UNDERSTANDING THE NONLINEAR BEHAVIOUR AND SYNCHRONIZING STABILITY OF A GRID-TIED VSC UNDER GRID VOLTAGE SAGS

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Abstract

Transients of a typical grid synchronizing VSC are closely associated with the over-current and over-voltage in circuits. Previous analysis in evaluating such transient phenomenon usually ignore the nonlinear control effects of the VSCs (e.g. phase-locked-loop, PLL). Although this assumption allows a simpler analysis of the transient process, it may overlook potential stability issues on which the nonlinear controls may have a great impact and the consequence of which is easily confused with the passive circuit transients. Therefore, this work aims to achieve a good understanding of the nonlinear control dynamics of the VSC and their impacts on the stability provoked by the grid voltage sags. To better reveal the mechanisms, the power control loop (PCL) and the PLL-dominant dynamics are analysed separately with corresponding reduced-order nonlinear models. From which the grid synchronizing stability of the VSC is revealed, and a quantitative study of stability margin is presented through the calculation and evaluation of the critical clearing time (CCT). Based on this, CCT under various PLL bandwidths are evaluated, the results of which could facilitate the parameter design of PLL from a stability viewpoint. All the analyses are verified by time-domain simulations in PSCAD/EMTDC.

1 Introduction

Voltage source converters (VSCs) are widely adopted in integrating renewable energy power generations (e.g. wind and solar power) with AC grid [1], as well as in interconnecting two asynchronous grids via the high-voltage-dc (HVDC) technology [2].

Despite its fast and flexible power control capability, recent experience shows that VSCs are susceptible to oscillate against a weak AC grid, e.g. a case of wind farm case in [3] and photovoltaic plant in [4], where the nonlinear dynamics of the phase-locked-loop (PLL) have drawn great attention. To explain and analyze these oscillation behaviors, efforts have been made (e.g. [5] and [6]), however in a small signal sense due to the consideration that most of the oscillations occurred are caused by a moderate change of system configurations. This allows linear-based methods to identify and predict the forward behavior around a steady state operating point. In this regard, many methods are applicable, and among which the impedance-based method becomes popular since the impedance can be either analytically modelled or practically measured.

Impedance models of a typical three-phase VSC can be in different formats according to the linearization methods employed. Typically, linearizing in sequence domain [7] results in a sequence impedance (e.g. [8] and [9]), whereas linearizing in dq domain [10] yields a dq impedance (e.g. [11] and [12]). Recently, other modeling methods, e.g. a phasor-based [13], a modified sequence domain based [14], a single-input and single-output based [15] and a complex transfer function based [16] methods are available, in which the properties of converters can be explained more intuitively, e.g. the mirror coupling effects [14] originated from the dq asymmetry of impedance matrices [17]. Despite different modeling techniques, all these models are capable of identifying oscillatory behaviors in a grid-tied VSC system by applying Nyquist-based stability analysis [18]. A useful finding is that PLL is of great importance for VSC small signal stability, particularly under a weak AC grid condition (e.g. [11] and [19]). Moreover, behaviors of PLL can be physically interpreted as a current-controlled-voltage-source (CCVS) in an equivalent RLC circuit of VSC-grid system [20], where the CCVS can exhibit negative damping to the equivalent RLC circuit if conditions in are met.

However, above discussions are conducted in the linear domain which can only predict the dynamics around a steady-state operating point, if the oscillation diverges, the subsequent behavior can no longer be predicted accurately, e.g. a limit cycle. In [21], a large signal impedance model is proposed to study the sustained oscillations, where the PWM saturation is properly modeled compared to the typical impedance model, e.g. [11]. In [22] and [23], a state space nonlinear model of VSC is proposed, where the bifurcation phenomenon is observed. However, these analyses focus on the memoryless nonlinearities (e.g. saturation), where the dynamical nonlinearities (e.g. PLL) are not discussed.
One step further, a phasor-based analysis of a wind farm with grid voltage dips is studied in [24] and [25], where the wind farm model is simplified to a current source. Requirements on the limitation of active current injection, in the presence of grid voltage dips are emphasized. This work provides an important implication that the current injection of VSC may lead to angle stability from a steady-state point of view. [26] moves further in this regard, where the same conclusion is drawn from the positive feedback effects of the PLL. Although [24]-[26] lack of a dynamical analysis, they are helpful and illuminated in reconsidering the origin of the VSC’s transients and its potential stability impact.

