are defined as:

$$\Pi_{i,t}(\xi, q_{i,t}, b_{i,t}) = p_{j,t}(\xi, \sum_{i_2 \in I_j} q_{i_2,t} + \sum_{i_2 \notin I_j} q'_{i_2,t}) q_{i,t} - c_{i,t}(q_{i,t}, b_{i,t}) \tag{2}$$

In Cournot competition, players calculate their individual profits based on assumptions representing decisions of other market participants, which are subsequently in this paper marked by $'$. As the model is aimed for short term applications, cost

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functions are assumed to be independent of uncertainty, thus limiting the profit impact of the stochastic parameter ξ on the components[2] of the price functions $p_{j,t}$. As hydropower cost components are generally considered to be negligible in short to medium term applications, the respective cost functions $c_{i,t}$ can be omitted, simplifying $\Pi_{i,t}(\xi, q_{i,t}, b_{i,t}) \rightarrow \Pi_{i,t}(\xi, q_{i,t}) \forall i \in I_j^{Hy}, t \in T$. In addition, this removal of the binary variables from the profit function also allows for a simplification of Eq.(1b) - since, for hydropower units, there are now no (negative) profit effects caused by the binary variables, they can be considered fixed as $b_{i,t} = 1 \forall i \in I_j^{Hy}, t \in T$. Furthermore, depending on the form of price and cost curves of the thermal units, the problem of a single player can take the form of a Mixed Integer Linear Program or a Mixed Integer Quadratic Program.

2.2. Market Clearing

A system operator would aim to clear the market by calculating a periodical demand, in the here presented framework with uniform pricing:

$$d_t = \sum_{i \in I_j} q_{i,t} + \sum_{i \notin I_j} q'_{i,t} \quad \forall j, t \tag{3}$$

To allow derivation of a definite market price, symmetric information on price curves has to be assumed:

$$p_{j_1,t}(\cdot) = p_{j_2,t}(\cdot) \quad \forall j_1 \in J, (j_2 \neq j_1) \in J, t \in T \tag{4}$$

As liberalized pool markets generally provide historical data publicly, such an assumption can be considered valid in practical applications. Thus, Eq.(2) can be simplified by:

$$p_{j,t}(\xi, \sum_{i_2 \in I_j} q_{i_2,t} + \sum_{i_2 \notin I_j} q'_{i_2,t}) \rightarrow p_t(\xi, d_t) \forall j \in J, t \in T \tag{5}$$

2.3. Model Limitations

As shown in Ref.[16], real-world hydropower short term unit commitment includes a large range of complicating factors such as interconnected inflows and technical characteristics (e.g. head and tail effects). In addition, due to increasing uncertainty through intermittent generation such as wind, unit commitment problems might require advanced techniques to cope with their tasks[17]. As mentioned below, in this paper however, uncertainty will be dealt with on an approximated level - through addition of slack variables and minimizing their weighted distance to a deterministic solution. Thus, the here presented problem gives a support tool to show the interaction between unit commitment and equilibrium problems and cannot replace optimal scheduling of units.

[2] In case of linear demand curves both slope and intercept would be subject to uncertainty.

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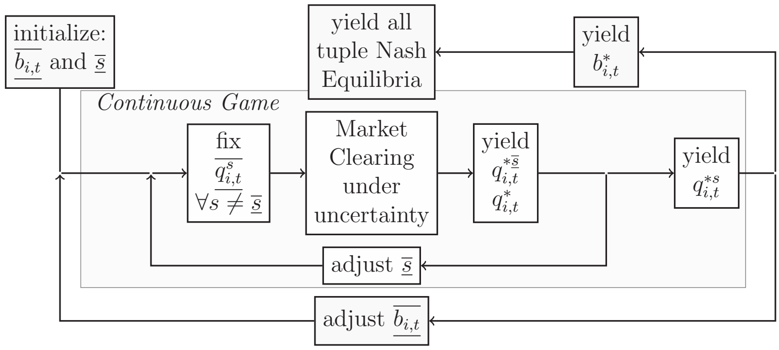
Figure 2: Solution Algorithm Flowchart (Discrete Game)



3. Solution Approaches

Even though the previously listed assumptions and simplifications support efficient solving of the model, several complications have still to be dealt with. Mainly, the previously mentioned duality gap caused by the binary scheduling variables that eliminates the possibility of straight-forward application of some of the conventional techniques shown in the introduction to this paper. Furthermore, the 'curse of dimensionality' related to problem size - i.e. amounts of scenarios, time stages, agents - has to be approached to allow for practical applications of the proposed framework. Thus, we propose a triple layer approach to derive multiple Nash Equilibria for the player decisions as shown in Eq.(1).

Regarding annotation: below we will mark fixed variables/parameters as $\overline{\underline{\cdot}}$ and optimal solutions of variables with $*$. The structure of the solution framework is shown in Fig.2.

3.1. Market Clearing under Uncertainty (Continuous Game part I)

The core problem is represented by the Karush Kuhn Tucker conditions of Eq.(1), with adjustments to cope with stochasticity and with initial presets for certain decision variables. Namely, the schedules - i.e. the binary decision variables - are given fixed values $\overline{\underline{b_{i,t}}}$, and a specific inflow source period $\underline{\overline{s}}$ is chosen, leading to all quantity decisions that are not related to this inflow period being considered as fixed parameters instead of variables, i.e. $q^s_{i,t} := \overline{\underline{q^s_{i,t}}} \forall (s \neq \underline{\overline{s}}) \in T$ with starting values of $\overline{\underline{q^s_{i,t}}} = 0$.

As presented in Fig.3 this decomposes the model in a number of smaller equilibria.

As with fixed schedules for thermal plants, price arbitrage between periods can only happen through hydropower units, considering a specific inflow period $\underline{\overline{s}}$ affects only the following periods. Thus a new period set can be defined:

$$T^{\underline{\overline{s}}} = \{t | t \in T, t \geq \underline{\overline{s}}\} \tag{6}$$

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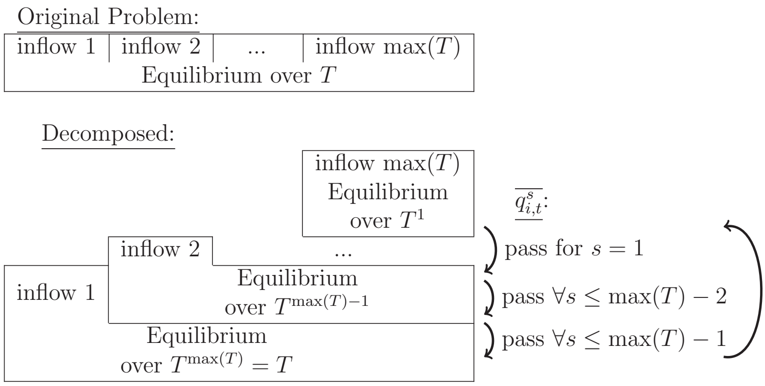
Figure 3: Continuous Game - Decomposition



This allows simplification and reformulation of Eq.(1):

$$
\begin{aligned}
\max_{q_{i,t},q^{\overline{\underline{s}}}_{i,t}} \quad & \Pi_j = \sum_{i\in I_j}\sum_{t\in T}\Pi_{i,t}(\xi, q_{i,t}, \underline{\overline{b_{i,t}}}) & & & \\
\text{s.t.} \quad & \underline{q}_i\underline{\overline{b_{i,t}}} \le q_{i,t} \le \bar{q}_i\underline{\overline{b_{i,t}}} & & \forall i \in I_j,\ t\in T^{\overline{\underline{s}}} & (\underline{\delta}_{i,t}, \bar{\delta}_{i,t}) \\
& \sum_{t\in T^{\overline{\underline{s}}}} q^{\overline{\underline{s}}}_{i,t} \le Q^{\overline{\underline{s}}}_i(\xi) & & \forall i \in I^{Hy}_j & (\sigma_i) \\
& \sum_{s=1}^{t}\begin{cases} q^{\overline{\underline{s}}}_{i,t}, & \text{if } s = \overline{\underline{s}} \\ \underline{\overline{q^{s}_{i,t}}}, & \text{if } s \neq \overline{\underline{s}} \end{cases} = q_{i,t} & & \forall i \in I^{Hy}_j,\ t\in T^{\overline{\underline{s}}} & (\gamma_{i,t}) \\
& \sum_{s=1}^{t-1}\sum_{t_2=t}^{\max(T)}\begin{cases} q^{\overline{\underline{s}}}_{i,t_2}, & \text{if } s = \overline{\underline{s}} \\ \underline{\overline{q^{s}_{i,t_2}}}, & \text{if } s \neq \overline{\underline{s}} \end{cases} \le \bar{R}_i & & \forall i \in I^{Hy}_j,\ (t > \overline{\underline{s}}) \in T^{\overline{\underline{s}}} & (\psi_{i,t}) \\
& q^{\overline{\underline{s}}}_{i,t} \ge 0 & & \forall i \in I^{Hy}_j,\ t\in T^{\overline{\underline{s}}} & (\mu_{i,t})
\end{aligned}
\tag{7}
$$

For a single scenario, taking the Karush Kuhn Tucker conditions of this problem allows derivation of an equilibrium solution. To clear the problem under several scenarios $\xi \in \Xi$ with individual scenario probabilities we apply a similar approach as found in Ref.[18], extending the model by two dual variables $\omega^{p,\xi}_{i,t}$ and $\omega^{Q,\xi}_{i}$ that represent the shadow prices of a scenario deviating from a selected deterministic solution, therefore minimizing the residuals between a deterministic solution and all scenarios. Discussion on this approach is provided in Appendix A.

Subsequently, the Karush Kuhn Tucker conditions are formulated and provided in Eq.(B.1) which can be found in Appendix B. The Lagrangians are shown in Eq.(B.1a) to (B.1c), the feasibility and complementarity conditions are found in Eq.(B.1d) to (B.1h). Eq.(B.1j) and (B.1k) respectively present the price and inflow consistency constraints that minimize the residuals between the different scenarios whose probability adjusted shadow prices can be found to affect Eq.(B.1a), Eq.(B.1b), and Eq.(B.1f).

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3.2. Backwards Pass (Continuous Game part II)

As shown in Fig.2 and Fig.3, readjustment of the fixed inflow source period $\underline{\overline{s}}$ is required to accommodate for all inflow periods. By nesting the single period problem into a backwards pass algorithm, the multi-period problem can be solved. After choosing a value for the convergence parameter this algorithm can be implemented in the following manner:

Algorithm 1.

0. *initialize* $\underline{\overline{s}} := \max(T)$, $run := 1$, $\Pi_j^0 := -\infty \; \forall j \in J$
1. *solve KKT conditions*
2. *update:* $\underline{\overline{q_{i,t}^s}} := q_{i,t}^{*\underline{\overline{s}}} \; \forall j \in J, i \in I_j^{hy}, t \in T, s = \underline{\overline{s}}$ *and* $\Pi_j^{run} := \Pi_j^* \; \forall j \in J$
3. *if* $\underline{\overline{s}} > 1$*:* $\underline{\overline{s}} := \underline{\overline{s}} - 1$ *and back to 1.*
4. *if* $\sum_{j \in J} |\Pi_j^{run-1} - \Pi_j^{run}| > k$ *:* $run := run + 1$ *and back to 1.*
5. ***converged:*** $q_{i,t}^{*s} = \underline{\overline{q_{i,t}^s}} \; \forall j \in J, i \in I_j^{hy}, t \in T, s \in T$

As mentioned in Ref.[13], such an equilibrium will only show one potential schedule for the hydropower units. Even with a convergence parameter of $k = 0$(as in the latter presented case study) and therefore requiring a global optimum, other schedules might result in similar profits making the optimum non-unique. Discussion on this matter can be found in Appendix C. However, as the derivation of the discontinuous Nash equilibrium only uses the information on objective values of the player problems (which are constant over all equilibria) and not the specific generation decisions (which might diverge), validity of the results still holds.

3.3. Discontinuous Nash Equilibrium (Discrete Game)

As displayed in Fig.2 the Backwards Pass algorithm determines optimal generation in the equilibrium, considering that unit commitment is predefined. Such fixed ramping schedules as utilized e.g. in Ref.[19, 20] which assume that binary schedules of the units are pre-established and treated as input parameters to the problem. However, as the here presented model aims to give the thermal plants the possibility to react to hydropower decisions (i.e. withdraw from time stages with peaks lowered through storage arbitrage). Fixing the ramping schedules and thus the binary variables $b_{i,t}$ eliminates those actions by the thermal players, thus weakening their model strategies in relation to their options in reality and thus distorting the model results and subsequently displaying a skewed representation of the equilibrium.

On the contrary to fixed schedules, all potential iterations of the binary variables would amount to a number of $2^{\sum_{j \in J} |I_j^{th}| \times \max(T)}$. Each of those iterations would in turn represent a 'Nash tuple', that is an equilibrium solution derived by the Continuous game as shown above.

Thus, fixing schedules to a single outcome might not represent the reality adequately, whilst keeping all iterations in the game introduces the problems traditionally related to Mixed Integer Programming: increasing complexity and the possibility of ending up in local maximums.

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To adequately address this issue, we apply an approach that can be considered a middle course between those mentioned. By using a reformulation similar to Ref.[28], we are able to reduce computational complexity whilst still keeping the core strategies of the players intact: instead of solving for the binary decision variable, we replace $b_{i,t} \rightarrow b_{i,t}(n_i)$. The function $b_{i,t}(n_i)$ represents preselected schedules indexed by the decision variable $n_i \in N_i$. This reduces complexity to a new number of iterations: $iter = 1 \leq 2^{\sum\limits_{j\in J}\sum\limits_{i\in I_j^{th}} |N_i|} \leq 2^{\sum\limits_{j\in J} |I_j^{th}|\times \max(T)}$. Even though this reformulation of the scheduling decisions results in a computationally less demanding problem setup, it still is of $\mathcal{NP}$-hard nature[29]. Modern techniques usually tend to work with various branch-and-cut approaches to derive solutions for such discontinuous problems[30]. However, comparing and evaluating different outcomes for Nash equilibria in their "validity" itself poses a problem that is not straightforward[27]. Furthermore, Nash equilibria are not necessarily globally optimal for the players, as demonstrated by famous examples such as the prisoners' dilemma. As a result, instead of applying an approach using bounds and risking jeopardizing potential viable equilibria, pre-selection of an adequately sized number of predefined schedules to enable brute-forcing all equilibrium tuples was conducted. This is made possible as every tuple (if feasible) can be solved for an equilibrium that defines profits for each player, allowing to determine dominant strategies (i.e. scheduling decisions). By defining a player j's assumption on other generation units' schedules as $n'_{i_2}\ \forall i_2 \in I_{j_2\neq j}$ a *Nash tuple equilibrium* can be defined as:

$$\begin{aligned} &\Phi = \langle q^*_{i,t}, q^{*s}_{i,t}, b_{i,t}(n^*_i)\rangle \\ &\text{where} \\ &\Pi^*_j = \max_{n_i} \sum_{i\in I_j}\sum_{t\in T} \Pi_{i,t}(\xi, q^*_{i,t}/q'^*_{i_2,t}, b_{i,t}(n_i)/b_{i_2,t}(n'_{i_2})) \quad \forall j \in J \end{aligned} \tag{8}$$

As each iteration can potentially represent an equilibrium, the number of tuple equilibria $|\Phi|$ will range within $0 \leq |\Phi| \leq iter$.

To derive the equilibria a search algorithm, e.g. in the following form, can be used:

Algorithm 2.

0. *initialize* $\phi(n_i) := \{0|\mathbb{N}^{\prod\limits_{i\in I_j} N_i}, j \in J\}$
1. *for* $j \in J$*:*
2. *initialize* $N'_j = \{n_{i_2}|n_{i_2} \in N_{i_2}, i_2 \in I_{j_2\neq j}\}$
3. *while* $N'_j \neq \{\varnothing\}$*:*
4. *choose any* $n'_{i_2} \in N'_j$
5. *solve* Π^*_j
6. *increment* $\phi(n_i/n'_{i_2}) := \phi(n_i/n'_{i_2}) + 1$
7. *remove* n'_{i_2} *from* N'_j
8. *for all* n_i *where* $\phi(n_i) = \max(J)$*:* $n_i = n^*_i$

This algorithm builds on the requirement of a computationally feasible set of available schedules. Adding more sophistication in terms of a larger set of unit

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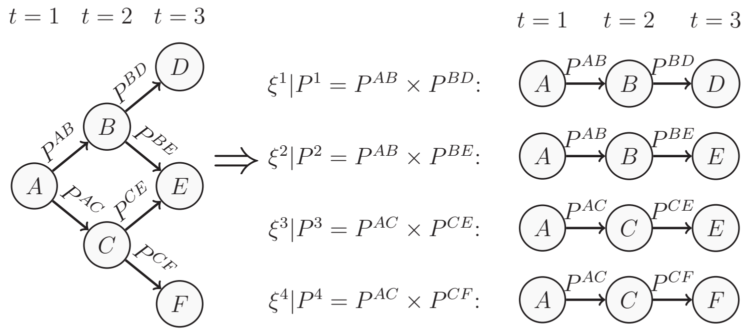
Figure 4: Obtaining Scenarios through Lattice Separation



schedules would require application of additional techniques, i.e. branch-and-bound or branch-and-cut as demonstrated in e.g. Ref.[31]. However, as the here presented model aims to focus on short term time frames, it is reasonable to assume that a range of potential unit commitment schedules is already established. As mentioned before, literature traditionally assumes a single such schedule, whereas we relax this by providing a set of potential schedules to our players to choose from.

4. Case Study

To validate the proposed model and methods, a test case representing the Scandinavian power system was designed to represent a late spring scenario in Southern Sweden. The aim of the case study is to provide a showcase of the capabilities of the demonstrated model framework.

Test System. Three oligopolistic players - respectively holding a hydropower, thermal and mixed generation portfolio, were considered competing over 7 periods. Out of the five thermal plants three heterogeneous plant types (Gas/Coal/Oil) were introduced, whereas units of a specific type were modeled in homogeneous manner in regards to generation capacities, up-/down-time limits and cost curves. The thermal units were given quadratic cost functions in the form of:

$$c_{i,t}(n_i, q_{i,t}) = c_{\text{fix}} b_{i,t}(n_i) + c^a_{\text{var}} q_{i,t} + c^b_{\text{var}} (q_{i,t})^2 \tag{9}$$

For the sake of simplicity in demonstration, ramp-up and -down cost were replaced with fixed rates for up-time.

The five hydropower units were modeled heterogeneous in regards to generation capacities, reservoir sizes, degree of regulation (relation of generation to reservoir size, where generally a low value can be used to represent a run-of-river unit and a large value a long term storage unit), inflow (base level, variability and trend). This can be observed in Fig.5 which shows the scenarios for latter[3]. The total

[3]The size of the outer rings and strength of the connection lines display the likelihood of the scenario.

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Figure 5: Inflow Scenarios for the Hydropower Units[3]

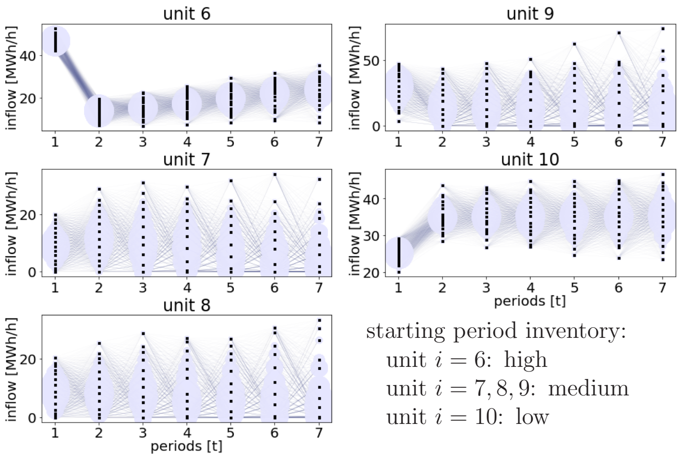
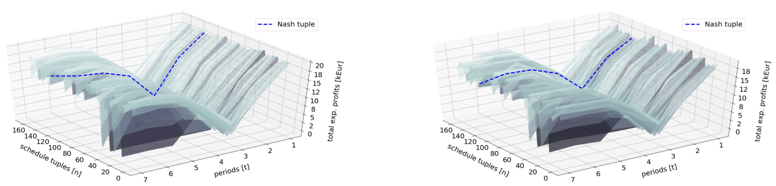


Figure 6: Nash Tuples

(a) Nash Tuple Hydro-Thermal Case (b) Nash Tuple Thermal-dominated Case



generation capacities for the thermal and hydropower plants were $1300MW$ and $280MW$ respectively.

Solvers. The model was coded in Python, using the multi solver interface Pyomo[32]. This interface provides nonlinear transformations for complementarity problems with continuous variables[33] based on a constrained optimization technique from Ref.[34], enabling the usage of freeware tools such as the nonlinear interior-point solver IPOPT[35]. Performance on an Intel i7-5600U was an average of 80.5 seconds to yield a tuple solution in each case, i.e. to solve the Continuous Game, for both cases running in parallel.

Base Case (Hydro-thermal competition). In the case study, the chosen form of representing uncertainty was using Markov processes transformed into samples. Techniques to obtain such scenario lattices that adequately represent the distribution exist plenty in literature, i.e. Ref.[36]. Therefore, we will assume that an adequate discrete representation of price and inflow distributions are given and can be transformed into scenarios as Fig.4 shows for a three period example. Historical price data was obtained from the public database of Scandinavian market operator Nordpool,

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Figure 7: Welfare Distribution

(a) Demand Side (b) Supplier Side

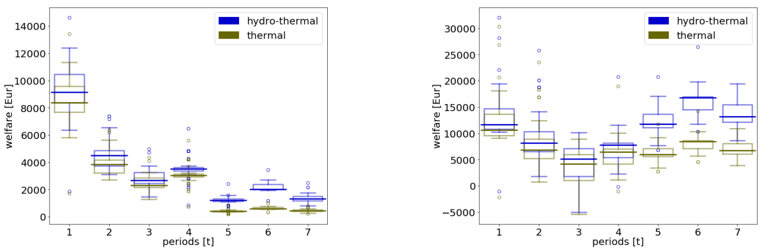
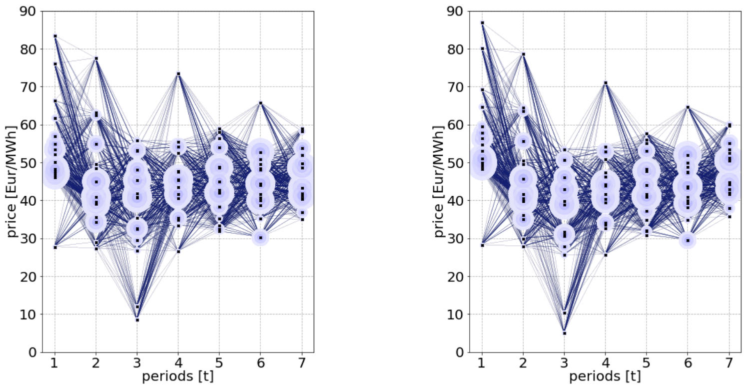


Figure 8: Price Scenarios for Nash Tuples[3]

(a) Hydro-Thermal Case (b) Thermal-dominated Case



whereas the elasticity of the linear price curves was extrapolated based on volume data[4]. 100 uniformly distributed initial scenarios were obtained and reduced to 15 normally distributed scenarios per time period.

162 schedule iterations(/tuples) of the thermal units were realized, whereas the per-period profits of every iteration as well as the tuple representing the (single) Nash equilibrium in schedules can be found in Fig.6a.

Thermal-dominated competition. For the second case analyzed, the hydropower player holding four out of five units (and thus holding a quasi-monopoly on storage, apart from unit $i = 6$) was removed, leading to a thermal dominated game between two players with a total capacity of $1350MW$ resulting in a different Nash equilibrium tuple found in Fig.6b.

Fig.7 shows that removing the hydropower player has a higher impact on the supply side welfare than the demand side, whereas this effect is amplified in periods

[4]The elasticity was adjusted accordingly for the thermal-dominated case presented below.

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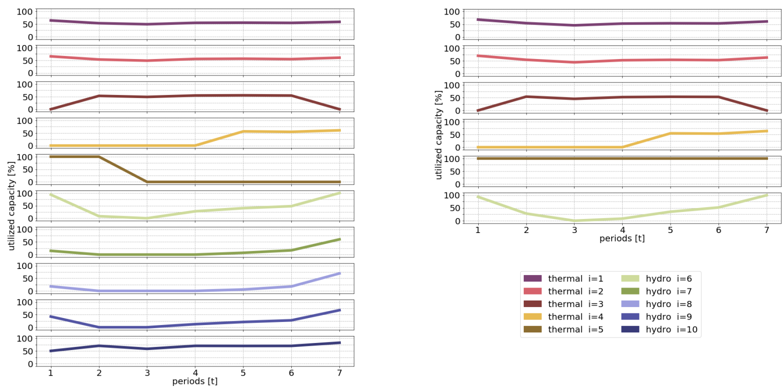
Figure 9: Expected Utilizations of Maximum Unit Capacity

(a) Hydro-Thermal Case (b) Thermal-dominated Case



with lower variance in prices. This suggests that peak skimming has not only strong positive effects on customer welfare but also benefits the generators, even though the introduction of additional storage shows dampening effects over the whole spectrum of price scenarios, as Fig.8 illustrates. Paradoxically, as seen in Fig.7b this drop of prices through addition of storage creates positive welfare effects on the supplier side. The reason herein lies in that the hydropower producer holds close to $\frac{3}{4}$ of the supplier profits by being able to generate at infinitesimally low cost. Further, this positive impact on the supplier side seems to be of greater extend than on the demand side, indicating that suppliers profit more from the cost savings of introducing storage technology. This can partially be explained by low elasticities of the price curves, as characteristic for electricity markets[21], but also is a result of the here presented model being able to accurately capture costs related to schedules.

Especially in period 3, another negative effect for the players holding thermal plants can be observed: due to plants aiming to schedule for upcoming high price periods, certain periods shows welfare potentially taking negative values, i.e. generators supplying under loss. In systems with storage inflow from the market (e.g. pumped hydro storage), capacity could be taken and transferred to later stages. In systems like in the here analyzed cases, no such possibility for moving capacity directly exists. Thus the thermal producers are given only two options: reduce the outputs with current schedule or change schedule. Either way, the thermal producers are forced away from their optimal point and punished for inflexibility. Thus, in systems with a large share of storage facilities with natural inflow (traditional hydropower plants) inflexible thermal units might see adverse effects of inflexibility amplified, whereas in systems with market inflow (e.g. pumped hydro storage), such effects might be dampened.

This change in schedule can be observed in Fig.9, which shows the adjustments in the output quantities of player $j = 1$ holding units $i = 1, 2$ and player $j = 2$ holding units $i = 3, 4, 5, 6$. All units apart from $i = 5$ react with slight adjustments of their

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Table 3: Fixed Cost $c_{\text{fix}} b_{i,t}(n_i)$ [€]

case	unit	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$	$t=6$	$t=7$
Hydro-Thermal	$i=1$	500	500	500	500	500	500	500
	$i=2$	2000	2000	2000	2000	2000	2000	2000
	$i=3$	0	500	500	500	500	500	0
	$i=4$	0	0	0	0	2000	2000	2000
	$i=5$	850	850	**0**	**0**	**0**	**0**	**0**
	$\sum_{i=1}^{5}$	3350	3850	**3000**	**3000**	**5000**	**5000**	**4500**
Thermal	$i=1$	500	500	500	500	500	500	500
	$i=2$	2000	2000	2000	2000	2000	2000	2000
	$i=3$	0	500	500	500	500	500	0
	$i=4$	0	0	0	0	2000	2000	2000
	$i=5$	850	850	**850**	**850**	**850**	**850**	**850**
	$\sum_{i=1}^{5}$	3350	3850	**3850**	**3850**	**5850**	**5850**	**5350**

Table 4: System Marginal Cost of Capacity

case		$t=1$	$t=2$	$t=3$	$t=4$	$t=5$	$t=6$	$t=7$
Hy.-Th.	marginal unit	$i=3$	$i=4$	$i=5$	$i=5$	$i=5$	$i=5$	$i=3$
	$mc_t^{\text{cap}}[€/MW]$	6.67	14.29	14.17	14.17	14.17	14.17	6.67
Th.	marginal unit	$i=3$	$i=4$	$i=4$	$i=4$	-	-	$i=3$
	$mc_t^{\text{cap}}[€/MW]$	6.67	14.29	14.29	14.29	∞	∞	6.67

∞ relates to no available capacity

output levels, withholding a minor amount of output in the thermal-dominated case and thus causing the price increase mentioned above. Reducing the amount of players leading to resuming players withholding quantity is an expected characteristic of Cournot models. However, in our presented model this effect is reduced by unit $i = 5$ switching the schedule in the thermal case in order to produce on maximum generation level. This leads to the conclusion that giving players the option to enter or leave a Cournot game (in our model through scheduling units for the respective time periods) dampens the effects of exercising market power. The impact on the total fixed cost in the system can be found in Tab.3.

This gives the possibility to formulate the system marginal cost of additional capacity by finding the minimum cost for adding an additional MW to the system:

$$mc_t^{\text{cap}} = \min_i \frac{\partial c_{i,t}(q_{i,t}, b_{i,t})}{\partial b_{i,t}} / \underline{q}_i \cdot (1 - b_{i,t}(n_i)) + \infty \cdot b_{i,t}(n_i) \tag{10}$$

The results of this for both cases can be found in Tab.4 which show an increase in cost for the case where more thermal capacity is procured.

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Table 5: Marginal Cost of Energy[€$/MWh$]

case	unit	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$	$t=6$	$t=7$
Hydro-Thermal	$i=1$	41.95	36.48	34.42	37.24	37.6	37.15	39.14
	$i=2$	37.95	33.61	32.00	34.37	34.72	34.09	36.14
	$i=3$	10	36.48	34.42	37.24	37.6	37.15	10
	$i=4$	15	15	15	15	34.72	34.09	36.14
	$i=5$	∞	∞	23	23	23	23	23
	system	37.95	33.61	32.00	34.37	34.72	34.09	36.14
Thermal	$i=1$	43.99	37.09	32.78	36.17	36.82	36.57	40.36
	$i=2$	39.62	34.10	30.63	33.48	34.06	33.61	37.17
	$i=3$	10	37.09	32.78	36.17	36.82	36.57	10
	$i=4$	15	15	15	15	34.06	33.61	37.17
	$i=5$	∞	∞	∞	∞	∞	∞	∞
	system	39.62	34.10	30.63	33.48	34.06	33.61	37.17

∞ relates to no available capacity

Further, the marginal costs of energy can be derived from Eq.(9):

$$p_{j,t}(\xi, \sum_{i_2 \in I_j} q_{i_2,t} + \sum_{i_2 \notin I_j} q'_{i_2,t}) \to p_t(\xi, d_t) \forall j \in J, t \in T \tag{11}$$

The results, presented in Fig.5 show that the cost effects of additional storage can not necessarily be found in reduction of cost, as the average marginal cost over all time periods is with 34.70€$/MWh$ in fact slightly higher in the hydro-thermal case than the 34.67€$/MWh$ in the thermal-dominated case. In addition, a flattening effect of storage can be observed. In traditional economic models that do not consider start-up and shut-down decisions, the system marginal costs would be given by $i=4,5$ in the hydrothermal case and by $i=4$ in the thermal case. In the here presented model however, those units are not participating in the market, as they are either not started in the first period (unit $i=4$) or actively decide to withdraw from the market (unit $i=5$ in the hydrothermal case). Thus, our model is able to capture both the indirect impact of changing capacity costs on energy prices as well as on marginal cost of energy. This plays a role regarding the modeling of otherwise homogeneous units. Units $i=1,3$ as well as units $i=2,4$ are assumed to be of similar types regarding cost curves and capacities. However, with different initial states (units $i=1,2$ running from the start of the time frame and units $i=3,4$ being off) and different proposed schedules, the unit commitment and scheduling decisions differ vastly in the Nash equilibrium tuples of both cases, as shown in Fig.9.

These scheduling tuples being Nash equilibria can also be supported by the results of the individual players. Fig.10 shows that player $j=2$ has an economic incentive to switch the schedule on unit $i=5$, with the chosen optimal schedules (taken from the Nash equilibrium tuple) showing a more beneficial outcome for a *risk-neutral* player.

The case studies indicate, that in a system relying on an energy only market, the increasing fixed cost from changes in schedule as presented in Fig.3 and thus the cost of optimal generation capacity are carried by both suppliers and generators.

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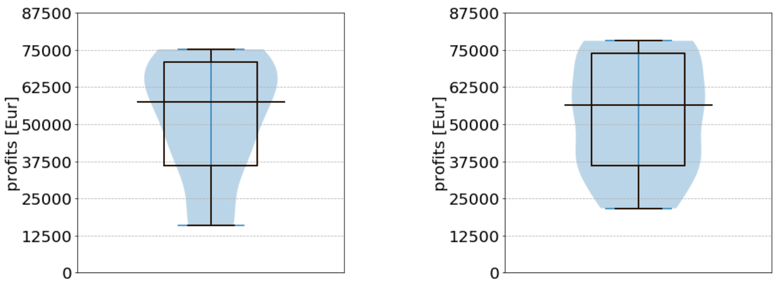
Figure 10: Profit Distributions of Player $j = 2$ in Thermal-dominated Case (probability distributions indicated in blue)

(a) Optimal Schedules (b) Schedules from Hydro-Thermal Case



Even in the here presented monopolistic cases, both suppliers and generators share the loss of welfare after losing flexibility (by removing the storage provider from the system).

However, adding additional capacities would require a payment in height of the marginal values presented in Tab.4. The derived marginal costs in the here presented case studies however lie significantly below the observed prices of regulating power in the analyzed area of Nordpool (commonly close to equal to energy prices), indicating high potential profit margins for participants in this market type. However, technical simplifications such as assuming a fixed cost rate instead of start-ups and shutdowns might play a role and thus we advise further research on this topic.

Model Discussion. The proposed approach presents a novelty in literature: solving an equilibrium model with two groups of agents that either actively choose to participate depending on the period or are able to distribute inventory over the time frame. This gives the possibility to quantify the impact of scheduling on market equilibria and thus allow to derive marginal cost of capacity, even for an energy-only market as the example of the case study demonstrates.

Initial tests hint that scalability of the model for larger applications in dimensions of time, player size and scenarios is of satisfying performance, especially considering the original problem is $\mathcal{NP}$-hard. The strictly disconnected nature of the Continuous and the Discrete Game also allows for partial adjustments of the model. Examples for former would be additional nodes and related transfer flows, more than a single energy markets or other uncertain parameters as the ones already considered. Examples for latter would be addition of capacity mechanisms or consideration of unit maintenance. As an extension of the model, we propose further discussion and research on replacing the Continuous Game by a Non-Convex Game and the resulting conditions to keep the Nash tuple equilibria yielded by the Discrete Game valid.

5. Conclusion

The presented base model shows a novel problem setup: players optimizing unit commitment and bidding simultaneously (in form of a Mixed Integer Lin-

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ear/Quadratic Problem) whilst competing against each other in a multi-period Cournot market under uncertainty. A nested equilibrium approach is proposed, first finding the Nash equilibria within a decomposed Continuous Game and subsequently comparing the resulting discrete decision tuples to yield what we refer to as Nash tuple equilibria.

Both the proposed Continuous and Discrete Game are complete novelties in literature and allow for a wide range of applications and adaptions. One such application is presented in form of a case study on the Scandinavian power market, where the quantitative influence of removing hydropower storage capacity is analyzed. Without having the hydropower generator in the system, sufficient capacities to conduct 'peak-skimming' are missing, leading to additional thermal unit start-ups. The welfare effects of those are observed and indicate that apart from the demand side also the supplier side is influenced negatively by a lack of storage. In addition, the effects on marginal cost on both continuous and discontinuous variables is calculated, which allows for derivation of a system marginal cost of capacity, even in the presented energy-only market. This allows for a variety of practical applications. In the here presented case study this is shown by being able to analyze the welfare losses of withdrawing flexible (storage) capacity. The results indicate that a loss of flexibility is shared by both suppliers and generators. However, increasing flexibility through additional dispatch over the point of optimal capacity has to be carried by the supply side (as e.g. through capacity or reserve payments to generators). In addition, through the possibility of deriving 'marginal cost of capacity' our results indicate that the approximate 1:1 relation of reserve and spot/intraday prices in Nordpool might be an overestimate in favor of reserve providers.

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References

[1] F. Sensfuß, M. Ragwitz, M. Genoese, The merit-order effect: A detailed analysis of the price effect of renewable electricity generation on spot market prices in Germany, Energy Policy 36 (8) (2008) 3076–3084. doi:10.1016/j.enpol.2008.03.035.

[2] S. Stoft, Power System Economics Designing Markets for Electricity, Wiley, New York, 2002.

[3] S.-E. Fleten, T. T. Lie, A Stochastic Game Model Applied to the Nordic Electricity Market, in: World Scientific Series in Finance, Vol. 4, 2013, pp. 421–441.

[4] B. F. Hobbs, Linear Complementarity Models of Nash Cournot Competition in Bilateral and POOLCO Power Markets, IEEE Transactions on Power Systems 16 (2) (2001) 194–202.

DOI: 10.1016/j.energy.2018.08.162

[5] S. De la Torre, J. Contreras, A. J. Conejo, Finding Multiperiod Nash Equilibria in Pool-Based Electricity Markets, IEEE Transactions on Power Systems 19 (1) (2004) 643–651. doi:10.1109/TPWRS.2003.820703.

[6] J. Villar, H. Rudnick, Hydrothermal market simulator using game theory: Assessment of market power, IEEE Transactions on Power Systems 18 (1) (2003) 91–98. doi:10.1109/TPWRS.2002.807061.

[7] K. C. Almeida, A. J. Conejo, Medium-term power dispatch in predominantly hydro systems: An equilibrium approach, IEEE Transactions on Power Systems 28 (3) (2013) 2384–2394. doi:10.1109/TPWRS.2012.2227838.

[8] T. Li, M. Shahidehpour, Price-Based Unit Commitment : A Case of Lagrangian Relaxation Versus Mixed Integer Programming, IEEE Transactions on Power Systems 20 (4) (2005) 2015–2025.

