Comparative Eigenvalue Analysis of Synchronous Machine Emulations and Synchronous Machines

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***Abstract*—This paper presents a comparative eigenvalue analysis of the stability characteristics and small signal dynamics of four different control strategies for Synchronous Machine Emulation (SME) by power electronic converters, considering a Synchronous Machine (SM) as the benchmark system. The four SME techniques are selected to represent the most established general approaches for emulating the inertial characteristics of SMs in the control of power electronic converters. The small-signal stability assessment is based on the analysis of system eigenvalues, including evaluation of participation factors and parametric sensitivities. All the investigated techniques can be tuned to obtain similar inertial dynamics under grid frequency variations, but exhibit differences in other small-signal characteristics due to the distinct control system implementations. Among the analyzed cases, the current-controlled virtual synchronous machine has the highest damping of the most oscillatory mode. However, the study shows that the most oscillatory modes of the other techniques are associated with the *LCL* impedance, and could be further attenuated by active damping techniques.**

***Index Terms*—Inertia Emulation, Frequency Control, Small-Signal Stability, Synchronous Machine Emulation, Swing Equation, Synchronverter, Virtual Synchronous Machine.**

I. INTRODUCTION

Increasing penetration of converter-interfaced generation systems in a power system may lead to challenges for the frequency control because power electronic converters do not inherently contribute to the inertia of the power system [1], [2]. To compensate for this emerging problem, power converters can be controlled to support the grid by providing inertial behavior and primary reserve. Therefore, several control strategies for synchronous machine emulation (SME) by grid-connected converters have emerged during the last decade, as for instance reviewed in [3]–[5].

Although the potential benefits of providing virtual inertia properties from power electronic converters are becoming well known, there is a lack of a clear criteria to choose the right implementation for a given application. This is mainly because the implementations reported in the literature have usually been studied separately for specific applications. Although some comparative studies have been recently published, they have mainly been based on internal comparison of various SME techniques under a specific range of operating conditions [6]–[8]. Thus, a comprehensive characterization or mapping of the different SME techniques against a common benchmark, and corresponding comparative results that can be used as a basis for choosing the right implementation for any given application, are not explicitly available in the literature.

As a step towards a generalized comparative evaluation, this paper presents an assessment of the performance and stability characteristics of four identifiable general classes of SMEs: frequency-derivative (df/dt)-based Inertia Emulation (IE), synchronverters (SV), current-controlled virtual synchronous machines (CCVSM) and voltage-controlled virtual synchronous machines (VCVSM). The comparative evaluation is presented with a grid-connected Synchronous Machine (SM) as a general benchmark, whose typical configuration can be observed in Fig. 1(a). As shown in the figure the SM is driven by a turbine, which is in turn controlled by a governor driven by an active power controller (APC). The reactive power controller (RPC) sets the reference for the exciter, which adapts the rotor voltage of the SM. Fig. 1(b), on the other hand, shows the configuration of a three-phase voltage-source converter (VSC) connected to the grid through a passive *LC* filter, and controlled by an SME technique to emulate the behaviour of an SM.

In the following, we first revisit in Section II the basic modeling and control of SMs and review the most relevant characteristics of the four evaluated types of SME strategies. Based on the presented configurations, Section III introduces the procedure for deriving small-signal models in the *dq*-domain, lists the most relevant conditions for these derivations and presents a validation of the derived models as a basis for subsequent analyses. The model validation is obtained by comparing the time-domain response of the linearized small-signal models with the original non-linear systems by simulation of small disturbances. In Section IV, the assessment of the stability of the evaluated systems is carried out by studying their eigenvalues and the corresponding properties, such as their damping, their participation factors and their sensitivity to parameter variations. Finally, Section V concludes with the most important remarks of the research.

