Optimal investment decisions in lice-fighting technologies: A case study in Norway

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Abstract

The Norwegian salmon farming industry is currently facing significant obstacles to future growth. The major challenge is posed by the high level of sea lice in the sea nets at the salmon farms. At the moment both salmon farmers and the supplying industries are working on developing ways to deal with the sea lice problem so that the industry can continue the growth observed during the last decade. This work attempts to fill the lack of investment studies dealing with risk management related to the lice challenge, as well as analysis of investments in new technologies used to abate this problem. Specifically, we address a two-fold investment problem of an aquaculture firm. First, we find the optimal adoption timing of disruptive innovation that has the potential to offer a long-term solution to the lice problem and is subject to uncertain technology development. Second, we find the optimal investment amount in temporary lice-fighting measures. Our results indicate that the high investment cost causes the firm to wait relatively long before adopting the disruptive technology. Consequently, it invests a relatively large amount in short-term methods.

1. Introduction

The challenge of sea lice (Lepeophtheirus salmonis) is currently considered a major threat for further industry growth in the Norwegian salmon aquaculture. Infestations with sea lice cause significant damages to the fish making them susceptible to infectious diseases (Costello, 2006). The high host-density has become a key determinant of an increase of the sea lice levels in salmon farming areas as a result of rapid production growth over the recent decades (Stien et al., 2005; Jansen et al., 2012; Torrissen et al., 2013). This has not only resulted in higher disease transmission rates, but has also negatively affected the welfare of wild stock in the neighborhoods of salmon farms due to the considerable risk of contamination (Johansen et al., 2011; Liu et al., 2011). As a result, the industry is experiencing significant challenges related to the increased costs of lice control (Costello, 2009; Abolafia et al., 2017), as well as the growing environmental concerns related to wild stock.

To ensure a sustainable production and avert lice infections spreading to wild salmon, the Norwegian Ministry of Trade, Industry and Fisheries has put a cap of 0.5 on the average number of grown female salmon lice permitted per fish in a farm (HesCh et al., 2005). If a farm exceeds the maximum allowed number of lice per fish it is forced to slaughter the salmon early, losing valuable increase in slaughter weight. In addition to lice regulations, all forms of aquaculture in Norway require a license and are subject to numerous other restrictions. A license is perpetual and assigns a maximum volume of fish allowed in the water at any given time (Asche and Bjørndal, 2011). Due to a political consensus that environmental aspects of any expansion are important, the Norwegian government has until recently been reluctant to allow for any production increase. One of the few possibilities was the trial development license scheme initialized by the Norwegian government in 2015 (Christiansen and Jakobsen, 2017). A development license allows farmers to produce fish for the purpose of testing new technologies that have the potential to benefit the industry. Development licenses can be converted into regular commercial licenses at the end of a 5-year testing period.

In order to control the lice and comply with the regulations, firms have traditionally used chemicals (Aaen et al., 2015). However, since the first sign of medical resistance in salmon lice was discovered, the industry was forced to rely on non-chemical methods that are more difficult to implement (Jones et al., 2013; Torrissen et al., 2013). The effectiveness of these methods varies for each firm and each use. Therefore, firms typically use a combination of several technologies to achieve the best effect (Abolafia et al., 2017). Nevertheless, the methods available in the market do not offer a long-term solution to the lice problem, and can rather be seen as stopgap measures to temporary mitigate it. We will, therefore, henceforth refer to these measures as short-term solutions. Because of the environmental issues and
regulatory pressure, investment decisions related to short-term technologies are being made rapidly and with incomplete information. Most of the firms do not follow any clear investment strategies and their decisions are mainly based on experiences from other fish farms who have previously tested the technology. Therefore, one of the contributions of this paper is that we explicitly address the short-term investment problem of an aquaculture firm, and determine how much capital a farm needs to allocate to short-term measures until the lice problem is under control.

A recent introduction of a development license scheme has facilitated an increase in technological innovation activity. The projects under consideration offer radical changes, such as offshore or land-based farming, that can potentially restructure the whole industry. These potentially disruptive technologies aim at reducing the sea lice to negligible levels, and preventing the lice problem in the long term. For firms adopting such a technology solution, the traditional short-term measures that are used currently would become obsolete. We will refer to such a disruptive technology solution as long-term technology solution in this paper. Since the development of these technologies is, however, at a very early stage, it is not clear whether and when they will be available in the market, and how quickly they are going to improve upon commercialization. In addition, the adoption of such technologies requires substantial sunk costs. In this situation, there exists a value of flexibility of waiting to receive more information about the development process of the newer technologies before undertaking an investment. As a result, it is not always optimal for the firm to adopt the technology immediately after it becomes available in the market. Therefore, in addition to the short-term investment problem, we determine the optimal adoption timing of the long-term technology solution, taking into account technological uncertainty.

The main contributions of this paper can be summarized as follows. We propose a model that allows to incorporate these recent developments in the Norwegian aquaculture industry. The model offers the means to support the aquaculture firms in their decisions to undertake investments in different types of lice fighting technologies. We use a real options approach to determine the optimal investment strategy in the long-term solution that is subject to uncertain technology development. In addition, we determine the optimal investment amount in short-term measures, given that they become obsolete at an unknown point in the future. Furthermore, we consider an extension of the basic model, where we relax the assumption that the firm can invest in the long-term solution only once, by allowing the firm to perform upgrades. We then conduct a case study of a fish farm in Norway to investigate the implications of the models to provide intuition on how uncertainty in technology affects an investment strategy of the firm.

The remainder of this paper is organized as follows. Section 2 gives an overview of the relevant literature. Section 3 presents the investment problem of an aquaculture firm. The results and sensitivity analysis are presented in Section 4. Section 5 concludes the paper and discusses suggestions for further work. The proofs of propositions and other relevant derivations are presented in the appendices.

2. Literature review

There is a large body of research investigating production efficiency in the aquaculture. Recent studies confirm that despite existing inefficiencies, the aquaculture industry has experienced a substantial productivity growth as a result of the R&D process and technological change (Asche et al., 2018). In light of the recent increase in technological innovation activity, there is a growing interest in the aquaculture literature towards investigating the performance of emerging technologies aimed at coping with the sea lice problem.

Several studies use the data from different sea sites to analyze the efficiency of the short-term methods, that can in general be categorized into three types: preventive, continuous and immediate. The former technologies prevent the lice from attaching to the fish, or entering the net. The studies on the efficiency of preventive technologies, focus on the improvements to traditional sea cages, such as, for example, lice skirts, snorkel barriers and closed floating cages (Stien et al., 2016; Whyte et al., 2016; Nilsen et al., 2017). Continuous methods incessantly keep the sea lice level down. Under this category falls lump fish and other types of cleaner fish that feed on sea lice (Skiftesvik et al., 2013; Imsland et al., 2014). Immediate technologies are acute treatment methods for removing lice when preventive or continuous methods have failed to work. These include, for example, bathing the fish in fresh water pools (Powell et al., 2015). Such treatments are typically performed when the amount of lice in a net is close to, or has exceeded, the legal limit.

Among the technologies, that can potentially be classified as long-term solutions are land-based facilities, that allow to produce salmon in highly controlled environments (Davidson et al., 2016). The studies related to the efficiency of land based systems, such as recirculating aquaculture systems (RAS), have gained relatively widespread attention in the literature, as the technology is currently used for production of smolt (Sandvold and Tvetenás, 2014; Sandvold, 2016) and several other fish species (Ngoc et al., 2016). With the exception of land based containment systems, the research related to the technologies that can offer a long-term solution to the lice problem is rather scarce. The reason is that most of these technologies are still at a very early development stage, and the information about their potential performance is rather limited.

The focus of the studies mentioned above is on the impact of technological improvements on the biological and technological efficiency rather than on the decisions of an individual firm. In turn, the literature on decision making of aquaculture firms addresses the issues related to production planning, rather than technology adoption. This includes, for example, Forsberg (1999) focusing on the optimal harvesting decisions under different management strategies, or Forsberg and Guttmersen (2006) investigating how the information about future price development alters the optimal production plan. However, there is a clear lack of studies that combine these two streams by considering the decision of an aquaculture firm to invest in new technologies. The few available studies that explicitly consider investment decisions are based on traditional capital budgeting methods and do not account for potential uncertainties (see, e.g., Bunting and Shpigel, 2009; Liu et al., 2016; Whitmarsh et al., 2006).