To bridge this gap, this paper aims to achieve a fundamental understanding of the nonlinear behavior of a typical VSC and its potential consequences on stability, all the transients are provoked by the grid voltage sags. This paper is organized as follows, in section 2, the nonlinear behavior of the power control loop (PCL) is analyzed firstly, where the PLL is assumed steady. Then, the PLL- dominant nonlinear behavior is explored in section 3, where the grid synchronizing stability (GSS) is identified and the mechanism of which is revealed. Based on the developed mechanism, stability margin of GSS is quantitatively evaluated through the calculation of the critical clearing time (CCT). Section 4 draws the main conclusion.

2. Analysis of the PCL dominant nonlinear dynamics

2.1 Description of the study system

Fig. 1 illustrates a typical grid-tied VSC, which is composed of a current control loop (CCL), a PCL and a PLL (Hc, Hs, HPLL are their PI controllers respectively). Usually, the VSC is connected to the bulk grid via two step-up transformers (T1 and T2) to boost voltage to the transmission level, hence the “equivalent grid” seen from the VSC (i.e. at the point of connection, PoC) is relatively weak, even though the impedance of the Thevenin grid (i.e. impedance seen from the point of common coupling, PCC) is small. This weak grid condition can lead to oscillation if the dynamics of the PLL and CCL are not decoupled in terms of time-scale, this small-signal-stability issue has been extensively studied in the literature as discussed before, e.g., in [20]. This work, therefore, assumes a properly designed CCL, which means CCL is much faster than PLL and PCL. Therefore, the CCL is assumed in a quasi-steady state if dynamics in the time scale of the PLL or PCL are analysed.

2.2 Modelling of the PCL dominant dynamics

Model reduction is necessary to allow an analytical study of nonlinear behaviours of a complex system. This section aims to analyse the PCL dominant dynamics, hence PLL is assumed steady temporarily. Based on this assumption, the voltage equation from the PoC (UPLL) to the Thevenin grid voltage (UHs) can be written as:

\[ U_{PLL}^H = j\omega_{PLL} L_s I_{PLL} + U_{PLL}^H \]

where, the output currents (IPLL) of VSC are assumed steady due to their fast dynamics, i.e. IPLL ≈ Iref. All the vectors are projected to the PLL reference frame (denoted by superscript ‘PLL’). Since the PLL is assumed steady in this case, αPLL = α is imposed. In PLL frame, the Thevenin grid voltage can be represented as:

\[ U_{PLL}^H = U_s\cos\theta_0 - jU_s\sin\theta_0 \]

where \( \theta_0 \) is the steady angle difference between \( U_s \) and \( U_c \). \( L_s \) is the lumped system inductance seen from PoC, it can be quantified by the short circuit ratio (SCR), i.e. in per unit format \( L_s = 1/SCR \).

The output power of VSC in complex format is:

\[ P + jQ = U_{PLL}^H (I_{PLL}^H)^* = j\omega_{PLL} L_s I_{PLL} (I_{PLL}^H)^* + U_{PLL}^H (I_{PLL}^H)^* \]

\[ \begin{align*}
P &= U_s\cos\theta_0 \cdot I_{cd} - U_s\sin\theta_0 \cdot I_{cq} \\
Q &= \omega_{PLL} I_{cd} (I_{cd}^2 + I_{cq}^2) - (U_s\cos\theta_0 I_{cq} + U_s\sin\theta_0 I_{cd})
\end{align*} \]

It can be observed from (2) that the active power \( P \) is a linear combination of the dq currents, whereas the reactive power \( Q \) is not. Next, the dynamical equation of PCL can be further derived from the control blocks in Fig. 1 as:

\[ \begin{align*}
\frac{dx_d}{dt} &= k_1 \left(P^* - P\right) \\
\frac{dx_q}{dt} &= k_1 \left(Q^* - Q\right) \\
I_{cd} &= x_d + k_p \left(P^* - P\right) \\
I_{cq} &= -x_q - k_p \left(Q^* - Q\right)
\end{align*} \]
In which, $x_q, x_i$ are the output states of PI integrators, while $k_p$ and $k_i$ are PI parameters of $H_i$. Consequently, (2) and (3) consist of the model of PCL-dominant dynamics.

2.3 Nonlinear behaviour analysis

Due to the developed PCL model is of second-order, phase portraits can, therefore, be adopted to study nonlinear its behaviours. In Fig.2, current phase portraits with varying PQ controller ($H_i$) parameters are plotted.

