Introducing Approximate Well Dynamics into Production Optimization for Operations Scheduling

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Abstract

Most of the literature on short-term production optimization concerns the computation of optimal system settings for steady-state operations. Such methodologies are applicable when the scales of time are faster than reservoir dynamics, and slower than the dynamics of top-side equipment. Effectively static problems are solved over time in response to changes in the prevailing conditions, which will remain persistent for long periods. However, when platform conditions change frequently or suddenly possibly due to reduced processing capacity, the dynamics of wells should not be neglected and well operations should be scheduled over time. To this end, this paper proposes a novel mathematical formulation for production optimization when dynamics matters, specifically when wells are shut-in (due to processing capacity drops) and restarted later as the normal conditions are recovered. The effectiveness of the methodology to schedule well operations is assessed by simulation of synthetic and field cases involving an offshore production platform.

Keywords: Short-term production optimization, startup well operation, approximate well dynamics, mixed-integer programming

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1. Introduction

Oil production in offshore fields is a complex enterprise that involves several disciplines and heterogeneous subsystems with rather different dynamics. At the upstream end of the production chain, the dynamics of fluid flow in the reservoir is usually slower than in the rest of the production system, remaining steady from weeks up to months. From the wells and gathering networks up to the topside facilities, the dynamics are faster and require new operating setpoints daily or even more than once a day. In this sense, reservoir dynamics can be assumed at steady-state conditions for the gathering network. Further down the production chain, in the separation and processing facilities of the platform, the dynamics are much faster and their control usually require automatic routines to compute new operating settings while ensuring stability and safety.

In order to cope with the complexity and sheer size of the production chain, the oil industry has adopted the decomposition of asset management into layers, each concerned with decisions that take place at different time scales. The layers are arranged in the control hierarchy depicted in Figure 1 [11], which illustrates the decisions taking place within the layers, along with the information and control signals communicated between them. The control hierarchy helps to position the works from the literature, and indicate the advancement towards integration and extension of the decision layers [9], such as is the case in the present work.

Despite the relevance of process dynamics to the choice of the model and methodology employed in optimization, the time scale and planning horizon are also relevant for decision making in each layer. One issue that naturally arises is whether a dynamic model should be adopted or a static model suffices, a topic that was the focus on an extensive discussion in [10] which based itself on realistic case studies from production optimization. In [10], Foss *et al.* claim that most production optimization problems can be solved using steady-state models and static optimization methods, with exceptions being the cases involving transients such as well startup and shut-in operations, cyclic behavior, and situations of fast reservoir dynamics found for instance in shale gas formations, among others.

In such situations near wellbore dynamics can matter, thus an appropriate description of pressures and flows require the use a reservoir model coupled to a dynamic well model. Nennie *et al.* [25] and more recently, da Silva *et al.* [6] have discussed the conditions when connecting both models is suitable. However, the increased complexity of such coupled models

can make difficult, if not intractable, their use in the solution of practical problems, for instance, well operations scheduling with integer decisions. Additionally, maintaining accurate reservoir and dynamic well models is by itself a challenging undertaking.

In this paper we focus on a practical approach to deal with well startup and shut-in operations in offshore production platforms, which are recurrent in the face of unexpected equipment failure, especially of rotary machines such as compressors. Such situations also arise in planning for preventive maintenance to reduce equipment wear and extend their lifetime. Moreover, oil wells cannot produce uninterruptedly during all field development phases and will invariably undergo shut-in and startup operations.
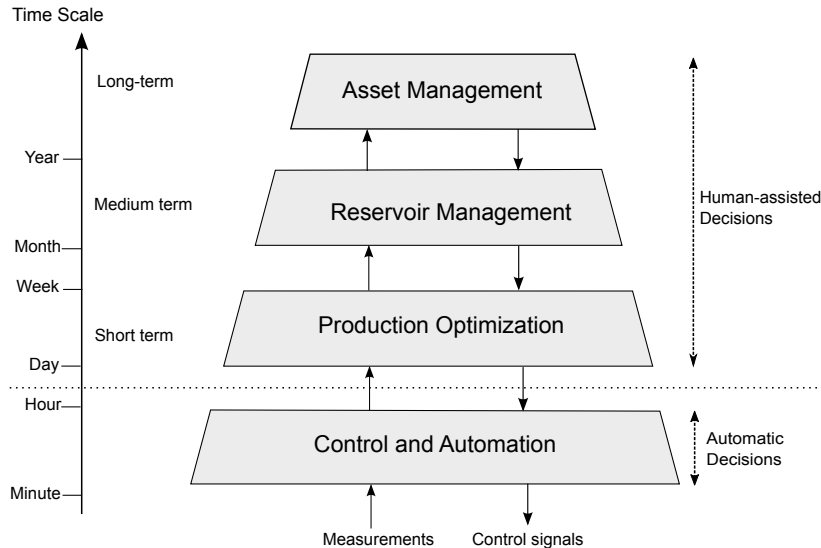

Figure 1: Multilevel control hierarchy, adapted from Foss and Jensen [11].

Motivation. Operators at offshore platforms are often faced with the failure of compressors, and other process equipment, that invariably reduces the handling capacity of fluids and gas. Such unexpected events require timely decisions to handle a sudden capacity reduction and the surplus of gas that arrives later, resulting from the return of lift-gas, which triggers a sequence of operations to reduce gas injection rates, constraint production valves, and shut in wells until the nominal system conditions are recovered. At that stage, the operators will be in a position to start up the wells that were previously shut in, a procedure that evolves in time according with

complex dynamics. The need of a software tool to support the scheduling of the operations motivated the development of a methodology to account for the well transients approximately, as a sequence of steady-state regimes, while maximizing total oil production. This methodology and software tools are the focus of this work.

Another use of the decision-support tool is in the planning of operations for preventive maintenance, which is scheduled to take place during a given time window. The analysis and synthesis of a course of action in response to simulated failures can also be addressed by the software tool.

Literature Review. Most of the literature and field applications in short-term production optimization focus on steady-state models to plan the daily and weekly operations. The optimization approaches vary depending on the existence of analytic models, simulation software, data, and whether or not discrete decisions are involved. In [14], Grimstad *et al.* present a framework for optimizing multiphase flow networks considering discrete decisions, in which B-splines and a MINLP solver are combined into a global solver for production optimization. In [4], Codas *et al.* rely on piecewise-linear approximation of fluid flow behavior and well production functions, implemented in simulation software, to convert a conceptual MINLP into a MILP for oil production in the Urucu Field. The strategy of using piecewise-linear approximation and MILP optimization has become popular in the industry, in part for its robustness but also for the ability of dealing with models available in simulation software [2]. At the other end, one can find applications of derivative-free algorithms that work directly with simulators of oil production systems [16]. Derivative-free approaches are rather flexible, but suffer from slow convergence and do not scale well with problem size. Owing to the inherently complexity of production optimization, most of the applications consist of advisory systems that recommend decisions and course of actions to human operators [9].

The sample of works above is small but representative of the literature on short-term production optimization. To the best of our knowledge, most of the works in the production optimization layer (see Figure 1) rely on steady-state models, arguably due to the difficulty of solving complex MINLP problems, let alone problems accounting for dynamics over time.

At the bottom of the control hierarchy, the focus is on the fast dynamics of processes and stabilization for safety operations. In [18], Jahanshahi and Skogestad develop an active control strategy to prevent severe slugging flow in wells and risers. Their strategy consists of model identification and

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PID controllers that proved to be effective at stabilizing well dynamics. In [27], Peixoto *et al.* develop an extremum-seeking controller to maintain oil production of a gas-lifted well around the optimum point. Their method injects periodic perturbations into the plant in order to obtain gradient information on the well-performance curve.

On the other hand, Codas *et al.* [5] attempt the system-wide optimization and control of production processing, considering multiple wells and surface facilities. Their work presents a nonlinear model predictive controller (NMPC) to stabilize and optimize an oil gathering network with two wells, riser, and separator. The approach consist of an economic NMPC derived from multiple shooting optimal control and Kalman filtering. The high computational cost of the multiple shooting NMPC motivated Diehl *et al.* [7] to develop a more practical NMPC, which performs dynamic linearization along the predicted trajectory in a manner akin, but simpler than sequential quadratic programming. In simulated experiments using the dynamic model developed in [8], the proposed NMPC was shown to improve the operation and production of an otherwise unstable oil well.

Attempts to integrate the layer of production optimization with the layer of automation and control are few and far between, and which typically result from the extension of dynamic control strategies to account for steady-state objectives from the upper layer. Put another way, the attempts are mostly on integrating real-time optimization (RTO) with advanced control strategies such as Model Predictive Control (MPC), but not concerning discrete decisions. An exception is [21, 20], in which Knudsen *et al.* derive a linear dynamic proxy model for shale wells to be embedded in the problem of scheduling operations, which accounts for pipeline routing and shut-in/startup well operations.

In [3], Campos *et al.* discuss the benefits of real-time optimization and advanced control for production optimization, however stress the technological challenges for effective and practical applications. In [22], Krishnamoorthy *et al.* propose a model updating for RTO using dynamic models and transient measurements, which improves model adaptation for real-time optimization while keeping complexity controlled by not solving dynamic optimization problems.

In [6], da Silva *et al.* present a review of coupled dynamic well-reservoir models. In [31, 25, 28], the authors discuss the interplay of well and reservoir phenomena in the context of several production and natural behaviors.

The fast dynamics are typical of shale-gas reservoirs and conventional

reservoirs with conning behavior, whereas shut-in and restart operations arise in the event of compressor failure or preventive maintenance in gas-lifted wells. With respect to shale-gas production systems, some recent works have contributed to the operations by considering fast dynamics and model reduction techinques. For instance, in [15], Gu and Hoo develop a model-based control strategy for the fracture geometry and proppant distribution of a hydraulic fracturing process. In [24], Narasingam and Kwon obtain reduced-order linear models for nonlinear dynamics, which enabled a feedback controller of a hydraulic fracturing process.

Interestingly, some applications in shale-gas relate to our work by considering the scheduling of operations. In [26], Ondeck *et al.* use a MILP formulation to schedule field-development operations, which are required to deploy a shale-gas pad and also optimize the production from the wells. In [1], Cafaro *et al.* propose a continuous-time model for the scheduling of multiple refracturing operations over the life span of shale gas wells.