The investment problems in lice fighting technologies are subject to numerous uncertainties - the largest being when, and if, any long-term solution to the lice problem arrives. Many of the technologies in the applications for the development licenses are eligible to be the first long-term technology solution to arrive. If the first batches of salmon farmed in the new facility are successful, commercial adoption may happen quickly. However, if adjustments must be made and further testing is required, commercial adoption will be further down the road. Moreover, the rate at which a disruptive technology develops after the arrival is also subject to uncertainty, as comparable technology development processes do not exist. In addition, the technologies, if commercialized, are expected to require significant sunk costs. These include the investment cost, as well as the costs related to training of the staff and transforming the way the firm produces salmon. When facing an investment opportunity with these characteristics, a salmon farm may wish to delay its investment decision, and first observe the further development of the technology. Real options valuation presents an approach that allows to correctly account for the flexibility of delaying an irreversible decision under uncertainty (Dixit and Pindyck, 1994).

There exists a number of real options studies that focus on the uncertainty in the technological development. An early contribution in this field is Grenadier and Weiss (1997) examining the technology adoption problem of a firm under a stochastic innovation process. They consider a setting, where the state of the technological process is represented by a geometric Brownian motion (GBM), and an arrival of the new technology is triggered when the process hits a fixed boundary.
The drawback of this approach is that in reality the technological progress can rarely be associated with negative shocks. Farzin et al. (1998) accounts for the non-declining nature of the technological progress by modeling it using a Poisson jump process. Huisman (2001) generalizes the model from Farzin et al. (1998) to account for different types of distributions for jump size and concludes that the probability distribution of the size of the jump does not influence the outcome of the model significantly. In this paper, we follow the approach of Huisman (2001) and Farzin et al. (1998) and model technological improvements using a Poisson process with a constant jump size. In addition, we extend their approach to account for the decision to invest in short-term methods before a long-term solution is launched to the market. In practice, when considering this problem, aquaculture firms do not have any incentives to postpone the investment decision despite the uncertainty about the performance of short-term methods. This is because delaying comes at a very high cost of violating the lice regulations and potentially slaughtering all the fish, and, thus, loosing all the future profits. Therefore, it is reasonable to model the decision to invest in short-term methods as a discounted cash flow (DCF) problem with stochastic ending time representing the arrival of the long-term technology. A recent contribution close to our model is Hagspiel et al. (2017) who use a real options approach to model the investment problem of an aquaculture firm under both profit and technology uncertainty. They develop a multi-factor real options model to find the optimal adoption timing of a post-smolt production technology. Unlike Hagspiel et al. (2017) who study the investment in a post-smolt production technology specifically, we model a general setting that can be applied to a wide range of technologies.

3. The investment problem

In this section, we propose a real options model aimed at supporting decision makers in the aquaculture industry when undertaking investment in different types of technologies. Consider a profit maximizing aquaculture firm with one license1 that has to undertake two decisions. First, it needs to determine how much capital to allocate to short-term solutions before the arrival of the long-term technology, which we denote by $l_s$. Second, it needs to decide whether and when to adopt the long-term technology if it becomes available on the market.

The development of the long-term technology is governed by an uncertain technological evolution. Let $\theta$ denote the state of the technological innovation process following a Poisson process, with rate parameter $\lambda_\theta$, i.e. the technology improvement arrival rate, and jump size $\beta$. For purposes of simplification, $\beta$ is assumed to be constant, which is a reasonable approximation to the steady progress made in innovation processes.2 Therefore, we have that $\theta_{t+dt} = \theta_t + d\theta$, where

$$d\theta = \begin{cases} \phi & \text{with probability } \lambda_\theta dt, \\ 0 & \text{with probability } (1 - \lambda_\theta) dt. \end{cases}$$ (1)

Let $\tau_\beta$ denote the arrival time of the long-term technology, and $\tau_0$ be the time that the firm waits with adoption upon its arrival. Then the instantaneous profits of the salmon farmer are equal to

$$p_t(\beta, L_s) = \begin{cases} p_0 - c_u(L_s) & \text{if } t < \tau_0 + \tau_\beta, \\ p_0 - c_\theta(\beta) & \text{if } t \geq \tau_0 + \tau_\beta, \end{cases}$$ (2)

where $p_0$ is the profit of one license net of lice-fighting costs, $c_u(L_s)$ and $c_\theta(\beta)$ denote the marginal operational lice-fighting costs for short-term methods and the long-term solution, respectively.

Here we let lice-fighting costs $c_\theta$ be a function of investment amount $l_\theta$, since we assume that the investments in short-term technologies reduce the level of marginal operational lice-fighting costs, due to increased protection against lice. The more a firm invests in an upgrade, the lower the resulting variable costs of lice are. However, the effect of increased investment amount is diminishing, and the costs will eventually approach a lower boundary $c_\theta$ as upgrades become redundant. Based on this, the properties required for the lice-fighting cost function $c_\theta(l_\theta)$ are: (i) $c_\theta(0) = c_\theta$ at zero investment, the cost function equals the initial lice-fighting costs $c_\theta$, (ii) $\frac{\partial c_\theta(l_\theta)}{\partial l_\theta} > 0$; the cost function is convex, (iii) the function decreases towards a lower boundary $c_\theta$. The properties are illustrated in Fig. 1.

We adopt the same functional form for the lice-fighting cost function as Majd and Pindyck (1989) and Mathews and Baroni (2013), who model the cost function as exponentially decreasing with respectively increasing production capacity and investment. The marginal operational lice-fighting costs are, therefore, given by

$$c_\theta(l_\theta) = c_s + (c_\theta - c_s)e^{-\alpha l_\theta},$$ (3)

where the parameter $\alpha$ is a cost reduction factor representing the rate at which the lice-fighting costs decrease in investment amount. Specifically, a higher $\alpha$ implies a larger decrease in $c_\theta(l_\theta)$. For simplification, we choose to model the lice-fighting costs as constant over time. In reality, the costs may change over time due to seasonal factors. However, the focus of this study is to provide general insights into the optimal investment strategy, which is why the seasonality component is considered out of scope of this paper.

Next, we define a long-term solution as a technology reducing lice-fighting costs down to a fraction, $\beta$, of the initial costs, $c_\theta$. We assume that the arrival of the technology at $\tau_\beta$ is stochastic, and follows an exponential distribution with arrival rate $\lambda_\beta$. In addition, after the long-term solution has arrived, the occurrence of technological improvements follows a Poisson process. According to Huisman (2001), the Poisson process is a natural choice to model technological improvements when the firm has no insight in the development process, which is representative for most companies in the aquaculture industry.3

The lice-fighting costs of a firm producing with a long-term technology $\theta$, are denoted $c_\theta(\theta)$. It is reasonable to assume that $c_\theta(\theta)$ has the same functional form as $c_\theta(l_\theta)$ of the short-term investment. Because the technology offers a reduction in lice-fighting costs, improvements of the technology will reduce lice-fighting costs further. The convexity of the curve is based on the assumption that when a new technology is launched commercially, the initial improvements will be the most effective in reducing lice-fighting costs. As the technology matures, improvements will be less effective and the reduction in lice-fighting costs will thus be decreasing. We therefore let the long-term technology lice-fighting cost be given as

$$c_\theta(\theta) = c_\theta e^{-\alpha \beta \theta}.$$ (4)

Note that at arrival ($\theta = 0$), the lice-fighting costs are a fraction $\beta$ of the original lice-fighting costs, $c_\theta$. As the technology develops and $\theta$ increases, $c_\theta(\theta)$ approaches zero.4

In order to adopt the long-term technology, a firm has to pay a sunk investment cost $I_p$. Because the current candidates to be the first commercialized long-term technology are all highly complex technologies, $I_p$ is assumed to be so high that the firm can only adopt the long-term solution once.

Similar to Huisman (2001), we solve this model by applying real

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1 Due to large variations in the size of aquaculture firms, we consider a general case where we optimize the value of the company per license of 780 t MAB.

2 For a study that considers stochastic jump size we refer to Huisman (2001).

3 The aquaculture industry is highly fragmented, and consists of a large number of smaller companies, and a few large actors (Kvaløy and Tveteras, 2008). It is only the latter that have the resources to perform R&D themselves.

4 This assumption is made for tractability, and can be relaxed to allow for positive lice-fighting costs when $\theta \rightarrow \infty$. 

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options valuation techniques. In the presented setting the salmon farmer holds a perpetual option to invest, but is under no obligation to do so. It can thus freely choose investment timing. When doing so, however, in the presence of the uncertainty about the technological process the firm is facing the following trade-off. On the one hand, it has an incentive to delay investment until until the long-term solution is sufficiently improved, which allows to decrease the lice-fighting costs, \( c_r(\theta) \). On the other hand, the investment is hastened as the company faces larger lice-fighting costs from waiting, as \( c_r(I_u) > c_r(\theta) \). Therefore, the problem we want to address concerns the timing of the firms investment. We seek to determine the optimal investment threshold that triggers the investment, and the optimal value function given the firm is subject to uncertainty in the technological progress. In what follows, we first present the solution for a given value of the short-term investment. We seek to determine the optimal investment threshold that this is not the case.