[9] O. Wolfgang, A. Haugstad, B. Mo, A. Gjelsvik, I. Wangensteen, G. Doorman, Hydro reservoir handling in Norway before and after deregulation, Energy 34 (10) (2009) 1642–1651. doi:10.1016/j.energy.2009.07.025.
URL http://dx.doi.org/10.1016/j.energy.2009.07.025

[10] P. Zamani-Dehkordi, S. Shafiee, L. Rakai, A. M. Knight, H. Zareipour, Price impact assessment for large-scale merchant energy storage facilities, Energy 125 (2017) 27–43. doi:10.1016/j.energy.2017.02.107.

[11] J. P. Molina, J. M. Zolezzi, J. Contreras, H. Rudnick, M. J. Reveco, Nash-Cournot Equilibria in Hydrothermal Electricity Markets, IEEE Transactions on Power Systems 26 (3) (2011) 1089–1101.

[12] M. Löschenbrand, M. Korpas, Multiple Nash Equilibria in Electricity Markets with price-making Hydrothermal Producers, IEEE Transactions on Power Systems (in press) (2018) 1–10. doi:10.1109/TPWRS.2018.2858574.
URL https://ieeexplore.ieee.org/document/8417933/

[13] E. Moiseeva, M. R. Hesamzadeh, Bayesian and Robust Nash Equilibria in Hydro-Dominated Systems under Uncertainty, IEEE Transactions on Sustainable Energy - (-) (2017) 1–12. doi:10.1109/TSTE.2017.2762086.
URL http://ieeexplore.ieee.org/document/8064663/

[14] J. Bushnell, A Mixed Complementarity Model of Hydrothermal Electricity Competition in the Western United States, Operations Research 51 (1) (2003) 80–93. doi:10.1287/opre.51.1.80.12800.

[15] J. Contreras, O. Candiles, J. I. De La Fuente, T. Gomez, A cobweb bidding model for competitive electricity markets, IEEE Transactions on Power Systems 17 (1) (2002) 148–153. doi:10.1109/59.982206.

[16] Y. Sahraoui, P. Bendotti, C. D'Ambrosio, Real-world hydro-power unit-commitment: Dealing with numerical errors and feasibility issues, Energy (in press) (2018) 1–14. doi:10.1016/j.energy.2017.11.064.
URL https://doi.org/10.1016/j.energy.2017.11.064

DOI: 10.1016/j.energy.2018.08.162

[17] T. Schulze, K. Mckinnon, The value of stochastic programming in day-ahead and intra-day generation unit commitment, Energy 101 (2016) 592–605. doi:10.1016/j.energy.2016.01.090.
URL http://dx.doi.org/10.1016/j.energy.2016.01.090

[18] K. A. Brekke, R. Golombek, M. Kaut, S. A. Kittelsen, S. W. Wallace, Stochastic energy market equilibrium modeling with multiple agents, Energy 134 (2017) 984–990. doi:10.1016/j.energy.2017.06.056.
URL http://dx.doi.org/10.1016/j.energy.2017.06.056

[19] A. Rahimi-kian, H. Haghighat, Gaming Analysis in Joint Energy and Spinning Reserve Markets, IEEE Transactions on Power Systems 22 (January 2015) (2007) 2074 – 2085. doi:10.1109/TPWRS.2007.907389.

[20] A. Baillo, M. Ventosa, M. Rivier, A. Ramos, Optimal Offering Strategies for Generation Companies Operating in Electricity Spot Markets, IEEE Transactions on Power Systems 19 (2) (2004) 745–753.

[21] S. A. Gabriel, A. J. Conejo, J. D. Fuller, B. F. Hobbs, C. Ruiz, Complementarity Modeling in Energy Markets, Springer, New York, 2013.

[22] A. Rudkevich, On the supply function equilibrium and its applications in electricity markets, Decision Support Systems 40 (3-4 SPEC. ISS.) (2005) 409–425. doi:10.1016/j.dss.2004.05.004.

[23] S. Leyffer, T. Munson, Solving multi-leader-common-follower games, Optimization Methods and Software 25 (4) (2010) 601–623. doi:10.1080/10556780903448052.

[24] G. Steeger, L. A. Barroso, S. Rebennack, Optimal Bidding Strategies for Hydro-Electric Producers: A Literature Survey, IEEE Transactions on Power Systems 29 (4) (2014) 1758–1766. doi:10.1109/TPWRS.2013.2296400.

[25] Y. Chen, B. F. Hobbs, An oligopolistic power market model with tradable NOx permits, IEEE Transactions on Power Systems 20 (1) (2005) 119–129. doi:10.1109/TPWRS.2004.840440.

[26] J. F. Bard, Short-Term Scheduling of Thermal-Electric Generators Using Lagrangian Relaxation, Operations Research 36 (5) (1988) 756–766. doi:10.1287/opre.36.5.756.
URL http://pubsonline.informs.org/doi/abs/10.1287/opre.36.5.756

[27] B. P. Bajari, H. Hong, S. P. Ryan, Identification and Estimation of a Discrete Game of Complete Information, Econometrica: Journal of the Econometric Society 78 (5) (2010) 1529–1568. doi:10.3982/ECTA5434.

[28] B. Zeng, Y. An, Solving Bilevel Mixed Integer Program by Reformulations and Decomposition, Optimization Online (2014) 1–34.

DOI: 10.1016/j.energy.2018.08.162

[29] K. G. Murty, F.-T. Yu, Linear Complementarity, Linear and Nonlinear Programming, Heldermann, Berlin, 1988.
URL http://www-personal.umich.edu/{~}murty/books/linear{_}complementarity{_}webbook/lcp-complete.pdf

[30] D. Pozo, E. Sauma, J. Contreras, Basic theoretical foundations and insights on bilevel models and their applications to power systems, Annals of Operations Research 254 (1) (2017) 303–334. doi:10.1007/s10479-017-2453-z.

[31] F. M. Kue, Solving discrete linear bilevel optimization problems using the optimal value reformulation, Journal of Global Optimization 68 (2) (2017) 255–277. doi:10.1007/s10898-016-0478-5.

[32] W. E. Hart, J. P. Watson, D. L. Woodruff, Pyomo: Modeling and solving mathematical programs in Python, Mathematical Programming Computation 3 (3) (2011) 219–260. doi:10.1007/s12532-011-0026-8.

[33] W. E. Hart, C. D. Laird, J.-P. Watson, D. L. Woodruff, G. A. Hackebeil, B. L. Nicholson, Pyomo - Optimization Modeling in Python, 2nd Edition, Springer Science and Business Media, 2017.

[34] M. C. Ferris, S. P. Dirkse, A. Meeraus, Mathematical Programs with Equilibrium Constraints: Automatic Reformulation and Solution via Constrained Optimization, Tech. rep. (2002).

[35] A. Wächter, L. T. Biegler, On the implementation of primal-dual interior point filter line search algorithm for large-scale nonlinear programming, in: Mathematical Programming, Vol. 106, Springer, 2006, pp. 25–57. doi:10.1007/s10107-004-0559-y.
URL http://link.springer.com/article/10.1007/s10107-004-0559-y

[36] N. Löhndorf, A. Shapiro, Modeling Time-dependent Randomness in Stochastic Dual Dynamic Programming (2010).

Appendix A. Market Equilibria under Uncertainty

In order to elaborate on the method used to deal with uncertainty, we will make use of an adjusted one-period example of the previously presented problem. In this example players maximize profits under uncertain output capacities:

$$\begin{array}{lll} \max\limits_{q_i} & \sum\limits_{i\in I_j} p(\xi, \sum\limits_{j\in J}\sum\limits_{i_2\in I_j} q_{i_2})q_i - c_i(q_i) & \\ \text{s.t.} & 0 \leq q_i \leq \bar{q}_i(\xi) & (\underline{\delta}_i, \bar{\delta}_i) \end{array} \tag{A.1}$$

The reader might notice that this resembles the bidding problem of a renewable energy producer such as wind or solar closer than the hydro-thermal example of this paper. This however is intended as the displayed example provides a problem under two different equations - an objective function and an inequality constraint - both under uncertainty. It also has to be noted that the market clearing condition under

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symmetric information is already included in the price function by using the other players actual bids q_i instead of using an assumption q_i'.

Assuming the players have symmetric information on the outcome of the uncertain parameter, thus fixing ξ as $\overline{\underline{\xi}}$ allows to formulate the KKT conditions as:

$$\begin{aligned}
\frac{\partial \mathscr{L}}{\partial q_i} &= -\frac{\partial p(\overline{\underline{\xi}}, \sum_{j\in J}\sum_{i_2\in I_j} q_{i_2})}{\partial q_i} q_i \\
&\quad -p(\overline{\underline{\xi}}, \sum_{j\in J}\sum_{i_2\in I_j} q_{i_2}) && \forall j \in J, i \in I_j \\
&\quad +\frac{\partial c_i(q_i)}{\partial q_i} - \underline{\delta}_i + \bar{\delta}_i = 0 \\
0 &\leq \underline{\delta}_i \perp -q_i \leq 0 && \forall j \in J, i \in I_j \\
0 &\leq \bar{\delta}_i \perp q_i - \bar{q}_i(\overline{\underline{\xi}}) \leq 0 && \forall j \in J, i \in I_j
\end{aligned} \tag{A.2}$$

Using uncertainty in form of scenarios $\xi \in \Xi$ however might result in complications, especially related to the objective function (which might be caused to not be able to clear). To deal with this issue, Ref.[18] propose introduction of a slack variable into the complementarity conditions. By denoting this variables as $\omega \in \mathbb{R}^i$ and assuming two scenarios ξ_1 and ξ_2 the uncertain generation maximum capacity constraint from the previous example can be reformulated as:

$$\begin{aligned}
0 \leq \bar{\delta}_i \perp q_i - \bar{q}_i(\xi_1) + \omega_i \leq 0 \quad \forall j \in J, i \in I_j \\
0 \leq \bar{\delta}_i \perp q_i - \bar{q}_i(\xi_2) - \omega_i \leq 0 \quad \forall j \in J, i \in I_j
\end{aligned} \tag{A.3}$$

This shows that the slack variable finds a single solution q_i that minimizes the residuals between the two different scenarios. Reformulation for an open number of scenarios leads to a similar formulation as in Ref.[18]:

$$\begin{aligned}
0 \leq \bar{\delta}_i \perp q_i - \bar{q}_i(\xi) + \omega_i^\xi \leq 0 &\quad \forall j \in J, i \in I_j, \xi \in \Xi \\
\sum_{\xi\in\Xi} \omega_i^\xi = 0 &\quad \forall j \in J, i \in I_j
\end{aligned} \tag{A.4}$$

Below, we extend this formulation by two characteristics.

Appendix A.1. Consideration of Probability Distributions

The original formulation does not explicitly consider probability distributions. Arguably, this could lead to distortions as outlier scenarios would be considered with similar priority as more likely outcomes. In theory, this can be circumvented by adding latter scenarios with a higher rate (i.e. scenario 2 is x times as likely as scenario 1, so add 1 scenario 1 and x scenario 2). In practical applications however, this would lead to an increase in model complexity, as every single additional scenario represents an additional complementarity constraint for each constraint affected by the uncertainty. Thus in this paper we decided to use the probabilities as weighting parameters to replace ω_i^ξ with $\frac{\omega_i^\xi}{P^\xi}$. Thus, the higher the likelihood of an outcome, the lesser the impact of the slack variable on diverging from that scenario. As a result, the solution will be closer to the scenario with higher likelihood and deviate more from the scenario with lower likelihood. It has to be noted, that the proposed formulation does not consider any risk preferences, making all participants *risk-neutral* players.

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Appendix A.2. Extension to Lagrangians

The Lagrangians can be relaxed in similar manner to the complimentarity constraints, yielding single solutions for the decision variables that allow clearing the market for objective functions under uncertainty. For the previously defined example this would result in the following KKT conditions:

$$\begin{array}{cc}
\begin{array}{c}\frac{\partial \mathscr{L}}{\partial q_i} = -\frac{\partial p(\xi, \sum\limits_{j\in J}\sum\limits_{i_2\in I_j} q_{i_2})}{\partial q_i} q_i \\ -p(\xi, \sum\limits_{j\in J}\sum\limits_{i_2\in I_j} q_{i_2}) \\ +\frac{\partial c_i(q_i)}{\partial q_i} - \underline{\delta}_i + \bar{\delta}_i + \frac{\omega_i^{p,\xi}}{P^{\xi}} = 0\end{array} & \begin{array}{c}\forall j \in J, i \in I_j, \\ \xi \in \Xi\end{array} \\
0 \leq \underline{\delta}_i \perp -q_i \leq 0 & \forall j \in J, i \in I_j \\
0 \leq \bar{\delta}_i \perp q_i - \bar{q}_i(\xi) + \frac{\omega_i^{\bar{q},\xi}}{P^{\xi}} \leq 0 & \begin{array}{c}\forall j \in J, i \in I_j, \\ \xi \in \Xi\end{array} \\
\sum\limits_{\xi\in\Xi} \omega_i^{p,\xi} = 0 & \forall j \in J, i \in I_j \\
\sum\limits_{\xi\in\Xi} \omega_i^{\bar{q},\xi} = 0 & \forall j \in J, i \in I_j
\end{array} \tag{A.5}$$

Again, this yields a single solution for the quantity decision of every generation unit and thus allows clearing the market similar to traditional (in this case: Cournot) clearing procedures.

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Appendix B. Karush Kuhn Tucker(KKT)-Conditions

The KKT-conditions in extend form are:

$$\frac{\partial \mathscr{L}}{\partial q_{i,t}} = -\frac{\partial p_t(\xi, \sum\limits_{j\in J}\sum\limits_{i_2\in I_j} q_{i_2,t})}{\partial q_{i,t}} q_{i,t} - p_t(\xi, \sum_{j\in J}\sum_{i_2\in I_j} q_{i_2,t}) + \frac{\partial c_{i,t}(q_{i,t},\overline{b_{i,t}})}{\partial q_{i,t}} - \underline{\delta}_{i,t} + \bar{\delta}_{i,t} + \frac{\omega^{p,\xi}_{i,t}}{P^\xi} = 0 \quad \forall j \in J, i \in I^{Th}_j, t \in T^{\overline{s}}, \xi \in \Xi \tag{B.1a}$$

$$\frac{\partial \mathscr{L}}{\partial q_{i,t}} = -\frac{\partial p_t(\xi, \sum\limits_{j\in J}\sum\limits_{i_2\in I_j} q_{i_2,t})}{\partial q_{i,t}} q_{i,t} - p_t(\xi, \sum_{j\in J}\sum_{i_2\in I_j} q_{i_2,t}) - \underline{\delta}_{i,t} + \bar{\delta}_{i,t} - \gamma_{i,t} + \frac{\omega^{p,\xi}_{i,t}}{P^\xi} = 0 \quad \forall j \in J, i \in I^{Hy}_j, t \in T^{\overline{s}}, \xi \in \Xi \tag{B.1b}$$

$$\frac{\partial \mathscr{L}}{\partial q^{\overline{s}}_{i,t}} = \sigma_i + \gamma_{i,t} + \sum_{t_2=\underline{s}+1}^{t} \psi_{i,t_2} - \mu_{i,t} = 0 \quad \forall j \in J, i \in I^{Hy}_j, t \in T^{\overline{s}} \tag{B.1c}$$

$$0 \le \underline{\delta}_{i,t} \perp \underline{q}_i \overline{b_{i,t}} - q_{i,t} \le 0 \quad \forall j \in J, i \in I_j, t \in T^{\overline{s}} \tag{B.1d}$$

$$0 \le \bar{\delta}_{i,t} \perp q_{i,t} - \bar{q}_i \overline{b_{i,t}} \le 0 \quad \forall j \in J, i \in I_j, t \in T^{\overline{s}} \tag{B.1e}$$

$$0 \le \sigma_i \perp \sum_{t\in T^{\overline{s}}} q^{\overline{s}}_{i,t} - Q^{\overline{s}}_i(\xi) + \frac{\omega^{Q,\xi}_i}{P^\xi} \le 0 \quad \forall j \in J, i \in I^{Hy}_j, \xi \in \Xi \tag{B.1f}$$

$$\sum_{s=1}^{t} \begin{cases} q^{\overline{s}}_{i,t}, & \text{if } s = \overline{s} \\ \underline{q^{s}_{i,t}}, & \text{if } s \neq \overline{s} \end{cases} - q_{i,t} = 0 \quad \forall j \in J, i \in I^{Hy}_j, t \in T^{\overline{s}} \tag{B.1g}$$

$$0 \le \psi_{i,t} \perp \sum_{s=1}^{t-1} \sum_{t_2=t}^{\max(T)} \begin{cases} q^{\overline{s}}_{i,t_2}, & \text{if } s = \overline{s} \\ \underline{q^{s}_{i,t_2}}, & \text{if } s \neq \overline{s} \end{cases} - \bar{R}_i \le 0 \quad \forall j \in J, i \in I^{Hy}_j, (t > \underline{s}) \in T^{\overline{s}} \tag{B.1h}$$

$$0 \le \mu_{i,t} \perp -q^{\overline{s}}_{i,t} \le 0 \quad \forall j \in J, i \in I^{Hy}_j, t \in T^{\overline{s}} \tag{B.1i}$$

$$\sum_{\xi\in\Xi} \omega^{p,\xi}_{i,t} = 0 \quad \forall j \in J, i \in I_j, t \in T^{\overline{s}} \tag{B.1j}$$

$$\sum_{\xi\in\Xi} \omega^{Q,\xi}_{i} = 0 \quad \forall j \in J, i \in I^{Hy}_j, t \in T^{\overline{s}} \tag{B.1k}$$

Appendix C. Multiple Solutions for Hydropower Commitment

Assumed be a game in two periods $t = 1, 2$ yields a player j holding two hydropower units $i = 1, 2$ an optimal profit of Π^*_j for clearing prices p^*_1 and p^*_2. As mentioned above, hydropower units are assumed to operate cost-neutral, thus the optimal profits cannot be decreased by changing commitment decisions as long as $p_t(q_{1,t} + q_{2,t} + \sum_{i_2\notin I_j} q'_{i_2,t}) = p_t(d^*_t) = p^*_t$ holds for both time periods and additional constraints such as reservoir and generation capacities are fulfilled. Assumed there is only a single deterministic inflow in period 1, denoted as Q_i and no end reservoir values are required (thus, the full inflow will be used in the two time periods), the

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previous condition can be reformulated as:

$$\begin{array}{cc} q_{1,t} + q_{2,t} = d_t^* - \sum_{i_2 \notin I_j} q'_{i_2,t} & \forall t = 1,2 \\ Q^i \geq q_{i,1} + q_{i,2} \geq 0 & \forall i = 1,2 \end{array} \tag{C.1}$$

Assuming constant quantities provided by other players, player j can choose i as either 1 or 2 and freely select any quantities $q_{i,t}$ as long as they fulfill:

$$\begin{array}{cc} 0 \leq q_{i,t} \leq d_t^* - \sum_{i_2 \notin I_j} q'_{i_2,t} & \forall t = 1,2 \\ q_{ii,t} = d_t^* - \sum_{i_2 \notin I_j} q'_{i_2,t} - q_{i,t} & \forall ii \neq i, t = 1,2 \\ Q^i \geq q_{i,1} + q_{i,2} \geq 0 & \forall t = 1,2 \\ Q^{ii} \geq q_{ii,1} + q_{ii,2} \geq 0 & \forall ii \neq i, t = 1,2 \end{array} \tag{C.2}$$

There is a range of potential commitment solutions that fulfill these conditions. They differ in reservoir held over the time stage as well as the periodical utilization of the generation units but yield the same (i.e. the optimal) profits for the player and end up in similar end reservoir values (here = 0).

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7.4 Multiple Nash Equilibria in Electricity Markets with price-making Hydrothermal Producers

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7.4.1 Extended Abstract

This paper formulates a deterministic hydro-thermal Cournot market with a minimum capacity component caused by requirements for inertial response. To solve this game and deal with the curse of dimensionality caused by the dimensions generation units/nodes/periods, the proposed technique splits the problem of deriving equilibrium solutions into a discontinuous and continuous problem. The continuous problem is solved via a stepwise Nikaido-Isoda convergence algorithm and the discontinous problem via a branch-and-cut methodology.

A special characteristic is that the here presented method does not require all continuous problems to be solved for their respective Nash solutions, making the application of such a stepwise procedure favorable to more commonly applied alternatives such as application of the Karush-Kuhn-Tucker conditions. This is based on the possibility of aborting the calculation of a specific equilibrium in favor of another equilibrium tuple on a similar branch, that promises faster convergence (i.e. a smaller value of the Nikaido-Isoda function).

By making problem-specific assumptions - which, in the presented case is 'only competition on the supply side amongst players aiming to maximize profits' - tailored cutting rules can be designed, further improving and supporting convergence to equilibria under dispatch decisions. Similar to traditional cutting plane methods, these rules can be based on feasibility and optimality.

A case study, with related data presented in its entirety, is introduced in the paper. The study aims to provide a showcase of the capabilities of the presented algorithm and to offer a starting point for models used on future decisions regarding new types of ancillary services under the assumption of market power. In the here presented case, hydropower producers in an adjacent network node suffered profit losses caused by the implementation of requirements for provision of inertial response (or similarly, spinning reserves restricted to a specific interconnected area). Further, the amount of potential unit schedules providing the Nash Equilibria was reduced, but nonetheless a larger spread in potential profit outcomes in these equilibria was observed.

Future usage of the proposed method may be manifold and not only restricted to electrical power systems but similar commodity markets under production and dispatch decisions. Further, the analyzed case study promises problem scalability and thus might allow for large-scale applications (with restrictions, as the reduced branching problem is, even though simplified, similar to the original problem of $\mathcal{NP}$-hard nature).

Multiple Nash Equilibria in Electricity Markets with price-making Hydrothermal Producers

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Abstract

This paper proposes a novel approach incorporating scheduling decisions into a multi-nodal multi-period Cournot game. Through applying the Nikaido-Isoda function, market clearing is conducted without dual values being required. Maps of Nash equilibria are obtained through a branch-and-cut algorithm, based on tailored cutting rules. A case study of inertial response requirements shows that these maps and the resulting range of potential player profits can be used to analyze the impacts of policy decisions influenced by discontinuous variables. The case study also shows the financial impact on neighboring producers to the node with applied inertial response requirements.

Index Terms

Nikaido-Isoda, Nash Equilibrium, Energy Markets, Cournot Competition, Nonlinear Optimization, Competitive Game, Inertial Response, Generation Dispatch, Hydropower

I. Introduction

Equilibrium modeling in power systems represents an established method for analyzing player behavior and their reactions to system changes [1]. These methods have been traditionally based on systems of conventional means of power production participating in single-period games [1], [2]. Changes in system generation portfolios have, however, led to greater integration of fluctuating renewable electricity generation such as wind and solar power and to electricity storage facilities being added to the grid. These developments can result in traditional market models not being fit to adequately deal with arising problems.

To address this, several approaches have been proposed in the literature to deal with multi-period setups: Ref. [3] implements an equilibrium model on an assumed, already conducted hydro power scheduling, extending a single period Cournot model to a short term time frame. Ref. [4] introduces storage operators that behave as price-takers in a natural gas market with gas inventory holding being simplified through a fixed overall period capacity. Ref. [5] analyzes, based on [3], the implications of market power in a system with large shares of hydropower generation using a two stage model that clears a Cournot market and then (re)schedules units. Another analysis of market power in systems under hydrostorage is given by [6], which embeds hydropower decisions into a game played within a dynamic program and solved via interpolating the best response functions. Ref. [7] shows a leader-follower framework in a stochastic *Equilibrium Problem with Equilibrium Constraints*. It circumvents reliance on Karush Kuhn Tucker-conditions by using strong duality constraints in its bi-level problem setup of clearing the market and maximizing profits. Ref. [8] uses the Nikadio-Isoda function to establish an *active set* algorithm to clear a multi-period hydro-thermal market. The method presented below also relies on such a Nikadio-Isoda equilibrium framework.

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Scheduling of generation units $i \in I$ is incorporated in the strategic decision problem by considering binary variables. Such scheduling over a finite time frame T creates a finite number of possible iterations. Each iteration consists of a problem setup similar to the one presented in [9] and each shows none or a (potentially) unique Nash equilibrium (NE) [10] and therefore results in a finite number of potential equilibria. Multiple NE can, as explained in [11] and [7], vary greatly in appearance. In [11] polynomial algebra is used to define the equilibrium space and to establish the formulation presented. The approach presented below, however, makes use of a branch-and-cut algorithm, due to the finite number of potential 'equilibrium tuples', with analytically derived *optimality* and *feasibility cuts* making mapping of such tuples a possibility.
Integer problems in power systems have various applications. One is given by the question of inertial system frequency response and its interaction with individual plants [12]. The problem is defined by a fixed contribution of inertial response (frequently referred to as 'inertia') that is related to the on/off states of generators and thus is closely related to other market problems such as the market for e.g. spinning reserves. There are, however, no implemented remuneration schemes for inertial response that we are aware of. Thus and to give a practical example, a case study, based on spot prices, on inertia will be presented below.

II. Nash Equilibria

Assumed are players J that own generation units I participate in a game where they receive a payoff(/profit) function $\pi(x)$ that depends on a set of q_i (quantity) decisions $x = \{q_i, i \in I\}$.

The set of collective actions from the perspective of a player, a generation company j, can be described as $(y_j|\overline{x}_j) \equiv \{q_{i \in I_j}\}$ where $\overline{x}_j$ defines player j's assumptions of decisions on units not controlled by player j, denoted as $\overline{x}_j = \{\overline{q}_{j,i}, i \notin I_j\}$. $\overline{q}_{j,i}$ thus specifies a single players assumption on the output of a specific unit held by a competitor. Denoting optimal solutions with $*$ and using X as the set of viable decisions allows the NE to be defined (similarly to [13] and [8]) as the point y^* that fulfills:

$$\pi_j(y^*) = \max_{(y_j|\overline{x}_j^*) \in X} \pi_j(y_j|\overline{x}_j^*) \quad \forall j \tag{1}$$

The multivariate Nikadio-Isoda function can be defined as follows [14]:

$$\Psi(x, y) = \sum_j [\pi_j(y_j|\overline{x}_j) - \pi_j(x)] \tag{2}$$

As shown in [9], this function is able to yield the distance to a (potentially unique) *NE* for (weakly) concave profit functions.

III. A Non-Cooperative, Non-Convex Game

Adding additional dimensions to the game, such as a network of multiple nodes n or several time periods, expressed through t and so leading to q_i becoming $q_{i,t}$, does not necessarily change the ability of the function proposed above to find the NE distance. Ref. [13] shows this e.g. by solving the multi-nodal example presented in [2]. However, it should be mentioned that the complexity of the approach could increase due to the need for techniques to extend the solution to other time periods, as later periods could bear uncertainty.
Generation scheduling is strongly related to binary decisions, as the on/off states of

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NOMENCLATURE

Indices:

$i, i_2 \in I$	generation unit
i_{hydro}, i_{wind}	hydropower/windpower unit
$j \in J$	producer
n	network node(/area/country)
n_s, n_d	source, destination $[node]$
t	period $[h]$
$s \in S$	branching tuple

Variables:

$y^q_{i,n,t} \in \mathbb{R}^+$	quantity decision $[MWh]$
$y^b_{i,t} \in \{0,1\}$	scheduling decision
$d_{n,t} \in \mathbb{R}^+$	energy demand $[MWh]$
$\overline{q}_{j,i,n,t} \in \mathbb{R}^+$	quantity assumption $[MWh]$

Fixed Variables:

$q_{i,n,t} \in \mathbb{R}^+$	quantity provided $[MWh]$
$b_{i,t} \in \{0,1\}$	dispatch decision

Functions:

$\overline{p}_{j,n,t}$	price estimation $[€/MWh]$
$c_{i,t}$	generation cost [€]
$p^*_{n,t}$	market clearing price $[€/MWh]$
π_j	profit function of producer j [€]

Parameters:

q^{min}_i, q^{max}_i	generation capacity $[MW]$
$c^{var}_{i,t}$	variable cost portion $[€/MWh]$
c^{fix}_i	fixed cost portion [€]
$l^{n_s,n_d}_{i,n}$	line flow from n_s to n_d $[\%]$
$l^{max}_{n_{so},n_{de}}$	line capacity $[MW]$
$w_{i_{hydro}}$	inventory end value $[€/MWh]$
$r_{i_{hydro}}$	available reservoir quota $[MWh]$
$q^{cap}_{i,t} \in [q^{min}_i, q^{max}_i]$	available wind capacity $[MWh]$
H_i	inertial response constant
H^d_n	inertial response requirement

Sets:

I_j	generation units of producer j
I_n	generation units in node n
S^N	set of branching tuples in NE

units are factors that have to be considered in startup cost, ramp rate limits, reserve constraints, and up and down time limits [15]. Using algorithms such as those proposed in [16] or [17] allows the scheduling problem to be solved as a mixed integer cost minimization problem for the optimal dispatch of thermal plants.

Adding storage technologies such as hydropower to such a game allows players to strategically dispatch their resources. Providers with storage capabilities will actively aim to provide in high price (i.e. peak) hours and to withhold in low price (i.e. base) hours. Ref. [3] shows this concept for a Cournot game (a game with competition in quantity) by binding the time stages by their *marginal value of water*. This concept, which is often termed *water value* is used in both scheduling and in the optimization of bidding in hydro power dominated systems [18]. Problem formulations based on water values however often neglect the strategic impact of other players. To strengthen their position on the market, these players might actively aim to withhold production from peak periods in which other players aim to produce. In non-cooperative games, this means a player might act as a leader in some time stages and as a follower in others [19], so making the multi-period game more dynamic than single-period approaches.

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Adding (binary) integer variables to the problem setup leads to non-convex, non-continuous payoff-functions and so breaches the definition of convex games from [14]. Ref. [8] extends the concept of [9] by using an *active set method* to yield a combination of decision variables (which we will later refer to as *tuple*) that define a NE. A binary scheduling problem with I generation units, T time periods would, however, show a possible number of $2^{I \times T}$ tuples, and as shown in [19], multiple equally viable tuple equilibria - ranging from 0 to $I \times T$ (which is also discussed briefly in the appendix). Therefore, deriving a single equilibrium tuple might give an incorrect perspective on the existing array of equilibrium tuples. Such a misrepresentation could prove problematic, particularly in the consideration of ancillary services/markets for reserve energy, where the number of "running" (i.e. committed) units is of significant importance.
We thus propose an algorithm based on the *Nikaido-Isoda equilibrium algorithm* that incorporates branching and cutting based on analytical rules. The Nikaido-Isoda function, first proposed in [14] presents an auxiliary function that defines whether a given player's solutions yield a Nash Equilibrium. A step-wise algorithm as in [9] allows such an equilibrium to be derived for a system with shared constraints (e.g. a network). Based on this concept, the algorithm presented below is meant to bridge the economical approach of determining market power effects and the technical aspect of deriving explicit schedules for providing the commodity.

IV. Selective Cutting

This section will briefly introduce solving a Cournot game with binary variables using the *Nikaido-Isoda function*. This problem is not unique to power systems. A more general formulation will therefore be used and will be extended in the following sections to problems specific to electrical power systems. As discussed above, Cournot games find broad application in power systems, as they are suitable solutions for commodity market problems [1]. Other games such as Bertrand competition might also be applicable. This would, however, require additional analysis of the *cutting rules* presented below. Other modes of competition therefore have been excluded from this paper. We define the profit function of a single player as:

$$\begin{aligned}
\pi_j(q_{i,n,t}, b_{i,t}) = \sum_t \sum_{i \in I_j} \sum_n [\overline{p}_{j,n,t}(\sum_{i \in I_j} q_{i,n,t} \\
+ \sum_{i_2 \notin I_j} \overline{q}_{j,i_2,n,t}) q_{i,n,t} - c_{i,t}(q_{i,n,t}, b_{i,t})] \\
\text{where:} \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \\
\overline{p}_{j,n,t}(\sum_{i \in I_j} q_{i,n,t} + \sum_{i_2 \notin I_j} \overline{q}_{j,i_2,n,t}) = p^*_{n,t}(d_{n,t}) \forall j, n, t \\
d_{n,t} = \sum_i q_{i,n,t} \qquad\qquad\qquad\qquad \forall n, t
\end{aligned} \tag{3}$$

This assumption of an existing market clearing quantity $d_{n,t}$ requires the underlying assumption of *information symmetry on price elasticity* among the competitors [1]. Thus, the expectations $\overline{p}_{j,n,t}$, $\overline{q}_{j,i_2,n,t}$ can be approximated as variables $p_{j,n,t}$, $q_{j,i_2,n,t}$ from the perspective of a player. To solve this problem, we establish the *Nikaido-Isoda function in the general form* as:

$$\begin{aligned}
\Psi((q_{i,n,t}, b_{i,t}), (y^q_{i,n,t}, y^b_{i,t})) = \\
\sum_j [\pi_j(y^q_{i,n,t}, y^b_{i,t}) - \pi_j(q_{i,n,t}, b_{i,t})]
\end{aligned} \tag{4}$$

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The total set of tuples $S = \{s^1, ..., s^{I\times T} | b^1 \neq ... \neq b^{I\times T}\}$ is established by all potential iterations of the binary variable $b_{i,t}$. By fixing $y^b_{i,t} := b_{i,t}$ it is possible to solve every tuple s for its equilibrium (assumed concave profit functions) through an iterative algorithm [9]:

Algorithm 1:

0) assume starting values for $(q_{i,n,t}, b_{i,t})$
1) solve for $\max\limits_{y^q_{i,n,t}} \Psi((q_{i,n,t}, b_{i,t}), (y^q_{i,n,t}, y^b_{i,t}))$
2) is $\Psi((q_{i,n,t}, b_{i,t}), (y^q_{i,n,t}, y^b_{i,t})) = 0$?
   yes - **end**, $(y^q_{i,n,t}, y^b_{i,t})$ is the NE point (i.e. tuple is solved);
   no - $(q_{i,n,t}, b_{i,t}) := (y^q_{i,n,t}, y^b_{i,t})$, back to 1)

Repeating this algorithm can be compared to individual players applying stepwise (profit-)maximization, resulting in (supply-side) welfare maximization, whilst operating under shared constraints. As mentioned before, problems in the form of (4) are non-convex. As shown in [9], every individual tuple s therefore offers a (potentially unique) NE as long as the set of constraints added to the problem is convex and allows a feasible solution [10].

We define the set of viable NE $S^N \subseteq S$ as being the set of tuples for which no player has an incentive to dispatch another unit (i.e. increase $\sum\limits_{i\in I_j} b_{i,t}$). The system is computationally efficient solved by making use of two characteristics of the Nikadio-Isoda equilibrium algorithm presented in [8], [9]:

- solving one step of the convergence algorithm is, depending on the cost function, a problem of linear/quadratic nature and thus solved computationally quickly using available commercial software.
- the objective function $\Psi((q_{i,n,t}, b_{i,t}), (y^q_{i,n,t}, y^b_{i,t}))$ provides a quantitative statement about the improvement in each step of the algorithm (as the NE is defined by a value of 0, i.e. 'no improvement potential for any participant'). It is therefore possible to rank tuples by their rate of convergence (lower value of the Nikaido-Isoda function) and select the tuples s that are solved computationally more quickly than others.

Our proposed algorithm labels all tuples s as either:

- **pending** - there can be no definite statement made about the tuple as the Nikaido-Isoda function still returns a value above 0.
- **solved** - the NE of the tuple was found (Nikaido-Isoda function returns 0) and might be considered to be a Nash tuple.
- **sorted out** - the tuple will not be a Nash tuple, irrespective of the value of the Nikaido-Isoda function.

All tuples start as **pending**. Three transitions are possible:
1) **pending** $\Rightarrow$ **solved**, 2) **pending** $\Rightarrow$ **sorted out**, 3) **solved** $\Rightarrow$ **sorted out**.

S^N is the set of (Nash) tuples, for which no such transitions are possible anymore (all tuples are thus either solved or sorted out). The proposed **Nash tuple mapping algorithm** can be formulated as:

Algorithm 2:

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0) **sort out** tuples that do not fulfill (non-convex) constraints associated with discontinuous variables (the *preliminary cutting rules* are presented later in this paper).
1) conduct algorithm 1 on a number of (randomly) selected tuples s that are still **pending**
2) use already **solved** tuples to **sort out** other tuples (i.e. apply *dynamic cutting rules*)
3) are there any **pending** tuples left?
no - proceed | yes - back to 1).
4) were any nodes **sorted out** in step 2)?
yes - back to 2) | no - **end** (S^N shows the "map of Nash Equilibria")

V. Cutting Rule Design

Even though the ranking of tuples heavily depends on starting values and in the proposed framework tuples to be solved were selected randomly, generally applicable rules can formulated to sort out unfavorable tuples. For one, they may relate to the set of constraints and thus must be specifically tailored to the application. As such, they mostly depend on pre-selecting iterations of the integer variable to sort out tuples that do not fulfill given constraints. The *Inertial Response Requirement Rule* mentioned later in this paper is such a type. For the other, rules can be defined, that dynamically declare branches of tuples as infeasible or unfavorable, after a single tuple is declared as such. The *Marginal Cost Rule* presented later is an example of such a rule. These rules can, furthermore, draw dynamic conclusions based on already solved tuples. We formulate one such rule here (referred later to as the *Payoff-Function Cutting Rule*) based on two assumptions:

Assumption 1: players will not schedule units if that leads to a decrease in payoff
Assumption 2: adding additional units to the schedule will not increase any market clearing prices

Assumption 1 is a straight forward economical decision and is valid for players that aim to maximize their outcome. Assumption 2 is valid as long as units solely operate on the supply side. Purchases (for example pumped hydro storage) would result in negative supply effects and would increase $d_{n,t}$ and result in higher prices. The model presented here is therefore limited to competition on the supply side. Non-concave, decreasing market price functions are also a necessity. The assumptions presented here rely on the concept of dominance in games [20]. No economically rational player j would choose to commit generation units if the new equilibrium point would not dominate the previous.