Now considering conning behavior, in [30], Siddhamshetty and Kwon derive a proxy model for the conning phenomena from simulation data, which is used in a MPC framework for NPV maximization with real-time measurements. In [23], Lerlertpakdee *et al.* develop a reduced-order model for production optimization preserving some of the underlying physics, but unlike our work concerns mid-term optimization. For startup operations of a single well, Schietz [29] coupled a reservoir model to a wellbore flow model in order to optimize its production.

The latest developments bring about the relevance of modeling dynamics in production optimization, a trend which is backed by the advances in technology and the increasing complexity of operations. To this end, Foss *et al.* [10] present an extensive study of the role of dynamics in production optimization. By showing two rather different cases, the authors argue that in most cases static optimization is sufficient for real-time optimization, with the exception being the cases where the reservoir dynamics change rapidly or the wells are shut-in and restarted within the optimization window.

Paper Contribution. This work contributes to the field of production optimization by introducing a proxy model for well transients that enables production planning of gas-lifted systems over a time horizon, accounting for well shut-in and startup operations. Such conditions result, for instance, from unexpected failures in compression units or when planning ahead for preventive maintenance of equipment, which invariably reduce processing capacity.

2. Problem Statement

This work aims to optimize the production of offshore platforms over a certain planning horizon considering the scheduling of well shut-in and restart operations. Such well operations occur relatively frequently in offshore platforms deployed to the Campos Basin, either in response to failure of compression units or in planning ahead for scheduled maintenance of equipment, which curtail the platform processing capacity.

In platforms of the Campos Basin, satellite gas-lifted wells produce most of the hydrocarbons which are gathered by the processing facilities, and then separated in oil, gas, and water streams. Water is treated before discharge or reinjected into the reservoir for pressure maintenance. The produced oil is transferred to onshore terminals by vessels. The gas exceeding the needs of electric turbines is exported by subsea pipelines or otherwise flared. The latter recourse is only used as a safety measure, when there is an excess of gas that cannot be compressed, and might be subjected to penalties by the regulating agency. Actually the sudden excess of produced gas in the event of a compressor failure, combined with the lack of it during well restart operations and the time lag from gas-lift injection and its return to the well head, makes the scheduling of operations a challenging problem.

The present work addresses the problem of scheduling operations, following a methodology that uses state variables to couple a sequence of static problems in time, and in which the dynamics of well shut-in and restart operations are approximated. The purpose is not to fully automate the processes, but rather assist engineers in making informed decisions at times of contingencies and in anticipation of preventive maintenance.

2.1. Assumptions

For managing the limited resources and scheduling the operations as a sequence of problems, the surrogate models for the dynamics of well startup should be restricted to the main production profiles, in face of the complexity of the problems involved. Aligned with this premise, the surrogate model was not derived from a dynamic well-reservoir model, but rather obtained from both the knowledge of engineers and up-to-date steady-state production curves to empirically reflect the main production profile of well startup. The assumptions of the mathematical model presented in the following sections are:

1) The well transients can be approximated with a static curve based on the experience and knowledge of operators about the process.

2) After a compression capacity drop, the return time to a steady-state operation is relatively short and is known.
3) In the event of a well shut-in, the platform has means and equipment to restart the well.
4) The planning horizon window is fixed and known a priori, during which the system limits and capacities are given for each period of time.
5) The well parameters such as productivity index, gas-oil ratio, and water cut do not change during the planning horizon.
6) At termination of the planning horizon the system conditions will reach a steady-state, meaning that the limits and capacities will no longer change.

3. Mathematical Formulation

The daily production optimization consists of computing the optimal operating setpoints for a set of wells, while managing the platform limited resources, and complying with operational and regulatory limits [17]. For the time scale of a day or couple of days it is reasonable to assume that the operating conditions will remain steady, thus the dynamics of flows and pressures can be neglected. Under such assumptions, operators are concerned with the static optimization problem which can be formulated as:

$$P_s: \quad \max_{\mathbf{x}, \mathbf{u}} \quad f_s(\mathbf{x}, \mathbf{u}; \boldsymbol{\theta}) \tag{1a}$$

$$\text{s.t.}: \quad \mathbf{x} \in \mathcal{X}, \ \mathbf{u} \in \mathcal{U} \tag{1b}$$

$$\mathbf{F}_s(\mathbf{x}, \mathbf{u}; \boldsymbol{\theta}) = \mathbf{0} \tag{1c}$$

$$\mathbf{G}_s(\mathbf{x}, \mathbf{u}; \boldsymbol{\theta}) \leq \mathbf{0} \tag{1d}$$

in which f_s defines the overall economic or production goal of the platform. State and control variables, $\mathbf{x}$ and $\mathbf{u}$ respectively, are vectors containing both pressures and flows, and potentially discrete decisions such as routing and well shut-in operations. The vector $\boldsymbol{\theta}$ denotes the parameters of the problem. The sets $\mathcal{X} \subseteq \mathbb{R}^{p_x} \times \mathbb{Z}^{q_x}$ and $\mathcal{U} \subseteq \mathbb{R}^{p_u} \times \mathbb{Z}^{q_u}$ are the domain for $\mathbf{x}$ and $\mathbf{u}$ respectively. Finally, the vector functions $\mathbf{F}_s$ and $\mathbf{G}_s$ collect the system equations and constraints.

For the short-term production optimization of an oilfield exploited through gas-lifted satellite wells, a typical representation of f_s is the aggregated oil

production:

$$f_s \equiv \sum_{n \in \mathcal{N}} q_o^n \tag{2a}$$

where q_o^n is the oil rate produced by well n. The vector function $\mathbf{F}_s$ aggregates well production curves and system constraints such as flow conservation and pressure balance. For instance,

$$\mathbf{F}_s \equiv \begin{cases} \sum_{n \in \mathcal{N}} q_g^n - q_{\text{exp}} - q_{\text{flare}} - q_{\text{turbine}} = 0 & \text{(2b)} \\[2mm] q_o^n - \widehat{q}_o^n(q_{\text{gl}}^n, p_{\text{wh}}^n) \cdot l^n = 0, \ \forall n \in \mathcal{N} & \text{(2c)} \\[2mm] q_w^n - \dfrac{bsw^n}{1 - bsw^n} \cdot q_o^n = 0, \ \forall n \in \mathcal{N} & \text{(2d)} \\[2mm] q_g^n - gor^n \cdot q_o^n = 0, \ \forall n \in \mathcal{N} & \text{(2e)} \end{cases}$$

in which q_g^n is the gas produced by well n, q_{exp} is the exported gas, q_{flare} is gas burned in the flare only to ensure the safety and integrity of the system, and q_{turbine} is the gas used to generate electricity for the platform. The oil and water produced by well n are q_o^n and q_w^n. This work considers that the wells are operated with gas-lift as the artificial lifting mechanism, which can be applied to improve production of surgent wells or enable production from mature wells. However, the methodology to be presented can cope with other lifting techniques.

The well production curve is then modeled by the function $\widehat{q}_o^n(q_{\text{gl}}^n, p_{\text{wh}}^n)$ with q_{gl}^n being the lift-gas injection rate and p_{wh}^n the well-head pressure. In order to account for the shutting in of a well, the binary variable l^n takes on value 1 when the well is producing and 0 otherwise. Under static conditions, the water cut bsw^n and the gas-oil ratio gor^n are assumed constant and known.

Completing the model, $\mathbf{G}_s$ imposes pressure and flow limits on the wells and the processing facilities. For example,

$$\mathbf{G}_s \equiv \begin{cases} \sum_{n \in \mathcal{N}} \left(q_g^n + q_{\text{gl}}^n \right) - q_{\text{flare}} - q_{\text{gtc}}^{\max} \leq 0 & \text{(2f)} \\[2mm] \sum_{n \in \mathcal{N}} \left(q_o^n + q_w^n \right) - q_{\text{l}}^{\max} \leq 0 & \text{(2g)} \\[2mm] q_{\text{flare}} - q_{\text{flare}}^{\max} \leq 0 & \text{(2h)} \end{cases}$$

in which $q_{\text{gtc}}^{\max}$ is the gas compression capacity, $q_{\text{l}}^{\max}$ is the limit on liquid handling by the platform, and $q_{\text{flare}}^{\max}$ is the limit of gas that can be flared

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which is imposed by the regulatory agency to comply with environmental legislation. Generally there are also upper and lower bounds on q_{gl}^n and p_{wh}^n and an upper bound on q_{exp}, that here are omitted for brevity.

The control variables can be lumped together in a vector $\mathbf{u} = (q_{\text{gl}}^n, p_{\text{wh}}^n, l^n : n \in \mathcal{N}) \cup (q_{\text{exp}}, q_{\text{flare}})$, and likewise the state variables in vector $\mathbf{x} = (q_{\text{o}}^n, q_{\text{g}}^n, q_{\text{w}}^n : n \in \mathcal{N})$. Finally, the parameters are given in vector $\boldsymbol{\theta} = (bsw^n, gor^n : n \in \mathcal{N}) \cup (q_{\text{turbine}}, q_{\text{gtc}}^{\max}, q_{\text{l}}^{\max}, q_{\text{flare}}^{\max})$.

3.1. Sequence of Static Problems

Very often short-term optimization can be treated with static models, provided that a well-defined interface between reservoir and production network is established. Since the dynamics in reservoirs are usually slower than in gathering networks, an inflow performance relationship can be adopted together with a steady-state model for the network. Under this assumption the well performance curves can be estimated by their productivity index, gas-oil ratio, and water cut at given reservoir conditions.

However, dynamics between reservoir and networks become relevant when the reservoir boundary conditions vary rapidly over time. Such an interplay between dynamics emerges in shale gas reservoirs, where there is gas or water coning near the production wells [19]. Also, from the network side, dynamics matter when transients are relevant for the advanced control of the wells, in particular when the wells need to be shut-in and restarted in response to maintenance operations or equipment failure.

Even when model dynamics are negligible, the schedule of decisions over time can become relevant in some industrial applications. In these settings the solution of a sequence of static optimization problems can be applied, particularly to handle constraints that span over time and which couple the problems, as exemplified below.

