



Attainable regions and fuzzy multi-criteria decisions: Modeling a novel configuration of methane bioreactor using experimental limits of operation

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ABSTRACT

This study sets out to develop an approach that couples attainable regions and fuzzy multicriteria decision methods for modeling optimal configurations of multistage digesters without using a kinetic model of the process. The approach is based on geometric analysis of methane curves as their shapes contain valuable insight into substrate biodegradability characteristics during anaerobic digestion. With the case study of abattoir waste, the results indicate that the optimal batch operation policy involves four anaerobic sequencing batch reactors operated in series with fresh feed being added at the second and the four stages (fed-batch systems). For continuous mode operation, the optimal configuration involves a continuous stirred tank digester with bypass from feed followed by an anaerobic baffled digester, which has been used to obtain a novel prototype. The methodological framework presented in this study can be adopted to enhance design of multistage anaerobic digesters especially when reliable kinetic models are unavailable.

1. Introduction

The anaerobic treatment process has increasingly been recognized as an efficient technology for sustainable nutrient recycling, renewable energy generation and waste sanitation, having a strong potential to mitigate current energy resource and climate change challenges. However, the success of an industrial-scale anaerobic digestion is only possible if the following two prerequisite factors are met: (1) Availability of a sustainable supply of organic feedstock and (2) Design of optimal process configurations that are well adapted to the characteristics of the feedstock of interest. Concerning the second requirement, a wide variety of anaerobic digester systems have been developed, which can be classified in to three groups: conventional digesters (e.g. ASBR, CSTR, and PFR), sludge retention digesters (e.g. ACR, UASB, UASSR, ABR and ICR) and membrane digesters (e.g. AF, EGSB and AFBR) (Mao et al., 2015). Recent studies continue to develop new digesters, which either modify the principle of an existing digester technology or present novel features, all geared towards improving process performance (Pan et al., 2019; Terboven et al., 2017; Xiong et al., 2019).

Although various digester systems exist, each with different physical and geometric characteristics, the hydrodynamic configurations of all

digesters can be derived from different combinations of three fundamental regimes: flow regime, mixing regime and sludge retention regime. Under flow regime, anaerobic digesters can be operated as batch, fed-batch or continuous; under mixing regime, they can be operated as completely mixed or with no axial mixing and under sludge retention regime, the operation can be with or without sludge retention. For example, a continuous flow regime operated with no axial mixing and no sludge retention gives a plug flow anaerobic digester (PFR) and when operated with sludge retention can result in either AF, ABR, UASB or EGSB. It is also important to mention that batch and fed-batch reactors can be operated as completely mixed or unmixed depending on the practical considerations.

Ming et al. (2016) illuminated critical aspects concerning a plug flow and a continuous stirred tank reactor focusing on mixing and reaction. By their analogy of a plug flow reactor (reactor with no axial mixing along the length) as a series of batch reactors (reacting vessels) travelling on a conveyor belt, the Plug flow reactor can be considered a reaction reactor. The authors also illustrated that CSTR operates directly opposite to the PFR with respect to mixing due to its perfect mixing assumption where conversion of reactants into product is assumed to occur as a result of mixing and dilution rather than from reaction alone. The PFR and CSTR are therefore at the extremes of mixing

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Nomenclature

V_g	volume of biogas produced (mL)
m_s	mass of substrate added to the digester (g)
y_i	cumulative biogas yield (mL/g)
ABR	Anaerobic Baffled Reactor
AF	Anaerobic Filter
AFBR	Anaerobic Fixed Bed Reactor
AHP	Analytical Hierarchy Process
APFR	Anaerobic Plug Flow Reactor
AR	Attainable Regions
ASBR	Anaerobic Sequencing Batch Reactor
CSTR	Continuous Stirred Tank Reactor
DSR	Differential Sidestream Reactor

EGSB	Expanded Granular Sludge Bed
F TOPSIS	Fuzzy Technique for Order of Preference by Similarity to Ideal Solution
FBDR	Fixed Bed Disc Reactor
ICR	Internal Circulating Reactor
MCDM	Multi-criteria decision-making
PFR	Plug Flow Reactor
TOPSIS	Technique for Order of Preference by Similarity to Ideal Solution
UASB	Up flow Anaerobic Sludge Bed
UASSR	Up flow Anaerobic Solid-State Reactor
α	mixing ratio (-)
τ	Residence time (days)

and reaction and different combinations of these digesters will provide different extents of mixing and reaction in a reactor system (or reactor network) made up of both reactor types.

Generally, digester technologies can be broadly classified into high-rate (having separate solids and hydraulic retention times) and low-rate (having coupled solid and hydraulic retention times systems (Mes et al., 2003). In essence, all high rate digesters (sludge retention and membrane reactors) provide a mechanism of sludge separation in addition to the mixing and/or reaction. For example the anaerobic contact reactor provides mixing (due to presence of a CSTR) and separation while the anaerobic baffled reactor provide reaction (because the ABR operates in plug flow mode) and separation (Abunde et al., 2019b). What differentiates the high rate digesters is the mechanism in which sludge separation is performed, which can either be through fixed microbial films on solid surfaces or through an external separation and recycle (Mes et al., 2003). As expected, the different mechanisms result in different extents of separation, each of which is more suited for different substrates and operational characteristics than others. Therefore, irrespective of the type of digester technology, the performance of the anaerobic treatment process depends on three fundamental processes, mixing (performed by CSTR) reaction (performed by PFR) and separation (performed by high rate systems). What this means is that instead of focusing attention to devise new or perhaps novel digesters with the aim of improving the systems performance, it would be more important to focus attention on optimally arranging combinations of PFR, CSTR and/or high rate systems, or integrating more fundamental processes to the anaerobic treatment process (e.g. reversed osmosis + anaerobic digestion). This is referred to as the so called multi-stage anaerobic digestion in which every step of the anaerobic treatment process is optimized by operating digesters in a network in a network (EPA, 2006).