The optimal investment threshold to invest in the long term technology is given by

\[
\theta^r(I_u) = \sup \left\{ \ln \left( \frac{\beta_c \lambda_p}{r (\alpha_p - c_r(I_u))} \right) \right\},
\]

for \( r \lambda_p > c_r(I_u) \), where \( r \) denotes the discount rate. Note that if \( r \lambda_p \geq c_r(I_u) \), it is never optimal to invest. If the operational lice-fighting costs \( c_r(I_u) \) from a short-term investment are sufficiently low, it would never be optimal to invest in a long-term solution. In what follows we assume this is the case. It is optimal for the firm to postpone the investment in the long-term technology for \( \theta < \theta^r \). The firm is then producing with costs \( c_r(I_u^*) \) from a short-term investment, and its value equals the value of the option to adopt at the level \( \theta \). For \( \theta \geq \theta^r \), it is optimal for the firm to undertake the investment and adopt a long-term technology. In this case, the value of the firm is equal to the perpetual profits of the firm producing with lice-fighting costs \( c_r(\theta^r) \), net of investment cost, \( \lambda_p \).

The value of the firm can be expressed in terms of the value in the three regions depending on \( \phi \) and \( \theta^r \) as follows:

where \( k(I_u) \) equals to

\[
k(I_u) = \frac{\alpha_0 - c_u(I_u)}{\lambda_p} \left( 1 - \frac{(r + \lambda_p)^2}{r} \right) + V(\theta^r(I_u)) - I_p.
\]

Since the technology adoption happens in the future, it needs to be properly discounted. In (6), the term \( \left( \frac{\lambda_p}{r + \lambda_p} \right) \) represents a stochastic discount factor. In particular \( \frac{\lambda_p}{r + \lambda_p} \) represents the discounted value of one unit of money after the next technology arrival. This factor needs to be corrected for the number of technology jumps before investment. Since the technology jumps are constant in this model, we can estimate exactly after how many technology arrivals, \( n^*(I_u) \), the firm is going to invest.\(^5\)

\[
n^*(I_u) = \left\lfloor \frac{\theta^r(I_u) - \theta^r}{\phi} \right\rfloor.
\]

The expected technology adoption time depends on the number of jumps needed to reach the optimal level, and the rate at which these jumps arrive. From (6), we see that the value of the firm is split into three regions. If \( \theta^r \) is large enough, \( \{ \theta^r \} \), it is optimal to undertake the investment immediately. If \( \theta^r \) is in the intermediate range, \( \{ \theta^r \} - \phi \leq \theta < \theta^r(I_u) \), the optimal decision is to postpone the investment until the next technology jump. If \( \theta^r \) is relatively small, \( \{ \theta^r \} \), the investment is not optimal even after the next technology jump. Note that the value of the firm in the continuation region is a function of both \( \theta \) and \( I_u \), whereas the value in the stopping region, only depends on \( \theta \). This is because the value of \( I_u \) affects the threshold \( \theta^r(I_u) \), and determines the profit flow in the continuation region.

With the technology level given in (5), the firm will produce with long-term lice fighting costs

\[
c_r(\theta^r(I_u)) = \frac{\alpha_0 - c_u(I_u)}{\lambda_p} \left( e^\phi - 1 \right) - \phi.
\]

Let \( \tau^r \) denote the expected time until the technology process reaches the technology adoption threshold. It can be further shown that \( \tau^r \) is given by

\[
\tau^r = \frac{n^*(I_u)}{\lambda_p}.
\]

The next step is to solve the optimal investment amount \( I_u^* \) in currently available technology to mitigate lice-fighting costs. We find the optimal investment amount numerically by maximizing the value of the firm with respect to the investment amount. The results are

\(^5\)In principle, \( \theta^r \) can take a value between the increments of the technology process. However, we should take into account that \( \theta \) changes in a discrete fashion, i.e. increases by \( \phi \) at each step. Therefore, the first time the technology level \( \theta^r \) is available to the firm is when the discrete jump process exceeds this value. Thus, we add a ceiling function \( \left\lfloor \frac{x}{\phi} \right\rfloor \) which gives the smallest integer larger, or equal to \( x/\phi \).
presented in Section 4.1. The current (i.e., at time \( t = 0 \)) optimal value of the firm is then equal to

\[
V^{*} = \frac{p_{0} - c_{u}(I^{*})}{r} - I^{*} + \frac{\lambda_{i}}{r + \lambda_{i}} \left( \frac{\theta_{r}}{r + \lambda_{p}} \right) \left( c_{p}(I^{*}) - c_{p}(p^{*}) - I_{p} \right)
\]

(11)

where \( \theta^{*} \) denotes the optimal investment threshold given the optimal investment amount \( I^{*} \):

\[
\theta^{*} = \theta^{*}(I^{*}).
\]

(12)

So far we have assumed that the firm undertakes an investment in the long-term technology without the possibility of preforming operational upgrades. The implication of this assumption is that the firm has larger incentives to wait for a better technology before adoption. This might result in an unrealistically long time until adoption. A more reasonable assumption is that improvements of the long-term technology will become available over time. These can be, for example, improved systems of temperature and water control or monitoring for land-based or offshore facilities that become available in the market over time. Therefore, we consider an extension of the model, where we allow the firm to invest in operational upgrades after the first investment in the long-term technology. The investment problem now consists of several investment decisions. The first decision concerns the question whether or not to undertake the investment in the long-term technology for a given level of technological progress, whereas the rest of the decisions concern the timing of operational upgrades of this technology given that it is adopted.

The detailed derivations of the optimal strategy and the description of the solution methods are presented in the Appendix.

4. Case study context

In order to estimate the parameters for the case study we have interviewed industry experts and analyzed the applications for the development licenses to the Norwegian government. Additionally, we have analyzed multiple reports by Nofima, a Norwegian research institution, that does extensive research on the topic of salmon lice and the cost drivers in the aquaculture industry. Specifically, Iversen et al. (2015) forms part of the basis for the quantification. For company specific parameters, we have consulted a fish farm in Flåtanger, Norway. Bjørøya Fiskeoppdrett AS is a relatively small aquaculture farm with 9.5 licenses for salmon aquaculture. The annual report from 2016 for Bjørøya is used as a base case scenario for our model. The parameters for the case study are given in Table 1.

In order to estimate the value for the initial lice-fighting cost a firm faces per license, \( c_{0} \), we find that the firm had lice-fighting costs of 27.391 million NOK before investments. However, the labor costs related to lice control are not included and thus, 20% of these costs should be added to the operational lice-fighting costs. As the labor costs for 2016 were 26.676 million NOK, we find that the labor costs related to lice control are not included and thus, 20% of these costs should be added to the operational lice-fighting costs. As the labor costs related to lice control are not included and thus, 20% of these costs should be added to the operational lice-fighting costs.

4.1. Baseline model results

By solving the optimal stopping problem for the baseline model numerically with the parameters from Table 1, we find the current value of the firm to be \( V^{*} = 193.816 \) million NOK/license. The optimal investment amount in short-term technologies that lead to this firm value is \( I_{p} = 3.360 \) million NOK/license. The operational lice-fighting costs will thus, be reduced from \( c_{0} = 6.890 \) million NOK/license to \( c_{p}(I_{p}) = 3.565 \) million NOK/license. The long-term technology adoption threshold is found to be \( \theta^{*} = 1.304. \) This implies that the operational lice-fighting costs when operating with the long-term technology will be at \( c_{p}(\theta^{*}) = 0.382 \) million NOK/license.

4.2. Upgrading decision results

Table 2 shows the numerical results for \( n \in \{1,2,3,4\} \) maximum number of switches. Unlike the baseline model, we present the optimal technology level to be adopted, \( \zeta_{k}^{*} \), and not the optimal investment threshold, \( \theta^{*} \), that can lie between two technology jumps. This is because the numerical approach does not solve for the optimal investment thresholds, but for the sequence of technologies it is optimal to adopt.

When relaxing the assumption of a one-time investment and allowing for \( n \) technology switches, the added flexibility increases the firm value, as seen in Table 2. The intuition is that after the adoption of the long-term technology, the firm does not have to operate with the same technology forever. It can afford adopting earlier and then upgrade the solution at a later point in time. Hence, the firm will operate with lower lice-fighting costs \( c_{p}(\zeta_{k}) \) for a longer period of time, which increases the firm value. However, the change in value is small because lice-fighting costs are a small fraction of the firm’s profits. Consequently, an additional upgrade of lice-fighting technologies does not greatly affect the firm’s value.