To execute the proposed **Payoff-Function Cutting Rule**, two solved tuples denoted as s^* and s^{**} are required. In addition, several conditions must be fulfilled by the equilibrium solutions of the tuples , denoted as $\langle y^{q*}_{i,n,t}, y^{b*}_{i,t}\rangle$ and $\langle y^{q**}_{i,n,t}, y^{b**}_{i,t}\rangle$ respectively:

$$b^{**}_{i,t} \geq b^{*}_{i,t} \quad \forall i,t \tag{5a}$$

$$\pi_{\bar{j}}(y^{q*}_{i,n,t}, y^{b*}_{i,t}) > \pi_{\bar{j}}(y^{q**}_{i,n,t}, y^{b**}_{i,t}) \quad \forall \bar{j} \tag{5b}$$

$$\sum_{i\in I_{\bar{j}}}\sum_{t} b^{**}_{i,t} > \sum_{i\in I_{\bar{j}}}\sum_{t} b^{*}_{i,t} \quad \forall \bar{j} \tag{5c}$$

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solved tuple s^*				
j	i	$b_{i,1}$	$b_{i,2}$	$b_{i,3}$
1	1	0	0	1
1	2	1	1	1
2	3	0	0	1
2	4	1	1	0
$\pi_1^* = 1000$, $\pi_2^* = 750$				

solved tuple s^{**}				
j	i	$b_{i,1}$	$b_{i,2}$	$b_{i,3}$
1	1	0	1	1
1	2	1	1	1
2	3	0	1	1
2	4	1	1	0
$\pi_1^{**} = 500$, $\pi_2^{**} = 1500$				

pending tuple s				
j	i	$b_{i,1}$	$b_{i,2}$	$b_{i,3}$
1	1	0	1	1
1	2	1	1	1
2	3	1	1	1
2	4	1	1	0
$\pi_1 =???$, $\pi_2 =???$				

As $\pi_1^* > \pi_1^{**}$ (i.e. π_1^* dominates), player 1 does not have an incentive to set $b_{1,2} := 1$. Thus, both the tuple s^{**} and its branch tuple s can be **sorted out**, independent of the profits of player 2.

Fig. 1: Numerical Cutting Example

$\bar{j}$ represents a specific player from the set of available players $\bar{j} \in J$. (5a) ensures, that tuple s^{**} is located on a branch of tuple s^*. (5b) holds where player $\bar{j}$ has a negative payoff effect from transitioning from tuple s^* to tuple s^{**}. Fulfilling requirement (5c) means that said player $\bar{j}$ made an active decision (committing an additional unit) that enabled this tree branch. According to assumption 1, no reasonable player $\bar{j}$ would choose such a decision. Thus, and according to assumption 2, the tree branch can be cut entirely: $\{s | b_{i,t} \geq b_{i,t}^{**}; \forall i, t\} :=$ **sorted out**

As one can see, this cutting method does not require the two tuples to be adjacent in the branching tree. The structure of the tree plays no role, as long as the stated conditions for s^* and s^{**} hold for the entire time frame. If the two assumptions hold, applicability to (Cournot) problems other than the case presented in this paper is given.

A numerical example of a cut is given by Figure 1. A practical application of the proposed algorithm with additional tailored cutting rules will now be presented.

VI. Multi Period Cournot Market Clearing

We developed an energy market clearing model based on problem (3) with affine cost functions:

$$c_{i,t}(q_{i,n,t}, b_{i,t}) = c_{i,t}^{var} \sum_n y_{i,n,t}^q + c_i^{fix} y_{i,t}^b \tag{6}$$

This allows the formulation of the *extended general form of the Nikaido-Isoda function for a single tuple* s:

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$$
\begin{aligned}
&\Psi((q_{i,n,t}, b_{i,t}), (y^q_{i,n,t}, y^b_{i,t})) = \\
&\sum_j \sum_t \sum_{i \in I_j} \sum_n \Big[\overline{p}_{j,n,t}(\sum_{i_2 \in I_j} y^q_{i_2,n,t} + \sum_{i_2 \notin I_j} q_{i_2,n,t}) y^q_{i,n,t} \\
&\qquad -c^{var}_{i,t} \sum_n y^q_{i,n,t} - c^{fix}_i y^b_{i,t}\Big] \\
&\qquad - \sum_{i \in I_j} \Big[\sum_n \overline{p}_{j,n,t}(d_{n,t}) q_{i,n,t} \\
&\qquad -c^{var}_i \sum_n q_{i,n,t} - c^{fix}_i b_{i,t}\Big]
\end{aligned}
\tag{7}
$$

where:

$$
\begin{aligned}
d_{n,t} &= \sum_i q_{i,n,t} \quad \forall n, t \\
y^b_{i,t} &= b_{i,t} \quad \forall i, t
\end{aligned}
$$

This function combines the objective functions of the players into a single optimization problem that allows conjoint optimization under consideration of previous optimization results entered in the form of previous tuple solutions $\langle q_{i,n,t}, b_{i,t}\rangle$. Using this function as the objective function (8) in an optimization problem and applying the shared constraints allows the distance to the Nash equilibrium for a specific tuple (i.e. a tuple with similar schedules $y^b_{i,t} = b_{i,t} \quad \forall i, t$) to be found.

The generation units show minimum and maximum output restrictions based on whether the unit is running or not. The constraint (9) therefore has to be added to the model. Line constraints connecting the different network nodes were also implemented, one in the positive and one in the negative direction: (10), (11). The concept proposed here extends the formulation proposed in [2], [13] by allowing the exclusion of specific generation units from participating in competing in certain market nodes n or using certain transfer lines $l^{n_s,n_d}_{i,n}$. A transmission system operator and arbitrageurs as independent players (as e.g. displayed in [2]) were excluded from the model for two reasons: 1.) the Nikaido-Isoda function would require additional complexity for such heterogeneous players to be incorporated, so increasing notational complexity unnecessarily; 2.) as shown in [9], the stepwise algorithm is capable of dealing with such shared constraints and can thus be used to assign line capacities shared by players. This comes as a result of the Nikaido-Isoda function allowing solving all players problems bundled within the single objective function (7) compared to other methods from literature such as derivation of the Karush-Kuhn-Tucker conditions. There will, however, be no direct result for wheeling fees, which can limit the applicability of the model in certain markets such as those found in the USA (which would require heterogeneous players). Furthermore, higher granularity of the problem (solving small scale problems within limited areas) would require additional technical specifications and thus additional (shared) constraints, both omitted in the here presented model. Assumptions such as demand curve elasticity can be considered valid assumptions for large scale problems. The case study was therefore chosen to represent an excerpt of cross-country trading within the European electricity market.

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$$\max_{y^q_{i,n,t}, y^b_{i,t}} \Psi((q_{i,n,t}, b_{i,t}), (y^q_{i,n,t}, y^b_{i,t})) \tag{8}$$

$$\text{s.t. } q^{min}_i y^b_{i,t} \leq \sum_n y^q_{i,n,t} \leq q^{max}_i y^b_{i,t} \forall i, t \tag{9}$$

$$\sum_i \sum_n (l^{n_s,n_d}_{i,n} - l^{n_d,n_s}_{i,n}) y^q_{i,n,t} \geq -l^{max}_{n_s,n_d} \forall t, n_s, n_d \tag{10}$$

$$\sum_i \sum_n (l^{n_s,n_d}_{i,n} - l^{n_d,n_s}_{i,n}) y^q_{i,n,t} \leq l^{max}_{n_s,n_d} \forall t, n_s, n_d \tag{11}$$

Solving the maximization problem (8) iteratively, as described above, would result in a NE point (i.e. $\Psi((q_{i,n,t}, b_{i,t}), (y^q_{i,n,t}, y^b_{i,t})) = 0$). As can be seen, this point fulfills the price clearing condition of (3): $\overline{p}_{j,n,t}(\sum_{i \in I_j} q_{i,n,t} + \sum_{i_2 \neq i} \overline{q}_{j,i_2,n,t}) = \overline{p}_{j,n,t}(d_{n,t}) = p^*_{n,t}(d_{n,t})$

Different constraints and parameter specifications must be added depending on plant type. It should be noted that this paper shows a limitation similar to the literature sources - the equilibrium considers only a deterministic representation. Uncertainty can affect a number of parameters including market prices, hydrological inflow, available wind power capacity, and fuel prices. Omitting stochastic representation, which was considered necessary to deal with model complexity - limits the model to shorter time frames that impose less uncertainty. To give an example, wind power is simulated through stochastic parameters in unit commitment models, see e.g. [21], [22]. Such an approach would, however, require additional techniques (i.e. *sampling*, *decomposition*) and therefore exceed the limits of this paper. It was therefore decided to instead use preselected wind capacity scenarios (i.e. *point forecasts* as presented in [23]).

A. Hydropower Plant

Hydropower plants show low cost profiles for production. Models therefore usually exclude the generation cost [18]. The opportunity cost of storing water is instead taken into consideration, defining the decision to generate or store in a single time period [24]. Due to applied formulation of the reservoir function, the approach presented in this paper manages the transition between time periods without[1] calculating the dual values of inventory that are commonly referred to as water value. It still, however, requires a finite set of time periods $t = 1, ..., T$ and an assumption of end values of variables, which are traditionally the end levels of reservoirs. This paper instead applies assumptions of the end value of stored hydrological inventory to demonstrate a different approach. The variable cost of the hydro units were therefore assigned the opportunity cost of stored water: $c^{var}_{i_{hydro},t} := w_{i_{hydro}} \forall t$.

The possibility of holding inventory effectively enables arbitrage over time stages. To incorporate this, a concept similar to [3], [4], [8] was implemented. Thus, a predetermined maximum allowance of available hydropower inventory for the total time frame being given as a parameter. This indirectly represents the hydrological inflow by approximating the state transition caused by reservoir storage as a capacity constraint over the total time frame. To realize this, additional constraints for each of the hydropower units are required:

$$\sum_n \sum_t y^q_{i_{hydro},n,t} \leq r_{i_{hydro}} \quad \forall i_{hydro} \tag{12}$$

[1]with the exception of an initial, fixed assumption of the value after the observed time frame

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In contrast to [3], [4], [8], the approach presented here is realized through an inequality constraint. This is made possible by assuming an end period water value instead of reservoir storage, whereas the storage level is now subject to player decision. In the case study presented here, the water values were considered to be the (assumed) spot price of electricity rounded down to 100s, in the next period after the analyzed time frame (i.e. $T+1$) within the location node of the respective plant.

Both of these changes to traditional hydropower equilibrium models, i.e. no requirement for dual values and alleviation of the inventory constraints, led to gains in computational efficiency that support the performance of the Nash tuple mapping algorithm. Similar to [5], spillage of hydrological inventory is not considered.

B. Wind Power Plant

Wind, unlike water, which can be physically stored, is a fluctuating resource, that cannot be transferred from time stage to time stage. Availability depends on external factors which the players have no control of (there is no market for the 'procurement' of wind). Wind curtailment can therefore be considered to be a parameter and requires additional constraints for the generation units of 'wind power' type:

$$\sum_{n} y^{q}_{i_{wind},n,t} \leq q^{cap}_{i,t} \quad \forall i_{wind}, t \tag{13}$$

C. Thermal Power Plant

An introduction of CO_2 caps or the ability to store coal or fossil fuels would add a constraint similar to (12) into the mix. However, such a constraint was omitted, for the sake of simplicity. It was considered sufficient for the case study to have higher variable and fixed cost factors than the renewable generation forms, which implicitly forces the players to minimize up-times and therefore CO_2 emissions. The here presented case also is focused on short term modeling. Thermal restrictions such as minimum and maximum downtime were therefore neglected. Constraints for the *contribution to nodal inertial response* were instead chosen to demonstrate a modern application which the algorithm shown here offers. Nonetheless, we propose future extensions to the model in the form of a more sophisticated representation of intertemporality in players' dispatch decisions. I.e. startup and stopping cost, maximum and minimum runtimes, etc.

D. Inertial Response Requirements

As debated in [12], evolving power systems shifting their production portfolio to higher shares of renewable generation, increases the demand for additional security services. One such service would be providing kinetic energy, or inertial response capabilities. The *inertia constant* (in the literature commonly denoted as H) was used to implement this characteristic in our presented market competition model and to rate the individual impact of generation units and formulate inertia "demand" constraints. Defining inertial response contribution as a parameter H_i supplied at an equal level as long as the unit is running (i.e. $y^{b}_{i,t} = 1$) and summation of those contributions to define the nodal/system inertia was considered to be an appropriate approximation [24]. This is given to create a realistic example to showcase the capabilities of the designed framework and is not necessarily aimed at providing a statement about quantitative impacts of inertial response that can be considered without further analysis.

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We assumed the fictional scenario in which nodes can be assigned minimum inertia requirements, relating to the model in the form of a nodal demand constraint:

$$\sum_{i \in I_n} y^b_{i,t} H_i \geq H^d_n \quad \forall t, n \tag{14}$$

E. Additional Cutting Rules

As mentioned in section V, analytically derived cutting rules are an integral component of the algorithm presented. In literature, cuts are commonly categorized into *feasibility* and *optimality* cuts. We also use the definitions *preliminary* and *dynamic* cuts. Preliminary refers to cuts that can be conducted before any tuple equilibria are obtained (i.e. relate to step 0 in the tuple sorting algorithm in III). Dynamic cuts require one or more already solved s^*(i.e. relate to step 2 in the Nash tuple mapping algorithm in III).

1) Feasibility Cuts: Some combinations of the binary variables $b_{i,t}$ cause infeasibility and thus yield no possible market equilibrium. Certain tuples therefore can and must be sorted out before calculating the tuple NE $\Psi((q_{i,n,t}, b_{i,t}), (y^{q*}_{i,n,t}, y^{b*}_{i,t})) = 0$.

- Inertial Response Requirement Cut (preliminary): Too few committed units in a certain node n in period t will result in a breach of (14). Tuples leading to such a situation can be sorted out before solving them. This can be formulated as:

$$\sum_{i \in I_n} b_{i,t} H_i < H^d_n \quad \text{for any } t, n \tag{15}$$

Tuples s that fulfill rule (15) must therefore be sorted out.

- Minimum Hydropower Output Cut (preliminary): Certain tuples can, due to their minimum outputs over the total time frame being higher than available reservoir volume, similarly show a constellation of binary values that breach constraint (12). The cutting rule reads:

$$\sum_t q^{min}_{i_{hydro}} b_{i_{hydro},t} > r_{i_{hydro}} \quad \text{for any } i_{hydro} \tag{16}$$

The affected s that breach (16) have to be (as for the other cuts) sorted out.

2) Optimality Cuts: One of the core aspects of the Nikaido-Isoda function is that some tuples converge faster than others. Therefore, (dynamic) optimality cuts can be conducted to stepwise decrease the amount of unsolved tuples.

- Payoff-Function Rule (dynamic): As explained above, two tuples are required to be in state **solved** for their NE whereas s^{**} has to be located on a branch of s^*.

- Marginal Cost Rule (dynamic): Players in Cournot competition are able to influence prices by varying their bidding quantity. It is therefore possible for prices to end up at a level where no production quantity could compensate for the involved cost. A tree branch, where the *Marginal Cost* of a unit i exceeds the *Market Clearing Price* is therefore not economically viable for the player controlling that unit. This means choosing the maximum price of all nodes n in a single period t as a benchmark clearing price:

$$\begin{gathered} \text{for a tuple } s^* \text{ cut all tuples } s \text{ where:} \\ b_{i,t} \geq b^*_{i,t} \ \forall i, t \\ \sum_{i,t} [b_{i,t} | p^{MC}_{i,t} > \max_n p^*_{n,t}] > 0 \\ p^{MC}_{i,t} = b_{i,t} c^{var}_{i,t} \end{gathered} \tag{17}$$

This shows that cuts might overlap. A tuple affected by the Marginal Cost Rule would show $q_{i,n,t} = 0$ for i and t where $p^{MC}_{i,t} > \max_n p^*_{n,t}$. Otherwise, the generator i

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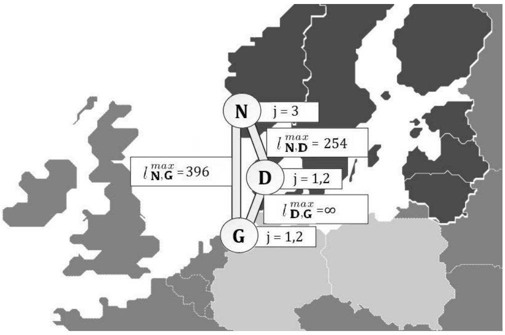

Fig. 2: Case Study Setup

would produce at a loss. However, as the cost portion c_i^{fix} must be paid by unit i due to $b_{i,t} = 1$, then the payoff-function will inherently yield a lower result $\pi_j(q_{i,n,t}, b_{i,t})$ for the owner of $i \in j$. Therefore, such a case would also be cut through the payoff-function rule presented above.

The following section introduces a case study to demonstrate a practical application of the framework presented here.

VII. Case Study

As illustrated by Figure 2, the case study is designed to relate to an excerpt of the European power system, nodes representing the countries of **N**orway, **D**enmark and **G**ermany. As discussed above, a representation of areas or countries in which little regard is paid to wheeling fees can be considered fitting for the model in the here proposed form. Further granularity would require adequate adjustment (i.e. the introduction of further agents). The test case resembles part of a week in late fall with medium to high available wind capacity (especially in the North Sea) and low to medium available hydropower capacities. The parameters can be found in the appendix. The importance of this case study is highlighted by the lack of literature on market power in hydrothermal competition and market power in the European system. Hydrothermal competition is based on the technical constraints related to the state variables, whereas analysis of market power is based on legislature aiming to hinder exercise of such (but not strategic bidding). We argue, particularly in the light of the introduction of new products such as commercialization of inertial response, that a careful analysis of market robustness to such actions should be incorporated in the design process.

Different types of generation units[2] (Table II) meet in a 3-bus network to conduct trade under the assumption of similar information on market clearing price elasticity (Table III). The power line flows $l_{i,n}^{n_s,n_d}$ found in Table IV were assumed to be similar to the P*ower Transfer Distribution Factors* (PTDF) presented in [2]. A single convergence criteria was added to the model:

$$\Psi((q_{i,n,t}, b_{i,t}), (y_{i,n,t}^{q*}, y_{i,t}^{b*})) \leq 0.00001 \equiv 0 \tag{18}$$

[2]note that the plants are assumed to be continuously running or idle for a whole day

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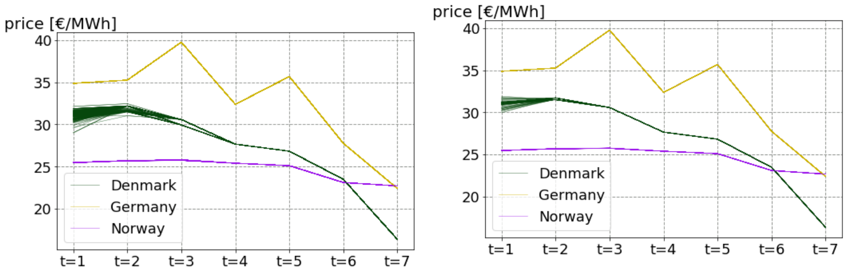

(a) case #1: no requirements for inertial response

(b) case #2: requirements for inertial response in node $n = D$

Fig. 3: Price curves for each Nash tuple $s \in S^N$

No further tolerances were added on the constraints, as they are not required due to tuple problems being represented by quadratic optimization problems (an advantage of the Nikaido-Isoda method compared to more traditional methods such as using Karush-Kuhn-Tucker conditions) that can be solved by most commercial solvers.

The case study aims to analyze the impacts of applying minimum requirements for inertial response in the wind power dominated node $n = D$. Therefore the case study can be adjusted to increase computational efficiency by analyzing generation schedules. Several plants show no fixed cost. No negative effects of the on/off states of the generation units on the profit functions can therefore be expected. These units can therefore be assumed to run continuously, i.e. $\bar{b}_{i,t} := 1 \forall i = \{1, 2, 5, 6, 8, 9, 10\}, t$. For hydropower plants, this is only possible as their minimum generation is assumed to be 0. Schedules would have to be included for minimum output capacities > 0 and the cut presented in (16) would have to be applied. However, as this is not the case, these tuples were removed, reducing the number of total tuples from $2^{10\times 7}$ to $2^{3\times 7}$.

The model does not consider the possibility of shared inertial response within the whole system but instead focuses on modeling nodal inertia demands. Thus, and as the scheduling of the thermal plant $i = 7$ does not affect the inertial response in node $n = D$, it was assumed to be predetermined as $\bar{b}_7 := [1, 1, 1, 1, 0, 0, 0]$. This led to a reduction in tuples from $2^{3\times 7}$ to $2^{2\times 7} = 16384$. This remaining set of tuples was solved twice:

#1: no requirements for inertial response:
$H_N^d = H_D^d = H_G^d = 0$

#2: requirements for inertial response in Denmark:
$H_N^d = H_G^d = 0, H_D^d = 1$

The Nash tuple mapping algorithm required solving 629 (randomly selected) tuples for case #1 and 385 tuples for case #2 until the mapping algorithm converged. Processing times on an Intel i7-5600 core @2.6 GHz were below 1 second for an iteration, with an average of 15 iterations until a single tuple converged. The resulting set of Nash tuples S^N contained 390 elements in case #1 and 128 elements in case #2. The model does not show a large range of infeasible states. Most cuts were therefore conducted dynamically.

Figure 3 shows a reduction in the ranges of price scenarios from case #1 to #2. Scheduling decisions however seem to mostly affect the node of the two plants with

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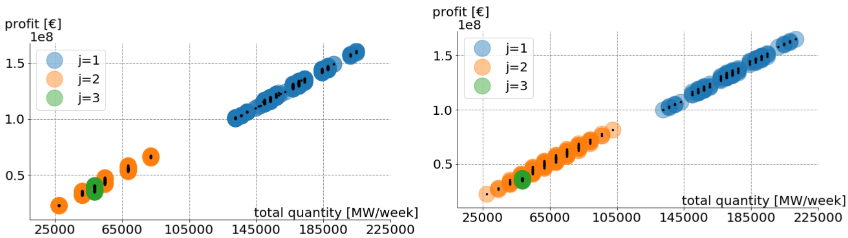
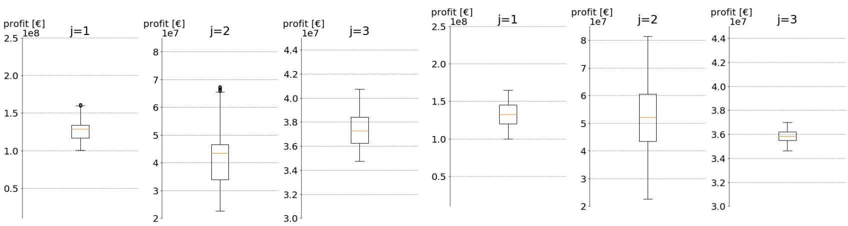


(a) case #1: no requirements for inertial response

(b) case #2: requirements for inertial response in node $n = D$

Fig. 4: Map of Nash tuples $s \in S^N$



(a) case #1: no requirements for inertial response

(b) case #2: requirements for inertial response in node $n = D$

Fig. 5: Firm profits for each Nash tuple $s \in S^N$

variable schedules. The map of NE is displayed in Figure 4. The figure demonstrates the importance of showing different equilibrium tuples. Every chosen schedule yields strongly different outputs and player profits, whereas each tuple is an equilibrium and thus represents a potential outcome. The figure shows that the range of equilibrium results increases slightly for player $j = 1$, strongly for $j = 2$ and decreases slightly for $j = 3$. This change in the range of profit scenarios is also displayed in Figure 5. Player $j = 1$ profits marginally and $j = 2$ strongly from apparent effects of "forced cooperation". This change in profit stems from that the respective owner would choose to not schedule in order to result in an alternate optimum (i.e. it would be profitable to shut them down, thus they would be sorted out by rule (5a) to (5c)).

Scheduling these unprofitable units however occupies transfer line capacities and thus reduces the possibility of the hydropower player $j = 3$ accessing other market nodes, resulting in lower profits across all scenarios. The effect of additional line congestion can be observed in the increase in average capacity in line $N \to D$, as displayed in result Table I. It shows that increasing exports and decreasing local production leads to lower impact of binary variables on the price 'spread'. This is shown by the wide gap of prices in $t = 1$ in Figure 3 and the low spread in $t = 7$. The case study demonstrates that influencing unit commitment decisions (as ancillary services such as primary reserves or the inertial response requirements discussed here do) has an impact on otherwise unaffected generators in the system - here represented by hydropower producer $j = 3$. This negative effect, i.e. a profit decrease, comes as a result of the market share that is shifted to generators that would choose to not schedule in the optimum, but who

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TABLE I: Transfer Results (averaged over all tuples)

line utilization	case #1	case #2
line $D \rightarrow G$	278.3 MW	330.8 MW
line $G \rightarrow N$	-194.4 MW	-191.8 MW
line $N \rightarrow D$	60.6 MW	113.3 MW

t =	day 1	day 2	day 3	day 4	day 5	day 6	day 7
Local generation $[MWh]$							
case #1	1281	1449	1274	549	81	243	0
case #2	1283	1620	1469	549	81	243	0
Exported generation $[MWh]$							
case #1	911	848	1048	633	823	716	1177
case #2	912	916	1055	709	907	759	1392

are, through system constraints, forced to participate in certain periods. This distorts competition by removing market share from more competitive players such as the time stage abitrageurs (hydro power producers) and assigning them to less competitive forms of generation such as thermal producers. This effect comes from the inertia requirements making certain equilibria from case #1 infeasible, thus effecting the tree and enabling branches that support less efficient equilibria. Averaged over the tuples, case #1 results in a generator welfare of $144M€$ whereas case #2 shows $224.6M€$, a welfare increase that would have to come at the expense of the demand side, i.e. consumers.

VIII. Conclusions

The proposed framework and case study in this paper presents a number of contributions:

The main contribution is the consideration of strategic scheduling decisions in a model with price-making generators and multiple interconnected time stages, a novelty in the literature [25]. Furthermore, the resulting mapping of a finite pool of Nash equilibrium tuples demonstrates a new view on discontinuous problems in energy systems, that have traditionally been occupied with converging towards single solution tuples (e.g. [9]) whilst disregarding potential other equilibria. This allows the discontinuous decisions of a player to relate to its market impact and vice versa, so determining the impact players have on each others' scheduling. In addition, the proposed cutting techniques and adjustments to other models proposed in the literature allows for a more computational efficient approach to model hydro-thermal(-renewable) systems. Finally, the proposed case study itself constitutes a novelty. It shows that introducing minimum requirements for committed units in single nodes has an effect on the profits of other participants in the system. The reason for this is found in transmission capacities being used by the newly committed units, occupying transmission lines that could be otherwise used by different actors to conduct nodal price arbitrage, resulting in a worse outcome for those arbitrageurs. This result and the framework proposed in the paper might aid future discussions of system design options, for example the analyzed requirements for inertial response. One limitation of the paper is demonstrated by the case study. The requirement of in-depth problem analysis does not allow for the plug-and-play of the solution framework. Tailored cuts and predetermining the generation units that are valid in active scheduling decisions requires a case-by-case analysis. As mentioned above, the problem in its current form might be applicable for similar large area applications such as the analysis of spinning reserves. However, and as for most equilibrium problems,

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real world uncertainty and resulting forecast volatility influence the outcome and thus provide an important starting point for future research.

REFERENCES

[1] S. A. Gabriel, A. J. Conejo, J. D. Fuller, B. F. Hobbs, and C. Ruiz, *Complementarity Modeling in Energy Markets*. New York: Springer, 2013.

[2] B. F. Hobbs, "Linear Complementarity Models of Nash Cournot Competition in Bilateral and POOLCO Power Markets," *IEEE Transactions on Power Systems*, vol. 16, no. 2, pp. 194–202, 2001.

[3] J. Bushnell, "A Mixed Complementarity Model of Hydrothermal Electricity Competition in the Western United States," *Operations Research*, vol. 51, no. 1, pp. 80–93, 2003.

[4] S. A. Gabriel, S. Kiet, and J. Zhuang, "A Mixed Complementarity-Based Equilibrium Model of Natural Gas Markets A Mixed Complementarity-Based Equilibrium Model of Natural Gas Markets," *Operation Research*, vol. 53, no. March 2017, pp. 799–818, 2005.

[5] S.-E. Fleten and T. T. Lie, "A Stochastic Game Model Applied to the Nordic Electricity Market," in *World Scientific Series in Finance*, 2013, vol. 4, pp. 421–441.

[6] G. Steeger and S. Rebennack, "Strategic bidding for multiple price-maker hydroelectric producers," *IIE Transactions (Institute of Industrial Engineers)*, vol. 47, no. 9, pp. 1013–1031, 2015.

[7] D. Pozo and J. Contreras, "Finding Multiple Nash Equilibria in Pool-Based Markets : A Stochastic EPEC Approach," *IEEE Transactions on Power Systems*, vol. 26, no. 3, pp. 1744–1752, 2011.

[8] J. P. Molina, J. M. Zolezzi, J. Contreras, H. Rudnick, and M. J. Reveco, "Nash-Cournot Equilibria in Hydrothermal Electricity Markets," *IEEE Transactions on Power Systems*, vol. 26, no. 3, pp. 1089–1101, 2011.

[9] J. B. Krawczyk and S. Uryasev, "Relaxation algorithms to find Nash equilibria with economic applications," *Environmental Modeling and Assessment*, vol. 5, pp. 63–73, 2000.

[10] J. Rosen, "Existence and Uniqueness of Equilibrium Points for Concave N-Person Games," *Econometrica*, vol. 33, no. 3, pp. 520–534, 1965.

[11] R. S. Datta, "Finding all Nash equilibria of a finite game using polynomial algebra," *Economic Theory*, vol. 42, no. 1, pp. 55–96, 2009.

[12] R. Doherty, G. Lalor, and M. O'Malley, "Frequency Control in Competitive Electricity Market Dispatch," *IEEE Transactions on Power Systems*, vol. 20, no. 3, pp. 1588–1596, 2005.

[13] J. Contreras, M. Klusch, and J. B. Krawczyk, "Numerical Solutions to Nash Cournot Equilibria in Coupled Constraint Electricity Markets," *IEEE Transactions on Power Systems*, vol. 19, no. 1, pp. 195–206, 2004.

[14] H. Nikaido and K. Isoda, "Note on non-cooperative convex game," *Pacific Journal of Mathematics*, vol. 5, no. 5, pp. 807–815, 1955.

[15] G. B. Sheble and G. N. Fahd, "Unit commitment literature synopsis," *IEEE Transactions on Power Systems*, vol. 9, no. 1, pp. 128–135, 1994.

[16] M. Carrión and J. M. Arroyo, "A computationally efficient mixed-integer linear formulation for the thermal unit commitment problem," *IEEE Transactions on Power Systems*, vol. 21, no. 3, pp. 1371–1378, 2006.

[17] Y. Ye, D. Papadaskalopoulos, and G. Strbac, "Factoring Flexible Demand Non-Convexities in Electricity Markets," *IEEE Transactions on Power Systems*, vol. 30, no. 4, pp. 2090–2099, 2015.

[18] S. E. Fleten and T. K. Kristoffersen, "Stochastic programming for optimizing bidding strategies of a Nordic hydropower producer," *European Journal of Operational Research*, vol. 181, pp. 916–928, 2007.

[19] L. Arvan, "Some Examples of Dynamic Cournot Duopoly with Inventory," *The RAND Journal of Economics*, vol. 16, no. 4, pp. 569–578, 1985.

[20] D. Pozo, E. Sauma, and J. Contreras, "Basic theoretical foundations and insights on bilevel models and their applications to power systems," *Annals of Operations Research*, vol. 254, no. 1, pp. 303–334, 2017.

[21] J. García-González, R. Moraga, R. De, and L. M. Santos, "Stochastic joint optimization of wind generation and pumped-storage units in an electricity market," *IEEE Transactions on Power Systems*, vol. 23, no. 2, pp. 460–468, 2008.

TABLE II: Generation Unit Parameters (prescheduled units marked with a *)

i	j	type	q_i^{min}	q_i^{max}	H_i	c_i^{fix}	$c_{i,t}^{var}$ / $w_{i_{hydro}}$	$r_{i_{hydro}}$/ $q_{i,t}^{cap}$
1*	1	wind	0	370	0	0	0,...,0	349, 337, 152, 128, 195, 256, 349
2*	1	wind	0	166	0	0	0,...,0	66, 91, 136, 78, 46, 62, 71
3	1	coal/fuel oil	145	715	2.8	4654	29.83, 30.29, 29.67, 29.67, 29.67, 29.67, 28.75	-
4	2	hard coal	285	570	3	3690	20.33, 37.25, 20.92, 43.0, 28.29, 23.5, 23.5	-
5*	1	wind	0	800	0	0	0,...,0	501, 661, 766, 325, 56, 200, 689
6*	2	wind	0	138	0	0	0,...,0	72, 58, 34, 19, 25, 43, 68
7*	2	coal/fuel oil	151	757	2.8	4921	26.67, 26.42, 26.04, 26.25, 25.38, 24.42, 23.29	-
8*	3	hydro	0	376	3.5	0	21	25265
9*	3	hydro	0	342	3.5	0	21	20110
10*	3	hydro	0	118	3.5	0	21	3092

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[22] A. Papavasiliou and S. Oren, "Multiarea Stochastic Unit Commitment for High Wind Penetration in a Transmission Constrained Network," *Operations Research*, vol. 61, no. 3, pp. 578–592, 2013.
[23] J. M. Morales, A. J. Conejo, H. Madsen, P. Pinson, and M. Zugno, "Renewable Energy Sources - Modeling and Forecasting," in *Integrating Renewables in Electricity Markets*, 2014, pp. 18–39.
[24] L. Pereira, "Active Power and Frequency Control," in *Handbook of Electrical Power System Dynamics*, 2013, pp. 291 – 338.
[25] G. Steeger, L. A. Barroso, and S. Rebennack, "Optimal Bidding Strategies for Hydro-Electric Producers: A Literature Survey," *IEEE Transactions on Power Systems*, vol. 29, no. 4, pp. 1758–1766, 2014.

Appendix

Note on Existence of Nash Equilibria

In the proposed framework, a stepwise Nikaido-Isoda convergence algorithm is applied to find a Nash Equilibrium for a tuple s that is defined by a fixed set of binary variables. This transforms the original Cournot Game of players optimizing Mixed Integer Problems into a number of individual continuous Cournot Games that are solved via a branch-and-cut algorithm. The existence of Nash Equilibria is represented by one of two forms: 0 **Nash tuples** - this situation can only occur due to infeasibility. The preliminary cuts proposed in this model will **sort out** all infeasible states, leaving no tuples that are able to transition towards the **solved** state. An infeasible problem also means that no Nash Equilibrium could be found by the Nikaido-Isoda optimization problem, thus leaving no result for the continuous problem that could represent a tuple. ≥ 1 **Nash tuples** - where the problem is feasible and due to the convexity of the continuous problem, each tuple can yield (at least one) Nash equilibrium (even if the solution is that every player produces at minimum/maximum levels) [9]. Multiple Nash equilibria might exist within one tuple (see e.g. [6]). However the Nikaido Isoda function is able to determine the optimal profits for a single equilibrium tuple which subsequently allows the comparison of tuple equilibria. It can be therefore stated that a Nash tuple represents a definite solution for the integer variables but can include a continuum of solutions for the continuous variables that yield similar player profits.

Data Sets

Table II lists plant types, specifications and related generation firms (i.e. players). Note that daily values were obtained through a factor of 24 on the parameters denoted in hours $[h]$ as the plant is assumed to consistently run/stand idle for a whole day. The selected data set is based on real world data from NordPool and from selected power plant data (slightly distorted to ensure anonymity). The fuel mixes are a hydro power generator in Norway, a representative Danish offshore wind/thermal mix and a representative slice of German generation in form of a large thermal plant, an onshore wind farm and an offshore wind farm. The remaining generation in the countries are expressed indirectly by the elasticity of the price curves given in Table III. These are based on the spot market volume obtained through NordPool. Table III shows the market price curves during the observed week. Table IV lists the locations and the PTDF associated with the plants adapted from the three-node case in [2]. It should be noted for $l_{i,n}^{n_s,n_d}$ that the superscript represents a lineflow $n_s \rightarrow n_d$ and the subscript represents the source unit i and the target node n.

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TABLE III: Market Price Parameters

$p^*_{n,t} =$	$n = N$	$n = D$	$n = G$
$t = 1$	$25.5 - 6\text{E}{-}6d_{n,t}$	$33.04 - 10\text{E}{-}5d_{n,t}$	$34.92 - 10\text{E}{-}6d_{n,t}$
$t = 2$	$25.7 - 6\text{E}{-}6d_{n,t}$	$33.63 - 7\text{E}{-}5d_{n,t}$	$35.29 - 10\text{E}{-}6d_{n,t}$
$t = 3$	$25.8 - 5\text{E}{-}6d_{n,t}$	$30.58 - 7\text{E}{-}5d_{n,t}$	$39.79 - 8\text{E}{-}5d_{n,t}$
$t = 4$	$25.4 - 5\text{E}{-}6d_{n,t}$	$27.67 - 7\text{E}{-}5d_{n,t}$	$32.42 - 10\text{E}{-}6d_{n,t}$
$t = 5$	$25.1 - 5\text{E}{-}6d_{n,t}$	$26.83 - 8\text{E}{-}5d_{n,t}$	$35.71 - 10\text{E}{-}6d_{n,t}$
$t = 6$	$23.1 - 8\text{E}{-}6d_{n,t}$	$23.5 - 6\text{E}{-}5d_{n,t}$	$27.79 - 10\text{E}{-}6d_{n,t}$
$t = 7$	$22.9 - 7\text{E}{-}6d_{n,t}$	$16.38 - 5\text{E}{-}5d_{n,t}$	$22.42 - 10\text{E}{-}6d_{n,t}$

TABLE IV: Plant locations and connections

$i = 1, 2, 3, 4$	location: $n = D$	$l^{D,G}_{i,G} = 67\%$, $l^{D,N}_{i,G} = l^{N,G}_{i,G} = 33\%$ $l^{D,N}_{i,N} = 67\%$,$l^{D,G}_{i,N} = l^{G,N}_{i,N} = 33\%$
$i = 5, 6, 7$	location: $n = G$	$l^{G,D}_{i,D} = 67\%$, $l^{G,N}_{i,D} = l^{N,D}_{i,D} = 33\%$ $l^{G,N}_{i,N} = 67\%$,$l^{G,D}_{i,N} = l^{D,N}_{i,N} = 33\%$
$i = 8, 9, 10$	location: $n = N$	$l^{N,D}_{i,D} = 67\%$, $l^{N,G}_{i,D} = l^{G,D}_{i,D} = 33\%$ $l^{N,G}_{i,G} = 67\%$,$l^{N,D}_{i,G} = l^{D,G}_{i,G} = 33\%$

Sensitivity of Water Values

Expectations of water values also impact the range of equilibrium tuples of non-hydro players. A low expectation of future prices and resulting low water values for the hydropower players leads to a higher range of potential schedules for the thermal players. A high price expectation furthermore leads to a reduction in potential tuples. This is a result of the additional flexibility of a hydropower producer having to shift production to another time stage if profitable.