Several studies exist operating anaerobic digestion by coupling two or more types of digesters in multiple stages. Some examples include: AF + UASB (Lew et al., 2004), UASB + AF + AF (Chernicharo and Machado, 1998) CSTR + UASB (Aslanzadeh et al., 2014), ABR + AF (Mang and Li, 2010) CSTR + CSTR (Gaby et al., 2017), etc. The cited studies and many other practical examples of coupled digester systems are usually designed using an empirical approach, where candidate digester configurations are predefined at the start followed by experimental evaluations to select the configuration that yields the best performance. This strategy is not only expensive and time-consuming, but also limited to series combination of digesters, without any systematic way to determine the number and type of digester subunits or how the individual digester subunits should be connected. In addition, other combinations (e.g. parallel or both parallel and series) of the fundamental anaerobic digester types can always be derived, which can have similar or even improved performance than the series combinations (hence problem of multiple solutions). The authors' recent studies, have been first to lay down the theoretical framework for use of attainable

regions (AR) in solving the problem of multiple solutions during synthesis of anaerobic digester networks (Abunde et al., 2019b,a). However, the attainable region approach used for modeling configurations of anaerobic digester networks provides a global optimal structure consisting of digesters operated in a plug flow or continuous stirred mode (sometimes involving bypass and or recycle streams) (Ming et al., 2013, Hildebrandt and Glasser, 1990) but provides no information about the type or nature of the individual digesters. This therefore poses another challenge on the choice of plug flow digester to use considering that there exist several digesters that can be considered to have a plug flow mode of operation. This study has therefore been designed to develop a novel methodological framework that couples attainable region theory (for reactor network synthesis) and a fuzzy multicriterial decision making method (for optimal selection of sub-units for a digester network configuration) for optimal synthesis of anaerobic digester structures. Another interesting aspect of the study is that unlike previous studies that require a reliable kinetic model before AR can be applied to synthesize anaerobic digesters, the framework presented in the current study only requires experimental data for synthesis of digester network configurations.

2. Methods

2.1. Attainable region synthesis of anaerobic digester networks

The Attainable Region (AR) theory is a technique that incorporates elements of geometry and mathematical optimization, to design and improve operation of chemical reactors (Hildebrandt and Glasser, 1990). The power of the AR approach to process optimization is that the answer to all possible optimization problems, even the ones not considered are first determined, before looking for ways of achieving that answer. In reactor operation knowledge of all possible reactor states for all possible reactor configurations, even those that have not yet been devised, is obtained. The convex hull for the set of all points achievable by all possible combinations of CSTR + PFR defines the attainable region. The convex hull is understood as the smallest subset of a set of points that can be used to generate all other points by reaction and mixing (Ming et al., 2016). Geometrically, a convex hull is a finite convex polytope enclosed by a finite number of hyperplanes, which is interpreted in a two-dimensional space as the smallest polygon enclosed by planar facets such that all of the elements lie on or in the interior of the polygon (Asiedu et al., 2015). Once the AR has been determined, the limits of achievability by the system for the given kinetics and feed point is known, which can then be used to answer different design or optimization questions related to the system.

Given a system, the following needs to be performed to do an AR analysis

- Define the fundamental processes occurring within the system

- Determine the state variables used to construct the AR
- Define the geometry of the fundamental reactor subunits
- Generate the AR using combinations of the fundamental processes
- Interpret the AR boundary in terms of reactor structures
- Define and overlay an objective function onto the AR boundary
- Determine the specific reactor configuration required to achieve the intersection point

The last two bullet points are essential if the attainable region is to be used to determine a specific design or optimization question. It is not the focus of the paper to present a deep theory of the AR concept. Interested readers can consult the cited literature for a more in-depth theoretical background (Ming et al., 2016).

2.2. Selection of plug for anaerobic digesters

2.2.1. Formulation of digester selection problem

As mentioned in Section 1, there are several anaerobic digesters (UASB, EGSB, AF, ABR, etc.) that can be considered to have a plug flow mode of operation (hence no axial mixing), selecting the appropriate plug flow digester becomes a challenging task. After a detailed literature survey, the most common plug flow anaerobic digesters were selected as candidates for the multicriteria decision making. These include: Anaerobic Fluidized Bed Reactor (AFBR), Anaerobic Plug Flow Reactor (APFR), Expanded Granular Sludge Bed (EGSB), Internal Circulating Reactor (ICR), Up flow Anaerobic Sludge Bed (UASB), Anaerobic Baffled Reactor (ABR) and Anaerobic Filter (AF). Therefore, the approach proposed in this paper relies on a modular coupling of the geometric technique of attainable regions followed by the multicriteria decision making tools. It is worth mentioning that the idea is not to explore all the existing types of plug flow digesters, but to present a framework for selection of plug flow digesters that will accompany the optimal structure defined by the attainable regions. Several criteria have been defined for use in evaluating the digester alternatives as described in Table 1. The next section of the paper will present the fuzzy multi-criteria decision-making process for selection of the most appropriate plug flow anaerobic digester using anaerobic treatment of abattoir waste as the case study.

2.2.2. Fuzzy multicriteria decision making process

The use of ordinary Multicriteria decision making (MCDM) tools for ranking of alternatives requires that the performance score of the alternatives with respect to each criterion is quantitative in nature (i.e. can be measured and attributed a crisp numerical value). An example is

the work of Karagiannidis and Perkoulidis (Karagiannidis and Perkoulidis, 2009), quantitative characteristics, which can be measured (such as greenhouse gas emitted, recovered energy, recovered nutrients, operating cost, etc.) for selection of anaerobic digester technologies. However, in this study the performance score of the alternatives with respect to each criterion did not have crisp numerical values and the ordinary MCDM cannot therefore be applied. The strength of this study is illustrated by extending the decision-making process to include fuzziness, where by ratings of alternatives versus criteria is done using linguistic variables represented as triangular fuzzy numbers (TFN). The linguistic variables and their corresponding TFN (presented in parenthesis) utilized include: very poor (1,0,0), poor (3,1,1), medium poor (5,3,3), fair (7,5,5), medium good (9,7,7), good (10,9,9) and very good (10,10,10). This provides an opportunity of the decision-making process to be performed even in cases where crisp numerical ratings of the alternatives with respect to the criteria is not available or even in cases of uncertainty.

The selection of the appropriate anaerobic plug flow digester was done using a hybrid of the Fuzzy Technique for Order Preference by Similarity to Ideal Solution (FTOPSIS) and the Analytical Hierarchy Process. At first, AHP is used to compute the criteria weights, which show the relative importance of the different attributes used for digester selection. Afterwards, the FTOPSIS method is applied to prioritize the different alternatives (plug flow digesters) based on the computed criteria weights (Esmaili Dooki et al., 2017, Balioti et al., 2018, Basahel and Taylan, 2016). Assuming there exist m alternatives and n criteria/attributes, the algorithm for the integrated AHP-Fuzzy TOPSIS method utilized in this study is summarized in the following 6 steps:

Step 1: Construct the decision matrix. The performance value of each alternative with respect to each criterion is determined using a fuzzified seven-point scale. The seven-point scale is fuzzified using a triangular membership function, where each linguistic term is expressed in positive triangular fuzzy numbers. In case of multiple decision makers, each decision maker attributes a linguistic label on all alternatives with respect to each criterion and Eq. (1) is used to compute the combined positive triangular fuzzy numbers for all the decision makers.

$$a_{ij} = \min_k \{a_{ij}^k\} \tag{1a}$$

$$b_{ij} = \frac{1}{K} \sum_{k=1}^K b_{ij}^k \tag{1b}$$

$$c_{ij} = \max_k \{c_{ij}^k\} \tag{1c}$$

Table 1
Set of decision criteria to evaluate plug flow anaerobic digesters.