The optimal investment amount, \( I_{p}^{*} \), decreases when we allow multiple switches, and \( n \geq 2 \). In the multiple switch case, the firm adopts the long-term solution earlier and the incentive to invest in short-term solutions is reduced. Hence, the firm invests less. Note that only the first adoption affects the investment amount, and not the number, nor timing of the following switches. This is because the short-term investment decision only affects the value of the firm until the adoption of the long-term technology. What happens after the adoption is not relevant when finding the optimal investment amount.

Moreover, we see from Table 2 that the optimal technology adoption level in the baseline model, is \( \zeta_{1} = 1.35 \), with expected adoption in 9 years. (Recall that the expected adoption time \( T_{ad} \) is given in years after arrival of the long-term technology.) When giving the firm more flexibility and allowing for \( n = 2 \) switches, the optimal adoption level decreases to \( \zeta_{1} = 1.20 \), and the expected adoption time to 8 years. The reason for the earlier adoption is that the incentive to wait for the technology process to reach a higher level is smaller, as it is no longer the firm’s only chance at obtaining low lice-fighting costs. At the same time, the firm is eager to invest to reduce lice-fighting costs from \( c_{p}(I_{p}) \) to \( c_{p}(\zeta_{1}) \). The firm therefore, makes the first investment earlier to benefit from the lower lice-fighting costs. When the technology level has increased sufficiently compared to when the firm first adopted the long-term technology, it upgrades to benefit from even lower lice-fighting costs \( c_{p}(\zeta_{2}) \). The upgrade is to technology adoption level \( \zeta_{2} = 2.40 \), and is expected to be done in 16 years. The intuition holds for the decrease of \( \zeta_{1} \) in the case of \( n = 3 \) switches. Note, however, that when we further increase flexibility by allowing \( n = 1 \) switches, the investment strategy remains unchanged from \( n = 3 \). With an investment cost of \( I_{p} = 26.7 \) million NOK/license, it is never optimal for the firm to make the first adoption earlier than \( T_{ad} = 7 \), regardless of the additional flexibility. The investment cost is then higher than the benefit from lower lice-fighting costs. Likewise, the switching cost \( I_{p} \) of 1 million NOK/license, constrains the second switch to be done no earlier than \( T_{ad} = 14 \), and the third switch no earlier than \( T_{ad} = 26 \).
The firm waits longer to make the switch from technology \( \pi \), in reducing lice-fighting cost as the technology improves. This implies the lice-fighting costs are reduced. However, the reduction is diminished from (4), when the technology process reaches a higher level, as these were difficult to estimate. We therefore do a sensitivity analysis on the parameter to see how the multiple switch results in Table 2 change as we vary \( \pi \). In order to do this, we look at a scenario where the investment cost is lower, as we considered an expensive scenario in the baseline case. It is reasonable to assume that the technologies being developed will have a lower cost once commercialized, compared to the cost of developing them the first time. We therefore test for an investment cost that is approximately 30% lower, at \( \pi = 20 \) million NOK/license. The results are presented in Table 3.

In this scenario, the optimal technology adoption level, \( \zeta_1 \), has decreased. An investment cost of \( \pi = 20 \) million NOK/license means a lower reduction in lice-fighting costs is required to justify the investment. As a result, the firm value increases when the firm benefits from lower lice-fighting costs \( \zeta_1 \) for a longer period. Note that also \( \zeta_2 \) and \( \zeta_4 \) have decreased, which is surprising as the cost of adopting these technologies is the switching cost \( \pi_1 \) that remains unchanged. This means they are affected by the earlier adoption of \( \zeta_1 \). The firm now operates with higher lice-fighting costs \( \zeta_1(\zeta_1) \) after the first adoption, as \( \zeta_1 \) is lower compared to when \( \pi_1 \) was larger. As the cost of waiting is higher and the firm is eager to invest, the threshold of the following switches will be lower, and the firm will invest in technologies of lower cost.

### Table 2

The value of the firm \( V \) in NOK/license, the optimal investment amount \( \pi_1 \) in NOK/license, the optimal technology adoption level \( \zeta_1 \) and the expected adoption time after arrival in years \( \tau_{\pi_1} \), for the maximum number of switches \( n \).

<p>| Parameters used: ( \pi_0 = 23.14 \cdot 10^6 ), ( c_0 = 6.89 \cdot 10^6 ), ( \zeta_2 = 3.45 \cdot 10^6 ), ( \pi_1 = 26.70 \cdot 10^6 ), ( \pi_2 = 1.00 \cdot 10^6 ), ( \lambda_1 = 0.2 ), ( \lambda_2 = 1.0 ), ( u = 0.15 ), ( \alpha = 1 \cdot 10^{-6} ), ( \beta = 0.2 ). |
|---|---|---|---|</p>
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<th>( \tau_{\pi_1} )</th>
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Table 2 shows that the difference in technology levels \( \zeta_2 \) and expected adoption timing \( \tau_{\pi_1} \) increases for each switch the firm makes. The firm waits longer to make the switch from \( \zeta_1 \) to \( \zeta_2 \). This might seem surprising as the switching cost \( \pi_1 \) is constant, and so is the reduction in lice-fighting costs required to justify the switch. The reason for this can be explained by recalling the lice-fighting cost function of the long-term technology, presented in the beginning of Chapter 3. As can be concluded from (4), when the technology process reaches a higher level, the lice-fighting costs are reduced. However, the reduction is diminishing, meaning that technological improvements become less effective in reducing lice-fighting cost as the technology improves. This implies that the number of technological improvements needed to justify the upgrade from \( \zeta_2 \) to \( \zeta_3 \) is larger than from \( \zeta_1 \) to \( \zeta_2 \). As a result, the firm waits longer after each switch.

### 4.3. Sensitivity analysis

In this section we present the sensitivity analysis of the result with respect to parameters \( \lambda_1 \) and \( \lambda_2 \) as these carry the highest uncertainties, as well as \( \alpha \), \( \lambda_1 \), and \( \lambda_2 \), as these were difficult to estimate. We start by checking the values for \( \pi_1 \) and \( \pi_2 \) for the results of the multiple switch model. As the models respond similarly to changes in \( \lambda_1 \) and \( \lambda_2 \), we present the effects of changes in these parameters only for the baseline model for the sake of tractability.

#### 4.3.1. Varying Investment cost \( \pi_1 \)

As described in Section 3, the investment cost \( \pi_1 \) of the long-term technology is difficult to quantify. Among the applicants for development licenses there are many different technology designs, and the investment cost depends on which technology proves to be most successful. We therefore do a sensitivity analysis on the parameter to see how the multiple switch results in Table 2 change as we vary \( \pi_1 \). In order to do this, we look at a scenario where the investment cost is lower, as we considered an expensive scenario in the baseline case. It is reasonable to assume that the technologies being developed will have a lower cost once commercialized, compared to the cost of developing them the first time. We therefore test for an investment cost that is approximately 30% lower, at \( \pi_1 = 20 \) million NOK/license. The results are presented in Table 3.

In this scenario, the optimal technology adoption level, \( \zeta_1 \), has decreased. An investment cost of \( \pi_1 = 20 \) million NOK/license means a lower reduction in lice-fighting costs is required to justify the investment. As a result, the firm value increases when the firm benefits from lower lice-fighting costs \( \zeta_1 \) for a longer period. Note that also \( \zeta_2 \), \( \zeta_3 \) and \( \zeta_4 \) have decreased, which is surprising as the cost of adopting these technologies is the switching cost \( \pi_1 \) that remains unchanged. This means they are affected by the earlier adoption of \( \zeta_1 \). The firm now operates with higher lice-fighting costs \( \zeta_1(\zeta_1) \) after the first adoption, as \( \zeta_1 \) is lower compared to when \( \pi_1 \) was larger. As the cost of waiting is higher and the firm is eager to invest, the threshold of the following switches will be lower, and the firm will invest in technologies of lower cost.

#### Table 3

The value of the firm \( V \) in NOK/license, the optimal investment amount \( \pi_1 \) in NOK/license, the optimal technology adoption level \( \zeta_1 \) and the expected adoption time after arrival in years \( \tau_{\pi_1} \), for the maximum number of switches \( n \).