This is shown by the result of setting water values to 16.67€ per hour (or 400€ per day) which increases the total number of equilibrium tuples to 412. Another extreme can be given by water values of 25€ per hour, which results in only a single equilibrium tuple. This indicates that flexibility in storage creates flexibility in the schedules in a system, even though the units do not belong to the same players.

Note on Performance

With increasing problem complexity, specifically additional actively scheduled units and extended time periods, decreasing performance can be expected. However, additional tests indicate that resource-efficient scaling is possible and the algorithm allows for more complex problem settings than the one presented. To provide an example, thermal unit $i = 7$ was considered with flexible schedule. The result was 15 Nash equilibrium tuples, that required solving 999 tuples (~ 15000 seconds) for a problem with a total of 2097152 tuples.

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7.5 Market Power in a Hydro-Thermal System under Uncertainty

This work has been presented at the *41st IAEE International Conference*.

7.5.1 Extended Abstract

This paper proposes a solution concept for aggregating uncertainty and unit commitment decisions within a hydro-thermal Cournot framework into a single decision vector and solving the resulting problem via robust optimization and interpolation.

The proposed concept reformulates binary dispatch decisions as functions of uncertainty, allowing for cases such as the following example:

- scenario 1 (e.g. high demand/hydropower inflow): unit always on
- scenario 2 (e.g. medium demand/hydropower inflow): unit either on or off
- scenario 3 (e.g. low demand/hydropower inflow): unit always off

A profit maximization problem of a single player holding several units under generation capacity and hydropower storage constraints operating under a number of such scenarios can be solved for its robust solution via a *Column and Constraint Generation* algorithm.

For two consecutive time periods, this single-player problem can be extended to a multi-player market clearing problem by using variables shared by several agents - here the market demand in the two time periods - as parameters in each individual players' decision problem. Through alternating this 'global' variable, a three dimensional supply curve can be established for both time periods, with the spaces between the solved variable combinations being filled via interpolation.

This is a result of robust optimization yielding a specific solution for both time periods, irrespective of the realized scenario, allowing to clear for Nash equilibria over both time periods.

The paper presents a case study on a three player problem under uncertain market prices and hydrological inflow uncertainty.

On the equilibrium results of this case, the Nikaido-Isoda function is applied as a measure of performance (i.e. closeness to a 'real' equilibrium for the interpolated solution). Based on this measure, the applied algorithm promises strong performance and allows for making active decisions in the tradeoff solution quality $\leftrightarrow$ solution time by alternating the number of different 'global' variable steps.

Extensions of the model to further time periods is possible, with each considered period adding a dimension to the supply curve of each period.

Market Power in a Hydro-Thermal System under Uncertainty

Markus Löschenbrand[*]

ABSTRACT

A method to find the Nash Equilibrium in a Cournot game for a number of players - generation companies - scheduling hydro-thermal units by robust optimization is proposed. Binary start/stop decisions are incorporated through approximation functions. Furthermore, an uncertain future time stage is considered, which effects hydrological inflows as well as price curve shape. The market in both time stages is cleared through interpolated three-dimensional supply functions. Finally, further discussion on improvement of the methodology and addition of further constraints to the players' optimization problems is provided.

Keywords: Game Theory, Cournot Game, Nash Equilibrium, Market Power, Nikaido-Isoda, Hydropower, Thermal Generation, Unit Commitment, Robust Optimization, Column & Constraint Generation

1. INTRODUCTION

Deregulation of power markets introduces competition aimed to decrease prices and increase service quality for the end customers. As in traditional commodity markets a concern regarding efficient market design is market power. As necessary infrastructure carries the risk of arising monopolies, the transmission side of power systems is traditionally exempt of regulation. On the contrary, the supply (i.e. generation) side of - especially European and U.S.American - power systems has seen constant deregulation, with all related risks such as arising asymmetric market power (Berry et al., 1999).

For conventional thermal generation established models are available, that consider power systems from an economic angle (Hobbs, 2001). Game theoretic methods similar to other commodity markets - such as for other forms of energy or resources - are established in literature (Gabriel et al., 2013). Those methods mostly consider single time periods which can show a distorted view of player decisions in systems under large scale storage, such as e.g. the Scandinavian power system with large hydropower capacities, as such systems tend to operate different from traditional thermal systems (Wolfgang et al., 2009).

Even though literature on such systems exists (e.g. (Førsund, 2015)), usually market power effects are dismissed due to lower price peaks and legislature aimed to prevent firms from exercising market power. In addition, network connections (such as e.g. to central Europe in the case of the Scandinavian system) introduce different generation setups, whereas the constellation of hydro-thermal plants in market clearing is not fixed as players can actively decide which markets to enter. (Bushnell, 2003) provides an approximation of hydropower units which can be included into basic deterministic thermal economic market models that are solved via tradtional techniques such as Karush-Kuhn-Tucker conditions. However, as shown in e.g. (Pereira and Pinto, 1991) uncertainty plays an important role in hydrological inflows and therefore requires consideration in hydropower dispatch (Wangensteen, 2012). Traditionally, dispatch problems are assumed to be external inputs to problems focused on the market side, an example is given by (Rahimi-kian and Haghighat, 2007).

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(Moiseeva and Hesamzadeh, 2017) establishes a methodology to obtain both a *Bayesian* and a *Robust Nash Equilibrium* for hydropower producers with a multitude of technical constraints such as a hydrological network, power flows and ramping constraints/costs.

(Steeger and Rebennack, 2015) uses interpolation to obtain revenue functions that are used to obtain Nash Equilibria. Furthermore it analyzes the potential number of Nash Equilibria, a characteristic the previously presented paper omits.

(Molina et al., 2011) uses a stepwise algorithm to distribute storage over a number of periods. The methodology is based on the concept of *Nikaido-Isoda* functions, which present a distance measure to an equilibrium point that was first presented in (Nikaido and Isoda, 1955). (Krawczyk and Uryasev, 2000) presents a relaxation algorithm that allows stepwise convergence to a Nash Equilibrium. This principle is especially useful in systems where players share constraints, as e.g. (Contreras et al., 2004) shows for an example of a traditional power market (also incorporating the numerical example provided in (Hobbs, 2001)).

The here presented methodology aims to provide a new angle. Assumed are hydro-thermal generation companies operating in a two-stage (i.e. two-period) single node/single market system under robust optimization (Zeng and Zhao, 2013). An interpolation method to establish three-dimensional supply functions is introduced and the market cleared via a sorting algorithm minimizing deviations. The Nikaido-Isoda function is used to determine the quality of the obtained Nash Equilibrium and further possibilities to increase the efficiency of the principle are introduced as well.

2. GENERATOR PROBLEM

This section introduces the decision process of a single hydro-thermal player, in specific a generation company (GenCo) j holding $i \in I^j$ generation units. Due to higher generation cost of thermal units, it can reasonably be assumed that thermal units provide the marginal units in market clearings. Thus, it is of importance for hydro-thermal producers to accurately depict the thermal optimization problem in their generation planning, or in specific: unit commitment decisions.

2.1 Unit Commitment

Solving optimization models with such active dispatch decisions - e.g. in the binary form of 'run unit'/'shut down unit' - traditionally makes use of techniques from the field of nonlinear programming and is strongly related to decision making of thermal power plants (Padhy, 2004), whereas some existing literature also extends its application to other forms of generation such as hydropower (Philpott et al., 2000).

Assumed be a time stage t: a generation unit i has maximum and minimum generation capacities of $q^i_{\max}[MW]$ and $q^i_{\min}[MW]$ respectively and makes a continuous decision on generated quantity $q^i_t[MWh] \in \mathbb{R}$ and a discontinuous decision on dispatch b^i_t[on/off] $\in \mathbb{Z}$. The resulting capacity constraint for every time stage thus reads:

$$q^i_{\min} b^i_t \leq q^i_t \leq q^i_{\max} b^i_t \tag{1}$$

Contrary to such models, approaches focused on optimization of bids traditionally tend to assume predetermined unit schedules, one such example can be found e.g. in (Baillo et al., 2004).

Assuming fixed schedules over longer time frames however might lead to distortion in models under uncertainty, as an extended time frame means extended leeway for a player to react to uncertain future events. Thus, this paper proposes an approximation (denoted by ˆ) that represents a balance between those approaches: instead of considering an active dispatch variable, unit commitment decisions are represented as a function of uncertainty ξ:

$$b^i_t \approx \hat{b}^i_t(\xi) \tag{2}$$

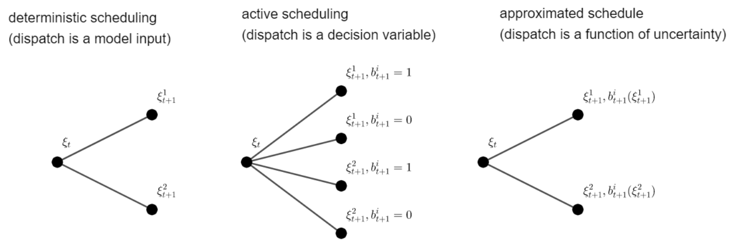

Figure 1: Different Implementations of Unit Dispatch for a single Generation Unit

This allows players to consider different schedules for e.g. low and high price scenarios without the requirement for additional discontinuous variables that introduce non-convexity to a decision problem.

Figure 1 shows a comparison between the two traditional methodologies - implementing scheduling as deterministic parameters or using decision variables - and the proposed approximation technique. A single generation unit i is given the possibilities to produce in the second stage ($b^i_{t+1} = 1$) or stay idle ($b^i_{t+1} = 0$). The first stage is supposed to be certain (scenario ξ_t) and the second stage is assumed to be one of two scenarios (xi^x_{t+1} or xi^x_{t+1}) with no known distribution.

It shows, that using approximations does not increase the size of the fan of potential outcomes but still provides more depth compared to the original deterministic approach.

2.2 Profit Maximization Problem

The underlying decision problem will be modeled in the form of a profit maximizing hydro-thermal GenCo that operates on a hourly cleared, uniformly priced market for energy (i.e. no capacity markets are included).

To further reduce model complexity several additional assumptions are made:

1. decisions in the current stage are assumed to be certain.
2. start-up and shutdown cost as well as down/uptime constraints for thermal plants are neglected.
3. no interaction between hydrological reservoirs is considered (i.e. no waterways).
4. due to low cost profiles, dispatch decisions of hydropower units are neglected.
5. start and end period reservoir levels are neglected.

Assuming a single player j holding a set of generation units/power plants denoted as I^j allows formulating the two-stage problem of maximizing the profits for a time period t with recourse in period $t+1$ as:

$$\max_{q,\phi} p_t(\xi_t, d_t) \sum_{i \in I^j} q_t^i - \sum_{i \in I^j} c_i(q_t^i) + \phi \tag{3a}$$

subject to

$$\phi \leq p_{t+1}(\xi_{t+1}^x, d_{t+1}) \sum_{i \in I^j} q_{t+1}^i - \sum_{i \in I^j} c_i(q_{t+1}^i) \quad \forall x \in R^j \tag{3b}$$

$$q_{\min}^i \hat{b}_t^i(\xi_t) \leq q_t^i \leq q_{\max}^i \hat{b}_t^i(\xi_t) \quad \forall i \in I_{\text{thermal}}^j \tag{3c}$$

$$q_{\min}^i \hat{b}_{t+1}^i(\xi_{t+1}^x) \leq q_t^i \leq q_{\max}^i \hat{b}_{t+1}^i(\xi_{t+1}^x) \quad \forall i \in I_{\text{thermal}}^j, x \in R^j \tag{3d}$$

$$q_{\min}^i \leq q_\tau^i \leq q_{\max}^i \quad \forall \tau \in \{t, t+1\}, i \in I_{\text{hydro}}^j \tag{3e}$$

$$q_t^i \leq l_t^i(\xi_t) \quad \forall i \in I_{\text{hydro}}^j \tag{3f}$$

$$q_t^i + q_{t+1}^i \leq l_t^i(\xi_t) + l_{t+1}^i(\xi_{t+1}^x) \quad \forall i \in I_{\text{hydro}}^j, x \in R^j \tag{3g}$$

$$l_t^i(\xi_t) - q_t^i \leq q_{\text{res}}^i \quad \forall i \in I_{\text{hydro}}^j \tag{3h}$$

$$l_t^i(\xi_t) + l_{t+1}^i(\xi_{t+1}^x) - q_t^i - q_{t+1}^i \leq q_{\text{res}}^i \quad \forall i \in I_{\text{hydro}}^j, x \in R^j \tag{3i}$$

The objective function (3a) presents a profit maximization. The first stage energy price function $p_t(\xi_t, d_t)[€/MWh]$ is dependent on a certain scenario ξ_t and a market demand $d_t[MWh]$ and is further multiplied by the generated quantity to give the revenues in period t. Subtracting the sum of the individual unit cost functions $c_i(q_t^i)[€]$ yields the profits.

Variable $\phi[€] \in \mathbb{R}$ represents an approximation of the profits in the recourse period $t+1$ and is defined by constraint (3b). The difference to the first stage profits lies in the scenarios being realized as xi_{t+1}^x, where x is a scenario index and $R^j \subseteq X$ a finite discrete set of recourse decisions. X represents the set of potential scenarios and is assumed to be player-independent, which will be further discussed below.

Capacity constraints (3c) and (3d) limit the generation of thermal units, whereas (3e) sets the capacities for the hydropower generation units.

Equation (3f) and state equation (3g) connect the first and second stage decisions by requiring the generation capacities to lie below the inflows of first and second stage, denoted as $l_t^i(\xi_t)[MWh]$ and $l_{t+1}^i(\xi_{t+1}^x)[MWh]$ respectively.

The capacities of the hydrological reservoirs are considered in constraint (3h) and (3i). The inflow in hydrological inventory minus the used hydrological inventory have to stay within the physical boundaries of the reservoir.

2.3 Obtaining a Robust Solution

This formulation allows applying a procedure similar to the technique presented in (Zeng and Zhao, 2013) - *Column and Constraint Generation* (CnCG).

Due to the reservoir limit constraint in the GenCo problem (3), decisions in stage 1 can lead to infeasibility in stage 2. Cutting plane algorithms like CnCG traditionally apply *feasibility cuts* to avoid consideration of such scenarios and cut the feasible area of the optimization problem to an area which does not incorporate infeasible scenarios. In reality however, such scenarios can happen: for example, a hydrological reservoir can reach its bounds and overflow. To consider this and therefore ensure complete recourse, a state constraint relaxation in form of a spillage variable $s_t^i[MWh] \in \mathbb{R}^+$ is introduced to the model. This removes the requirement for feasibility cuts in the later presented algorithm and therefore increases computational performance.

$$
\begin{aligned}
&\max_{q_t, s_t, \phi} p_t(\xi_t, d_t) \sum_{i \in I^j} q_t^i - \sum_{i \in I^j} c_i(q_t^i) + \phi \\
&\text{subject to} \\
&\phi \leq p_{t+1}(\xi_{t+1}^x, d_{t+1}) \sum_{i \in I^j} q_{t+1}^i - \sum_{i \in I^j} c_i(q_{t+1}^i) \quad \forall x \in R^j \\
&q_{\min}^i \hat{b}_t^i(\xi_t) \leq q_t^i \leq q_{\max}^i \hat{b}_t^i(\xi_t) \quad \forall i \in I_{\text{thermal}}^j \\
&q_{\min}^i \leq q_t^i \leq q_{\max}^i \quad \forall i \in I_{\text{hydro}}^j \\
&0 \leq l_t^i(\xi_t) - q_t^i - s_t^i \leq q_{\text{res}}^i \quad \forall i \in I_{\text{hydro}}^j \\
&0 \leq l_t^i(\xi_t) + l_{t+1}^i(\xi_{t+1}^x) - q_t^i - q_{t+1}^i - s_t^i - s_{t+1}^i \leq q_{\text{res}}^i \quad \forall i \in I_{\text{hydro}}^j, x \in R^j
\end{aligned} \tag{4}
$$

Thus, a (relaxed) *master problem* is established. Furthermore and for the sake of simplicity, constraints (3f), (3g) and (3h), (3i) are gathered into single constraints.

$$
\begin{aligned}
&\Phi(q_t^i, s_t^i) = \min_{x \in X} \max_{q_{t+1}, s_{t+1}} p_{t+1}(\xi_{t+1}^x, d_{t+1}) \sum_{i \in I^j} q_{t+1}^i - \sum_{i \in I^j} c_i(q_{t+1}^i) \\
&\text{subject to} \\
&q_{\min}^i \hat{b}_{t+1}^i(\xi_{t+1}^x) \leq q_t^i \leq q_{\max}^i \hat{b}_{t+1}^i(\xi_{t+1}^x) \quad \forall i \in I_{\text{thermal}}^j \\
&q_{\min}^i \leq q_{t+1}^i \leq q_{\max}^i \quad \forall i \in I_{\text{hydro}}^j \\
&0 \leq l_t^i(\xi_t) + l_{t+1}^i(\xi_{t+1}^x) - q_t^i - q_{t+1}^i - s_t^i - s_{t+1}^i \leq q_{\text{res}}^i \quad \forall i \in I_{\text{hydro}}^j
\end{aligned} \tag{5}
$$

Next, a *sub problem* is established that returns the approximation of the second stage as a function of the first stage decision $\Phi(q_t^i, s_t^i)$[€]. It is represented by a profit maximization problem nested in a minimization problem, whereas latter returns the worst case scenario x (selecting the scenario with minimal profits).

Algorithm 1 Column-and-Constraint Generation

initialize upper bound $UB = +\infty$, lower bound $LB = -\infty$, recourse decisions $R^j = \{\varnothing\}$;
for $UB - LB \leq conv$ and $R^j \neq X$ **do**
  [MP] solve master problem (4) and derive optimal solutions $q_t^{i*}, s_t^{i*}, \phi^*$;
    update upper bound $UB := p_t(\xi_t, d_t) \sum_{i \in I^j} q_t^{i*} - \sum_{i \in I^j} c_i(q_t^{i*}) + \phi^*$
  [SP] solve sub problem (5) and derive optimal solution $\Phi(q_{t+1}^{i*}, s_t^{i*})$ and derive scenario x^*;
    derive q_{t+1}^{i*} and s_t^{i*}
    update lower bound $LB := \max\{LB, p_t(\xi_t, d_t) \sum_{i \in I^j} q_t^{i*} - \sum_{i \in I^j} c_i(q_t^{i*}) + \Phi(q_t^{i*}, s_t^{i*})\}$
  [cut] optimality cut: add scenario x^* to R^j
[fin] converged

Assuming a convergence coefficient denoted as *conv* allows to formulate the CnCG algorithm in pseudo-code as shown in algorithm 1.

The result is a generation schedule that holds for the maximization problem in period 1 as well as the min-max problem in period 2.

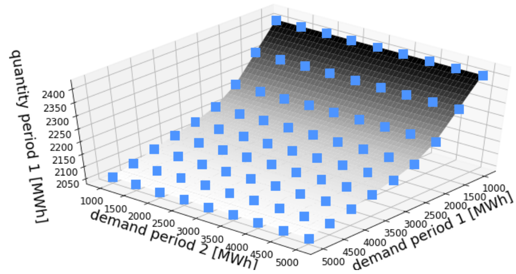

Figure 2: Supply Function Interpolation in t

3. MARKET CLEARING PROBLEM

Similar to literature, the underlying game formulation is that of Cournot competition. The price being defined as a function of demand allows establishing a trajectory of a price curve. Market clearing, i.e. matching the demand with the supplied quantity, cannot however be achieved in similar manner to traditional models, which would rely on establishing necessary and sufficient conditions and e.g. solve it as a complementarity problem by formulating the Karush Kuhn Tucker conditions(Hobbs, 2001). The reason herein lies in that the player problems are solved via CnCG, a non-convex function due to the recourse scenario selection decision being an integer decision. (Steeger and Rebennack, 2015) solve a similar problem with interpolation, which will also be the method of choice to conduct the market clearing in this paper.

Applying the discussed robust optimization method for a specific demand tuple $\langle d_t, d_{t+1}\rangle$ and player j results in quantity solutions $q_t^{i*}, q_{t+1}^{i*}\ \forall i \in I^j$. After all players are solved to convergence, the solutions for all generation units participating in the game, denoted as $I \supseteq I^j$, can be defined as the market supply tuple $\langle q_t^m, q_{t+1}^m\rangle$, where $q_t^m = \sum_{i \in I} q_t^{i*}$ and $q_{t+1}^m = \sum_{i \in I} q_{t+1}^{i*}$.

Choosing a number n of demand tuples $D = \{\langle d_t^1, d_{t+1}^1\rangle, ..., \langle d_t^n, d_{t+1}^n\rangle\}$ and solving them via CnCG yields a similar number of supply tuples $S(D) = \{\langle q_t^{m1}, q_{t+1}^{m1}\rangle, ..., \langle q_t^{mn}, q_{t+1}^{mn}\rangle\}$, which can be done by defining a demand range (between which the market clearing quantities can be assumed) and solving for a pre-defined step size. As discussed previously, solving the robust problem is faster than other techniques such as applying Benders' cuts, but in most realistic problem setups can be still too computationally demanding to cope with small step sizes. Thus, as proposed in (Steeger and Rebennack, 2015) and mentioned earlier, (linear) interpolation is applied to define additional steps that lie between the tuples derived by CnCG.

The interpolation results for the case study proposed later in this paper is shown in figures 2 and 3. 81 demand scenarios (shown in blue) are solved and extended by 5000 additional values obtained by interpolation to obtain a three-dimensional supply function for both periods. The discontinuity observed in figure 3 is analyzed in the result discussion below.

After these sets of solved tuples D and $S(D)$ are established, the market clearing tuple that fulfills $\langle d_t^{m*}, d_{t+1}^{m*}\rangle \approx \langle q_t^{m*}, q_{t+1}^{m*}\rangle$ can be found by solving an integer program:

$$\begin{aligned} \min_{d_t^m, d_{t+1}^m} \ & (d_t^m - q_t^m)^2 + (d_{t+1}^m - q_{t+1}^m)^2 \\ \text{subject to} \ & \\ & \langle d_t^m, d_{t+1}^m\rangle \in D \\ & \langle q_t^m, q_{t+1}^m\rangle \in S(D) \end{aligned} \tag{6}$$

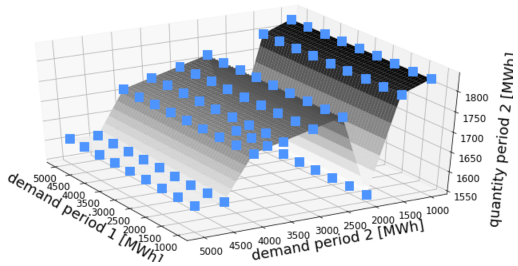

Figure 3: Supply Function Interpolation in $t+1$

The objective function shows the minimization of the Mean Squared Errors, whilst selecting from the finite set of demand tuples. Due to the calculation times of the iterations[1] and CnCG tuple solutions increasing to greater extend than the integer clearing problem, it can be assumed that the problem can be solved for a global optimum via brute-force search.

4. NASH EQUILIBRIUM

A state of a game where no players can take actions to increase their payoffs considering the rest of the players actions remains unchanged, is defined as a *Nash Equilibrium*. In the here presented game the player payoffs can be denoted as profit functions $\Pi^{x,j}(d_t, d_{t+1})[€]$. For the recourse problem (3) this reads:

$$\Pi^{x,j}(d_t, d_{t+1}) = \quad p_t(\xi_t, d_t) \sum_{i \in I^j} q_t^i - \sum_{i \in I^j} c_i(q_t^i) + p_{t+1}(\xi_{t+1}^x, d_{t+1}) \sum_{i \in I^j} q_{t+1}^i - \sum_{i \in I^j} c_i(q_{t+1}^i) \tag{7}$$

Denoting the (quantity) decisions of other players with $'$ and assuming the market clearing demand equals the generation decisions leads to a market clearing condition:

$$\begin{aligned} d_t &= \sum_{i \in I^j} q_t^i + \sum_{i \notin I^j} q_t'^i \qquad \forall j \\ d_{t+1} &= \sum_{i \in I^j} q_{t+1}^i + \sum_{i \notin I^j} q_{t+1}'^i \quad \forall j \end{aligned} \tag{8}$$

As discussed above, the market can be assumed to be cleared, meaning this condition is assumed to hold. Thus, the profit functions can be reformulated as:

$$\Pi^{x,j}(d_t, d_{t+1}) = \Pi^{x,j}(\sum_{i \in I} q_t^i, \sum_{i \in I} q_{t+1}^i) = \Pi^{x,j}(\sum_{i \in I^j} q_t^i + \sum_{i \notin I^j} q_t'^i, \sum_{i \in I^j} q_{t+1}^i + \sum_{i \notin I^j} q_{t+1}'^i) \tag{9}$$

This allows to state the previously introduced Nash Equilibrium as a solution tuple $\langle \sum_{i \in I} q_t^{i*}, \sum_{i \in I} q_{t+1}^{i*} \rangle$ that fulfills the Nash condition that no player can increase their payoff by departing from the Nash solution:

$$\Pi^{x,j}(\sum_{i \in I} q_t^{i*}, \sum_{i \in I} q_{t+1}^{i*}) = \max_{q_t^i, q_{t+1}^i \forall i \in I^j} \Pi^{x,j}(\sum_{i \in I^j} q_t^i + \sum_{i \notin I^j} q_t'^{i*}, \sum_{i \in I^j} q_{t+1}^i + \sum_{i \notin I^j} q_{t+1}'^{i*}) \quad \forall j \tag{10}$$

(Nikaido and Isoda, 1955) proposes a distance measure that defines the difference between a Nash Equilibrium and a current solution. For the here proposed two-stage problem this concept,

[1] in the here proposed approach conducted via the scipy.interpolate library function 'griddata' that allows three-dimensional linear interpolation and shows adequate performance (SciPy.org, 2018)

Table 1: Case Study Parameters

player	unit	type	$q^i_{\min}$	$q^i_{\max}$	q^i_{res}	cost curve
$j = 1$	$i = 1$	biofuel	$80MW$	$800MW$	-	$10q^i_t + 0.075(q^i_t)^2$
$j = 1$	$i = 2$	oil	$87.5MW$	$350MW$	-	$25q^i_t + 0.1(q^i_t)^2$
$j = 1$	$i = 3$	gas	$120MW$	$400MW$	-	$30q^i_t + 0.05(q^i_t)^2$
$j = 2$	$i = 4$	coal	$400MW$	$1000MW$	-	$30q^i_t + 0.035(q^i_t)^2$
$j = 2$	$i = 5$	coal	$320MW$	$800MW$	-	$32q^i_t + 0.03(q^i_t)^2$
$j = 2$	$i = 6$	coal	$200MW$	$500MW$	-	$34q^i_t + 0.03(q^i_t)^2$
$j = 3$	$i = 7$	hydro	$0MW$	$950MW$	$500MW$	$0.005q^i_t + 0.0001(q^i_t)^2$
$j = 3$	$i = 8$	hydro	$0MW$	$300MW$	$300MW$	$0.005q^i_t + 0.0001(q^i_t)^2$

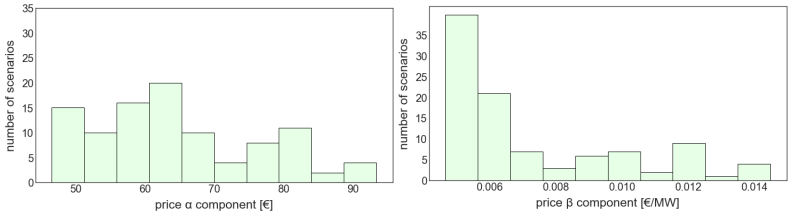

Figure 4: Components of price in period $t + 1$

referred to in literature as Nikaido-Isoda function (Molina et al., 2011; Krawczyk and Uryasev, 2000), can be formulated in similar form.

To do so, q^{m*} is defined as the market clearing decisions that fulfill constraint (8) and q^{j*} as the individual profit maximizing quantities that solve (3) for a specific player (through robust optimization as shown above). This results in the problem-specific formulation of the Nikaido-Isoda function as:

$$\Psi^x(q^{m*}, q^{j*}) = \sum_j \Big[\Pi^{x,j}\big(\sum_{i\in I^j} q_t^{j,i*} + \sum_{i\notin I^j} q_t^{\prime m,i*}, \sum_{i\in I^j} q_{t+1}^{j,i*} + \sum_{i\notin I^j} q_{t+1}^{\prime m,i*}\big) - \Pi^{x,j}\big(\sum_{i\in I} q_t^{m,i*}, \sum_{i\in I} q_{t+1}^{m,i*}\big)\Big] \tag{11}$$

As per definition the market clearing supply has to equal the clearing demand, i.e. $q^{m*} = d^{m*}$. Section 3 showed how to derive those clearing quantities. Equation (7) can thus be used to yield the values of the term $\Pi^{x,j}(\sum_{i\in I} q_t^{m,i*}, \sum_{i\in I} q_{t+1}^{m,i*})$ within the Nikaido-Isoda function.

However, and as shown above, the market clearing condition $q^{m*} = d^{m*}$ will not hold due to the values being interpolations of instead of actual solution to the robust optimization problem. Thus, taking the tuple $\langle d_t^{m*}, d_{t+1}^{m*}\rangle$ allows solving the two stage problem (3) from the perspective of a single case player. It yields the tuple $\langle q_t^{j*}, q_{t+1}^{j*}\rangle$ which then in turn results in a quantitative solution of the Nikaido-Isoda function.

The value of this solution then allows to give a statement about the 'quality' of the interpolation: $\Psi^x(q^{m*}, q^{j*})$[€] denotes the total improvement in payoff (i.e. profit) that all individual players could gain by individually departing from their strategies given that the second stage scenario is ξ^x_{t+1}.

5. CASE STUDY

Table 1 shows the case study setup: a generation portfolio of 8 generation units held by 3 players - 2 thermal players $j = 1, 2$ and 1 hydropower player $j = 3$.

Market data was obtained from the public data platform of the Scandinavian electricity

Table 2: Thermal Unit Commitment Decisions

	generation unit:	$i=1$	$i=2$	$i=3$	$i=4$	$i=5$	$i=6$
period t generation unit state:	$\hat{b}_t^i(\xi_t) =$	1	1	0	1	0	1
period $t+1$ availability:	$\frac{\sum_{x \in X} \hat{b}_{t+1}^i(\xi_{t+1}^x)}{\text{card}(X)} =$	0.95	0.88	0.76	0.90	0.88	0.92

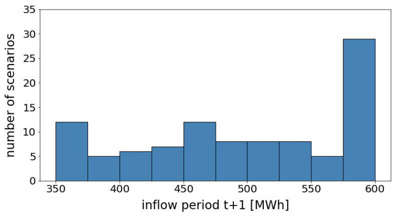

Figure 5: Inflow Scenarios Unit $i = 7$

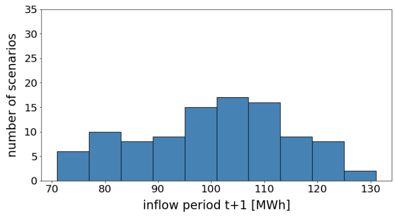

Figure 6: Inflow Scenarios Unit $i = 8$

market operator *Nord Pool*. The case study considers a number of card$(X) = 100$ scenarios in the second stage.

The price in the first stage was considered as the function $p_t(\xi_t, d_t) = 60 - 0.0065d_t$ for a certain scenario ξ_t. Second stage prices were formulated as $p_t(\xi_t^x, d_{t+1}) = \alpha^x - \beta^x d_{t+1}$ whereas the values for the vectors α and β can be found in figure 4. β is based on the elasticity of the total market (i.e. the volume of the units within the case study related to the total market volume obtained from the public data platform). It can be observed that the main assumption is low to no elasticity in the second stage ($\alpha^x \leq 0.006$) which aligns with the general assumption on market power Scandinavian electricity markets(Wangensteen, 2012). However, the here proposed model includes a number of scenarios where market power does exist (even though to a limited degree). Such scenarios and the resulting counter-effects with other players are neglected in traditional optimization models that focus on individual players.

The thermal unit commitment decisions can be found in table 2. Even though startup and shutdown cost were neglected in the here presented simplified model formulation, negative payoff effects due to units running on minimum generation with cost curves above the market price might be observed.

Figures 5 and 6 show the distributions of the hydrological inflows into the reservoirs of the two hydropower units in stage 2. The deterministic inflows in stage 1 were $1000MWh$ and $300MWh$ for unit $i = 7$ and $i = 8$ respectively.

5.1 Results

Conducting the previously introduced algorithm on a number of 81 original and $25E6$ interpolated tuples(as shown in figures 2 and 3) required ~37 minutes on an Intel i7-5600U @ 2.60 Ghz to solve to an equilibrium solution.

The equilibrium points within the supply curves can be found indicated in blue in figure 7 for the first period and figure 8 for the second.

These equilibria result in potential payoffs for the three players which are displayed in figure 9. It shows that player $j = 1$, i.e. the GenCo with the thermal units with lower cost curves (shown in figure 11), also has the lowest spread of payoff over the scenarios (indicated in grey in figure 9). Hydropower player $j = 3$ shows the largest spread of profit over the analyzed scenarios. In addition, the range of potential profits can also range into negative outcomes, which is the case for certain scenarios for both thermal players, especially for $j = 2$.

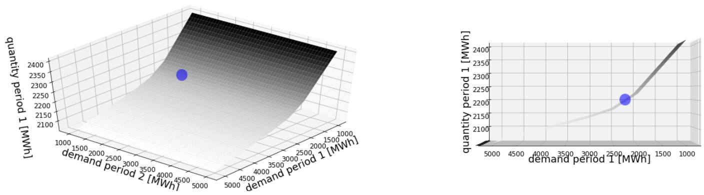

Figure 7: Market Clearing in Stage t

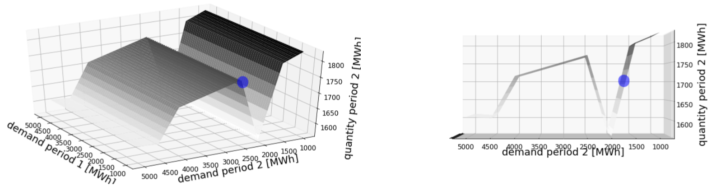

Figure 8: Market Clearing in Stage $t + 1$

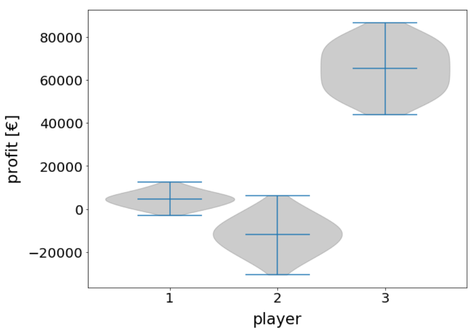

Figure 9: Total Profits for potential Second Stage Scenarios ξ^x_{t+1}

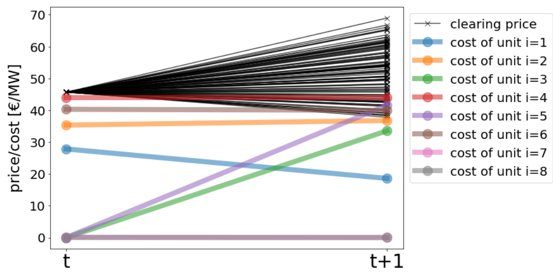

Figure 10: Clearing Price Scenarios and Generation Cost

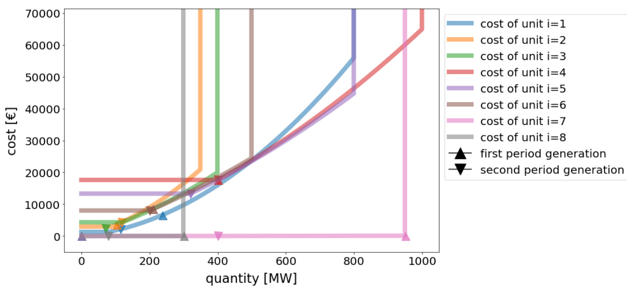

Figure 11: Generation Cost Functions

This indicates that there exists a number of scenarios where players operate units under a loss. This would be amplified by cost curves that are functions of the binary variables (e.g. by incorporating startup/shutdown cost). The appearance of losses are a result of minimum generation capacities of running thermal units, which for the marginal units seems to lie above the market price for certain scenarios.

This effect is also displayed in figure 10 that displays the market clearing price scenarios and the generation costs for the individual units. Units $i = 2, 4, 5, 6$ overlap with the scenario fan in stage $t + 1$, showing that there exist potential scenarios where the units operate under a loss.

To make a statement about the validity of the Nash Equilibrium, the previously introduced Nikaido-Isoda function $\Psi^x(q^{m*}, q^{j*})$ is related to the total profits and averaged over the scenarios:

$$\Psi(q^{m*}, q^{j*}) = \sum_{x \in X} \frac{\Psi^x(q^{m*}, q^{j*})}{\sum_j \left[p_t(\xi_t, d_t^{m*}) \sum_{i \in I^j} q_t^{m,i*} - \sum_{i \in I^j} c_i(q_t^{m,i*}) + p_{t+1}(\xi_{t+1}^x, d_{t+1}^{m*}) \sum_{i \in I^j} q_{t+1}^{m,i*} - \sum_{i \in I^j} c_i(q_{t+1}^{m,i*}) \right]} / \mathrm{card}(X) \tag{12}$$

The resulting distance measure of ~0.017 can be interpreted as that all players together have an incentive to depart from the market quantities yielded by the algorithm by an average of 1.7% per scenario.