Let us recall the gas-lift distribution problem stated as P_{s} in Eq. (1). Suppose that the operations engineers aim to determine the best setpoints for the platform, in anticipation to a compression capacity drop that would extend for a whole week for preventive maintenance. In this context, a solution can be obtained by solving a sequence of static problems. Although dynamic effects could be disregarded, a conventional coupling constraint that ties the problems all together is the total volume of gas that can be flared monthly according to environmental regulations. A reasonable policy is to consider varying bounds for flaring that respect the monthly limit. According to this practice a sequence of static problems P_{s}, one for each

time step $\tau \in \mathcal{T}$, can be defined by P_τ as follows:

$$P_\tau : \quad \max_{\mathbf{x}_\tau, \mathbf{u}_\tau} \quad f_{\mathrm{s}}(\mathbf{x}_\tau, \mathbf{u}_\tau; \boldsymbol{\theta}_\tau) \tag{3a}$$

$$\text{s.t.}: \quad \mathbf{x}_\tau \in \mathcal{X}, \ \mathbf{u}_\tau \in \mathcal{U} \tag{3b}$$

$$\mathbf{F}_{\mathrm{s}}(\mathbf{x}_\tau, \mathbf{u}_\tau; \boldsymbol{\theta}_\tau) = \mathbf{0} \tag{3c}$$

$$\mathbf{G}_{\mathrm{s}}(\mathbf{x}_\tau, \mathbf{u}_\tau; \boldsymbol{\theta}_\tau) \leq \mathbf{0} \tag{3d}$$

where $\mathbf{x}_\tau$, $\mathbf{u}_\tau$, and $\boldsymbol{\theta}_\tau$ are the states, controls, and parameters within time step τ. The same functions f_{s}, $\mathbf{F}_{\mathrm{s}}$, and $\mathbf{G}_{\mathrm{s}}$ defined in the static problem P_{s} are applied for different states, controls and parameters in each time step $\tau \in \mathcal{T}$. In regards to the monthly total volume of gas that can be flared, the operations engineers would have to determine the maximum daily flare rate over time, which translates into setting in $\mathcal{U}_\tau$ the value $q_{\mathrm{flare},\tau}^{\max}$, for each time step τ.

Despite this strategy being effective in practice, the imposition of bounds on flaring invariably prevents the outcome from being optimal. Instead the bounds in each period should be tied together by the original coupling constraint, allowing the optimization of the profile of the gas burned in the flare. In this context, the solution of a sequence of static problems with a coupling constraint is more realistic and can mitigate economic losses.

3.2. Pseudo-Dynamic Model

This section proposes a *pseudo-dynamic model* to cope with coupling constraints, and further account for dynamic effects that might impact the decision process, such as well shut-in and restart operations.

Besides the coupling constraints, in some situations the effects of transients become relevant to the extent that the solutions obtained by solving the sequence of *uncoupled* static problems P_τ can be suboptimal, or even infeasible. For instance, in the event of a compression maintenance planned over a short horizon, some wells may have to be shut-in and then restarted later when the system is brought back to full capacity. During a restart operation, a well will undergo complex dynamic behavior for several time periods until it reaches a steady state. In this context, the shut-in and restart operations should be scheduled in order to mitigate the effects of transients, such as the delay between the time lift-gas is injected and the moment it returns to the well head. Transients can also impact the coupling constraint on the total volume of flared gas, as the surplus of lift-gas not reinjected due to a shut-in operation may have to be flared over a short period of time.

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When a production system undergoes transients due to shut-in and restart operations, the decisions taken now may have severe impact in the future, particularly so on delayed operations and coupling constraints. As previously mentioned, an alternative to handle such effects is to incorporate detailed dynamics of the transients into the optimization model. The drawback is the complexity of the resulting model, which according to Foss *et al.* [10] can limit the scalability of the problems that can be solved.

Inspired from a practical application from assets in Petrobras, we propose a different approach to handle the critical effects of dynamics. Our proposal consists in keeping whenever possible the static models and adding surrogate models to approximate the transients, leading to a *pseudo-dynamic model* for planing operations. The advantages include the ability to handle coupling constraints in a more realistic manner while retaining model scalability.

By incorporating the transients, each step becomes dependent of other time steps which couples the sequence of static problems P_τ. To this end, state vectors for each time step should be augmented with new entries to capture the relevant dynamics between consecutive periods. When modeling dynamics, new degrees of freedom appear in the formulation, which may require the control space to be extended with additional variables. Another difference is that the model response becomes dependent of the system initial conditions $\mathbf{x}_0$, since previous states also impact in the current state. For instance, a shut-in well cannot be brought back to full production immediately, but rather the well will undergo dynamics to restart operations until reaching a steady state. In a general form, the pseudo-dynamic model can be cast as:

$$P_{\mathrm{d}} : \quad \max_{\mathbf{X},\mathbf{U}} \ f_{\mathrm{d}}(\mathbf{X},\mathbf{U};\mathbf{\Theta}) \tag{4a}$$

$$\text{s.t.} : \ (\mathbf{X},\mathbf{U}) \in \mathcal{D}_{\mathrm{d}} \tag{4b}$$

$$\mathbf{C}(\mathbf{X},\mathbf{U};\mathbf{\Theta}) \leq \mathbf{0} \tag{4c}$$

where:

- $\mathbf{X} \in \mathcal{X}_{\mathrm{d}} := \widetilde{\mathcal{X}}^{|\mathcal{T}|}$ is an augmented matrix spanning the vector of states over the planing horizon, where $\widetilde{\mathcal{X}} := \mathcal{X} \times (\mathbb{R}^{nc_x} \times \mathbb{Z}^{ni_x})$. The columns of $\mathbf{X}$ contain the state variables $\mathbf{x}_\tau$ in all time periods $\tau \in \mathcal{T}$, augmented with nc_x continuous and ni_x integer variables to capture the dynamics of transients. For instance, the startup stage of a well is an additional variable required to compute the pressure and flow rates during a restart operation.

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- $\mathbf{U} \in \mathcal{U}_d := \widetilde{\mathcal{U}}^{|\mathcal{T}|}$ is an extended matrix containing the controls $\mathbf{u}_\tau$ for all time periods, along with additional controls that play a part when dynamics are involved, where $\widetilde{\mathcal{U}} := \mathcal{U} \times (\mathbb{R}^{nc_u} \times \mathbb{Z}^{ni_u})$. When a well is shut-in, hydrates may arise which requires pipeline cleaning within a critical time frame, a process that must be controlled with additional variables.

- $\mathbf{\Theta} \in \mathbb{R}^{(|\boldsymbol{\theta}|+n_{\boldsymbol{\theta}}) \times |\mathcal{T}|}$ is the augmented matrix with $n_{\boldsymbol{\theta}}$ additional parameters over the planing horizon. To account for variations from the dynamics, the parameters become time dependent and embedded in the matrix $\mathbf{\Theta}$ such as the maximum compression capacity $q_{\mathrm{gtc}}^{\max}(\tau)$. Also $\mathbf{\Theta}$ can contain additional parameters, such as the system initial condition $\mathbf{x}_0$.

- $\mathcal{D}_d$ is the feasible domain for the states and control variables considering both static and dynamic behavior. To this end,

  - the feasible domain for the static behavior is $\mathcal{D}_d^{\mathrm{st}} := \{(\mathbf{X}, \mathbf{U}) \in \prod_{\tau \in \mathcal{T}} \mathcal{D}_\tau^{\mathrm{st}}\}$, with $\mathcal{D}_\tau^{\mathrm{st}} := \{(\mathbf{x}_\tau, \mathbf{u}_\tau) \in \mathcal{X} \times \mathcal{U} : \mathbf{F}_s^{\mathrm{st}}(\mathbf{x}_\tau, \mathbf{u}_\tau; \boldsymbol{\theta}_\tau) = \mathbf{0}, \mathbf{G}_s^{\mathrm{st}}(\mathbf{x}_\tau, \mathbf{u}_\tau; \boldsymbol{\theta}_\tau) \leq \mathbf{0}\}$ in which $\mathbf{F}_s^{\mathrm{st}}$ and $\mathbf{G}_s^{\mathrm{st}}$ are derived from $\mathbf{F}_s$ and $\mathbf{G}_s$ by discarding the equations that no longer comply with steady state conditions. For instance, the static gas balance equation (2b) should be dropped in order to account for dynamic effects, such as the delay from the time gas-lift is injected and returned to the processing facility, which can span over multiple time steps.

  - the feasible domain for the dynamic behavior $\mathcal{D}_d^{\mathrm{dyn}} := \{(\mathbf{X}, \mathbf{U}) \in \mathcal{X}_d \times \mathcal{U}_d : \mathbf{F}_s^{\mathrm{dyn}}(\mathbf{X}, \mathbf{U}, \mathbf{\Theta}) = 0, \mathbf{G}_s^{\mathrm{dyn}}(\mathbf{X}, \mathbf{U}, \mathbf{\Theta}) \leq 0\}$, where $\mathbf{F}_s^{\mathrm{dyn}}$ and $\mathbf{G}_s^{\mathrm{dyn}}$ yield the modeling of dynamic behaviors, which were previously in steady-state and dropped from $\mathbf{F}_s$ and $\mathbf{G}_s$. Further, $\mathbf{F}_s^{\mathrm{dyn}}$ and $\mathbf{G}_s^{\mathrm{dyn}}$ also contain additional equations with regards to the transients. The production function of a well during a startup operation requires additional equations to capture the transients involved until reaching a steady-state.

  Having introduced the above notation and sets, the feasible domain for the pseudo-dynamic model is defined as $\mathcal{D}_d = \mathcal{D}_d^{\mathrm{st}} \cap \mathcal{D}_d^{\mathrm{dyn}}$.

- $\mathbf{C}(\mathbf{X}, \mathbf{U}; \mathbf{\Theta})$ is a vector function that contains the coupling constraints not represented in the dynamic modeling.

- Finally, the objective function $f_{\mathrm{d}}(\mathbf{X}, \mathbf{U}; \boldsymbol{\Theta})$ can be defined for our problem as $f_{\mathrm{d}}(\mathbf{X}, \mathbf{U}; \boldsymbol{\Theta}) = \sum_{\tau \in \mathcal{T}} f_{\mathrm{s}}(\mathbf{x}_\tau, \mathbf{u}_\tau; \boldsymbol{\theta}_\tau)$, but more general functions are admissible.

Formulation (4) represents the framework that combines surrogate dynamic models with static models in a sequence over a planning horizon. This general framework holds under the assumptions stated in Section 2.1. For typical instances of short-term production optimization, such as the case study to be developed below, the static models are considerably complex given the nonlinear nature of the process functions and discrete decisions. Introducing detailed well-reservoir dynamics for scheduling well operations under varying process conditions would render the problem intractable.

Instead, by using an empirical model to capture the main profile of well startup operations, a practical optimization formulation is derived for scheduling operations which can be efficiently solved. In what follows, a case study is presented for the particular application of production optimization of an offshore platform, under conditions of equipment maintenance with well shut-in and restart operations. Derived by field engineers and using information from up-to-date steady-state production models, the surrogate model approximates the production profile with a static curve that consists of a ramp-up curve to render the problem tractable.