<p>| Parameters used: ( \pi_0 = 23.14 \cdot 10^6 ), ( c_0 = 6.89 \cdot 10^6 ), ( \zeta_2 = 3.45 \cdot 10^6 ), ( \pi_1 = 20.00 \cdot 10^6 ), ( \pi_2 = 1.00 \cdot 10^6 ), ( \lambda_1 = 0.2 ), ( \lambda_2 = 1.0 ), ( u = 0.15 ), ( \alpha = 1 \cdot 10^{-6} ), ( \beta = 0.2 ). |
|---|---|---|---|---|</p>
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<td>4</td>
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<td>2.891 \cdot 10^6</td>
<td>0.45</td>
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</tbody>
</table>

In addition, for \( n = 4 \), we see that the firm only makes three switches, even though it has not reached its maximum number of switches. This is simply because the benefit from upgrading is now so small that it does not make up for the switching costs. As the firm will never make any additional switches as long as \( \pi_1 = 1 \) million NOK/license, we do not run any results for \( n > 4 \).
optimal adoption level.

This explains why the firm now makes all four switches in $n = 4$. In the original results, the value of making the fourth switch was too small to justify the switching cost of $I_p = 1.0$ million NOK/license. However, in this case, the firm makes the switch although $I_p$ is unchanged, which implies that the value of the switch has increased. Again, this is because $c_n(c')$ is higher as a result of reduced investment cost $I_p$, meaning that the cost reduction of an extra switch is sufficiently large to justify the cost. In Fig. 2, we illustrate the effect of $I_p$ on the technology levels $c$ for $n = 3$.

Using Fig. 2 to compare the optimal technology adoption levels for $I_p = 26.7$ million NOK/license and $I_p = 20$ million NOK/license, we conclude that the optimal technology levels are much higher for $I_p = 26.7$ million NOK/license. This is because a higher investment cost increases the incentive to wait before adopting a technology, in order for the technology process to reach a higher level.

At some point, $I_p$ reaches levels where it is no longer optimal to switch the technology, or adopt at all. This is illustrated by the peaks in the figure. Recall from (5) for the optimal adoption threshold, that the condition for adoption is $I_p < \frac{c_n(c')}{a}$. When the investment cost becomes so high that it is more optimal to operate with lice-fighting costs $c_n(I_p')$ than to undertake the investment, the firm will never adopt the long-term technology. This is illustrated by the last peak in the figure. The first and second peak show the threshold for $I_p$, where it is never optimal to make the third and second switch, respectively.

For $I_p < 12$ million NOK/license, the firm adopts immediately because the value of adopting the long-term technology is higher than the investment cost, thus, $c_1 = 0$. However, the optimal technology adoption level for the second and third switch are determined by the switching cost $I_p = 1$ million NOK/license, and are therefore higher than zero.

4.3.2. Varying switching cost $I_p$

Similar to the investment cost, there is high uncertainty related to the switching cost of the long-term technology. Therefore, we will check the results of the multiple switch model for a switching cost that is half of what was estimated in Section 4, for the same reason we tested a lower $I_p$. We set $I_p = 0.5$ million NOK/license and present the results in Table 4.

Many of the same effects from changing $I_p$ are visible for a lower $I_p$. Earlier adoption results in an increase in firm value and decrease in investment amount. Compared to the original multiple switch results in Section 4.2, the fourth switch is now made due to both a higher value of the switch, and a lower switching cost. Note that the optimal adoption technology level $c_4$ reaches its minimum at $n = 2$ for the same value as in the original results, $c_1 = 1.05$. This is because $c_4$ is determined by the investment cost $I_p$ of the first adoption of the long-term technology, and all decisions made after $c_1$. Consequently, it is less sensitive to the switching cost $I_p'$ than $c_2$ and $c_3$, which are directly dependent on $I_p$.

Fig. 3 illustrates the effect of $I_p'$ on the optimal technology adoption level for $n = 3$.

Fig. 3 confirms that $c_1$, $c_2$ and $c_3$ increase in $I_p'$. Note also that $c_1$ is the least sensitive to changes in $I_p'$, as seen in Table 4. However, when $c_1$ does increase for a higher $I_p'$, it is because the threshold of adoption has increased due to more costly upgrades. On the other hand, $c_3$ is the most sensitive to change. This is because the effects of a change in $I_p'$ accumulate with every switch, and are therefore higher the later the switch. Further, we compare Fig. 3 to Fig. 2 where $I_p$ was varied, and note that the optimal technology adoption levels behave differently despite increasing levels in both. When varying $I_p'$ it is the change in the first switch that delays the second and third switch. Therefore, the difference between $c_1$, $c_2$ and $c_3$ stays relatively similar for all $I_p'$.

![Fig. 2](image2.png)  
Fig. 2. Optimal technology adoption levels $c_1$, $c_2$, $c_3$, after first, second and third switch, as a function of the investment cost of the long-term technology, $I_p$. Parameters used: $\pi_0 = 23.14 \cdot 10^6$, $c_0 = 6.89 \cdot 10^6$, $c_l = 1.45 \cdot 10^5$, $I_p = 1.00 \cdot 10^6$, $\lambda_l = 0.2$, $\lambda_p = 1.0$, $u = 0.15$, $r = 0.1$, $a = 1 \cdot 10^{-6}$, $\beta = 0.2$.

![Fig. 3](image3.png)  
Fig. 3. Technology levels adopted after first, second and third switch, $c_1$, $c_2$ and $c_3$, as a function of the switching cost of the long-term technology, $I_p$. Parameters used: $\pi_0 = 23.14 \cdot 10^6$, $c_0 = 6.89 \cdot 10^6$, $c_l = 3.45 \cdot 10^5$, $I_p = 26.70 \cdot 10^6$, $\lambda_l = 0.2$, $\lambda_p = 1.0$, $u = 0.15$, $r = 0.1$, $a = 1 \cdot 10^{-6}$, $\beta = 0.2$.

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Table 4

The value of the firm $V^*$ in NOK/license, the optimal investment amount $I_p^*$ in NOK/license, the optimal technology adoption level $c_i$ and the expected adoption time after arrival in years $\tau_{pi}$, for the maximum number of switches $n$. Parameters used: $\pi_0 = 23.14 \cdot 10^6$, $c_0 = 6.89 \cdot 10^6$, $c_l = 3.45 \cdot 10^5$, $I_p = 26.70 \cdot 10^6$, $\lambda_l = 0.2$, $\lambda_p = 1.0$, $u = 0.15$, $r = 0.1$, $a = 1 \cdot 10^{-6}$, $\beta = 0.2$. 
However, when varying $I_p^*$, the difference between the technology levels increases for higher $I_p^*$. This is caused by the accumulating effect as a result of the second and third switch both being directly affected by $I_p$. Finally, we note that the peaks in the figure represent the situation where the upgrading cost $I_p^*$ has become so high that the firm will not undertake the respective switch.

4.3.3. Varying arrival rates $\lambda_l$ and $\lambda_p$

To understand the effects a change in arrival rates $\lambda_l$ and $\lambda_p$ on has on our model, we present a sensitivity analysis with respect to these two variables. Recall that these analyses are only done for the baseline model. In the analysis we will focus on the optimal investment threshold $\theta^*$, rather than the optimal technology adoption level, $\varsigma_1$, as $\varsigma_1$ is a function of $\theta$. Similarly, we focus on the expected time until the investment threshold, $r_{\theta^*}$.

Fig. 4 displays the sensitivity of the investment threshold, $\theta^*$, and the expected time until the investment threshold, $r_{\theta^*}$, with respect to the arrival rate of improvements of the long-term technology, $\lambda_p$.

As seen from Fig. 4, $\theta^*$ increases in $\lambda_p$. To understand this, we look at the expression for $c_p(\theta^*)$ in (9). The optimal lice-fighting costs from the long-term solution, $c_p(\theta^*)$, decrease in $\lambda_p$. The intuition behind this is that if technology improvements arrive sooner, the lice-fighting costs can reach a lower level before the firm chooses to invest. Consequently, the optimal threshold $\theta^*$ is higher for a higher arrival rate. It is worth noting that this only holds if $I_p < \frac{\alpha(\theta^*)}{\lambda_p}$. If this condition is not satisfied, the investment cost is so high compared to the operational lice-fighting costs, that an investment will never become profitable, as seen in (5).

The second graph in Fig. 4 shows the change in expected time until the investment threshold is reached, $r_{\theta^*}$, as a result of the increase in $\lambda_p$. We can observe a general declining trend as result of increase in the arrival rate of the long-term technology. The intuition is that when improvements of the long-term technology arrive more often, the long-term solution becomes more attractive. Hence, the firm is more eager to invest in order to benefit from the lower lice-fighting costs sooner.

Note, however, that the increase in $\lambda_p$ increases the optimal adoption threshold and, as a result, the number of jumps needed to reach this threshold. Due to the fact that $n'(I_u)$ changes in discrete fashion, we observe increasing spikes in the expected time until investment. In the long run, however, this effect is dominated by the decrease of $r_\theta$ for a higher arrival rate of long-term technology improvements.