Symbol	Name of Criteria	Objective	Description
C1	COD/VS Reduction Efficiency	Maximize	This measures the ability of an anaerobic digester to reduce organic pollution
C2	Retention of Residual Nutrients	Minimize	High nutrient retention by anaerobic digestate can result in eutrophication when disposed to the environment. The objective is to maximize biogas production and not nutrient recovery
C3	Total Solids content in the Digester	Minimize	This parameter differentiates between wet fermentation (15–25%T) and dry fermentation (> 30%). Wet digesters that are more adapted to minimal TS because the substrate (abattoir waste) fall in the range of wet fermentation (TS = 17.5%)
C4	Organic Loading Capacity	Maximize	This measures the processing rate of organic matter for a given anaerobic digester type. Higher values are economically attractive
C5	Axial Mixing	Minimize	Digesters with plug flow operation (PFRs, UASB, AFs, ABRs, etc.) offer a higher processing capacity to microorganisms and hence higher degree of biodegradation. This is because such systems present little or no axial mixing of the digester content during operation
C6	Biogas Yield	Maximize	Biogas is a renewable energy, which can be used as substitute for fossil-based fuels
C7	Stage of Treatment (Primary or Secondary)	Maximize	Nutrient removal, hygienisation and COD reduction mostly occur in the secondary treatment
C8	Thermal stability of the system	Maximize	Small-scaled digester systems mostly operate under non-isothermal conditions. Hence system that are less sensitive to temperature variations are more attractive

Step 2: Construct a normalized fuzzy decision matrix. This was done by calculating a normalized fuzzy performance value of each alternative with respect to each criterion. For the benefit criteria, a maximum value is desired while for the cost criteria, a minimum value is desired. The normalized fuzzy performance ratings for the alternatives with respect to the benefit and cost criteria was done using Eqs. (2) and (3) respectively.

$$\tilde{r}_{ij} = \left[\frac{a_{ij}}{c_j^*}, \frac{b_{ij}}{c_j^*}, \frac{c_{ij}}{c_j^*} \right], \quad c_j^* = \max_i \{c_{ij}\} (\text{benefit criteria}) \tag{2}$$

$$\tilde{r}_{ij} = \left[\frac{a_j^-}{c_{ij}}, \frac{a_j^-}{b_{ij}}, \frac{a_j^-}{a_{ij}} \right], \quad a_j^- = \min_i \{a_{ij}\} (\text{cost criteria}) \tag{3}$$

where $j = 1, 2, 3, \dots, n$ and $i = 1, 2, 3, \dots, m$

Step 3: Construct a weighted normalized fuzzy decision matrix by multiplying the normalized TFNs with the weight of each criteria as shown in Eq. (4)

$$\tilde{v}_{ij} = \tilde{r}_{ij} \times w_j \tag{4}$$

The weights of relative importance (w_j) of each of each criterion were determined using the AHP. A pairwise comparison matrix, A using a scale of relative importance was then constructed whereby an attribute compared with itself is always assigned the value 1. The numbers 3, 5, 7, and 9 correspond to the verbal judgments “moderate importance”, “strong importance”, “very strong importance”, and “absolute importance”.

The criteria weight vector $W = [W_1, W_2, \dots, W_N]$ was determined using these two steps:

- Normalize the pair-wise comparison matrix, A_{norm} by dividing each entry in A_{norm} column i of A by the sum of the entries in column i .
- The W_i was estimated as the average of the entries in row i of A_{norm} .

The pair-wise comparison matrix is then subjected to consistency check, which involves determination of the maximum Eigen value, Eq. (5) and the consistency Index (CI), Eq. (6).

$$\lambda_{max} = 1/n \sum_{i=1}^n \frac{i^{th} \text{ entry in } AW^T}{i^{th} \text{ entry in } W^T} \tag{5}$$

where

λ_{max} = maximum Eigen value

n = number of attributes

A = pairwise comparison matrix

W = The estimate of the decision-maker’s weight

$$CI = \frac{\lambda_{max} - n}{n - 1} \tag{6}$$

Consistency is checked by comparing the consistency Index to the Random Index (RI) for the appropriate value of n , used in decision-making (Saaty, 2000). If $(CI/RI) < 0.10$, the degree of consistency is satisfactory, but if $(CI/RI) > 0.10$, serious inconsistencies may exist, and the results produced by AHP may not be meaningful.

Step 4: Determine the fuzzy positive ideal solution (FPIS) and fuzzy negative ideal solutions (FNIS). For benefit attributes, the ideal best value of all alternatives with respect to a given attribute is the maximum while negative ideal is the minimum weighted normalized fuzzy performance value. The FPIS and FNIS are computed by Eqs. (7) and (8) respectively.

$$A^+ = (\tilde{v}_1^*, \tilde{v}_2^*, \dots, \tilde{v}_n^*) \quad \text{where } \tilde{v}_j^* = \max_i \{v_{ij}\} \tag{7}$$

$$A^- = (\tilde{v}_1^-, \tilde{v}_2^-, \dots, \tilde{v}_n^-) \quad \text{where } \tilde{v}_j^- = \min_i \{v_{ij}\} \tag{8}$$

Step 5: Calculate the separation measurement Euclidean distance of each alternative from the FPIS and FNIS. The distance from FPIS (S_i^+) is computed using Eq. (9) while the distance from FNIS (S_i^-) is computed using Eq. (10). The Euclidean distance between two triangular fuzzy numbers, $\tilde{a}_{ij} = (a_{1ij}, a_{2ij}, a_{3ij})$ and $\tilde{b}_{ij} = (b_{1ij}, b_{2ij}, b_{3ij})$ is given by Eq. (11)

$$S_i^+ = \sum_{j=1}^n d(\tilde{v}_{ij}, \tilde{v}_j^*) \tag{9}$$

$$S_i^- = \sum_{j=1}^n d(\tilde{v}_{ij}, \tilde{v}_j^-) \tag{10}$$

$$d(\tilde{a}_{ij}, \tilde{b}_{ij}) = \sqrt{\frac{1}{3}(a_{1ij} - b_{1ij})^2 + (a_{2ij} - b_{2ij})^2 + (a_{3ij} - b_{3ij})^2} \tag{11}$$

Step 6: Determine the relative closeness or performance score (CC^*) of each alternative, Eq. (12). The alternatives are then ranked based on their performance score with respect to the ideal solution.

$$CC^* = \frac{S^-}{S^+ + S^-} \tag{12}$$

2.3. Experimental edge

2.3.1. Substrate sampling and characterization

The Anaerobic Digestion experiment was conducted at the Department of Agricultural and Biosystems Engineering, Kwame Nkrumah University of Science and Technology (KNUST), Ashanti Region of Ghana. It is located within 06° 41’ 5.67” N 01° 34’ 13.87” W. The abattoir waste was obtained from the Kumasi central abattoir. Seed

Table 2
Characteristics of abattoir effluent used for anaerobic treatability studies.