4. Case Study: Problem Setup

Herein, we develop a concrete instance of the pseudo-dynamic model P_{d} to optimize the production of an offshore platform under conditions of short-term compression maintenance. Over a prediction horizon, the optimization consists in determining the best operating settings for the wells, such as lift-gas injection rates, well-head pressures, and well shut-in and restart times. As stated in Assumptions 4 and 6, the planning horizon $\mathcal{T}$ is assumed known and sufficiently long to cover the period during which the production system undergoes deviations from the nominal operation. In this sense, a steady state should be reached at the end of the horizon, when the production system has recovered from the shortage in processing capacity. The short-term optimization problem in the static form is the one given by Equations (2).

Let us recall how the pseudo-dynamic problem P_{d} is derived from the static optimization problem P_{s}. The well variables and parameters of the static model P_{s} are spanned over time and the constraints cast in the pseudo-dynamic model. As discussed above, some constraints remain in the feasible

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subset of static constraints, while others are redesigned to incorporate the transients and additional equations that represent the dynamics.

The controls at one time step τ are defined as $\mathbf{u}_\tau := [\mathbf{q}_{gl}, \mathbf{p}_{wh}, \mathbf{l}, q_{exp}, q_{flare}]$ where the first three entries have the gas-lift rates, the well-head pressures and the status for all wells $n \in \mathcal{N}$, and the last two are the gas exported and flared, respectively. The one-step states are defined as $\mathbf{x}_\tau := [\mathbf{q}_o, \mathbf{q}_g, \mathbf{q}_w]$ where each entry has the oil, gas and water flow rates for all wells. The parameters at one step τ are stated as $\boldsymbol{\theta}_\tau := [\boldsymbol{bsw}, \boldsymbol{gor}, q_{turbine}, q_{flare}^{max}, q_l^{max}, q_{gtc}^{max}, \mathbf{x}_0]$ where the entries are the base sediment and water ratio and the gas-oil ratio for all the wells, the rate of gas used for generating energy, and the limits on gas to be flared, liquid to be processed, and gas to be compressed, as well as the system initial conditions. Notice that Assumption 4 is met given that the parameters $\boldsymbol{\theta}_\tau$ are known at each time step of the planing horizon. For this case study, the parameters in $\boldsymbol{\theta}_\tau$ remain the same for all τ, apart from q_{gtc}^{max}, which actually varies through the horizon of the experiment. The duration of the time steps τ are typically given in hours or fractions. Yet their length remain the same for the entire horizon, which should not extend for more than a day.

The problem goal is modified to account for the overall oil production over the planning horizon:

$$f_{time} = \max \sum_{\tau \in \mathcal{T}} \sum_{n \in \mathcal{N}} q_o^n(\tau) \tag{5}$$

Now let us define the four blocks of constraints $\mathbf{F}_d^{st}$, $\mathbf{G}_d^{st}$, $\mathbf{F}_d^{dyn}$, $\mathbf{G}_d^{dyn}$ that determine the feasible space $\mathcal{D}_d$ for the states and controls. The vector function $\mathbf{F}_d^{st}$ contains the static equality functions from $\mathbf{F}_s$ that remain in their original form in all time steps $\tau \in \mathcal{T}$:

$$\begin{cases} q_w^n(\tau) - \dfrac{bsw^n}{1 - bsw^n} \cdot q_o^n(\tau) = 0 & \text{(6a)} \\[4mm] q_g^n(\tau) - gor^n \cdot q_o^n(\tau) = 0 & \text{(6b)} \end{cases}$$

in which the parameters bsw^n and gor^n are assumed time-invariant over the horizon, as stipulated by Assumption 5. Likewise $\mathbf{G}_d^{st}$ has the static inequalities from $\mathbf{G}_s$ expanded for all time steps $\tau \in \mathcal{T}$:

$$\begin{cases} \displaystyle\sum_{n \in \mathcal{N}} (q_o^n(\tau) + q_w^n(\tau)) - q_l^{max} \leq 0 & \text{(7a)} \\[4mm] q_{flare}(\tau) - q_{flare}^{max} \leq 0 & \text{(7b)} \end{cases}$$

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and like before the bounds on flaring $q_{\text{flare}}^{\max}$ and liquid handling $q_{\text{l}}^{\max}$ remain fixed during the horizon. The constraints on liquid handling and flare capacity must reflect Assumption 3, meaning that the platform has the resources necessary to restart wells.

The dynamic blocks of constraints $\mathbf{F}_{\text{d}}^{\text{dyn}}$ and $\mathbf{G}_{\text{d}}^{\text{dyn}}$ have the equations from the static blocks that become dynamic and some additional equations to represent the well transients. First let us introduce a new state variable, $q_{\text{gl,rec}}^n(\tau)$, that represents the rate of gas-lift being recovered from a given well n at time step τ:

$$q_{\text{gl,rec}}^n(\tau) = q_{\text{gl}}^n(\tau - \tau_{\text{dyn}}^n) \qquad (8)$$

where τ_{dyn}^n is a new parameter that measures the average time for the gas-lift to travel through the production system of well n, thereby introducing a time lag. The above equation means that the gas injected at given time step τ will return to the processing facility at a future time step $(\tau + \tau_{\text{dyn}}^n)$.

4.1. Platform Constraints

The constraints that model flow conservation, pressures and operational conditions of the platform are presented below. The equations (2b) and (2f), which respectively model flow balance and gas-compression capacity, are recast in dynamic form:

$$\sum_{n\in\mathcal{N}} [q_{\text{g}}^n(\tau) + q_{\text{gl,rec}}^n(\tau) - q_{\text{gl}}^n(\tau)] - q_{\text{exp}}(\tau) - q_{\text{flare}}(\tau) - q_{\text{turbine}} = 0 \qquad (9a)$$

$$\sum_{n\in\mathcal{N}} \left(q_{\text{g}}^n(\tau) + q_{\text{gl,rec}}^n(\tau)\right) - q_{\text{flare}}(\tau) - q_{\text{gtc}}^{\max}(\tau) \leq 0 \qquad (9b)$$

The former equation differs from its static counter-part, as the flow of gas-lift received at time τ is not equal to the injected gas due to the dynamic effects. The latter equation is as before but replicated over time. As stated in Assumption 3, the demand on gas for electric power generation and the total compression capacity are assumed sufficient for well startup. Any drop in compression capacity expressed by $q_{\text{gtc}}^{\max}(\tau)$ is expected to be short and known, according with Assumption 2.

4.2. Well Surrogate Dynamic Model

This work represents well restart operations with a surrogate model that captures that trends in flows as a function of time, according with the expertise of field operators. This empirical model presented below serves for

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the scheduling of operations while accouting from limited resources. The surrogate model has two phases: the first consists of a period of zero production ("dead time"), whereas during the second phase the production starts and increases until reaching the steady state ("ramp-up"), which is defined by the final operating conditions. This model complies with Assumption 1 by enabling the production engineer to approximate the production profile with any static curve which, in the case of the work at hand, consists of a linear ramp-up curve. Notice that from the flexible nature of the MILP formulation, any other shape of the production profile could be modeled. Figure 2 illustrates the surrogate model for an arbitrary well n. Starting as a shut-in well, the restart operation is initiated at time step τ_1 when the lift-gas rate q_{gl}^n is injected (the target condition). After τ_{dyn}^n units of time have elapsed, the well starts producing at time τ_2 and increases steadily for τ_{ramp}^n unit steps until reaching the steady state at time τ_3.
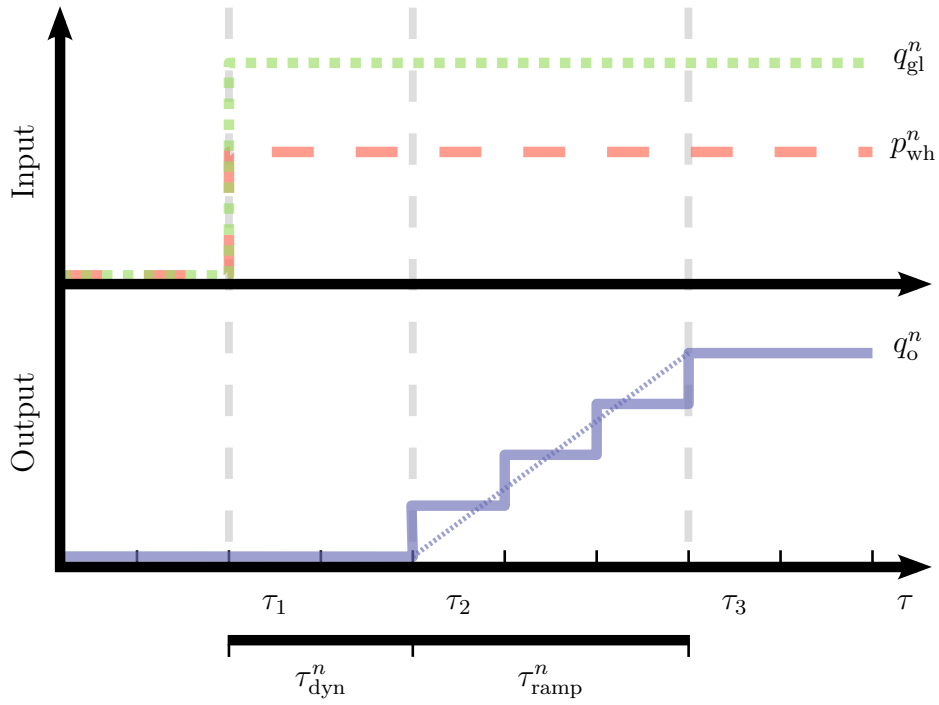
Figure 2: Well startup model.

In order to capture the dynamics of operational states of wells, which may undergo shut-in and restart phases, a set of logic dynamic variables are

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introduced. During each time period $\tau \in \mathcal{T}$, a well $n \in \mathcal{N}$ must be in one of the following conditions $c \in \mathcal{C}$:

- $[c = 1]$: Steady state condition before a shut-in operation;

- $[c = 2]$: Shut-in;

- $[c = 3]$: Restart operation;

- $[c = 4]$: Steady state condition after the completion of a restart operation.