In a more intuitive way, this can be explained as two contradicting incentives. On one hand, the firm is eager to reduce the time it operates with lice-fighting costs $c_p(I_u^*)$, when improvements arrive more often. On the other hand, it has an incentive to wait before adopting in order to let the technology reach a level that results in even lower long-term lice-fighting costs. The dominating effect is the incentive to invest earlier. This is because it is more costly to operate a longer period until adoption with $c_p(I_u^*)$, than to operate from $r_\theta$ to infinity with slightly lower $c_p(\theta^*)$. The reason for why the long-term lice-fighting costs are only slightly lower for higher arrival rate $\lambda_p$ is the diminishing reduction in lice-fighting costs for higher technology level $\theta$.

Summarizing Fig. 4, a higher arrival rate of the improvements leads to the firm investing earlier in time, and at a higher technology level.

We now vary the arrival rate of the long-term technology, $\lambda_l$, to check the sensitivity for $\theta^*$ and $r_{\theta^*}$. The analyses are done for four different arrival rates of the long-term solution. $\lambda_l = 0.1$ translates to an expected arrival time of 10 years, $\lambda_l = 0.2$ implies arrival in 5 years similar to the baseline case, $\lambda_l = 0.3$ means the technology arrives in approximately 3 years, and $\lambda_l = 0.4$ implies arrival in 2.5 years. Based on our knowledge about the technologies, we find these estimates to be realistic. The results are presented in the Table 5 below.

Note that an increase in $\lambda_l$ means a slightly lower investment threshold. This is because a firm expecting an early arrival of the long-term solution will invest less in short-term solutions. Consequently, the firm has higher costs while waiting to adopt the long-term solution, and is therefore more eager to undertake the investment. A firm expecting a late arrival, on the other hand, will invest more in short-term solutions, and have a smaller cost of waiting. In this case, the investment threshold is higher. Also the optimal adoption timing $r_\theta$ decreases marginally in $\lambda_l$. The intuition is the same as for $\theta^*$. The low sensitivity to $\lambda_l$ implies that the base case is very robust to changes in the arrival rate.

Fig. 5 shows that the optimal investment amount $I_u^*$ changes non-monotonically as $\lambda_l$ increases. This is because of the non-monotonic behavior of the expected time until adoption. First note, that on the one hand, we observe a general declining trend in $I_u^*$ if improvements move forward. In a more intuitive way, this can be explained as two contradicting incentives. On one hand, the firm is eager to reduce the time it operates with lice-fighting costs $c_p(I_u^*)$, when improvements arrive more often. On the other hand, it has an incentive to wait before adopting in order to let the technology reach a level that results in even lower long-term lice-fighting costs. The dominating effect is the incentive to invest earlier. This is because it is more costly to operate a longer period until adoption with $c_p(I_u^*)$, than to operate from $r_\theta$ to infinity with slightly lower $c_p(\theta^*)$. The reason for why the long-term lice-fighting costs are only slightly lower for higher arrival rate $\lambda_l$ is the diminishing reduction in lice-fighting costs for higher technology level $\theta$.

Summarizing Fig. 4, a higher arrival rate of the improvements leads to the firm investing earlier in time, and at a higher technology level.

We now vary the arrival rate of the long-term technology, $\lambda_l$, to check the sensitivity for $\theta^*$ and $r_{\theta^*}$. The analyses are done for four different arrival rates of the long-term solution. $\lambda_l = 0.1$ translates to an expected arrival time of 10 years, $\lambda_l = 0.2$ implies arrival in 5 years similar to the baseline case, $\lambda_l = 0.3$ means the technology arrives in approximately 3 years, and $\lambda_l = 0.4$ implies arrival in 2.5 years. Based on our knowledge about the technologies, we find these estimates to be realistic. The results are presented in the Table 5 below.

Note that an increase in $\lambda_l$ means a slightly lower investment threshold. This is because a firm expecting an early arrival of the long-term solution will invest less in short-term solutions. Consequently, the firm has higher costs while waiting to adopt the long-term solution, and is therefore more eager to undertake the investment. A firm expecting a late arrival, on the other hand, will invest more in short-term solutions, and have a smaller cost of waiting. In this case, the investment threshold is higher. Also the optimal adoption timing $r_\theta$ decreases marginally in $\lambda_l$. The intuition is the same as for $\theta^*$. The low sensitivity to $\lambda_l$ implies that the base case is very robust to changes in the arrival rate.

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### Table 5

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arrive more often, we concluded that the firm would invest earlier in the long run, at a higher technology level $\theta^*$. This decreases the incentive to invest in short-term solutions, as adopting a long-term technology makes these solutions obsolete. Hence, the optimal investment amount is lower. On the other hand, the expected time until investment is increasing in $\lambda_p$, when the effect of an increase in the number of technology arrivals before adoption dominates the general declining trend. This causes the optimal investment amount $I_u^*$ to increase as well, as the firm expects to operate longer with short-term technology.

From Fig. 5, we also see that $I_u^*$ decreases in $\lambda_p$. The intuition is that if the long-term solution arrives sooner, the incentive to invest in temporary, short-term solutions decreases. This effect is strengthened if the arrival rate of long-term improvements $\lambda_p$ is large, making the long-term solution more attractive.

Fig. 6 illustrates how the maximum firm value $V^*$ is affected by $\lambda_p$. The maximum value of the firm $V^*$ changes non-monotonically in $\lambda_p$, but a general increasing trend is evident when the arrival rate of improvements in long-term technology is higher. This is a result of how an increase in $\lambda_p$ affects $\theta^*$, $\tau_{\theta^*}$, and $I_u^*$. As can be seen from Figs. 4 and 5, both expected investment timing, $\tau_{\theta^*}$, and the optimal investment amount in short-term solutions, $I_u^*$, exhibit non-monotonic behavior as a result of an increase in $\lambda_p$, which in turn affects the value function. On the one hand, if a higher $\lambda_p$ leads to an earlier investment timing, and as a result, lower investment amount $I_u^*$, the firm starts benefit from low lice -costs $c_p(\theta^*)$ of the long-term solution earlier. This increases the value of the firm. On the other hand, if it is optimal for a firm to invest later as a result on an increase in $\lambda_p$, the opposite effect is observed. The firm operates longer with short-term technologies, and, therefore, the value in Fig. 6 declines.

We also see that $V^*$ increases in $\lambda_p$. If a long-term technology arrives sooner, the firm operates with lice-fighting costs $c_p(I_u^*)$ for a shorter period. Because $c_p(I_u^*) > c_p(\theta^*)$, this increases the firm value.

5. Concluding remarks

In this paper we have investigated the investment strategy of an aquaculture firm in Norway that faces uncertain technology development. Our study provides several interesting findings. We show that the high investment cost of the long-term solution creates incentives to wait a relatively long time after the arrival of a long-term solution, before adopting it. This results in a relatively large investment amount in currently available technologies in order to lower the lice-fighting cost until it is optimal to adopt a long-term technology. This is particularly interesting in light of the fact that aquaculture firms currently tend to invest rapidly in new and unproven technologies once they become available on the market. Our results, thus, indicate that the firms need to revisit their investment strategies when facing an opportunity to invest in disruptive revolutionary technologies.

We find that the possibility to upgrade increases the firm value due to the additional flexibility. The firm will then adopt the long-term technology earlier, and upgrade it at a lower cost. However, because of diminishing reduction of lice-fighting cost, the firm only performs a limited number of upgrades before it is no longer profitable to do so. That said, the number of upgrades are highly dependent on the investment and switching cost - the smaller the costs, the more upgrades the firm performs.

In what follows we propose several suggestions for future research. First, it would be interesting to investigate the effect of increased operating costs when the first adoption of the long-term technology is made. All current suggestions for long-term solutions to the lice problem constitute far more complex structures than the pens used in the industry today. More complex technologies imply higher operating costs, which are however, also likely to decline as the technology matures. Furthermore, as discussed in Section 1, there are high, unrecorded costs related to treatment of fish, as a result of diseases, death, and lost growth due to starvation prior to the delousing treatment. However, these costs can vary with the technology used, the fish health, and with the number of previous treatments. To include these factors in the model, a component can be added to the lice-fighting cost function $c_p(I_u)$. Finally, an interesting extension would be to consider a stochastic jump size in technological progress, which allows a more realistic modeling of technological uncertainty.