The resulting productions are displayed in figure 11: the hydropower units are running on full capacity in stage t, whereas the thermal units adjust their production (upwards or downwards) within the time stage transition. The results consider all units running in the second stage, indicating that players applying robust optimization is similar to an assumption of strong competition, as shutting generation units down would result in higher prices and therefore better conditions for cooperating players. However, such analysis is not considered in this paper but might provide a promising starting

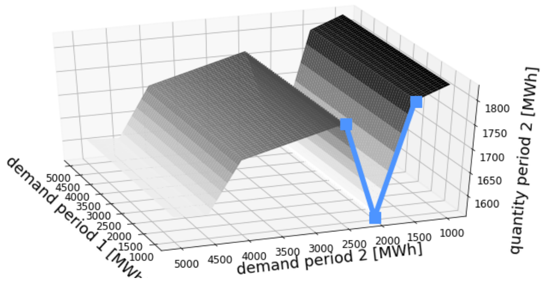

Figure 12: Discontinuity in Stage $t + 1$

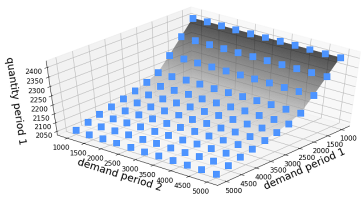

Figure 13: Stage t solved for 121 original Tuples

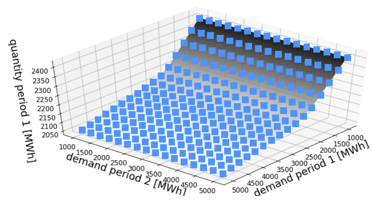

Figure 14: Stage t solved for 256 original Tuples

point for future research. The market clearing results shows hydropower to have the greatest benefits from uniform pricing, as the low marginal cost allow for large supply side welfare, whereas the player holding the marginal thermal unit(s) $i = 4$ (shortly followed by $i = 5$) shows the lowest welfare.

The market clearing results for the analyzed case study are shown in table 3 attached in the appendix. Due to the quantity results being obtained by point interpolation instead of continuous functions, a gap between clearing supply q_t^{i*}, q_{t+1}^{i*} and clearing demand d_t^*, d_{t+1}^* can be found. Sensitivity analysis (introduced below) showed this gap to close for higher number of tuples used to conduct the algorithm.

As observed in figure 3 and highlighted in figure 12 a distinctive discontinuity close to the market clearing demand is observed in stage $t + 1$.

Table 4 in the appendix shows the reasons for this nonconvexity: a transition of second stage values from $d_{t+1} = 1500MWh \rightarrow 2000MWh \rightarrow 2500MWh$ results in the binary decisions for units $i = 1, 2 : \hat{b}_{t+1}^i(\xi_{t+1}^x) = 1$(i.e. on) $\rightarrow 0$(i.e. off) $\rightarrow 1$. As the first and second tuple are close to the market clearing quantities in the second period(see: figure 8), additional granularity might yield a more stable Nash Equilibrium. Below this will be further analyzed.

5.2 Sensitivity

Increasing the resolution of the solution procedure comes in two different aspects: increasing the number of original tuples and increasing the number of interpolated tuples. For the sake of simplicity, in this paper both will be scaled simultaneously, thus not allowing a statement about which provides a larger contribution to result quality. This quality is defined as the previously introduced distance measure $\Psi(q^{m*}, q^{j*})$ - the lower the number the closer the result to a Nash Equilibrium.

Increasing the number of tuples - displayed for stage t in figures 13 and 14 and for $t + 1$ in figures 15 and 16 - showed a positive effect on this Nash Equilibrium quality. The reason can be

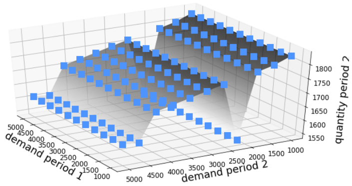

Figure 15: Stage $t + 1$ solved for 121 original Tuples

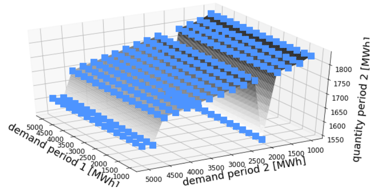

Figure 16: Stage $t + 1$ solved for 256 original Tuples

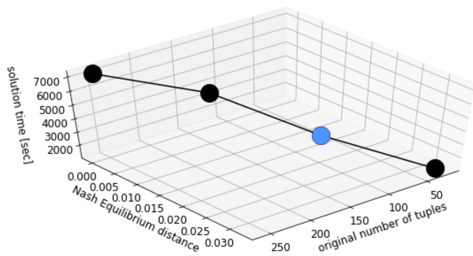

Figure 17: Performance for different Tuple Resolutions

applied tuple resolutions:

1. 25 original/ $6.25E6$ interpolated
2. 81 original/ $25E6$ interpolated

   (analyzed case, shown in blue)
3. 121 original/ $100E6$ interpolated
4. 256 original/ $225E6$ interpolated

attributed to the location of the clearing points: close to the 'bend' of the plane in stage t and at the edge of the 'gap' in stage $t + 1$, both which gain additional resolution and are more 'defined' in the cases incorporating a higher number of tuples.

Figure 17 shows the increase in quality/decrease in computational performance due to an increase in tuple resolution.

6. DISCUSSION

The provided case and subsequent case study illustrate that the proposed framework can yield schedules for a 3 player model with 2 time periods and 100 second stage scenarios under uncertain distribution. These results can then used to create 3-dimensional supply functions and subsequently be interpreted as approximate Nash Equilibria that fulfill the Nash condition with minimal deviations. Despite the novelty of the proposed concept, model limitations exist and provide potential starting points for future research. Those limitations and proposals for future potential model extensions will be discussed below.

6.1 Limitations

The main limitation of the model is the computational performance. By applying approximation functions to replace integer variables in the core problem as well as choosing the technique of CnCG over other more resource demanding cutting-plane techniques such as Benders' decomposition the performance can be increased, the model performance still indicates that large-scale application requires further adjustments. This could play an especially important role in applications that extend the model to a larger time frame than the original two-stage solution. Similar to other economic applications this can be circumvented via adequate problem analysis and design - e.g. pooling similar plant types in single plants, collapsing the scenario tree, etc. However, this could decrease the possibility of dynamic application as it would require potentially extensive preparation and tailoring of the applied cases instead of simple 'plug-and-play'.

Another limitation is given in the assumption of symmetric information that the market clearing requires. As both the tuple solutions as well as the analysis of the equilibrium points are implicitely based on this assumption, the model could prove problematic in situations with strong information asymmetry. Such cases can appear when a large number of technical constraints is incorporated into the players' base model, which the proposed framework allows. However, the yielded Nash Equilibrium might a) not exist or b) exist but might never be reached in reality due to a lack of player information. Thus, asymmetric information has been left out of the analysis conducted in this paper.

Not only that the market clearing algorithm cannot ensure existence of a solution, multiple Nash Equilibria can also not be ruled out. Additionally, due to the non-convexity of the problem (as e.g. shown by the 'gap' in the supply function of stage $t + 1$) an assumed solution method that yields more than one equilibrium could not provide any information on the number of Nash Equilibria[2].

6.2 Potential Extensions

Three different areas of improvement can be defined for the proposed model - increase the time frame, increase the technical complexity of the model, increase the efficiency:

6.2.1 Extend Model Time Frame

The core problem of the framework is formulated as a two-stage problem with recourse. In principle this would allow for application of similar dynamic models as are traditionally applied on problems from the field of hydropower optimization (Pereira and Pinto, 1991). Available literature exists on how to apply such multi-time stage mechanisms on single players under robust optimization (see e.g. (Jiang et al., 2012)). In theory, expanding the proposed framework of this paper can be done in similar manner. However, incorporating more time periods could increase computational complexity to a level where additional approximation techniques and solution methods are required to ensure efficient solving of the problem.

6.2.2 Increase Model Complexity

Literature on incorporating price-making effects into traditional hydropower optimization is rare. Examples of such are (Moiseeva and Hesamzadeh, 2017) and (Steeger and Rebennack, 2015) which both discuss derivation of Nash Equilibria. Further research could be conducted on incorporating technical specifications that the models proposed in these papers consider but which were left out in the here presented framework for the sake of simplicity. Those include: waterway connections, power flows, ramping costs, capacity bids. In addition, more electrical network nodes and minimum/maximum up- and down-times could be incorporated.

6.2.3 Increase Model Efficiency

Decreasing the computational complexity relates to the scaling of the model presented in the previous subsections. Higher complexity of the player models and larger time periods require more efficient solution methods.

[2]as for convex problems this number can be either 0,1 or ∞; for non-convex problems however ≥ 0 and $\leq \infty$ (Gabriel et al., 2013)

As shown in section 5.2 solutions for low amounts of tuples (25 original tuples) can already result in a Nash Equilibrium distance of ~0.032. Thus, a search algorithm increasing granularity around the area that surrounds the equilibrium might prove more efficient than evenly distributing the tuples as it was done in the sensitivity analysis above. Furthermore, brute-forcing the minimization problem within the second stage min-max problem could potentially be replaced by an approximation and scenario-fan collapsing techniques could be used to increase solution times for the individual player problems.

7. CONCLUSION

This paper proposed a novel algorithm to locate and subsequently evaluate the quality of a Nash Equilibrium in a hydro-thermal power system under price-making competition. Through approximation of binary variables a (potentially) convex two-stage player problem is obtained. Similar to (Zeng and Zhao, 2013) and (Jiang et al., 2012) the problem is solved distributional robust. Applying a brute-force algorithm allows interpolation of three-dimensional supply functions that can be used to derive a tuple solution which can further be evaluated for its distance to a Nash Equilibrium. Similar to literature (e.g. (Contreras et al., 2004), (Molina et al., 2011)) this is done via applying Nikaido-Isoda functions which are then averaged over the scenarios. The case study and following sensitivity analysis shows promising results that hint for practical application of the proposed framework.

8. APPENDIX

Table 3: Market Clearing Results for the Case Study

	q_t^{i*}	q_{t+1}^{i*}
$i = 1$	$237.96MWh$	$114.52MWh$
$i = 2$	$103.47MWh$	$117.71MWh$
$i = 3$	$0MWh$	$71.39MWh$
$i = 4$	$400MWh$	$400MWh$
$i = 5$	$0MWh$	$320MWh$
$i = 6$	$209.97MWh$	$200MWh$
$i = 7$	$950MWh$	$400MWh$
$i = 8$	$300MWh$	$79MWh$
total:	$2201.39MWh$	$1702.63MWh$
	d_t^*	d_{t+1}^*
	$2201.04MWh$	$1702.54MWh$

Table 4: Discontinuity - Point Results

tuple:	$d_t = 1000, d_{t+1} = 1500$	$d_t = 1000, d_{t+1} = 2000$	$d_t = 1000, d_{t+1} = 2500$
$i = 1$	$192.5MWh$	$0MWh$	$161.02MWh$
$i = 2$	$87.5MWh$	$162.09MWh$	$87.5MWh$
$i = 3$	$120MWh$	$0MWh$	$120MWh$
$i = 4$	$400MWh$	$400MWh$	$400MWh$
$i = 5$	$320MWh$	$320MWh$	$320MWh$
$i = 6$	$200MWh$	$200MWh$	$200MWh$
$i = 7$	$400MWh$	$400MWh$	$400MWh$
$i = 8$	$79MWh$	$79MWh$	$79MWh$
total:	$1799MWh$	$1561.09MWh$	$1767.52MWh$

Table 3 shows the case study quantity clearing results. Table 4 displays the numerical values of the 'gap'/non-convexity in the second period of the analyzed case study.

8.1 Note on the Integer Approximation applied on Column-and-Constraint-Generation

Assumed be a single player holding a single thermal generation unit i that optimizes profits in accordance to the recourse problem (3) under anticipation of two possible scenarios. Considering no demand effects on prices, the simplified Mixed Integer (Bidding) Problem (MIP) can be defined as:

$$\begin{aligned} &\text{for } x = 1, 2 : \\ &\qquad \max_{q^i, b^i_{t+1}} \; p_t q^i_t - c_i(q^i_t) + p^x_{t+1} q^i_{t+1} - c_i(q^i_{t+1}) \\ &\text{subject to} \\ &\qquad q^i_{\min} b^i_t \leq q^i_t \leq q^i_{\max} b^i_t \\ &\qquad q^i_{\min} b^i_{t+1} \leq q^i_{t+1} \leq q^i_{\max} b^i_{t+1} \\ &\qquad q^i_t, q^i_{t+1} \in \mathbb{R}, b^i_{t+1} \in \mathbb{Z}^2 \end{aligned} \tag{13}$$

Assuming the player does not want to estimate scheduling functions as proposed in section 2.1, the two Mixed Integer Problems (13) can be reformulated as four equivalent continuous problems:

$$\text{for } \langle p^x_{t+1}, \hat{b}^i_{t+1}(\xi^x_{t+1})\rangle \in \left[\langle p^1_{t+1}, 0\rangle, \langle p^1_{t+1}, 1\rangle, \langle p^2_{t+1}, 0\rangle, \langle p^2_{t+1}, 1\rangle\right] : \tag{14}$$

$$\begin{aligned} &\max_{q^i} p_t q^i_t - c_i(q^i_t) + p^x_{t+1} q^i_{t+1} - c_i(q^i_{t+1}) \\ &\text{subject to} \\ &\qquad q^i_{\min} b^i_t \leq q^i_t \leq q^i_{\max} b^i_t \\ &\qquad q^i_{\min} \hat{b}^i_{t+1}(\xi^x_{t+1}) \leq q^i_{t+1} \leq q^i_{\max} \hat{b}^i_{t+1}(\xi^x_{t+1}) \\ &\qquad q^i_t, q^i_{t+1} \in \mathbb{R} \end{aligned} \tag{15}$$

Considering a similar approach as shown in section 2.3 results in two different subproblems:

1. problem 13 shows a maximization problem over two scenarios with a nested non-convex MIP problem.
2. problem 14 shows a maximization problem over four scenarios with a nested linear/quadratic/... and potentially convex[3] problem.

It can be assumed that for a larger scale (more units, thus more binary variables in problem 13 and more scenarios in problem 14) problem 14 solves faster as it can be conducted with techniques from linear programming. In addition with the potential of reducing the scenario tree by assuming predetermined schedules as functions of scenarios (as was done in the presented method above), a significant reduction in computation times can be expected by applying this reformulation combined with the method of Column-and-Constraint-Generation.

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REFERENCES

Carolyn A. Berry, Benjamin F. Hobbs, William A. Meroney, Richard P. O'Neill, and William R. Stewart. Understanding how market power can arise in network competition: A game theoretic approach. *Utilities Policy*, 8(3):139–158, 1999. ISSN 09571787. doi: 10.1016/S0957-1787(99)00016-8.

[3] depending on price and cost functions

Benjamin F Hobbs. Linear Complementarity Models of Nash – Cournot Competition in Bilateral and POOLCO Power Markets. *IEEE Transactions on Power Systems*, 16(2):194–202, 2001.

Steven A Gabriel, Antonio J. Conejo, J. David Fuller, Benjamin F. Hobbs, and Carlos Ruiz. *Complementarity Modeling in Energy Markets*. Springer, New York, 2013.

Ove Wolfgang, Arne Haugstad, Birger Mo, Anders Gjelsvik, Ivar Wangensteen, and Gerard Doorman. Hydro reservoir handling in Norway before and after deregulation. *Energy*, 34(10):1642–1651, 2009. ISSN 0360-5442. doi: 10.1016/j.energy.2009.07.025. URL `http://dx.doi.org/10.1016/j.energy.2009.07.025`.

Finn R. Førsund. *Hydropower Economics*. Springer, Oslo, 2 edition, 2015. ISBN 978-1-4899-7518-8. doi: 10.1007/978-1-4899-7519-5.

James Bushnell. A Mixed Complementarity Model of Hydrothermal Electricity Competition in the Western United States. *Operations Research*, 51(1):80–93, 2003. ISSN 0030-364X. doi: 10.1287/opre.51.1.80.12800.

M.V.F. Pereira and L.M.V.G Pinto. Multi-stage stochastic optimization applied to energy planning. *Mathematical Programming*, 52(1-3):359–375, 1991. ISSN 0025-5610. doi: 10.1007/BF01582895. URL `http://link.springer.com/article/10.1007/BF01582895`.

Ivar Wangensteen. *Power system economics the Nordic electricity market*. Tapir academic press, 2012.

Ashkan Rahimi-kian and Hossein Haghighat. Gaming Analysis in Joint Energy and Spinning Reserve Markets. *IEEE Transactions on Power Systems*, 22(January 2015):2074 – 2085, 2007. doi: 10.1109/TPWRS.2007.907389.

Ekaterina Moiseeva and Mohammad Reza Hesamzadeh. Bayesian and Robust Nash Equilibria in Hydro-Dominated Systems under Uncertainty. *IEEE Transactions on Sustainable Energy*, -(-):1–12, 2017. ISSN 1949-3029. doi: 10.1109/TSTE.2017.2762086. URL `http://ieeexplore.ieee.org/document/8064663/`.

Gregory Steeger and Steffen Rebennack. Strategic bidding for multiple price-maker hydroelectric producers. *IIE Transactions (Institute of Industrial Engineers)*, 47(9):1013–1031, 2015. ISSN 15458830. doi: 10.1080/0740817X.2014.1001928.

Juan Pablo Molina, Juan Manuel Zolezzi, Javier Contreras, Hugh Rudnick, and María José Reveco. Nash-Cournot Equilibria in Hydrothermal Electricity Markets. *IEEE Transactions on Power Systems*, 26(3):1089–1101, 2011.

Hukukane Nikaido and Kazuo Isoda. Note on non-cooperative convex game. *Pacific Journal of Mathematics*, 5(5):807–815, 1955.

Jacek B Krawczyk and Stanislav Uryasev. Relaxation algorithms to find Nash equilibria with economic applications. *Environmental Modeling and Assessment*, 5:63–73, 2000.

Javier Contreras, Matthias Klusch, and Jacek B Krawczyk. Numerical Solutions to Nash – Cournot Equilibria in Coupled Constraint Electricity Markets. *IEEE Transactions on Power Systems*, 19(1):195–206, 2004.

Bo Zeng and Long Zhao. Solving two-stage robust optimization problems using a column-and- constraint generation method. *Operations Research Letters*, 41(5):457–461, 2013. ISSN 01676377. doi: 10.1016/j.orl.2013.05.003. URL `http://dx.doi.org/10.1016/j.orl.2013.05.003`.

Narayana Prasad Padhy. Unit commitment - A bibliographical survey. *IEEE Transactions on Power Systems*, 19(2):1196–1205, 2004. ISSN 08858950. doi: 10.1109/TPWRS.2003.821611.

A. B. Philpott, M. Craddock, and H. Waterer. Hydro-electric unit commitment subject to uncertain demand. *European Journal of Operational Research*, 125(2):410–424, 2000. ISSN 03772217. doi: 10.1016/S0377-2217(99)00172-1.

Alvaro Baillo, Mariano Ventosa, Michel Rivier, and Andres Ramos. Optimal Offering Strategies for Generation Companies Operating in Electricity Spot Markets. *IEEE Transactions on Power Systems*, 19(2):745–753, 2004.

SciPy.org. Interpolation of unstructured grid data, 2018. URL `https://docs.scipy.org/doc/scipy/reference/generated/scipy.interpolate.griddata.html`.

Ruiwei Jiang, Jianhui Wang, and Yongpei Guan. Robust Unit Commitment With Wind Power and Pumped Storage Hydro. *IEEE Transactions on Power Systems*, 27(2):800–810, 2012.

7.6 Market Power in Hydro-Thermal Systems with Marginal Cost Bidding

This work has been presented at the *15th International Conference on the European Energy Market.*

7.6.1 Extended Abstract

This paper presents an analysis of the potential to use market power to increase profits in a multi-player game with storage capacities, unit dispatch and marginal cost bidding.

The problem is formulated as a supply function equilibrium problem and the storage capacities are assigned via a stepwise convergence algorithm. Unit commitment schedules for the binary dispatch variables are reformulated as functions of a scheduling index to reduce model complexity.

The result is a four-period case study that analyzes the optimal decision (matrix) of hydro-thermal players. It observes that storage capacities allows for opportunistic tacit collusion between profit/welfare-maximizing hydro-thermal players by strategically changing unit dispatches.

This collusion in turn leads to system welfare losses at the expense of consumers, even though the traditional definition of market power abuse - bidding above marginal cost and withholding generation - are not breached in traditional sense. Instead, supply is withheld indirectly by shutting down units and by not utilizing the full available storage capacities.

The results are higher prices as in the comparison case of a less market power per participating firm, which is established by splitting the hydropower producer in a larger number of smaller producers.

Market Power in Hydro-Thermal Systems with Marginal Cost Bidding

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Abstract

Traditionally, electricity markets have been designed with the intention of disabling producer side market power or prohibiting exercising it. Nonetheless it can be assumed that players participating in pool markets and aiming to maximize their individual benefits might depart from the optimum in terms of total system welfare. To recognize and analyze such behavior, system operators have a wide range of methods available. In the here presented paper, one of those methods - deriving a supply function equilibrium - is used and nested in a traditional discontinuous Nash game. The result is a case study that shows that marginal cost bidding thermal producers have an incentive to collaborate on scheduling in order to cause similar effects to tacit collusion.

Index Terms

hydro power, thermal power, market power, nash equilibrium

I. Introduction

Literature shows various examples in which supply functions were applied to analyze market power in electricity markets. [1] models the effects of consumer pricing schemes on market power in electricity spot markets, concluding that variable end consumer billing has collusion-reducing effects compared to fixed rates. [2] use supply functions individual to each participant to cope with the downsides of Cournot competition, namely impossibility of demand curves without any elasticity and the resulting demand distortion for extreme price scenarios. [3] nests the supply function equilibrium in a *prisoners game*, aiming to explain price spikes created through collusion. It concludes that suppliers have incentives to withhold and selectively place supply in order to increase profits. [4] considers individual welfare maximization as well as an aggregated supply function in order to find market power in an optimal power flow setup and uses a similar stepwise convergence algorithm as [5] to derive a *Nash equilibrium*. [6] establishes affine supply functions that are turned piece-wise by incorporation of capacity constraints in order to analyze various games with different generation unit and firm(i.e. player) setups. The approach presented in the following sections will introduce the coupling of time periods as well as active scheduling decisions to a supply function market clearing. No existing literature on combining those concepts in a single model is known to the authors.

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II. Supply Function Market Clearing Model

Considering a pool market, every clearing period t participating generation units i will submit bids consisting of both price $b^p_{i,t}[\$]$ and quantity $b^q_{i,t}[MW/t]$. The price curves symbolize the *individual supply function* for each generation unit. After receiving the bids, a market operator here assumed to be clearing for a *uniform price* in a *pool market* will create a supply curve, denoting the period supply curve as a $S_t(d_t)[\$]$, a function depending on the periods' demand d_t:

$$\begin{aligned} S_t(d_t) = \quad & \min_{q_{i,t}} \sum_i b^p_{i,t}(q_{i,t})q_{i,t} \\ \text{s.t.} \quad & q_{i,t} \leq b^q_{i,t} \qquad \forall i \\ & d_t = \sum_i q_{i,t} \\ & q_{i,t} \in \mathbb{R}^+ \qquad \forall i \end{aligned} \tag{1}$$

As long as the objective function is convex and the quantity bids $b^q_{i,t}$ are enough to fulfill the demand clearing (i.e. $\sum\limits_i b^q_{i,t} \geq d_t \forall t$) constraint, this *market supply curve* will yield a finite global result. Assuming quadratic cost functions $C_i(q_{i,t})$ and generators bidding at *marginal cost* ($MC_i(q_{i,t})$) level yields the following formulation for the price bidding function:

$$\begin{aligned} C_i(q_{i,t}) = a_i + b_i q_{i,t} + c_i q^2_{i,t} \\ b^p_{i,t}(q_{i,t}) = MC_i(q_{i,t}) = \tfrac{\partial C_i(q_{i,t})}{\partial q_{i,t}} = b_i + 2c_i q_{i,t} \end{aligned} \tag{2}$$

These price bids as well as the size of the offered quantity are assumed to be a result of an interal optimization process of each player j which owns a number of generation units. For simplicity's sake we set the number of potential bids per player equal to the number of generation units $i \in j$. It will also be assumed, that these bidding curves $b^q_{i,t}(q_{i,t})$ are predefined and will not be altered depending on factors such as amount of players and will stay time-consistent disregarding external influences such as the fuel prices. As the analyzed time frame will be of short term, this approximation can be considered valid.

Furthermore, affine demand functions are assumed:

$$D_t(d_t) = \alpha_t - \beta_t d_t \tag{3}$$

Generally, demand in electricity markets offers low elasticity, requiring this curve to be sufficiently steep.

The market clearing price p^*_t can be formulated as the intersection of demand and supply curves at a clearing quantity d^*_t:

$$p^*_t = D_t(d^*_t) = S_t(d^*_t) \tag{4}$$

As the here presented model follows a *pool based auction with uniform pricing*, every player will have its supplied quantity $\sum\limits_{i \in j} q_{i,t}$ remunerated at this market clearing price.

III. Hydro-Thermal Model

Storage technology allows binding in the dimension of time, giving storage facility operators(in the here presented case: hydro power plants $i \in I^{\text{hy}}$) the chance to transfer quantity $r_{i,t}$ from a time stage t to the next. In addition, due to maximum uptimes and cooling down periods, thermal plants can be considered to not operate during the

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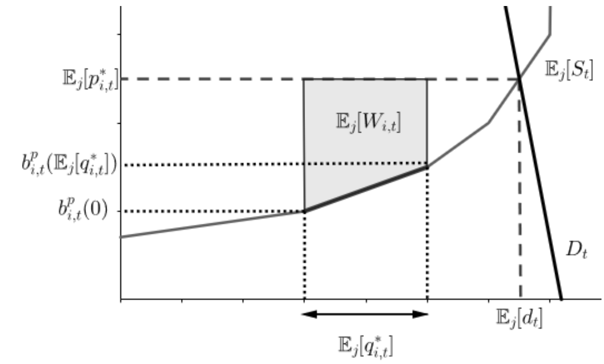

Fig. 1. Generator Surplus for Unit $i \in I_j$

entire duration $t = 1, ..., T$. In a stateless 1-period model scheduling decisions would be implemented through altering the pool of generation units $i \in I^{\text{th}}$. In the case of storage however, time periods are connected and thus scheduling variables $s^{n_i}_{i,t}$ that determine the on/off-states ($b^q_{i,t}$/0) of the thermal plants are required to ensure fluctuating player pools over the time frame. This results in the following formulation for the *hydro-thermal supply function*:

$$
\begin{aligned}
S_t(d_t, r_{i,t}, s^{n_i}_{i,t}) = \min_{q_{i,t}} \sum_i & b^p_{i,t}(q_{i,t}) q_{i,t} \\
\text{s.t. } q_{i,t} \leq b^q_{i,t} & + r_{i,t-1} - r_{i,t} && \forall i \in I^{\text{hy}} \\
q_{i,t} \leq & s^{n_i}_{i,t} && \forall i \in I^{\text{th}} \\
d_t = & \sum_i q_{i,t} \\
q_{i,t} \in & \mathbb{R}^+ && \forall i
\end{aligned}
\tag{5}
$$

A player j would aim to maximize its supply side surplus W_i as seen in figure 1 for all of its units $i \in I_j$. This requires initial assumptions on market clearing price ($\mathbb{E}_j[p^*_t]$) and on procured quantity ($\mathbb{E}_j[q^*_{i,t}]$). Every producer would then alter the stored hydropower inventory $r_{i,t}[MW/t]$ indirectly (through adjusting generation) for all of its storage units in addition to choosing a production schedule $[s^{n_i}_{i,1}, ..., s^{n_i}_{i,T}]$ for its thermal units. It can be considered that the player defines a limited set of predefined schedules s which leads to a reformulation in the form of:

$$
\begin{aligned}
s = & \begin{bmatrix} \begin{bmatrix} [s^1_{1,1}, ..., s^1_{1,T}] \\ ... \\ [s^1_{I^{\text{th}},1}, ..., s^1_{I^{\text{th}},T}] \end{bmatrix} \\ ... \\ \begin{bmatrix} [s^{N_i}_{1,1}, ..., s^{N_i}_{1,T}] \\ ... \\ [s^{N_i}_{I^{\text{th}},1}, ..., s^{N_i}_{I^{\text{th}},T}] \end{bmatrix} \end{bmatrix} \\
s^{n_i}_{i,t} = & \{0, b^q_{i,t}\} \qquad \forall i \in I^{\text{th}}, t
\end{aligned}
\tag{6}
$$

Scheduling of thermal units is thereby conducted through making index n_i, which refers to the prospective schedule of generation unit i, a decision variable and thus

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reformulating the players' previous decision problem over the whole time stage as:

$$
\begin{array}{rll}
& W_j = \max\limits_{r_{i,t},n_i} \sum\limits_{t} \sum\limits_{i \in j} \mathbb{E}_j[W_{i,t}] & \\
& \mathbb{E}_j[W_{i,t}] = \begin{array}{l} \left(\mathbb{E}_j[p^*_t] - b^p_{i,t}(\mathbb{E}_j[q^*_{i,t}])\right) \cdot \mathbb{E}_j[q^*_{i,t}] \\ + \frac{b^p_{i,t}(\mathbb{E}_j[q^*_{i,t}]) - b^p_{i,t}(0)}{2} \cdot \mathbb{E}_j[q^*_{i,t}] \end{array} & \forall i \in I_j, t \\
\text{s.t.} & S_t(d_t, r_{i,t}, s^{n_i}_{i,t}) = S_t(d^*_t, r_{i,t}, s^{n_i}_{i,t}) & \forall t \\
& r_{i,t} \leq \bar{r}_i & \forall i \in \{I^{\text{hy}} \cap I_j\}, t \\
& r_{i,t} \in \mathbb{R}^+ & \forall i \in \{I^{\text{hy}} \cap I_j\}, t \\
& n_i \in \mathbb{Z}^+ & \forall i \in \{I^{\text{th}} \cap I_j\}, t
\end{array}
\tag{7}
$$

Reservoir storage from one period to the next has an imposed upper capacity of $\bar{r}_i[MW]$.

Defining the set of strategies that a player chooses depending on the assumption on other players strategies as $\langle r_{i,t}, n_i \forall i \in I_j; \mathbb{E}[r_{i,t}], \mathbb{E}[n_i] \forall i \notin I_j \rangle$, the resulting *Nash-Equilibrium* can be formulated as:

$$
W_j = \max_{\langle r^*_{i,t}, n^*_i \forall i \in I_{j_2}; \mathbb{E}[r_{i,t}], \mathbb{E}[n_i] \forall i \notin I_{j_2} \rangle} W_{j_2} \quad \forall j, j_2 \neq j \tag{8}
$$

In words, a Nash equilibrium is reached if no player has an incentive to store more/less water or change the schedule for the thermal units.

The assumption of complete and symmetric information gives the possibility to solve for this equilibrium:

$$
\begin{array}{ll}
p^*_t = \mathbb{E}_j[p^*_t] = \mathbb{E}_{j_2}[p^*_t] & \forall j, j_2 \neq j \\
q^*_{i,t} = \mathbb{E}_j[q^*_{i,t}] = \mathbb{E}_{j_2}[q^*_{i,t}] & \forall j, j_2 \neq j
\end{array}
\tag{9}
$$

Symmetric information in this case refers to the players having knowledge of each others individual supply functions/bidding curves. As mentioned above, marginal cost bidding was assumed and due to the availability of a large range of historical data within power markets, player knowledge about competitors' marginal cost functions can be considered valid.

Assuming a fixed schedule for all thermal units allows this welfare game to be solved for its equilibrium point. Market clearing condition (4) presents a sub-problem in the welfare-maximization of every player. Taking advantage of the fact that every producer has an incentive to shift generation from time step $t-1$ to t as long as both the capacity constraint for maximum reservoir inventory is not breached and there are generator side surplus gains to be made for this producer, time stage transfer of inventory (i.e. arbitrage) will happen. This can be solved by selecting an adequately small step size r^{step} and applying a convergence algorithm:

0) `initialize` $r^{\text{original}} = \begin{bmatrix} [0,..,0] \\ ... \\ [0,..,0] \end{bmatrix}$
1) `calculate` (4) $\forall t$ `to establish sorted set`
   $\tau = \{\tau_1, ..., \tau_T | p^*_{\tau_1} \geq ... \geq p^*_{\tau_T}\}$ `where` $\tau_t \in \{1, ..., T\}$ `and` $\tau_1 \neq ... \neq \tau_T$
2) `remove` $\max(\tau_t \in \tau)$
3) `if last element in` τ `is equal to` $\max(\tau)$ `:`
   `remove it and` **`back to 3)`**
4) `if` $\tau = \{\emptyset\}$ **`: finished`** `-`
   `i.e. converged to equilibrium as shown in` (8)
5) `select first element` τ_t `from` τ

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TABLE I
GAME SETUP

Generator j	i	type	$b^q_{i,t}\forall t$	$b^p_{i,t}(q_{i,t})\forall t$
1	1	hydro	$4MW/t$	$0+0\cdot 2\cdot q_{i,t}$
	2	hydro	$4MW/t$	$0+0\cdot 2\cdot q_{i,t}$
2	3	hydro	$4MW/t$	$0+0\cdot 2\cdot q_{i,t}$
	4	thermal	$5MW/t$	$1+0.02\cdot 2\cdot q_{i,t}$
3	5	thermal	$5MW/t$	$1.5+0.02\cdot 2\cdot q_{i,t}$
	6	thermal	$8MW/t$	$3+0.02\cdot 2\cdot q_{i,t}$

period t	Customer Demand
1	$D_t(d_t) = 150000 - 10000\cdot d_t$
2	$D_t(d_t) = 160000 - 10000\cdot d_t$
3	$D_t(d_t) = 180000 - 10000\cdot d_t$
4	$D_t(d_t) = 170000 - 10000\cdot d_t$

6) `solve objective function of` (7) `to receive:`
   W_j^{original} `for` r^{original} `and`
   W_j^{new} `for`

$$r_j^{\text{new}} = \begin{bmatrix} r_{i,t}^{\text{original}} & \forall i \in \{I^{\text{hy}} \cap I_j\}, t \neq \tau_t, \\ r_{i,t}^{\text{original}} & \forall i \in \{I^{\text{hy}} \cup I_j\}, t \neq \tau_t, \\ \min\{r_{i,t}^{\text{original}} + r^{\text{step}}, \bar{r}_i\} & \forall i \in \{I^{\text{hy}} \cup I_j\}, t = \tau_t \end{bmatrix}$$

7) `for each` j `where` $W_j^{\text{new}} > W_j^{\text{original}}$ `:`
   `set` $r_{i,t}^{\text{original}} := r_{j,i,t}^{\text{new}} \forall i \in \{I^{\text{hy}} \cup I_j\}$
   `if` r^{original} `is unchanged: remove` τ_t `from` τ `and` **`back to 4)`**
8) `and` **`back to 1)`**

The algorithm increases the held inventory continuously by step size r^{step} until no player can increase their individual welfare by increasing the inventory, which corresponds with the definition of a Nash equilibrium. Similar convergence algorithms were used to derive equilibrium points in bidding problems for power-flow based problems [4] and within hydro-storage convergence algorithms [5]. As the here presented algorithm solves the market clearing problem via supply curve matching, the convergence algorithm is solely concerned with matching the held inventory.

As mentioned before, this can be only conducted by assuming fixed n_i for all players. Thus, the algorithm has to be brute-forced for all possible iterations that the schedules allow, in other words for all combinations of $s_i^{n_i}$ that s allows.

IV. CASE STUDY

The presented cases will analyze the game setup shown in table I: six generation units owned by three players (one hydro player, one thermal player, one mixed) compete for a (nearly) inelastic market demand ranging between 16 to $20MW/t$ depending on the period. Such demand curve shifts can stem from a variety of factors such as fluctuating base-load caused by renewable generation or consumer behavior.

Case 1, as shown in table II proposes the base case with no available storage capacity and all thermal units running continuously in every period.

The results found in figure 2 show the impact of a lack of period price balancing effects that holding inventory provides: producers owning storage units would have an incentive to increase their surplus by shifting generation into successive periods and

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TABLE II
IMPLEMENTED CASES

case #1	$\bar{r_1} = 0$ $\bar{r_2} = 0$ $\bar{r_3} = 0$	$s_4 = [[1, 1, 1, 1]]$ $s_5 = [[1, 1, 1, 1]]$ $s_6 = [[1, 1, 1, 1]]$
case #2	$\bar{r_1} = 0$ $\bar{r_2} = 0$ $\bar{r_3} = 0$	$s_4 = \begin{bmatrix} s_4^1 = [1, 1, 1, 1] \\ s_4^2 = [1, 1, \mathbf{0}, \mathbf{0}] \end{bmatrix}$ $s_5 = \begin{bmatrix} s_5^1 = [1, 1, 1, 1] \\ s_5^2 = [1, 1, \mathbf{0}, \mathbf{0}] \end{bmatrix}$ $s_6 = [[1, 1, 1, 1]]$
case #3	$\bar{r_1} = 1.5$ $\bar{r_2} = 1.5$ $\bar{r_3} = 1.5$	$s_4 = \begin{bmatrix} s_4^1 = [1, 1, 1, 1] \\ s_4^2 = [1, 1, \mathbf{0}, \mathbf{0}] \end{bmatrix}$ $s_5 = \begin{bmatrix} s_5^1 = [1, 1, 1, 1] \\ s_5^2 = [1, 1, \mathbf{0}, \mathbf{0}] \end{bmatrix}$ $s_6 = [[1, 1, 1, 1]]$

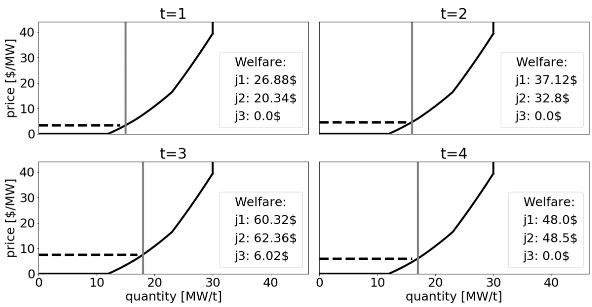

Fig. 2. Result Case 1

thus skimming the price peaks. As there is no storage capacity, high deviations in price are a result of the demand changes.

Case 2 assumes the possibility of schedule changes - the two thermal units $i = 4, 5$ might be shut down in period 3 by choosing another schedule (i.e. setting $n_i = 2$ respectively). This can result in 4 different states, depending on which schedule is chosen by which player.