II. BACKGROUND AND OVERVIEW OF SME TECHNIQUES

SME control strategies are inspired by the operational characteristics of classical synchronous generators employed to regulate the frequency and voltage of electric grids. Therefore, these techniques are generally designed to emulate the *swing equation* that determines the electromechanical behaviour of

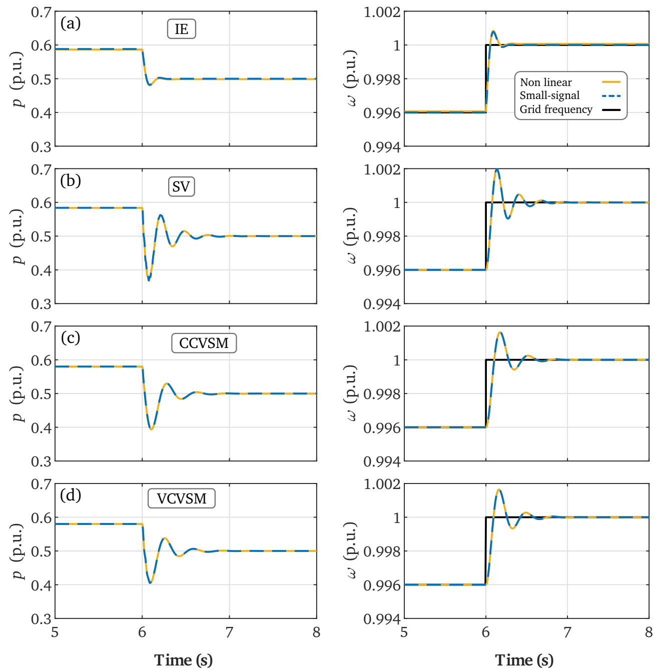

Fig. 6: Dynamic response of grid-connected SME controlled inverters over a grid frequency variation: output power in the left and controller frequency in the right