Appendix

Investment model solution

The maximization problem of the fish farmer is then given by

$$\sup_{I_u \geq 0} \left\{ I_u^{\Pi_\theta} (m - c_p(I_u)) \sigma - I_u + \int_{I_u} I_u^{\Pi_\theta} (m - c_p(\theta)) \sigma - I_u + \int_{I_u}^{\Pi_\theta} (m - c_p(\theta)) \sigma - I_u + \int_{I_u}^{\Pi_\theta} (m - c_p(\theta)) \sigma - I_u \right\} = 0 \right\}$$

where the first term in represents the value in the period before the
arrival of the permanent technology, whereas the last two terms are the values before and after its adoption, respectively. The expectation is conditional on $\theta_0 = 0$, since the evolution of technology process, $\theta_t$, is assumed to start at zero after it becomes available in the market.

In what follows, we solve the problem in (13) backwards. We first consider the case when the long-term technology is already available in the market at the level $\theta$ and find its optimal adoption time $c_\theta$. Then we look at the problem before the arrival of the long-term solution, and solve for the optimal investment amount in short-term technologies $I^*_u$.

The optimal stopping problem of a firm after the long-term technology has arrived is given by

$$\sup_{t_0} \mathbb{E} \left[ \int_{t_0}^\infty (\pi_0 - c_\theta) e^{-r(s-t_0)} ds + \int_{t_0}^\infty (\pi_0 - c_\theta) e^{-r(s-t_0)} ds - I_p e^{-r_0} \right] = \hat{\theta}.$$

(14)

The state space of this problem consists of a stopping region, where it is optimal to undertake an investment, and a continuation region, where it is optimal to wait. According to Proposition 2 in Chapter 2 of Huisman (2001), there exists a unique threshold $\theta^*$ that separates these regions, if the profit function is concave in $\theta$. In our model it holds that

$$\frac{\partial^2 \pi (\theta)}{\partial \theta^2} = \frac{\partial^2 (\pi_0 - \beta_0 e^{-\theta})}{\partial \theta^2} = -\beta_0 e^{-\theta} < 0,$$

which means that the profit function is concave, and consequently, a unique threshold $\theta^*$ exists.

If the current value of $\theta_0$, i.e. the technology available in the market, denoted by $\theta$, is in the stopping region, then it is optimal to undertake the investment immediately. If $\theta$ is such that it is still optimal to wait with investment, this value of $\theta$ belongs to the continuation region. The optimal investment threshold, therefore, must be such value of $\theta$, for which the firm is indifferent between investing and waiting. Therefore, the value of stopping, i.e. investing immediately for this level of $\theta$, denoted by $V(\theta)$ and the value of waiting with investment, denoted by $F(\theta, I_0)$, must be equal.

We start by defining the value in the stopping region, where the firm receives the profit flow $\pi_0 - c_\theta(\theta)$ from producing with a long-term technology $\theta$. Note that upon investment the technology level adopted by the firm is fixed, and the evolution of $\theta_t$ does not play a role. The value of the firm upon adoption a long-term technology $\theta$ is, therefore, equal to

$$V(\theta) = \int_0^\infty (\pi_0 - c_\theta(\theta)) e^{-r_0} dt - I_p,$$

(15)

where $I_p$ is the investment cost of adopting the technology.

In order to find the value in the continuation region we divide the continuation region into two parts. In the first part investing is optimal after the next jump, i.e. $(\theta \theta^* - \phi \leq \theta < \theta^*)$, while in the second part investing is not optimal even after the next jump, i.e. $(\theta \theta^* - \phi \geq \theta)$. We let $F_0(\theta, I_0)$ and $F_1(\theta, I_0)$ denote the value of the option in the first and second part of the continuation region, respectively. In both parts of continuation region, the value of the firm, $F(\theta, I_0)$, must satisfy the Bellman equation given by

$$r_0 F(\theta, I_0) = \pi_0 - c_\theta(\theta) + \lim_{\theta \to \theta^*} \frac{1}{\theta^* - \theta} \mathbb{E} \left[ dF^*(\theta, I_0) \right].$$

(16)

For the first part of the continuation region, $\theta < \theta^* - \phi$, we find the value by applying Ito’s lemma to $\lim_{\theta \to \theta^*} \frac{1}{\theta^* - \theta} \mathbb{E} \left[ dF^*(\theta, I_0) \right]$ and inserting into (16). This gives the differential equation

$$r_0 F_0(\theta, I_0) = \pi_0 - c_\theta(\theta) + \lambda_0 \left[ F(\theta + \phi, I_0) - F_0(\theta, I_0) \right].$$

(17)

By trial and error, we find the solution to (17) to be given by

$$F_0(\theta, I_0) = K \left( \frac{\lambda_0}{r + \lambda_0} \right) \left( \frac{\theta - \theta^*}{\theta^* - \theta} \right) + \frac{\pi_0 - c_\theta(\theta)(r + \lambda_0)}{r} \ .$$

(18)

where $K$ is a constant found by value matching at the boundary between the two parts of the continuation region, $\theta = \theta^* - \phi$. This solution can be verified by substitution.

We now consider the second part of the continuation region, where $\theta^* - \phi \leq \theta < \theta^*$. We again apply Ito’s lemma to $\lim_{\theta \to \theta^*} \frac{1}{\theta^* - \theta} \mathbb{E} \left[ dF^*(\theta, I_0) \right]$. Combining the result with (16) gives

$$r_0 F_1(\theta, I_0) = \pi_0 - c_\theta(\theta) + \lambda_0 \left[ F(\theta + \phi, I_0) - F_1(\theta, I_0) \right].$$

(19)

Rearranging gives the following value of the firm in the second part of the continuation region:

$$F_1(\theta, I_0) = \frac{\pi_0 - c_\theta(\theta)}{r + \lambda_0} + \frac{\lambda_0}{r + \lambda_0} \left[ V(\theta + \phi) - I_p \right].$$

(20)

By value matching at $\theta = \theta^* - \phi$, we have that $F_0(\theta^* - \phi, I_0) = F_1(\theta^* - \phi, I_0)$. By setting (18) equal to (20), we find that

$$F(\theta, I_0) = \left\{ \begin{array}{ll} k(I_0) \left( \frac{\lambda_0}{r + \lambda_0} \right) \left[ F^*(\theta) \right] & \text{if } \theta < \theta^*(I_0) - \phi, \\ \frac{\pi_0 - c_\theta(\theta)(r + \lambda_0)}{r} & \text{if } \theta^*(I_0) - \phi \leq \theta < \theta^*(I_0), \\ V(\theta) - I_p, & \text{if } \theta \geq \theta^*(I_0), \end{array} \right.$$
The optimal investment threshold is then given by

$$
\theta^*(I_u) = \sup \left\{ \ln \left( \frac{c_u(I_u)}{r(l - c_u(I_u))} \right) \mid 0 \leq \theta \leq \theta^*(I_u) \right\},
$$

(23)

for \( r \ell_p < c_u(I_u) \).

From (21), we see that the value of the firm is split into three regions. If \( \theta \) is large enough, \( \theta \theta^*(I_u) \), it is optimal to undertake the investment immediately. If \( \theta \) is in the intermediate range, \( \theta^*(I_u) - \phi \leq \theta < \theta^*(I_u) \), the optimal decision is to postpone the investment until the next technology jump. If \( \theta \) is relatively small, \( \theta < \theta^*(I_u) - \phi \), the investment is not optimal even after the next technology jump.

Note that the value of the firm in the continuation region is a function of both \( \theta \) and \( c_u \), whereas the value in the stopping region, only depends on \( \theta \). This is because the value of \( c_u \) affects the threshold \( \theta^* \), and determines the profit flow in the continuation region.

From (23), note that if \( r \ell_p \geq c_u(I_u) \), it is never optimal to invest. If the operational lice-fighting costs \( c_u(I_u) \) from a short-term investment are sufficiently low, it would never be optimal to invest in a long-term solution. In what follows we assume this is not the case. Now we solve for the optimal investment amount, \( I_u^* \) to find the solution to (13). We rewrite (13) as

$$
\sup_{I_u} \left\{ \int_0^{I_u^{l,n} + \phi} (\alpha_0 - \alpha(I_u)) e^{-\gamma s} ds - \lambda_p I_u + \int_{I_u^{l,n} + \phi}^\infty (\alpha_0 - \alpha(I) + \lambda_p e^{\alpha(I)} - I_p) e^{-\gamma (s + l) - \phi} ds \mid \theta = 0 \right\}.
$$

(24)

By inserting for (23) and maximizing (24) with respect to \( I_u \) and \( \theta \), we find that

$$
V^* = \frac{\alpha_0 - \alpha_c(I_u^*)}{r} - \lambda_p I_u + \frac{\lambda_p}{r + \lambda_p} \left\{ \left( \frac{\alpha_0 - \alpha_c(I_u^*)}{r} - \frac{(r + \lambda_p)^2}{r + \lambda_p} \right) - \frac{F_2(\theta^*, \phi + \phi) - I_p}{\phi} \right\}.
$$

(25)

We find the optimal investment amount by numerically maximizing the value of the firm with respect to the investment amount.