Elemental characteristics (ppm or mg/L)										
Ca	Mg	S	P	Fe	Cu	Zn	Ni	Mn	K	N
0.10	0.74	0.50	0.4	114.6	9.1	39.19	0.04	22.9	1.25	2.02
Biochemical characteristics										
Protein (%) (DM)	Crude fiber (%DM)		Carbohydrates (%) (DM)			Total Ash (%DM)	Fats (%) (DM)		BOD (mg/L)	
27.6	13.96		44.48			3.926	2.25		520	
Physicochemical characteristics										
Volatile solids (%)		Total solids (%)	Moisture (%)		Total alkalinity (mgCaCO ₃ /L)		Total dissolve solids (mg/L)		COD (mg/L)	
87.41		17.515	82.49		1650		220		740	

sludge used to facilitate start-up of the digestion process was obtained from a 40 m³ fixed dome digester fed with faecal matter and located at the Kumasi Institute of Tropical Agriculture. To better understand the intrinsic nature of the abattoir waste, the elemental, biochemical and physiochemical characteristics were determined as presented in Table 2. Moisture content was determined using oven drying method 105 °C while the Total solids (TS) and volatile solids (VS), total dissolved solids (TDS) chemical oxygen demand (COD) biological oxygen demand (BOD) and total alkalinity were determined following the standard methods (APHA, 1998). The analyses of crude fiber (CF), crude protein (CP), crude fat (ether extract) (TF), ash and nitrogen free extract (NFE) were performed following the methods detailed in (AOAC, 1990). Total carbohydrate (TC) was calculated by using values obtained for CF and NFE (TC = CF + NFE). The quantification of heavy metals was done using an absorption spectrophotometer located at the crop research institute, Kumasi, Ghana.

2.3.2. Experimental setup and procedure

4.5 kg of substrate and 0.5 kg of inoculum and 1 L of water was mixed using a paddle and fed into the digester. Anaerobic digestion was performed in a 5 L batch reactor with a total digestion time of 30 days. The digester was insulated with a black polyethene sheet and the system was operated under an average room temperature of 31 °C. A 0.5 L changeable gas collection bag was connected to the digester using a drip set and a silicone sealant was used to make the connection airtight in order to ensure anaerobic conditions exist in the system. The digester was agitated everyday by shaking in order to prevent the formation of surface crust which may prevent contact between the anaerobic microorganisms and the substrate. The daily volumetric gas production was measured everyday using the water displacement method.

3. Results and discussion

3.1. Experimental studies and attainable region construction

Fig. 1 presents the dynamics of cumulative biogas yield obtained from anaerobic treatment of abattoir waste. The curve has a sigmoidal shape, which is characteristic of easily degradable substrates that are prone to some degree of inhibition (Labatut et al., 2011). The interest is not necessarily on the shape of the biogas yield curve, but on how the authors use the curve to synthesize digester structures to minimize digestion time. The design of the optimal digester structure to minimize digestion time involves three main aspects: (1) Construction of attainable regions using geometric techniques, (2) scheduling of batch operation from the attainable regions, and (3) interpretation of continuous mode operation structures from the batch operation.

3.1.1. Construction of attainable regions

The optimization of digestion time using the attainable region technique is done using three major steps and Fig. 2 presents the plots obtained at the different stages of the construction process.

Step 1: Construction of base trajectory

In AR convention, when dealing with data involving residence time space, it is often conventional to plot residence time on the vertical axis while concentration or yield is plotted on the horizontal axis. Fig. 2(a) presents the cumulative biogas yield curve plotted in AR convention and the curve ABCD is called the base anaerobic digestion trajectory.

Step 2: Determine and bypass concavity using a mixing line

Observe that the base anaerobic digestion trajectory given by curve ABCD, is concave with respect to residence time axis, which may be filled by joining points A and C with a mixing line as shown in Fig. 2(b). The location of ABCD on the curve is done as follows: Firstly, determine point A (usually the starting point or feed point). Secondly, identify the region of concavity (on the lower side of the residence time axis) and locate another point, C such that a line drawn from A to the point C fills

the concavity. Thirdly, the segment of the curve between A and C is called B.

The straight-line AC has a very significant property. A key criteria for selecting variables in AR is that they must obey the linear mixing law (Hildebrandt et al., 1990). It can be shown that the residence time of a system must lie in a straight line between the residence times of the individual reactors, τ_1 and τ_2 comprising the system (Ming et al., 2016). This implies the residence time obeys the linear mixing law, Eq. (13)

$$\tau_{mix} = \alpha\tau_1 + (1 - \alpha)\tau_2 \quad (13)$$

The cumulative biogas yield (y_i) is given by the volume of biogas produced (mL) per mass of substrate added to the digester (g). $y_i = V_g/m_s$. Consider two digesters of known biogas yield, the actual volume of biogas produced for digesters 1 and 2 can be obtained by $V_{g1} = y_{i1}m_{s1}$ and $V_{g2} = y_{i2}m_{s2}$ respectively. Conservation of mass may be used to calculate the total cumulative biogas yield for both digesters. Conservation of mass ensures that the total mass of substrate in the mixture is equal to the sum of the individual substrate masses contained in digesters 1 and 2, which is given by $m_{ST} = m_{s1} + m_{s2}$. Computing the biogas yield of the entire system is equivalent to determining the biogas yield for a mixture of digesters 1 and 2 because the density of the liquid phase of the digester can be assumed constant. The biogas yield of the mixture is given by the ratio of the total volume of biogas produced to the total mass of organic substrate added as shown by Eq. (14).

$$y_{IM} = \frac{y_{i1}m_{s1} + y_{i2}m_{s2}}{m_{ST}} \quad (14)$$

By setting $\alpha = m_{s1}/m_{ST}$ then Eq. (14) can be written as Eq. (15), which is similar to the linear mixing law. What this means practically is that by mixing the contents of the liquid phase of two digesters, each of which contains a given mass of organic substrate, then the total cumulative biogas yield of the mixture will lie in a straight line joining that of both digesters.

$$y_{IM} = \alpha y_{i1} + (1 - \alpha)y_{i2} \quad (15)$$

This is known as the lever-arm rule and the process of combining the contents of two parallel digesters of different substrate masses results in a linear mixing law (where α is known as the mixing ratio) measured in term of cumulative biogas yield.