From Fig.2 (a) one could observe that the states ($I_{da}$, $I_{dq}$) can be either attracted by the stable manifolds or driven away by the unstable manifolds, depending on their initial values. In other words, the equilibrium ($0.5, 0$) in this case has a limited region of attraction, consequently, any initial states that are close enough to the equilibrium can be attracted, otherwise a divergent of states can occur. On the other side, comparing the dotted line and solid line we can identify that, increasing the proportional gain of PQ controller ($H_i$) has a negative impacts on stability (i.e. region of attraction becomes smaller), whereas the integral gain has negligible effects on the phase portraits as depicted in Fig.2 (b).

To validate the analysis, time domain simulations are presented in Fig.2 (c), where the transient responses of currents ($I_{dq}$) and active/reactive power ($PQ$) under a symmetrical grid fault are measured. In accordance with the model assumption, PLL frequency is locked to 1.0 p.u. after VSC is synchronized (i.e. at 1.8s). As shown in Fig.2 (c), if the currents are limited to a relatively small value (left columns), the system can converge to the original equilibrium (i.e. ($Id, Iq$) = (0.5, 0)) after grid fault is cleared, this case is consistent with the analysis in Fig.2 (a) with initial states within the region of attraction.

However, if the currents are limited to a relatively large value (right column in Fig.2 (c)), the system cannot converge to its original equilibrium but to another “steady” point. It should be noted that this “steady” point is not an equilibrium because the reactive power does not converge to its set-point, which is $Q = 0$. Hence, a condition: $x_q = 0$, $\dot{x}_q \neq 0$ can be obtained from (3). Further based on (2) we can notice that, it has three unknowns (i.e. $Q, I_d, I_q$) but only with two independent equations, thus the solution is not unique, and the “steady” point shown in Fig.2 (c) (right column) is essentially one of them. Apparently, this steady point is not an equilibrium since the condition $\dot{x}_d = 0, \dot{x}_q = 0$ is not satisfied.

Another finding should be addressed is that, although the theoretical analysis can be unstable, this may be unachievable due to the current limits in practice should be small for protection purpose (e.g. typical values are Imax = 1.1), therefore, all the initial states of currents can be constrained in the region of attraction (red arcs in Fig.2 (a) and (b)), which means the nonlinear dynamics of the PCL could be absolute stable under such practical constraints.

3. Analysis of the PLL dominant nonlinear dynamics

Previous section analysed the PCL dominant nonlinear behaviours, where the PLL is assumed steady. In this section, for better revealing the GSS, the nonlinear behaviour of PLL-dominant dynamics will be analysed in detail, likewise, a slow regulation of the PLL is assumed, thus the current reference of the CCL is assumed constant.
3.1 Modelling of PLL dominant dynamics

Based on the above assumption, in the time-scale of PLL dynamics, \( I_{\text{PLL}} \approx I_{\text{ref}} \) is imposed, i.e. a fast CCL and \( I_{\text{PLL}} \) remains steady due to the assumption of a slow PCL.

Then, the input of PLL is the \( q \) axis voltage at PoC, i.e. \( u_{\text{PLL}} = \text{Im} \{ u_{\text{PLL}} \} \) from (1). In combination with PLL control blocks in Fig. 1, the following equations can be obtained:

\[
\begin{align*}
\frac{d\Delta \omega_{\text{PLL}}}{dt} &= k_{p} \Delta i_{\text{PLL}} + k_{i} \Delta u_{\text{PLL}} \\
\frac{d\delta_{\text{PLL}}}{dt} &= \Delta \omega_{\text{PLL}}
\end{align*}
\]

In which, \( u_{\text{PLL}} = \omega_{\text{PLL}} L_{\text{cd}} I_{\text{ref}} - U_{s} \sin \delta_{\text{PLL}}, \quad \delta_{\text{PLL}} = \theta_{\text{PLL}} - \theta_{s}, \quad k_{p} \) and \( k_{i} \) are the PI parameters of \( H_{\text{PLL}} \). In the later analysis, PLL bandwidth \( (\alpha_{\text{PLL}}) \) is frequently used instead of PI parameters, in which \( k_{p} = 2\alpha_{\text{PLL}} / U_{s} \) and \( k_{i} = 2\alpha_{\text{PLL}}^{2} / U_{s} \) are adopted [20].