Those states are shown in form of a decision matrix in table III. This shows that individually, player $j = 2$ has no incentive to shut unit $i = 4$ down, combined with the thermal player committing to a shutdown, both are however able to increase their surplus. Those effects are displayed in figure 3. Thus, curtailing available supply and

TABLE III
DECISION MATRIX CASE 2

		$j = 3$	
		$n_5 = 1$	$n_5 = 2$
$j = 2$	$n_4 = 1$	$W_1 = 172.32$ $W_2 = 164.0$ $W_3 = 6.02$	$W_1 = 184.32$ $W_2 = 177.5$ $W_3 = 6.02$
	$n_4 = 2$	$W_1 = 215.52$ $W_2 = 128.9$ $W_3 = 87.42$	$W_1 = 347.52$ $W_2 = 194.9$ $W_3 = 162.42$

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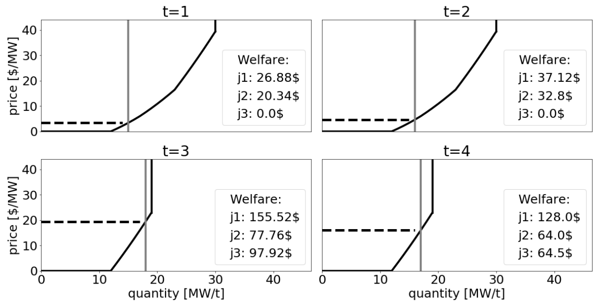

Fig. 3. Result Case 2, with $n_4^* = 2, n_5^* = 2$

TABLE IV
DECISION MATRIX CASE 3

		$j = 3$	
		$n_5 = 1$	$n_5 = 2$
$j = 2$	$n_4 = 1$	$W_1 = 169.556$ $W_2 = 159.469$ $W_3 = 2.625$	$W_1 = 178.775$ $W_2 = 175.19$ $W_3 = 12.04$
	$n_4 = 2$	$W_1 = 208.806$ $W_2 = 141.872$ $W_3 = 62.349$	$W_1 = 322.882$ $W_2 = 209.049$ $W_3 = 117.546$

thus increasing the price to maximize individual producer surplus can be enabled through cooperation, resulting in the equilibrium defined by $n_4^* = 2, n_5^* = 2$. This phenomenon referred to as *opportunistic tacit collusion* by [1] and [3] seems to be amplified by collaboration amongst colluding players. As [7] illustrates - a supply side increase in surplus would come at amplified expenses on the demand side, resulting in a loss of total welfare. Thus, avoiding such coordination is in the interest of a welfare maximizing system operator. However, the result of case 1 - $n_4^* = 1, n_5^* = 1$ - still fulfills the definition of the Nash equilibrium shown in (8). Thus, the standard case with no shutdowns still offers an equilibrium situation, that could similarly be achieved due to the indifference of player $j = 3$ between shutdown or not in case player $j = 2$ chooses no shutdown.

Case 3 removes this indifference by adding hydro storage capacity to the respective units. The resulting scheduling decision matrix is shown in figure IV. There now only exists a single Nash equilibrium where both players owning thermal units collaborate in order to increase supply side surplus. Total transfers from period to period accumulate to $0.9MW, 1.5MW, 1MW$ respectively. The reason for this (seemingly) low transfer is the small pool of players and the resulting high impact of transfer decisions. Players will aim to maximize their relation of $\sum_{i \in I_j} q_{i,t}^*$ to $\frac{\partial S_t(d_t)}{\partial d_t}$ (Appendix B analyzes this). In the specific case, the hydro power operator does not have a strong incentive for peak skimming as it would hurt this balance, especially since the mixed hydro-thermal operator actively shifts units (with full capacity of $1.5MW$) from period 2 into period 3, to actively create peaks. Appendix A introduces the game from case 3, all hydro power units taken from their respective firms and distributed equally amongst a larger

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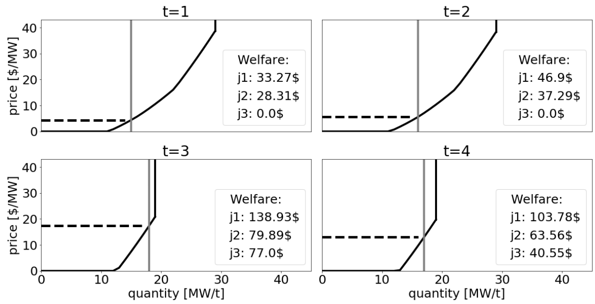

Fig. 4. Result Case 3, with $n_4^* = 2, n_5^* = 2$

pool of smaller firms. It clearly shows a loss of the ”strategic value” of hydro power for the mixed generator.

V. Conclusion

This paper presented a novel aspect of electrical systems: the possibility of exercising market power whilst bidding marginal through colluding on unit commitment. A supply function was applied on a hydro-thermal system spanning over several time periods and individual player (generation firm) surpluses were calculated. Two forms of exercise of market power were noted in thermal/hydro-thermal operators: tacit collusion through alignment of schedules of thermal units, creating price peaks through withholding low cost units; and active creation of price peaks through strategic supply shifts over periods. It has to be noted, however, that the here presented concepts of influencing the market require a large impact of single generation firms. It shows however, that monopolists are still able to bid at marginal cost, which is traditionally considered as system welfare-maximizing, and still exercise market power, resulting in welfare-losses on customer side. This however requires information on scheduling decisions of other (thermal) players. As shared knowledge of the game participants’ schedules might not be complete, future research on this topic is proposed.

Acknowledgment

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References

[1] F. Bolle, “Supply function equilibria and the danger of tacit collusion. The case of spot markets for electricity,” *Energy Economics*, vol. 14, no. 2, pp. 94–102, 1992.

[2] C. J. Day, B. F. Hobbs, and J.-S. Pang, “Oligopolistic Competition in Power Networks: A Conjectured Supply Function Approach,” 2002.

[3] X. Guan, Y. C. Ho, and D. L. Pepyne, “Gaming and price spikes in electric power markets,” *IEEE Transactions on Power Systems*, vol. 16, no. 3, pp. 402–408, 2001.

[4] J. D. Weber, T. J. Overbye, and S. Member, “An Individual Welfare Maximation Algorithm for Electricity Markets,” *IEEE Transactions on Power Systems*, vol. 17, no. 3, pp. 590 – 596, 2002.

[5] J. P. Molina, J. M. Zolezzi, J. Contreras, H. Rudnick, and M. J. Reveco, “Nash-Cournot Equilibria in Hydrothermal Electricity Markets,” *IEEE Transactions on Power Systems*, vol. 26, no. 3, pp. 1089–1101, 2011.

DOI: 10.1109/EEM.2018.8469984

[6] R. Baldick, R. Grant, and E. Kahn, "Theory and application of linear supply function equilibrium in electricity markets," *Journal of Regulatory Economics*, vol. 25, no. 2, pp. 143–167, 2004. [Online]. Available: isi:000188212700002

[7] R. Sioshansi, P. Denholm, T. Jenkin, and J. Weiss, "Estimating the value of electricity storage in PJM: Arbitrage and some welfare effects," *Energy Economics*, vol. 31, no. 2, pp. 269–277, 2009. [Online]. Available: http://dx.doi.org/10.1016/j.eneco.2008.10.005

APPENDIX A

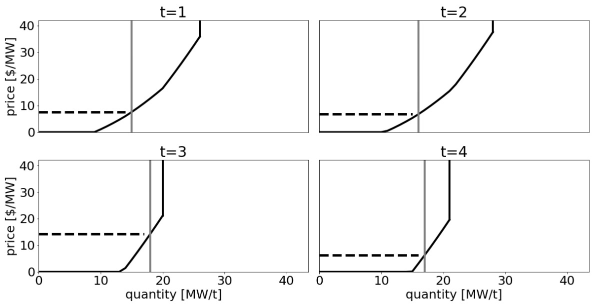

Fig. 5. Result Case 3, with 52 players

Figure 5 shows the results from figure 4 where the hydro power units are taken from the participants and split up into 50 firms with equal capacities. All of the smaller players thus inherit less market power, resulting in them bidding more competitively with reservoir transfers of $4MW, 4.5MW, 4MW$.

APPENDIX B

Assuming a hydro power producer considers shifting $r_{i,t}MW$ from period t to $t+1$, the resulting period surplus changes for unit i are:

$$\begin{aligned} \Delta W_{i,t} = & \frac{\partial S_t(b^q_{i,t} - r_{i,t}, \sum_{i_2 \neq i} b^q_{i_2,t})}{\partial d_t} q_{i,t}(b^q_{i,t} - r_{i,t}) \\ \Delta W_{i,t+1} = & \frac{\partial S_{t+1}(b^q_{i,t+1} - r_{i,t+1}, \sum_{i_2 \neq i} b^q_{i_2,t})}{\partial d_t} q_{i,t+1}(b^q_{i,t+1} + r_{i,t+1}) \end{aligned} \tag{10}$$

Further considering all of the transferred capacity will be acquired by the market results in:

$$\begin{aligned} \Delta W_{i,t} = & -\frac{\partial S_t(b^q_{i,t} - r_{i,t}, \sum_{i_2 \neq i} b^q_{i_2,t})}{\partial d_t} r_{i,t} \\ \Delta W_{i,t+1} = & +\frac{\partial S_{t+1}(b^q_{i,t+1} - r_{i,t+1}, \sum_{i_2 \neq i} b^q_{i_2,t})}{\partial d_t} r_{i,t+1} \end{aligned} \tag{11}$$

Thus, in case there is sufficient demand for the shifted capacity, only the resulting slopes of the demand functions in relation to the chosen step size will define if a period shift is conducted. Thus, players generally do not have an incentive to shift capacity if the steepness in period $t+1$ does not outweight period t.

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7.7 Impact of Inertial Response Requirements on a Multi Area Renewable Network

This work has been presented at the *12th IEEE PES PowerTech Conference*.

7.7.1 Extended Abstract

This paper analyzes the impact of transfer lines capable of sharing inertial response on unit commitment decisions. Three types of generation - thermal, wind and hydropower - are considered operating over a limited number of periods in two areas connected by a transfer line.

The presented model considers additional constraints on binary variables such as minimum up-times and down-times. Optimal dispatch is conducted via a cutting plane algorithm.

Further, it discusses the issues specific to determining a 'monetary value of inertia'. Due to no differentiability of functions containing integer variables, there can be no clear marginal cost calculation that would yield a price for inertia. However, the presented analysis of the stepwise structure of provision of inertial response might support future decisions on market structure for such ancillary services.

Impact of Inertial Response Requirements on a Multi Area Renewable Network

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Abstract

Developments in renewable integration are continuously changing power system portfolios globally. Higher volatility of the networks might pose a threat to grid stability and thus increase the need for ancillary services. In this paper one such service - the provision of inertial frequency response (in short referred to as *inertia*) - is analyzed. An additional demand constraint is added to a SMIP (Stochastic Mixed Integer Problem) formulation of an interconnected two-area system consisting of wind, hydro and conventional thermal plants. Environmental stochastic influences - wind curtailment and hydrological inflow - as well as demand fluctuation, forecasting errors and inter-area congestion are incorporated. The potential of cross-border trade of inertial response such as the impact of inertia requirements on traditional scheduling is analyzed and discussed.

Index Terms

Inertia, MIP, Stochastic Programming, Generation Scheduling, Ancillary Service, Cutting Plane

I. Introduction

Growing integration of 'green' generation into power grids lead to an increase in demand for ancillary and balancing services. Those services are a necessity to stabilize grids with high shares of renewable energy, a form of generation more prone to deviation [1]. One factor describing grid quality - in terms of stability - is the reaction time to frequency deviations, also referred to as inertial (frequency) response or *'inertia'* [2], [3]. A wide range of research on this topic has been carried out over recent years, examples include the impact of frequency control on market dispatch [4] or the impact of wind power integration on grid stability [5]. However, no models to quantify the cost impact on a power system have been analyzed, which this paper aims to provide in a novel approach. The chosen method was a scheduling model, a method with long history, initially used in deterministic single unit systems [6] and recently focused on stochastic influences such as presented in [7], [8] and [9] for hydro-thermal, [10] for wind-hydro or [11] for wind-hydro. This paper aims to gather the ideas proposed here and add various components to show the impact of inertial response.

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NOMENCLATURE

$i = 1, ..., I$	wind units
$j = 1, ..., J$	hydro units
$k = 1, ..., K$	conventional (thermal) units
$t = 1, ..., T$	time period
$a = [1, 2]$	areas
$P = \{i, j, k\}$	total available plants
$P_1 = \{i, k\}$	available plants in area 1
$o_{P,t} \in \{0, 1\}$	plant state: off/on
$p_{P,t} \in \mathbb{R}^+$	unit output level
$r_{j,t} \in \mathbb{R}^+$	hydro inventory
$F_t \in \mathbb{R}$	(bidirectional) area transfer flow
τ	current time period
$p_h^{min/max}$, $p_k^{min/max}$	min/max output level hydro and thermal
p_P^n	nominal power plant output
p_i^{min}, $p_{i,t}^{max}$	min/max output level wind
r_j^{max}	hydro reservoir size
$e_{j,t}$	hydro reservoir inflow
$\theta_{k,t}$	current thermal runtime
θ_k^{max}	max thermal uptime (consecutive)
θ_k^{min}	min thermal downtime (consecutive)
α_P, β_P, γ_{P_1}	cost factors
$c_k^{up/down}$	thermal start/stop cost
$D_{a,t}^p$	electricity demand
C_t^F	flow capacity
H_P	inertia constant
D_a^H	inertia demand
Λ_t	hydro inventory coefficient
$W(t)$	social welfare function
$O_{t,k}^{up}$	minimum amount of starts over total time period T
$O_{t,k}^{downt}$	minimum amount of downtime over total time period T
$\mathbf{E}[D_{a,t}^p]$	electricity demand forecast
$\mathbf{E}[C_t^F]$	expected flow capacity
$\mathbf{E}[p_{i,t}^{max}]$	wind curtailment forecast
$\mathbf{E}[e_{j,t}]$	reservoir inflow forecast

II. MODEL FORMULATION

In the here presented model, the plants participating in the scheduling will be separated into two adjacent areas by a flow constraint. Fig. 1 illustrates this setup: I wind power plants and K conventional (thermal) power plants meet in a node, from here on referred to as *area*, and are connected by a transfer line to a second area with an array of J hydro power plants. The chosen representation for the reservoirs of the hydro power plants was the aggregated single reservoir form [12], [13]. This allowed to neglect interactions between the hydro facilities (one might think of outflow influencing

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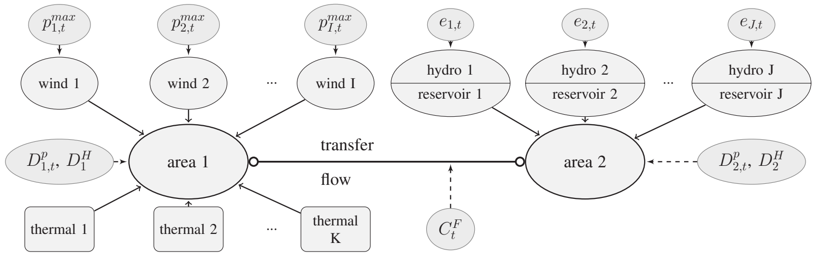
Fig. 1. Model Setup



the inflow of others) as well as different reservoir constellations which would otherwise increase the complexity of the model unnecessarily[1].

Objective function:

$$\begin{aligned}
&\underset{o_{i/j/k,t}\ p_{i/j/k,t}\ r_{j,t}\ F_t}{\text{minimize}} W(t) = \sum_{t=\tau}^{T} -\Lambda_t \sum_{j=1}^{J} r_{j,t} \\
&+ \sum_{t=\tau}^{T} \sum_{i=1}^{I} \alpha_i o_{i,t} + \beta_i p_{i,t} \\
&+ \sum_{t=\tau}^{T} \sum_{j=1}^{J} \alpha_j o_{j,t} + \beta_j p_{j,t} + \gamma_j p_{j,t}^2 \\
&+ \sum_{t=\tau}^{T} \sum_{k=1}^{K} \alpha_k o_{k,t} + \beta_k p_{k,t} + \gamma_j p_{k,t}^2 \\
&+ \sum_{t=\tau}^{T} \sum_{k=1}^{K} o_{k,t}(1 - o_{k,t+1}) c_k^{down} \\
&+ \sum_{t=\tau}^{T} \sum_{k=1}^{K} o_{k,t}(1 - o_{k,t-1}) c_k^{up}
\end{aligned} \tag{1}$$

The chosen goal was to maximize welfare by fulfilling the inelastic demand in both areas. The objective function is shown in (1), it consists of the targets to minimize total cost (wind farms have linear cost curves, thermal and hydro quadratic curves) and maximimize reservoir inventory. Starting costs for conventional plants were included, but the low extend of those factors for hydro and wind power plants led to them not being included in the model. The initial plant states were considered as off ($o_{i/j/k,0} = 0$). As shown in [12], to incentivize saving water in hydro power plants, the value of the reservoir inventory (the so-called 'water value' Λ) can be assumed to have an increasing value over time. As shown in [12], the monetary difference in reservoir inventory ('water value' Λ_t) has to fulfill the condition $\Lambda_t < \Lambda_{t+1}$ in each period[2]. However, as the focus of this paper did not lie on determining this value, it was set to a static, very minor number($\sum_t \Lambda_t \to 0$). The aim of the here presented model was to schedule the

[1]pumping was not included in the here presented model

[2]other cases like the risk of spilling in high inflow scenarios which were chosen to be neglected in this model

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generators in every single period t starting from the current period τ until the final period T, under consideration of the already committed resources in $t < \tau$ such as the reservoir levels and the total thermal uptime.

$$\begin{aligned} D^p_{1,t} &\leq \sum_{i=1}^{I} p_{i,t} + \sum_{k=1}^{K} p_{k,t} + F_t \ \ \forall t = \tau \\ \mathbf{E}[D^p_{1,t}] &\leq \sum_{i=1}^{I} p_{i,t} + \sum_{k=1}^{K} p_{k,t} + F_t \ \ \forall t = \tau + 1, ..., T \end{aligned} \tag{2}$$

(2) and (3) shows the electricity demand fulfillment constraints. The flow in between was restricted through the time variable flow capacity constraint presented in (4)[3].

$$\begin{aligned} D^p_{2,t} &\leq \sum_{j=1}^{J} p_{j,t} - F_t \ \ \forall t = \tau \\ \mathbf{E}[D^p_{2,t}] &\leq \sum_{j=1}^{J} p_{j,t} - F_t \ \ \forall t = \tau + 1, ..., T \end{aligned} \tag{3}$$

$$\begin{aligned} -C^F_t \leq F_t &\leq C^F_t \ \ \forall t = \tau \\ -\mathbf{E}[C^F_t] \leq F_t &\leq \mathbf{E}[C^F_t] \ \ \forall t = \tau + 1, ..., T \end{aligned} \tag{4}$$

$$\begin{aligned} &\sum_{i=1}^{I} o_{i,t} \times p^n_i \times H_i + \sum_{j=1}^{J} o_{j,t} \times p^n_j \times H_j \\ &+ \sum_{k=1}^{K} o_{k,t} \times p^n_k \times H_k \geq D^H \ \ \forall t = \tau, ..., T \end{aligned} \tag{5a}$$

in case the inertial response cannot be shared:

$$\begin{aligned} &\sum_{i=1}^{I} o_{i,t} \times p^n_i \times H_i \\ &+ \sum_{k=1}^{K} o_{k,t} \times p^n_k \times H_k \geq D^H_1 \ \ \forall t = \tau, ..., T \\ &\sum_{j=1}^{J} o_{j,t} \times p^n_j \times H_j \geq D^H_2 \ \ \forall t = \tau, ..., T \end{aligned} \tag{5b}$$

In addition to the demand fulfillment, the market areas also impose inertia requirements on the plant schedules, as shown in (5). The calculation method was derived from the formulation of total system inertia in [14]. Both cases of shared eqrefeq:inertiademandA and separated fulfillment (5b) can be studied with the model. This separation solely focuses on catering to an inertial response requirement in an AC network, demand fulfillment is considered pooled in any of the presented cases. Inertia was considered

[3] It has to be noted that the flow capacity has a negative lower bound. In solvers unable to compute this, two variables with opposing directions achieve a similar outcome in summation.

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to be able to be provided by all involved means of generation, in case of wind plants through additional curtailment [5].

$$\begin{aligned} o_{i,t} \times p_i^{min} &\leq p_{i,t} \\ p_{i,t} &\leq o_{i,t} \times p_{i,t}^{max} \end{aligned} \quad \forall i = 1, ..., I;\ t = \tau$$
$$\begin{aligned} o_{i,t} \times p_i^{min} &\leq p_{i,t} \\ p_{i,t} &\leq o_{i,t} \times \mathbf{E}[p_{i,t}^{max}] \end{aligned} \quad \forall i = 1, ..., I;\ t = \tau + 1, ..., T \tag{6}$$

(6) realizes the wind capacity constraints, whereas the maximum possible output is variable over time. The reason lies in the curtailment of wind.

$$\begin{aligned} o_{j,t} \times p_j^{min} &\leq p_{j,t} \\ p_{j,t} &\leq o_{j,t} \times p_j^{max} \end{aligned} \quad \forall j = 1, ..., J;\ t = \tau, ..., T \tag{7}$$

$$\begin{aligned} o_{k,t} \times p_k^{min} &\leq p_{k,t} \\ p_{k,t} &\leq o_{k,t} \times p_k^{max} \end{aligned} \quad \forall k = 1, ..., K;\ t = \tau, ..., T \tag{8}$$

(7) and (8) show the capacity constraints for the other plant types.

$$\begin{aligned} \theta_{k,t} \geq o_{k,t} + o_{k,t} \times \theta_{k,t-1} &\ \ \forall k = 1, ..., K;\ t = 1, ..., T \\ 0 \leq \theta_{k,t} \leq \theta_k^{max} &\ \ \forall k = 1, ..., K;\ t = 1, ..., T \end{aligned} \tag{9}$$

As mentioned above, there has to be a maximum thermal uptime imposed, realized through (9). This constraint is based on the potential of overheating of units, which has to be avoided through forcing the units to stop to cool off. It has to be mentioned that the nested variable $\theta_{k,t-1}$ in itself is limited by $o_{k,t-1} + o_{k,t-1} \times \theta_{k,t-2}$ with the emerging row of constraints proceeding until $o_{k,0} = 0$.

$$\theta_{k,t} = \sum_{y=1}^{T} \prod_{t=y}^{T} o_{k,t} \ \ \forall k = 1, ..., K \tag{10}$$

The series notation shown in (10) demonstrates that the constraint is of quadratic nature and thus would transform the existing problem from a MIP to a MIQCP (Mixed Integer Quadratic Constrained Problem), thus result in avoidable complications (as described in [15]) - as the conventional plants constitute the most expensive plants and thus will be run in support (as a peak load plant) to the other means of production, thus in most cases already fulfilling the required downtime constraint without the need to have it imposed strictly [1]. Thus a cutting plane algorithm, as shown in Fig. 2 and the necessary dynamic capacity constraint (11) was determined a fitting solution and introduced to the model to enable it to deal with and decrease the level of complexity.

$$\sum_{t=\tau}^{T} o_{k,t} \times (1 - o_{k,t-1}) \geq O_{\tau,k}^{up} \ \ \forall k = 1, ..., K \tag{11}$$

In a similar manner, the minimum downtime restriction (12) - required to give the thermal units the necessary time to cool down - had to be implemented.

$$\begin{gathered} \sum_{t_2=0}^{\theta_k^{min}-1} o_{k,t+t_2} \leq 0 \ \ \forall t = 3, ..., T - \theta_k^{min} \\ \text{where } o_{k,t-2} \times (1 - o_{k,t-1}) = 1 \end{gathered} \tag{12}$$

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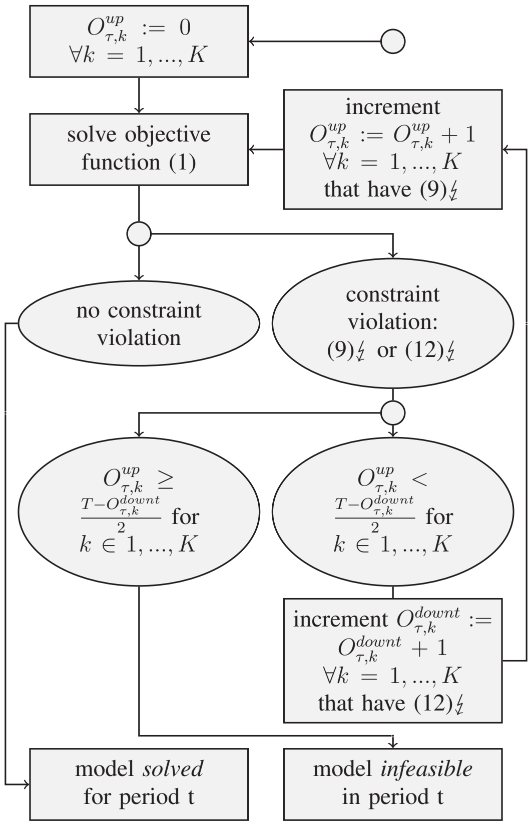
Fig. 2. Cutting Plane Algorithm for Time Period t



This was realized by adding another dynamic constraint with the aim of increasing the total downtime (or, in other words, decreasing the total uptime) of the units[4] until either a feasible solution was reached or infeasibility determined, as seen in (13).

$$\sum_{t=\tau}^{T} o_{k,t} \leq T - O^{downt}_{\tau,k} \quad \forall k = 1, ..., K \tag{13}$$

$$\begin{aligned} r_{j,t} &\leq r_{j,t-1} + e_{j,t} - p_{j,t} && \begin{matrix} \forall j = 1, ..., J; \\ t = 1, ..., \tau \end{matrix} \\ r_{j,t} &\leq r_{j,t-1} + \mathbf{E}[e_{j,t}] - p_{j,t} && \begin{matrix} \forall j = 1, ..., J; \\ t = \tau + 1, ..., T \end{matrix} \end{aligned} \tag{14}$$

[4]not to be mistaken with the total startups/stops

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Fig. 3. Inertia Demand Scenario with $RH = LH$

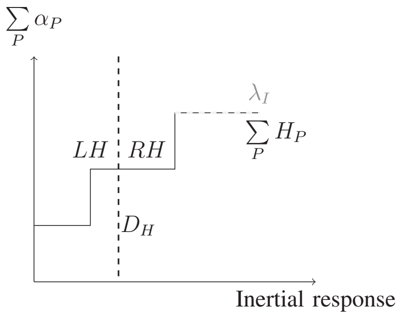
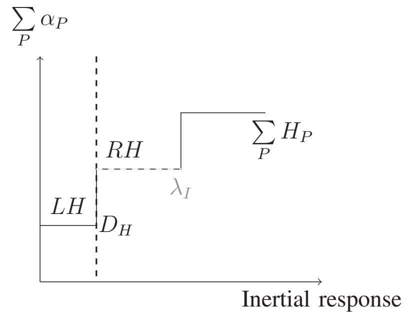


Fig. 4. Inertia Demand Scenario with $RH \neq LH$



(14) displays the inventory function of the reservoirs. Spilling is indirectly considered through the opportunity costs of losing one unit of Λ for each unit of $e_{j,t}$ spilled. (15) defines the reservoir size.

$$r_{j,t} \leq r_j^{max} \forall j = 1, ..., J;\ t = 1, ..., T \tag{15}$$

Determining a price for Inertia through dual values of the inertia constant demand constraint (5) is restricted by the fact that Integer Problems (IPs) do not offer a straight forward approach for the determination of shadow prices. In consideration of the constraint setup however, it can be seen that there only exists one direct impact lever for an increase of inertial response - constraint (set) (5), meaning that Inertia as such should have a marginal cost function of 0, as there are no variable cost components necessary to consider in increasing total system inertia, causing the sole existance of long term implications of inertia requirements, which should be included in the investment decision rather than the day to day expenses. Another implication demonstrated in [16], the fallacy of strongly inelastic demand - as assumed in the here proposed model - is given by the (potential) difference in right-hand (RH) and left-hand (LH) side values, as shown in Fig. 3 and Fig. 4.

$$\begin{aligned} \lambda_{I,\tau} =& \min\{\frac{\alpha_P}{p_P^n \times H_P} + \psi_\tau \,|\, o_{P,\tau} = 0\} \\ & \text{with new objective function:} \\ W(t)+& \begin{vmatrix} \psi_1 \\ ... \\ \psi_T \end{vmatrix}^T \begin{vmatrix} C_T^F - F_1 \\ ... \\ C_T^F - F_T \end{vmatrix} \\ \text{and} \quad & \psi_t \geq 0 \end{aligned} \tag{16a}$$

$$\begin{aligned} & \text{or in case of inertia-segregated areas:} \\ \lambda_{I,\tau,1} =& \min\{\frac{\alpha_{P_1}}{p_{P_1}^n \times H_{P_1}} \,|\, o_{P_1,\tau} = 0\} \\ \lambda_{I,\tau,2} =& \min\{\frac{\alpha_j}{p_j^n \times H_j} \,|\, o_{j,\tau} = 0\} \end{aligned} \tag{16b}$$

However, it was still deemed possible to give a quantiative estimation for the value of the short term impact of inertia. The 'shadow price' of inertial response - $\lambda_{I,\tau}$ (or

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Fig. 5. Hydro Plant Reservoir Inflow Forecast Accuracy

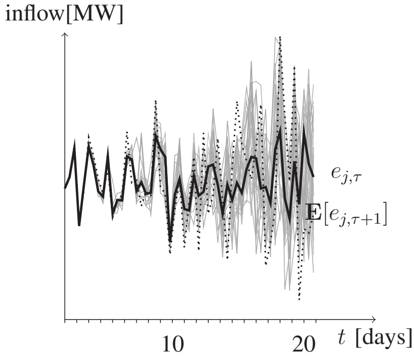
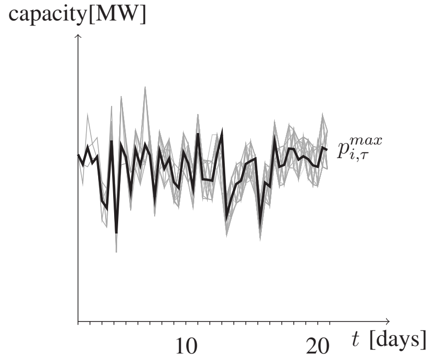


Fig. 6. Wind Power Forecast Accuracy



$\lambda_{I,\tau,1/2}$ in case of separated areas) - was thus selected to appear in the next committed unit with the lowest fixed cost component. It has to be noted that the distribution shown in Fig. 3 and Fig. 4 does not have to follow a merit order, as the units on the RH side might carry a lower fixed cost portion than on the LH, as the units are committed in regards of their lowest total cost and not only their fixed cost. However, the next unit committed solely for the purpose of providing inertial response has to show the lowest fixed component of the available units. (16) depicts this. In the case of (16a) with the possibility of transfer of inertial response, the shadow prices ψ_τ of the line flow, i.e. the dual value of (4) have to be defined. A two-stage approach (solving the original problem and then assigning the resulting schedule as deterministic parameters to the binary variables $o_{P,t}$) proved successful. Due to the nature of dual values, no congestion would therefore set the shadow price to 0 and the price for inertial response to a fraction of the fixed cost of the cheapest extra-marginal unit, such as in the case of two areas without shared inertia as shown in (16b).

III. Case Study

A system consisting of two small and one medium wind power plant, two medium sized hydro power plants and a single thermal plant was used to evaluate the model(inertial response impact was held constant for the different plant types.). The total potential power output without wind curtailment and with full reservoirs was set to 25 MWh in total. The hydro reservoirs were sized to be able to cater demand of one area for 3 periods and the transmission line was considered big enough to transmit the nearly the full demand of one area to another (nearly no congestion). There was a seasonality in form of a sinus curve with a 20% change over the 3 week period applied to both areas. To create expected values used in the system forecast, an exponential error term was added to the deterministic data set. The expected value of a function f in time $t2$, as observed in time t was defined as $E[f(t2)]_t = \min[f(t2) \times \Gamma(t), 0]$ where the uniformly distributed error term used in the equation was defined as $\Gamma(t) \in \mathrm{U}\{2 - e^{(t2/s)}, e^{(t2/s)}\}$ and $-\Gamma(t)^{max} <= \Gamma(t) <= \Gamma(t)^{max}$. Thus, being closer in time to a period gave a more accurate depiction of the situation to incorporate into the planning - Fig. 5 and Fig. 6 show the simulated forecast scenarios for wind and water [1]. Fig. 7 shows the scheduled output aggregated on form of generation for a period of three weeks and

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Fig. 7. No Inertia Requirement

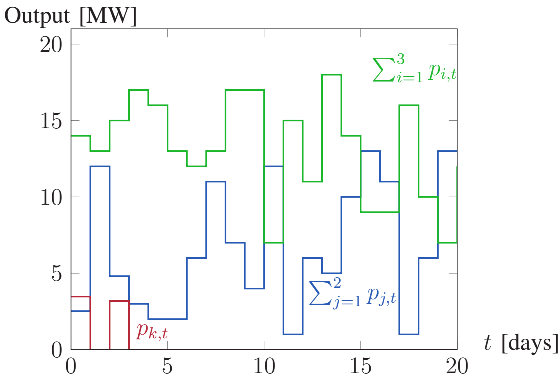

Fig. 8. Shared Inertia Requirement

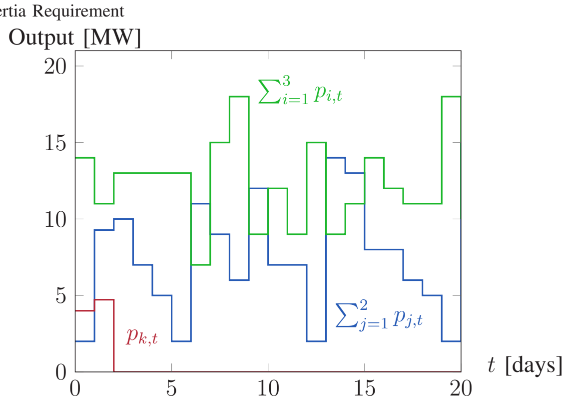

Fig. 9. Separated Inertia Requirement

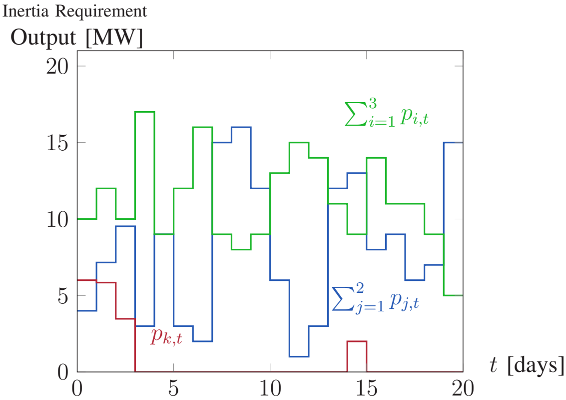

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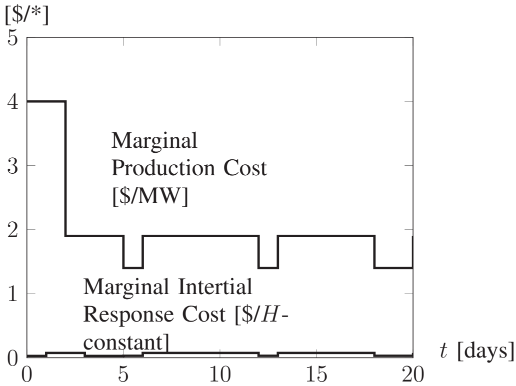
Fig. 10. Cost Curves for Shared Inertia Obligation (Fig.8)



in case of no inertia requirements. Wind power provided the main generation to the system, supported by hydro in periods of low wind supply. Thermal production was used to supply the demand in the initial filling period, balancing the starting level of the hydro plants. Adding a shared inertia constraint on both areas, increased the on-time of the hydro plants, as shown in 8. Decoupling of the trade of inertia as shown in 9 lead to the necessity of starting the thermal plant to supply the necessary inertia and thus to an even greater total price increase. Consequently, it can be stated that increasing the inertia requirement causes plants to start redundantly (and thus to pay their fixed cost portion α_P), which would otherwise not have been scheduled. Fig. 10 shows the difference in *Market Cutoff Price* and price of inertia $\lambda_{I,\tau}$ for the case of an area with shared inertia fulfillment[5].

IV. Conclusion

In a novel modeling approach, a 'demand' for inertial response was imposed on the system and its impact quantified. This is based on the fact that as this inelastic requirement was realized through a cut in the solution set, a quantitative difference to the initial, optimal social welfare situation can be expected. Furthermore, this paper analyzed the impact of inertial response requirements on a two-node/are system characterized by renewable generation forms. It was demonstrated, that inertia intuitively behaves like a traditional capacity payment but differs on short term from the price of capacity as the cost of providing inertial response are carried by the cost of production; however, still influenced by area congestion. Thus follows, that on a long term average, capacity price should be considered $\geq$ inertia price. For future work, a long term analysis via a rolling time horizon and more (both in number and generation form diversity) areas could be included in the model, as well as additional work on the process of inertia-pricing might be advised.

[5]the price curve of the test system for separated areas of inertia was omitted as it showed a nearly steady level due to the low amount of participating plants

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References

[1] J. M. Morales, A. J. Conejo, H. Madsen, P. Pinson, and M. Zugno, *Integrating Renewables in Electricity Markets*. Springer, 2014.

[2] A. Mullane, G. Bryanst, and M. O'Malley, "Kinetic energy and frequency response comparisons for renewable generation systems," in *International Conference on Future Power Systems*, no. 1, 2005, pp. 1–6.

[3] J. Morren, S. W. H. de Haan, W. L. Kling, and J. A. Ferreira, "Wind turbines emulating inertia and supporting primary frequency control," *IEEE Transactions on Power Systems*, vol. 21, no. 1, pp. 433–434, 2006.

[4] R. Doherty, G. Lalor, and M. O'Malley, "Frequency Control in Competitive Electricity Market Dispatch," *IEEE Transactions on Power Systems*, vol. 20, no. 3, pp. 1588–1596, 2005. [Online]. Available: http://ieeexplore.ieee.org/lpdocs/epic03/wrapper.htm?arnumber=1490613

[5] N. W. Miller, K. Clark, and M. Shao, "Frequency responsive wind plant controls: Impacts on grid performance," in *IEEE Power and Energy Society General Meeting*, 2011, pp. 1–8.