The point A on the curve represents a digester condition where a fresh mass of substrate has just been added and no biogas has been produced. The straight-line AC therefore represents a batch digester, which is run up to a certain residence time then the content is mixed with fresh substrate. Because the base anaerobic trajectory lies higher up on the residence time axis than the mixing line AC, bypassing fresh

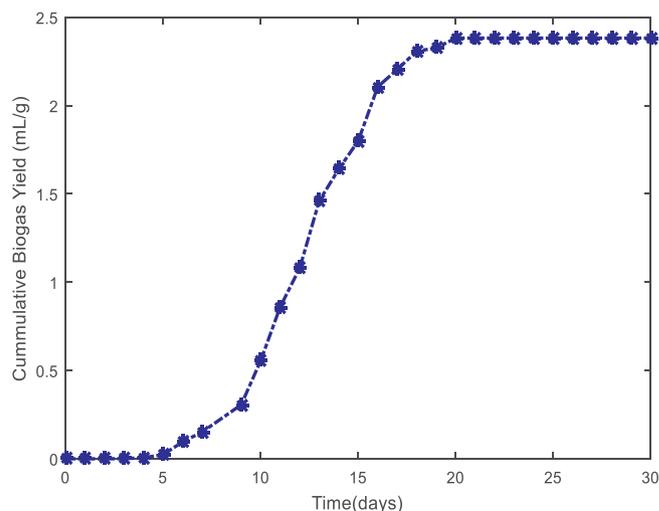


Fig. 1. Cumulative biogas yield curve for anaerobic digestion of abattoir waste.

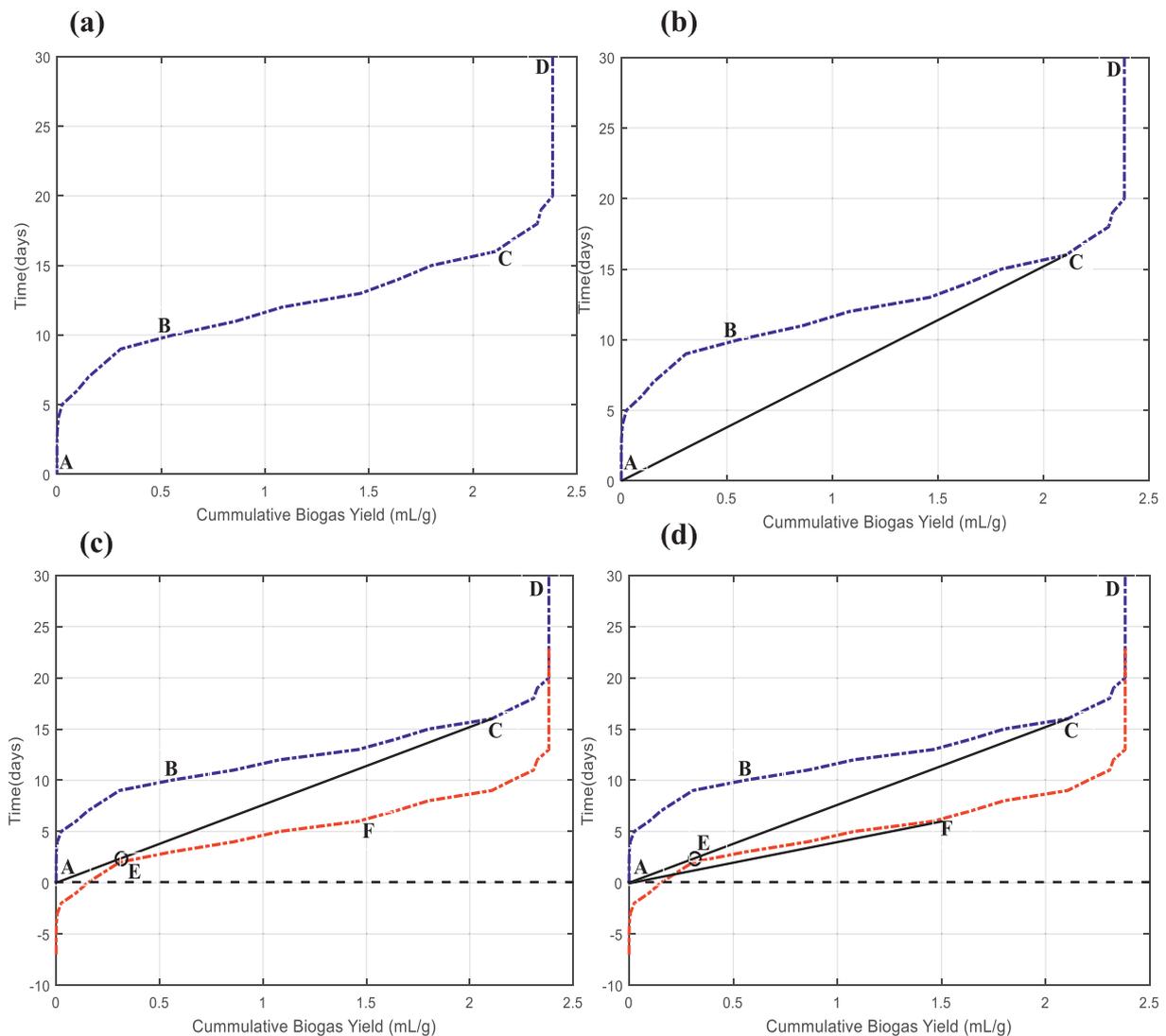


Fig. 2. Attainable region construction process (a) Base anaerobic digestion trajectory in AR convention. (b) Base anaerobic digestion trajectory showing mixing line. (c) Moving down the based trajectory until it touches the mixing line. (d) Generating a candidate AR using only PFRs and a base trajectory.

organic substrate reduces the overall residence for the same cumulative biogas yield (this is only for yields between points A and C). For example, on the initial anaerobic digestion trajectory, observe that a residence time of 10 days is needed to achieve a cumulative biogas yield of 0.5 mL/g, meanwhile the same yield can be achieved at 5 days using the mixing line. This is possible by operating the batch digester up to point C and then mixing fresh substrate with this stream to obtain the desired overall yield. Note that this optimization is only possible because of the concavity in the original anaerobic digestion trajectory, and hence regions of low digestion rate in the digester are to be bypassed by the use of mixing. This phenomenon can be attributed to the fact that adding fresh substrate increases nutrient bioavailability for the anaerobic microorganisms thereby increasing growth and hence production of the desired biogas