Converting (4) into per unit format, yields:

\[
\begin{align*}
T_{\text{PLL}} \frac{d\Delta \omega_{\text{PLL}}}{dt} &= -D_{\text{PLL}} \Delta \omega_{\text{PLL}} + \bar{T}_{m} - \bar{T}_{e} \\
\frac{d\delta_{\text{PLL}}}{dt} &= \alpha_{h} \Delta \omega_{\text{PLL}} \\
D_{\text{PLL}}(\delta_{\text{PLL}}) &= \frac{k_{p}}{k_{i}} \left( \bar{U}_{s} \cos \delta_{\text{PLL}} \alpha_{h} - L_{\text{cd}} I_{\text{ref}} \right) \\
T_{\text{PLL}} &= \frac{\alpha_{h} - k_{p} \bar{T}_{m} \bar{T}_{c}}{k_{i}} \\
\bar{T}_{m} &= \bar{\omega}_{h} L_{\text{cd}} I_{\text{ref}} - \bar{T}_{e} = \bar{U}_{s} \sin \delta_{\text{PLL}}
\end{align*}
\]

where \( \omega_{h} \) is the base value of the angular frequency. \( T_{\text{PLL}} \) is a constant, whereas \( D_{\text{PLL}} \) is \( \delta_{\text{PLL}} \) dependent. For a small value of \( \delta_{\text{PLL}}, \) \( D_{\text{PLL}}(\delta_{\text{PLL}}) \) is positive; otherwise, it can be negative. \( \bar{T}_{m} \) is constant input, \( \bar{T}_{e} \) is the nonlinear state feedback to the frequency dynamics.

Consequently, the nonlinear model of PLL–dominant dynamics is developed in (5). One may observe that this model resembles the motion equation of a synchronous generator (SG), which means the well-known SG-based transient angle analysis methods are applicable, e.g. the equal area principle (EAP). Due to this similarity, the variable notation of an SG is adopted to the definition of the variables of the PLL dynamics, e.g. \( \bar{T}_{m}, \bar{T}_{e}, \) though \( \bar{T}_{m} \) and \( \bar{T}_{e} \) are essentially voltages in physics.

3.2 Nonlinear Behaviour analysis

Similar to the analysis of the PCL dominant dynamics, the PLL dominant dynamics is characterized as a second-order nonlinear system as well, e.g., (5), rendering a graphical analysis of dynamical behaviours through the phase portrait, where \( (\Delta \omega_{\text{PLL}}, \delta_{\text{PLL}}) \) are the two state variables.

Fig. 3 (a) \( D_{\text{PLL}} \) approximated by a constant at \( \delta_{\text{PLL}} \); (b) \( D_{\text{PLL}} \) varies with angle \( \delta_{\text{PLL}} \); (c) Time domain simulations; PLL bandwidth is 20 Hz, i.e. \( k_{p} = 20, \) \( k_{i} = 800, \) SCR = 4, \( T_{\text{ref}} = 1 \) pu; in (c), a symmetrical grid fault is applied at PCC with a duration of 500ms

First, in Fig. 3 (a), the phase portraits of (5) with varying initial states \( (\Delta \omega_{\text{PLL}}, \delta_{\text{PLL}}) \) are plotted, where the damping term is approximated by a positive constant \( D_{\text{PLL}}(\delta_{\text{PLL}}) \) (i.e. evaluated at \( \delta_{\text{PLL}} \)). One could observe that for small initial
states, the state trajectory can converge to the original point after several cycles’ motion. However, if the initial states are far away from this point, they can converge as well, but to another equilibrium that separated by $2\pi$. This is an intrinsic property of the nonlinear system (5) that, there is an equilibrium set (i.e. $\delta_{\text{eq0}} + 2k\pi$, $k = \pm 1, \pm 2...$) that is locally stable, and states can always converge to the equilibrium set due to $D_{\text{PLL}}(\delta_{\text{eq0}})$.

Further, as emphasized before, the damping term $D_{\text{PLL}}$ is a nonlinear function of $\delta_{\text{PLL}}$, which means it can be negative if $\delta_{\text{PLL}}$ is large (e.g. $\delta_{\text{PLL}} \in \left[2k\pi + \frac{\pi}{2}, 2k\pi + \frac{3\pi}{2}\right]$, $k = 0, \pm 1...$).

Therefore, a detailed phase portrait including the effects of $D_{\text{PLL}}(\delta_{\text{PLL}})$ is plotted in Fig. 3 (b). Clearly, the system can be unstable (see the dotted and red line) if the initial states ($\Delta\omega_{\text{PLL}}, \delta_{\text{PLL}}$) are large. This feature cannot be captured by Fig. 3 (a) since the approximated constant $D_{\text{PLL}}(\delta_{\text{PLL}})$ is positive.