[6] J. F. Bard, "Short-Term Scheduling of Thermal-Electric Generators Using Lagrangian Relaxation," *Operations Research*, vol. 36, no. 5, pp. 756–766, 1988. [Online]. Available: http://pubsonline.informs.org/doi/abs/10.1287/opre.36.5.756

[7] A. R. de Queiroz, "Stochastic hydro-thermal scheduling optimization: An overview," *Renewable and Sustainable Energy Reviews*, vol. 62, pp. 382–395, 2016. [Online]. Available: http://linkinghub.elsevier.com/retrieve/pii/S1364032116300983

[8] M. P. Nowak, R. Nürnberg, W. Römisch, R. Schultz, and M. Westphalen, "Stochastic programming for power production and trading under uncertainty," in *MathematicsKey Technology for the Future*. Berlin - Heidelberg: Springer, 2003, pp. 623–636.

[9] R. Nürnberg and W. Römisch, "A Two-Stage Planning Model for Power Scheduling in a Hydro-Thermal System Under Uncertainty," *Optimization and Engineering*, vol. 3, no. 4, pp. 355–378, 2003.

[10] E. D. Castronuovo and J. A. P. Lopes, "On the optimization of the daily operation of a wind-hydro power plant," *IEEE Transactions on Power Systems*, vol. 19, no. 3, pp. 1599–1606, 2004.

[11] J. García-gonzález, R. Moraga, R. De, and L. M. Santos, "Stochastic joint optimization of wind generation and pumped-s torage units in an electricity market," *IEEE TRANSACTIONS ON POWER SYSTEMS*, vol. 23, no. 2, pp. 460–468, 2008.

[12] F. R. Førsund, *Hydropower Economics*, 2nd ed. Springer, 2015.

[13] ——, "Hveding's Conjecture: On the Aggregation of a Hydroelectric Multiplant Multireservoir System," University of Oslo, Tech. Rep., 2014.

[14] E. Ørum, M. Laasonen, and E. al, "Future system inertia," entsoe, Tech. Rep., 2015. [Online]. Available: https://www.entsoe.eu/Documents/Publications/ SOC/Nordic/Nordic report Future System Inertia.pdf

[15] G. Warland and A. Haugstad, "Including thermal unit start-up costs in a long-term hydro-thermal scheduling model," *Proceedings of the 16th Power Systems Computation Conference*, pp. 1–7, 2008.

[16] S. Stoft, "Marginal Cost in a Power Market," in *Power System Economics: Designing Markets for Electricity*. IEEE Press, 2002, pp. 60–73.

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7.8 Errata

For the sake of critical discussion of the presented publications, a thorough list of weaknesses in modeling and presentation shall be given here. These weaknesses are the following:

[POLICY]: the case study 2 on page 11 should be titled 'lower elasticity'. Thus, the text on page 10 should read 'decreasing the elasticity' and 'lower price elasticity' instead of 'increasing the elasticity' and 'higher price elasticity'.

[EJOR]: instead of 'other techniques from the field of reinforcement learning' should read 'other optimizers'.

[ENERGY]: the case study results will be misrepresented due to a missing constraint interlinking player quantities. This would require an aggregation quantity in the form of $q_j = \sum\limits_{i \in I_j} q_i$ for every single player j. In the current representation, each individual units stands in competition with all other units.

[ITRANS]: the Nikaido-Isoda convergence algorithm has no parameter assigned to determine its learning rate. This means that a learning rate of $u = 1$ is used, whereas the range in the original presentation of the algorithm is stated as $0 < u <= 1$. A different learning rate would support convergence and thus would improve algorithm performance.

[IAEE]: equation (3d) and equation (5) use q_t^i which should read q_{t+1}^i in both cases.

[EEM]: generator welfare maximization is utilized. However, there is no difference to the principle of profit maximization.

[POWERT]: the problem is continuously referred to as 'stochastic'. However, instead of a stochastic representation a deterministic equivalent is used.

[POWERT]: the nomenclature uses $p_h^{min/max}$ which instead should read $p_j^{min/max}$".

None of the presented points change the core principles of the presented methods and the validity of the provided core statements in the paper.

Chapter 8

Further potential Applications

As the research question and main assumptions in Chapter 5 were kept intentionally general, a wide range of applications of the proposed models and methods from Chapter 7 can be expected. Proposals for future research and applications of the presented concepts will therefore be introduced in this chapter.

On one hand, increasing importance of similar models in electric power systems can be expected, e.g. based on the decreasing generation portfolio shares of flexible means of generation [53].

On the other hand, various other industries might encounter problems that are in a similar manner described as *'games under storage and dispatch decisions'*. As the novel models and methods presented in this dissertation are able to approach these problems, possible extensions to other fields will also be discussed in this chapter.

8.1 Applications in Electric Power Systems

There exist various possible additional starting points for future research using the presented concepts:

8.1.1 Model Extensions

As presented in References [1, 36, 95, 111], various additional factors such as hydrological typologies, conversion rates, bidding blocks, different bidding time frames and various market types could be incorporated in the proposed market models. Even though these factors have not been considered in depth in the presented publications, extension of the introduced models to incorporate such features would allow to analyze a broader range of practical cases.

8.1.2 Different Means of Storage

Existing studies highlight the feasibility of applying small-scale storage to buffer intermittency of renewable sources [14, 29]. An operator of a network of small-scale storage units would be able to conduct arbitrage similar to traditional large-scale storage providers. Considering that such applications are also intended to reduce the strains on the transmission networks [29], local price-maker areas/nodes might become a growing topic of analysis [16]. Here, the presented methods might provide a starting point for future research.

8.1.3 Multi-level Games

Multi-level games such as games under investment decisions [84] or leader-follower games [98] could be reformulated as single-stage games by adding state constraints as proposed above. Connection between the individual levels of the model would be similar to storage decisions, whereas constraints denying temporal regression, i.e. holding inventory backwards in time, could be removed. This would allow lower ranked levels to exercise impact on the higher ranked levels of such a model.

8.2 Applications in other Fields

Due to the generality of the problem, applications beyond the analyzed areas can be expected. These include the following:

8.2.1 Non-electric Energy Markets

Formulation of competition as complementarity models and solution techniques presented in Chapter 2 have been proposed for other energy markets such as gas markets [22, 51], coal [73], crude oil [85], district heating [159] or emission allowance schemes [166]. State decisions such as network routing and (pipeline) inventories and uncertainty in demand, supply, weather effects, etc. could be considered viable additions to such models. These additions could be made in similar manner to the previously proposed concepts.

8.2.2 Other Applications from Operations Research

The above presented methods share various characteristics with other markets for commodity goods: players acting under consideration of limitations (e.g. production capacities, conversion efficiencies, state-dependent constraints,...), a distribution network (e.g. electricity grid, road network, railway network, storage facility pathways), state decisions (e.g. inventory, dispatch) and varying levels of competition as well as several market/contract types.

Due to this, the range of practical research applications is broad. The models presented in the publications above allow to incorporate competition between multiple players to multi-period *newsvendor models* [31, 107], *vehicle routing problems* [4], (storage facility) *allocation problems* [55] or *queuing problems* [101].

Chapter 9

Conclusions

The presented work attempts to introduce novel market models in electric power systems and subsequently design solution techniques to approach those. The main focus of the work is directed on competition between peak-load units. These units are assumed to be provided by thermal power plants and large-scale storage plants. In practical applications, the latter can be assumed to be provided by hydropower units[1]. In the presented applications, the applied models focus mainly on storage plants with natural (hydrological) inflow and no pumping/purchasing. Latter would be an important trait in analyzing other means of storage. However, due to the simplifications of the state constraints and methods, extension to other storage types such as large scale battery storage is considered a possibility.

Due to the focus on player interaction, models from the field of game-theory were applied. Competition was mainly formulated via Cournot models, due to the importance of 'quantity games' in commodity markets such as the electric power system.

The solution and modeling techniques developed during the work on this dissertation mainly focus on a model with two problem "dimensions":

- j - Several players are assumed to participate in the proposed market models. Those players are assumed to have the means to impact the decisions of other players. This is done through actions such as changing the decisions on generation and thus impacting market prices.

- t - Several stages/time periods are considered. These periods allow the players to alternate their states via such decisions as dispatching units and storing inventory.

[1] It has to be noted that hydropower generation do not necessarily act only as peak load providers. The ratio of inflow to storage capacity influences the range of viable storage decisions (and might vary over the year/s). If this number is high, operation similar to traditional base load units is inevitable. However, this dissertation focuses on plants that are assumed to have sufficient freedom in their storage decisions.

The previous problem "dimension" implies that these state decisions are therefore also able to influence other players' state decisions.

Literature on this intersection of traditional *competition models* and *unit commitment models* was found to be rare. In addition, both of these models individually suffer from the 'curse of dimensionality' that the required non-linear solution methods offer.

Because of these problem characteristics, the choice to design suitable methods and techniques to solve such problems efficiently was made. Rather than direct the focus of the work on large-scale case studies, the focus was directed on suitable small-scale examples intended to display the viability of the applied approaches. These models were aimed on the Northern European power system due to the high share of hydropower in this part of the European grid. However, the applications were designed with the intend of supporting execution of studies of greater extent. Thus, computational complexity and resource efficiency were a core topic of the presented algorithms.

Further importance is found in the appearance of solutions, as both finite and infinite ranges of multiple Nash equilibria are possible results to the proposed models. Careful selection and analysis of those potential outcomes is therefore both advised and conducted in the presented case studies. One result of this analysis is the mapping of Nash equilibria, as conducted in various of the publications presented in this dissertation.

The main research results fulfill the originally formulated research question from Chapter 5: *"How would a model need to be designed to accurately depict the decision process of a price-making storage operator?"*[2] Considering the number of published works as a result of this dissertation, this research question can be considered approached in multiple ways by the work presented in Chapter 7. Several techniques and models were designed and discussed:

- Publication [EJOR] presents a dynamic approximation scheme for a non-symmetric/ non-convex game solved via Gröbner basis formulation.
- Publication [ENERGY] introduces a discontinuous game under uncertainty that is solved via reformulation, a nested discontinuous-continuous algorithm and a weighted residual approach.
- Publication [ITRANS] provides a solution methodology based on a stepwise Nikaido-Isoda convergence algorithm that is nested in a tailored cut-and-branch methodology.
- Publication [IAEE] solves a discontinuous game under uncertainty via a method based on multi-dimensional price curves approximated via interpolation that is nested in a Column-and-Constraint-Generation algorithm.

[2] Further assumptions such as a focus on non-pumped hydropower storage and reserve markets/ancillary services are formulated in Chapter 5. The previous chapters present discussion leading to those assumptions.

Nonetheless, additional results and finds beyond this core question were made and presented in the work. These research contributions can be summarized the following:

# 1	An overview over existing publications on competition under state-constraints was provided. Due to the computational ramifications of including such traits, these models proved rare in literature. This however allowed for a rather complete mapping of the existing work in the field that is provided within Chapter 6 of this dissertation.
# 2	Additional solution techniques beyond the traditional Karush-Kuhn-Tucker for equilibrium models were applied. Applications based on Nikaido-Isoda functions and Gröbner bases were developed. Additional convergence algorithms and supply function equilibria were discussed. Further, dynamic approximations via machine-learning/metaheuristical techniques aimed at traditionally neglected problems from game-theory were introduced.
# 3	Practical applications of the developed methodologies were shown. These applications such as strategic unit commitment or markets for running reserves/inertial response could prove valuable starting points for future research.
# 4	Various methods to find and select multiple Nash equilibria are proposed. These range from polynomial formulations to reformulation techniques that reduce the complexity of the discontinuous decisions. As a starting point for future research, large-scale cases could test and assess those approaches in their practical validity.
# 5	Modular methodologies were designed. Even though problems were solved entirely on single-level[3], discontinuous and continuous decisions were most often separated. This allows for adjustments of individual parts without changing the concept of another. For example could the branch-and-cut technique proposed in Publication [ITRANS] be used to solve the discontinuous problem in Publication [ENERGY], or the approximation technique in Publication [EJOR] to solve a Nikaido-Isoda convergence problem as in Publication [ITRANS] or a traditional KKT problem as in Publication [ENERGY].

[3]Opposed to traditional approaches, which often solve bi-level problems in the form of e.g. 'solve scheduling/storage decision' $\rightarrow$ 'solve market equilibrium'.

# 6	Concepts for including uncertainty in electricity market games were analyzed. This matter was considered of vital importance, as hydropower optimization is strongly influenced by uncertainty in inflow caused by precipitation. The current best practice for this application is provided by techniques from stochastic optimization. Such techniques, however, result in a branched tree instead of single (expected) solution states for future periods (this is extended on in Chapter 3). Therefore, the developed methods were mostly focused on robust/interior point solutions.
# 7	The traditional definition of market power, i.e. *'bidding above marginal cost'* was challenged. It was shown that systems under marginal cost bidding still offer strategic players options to influence market prices. The here considered strategies were: a) for hydropower/storage units - not conduct storage even though sufficient capacity is available, releasing available capacities prior to peak periods in order to create higher peaks. b) for thermal units - strategically dispatch units by planning in order to have units with higher marginal cost available in peak demand periods. Both of these strategies could be regarded as the traditional monopolistic/oligopolistic strategy of *'withholding supply'* but are conducted indirectly by players limiting their own generation capacities in subsequent periods. In Publication [POLICY] the future impact of such decisions was projected to continue growing due to a reduced share of peak load units and reduced supply elasticities.

By providing a review on existing literature in Chapter 6, the presented dissertation highlights the novelty of the presented methods. Techniques to reduce problem complexity were presented in multiple forms and include reformulations, decomposition, approximations and problem-specific analysis. However, the essential core problem of non-convex games still is $\mathcal{NP}$-hard. Extending this problem to multiple time stages further increases this complexity. The developed methodologies thus offer a useful starting point for future developments and improvements that might show a growing importance in changing future power systems.

9.1 Closing Words

With the work on this dissertation and the publications presented in Chapter 7, the author hopes to contribute to a new angle of view on competition under storage. In electric power systems, models from game theory were traditionally intended to analyze the interaction between flexible agents in selected, representative time periods/scenarios. Recent developments have, however, led to drastic changes in the system.

These changes of the electric power system include: intermittency and uncertainty caused by a higher share of renewable generation in the system, end customers actively seeking participation in markets, both via decisions on prices but also via voicing preferences for certain generation portfolios; higher shares of small-scale storage also in the form of electric vehicles), shorter investment cycles for generation units, increases of large-scale cooperation projects[4] and increasing complexity in financial markets.

Considering these changes, reassessment of traditional techniques can be considered a crucial step in preparation for future challenges. The author of this dissertation aimed to provide valid concepts and ideas to detect and analyze inefficiencies in such future electric power systems.

The motivation was to support the responsible decision makers to implement adequate solutions to deal with such inefficiencies in order to fulfill their mission imposed by society: supplying consumers economically and ecologically efficient with the commodity that is electricity.

[4]Examples are provided in e.g. Reference [52]

Chapter 10

Bibliography

[1] Aasgård, E. K., Fleten, S.-E., Kaut, M., Midthun, K. and Perez-Valdes, G. A. [2018], 'Hydropower bidding in a multi-market setting', *Energy Systems* .

[2] Aghassi, M. and Bertsimas, D. [2006], 'Robust game theory', *Mathematical Programming* **107**(1), 231–273.

[3] Almeida, K. C. and Conejo, A. J. [2013], 'Medium-term power dispatch in predominantly hydro systems: An equilibrium approach', *IEEE Transactions on Power Systems* **28**(3), 2384–2394.

[4] Archetti, C., Speranza, M. and Vigo, D. [2013], Vehicle Routing Problems with Profits, *in* 'Vehicle Routing', chapter 10, pp. 273–297.

[5] Arvan, L. [1985], 'Some Examples of Dynamic Cournot Duopoly with Inventory', *The RAND Journal of Economics* **16**(4), 569–578.

[6] Aussel, D., Bendotti, P. and Pištěk, M. [2017*a*], 'Nash equilibrium in a pay-as-bid electricity market: Part 1 existence and characterization', *Optimization* **66**(6), 1013–1025.

[7] Aussel, D., Bendotti, P. and Pištěk, M. [2017*b*], 'Nash equilibrium in a pay-as-bid electricity market Part 2 - best response of a producer', *Optimization* **66**(6), 1027–1053.

[8] Baillo, A., Ventosa, M., Rivier, M. and Ramos, A. [2004], 'Optimal Offering Strategies for Generation Companies Operating in Electricity Spot Markets', *IEEE Transactions on Power Systems* **19**(2), 745–753.

[9] Bajari, B. P., Hong, H. and Ryan, S. P. [2010], 'Identification and Estimation of a Discrete Game of Complete Information', *Econometrica: Journal of the Econometric Society* **78**(5), 1529–1568.

[10] Baldick, R., Grant, R. and Kahn, E. [2004], 'Theory and application of linear supply function equilibrium in electricity markets', *Journal of Regulatory Economics* **25**(2), 143–167.

[11] Baños, R., Manzano-Agugliaro, F., Montoya, F. G., Gil, C., Alcayde, A. and Gómez, J. [2011], 'Optimization methods applied to renewable and sustainable energy: A review', *Renewable and Sustainable Energy Reviews* **15**(4), 1753–1766.

[12] Bard, J. F. [1988], 'Short-Term Scheduling of Thermal-Electric Generators Using Lagrangian Relaxation', *Operations Research* **36**(5), 756–766.

[13] Bardet, M., Faugère, J. C. and Salvy, B. [2015], 'On the complexity of the F5 Gröbner basis algorithm', *Journal of Symbolic Computation* **70**, 49–70.

[14] Barton, J. P. and Infield, D. G. [2004], 'Energy storage and its use with intermittent renewable energy', *IEEE Transactions on Energy Conversion* **19**(2), 441–448.

[15] Berrada, A., Loudiyi, K. and Zorkani, I. [2016], 'Valuation of energy storage in energy and regulation markets', *Energy* **115**, 1109–1118.

[16] Berry, C. A., Hobbs, B. F., Meroney, W. A., O'Neill, R. P. and Stewart, W. R. [1999], 'Understanding how market power can arise in network competition: A game theoretic approach', *Utilities Policy* **8**(3), 139–158.

[17] Bertsekas, D. P. [1999], *Nonlinear Programming*, 2 edn, Athena Scientific, Belmont, Massachusetts.

[18] Bertsekas, D. P. [2012], *Dynamic Programming and Optimal Control Volume II*, 4th edn, Belmont, Massachusetts.

[19] Bertsekas, D. P. [2017], *Dynamic Programming and Optimal Control Volume I*, 4th edn, Athena Scientific, Belmont, Massachusetts.

[20] Birge, J. R. and Louveaux, F. [2011], *Introduction to stochastic programming*, 2nd edn, Springer.

[21] Bolle, F. [1992], 'Supply function equilibria and the danger of tacit collusion. The case of spot markets for electricity', *Energy Economics* **14**(2), 94–102.

[22] Boots, M. G., Rijkers, F. A. M. and Hobbs, B. F. [2004], 'Trading in the Downstream European Gas Market: A Successive Oligopoly Approach', *The Energy Journal* **25**(3), 73–102.

[23] Borenstein, S., Bushnell, J., Knittel, C. R. and Wolfram, C. [2008], 'Inefficiencies and Market Power in Financial Arbitrage : A Study of California ' s Electricity Markets', *The Journal of Industrial Economics* **56**(2), 347 – 378.

[24] Boyd, S. and Vandenberghe, L. [2009], *Convex Optimization*, 7 edn, Cambridge University Press, Cambridge.

[25] Brekke, K. A., Golombek, R., Kaut, M., Kittelsen, S. A. and Wallace, S. W. [2017], 'Stochastic energy market equilibrium modeling with multiple agents', *Energy* **134**, 984–990.

[26] Buchberger, B. [2006], 'Bruno Buchberger's PhD thesis 1965: An algorithm for finding the basis elements of the residue class ring of a zero dimensional polynomial ideal', *Journal of Symbolic Computation* **41**(3-4), 475–511.

[27] Bushnell, J. [2003], 'A Mixed Complementarity Model of Hydrothermal Electricity Competition in the Western United States', *Operations Research* **51**(1), 80–93.

[28] Catalão, J. P. S., Pousinho, H. M. I. and Contreras, J. [2012], 'Optimal hydro scheduling and offering strategies considering price uncertainty and risk management', *Energy* **37**(1), 237–244.

[29] Cavallo, A. [2007], 'Controllable and affordable utility-scale electricity from intermittent wind resources and compressed air energy storage (CAES)', *Energy* **32**(2), 120–127.

[30] Chang, G. W., Aganagic, M., Waight, J. G., Medina, J., Burton, T., Reeves, S. and Christoforidis, M. [2001], 'Experiences with mixed integer linear programming based approaches on short-term hydro scheduling', *IEEE Transactions on Power Systems* **16**(4), 743–749.

[31] Chen, F. Y., Yan, H. and Yao, L. [2004], 'A newsvendor pricing game', *IEEE Transactions on Systems, Man, and Cybernetics Part A:Systems and Humans.* **34**(4), 450–456.

[32] Chen, X. and Simchi-Levi, D. [2012], 'Pricing and Inventory Management', *The Oxford Handbook of Pricing Management* pp. 1–55.

[33] Chen, Y. and Hobbs, B. F. [2005], 'An oligopolistic power market model with tradable NOx permits', *IEEE Transactions on Power Systems* **20**(1), 119–129.

[34] Chen, Y., Wei, W., Liu, F. and Mei, S. [2016], 'Distributionally robust hydro-thermal-wind economic dispatch', *Applied Energy* **173**, 511–519.

[35] Commission, E. [2017], 'Commission Regulation on Electricity Balancing'.

[36] Conejo, A. J., Arroyo, J. M., Contreras, J. and Villamor, F. A. [2002], 'Self-scheduling of a hydro producer in a pool-based electricity market', *IEEE Transactions on Power Systems* **17**(4), 1265–1271.

[37] Conforti, M., Cornuéjols, G. and Zambelli, G. [2014], *Integer Programming*, Springer, Padova.

[38] Contreras, J., Candiles, O., De La Fuente, J. I. and Gomez, T. [2002], 'A cobweb bidding model for competitive electricity markets', *IEEE Transactions on Power Systems* **17**(1), 148–153.

[39] Contreras, J., Klusch, M. and Krawczyk, J. B. [2004], 'Numerical Solutions to Nash Cournot Equilibria in Coupled Constraint Electricity Markets', *IEEE Transactions on Power Systems* **19**(1), 195–206.

[40] Correia, P. F., Overbye, T. J. and Hiskens, I. A. [2003], 'Searching for Noncooperative Equilibria in Centralized Electricity Markets', *IEEE Transactions on Power Systems* **18**(4), 1417–1424.

[41] Crampes, C. and Moreaux, M. [2001], 'Water resource and power generation', *International Journal of Industrial Organization* **19**(6), 975–997.

[42] Crampes, C. and Moreaux, M. [2008], 'Pumping water to compete in electricity markets', *IDEI Working Paper* pp. 1–45.

[43] Datta, R. S. [2009], 'Finding all Nash equilibria of a finite game using polynomial algebra', *Economic Theory* **42**(1), 55–96.

[44] Day, C. J., Hobbs, B. F. and Pang, J.-S. [2002], *Oligopolistic Competition in Power Networks: A Conjectured Supply Function Approach*, University of California Energy Institute.

[45] De la Torre, S., Contreras, J. and Conejo, A. J. [2004], 'Finding Multiperiod Nash Equilibria in Pool-Based Electricity Markets', *IEEE Transactions on Power Systems* **19**(1), 643–651.

[46] De Ladurantaye, D., Gendreau, M. and Potvin, J. Y. [2007], 'Strategic bidding for price-taker hydroelectricity producers', *IEEE Transactions on Power Systems* **22**(4), 2187–2203.

[47] De Queiroz, A. R. [2016], 'Stochastic hydro-thermal scheduling optimization: An overview', *Renewable and Sustainable Energy Reviews* **62**, 382–395.

[48] Denholm, P., Ela, E., Kirby, B. and Milligan, M. [2010], The Role of Energy Storage with Renewable Electricity Generation, Technical Report 1.

[49] Dimensions [2019], 'Dimensions.ai'.
URL: *https://app.dimensions.ai/discover/publication*

[50] Dirkse, S. P. and Ferris, M. C. [1995], 'The path solver: A nonmonotone stabilization scheme for mixed complementarity problems', *Optimization Methods and Software* **5**(2), 123–156.

[51] Egging, R. G. and Gabriel, S. A. [2006], 'Examining market power in the European natural gas market', *Energy Policy* **34**(17), 2762–2778.

[52] ENTSO-E [2018], TYNDP 2018 Executive Report, Technical report.

[53] European Commission [2016], EU Reference Scenario 2016 - Energy, Transport and GHG Emissions - Trends to 2050, Technical report.

[54] Eyer, J. and Corey, G. [2010], Energy Storage for the Electricity Grid : Benefits and Market Potential Assessment Guide, Technical Report February, Sandia National Laboratories, Albuquerque.

[55] Farahani, R. Z., Asgari, N., Heidari, N., Hosseininia, M. and Goh, M. [2012], 'Covering problems in facility location: A review', *Computers and Industrial Engineering* **62**(1), 368–407.

[56] Faugère, J. C. [2002], 'A new efficient algorithm for computing Gröbner bases without reduction to zero (F5)', *Proceedings of the 2002 international symposium on Symbolic and algebraic computation - ISSAC '02* **-**(-), 75–83.

[57] Ferris, M. C., Dirkse, S. P. and Meeraus, A. [2002], Mathematical Programs with Equilibrium Constraints: Automatic Reformulation and Solution via Constrained Optimization, Technical report, Oxford University Computing Laboratory.

[58] Fleten, S. E. and Kristoffersen, T. K. [2007], 'Stochastic programming for optimizing bidding strategies of a Nordic hydropower producer', *European Journal of Operational Research* **181**, 916–928.

[59] Fleten, S. E. and Kristoffersen, T. K. [2008], 'Short-term hydropower production planning by stochastic programming', *Computers and Operations Research* **35**(8), 2656–2671.

[60] Fleten, S.-E. and Lie, T. T. [2013], A Stochastic Game Model Applied to the Nordic Electricity Market, *in* 'World Scientific Series in Finance', Vol. 4, pp. 421–441.

[61] Fodstad, M., Helseth, A. and Henden, A. L. [2016], Modeling start/stop in short-term multi-market hydropower scheduling, *in* 'International Conference on the European Energy Market, EEM', Vol. 5.

[62] Førsund, F. R. [2014], 'Hveding's Conjecture: On the Aggregation of a Hydroelectric Multiplant Multireservoir System', *CREE working paper* pp. 1–30.

[63] Førsund, F. R. [2015], *Hydropower Economics*, 2 edn, Springer.

[64] Fortuny-Amat, J. and McCarl, B. [1981], 'A Representation and Economic Interpretation of a Two-Level Programming Problem', *The Journal of the Operational Research Society* **32**, 783 – 792.

[65] Gabriel, S. A., Conejo, A. J., Fuller, J. D., Hobbs, B. F. and Ruiz, C. [2013], *Complementarity Modeling in Energy Markets*, Springer, New York.

[66] Gabriel, S. A., Kiet, S. and Zhuang, J. [2005], 'A Mixed Complementarity-Based Equilibrium Model of Natural Gas Markets', *Operation Research* **53**(March 2017), 799–818.

[67] Gabriel, S. A., Siddiqui, S. A., Conejo, A. J. and Ruiz, C. [2013], 'Solving Discretely-Constrained Nash-Cournot Games with an Application to Power Markets', *Networks and Spatial Economics* **13**(3), 307–326.

[68] Gaudard, L. and Romerio, F. [2013], 'The future of hydropower in Europe: Interconnecting climate, markets and policies.', *Environmental Science and Policy* **37**, 1–10.

[69] González, P., Villar, J., Díaz, C. A. and Campos, F. A. [2014], 'Joint energy and reserve markets : Current implementations and modeling trends', *Electric Power Systems Research* **109**, 101–111.

[70] Gribik, P., Hogan, W. and Pope, S. [2007], Market-clearing electricity prices and energy uplift.

[71] Guan, X., Ho, Y. C. and Pepyne, D. L. [2001], 'Gaming and price spikes in electric power markets', *IEEE Transactions on Power Systems* **16**(3), 402–408.

[72] Guo, H., Chen, Q., Xia, Q. and Kang, C. [2018], 'Market equilibrium analysis with high penetration of renewables and gas-fired generation: An empirical case of the Beijing-Tianjin-Tangshan power system', *Applied Energy* **227**(January 2017), 384–392.

[73] Haftendorn, C. and Holz, F. [2008], Analysis of the world market for steam coal using a complementarity model.

[74] Hagglof, K., Lindberg, P. O. and Svensson, L. [1995], 'Computing Global Minima to Polynomial Optimization Problems Using Groebner Bases', *Journal of Global Optimization* pp. 115–125.

[75] Harsanyi, J. C. [1967], 'Games with Incomplete Information Played by Bayesian Players, IIII Part I. The Basic Model', *Management Science* **14**(3), 159–182.

[76] Hart, W. E., Laird, C. D., Watson, J.-P., Woodruff, D. L., Hackebeil, G. A. and Nicholson, B. L. [2017], *Pyomo - Optimization Modeling in Python*, second edn, Springer Science and Business Media.

[77] Hesamzadeh, M. R. and Biggar, D. R. [2012], 'Computation of extremal-nash equilibria in a wholesale power market using a single-stage MILP', *IEEE Transactions on Power Systems* **27**(3), 1706–1707.

[78] Hobbs, B. F. [2001], 'Linear Complementarity Models of Nash Cournot Competition in Bilateral and POOLCO Power Markets', *IEEE Transactions on Power Systems* **16**(2), 194–202.

[79] Hobbs, B. F., Metzler, C. B. and Pang, J. S. [2000], 'Strategic gaming analysis for electric power systems: An MPEC approach', *IEEE Transactions on Power Systems* **15**(2), 638–645.

[80] Hosoe, N., Gasawa, K. and Hashimoto, H. [2010], *Textbook of Computable General Equilibrium Modelling*, Springer.

[81] Hotelling, H. [1929], 'Stability in Competition', *The Economic Journal* **39**(-), 41–57.

[82] Hua, B. and Baldick, R. [2016], 'A Convex Primal Formulation for Convex Hull Pricing', *IEEE Transactions on Power Systems* **8950**(c), 1–10.

[83] Huang, Y., Pardalos, P. M. and Zheng, Q. P. [2017], *Electrical Power Unit Commitment*, Springer, Orlando.

[84] Huppmann, D. and Egerer, J. [2015], 'National-strategic investment in European power transmission capacity', *European Journal of Operational Research* **247**(1), 191–203.

[85] Huppmann, D. and Holz, F. [2012], 'Crude Oil Market Power A Shift in Recent Years?', *The Energy Journal* **33**(4), 1–22.

[86] Huppmann, D. and Siddiqui, S. [2018], 'An exact solution method for binary equilibrium problems with compensation and the power market uplift problem', *European Journal of Operational Research* **266**(2), 622–638.

[87] International Energy Agency [2012], Technology Roadmap Hydropower, Technical report.

[88] Ito, K. and Reguant, M. [2016], 'Sequential Markets, Market Power, and Arbitrage', *American Economic Review* **106**(7), 1921–1957.

[89] Joskow, P. L. [1997], 'Restructuring, Competition and Regulatory Reform in the U.S. Electricity Sector', *Journal of Economic Perspectives* **11**(3), 119–138.

[90] Just, S. and Weber, C. [2008], 'Pricing of reserves : Valuing system reserve capacity against spot prices in electricity markets', *Energy Economics* **30**, 3198–3221.

[91] Kirschen, D. and Strbac, G. [2004], *Fundamentals of Power System Economics*, West Sussex.

[92] Klemperer, B. Y. P. D. and Meyer, M. A. [1989], 'Supply Function Equilibria in Oligopoly under Uncertainty', *Econometrica: Journal of the Econometric Society* **57**(6), 1243–1277.

[93] Krawczyk, J. B. and Uryasev, S. [2000], 'Relaxation algorithms to find Nash equilibria with economic applications', *Environmental Modeling and Assessment* **5**, 63–73.

[94] Kubler, F. and Schmedders, K. [2010], 'Tackling Multiplicity of Equilibria with Gröbner Bases', *Operations Research* **58**(4-part-2), 1037–1050.

[95] Labadie, J. W. [2004], 'Optimal Operation of Multireservoir Systems: State-of-the-Art Review', *Journal of Water Resources Planning and Management* **130**(2), 93–111.

[96] Larsen, E. R., van Ackere, A. and Osorio, S. [2018], 'Can electricity companies be too big to fail?', *Energy Policy* **119**(May), 696–703.

[97] Lehner, B., Liermann, C. R., Revenga, C., Vörömsmarty, C., Fekete, B., Crouzet, P., Döll, P., Endejan, M., Frenken, K., Magome, J., Nilsson, C., Robertson, J. C., Rödel, R., Sindorf, N. and Wisser, D. [2011], 'High-resolution mapping of the world's reservoirs and dams for sustainable river-flow management', *Frontiers in Ecology and the Environment* **9**(9), 494–502.

[98] Leyffer, S. and Munson, T. [2010], 'Solving multi-leader-common-follower games', *Optimization Methods and Software* **25**(4), 601–623.

[99] Li, T. and Shahidehpour, M. [2005], 'Strategic bidding of transmission-constrained GENCOs with incomplete information', *IEEE Transactions on Power Systems* **20**(1), 437–447.

[100] Liberopoulos, G. and Andrianesis, P. [2016], 'Critical Review of Pricing Schemes in Markets with Non-Convex Costs', *Operations Research* (February), opre.2015.1451.

[101] Liu, H. H. [2009], Introduction to Queuing Theor, *in* 'Software Performance and Scalability', chapter 4, pp. 137–176.

[102] Löhndorf, N., Wozabal, D. and Minner, S. [2013], 'Optimizing Trading Decisions for Hydro Storage Systems Using Approximate Dual Dynamic Programming', *Operation Research* **61**(September 2014), 810–823.

[103] Löschenbrand, M. and Korpås, M. [2017], 'Hydro power reservoir aggregation via genetic algorithms', *Energies* **10**(12).

[104] Lumbroso, D., Hurford, A., Winpenny, J. and Wade, S. [2014], Harnessing hydropower: Literature review, Technical Report August, Evidence on Demand, Wallingford.

[105] Luo, Z.-Q., Pang, J.-S. and Ralph, D. [1996], *Mathematical Programs with Equilibrium Constraints*, 1 edn, Cambridge University Press, Cambridge.

[106] Matsumoto, A. and Szidarovszky, F. [2016], *Game Theory and Its Applications*, Springer, Tokyo.

[107] Matsuyama, K. [2006], 'The multi-period newsboy problem', *European Journal of Operational Research* **171**(1), 170–188.

[108] Mei, S., Wei, W. and Liu, F. [2017], 'On engineering game theory with its application in power systems', *Control Theory Tech* **15**(1), 1–12.

[109] Meurer, A., Smith, C. P., Paprocki, M., Certik, O. and Kirpichev, S. B. [2017], SymPy: symbolic computing in Python.

[110] Moarefdoost, M. M., Lamadrid, A. J. and Zuluaga, L. F. [2016], 'A robust model for the ramp-constrained economic dispatch problem with uncertain renewable energy', *Energy Economics* **56**, 310–325.

[111] Moiseeva, E. and Hesamzadeh, M. R. [2017], 'Bayesian and Robust Nash Equilibria in Hydro-Dominated Systems under Uncertainty', *IEEE Transactions on Sustainable Energy* **-**(-), 1–12.

[112] Moiseeva, E., Hesamzadeh, M. R. and Biggar, D. R. [2015], 'Exercise of Market Power on Ramp Rate in Wind-Integrated Power Systems', *IEEE Transactions on Power Systems* **30**(3), 1614–1623.

[113] Moiseeva, E., Wogrin, S. and Hesamzadeh, M. R. [2017], 'Generation flexibility in ramp rates: Strategic behavior and lessons for electricity market design', *European Journal of Operational Research* **261**(2), 755–771.

[114] Molina, J. P., Zolezzi, J. M., Contreras, J., Rudnick, H. and Reveco, M. J. [2011], 'Nash-Cournot Equilibria in Hydrothermal Electricity Markets', *IEEE Transactions on Power Systems* **26**(3), 1089–1101.

[115] Morales, J. M., Conejo, A. J., Madsen, H., Pinson, P. and Zugno, M. [2014], *Integrating Renewables in Electricity Markets*, Springer.

[116] Munoz, F. D., Wogrin, S., Oren, S. S. and Hobbs, B. F. [2017], Economic Inefficiencies of Cost-Based Electricity Market Designs.

[117] Nash, J. [1951], 'Non-Cooperative Games', *Annals of Mathematics* **54**(2), 286–295.

[118] Newbery, D. M. [1997], 'Privatisation and liberalisation of network utilities', *European Economic Review* **41**(3-5), 357–383.

[119] Newbery, D. M. [2002], 'Problems of liberalising the electricity industry', *European Economic Review* **46**(4-5), 919–927.

[120] Nikaido, H. and Isoda, K. [1955], 'Note on non-cooperative convex game', *Pacific Journal of Mathematics* **5**(5), 807–815.

[121] Nürnberg, R. and Römisch, W. [2003], 'A Two-Stage Planning Model for Power Scheduling in a Hydro-Thermal System Under Uncertainty', *Optimization and Engineering* **3**(4), 355–378.

[122] Padhy, N. P. [2004], 'Unit commitment - A bibliographical survey', *IEEE Transactions on Power Systems* **19**(2), 1196–1205.

[123] Percebois, J. [2008], 'Electricity Liberalization in the European Union: Balancing Benefits and Risks', *The Energy Journal* **29**(1), 1–19.

[124] Pereira-Cardenal, S. J., Mo, B., Gjelsvik, A., Riegels, N. D., Arnbjerg-Nielsen, K. and Bauer-Gottwein, P. [2016], 'Joint optimization of regional water-power systems', *Advances in Water Resources* **92**, 200–207.

[125] Pereira, M. and Pinto, L. [1991], 'Multi-stage stochastic optimization applied to energy planning', *Mathematical Programming* **52**(1-3), 359–375.