These conditions are regulated by a set of binary variables, whereby $z_c^n(\tau) = 1$ if, and only if, well n is operating according with condition c at time step τ. The logic variables $z_c^n(\tau)$ signal the dynamic conditions of wells, which follow a certain precedence triggered by events to be explained below. Figure 3 illustrates the signal profile of a well that goes through all conditions $c \in \mathcal{C}$: a) starting from steady state conditions for $\tau \in \{1, 2, 3\}$, $z_1^c(\tau) = 1$; b) the well is shut-in during $\tau \in \{4, 5\}$, $z_2^c(\tau) = 1$; c) undergoes a restarting operation during $\tau \in \{6, \ldots, 10\}$, $z_3^c(\tau) = 1$; and d) returns to a steady state condition at time $\tau \geq 11$, $z_4^c(\tau) = 1$.

For each well $n \in \mathcal{N}$, the logic conditions on these binary variables are established by the following equations:

- First, all variables are binary:

$$z_c^n(\tau) \in \{0, 1\}, \tau \in \mathcal{T},\, c \in \mathcal{C}. \tag{10}$$

- Exactly one of the signals should assume a non-zero value at each time period:

$$\sum_{c \in \mathcal{C}} z_c^n(\tau) = 1,\, \tau \in \mathcal{T}. \tag{11}$$

- The signal $z_1^n(\tau)$ starts with 0 or 1 depending on the previous state $\zeta^n \in \{0, 1\}$ of well n, and if it assumes value 0 at some time period τ, it then remains zero until the end of the horizon:

$$z_1^n(1) \leq \zeta^n, \tag{12a}$$
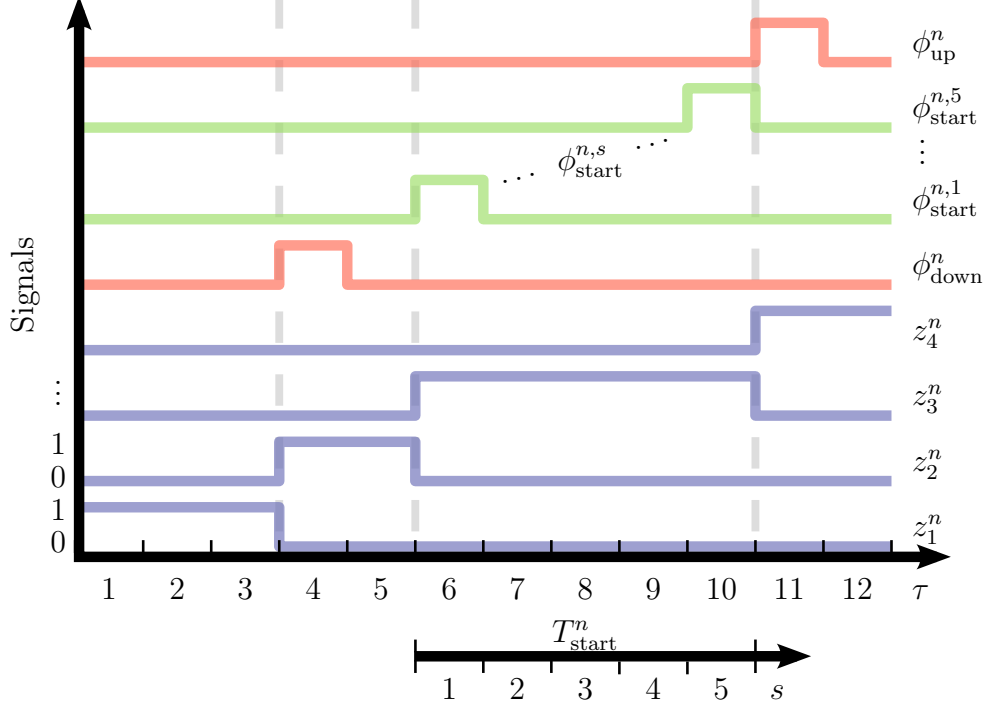$$z_1^n(\tau) \geq z_1^n(\tau + 1),\, \tau = 1, \ldots, T - 1, \tag{12b}$$

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Figure 3: Signal profile $z_c^n(\tau)$ for a given well n and conditions c. Also all the indicator signals $\phi^n(\tau)$, which flag the first time steps a well n is *down*, the time step the well is at each startup phase s, and when the well is back *up*, producing at steady-state

- The behavior of the well shut-in signal $z_2^n(\tau)$ is ensured by the following equations:

$$\phi_{\text{down}}^n(\tau) \in \{0,1\},\ \tau \in \mathcal{T} \tag{13a}$$

$$\phi_{\text{down}}^n(1) = 1 - z_1^n(1) \tag{13b}$$

$$\phi_{\text{down}}^n(\tau) = z_1^n(\tau-1) - z_1^n(\tau),\ \tau = 2,\ldots,T \tag{13c}$$

$$z_2^n(\tau) \geq \phi_{\text{down}}^n(\tau),\ \tau \in \mathcal{T} \tag{13d}$$

$$z_2^n(\tau) \geq z_2^n(\tau+1) - z_1^n(\tau),\ \tau = 1,\ldots,T \tag{13e}$$

according to which $\phi_{\text{down}}^n(\tau)$ takes on value 1 precisely at the first time period τ when $z_1^n(\tau)$ assumes value 0, a condition enforced by Eqs. (13b)-(13c). Eq. (13d) indicates that $z_2^n(\tau)$ becomes 1 at the time period τ when $\phi_{\text{down}}^n(\tau) = 1$. In Figure 3, notice that $\phi_{\text{down}}^n(\tau) = 1$ at the same time $\tau = 4$, thereby flagging the transition of $z_2^n(\tau)$ from 0 to 1.

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Finally, Equation (13e) ensures that once $z_2^n(\tau)$ drops to 0, after being on state 1, it will remain 0 until the end of the planning horizon. However, $z_1^n(\tau)$ is subtracted on the right-hand size to render the inequality innocuous when the well is initially operating in steady-state, which allows $z_2^n(\tau)$ to switch from 0 to 1 (start a shut-in operation).

- The behavior of the restart signal $z_3^n(\tau)$ is given by the following equations:

$$\phi_{\text{start}}^{n,s}(\tau) \in \{0,1\}, \ \tau \in \mathcal{T}, \ s = 1, \ldots, T_{\text{start}}^n \tag{14a}$$

$$\phi_{\text{start}}^{n,s}(1) = 0, \ s = 1, \ldots, T_{\text{start}}^n \tag{14b}$$

$$\phi_{\text{start}}^{n,1}(\tau) = z_2^n(\tau - 1) - z_2^n(\tau) + \phi_{\text{down}}^n(\tau), \tau = 2, \ldots, T \tag{14c}$$

$$z_3^n(\tau) \geq \phi_{\text{start}}^{n,s}(\tau), \ s = 1, \ldots, T_{\text{start}}^n, \ \tau \in \mathcal{T} \tag{14d}$$

$$z_3^n(\tau) \geq z_3^n(\tau + 1) - z_2^n(\tau), \ \tau = 1, \ldots, T-1 \tag{14e}$$

$$\phi_{\text{start}}^{n,s}(\tau) = \phi_{\text{start}}^{n,s-1}(\tau - 1), \ \tau = 2, \ldots, T, \ s = 2, \ldots, T_{\text{start}}^n \tag{14f}$$

where $T_{\text{start}}^n = (\tau_{\text{dyn}}^n + \tau_{\text{ramp}}^n)$ is the number of periods to perform a start up operation until well n reaches a steady state, and $\phi_{\text{start}}^{n,s}(\tau) = 1$ if and only if the start up operation of well n is at stage s during period τ. Notice that $\phi_{\text{start}}^{n,s}(1) = 0$ to indicate that the well is in the steady state or shut-in condition at the beginning of the planing horizon, as imposed by Eq. (14b). From Eq. (14c), variable $\phi_{\text{start}}^{n,1}(\tau)$ takes value 1 when $z_2^n(\tau)$ switches to value 0 at time τ from value 1 at time $\tau - 1$, namely when $z_3^n(\tau) = 1$, as modeled by Equation (14c). (The variable $\phi_{\text{down}}^n(\tau)$ is added to the right-hand side to ensure consistency when the well is shut-in at time τ but operating at time $\tau - 1$.)

Equation (14d) establishes the consistency between the condition variable $z_3^n(\tau)$ and the variables $\phi_{\text{start}}^{n,s}(\tau)$ that keep track of the well startup stage. Equation (14e) states that once the well terminates a startup operation at time τ, as signaled by $z_3^n(\tau) = 0$, the signal variables $z_3^n(\tau') = 0$ for $\tau' > \tau$. (The variable $z_2^n(\tau)$ is deducted in right-hand side to not enforce this condition when the well is undergoing a shut-in operation. Otherwise the equation would prevent $z_3^n(\tau + 1)$ to switch to 1 from $z_3^n(\tau) = 0$ if the well was shut-in at period τ, as signaled by $z_2^n(\tau) = 1$.)

Equation (14f) guarantees the sequence of restart signals from the well. If a startup operation begins in stage $s = 1$ at period τ,

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then the well will progress to the next stages in a sequence for T_{start} periods.

- If the well initiates a restart phase, the operation must be carried out during a specified number of time steps:

$$\sum_{\tau \in \mathcal{T}} z_3^n(\tau) = T_{\text{start}}^n \cdot \sum_{\tau \in \mathcal{T}} \phi_{\text{start}}^{n,1}(\tau) \tag{15}$$

Notice that, if the well n is reopened, $\phi_{\text{start}}^{n,1}(\tau)$ assumes value 1 exactly once at the first instant τ at which $z_3^n(\tau) = 1$.

- The behavior of the signal $z_4^n(\tau)$, which regulates the steady state reached after a startup operation, is ensured by the following equations:

$$\phi_{\text{up}}^n(\tau) \in \{0, 1\}, \ \tau \in \mathcal{T} \tag{16a}$$
$$z_4^n(\tau) \leq z_4^n(\tau + 1), \ \tau = 1, \ldots, T - 1, \tag{16b}$$
$$\phi_{\text{up}}^n(1) = 0 \tag{16c}$$
$$\phi_{\text{up}}^n(\tau) = z_3^n(\tau - 1) - z_3^n(\tau) + \phi_{\text{start}}^{n,1}(\tau), \tau = 2, \ldots, T \tag{16d}$$

whereby $\phi_{\text{up}}^n(\tau)$ takes on value 1 at the time τ when the well reaches the steady state after a startup operation, when $z_3^n(\tau - 1) = 1$ and $z_3^n(\tau) = 0$. In Figure 3, the signal variable $\phi_{\text{up}}^n(\tau) = 1$ at time step $\tau = 11$ when $z_3^n(\tau)$ is switched to 0 and well n arrives at a steady state.