A time domain simulation is conducted and presented in Fig. 3 (c), where the PLL frequency and the magnitude of PCC voltage are measured. In order to have a large deviation of initial states, a symmetrical grid fault is applied at PCC with a duration of 500ms, it can be clearly identified that the PLL lost synchronization after the grid fault is cleared, and it exhibits a similar manner as the phase portrait in Fig. 3 (b).

3.3 Mechanism analysis of the GSS

As mentioned before, the EAP for the transient stability analysis of an SG [27] can be applied to the analysis of PLL dominant dynamics due to the resemblance.

According to (5), characteristics of $\bar{T}_m = \bar{a}_m\bar{L}_m\bar{T}_{\text{ref}}$ and $\bar{T}_e = \bar{U}_s\sin\delta_{\text{PLL}}$ can be illustrated by curves in Fig. 4 (a). It can be observed that a grid fault can change the characteristic of $\bar{T}_e$ abruptly, whereas $\bar{T}_m$ remains constant (i.e. $\bar{T}_m^{00} = \bar{T}_m^{00}$) due to the current is assumed steady. The magnitude differences between $\bar{T}_e^{00}$ and $\bar{T}_m^{00}$ can result in the acceleration of system states: $\left(\Delta\omega_{\text{PLL}}, \delta_{\text{PLL}}\right)$. Then, after a short period, the grid fault is cleared (e.g. at point C), where characteristic curve of $\bar{T}_e^{00}$ changes back to its pre-fault value ($\bar{T}_e^{00}$), so that $\Delta\omega_{\text{PLL}}$ starts decelerating due to $\bar{T}_e^{00}$ is greater than $\bar{T}_m^{00}$. However, $\delta_{\text{PLL}}$ will remain increasing until $\Delta\omega_{\text{PLL}}$ becomes zero again.

The EAP claims that if $\exists S_n\left(\Delta\omega_{\text{PLL}} = 0, \delta_{\text{PLL}} \leq \delta_{\text{PLL}}^B\right); S_1 = S_n$, then the system is referred to as first swing stable. It should be noted that, although the nonlinear system (5) can converge to an equilibrium set (e.g. in Fig. 3 (a)) as discussed before, only the principal one i.e., $\Delta\omega_{\text{PLL}} = 0, \delta_{\text{PLL}} = \delta_{\text{PLL}}^B$ is of interests or physical significance. Because the transition from one equilibrium to another can exhibit large transients in currents or voltages, which are not allowed due to the limited stress of physical components. For an SG, this also indicates a “pole slip” operation, which is detrimental and not allowed.

From the EAP it is obtained that, if the fault clearing angle $\delta_{\text{PLL}}^C$ is small (i.e. fault is cleared fast), then there is more margin for the deceleration area that stability can be assured. To illustrate this character, a time domain simulation is shown in Fig. 4 (b), where the fault is cleared faster compared to the one in Fig. 3 (c). Clearly, the frequency of PLL is stable under such a condition, indicating that a small $\delta_{\text{PLL}}^C$ is helpful for stability.

Based on the above analysis, it is seen that the fault clearing angle can be a metric of the GSS margin. Specifically, the critical condition that ensuring a first swing stable system is of most interest, i.e. there exists a critical angle that EAP is met, i.e. $\exists \delta_{\text{PLL}}^{\text{CCA}}: S_1 = S_n^{\text{max}}\left(\Delta\omega_{\text{PLL}} = 0, \delta_{\text{PLL}}^{\text{max}} = \delta_{\text{PLL}}^B\right)$. This angle is referred to as Critical Clearing Angle (CCA), its calculation will be shown subsequently.
3.4 Analysis of the GSS margin

The CCA can be calculated by numerical method if the analytical model of \( \tilde{T}_e^{0+}, \tilde{T}_e^{0+} \) and \( \tilde{T}_m \) are known, in which \( \tilde{T}_e^{0+} \) and \( \tilde{T}_m \) can be obtained from (5), whereas \( \tilde{T}_e^{0+} \) is fault dependent.

To calculate the post-fault \( \tilde{T}_e^{0+} \), circuit analysis of the grid fault is necessary. In accordance with PLL modelling, the equivalent circuit of Fig.1 can be drawn in Fig. 5 (a), where \( Z_{\text{ref}} \) is the lumped line impedance seen from PoC, \( Z_s \) is the source impedance seen from PCC, \( Z_f \) is the short circuit impedance applied at PCC.