[126] Philpott, A. B., Craddock, M. and Waterer, H. [2000], 'Hydro-electric unit commitment subject to uncertain demand', *European Journal of Operational Research* **125**(2), 410–424.

[127] Philpott, A., Ferris, M. and Wets, R. [2016], *Equilibrium, uncertainty and risk in hydro-thermal electricity systems*, Vol. 157, Springer Berlin Heidelberg.

[128] Pozo, D. and Contreras, J. [2011], 'Finding Multiple Nash Equilibria in Pool-Based Markets : A Stochastic EPEC Approach', *IEEE Transactions on Power Systems* **26**(3), 1744–1752.

[129] Rahimi-kian, A. and Haghighat, H. [2007], 'Gaming Analysis in Joint Energy and Spinning Reserve Markets', *IEEE Transactions on Power Systems* **22**(January 2015), 2074 – 2085.

[130] Raineri, R., Ríos, S. and Schiele, D. [2006], 'Technical and economic aspects of ancillary services markets in the electric power industry : an international comparison', *Energy Policy* **34**(1010750), 1540–1555.

[131] Reny, P. J. [1999], 'On the existence of nash equilibria in discontinuous and qualitative games', *Econometrica* **67**(5), 1029–1056.

[132] Ruiz, C., Conejo, A. J., Fuller, J. D., Gabriel, S. A. and Hobbs, B. F. [2014], 'A tutorial review of complementarity models for decision-making in energy markets', *European Journal of Decision Processes* pp. 91–120.

[133] Schill, W.-P. and Kemfert, C. [2010], 'Modeling Strategic Electricity Storage: The Case of Pumped Hydro Storage in Germany', *The Energy Journal* **32**(3), 59 – 87.

[134] Schulze, T. and Mckinnon, K. [2016], 'The value of stochastic programming in day-ahead and intra-day generation unit commitment', *Energy* **101**, 592–605.

[135] Sensfuß, F., Ragwitz, M. and Genoese, M. [2008], 'The merit-order effect: A detailed analysis of the price effect of renewable electricity generation on spot market prices in Germany', *Energy Policy* **36**(8), 3076–3084.

[136] Shapiro, A. [2011], 'Analysis of stochastic dual dynamic programming method', *European Journal of Operational Research* **209**(1), 63–72.

[137] Shayesteh, E., Amelin, M. and Soder, L. [2016], 'Multi-Station equivalents for Short-term hydropower scheduling', *IEEE Transactions on Power Systems* **31**(6), 4616–4625.

[138] Sheble, G. B. and Fahd, G. N. [1994], 'Unit commitment literature synopsis', *IEEE Transactions on Power Systems* **9**(1), 128–135.

[139] Singh, V. K. and Singal, S. K. [2017], 'Operation of hydro power plants-a review', *Renewable and Sustainable Energy Reviews* **69**(August 2016), 610–619.

[140] Sioshansi, R. [2010], 'Welfare Impacts of Electricity Storage and the Implications of Ownership Structure', *Energy Journal* **31**(2), 173–198.

[141] Sioshansi, R., Denholm, P., Jenkin, T. and Weiss, J. [2009], 'Estimating the value of electricity storage in PJM: Arbitrage and some welfare effects', *Energy Economics* **31**(2), 269–277.

[142] Steeger, G., Barroso, L. A. and Rebennack, S. [2014], 'Optimal Bidding Strategies for Hydro-Electric Producers: A Literature Survey', *IEEE Transactions on Power Systems* **29**(4), 1758–1766.

[143] Steeger, G. and Rebennack, S. [2015], 'Strategic bidding for multiple price-maker hydroelectric producers', *IIE Transactions (Institute of Industrial Engineers)* **47**(9), 1013–1031.

[144] Stoft, S. [2002], *Power System Economics - Designing Markets for Electricity*, Wiley, New York.

[145] Sturmfels, B. [2005], 'What is a Gröbner Basis', *Notices-American Mathematical Society* **52**(10), 2–3.

[146] Su, C.-L. [2005], Equilibrium problems with equilibrium constraints: Stationarities, algorithms, and applications, PhD thesis, Stanford University.

[147] Swider, D. J. and Weber, C. [2007], ‘Bidding under price uncertainty in multi-unit pay-as-bid procurement auctions for power systems reserve’, *European Journal of Operational Research* **181**(3), 1297–1308.

[148] Todd, M. J. [2014], ‘Computing a Cournot equilibrium in integers’, *School of Operations Research and Information Engineering, Cornell University* pp. 1–10.

[149] Turgeon, A. [2005], ‘Solving reservoir management problems with serially correlated inflows’, *River Basin Management III* **83**, 247–255.

[150] Turgeon, A. [2007], ‘Optimal rule curves for interconnected reservoirs’, *River Basin Management IV* **I**, 87–96.

[151] Turgeon, A. and Charbonneau, R. [1998], ‘An aggregation-disaggregation approach to long-term reservoir management’, *Water Resources Research* **34**(12), 3585–3594.

[152] Ventosa, M., Baíllo, Á., Ramos, A. and Rivier, M. [2005], ‘Electricity market modeling trends’, *Energy Policy* **33**(7), 897–913.

[153] Verseille, J. and Staschus, K. [2015], ‘The mesh-up: ENTSO-E and European TSO cooperation in operations, planning, and R&D’, *IEEE Power and Energy Magazine* **13**(1), 20–29.

[154] Villar, J. and Rudnick, H. [2003], ‘Hydrothermal market simulator using game theory: Assessment of market power’, *IEEE Transactions on Power Systems* **18**(1), 91–98.

[155] Wang, Q. and Chen, X. [2012], ‘China’s electricity market-oriented reform: From an absolute to a relative monopoly’, *Energy Policy* **51**, 143–148.

[156] Wei, W., Feng, L. and Shengwei, M. [2015], ‘Nash Bargain and Complementarity Approach Based Environmental/Economic Dispatch’, *IEEE Transactions on Power Systems* **30**(3), 1548–1549.

[157] Wei, W., Wang, J. and Mei, S. [2016], ‘Convexification of the Nash Bargaining Based Environmental-Economic Dispatch’, *IEEE Transactions on Power Systems* **31**(6), 5208–5209.

[158] Wolfgang, O., Haugstad, A., Mo, B., Gjelsvik, A., Wangensteen, I. and Doorman, G. [2009], ‘Hydro reservoir handling in Norway before and after deregulation’, *Energy* **34**(10), 1642–1651.

[159] Wu, Y. J. and Rosen, M. A. [1999], ‘Assessing and optimizing the economic and environmental impacts of cogeneration/district energy systems using an energy equilibrium model’, *Applied Energy* **62**(3), 141–154.

[160] Xian, W., Yuzeng, L. and Shaohua, Z. [2004], 'Oligopolistic equilibrium analysis for electricity markets: A nonlinear complementarity approach', *IEEE Transactions on Power Systems* **19**(3), 1348–1355.

[161] Yakowitz, S. [1983], 'Dynamic programming applications in water resources', *Water Resources Research* **18**(4), 673–696.

[162] Yang, Y., Zhang, Y., Li, F. and Chen, H. [2012], 'Computing all Nash equilibria of multiplayer games in electricity markets by solving polynomial equations', *IEEE Transactions on Power Systems* **27**(1), 81–91.

[163] Yao, J., Adler, I. and Oren, S. S. [2008], 'Modeling and Computing Two-Settlement Oligopolistic Equilibrium in a Congested Electricity Network', *Operations Research* **56**(1), 34–47.

[164] Zeng, B. and An, Y. [2014], 'Solving Bilevel Mixed Integer Program by Reformulations and Decomposition', *Optimization Online* pp. 1–34.

[165] Zeng, B. and Zhao, L. [2013], 'Solving Two-stage Robust Optimization Problems Using a Column-and-Constraint Generation Method', *Operations Research Letters* **41**(5), 457–461.

[166] Zhao, J., Hobbs, B. F. and Pang, J.-S. [2010], 'Long-Run Equilibrium Modeling of Emissions Allowance Allocation Systems in Electric Power Markets', *Operations Research* **58**(3), 529–548.

Appendix A

Solving Equilibria in Code

This chapter will introduce examples of various methods formulated in the *Python Programming Language* used to yield Nash equilibria for a selected problem setup. Since the methods presented in Chapter 7 are developed as extensions to the techniques presented in Chapter 2, this chapter aims to provide a practical example for implementation of the basis market clearing algorithms. Due to its importance in the publications presented above, the hydro-thermal bidding problem setup presented in Reference [27] is taken as an example to demonstrate the various solution techniques:

$$\begin{aligned}
&\underline{\text{player optimization problem}}\ \forall j \in J: \\
&\max_{q_i \forall i \in I_j} \sum_{i \in I_j} \sum_t p_{j,t}\Big(\sum_{i2 \in I_j} q_{i2,t} + \sum_{i3 \notin I_j} q'_{j,i3,t}\Big) q_{i,t} - \sum_{i \in I_j^{\mathrm{Th}}} \sum_t c_i(q_{i,t}) \\
&\text{s.t.}\quad \underline{q}_i \le q_{i,t} \le \bar{q}_i \quad \forall i \in I_j, t \\
&\qquad \sum_t q_{i,t} = r_i \quad \forall i \in I_j^{\mathrm{Hy}}
\end{aligned} \tag{A.1a}$$

$$\begin{aligned}
&\underline{\text{market clearing condition}}: \\
&\sum_{i \in I_j} q_{i,t} + \sum_{i \notin I_j} q'_{j,i,t} = \sum_{i \in I} q_{i,t} \quad \forall j, t
\end{aligned} \tag{A.1b}$$

The individual player decision problems presented in Equation Set (A.1a) formulate competition on a Cournot market over several time stages t. Players hold a number of generation units I_j which can be either of the types 'thermal' or 'hydropower'. All units are constrained by minimum and maximum generation, denoted as $\underline{q}_i$ and $\bar{q}_i$ respectively. Hydropower units further have a limited available reservoir inventory over all considered time frames, which is denoted by r_i. As mentioned above, the market clearing condition (A.1b) is required to ensure convergence towards a Nash equilibrium. In the original paper, Bushnell uses linear demand functions and piecewise linear cost functions. For the sake of simplicity, cost functions will also be assumed linear in the here presented example. Further, complete information on demand functions is modeled as $p_{j,t} \approx p_t \forall j$. The object preamble defining the data set and results are:

```
class Parameter(object):
    def __init__(self):
        self.J = None # an integer
        self.I = None # an integer
        self.T = None # an integer
        self.i = {} # .i[j]=[units]
        self.Ihy = [] # integers
        self.Ith = [] # integers
        self.p_a = []  # price = p_a[t] - p_b[t] * q_sum[t]
        self.p_b = []  # ---
        self.c_a = []  # cost = c_a[i] + c_b[i] * q_th[i]
        self.c_b = []  # (c_a = c_b = 0 for hydro)
        self.qmin = []  # lower cap
        self.qmax = []  # upper cap
        self.r = [] # (.r[i]=0 for thermal)
class Variables(object):
    def __init__(self):
        self.q = [] # .q[j] = float
        self.q_sum = None # market clearing quantity
```

A.1 (Linearized) Karush-Kuhn-Tucker Conditions

The KKT-conditions for the presented problem can be formulated similar to Equation Set (2.16). In extended form they read:

$$
\begin{aligned}
-p_t(\sum_{j2\in J} q_{j2,t}) - \frac{\partial p_t(\sum_{j2\in J} q_{j2,t})}{\partial q_{j,t}} q_{j,t} - \sigma_{j,t} = 0 \qquad & \forall j,t \\
\begin{cases} 0 & \text{if } i \in I^{\text{Hy}} \\ \frac{\partial c_i(q_{i,t})}{\partial q_{i,t}} & \text{if } i \in I^{\text{Th}} \end{cases} - \underline{\lambda}_{i,t} + \bar{\lambda}_{i,t} + \begin{cases} \sigma_i^r & \text{if } i \in I^{\text{Hy}} \\ 0 & \text{if } i \in I^{\text{Th}} \end{cases} + \sigma_{j,t} = 0 \qquad & \forall j, i \in I_j, t \\
0 \le \underline{\lambda}_{i,t} \perp \underline{q}_i - q_{i,t} \le 0 \qquad & \forall i,t \\
0 \le \bar{\lambda}_{i,t} \perp q_{i,t} - \bar{q}_i \le 0 \qquad & \forall i,t \\
\sum_t q_{i,t} - r_i = 0 \qquad & \forall i \in I^{\text{Hy}} \\
\sum_{i\in I_j} q_{i,t} - q_{j,t} = 0 \qquad & \forall j,t \\
\sigma_i^r, \sigma_{j,t} \in \mathbb{R} \qquad & \forall i \in I^{\text{Hy}}, t \\
\underline{\lambda}_{i,t}, \bar{\lambda}_{i,t} \in \mathbb{R}^+ \qquad & \forall i,t
\end{aligned}
\tag{A.2}
$$

The variables to be solved for are the two decisions q_i (generation per plant) and q_j (total generation) made by a player and the four dual variables $\underline{\lambda}, \bar{\lambda}, \sigma^r, \sigma$.

The Python plugin *Pyomo* has several linear transformation methods included as callable functions [76]. The KKT conditions in linearized form can thus be implemented via the following code:

```
from pyomo.environ import *
from pyomo.opt import SolverFactory
from coopr.pyomo import value
from pyomo.mpec import *
import random
def linearizedKKT(Parameters):
    par = Parameters
    kkt = ConcreteModel()
    # sets:
    kkt.J = Set(initialize = range(par.J))
    kkt.I = Set(initialize = range(par.I))
    kkt.T = Set(initialize = range(par.T))
    # decision variables:
    kkt.q = Var(kkt.I,kkt.T,within=Reals, \
    initialize = random.randrange(min(par.qmin),max(par.qmax)))
    kkt.qj = Var(kkt.J,kkt.T,within=Reals)
    kkt.qsum = Var(kkt.T,within=Reals)
    # dual variables:
    kkt.lambda_d = Var(kkt.I,kkt.T,within=NonNegativeReals)
    kkt.lambda_u = Var(kkt.I,kkt.T,within=NonNegativeReals)
    kkt.sigmar = Var(kkt.I,within=Reals)
    kkt.sigma = Var(kkt.J,kkt.T,within=Reals)
    # KKT conditions:
    def lagrangian1(kkt,j,t):
        return -(par.p_a[t]-par.p_b[t]*kkt.qsum[t])-\
        (-par.p_b[t]*kkt.qj[j,t])-kkt.sigma[j,t]==0
    kkt.lagrangian1 = Constraint(kkt.J,kkt.T,rule=lagrangian1)
    def lagrangian2(kkt,i,t):
        for j2 in range(par.J):
            if i in par.i[j2]:
                j = j2
        return par.c_b[i]-kkt.lambda_d[i,t]+\
        kkt.lambda_u[i,t]+kkt.sigmar[i]+kkt.sigma[j,t]==0
    kkt.lagrangia2n = Constraint(kkt.I,kkt.T,rule=lagrangian2)
    def capacityDown(kkt,i,t):
        return complements(0<=kkt.lambda_d[i,t],\
                           par.qmin[i]-kkt.q[i,t]<=0)
    kkt.capdown = Complementarity(kkt.I,kkt.T,\
                                  rule=capacityDown)
```

```
    def capacityUp(kkt,i,t):
        return complements(0<=kkt.lambda_u[i,t],\
                           kkt.q[i,t]-par.qmax[i]<=0)
    kkt.capup = Complementarity(kkt.I,kkt.T,rule=capacityUp)
    def stateequation(kkt,i):
        return sum(kkt.q[i,t] for t in kkt.T)-par.r[i]==0
    kkt.reservoirs = Constraint(par.Ihy,rule=stateequation)
    def firmbid(kkt,j,t):
        return sum(kkt.q[i,t] for i in par.i[j])-kkt.qj[j,t]==0
    kkt.firm = Constraint(kkt.J,kkt.T,rule=firmbid)
    def marketclear(kkt,t):
        return sum(kkt.q[i,t] for i in kkt.I)-kkt.qsum[t]==0
    kkt.marketclear = Constraint(kkt.T,rule=marketclear)
    # thermal plants have no reservoirs and thus no sigmas:
    def sigmafixer(kkt,i):
        return kkt.sigmar[i]==0
    kkt.sigmafix = Constraint(par.Ith,rule=sigmafixer)
    transformations = ['gdp.bigm','mpec.simple_nonlinear',\
                       'mpec.simple_disjunction','gdp.chull']
    # choose linearization method
    xfrm = TransformationFactory(transformations[2])
    xfrm.apply_to(kkt)
    def SolveIt():
        return SolverFactory("ipopt")   #define solver
    opt = SolveIt()
    results = opt.solve(kkt,tee=True)   #solve
    var_result = Variables()
    var_result.q = [[round(value(kkt.q[i,t]),5)\
                for t in kkt.T] for i in kkt.I]
    var_result.qsum = [round(value(kkt.qsum[t]),5)\
                    for t in kkt.T]
    return var_result
```

A.2 Nikaido-Isoda Convergence Algorithm

As described in section 2.5.3, applying the Nikaido-Isoda Convergence algorithm requires establishing the Nikaido-Isoda function. For the prior established problem, this function reads:

$$\begin{aligned}\Psi(q^s,q^y) = \quad & \textstyle\sum_j \Big(\big(\sum_{i\in I_j} \sum_t p_t (\sum_{i2\in I_j} q^y_{i2,t} + \sum_{i3\notin I_j} q^s_{i3,t}) q^y_{i,t} - \sum_{i\in I_j^{\mathrm{Th}}} \sum_t c_i(q^y_{i,t}) \big) \\ & - \big(\sum_{i\in I_j} \sum_t p_t (\sum_{i2\in I} q^s_{i2,t}) q^s_{i,t} - \sum_{i\in I_j^{\mathrm{Th}}} \sum_t c_i(q^s_{i,t}) \big) \Big)\end{aligned} \tag{A.3}$$

The sub problem of the convergence step can in turn be formulated as:

$$\begin{aligned} & \max_{q^y} \Psi(q^s,q^y) \\ \text{s.t.} \quad & \underline{q}_i \le q^y_{i,t} \le \bar{q}_i \quad \forall i,t \\ & \sum_t q^y_{i,t} = r_i \qquad \forall i \end{aligned} \tag{A.4}$$

In Python-code this can be formulated the following:

```
from pyomo.environ import *
from pyomo.opt import SolverFactory
from coopr.pyomo import value
def NikaidoIsodaFunction(Parameters,qs,qy):
    # the Nikaido-Isoda function = sum_j(profitL - profitR)
    par = Parameters
    def qsum1(j,t):
        return sum(qy[i,t]-qs[i][t] for i in par.i[j]) +\
            sum(qs[i][t] for i in range(par.I))
    def qsum2(j,t):
        return sum(qs[i][t] for i in range(par.I))
    profitL = sum(sum(sum((par.p_a[t]-\
        par.p_b[t]*qsum1(j,t))*qy[i,t]\
    for t in range(par.T))for i in par.i[j]) -\
    sum(sum(par.c_a[i]+par.c_b[i]*qy[i,t]\
    for t in range(par.T))for i in par.i[j])\
        for j in range(par.J))
    profitR = sum(sum(sum((par.p_a[t]-\
        par.p_b[t]*qsum2(j,t))*qs[i][t]\
    for t in range(par.T))for i in par.i[j]) -\
    sum(sum(par.c_a[i]+par.c_b[i]*qs[i][t]\
    for t in range(par.T))for i in par.i[j])\
        for j in range(par.J))
    return profitL - profitR
def NiIs_SubProblem(Parameters,qs,u):
    par = Parameters
    m = ConcreteModel()
    m.I = Set(initialize = range(par.I))
```

```
    m.T = Set(initialize = range(par.T))
    # decision variables:
    m.qy = Var(m.I,m.T, within=NonNegativeReals)
    # objective function:
    m.obj=Objective(expr=NikaidoIsodaFunction(par,qs,m.qy),\
                    sense=maximize)
    # constraints:
    def LowerCap(m,i,t):
        return par.qmin[i]<=m.qy[i,t]
    m.lowcap = Constraint(m.I,m.T, rule=LowerCap)
    def UpperCap(m,i,t):
        return m.qy[i,t]<=par.qmax[i]
    m.upcap = Constraint(m.I,m.T, rule=UpperCap)
    def HydroRes(m,i):
        return sum(m.qy[i,t] for t in m.T)==par.r[i]
    m.res = Constraint(par.Ihy, rule=HydroRes)
    # solver:
    def SolveIt():
        return SolverFactory("ipopt")
    opt = SolveIt()
    results = opt.solve(m, tee=False)
    print("nikaido-isoda-function value:",m.obj())
    var_result = Variables()
    var_result.q = [[(1-u)*qs[i][t]+u*value(m.qy[i,t])\
                for t in m.T] for i in m.I]
    var_result.qsum = [sum(var_result.q[i][t]\
                for i in m.I) for t in m.T]
    return var_result,m.obj()
def NiIs_ConvStep(Parameters,Variables,u):
    # u defines the learning rate 0<u<=1
    par,var = Parameters,Variables
    try:
        results,nikiso = NiIs_SubProblem(Parameters,var.q,u)
    except:
        #initial estimate for q
        q=[[0 for t in range(par.T)] for i in range(par.I)]
        results,nikiso = NiIs_SubProblem(Parameters,q,u)
    return results,nikiso
```

A.3 Gröbner Basis Reformulation

To apply the above presented Gröbner basis reformulation, the KKT conditions (A.2) can be reformulated:

$$
G\left(\begin{array}{lr}
-p_t(\sum_{j2\in J} q_{j2,t}) - \frac{\partial p_t(\sum_{j2\in J} q_{j2,t})}{\partial q_{j,t}} q_{j,t} - \sigma_{j,t} = 0 & \forall j,t \\
\begin{cases} 0 & \text{if } i \in I^{\text{Hy}} \\ \frac{\partial c_i(q_{i,t})}{\partial q_{i,t}} & \text{if } i \in I^{\text{Th}} \end{cases} - \underline{\lambda}_{i,t} + \bar{\lambda}_{i,t} + \begin{cases} \sigma_i^r & \text{if } i \in I^{\text{Hy}} \\ 0 & \text{if } i \in I^{\text{Th}} \end{cases} + \sigma_{j,t} = 0 & \forall j, i \in I_j, t \\
\underline{\lambda}_{i,t}(\underline{q}_i - q_{i,t}) = 0 & \forall i,t \\
\bar{\lambda}_{i,t}(q_{i,t} - \bar{q}_i) = 0 & \forall i,t \\
\sum_t q_{i,t} - r_i = 0 & \forall i \in I^{\text{Hy}} \\
\sum_{i\in I_j} q_{i,t} - q_{j,t} = 0 & \forall j,t
\end{array}\right)
$$
$$
\begin{array}{ll}
\underline{q}_i - q_{i,t} \leq 0 & \forall i,t \\
q_{i,t} - \bar{q}_i \leq 0 & \forall i,t \\
\sigma_i^r, \sigma_{j,t} \in \mathbb{R} & \forall i \in I^{\text{Hy}}, t \\
\underline{\lambda}_{i,t}, \bar{\lambda}_{i,t} \in \mathbb{R}^+ & \forall i,t
\end{array}
\tag{A.5}
$$

The respective Python code is the following:

```
from sympy import *
import time
def groebnerReformulation(Parameters):
    par = Parameters
    # define the primal variables:
    q = [[symbols("q"+str(i)+"_"+str(t)) \
    for i in range(par.I)] for t in range(par.T)]
    qj = [[symbols("qj"+str(j)+"_"+str(t)) \
    for j in range(par.J)] for t in range(par.T)]
    qsum = [symbols("qsum"+str(t)) \
    for t in range(par.T)]
    # define the dual variables:
    lambda_d = [[symbols("ld"+str(i)+"_"+str(t)) \
    for i in range(par.I)] for t in range(par.T)]
    lambda_u = [[symbols("lu"+str(i)+"_"+str(t)) \
    for i in range(par.I)] for t in range(par.T)]
    def sigr(i):
        if i in par.Ihy:
            return symbols("sigr"+str(i))
        else:
            return 0
    sigmar = [sigr(i) for i in range(par.I)]
```

```
sigma = [[symbols("sgj"+str(j)+"_"+str(t)) \
for j in range(par.J)] for t in range(par.T)]
# the marginal cost:
def marginalcost(i):
    if i in par.Ith:
        return par.c_b[i]
    else:
        return 0
mc = [marginalcost(i) for i in range(par.I)]
# define the polynomials:
equations = []
for j in range(par.J):
    for t in range(par.T):
        equations.append(-(par.p_a[t] - par.p_b[t] *\
        sum(qj[t][j2] for j2 in range(par.J)))-\
        (-par.p_b[t])*qj[t][j] - sigma[t][j])
for j in range(par.J):
    for i in par.i[j]:
        for t in range(par.T):
            equations.append(mc[i]-lambda_d[t][i]+\
            lambda_u[t][i]+sigmar[i]+sigma[t][j])
for i in range(par.I):
    for t in range(par.T):
        equations.append(lambda_d[t][i]*\
                    (par.qmin[i]-q[t][i]))
        equations.append(lambda_u[t][i]*\
                    (q[t][i]-par.qmax[i]))
for i in par.Ihy:
    equations.append(sum(q[t][i]\
            for t in range(par.T))-par.r[i])
for j in range(par.J):
    for t in range(par.T):
        equations.append(sum(q[t][i]\
                    for i in par.i[j])-qj[t][j])
print("start groebner with "+\
    str(len(equations))+" equations")
starttime = time.time()
marketclear = groebner(equations,\
    order='grevlex', method="F5B")
print("solution time:",time.time()-starttime)
groebnerkkt = [str(marketclear[l])+"=0"\
                for l in range(len(marketclear))]
```

```
    for i in range(par.I):
        for t in range(par.T):
            groebnerkkt.append(str(par.qmin[i]-q[t][i])+"<=0")
            groebnerkkt.append(str(q[t][i]-par.qmax[i])+"<=0")
            groebnerkkt.append(str(lambda_d[t][i])+">=0")
            groebnerkkt.append(str(lambda_u[t][i])+">=0")
    print("reduced set:")
    print(groebnerkkt)
    # optionally sympy supplies an algebraic solver:
    # solve(marketclear)
```

Here, it has to be noted that executing the proposed code fragment might take a significant time duration to solve. Thus, an approach similar to the one proposed in publication [EJOR] might be the method of choice when applying Gröbner bases on this or similar problems.

Appendix B

General Formulation of the Storage State Constraint

The previous example of Equation A.2 already allows for a more general formulation of the storage problem that is not explicitly discussed in Reference [27]: assigning negative lower generation capacity limits $\underline{q}_i$ to storage operators allows them to act as consumers on the market, whereas this consumption is stored to be used in other periods.

The following code establishes a test case that explores this with the previously introduced model:

```
def casestudy(storage):
    case = Parameter()
    case.J = 2
    case.I = 2
    case.T = 3
    case.i = {0:[0],1:[1]}
    case.Ihy = [0]
    case.Ith = [1]
    case.p_a = [30,100,90]
    case.p_b = [1,0.75,0.75]
    case.c_a = [0,50]
    case.c_b = [0,15]
    if storage == True:
        case.qmin = [-25,15]
    else:
        case.qmin = [0,15]
    case.qmax = [25,55]
    case.r = [0,0]
```

```
    return case
```

In case of `storage = False` there is no available storage capacities to the storage operator as the inflow is equal to 0, resulting in no output for this player:

```
EXIT: Optimal Solution Found.
quantity unit 1 : [0.0, 0.0, 0.0]
quantity unit 2 : [15.0, 55.0, 50.0]
quantity total: [15.0, 55.0, 50.0]
prices: [15.0, 58.75, 52.5]
```

Adding the possibility for a storage decision by setting `storage = True` results in negative quantity decisions and prices rising above the price-intercept in the first period:

```
EXIT: Optimal Solution Found.
quantity unit 1 : [-15.75758, 10.10101, 5.65657]
quantity unit 2 : [15.37879, 51.61616, 47.17172]
quantity total: [-0.37879, 61.71717, 52.82828]
prices: [30.37879, 53.7121225, 50.37879]
```

However, the presented model would be better suited for traditional inventory problems as for applications in pumped hydropower storage, as an essential technical trait of electricity storage problems is not included - the efficiency loss of conducting such storage.

Thus, a more general formulation of the traditional optimal (inventory) control problem can be formulated the following[17, 19, 32]:

$$r_{i,t+1} = r_{i,t} - q_{i,t}^{\text{out}} + q_{i,t}^{\text{in}} \tag{B.1}$$

This formulation can be considered a generalization of the previously introduced hydropower reservoir functions (deterministic - Equation (3.2), stochastic - Equation Equation (3.3)).

In literature, distinguishing between quantities offered $q_{i,t}^{\text{out}} \in \mathbb{R}^+$ and quantities demanded $q_{i,t}^{\text{in}} \in \mathbb{R}^+$ contrary to applying a single quantity decision $q_{i,t} \in \mathbb{R}$ is of importance. In examples from traditional supply chain models, this could e.g. mean that producing in-house might be cheaper than purchasing from the market, leading to two different cost curves $c_i^{\text{out}}(\cdot) \neq c_i^{\text{in}}(\cdot)$ depending on their respective quantities.

For the example of pumped hydropower storage, efficiency losses play a key role in regards to this control problem[133], as it cannot be expected that 100 % of the energy required for pumping can be conserved and shed again at a later stage.

Thus, effiency constants $\eta_{i,t}[\%]$ can be considered, leading to an inventory function in the form of:

$$r_{i,t+1} = r_{i,t} - q_{i,t}^{\text{out}} + \eta_{i,t} q_{i,t}^{\text{in}} + l(\xi)_{i,t} \tag{B.2}$$

As presented in Publication [EJOR], efficiencies of storage can themselves be based on the current reservoir level, which in this case would lead to an efficiency function $\eta_{i,t}(r_{i,t})[\%]$.

Due to the limited available literature on games combined with such inventory control problems, extensions beyond the basic inventory functions were considered out of scope of this work and not considered in the presented publications. However, the presented models and methods were designed to allow for such extended models, which also shows in the publications' focus on performance and computational efficiency.

Appendix C

Market Power and Storage: the Welfare Gap

Assumed be a price-making storage operator j with a single-period profit function:

$$\Pi_{j,t}(q_{j,t}) = p_{j,t}(q_{j,t})q_{j,t} - c_j(q_{j,t}) \tag{C.1}$$

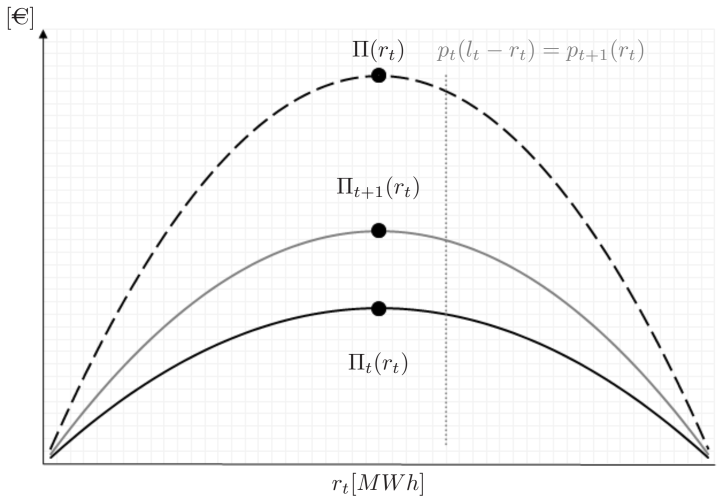
Further, cost functions are assumed to be negligible minor [58], i.e. $c_j(q_{j,t}) \approx 0$, and the price function $p_{j,t}(q_{j,t})$ is considered strictly decreasing. In addition, it shall be assumed that a single firm (thus dropping the j) optimizes storage over two periods t and $t+1$. The profit functions for the whole time frame and the individual periods are:

$$\begin{gathered} \Pi(r_t) = p_t(l_t - r_t)(l_t - r_t) + p_{t+1}(r_t)r_t \\ \Pi_t(r_t) = p_t(l_t - r_t)(l_t - r_t) \\ \Pi_{t+1}(r_t) = p_{t+1}(r_t)r_t \end{gathered} \tag{C.2}$$

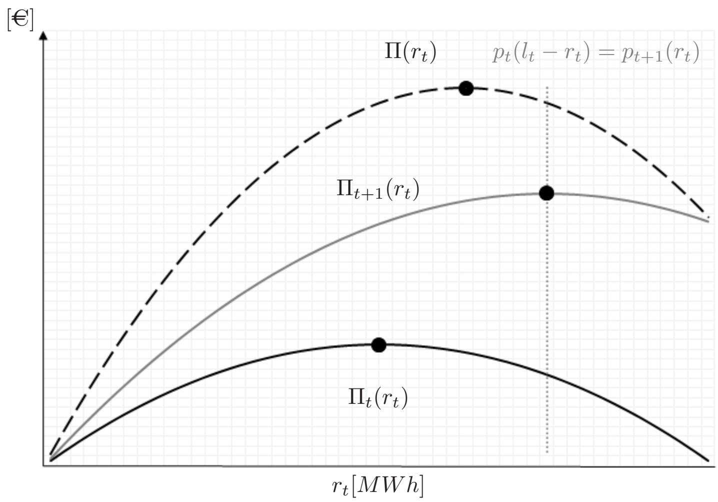
In this case it is assumed, that there exists only a single inflow l_t in the first period and no capacity restrictions on storage. Further, it is assumed that the entire inflow has to be used up in the considered time frame. It can be assumed that there exist stationary points that provide global optima for the respective profit functions:

$$\begin{gathered} \frac{\partial \Pi(r_t)}{\partial r_t} = 0 \\ \frac{\partial \Pi_t(r_t)}{\partial r_t} = 0 \\ \frac{\partial \Pi_{t+1}(r_t)}{\partial r_t} = 0 \end{gathered} \tag{C.3}$$

However, a reservoir decision r_t that provides a stationary point for one of the profit functions does not necessarily provide optima for the others. This can be interpreted as a firm having to commit to tradeoffs between the individual time periods in order to maximize profits over the entire time frame.

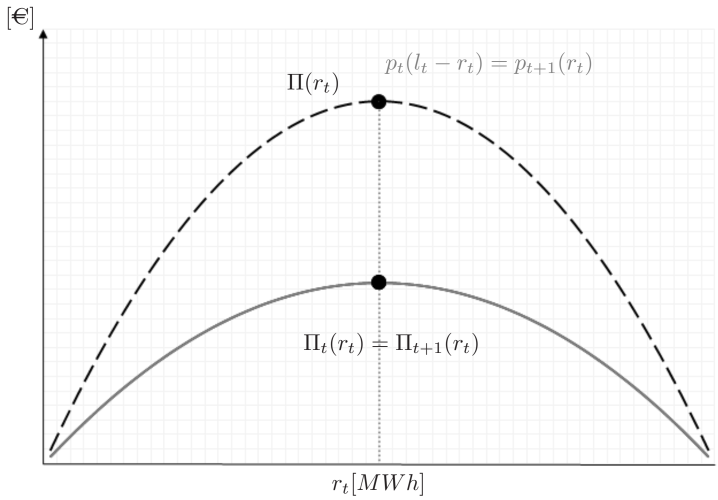


(a) $p_t^b = 0.01 p_t^a$, $p_{t+1}^a = 1.5 p_t^a$, $p_{t+1}^b = 0.015 p_t^a$

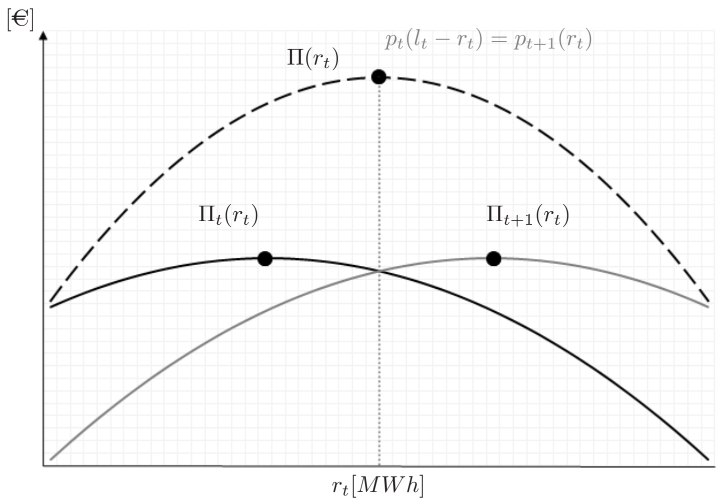


(b) $p_t^b = p_{t+1}^b = 0.01 p_t^a$, $p_{t+1}^a = 1.5 p_t^a$

Figure C.1: Optimal Profits for linear Price Functions



(c) $p_t^b = p_{t+1}^b = 0.01p_t^a$, $p_{t+1}^a = p_t^a$



(d) $p_t^b = p_{t+1}^b = 0.0075p_t^a$, $p_{t+1}^a = p_t^a$

Figure C.1: Optimal Profits for linear Price Functions

It cannot be expected that these optimal points equalize prices over the periods. Figure C.1 demonstrate this under usage of linear price functions based on the intercept p^a and slope p^b:

$$\begin{aligned} p_t(l_t - r_t) &= p_t^a - p_t^b(l_t - r_t) \\ p_{t+1}(r_t) &= p_{t+1}^a - p_{t+1}^b r_t \end{aligned} \tag{C.4}$$

Figure C.1a demonstrates that overlapping stationary points, i.e. a single result for r_t fulfilling all conditions from Equation set (C.3), do not necessarily mean no price differences (dotted line indicated in gray) and thus a persisting welfare gap. This welfare gap is illustrated in Reference [63], which shows by the example of the Hotelling rule that with sufficient storage capacities, the welfare-maximizing outcome is that of price equalization.

Figure C.1b illustrates that even though equal prices would be the optimal choice for the more profitable period $t+1$, a monopolist would still choose to deviate from this point by storing less.

Figure C.1c shows the case of a monopolist deciding to equalize prices due to similar cost functions in both periods.

Figure C.1d shows another case of similar cost functions but demonstrates that the optimal points do not necessarily have to overlap for the individual periods.

In real cases, additional details like less market power e.g. through more competitors, less price elasticity; or complicating factors such as limited reservoir capacity and uncertainty might influence the optimal points. However, this example demonstrates the weakness of the price-taker assumption (i.e. prices are matched over the periods) especially in systems with large storage capacity in relation to the system size.