Equation (16b) ensures that once the well reaches a steady state after a startup operation, at time τ, it remains in this state thereafter as signaled by $z_4^n(\tau) = 1$. Equation (16c) states the initial condition of $\phi_{\text{up}}^n(\tau)$. Equation (16d) enforces $\phi_{\text{up}}^n(\tau)$ to assume value 1, at time τ, when the well reaches a steady state condition after a startup operation. The term $\phi_{\text{start}}^{n,1}(\tau)$ is added to the right-hand side of the equation for consistency when the well leaves the shut-in condition and begins a startup operation.

4.3. Well Production

The oil produced by a well n at a given time-step τ depends on the operating condition $c \in \mathcal{C}$. For well $n \in \mathcal{N}$ and time $\tau \in \mathcal{T}$, the production

is given by the following equations:

$$q_o^n(\tau) = \sum_{c \in \mathcal{C}} q_o^{n,c}(\tau) \tag{17a}$$

$$q_o^{n,c}(\tau) = \widehat{q}_o^n(q_{gl}^n(\tau), p_{wh}^n(\tau)) \cdot z_c^n(\tau), \ c \in \{1, 4\} \tag{17b}$$

$$q_o^{n,2}(\tau) = 0 \tag{17c}$$

$$q_o^{n,3}(\tau) = \sum_{s \in \mathcal{S}^n} \widehat{q}_o^{n,s}(q_{gl}^n(\tau), p_{wh}^n(\tau)) \cdot \phi_{start}^{n,s}(\tau) \tag{17d}$$

$$l^n(\tau) = z_1^n(\tau) + z_4^n(\tau) + \sum_{s \in \mathcal{S}} \phi_{start}^{n,s}(\tau) \tag{17e}$$

The oil production results from the sum of production in all conditions c, as stated in Eq. (17a). Steady state production is induced by the function $\widehat{q}_o^n(q_{gl}^n(\tau), p_{wh}^n(\tau))$ of the lift-gas injected and well-head pressure in Eq. (17b). Equation (17c) ensures zero production during well shut-in. Equation (17d) gives the production achieved during each stage of a startup operation. Equation (17e) defines the variable $l^n(\tau)$, which flags oil production by well n during period τ, as a function of the condition signals.

4.4. Additional Behavior Modeling

This section presents constraints that impose desired behaviors on transients, which arise from the modeling of dynamics. Of relevance for the case study, limits on variation of the lift-gas injection and well-head pressure are imposed by the following equations:

$$q_{gl}^n(\tau) - q_{gl}^n(\tau - 1) = \Delta q_{gl}^n(\tau), \ \tau = 2, \dots, T \tag{18a}$$

$$-q_{gl}^{max,n} \cdot \delta^n(\tau) \leq \Delta q_{gl}^n(\tau) \leq \delta^n(\tau) \cdot q_{gl}^{max,n}, \ \tau = 2, \dots, T \tag{18b}$$

$$p_{wh}^n(\tau) - p_{wh}^n(\tau - 1) = \Delta p_{wh}^n(\tau), \ \tau = 2, \dots, T \tag{18c}$$

$$-p_{wh}^{max,n} \cdot \delta^n(\tau) \leq \Delta p_{wh}^n(\tau) \leq \delta^n(\tau) \cdot p_{wh}^{max,n}, \ \tau = 2, \dots, T \tag{18d}$$

$$\sum_{t=2}^{T} \delta^n(\tau) \leq \delta^{max,n} \tag{18e}$$

in which $\Delta q_{ql}^n(\tau)$ and $\Delta p_{wh}^n(\tau)$ are the variation in lift-gas rate and well-head pressure at time step τ, $\delta^{max,n}$ is the maximum number of changes to well controls for the prediction horizon, and $\delta^n(\tau)$ is a binary variable that flags a change in well control at time step τ.

4.5. MILP Formulation for Pseudo-Dynamic Optimization

Having introduced the functions and process models, the pseudo-dynamic problem P_{d} can be obtained by putting together the objective (5) and the constraint equations (6) through (18). The resulting P_{d} is a *conceptual* problem in the sense that the model functions are not explicitly given, for instance the functions $\widehat{q}_{\mathrm{o}}^{n}$ and $\widehat{q}_{\mathrm{o}}^{n,s}$ that model well production. A concrete MINLP formulation would arise by expressing the process functions from first-principles or fitting nonlinear curves to simulated or real data [13]. This approach is rather general and can benefit from existing models, however the solutions approaches are complex and prone to terminate at a local solution that can be far from the optimum.

Derivative-free methods have been applied when simulation models are available for the oil production processes [16, 12]. These methods bring about great flexibly for making mild assumptions, particularly so the model-free algorithms that rely on sampling of the simulated functions. This flexibility comes with a price though, namely high computational cost and slow convergence to local solutions.

A popular alternative is the MILP formulation resulting from piecewise-linear approximation with the given process data points, which can be obtained from real measurements and simulation analysis [17]. The piecewise-linear approximation proved to be very effective in academic and real-world applications, in part for not depending on model fitting but also for the robustness of MILP technology. For these desirable features, Petrobras has adopted such strategies for its in-house production optimization system (BR-SiOP), which is routinely used by engineers to optimize diverse operations [2].

Since the case study considers scenarios from BR-SiOP, the pseudo-dynamic model is approximated with a MILP problem resulting from the piecewise-linear approximation of the well production functions $\widehat{q}_{\mathrm{o}}$ (in steady state) and $\widehat{q}_{\mathrm{o}}^{n,s}$ (during startup). The resulting MILP is denoted hereafter by $\widetilde{P}_{\mathrm{d}}$.

4.6. Remarks and Extensions

In order to capture relevant static and dynamic behavior in a real-world, the resulting pseudo-dynamic model can be rather complex such as the one considered in the case study. Actual operations may account for additional behaviors that can be handled by existing framework, a direct result of the flexibility of the pseudo-dynamic model and MILP optimization.

This paper restricted itself to the most representative behaviors, such as the scheduling of shut-in and startup operation, in order to keep the model manageable and the presentation as succinct as possible. Some additional features and processes that can be of concern include: multiple production streams that are handled by independent compressors and separators; production wells with dual completion, typically with ESPs and gas-lift; subsea layout with a mix of satellite wells and wells that share a manifold and riser; and the scheduling of clean-up operations to prevent the formation of hydrates when a well is shut-in for prolonged periods.

5. Case Study: Analysis

This section is devoted to the analysis of the pseudo-dynamic optimization methodology for scheduling operations in production platforms. The resulting problem $\widetilde{P}_{\mathrm{d}}$ is instantiated for the case study introduced in the previous section, considering data from an oilfield and also sampled from simulation models. The analysis is divided in two parts to better elicit the features of pseudo-dynamic optimization. The first part relies on a compact, and representative scenario derived from the real-world case to illustrate key features and behaviors that are captured by the proposed methodology. The second part analyzes the results obtained with the pseudo-dynamic methodology against field data from the real-world operations of a platform that undergoes a shortfall in gas compression capacity.

5.1. Production Platform

The platform of interest is a Floating Production Storage and Offloading (FPSO) operating in the Campos Basin, Brazil. Figure 4 has a simplified schematic of the production system. At the topside, three compressors supply high-pressured gas to thirteen gas-lifted oil wells. The gas-injection rate and well-head pressure can be used to control the well production over time. Though every well is equipped with a gas-lift valve some of them could operate in surgency. The mixed stream of oil, water, and gas produced by each satellite well converge to a separator at the processing facilities. After treatment, the water is disposed while oil is stored. For the gas stream, a limited amount can be flared, the remaining going through the compression system. The pressurized gas that is not used for power generation or as lift-gas is then exported onshore through a pipeline network.

At steady-state, when gas compression and exportation are at full capacity, not all of the wells can produce at their maximum. Thus, the lift-gas has
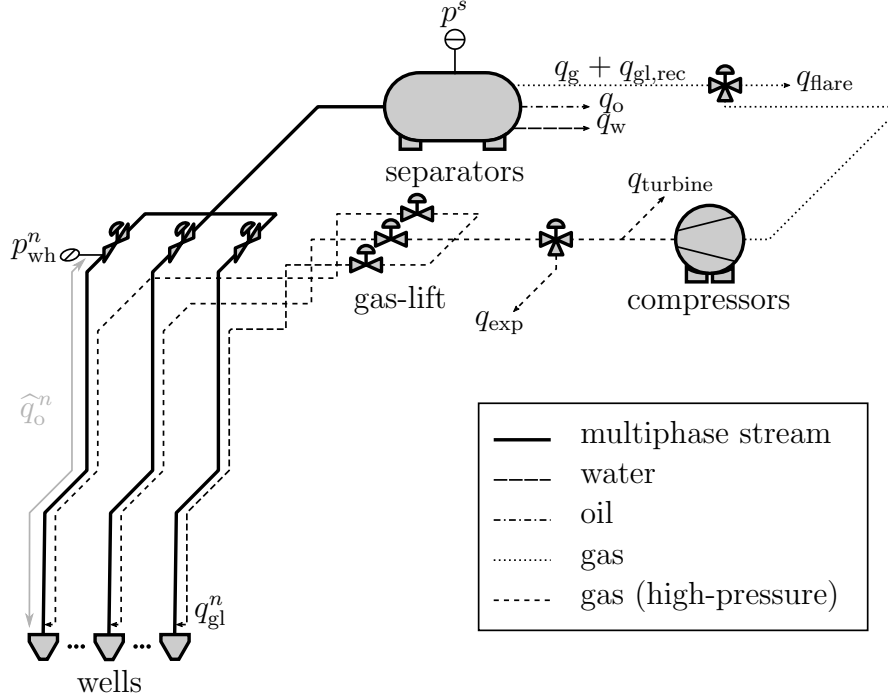
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Figure 4: Simplified schematic of the production platform.

to be properly allocated in order to optimize production. In that regard, engineers are advised by an in-house optimization software (BR-SiOP), which uses data of well performance curves from simulators that are routinely calibrated with test data. As mentioned in Section 4.5, the same modeling strategy is employed to obtain the data needed for the studies, and cast the MILP production optimization problem $\widetilde{P}_{\mathrm{d}}$. The remaining well parameters concern their dynamic behaviors, namely dead-time and average ramp time which are estimated by analyzing historical production records during startup procedures and combining it with the knowledge of the engineers about the field.