Step 3: Expansion of candidate attainable regions using batch trajectory and the mixing line

Notice from step 2 how graphical techniques have been applied to expand the total set biogas yields that is achievable in the anaerobic digester by making use of concavities in cumulative biogas yield curves. Furthermore, from the principles of differential algebra, process trajectories from batch reactors are directional. Geometrically, the reaction rate vectors of batch processes have a unique nature, which ensures that different batch trajectories progress in a manner that they do not

cross one another (Asiedu et al., 2014). For a given feed point there exists a unique trajectory for a process operated in batch mode. The overall residence time for anaerobic digestion can be decreased by using the base anaerobic digestion trajectory. This is done by moving the trajectory down until it touches the mixing line at a unique point (point E) as shown in Fig. 2(c). This point of contact (point E) has a significant geometric and practical meaning for further optimization of the anaerobic digestion process. Geometrically, it represents the point where the reaction rate vectors point out of the boundary of the attainable region, which means the region can further be expanded from that point in order to meet the necessary condition of convexity (Glasser et al., 1993, Hildebrandt et al., 1990). Practically, it represents the lowest digestion time on the boundary of the candidate attainable region where from an additional batch digester can be initiated to further expand the region and minimize the residence time. By translating the curve downwards, the direction of the reaction rate vectors vary along the length of the mixing line. Observe that the shifted trajectory this has some small concavity with respect to residence time axis, which may be filled by joining points A and F with a mixing line as shown in Fig. 2(d). By translating the curve downwards, the direction of the reaction rate vectors varies along the length of the mixing line. When the attainable region becomes convex, it implies there is no part on the boundary of the attainable region where the rate vectors point outward, and this

implies that the true attainable region has been obtained. The convex curve AFD represents therefore represents attainable region for the anaerobic treatment process. The attainable region represents all possible outputs that can be achieved for all possible reactor designs by interpreting chemical processes as geometric objects. Geometrically, it represents the convex hull for the set of points achievable by a given system.

3.1.2. Modeling process configurations

Fig. 3 presents the optimal process configurations of the anaerobic treatment process for both batch and continuous mode operation Sections 3.1.2.1 and 3.1.2.2 provides a detailed description of how the process structures have been obtained.

3.1.2.1. Scheduling of batch operation policy. After the attainable region has been obtained, the boundary of the attainable region can be used to schedule an operating policy for batch anaerobic digestion, which can be used to achieve the limits defined by the system Point E is obtained by running a batch with fresh feed, up to the point C (stage 1) then mixing the content into another batch digester, which is then mixed with fresh organic waste (stage 1). This batch operation policy is illustrated in Fig. 3(a). Another batch is run with the contents of stage 2, stage 3 (from point E) to obtain the point F found on the EFD trajectory then mixed with fresh organic waste (stage 4) to obtain points located on line AF.

3.1.2.2. Continuous mode operation. The boundary of the attainable region can also be interpreted into continuous process configurations, which can be used to attain the same achievable limits defined by the attainable region (Hildebrandt and Glasser, 1990). This will be based on the final structure of the attainable region boundary defined by the curve AFD. The interpretation of the AR boundary is based on three key fundamental results of two-dimensional AR used in everyday practice (Ming et al., 2016). (1) The AR is composed of reaction and mixing surfaces only. Reaction surfaces are always convex. (2) Points that form convex sections of the AR boundary arise from effluent concentrations specifically from PFR trajectories. (3) Points on the AR boundary that initiate these convex PFR trajectories (from point 2 above) arise from specialized CSTRs for two-dimensional constructions. This implies that

the point (F), arise from a CSTR and is used to imitate convex PFR trajectories to form the AFD trajectory. The mixing line AF, which eliminates the concavity in the system is represented structurally by a CSTR with a bypass from point A. The final structure of the continuous digester structure is shown in Fig. 3(b).

It is interesting to compare the results of this study with that of the authors' recent studies using attainable regions to synthesize anaerobic digester structures (Abunde et al., 2019a,b). The studies developed a simplified kinetic model of the anaerobic treatment process and applied the kinetic models to construct the attainable regions. The reliability of this approach depends on the availability of a suitable kinetic model as well as kinetic coefficients of the process. In the current study, the construction of attainable regions using only experimental data has been presented. This implies that even without having a kinetic model of the process, it is still possible to design optimal digester systems.

3.2. Selection of digester subunits and definition of optimal process configurations

As mentioned in Section 3.1, the attainable region defines optimal digester structures in terms of the mode of operation, which can be plug flow (with no axial mixing) or continuous (with mixing). Also, Fig. 1 illustrates that there exist several anaerobic digesters, which can be considered to have a plug flow operation. A list of criteria used to select the appropriate plug flow digester to enhance biogas generation has been presented in Table 1. Fig. 4 presents a spider web diagram showing the weights of the criteria (obtained using the analytical hierarchy process), which indicates the extent to which each criterion has on the selection of an anaerobic plug flow digester. Compared to other multi-criteria decision-making methods, the AHP is well known for its strength of weighting criteria, which is why the authors chose it for criteria weighting. The AHP determines the weight of importance of each criterion by using pair-wise comparison matrix that uses the scale of relative importance proposed by Saaty.

It should be noted that out of the three main applications of anaerobic digestion (waste hygienisation, renewable generation and nutrient recycling), the weighting has been focused on maximizing renewable energy generation potential of the process. Fig. 4 shows that biogas yield and stage of treatment carry the highest weights, followed