**Upgrading problem solution**

We now let \( \zeta_i \) denote the optimal technology level adopted by the company after the \( i \)-th switch for \( i \in \{1, \ldots, n\} \). Note that because it is not possible to adopt a partial technology level, \( \zeta_i \) is a discrete variable with increments of jump size \( \phi \). Furthermore, \( c_p(\zeta_{i-1}) \) now denotes the lice-fighting costs of the company from the previous switch. However, before the first switch when \( i = 0 \), the lice-fighting costs are \( c_u \) from the investment \( I_u^* \) in short-term solutions. The value of the firm before the first switch is therefore given by

$$
F_1(\theta, I_u) = \frac{\rho_0 - \rho_c(I_u)}{\rho} \left( 1 - \frac{(r + \rho)^2}{r + \rho} \right) + F_2(\theta, \phi + \phi) - I_p
$$

if \( \theta > \theta^*_1 - \phi \),

$$
\frac{\rho_0 - \rho_c(I_u)}{\rho} + \frac{\rho}{\rho + \rho_p} (F_2(\theta + \phi, \phi + \phi) - I_p)
$$

if \( \theta^*_1 - \phi \leq \theta < \theta^*_1 \),

$$
F_2(\theta, \phi) - I_p
$$

if \( \theta \geq \theta^*_1 \).

(26)

The last stopping problem is equal to the problem solved earlier. Therefore, the value of the firm before the last technology switch is given by

$$
F_n(\theta, \zeta_{n-1}) = \frac{\rho_0 - \rho_c(\zeta_{n-1})}{\rho} \left( 1 - \frac{(r + \rho)^2}{r + \rho} \right) + \frac{\rho_0 - \rho_c(\zeta_{n-1})}{\rho} (r + \rho)\left( F_2(\theta + \phi) - I_p \right)
$$

if \( \theta < \theta^*_n - \phi \),

$$
\frac{\rho_0 - \rho_c(\zeta_{n-1})}{r + \rho_p} + \frac{\rho}{r + \rho_p} (F_2(\theta + \phi) - I_p)
$$

if \( \theta^*_n - \phi \leq \theta < \theta^*_n \),

$$
\rho_0 - \rho_c(\zeta_{n-1}) + (V(\theta) - I_p)
$$

if \( \theta \geq \theta^*_n \).

(27)

where \( I_p^* \) is the switching cost. After the first investment, we assume the upgrades of the adopted, long-term technology are done at a substantially lower cost than the initial investment cost of \( I_p \). Furthermore, for the \( i \)-th optimal stopping problem where \( i \in \{2, \ldots, n - 1\} \), the value of the firm before the \( i \)-th switch is given by
Eq. (28) is a generalization of (26) that accounts for \( n \) switches. It also differs from (27) because \( F_i(\zeta_{i-1}) \) depends on the value before the next switch at \( i + 1 \), which again includes the value of future switches. However, when \( i = n \) there are no more switches to be made and the value of the firm is therefore deterministic.

The value matching equations at \( \theta = \theta_i^{*} \) cannot be solve analytically. This is because \( \zeta_i \) a discrete variable, meaning that a value that satisfies the equality cannot be found. We therefore present an algorithm to determine a numerical solution to the investment problem.

Recall that \( \theta_t \) is a discrete variable with increments of jump size \( \phi \). Each value \( \theta \) represents a technology available, and to give the numerical algorithm a stopping criteria, the number of technology jumps is now limited to \( N \). It is, however, essential that \( N \) is set sufficiently high so that it does not constrain the technology adoption strategy. Further, let \( i \) be the number of switches the firm has already done, and \( n \) be the maximum number of switches a firm can do. A solution is found by exploring all possible investment strategies, comparing them and choosing the one with the highest value.

To find the value of the firm, we define a numerical approximation of the value function, similar to Huisman (2001). Let \( j_i \) denote the number of the technology used by the firm before switch \( i \), and \( m_i \) the number of the technology the firm switches to at switch \( i \). As there are no incentives to adopt an old technology, we assume the firm always adopts the best technology available after a switch. The firm’s value function is given by

\[
F_i(\zeta_{i-1}) = \begin{cases} 
\left( \frac{\lambda_p}{r + \lambda_p} \right)^{\frac{n_i - 1}{r}} \left( \frac{\lambda_p - c_p(\zeta_{i-1})}{r} \right) \left( 1 - \frac{(r + \lambda_p)^i}{r} \right) & \text{if } \theta < \theta_i^{*} - \phi, \\
\pi_0 - c_p(\zeta_{i-1}) + \frac{\lambda_p}{r + \lambda_p} (F_{i+1}(\theta + \phi, \theta + \phi) - \pi_i) & \text{if } \theta_i^{*} - \phi \leq \theta < \theta_i^{*}, \\
F_{i+1}(\theta, \theta) - \pi_i & \text{if } \theta \geq \theta_i^{*}.
\end{cases}
\]  

(28)

For \( i \in \{0, \ldots, n - 1\} \) we have that,

\[
F_{i+1}(\theta, \theta) = V(c_p(\theta_j)) + E[e^{-(\gamma - \delta)\tau_j}(g_{i+1}(m_{i+1}) - V(c_p(\theta_j)))].
\]

(30)
The first term on the right hand side in (30) is the value of the firm when producing with the current technology. The second term is the expected gain from upgrading from technology \( j_i \) to \( m_i \), where \( g_i(m_i, m_{i-1}) \) is the value of the firm in the next switch. The expected gain is discounted by rate \( r \) from the adoption time of technology \( m_i \), \( T_{mi} \), to the adoption time of technology \( j_i, T_{ji} \). From (29) we see that when the firm has made \( n \) switches, the value of the firm is simply the value of producing with its current technology.

Fig. 7 illustrates how the algorithm finds the firm value and the optimal technology adoption strategy by comparing all possible combinations and choosing the one with the highest value. Each branch in the tree represents a sequence of technology adoptions. These range from the leftmost branch, where the firm does not adopt the long-term solution at all, to the rightmost branch, where the firm waits until the last technology arrives and adopts this when it arrives. In each node, the algorithm uses (29) to calculate both the value of producing with the current technology, and the expected gain of future switches. To find the expected gain, the value of the firm in all child nodes is calculated by the algorithm, which chooses the one with the highest value. This creates a recursive call that propagates throughout the tree until the base case is reached in each leaf node.

More formally, we can define the optimal investment strategy by letting the technology levels \( \theta_i \) for \( i \in \{0, \ldots, N\} \) be given. Let \( m_i^* \) denote the optimal number of the technology adopted in switch \( i = 1 \), so that the switch maximizes Eq. (30). The firm must then choose the \( m_i \) that maximizes the value of a firm switching from \( m_{i-1} \) to \( m_i \). It is then optimal for the firm to adopt the technology levels \( \theta_i = \theta_{m_i^*} \) for \( i \in \{1, \ldots, n\} \) at the time they arrive, where

\[
m_i^* = \arg\sup_{m_i \in m_{m_i-1}} \left( f_{m_i}(m_i^*, m_i), i \in \{1, \ldots, n\}. \right)
\]

This gives a solution to the optimal stopping problems of our model. The next step is to determine the optimal investment amount \( L_i^* \) in short-term solutions. As in the single-switch case, \( L_i^* \) impacts the solutions to the optimal stopping problems. Therefore, the optimal stopping problems and the optimal investment amount must be solved simultaneously.

Similarly to the baseline case, we must now solve for the optimal investment amount, \( L_i^* \). We find that the maximum value of the firm is given by

\[
V^* = \sup_{L_i} \lim_{T_i \to -\infty} \left[ \int_0^{T_i+r \theta_i} (x_0 - c_i(L_i))e^{-r \theta_i} ds - I_0 \right] + \sum_{i=1}^{n-1} \int_{T_{ji} + r \theta_i}^{T_{j_i} + r \theta_i} (x_0 - c_i(\theta_i))e^{-r \theta_i} ds - I_0 e^{-r(T_j + r \theta_j)} + \sum_{i=1}^{n} \int_{T_{ji} + r \theta_i}^\infty (x_0 - c_i(\theta_i))e^{-r \theta_i} ds \right].
\]

The optimal investment amount, \( L_i^* \), and the expected optimal adoption timing, \( T_{mi} \), can only be found numerically.

References

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