TABLE IV: Eigenvalue analysis of the SM and SME-controlled converters

Case	i	Eigenvalue λ_i	Damp. ζ_i	Freq. f_i (Hz)	Param. Sen. $\sigma_{i_{\max}}$
SM	1 ⋮ 10	-0.19	1	-	$K_{i_{ex}}$, L_l, L_{m_d}
		-1.86	1	-	T_{gt}, L_{m_d}, L_{m_q}
		-5.83	1	-	L_{m_d}, L_{m_q}, $R_{k_{q1}}$
		$-4.98 \pm j3.39$	0.83	0.54	T_{ex}, L_{m_d}, L_{m_q}
		$-0.28 \pm j14.89$	0.02	2.37	L_l, L_{m_d}, L_{m_q}
		-32.85	1	-	L_{m_d}, R_{k_d}, L_{f1_d}
		$-9.87 \pm j314.08$	0.03	49.99	R_g, L_{m_d}, L_{f1_d}
IE	1 ⋮ 14	$-11.2 \pm j0.59$	1	0.09	K_{i_c}, K_{p_i}, T_s
		$-16.27 \pm j27.86$	0.5	4.43	$K_{i_{\text{PLL}}}$, $K_{p_{\text{PLL}}}$, K_ω
		-1689.04	1	-	K_{p_i}, L_g, L_f
		$-157.13 \pm j2227.21$	0.07	354.47	K_{p_i}, C_f, $K_{p_{\text{PLL}}}$
		-6238.67	1	-	K_{p_i}, T_s, L_f
		$-1121.42 \pm j7210.9$	0.15	1147.65	T_s, L_f, C_f
		$-2213.96 \pm j8809.25$	0.24	1402.04	T_s, L_f, K_{p_i}
		-31821.78	1	-	T_s, K_{p_i}, L_f
		-33292.45	1	-	T_s, K_{p_i}, L_f
SV	1 ⋮ 13	$-20000 \pm j0$	1	-	T_s
		-0.02	1	-	K, L_g, L_f,
		$-5.3 \pm j23.25$	0.22	3.7	H, K_ω, K_D
		$-14.58 \pm j314.16$	0.046	50	R_g, L_g, L_f
		$-20000.11 \pm j59.14$	1	9.41	T_s, H, L_f
		$-6.37 \pm j4517.06$	0.0014	718.91	R_f, L_f, L_g
		$-6.38 \pm j5145.35$	0.0012	818.91	R_f, L_f, L_g
CCVSM	1 ⋮ 18	-5.2	1	-	ω_d, K_D, L_s
		-9.73	1	-	K_{i_c}, K_{p_i}, L_s
		-11.79	1	-	K_{p_i}, K_{i_c}, L_s
		$-5.54 \pm j18.34$	0.29	2.92	H, K_ω, K_D
		-36.2	1	-	K_{i_c}, K_{p_i}, L_s
		$-73.27 \pm j308.52$	0.23	49.10	R_s, L_s, L_g
		$-1578.44 \pm j3101.65$	0.45	493.64	K_{p_i}, C_f, L_f
		$-1313.05 \pm j3804.04$	0.33	605.43	K_{p_i}, C_f, L_s
		$-2107.2 \pm j6855.36$	0.29	1091.06	T_s, L_f, C_f
		$-2408.36 \pm j7477.7$	0.31	1190.11	T_s, L_f, C_f
		$-32644.58 \pm j312.57$	1	49.75	T_s, K_{p_i}, L_f
VCVSM	1 ⋮ 18	$-5.44 \pm j19.09$	0.27	3.04	H, K_ω, L_v
		-11.26	1	-	K_{p_i}, K_{i_c}, K_{i_v}
		-11.34	1	-	K_{p_i}, K_{i_c}, T_s
		$-14.16 \pm j35.61$	0.37	5.67	$K_{p_{\text{PLL}}}$, H, K_D
		$-29.51 \pm j196.87$	0.15	31.32	K_{p_v}, K_{i_v}, L_v
		$-2122.76 \pm j550.59$	0.97	87.63	K_{p_i}, K_{p_v}, L_g
		$-4934.58 \pm j1131.71$	0.98	180.12	K_{p_i}, T_s, L_f
		$-663.77 \pm j6912.45$	0.096	1100.15	T_s, C_f, K_{p_v}
		$-104.64 \pm j7812.09$	0.013	1243.33	T_s, C_f, K_{p_v}
		$-32160.9 \pm j426.17$	1	67.83	T_s, K_{p_i}, L_f

Less-damped modes
Modes associated to the inertial response

and facilitate the design/adaptation of controllers to improve these properties [9].

For the investigated operating condition, the numerical values of the eigenvalues for all the studies cases and their most relevant properties are listed in Table IV. The rows highlighted in grey represent the eigenvalues with the lowest damping factors. If excited, these dominant eigenvalues will determine the most poorly damped dynamic behaviour of that system around the linearization point. Although the results in Table IV are obtained for a single operation point, results from studies of individual cases in previous literature indicate that the general characteristics will still be valid for a reasonable range of combinations of controller and/or physical parameters [7], [19], [20], [23]. Taking that into account, and due to space constraints, the results are limited to a single set of parameters.

Apart from the damping factors and the frequencies of the eigenvalues, we have also calculated the sensitivity of the eigenvalue locations in the imaginary plane with respect to the parameters of the system (ρ_k). This information is useful to identify which parameters are related to instabilities or poorly damped oscillations, and could be modified to improve the stability margins. In this case, we have evaluated the normalized parametric sensitivity as in [7]

The results from Table IV show that all the eigenvalues of the systems have a negative real part. This corroborates the conclusions from the time-domain simulations, where all the analyzed systems were shown to be stable. From these values we can also see that, apart from the CCVSM, the rest of systems contain some eigenvalue with a low or very low damping factor. This implies that even if all the systems are stable, they contain modes that can cause poorly damped oscillations.