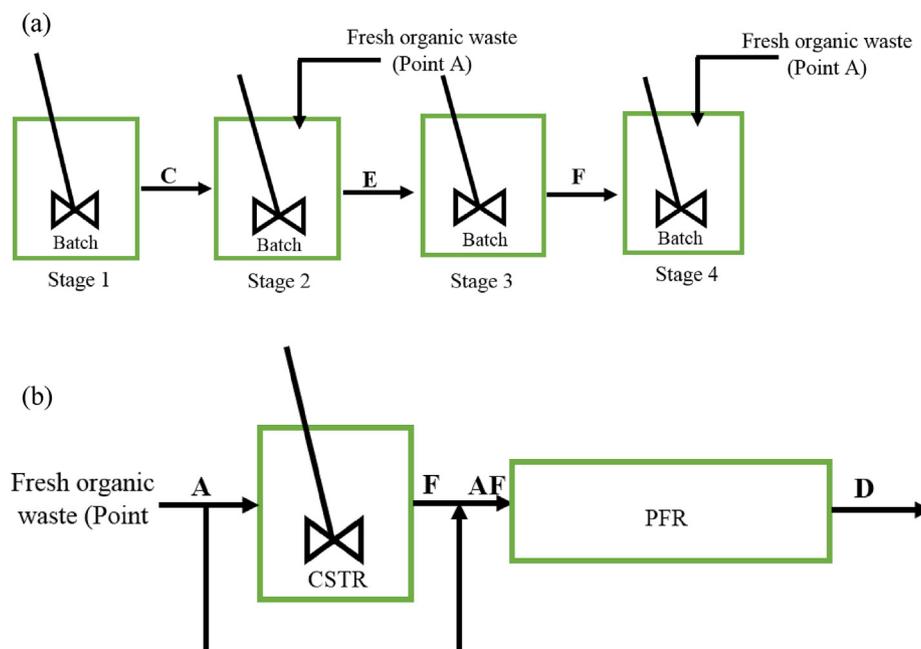


Fig. 3. Optimal process configurations of anaerobic treatment process (a) Optimal scheduling of operating policy for anaerobic batch digesters. (b) Optimal continuous digester structure of treatment of abattoir waste.

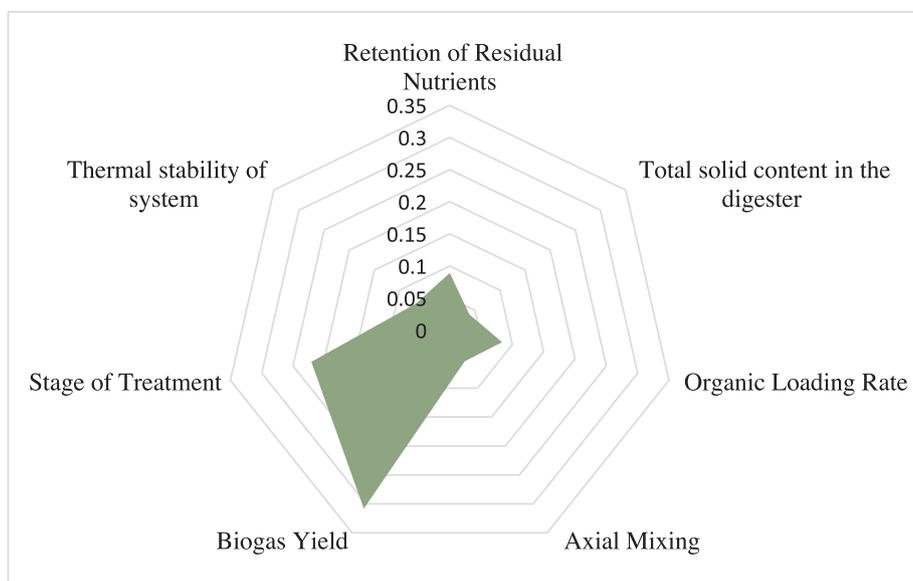


Fig. 4. Criteria weights for selection of anaerobic plug flow digester.

by organic loading rate and retention of residual nutrients while axial mixing, thermal stability and total solids content carry the smallest weights. This can be explained as follows: biogas yield is highest since the objective is to generate renewable energy and hence plug flow digesters more adapted to produce more biogas per gram of substrate are given priority. Regarding stage of treatment, anaerobic digestion can be operated as a primary or secondary treatment system and the primary systems are more adapted to biogas generation while the secondary systems are more adapted to waste hygienisation of nutrient recovery (Mang and Li, 2010). It is therefore important to select digesters that are more adapted to primary treatment. Organic loading rate carries relatively less weight because when anaerobic digestion is used for renewable energy generation, the system is dimensioned based on the energy requirements of the users and not on the flowrate of effluent available. The goal is not to digest all the effluent, but to digest the effluent that will produce the required quantity of energy. However, if the system is to be designed mainly for treatment of effluent, the dimensioning would base on the flow rate of effluent available, which must be treated to meet a given discharge standard. In the same light, the retention of residual nutrients is relatively less important as the goal is to maximise biogas generation. For the case of thermal stability, the relatively low weight is attributed to the fact that the digester will be operated under isothermal conditions and for axial mixing, all the plug flow digesters are assumed to have no axial mixing, with little variations. Finally, for the case of total solids content, the digester is to be used for the treatment of effluent from abattoir with a defined solids content. It is important for readers to note that the authors are not saying some of the criteria are not important but are just explaining why some of the criteria are considered more important than others in the selection of the appropriate plug flow digester.

Therefore, according to the criteria weights, the ranking order of all the plug anaerobic digesters according to importance was determined using the fuzzy TOPSIS approach and the best alternative was selected among seven alternative anaerobic digesters. The ranking order of the anaerobic digesters based on the closeness coefficient to the ideal solution is given in Table 3. Table 3 indicates that out of the seven anaerobic plug flow digesters considered, the Anaerobic Baffled Reactor (ABR) had the best performance. The results are further strengthened by the findings presented by other researchers concerning the operational characteristics of the ABR. The ABR is a high rate anaerobic plug flow digester having a decoupled sludge and hydraulic retention times enabled by a series of vertical baffles through which effluent flows (Mao

et al., 2015). The baffles divide the reactor into a series of compartments and forces incoming effluent to flow axially through a series of blanketed sludge trapped in each compartment. The ABR is therefore considered to be a multi-stage system consisting of several UASB connected in series. The separation of the biological steps within the system ensures overall improvement in performance, as each of the steps can be allowed to operate at their optimal conditions there by minimizing issues of toxicity (Bachmann et al., 1985, Barber and Stuckey, 1999). Recall that the optimal digester structure (Fig. 4b) obtained for the treatment of abattoir effluent consist of a CSTR with bypass of feed followed by a PFR, which will now be a CSTR with bypass of feed followed by an ABR. The optimal reactor configuration has been modelled in 3D as presented in Fig. 5(a). Fig. 5(b) shows a cross sectional view of the system indicating how the baffles have been designed while Fig. 5(c) shows a transparent view of the novel system.

Studies have shown that the optimal application of ABR is post-treatment after a primary treatment step (Mang and Li, 2010). On the other hand, the operation of a single CSTR is less efficient in terms of the biogas yield and hence effluent quality (Boe and Angelidaki, 2009). This further supports why the ABR coming after a primary digestion step using a CSTR is an optimal reactor structure. The novel prototype combines the advantages of a continuous stirred tank anaerobic reactor and an anaerobic baffled reactor.