Fig. 5 (a) Equivalent circuit of Fig. 1; (b) Post-fault circuit

A Thévenin equivalent circuit seen from the fault branch can be further developed in Fig. 5 (b), in which the post-fault PCC voltage can be calculated as: \( U_{\text{PCC}}^{0} = k_f U_{\text{PCC}}^{0} \), where \( U_{\text{PCC}}^{0} = U_{s}^{0+} + I_{f}^{0+} Z_s \) and \( k_f = (Z_f + Z_s) = k_f e^{j\phi} \).

Therefore, the post-fault characteristic of \( \tilde{T}_e^{0+} \) is obtained:

\[
\tilde{T}_e^{0+} = \text{Im}\left\{U_{\text{PCC}}^{0}\right\} = k_f U_{\text{PCC}}^{0} \sin\left(\delta_{\text{PCC}} - \phi_1\right) \tag{6}
\]

where, \( \delta_{\text{PCC}} \) is the phase angle between \( U_{\text{PCC}}^{0} \) and \( U_{\text{PCC}}^{0} \), and \( \tilde{T}_m \) is modified to \( \tilde{T}_m = \tilde{T}_m^{\text{ref}} T_{\text{cd}} + \tilde{R}_{\text{ref}} T_{\text{ref}} \).

From (6) it is observed that, in geometry, \( \tilde{T}_e^{0+} \) is essentially a curve deformed by \( \tilde{T}_e^{0+} \), consequently \( \tilde{T}_e^{0+} \) can be easily drawn in Fig. 4 (a) by shifting and compressing curve \( \tilde{T}_e^{0+} \). As the grid impedance \( Z_s \) is mostly inductive, and if \( Z_f \) is inductive as well, there exists no phase shift according to the expression of \( k_f \). Otherwise, for a resistive short circuit branch, the phase shift can be \( \phi_1 \in \left[-\frac{\pi}{2}, 0\right] \).

Usually, effects of \( \phi_1 \) is negligible since the acceleration and deceleration areas (\( S_1 \) and \( S_2 \)) are primarily determined by the magnitude of \( \tilde{T}_e^{0+} \), particularly under severe grid sags.

Based on the models of \( \tilde{T}_e^{0+}, \tilde{T}_e^{0+} \) and \( \tilde{T}_m \), the CCA can be calculated numerically from the nonlinear algebraic equation:

\[
S_1 = S_1^{\text{max}} \rightarrow \int_{0}^{\delta_{\text{pcc}}^{\text{max}} / \omega_{\text{pcc}}} (\tilde{T}_m - \tilde{T}_e^{0+}) d\delta_{\text{pl}} = -\int_{\delta_{\text{pcc}}^{\text{min}} / \omega_{\text{pcc}}}^{\delta_{\text{pcc}}^{\text{max}} / \omega_{\text{pcc}}} (\tilde{T}_m - \tilde{T}_e^{0+}) d\delta_{\text{pl}}
\]

If resubstituting the numerical results of CCA into the dynamical equation of \( \Delta\omega_{\text{pl}} \) in (5), the corresponding Critical Clearing Time (CCT) can be estimated as:

\[
t_{\text{CCT}} = \frac{(\delta_{\text{pl}}^{\text{CCA}} - \delta_{\text{pl}}^{\text{S}})}{k_0} \sqrt{\frac{\tilde{T}_{\text{pl}}}{} 2S_{\text{pl}} \omega_{\text{pl}}}
\]

where \( k_0 \) is a coefficient that used to estimate the integral of \( \int_{0}^{t_{\text{CCT}}} \Delta\omega_{\text{pl}} dt \) by \( (k_0 \omega_{\text{pl}} \Delta\omega_{\text{pl}}) \cdot t_{\text{CCT}} \), in this work \( k_0 = 2/3 \) is adopted. The CCT is a more intuitive metric since the time can be more easily measured than the CCA.

To illustrate the feasibility of CCT in evaluating stability margin, a numerical evaluation of the CCT with varying PLL bandwidth is conducted and the results are plotted in Fig. 6 (a). It is seen that by increasing the PLL bandwidth, the CCTs are reducing, which means stability margin is deteriorated. On the other hand, the magnitudes of \( \tilde{T}_e^{0+} \) (i.e.
This numerical calculation of CCT can also be employed for studying the stability impacts of other parameters, e.g. the phase jump of grid voltage.

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6 References