In the case of the SM, the results show that the oscillation frequency of 2.37 Hz of the dominant eigenvalues ($\lambda_{6,7}$) corresponds to the frequency of the oscillation identified in Fig. 5. In fact, the parametric sensitivity shows that these eigenvalues are mainly influenced by the physical inductances L_l, L_{m_d} and L_{m_q} of the machine, meaning that the stability margins of this system are primarily determined by the design of the machine itself. From Table IV, we can also notice that the SM has a pair of poorly damped eigenvalues at a frequency of nearly 50 Hz. According to previous studies, this oscillation mode is usually neglected for SMs because of the slow mechanical reaction of the machine [26]. However, such “synchronous frequency resonances" can be also observed in the case of the SV and the CCVSM, and due to the fast regulation capabilities of power converters they might cause power oscillations and should be damped by adapting the controllers as discussed in [7], [26]. In the case of the SV, these modes have a relatively low damping factor, and can significantly influence the dynamic response to certain perturbations. In the case of the CCVSM these oscillation modes are dominant—they have the lowest damping factor—but they can be effectively damped by the virtual resistance (R_s) [7]. At this point it should be highlighted that unlike the SV and the CCVSM, the IE- and VCVSM-controlled converters do not exhibit these "synchronous frequency resonances".

The most critical eigenvalues of the VCVSM- and IE-controlled systems are affected mainly by the filter capacitor C_f and the controller proportional gains $K_{p_{\mathrm{PLL}}}$, K_{p_i} and K_{p_v}, associated with the PLL, the current control and voltage control loops, respectively. Regarding the SV and the CCVSM, from Table IV we can deduce that their dominant eigenvalues are primarily affected by the grid-side inductance L_g as well as the filter inductance with its parasitic resistance (L_f and R_f) in the case of the SV and the virtual machine windings (L_s and R_s) in the case of the CCVSM. This means that the stability margins of the SV are mainly determined by the parameters of the electrical circuit whereas the CCVSM is more influenced by the control parameters.

From Table IV we can also identify the modes that are related to the inertial behaviour of SME-controlled inverters (highlighted in red), which determine the inertial response of the system. In the case of the SV, the CCVSM and the VCVSM technique we can see that these modes have a very low oscillation frequency ($f_{6,7} = 3.7$ Hz, $f_{7,8} = 2.92$ Hz and $f_{5,6} = 5.67$ Hz, respectively). Regarding the IE technique, the damping term (K_D) and the virtual inertia (H) do not significantly influence any of the modes of the system. However, we can see from the results that the gains of the PLL, from which the inertia is emulated, have a strong influence in two modes with a low oscillation frequency ($f_{2,3} = 4.43$ Hz). From this analysis we can conclude that all the SME techniques have low frequency modes directly or indirectly associated with the emulation of inertial behaviour of the virtual swing equation. In fact, the frequency and power perturbations carried out in Section III cause low frequency oscillations that are directly related to these dynamic modes, meaning that for these perturbations the most influential modes are the ones associated to the emulation of inertia. Although the discussions above are based on the eigenvalues at a particular point of operation (0.5 p.u.), the eigenvalue trajectories in Figure 7 show that, except in the IE case, changes in the operation point do not significantly influence the modes of the system. The conclusions from the paper can be hence extrapolated to the entire range of operation. Due to the lack of space, a more in depth analysis of the IE controller is left for future research.

As a means to gain more information about the relation between the states of the system and the eigenvalues, we have also carried out a participation factor analysis according to [9].

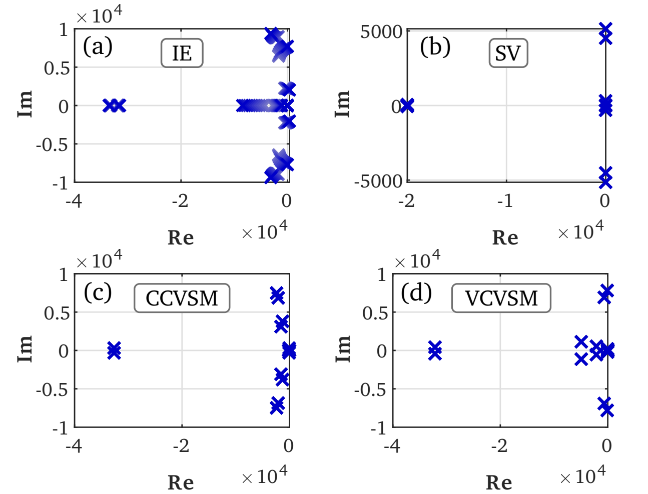

Fig. 7: Location of eigenvalues for different operation points: output power from 0 p.u. (light blue) to 1 p.u. (dark blue)