The system is envisaged to operate as in three stages as follows: In stage 1, effluent is mixed and homogenized in a continuous stirred tank for a given retention time. This first stage has the advantage of rapid acidification due to mixing from continuous stirring, resulting in the production of high quantities of volatile fatty acids. The second stage involves bypass of fresh effluent to mix with the effluent from CSTR. As

Table 3 Importance ranks of anaerobic plug flow digesters fuzzy AHP-TOPSIS method.

	Positive ideal Solution (S^+)	Negative Ideal solution (S^-)	Relative closeness to ideal solution $S^-/(S^- + S^+)$	Rank	Digesters
A_1	2.301	1.910	0.453	6	AFBR
A_2	0.718	2.888	0.800	2	UPFR
A_3	2.254	1.304	0.366	7	EGSB
A_4	1.292	3.543	0.732	3	ICR
A_5	1.895	2.459	0.564	5	UASB
A_6	0.146	3.855	0.952	1	ABR
A_7	1.307	3.675	0.677	4	AF

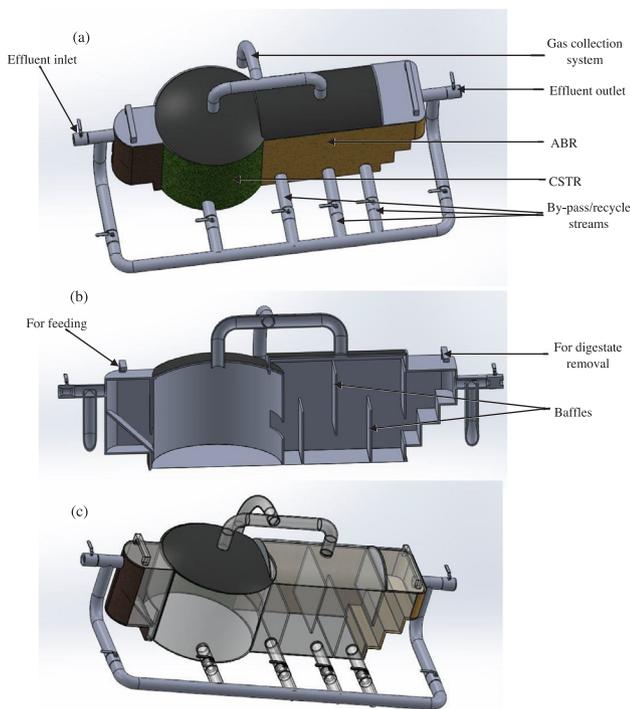


Fig. 5. Attainable region inspired novel prototype of an anaerobic digester.

demonstrated in Section 3.1, the bypass valve from feed has been systematically added based on the attainable region process to bypass regions of slow biodegradation increases the overall efficiency of the process. The third stage (ABR), which retains high amounts of sludge rapidly converts the volatile acids into biogas. Observe from Fig. 5 that other valves have been included in the system meanwhile the optimal system for treatment of abattoir effluent includes only a single bypass stream. The authors' previous study on attainable regions indicated that the attainable region and hence the optimal digester structure is unique for each digested substrate (Abunde et al., 2019b). Hence a different organic substrate might require a change in the position of the bypass stream or the addition of a recycle stream. In such cases, the network configuration can be changed by simply opening and closing certain valves, which ensure robustness in the reactor structure for different substrates. This will be very helpful during experimental testing of the system where multiple substrates can be tested using the same prototype without the need to redesign a completely new system in cases where the configuration changes due to change in substrate. It is interesting to compare the conceptual operation of the novel prototype presented in this study with some of the multistage studies presented in the literature involving CSTR in the primary stage. Boe and Angelidaki (2009) confirmed that using a multi-stage system involving two CSTRs, an improvement in biodegradation efficiency and biogas generation is seen mainly after addition of the second stage. In another study using a CSTR as a primary stage and an up-flow anaerobic sludge blanket reactor as the second stage, the results showed that the two stage system is more stable at higher organic loading rates compared to a single stage involving only a CSTR (Aslanzadeh et al., 2014). Observe that in both cases, CSTR performs optimally when used as a first stage. However, a major drawback with the aforementioned and many other studies involving multistage digestion is that the digester configuration is often predefined at the start of the study with no systematic rule for answering the following key questions: (1) what type of digester subunits to include in the network (2) how many individual digester subunits should be included (3) should the subunits be connected in series, parallel or both (4) should bypass or recycle streams be included and if yes where within the system? The main advantage of the presented

prototype compared to other multistage systems in that it has been designed based on a systematic framework that uses experimental data, which contains necessary information about the kinetics of the process. In addition, by being a compact multistage system, the prototype can separate the acidogenic and methanogenic phases axially within the reactor (as with other multistage systems) but without the high cost and control problems normally associated with multistage systems.

Although the prototype is still to be subjected to experimental validation, it can be theorised to have the following advantages: simple design (relative to other multistage systems), low sludge generation (as much of the sludge is retained in the system), no requirement of biomass with special settling properties, no requirement of a special gas or sludge separation system as well as stability to organic shocks. A natural progression of the study will be to subject the prototype to experimental testing whereby it will be constructed and operated simultaneously with a conventional fixed dome system under similar experimental conditions. This will allow for the determination of optimal flow rates for the feed stream, bypass stream and effluent stream from the primary treatment stage.

A very interesting continuation of the current study with respect to the fuzzy decision-making aspect will be to consider other scenarios for use of anaerobic digestion technologies. The anaerobic digestion technology can be used for three main applications: Renewable energy generation, sustainable nutrient recycling as well as waste sanitation and different digester technologies are more adapted for one application than the other. This implies the ranking of the digester technologies using the fuzzy method will be different if the application of anaerobic digestion technology changes. This study has only focused on use of anaerobic digestion for renewable energy generation. It will be interesting if further studies could expand the fuzzy multicriteria decision to the other two applications of anaerobic digestion and compare the results for all three cases. More interestingly, because the method is novel and not very common in the field of anaerobic digestion, the ultimate research goal should be to integrate the methodological framework presented in this study into a web-based application, which can serve industry practitioners and researchers involved in design of anaerobic digester systems

4. Conclusion

A framework that couples attainable regions and fuzzy multicriteria decision making for modeling configurations of anaerobic digesters without use of a kinetic model has been developed. Taking a case study of anaerobic treatment of abattoir effluent, the optimal batch policy involves four anaerobic sequencing batch reactors operated in series with fresh feed being added at the second and the fourth stages (fed-batch systems). In the case of a continuous mode operation, the optimal digester structure involves a continuous stirred tank digester with bypass from feed followed by an anaerobic baffled digester, which has been modelled as a compact three-dimensional prototype.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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