5.2. Numerical Studies

By simulation analysis, the benefits of the proposed optimization methodology is assessed in the scheduling of well operations under full and partial compression capacity due to unexpected contingencies. The simulation testbed is composed of a subset of 3 satellite wells, with platform process-

ing constraints adjusted to fit the overall production. The scenarios are illustrative and aim to highlight operational circumstances in which the optimization tool could be beneficial. Although the scenarios are synthetic, the well production curves and other well parameters are realistic since they were drawn from an in-house simulator currently in use at Petrobras, which is tuned to reproduce the field measurements.

Three scenarios are presented in this study: in the first two the compressor undergoes a partial capacity drop, whereas the last consists of a full capacity drop. In the first scenario, the drop in compression capacity is not sufficiently severe to cause the shut-in of wells.

Figure 5 shows normalized values of the optimal oil production, lift-gas injection rates and well-head pressures of the wells. The capacity drop occurs in the beginning of the optimization window and remains until the third time period. Because the gas handling capacity drops considerably, the lift-gas rate is decreased in wells 1 and 3 to keep the wells opened, while well 2 produces without artificial-lift support until the compressor returns to normal operating conditions at time period 4. Because there is no shut-in of wells, the optimal solution is equivalent to the one that would have been obtained by solving a sequence of static problems, one for each time period.

In the second scenario, the compression capacity drops to the extent that it becomes infeasible to process the gas produced from all the wells. The normalized oil production, lift-gas rates and well-head pressures of the wells are plotted in Figure 6. According to the optimal solution, wells 1 and 2 are shut-in during the gas-compression capacity shortfall, being restarted at the forth and third time steps, respectively. Because it can operate in surgency, well 2 is restarted one time step before the compression system returns to full capacity, unlike well 1 which requires lift-gas to operate. On the other hand, well 3 is not shut-in but its production is decreased by approximately 10% during the capacity drop, after which both the lift-gas injection and oil production rates return to their maximum values.

The third and last scenario addresses the problem of well operations scheduling after a full compression capacity drop, as can be seen in Figure 7. All wells remain shut-in for the first two time periods, since the compressor is down and thereby no produced gas can be processed by the facilities. Well 2 restarted in surgency mode one time period ahead of the other wells, as they require lift-gas injection to operate. The platform reaches full production at the seventh time period, when the transients are damped and the wells return to their steady-state operation, which were the prevailing conditions
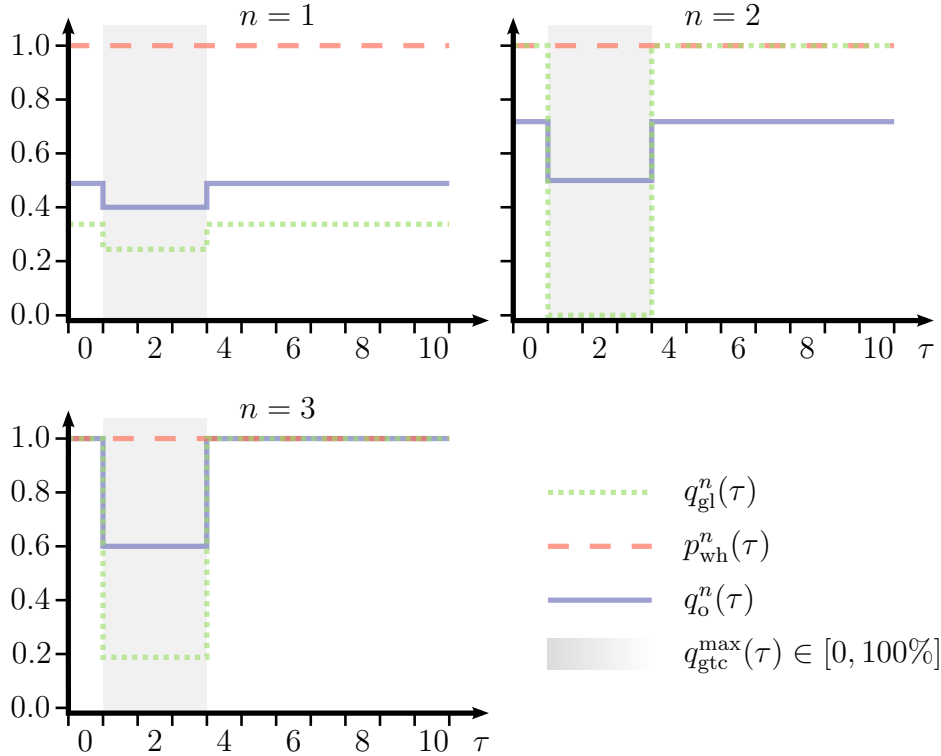
Figure 5: Operations schedule with partial compression capacity drop.

before the compressor shutdown. It is apposite to remark the difficulty, even for an experienced operator, to make the best decisions under such distinct scenarios without the support of such a mathematical optimization tools.

5.3. Field Studies

In order to further assess the use of the proposed methodology in a real application, field measurements obtained during an unexpected failure of the entire compression system are compared against optimization results subjected to the same sequence of events. The scenario comprises the shutdown of the three gas compressors. The high-pressure gas shortage extends for 6 hours, after which one equipment is release for restart. Two hours later the compression system returns to operate at full capacity. At the moment of shutdown all the wells are shut-in. Afterwards, as compressors become ready for use, the wells are started according to a pre-established schedule defined by the operators.
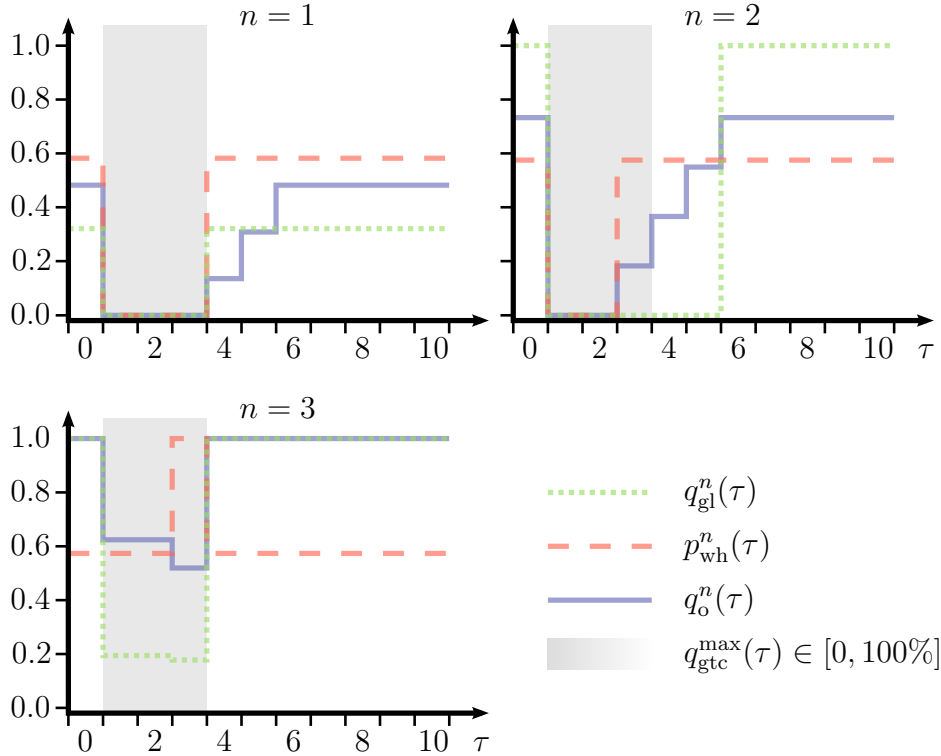
Figure 6: Operations schedule with partial compression capacity drop and shut-in of wells.

On-site measurements over time are available for the well inputs, i.e. well-head pressure and lift-gas rate, and the aggregated of oil, water and gas. Figure 8 has the time line of events from one hour before the shutdown up to 16 hours after the event. The gas compression capacity over time, $q_{gtc}^{max}(\tau)$, is represented by shades of gray at the back of the charts. The normalized controls for 4 of the 13 wells are depicted in Figure 8. The data is averaged by 10 minutes to reduce measurement noise and make the trends clearer.

Notice that some operational practices applied to the well controls deviate from the proposed controls in surrogate dynamic model, in Figure 2). The differences are due to neglected startup dynamics, disregarded for building a simpler representation. For instance, unlike the model, which assumes wells are brought up using a single gas-lift injection step, operators might increase the gas rate gradually as the wells start producing, as
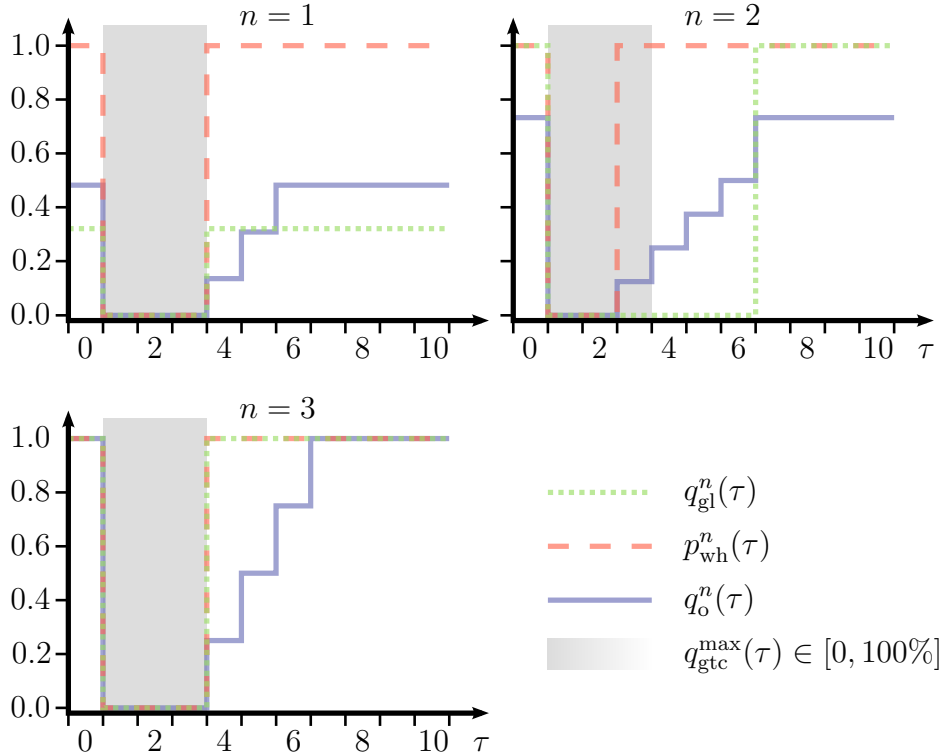
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Figure 7: Operations schedule to recover from a full compression capacity drop.

depicted in the third plot, $n = 7$. Also, they might overshoot the target gas injection rate as shown in the second and fourth plots of Figure 8, $n = 6$ and 13. Another operational behavior that differs from the dynamic model regards the well-head pressure. At startup the wells are closed at the tree level and not opened until pressure exceeds a threshold. Then, at each well-head valve the flow is initially limited, imposing a higher pressure before the target well-head pressure is set. Evidently, such procedures are not expected to be present in the result of the optimization problem, since they were not considered in the surrogate model.