In order to represent this relationship in a percentage scale, we have also weighted the participation factors as in [27]. In Fig. 8 we show the states that have a weighted participation in the critical mode that is higher than a 10%. These charts show that in most cases the output current of the converter ($\tilde{\mathbf{i}}_c$), the grid-side current ($\tilde{\mathbf{i}}_o$) and the filter capacitor voltage ($\tilde{\mathbf{v}}_o$) are the states with a highest participation factor. The CCVSM is the only exception as the most influencing state is the current flowing through the virtual SM windings ($\tilde{\mathbf{i}}_s$), but the grid-side currents also have a high participation factor in this case. In the case of the IE- and the VCVSM-controlled inverter we can also see that the states associated with the delay of the PWM ($\tilde{\boldsymbol{\beta}}_2$) have a significant influence in the dominant eigenvalues.

Based on the study of the eigenvalues and their properties, the CCVSM provides the highest stability margins among the studied approaches. However, the oscillation frequencies of the critical modes for the other VSM-based cases, i.e. the SV and the VCVSM, are near the resonance frequency of the *LCL* impedance between the converter and the grid. Thus, it can be expected that utilization of active damping techniques for this frequency range might significantly improve the margins of these cases. This consideration is also in accordance with the results obtained in [8].

V. Conclusion

This paper has presented a comparative eigenvalue-based evaluation of four different synchronous machine emulation (SME) techniques in terms small-signal dynamics, taking a synchronous machine (SM) as the benchmark system. The four SME techniques are selected to represent the main general classes of implementations established in literature, and the results show how all these control schemes can provide an inertial response with higher damping than the SM. This is mainly because the SME techniques can be implemented with a higher damping coefficient in the emulated swing equation than what results from the practical design of a SM, but also because the SME techniques can be designed without

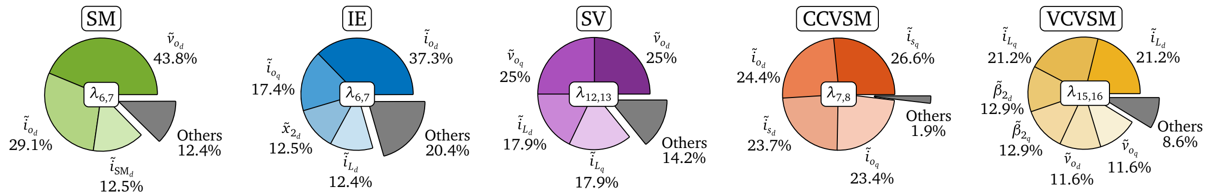

Fig. 8: Participation factors of the most critical eigenvalues

the slow response of a mechanical governor. Among the four investigated SME techniques, the SV, CCVSM and VCVSM provide a similar inertial response under perturbations in the grid frequency as well as in the power reference. While the IE can provide inertial response to frequency variations, it does not exhibit similar dynamics as the other techniques when exposed to power reference variations, since it does not explicitly emulate a SM swing equation and only responds to the variations in the estimated grid frequency.

The eigenvalue analysis has also shown that the CCVSM has the highest stability margins for the investigated operating conditions. However, the eigenvalues of the SV and the VCVSM with the lowest damping are mainly associated with the *LCL* filter dynamics. Thus, active damping techniques designed for attenuating *LC*-oscillations can significantly improve the stability margins of these control techniques. Thus, it is expected that all the schemes can be designed to ensure sufficient stability margins, but with slightly different small-signal dynamics due to the different control system configurations.

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