The MILP problem is modeled using GAMS modeling language and solved with CPLEX 12.7.1. The instance has over 4M constraints, 3.5M continuous variables and 17k integer variables. The optimal solution, for a 0.01% gap, is retrieved in less than 1 hour using an Intel® Xeon® CPU E5-2630 v4 @ 2.20GHz, and 4GB RAM. Figure 9 has the operational schedules in the field study. For each well, the upper bar indicates the operations
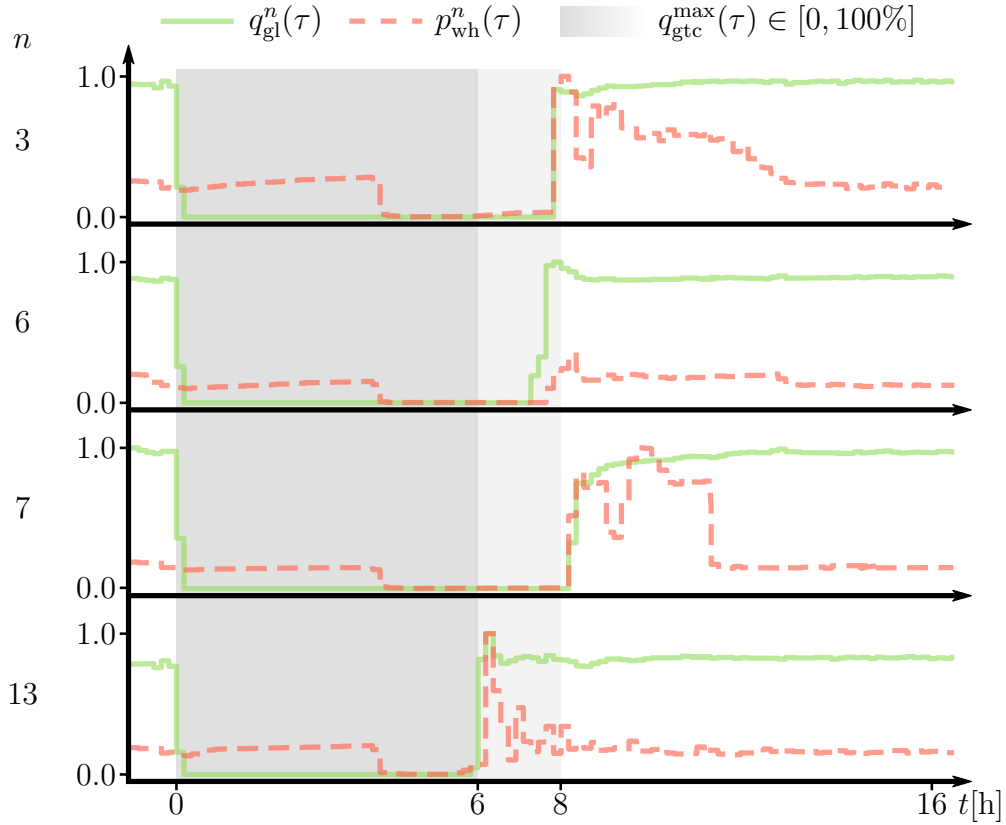
Figure 8: Well input measurements.

executed over time at the platform, while the lower bar has the ones obtained with the optimization model. A well can be either operating at steady state (green), shut-in (red), or at startup (yellow). Comparing the results, the proposed methodology suggest 3 wells to keep the same procedures, 3 should be postponed, and 7 anticipated.

Finally, Figure 10 compares the trend of the aggregated oil production measured at the platform, the blue line, with the estimated production to be obtained if the optimization controls are to be employed, in yellow. The volume of oil generated by the optimization results is 0.83% higher than the volume measured in the field during the time window considered. It is worth mentioning that in practice this percentage should be higher because the production engineer is only given guidelines by the optimization model. The engineer can implement adjustments that will enhance production.
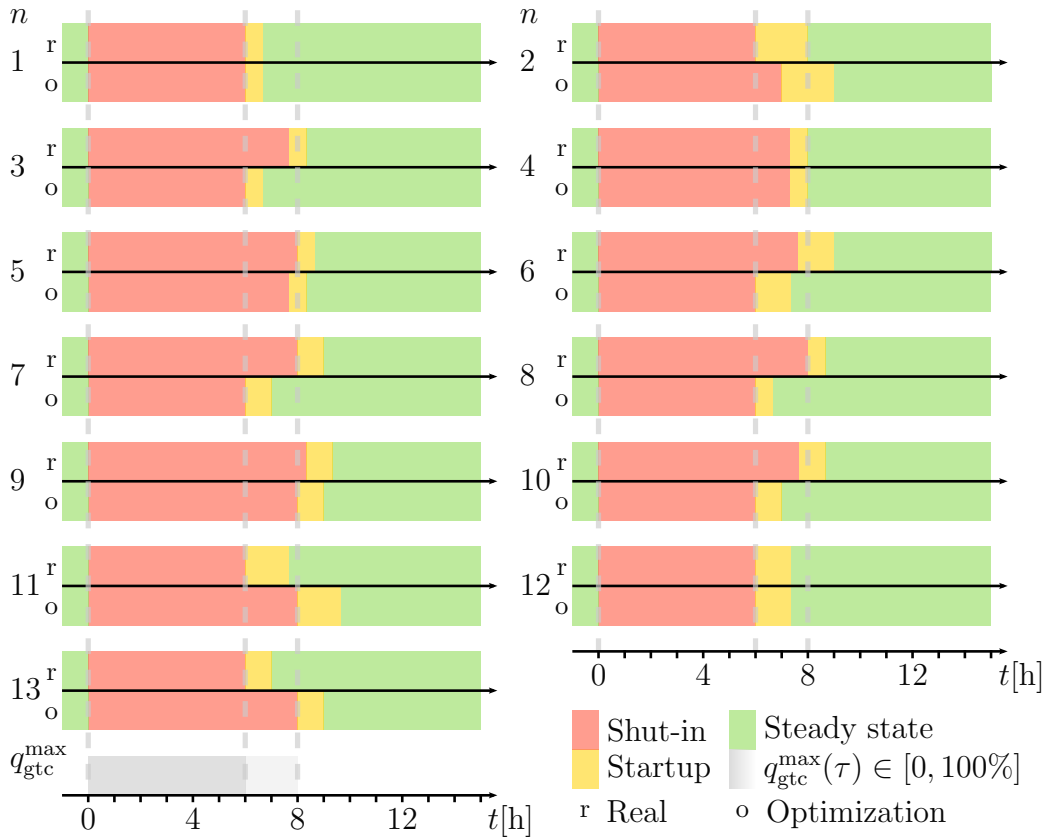
Figure 9: Comparison of the operations schedule applied by operators (upper bars) and the ones obtained with the optimization model (lower bars). The platform is recovering from a full compression capacity drop.

6. Conclusions

By modeling the approximate dynamics of well startup, we are able to determine the best operations schedule of a platform when some wells are shut-in and restarted due to a varying compression capacity. Therefore, the proposed formulation can be employed to support operations both in response to unexpected contingencies or in the case of a planned shutdown of the compression system for preventive maintenance. Additionally, with the use of the proposed formulation, it is possible to handle operational issues that are not typically represented in optimization models, such as well cleaning, restart scheduling priority or even production optimization during the transients.

The simulated results demonstrated that the proposed methodology pro-
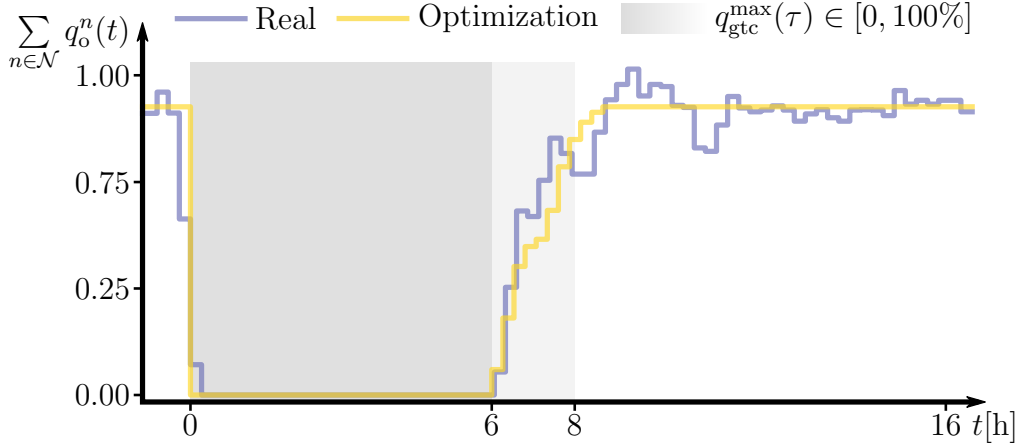
31

Figure 10: Comparison of the aggregated oil production over time between platform measurements and optimization results.

duces consistent operations plans, which reflect chief behavior involving well transients that impact the operations scheduling. The production volumes achieved with the optimized schedule were slightly above the volumes observed by a baseline schedule implemented in practice though. This experiment indicates that better results can be achieved when the methodology is supported by more accurate simulation models and systems parameters. Nevertheless, given the high number of decisions involved in this kind of problem, even for small instances (with few wells and time steps), the proposed decision-support tool can help even skilled engineers to optimize oil production.

In view of the related literature, future work could look into field applications in which dynamic well-reservoir models are involved. Such an investigation would attempt to derive approximate models from the dynamic models, aiming to augment the proposed framework to better capture relevant behavior, while not compromising problem complexity.

Acknowledgments

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